MATRIX SOLUTION FOR LINEAR AND NONLINEAR BUCKLING

OF HYPERBOLOIDS OF REVOLUTION

by

CHEN PAN

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INTRODUCTION

From the first cooling tower built in 1963 (1), because of the increasing needs for electric power, the hyperboloid of revolution shell cooling tower for the nuclear power plant has been built bigger and the thickness of the shell has become thinner. The stability of the shell is becoming of great importance in the designing of the hyperboloid cooling tower. By considering the safety of the shell, the buckling behavior of the shell was examined in reference (2). Generally, three methods of analysis such as the analytical method, the experiment method and the numerical method have been used in these investigations.

The third kind of the above method is used in this research to predict the buckling behavior hyperboloid shells. A simplified approximate mathematical model based on structural mechanics has been developed and is constructed by the use of Sylvester's Operator (6) with the aid of the finite difference method (7), although it can be viewed as a finite strip method (8). By partitioning the continuous Hyperboloid shell into vertical strip and ring strips, the intersection of these partition lines gives a discrete mesh. By the relationship between deformation and applied forces derived by (9), the Sylvester's Operator can naturally describe this discrete system; it reads, $AW\alpha + \beta WB = \lambda(CW + WD)$. The matrices A and B relate the finite difference operators as approximate formulations of differential operators in the differential equation which characterize the relationship between forces and deformation. The terms α and β are weightings of geometry and stiffness, and λ is the eigenvalue of interest with the buckling mode charterized by matrix W. By the iteration method one can find the smallest eigenvalue, corresponding to the linear buckling load, and the eigenmatrix W, such that W(I, J) equals the value of the mode shape function at $\phi(I)$, $\theta(J)$ of the first mode of buckling. The applied load considered in this analysis consists of selfweight, uniform external pressure and axial load imposed upon the top of the shell. A computer program based on this method has been developed and used to complete the calculation using the ITEL computer, this is presented in Appendix A.

The application of this research is the following:

1. It can give a specific idea for the designer to design a more safe hyperboloid shell.
2. It can give direction on experimental testing for buckling of these shells.
3. It can be developed into a nonlinear analysis based on a mathematical model.

Some results and possible applications of the research are summerized in the concluding remarks.

CHAPTER ONE

Derivation of Basic Relationship of Geometry and Equilibrium

1-1 Differential Geometry of a surface

The thickness of the shell is denoted by h. For a thin shell, h is small compared with the other dimensions of the shell and its radii of curvature. The surface that bisects the thickness of the shell is the middle surface. The geometry of a shell is entirely defined by specifying the form of the middle surface and the thickness of the shell at each point. The notations of vector analysis are used frequently to describe some geometric properties of a surface. A surface is defined as the locus of points each of whose position vector $\vec{r}$, relative to some fixed origin o, is a function of two independent parameters c_1, c_2.

Thus, the cartesian coordinates (x, y, z) of a point on a surface are known functions of c_1, c_2 and can be written as

$$\{\vec{r}\ (c_1,\ c_2)\} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} f_1\ (c_1,\ c_2) \\ f_2\ (c_1,\ c_2) \\ f_3\ (c_1,\ c_2) \end{Bmatrix} \tag{1-1}$$

or $\vec{r}(c_1,\ c_2) = r(c_1,\ c_2)\ \vec{n}$, $\vec{n}$ is a unit vector along $\vec{r}$. Equation (1-1) is the parametric equation of a surface. If c_1 and c_2 are eliminated in these equation, the following form is

$$F(x,\ y,\ z) = 0 \tag{1-2}$$

is for the surface defined by equation (1-1).

Once a relationship between parameters, say $g(c_1,\ c_2) = 0$, is given, $\vec{r}$ becomes a function of only one independent parameter, and the locus is reduced to a curve. Curves on a surface along which a parameter remains constant are called parametric curves. The parameters c_1 and c_2 thus constitute a system of curvilinear coordinates for points on the surface. The position of any point on the surface being determined by the value c_1 and c_2 is illustrated in Fig. 1-1.

As an example of this method of description, one can consider the surface in terms of the usual spherical coordinates (R, ϕ, θ) as shown in Fig. 1-2. If R is the radius of the sphere, the cartesian coordinates of a point on the sphere are

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} R \sin\phi \cos\theta \\ R \cos\phi \\ R \sin\phi \sin\theta \end{Bmatrix} \tag{1-3}$$

In this case, parameters θ and ϕ may be considered as the general parameters c_1 and c_2.

1-2 Membrane Forces for Shells in the form of Revolution

In many problems of thin shells, the deformation due to the load effects follow the shape when resisting the external distributed load, and stresses in the shell are mainly due to membrane forces N_ϕ, N_θ and $N_{\phi\theta}$. That is the effects due to bending are very small (10). In such cases, a good approximation for stresses under the application of external loads can be obtained by neglecting the influence of the bending moments and the related shearing forces. Thus, one can determine N_ϕ, N_θ and $N_{\phi\theta}$ from the solutions to the differential equations obtained from the equilibrium conditions. In the other words, if the external forces acting on the shell are given, the problem becomes statically determinated, and the unknown forces can be determined without using the strain relations. The problem of stress analysis is therefore greatly simplified. The forces N_ϕ, N_θ and $N_{\phi\theta}$ obtained in this manner are called membrane forces, and the theory based on this assumption is called membrane theory.

For shells in the form of surfaces of revolution, a surface is generated by rotating a meridinal curve about an axis of symmetry. A natural selection for the curves c_1 = constant are the meridians and c_2 = constant are the circles in the planes perpendicular to the axis of revolution. Using notations as shown in Fig. 1-3a, $c_1 = \theta$ and $c_2 = \phi$ can be defined. The tangents of the curvilinear coordinate curves ϕ and θ passing through the point A are denoted by x and y, respectively.

In the case of a surface of revolution, it is clear that the meridianal plane contains one of the principal radii of curvature, and the other principal radius of curvature will be that of the curve which is contained by the plane perpendicular to the meridian at the point of intersection. These two radii lie on the same line, but in general, have different lengths associated with the different cutting planes. Thus, the length of a line element on the middle surface of the shell is given by the formulation.

$$ds^2 = (r_2 d\theta_1)^2 + (r_1 d\phi)^2 \tag{1-5}$$

For convenience, equation (1-5) can be written as

$$ds^2 = (r d\theta)^2 + (r_1 d\phi)^2 \tag{1-5'}$$

It can be observed that $r_2 \sin\phi = r$. If the bending stresses in the shell can be neglected, one may assume that $M_\theta = M_\phi = M_{\theta\phi} = M_{\phi\theta} = 0$.

Furthermore, if a hyperboloid of revolution is loaded symmetrically with respect to its axis, the membrane forces N_ϕ^g, N_θ^g, N_ϕ^q, N_θ^q, N_ϕ^p, N_θ^p (Fig. 1-3b) (with $N_{\theta\phi} \cong N_{\phi\theta} = 0$) due to the selfweight, external uniform pressure and axial load can be calculated as follows:

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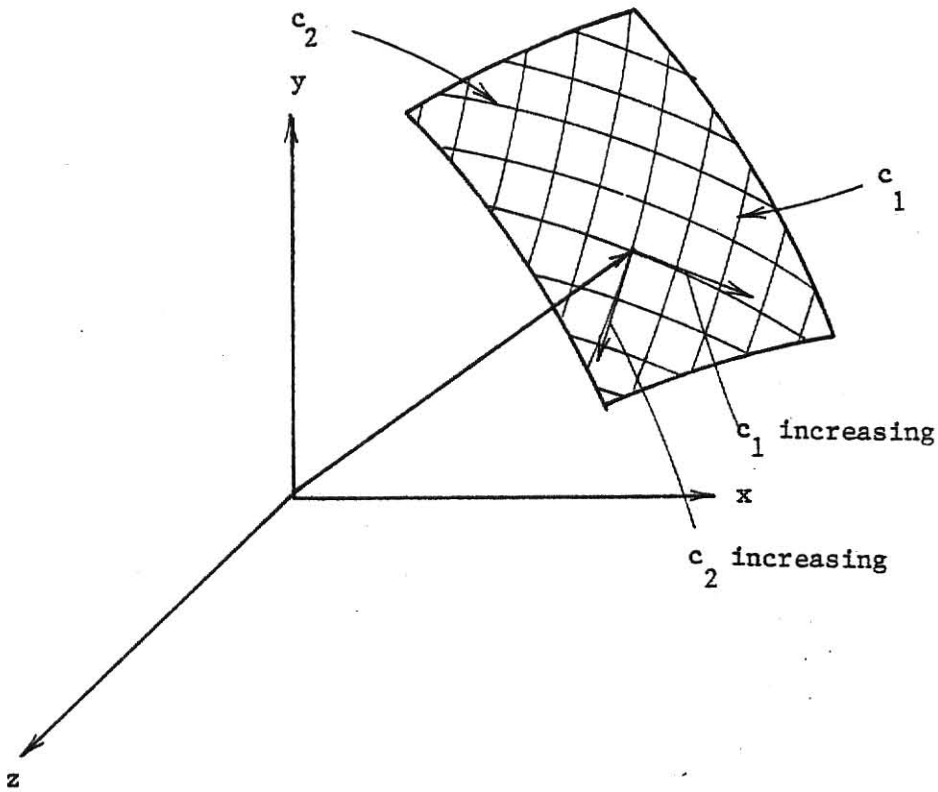


Fig. 1-1 Parameters of the constitutive system

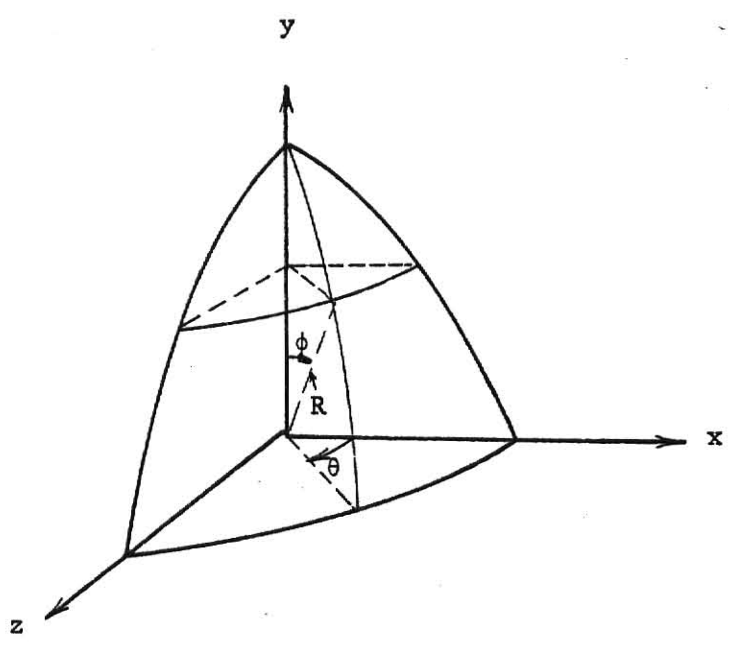


Fig. 1-2 Rectangular and Spherical coordinate system

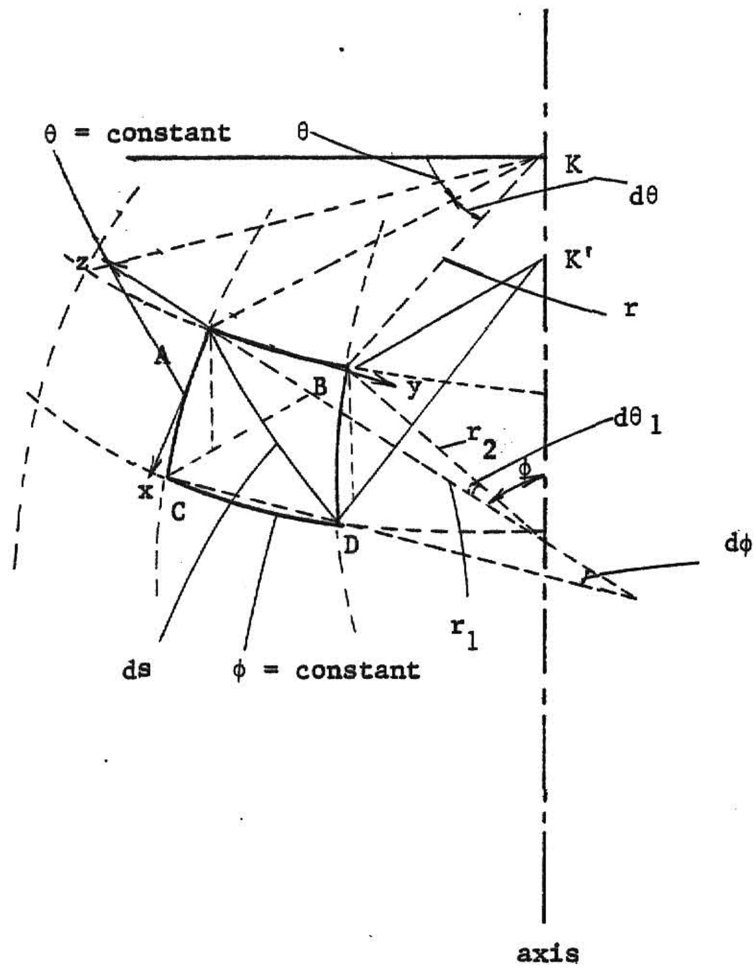


Fig. 1-3-a Geometry of the element in the shell of revolution

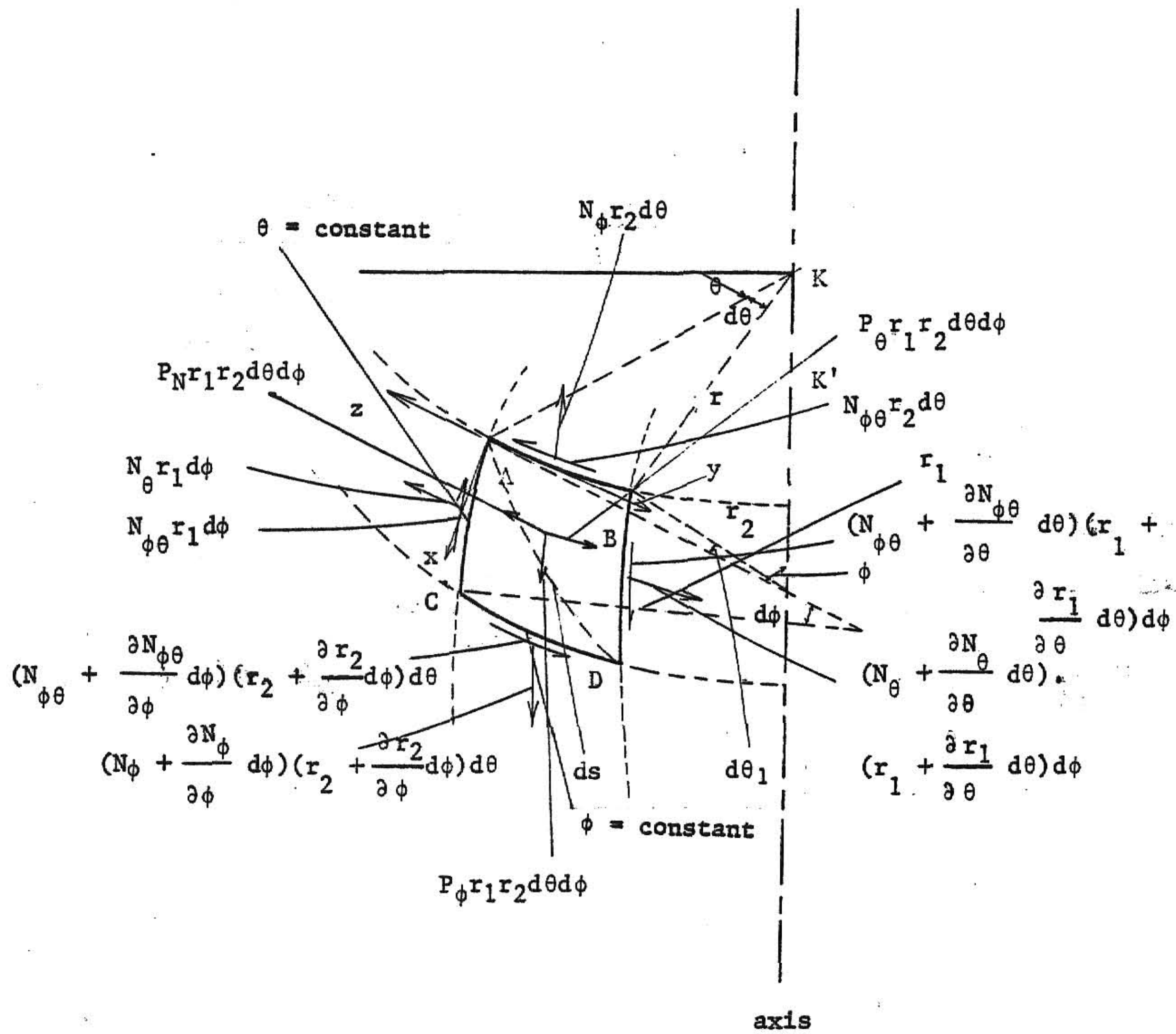


Fig. 1-3-b Membrane forces acting on the shell of revolution

a) Due to selfweight g:

$$N_\phi^g = -\frac{g}{4} b^2 \sqrt{a^2 + b^2} \frac{\sqrt{1 - \xi^2}}{(a^2 + b^2 - a^2\xi^2)} (f(\xi) - f(\xi_T)) \quad (1\text{-}6)$$

$$N_\theta^g = \frac{-ga^2}{\sqrt{a^2 + b^2}} \frac{\xi}{\sqrt{1 - \xi^2}} + N_\phi^g \frac{a^2}{b^2} (1 - \xi^2) \quad (1\text{-}7)$$

with

$$f(\xi) = \frac{2\xi}{1 - \xi^2} + \ell n \frac{1 + \xi}{1 - \xi}$$

and

$$\xi = \sqrt{1 + (\frac{b}{a})^2} \cos\phi$$

where the geometric parameters a and b are defined in Fig. 2-1.

b) Due to uniform external pressure q:

$$N_\phi^q = \frac{-qa^2}{2b} \frac{\sqrt{\tau}}{1 + \tau} (\tau - \tau_T) \quad (1\text{-}8)$$

$$N_\theta^q = \frac{-r_2}{r_1} N_\phi^q - r_2 q \quad (1\text{-}9)$$

with

$$\tau = \frac{b^2}{a^2 \sin^2\phi - b^2 \cos^2\phi}$$

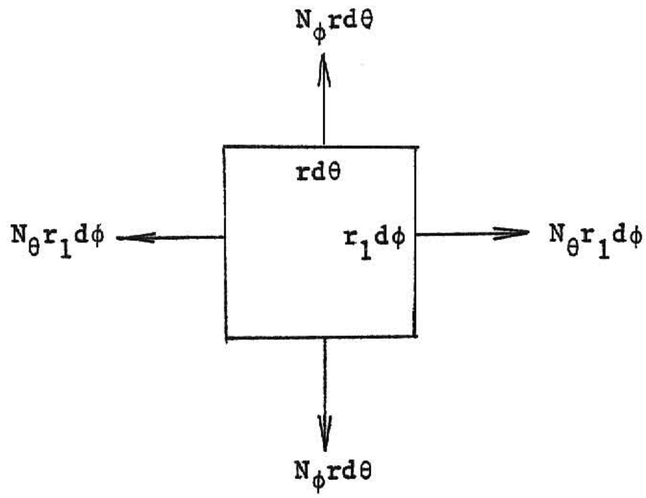
and subscript T of τ_T indicates its value of τ at the top of the shell.

c) Due to downward axially distributed load p per unit length of circumference applied at the top of the shell

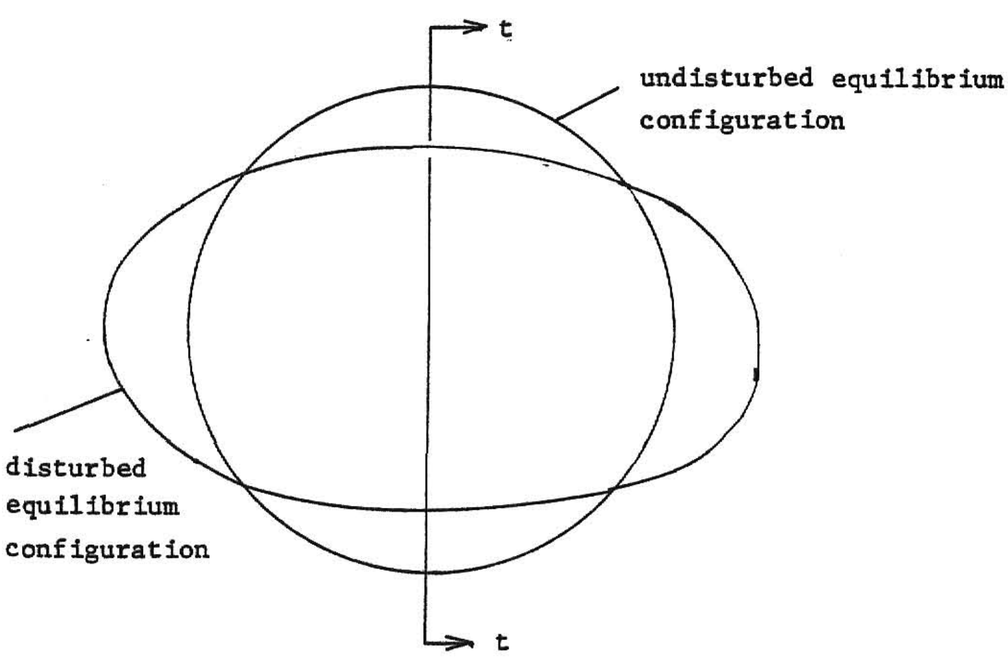
$$N_\theta^p = \frac{-r_2}{r_1} N_\phi^p \quad (1\text{-}10)$$

$$N_\phi^p = \frac{-R_{Top}\ p}{r \sin\phi} \quad (1\text{-}11)$$

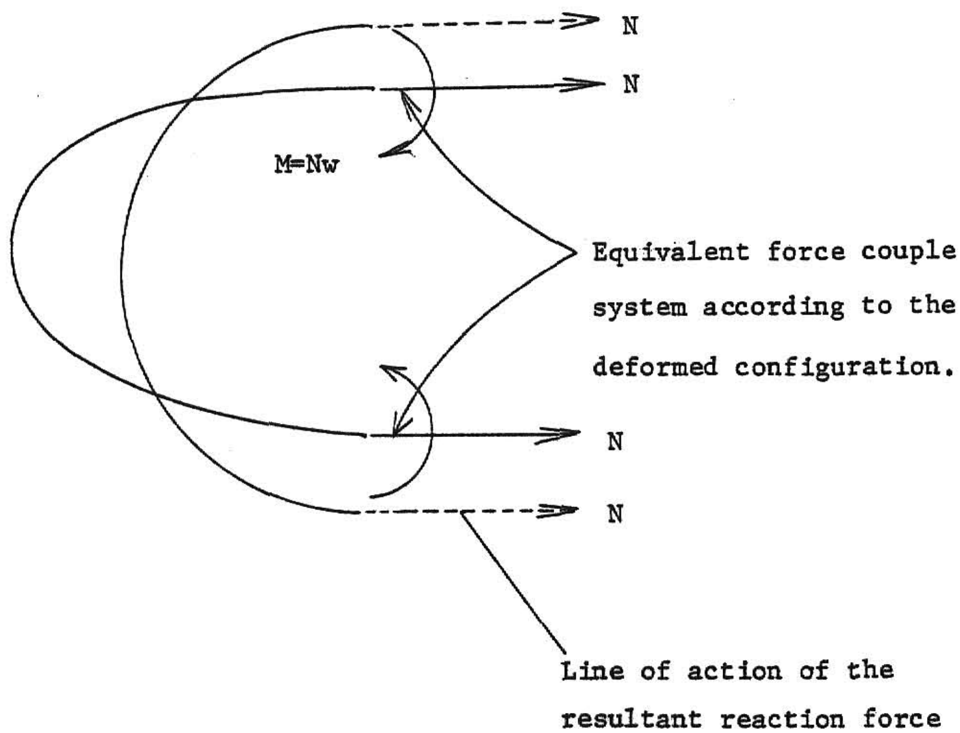
The uniform load applied by suction in the top of the shell (2), (12). Therefore the total axial load taken by the shell wall is 0.56 R_{Top}^2 q



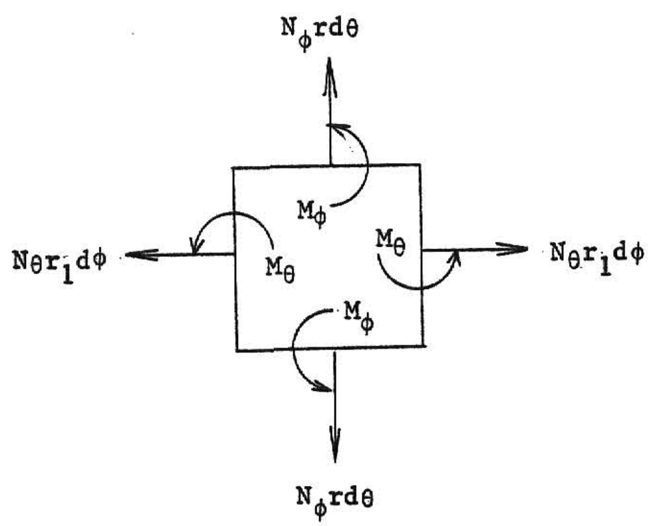
1-4-a A differential shell element



1-4-b Deformation from the undistributed equilibrium position



1-4-c. Bending moment due to the deformation



1-4-d. M_ϕ and M_θ on a differential shell element

Fig. 1-4 Relation between Forces and Deformation because the column will take approximately 44% of the total load based on plate bending theory (22). Thus, the load per unit length of the shell wall is

$$P = 0.28\ R_{Top}\ q \tag{1-12}$$

1-3 Bending Moment due to Elastic Buckling Deformation

Both r and r_1 can be expressed as functions of ϕ. A differential element of the shell shown in Fig. 1-4-a shows the sign convention for positive N_ϕ and N_θ. The material of the shell is assumed to be homogeneous and isotropic before buckling. The buckling deformation of the location of interest is measured from the equilibrium position of a loaded shell before buckling. Fig. 1-4-b is a cross section of the shell showing the membrane deformation from the equilibrium position due to buckling. The diagram in Fig. 1-4-c illustrates the bending moment produced by N due to the lateral deformation w which has the same effect as shifting the line of action to a new position, and the outward direction of w will be assumed to be positive. Therefore,

$$M_\phi = N_\phi\ w \tag{1-13-a}$$

and

$$M_\theta = N_\theta\ w \tag{1-13-b}$$

The moment due to buckling deformation w is calculated by the product of the membrane force and the perpendicular component to the deformation at any point of the shell. The bending moments M_ϕ and M_θ also can be obtained by the curvature changes which are

$$M_\phi = K(K_\phi + \mu K_\theta)$$

$$M_\theta = K(K_\theta + \mu K_\phi)$$

K_ϕ and K_θ are the curvatures, and K is stiffness of the shell. According to reference (13), the second derivative of the moment will give the equivalent lateral load associated with the buckling deformation of the structure. By taking the derivate of M_ϕ with respect to s_ϕ twice, equation (1-13-a) becomes to

$$\frac{\partial^2 M_\phi}{\partial s_\phi^2} = \frac{\partial^2}{\partial s_\phi^2}(N_\phi w) \qquad (1\text{-}14)$$

Using Q_ϕ and Q_θ to represent the equivalent lateral load due to M_ϕ and M_θ, respectively, then

$$\frac{\partial^2 M_\phi}{\partial s_\phi^2} = \frac{\partial^2}{\partial s_\phi^2}(N_\phi w) = Q_\phi \qquad (1\text{-}15)$$

Similarly, taking second derivative of M_θ with respect to s_θ from equation (1-13-b), yields

$$\frac{\partial^2 M_\phi}{\partial s_\theta^2} = \frac{\partial^2}{\partial s_\theta^2}(N_\theta w) = Q_\theta \qquad (1\text{-}16)$$

Note that $r_1 d\phi = ds_\phi$ and $r d\theta = ds_\theta$. Thus, the first and second derivative can be carried out in the following form,

$$\begin{aligned} \frac{\partial w}{\partial s_\theta} &= \frac{1}{r}\frac{\partial w}{\partial \theta}, \quad \frac{\partial^2 w}{\partial s_\theta^2} = \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}\right); \\ \frac{\partial w}{\partial s_\phi} &= \frac{1}{r_1}\frac{\partial w}{\partial \phi}, \quad \frac{\partial^2 w}{\partial s_\phi^2} = \frac{1}{r}\frac{\partial}{\partial \phi}\left(\frac{1}{r_1}\frac{\partial w}{\partial \phi}\right) . \end{aligned} \qquad (1\text{-}17)$$

Substituting into Equations (1-15) and (1-16) one has

$$\frac{1}{r_1}\frac{\partial}{\partial \phi}\left(\frac{1}{r_1}\frac{\partial}{\partial \phi}\right) M_\phi = \frac{1}{r_1}\frac{\partial}{\partial \phi}\left(\frac{1}{r_1}\frac{\partial}{\partial \phi}\right)(N_\phi w) = Q_\phi; \qquad (1\text{-}18a)$$

and

$$\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{r}\frac{\partial}{\partial \theta}\right) M = \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{r}\frac{\partial}{\partial \theta}\right)(N_\theta w) = Q_\theta \quad . \qquad (1\text{-}18b)$$

Due to the fact that r is independent of θ, the second differential equation of (1-18) can be simplified to

$$\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} M_\theta = \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}(N_\theta\ w) = Q_\theta \qquad (1\text{-}18\text{-}b')$$

These differential relationship between the product of membrane forces and normal deflection and bending moments will be used in deriving the matrix equation which characterizes the stability of the shell.

1-4 Bending Moments, Curvature Changes and the Characteristic Equation

The main object of this section is to use the differential relationship between bending moments and curvature changes, and the curvature changes to the displacement function to derive the characteristic equation of the cooling tower. The expression for bending moment derived by W. Flugge (10) has been used and listed here:

$$M_\phi = K\{\frac{1}{r_1}(\frac{\dot{w}}{r_1})^{\bullet} + \frac{\mu}{r}(\frac{w''}{r} + \frac{\dot{w}}{r_1}\cos\phi)\}$$

and

$$M_\theta = K\{\frac{1}{r}(\frac{w''}{r} + \frac{\dot{w}}{r_1}\cos\phi) + \frac{\mu}{r_1}(\frac{\dot{w}}{r_1})^{\bullet}\} \qquad (1\text{-}19)$$

According to these equations M_ϕ and M_θ are expressed as functions of the buckling deformation w. In equation (1-19) the derivatives with respect to ϕ and θ are defined as

$$\dot{w} = \frac{\partial w}{\partial\phi}\ ,\quad w' = \frac{\partial w}{\partial\theta} \quad \text{and} \quad (\frac{\dot{w}}{r_1})^{\bullet} = \frac{\partial}{\partial\phi}(\frac{\dot{w}}{r_1}) \qquad (1\text{-}20)$$

The factor K is the flexural rigidity of the shell, and is

$$K = \frac{Eh^3}{12(1-\mu^2)}$$

and E is Young's modulus, h is the thickness of the shell and μ is Poisson's ratio.

From equation (1-19) the second derivative of M_ϕ and M_θ can be calculated by the following expressions.

$$\frac{\partial^2}{\partial s_\phi^2} M_\phi = \frac{1}{r_1}\frac{\partial}{\partial\phi}\left(\frac{1}{r_1}\frac{\partial M_\phi}{\partial \phi}\right) = \frac{1}{r_1}\frac{\partial}{\partial\phi}\left(\frac{1}{r_1}\frac{\partial}{\partial\phi}\right)\{K\frac{1}{r_1^2}\left(\frac{\partial^2}{\partial\phi} - \frac{1}{r_1}\frac{\partial r_1}{\partial\phi}\frac{\partial}{\partial\phi}\right) w\}$$

$$+ \{\frac{1}{r_1}\frac{\partial}{\partial\phi}\left(\frac{1}{r_1}\frac{\partial}{\partial\phi}\right)(K\frac{\mu}{r^2}\frac{\partial^2 w}{\partial\theta^2})\} + \{\frac{1}{r_1}\frac{\partial}{\partial\phi}\left(\frac{1}{r_1}\frac{\partial}{\partial\phi}\right)\cdot$$

$$(K\frac{\mu}{r\, r_1}\frac{\partial w}{\partial\phi}\cos\phi)\} \tag{1-21}$$

and

$$\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} M_\theta = \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} K\ \{(\frac{1}{r^2}\frac{\partial w^2}{\partial\theta^2} + \frac{1}{r\, r_1}\frac{\partial w}{\partial\phi}\cos\phi)$$

$$+ (\frac{\mu}{r_1^2}(\frac{\partial^2}{\partial\phi^2} - \frac{1}{r_1}\frac{\partial r_1}{\partial\phi}\frac{\partial}{\partial\phi})\ w)\} \tag{1-22}$$

$$= K\{\frac{1}{r^4}\frac{\partial^4}{\partial\theta^4} w + \frac{\cos\phi}{r^3}\frac{1}{r_1}\frac{\partial^2}{\partial\theta^2}\frac{\partial w}{\partial\phi} +$$

$$\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}(\frac{\mu}{r_1^2}(\frac{\partial^2 w}{\partial\phi^2} - \frac{1}{r_1}\frac{\partial r_1}{\partial\phi}\frac{\partial w}{\partial\phi}))\}$$

Note that the equivalent lateral load is $Q = Q_\phi + Q_\theta$ which can be calculated by

$$\frac{\partial^2 M_\phi}{\partial s_\phi^2} + \frac{\partial^2 M_\theta}{\partial s_\phi^2} = \frac{\partial^2 (N_\phi w)}{\partial s_\phi^2} + \frac{\partial^2 (N_\theta w)}{\partial s_\phi^2} \tag{1-23}$$

The substitution of equations (1-18), (1-21) and (1-22) into (1-23), yields

$$\frac{1}{r_1}\frac{\partial}{\partial\phi}\left(\frac{1}{r_1}\frac{\partial}{\partial\phi}\right)\{K(\frac{1}{r_1}\frac{\partial}{\partial\phi}\left(\frac{1}{r_1}\frac{\partial}{\partial\phi}\right) w\} + \{\left(\frac{1}{r_1}\frac{\partial}{\partial\phi}\left(\frac{1}{r_1}\frac{\partial}{\partial\phi}\right)(K\frac{\mu}{r^2}\frac{\partial^2 w}{\partial\theta^2})\right\} +$$

$$\{\frac{1}{r_1}\frac{\partial}{\partial\phi}\left(\frac{1}{r_1}\frac{\partial}{\partial\phi}\right)(K\frac{\mu}{r}\frac{1}{r_1}\frac{\partial w}{\partial\phi}\cos\phi)\} + K\ \{\frac{1}{r^4}\frac{\partial^4}{\partial\theta^4} w + \frac{\cos\phi}{r^3 r_1}\frac{\partial^2}{\partial\theta^2}(\frac{\partial w}{\partial\phi})\} +$$

$$\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} K\ \{\frac{\mu}{r_1^2}(\frac{\partial^2 w}{\partial\phi^2} - \frac{1}{r_1}\frac{\partial r_1}{\partial\phi}\frac{\partial w}{\partial\phi}) = \frac{1}{r_1}\frac{\partial}{\partial\phi}\left(\frac{1}{r_1}\frac{\partial}{\partial\phi}\right) N_\phi\ w + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} N_\theta\ w \tag{1-24}$$

When N_ϕ and N_θ reach a certain critical value equation (1-24) can be satisfied by a nontrivial solution for w. Therefore, the equation is simplified into a standard form in the next section to obtain a characteristic equation of buckling of the shell.

1-5 Reduction of a Continuous System into a Discrete System

Equation (1-24) can be rearranged into the following form.

$$
\begin{aligned}
& K\{\frac{1}{r_1^2}(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi})(\frac{1}{r_1^2}(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi})\, w + \\
& \frac{1}{r_1^2}(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi})(\frac{\mu\cos\phi}{r\, r_1}\frac{\partial}{\partial\phi} w) + \mu(\frac{1}{r_1^2}(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi})(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2})w \\
& + \frac{1}{r^4}\frac{\partial^4 w}{\partial\theta^4} + \frac{1}{r^3}\frac{\cos\phi}{r_1}\frac{\partial^2}{\partial\theta^2}\frac{\partial w}{\partial\phi} + \\
& \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}(\frac{\mu}{r_1^2}(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi}))\, w\} \qquad (1\text{-}25) \\
& = \lambda\{\frac{1}{r_1^2}(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi})(N_\phi^{q+p}\, w) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}(N_\theta^{q+p} w)\} + \\
& \frac{1}{r_1^2}(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi})(N_\phi^g\, w) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}(N_\theta^g\, w)
\end{aligned}
$$

Where the superscripts q and p for the membrane forces refer to symmetrical pressure and axial load, respectively, and g refers to the dead load on the shell. Note that later N_ϕ^* and N_θ^* are used to replace N_ϕ^{q+p} and N_θ^{q+p} since they are results of a unit pressure.

Note also that the product of the eigenvalue λ and N_ϕ^* and N_θ^* will be the corresponding membrane forces due to the applied load, and N_ϕ^g and N_θ^g remain unchanged. If all the differential operators and geometric datas of equation (1-25) are replaced by matrix operators, equation (1-25) can be written as

$$(B)\ (W) + (C)\ (W)\ (D) + (BE5)\ (W) + (\alpha)\ (W)\ (A) =$$

$$\lambda(\ (E)\ (W) + (\beta)\ (W)\ (F)\ ) + (G)\ (W) + (\Gamma)\ (W)\ (H) \qquad (1\text{-}26)$$

Although many different approaches may be used to construct these matrices and matrix operators, those in equation (1-26) are constructed by the use of the finite difference method here. All matrices except (W) in equation (1-26) are square matrices. Note that some of these matrices are premultiplications and others are postmultiplications to the operand (W) matrix, according to whether the partial derivative operator is with respect to ϕ or θ. This equation has the form of Sylvester's *matrix operator* (15) and approximately preserves the relation between force and deformation as characterized by the partial differential equation of (1-25). The parameter λ is an eigenvalue that magnifies the effect due to an assumed unit external load. The matrix (W) specifies values of the buckling mode at nodal points which are formed by intersections of the partition lines. In a row-wise sense (W) give values of the buckled shape at equally spaced sample points of the corresponding ring. It can be seen that the matrix (W) provides a natural representation of the buckled shape at grid points. For example, the element W_{ij} equals the value of the normalized buckling mode at the intersection of ith (ϕ = constant) horizontal partition line and jth (θ = constant) axial partition line. Comparing equation (1-25) to equation (1-26), one can find the definition of each matrix as stated in the following list:

$$(B)(W) = K \left[\left(\frac{1}{r_1^2}\left(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi}\right)\left(\frac{1}{r_1^2}\left(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi}\right)\right) w \right]$$

$$(W)(A) = \left[K \frac{\partial^4}{\partial\theta^4} w \right]$$

$$(\alpha) = \left(\frac{1}{r^4}\right)$$

$$(C_1)(W) = \left[K \left(\frac{\cos\phi}{r_1 r^3}\frac{\partial}{\partial\phi}\right) w\right]$$

$$(C_2)(W) = \left[K \left(\left(\frac{1}{r_1^2}\left(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi}\right)\frac{\mu}{r^2}\right) w\right] \qquad (1\text{-}27)$$

$$(C_3)(W) = \left[\frac{\mu}{r^2}\frac{K}{r^2}\left(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi}\right) w\right]$$

$$(C)(W) = \left[\left((C_1) + (C_2) + (C_3) \right) w\right]$$

$$(BE5)(W) = \left[\mu \frac{K}{r_1^2}\left(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi}\right)\left(\frac{\cos\phi}{r_1\, r}\frac{\partial}{\partial\phi} w\right)\right]$$

$$(W)^* = [(C)\ w]$$

$$(W)^*(D) = \left[\left(\frac{\partial^2}{\partial\theta^2}\right)(W)^*\right]$$

$$(E)(W) = \left[\left(\frac{1}{r_1^2}\left(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi}\right)\right)(N_\phi^*\ w)\right]$$

$$(\beta)(W) = \left[\left(\frac{1}{r^2}\right) N_\theta^*\ w\right]$$

$$(W)^{**} = [(\beta)\ (W)]$$

$$[(W)^{**}(F)] = \left(\frac{\partial^2}{\partial\theta^2}\right)(W)^{**}$$

$$(G)(W) = \left[\frac{1}{r_1^2}\left(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi}\right) N_\phi^g\ w\right]$$

$$(W)^+ = [(\Gamma)(W)] = \left[\frac{1}{r^2} N_\theta^g\ w\right]$$

$$(W)^*(H) = \frac{\partial^2}{\partial\theta^2}(W)^+$$

As mentioned earlier, N^*_ϕ and N^*_θ are membrane forces due to unit load in the corresponding direction as defined by the subscripts ϕ and θ. For simplicity in the construction of an iterative form the matrices on the right hand side of the generalized matrix equation (1-26) are redefined as

$$Q(W) = \lambda(\ (E)\ (W) + (\beta)\ (W)\ (F)\ ) + (G)\ (W) + (\Gamma)\ (W)\ (H)$$

Let

$$Q^g(W) = (G)\ (W) + (\Gamma)\ (W)\ (H)$$

$$Q^p(W) = (E)^p\ (W) + (\beta)^p\ (W)\ (F)$$

$$Q^q(W) = (E)^q\ (W) + (\beta)^q\ (W)\ (F)$$

the the above equation gives

$$Q(W) = \lambda\{Q^q(W) + Q^p(W)\} + Q^g(W) \qquad (1\text{-}28)$$

CHAPTER TWO

Numerical Data for the Gridwork of Hyperboloid Shells

2-1 Geometry of the Hyperboloid Shell

In Fig. 2-1 a hyperboloid shell of revolution is shown with standard notations (12). The relationship between z and r of the shell in the cylindrical coordinate system reads

$$r^2 = a^2 \left(1 + \frac{z^2}{b^2}\right) \tag{2-1}$$

As shown in Fig. 2-1 r is the polar distance of a point for given distance z from the central axis coordinate measured from the throat, downward as positive. And

$$b = \frac{aH_T}{\sqrt{R_B^2 - a^2}}$$

The principal radii r_1, r_2 can be written such as

$$r_1 = \frac{-a^2 b^2}{(a^2 \sin^2\phi - b^2 \cos^2\phi)^{\frac{3}{2}}}$$

$$r_2 = \frac{a^2}{(a^2 \sin^2\phi - b^2 \cos^2\phi)^{\frac{1}{2}}} \quad , \; r = r_2 \sin\phi \tag{2-2}$$

Note that r_1 and r_2 have different signs, it is due to the fact that the centers of the principal curvatures are on different sides of the shell. The geometry of the ruling line can be obtained by the intersection of a plane tangent to the shell at the throat and the shell surface. The radii of these lines are

$$r = a + (R_B - a)\left(\frac{z}{H_T}\right)$$

Taking the derivative of equation (2-1) on both side with respect to z, yields

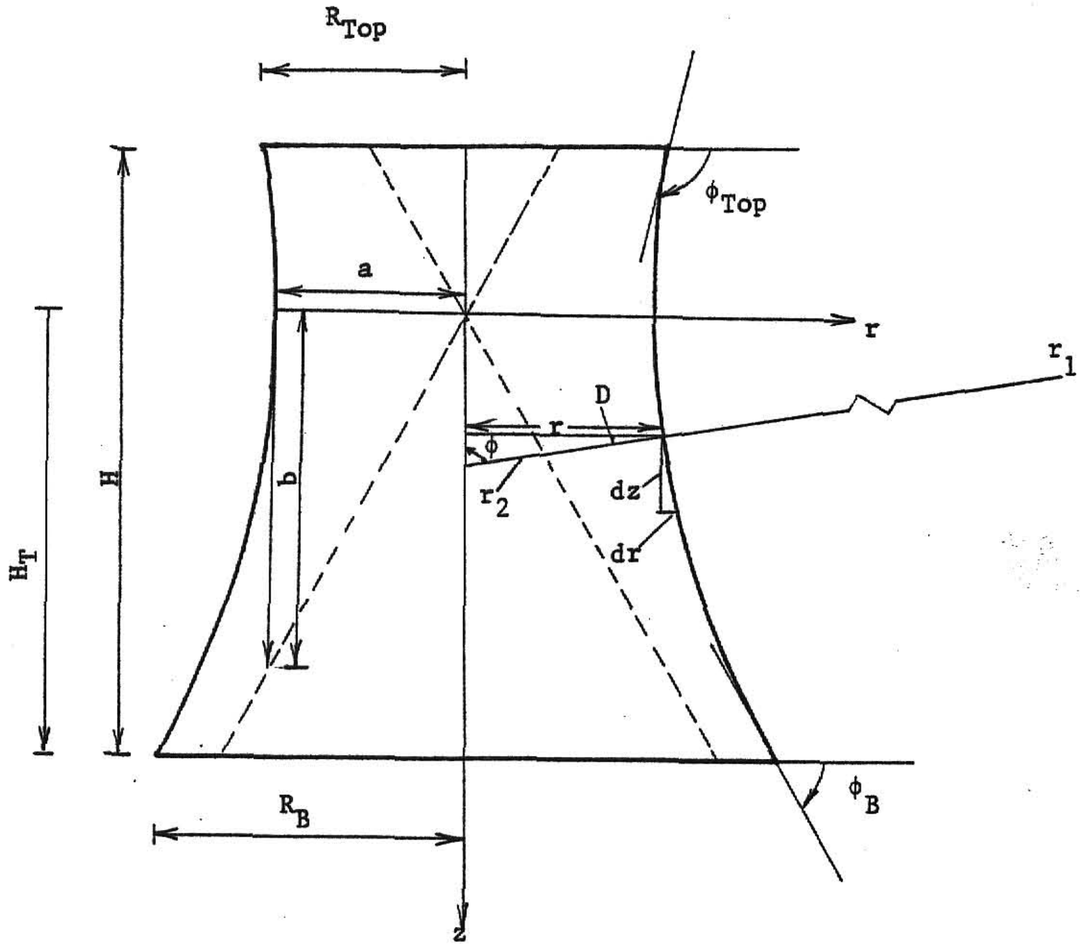


Fig. 2-1 Hyperboloid of Revolution Geometry

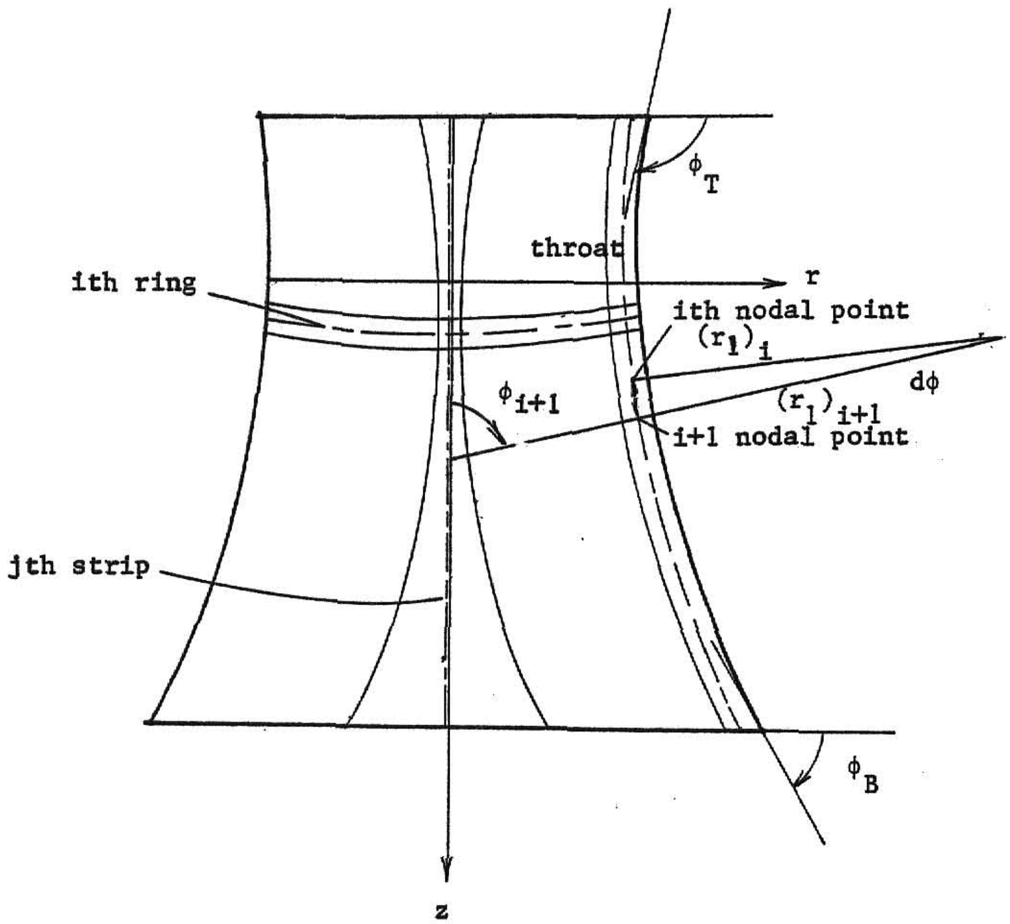


Fig. 2-2 Typical partition lines of a discrete system

$$\frac{dr}{dz} = \frac{a^2}{b^2}\frac{z}{r}$$

or

$$\frac{dr}{dz} = \tan D = \frac{az}{b^2\sqrt{1 + (z/b)^2}}\,, \qquad D = \frac{\pi}{2} - \phi \tag{2-3}$$

Let

$$\phi = \frac{\pi}{2} - D$$

ϕ can be obtained as

$$\phi = \frac{\pi}{2} - \tan^{-1}\left(\frac{az}{b^2\sqrt{1 + (z/b)^2}}\right) \tag{2-4}$$

which is the only variable necessary to express every geometric function, i.e., r_1, r and z are expressible in terms of ϕ.

2-2 Nodal points on the Hyperboloid Cooling Tower

By cutting the hyperboloid shell into finite segments shown in Fig. 2-2 along θ = constant and ϕ = constant, respectively, one can form a grid of nodal points. The procedure for finding the partition lines of equally spaced ϕ, i.e., $\Delta\phi$ = constant, is considered below. As shown in Fig. 2-3 the values of the ϕ curve at m+1 sample points are calculated according to equation (2-4) with $\Delta z = \frac{(z_B - z_T)}{m}$. Using $\frac{\phi_T - \phi_B}{m} = \Delta\phi$, $\phi_i = \phi_i + i\Delta\phi$, $i = 1,2,\ldots,m$. One can use any interpolation method to find a set of z_1, z_2, ..., z_{m+1} such that $\phi(z_{i+1}) - \phi(z_i) = \Delta\phi$ can be satisfied. By substituting a set of properly selected z_i values into equations (2-2) and (2-4), the principal radii r_1 and r_2 at each nodal point of equally spaced $\Delta\phi$ can be obtained.

In Fig. 2-4 a typical jth vertical strip is used to show the index i of the nodal points along its center line, where $i = 1,2,\ldots,m$. The index j with $j = 1,2,\ldots,n$ are shown in Fig. 2-5 for the ith typical ring section of the shell.

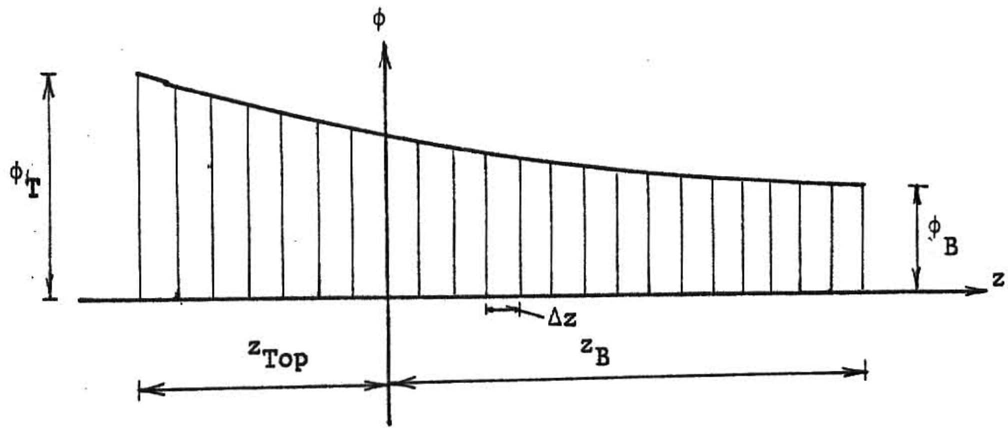


Fig. 2-3

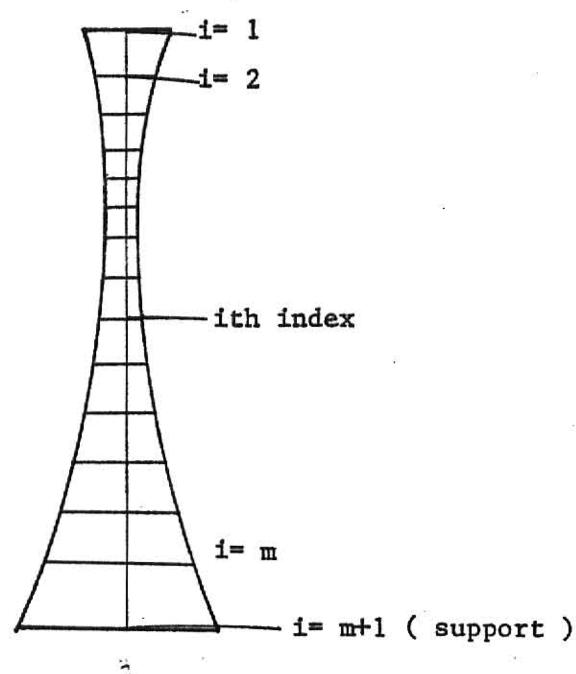


Fig. 2-4 jth vertical strip with i index of the partition number

The transformation of the gridwork on the shell into a rectangular gridwork in the $\phi\theta$-plane is shown in Fig. 2-6. The indices, $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$, are used in the matrix (W_{ij}) to express the value of the normalized buckling deformation at each nodal point $W(\phi_i, \theta_j)$. Let the matrix W be partitioned columnwisely.

$$(W) = (S_1 \mid S_2 \mid S_3 \mid \cdots \mid S_j \mid \cdots \mid S_n)$$

Then the jth column

$$S_j = \begin{Bmatrix} W_{1_j} \\ W_{2_j} \\ \vdots \\ W_{i_j} \\ \vdots \\ W_{m_j} \end{Bmatrix} \tag{2-5}$$

which defines the values of W at $\theta = \theta_j$ and $\phi = \phi_1, \phi_2, \ldots, \phi_m$, respectively.

In equation (1-25) the differential operator was substituted by the premultiplication and postmultiplcation matrix operator of equation (1-26). The main reason for these matrix operators is because of the following differential order.

$$\frac{\partial^2}{\partial s_\phi^2}(M_\phi) = \frac{\partial^2}{\partial s_\phi^2}(N_\phi W)$$

$$\frac{\partial^2}{\partial s_\theta^2}(M_\theta) = \frac{\partial^2}{\partial s_\theta^2}(N_\theta W) \tag{2-6}$$

and one can combine the differential operator and geometric data of each ring or vertical strip into a single matrix operator such as (Ω), (H), (Z) and (D). These are the form as

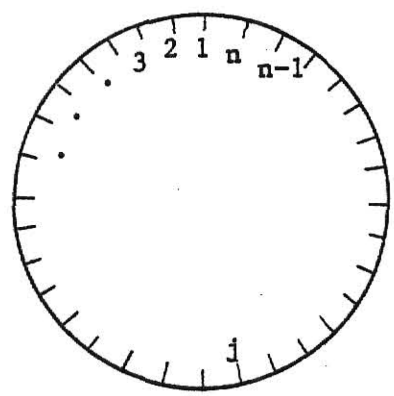


Fig. 2-5 ith ring with j index of the partition number

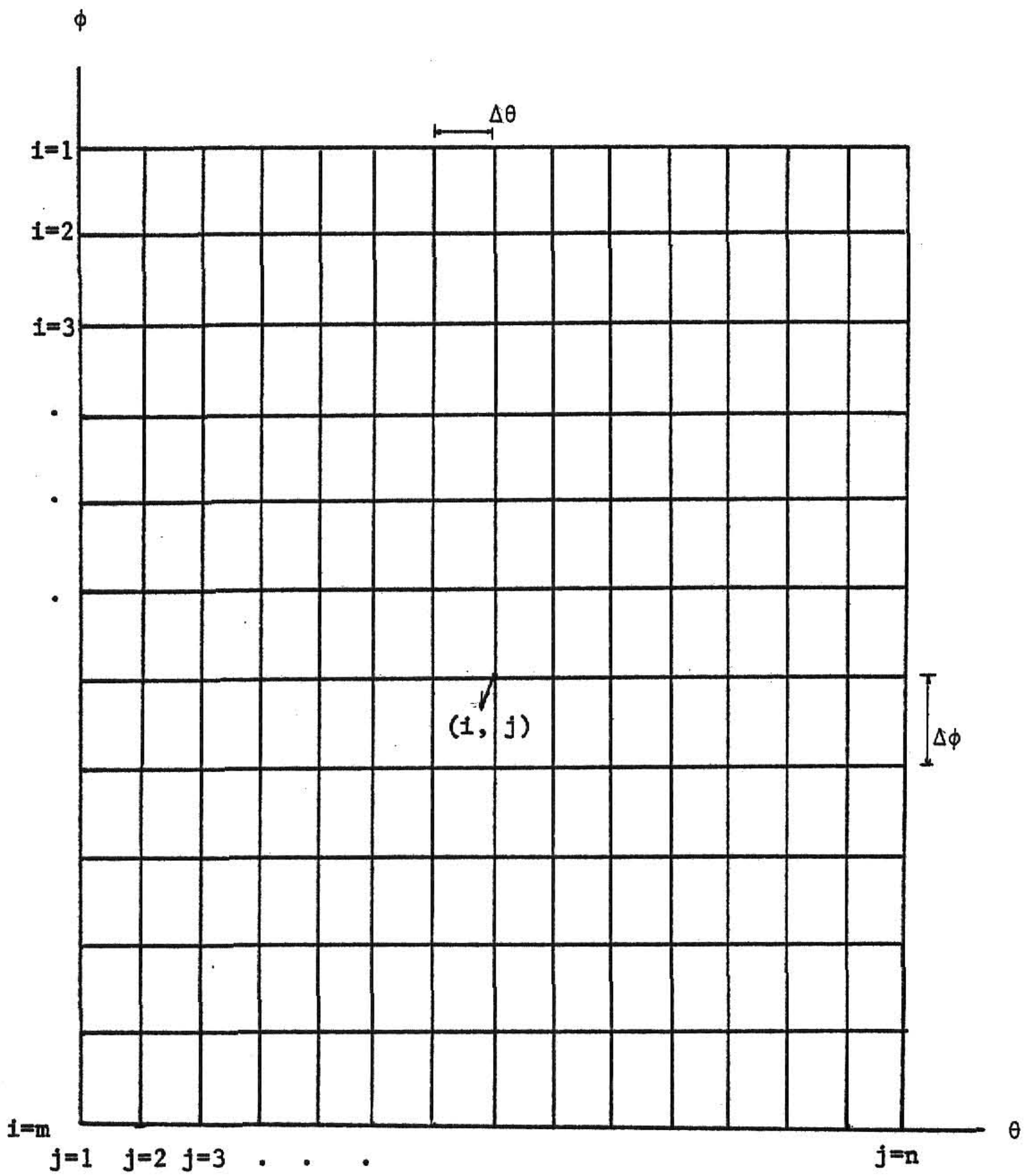


Fig. 2-6 A mapping of the gridwork on shell into $\phi\theta$-plane

$$(\Omega)(W_{\theta\phi}) = \lambda(H)\ (W_{\theta\phi}) \tag{2-7-a}$$

$$(Z)(W_{\phi\theta}) = \lambda(D)(W_{\phi\theta}) \tag{2-7-b}$$

The transpose of both sides of equation (2-7-a) will yield

$$(W_{\theta\phi})^T(\Omega)^T = \lambda(W_{\theta\phi})^T(H)^T \tag{2-8}$$

and (Ω) and (H) are symmetrical matrix operators. Therefore $(\Omega)^T = (\Omega)$ and $(H)^T = (H)$. The additon of equation (2-7-b) and (2-8) with $(W_{\theta\phi})^T=(W_{\theta\phi})$ is equation (2-9). This is the same form as equations (1-26). The CwD part and the other part in the R.H.S. related to the dead load are combined into equation (2-9) for simpler expression.

$$(Z)(W_{\phi\theta}) + (W_{\phi\theta})(\Omega) = \lambda((D)(W_{\phi\theta}) + (W_{\phi\theta})(H)) \tag{2-9}$$

CHAPTER THREE

Solution of the Elastic Buckling of the Cooling Tower

3-1 Matrix Operators with Respect to Variable ϕ and θ

The operator (B) performs the partial derivatives of its operand w with respect to ϕ as defined in equation (1-26). It is listed below for reference.

$$(B)(W) = K \left(\frac{1}{r_1^2}\left(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi}\right)\right)\left(\frac{1}{r_1^2}\left(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi}\right)\right) W(\phi,\ \theta)\Big|_{(\phi_i,\ \theta_j)} \tag{3-1}$$

Let $(B) = K(B_2)(B_1)$, such that the result of the operator (B_1) which is applied to (W) reads

$$(B_1)(W) = \left(\frac{1}{r_1^2}\left(\frac{\partial^2}{\partial\phi^2} - \frac{\dot{r}_1}{r_1}\frac{\partial}{\partial\phi}\right)\right) W(\phi,\ \theta)\Big|_{(\phi_i,\ \theta_j)} \tag{3-2}$$

In order to construct (B_1) and (B_2), a matrix $(D_\phi)^{(1)}$ of (m+2, m+2), is introduced as an approximate operator of the first partial derivative with respect to ϕ at the sample points. This matrix can be constructed by the use of central difference quotients and is defined as follows:

$$(D_\phi)^{(1)} \underset{\text{jth column of (W)}}{\begin{Bmatrix} W_{-2} \\ W_{-1} \\ W_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ W_{m-1} \\ W_m \end{Bmatrix}} = \frac{1}{2\Delta\phi} \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & -1 & 0 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & 0 & 1 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & -1 & 0 \end{bmatrix} \underset{\text{jth column of (W)}}{\begin{Bmatrix} W_{-2} \\ W_{-1} \\ W_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ W_{m-1} \\ W_m \end{Bmatrix}} \tag{3-3}$$

$$(D_{\phi\phi})^{(1)} \begin{Bmatrix} W_{-2} \\ W_{-1} \\ W_1 \\ \cdot \\ \cdot \\ \cdot \\ W_{m-1} \\ W_m \end{Bmatrix}_{\substack{\text{jth column} \\ \text{of (W)}}} = \frac{1}{2\Delta\phi} \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 1 & -2 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & -2 & 1 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 & -2 \end{bmatrix} \begin{Bmatrix} W_{-2} \\ W_{-1} \\ W_1 \\ \cdot \\ \vdots \\ \cdot \\ W_{m-1} \\ W_m \end{Bmatrix}_{\substack{\text{jth column of} \\ \text{(W)}}} \tag{3-4}$$

Where $(\frac{1}{r_1^2})$ and $(\frac{1}{r_1})$ are diagonal matrices according to the changing radius of r_1. Therefore, the operator (B_1) becomes to

$$(B_1) = (\frac{1}{r_1^2})((D_{\phi\phi})^{(1)} - (\frac{1}{r_1})(\frac{\partial r_1}{\partial \phi})(D_\phi)^{(1)}) \tag{3-5}$$

Matrix $(\frac{\partial r_1}{\partial \phi})$ can be obtained directly by the use of central difference quotients of r_1 of each nodal point in the vertical strip to form a diagonal matrix operator such as

$$(\frac{\partial r_1}{\partial \phi})\bigg|_{(i,i)} = \frac{-r_{1_{i-1}} + r_{1_{i+1}}}{\Delta\phi} \quad , \qquad i = 3, \ldots, m \tag{3-6}$$

Similarly (B_2) can be expressed by

$$(B_2) = (\frac{1}{r_1^2})((D_{\phi\phi})^{(2)} - (\frac{1}{r_1})(\frac{\partial r_1}{\partial \phi})(D_\phi)^{(2)}) \tag{3-7}$$

Note that in equations (3-5) and (3-7) superscripts (1) and (2) are used to show that it is part of (B_1) or (B_2). Except in the third row of matrices $(D_\phi)^{(2)}$ and $(D_{\phi\phi})^{(2)}$ are constructed by forward difference quotients, Note that the first two rows of $(D_\phi)^{(1)}$ are corresponding to the additional parameters to take care of force boundary conditions at the upper edge of the shell, and they will be derived in section (3-2) seperately.

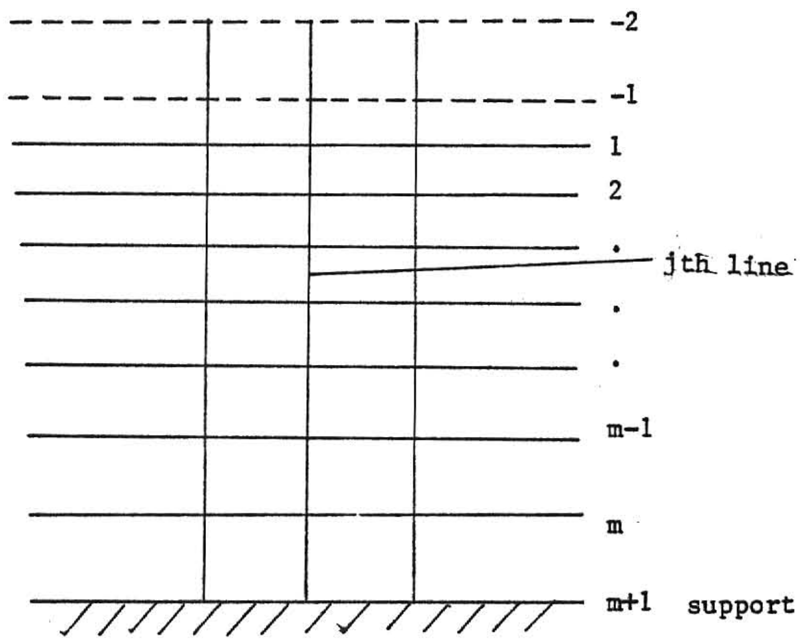


Fig. 3-1 jth vertical line of the shell

Note also that the last row of the operator has its weighting corresponding to the adjacent rows of the W matrix on the mth partition line of the shell which are -1 and 0, and it is because of zero value of W at the bottom of the shell since it is either a fixed or a hinged support.

Another operator of second derivative with respect to ϕ defined in equation (3-4) is indicated by $(D_{\phi\phi})^{(1)}$. It is obtained by the use of central difference quotients in a similar manner as $(D_\phi)^{(1)}$:

$$(D_\phi)^{(2)} \underbrace{\left\{ \begin{matrix} W_{-2} \\ W_{-1} \\ \cdot \\ \cdot \\ \cdot \\ W_{m-1} \\ W_m \end{matrix} \right\}}_{\text{jth column of (W)}} = \frac{1}{2\Delta\phi} \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & -1 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdots & -1 & 0 \end{bmatrix} \underbrace{\left\{ \begin{matrix} W_{-2} \\ W_{-1} \\ \cdot \\ \cdot \\ \cdot \\ W_{m-1} \\ W_m \end{matrix} \right\}}_{\text{jth column of (W)}} \quad (3\text{-}8)$$

and

$$(D_{\phi\phi})^{(2)} \underbrace{\left\{ \begin{matrix} W_{-2} \\ W_{-1} \\ \cdot \\ \cdot \\ \cdot \\ W_{m-1} \\ W_m \end{matrix} \right\}}_{\text{jth column of (W)}} = \frac{1}{\Delta\phi^2} \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 1 & -2 & 1 & 0 & \cdot & \cdots & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & \cdot & \cdots & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & 1 & -2 \end{bmatrix} \underbrace{\left\{ \begin{matrix} W_{-2} \\ W_{-1} \\ \cdot \\ \cdot \\ \cdot \\ W_{m-1} \\ W_m \end{matrix} \right\}}_{\text{jth column of (W)}} \quad (3\text{-}9)$$

So far, the product of K, (B_2) and (B_1) completes the construction of the interior part of operator (B). For this part of (B) yields the fourth partial derivative of w with respect to ϕ. The other fourth and second partial derivative operators with respect to θ are $\frac{\partial^4}{\partial\theta^4}$ and $\frac{\partial^2}{\partial\theta^2}$. They can be constructed into matrices by central difference quotients and are denoted as (A) and (A1). For example, the values of the second partial derivative of w with respect to

$r\,d\theta$ of the ith ring at the sample points shown in Fig. 3-2 can be calculated approximately by the postmultiplication of row matrix $\lfloor W \rfloor_i$ by $(\frac{1}{r^2})$ and (A1) matrix as follows:

$$(\frac{1}{r^2})_{ii} \lfloor W_1, W_2, \cdots, W_n \rfloor_{\text{ith row of } (W)} \quad \text{(A1)}$$

$$= (\frac{1}{r^2})_{ii} \lfloor W_1, W_2, \cdots, W_n \rfloor_{\text{ith row of } (W)} \frac{1}{\Delta\theta^2} \begin{bmatrix} -2 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 \\ 1 & -2 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & -2 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & -2 & 1 \\ 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 & -2 \end{bmatrix}$$

$$\simeq \left\lfloor \frac{\partial^2 W(\phi_i, \theta_1)}{(r(\phi_i)\partial\theta)^2}, \ldots, \frac{\partial^2 W(\phi_i, \theta_n)}{(r(\phi_i)\partial\theta)^2} \right\rfloor \qquad (3\text{-}10)$$

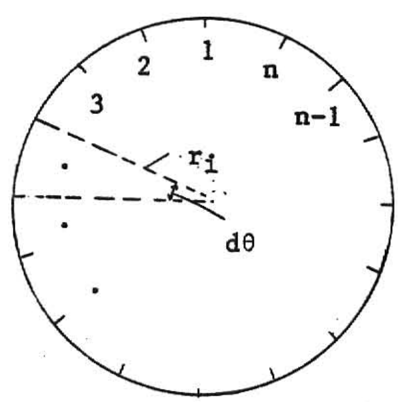


Fig. 3-2 ith ring of the discrete system

In this expression each row of (A1) gives the weighting factors for deformation W of related nodal points, and $(\frac{1}{r^2})_{ii}$ is the ith element of $(\frac{1}{r^2})$. Using a similar method as the second derivative operator with respect to θ, an approximate fourth partial derivative with respect to θ of the ith ring can be obtained by the postmultiplication of W_i by $(\frac{1}{r^4})_{ii}$ and the matrix (A) defined below:

$$(\frac{1}{r^4})_{ii} \llcorner W_1, W_2, \ldots\ldots, W_n \lrcorner \qquad \underset{\text{ith row}}{(A)}$$

$$= (\frac{1}{r^4})_{ii} \llcorner W_1, W_2, \ldots\ldots, W_n \lrcorner_{\text{ith row}} \frac{1}{\Delta\theta^4}$$

$$= \llcorner \frac{\partial^4 W(\phi_i, \theta_1)}{(r(\phi_i)\partial\theta)^4}, \ldots\ldots, \frac{\partial^4 W(\phi_i, \theta_n)}{(r(\phi_i)\partial\theta)^4} \lrcorner \qquad (3\text{-}11)$$

Note that the diagonal matrices $(\frac{1}{r^4})$ and $(\frac{1}{r^2})$ when applied to matrix (W) use premultiply of (W).

3-2 Boundary Conditions on the Upper Edge

The boundary conditions on the upper edge of the shell are $M_\phi = 0$ and $V_\phi = 0$ which means that both the bending moment and the radial shear force around the edge of top ring are equal to zero. The weighting corresponding to the deflections of the shell according to the moment boundary condition on the edge is computed as

$$\llcorner M_\phi \lrcorner_1 = \llcorner K(\kappa_\phi + \mu\kappa_\theta)_j \lrcorner_1 = \llcorner 0 \lrcorner_1 \qquad (3\text{-}12)$$

According to equation (1-19) M_ϕ can be expressed as

$$M_\phi = K\{\frac{1}{r_1}(\frac{\dot{w}}{r_1})^{\cdot} + \frac{\mu}{r}(\frac{w''}{r} + \frac{\dot{w}}{r_1}\cos\phi)\} \qquad (1\text{-}19)$$

Note that subscript 1 of equation (3-12) represent the position of the nodal point of the first ring and j represent the jth central difference for the jth nodal point at the top ring. $\kappa_{\phi 1}$ and $\kappa_{\theta 1}$ are curvatures of w with respect to ϕ and θ at the nodal points of the top ring.

$$\kappa_{\phi 1} = \frac{1}{(r_1)_1}(\frac{\dot{w}}{(r_1)_1})^{\cdot}, \quad \kappa_{\theta 1} = \kappa'_{\theta 1} + \kappa''_{\theta 1}, \quad \kappa'_{\theta 1} = \frac{\mu}{(r)_1}\ \frac{\dot{w}}{(r_1)_1}\cos\phi_1,$$

$$\kappa''_{\theta 1} = \frac{\mu}{(r)_1^2} w''. \qquad (3\text{-}13)$$

Selecting deflections of the extended vertical strips, the rows W_{-2}, W_{-1}, W_1, W_2 and W_3, are the operand of the operators $\frac{\partial^4}{\partial\theta^4}$ and $\frac{\partial^2}{\partial\theta^2}$ in this stage. By central difference quotients one has:

$$\frac{\partial^2}{\partial\phi^2} w = \frac{W_{-1} - 2W_1 + W_2}{\Delta\phi^2} \qquad (3\text{-}14)$$

and

$$\frac{\partial}{\partial\phi} w = \frac{-W_{-1} + W_2}{2\Delta\phi}$$

Therefore,

$$\kappa_{\phi 1} = \frac{1}{(r_1)_1^2}(\frac{W_{-1} - 2W_1 + W_2}{\Delta\phi^2}) - \frac{1}{(r_1)_1^3}(\frac{-(r_1)_{-1} + (r_1)_2}{2\Delta\phi})(\frac{-W_{-1} + W_2}{2\Delta\phi})$$

$$= (\frac{1}{(r_1)_1^2\Delta\phi^2} - \frac{-(r_1)_{-1} + (r_1)_2}{4\Delta\phi^2 (r_1)_1^3}) W_{-1} + \frac{-2}{(r_1)_1^2\Delta\phi^2} W_1 +$$

$$(\frac{1}{(r_1)_1^2\Delta\phi^2} - \frac{-(r_1)_{-1} + (r_1)_2}{4\Delta\phi^2 (r_1)_1^3}) W_2 \qquad (3\text{-}15)$$

$(r_1)_{-1}$, $(r_1)_1$ and $(r_1)_2$ are the principal radii at nodal points of the ring number -1, 1 and 2, respectively. κ'_θ can be calculated as

$$\kappa'_{\theta 1} = \frac{\mu}{(r)_1 (r_1)_1} \; \frac{-W_{-1} + W_2}{2\Delta\phi} \cos\phi_1$$

$$= \left(\frac{\mu\cos\phi_1}{2(r)_1 (r_1)_1 \Delta\phi}\right) W_{-1} + \left(\frac{\mu\cos\phi_1}{2(r)_1 (r_1)_1} \; \frac{1}{\Delta\phi}\right) W_2$$

for $\kappa''_{\theta 1}$

$$\kappa''_{\theta 1} = \frac{1}{((r)_1)^2} \; \frac{\partial^2}{\partial\theta^2} W$$

κ''_θ can be input to the operator which belong to the CWD part (i=1, j=3) with postmultiplication operator (A1). The completed boundary condition $\llcorner M_\phi \lrcorner_1 = \llcorner 0 \lrcorner_1$ for any $\theta = \theta_i$ can be expressed such as

$$\begin{aligned}
\llcorner M_\phi \lrcorner_1 = {} & \left(\frac{1}{((r_1)_1)^2 \Delta\phi^2} - \frac{-(r_1)_{-1} + (r_1)_2}{4\Delta\phi^2 ((r_1)_1)^3}\right) \llcorner W_{-1} \lrcorner + \left(\frac{-2}{((r_1)_1)^2 \Delta\phi^2}\right) \llcorner W_1 \lrcorner \\
& + \left(\frac{1}{((r_1)_1)^2 \Delta\phi^2} - \frac{-(r_1)_{-1} + (r_1)_2}{4\Delta\phi^2 ((r_1)_1)^3}\right) \llcorner W_2 \lrcorner \\
& + \left[\left(\frac{\mu\cos\phi_1}{2(r)_1 (r_1)_1 \Delta\phi}\right) \llcorner W_{-1} \lrcorner + \left(\frac{\mu\cos\phi_1}{2(r)_1 (r_1)_1 \Delta\phi}\right) \llcorner W_2 \lrcorner + \frac{\mu}{((r)_1)^2} \llcorner W_1 \lrcorner \right] \left\{\frac{\partial^2}{\partial\theta^2}\right\}_i \\
= {} & \llcorner 0 \lrcorner_1
\end{aligned}$$

Therefore, the nonzero terms of the first row of (B) due to this boundary condition are

$$B(1,2) = \left(\frac{1}{((r_1)_1)^2 \Delta\phi^2} - \frac{-(r_1)_{-1} + (r_1)_2}{4\Delta\phi^2 ((r_1)_1)^3}\right)$$

$$B(1,3) = \frac{-2}{((r_1)_1)^2 \Delta\phi^2}$$

$$B(1,4) = \left(\frac{1}{((r_1)_1)^2 \Delta\phi^2} - \frac{-(r_1)_{-1} + (r_1)_2}{4\Delta\phi^2 ((r_1)_1)^3}\right)$$

According to $M_\phi = 0$, three terms related to μ value have to be put into the matrix operator such as (BE5) and (AP) (refer to equations 1-26 and 3-19). The nonzero terms of (BE5) and (AP) are

$$BE5(1,2) = \frac{\mu\cos\phi_1}{2(r)_1(r_1)_1\Delta\phi}$$

$$BE5(1,4) = \frac{\mu\cos\phi_1}{2(r)_1(r_1)_1\Delta\phi}$$

and

$$AP(1,3) = \frac{\mu}{((r)_1)^2}$$

At the upper edge the radial shear is computed by taking the derivative of M_ϕ with respect to $r_1 d\phi$. Using the central difference quotients, this can be obtained by directly finding the curvature changes at the nodal points of -1 and 2.

$$V_{\phi_1} = \frac{1}{(r_1)_1}\frac{\partial}{\partial\phi} M_{\phi 1} = K(V'_{\phi_1} + V''_{\phi_1})$$

where

$$V'_{\phi_1} = \frac{-\kappa_{\phi_{-1}} + \kappa_{\phi_2}}{2(r_1)_1\Delta\phi}$$

$$V''_{\phi_1} = \mu\,\frac{-\kappa_{\theta_{-1}} + \kappa_{\theta_2}}{2(r_1)_1\Delta\phi}$$

are obtained by the use of central difference quotion.

Specifically, $\kappa_{\phi_{-1}}$, $\kappa_{\phi 2}$, $\kappa_{\theta_{-1}}$ and κ_{θ_2} are calculated in a similar manner as in equation (3-14) except the indices are changed into -1 or 2 along the extended vertical strips. The weighting corresponding to the elements of the operand, the deflection matrix (W), to enforce $V_{\phi_1} = 0$, are put in the second row of the operator (B), (AP) and (BE5) in a similar manner as weighting factors in the first row. Note that the top two rows of (B) break down the almost symmetrical form of (B). The addition of (B) and $(B)^T$, transpose of (B), form a new symmetrical matrix operator (BT1), and the difference between (B) and $(B)^T$ give a skew matrix operator which is used as a perterbation in the iteration process.

$$(BT1) = \frac{(B) + (B)^T}{2}$$

$$(BT2) = \frac{(B) - (B)^T}{2} .$$

Due to the two extra row vectors of deflection matrix (W) introduced for taking care of boundary conditions, the result of (W)(A) involves more terms beyond what are required by the original system. These are corrected by adding the same resultant matrix at the right hand side of equation (1-26). for this (W)' is defined to have its first two rows be those of the deflections matrix of (W) and the rest are zero elements such that (W)'(A) is listed below

$$(W)'(A) = \begin{bmatrix} W_{-21} & W_{-22} \cdot & \cdot & \cdot & W_{-2n} \\ W_{-11} & W_{-12} \cdot & \cdot & \cdot & W_{-1n} \\ 0 & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix} (A) \qquad (3\text{-}18)$$

will be added to the right hand side of equation (1-26). It reads

$$(B)(W) + (C)(W)(D) + (BE5)(W) + (\alpha)(W)(A) = \lambda\{(E)(W) + (\beta)(W)(F)\} + (G)(W) + (\Gamma)(W)(H) + (\alpha)(W)'(A) \qquad (1\text{-}26)'$$

The part of the mixed operator (C)(W)(D) is mainly due to a nonzero Poisson value μ, and most parts of this mixed operator of equations (1-25) and (1-26) had been defined already. They are listed as

$$(C)(W)(D) = \mu\{(B_2)(\frac{\cos\phi}{r\ r_1})(D_\phi)^{(1)}(W) + (B_1)(\frac{1}{r^2})(W)(A1) +$$

$$(B_1)(\frac{1}{r^2})(W)(A1) + (\frac{1}{r^3})(\frac{\cos\phi}{r_1})(D_\phi)^{(1)}(W)(A1)\}$$

$$= (BE5)(W) + (AP)(W)(A1) + (AP1)(W)(A1) +$$

$$(AP2)(W)(A1) \qquad (3\text{-}19)$$

where (B_2) and (B_1) can be found from equations (3-5) and (3-7)

3-3 A Method of Solution for a Simple Operator Equation $(\alpha)^{-1}(BT1)(W) + (W)(A) = (CR)$

In order to solve the Sylvester's generalized matrix equation (6) eigenvalues and eigenvectors of the stiffness matrices (A) and $(\alpha)^{-1}(BT1)$ should be obtained. Note that both matrix (A) and (BT1) are symmetric matrices. Therefore, their eigenvalues and eigenvectors can be computed by the use of the Jacobian method of successive rotational transformations. However the matrix of $(\alpha)^{-1}(BT1)$ is not a symmetric matrix. Since $(\alpha)^{-1}$ is a diagonal matrix, it is relatively simple to have the symmetric matrix

$$(B)' = (\alpha)^{-\frac{1}{2}}(BT1)(\alpha)^{-\frac{1}{2}} \qquad (3\text{-}20)$$

The eigenvalues and eigenvectors of this transformed matrix can be constructed by Jacobian method and expressed in the following form

$$(B)'(\Xi) = (\Xi)(\Lambda) \qquad (3\text{-}21)$$

where

$$(\Xi) = (\xi_1 \mid \xi_2 \mid \xi_3 \mid \cdots \mid \xi_{m+2})$$

and

$$(\Lambda) = \begin{pmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_{m+2} \end{pmatrix}$$

are the matrix of eigenvectors and the diagonal matrix of eigenvalues of (B)'. Using these results, one can construct the eigenvectors of the operator $(\alpha)^{-1}$ (BT1) (or (B)").

Let

$$V = (v_1 \mid v_2 \mid \cdots \mid v_{m+2})$$

be the eigenmatrix, then

$$(V) = (\alpha)^{-\frac{1}{2}}(\Xi) \tag{3-22}$$

or $$(V)\ (\Lambda) = (\alpha)^{-\frac{1}{2}}(\Xi)\ (\Lambda)$$

In order to show (V) is the matrix of eigenvectors of $(\alpha)^{-1}$(BT1), considering the following products of matrices:

$$(\alpha)^{-1}(BT1)\ (V) = (\alpha)^{-1}(BT1)\ (\alpha)^{-\frac{1}{2}}\ (\Xi)$$

$$= (\alpha)^{-\frac{1}{2}} (\alpha)^{-\frac{1}{2}} (BT1) (\alpha)^{-\frac{1}{2}} (\Xi)$$

$$= (\alpha)^{-\frac{1}{2}} (\Xi) (\Lambda)$$

$$= (V) (\Lambda)$$

This shows that $(B)'$ and $(\alpha)^{-1}(BT1)$ have the same set of eigenvalues. Similarly, one can find matrix (Γ) for eigenvalues and matrix (H) for eigenvectors of matrix (A) such that

$$(A)'(H) = (H) (\Gamma) \tag{3-24}$$

where

$$(\Gamma) = \begin{pmatrix} \gamma_1 & & & 0 \\ & \gamma_2 & & \\ & & \ddots & \\ 0 & & & \gamma_n \end{pmatrix}$$

and

$$(H) = (\eta_1 \mid \eta_2 \mid \cdots \mid \eta_n)$$

For a general matrix equation $(T_o)(W) = (\alpha)^{-1}(BT1)(W) + (W)(A) = (CR)$, the inverse of the operator T_o can be completed by the use of eigenvalues and and eigenvectors of $(\alpha)^{-1}(BT1)$ and those of (A)'s.

Note that

$$(B)'' = (V) (\Lambda) (V)^{-1} = (\alpha)^{-\frac{1}{2}}(\Xi) (\Lambda) (\Xi)^T(\alpha)^{\frac{1}{2}} \tag{3-25}$$

and

$$(A) = (H)\ (\Lambda)\ (H)^{-1} = (H)\ (\Gamma)\ (H)^{T} \tag{3-26}$$

Since both (Ξ) and (H) are orthonormal eigenvectors of symmetric matrices. $(\Xi)^T$ and $(H)^T$ are the transpose of (Ξ) and (H), respectively. Therefore, by substitution, the matrix equation $(T_o)(W) = (CR)$ becomes

$$(V)\ (\Lambda)\ (V)^{-1}(W) + (W)\ (H)\ (\Gamma)\ (H)^{T} = (CR) \tag{3-27}$$

Postmultiplied by (H) and premultiplied by $(V)^{-1}$, equation (3-27) turns into

$$(\Lambda)\ (V)^{-1}(W)\ (H) + (V)^{-1}(W)\ (H)\ (\Gamma) = (V)^{-1}(CR)\ (H) \tag{3-28}$$

Let

$$(Y) = (V)^{-1}(W)\ (H)$$

and

$$(Z) = (V)^{-1}(CR)\ (H) = (\Xi)^{T}(\alpha)^{\frac{1}{2}}(CR)(H)$$

The equation (3-27) is reduced into the following generalized matrix equation of canonical form.

$$(\Lambda)\ (Y) + (Y)\ (\Gamma) = (Z) \tag{3-29}$$

Since (Λ) and (Γ) are diagonal matrices, the unknown matrix (Y) can be solved

$$Y_{ij} = \frac{Z_{ij}}{(\ \lambda_i + \gamma_j\ )} \quad \text{, so } (W) = (V)(Y)(H)^T \tag{3-30}$$

3-4 Iterative Solution for the Buckling of Cooling Tower

A matrix equation which characterizes the buckling of a cooling tower that satisfies all boundary conditions on the edges and base is written as

$$(BT1)(W) + (AP1)(W)(A1) + (AP)(W)(A1) + (AP2)(W)(A1) + (BE5)(W) + (BT2)(W)$$

$$(\alpha)(W)(A) = (\alpha)(W)'(A) + \lambda\{(B_2)(N_\phi^q + N_\phi^p)(W) + (\beta)(N_\theta^q + N_\theta^p)(W)(A1)\} + (B_2)(N_\phi^g)\cdot$$

$$(W)(A1) + (\beta)(N_\theta^g)(W)(A1) \tag{3-31}$$

Let $(T_p)(W)$ and $(T_s)(W)$ be used to represent the primary parts and secondary parts of equation (3-31) defined as

$$(T_p)(W) = (B)''(W) + (W)(A) \tag{3-32}$$

$$\begin{aligned}(T_s)(W) = (\alpha)^{-1}\{&(AP)(W)(A1) + (AP2)(W)(A1)\\ &+ (AP1)(W)(A1) + (BE5)(W)\\ &+ (BT2)(W) - (B_2)(N_\phi^g)(W) - (\beta)(N_\theta^g)(W)(A1)\\ &- (\alpha)(W)'(A)\}\end{aligned} \tag{3-33}$$

$$(CR) = (T_p)\ (W) + (T_s)\ (W)$$

$$= (\alpha)^{-1}(B_2)\ (N_\phi^{q+p})\ (W) + (\beta)\ (N_\theta^{q+p})\ (W)\ (A1) \tag{3-34}$$

The ratio of the two norm of (T_s) and (T_p) will determine the rate of convergence of the iteration.

Using the power method presented, one can find the linear buckling load λ and associated buckling mode (W) at selected nodal points. As mentioned before, λ is a scalar multiplier of unit load q.

The iteration starts with an assumed (W_o) to match the approximate buckling mode according to engineering judgement. Subscript o stands for the first (W), substitute (W_o) into (T_p) (W) and let

$$(T_p)\ (W_o) = Q^q\ (W_o) + Q^p\ (W_o) = (CR) \tag{3-35}$$

Q^q and Q^p are defined in equation (1-28). According to the method stated in section (3-3) to find $(T_p)^{-1}$, i.e., (T_p) inverse, (W_1) can be obtained as

$$(W_1) = (T_p)^{-1}(W_o)\ (CR) \tag{3-36}$$

Then, (W_1) is normalized as

$$(\hat{W}_1) = \frac{W_{1_{ij}}}{\sqrt{W_{1_{11}}^2 + W_{1_{12}}^2 + \cdots + W_{1_{ij}}^2 + \cdots + W_{1_{mn}}^2}} \tag{3-37}$$

Substituting $(\hat{W}_1)$ into equation (3-33), one can obtain

$$(T_p)\ (W_1) + (T_s)\ (W_1) = \lambda_1\ (Q^q(W_1) + Q^p(W_1)) \tag{3-38}$$

The norm of the left hand side divided by right hand side of equation (3-37) will give the first approximation of λ, say λ_1. Therefore,

$$\lambda_1 = \frac{(T_p)\ (\hat{W}_1) + (T_s)\ (\hat{W}_1)}{Q^q(\hat{W}_1) + Q^p(\hat{W}_1)} \tag{3-39}$$

Using $(\hat{W}_i)$ one can improve the mode shape (W_{i+1}) by solving (W_i) from

$$(T_p)\ (W_{i+1}) = \lambda_i\ (Q^q(\hat{W}_i) + Q^p(\hat{W}_i)) - (T_s)\ (\hat{W}_i) \tag{3-40}$$

yields

$$(W_{i+1}) = ((T_p)\ (\hat{W}_i))^{-1}(\lambda_i\ (Q^q(\hat{W}_i) + Q^p(\hat{W}_i) - (T_s)\ (\hat{W}_i)) \tag{3-41}$$

so

$$\lambda_{i+1} = \frac{(T_p)\ (\hat{W}_i) + (T_s)\ (\hat{W}_i)}{Q^q(\hat{W}_i) + Q^p(\hat{W}_i)} \tag{3-42}$$

for iteration order number

$$i = 1,\ 2,\ \cdots,\ N$$

When the relative error for the lowest eigenvalue λ from iteration relative error $= \left| \dfrac{\lambda_i - \lambda_{i-1}}{\lambda_{i-1}} \right| \leq 0.001$ is considered that the error is small enough to terminate the iterative process and the elastic buckling load and the corresponding mode of buckling are obtained.

The next section will discuss a method to find the nonlinear buckling pressure of a cooling tower due to the nonlinearity of the material.

3-5 Buckling of the Shell due to Nonlinear Materials

In view that the matrix equation for buckling of nonlinear materials is

$$T(\lambda)\ (W) = \lambda\ (P)\ (W) + (Q)\ (W)\ . \qquad (3\text{-}43)$$

T, P and Q are generalized matrix operators and $T(\lambda)$ depends on the value of λ, it is clear if λ value is known, say λ_o, then $T(\lambda_o) = T^*$ is a well defined operator. With this operator the matrix equation turns into

$$(T^*)\ (W) = \lambda\ (P)\ (W) + (Q)\ (W) \qquad (3\text{-}44)$$

and one of the obtained eigenvalues should be λ . This observation suggests that nonlinear buckling problem can be solved by a set of linearized operator equations. Furthermore, the stiffness of concrete decrease with increasing stresses and the buckling load is related to both geometric shape and the stiffness of a structure. This implies that the computed buckling value obtained based on the assumption that material maintain a constant Young's modulus as that of zero stress level during a test up to buckling load will give an upper bound of the true buckling load.

Let λ_a be some assumed buckling load, from which the membrane force N_ϕ and N_θ can be obtained to calculate the operator $T(\lambda_a)$. The smallest computed eigenvalue λ_c according to

$$(T(\lambda_a))\ (W) = \lambda_c (P)\ (W) + (Q)\ (W) \tag{3-45}$$

can be solved. Thus for $0 \leq \lambda_a \leq \lambda_e$, with λ_e as the computed λ_c which is corresponding to the case when $\lambda_a = 0$. One can construct a function of $\lambda_c = \lambda_c(\lambda_a)$. The true solution for λ is when $\lambda_c = \lambda_a$ (21).

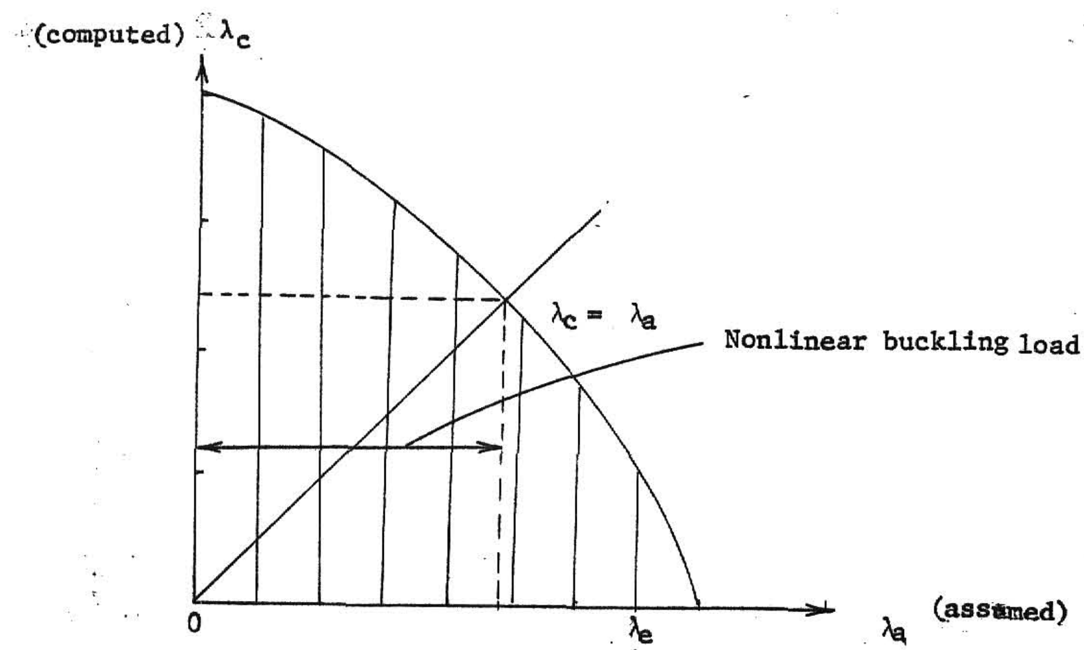


Fig. 3-3 Relation between λ_c and λ_a

A set of sample points can be selected by the use of λ_e and desired value λ can be calculated by some method of interpolation.

The stresses due to assumed λ_a are calculated by

$$\sigma_\phi = \frac{\lambda_a (N_\phi^p + N_\phi^q) + N_\phi^g}{t}$$

(3-46)

$$\sigma_\theta = \frac{\lambda_a (N_\theta^p + N_\theta^q) + N_\theta^g}{t}$$

and the Young's modulus can be calculated by either

$$E_T = \frac{2f_c'}{\varepsilon_o}(1 - \frac{\varepsilon_c}{\varepsilon_o})$$

(3-47)

or

$$E_T(\sigma(\varepsilon_c)) = E_b + \frac{(E_T(\sigma(\varepsilon_o)) - E_b)}{\sigma^2(\varepsilon_o)}\sigma^2(\varepsilon_o)$$

Note that E_ϕ and E_θ are not that same when $\sigma_\phi = \sigma_\theta$. Thus it is an orthotropic shell due to the consideration of nonlinearity of the materials. The E values for mixed operator $E_{\phi\theta} = \sqrt{E_\phi E_\theta}$ is used at each point.

CHAPTER FOUR

Application of the Method and Numerical Examples

4-1 The Prediction of the Buckling Behavior of the Prototype Model

The prototype model is being built in Kansas State University and is studied by the method stated in Chapter Three. The material for the model is assumed to be isotropic and homogeneous for the elastic buckling analysis. The boundary conditions for this model are assumed to be clamped at the base of the shell and free from constraints at the top. The shell will be tested by a combination of external loads namely the uniform pressure, axial load on top of the shell and the body force of the shell itself. The unit dead load for this microconcrete is estimated about 150 pcf which leads to a load intensity of 0.0434 psi. The geometric parameters for the model are: total height is 144 inches, the distance of the throat is 108 inches, the radii at the top, throat and base are 38.7, 36.0 and 56 inches, respectively, and the constant thickness of the shell is 0.5 inches shown in Fig. 4-1.

By using partition size (16x16) for the matrix method and the computer program, the linear elastic buckling pressure obtained is 3.76 psi. The non-linear buckling pressure is about 3.59 psi (see Fig. 4-6). According to the analysis, the associated buckling shape of ring section shown in Fig. 4-2 is for those at the location of maximum deformation. The location of the maximum deformation along the vertical strip is about 67 inches below the throat. In order to see the influence of the partion size to the accuracy, two different partition patterns (20x20) and (30x30) also have been computed. The linear buckling load P_{cr} changed from 3.76 to 3.65 and 3.151 psi, respectively. The relationship between $\Delta\phi$ and the linear buckling pressure P_{cr} is shown in Fig. 4-7.

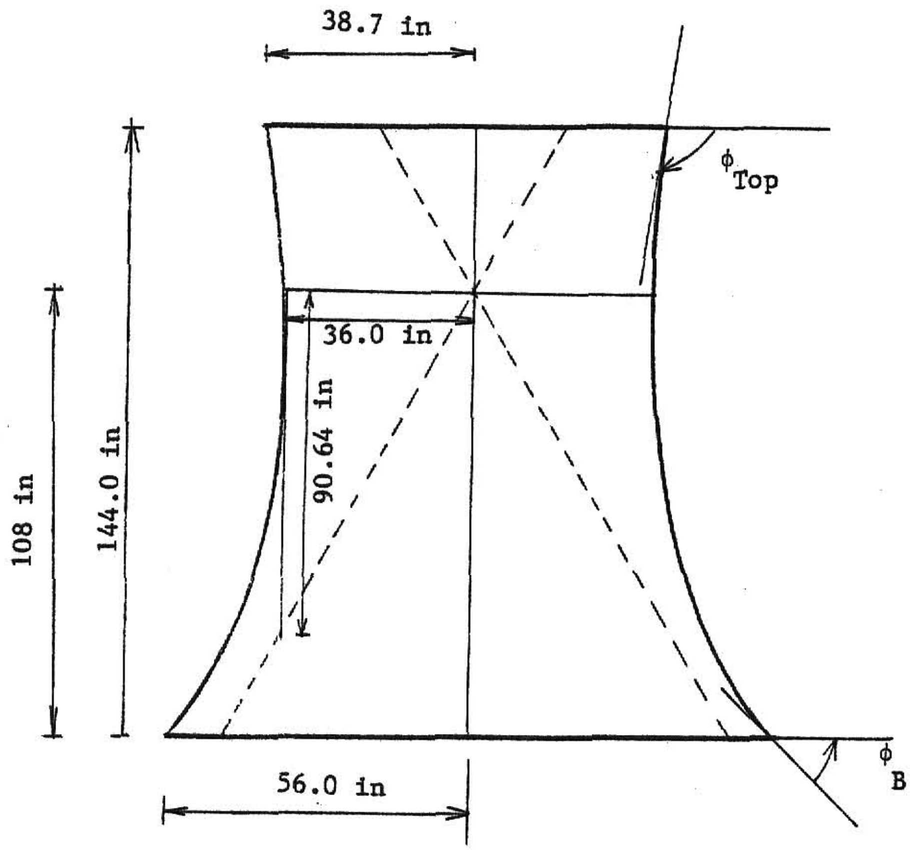
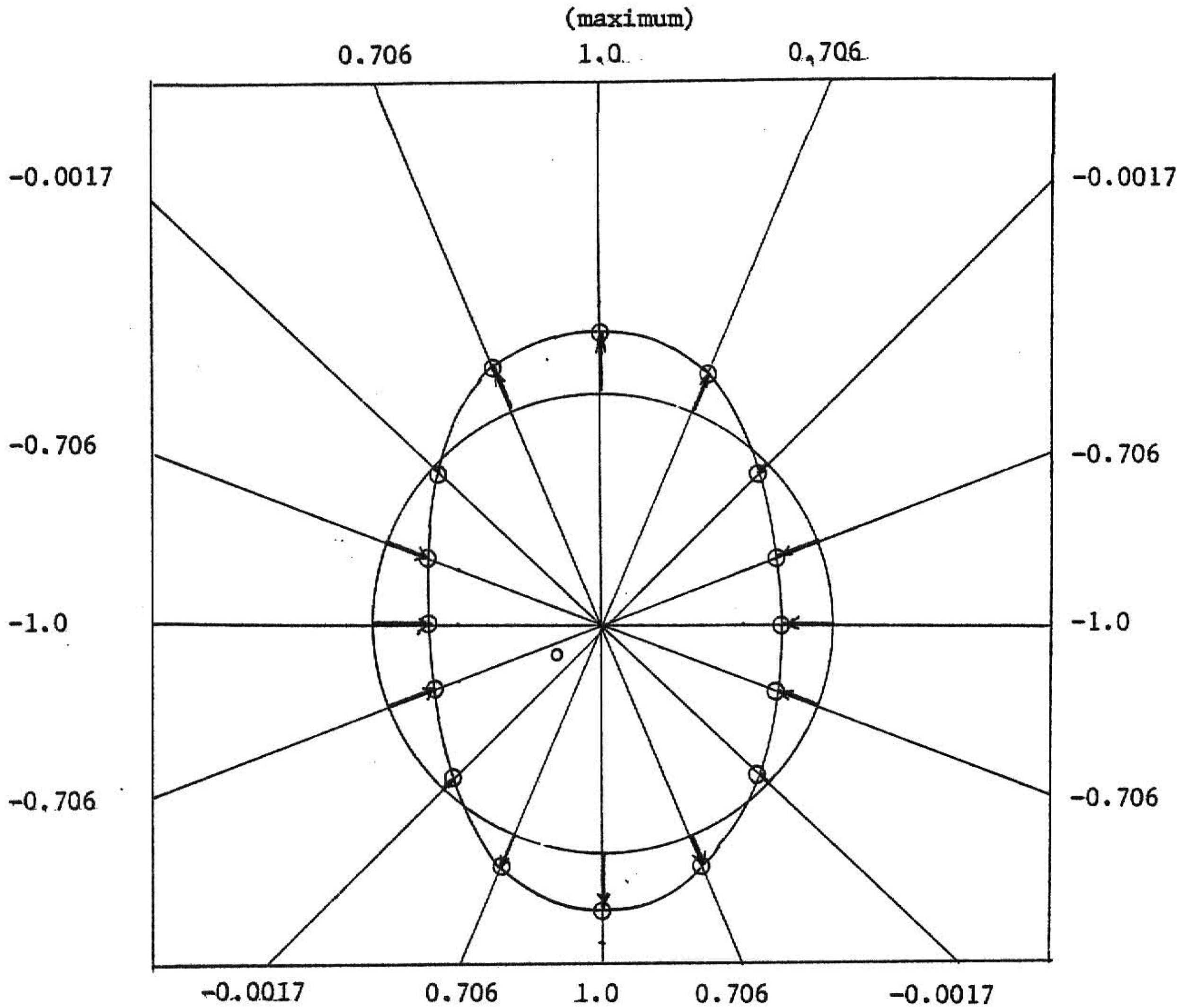


Fig. 4-1 Geometric Parameters of the Prototype Model



——— represent the deformation direction (outward is positive)

Fig. 4-2 Normalized Deformation Ratio of the Ring Strip

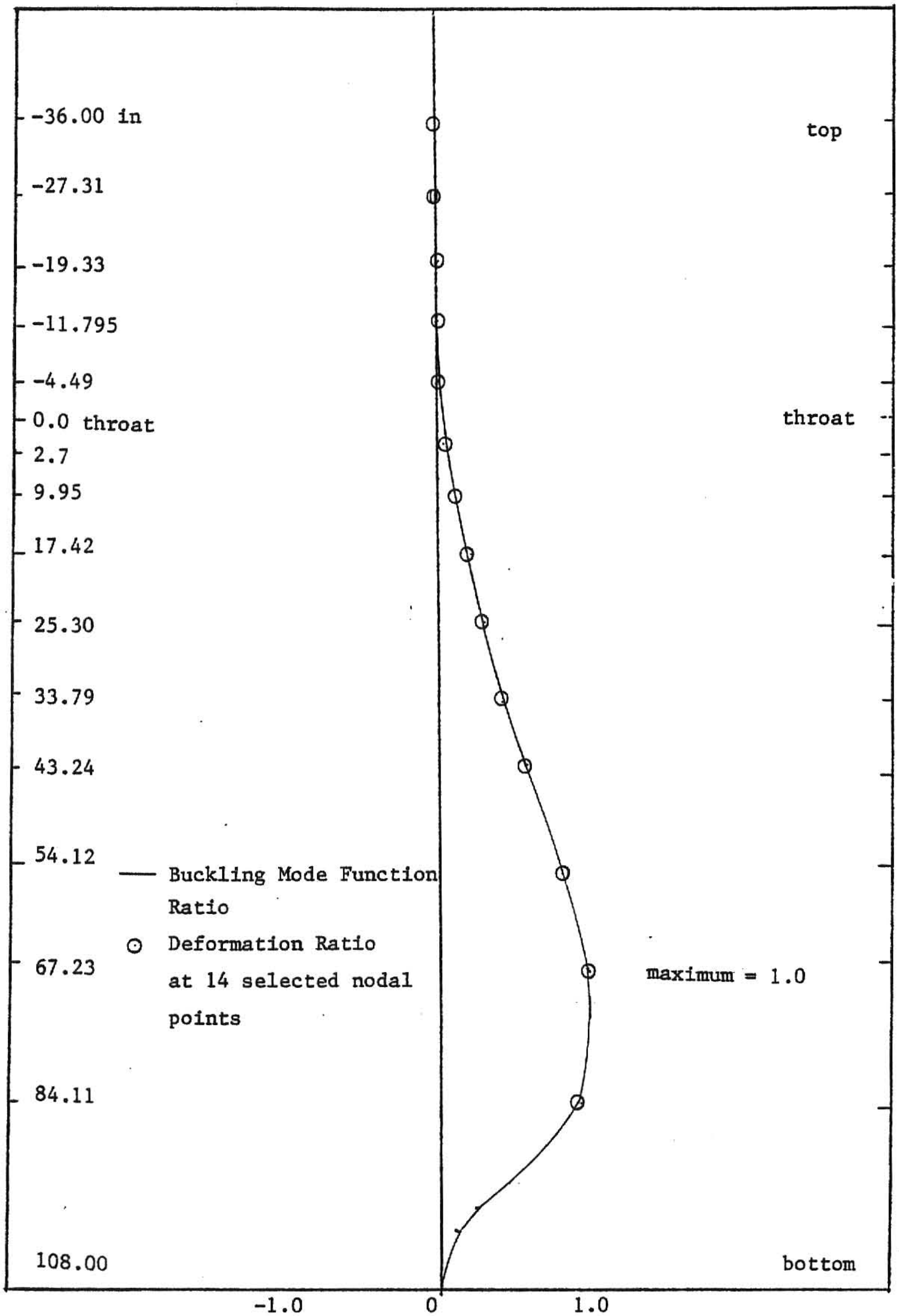


Fig. 4-3 Vertical Strip Deformation Ratio (Normalized)

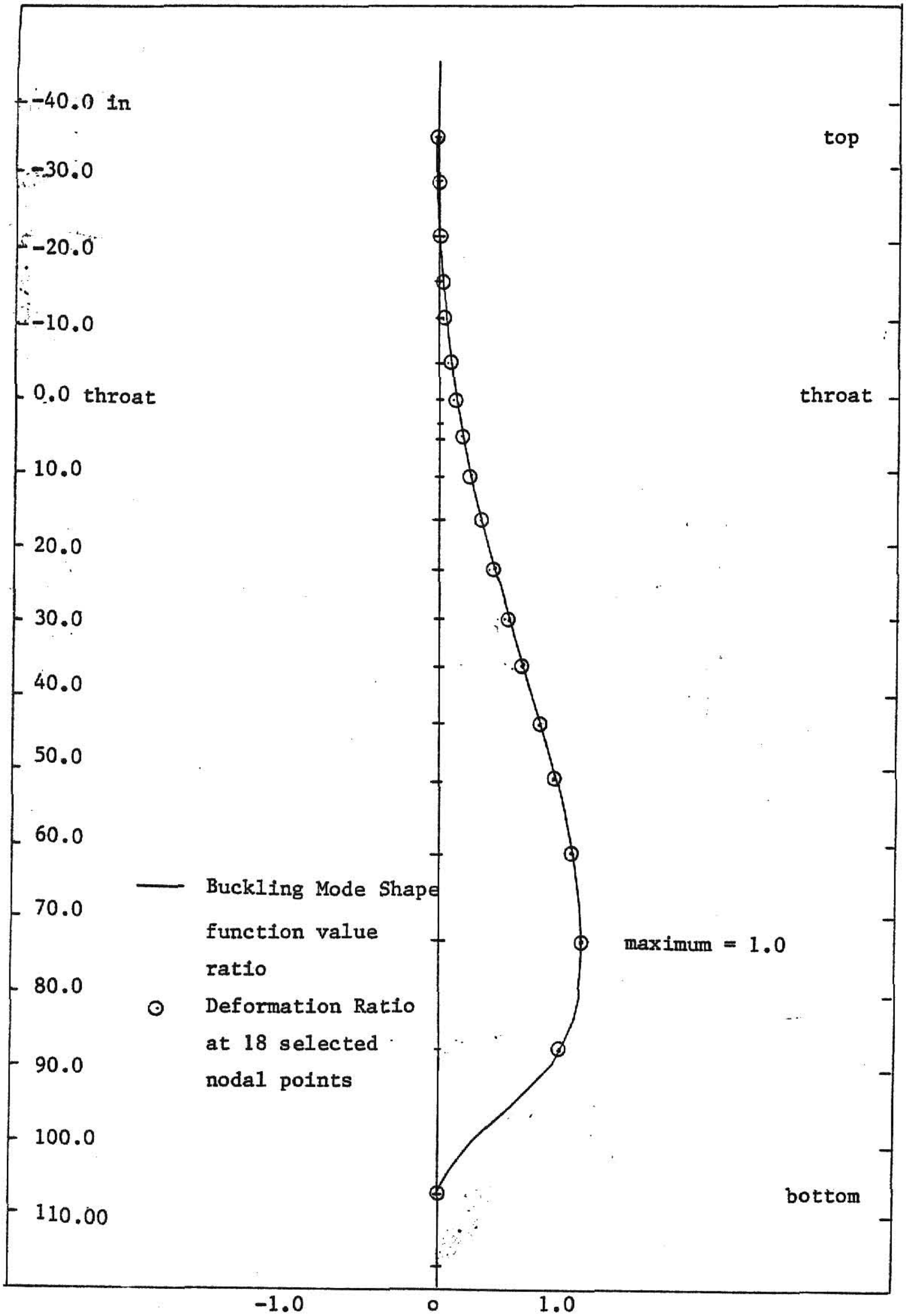


Fig. 4-4 Vertical Strip Normalized Deformation Ratio for (20x20) Partition

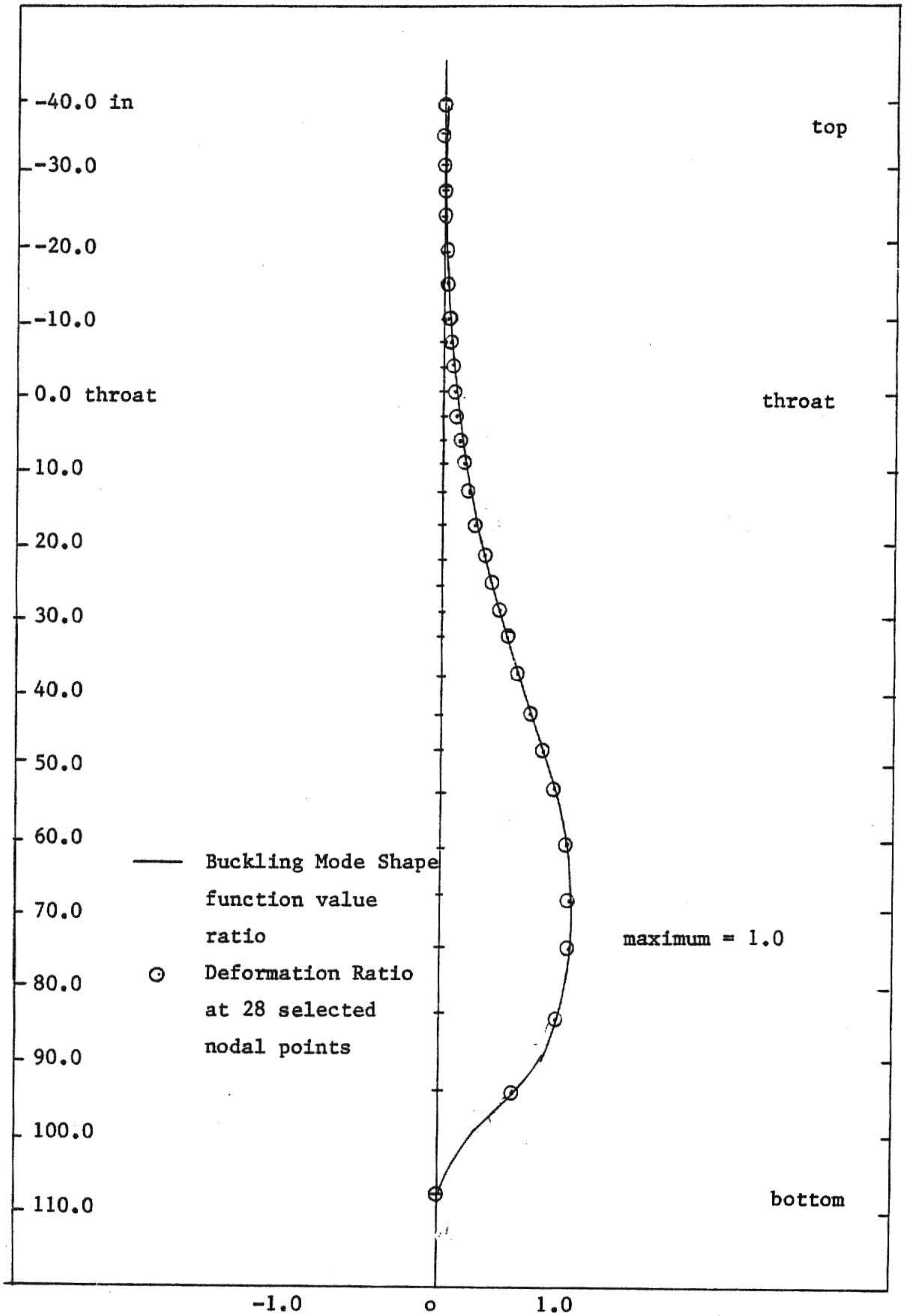


Fig. 4-5 Vertical Strip Normalized Deformation Ratio for (30x30) Partition

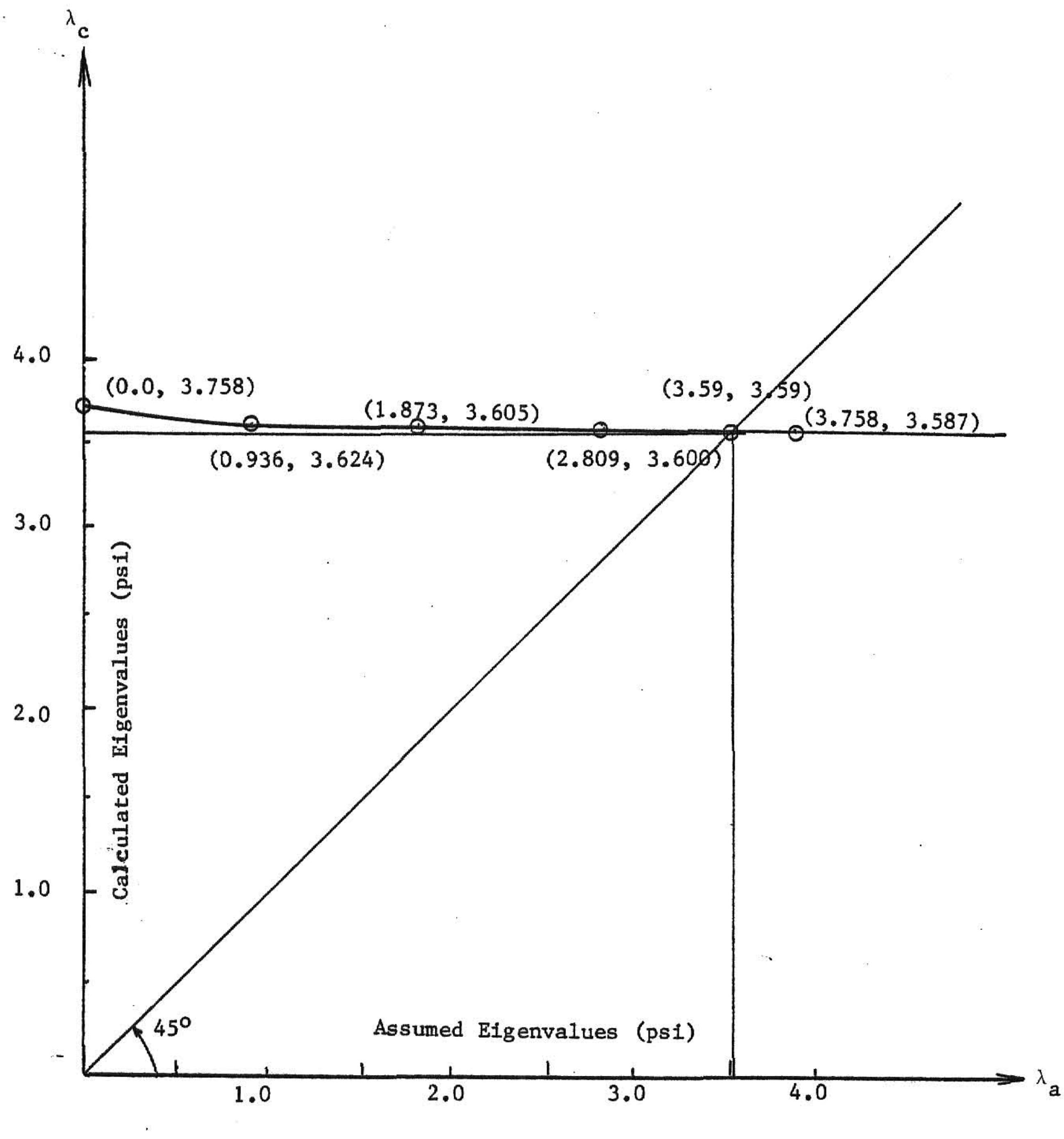


Fig. 4-6 Relationship between λ_a and λ_c for (16, 16) example

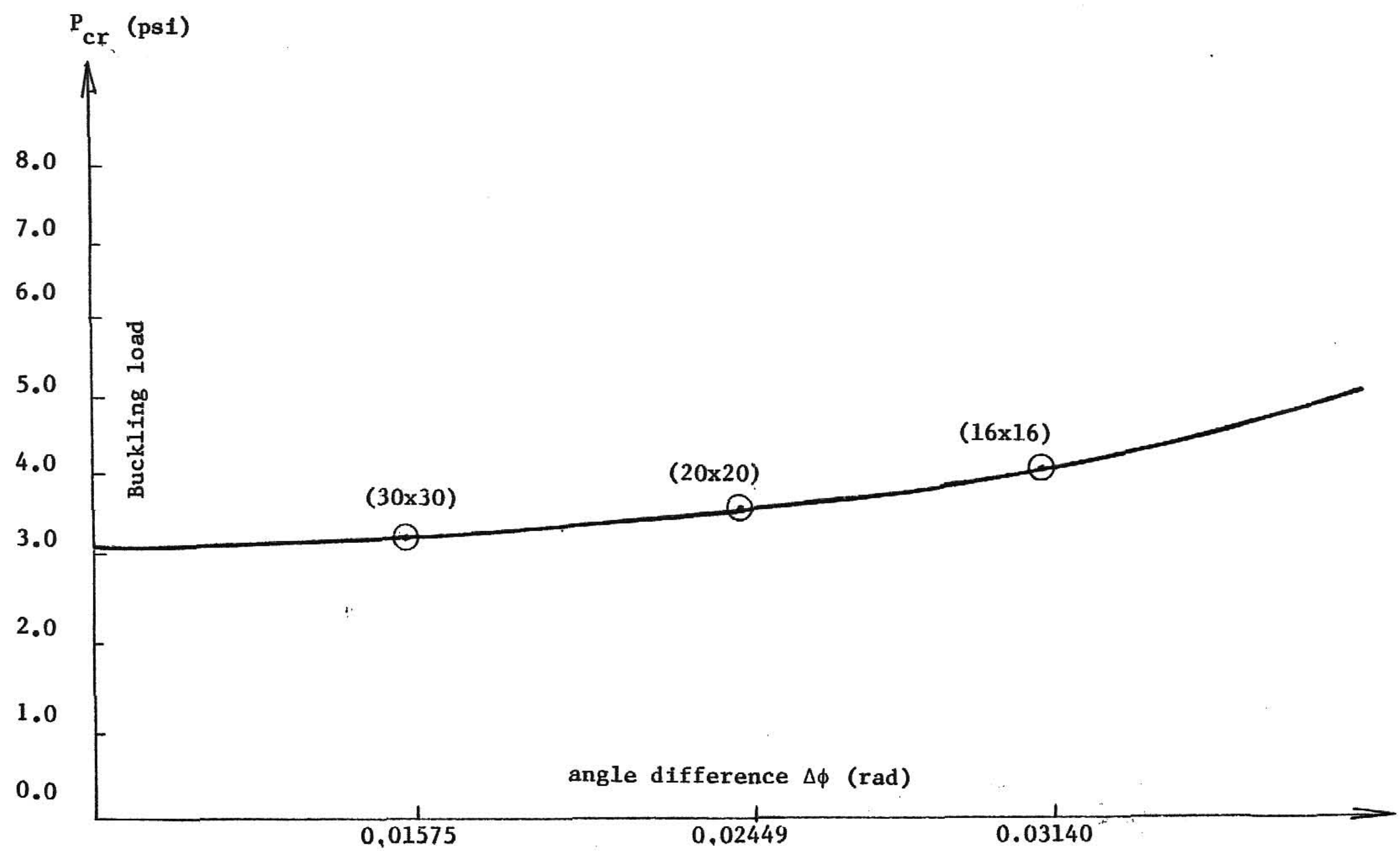


Fig. 4-7 Relationship between $\Delta\phi$ and buckling load (linear)

By the use of Richardson extropolation (23) base on that error of central difference in order of $(\Delta\phi)^3$, one can find an improved P_{cr} which is obtained as

$T_1 = 3.758$, $T_2 = 3.650$, $T_3 = 3.151$

$$\begin{cases} T_{imp} - T_1 = C\left(\frac{1}{16}\right)^3 \\ T_{imp} - T_2 = C\left(\frac{1}{20}\right)^3 \end{cases} \qquad T_{imp} = 3.637 \qquad P_{cr} = 3.637 \text{ psi}$$

$$\begin{cases} T_{imp} - T_2 = C\left(\frac{1}{20}\right)^3 \\ T_{imp} - T_3 = C\left(\frac{1}{30}\right)^3 \end{cases} \qquad T_{imp} = 2.9409 \qquad P_{cr} = 2.9409 \text{ psi}$$

To show the variation of the buckling mode shape function due to various partition sizes. Figures for the relative buckling deformation along the vertical strip with maximum w values are shown in Fig. 4-3, 4-4, 4-5. The detailed deformation from the computer output are listed in Appendix B. Note that due to the nonlinearity of the material such as concrete, the shell becomes nonhomogeneous and orthotropic during the nonlinear buckling analysis as discussed in Chapter Three.

4-2 The Analysis of Fort Martin Tower

The studies of the Fort Martin Tower had been examined by Langhaar (17) and Cole (2). The material is assumed to be isotropic and homogeneous in the present analysis (linear), the Young's Modulus is 3.09×10^8 psf, the Poisson's ratio is 0.197. The boundary conditions are assumed to be clamped at the support and free from constraints at the top of the shell. The external load is uniform pressure. The constant thickness for the shell is 5.5 inches. The total height is 310.0 ft, the distance of the throat is 216 ft, the radii at the top, throat and base are 90, 80.25 and 124.3 ft, and b = 183.92 ft shown in Fig. 4-8

The linear buckling load P_{cr} obtained is 143.38 psf. A comparison of the results to the published results are listed in Table 4-1. The ratio of

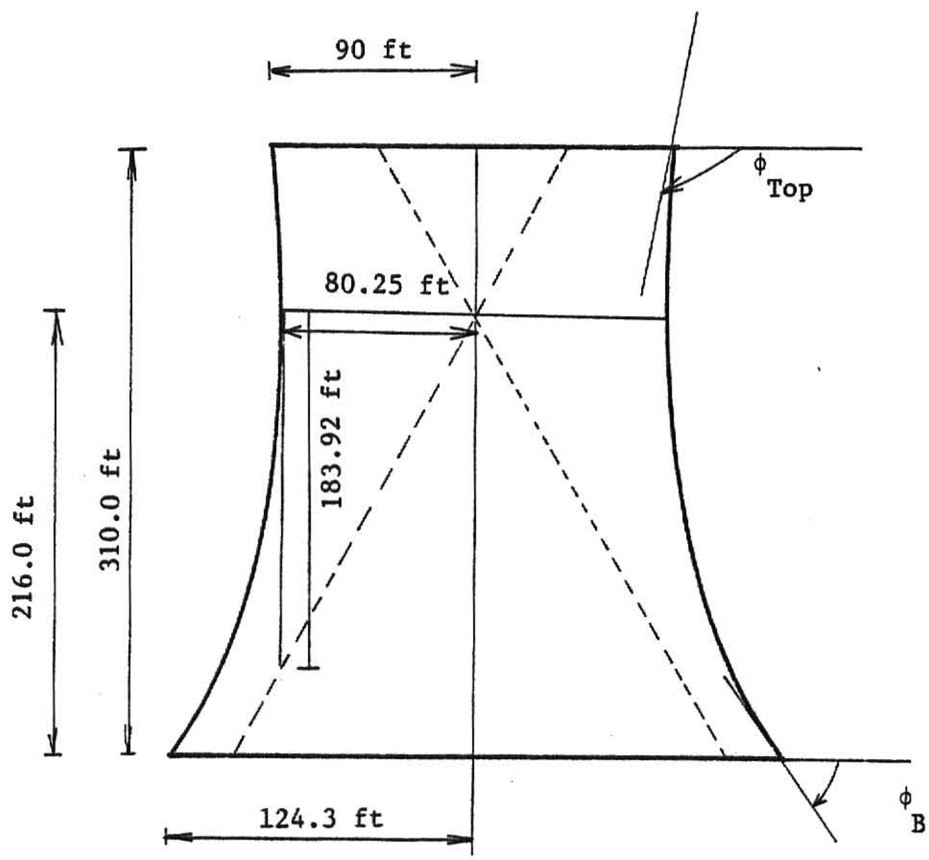


Fig. 4-8 Geometric Parameters of Fort Martin Tower

Support	P_{cr}(psf)		
	P. P. Cole	Langhaar	Present
Clamped	307.01	313.01	143.38
No. of waves	————	7	2
P_{cr} for seven waves mode shape is about 317. psf by using the present method.			

Table 4-1 Buckling Load (linear) for Fort Martin Tower

of the present P_{cr} to those published results range from 46% to 47%, which the present results are more conservative than the previous analysis results.

Note that the number of the waves of the buckling mode along the circumferential direction is two according to the present analysis and there are seven waves according to other investigators for this model.

4-3 Proposed Method of Solving the Geometric Imperfection

When the geometrical imperfection is of importance, the problem can be solved by the classical method of local linearization which can be viewed as a kind of incremental techanique.

This start with

$$T_o(W_o) = Q(W_o) + Q(\delta)$$

where δ is a imperfection matrix

$$T_1(W_1) = (T_o + W_o)$$

$$= \Delta\lambda P(\delta + W_o + \Delta W_o) + Q(\delta + W_o + \Delta W_o)$$

after simplification, this reads

$$T_o(\Delta W_o) = \Delta\lambda P(\delta + W_o) - \Delta T_o(W_o) + \Delta\lambda P(\Delta W_o) + (T_1 - T_o)(\Delta W_o) + Q(\Delta W_o)$$

with ΔW_o as the unknown matrix.

Let $\lambda_n = n\Delta\lambda$, and the corresponding left hand operator be T_n, then the increment of W at the (n + 1)th iteration is characterized by the following linearized form.

$$T_o(\Delta W_n) = \Delta\lambda P(W_n + \delta) - (T_{n+1} - T_n)(W_n) + \lambda_{n+1}P(\Delta W_n) + Q(W_n) -$$
$$(T_{n+1} - T_o)(\Delta W_n)$$

Note that one only need to construct the inverse operator T_o^{-1} once for all iterations.

CHAPTER FIVE

CONCLUSION

According to the numerical analysis of the concrete prototype model shell constructed at KSU the linear buckling load is about 3.76 psi. The buckling mode shape in the circumferential direction in general is quite agreeable to the elastic buckling mode shape for a cylindrical shell under uniform lateral pressure (1). The buckling load obtained from the nonlinear elastic buckling analysis has little difference in the case of linear elastic buckling analysis for the prototype shell. This can be explained due to the buckling that occurs at a low stress level to show the significance of the consideration of the nonlinearity of the material.

Through the investigation of the numerical experiment, the following observations can be made.

1) The organization of the elements of the matrix W can fit the natural arrangement of the nodal points.
2) The use of Sylvester's operator can reduce the matrix equation into the smallest possible dimension.
3) Seven iterations for the method used can reach a relative accuracy of 0.1% for mode shape function and 0.01% for eigenvalues.
4) Partition size of 16x16 for the meridianal and circumferential direction in the numerical analysis is adequate for the practical use.
5) The buckling load obtained is only about 25% of that predicted by the use of the formula of Mungan (12), (24). The values will be verified by experiment test of the concrete model in the near future.
6) The result of the analysis compared to that predicted by Langhaar is about 50% lower. It shows that the method provides conservative prediction.

7) The computer program developed can be used to find the buckling behavior of the geometrical imperfect shell.

8) The boundary condition also can be changed by modification of the method presented to solve general cases.

9) With modification the computer program developed can be used to find the lowest natural mode of vibration.

10) Further developments are needed to find higher modes of vibration as desired in the dynamic analysis of the shell structures.

NOTATION

The following symbols are used in this paper

a	Radius of the throat
b	Geometric parameters related to ruling line
c_1, c_2	Independent parameters to describe the constitutive system
c_j	Column vector of (W_ϕ)
$d\theta$, $d\phi$, dr, dz	Discrete angle and length for θ, ϕ, r and z
E	Young's modulus
E_ϕ, E_θ	Young's modulus, subscripts refer to directions
H_T	Height of the tower
H	Length from throat to the support
K	Plate or shell stiffness
M_θ, M_ϕ	Bending moment (equations (1-15) and (1-16))
N_θ, N_ϕ, $N_{\phi\theta}$	Membrane forces subscripts refer to directions
N_ϕ^q, N_θ^q	Membrane forces due to the uniform lateral pressure
N_ϕ^p, N_θ^p	Membrane forces due to axial load
N_ϕ^g, N_θ^g	Membrane forces due to selfweight
N_ϕ^*, N_θ^*	$N_\phi^* = N_\phi^{q+p}$, $N_\theta^* = N_\theta^{q+p}$
Q_ϕ, Q_θ	Equivalent lateral force
Q^q, Q^p	Related to equation (1-28)
q	Unit external load
r, r_1, r_2	Radii
R_T, R_B	Radius at top and support

$(T(\lambda))$	Generalized operator
$(T(\lambda_o))$, T^*	Generalized operator with linearized eigenvalue
h	Thickness of the shell
(T_p)	Principal operator
(T_s)	Seccndary operator
W	Buckling deformation (matrix)
$(\hat{W})$	Normalized buckling matrix
w	Buckling deformation in differential equation
x, y, z	Cartesian coordinates
λ	Eigenvalue
λ_a	Assumed eigenvalue
λ_c	Calculated eigenvalue
σ_ϕ, σ_θ	Stresses, subscripts refer to directions
ϵ_ϕ ϵ_θ	Strains, subscripts refer to direction
ξ_i	Column eigenvectors for (B)'
η_i	Column eigenvectors for (A)
γ_i	Eigenvalue for (A)
λ_i	Eigenvalue for (B)'
(Ξ)	Eigenvectors for (B)'
(H)	Eigenvectors for (A)
μ	Poisson value
(Λ)	Eigenvalues for (B)'
(Γ)	Eigenvalues for (A)

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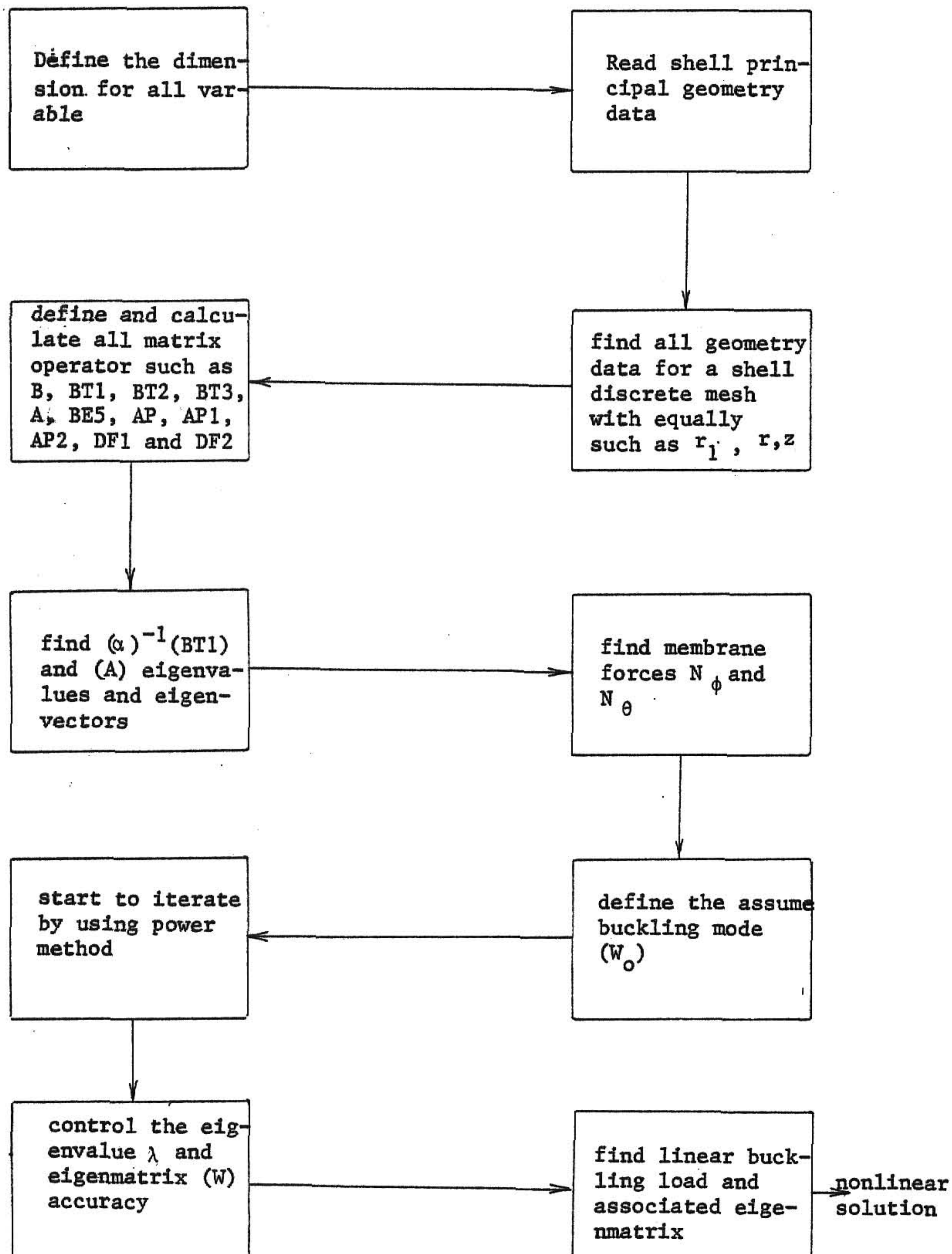
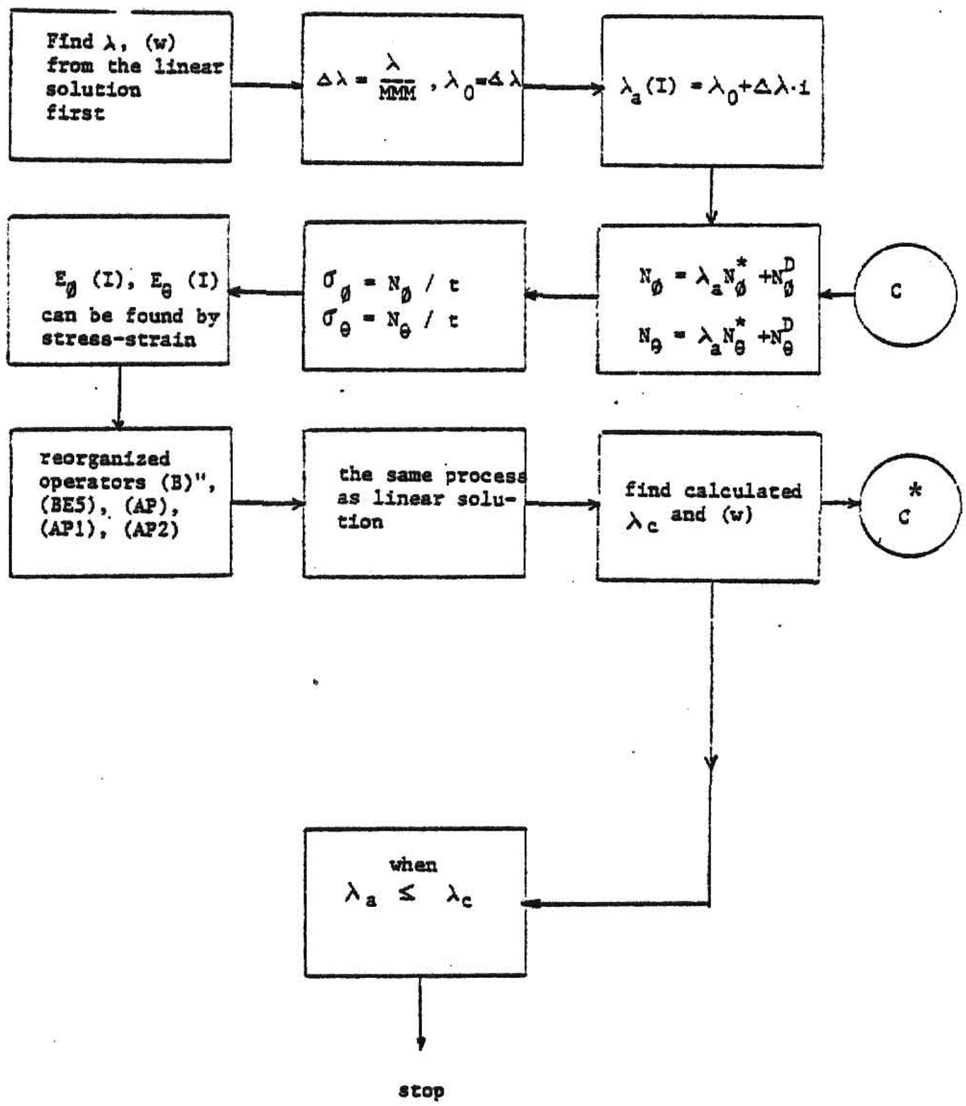
APPENDIX A

A-1 Skeletal flow chart for SHEL1 stability computer program



A-1-1 Nonlinear solution procedures



Flow chart for nonlinear solution applied on SHEL1 program

* process of the program

A-2 SHEL 1 & 2 Computer Program

```
$JOB            ,P=80,TIME=(0,30)
C       SHEL1 PROGRAM AND SHEL2 PROGRAM
C       THIS PROGRAM IS BUILT BECAUSE THE BUCKLING LOAD AND
C       MODE OF
C       HYPERBOLIC PARABOLOID  COOLING TOWER SHELL ANALYSIS
C       , IT CON-
C       TAINS LINEAR ELASTIC BUCKLING ANALYSIS AND NONLINEAR
C       ELASTIC BUBUCKLING
C       ANALYSIS.
C       THIS PROGRAM IS MAINLY BASED ON THE SYLVESTER
C       AND FINITE
C       DIFFERENCE AND ITERATION METHOD
C       DIMENSION OF EACH TERM IN THE PROGRAM AND  THE
C       SIMPLE STATEMENT
C       RI(M3)=RADIOUS OF EACH RING
C       WA(M2)=1/RI**4
C       Z1(M3)=COORDINATE OF LONGITUDINAL DIRECTION
C       A(N,N)=4TH DERIVITIVE WITH PHI
C       RPHI(M3)=RADIOUS OF HYPERBOLIC SHELL CURVE
C       BM(M2,M2)
C       WAB(M2)=1/WA
C       WABS(M2,M2)=WABS(I)*BT1*WABS(J)
C       VA(N,N)=EIGENVECTOR FOR A
C       EGNA(N)=EIGENVALUE FOR A
C       PHI(M3)=ANGLE ALONG WITH RPHI
C       Z2(M3)=CORRECTION COORDINATE OF Z  WITH EQUAL
C       DISCRETE PHI
C       VB(M2,M2)=EIGENVECTER FOR CORRECTION B
C       EGNB(M2)=EIGENVALUES FOR BT3
C       DF1(M2)=FIRST ORDER DIFFERENTIAL OPERATOR WITH PHI
C       DS1((M2)=SECOND ORDER DIFFERENTIAL
C       OPERATOR IN B1
C       DF2(M2)=FIRST ORDER DIFFERENTIAL OPERATOR WITH PHI IN B3
C       DS2(M2)=SECOND DIFFERTIAL OPERATOR WITH PHI IN B3
C       B1(M2,M2)=OPERATOR ON (W)
C       B2(M2,M2)=CONNECTION MATRICES
C       WB2(M2)
C       B3(M2,M2)=OPERATOR ON BI
C       B4(M2,M2)==ONE OPERATOR OF THE CXD PART
C       B(M2,M2)= B2*BI UNSYMMETRICAL
C       B5(M2,M2)= THE CHANGE FORM OF B4 BY ADDING FORCE
C       BOUNDARY CONDI-
C       TIONS IN
C       WA2((M2)= WEIGHTING MATRICES OF RADIOUS
C       AP(M2,M2)=ONE OPERATOR OF CXD PART
C       AP1(M2,M2)
C       AP2(M2,M2)
C       BT1(M2,M2)=SYMMERTRICAL PART OF B
C       ST(DIMENSION DEPEND ON THE BT1 DIMENSION)
C       AS(DIMENSION DEPEND ON THE BT1 DIMENSION)
C       BT(M2,M2)= B TRANSPOSE
C       A1(N,N)=SECOND DIFFERENTIAL OPERATOR WITH THITA
C       SNPHI(M2)= MEMBRANE FORCE WITH PHI DUE TO DEAD LOAD
C       SNTHIT(M2)=MEMBRANE FORCE WITH THITA DUE TO DEAD LOAD
C       ENPHI(M2)=MEMBRANE FORCE WITH PHI DUE TO EXTERNAL
C       PRESSURE
C       ENTHI(M2)=MEMBRANE FORCE WITH THITA DUE TO EXTERNAL
C       PRESSURE
C       X(M2,M2)=ONE CONNECTION MARRICES OF BUCKLING MODE
C       T3(M2,M2)==SUMMATION OF MATRICES
```

```
C       QD(M2,M2)=DEAD LOAD FINAL RESULT MATRICES WITH PHI
C       QD2(M2,M2)=DEAD LOAD FINAL RESULT MATRICES WITH PHI
C       QDT1(M,M2)=DEAD LOAD FINAL RESULT MATRICES WITH THITA
C       QDT2(M2,M2)=DEAD LOAD FINAL RESULT MATRICES WITH THITA
C       QP(M2,M2)=FINAL RESULT MATRICES WITH PHI DUE
C       TO EXTERNAL PRESSURE
C       QP2(M2,M2)=FINAL RESULT MATRICES WITH PHI DUE
C       TO EXTERNAL PRESSURE
C       QPT(M2,M2)=FINAL RESULT MATRICES WITHTHITA EXTERNAL LOAD
C       QPT2(M2,M2)=FINAL RESULT MATRICES WITHTHITA EXTERNAL LOAD
C       QPT1(M,M2)=FINAL RESULT MATRICES WITHTHITA EXTERNAL LOAD
C       BT2(M2,M2)=UNSYMMETRICAL PART OF B
C       RR1(M2,M2)
C       RRR1(M2,M2)
C       RRR2(M2,M2)
C       RRR3(M2,M2)
C       RR3(M2,M2)
C       RRR5(M2,M2)
C       RRR7(M2,M2)
C       TS(M2,M2)=SUMMATICN OF THE LEFT HAND SIDE SYMMETRICAL PART
C       C(M2,M2)=SUMMATION OF RIGHT HAND SIDE MATRICES RELATE TO EI
C       E
C       VBT(M2,M2)=TRANSPOSE OF THE EIGENVECTOR MATRICES OF BT3
C       DI1(M2,M2)
C       D(M2,M2)
C       XR(M2,M2)
C       W1(M2,M2)
C       PHI1(M3)=CONNECTION  VECTOR
C       SNPDA(M3)=MEMBRANE FORCE OF AXIAL LOAD WITH PHI
C       SNTDA(M3)=MEMBRANE FORCE OF AXIAL LOAD WITH THITA
C       WA3(M3)=WEIGHTING MATRICES OF RADIOUS
1       DIMENSION R1(22),WA(19),Z1(21),A(16,16)
       C,RPHI(21),BM(16,16),WAB(19),WABS(19),VA(16,16),EGNA(18),
       C PHI(19),Z2(19),VB(16,16),EGNB(19)
       C,DF1(16,16),DS1(16,16),DF2(16,16),DS2(16,16)
       C,B1(16,16),B2(16,16),WB2(19),B3(16,16)
       C,B4(16,16),B(16,16),B5(16,16),WA2(19),AP(16,16)
       C,AP1(16,16),AP2(
       C16,16),BT1(16,16),ST(300),AS(300),BT(16,16),A1(16,16)
       C,SNPHI(16),SNTHIT(16),ENPHI(16),ENTHI(16),X(16,16),
       CT3(16,16),
       C QD(16,16),QD2(16,16),QDT(14,16),QDT1(14,16),QDT2(16,16)
       C,QP(16,16),QP2(16,16),QPT(14,16),QPT2(16,16),QPT1(14,16)
       C,BT2(16,16),RRR1(16,16),RRR2(16,16),RR1(16,16)
       C,RRR3(16,16)
       C,RR3(16,16),RRR5(16,16),RRR7(16,16),TS(16,16)
       C,C(16,16),VBT(16,16),DI1(16,16),D(16,16),XR(16,16)
       C,W1(16,16),
       CVAT(16,16),W(16,16),E(16,16),TP(16,16),TP1(16,16)
       C,EIG(50),EIGE(50)
       C,Z3(19),PHI1(19),SNPDA(18),R2(18),WA3(19),BT3(16,16)
       C,XER(16,16)
       C,SNTDA(19)
       C,RR5(16,16)
       C,VBT1(16,16),FE(16,16),A2(16,16)
       C,EP(16),ET(16),EPT(16),EI(20),STR(16,2),B8(16,16),CD(16,16)
       C,CD1(16,16),WM(16,16)
       C,CT1(7)
       C,CONST(16),RD1(16),RD2(16),RD3(16)
2       III=1
```

```
      JJJ=0
      LK1=0.
      LK2=0.
      LK3=1.
      LK4=1.
      CT1(1)=1.
      CT1(2)=1.
      CT1(3)=1.
      CT1(4)=2.
      CT1(5)=2.
      CT1(6)=1.
      CT1(7)=1.
C
C.....READ THE GEOMETRY DATA AND LOADING TERM
C
      READ,M,N,MMM,QT1,QT2,QS1,TS3,TS1,TS2,PRS
      YE1=TS3/TS2
      QC3=TS1/TS2
      QC4=(YE1-QC3)/TS3**2
      QC1=QS1/QT2
      QC2=-QC1/QS1**2
      M1=M+1
      M2=M+2
      M3=M+3
      READ,NEND,Z1(1),Z1(M1),QL,GL,PO,THICK,YOUNG
      READ,AL,BL,RT,RB,HT
150   FORMAT(1X,8E13.6)
      FR=YOUNG*THICK**3/(12.*(1.-PO**2))
      AXIAL=0.28*RT*QL*PRS
      STIFF=FR/YOUNG
      DO 1505 I=1,M2
      FP(I)=FR
      ET(I)=FR
1505  EPT(I)=FR
      WRITE(6,1743)
1743  FORMAT('1')
      PRINT,'Z AT TOP=',Z1(1),'ZAT BOTTOM=',Z1(M1),'Q=',QL,
     C'G=',GL
      PRINT,'POISSON VALUE=',PO,'THICKNESS='
     C,THICK,'YOUNGS MODULUS=',Y
     COUNG,'A=',AL
      PRINT,'B=',BL,'RT=',RT,'RB=',RB,'HT=',HT
      PRINT,'FLEXTURAL RIGIDITY=',FR,'UNIT AXIAL LOAD=',AXIAL
C
C.....GEOMETRY DATA FOR COOLING TOWER SHELL CALCULATION
C
      ANGLE=3.14159265/2.
      XX=(AL/BL**2)*Z1(1)/(1+(Z1(1)/BL)**2)**0.5
      XX1=ATAN(XX)
      PT=ANGLE-XX1
      XX=(AL/BL**2)*Z1(M1)/(1+(Z1(M1)/BL)**2)**0.5
      XX1=ATAN(XX)
      PB=ANGLE-XX1
      DP=(PT-PB)/M
      PHI(1)=PT+2.*DP
      PHI(2)=PT+DP
      PHI(3)=PT
      DO 22 I=4,M3
      PT=PT-DP
22    PHI(I)=PT
```

```
      DO 77 I=1,M3
      IF(PHI(I).LT.ANGLE) GO TO 80
      XX=TAN(ANGLE-PHI(I))
      DDD=(XX/(AL/BL**2))**2
      Z1(I)=-SQRT(DDD/(1.-DDD/BL**2))
      GO TO 77
   80 XX1=TAN(ANGLE-PHI(I))
      DDD=(XX1/(AL/BL**2))**2
      Z1(I)=SQRT(DDD/(1.-DDD/BL**2))
   77 CONTINUE
      DO 86 I=1,M3
      RPHI(I)=-(AL**2*BL**2)/(AL**2*SIN(PHI(I))**2-BL**2
     C **2)**1.5
   86 R1(I)=AL*(1.+(Z1(I)/BL)**2)**0.5
      PRINT,(RPHI(I),I=1,M3)
      PRINT,(R1(I),I=1,M3)
      PRINT,(Z1(I),I=1,M3)
      PRINT,(PHI(I),I=1,M3)
      DO 1220 I=1,M3
 1220 R2(I)=R1(I)/SIN(PHI(I))
      WRITE(6,1641)
 1641 FORMAT(5X,'POINT',5X,'PHI',16X,'RPHI'
     C,13X,'RTHITA',10X,'Z')
      DO 1304 I=3,M3
      I3=I-2
 1305 FORMAT(5X,I3,3X,E13.6,7X,E13.6,3X,E13.6,3X,E13.6)
      WRITE(6,1305) I3,PHI(I),RPHI(I),R1(I),Z1(I)
 1304 CONTINUE
C
C.....FINISH CALCULATING THE GEOMETRY DATA FOR
C.....THE ENTIRELY HYPERBOLIC PARABOLOID SHELL
C
C
C.....CONSTRUCT THE FOURTH DERIVITIVE OPERATER WITH THITA
C
      AN=N
      D1=2.*3.141592653/AN
      D2=D1**2
      D4=D1**4
      DO 70 I=1,N
      DO 70 J=1,N
      A1(I,J)=0.
   70 A(I,J)=0.0
      DO 71 I=1,N
   71 A(I,I)=6./D4
      N1=N-1
      N2=N-2
      DO 72 I=1,N1
      I1=I+1
      A(I,I1)=-4.0/D4
   72 A(I1,I)=-4.0/D4
      DO 73 I=1,N2
      I2=I+2
      A(I,I2)=1.0/D4
   73 A(I2,I)=1.0/D4
      A(1,N-1)=1.0/D4
      A(1,N)=-4.0/D4
      A(2,N)=1.0/D4
      A(N-1,1)=1/D4
      A(N,1)=-4.0/D4
```

```
      A(N,2)=1.0/D4
      DO 14101 I=1,N
14101 A1(I,I)=-2./D2
      DO 142 I=1,N1
      I1=I+1
      A1(I,I1)=1./D2
142   A1(I1,I)=1./D2
      A1(1,N)=1.0/D2
      A1(N,1)=A1(1,N)
C
      M2=M+2
      M1=M+1
1506  DO 1234 I=1,M2
      WA2(I)=R1(I)**2
1234  WB2(I)=RPHI(I)**2
      DO 1235 I=1,M2
      WA(I)=ET(I)/(R1(I)**4)
1235  WA3(I)=1./WA(I)
      IF(III.GT.1) GO TO 4170
C
      DP4=DP**4
      DP3=2*DP**3
      DP2=DP**2
      DP1=2.*DP
C
      DO 401 I=1,M2
      DO 401 J=1,M2
      B(I,J)=0.
      B1(I,J)=0.
      B2(I,J)=0.
      B3(I,J)=0.
      B4(I,J)=0.
      DS2(I,J)=0.0
      DF2(I,J)=0.0
      DS1(I,J)=0.0
401   DF1(I,J)=0.
      DO 402 I=3,M2
402   DF1(I,I-1)=-1.0/(2.*DP)
      DO 403 I=3,M1
403   DF1(I,I+1)=1.0/(2.*DP)
      DO 405 I=3,M2
405   DS1(I,I-1)=1.0/DP2
      DO 406 I=3,M2
406   DS1(I,I)=-2.0/DP2
      DO 407 I=3,M1
407   DS1(I,I+1)=1.0/DP2
      DO 409 I=4,M2
409   DF2(I,I-1)=-1.0/(2.*DP)
      DO 460 I=4,M1
460   DF2(I,I+1)=1.0/(2.*DP)
      DO 411 I=4,M2
      DS2(I,I-1)=1.0/DP2
411   DS2(I,I)=-2.0/DP2
      DO 461 I=4,M1
461   DS2(I,I+1)=1.0/DP2
      DF2(3,2)=-1./(2.*DP)
      DF2(3,4)=1./(2.*DP)
      DS2(3,2)=1./DP2
      DS2(3,3)=-2./DP2
      DS2(3,4)=DS2(3,2)
```

```
      DO 414 I=2,M1
414   B1(I+1,I+1)=(-RPHI(I)+RPHI(I+2))/(2.*DP*RPHI(I+1))
      DO 415 I=1,M2
      DO 415 J=1,M2
415   B2(I,J)=B1(I,I)*DF1(I,J)
4170  DO 417 I=1,M2
      DO 417 J=1,M2
417   B1(I,J)=EP(I)*(DS1(I,J)-B2(I,J))/WB2(I)
      IF(III.GT.1) GO TO 4251
      DO 419 I=2,M1
      DO 419 J=1,M2
419   B3(I+1,I+1)=(-RPHI(I)+RPHI(I+2))/(2.*DP*RPHI(I+1))
      DO 422 I=1,M2
      DO 422 J=1,M2
422   B8(I,J)=B3(I,I)*DF2(I,J)
      DO 424 I=1,M2
      DO 424 J=1,M2
424   B3(I,J)=(DS2(I,J)-B8(I,J))/WB2(I)
4251  DO 425 I=1,M2
      DO 425 J=1,M2
      B(I,J)=0.
      DO 425 K=1,M2
425   B(I,J)=B(I,J)+B3(I,K)*B1(K,J)
      DO 451 I=1,2
      DO 451 J=1,2
451   B(I,J)=0.0
      SSS=RPHI(3)**2*DP**2
      TTT=SSS*RPHI(3)*4.
      UUU=-RPHI(2)+RPHI(4)
      B(1,2)=(1./SSS+UUU/TTT)
      B(1,3)=(-2./SSS)
      B(1,4)=(1./SSS-UUU/TTT)
      UK=-2.*DP*RPHI(3)
      UK1=-UK
      SSS=RPHI(2)**2*DP**2
      TTT=SSS*RPHI(2)*4.
      UUU=-RPHI(1)+RPHI(3)
      B(2,1)=(1./SSS+UUU/TTT)/UK
      B(2,2)=(-2./SSS)/UK
      B(2,3)=(1./SSS-UUU/TTT)/UK
      SSS=RPHI(4)**2*DP**2
      TTT=SSS*RPHI(4)*4.
      UUU=-RPHI(3)+RPHI(5)
      B(2,3)=B(2,3)+(1./SSS+UUU/TTT)/UK1
      B(2,4)=(-2./SSS)/UK1
      B(2,5)=(1./SSS-UUU/TTT)/UK1
      DO 4250 I=1,2
      DO 4250 J=1,M2
4250  B(I,J)=EP(3)*B(I,J)
      DO 453 I=1,M2
      DO 453 J=1,M2
      BT2(I,J)=(B(I,J)-B(J,I))/2.0
453   BT1(I,J)=(B(J,I)+B(I,J))/2.
      DO 2002 I=1,M2
      DO 2002 J=1,M2
2002  BT3(I,J)=BT1(I,J)*WA3(I)
      DO 4540 I=1,M2
      DO 4540 J=1,M2
4540  BT2(I,J)=WA3(I)*BT2(I,J)
      DO 426 I=1,M2
```

```
      DO 426 J=1,M2
  426 B4(I,J)=EP(J)*B3(I,J)*PC*COS(PHI(J))/(R1(J)*RPHI(J))
      DO 427 I=1,M2
      DO 427 J=1,M2
      B5(I,J)=0.
      DO 427 K=1,M2
  427 B5(I,J)=B5(I,J)+B4(I,K)*DF1(K,J)
      DO 4270 I=1,2
      DO 4270 J=1,M2
 4270 B5(I,J)=0.0
      QQ=COS(PHI(3))/(2.*DP*R1(3)*RPHI(3))
      B5(1,2)=-QQ
      B5(1,4)=QQ
      QQ=COS(PHI(2))/(2.*DP*R1(2)*RPHI(2))
      QQ1=COS(PHI(4))/(2.*DP*R1(4)*RPHI(4))
      B5(2,1)=QQ/(2.*DP*RPHI(3))
      B5(2,3)=(-QQ-QQ1)/(2.*DP*RPHI(3))
      B5(2,5)=QQ1/(2.*DP*RPHI(3))
      DO 801 J=1,M2
      B5(1,J)=PO*B5(1,J)*EP(3)
  801 B5(2,J)=PO*B5(2,J)*EP(3)
      DO 6310 I=1,M2
      DO 6310J=1,M2
 6310 B5(I,J)=B5(I,J)*WA3(I)
      DO 429 I=1,M2
      DO 429 J=1,M2
  429 AP(I,J)=WA3(I)*B3(I,J)*PC*EPT(J)/(WA2(J))
      AP(1,3)=WA3(1)*EPT(3)*PC/(R1(3)**2)
      AP(2,2)=-WA3(2)*EPT(3)*PC/(R1(2)**2*2.*DP*RPHI(3))
      AP(2,4)=WA3(2)*EPT(3)*PC/(R1(4)**2*2.*DP*RPHI(3))
      DO 431 I=1,M2
      DO 431 J=1,M2
  431 AP1(I,J)=WA3(I)*B3(I,J)*PC*EPT(I)/(WA2(I))
      DO 432 I=1,M2
      DO 432 J=1,M2
  432 AP2(I,J)=WA3(I)*EPT(I)*DF1(I,J)*COS(PHI(I))
     C/(RPHI(I)*R1(I)**3)
C
C.....BT+B/2*(RTHITA**4) TO FIND ITS EIGENVALUE AND EIGEN\
C.....B*B CONSTRUCTION
C
      DO 25 I=1,M2
      WAB(I)=WA3(I)
   25 WABS(I)=SQRT(WAB(I))
      DO 27 I=1,M2
      DO 27 J=1,M2
   27 BM(I,J)=WABS(I)*WABS(J)*BT1(I,J)
      K=0
      DO 28 J=1,M2
      DO 28 I=1,J
      K=K+1
   28 ST(K)=BM(I,J)
      NA=M2
      MV=0
      CALL EIGEN(ST,AS,NA,MV)
      K=0
      DO 61 J=1,M2
      JS=(J-1)*M2
      DO 61 I=1,M2
      IJ=JS+I
```

```
      61     VB(I,J)=AS(IJ)
             IJ=0
             DO 62 I=1,M2
             IJ=IJ+I
      62     EGNB(I)=ST(IJ)
             DO 7021 I=1,M2
             DO 7021 J=1,M2
      7021   VBT(I,J)=VB(J,I)
             DO 7567 I=1,M2
             DO 7567 J=1,M2
      7567   VBT(I,J)=VBT(I,J)/WABS(J)
             DO 702 I=1,M2
             DO 702 J=1,M2
      702    VB(I,J)=WABS(I)*VB(I,J)
      C
      C.....FIND THE EIGENVALUE AND EIGENVECTOR OF A
             IF(III.GT.1) GO TO 7140
             NA=N
             MV=0
             K=0
             DO 30 J=1,N
             DO 30 I=1,J
             K=K+1
      30     ST(K)=A(I,J)
             CALL EIGEN(ST,AS,NA,MV)
             K=0
             DO 29 J=1,N
             JS=(J-1)*N
             DO 29 I=1,N
             IJ=JS+I
      29     VA(I,J)=AS(IJ)
             IJ=0
             DO 37 I=1,N
             IJ=IJ+I
      37     EGNA(I)=ST(IJ)
             DO 78357 I=1,N
             DO 78357 J=1,N
      78357  VAT(I,J)=VA(J,I)
      C
      C.....FIND THE MEMBRANE FORCES BY UNIFORM PRESSURE
      C....., AXIAL LOAD AND SELFWEIGHT
      C
      7140   IF(III.GT.1) GO TO 17356
      7178   IF(III.EQ.1) GO TO 7171
             IF(III.EQ.2) GO TO 7089
             IF(III.GT.2) GO TO 7099
      7089   EICR=EIG(KK)/MMM
             PRINT,'        INCREMENT OF EIGENVALUE=   ',EICR
             EI(1)=EICR
             DO 7072 I=1,MMM
             I1=I+1
      7072   EI(I1)=EICR+EI(I)
             IF(III.GT.1) GO TO 7099
      7171   BL3=AL**2
             BL4=BL**2
             BL1=BL3+BL4
             BL2=BL4*SQRT(BL1)
             Q1=-QL*BL3/(2.*BL)
             SK=SQRT(1.+(BL/AL)**2)
             FF=SK*COS(PHI(3))
```

```
          FF=2.*FF/(1.-FF**2)+ALCG((1.+FF)/(1.-FF))
          DO 190 I=3,M2
          S1=SK*COS(PHI(I))
          S2=S1**2
          S3=1.-S2
          FD=2.*S1/S3+ALCG((1+S1)/(1.-S1))
          SNPHI(I)=-GL/4.*BL2*SQRT(S3)*(FD-FF)/(BL1-BL3*S2)
          SNTHIT(I)=(-GL*BL3/SQRT(BL1))*S1/SQRT(S3)+
         CSNPHI(I)*(AL/BL)**2*S3
  190     CONTINUE
          WRITE(6,1902)
 1902     FORMAT(5X,'MEMBRANE FORCES DUE TO DEAD LCAD (P/IN)',14X,
         C'NPHI',13X,'NTHITA')
          DO 1901 I=3,M2
          I2=I-2
          WRITE(6,1903)I2,SNPHI(I),SNTHIT(I)
 1903     FORMAT(49X,I3,2X,E13.6,4X,E13.6)
 1901     CONTINUE
          SNPHI(1)=0.
          SNTHIT(1)=0.
          SNPHI(2)=0.
          SNTHIT(2)=0.
          ETO=BL4/((BL3*SIN(PHI(3))**2)-(BL4*CCS(PHI(3))**2))
          DO 191 I=3,M2
          ED=BL4/(BL3*SIN(PHI(I))**2-BL4*COS(PHI(I))**2)
          E1=1+ED
          E2=SQRT(ED)
          ENPHI(I)=Q1*(E2/E1)*(ED-ETO)
          ENTHI(I)=-(R2(I)/RPHI(I))*ENPHI(I)-R2(I)*GL
  191     CONTINUE
          WRITE(6,1912)
 1912     FORMAT(5X,
         C'MEMBRANE FORCES DUE TO EXTERNAL  PRESSURE (P/IN)',
         C5X,'NPHI',13X,'NTHITA')
          DO 1911 I=3,M2
          I2=I-2
          WRITE(6,1913) I2,ENPHI(I),ENTHI(I)
 1913     FORMAT(49X,I3,2X,E13.6,4X,E13.6)
 1911     CONTINUE
          ENPHI(1)=0.
          ENPHI(2)=0.
          ENTHI(2)=0.
          ENTHI(1)=0.
          SNPDA(1)=0.
          SNPDA(2)=0.
          SNTDA(1)=0.
          SNTDA(2)=0.
          DO 1003 I=1,M
          I2=I+2
          SNPDA(I2)=-R1(3)*AXIAL/(R1(I2)*SIN(PHI(I2)))
 1003     SNTDA(I2)=-(R2(I2)/RPHI(I2))*SNPDA(I2)
          WRITE(6,1874)
 1874     FORMAT(5X,'MEMBRANE FORCES DUE TO AXIAL LCAD (P/IN)'
         C,10X,'NPHI',16
         CX,'NTHITA')
          DO 1005 I=1,M
          I2=I+2
          WRITE(6,1875) I,SNPDA(I2),SNTDA(I2)
 1875     FORMAT(49X,I3,2X,E13.6,4X,E13.6)
 1005     CONTINUE
```

```
               DO 1006 I=1,M2
               ENPHI(I)=ENPHI(I)+SNPDA(I)
       1006    ENTHI(I)=ENTHI(I)+SNTDA(I)
               IF(III.EQ.1) GO TO 7172
       7099    IIJ=III-1
               PRINT,'       ASSUMED EIGENVALUE=          ',EI(IIJ)
               DO 1038 I=1,M2
               STR(I,1)=(ENPHI(I)*EI(IIJ)+SNPHI(I))/THICK
       1038    STR(I,2)=(ENTHI(I)*EI(IIJ)+SNTHIT(I))/THICK
               PRINT,' PHI            THITA    ',' STRESS DISTRI
               WRITE(6,9039)((STR(I,J),J=1,2),I=1,M2)
       9039    FORMAT(1X,2E13.6)
               DO 6012 J=1,2
               DO 6012 I=1,M2
       6013    IF(J.EQ.2) GO TO 6014
               IF(STR(I,J).GT.0.) GO TO 6060
               GO TO 6059
       6060    EP(I)=QC1+QC2*STR(I,J)**2
               GO TO 6012
       6059    EP(I)=QC3+QC4*STR(I,J)**2
               GO TO 6012
       6014    IF(STR(I,J).GE.0.) GO TO 6061
               GO TO 6062
       6061    ET(I)=QC1+QC2*STR(I,J)**2
               GO TO 6012
       6062    ET(I)=QC3+QC4*STR(I,J)**2
       6012    CONTINUE
               IF(EIG(KK).LT.EI(IIJ)) GO TO 10000
               DO 6015 I=1,M2
               EP(I)=EP(I)*STIFF
               ET(I)=ET(I)*STIFF
       6015    EPT(I)=SQRT(EP(I)*ET(I))
               GO TO 1506
      C
      C.....ASSUMED BUCKLING MODE
      C
       7172    AM=Z1(M3)**2
               BM1=Z1(M3)**3
               AMP=Z1(1)**2
               BMP=Z1(1)**3
               DET=AM*BMP-AMP*BM1
               BMP=BMP/DET
               AMP=-AMP/DET
               BW=-BMP
               CW=AMP
       17356   DO 193 I=1,M2
       193     X(I,1)=1.+BW*Z1(I)**2+CW*Z1(I)**3
               T=0.
               DO 194 I=2,M2
               T=T+D1
               DO 194 J=1,M2
       194     X(J,I)=X(J,1)*COS(2.*T)
               S=0.
               DO 195 I=1,M2
               DO 195 J=1,N
       195     S=S+X(I,J)**2
               S=SQRT(S)
               DO 1758 I=1,M2
               DO 1758 J=1,N
       1758    X(I,J)=X(I,J)/S
```

```
7830  KK=1
      TR=1.
      FA=1.
      EMAX=1.
      EIGE(1)=20.
65432 EIG(KK)=20.
783   DO 610 I=1,M2
      DO 610 J=1,N
610   QP(I,J)=X(I,J)*ENPHI(I)
      DO 611 I=1,M2
      DO 611 J=1,N
      QP2(I,J)=0.
      DO 611 K=1,M2
611   QP2(I,J)=QP2(I,J)+B3(I,K)*QP(K,J)
      DO 6135 I=1,2
      DO 6135 J=1,N
6135  QP2(I,J)=0.
      DO 6130, I=1,M2
      DO 6130 J=1,N
6130  QP2(I,J)=WA3(I)*QP2(I,J)
6140  DO 614 I=1,M
      DO 614 J=1,N
614   QPT(I,J)=ENTHI(I+2)*X(I+2,J)/WA2(I+2)
      DO 6150 I=1,M
      DO 6150 J=1,N
      QPT1(I,J)=0.
      DO 6150 K=1,N
6150  QPT1(I,J)=QPT1(I,J)+QPT(I,K)*A1(K,J)
6170  DO 617 I=1,M
      DO 617 J=1,N
      I2=I+2
617   QPT2(I2,J)=QPT1(I,J)
      DO 618 I=1,2
      DO 618 J=1,N
618   QPT2(I,J)=0.
      DO 6180 I=3,M2
      DO 6180 J=1,N
6180  QPT2(I,J)=WA3(I)*QPT2(I,J)
      DO 60800 I=1,M2
      DO 60800 J=1,N
60800 CO(I,J)=QP2(I,J)+QPT2(I,J)
62720 DO 600 I=1,M2
      DO 600 J=1,N
600   QD(I,J)=X(I,J)*SNPHI(I)
C     B(M,M+1)*NPHI*W=QD1(M,N)
      DO 601 I=1,M2
      DO 601 J=1,N
      QD2(I,J)=0.
      DO 601 K=1,M2
601   QD2(I,J)=QD2(I,J)+B3(I,K)*QD(K,J)
      DO 2523 I=1,2
      DO 2523 J=1,N
2523  QD2(I,J)=0.
      DO 6030 I=1,M2
      DO 6030 J=1,N
6030  QD2(I,J)=QD2(I,J)*WA3(I)
C     SNTHITA*W
C     QDT2(M+2,N)
60500 DO 604 I=1,M
      DO 604 J=1,N
```

```
604     QDT(I,J)=SNTHIT(I+2)*X(I+2,J)/WA2(I+2)
        DO 60501 I=1,M
        DO 60501 J=1,N
        QDT1(I,J)=0.
        DO 60501 K=1,N
60501   QDT1(I,J)=QDT(I,K)*A1(K,J)+QDT1(I,J)
60700   DO 607 I=1,M
        DO 607 J=1,N
        I2=I+2
607     QDT2(I2,J)=QDT1(I,J)
        DO 608 I=1,2
        DO 608 J=1,N
608     QDT2(I,J)=0.
        DO 6080 I=1,M2
        DO 6080 J=1,N
6080    QDT2(I,J)=QDT2(I,J)*WA3(I)
C       (B-BT)/2*X
        DO 620 I=1,M2
        DO 620 J=1,N
        RRR1(I,J)=0.
        DO 620 K=1,M2
620     RRR1(I,J)=RRR1(I,J)+BT2(I,K)*X(K,J)
C       -BES*X
        DO 632 I=1,M2
        DO 632 J=1,N
        RRR2(I,J)=0.
        DO 632 K=1,M2
632     RRR2(I,J)=RRR2(I,J)+BS(I,K)*X(K,J)
        DO 633 I=1,M2
        DO 633 J=1,N
633     RRR2(I,J)=-RRR2(I,J)
63500   DO 634 I=1,M2
        DO 634 J=1,N
        RR1(I,J)=0.
        DO 634 K=1,M2
634     RR1(I,J)=RR1(I,J)+AP(I,K)*X(K,J)
        DO 63501 I=1,M2
        DO 63501 J=1,N
        RRR3(I,J)=0.
        DO 63501 K=1,N
63501   RRR3(I,J)=RRR3(I,J)+RR1(I,K)*A1(K,J)
64100   DO 641 I=1,M2
        DO 641 J=1,N
641     RRR3(I,J)=-RRR3(I,J)
63800   DO 638 I=1,M2
        DO 638 J=1,N
        RR3(I,J)=0.
        DO 638 K=1,M2
638     RR3(I,J)=RR3(I,J)+AP2(I,K)*X(K,J)
        DO 63801 I=1,M2
        DO 63801 J=1,N
        RRR5(I,J)=0.
        DO 63801 K=1,N
63801   RRR5(I,J)=RRR5(I,J)+RR3(I,K)*A1(K,J)
63900   DO 645 I=1,M2
        DO 645 J=1,N
645     RRR5(I,J)=-RRR5(I,J)
64400   DO 640 I=1,M2
        DO 640 J=1,N
        RR5(I,J)=0.
```

```
      DO 640 K=1,M2
  640 RR5(I,J)=RR5(I,J)+AP1(I,K)*X(K,J)
      DO 64401 I=1,M2
      DO 64401 J=1,N
      RRR7(I,J)=0.
      DO 64401 K=1,N
64401 RRR7(I,J)=RRR7(I,J)+RR5(I,K)*A1(K,J)
64800 DO 648 I=1,M2
      DO 648 J=1,N
  648 RRR7(I,J)=-RRR7(I,J)
      DO 62003 I=1,2
      DO 62003 J=1,N
      XER(I,J)=0.
      DO 62003 K=1,N
62003 XER(I,J)=XER(I,J)+X(I,K)*A(K,J)
      DO 62437 I=3,M2
      DO 62437 J=1,N
62437 XER(I,J)=0.
      DO 649 I=1,M2
      DO 649 J=1,N
  649 TS(I,J)=-RRR1(I,J)+RRR2(I,J)+RRR3(I,J)+RRR5(I,J)
     C +XER(I,J)
      IF(EMAX.LE.0.01) GO TO 760
 7010   DO 701 I=1,M2
      DO 701 J=1,N
  701 C(I,J)=CO(I,J)*EIG(KK)+TS(I,J)+OD2(I,J)+CDT2(I,J)
  785 DO 703 I=1,M2
      DO 703 J=1,N
      D(I,J)=0.
      DO 703 K=1,M2
  703 D(I,J)=D(I,J)+VBT(I,K)*C(K,J)
      DO 704 I=1,M2
      DO 704 J=1,N
      D11(I,J)=0.
      DO 704 K=1,N
  704 D11(I,J)=D11(I,J)+D(I,K)*VA(K,J)
      DO 705 I=1,M2
      DO 705 J=1,N
  705 XR(I,J)=D11(I,J)/(EGNB(I)+EGNA(J))
      DO 706 I=1,M2
      DO 706 J=1,N
      W1(I,J)=0.
      DO 706 K=1,M2
  706 W1(I,J)=W1(I,J)+VB(I,K)*XR(K,J)
      DO 708 I=1,M2
      DO 708 J=1,N
      W(I,J)=0.
      DO 708 K=1,N
  708 W(I,J)=W(I,J)+W1(I,K)*VAT(K,J)
      S=0.
      DO 770 I=1,M2
      DO 770 J=1,N
  770 S=S+W(I,J)**2
      SUM=SQRT(S)
      DO 76150 I=1,M2
      DO 76150 J=1,N
76150 W(I,J)=W(I,J)/SUM
      DO 711  I=1,M2
      DO 711  J=1,N
  711 E(I,J)=(W(I,J)-X(I,J))
```

```
                EMAX=ABS(E(1,1))
                DO 712 I=1,M2
                DO 712 J=1,N
                IF(ABS(E(I,J))-EMAX) 712,713,713
      713       EMAX=ABS(E(I,J))
      712       CONTINUE
      750       DO 751 I=1,M2
                DO 751 J=1,N
      751       X(I,J)=W(I,J)
                TR=TR+1.
                IF(EMAX.LE.0.01) GO TO 783
                GO TO 62720
      760       DO 7708 I=1,M2
                DO 7708 J=1,N
                TP(I,J)=0.
                DO 7708 K=1,M2
      7708      TP(I,J)=TP(I,J)+BT3(I,K)*X(I,J)
                DO 7718 I=1,M2
                DO 7718 J=1,N
                TP1(I,J)=0.
                DO 7718 K=1,N
      7718      TP1(I,J)=TP1(I,J)+X(I,K)*A(K,J)
                DO 772 I=1,M2
                DO 772 J=1,N
      772       TP(I,J)=TP(I,J)+TP1(I,J)
                DO 781 I=1,M2
                DO 781 J=1,N
                T3(I,J)=TP(I,J)-TS(I,J)-GD2(I,J)-QDT2(I,J)
      781       FE(I,J)=CO(I,J)
                SUM=0.
                DO 782 I=1,M2
                DO 782 J=1,N
      782       SUM=SUM+T3(I,J)**2
                SUM=SQRT(SUM)
      786       SUM1=0.
                DO 784 I=1,M2
                DO 784 J=1,N
      784       SUM1=SUM1+FE(I,J)**2
                SUM1=SQRT(SUM1)
                KK=KK+1
                EIG(KK)=SUM/SUM1
                EIGE(KK)=EIG(KK)
                ERROR=ABS((EIGE(KK-1)-EIGE(KK))/EIGE(KK-1))
                IF(ABS(ERROR).GT.0.15) GO TO 790
      7510      WRITE(6,7035)EMAX
      7035      FORMAT('1','ERROR BETWEEN TWO ITERATION MODE=',E13.6)
                WRITE(6,7036)
      7036      FORMAT('0',25X,'BUCKLING MODE OF SHELL OF SELECTED NC
                DO 15032 I=3,M2
                I2=I-2
                PRINT,'RING NUMBER = ',I2
      15032     PRINT 150,(W(I,J),J=1,N)
                WRITE(6,7037)EIG(KK)
      7037      FORMAT('0','BUCKLING LOAD=',E13.6,' (PSI)')
                III=III+1
                IF(III-MMM) 7178,7178,10000
      790       EMAX=1.
                TR=1.
                GO TO 783
      10000     STOP
```

```
      END
      SUBROUTINE EIGEN (ST,AS,N,MV)
      DOUBLE PRECISION A,R,ANORM,ANRMX,THR,X,Y,SINX,SINX2,
     1                    COSX2,SINCS,RANGE,FLOAT
      DIMENSION A(140),R(300),ST(140),AS(300)
      FLOAT(N)=N
      NCC=N*(N+1)/2
      DO 3 I=1,NCC
    3 A(I)=ST(I)
    5 RANGE=1.0D-12
      IF(MV-1) 10,25,10
   10 IQ=-N
      DO 20 J=1,N
      IQ=IQ+N
      DO 20 I=1,N
      IJ=IQ+I
      R(IJ)=0.0
      IF(I-J) 20,15,20
   15 R(IJ)=1.0
   20 CONTINUE
C
C        COMPUTE INITIAL AND FINAL NORMS (ANORM AND ANORMX
C
   25 ANORM=0.0
      DO 35 I=1,N
      DO 35 J=I,N
      IF(I-J) 30,35,30
   30 IA=I+(J*J-J)/2
      ANORM=ANORM+A(IA)*A(IA)
   35 CONTINUE
      IF(ANORM) 165,165,40
   40 ANORM=1.414*DSQRT(ANORM)
      ANRMX=ANORM*RANGE/FLOAT(N)
C
C        INITIALIZE INDICATORS AND COMPUTE THRESHOLD, THR
C
      IND=0
      THR=ANORM
   45 THR=THR/FLOAT(N)
   50 L=1
   55 M=L+1
C
C        COMPUTE SIN AND COS
C
   60 MQ=(M*M-M)/2
      LQ=(L*L-L)/2
      LM=L+MQ
   62 IF(DABS(A(LM))-THR) 130,65,65
   65 IND=1
      LL=L+LQ
      MM=M+MQ
      X=0.5*(A(LL)-A(MM))
   68 Y=-A(LM)/DSQRT(A(LM)*A(LM)+X*X)
      IF(X) 70,75,75
   70 Y=-Y
   75 SINX=Y/DSQRT(2.0*(1.0+(DSQRT(1.0-Y*Y))))
      SINX2=SINX*SINX
   78 COSX=DSQRT(1.0-SINX2)
      COSX2=COSX*COSX
```

```
      SINCS =SINX*COSX
C
C        ROTATE L AND M COLUMNS
C
      ILQ=N*(L-1)
      IMQ=N*(M-1)
      DO 125 I=1,N
      IQ=(I*I-I)/2
      IF(I-L) 80,115,80
   80 IF(I-M) 85,115,90
   85 IM=I+MQ
      GO TO 95
   90 IM=M+IQ
   95 IF(I-L) 100,105,105
  100 IL=I+LQ
      GO TO 110
  105 IL=L+IQ
  110 X=A(IL)*COSX-A(IM)*SINX
      A(IM)=A(IL)*SINX+A(IM)*COSX
      A(IL)=X
  115 IF(MV-1) 120,125,120
  120 ILR=ILQ+I
      IMR=IMQ+I
      X=R(ILR)*COSX-R(IMR)*SINX
      R(IMR)=R(ILR)*SINX+R(IMR)*COSX
      R(ILR)=X
  125 CONTINUE
      X=2.0*A(LM)*SINCS
      Y=A(LL)*COSX2+A(MM)*SINX2-X
      X=A(LL)*SINX2+A(MM)*COSX2+X
      A(LM)=(A(LL)-A(MM))*SINCS+A(LM)*(COSX2-SINX2)
      A(LL)=Y
      A(MM)=X
C
C        TESTS FOR COMPLETION
C
C        TEST FOR M = LAST COLUMN
C
  130 IF(M-N) 135,140,135
  135 M=M+1
      GO TO 60
C
C        TEST FOR L = SECOND FROM LAST COLUMN
C
  140 IF(L-(N-1)) 145,150,145
  145 L=L+1
      GO TO 55
  150 IF(IND-1) 160,155,160
  155 IND=0
      GO TO 50
C
C        COMPARE THRESHOLD WITH FINAL NORM
C
  160 IF(THR-ANRMX) 165,165,45
C
C        SORT EIGENVALUES AND EIGENVECTORS
C
  165 IQ=-N
      DO 185 I=1,N
      IQ=IQ+N
```

```
      LL=I+(I*I-I)/2
      JQ=N*(I-2)
      DO 185 J=I,N
      JQ=JQ+N
      MM=J+(J*J-J)/2
      IF(A(LL)-A(MM)) 170,185,185
  170 X=A(LL)
      A(LL)=A(MM)
      A(MM)=X
      IF(MV-1) 175,185,175
  175 DO 180 K=1,N
      ILR=IQ+K
      IMR=JQ+K
      X=R(ILR)
      R(ILR)=R(IMR)
  180 R(IMR)=X
  185 CONTINUE
      DO 192 I=1,NCC
  192 ST(I)=A(I)
      NN2=N*N
      DO 181 I=1,NN2
  181 AS(I)=R(I)
      RETURN
       END
```

$ENTRY

APPENDIX B

```
Z AT TOP=    -0.3600000E 02 ZAT BOTTOM=     0.1080000E 03 Q=     0.1000000E 01 G=     0.4340000E-01
POISSON VALUE=     0.1700000E 00 THICKNESS=     0.5000000E 00 YOUNGS MODULUS=     0.3600000E 07 A=     0.3600000E 02
B=     0.9064000E 02 RT=     0.3870000E 02 RB=     0.5600000E 02 HT=     0.1080000E 03
FLEXTURAL RIGIDITY=     0.3861600E 05 UNIT AXIAL LOAD=     0.1083600E 02
```

POINT	PHI	RPHI	RTHETA	Z
1	0.171637E 01	-0.293502E 03	0.387355E 02	-0.359999E 02
2	0.168487E 01	-0.265115E 03	0.375984E 02	-0.273082E 02
3	0.165338E 01	-0.246440E 03	0.368082E 02	-0.193137E 02
4	0.162189E 01	-0.234924E 03	0.363021E 02	-0.117677E 02
5	0.159039E 01	-0.229180E 03	0.360439E 02	-0.447851E 01
6	0.155890E 01	-0.228567E 03	0.360161E 02	0.271596E 01
7	0.152741E 01	-0.233021E 03	0.362170E 02	0.996766E 01
8	0.149591E 01	-0.243031E 03	0.366599E 02	0.174346E 02
9	0.146442E 01	-0.259774E 03	0.373760E 02	0.252993E 02
10	0.143293E 01	-0.285460E 03	0.384206E 02	0.337929E 02
11	0.140143E 01	-0.324091E 03	0.398860E 02	0.432363E 02
12	0.136994E 01	-0.383177E 03	0.419279E 02	0.541146E 02
13	0.133845E 01	-0.477987E 03	0.448227E 02	0.672336E 02
14	0.130696E 01	-0.643839E 03	0.491082E 02	0.840959E 02
15	0.127546E 01	-0.980981E 03	0.559999E 02	0.108001E 03

```
ERROR BETWEEN TWO ITERATION MODE= 0.543982E-04

                         BUCKLING MODE OF SHELL OF SELECTED NODAL POINTS
RING NUMBER =             1
-0.973284E-04-0.702273E-04-0.266554E-04 0.209258E-04 0.544643E-04 0.643758E-04 0.536298E-04 0.346755E-04
 0.212122E-04 0.197679E-04 0.260854E-04 0.283886E-04 0.155125E-04-0.150609E-04-0.541995E-04-0.851295E-04
RING NUMBER =             2
 0.163533E-03 0.107133E-03-0.459302E-04-0.192076E-03-0.228779E-03-0.117197E-03 0.924306E-04 0.287914E-03
 0.359215E-03 0.262227E-03 0.449657E-04-0.179216E-03-0.295912E-03-0.254105E-03-0.934095E-04 0.814344E-04
RING NUMBER =             3
 0.113837E-02 0.791484E-03-0.727473E-04-0.926027E-03-0.124172E-02-0.807422E-03 0.146415E-03 0.107785E-02
 0.144835E-02 0.103717E-02 0.712390E-04-0.905657E-03-0.134807E-02-0.102430E-02-0.147976E-03 0.750765E-03
RING NUMBER =             4
 0.370545E-02 0.260179E-02-0.992446E-04-0.278534E-02-0.384647E-02-0.262354E-02 0.199751E-03 0.299249E-02
 0.412839E-02 0.293700E-02 0.972108E-04-0.275756E-02-0.399157E-02-0.291946E-02-0.201924E-03 0.254621E-02
RING NUMBER =             5
 0.822493E-02 0.579264E-02-0.125848E-03-0.602541E-02-0.840380E-02-0.582024E-02 0.253305E-03 0.628811E-02
 0.876134E-02 0.621778E-02 0.123310E-03-0.599017E-02-0.858784E-02-0.619555E-02-0.256136E-03 0.572210E-02
RING NUMBER =             6
 0.151780E-01 0.107041E-01-0.153133E-03-0.109874E-01-0.153957E-01-0.107378E-01 0.308237E-03 0.113071E-01
 0.158308E-01 0.112216E-01 0.150104E-03-0.109445E-01-0.156197E-01-0.111945E-01-0.311782E-03 0.106182E-01
RING NUMBER =             7
 0.252420E-01 0.178153E-01-0.181668E-03-0.181514E-01-0.255004E-01-0.178552E-01 0.365709E-03 0.185307E-01
 0.260168E-01 0.184293E-01 0.178180E-03-0.181005E-01-0.257662E-01-0.183973E-01-0.370073E-03 0.177132E-01
RING NUMBER =             8
 0.393516E-01 0.277867E-01-0.211951E-03-0.281788E-01-0.396531E-01-0.278333E-01 0.426714E-03 0.286216E-01
 0.402557E-01 0.285033E-01 0.208006E-03-0.281195E-01-0.399633E-01-0.284659E-01-0.431982E-03 0.276675E-01
RING NUMBER =             9
 0.587257E-01 0.414803E-01-0.244153E-03-0.419321E-01-0.590732E-01-0.415341E-01 0.491614E-03 0.424422E-01
 0.597676E-01 0.423062E-01 0.239795E-03-0.418637E-01-0.594308E-01-0.422631E-01-0.497930E-03 0.413428E-01
RING NUMBER =             10
 0.847655E-01 0.598872E-01-0.277736E-03-0.604011E-01-0.851610E-01-0.599483E-01 0.559339E-03 0.609817E-01
 0.855512E-01 0.608270E-01 0.273040E-03-0.603232E-01-0.855679E-01-0.607780E-01-0.566854E-03 0.597305E-01
RING NUMBER =             11
 0.118496E 00 0.837324E-01-0.310411E-03-0.843069E-01-0.118939E 00-0.838009E-01 0.625289E-03 0.849562E-01
 0.119822E 00 0.847834E-01 0.305523E-03-0.842199E-01-0.119394E 00-0.847288E-01-0.634129E-03 0.835570E-01
RING NUMBER =             12
 0.158523E 00 0.112031E 00-0.335602E-03-0.112652E 00-0.159001E 00-0.112105E 00 0.676206E-03 0.113354E 00
 0.159957E 00 0.113168E 00 0.330762E-03-0.112558E 00-0.159493E 00-0.113109E 00-0.686308E-03 0.111841E 00
RING NUMBER =             13
 0.194208E 00 0.137264E 00-0.335766E-03-0.137886E 00-0.194687E 00-0.137339E 00 0.676754E-03 0.138589E 00
 0.195645E 00 0.138403E 00 0.331460E-03-0.137792E 00-0.195180E 00-0.138344E 00-0.687481E-03 0.137074E 00
RING NUMBER =             14
 0.184020E 00 0.130073E 00-0.264459E-03-0.130563E 00-0.184397E 00-0.130132E 00 0.533179E-03 0.131117E 00
 0.185152E 00 0.130971E 00 0.261470E-03-0.130489E 00-0.184786E 00-0.130924E 00-0.542132E-03 0.129922E 00

BUCKLING LOAD= 0.375834E 01 (PSI)
```

Table B-1 The Buckling Mode Shape Function Values at (16x16) Grid Points for the Prototype Model

POINT	PHI	RPHI	RTHITA	Z
1	0.171637E 01	-0.293502E 03	0.387355E 02	-0.359999E 02
2	0.169187E 01	-0.270462E 03	0.378182E 02	-0.291690E 02
3	0.166738E 01	-0.253738E 03	0.371212E 02	-0.227982E 02
4	0.164288E 01	-0.241892E 03	0.366101E 02	-0.167585E 02
5	0.161839E 01	-0.234017E 03	0.362616E 02	-0.109478E 02
6	0.159389E 01	-0.229558E 03	0.360610E 02	-0.528044E 01
7	0.156940E 01	-0.228216E 03	0.360002E 02	0.319497E 00
8	0.154490E 01	-0.229906E 03	0.360768E 02	0.592349E 01
9	0.152041E 01	-0.234737E 03	0.362937E 02	0.116033E 02
10	0.149591E 01	-0.243032E 03	0.366600E 02	0.174356E 02
11	0.147142E 01	-0.255382E 03	0.371910E 02	0.235075E 02
12	0.144692E 01	-0.272742E 03	0.379111E 02	0.299238E 02
13	0.142243E 01	-0.296630E 03	0.388565E 02	0.368171E 02
14	0.139793E 01	-0.329466E 03	0.400810E 02	0.443648E 02
15	0.137344E 01	-0.375244E 03	0.416659E 02	0.528162E 02
16	0.134894E 01	-0.440911E 03	0.437384E 02	0.625428E 02
17	0.132444E 01	-0.539465E 03	0.465082E 02	0.741364E 02
18	0.129995E 01	-0.697878E 03	0.503486E 02	0.886238E 02
19	0.127545E 01	-0.981103E 03	0.560021E 02	0.108008E 03

```
ERROR BETWEEN TWO ITERATION MODE= 0.173792E-04

                         BUCKLING MODE OF SHELL OF SELECTED NODAL POINTS
RING NUMBER =              1
-0.100849E-03-0.826302E-04-0.368496E-04 0.199670E-04 0.673718E-04 0.886812E-04 0.772091E-04 0.386784E-04
-0.110961E-04-0.523554E-04-0.690169E-04-0.548490E-04-0.158379E-04 0.321537E-04 0.695408E-04 0.806193E-04
 0.597044E-04 0.134436E-04-0.415905E-04-0.851231E-04
RING NUMBER =              2
-0.763482E-04-0.635692E-04-0.334338E-04 0.421097E-05 0.371572E-04 0.552857E-04 0.541873E-04 0.366034E-04
 0.111502E-04-0.111569E-04-0.212386E-04-0.154729E-04 0.294232E-05 0.253088E-04 0.409120E-04 0.413281E-04
 0.238820E-04-0.708426E-05-0.416431E-04-0.678858E-04
RING NUMBER =              3
 0.250036E-03 0.199419E-03 0.616108E-04-0.108101E-03-0.241440E-03-0.283551E-03-0.214347E-03-0.565684E-04
 0.132543E-03 0.282811E-03 0.337726E-03 0.275949E-03 0.119493E-03-0.745302E-04-0.235465E-03-0.305760E-03
-0.262568E-03-0.126083E-03 0.485403E-04 0.192545E-03
RING NUMBER =              4
 0.120201E-02 0.968517E-03 0.349933E-03-0.413818E-03-0.102626E-02-0.124808E-02-0.989040E-03-0.343011E-03
 0.447401E-03 0.108311E-02 0.132253E-02 0.107369E-02 0.429483E-03-0.367679E-03-0.101805E-02-0.127860E-02
-0.105531E-02-0.438542E-03 0.331958E-03 0.959063E-03
RING NUMBER =              5
 0.289967E-02 0.234087E-02 0.868636E-03-0.950054E-03-0.241447E-02-0.295839E-02-0.236704E-02-0.859826E-03
 0.992848E-03 0.248693E-02 0.305328E-02 0.247494E-02 0.970038E-03-0.891240E-03-0.240400E-02-0.299730E-02
-0.245151E-02-0.981590E-03 0.845704E-03 0.232881E-02
RING NUMBER =              6
 0.548067E-02 0.442786E-02 0.166021E-02-0.175945E-02-0.451758E-02-0.555228E-02-0.445978E-02-0.164949E-02
 0.181161E-02 0.460589E-02 0.566794E-02 0.459130E-02 0.178384E-02-0.168775E-02-0.450481E-02-0.559970E-02
-0.456275E-02-0.179793E-02 0.163223E-02 0.441314E-02
RING NUMBER =              7
 0.912340E-02 0.737377E-02 0.277972E-02-0.289728E-02-0.748007E-02-0.920826E-02-0.741161E-02-0.276703E-02
 0.295906E-02 0.758470E-02 0.934530E-02 0.756744E-02 0.292621E-02-0.281231E-02-0.746494E-02-0.926445E-02
-0.753362E-02-0.294291E-02 0.274651E-02 0.735630E-02
RING NUMBER =              8
 0.140647E-01 0.113702E-01 0.430026E-02-0.443688E-02-0.114938E-01-0.141634E-01-0.114142E-01-0.428553E-02
 0.450867E-02 0.116154E-01 0.143226E-01 0.115954E-01 0.447055E-02-0.433809E-02-0.114762E-01-0.142287E-01
-0.115560E-01-0.448998E-02 0.426160E-02 0.113499E-01
RING NUMBER =              9
 0.206172E-01 0.166701E-01 0.631838E-02-0.647504E-02-0.168118E-01-0.207304E-01-0.167206E-01-0.630151E-02
 0.655736E-02 0.169513E-01 0.209131E-01 0.169284E-01 0.651373E-02-0.636172E-02-0.167916E-01-0.208053E-01
-0.168833E-01-0.653604E-02 0.627396E-02 0.166467E-01
RING NUMBER =             10
 0.291859E-01 0.236010E-01 0.895912E-02-0.913701E-02-0.237619E-01-0.293145E-01-0.236585E-01-0.894000E-02
 0.923049E-02 0.239204E-01 0.295220E-01 0.238944E-01 0.918105E-02-0.900827E-02-0.237390E-01-0.293996E-01
-0.238432E-01-0.920640E-02 0.890857E-02 0.235744E-01
RING NUMBER =             11
 0.402775E-01 0.325730E-01 0.123791E-01-0.125795E-01-0.327543E-01-0.404225E-01-0.326377E-01-0.123575E-01
 0.126848E-01 0.329328E-01 0.406564E-01 0.329037E-01 0.126292E-01-0.124344E-01-0.327285E-01-0.405184E-01
-0.328460E-01-0.126578E-01 0.123220E-01 0.325429E-01
RING NUMBER =             12
 0.544884E-01 0.440685E-01 0.167625E-01-0.169866E-01-0.442713E-01-0.546507E-01-0.441409E-01-0.167385E-01
 0.171044E-01 0.444710E-01 0.549123E-01 0.444385E-01 0.170425E-01-0.168242E-01-0.442423E-01-0.547580E-01
-0.443740E-01-0.170745E-01 0.166985E-01 0.440347E-01
RING NUMBER =             13
 0.724265E-01 0.585793E-01 0.222975E-01-0.225460E-01-0.588043E-01-0.726066E-01-0.586598E-01-0.222709E-01
 0.226767E-01 0.590258E-01 0.728969E-01 0.589898E-01 0.226081E-01-0.223658E-01-0.587721E-01-0.727257E-01
-0.589183E-01-0.226437E-01 0.222262E-01 0.585417E-01
RING NUMBER =             14
 0.944568E-01 0.764007E-01 0.290973E-01-0.293695E-01-0.766473E-01-0.946543E-01-0.764890E-01-0.290683E-01
 0.295127E-01 0.768900E-01 0.949724E-01 0.768508E-01 0.294378E-01-0.291719E-01-0.766120E-01-0.947848E-01
-0.767725E-01-0.294768E-01 0.290189E-01 0.763593E-01
RING NUMBER =             15
```

```
 0.119992E 00 0.970577E-01 0.369814E-01-0.372734E-01-0.973223E-01-0.120204E 00-0.971526E-01-0.369503E-01
 0.374270E-01 0.975828E-01 0.120545E 00 0.975410E-01 0.373471E-01-0.370613E-01-0.972844E-01-0.120344E 00
-0.974569E-01-0.373890E-01 0.368969E-01 0.970131E-01
RING NUMBER =                16
-0.145611E 00 0.117784E 00 0.448952E-01-0.451961E-01-0.118056E 00-0.145830E 00-0.117882E 00-0.448632E-01
 0.453544E-01 0.118325E 00 0.146182E 00 0.118282E 00 0.452724E-01-0.449773E-01-0.118017E 00-0.145974E 00
-0.118195E 00-0.453157E-01 0.448077E-01 0.117737E 00
RING NUMBER =                17
 0.160256E 00 0.129633E 00 0.494269E-01-0.497104E-01-0.129890E 00-0.160463E 00-0.129726E 00-0.493968E-01
 0.498596E-01 0.130144E 00 0.160795E 00 0.130104E 00 0.497827E-01-0.495039E-01-0.129854E 00-0.160599E 00
-0.130022E 00-0.498236E-01 0.493440E-01 0.129589E 00
RING NUMBER =                18
-0.133876E 00-0.108296E 00 0.413009E-01-0.415072E-01-0.108483E 00-0.134027E 00-0.108363E 00-0.412790E-01
 0.416158E-01 0.108668E 00 0.134269E 00 0.108639E 00 0.415602E-01-0.413567E-01-0.108456E 00-0.134126E 00
-0.108579E 00-0.415900E-01 0.412402E-01 0.108264E 00

BUCKLING LOAD= 0.364977E 01 (PSI)

CORE USAGE        OBJECT CODE=    42232 BYTES,ARRAY AREA=  111824 BYTES,TOTAL AREA AVAILABLE=  159840  BYTES

DIAGNOSTICS        NUMBER OF ERRORS=      0, NUMBER OF WARNINGS=       1, NUMBER OF EXTENSIONS=       1

COMPILE TIME=     3.24 SEC,EXECUTION TIME=   305.28 SEC.      0.13.45   SATURDAY     18 APR 81     WATFIV -
```

Table B-2 The Buckling Mode Shape Function Values at (20x20) Grid Points

for the Prototype Model

POINT	PHI	RPHI	RTHITA	Z
1	0.171637E 01	-0.293502E 03	0.387355E 02	-0.359999E 02
2	0.170062E 01	-0.277869E 03	0.381182E 02	-0.315479E 02
3	0.168487E 01	-0.265114E 03	0.375983E 02	-0.273080E 02
4	0.166913E 01	-0.254757E 03	0.371646E 02	-0.232408E 02
5	0.165338E 01	-0.246439E 03	0.368081E 02	-0.193132E 02
6	0.163763E 01	-0.239894E 03	0.365224E 02	-0.154970E 02
7	0.162188E 01	-0.234923E 03	0.363021E 02	-0.117670E 02
8	0.160614E 01	-0.231384E 03	0.361435E 02	-0.810080E 01
9	0.159039E 01	-0.229179E 03	0.360439E 02	-0.447764E 01
10	0.157464E 01	-0.228249E 03	0.360016E 02	-0.878002E 00
11	0.155890E 01	-0.228568E 03	0.360161E 02	0.271705E 01
12	0.154315E 01	-0.230145E 03	0.360875E 02	0.632632E 01
13	0.152740E 01	-0.233022E 03	0.362170E 02	0.996900E 01
14	0.151165E 01	-0.237279E 03	0.364068E 02	0.136651E 02
15	0.149591E 01	-0.243033E 03	0.366600E 02	0.174363E 02
16	0.148016E 01	-0.250457E 03	0.369812E 02	0.213061E 02
17	0.146441E 01	-0.259779E 03	0.373762E 02	0.253013E 02
18	0.144867E 01	-0.271311E 03	0.378528E 02	0.294523E 02
19	0.143292E 01	-0.285468E 03	0.384209E 02	0.337954E 02
20	0.141717E 01	-0.302812E 03	0.390933E 02	0.383734E 02
21	0.140143E 01	-0.324105E 03	0.398865E 02	0.432394E 02
22	0.138568E 01	-0.350404E 03	0.408221E 02	0.484595E 02
23	0.136993E 01	-0.383202E 03	0.419287E 02	0.541186E 02
24	0.135418E 01	-0.424663E 03	0.432449E 02	0.603281E 02
25	0.133844E 01	-0.478031E 03	0.448240E 02	0.672389E 02
26	0.132269E 01	-0.548362E 03	0.467418E 02	0.750618E 02
27	0.130694E 01	-0.643928E 03	0.491103E 02	0.841036E 02
28	0.129120E 01	-0.779139E 03	0.521025E 02	0.948330E 02
29	0.127545E 01	-0.981194E 03	0.560037E 02	0.108013E 03

```
ERROR BETWEEN TWO ITERATION MODE= 0.259366E-04

                    BUCKLING MODE OF SHELL OF SELECTED NODAL POINTS
RING NUMBER =            1
 0.225244E-04 0.206491E-04 0.150529E-04 0.673455E-05-0.283153E-05-0.119517E-04-0.190072E-04-0.227365E-04
-0.224544E-04-0.181733E-04-0.106019E-04-0.102441E-05 0.892034E-05 0.175220E-04 0.232935E-04 0.252286E-04
 0.229760E-04 0.169007E-04 0.802237E-05-0.216011E-05-0.119259E-04-0.196283E-04-0.239769E-04-0.242600E-04
-0.204650E-04-0.132797E-04-0.397127E-05 0.583306E-05 0.144291E-04 0.203302E-04
RING NUMBER =            2
 0.496934E-04 0.455212E-04 0.332191E-04 0.149676E-04-0.601533E-05-0.260331E-04-0.415530E-04-0.498195E-04
-0.493343E-04-0.401181E-04-0.237105E-04-0.290568E-05 0.187289E-04 0.374679E-04 0.500720E-04 0.543473E-04
 0.495257E-04 0.363991E-04 0.171838E-04-0.485987E-05-0.259889E-04-0.426218E-04-0.519543E-04-0.524415E-04
-0.440620E-04-0.283188E-04-0.797708E-05 0.134159E-04 0.321453E-04 0.449723E-04
RING NUMBER =            3
 0.161265E-03 0.147520E-03 0.107857E-03 0.492188E-04-0.181568E-04-0.825118E-04-0.132605E-03-0.159661E-03
-0.158893E-03-0.130333E-03-0.788330E-04-0.132312E-04 0.551770E-04 0.114588E-03 0.154730E-03 0.168639E-03
 0.153864E-03 0.112895E-03 0.527298E-04-0.163264E-04-0.824420E-04-0.134299E-03-0.163044E-03-0.163816E-03
-0.136581E-03-0.861343E-04-0.212662E-04 0.467590E-04 0.106155E-03 0.146650E-03
RING NUMBER =            4
 0.397911E-03 0.363783E-03 0.266186E-03 0.122112E-03-0.433909E-04-0.201555E-03-0.324876E-03-0.391872E-03
-0.390809E-03-0.321732E-03-0.196466E-03-0.365764E-04 0.130355E-03 0.275498E-03 0.373759E-03 0.408114E-03
 0.372563E-03 0.273157E-03 0.126970E-03-0.408584E-04-0.201459E-03-0.327220E-03-0.396553E-03-0.397622E-03
-0.330380E-03-0.206570E-03-0.476959E-04 0.118706E-03 0.263829E-03 0.362578E-03
RING NUMBER =            5
 0.781580E-03 0.714360E-03 0.522892E-03 0.240432E-03-0.840054E-04-0.394128E-03-0.636111E-03-0.767913E-03
-0.766548E-03-0.632073E-03-0.387591E-03-0.752526E-04 0.251020E-03 0.534855E-03 0.727177E-03 0.794689E-03
 0.725640E-03 0.531848E-03 0.246673E-03-0.807518E-04-0.394005E-03-0.639124E-03-0.773928E-03-0.775302E-03
-0.643185E-03-0.400574E-03-0.895407E-04 0.236052E-03 0.519860E-03 0.712810E-03
RING NUMBER =            6
 0.133225E-02 0.121750E-02 0.891341E-03 0.410351E-03-0.142086E-03-0.670214E-03-0.108247E-02-0.130732E-02
-0.130564E-02-0.107752E-02-0.662200E-03-0.131355E-03 0.423334E-03 0.906010E-03 0.123322E-02 0.134832E-02
 0.123133E-02 0.902325E-03 0.418006E-03-0.138096E-03-0.670063E-03-0.108616E-02-0.131469E-02-0.131638E-02
-0.109115E-02-0.678125E-03-0.148879E-03 0.404975E-03 0.887619E-03 0.121560E-02
RING NUMBER =            7
 0.207082E-02 0.189231E-02 0.138553E-02 0.638337E-03-0.219817E-03-0.104027E-02-0.168087E-02-0.203054E-02
-0.202855E-02-0.167499E-02-0.103075E-02-0.207072E-03 0.653758E-03 0.140296E-02 0.191098E-02 0.208993E-02
 0.190874E-02 0.139858E-02 0.647432E-03-0.215077E-03-0.104009E-02-0.168526E-02-0.203931E-02-0.204131E-02
-0.169118E-02-0.104968E-02-0.227897E-03 0.631942E-03 0.138110E-02 0.189004E-02
RING NUMBER =            8
 0.302100E-02 0.276042E-02 0.202131E-02 0.931705E-03-0.319677E-03-0.151614E-02-0.245047E-02-0.296076E-02
-0.295845E-02-0.244364E-02-0.150509E-02-0.304876E-03 0.949615E-03 0.204155E-02 0.278211E-02 0.304320E-02
 0.277952E-02 0.203647E-02 0.942271E-03-0.314170E-03-0.151594E-02-0.245557E-02-0.297094E-02-0.297328E-02
-0.246246E-02-0.152708E-02-0.329071E-03 0.924268E-03 0.201615E-02 0.275779E-02
RING NUMBER =            9
 0.421088E-02 0.384752E-02 0.281747E-02 0.129914E-02-0.444603E-03-0.211188E-02-0.341401E-02-0.412565E-02
-0.412282E-02-0.340622E-02-0.209926E-02-0.427700E-03 0.131960E-02 0.284059E-02 0.387230E-02 0.423624E-02
 0.386934E-02 0.283480E-02 0.131121E-02-0.438312E-03-0.211164E-02-0.341984E-02-0.413709E-02-0.413976E-02
-0.342772E-02-0.212438E-02-0.455346E-03 0.129063E-02 0.281158E-02 0.384450E-02
RING NUMBER =           10
 0.567427E-02 0.518448E-02 0.379666E-02 0.175109E-02-0.598134E-03-0.284440E-02-0.459886E-02-0.555771E-02
-0.555474E-02-0.459007E-02-0.283017E-02-0.579074E-03 0.177416E-02 0.382274E-02 0.521244E-02 0.570289E-02
 0.520910E-02 0.381621E-02 0.176471E-02-0.591036E-03-0.284413E-02-0.460544E-02-0.557085E-02-0.557386E-02
-0.461433E-02-0.285851E-02-0.610268E-03 0.174148E-02 0.379000E-02 0.518108E-02
RING NUMBER =           11
 0.745197E-02 0.680858E-02 0.498616E-02 0.230016E-02-0.784537E-03-0.373410E-02-0.603803E-02-0.729743E-02
-0.729411E-02-0.602821E-02-0.371821E-02-0.763251E-03 0.232593E-02 0.501529E-02 0.683982E-02 0.748394E-02
 0.683609E-02 0.500800E-02 0.231538E-02-0.776606E-03-0.373380E-02-0.604538E-02-0.731211E-02-0.731547E-02
-0.605532E-02-0.374988E-02-0.798107E-03 0.228941E-02 0.497871E-02 0.680478E-02
RING NUMBER =           12
 0.959319E-02 0.876478E-02 0.641889E-02 0.296156E-02-0.100896E-02-0.480558E-02-0.777129E-02-0.939276E-02
```

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-0.938908E-02-0.776041E-02-0.478797E-02-0.985368E-03 0.299011E-02 0.645118E-02 0.879941E-02 0.962864E-02
 0.876529E-02 0.644311E-02 0.297843E-02-0.100016E-02-0.480525E-02-0.777943E-02-0.940904E-02-0.941277E-02
-0.779048E-02-0.482310E-02-0.102402E-02 0.294962E-02 0.641062E-02 0.876055E-02
RING NUMBER =         13
 0.121568E-01 0.111069E-01 0.813427E-02 0.375347E-02-0.127755E-02-0.608830E-02-0.984634E-02-0.119013E-01
-0.118972E-01-0.983436E-02-0.606890E-02-0.125157E-02 0.378492E-02 0.816984E-02 0.111450E-01 0.121959E-01
 0.111405E-01 0.816096E-02 0.377207E-02-0.126786E-02-0.608794E-02-0.985532E-02-0.119192E-01-0.119233E-01
-0.986750E-02-0.610762E-02-0.129417E-02 0.374030E-02 0.812514E-02 0.111022E-01
RING NUMBER =         14
 0.152130E-01 0.138989E-01 0.101792E-01 0.469758E-02-0.159766E-02-0.761732E-02-0.123199E-01-0.148916E-01
-0.148872E-01-0.123068E-01-0.759607E-02-0.156918E-02 0.473205E-02 0.102182E-01 0.139407E-01 0.152558E-01
 0.135358E-01 0.102085E-01 0.471798E-02-0.158702E-02-0.761691E-02-0.123298E-01-0.149113E-01-0.149158E-01
-0.123431E-01-0.763853E-02-0.161591E-02 0.468311E-02 0.101692E-01 0.138938E-01
RING NUMBER =         15
 0.188444E-01 0.172165E-01 0.126091E-01 0.581944E-02-0.197792E-02-0.943403E-02-0.152589E-01-0.184448E-01
-0.184399E-01-0.152446E-01-0.941084E-02-0.194685E-02 0.585708E-02 0.126517E-01 0.172672E-01 0.188912E-01
 0.172568E-01 0.126411E-01 0.584172E-02-0.196631E-02-0.943360E-02-0.152697E-01-0.184663E-01-0.184712E-01
-0.152843E-01-0.945722E-02-0.199788E-02 0.580361E-02 0.125982E-01 0.172109E-01
RING NUMBER =         16
 0.231471E-01 0.211474E-01 0.154882E-01 0.714872E-02-0.242839E-02-0.115864E-01-0.187411E-01-0.226545E-01
-0.226493E-01-0.187255E-01-0.115612E-01-0.239459E-02 0.718966E-02 0.155345E-01 0.211971E-01 0.231981E-01
 0.211912E-01 0.155230E-01 0.717298E-02-0.241574E-02-0.115859E-01-0.187528E-01-0.226779E-01-0.226833E-01
-0.187688E-01-0.116117E-01-0.245014E-02 0.713147E-02 0.154762E-01 0.211413E-01
RING NUMBER =         17
 0.282301E-01 0.257910E-01 0.188893E-01 0.871910E-02-0.296043E-02-0.141290E-01-0.228545E-01-0.276275E-01
-0.276218E-01-0.228376E-01-0.141016E-01-0.292380E-02 0.876349E-02 0.189396E-01 0.258449E-01 0.282854E-01
 0.258386E-01 0.189271E-01 0.874542E-02-0.294671E-02-0.141284E-01-0.228672E-01-0.276529E-01-0.276587E-01
-0.228845E-01-0.141564E-01-0.298407E-02 0.870034E-02 0.188763E-01 0.257843E-01
RING NUMBER =         18
 0.342124E-01 0.312563E-01 0.228923E-01 0.105674E-01-0.358653E-02-0.171213E-01-0.276957E-01-0.334803E-01
-0.334742E-01-0.276774E-01-0.170918E-01-0.354694E-02 0.106154E-01 0.229466E-01 0.313146E-01 0.342723E-01
 0.313078E-01 0.229332E-01 0.105959E-01-0.357168E-02-0.171207E-01-0.277094E-01-0.335078E-01-0.335141E-01
-0.277282E-01-0.171510E-01-0.361214E-02 0.105471E-01 0.228782E-01 0.312491E-01
RING NUMBER =         19
 0.412160E-01 0.376545E-01 0.275786E-01 0.127313E-01-0.431939E-02-0.206242E-01-0.333630E-01-0.403321E-01
-0.403255E-01-0.333434E-01-0.205925E-01-0.427676E-02 0.127829E-01 0.276372E-01 0.377174E-01 0.412805E-01
 0.377100E-01 0.276227E-01 0.127620E-01-0.430338E-02-0.206236E-01-0.333778E-01-0.403617E-01-0.403685E-01
-0.333981E-01-0.206563E-01-0.434704E-02 0.127093E-01 0.275634E-01 0.376467E-01
RING NUMBER =         20
 0.493482E-01 0.450838E-01 0.330201E-01 0.152439E-01-0.517023E-02-0.246915E-01-0.399435E-01-0.482878E-01
-0.482807E-01-0.399224E-01-0.246574E-01-0.512452E-02 0.152993E-01 0.330829E-01 0.451513E-01 0.494175E-01
 0.451434E-01 0.330674E-01 0.152769E-01-0.515303E-02-0.246908E-01-0.399594E-01-0.483196E-01-0.483269E-01
-0.399811E-01-0.247260E-01-0.519997E-02 0.152203E-01 0.330038E-01 0.450754E-01
RING NUMBER =         21
 0.586679E-01 0.535979E-01 0.392562E-01 0.181235E-01-0.614518E-02-0.293525E-01-0.474846E-01-0.574050E-01
-0.573974E-01-0.474621E-01-0.293162E-01-0.609644E-02 0.181826E-01 0.393232E-01 0.536699E-01 0.587418E-01
 0.536615E-01 0.393067E-01 0.181587E-01-0.612681E-02-0.293518E-01-0.475015E-01-0.574390E-01-0.574468E-01
-0.475248E-01-0.293894E-01-0.617700E-02 0.180982E-01 0.392387E-01 0.535889E-01
RING NUMBER =         22
 0.691178E-01 0.631446E-01 0.462486E-01 0.213524E-01-0.723821E-02-0.345785E-01-0.559400E-01-0.676277E-01
-0.676197E-01-0.559163E-01-0.345401E-01-0.718664E-02 0.214150E-01 0.463195E-01 0.632208E-01 0.691962E-01
 0.632120E-01 0.463022E-01 0.213897E-01-0.721874E-02-0.345778E-01-0.559580E-01-0.676637E-01-0.676721E-01
-0.559827E-01-0.346177E-01-0.727201E-02 0.213256E-01 0.462300E-01 0.631350E-01
RING NUMBER =         23
 0.803967E-01 0.734484E-01 0.537956E-01 0.248375E-01-0.841778E-02-0.402189E-01-0.650658E-01-0.786611E-01
-0.786527E-01-0.650410E-01-0.401787E-01-0.836383E-02 0.249030E-01 0.538699E-01 0.735283E-01 0.804787E-01
 0.735190E-01 0.538518E-01 0.248766E-01-0.839737E-02-0.402181E-01-0.650846E-01-0.786988E-01-0.787075E-01
-0.651106E-01-0.402600E-01-0.845328E-02 0.248093E-01 0.537760E-01 0.734383E-01
RING NUMBER =         24
 0.917167E-01 0.837898E-01 0.613702E-01 0.283354E-01-0.960141E-02-0.458794E-01-0.742245E-01-0.897343E-01
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-0.897257E-01-0.741991E-01-0.458382E-01-0.954596E-02 0.284027E-01 0.614466E-01 0.838720E-01 0.918012E-01
 0.838625E-01 0.614280E-01 0.283757E-01-0.958039E-02-0.458786E-01-0.742440E-01-0.897732E-01-0.897822E-01
-0.742708E-01-0.459219E-01-0.963806E-02 0.283063E-01 0.613500E-01 0.837795E-01
RING NUMBER =          25
 0.101356E 00 0.925962E-01 0.678205E-01 0.313143E-01-0.106090E-01-0.506993E-01-0.820233E-01-0.991635E-01
-0.991549E-01-0.819979E-01-0.506581E-01-0.105537E-01 0.313815E-01 0.678968E-01 0.926784E-01 0.101441E 00
 0.926690E-01 0.678784E-01 0.313547E-01-0.105880E-01-0.506985E-01-0.820428E-01-0.992023E-01-0.992115E-01
-0.820696E-01-0.507419E-01-0.106458E-01 0.312851E-01 0.678002E-01 0.925859E-01
RING NUMBER =          26
 0.105858E 00 0.967086E-01 0.708328E-01 0.327058E-01-0.110788E-01-0.529490E-01-0.856640E-01-0.103565E 00
-0.103557E 00-0.856400E-01-0.529100E-01-0.110264E-01 0.327695E-01 0.709051E-01 0.967866E-01 0.105938E 00
 0.967777E-01 0.708877E-01 0.327441E-01-0.110588E-01-0.529483E-01-0.856825E-01-0.103602E 00-0.103611E 00
-0.857080E-01-0.529895E-01-0.111138E-01 0.326780E-01 0.708134E-01 0.966988E-01
RING NUMBER =          27
 0.986947E-01 0.901641E-01 0.660395E-01 0.304931E-01-0.103280E-01-0.493644E-01-0.798652E-01-0.965557E-01
-0.965488E-01-0.798448E-01-0.493311E-01-0.102833E-01 0.305474E-01 0.661013E-01 0.902306E-01 0.987632E-01
 0.902231E-01 0.660864E-01 0.305258E-01-0.103109E-01-0.493637E-01-0.798810E-01-0.965872E-01-0.965946E-01
-0.799029E-01-0.493990E-01-0.103579E-01 0.304693E-01 0.660230E-01 0.901557E-01
RI G NUMBER =          28
 0.685805E-01 0.626527E-01 0.458893E-01 0.211892E-01-0.717606E-02-0.343012E-01-0.554954E-01-0.670933E-01
-0.670888E-01-0.554822E-01-0.342798E-01-0.714724E-02 0.212242E-01 0.459291E-01 0.626957E-01 0.686247E-01
 0.626909E-01 0.459196E-01 0.212103E-01-0.716501E-02-0.343008E-01-0.555056E-01-0.671136E-01-0.671185E-01
-0.555198E-01-0.343237E-01-0.719548E-02 0.211737E-01 0.458785E-01 0.626472E-01
BUCKLING LOAD= 0.315098E 01 (PSI)
CORE USAGE        OBJECT CODE=    42168 BYTES,ARRAY AREA=  249800 BYTES,TOTAL AREA AVAILABLE=  307296  BYTES
DIAGNOSTICS           NUMBER OF ERRORS=        0, NUMBER OF WARNINGS=        1, NUMBER OF EXTENSIONS=        1
COMPILE TIME=    3.35 SEC,EXECUTION TIME=   813.02 SEC,    3.14.56   SATURDAY    18 APR 81    WATFIV -
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Table B-3 The Buckling Mode Shape Function Values at (30x30) Grid Points

for the Prototype Model

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ERROR BETWEEN TWO ITERATION MODE= 0.274116E-03

                          BUCKLING MODE OF SHELL OF SELECTED NODAL POINTS
RING NUMBER =                 1
 0.673208E-04-0.969687E-05-0.219561E-03-0.396002E-03-0.377834E-03-0.112184E-03 0.304870E-03 0.675516E-03
 0.809001E-03 0.629364E-03 0.219590E-03-0.223616E-03-0.498458E-03-0.507455E-03-0.304871E-03-0.558689E-04
RING NUMBER =                 2
-0.356131E-03 0.151188E-03-0.385614E-03-0.863714E-03-0.901486E-03-0.365247E-03 0.535442E-03 0.135462E-02
 0.165874E-02 0.127357E-02 0.385664E-03-0.560955E-03-0.111334E-02-0.105946E-02-0.535444E-03 0.700950E-04
RING NUMBER =                 3
 0.121670E-02 0.699635E-03-0.615787E-03-0.183747E-02-0.208758E-02-0.104147E-02 0.855048E-03 0.262141E-02
 0.329685E-02 0.249197E-02 0.615871E-03-0.135399E-02-0.242589E-02-0.215006E-02-0.855058E-03 0.570133E-03
RING NUMBER =                 4
 0.394075E-02 0.257115E-02-0.824010E-03-0.409374E-02-0.510564E-02-0.302859E-02 0.114418E-02 0.514277E-02
 0.672383E-02 0.496960E-02 0.824158E-03-0.344677E-02-0.555836E-02-0.451208E-02-0.114424E-02 0.239783E-02
RING NUMBER =                 5
 0.864554E-02 0.584738E-02-0.101916E-02-0.773057E-02-0.100870E-01-0.641317E-02 0.141515E-02 0.902807E-02
 0.120885E-01 0.881392E-02 0.101940E-02-0.693035E-02-0.106469E-01-0.824804E-02-0.141530E-02 0.563294E-02
RING NUMBER =                 6
 0.156175E-01 0.107275E-01-0.120988E-02-0.129632E-01-0.173287E-01-0.113992E-01 0.167999E-02 0.145035E-01
 0.197048E-01 0.142494E-01 0.121026E-02-0.120132E-01-0.179935E-01-0.135776E-01-0.168028E-02 0.104729E-01
RING NUMBER =                 7
 0.253717E-01 0.175743E-01-0.140376E-02-0.201682E-01-0.273573E-01-0.183537E-01 0.194922E-02 0.219555E-01
 0.301143E-01 0.216607E-01 0.140432E-02-0.190659E-01-0.281286E-01-0.208812E-01-0.194969E-02 0.172787E-01
RING NUMBER =                 8
 0.387077E-01 0.269513E-01-0.160696E-02-0.299207E-01-0.409808E-01-0.278435E-01 0.223138E-02 0.319667E-01
 0.441370E-01 0.316293E-01 0.160772E-02-0.286588E-01-0.418638E-01-0.307370E-01-0.223207E-02 0.266128E-01
RING NUMBER =                 9
 0.567557E-01 0.396567E-01-0.182333E-02-0.430260E-01-0.593350E-01-0.406692E-01 0.253184E-02 0.453476E-01
 0.625163E-01 0.449649E-01 0.182441E-02-0.415941E-01-0.603370E-01-0.439525E-01-0.253289E-02 0.392725E-01
RING NUMBER =                10
 0.805463E-01 0.567025E-01-0.205206E-02-0.604945E-01-0.838493E-01-0.578421E-01 0.284950E-02 0.631076E-01
 0.878802E-01 0.626770E-01 0.205355E-02-0.588829E-01-0.849771E-01-0.615375E-01-0.285099E-02 0.562699E-01
RING NUMBER =                11
 0.112642E 00 0.790550E-01-0.228088E-02-0.832700E-01-0.115869E 00-0.803217E-01 0.316728E-02 0.861745E-01
 0.120350E 00 0.856962E-01 0.228292E-02-0.814785E-01-0.117123E 00-0.844295E-01-0.316938E-02 0.785738E-01
RING NUMBER =                12
 0.151490E 00 0.106476E 00-0.246816E-02-0.111037E 00-0.154982E 00-0.107847E 00 0.342739E-02 0.114180E 00
 0.159831E 00 0.113663E 00 0.247086E-02-0.109098E 00-0.156339E 00-0.112293E 00-0.343023E-02 0.105955E 00
RING NUMBER =                13
 0.189160E 00 0.133107E 00-0.249289E-02-0.137713E 00-0.192688E 00-0.134491E 00 0.346180E-02 0.140889E 00
 0.197586E 00 0.140367E 00 0.249624E-02-0.135755E 00-0.194059E 00-0.138982E 00-0.346537E-02 0.132580E 00
RING NUMBER =                14
 0.186477E 00 0.131336E 00-0.200716E-02-0.135045E 00-0.189318E 00-0.132451E 00 0.278736E-02 0.137602E 00
 0.193262E 00 0.137187E 00 0.201043E-02-0.133468E 00-0.190427E 00-0.136067E 00-0.279087E-02 0.130911E 00

BUCKLING LOAD= 0.143376E 03
```

Table B-4 The Buckling Mode Shape Function Values at (16x16) Grid Points

for the Fort Martin Tower

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MATRIX SOLUTION FOR LINEAR AND NONLINEAR BUCKLING OF HYPERBOLOIDS OF REVOLUTION

by

CHEN PAN

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MASTER OF SCIENCE

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ABSTRACT

A matrix method for the approximate solution of buckling of cooling towers is developed which is based on the differential equations of deformations of shells with the use of Sylvester's generalized matrix equation. Finite difference techniques have been employed in the construction of the operators for linear and nonlinear elastic buckling of the hyperboloid of revolution under the application of axisymmetric loads. A computer program has been developed according to the method of solution presented.

The influence of different step sizes of the partitioning on the eigenvalues has been studied and used with an extrapolation to improve the values obtained.

Nonlinear buckling is solved by a quasi-linearization technique which converts a nonlinear problem into a set of linear ones and the desired eigenvalue is calculated by an interpolation method.

Numerical examples based on the data for a model of a cooling tower under construction, as well as data for the Fort Martin Tower published are used.