LABORATORY EXPERIMENTS FOR SENIOR HIGH GENERAL MATHEMATICS

by

GLENDA GAYLE TRAVIS

B.S., Fort Hays Kansas State College, 1965

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

College of Education

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

Approved by:

Major Professor

This report is affectionately dedicated to

my parents

Mr. and Mrs. Glenn W. Cleveland

TABLE OF CONTENTS

CHAPTER		PAGE
I.	INTRODUCTION	1
	The Problem	1
	Statement of the problem	1
	Importance of the study	2
	Definitions of Terms Used	2
	Computing, Managing, and Spending your Income	2
	General Mathematics	3
	Guide sheets	3
	Informal Geometry	3
	Laboratory experiments	3
	Laboratory	3
	Senior high school	3
	Statistics and Probability	3
	Terminal student	4
	Procedures Employed in the Study	4
II.	REVIEW OF THE LITERATURE	6
	Studies Concerning the Laboratory Method of Teaching Mathematics	6
	How the Laboratory Method Makes Use of Principles of Learning	6
	Educators Who Recommend Using the Laboratory Method of Teaching Mathematics	9
	Organizing a Mathematics Laboratory	13

CHAPTER PAGE

Level at Which the Terminal Student Should Enroll in General Mathematics 15

III. SUGGESTED LABORATORY EXERCISES FOR PROPOSED UNITS 17

Exercises for Informal Geometry 18

Exercises for Statistics and Probability 29

Exercises for Computing, Managing, and Spending Your Income 36

ACKNOWLEDGMENTS 42

BIBLIOGRAPHY 43

APPENDIX A 47

APPENDIX B 49

APPENDIX C 51

CHAPTER I

INTRODUCTION

The terminal student usually finds mathematics difficult and uninteresting. Generally, he only takes a course in mathematics to gain a credit for graduation.

The terminal student's dislike of mathematics many times is a result of inadequate teaching methods. Professional journals and mathematics method books indicate that the traditional lecture-demonstration-drill approach does not meet the needs of this type of student. Specialists in the field of mathematics methodology, such as Donovan A. Johnson and Charles Butler, are suggesting that teachers use the laboratory method. Title III of the National Defense Education Act of 1958 is promoting this method by enabling mathematics departments to remodel facilities and to purchase equipment and materials. In addition to facilities, equipment, and materials, the teacher needs to have guide sheets for the students to use in the laboratory. This study deals with the development of guide sheets for the teacher to use.

I. THE PROBLEM

Statement of the problem. This study was designed to develop three sets of laboratory experiments for teachers which would enable them to make use of the laboratory method in the classroom with a minimum of expensive equipment. They are presented in the form of guide sheets. The experiments were designed for an eleventh or twelfth grade general mathematics course.

Specifically, the three sets of experiments will deal with these three units of study:

1. Informal Geometry
2. Statistics and Probability
3. Computing, Managing, and Spending Your Income.

It is hoped that the writer and other teachers will use these experiments, and design studies to answer the following questions:

Do terminal students achieve more under the laboratory method than under the lecture-demonstration-drill method in a general mathematics course?

Does the terminal student's attitude improve under the laboratory method in a general mathematics course?

Importance of the study. There is a dearth of well-organized experiments for teachers; however, many teachers lack the ideas, the time, or the material to develop them. Thus, the development of specific laboratory experiments to be used in specific units of study should prove useful to teachers of general mathematics classes.

II. DEFINITIONS OF TERMS USED

Certain educational terms are defined so the reader will have a common understanding of the way in which they are used in this paper.

Computing, Managing, and Spending Your Income. A unit of study in general mathematics dealing with the following topics: budgets, buying wisely, household expenses, kitchen problems, parcel post, services of a bank, installment buying, credit, discounts, insurance, investments, Social Security, and taxes.

General mathematics. An academic course for the terminal student in senior high school. This course is taken during the junior or senior year.

Guide sheet. The guide sheet contains essential information which the student must have in order to carry out the experiment. Specifically, the guide sheet contains a list of objectives, equipment needed, directions, and questions which lead the student to the conclusions.

Informal Geometry. A unit of study taught in a general mathematics course. The topics covered in this unit are: perimeters and areas of quadrilaterals and polygons, circumference and area of circles, volumes, points, lines, angles, planes, spaces, scales, triangles, and geometric constructions.

Laboratory experiments. Activities that students can perform individually or in groups covering material related to the topics they are studying, will study, or have studied.

Laboratory. A room which is properly equipped for the experiments to be performed. This may be the classroom or a special room equipped for the purpose of carrying on laboratory experiments.

Senior high school. A school containing grades ten, eleven, and twelve.

Statistics and Probability. A unit of study taught in a general mathematics course. The topics covered in this unit are: collecting data, arranging and presenting data, interpreting statistics, measures of central

tendency, percentiles, quartiles, measures of dispersion, frequency distributions, permutations, combinations, and probability.

Terminal student. A student enrolled in high school who would be classified as one of the following types of students:

1. A number of college-capable students whose mathematical talent has as yet not been discovered.
2. Those who have been insufficiently challenged.
3. Those who want to prepare for an occupation that does not require them to take algebra or geometry, although they could be successful in these subjects if they took them.
4. Students from economically depressed areas or from disadvantaged homes.
5. Socially maladjusted children.
6. Those with emotional problems that prevent them from functioning at their level of ability.
7. Lazy or reluctant students.
8. Unmotivated students.
9. Those with low native ability.[1]

Usually, the term terminal student refers only to those students who expect to terminate their formal education upon graduation from high school. The number one type of student was included because he is a potential terminal student unless his talents are discovered.

III. PROCEDURES EMPLOYED IN THE STUDY

The procedures employed in this study consisted of (1) an analysis of general mathematics textbooks available for use at Kansas State University

---

[1]Sol Weiss, "Innovations and Research in the Teaching of Mathematics to the Terminal Student," The Mathematics Teacher, 60:611-2, October, 1967.

to determine the units and topics to be included in the experiments,[2] (2) development of a format for guide sheets used by students in the laboratory, and (3) designing the experiments to be used in the three units of study.

[2]Appendix A contains a bibliography of these textbooks.

CHAPTER II

REVIEW OF THE LITERATURE

A review of the literature pertaining to the laboratory method of teaching mathematics was carried out to answer the following questions:

1. Have studies concerning the laboratory method of teaching mathematics been conducted?
2. Does the laboratory method put into practice principles of learning that are commonly accepted by psychologists?
3. Do specialists in the field of mathematics methodology, mathematics consultants, and teachers advocate the use of the laboratory method?
4. How should the mathematics laboratory be organized?
5. When should general mathematics be taught to the terminal student?

Each of these questions is dealt with in the following sections of this chapter.

I. STUDIES CONCERNING THE LABORATORY METHOD OF TEACHING MATHEMATICS

A review of the literature revealed that there have not been any studies conducted concerning the laboratory method of teaching mathematics.

II. HOW THE LABORATORY METHOD MAKES USE OF ACCEPTED PRINCIPLES OF LEARNING

J. S. Frame,[1] Donovan A. Johnson,[2] Larry K. Johnson,[3] and Lloyd V.

---

[1]J. S. Frame, "Facilities for Secondary School Mathematics," The Mathematics Teacher, 57:388, October, 1964.

[2]Donovan A. Johnson, Guidelines for Teaching Mathematics (Belmont, California: Wadsworth Publishing Company, Inc., 1967), pp. 301-2.

[3]Larry K. Johnson, "The Mathematics Laboratory in Today's Schools," School Science and Mathematics, 62:586-7, November, 1962.

Manwiller[4] cited principles of learning which the laboratory method puts into practice. All of these writers mentioned the principle--students learn better by doing and applying. They discussed how this principle is used in the laboratory. In the laboratory each student is actively involved in the learning process as he participates in laboratory experiences--collecting data, estimating, comparing, analyzing, classifying, checking--and as he thinks while doing these experiments. Larry Johnson refers to this principle as "the basic philosophy underlying the laboratory method."[5]

Morris Bigge[6] listed the principle--motivation plays a central role in learning--as one that is commonly accepted by modern psychologists. He indicated that this principle is important because "students who are motivated work purposefully and energetically. They display few if any 'discipline' problems."[7] Donovan Johnson[8] and Manwiller[9] discussed ways in which the laboratory method generates motivation. To them, the relaxed independence of the laboratory, tangible evidence of progress, meaningful problems, and the wide variety of materials and equipment used are all significant motivators.

---

[4]Lloyd V. Manwiller, "Laboratory Activities--Why?" School Science and Mathematics, 61:86, February, 1956.

[5]Larry Johnson, op. cit., p. 586.

[6]Morris L. Bigge, Learning Theories for Teachers (New York: Harper and Row, Publishers, 1964), pp. 289, 294.

[7]Ibid., p. 289.

[8]Donovan Johnson, op. cit., p. 302.

[9]Manwiller, op. cit., p. 86.

Bigge,[10] also, stated that readiness for learning is important if learning is to take place. According to Donovan Johnson[11] and Manwiller,[12] laboratory activities recognize the difference in abilities and readiness within a class. Laboratory experiments can be chosen that are commensurate with the students' abilities and interests. Thus, laboratory experiments are particularly useful in working with the culturally deprived, in preventing dropouts, in providing for creative, talented students, and in helping the slow learner.

Manwiller stated another principle of learning that the laboratory method puts into practice--"Learning follows a developmental sequence(...). The learning of concepts and principles is promoted to the extent that new experiences find a place in the over-all pattern of experience and take on meaning."[13] This learning takes place while doing a laboratory experiment, as the student procedes from the concrete to the abstract and from the known to the unknown. Also, laboratory experiences take on meaning as the student conducts experiments dealing with situations he meets or will meet in daily life.

This review of literature definitely indicated that the laboratory method puts into practice principles of learning which are commonly accepted by psychologists.

---

[10]Bigge, op. cit., p. 294.

[11]Donovan Johnson, op. cit., p. 302.

[12]Manwiller, op. cit., p. 86.

[13]Ibid.

III. EDUCATORS WHO RECOMMEND USING THE LABORATORY METHOD OF TEACHING MATHEMATICS

In the early 1900's educators began recommending that mathematics teachers use the laboratory method. In 1935, J. A. Ramseyer[14] advised teachers to make use of a laboratory. He suggested that they equip it with instruments and devices which would enable "pupils to see the part that mathematics has played in changing civilization."[15] Today, educators are still advocating the use of a mathematics laboratory.

Eldert Groenendyk,[16] mathematics consultant for the Iowa State Department of Public Instruction, supported the use of the mathematics laboratory. Mathematics is known as the "queen of the sciences." Therefore, he felt that teachers should consider the scientific approach since mathematics is a science. Students should be provided with a mathematics laboratory where they can experiment and make basic mathematical discoveries.

Several specialists in the field of mathematics methodology recommended that teachers use the laboratory method. Donovan A. Johnson[17] devoted an entire chapter of his book, Guidelines for Teaching Mathematics, to discussing laboratory lessons. The following statement reveals why

---

[14]R. A. Ramsmeyer, "The Mathematics Laboratory," The Mathematics Teacher, 28:229, 231, April, 1935.

[15]Ibid., 231.

[16]Eldert Groenendyk, "The Mathematics Laboratory," Midland Schools, 77:11, January, 1963.

[17]Donovan Johnson, op. cit., pp. 301-20.

Johnson felt the laboratory method should be used in teaching mathematics:

> Mathematics is a study of patterns, a language for expressing relationships, and a tool for dealing with approximate data. Consequently, mathematics is uniquely suited for exploring ideas in a laboratory setting.[18]

Charles Butler and Lynwood Wren[19] advocated laboratory work in mathematics because experiments serve a double purpose: (1) they stimulate interest and (2) they clarify many mathematical concepts and relations as students associate the concepts and relations with physical things. William Reeve[20] felt that recitation rooms need to be transformed into laboratories and workshops because they facilitate the proper use of mult-sensory aids.

College professors, Foster Grossnickle, Charlotte Junge, and William Metzner,[21] Larry Johnson,[22] and Raymond Sweet[23] have written articles supporting the laboratory method in teaching mathematics. Grossnickle, Junge, and Metzner[24] said a period of experimentation and discovery is an important step in the learning process of arithmetic. If effective learning of generalizations takes place, the student, under the guidance of the

---

[18] Ibid., p. 302.

[19] Charles H. Butler and F. Lynwood Wren, The Teaching of Secondary Mathematics (New York: McGraw-Hill Book Company, 1965), p. 136.

[20] William David Reeve, Mathematics for the Secondary School (New York: Henry Holt and Company, 1954), p. 504.

[21] Foster E. Grossnickle, Charlotte Junge, and William Metzner, "Instructional Material for Teaching Arithmetic," The Teaching of Arithmetic, National Society for the Study of Education Fiftieth Yearbook, Part II (Chicago, Illinois: The University of Chicago Press, 1951), pp. 156-8.

[22] Larry Johnson, op. cit., p. 587.

[23] Raymond Sweet, "Organizing a Mathematics Laboratory," The Mathematics Teacher, 60:117, February, 1967.

[24] Grossnickle, loc. cit.

teacher, should discover each generalization for himself. These discoveries can be either individual discoveries or group discoveries. Johnson[25] suggested this method because, under a laboratory situation, students are able to experiment and deal with concrete situations. As a result of these experiments, students know why certain methods work and can apply them to real and practical problems. Sweet[26] mentioned a number of reasons for operating a mathematics laboratory:

1. Each student participates.
2. Each student is allowed to work at his own rate.
3. Instructors are left free to provide more individual help.
4. Students have an opportunity for discussion: thus, they learn from one another.
5. Variety is added to the classroom routine.
6. Students base their conclusions on their own observations instead of being spoon-fed facts.

Many teachers using the laboratory method of teaching advocate its use. The Secondary-School Curriculum Committee of the National Council of Teachers of Mathematics[27] recommended that teachers use laboratory techniques when teaching the slow learner. Harry Phillips,[28] a specialist for mathematics with the U. S. Office of Education, reported that many schools are setting up mathematics laboratories. In 1965, over half of

---

[25]Larry Johnson, op. cit., p. 587.

[26]Sweet, op. cit., p. 117.

[27]"The Secondary Mathematics Curriculum," The Mathematics Teacher, 52:408, May, 1959.

[28]Harry L. Phillips, "The Mathematics Laboratory," American Education, 1:1-2, March, 1965.

Georgia's school districts had mathematics laboratories. Another school is Fridley High School in Minneapolis which has a mathematics laboratory between every two mathematics classrooms. They are equipped with such things as plastic models, overhead projectors, calculators, and a typewriter with mathematical symbols.

Eldert Groenendyk[29] reported that the Marion and Carlisle high schools in Iowa have established mathematics laboratories. Their establishment was made possible by the National Defense Act of 1958. The schools feel that these laboratories have been successful. The pupils say they learn better and understand mathematical concepts more clearly. These laboratories have helped some students find a new interest in mathematics, and they went on to take a first course in algebra. Teachers noticed that discipline problems and truancy receded.

Teaneck High School[30] in Teaneck, New Jersey, began using the laboratory method with applied and advanced mathematics in 1966. The students in their laboratories were learning to control calculators and computers. In setting up their laboratories, they were assisted by funds supplied under the National Defense Act of 1958. The students in their program reacted favorably to the mathematics laboratory. Their experienced teachers felt that the calculators helped in motivating and inspiring students. The teachers, also, were of the opinion that their classroom time was made more meaningful.

---

[29]Groenendyk, op. cit., pp. 12-3.

[30]"Confronting the New Technology," Business Education World, 48:11, December, 1967.

IV. ORGANIZING A MATHEMATICS LABORATORY

Before a teacher can use the laboratory method, he must know how to organize a mathematics laboratory. Raymond Sweet[31] noted an important fact that teachers should realize before they begin organizing a mathematics laboratory. The laboratory should not replace any of the classroom activities that educators know are successful learning activities. Instead, it should be used to teach those concepts which the student can learn best through the laboratory method.

Larry Johnson,[32] Charles Butler and Lynwood Wren[33] stressed the point that it is the teacher's responsibility to set up a successful laboratory. If the laboratory experiments are not carefully planned and co-ordinated with the classroom activities, closely supervised, and guided toward definite ends, they will probably be no more than busywork. This busywork will probably contribute nothing toward the student's understanding of concepts, and waste both the students' and teacher's time.

In order to have successful laboratory experiments, the teacher should follow specific procedures. Donovan Johnson[34] lists five procedures to follow:

1. Prepare guide sheets for the students so that they will know what material they need and what they are to investigate.
2. The laboratory lesson needs adequate materials.

---

[31]Sweet, op. cit., p. 117.

[32]Larry Johnson, op. cit., p. 588.

[33]Butler, op. cit., p. 137.

[34]Donovan Johnson, op. cit., pp. 307-10.

3. The laboratory lesson needs a classroom with some flexibility and certain special equipment.

4. Each student should participate in an activity in which he can have some success.

5. The students must be properly prepared for the laboratory lesson.

The teacher may use several types of laboratory experiments. These types range from data collecting experiments to using mathematical games. A list of eighteen types of experiments as given by Donovan A. Johnson may be found in Appendix B.

Several types of laboratories are found in high schools. The type used depends on what facilities and materials the teacher has at his disposal. Four types of laboratories are described by J. S. Frame.[35] The simplest type is an alcove of a classroom where students can conduct laboratory experiments. Some high schools have a small room built between two classrooms which are used by students when conducting experiments. A third type that several schools have is a computation laboratory. In this type of laboratory, the students deal with the field of digital computers. The last type mentioned by Frame is equipped with several desk calculators and also a high-speed electronic digital computer. This type of laboratory is used with advanced students.

Marguerite Kluttz says the most common interpretation of a laboratory is "a classroom which contains an abundance of specialized mathematics equipment and materials, and where frequent use is made of the laboratory method of teaching."[36] Some teachers don't think they have the proper

---

[35] J. S. Frame, "Facilities for Secondary School Mathematics," The Mathematics Teacher, 57:388-9, October, 1964.

[36] Marguerite Kluttz, "The Mathematics Laboratory--A Meaningful Approach to Mathematics Instruction," The Mathematics Teacher, 56:141, March, 1963.

facilities for a laboratory. But Harry L. Phillips[37] says many teachers, fortunately, are using the equipment that is available to them; instead of waiting until they can have a perfected laboratory.

The laboratory materials found in mathematics laboratories varies from school to school. A list of items that would be useful in an up-to-date mathematics laboratory may be found in Appendix C.

V. LEVEL AT WHICH THE TERMINAL STUDENT SHOULD ENROLL IN GENERAL MATHEMATICS

Sol Weiss[38] believed that the terminal student has been neglected by mathematics teachers. Many times, those who need the greatest help are taught by the poorest teachers, with inadequate materials, and the wrong content. This is unfortunate when one considers that two-thirds of the skilled and semi-skilled jobs available to these students require an understanding of the basic principles of arithmetic, geometry and algebra.

H. Van Engen,[39] Edward Krug,[40] Ida Ostrander,[41] William Reeve,[42]

---

[37]Phillips, op. cit., p. 1.

[38]Sol Weiss, "Innovations and Research in the Teaching of Mathematics to the Terminal Student," The Mathematics Teacher, 60:611, October, 1967.

[39]H. Van Engen, "Arithmetic in the Junior-Senior High School, "The Teaching of Arithmetic, National Society for the Study of Education Fiftieth Yearbook, Part II (Chicago, Illinois: The University of Chicago Press, 1951), pp. 115-6.

[40]Edward A. Krug, Curriculum Planning (New York: Harper and Brothers Publishers, 1957), pp. 147-8.

[41]Ida A. Ostrander, "A General Mathematics Program for a Large School," The Bulletin of the National Association of Secondary-School Principals, 43:49, May, 1959.

[42]Reeve, op. cit., p. 438.

and Robert Strom[43] recommended that a general mathematics course for the terminal student be taken during the junior and senior years of high school. Engen and Strom felt that junior high students aren't ready for or interested in topics such as installment buying, taxation, banking, interest, and insurance. The five authors mentioned above recommended that terminal students be taught topics they will need as citizens and on the job during their later secondary years. This is when there is a greater likelihood that the terminal student will realize he needs a basic mathematical background to live successfully in our world today.

The investigator felt the review of literature indicated that the laboratory method would be extremely helpful when teaching a general mathematics course for terminal students. The laboratory method would enable the teacher to better meet the needs of this type of student. The teacher would be supported in his use of this method because it puts into practice principles of learning that are commonly accepted by psychologists. And its use is advocated by teachers, mathematics consultants, and specialists in the field of mathematics methodology.

---

[43]Robert D. Strom, "A Realistic Curriculum for the Predictive Dropout," The Clearing House, 39:105-6, October, 1964.

CHAPTER III

SUGGESTED LABORATORY EXERCISES FOR PROPOSED UNITS

This section contains eighteen experiments designed for an eleventh or twelfth grade general mathematics class. The experiments included for each unit are representative of experiments that can be used in this unit. Teachers teaching these units would use these experiments as a guide to design similar experiments for each unit that are not included in this chapter.

The experiments presented are flexible. For some classes more information may need to be supplied or careful step-by-step instructions given by the teacher during the laboratory period. For other classes the teacher may need to omit some experiments and add experiments that are more challenging.

INFORMAL GEOMETRY

A. Laboratory Exercise: Lines

OBJECTIVE: To list the properties of a line.

EQUIPMENT:

Ruler, string.

EXERCISES:

a. Draw a straight line, a line segment, a curved line, and a broken line.

b. Make 2 dots on a piece of paper. Label them A and B. Hold a string tightly over the paper so that the string is on each of the points A and B. What geometric idea do you think of when you look at the part of the string between A and B?

c. Fold a piece of paper in half and then look at the edge of the crease. What geometric idea is represented by the edge formed by the crease?

d. Give several examples of objects that represent lines.

e. Mark 2 dots on a piece of paper. See how many lines you can draw through both of those points. Through 2 points you can draw _____ lines.

f. Pick a point in the corner of the classroom and another point in the opposite corner. How many lines contain these two points? _____ How far does this line extend outside the classroom? _____

g. Draw a plane figure, if possible, in which the intersection of a line and a triangle is

1. one point
2. two points
3. three points
4. the empty set
5. a line
6. a segment

h. Draw a plane figure, if possible, in which the intersection of a line and a circle is

1. one point
2. two points
3. three points
4. the empty set
5. a line
6. a segment

B. Laboratory Exercise: The Circumference of a Circle

OBJECTIVES:

a. To collect data on the circumference and diameters of several round objects.

b. To investigate whether there is a pattern in the data which can be expressed by a simple formula.

EQUIPMENT:

Ruler, tape measurer, wheels, wastebaskets, tin cans, other round objects.

DIRECTIONS:

a. Measure the circumference and diameter of each round object.

b. Record both the circumference and diameter of each round object in your table.

c. Compute the sum, difference, product, and quotient of each pair of measures.

d. Record the sum, difference, product, and quotient of each pair of measures in your table.

TABLE

Object	Circumference (C)	Diameter (D)	C+D	C-D	CXD	C÷D
Large wheel						
Small wheel						
Wastepaper basket						
Small can						
Large can						

RESULTS:

a. What sums, differences, products, or quotients appear to be related?

b. What formula expresses this relationship?

c. From the above formula can you derive a formula expressing the circumference of any circle?

d. What are some applications of this formula?

INFORMAL GEOMETRY

C. Laboratory Exercise: Area of a Rectangle

OBJECTIVES:

a. To collect data on the length, width, and area of a rectangle.

b. To investigate whether there is a relationship in the data which can be expressed by a formula.

EQUIPMENT:

Ruler, several books, pieces of paper, piece of cardboard.

DIRECTIONS:

a. Measure the length and width of each book and piece of paper.

b. Cut a 1" square out of cardboard.

c. Use this 1" square to measure the area of each book and piece of paper.

TABLE

Object	Length (L)	Width (W)	Area (A)	L+W	L-W	LXW	L÷W
Book No. 1							
Book No. 2							
Book No. 3							
Paper No. 1							
Paper No. 2							

RESULTS:

a. Is the length and width of a rectangle related to its area?

b. If answer to (a) is yes, write a formula expressing this relationship.

c. If the answer to (a) is yes, what are some applications of the formula obtained in (b)?

D. Laboratory Exercise: Volume of a Rectangular Solid

OBJECTIVES:

a. To collect data on the length, width, height, and volume of a rectangular solid.

b. To investigate whether there is a relationship in the data which can be expressed by a simple formula.

EQUIPMENT:

Ruler, cardboard boxes, 1" cubes (blocks).

DIRECTIONS:

a. Measure the length, width, and height of each box.

b. Determine the volume of each box by using 1" cubes. (The volume is equal to the number of blocks the box will hold since each block is a cubic inch.)

TABLE

Object	Length (L)	Width (W)	Height (H)	Volume (V)	L+W+H	LXWXH
Box No. 1						
Box No. 2						
Box No. 3						

RESULTS:

a. Is the length, width, and height of a rectangular solid related to its volume?

b. If answer to (a) is yes, write a formula expressing this relationship.

c. If the answer to (a) is yes, what are some applications of the formula obtained in (b)?

INFORMAL GEOMETRY

E. Laboratory Exercise: Sum of the Angles of a Triangle

OBJECTIVES:

a. To measure the sum of the angles of various triangles.

b. To identify similarities between the sums of these triangles.

EQUIPMENT:

Ruler, protractor, triangles made from cardboard.

DIRECTIONS:

a. Draw three triangles.

b. Measure their three angles.

c. Find the sum of the measures of their three angles.

d. Determine the measures of the angles of the triangles provided for you.

e. Find the sum of the measures of their three angles.

TABLE

Object	∠1	∠2	∠3	∠1 + ∠2 + ∠3
Triangle No. 1				
Triangle No. 2				
Triangle No. 3				
Triangle No. 4				
Triangle No. 5				
Triangle No. 6				

RESULTS:

a. Does the sum of the three angles of a triangle vary from triangle to triangle?

b. Can you determine the measure of the third angle of a triangle when the other two angles measure:
   1. 53° and 64°
   2. 19° and 48°
   3. 104° and 33°

F. Laboratory Exercise: Constructing Polygons

OBJECTIVES:

a. To learn to construct triangles when the lengths of their sides are specified.

b. To learn to construct a regular polygon.

EQUIPMENT:

Ruler, compass.

DIRECTIONS:

a. Constructing a triangle with sides "a" inches, "b" inches, and "c" inches.

1. Lay off a line segment "a" inches long.

2. Set the compass so that a circle of radius "b" inches could be drawn.

3. With one end point as center draw an arc with the compass set as in (2).

4. Set the compass so that a circle of radius "c" inches could be drawn.

5. With the other end point as center draw an arc with the compass set as in (4) so that it crosses the first arc.

6. From the point of intersection of the two arcs draw line segments to the end points of the base line.

b. Constructing a regular polygon with "d" sides.

1. Draw a circle.

2. Divide it into "d" equal arcs by drawing "d" equal central angles.

3. Draw line segments connecting the points where the central angles intersect the circle.

EXERCISES:

I. Construct the following triangles using directions (a) above.

A. Construct a triangle with sides 3", 4", 5".

B. Construct a triangle with sides 1 3/4", 1 1/4", 1 1/2".

C. Construct an equilateral triangle.

D. Construct an isoceles triangle.

II. Construct the following polygons using directions (b) above.

A. Construct a square.

B. Construct a pentagon.

C. Construct a hexagon.

D. Construct an equilateral triangle.

E. Construct a decagon.

F. Construct a dodecagon.

INFORMAL GEOMETRY

G. Laboratory Exercise: Pythagorean Theorem

OBJECTIVE:

To find a relationship between the sides of a right triangle.

EQUIPMENT:

Ruler, right triangles made of cardboard.

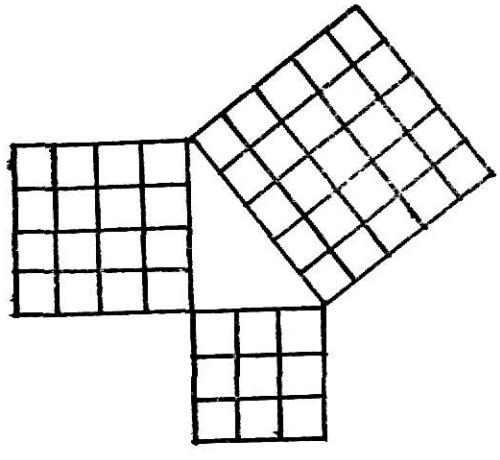
DIRECTIONS:

a. A right triangle always has a right angle. One of the sides of a right triangle is always longer than the other two sides. The longest side is called the hypotenuse.

b. Measure the sides of the triangles provided for you.

c. Give a rule that will tell how to determine the hypotenuse of a right triangle without measuring the sides.

d. Below you see a right triangle, $C = 90^{\circ}$, $AB = 5$ units, $AC = 4$ units, $BC = 3$ units. A square has been drawn on each side of the triangle.



How many square units are in the square of the hypotenuse? How many square units are there in the sum of the squares of the other two sides?

TABLE

Object	length of hypotenuse (c)	length of one side (a)	length of other side (b)	a^2	b^2	c^2
Δ 1						
Δ 2						
Δ 3						
Δ 4						
Δ 5						

RESULTS:

a. How are a^2, b^2, and c^2 related?

b. What formula expresses this relationship?

c. The above formula is called Pythagorean's Theorem's. Use Pythagorean's Theorem's to determine the unknown side in the following problems, where "c" is the hypotenuse and "a" and "b" are the other two sides.

1. a = 2", b = 3", c = ?

2. a = 12', b = 5', c = ?

3. a = 3", b = ?, c = 5".

4. a = 10', b = 3', c = ?

5. a = ?, b = 2", c = 7.28".

STATISTICS AND PROBABILITY

A. Laboratory Exercise: Gathering Statistical Data

OBJECTIVE:

To identify statistical data that can be used to determine dependable conclusions.

DIRECTIONS:

a. Record the kinds of cars parked in our school parking lot.

b. Use these results to approximate the number of Fords, Chevrolets, etc. in the U. S.

c. Using the World Almanac determine the number of Fords, Chevrolets, etc. in the U. S.

d. How closely do your figures agree with those found in the World Almanac?

e. What are some reasons why the suggested sampling may not be a proper guide?

f. What determines how dependable your conclusions (those you arrive at from your sampling) are?

STATISTICS AND PROBABILITY

B. Laboratory Exercise: Statistics

OBJECTIVE:

To learn to collect and organize figures.

EQUIPMENT:

Yard stick

DIRECTIONS:

a. Each student's height will be measured today. Record each of these heights in inches.

b. Arrange the figures in a frequency distribution.

c. Determine the range.

d. Determine the mean.

e. Determine the median.

f. Determine the mode.

STATISTICS AND PROBABILITY

C. Laboratory Exercise: Circle Graph

OBJECTIVE:

To construct a circle graph.

EQUIPMENT:

Protractor, compass, ruler.

DIRECTIONS:

a. Determine the amount of time you spend for sleep, for school, for study, for recreation, for miscellaneous activities.

b. Make a table showing: (1) given facts, (2) per cent each quantity is of the whole, (3) the number of degrees representing each per cent, obtained by multiplying the per cent by 360°.

c. Draw a convenient circle. With your protractor construct successive central angles, using the number of degrees representing each part.

d. Label each part of the circle.

e. Select and print an appropriate title.

STATISTICS AND PROBABILITY

D. Laboratory Exercise: Permutations

OBJECTIVE:

To be able to identify when to use the formula ${}_nP_r$ and to use the formula correctly.

EQUIPMENT:

4 different books, 2 book ends.

DIRECTIONS:

a. Arrange 3 different books between two book ends in as many ways as you can, keeping record of the number of different ways.

b. Arrange 4 different books between two book ends in as many ways as you can, keeping record of the number of different ways.

c. Using the formula ${}_nP_r = n(n-1)(n-2)\ldots(n-r+1)$, calculate the number of ways 3 different books can be arranged between two book ends.

d. Using the formula in "c", calculate the number of ways 4 different books can be arranged between two book ends.

e. Compare your experimental results with your calculations in "c" and "d".

DATA:

---

number of ways
of arranging 3
different books

---

number of ways
of arranging 4
different books

CALCULATIONS:

	experimental values	values from formula
ways-3 books		
ways-4 books		

STATISTICS AND PROBABILITY

E. Laboratory Exercise: Probability

OBJECTIVE:

To be able to calculate probability.

EQUIPMENT:

3 pennies.

DIRECTIONS:

a. Toss three pennies 16 times, keeping record of when you get 2H-1T, 3H, 3T, 1H-2T.

b. Toss three pennies 32 times, keeping record of when you get 2H-1T, 3H, 3T, 1H-2T.

c. Calculate the number of outcomes in tossing 3 coins.

d. Calculate the probability of getting 2H-1T, 3H, 3T, 1H-2T.

e. Put the number of times you get 2H-1T over the total number of outcomes (16 in case "a", 32 in case "b"). Repeat this process for 3H, 3T, 1H-2T. These are your experimental probabilities.

f. Compare the ratios you obtained in "e" to the probabilities you calculated in "d".

g. Were your experimental probabilities closer to the calculated probabilities when you tossed the coins 32 times than when you tossed the coins 16 times?

h. If you tossed the coins 200 times, would your experimental probabilities be closer to the calculated probabilities?

DATA:

	2H-1T	3H	3T	1H-2T
16 times				
32 times				

CALCULATIONS:

	Case "a" Probability	Case "b" Probability	Calculated Probability
2H-1T			
3H			
3T			
1H-2T			

COMPUTING, MANAGING, AND SPENDING YOUR INCOME

A. Laboratory Exercise: Buying Wisely

OBJECTIVE:

To learn to determine which products are the best buy and whether a buy is a bargain.

EQUIPMENT:

Can goods, bottles of liquids, etc., sale notices from newspapers.

DIRECTIONS:

a. At various places in the room there are collections of the same item. The items can be bought in various shapes and quantities and prices. Determine which of the items you think is the best buy. Give your reasons for choosing it.

b. Choose one of the sale notices provided.

c. Tell why each item is or is not a bargain.

COMPUTING, MANAGING, AND SPENDING YOUR INCOME

B. Laboratory Exercise: Writing and Cashing Checks and Balancing the Check Book

OBJECTIVES:

a. To learn how to write checks correctly.

b. To learn how to properly endorse checks.

c. To learn how to make out deposit slips.

d. To learn how to balance a check book.

EQUIPMENT:

Check blanks, check registers, deposit slips.

DIRECTIONS:

a. Make out a check to your parents.

b. Endorse this check as your parents would endorse it in blank.

c. When should a check be endorsed in this manner?

d. Make out a check to your English instructor.

e. Assume that your instructor gives the check to your math instructor. This is called endorsement in full. Endorse the check as it should be endorsed so that your math instructor can cash it.

f. Make out a check to yourself from your employer.

g. Assume that you want this check to be deposited only. Endorse this check as it should be endorsed. This is called restricted endorsement.

h. When should a check be endorsed in this manner?

i. Make out a deposit slip for this check.

j. Using your check register record your deposit and the amount of each check written. What is your balance?

COMPUTING, MANAGING, AND SPENDING YOUR INCOME

C. Laboratory Exercise: Sales Tax

OBJECTIVE:

To learn to determine sales tax on items one purchases.

EQUIPMENT:

Catalog.

DIRECTIONS:

a. Look through the catalog and decide on ten items you would like to purchase.

b. What is the sales tax rate in Kansas?

c. Using the rate in "b" determine the amount of sales tax you would pay on each item.

COMPUTING, MANAGING, AND SPENDING YOUR INCOME

D. Laboratory Exercise: Installment Buying

OBJECTIVE:

To be able to calculate the amount of money one spends for an item when buying on time and to compare this sum with the original price of the item.

EQUIPMENT:

Newspaper advertisements.

DIRECTIONS:

a. From the advertisements available choose eight items you would like to purchase.

b. Determine the amount you would pay if you bought the item "on time".

c. Determine how much you would save by paying cash for your purchases.

DISCUSSION:

a. Assume you have just agreed to buy one of the eight items above. When does the item actually belong to you: when you leave the store with it, or when you have paid the full price plus service charge? Why?

b. At your company credit union you can borrow the money to pay for item number _____ (your instructor will select one for you) above at the rate of 1 per cent per month on the unpaid balance.

   1. Find the total amount of each monthly payment if you repaid the amount in the same number of monthly installments as the store asked.

   2. Find the total amount you would pay the credit union.

c. Compare the amount you would pay the store and the amount you would pay the credit union.

COMPUTING, MANAGING, AND SPENDING YOUR INCOME

E. Laboratory Exercise: Catalog Buying

OBJECTIVE:

To be able to order from a catalog correctly.

EQUIPMENT:

Catalog, ordering forms.

DIRECTIONS:

a. Look through the catalog and decide on ten articles from the following sections of the catalog:

1. Clothing
2. Fabric
3. Automotive
4. Sporting goods
5. Appliances
6. Bedding
7. Furniture

b. If practical, order more than one of the article.

c. Fill out your order form in its entirety.

d. Make out a check or money order for the amount you would pay for the merchandise.

COMPUTING, MANAGING, AND SPENDING YOUR INCOME

F. Laboratory Exercise: Budget Preparation

OBJECTIVE:

To be able to prepare a budget and write checks and balance the check book as one spends the money as indicated in the budget.

EQUIPMENT:

Catalog, check blanks, check registers.

DIRECTIONS:

a. Assume that ten years from now you have a job. What would you like that job to be?

b. Using the job you mentioned in "a" determine your pay by using the Encyclopedia of Careers and Vocational Guidance.

c. Prepare a budget for one month on the basis of this salary.

d. Write checks as if you were spending this salary for one month.

e. Balance your check book.

ACKNOWLEDGMENTS

The writer would like to express her appreciation to Dr. Charles M. Peccolo, major advisor, for his guidance and advice. The writer also wishes to express her thanks to her husband, David, for his cooperation and understanding during the writing of this report.

BIBLIOGRAPHY

Johnson, Donovan A. Guidelines for Teaching Mathematics. Belmont, California: Wadsworth Publishing Company, Inc., 1967. 446 pp.
A general text on the methods of teaching mathematics. It is an excellent source for an overview of mathematics laboratories and suggestions for experiments.

Johnson, Larry K. "The Mathematics Laboratory in Today's Schools," School Science and Mathematics, 62:586-92, November, 1962.
A thorough discussion of the mathematics laboratory.

Kluttz, Marguerite. "The Mathematics Laboratory--a Meaningful Approach to Mathematics Instruction," The Mathematics Teacher, 56:141-5, March, 1963.
A general discussion of the mathematics laboratory and its value.

Krug, Edward A. Curriculum Planning. New York: Harper and Brothers Publishers, 1957. 336 pp.
A general text on curriculum planning. General mathematics and the terminal student is covered.

Manwiller, Lloyd V. "Laboratory Activities--Why?" School Science and Mathematics, 61:85-9, February, 1956.
A discussion of the rationale of the laboratory method.

Ostrander, Ida A. "A General Mathematics Program for a Large School, The Bulletin of the National Association of Secondary-School Principals, 43:46-50, May, 1959.
A description of a mathematical curriculum which emphasizes general mathematics in a comprehensive high school with a population of a thousand.

Phillips, Harry L. "The Mathematics Laboratory," American Education, 1:1-3, March, 1965.
An excellent report on mathematics laboratories as used in the schools of America.

Ramsmeyer, J. A. "The Mathematics Laboratory," The Mathematics Teacher, 28:228-33, April, 1935.
A discussion of the mathematics laboratory.

Reeve, William David. Mathematics for the Secondary School. New York: Henry Holt and Company, 1954. 547 pp.
A methods book concerning the content and methods of teaching and learning mathematics.

Sims, Weldon and Albert Oliver. "The Laboratory Approach to Mathematics," School Science and Mathematics, 50:621-7, November, 1950.
A general discussion of the laboratory concept.

Sweet, Raymond. "Organizing a Mathematics Laboratory," The Mathematics Teacher, 60:117-20, February, 1967.
A thorough discussion of how to organize a mathematics laboratory.

Bigge, Morris L. Learning Theories for Teachers. New York: Harper and Row, Publishers, 1965. 366 pp.
An excellent book concerning the explanation of various learning theories. Principles of learning that are commonly accepted by psychologists was covered.

Butler, Charles H. and F. Lynwood Wren. The Teaching of Secondary Mathematics. New York: McGraw-Hill Book Company, 1965. 613 pp.
A thorough text on mathematics methodology.

"Confronting the New Technology," Business Education World, 48:11, 27-8, December, 1967.
A report of a laboratory approach to applied and advanced mathematics as used in Teaneck High School in Teaneck, New Jersey. Their laboratories are equipped with calculators and computers.

Engen, H. Van. "Arithmetic in the Junior-Senior High School," The Teaching of Arithmetic, National Society for the Study of Education Fiftieth Yearbook, Part II. Chicago, Illinois: The University of Chicago Press, 1951. 302 pp.
The author discusses why applied mathematics courses should be taught at the high school level in the eleventh or twelfth grade.

Frame, J. S. "Facilities for Secondary School Mathematics," The Mathematics Teacher, 62:379-91, October, 1964.
A discussion of facilities for a mathematics classroom. Types of laboratories are covered.

Groenendyk, Eldert A. "General Math for Terminal Students," Midland Schools, 78:12-3, May, 1964.
A brief discussion of the use of the mathematics laboratory.

_____. "Lab Suited for Advanced General Math," Midland Schools, 78:14-5, May, 1964.
A report of a mathematics laboratory in Carlisle High School in Warren County, Iowa. A General Mathematics II course is taught using printing calculators.

_____. "The Mathematics Laboratory," Midland Schools, 77:11-3, January, 1963.
A report of mathematics laboratories as used in Carlisle and Marion high schools in Iowa.

Grossnickle, Foster, Charlotte Junge and William Metzner. "Instructional Material for Teaching Arithmetic," The Teaching of Arithmetic, National Society for the Study of Education Fiftieth Yearbook, Part II. Chicago, Illinois: The University of Chicago Press, 1951. 302 pp.
The authors emphasize the use of the mathematics laboratory for effective learning of generalizations.

"The Secondary Mathematics Curriculum," The Mathematics Teacher, 52:389-417, May, 1959.

A report by the Secondary-School Curriculum Committee of the National Council of Teachers of Mathematics on needed mathematics curriculums in our schools.

Weiss, Sol. "Innovations and Research in the Teaching of Mathematics to the Terminal Student," The Mathematics Teacher, 60:611-8, October, 1967.

An excellent discussion concerning who the terminal student is and what to do with this student in mathematics.

APPENDIX A

BIBLIOGRAPHY OF SELECTED TEXTBOOKS

Bernstein, Allen L. and David W. Wells. Trouble Shooting Mathematics Skills. New York: Holt, Rinehart and Winston, Inc., 1963. 358 pp.

Brown, Kenneth E. and Daniel W. Snader. General Mathematics--Book One. River Forest, Illinois: Laidlaw Brothers Publisher, 1963. 640 pp.

Brown, Kenneth E., Leonard Simon, and Daniel W. Snader. General Mathematics--Book Two. River Forest, Illinois: Laidlaw Brothers Publisher, 1963. 640 pp.

Crocks, Thomas C. and Harry L. Hancock. Contemporary Arithmetic. New York: The MacMillan Company, 1968. 336 pp.

Haag, Vincent H. and Edwin A. Dudley. Introduction to Secondary Mathematics--1. Boston: D. C. Heath and Company, 1967. 306 pp.

Hart, Walter W., Veryl Schult and Lee Irvin. Mathematics in Daily Use. Boston: D. C. Heath and Company, 1966. 372 pp.

Herrick, Marian C. Modern Mathematics for Achievement. Boston: Houghton Mifflin Company, 1966. 368 pp.

Herrick, Marian Cliffe, and others. Modern Mathematics for Achievement, Second Course. Boston: Houghton Mifflin Company, 1967. 368 pp.

Lankford, Frances G., Jr., James L. Ulrich and John R. Clark. Essential Mathematics. New York: World Book Company, 1961. 536 pp.

Lankford, Frances G., Jr., Raleigh Schorling and John R. Clark. Mathematics for the Consumer. New York: World Book Company, 1953. 438 pp.

Peters, M. Going Places with Mathematics. Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1956. 358 pp.

Stein, Edwin I. First Course in Fundamentals of Mathematics. Boston: Allyn and Bacon, Inc., 1966. 290 pp.

Stein, Edwin I. Second Course in Fundamentals of Mathematics. Boston: Allyn and Bacon, Inc., 1966. 354 pp.

_____. Fundamentals of Mathematics, Modern Edition. Boston: Allyn and Bacon, Inc., 1968. 468 pp.

_____. Refresher Mathematics. Boston: Allyn and Bacon, Inc., 1966. 655 pp.

Thordarson, T. W. and R. Perry Anderson. Mathematics for High Schools. Boston: Allyn and Bacon, Inc., 1965. 310 pp.

APPENDIX B

TYPES OF EXPERIMENTS[1]

1. Make measurements of two variables in an everyday setting to determine the relationship between them.
2. Collect data by performing an operation.
3. Find a pattern by drawings or constructions.
4. Discover a scientific principle by performing an experiment with simple equipment.
5. Explore a relationship by manipulating simple objects.
6. Collect original data from a survey to determine what variables are related.
7. Perform indirect measurements to determine unknown distances.
8. Extend the solution of a specific problem to a generalization.
9. Represent a mathematical idea by building a simple model.
10. Complete an individual project or report.
11. Illustrate mathematical ideas by folding paper.
12. Build exhibits, charts, bulletin board displays.
13. Use commercial laboratory devices to perform experiments.
14. Use laboratory kits for laboratory work.
15. Plan, make, and use mathematical games, puzzles, stunts.
16. Operate calculators and program computers.
17. Organize and operate a business enterprise.
18. Use an audio-visual aid as the basis for laboratory work.

---

[1]Donovan A. Johnson, Guidelines for Teaching Mathematics (Belmont, California: Wadsworth Publishing Company, Inc., 1967), pp. 307-10.

APPENDIX C

LABORATORY MATERIALS[1]

The following list of items is considered useful in an up-to-date mathematics laboratory:

1. An overhead projector complete with an acetate roll, transparency sheets, pencils, and other accessories.

2. Wall screens of appropriate size for use with the overhead projector and other audio-visual machines.

3. A complete library, including a large variety of books at various ability levels, magazines, experimental mathematics materials, supplemental textbooks, recreational mathematics materials, pictures, poster, charts, programmed teaching materials, information on mathematics films and filmstrips, monographs, enrichment booklets, collateral reading materials, occupational information relative to mathematics, and other reference materials.

4. Rigid models of various kinds to help in understanding the geometric designs and shapes, as well as becoming familiar with certain measuring instruments.

5. Storage cabinets and bookcases to adequately store and protect the equipment and materials.

6. A tape recorder that may be used for individual or group instruction.

7. A combination slide and filmstrip projector accompanied with appropriate slides and filmstrips that may be used to clarify situations that arise in the course.

8. A controlled reading machine or tachistoscope accompanied with appropriate filmstrips that may be used to improve mental reading and computation abilities.

9. An electric calculator that may be used to perform lengthy manipulations.

10. A few simple hand adding machines that may be used, at first, to understand the concept of addition, and later, to free the pupil from boring computations.

11. Work tables that may be used during the experimental portions of the class period.

---

[1]Eldert Groenendyk, "The Mathematics Laboratory," Midland Schools, 77:12, January, 1963.

12. A variety of adjustable figures and models that are helpful in explaining geometric theorems.

13. A variety of instruments, such as pantographs, compasses, rulers, meter sticks, slide rules, drawing pencils, protractors, and T-squares that may be used in appropriate situations.

14. A plainimeter to help create understanding of formulas for areas of regular and irregular figures.

15. A complete transit set that may be used in outdoor projects where angle measurement is necessary.

16. A stop watch that may be used in collecting data used in deriving generalizations and formulas.

17. A variety of manipulative devices and games to stimulate interest and help in clarifying basic concepts.

18. A magnetic chalkboard accompanied with magnetic devices that may be used in explaining set theory, number line, and numeration systems.

19. An adequate supply of chalkboard instruments that may be used in chalkboard work and demonstrations.

20. An opaque projector for showing materials that cannot be reproduced easily on transparencies.

21. The regular equipment and furniture required in classrooms, such as the teacher's desk, pupil desks, filing cabinets, typewriter, closet space, chalkboards, extension cords, convenient electrical outlets, proper light control for audio-visual equipment, and other minor items.

22. Special enrichment items, such as simple teaching machines, record player, simple tools, portable screen, and mathematics kits provide many enrichment possibilities.

23. Headphone sets may also be used for individual or group study.

24. The large electronic teaching machines with special booths may be used with prepared filmstrips for remedial and enrichment purposes.

25. A few drawing boards and instruments may be available for work in scale drawings.

LABORATORY EXPERIMENTS FOR SENIOR HIGH GENERAL MATHEMATICS

by

GLENDA GAYLE TRAVIS

B.S., Fort Hays Kansas State College, 1965

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

College of Education

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

This study was designed to develop three sets of laboratory experiments for teachers which would enable them to make use of the laboratory method in the classroom with a minimum of expensive equipment. The experiments are presented in the form of guide sheets. They were designed for an eleventh or twelfth grade general mathematics course.

Specifically, the three sets of experiments deal with these three units of study:

1. Informal Geometry
2. Statistics and Probability
3. Computing, Managing, and Spending Your Income.

The procedures employed in this study consisted of (1) an analysis of general mathematics textbooks available for use at Kansas State University to determine the units and topics to be included in the experiments, (2) development of a format for guide sheets used by students in the laboratory, and (3) designing the experiments to be used in the three units of study.

The review of literature covered the following topics:

1. How the laboratory method makes use of accepted principles of learning.
2. Educators who recommend using the laboratory method of teaching mathematics.
3. Organizing a mathematics laboratory.
4. Level at which the terminal student should enroll in general mathematics.