STATISTICALLY MONITORING INVENTORY ACCURACY IN LARGE WAREHOUSE AND RETAIL ENVIROMENTS

by

ANDREW HUSCHKA

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Department of Industrial and Manufacturing Systems Engineering
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Approved By:
Major Professor
John English

Abstract

This research builds upon previous efforts to explore the use of Statistical Process Control (SPC) in lieu of cycle counting. Specifically a three pronged effort is developed. First, in the work of Huschka (2009) and Miller (2008), a mixture distribution is proposed to model the complexities of multiple Stock Keeping Units (SKU) within an operating department. We have gained access to data set from a large retailer and have analyzed the data in an effort to validate the core models. Secondly, we develop a recursive relationship that enables large samples of SKUs to be evaluated with appropriately with the SPC approach. Finally, we present a comprehensive set of type I and type II error rates for the SPC approach to inventory accuracy monitoring.

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Chapter 1: Introduction and Objectives

1.1 Background

Inventory record accuracy is vital to companies with high numbers of products. For a company to have accurate records, actual on hand inventory should equal their recorded inventory. As retail stores become larger and distribution centers service larger regions, assurance of accurate inventory records becomes much more significant and a more challenging task. Retail environments (e.g. large retail stores, distribution centers, etc.) often have thousands of different stock keeping units (SKU's) in their inventory. Brooks and Wilson (2005) state that failure to keep accurate inventory records can result in loss of product, time wasted correcting records, product not in stock for consumers, and overstock of items.

Cycle counting is currently the most common and established method used by companies to keep inventory record accuracy as described in Dehoratius and Raman (2008). Cycle counting has generally replaced periodic physical inventory checks. Cycle counting is accepted as a better method as it doesn't require the entire store or warehouse to shut down to count SKU's. Physical inventory checks are not only tedious and stressful, but they usually result in errors due to the time constraints on the availability of the facility. With cycle counting, subsets of the SKU's within inventory are examined to see if the actual on hand inventory equals the recorded inventory. If there are differences between the two, errors are corrected. Cycle counting is found to be less disruptive to daily operations, provides an ongoing measure of inventory accuracy, and can be adapted to focus on items with higher value.

Brooks and Wilson (2005) explain that with the correct execution of cycle counting, a company can have "95% or better accuracy." The dilemma for a large company is that it takes a large amount of resources, labor hours, and money to ensure that cycle counting is implemented correctly.

Consequently, for large retail environments, there is a need for a method to keep high levels of inventory accuracy without the large amount of time and resources that cycle counting requires. Furthermore, a more feasible approach would be one that is simply a monitoring approach and can be added to the periodic activities of operational personnel. As companies strive to be more efficient, the cost competitive pressures mount on the effective use of resources.

Statistical Process Control (SPC) is a proven statistical method used to monitor processes and improve quality using variance reduction. SPC utilizes random samples to monitor and control a process to ensure it is operating correctly and producing parts in accordance to its stochastic nature. In the inventory accuracy domain there is an opportunity to utilize random samples rather than the prescribed selection of SKU's as implemented in varied approaches of cycle counting so that Type I and Type II errors are controlled. As such, statistical process control is an ideal application for monitoring inventory accuracy as the total sampled number of SKU's can be dramatically reduced. There are two SPC tools that could be used to monitor inventory record accuracy.

The first method is a P-chart. A P-chart can be used to monitor the percent of SKU's in a sample for which the observed inventory level that matches the recorded inventory level. This means a random sample of *n* SKU's is selected and each SKU is checked to see if the actual on hand inventory exactly equals the recorded inventory. The number of SKU's for which the observed quantity matches the recorded inventory is divided by the total sample size. That provides a point estimate of the inventory accuracy, or P. Over time, P, is plotted on a P chart as seen in Cozzucoli (2009). The second method is a C-chart. C-charts can be used to monitor the collective number of item adjustments for a set of randomly observed SKU's where the on hand inventory failed to match the recorded inventory. That is, when a given SKU is sampled and the on-hand inventory does not match the recorded inventory level, the on-hand inventory or the recorded inventory will be adjusted in order to match the two. This means that either items will be ordered or the on-hand inventory will be adjusted. For the C-chart application to inventory accuracy an inspection unit of size *n* is sampled and the observed number of inventory adjustments is plotted in relationship to time as seen in Huschka (2009).

Huschka (2009) presents an analytical approach for small sample sizes and a simulation to provide suitable estimates of Type I and Type II errors for larger sample sizes as suggested in Yu (2007). In this research, we establish an efficient approach so that Huschka's (2009) analytical approach can be extended to a wide set of real world scenarios. A program is created that allows for the examination of type I and type II error rates of the C-chart with such populations. Conditions considered are typical of populations found in the industry.

1.2 Objective

Recent advances in Huschka (2009) provide much of the motivation for this research. Huschka (2009) presents an analytical approach to determining the Type I and Type II error rates for C-charts used to detect shifts in the number of inventory adjustments. In this thesis, the objective is to comprehensively explore the use of a C-chart to manage inventory adjustments that are required when the recorded inventory fails to match the number of inventory on the shelf. The first part of our research will verify the analytical model created by Huschka (2009) with real world data provided by a large international retailer. As the work of Huschka (2009) is limited to small samples, we extend that work to examine the impact of the C-chart approach in a real world setting.

1.3 Tasks

The following tasks are a summary of the goals of this research:

1. Verify Analytical Model

    1.1. Work with a national leader in the retail environment to secure data that supports or refutes the modeling concepts in Huschka (2009)

    1.2. Analyze the data to determine "reasonable" modeling parameter values

    1.3. Report findings of 1.1 and 1.2 in a report to document the usefulness of assuming the environmental conditions as prescribed in Huschka (2009)

2. Create a program able to calculate any sample size

    2.1. Define an equation able to calculate any sample size

    2.2. Create program that allows for the evaluation of type I and type II error rates for different values of the c chart as defined below. α is the proportion of the population represented by the PDF, λ is the Poisson arrival rate of the population, n is number of SKU's sampled, and m is the number of sub-populations

    2.2.1. $.01 \leq \alpha \leq .5$

    2.2.2. $.5 \leq \lambda \leq 5$

    2.2.3. $1 \leq n \leq 150$

    2.2.4. $1 \leq m \leq 15$

    2.3. Using the large retail environment data to present observations and conclusions

Chapter 2: Literature Review

To understand this research, there are two fields of work that are important: statistical quality control and inventory control. Specifically, we pay particular attention to the advances in the use and development of statistical process control (SPC) and cycle counting. We provide basic descriptions of the two areas and some of the more recent advances. It will be our observation that SPC can be used to efficiently monitor inventory accuracy.

2.1 Inventory Control Systems

Inventory record accuracy is vital to any company with high levels of inventory. For a company to keep accurate records, on hand inventory should equal recorded inventory. This has become a challenging task for some environments (e.g. large retail stores, distribution centers, etc.) because they often have thousands of different stock keeping units (SKU's) in their inventory. Piasecki (2003) indicates that there are several causes of discrepancies between actual on hand inventory and recorded inventory: stock loss or shrinkage, transaction errors, and product misplacement. Kok and Shang (2007) report that there are two problems that occur when inventory accuracy is poor. The first happens when an out of stock item is reported as in stock. This prevents the replenishment system from ordering more of the product. This results in higher backorder penalties and lost sales. The second happens when the recorded inventory shows fewer items than are in the physical inventory. This causes more products to be ordered and leads to higher inventory costs. To find and fix these discrepancies, there needs to be a system to monitor and make the required changes to inventory records.

2.1.1 Cycle Counting

Dehoratius and Raman (2008) examine 370,000 inventory records from 37 different stores and found that 65% of their inventory was inaccurate. This is not uncommon, and it is the reason that many companies have implemented strategies to keep track of the inventory records. Cycle counting is currently the most common and established method used by companies to keep inventory record accuracy. Cycle counting generally replaces annual physical inventory checks. Cycle counting is accepted as a better method, because it doesn't require the entire store to shut down to count SKUs as often required with physical inventory checks. Physical inventory checks are tedious and usually result in errors due to the time constraints on counting the SKUs. With cycle counting, subsets of inventory are counted to check that the actual on hand inventory equals the recorded inventory. If there are differences between the two, errors are corrected. When compared to inventory checking where a facility is closed and all SKUs are checked for accuracy, cycle counting is less disruptive to daily operations, provides ongoing measure of inventory accuracy, and can be enhanced to focus on items with higher monetary value.

Brooks and Wilson (2005) stated that "through the proper use of cycle counting, inventory record accuracy above 95% can be consistently maintained." As suggested, the dilemma for a large company is that it takes a large amount of resources, labor hours, and money to ensure that cycle counting is implemented correctly. For large environments, there is a need for a method to keep high levels of inventory accuracy that does not require the large amount of time, structure of operation, and large resources required by cycle counting. We further assume that basic concepts of statistical inference can be accepted as basic knowledge of the work force. As companies strive to be more efficient, cost competitive pressures mount on the effective use of resources. The next section provides an overview of some of the more common ways that cycle counting is being performed. Most of the concepts presented are found in Brooks and Wilson (2005).

2.1.1.1 Random Sample Cycle Counting

Random sample cycle counting is one of the more basic forms of cycle counting. SKU's are randomly selected for a given inventory such that each SKU of the population has an equal opportunity of being selected. There are two ways that random cycle counting can be carried out. They are called constant population counting and diminishing population counting.

Constant population technique implements sampling such that any SKU can be selected for any sampling period. In essence, this is sampling with replacement and is similar to the concept of sampling for SPC. This means, that if a SKU is picked for one sampling interval, it could be picked at the exact same likelihood the next period. It is also called "sampling with replacement." Diminishing population implements sampling such that after a SKU is picked it isn't returned to the population until all the other SKU's have been chosen. In essence this is "sampling without replacement." Probabilistically, this is the sampling approach connected to distributions such as the hypergeometric distribution.

2.1.1.2 Geographic Cycle Counting

Schreibfeder (2005) describes geographic cycle counting as starting at one end of the warehouse and counting a certain number of products each day until you reach the other end of the building. This method is considered the simplest form of cycle counting. This method allows a methodical approach to counting all materials in a warehouse, and it is not confusing to implement. Schreibfeder (2005) recommends that all items be counted at least once every 3 months.

2.1.1.3 Process Control Cycle Counting

Brooks and Wilson (2005) first introduced process control cycle counting. It is built upon the examination of SKUs that are convenient to count. This method is considered controversial in theory but effective in practice. To perform this method, the inventory records must have counts for each SKU at each location where the SKU is stored. The employee is then sent to a specific location to perform the cycle counting. They check the parts in every location, but they only spend time counting parts that will be easy to count. They then make adjustments necessary as overages or shortages are discovered.

If the parts are not easy to count, the counter checks the part identification, location, and order size. The employee then “eye balls” or estimates the number in a given the bin to see if it looks similar to the number of parts recorded in the system. If there is a large discrepancy between the numbers of parts in the bin compared to what the recorded inventory shows then a precise count is attained and an adjustment is made. For example, if the bin has about 10 parts in it and the system records an inventory of 100, the employee denotes the discrepancy, makes the exact count and the necessary adjustment. Brooks and Wilson (2005) state that the advantage of this method is that you can count 10-20 times the part numbers in a given time period with no extra cost. The disadvantage is having the employees determine what is “easy to count”.

2.1.1.4 ABC Method

The ABC method, also known as the Ranking Method, is based on the Pareto Principle and is a common way to perform cycle counting. The Pareto Principle has its root in economics where it is known that a majority of the wealth is held by a few number of people. For application inventory accuracy, it is assumed that only a few number of SKUs drive the bulk of inventory inaccuracy. An example of the Pareto Principle can be seen below in Figure 1.

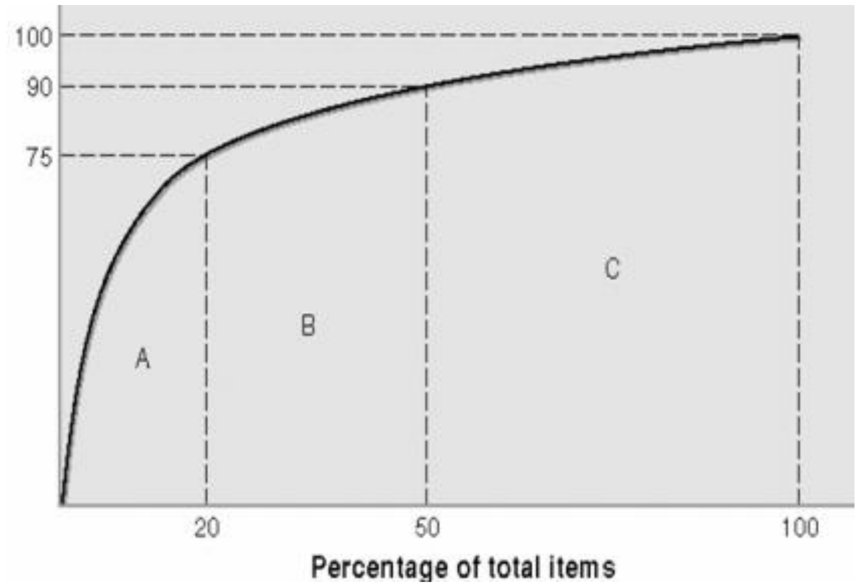

Figure 1: Pareto Principle Example (Reproduced from Inventory and Demand Analysis. <http://www.resourcesystemsconsulting.com/blog/archives/37>)

In Figure 1., inventory items are divided into three different categories: A items, B items, and C items. The ABC method places emphasis on parts that have a history of poor accuracy levels. Figure 1 shows that although A items are only 20% of the total inventory, they result in 75% of the errors. It can also be seen that B items are 30% of the total inventory and result in 15% of the total errors. Finally, C items are 50% of the total inventory but result in only 10% of the total errors.

Rossetti et al. (2001) states that the ranking method can be tailored to the specific priorities of the organization (e.g., accuracy levels, inventory cost, etc.). The company must establish the ranking of each SKU and design sampling accordingly. Rossetti et al. (2001) warns that the ABC method has a disadvantage in that the category classifications are primarily based on financial considerations. When considering delaying production or shipments, inexpensive items are as important as expensive items.

For example in an automobile assembly plant, the engine is much more expensive than the motor mounts, but absence of either stops production. Thus, it is important to consider lead-time, amount of usage, and bill of material (BOM) level when employing ABC cycle counting.

2.2 Statistical Process Control (SPC)

Montgomery (2009) states that “a company must continuously seek to improve process performance and reduce variability in key parameters.” He goes further to state “Statistical Process Control (SPC) is a primary tool for achieving this objective.” SPC is a powerful collection of problem solving tools useful in achieving process stability and improving capability through the reduction of variability. SPC has become such a powerful tool because it is easy to use, it has a significant impact, and can be applied to virtually any process. The tools of SPC are often called the “the magnificent seven”, they are:

1. Histogram
2. Check Sheet
3. Pareto Chart
4. Cause and Effect Diagram
5. Defect Concentration Diagram
6. Scatter Diagram
7. Control Chart

For this research, we use control charts as a method to monitor inventory accuracy. Control Charts utilize random samples to monitor a process to ensure it is operating in accordance to its natural stochastic behavior. In inventory accuracy domain, there is opportunity to utilize random samples rather than the various approaches of cycle counting. As eluded before, the random sample approach of cycle counting hints at the procedures used in control charts. However, there is no statistical approach to draw inference on the entire inventory. Control charts are statistically valid approaches that control underlying type I and type II errors. Statistical process control is an ideal application for monitoring inventory accuracy.

2.2.1 Control Charts

Control charts were first proposed in Shewart (1926, 1927), and they are considered one of the primary techniques of SPC. A control chart essentially plots measurements of a quality characteristic versus time. The chart consists of a centerline (CL) upper control limit (UCL) and lower control limit (LCL). The centerline is used to describe the central tendency or estimate average of the sampled statistic. The UCL and LCL denote the upper and lower bounds where the sampled statistic should fall given the process is operating in its normal stationary way (also called "in-control"). The UCL and LCL are estimated differently depending on the sampled statistic, but they all follow a similar formula which can be seen below in equation (1) and (2):

$$UCL = \mu_w + L\sigma_w \tag{1}$$

$$LCL = \mu_w - L\sigma_w \tag{2}$$

In this case, w is the sampled statistic that measures a given quality characteristic. The mean of w is μ_w, and the variance of w is ${\sigma_w}^2$. L is the distance of the control limits from the center line in multiples of the standard deviation of w and is often assumed to be 3. From these equations it is noted that the mean and variance are never known in practice and can only be estimated.

2.2.2 Type I and Type II Error Rates for Control Charts

Control limits are generally set at three (3) standard deviations away from the mean of the population. When a data point falls out of these limits, it indicates that the process is not stationary or out of control. There are two types of errors that are associated with control charts. They are type I and type II errors. When predicting type I and type II errors, there is a null hypothesis (H_0) and an alternative hypothesis (H_1). In our retail and warehouse domain the null hypothesis will be the mixture distribution is representative of the population. The alternative hypothesis will be that the mixture distribution is not representative of the population.

The type I (α') errors are known as the false alarm rate and occurs when the null hypothesis is rejected, but it is actually true. In application, this would happen if the operator concludes that the process is out of control when it is in fact in control. Type II (β) errors happen when we fail to reject the null hypothesis but the alternative is actually true. This means that the operator concludes the process is in control when it is in fact out of control. The common probabilistic statements defining α' and β are shown below in equation (3) and (4):

$$\alpha_n{}' = P\{reject\ H_0|H_0\ is\ true\} \tag{3}$$

$$\beta = P\{fail\ to\ reject\ H_0|H_0\ is\ false\} \tag{4}$$

2.2.3 Average Run Length (ARL)

The average run length (ARL) is the average number of points that must be plotted before a point indicates an out of control condition. For Shewart control chart, the ARL can be calculated from the equation (5):

$$ARL = \frac{1}{p} \tag{5}$$

The probability that any point exceeds the upper control limit or falls below the lower control limit is *P*. The in control ARL is the inverse of the probability of a type I error, α. The out of control ARL is the inverse of (1-P(β)). The out of control ARL is the average number of points needed to detect a process shift when one has occurred. The formulas to find type I and type II error rates are below in equation (6) and (7):

$$\alpha_n{}' = 1 - (P[W \leq UCL \mid \text{Ho is True}] - P[W \leq LCL \mid \text{Ho is True}]) \tag{6}$$

$$\beta_n = (P[W \leq UCL \mid \text{Ho is not True}] - P[W \leq LCL \mid \text{Ho is not True}]) \tag{7}$$

In equation (6) and (7) W is the sampled statistic. The ARL is used in many research advances to evaluate the performance of control charts. Crowder (1987) shows a numerical procedure using integral equations for the tabulation of moments of run lengths of exponentially weighted moving averages (EWMA). Gan (1993) presents a computer program for computing the probability of a function and percentiles of run length for a CUSUM control chart. Calzada and Scariano (2003) study the integral equation and Markov chain approaches for computing average run lengths for two-sided EWMA control charts.

Crowder (1987) tabulates α's for the EWMA control chart. Champ and Woodall (1987) use Markov chains to compute the ARL's for the $\overline{X}$ chart while embedding various run rules. Marcellus (2008) compares Bayesian analogue of Shewhart $\overline{X}$ chart to cumulative sum charts. He found that Bayesian offered better results, but it required more information which may be difficult to obtain.

Burroughs et al. (2003) studied the effect of using run rules on $\overline{X}$ charts and determined that they improve the sensitivity of the charts. There are hundreds of such advances in the literature, and they point to the conclusion that controls charts, if designed correctly, can be useful in monitoring performance. The ultimate goal of a control chart is to have a large in control ARL and small out of control ARL's.

2.2.4 Variable Control charts

Variable control charts are used when quality characteristics are expressed in terms of numerical measurements. This can include any single measurable quality characteristic such as length, weight, diameter, or volume. The three common control charts that are used for variable data are the $\overline{X}$, R, and S control Charts. The $\overline{X}$ control chart is used to monitor the process average or mean quality level and is also known as the control chart for means. The R control chart is used to monitor the range, while the S control chart is used to monitor the standard deviation. The range is the difference between the max and minimum values found in a given sample of n observations. Either the R or S control chart can be used to monitor the process variability.

2.2.4.1 $\overline{X}$ Control Chart

For a process, sampled observations, x_i , can be collected. The observations are assumed to follow a normal distribution with mean μ and variance σ_X^2. The sample average, called $\bar{x}$, (an unbiased estimator of μ) is calculated as:

$$\overline{x} = \frac{X_1+X_2+X_3+\cdots+X_n}{n} \tag{8}$$

It is well known that the resulting population of $\bar{x}'$s follows the normal distribution with mean μ and variance $\sigma_{\bar{x}}^2$ ($\frac{\sigma_{\bar{x}}{}^2}{n}$). The $\overline{X}$ control chart is used to monitor the $\bar{x}'$s and gain inference on the stability of the central tendency of the process. Using equations (1) and (2) as a basis, the theoretical control limits for the $\bar{x}$ chart are as follows:

$$UCL = \mu_X + 3\sigma_{\bar{X}} \tag{9}$$

$$CL = \mu_X \tag{10}$$

$$UCL = \mu_X - 3\sigma_{\bar{X}} \tag{11}$$

Clearly, μ and σ_X^2 are never known with certainty, so in practice they must be estimated with unbiased estimators as shown below:

$$\bar{\bar{X}} = \hat{\mu} = \frac{\sum_{i=1}^{m} \bar{X}_i}{m} \tag{12}$$

$$\hat{\sigma}_X^2 = \frac{\overline{R}}{d_2} \tag{13}$$

In equation (12) m = number of subgroups observed. d_2 is one of many control chart constants and are tabled for various subgroup sizes in all basic texts in quality control (e.g., Montgomery (2009)). $R_i = \max(X_1, X_2, \ldots, X_n) - min\ (X_1, X_2, \ldots, X_n)$ for each subgroup *i*, also called the range. The resulting control limits used in practice are as follows:

$$UCL = \bar{\bar{x}} + 3\bar{R}/d_2\sqrt{n} \tag{14}$$

$$CL = \bar{\bar{x}} \tag{15}$$

$$LCL = \bar{\bar{x}} - 3\bar{R}/d_2\sqrt{n} \tag{16}$$

Making the substitution, $A_2 = \frac{3}{d_2\sqrt{n}}$, (also a standard tabled value for control charts), the resulting control chart limits are classically estimated as:

$$UCL = \bar{\bar{X}} + A_2\bar{R} \tag{17}$$

$$CL = \bar{\bar{x}} \tag{18}$$

$$UCL = \bar{\bar{X}} - A_2\bar{R} \tag{19}$$

2.2.4.2 R Control Chart

R and S charts are used to monitor process variability. The R chart does this by plotting the range while the S chart uses the sample standard deviation. For this research, we are going to concentrate on the R chart as a method to monitor process variability. As described earlier R_i is simply the difference between the largest and smallest observation and can be easily collected. The center line of an R chart is the average range which can be calculated below:

$$\overline{R} = \frac{R_1+R_2+R_3+\cdots+R_n}{n} \tag{20}$$

The R chart follows the basic 3-sigma control limit approach, where μ_R and ${\sigma_R}^2$ are the mean and variance of the range statistic R as described in Montgomery (2009). Again using the theoretical control limits from equation (1) and (2) the control limits are defined as:

$$UCL = \mu_R + 3\sigma_R \tag{21}$$

$$CL = \mu_R \tag{22}$$

$$UCL = \mu_R - 3\sigma_R \tag{23}$$

Because, μ and σ_X^2 are never known with certainty, so in practice they must be estimated with unbiased estimators as shown below:

$$\bar{R} = \hat{\mu} = \frac{\sum_{i=1}^{m} \bar{R}_i}{m} \tag{24}$$

$$\hat{\sigma}_r^2 = d_3 \left(\frac{\overline{R}}{d_2}\right) \tag{25}$$

d_3 is another control chart constant and can be found in all basic texts in quality control (e.g., Montgomery (2009)). Substituting these unbiased estimators the resulting control limits used in practice are as follows:

$$UCL = \bar{R} + 3d_3 \left(\frac{\overline{R}}{d_2}\right) \tag{26}$$

$$CL = \bar{R} \tag{27}$$

$$LCL = \bar{R} - 3d_3 \left(\frac{\overline{R}}{d_2}\right) \tag{28}$$

d_2 and d_3 are control chart constants whose values depend on the sample size. To ease the computations equation (29) and (30) can be defined:

$$d_3 = 1 - \frac{3d_3}{d_2} \tag{29}$$

$$d_4 = 1 + \frac{3d_3}{d_2} \tag{30}$$

Substituting in these equations the resulting control chart limits are classically estimated as:

$$UCL = d_4\bar{R} \tag{31}$$

$$CL = \bar{R} \tag{32}$$

$$UCL = d_3\bar{R} \tag{33}$$

The R chart has been a very commonly used method for monitoring process variability. Wang (2009) identifies the R chart as a hybrid approach that allows you to control chart concurrent patterns at once. Castagliola (2005) found that the R chart can be used in tandem with a EWMA control chart to better monitor the process range. Costa and Magalhaes (2007) shows how joint X and R charts with varying sample sizes and variable intervals improves the control chart performance in terms of the speed with which shifts can be detected.

2.2.5 Attribute Control Charts

Attribute control charts are the second way that data can be monitored by control charts. Unlike variable control charts that are used to measure numerical data, attribute control charts are used to measure values that are determined by a discrete response. Some examples of attribute control charts would be conforming/nonconforming, pass/fail, go/no go, and good/bad. There are four different types of attribute control charts that are commonly used and they are P, NP, C and U control charts. Each of these charts will be explained in more detail in the following sections.

There are numerous other control charts that have been developed. Burke (1992) examines the use of a G chart and H chart to monitor the total number of defects and average number of defects based on the geometric distribution. Taleb (2009) looks at attribute control charts based on average run length with a pre defined process shift. Rudisill et al. (2004) uses a method of modifying U charts to monitor Poisson attribute processes. Ou et al. (2009) looks at using CUSUM as a method of control chart for attribute controls. Woodall (1997) gives a good summary of substitutes that have been tried for P, NP, C, and U charts. He goes further to explain why P, NP, C, and U are the widely accepted choice for attribute charts. In this research we will concentrate on P and C control charts.

2.2.5.1 P Control Chart

The P chart is commonly called the fraction nonconforming control chart. A part is considered nonconforming when it doesn't "conform" to the standard or requirement of one or more characteristics. Let us assume that the probability a unit will not conform to specifications is p. The likelihood of producing a defect is descriptive of a Bernoulli random variable. If one desires to describe the, D_i, number of independent defective units in a sample of size n, the resulting number of successes describes the binomial distribution shown in equation (34):

$$p[D_i] = \binom{n}{D_i} p^{D_i}(1-p)^{n-D_i} \tag{34}$$

In equation (34) $\mu_{D_i} = np$ and $\sigma_{D_i}^2 = np(1-p)$. The number of units that are nonconforming is *D* and *n* is the total sample size. *D* follows a binomial distribution with parameters *n* and *p*. The sample fraction nonconforming, $\hat{p}$, is defined as the ratio of the number of nonconforming units in the sample D to the sample size n. From a sampling perspective, the underlying Bernoulli parameter, *p*, can be estimated for sample i in equation (35):

$$\hat{p} = \frac{D}{n} \tag{35}$$

The P chart essentially plots sample estimates of p for successive samples and plots them on a control chart as seen in equations (36) – (38):

$$UCL = \mu_P + 3\sigma_P \tag{36}$$

$$CL = \mu_P \tag{37}$$

$$UCL = \mu_P - 3\sigma_P \tag{38}$$

Obviously the population parameter, *p*, is never known with certainty. Therefore the Bernoulli parameter, *p*, is estimated for *m* samples as:

$$\hat{p} = \overline{p} = \frac{\sum_{i=1}^{m} \widehat{p_i}}{m} \tag{39}$$

Equation (39) produces an unbiased estimate of p. To calculate the control limits, the average, μ, and variance, σ_p^2, are estimated as follows:

$$\overline{P} = \hat{\mu}_p = \frac{\sum_{i=1}^{m} p_i}{m} \tag{40}$$

$$\hat{\sigma}_p{}^2 = \frac{p(1-p)}{n} \tag{41}$$

Using equations (36), (37), and (38) we are then able to make simple substitutions to estimate the control limits for the P chart:

$$UCL = \overline{P} + 3\sqrt{\frac{\overline{P}\,(1-\overline{P}\,)}{n}} \tag{42}$$

$$Center\ line = \overline{P} \tag{43}$$

$$LCL = \overline{P} - 3\sqrt{\frac{\overline{P}\,(1-\overline{P}\,)}{n}} \tag{44}$$

The NP chart is very similar to the P chart in the fact that it is used to monitor nonconforming parts. The difference between the two charts is that NP monitors the number of nonconforming parts instead of the fraction of nonconforming parts. The NP chart is used when it is easier to interpret process performance as the actual number of defective units plotted. There are many advances in P charts over the years. More recently, Spliid (2010) looked at EWMA control chart for Bernoulli data. Cozzucoli (2009) used a P chart to monitor multivariate processes.

2.2.5.2 C Control Chart

When looking at parts or items there can be many nonconformities on a given unit (e.g. scratches, dents, etc.). If the total number of nonconformities becomes excessive, then given units can be judged defective or nonconforming. The C chart is used to measure the number of nonconformities per inspection unit. Unlike the P chart the C chart uses the Poisson distribution to describe it, which can be seen in equation (45):

$$p(x) = \frac{e^{-c}c^{x}}{x!} \tag{45}$$

Using the theoretical control limits from equations (1) and (2) the control limits for a c chart are defined:

$$UCL = \mu_c + 3\sigma_c \tag{46}$$

$$CL = \mu_c \tag{47}$$

$$UCL = \mu_c - 3\sigma_c \tag{48}$$

The next step is to define the mean and variance for a c chart which is shown in the equations below:

$$\overline{c} = \hat{\mu} = \frac{\sum_{i=1}^{m} c_i}{m} \tag{49}$$

$$\hat{\sigma}_c{}^2 = \bar{c} \tag{50}$$

Then by combining these equations the commonly used control limits can be defined:

$$UCL = \bar{c} + 3\sqrt{\bar{c}} \tag{51}$$

$$Center\ line = \bar{c} \tag{52}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} \tag{53}$$

A commonly used attribute chart, the C chart, has had many advances over the years so I have included a few more recent advances. Lapinski and Dessouky (1994) look at methods to improve the C chart to optimize the control limits. Specifically they look at ways to choose the locations to sample, the size of the sample, and frequency of sampling. Through this work the authors found that C charts can be slow in detecting small shifts. Khoo (2004) identified an efficient alternative that constructs a Poisson moving average chart for the number of nonconformities.

2.3 Background of SPC Approach for Inventory Control

Since our research deals with SPC as an approach to monitor inventory accuracy, it is important to look at previous work. Huschka (2009) developed an analytical model to monitor inventory adjustments with an SPC approach. This model is a logical extension of Miller (2008). Huschka integrates SPC as a means to improve inventory control and management. In his work, he shows that the C-chart is a reasonable approach to monitor inventory adjustments for sample sizes, and it is likely effective in real world applications. The effort uses complete enumeration to develop a baseline for model validation and concludes with some preliminary simulation findings.

2.3.1 Application and Notation

Setting up a background of SPC and inventory control methods is important to understand how these two methods can be used together to improve current industry standards. Up to this point our research has been based on a theoretical problem and has not been defined in practical terms. This section will look to show how these methods could be used in a real large retail environment. The first step to make this happen is to define a retail environment to apply these methods too. This research examines discount department stores which include stores like Costco, K-Mart, Meijer, Target, and Wal-Mart. Below, in Figure 2, we have set up a discount department store that is broken up into departments that are commonly found in real world discount department stores.

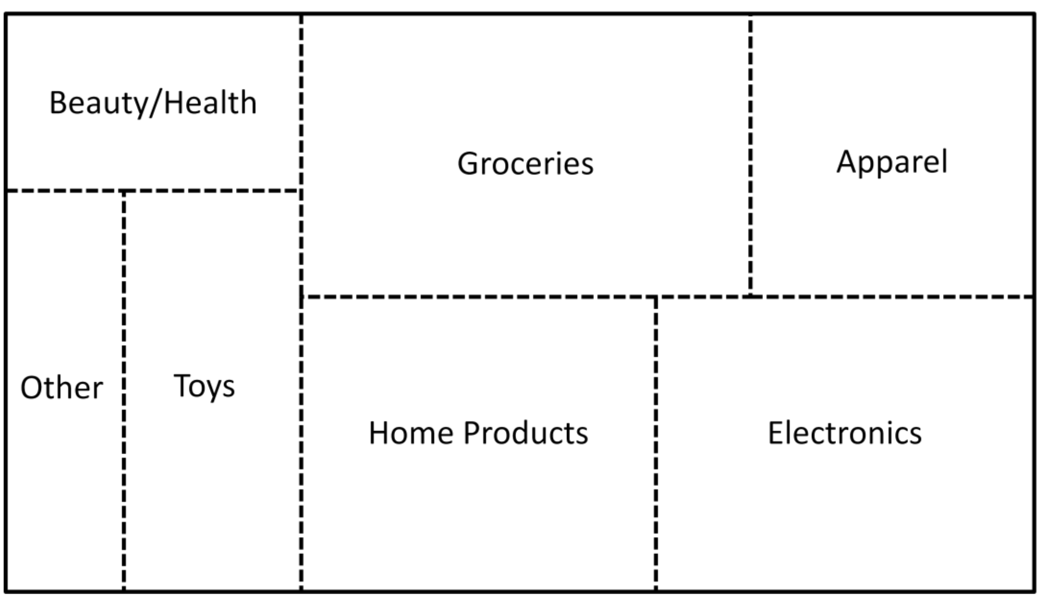

Figure 2: Discount Department Store

From Figure 2 it is easy to see that there are a total of 7 departments within this store. They are Apparel, Beauty/Health, Electronics, Groceries, Home Products, Toys, and Other. The other section could represent multiple different areas such as outdoor living, sports/recreation, automotive, and seasonal products to name a few examples.

For a given department, Huschka (2009) assumes that the number of inventory adjustments for a SKU within a department (say department i) follows a Poisson distribution with parameter λ_i and that each SKU in the department follows the same Poisson distribution. In essence, a given discount retail environment can be thought of as a collection of Poisson sub-populations with parameter λ_i, where each sub-population makes up α_i of the entire population as shown in Figure 3.

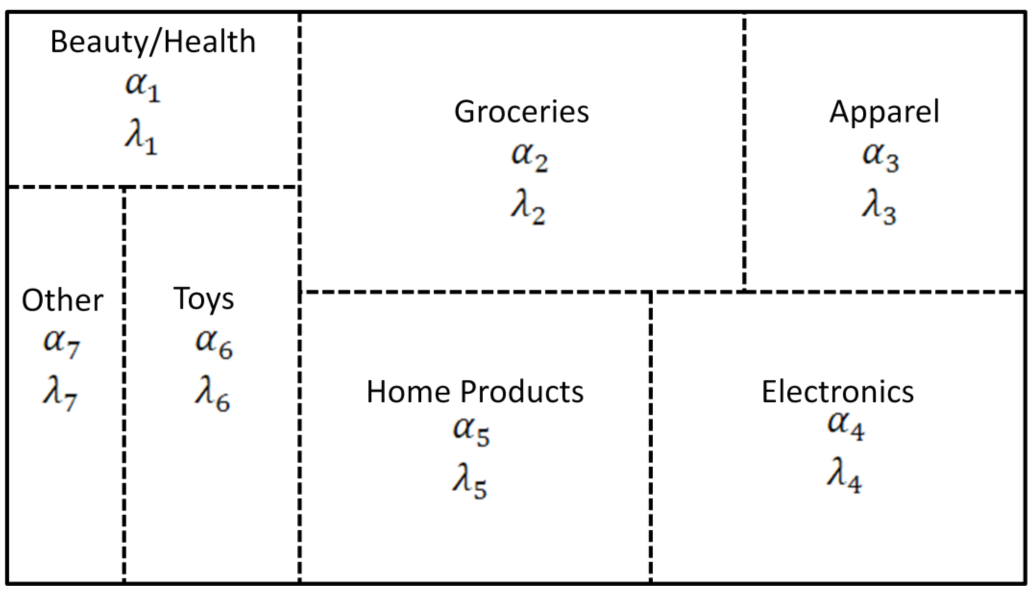

Figure 3: Discount Department Stores (α_i and λ_i)

This concept gives rise to the following nomenclature:

α_i = proportion of the population represented by pdf i

λ_i = Poisson arrival rate of the i^{th} population

$n =$ number of SKU's sampled to assess inventory adjustments for the population

$m =$ number of sub-populations

2.3.2 SPC Approach to Inventory Accuracy Monitoring

In this section the same complete enumeration concept developed by Huschka (2009) was used as validation points for the new approach of the thesis. Complete enumeration looks at all possible events and the probabilities associated with the events. To set up this problem the PDF is first introduced which can be seen in equation (54), below:

$$f_z = \alpha_1 * f_{z1} + \alpha_2 * f_{z2} + \cdots + \alpha_m * f_{zm} \tag{54}$$

Using this notation, Huschka (2009) describes Z as the variable that is observed from a population that results from a mixture of m sub-populations. Using a mixture of m Poisson distributions the resulting expected value, E(Z) and variance, Var(Z) can be seen in equations (55) and (56):

$$E(Z) = \sum_{i=1}^{m} \alpha_i \, \lambda_i \tag{55}$$

$$Var(Z) = \sum_{i=1}^{m} \alpha_i [(\lambda_i)^2 + \lambda_i] - (\sum_{i=1}^{m} \alpha_i \, \lambda_i)^2 \tag{56}$$

With the expected value and variance known control limits are easily described. Huschka (2009) uses equations (57) and (58) to create the control limits:

$$UCL_{\mathrm{Y}} = nE[Z] + 3\sqrt{nVar[Z]} \tag{57}$$

$$LCL_{\mathrm{Y}} = nE[Z] - 3\sqrt{nVar[Z]} \tag{58}$$

Where $Y = \sum_{i=1}^{n} X_i$.

After determining the expected value, variance, and control limits the next logical step for Huschka (2009) was to figure out the type I and type II error rates which can be seen in equations (59) and (60):

$$\alpha' = 1 - (P[\sum_{i=1}^{n} X_i \leq UCL \vdots \text{ Ho is True}] - P[\sum_{i=1}^{n} X_i \leq LCL \vdots \text{ Ho is True}]) \tag{59}$$

$$\beta = (P[\sum_{i=1}^{n} X_i \leq UCL \vdots \text{ Ho is not True}] - P[\sum_{i=1}^{n} X_i \leq LCL \vdots \text{ Ho is not True}] \tag{60}$$

2.3.3 Constructing an Example

After presenting this equation Huschka (2009) gives an example with m=5 sub-populations and n=2 SKUs with the following parameters:

λ_i = 0.25, 0.50, 0.75, 0.20, 0.30

$\propto_i$ = 0.15, 0.20, 0.05, 0.40, 0.20

With these parameters in place Huschka (2009) was able to easily calculate the expected value and variance using equations (55) and (56) which are respectively 0.315 and 0.337. Huschka (2009) was then able to compute the UCL and LCL from equations (57) and (58) to be 4.74 and -2.22, respectively. From this the next step is to utilize the PDF and complete enumeration to come up with the expected probabilities at each state. Table 1, on the following page, provides the final probabilities at each state:

Table 1: Complete Enumeration Example

Y= X1 + X2	Prob(Y)
0	0.54375953
1	0.32079783
2	0.104350788
3	0.025076815
4	0.004989009
5	0.000867797
6	0.000135831
7	1.94586E-05
8	2.58001E-06
9	3.19167E-07
10	3.7063E-08
11	4.0588E-09
Sum	0.999999999

From Table 1, it is seen that once at 11 errors the sum of the probabilities is close enough to 1 to stop calculating past this point. With the cumulative probabilities in place the next step is to calculate the type I error rate by utilizing the calculated UCL, LCL, and equation (59), which turns out to be .001026.

Chapter 3: Defining Sub-populations of SKUs

This chapter provides analysis of data from a national leader in retail. Due to time constraints and the large amount of data that were received, only the electronics department was used for analysis. All of the work that is done with the electronics department could easily be reapplied to the other departments within this retail environment.

3.1 Defining Sub-Populations

Before analysis on the data could be performed there needed to be defined sub-populations with SKU's in each sub-population that were a good fit. To do this, SKUs were broken into logical categories that were believed to follow similar distributions. For example a 46" TV is assumed to follow a similar pattern as a 32" TV, therefore, these two would be grouped together. Once logical groups were assigned, Minitab software was utilized to complete the statistical analysis to see if every combination of SKUs within the sub-population followed a similar distribution. To do this, a two sample t-test was run for each SKU within the population against every other SKU within the population. A 95% confidence interval was examined for the difference between these SKUs. The equations that were used to perform the two sample t-test can be seen below in formulas (61), (62), and (63):

$$C_1 - C_2 = \frac{Y_1+1}{n_1+2} - \frac{Y_2+1}{n_2+2} \tag{61}$$

$$SE_{C1-C2} = \sqrt{\frac{C_1(1-C_1)}{n_1+2} + \frac{C_2(1-C_2)}{n_2+2}} \tag{62}$$

$$C_1 - C_2 \pm Z_{\alpha/2}(SE_{C1-C2}) \tag{63}$$

In these equations, Y represents the number of errors found within the SKU while *n* represents the total number of observations for each SKU. If "0" was found to be within the confidence interval then it was determined for the purposes of this research that these two SKU's were similar enough to be in the same sub-population. For each sub-population that was looked at, a data matrix was completed to show the confidence interval of each SKU with every other SKU within the sub-population. An example of one of these matrixes can be seen below in Figure 4.

	375	376	3405	4825	5235	5240	5615	6025
375: Home Phone		(-2.342, 1.136)	(-1.757, 1.107)	(-1.780, 1.056)	(-1.552, 1.056)	(-1.747, 1.141)	(-2.319, 1.456)	(-2.195, 0.307)
376: Home Phone			(-1.275, 1.830)	(-1.299, 1.780)	(-1.083, 1.792)	(-1.263, 1.863)	(-1.802, 2.144)	(-1.732, 1.050)
3405: GPS				(-1.212, 1.137)	(-0.952, 1.106)	(-1.182, 1.226)	(-1.830, 1.617)	(-1.580, 0.342)
4825: Antenna					(-0.898, 1.126)	(-1.129, 1.247)	(-1.780, 1.642)	(-1.526, 0.362)
5235: MP3 Min. store						(-1.098, 0.989)	(-1.808, 1.442)	(-1.439, 0.047)
5240: MP3 Player							(-1.860, 1.604)	(-1.617, 0.335)
5615: CD Player								(-2.103, 1.077)
6025: Home Theatre								

Figure 4: Home Electronics t-test Matrix

Figure 4 shows that every SKU within this sub-population has a similar mean. Five additional sub-populations were similarly considered. In each case, games, home electronics, accessories, computer, ink, and TV/DVD, similar mean characteristics were found for those sub-populations; therefore, the assumption that similar SKUs follow the same mean error rate is supported. Tables 2-7, on the following page, provides the SKUs and final groupings for each sub-population.

Table 2: Game Sub-Population

SKU	Description
1300	PC Game
1315	PC Game
1325	PC Game
1350	PC Game
2010	PC Software
2015	PC Software

Table 3: Home Electronics Sub-Population

SKU	Description
375	Home Phone
376	Home Phone
3405	GPS
4825	Antenna
5235	MP3 Min. store
5240	MP3 Player
5615	CD Player
6025	Home Theatre

Table 4: Accessories Sub-Population

SKU	Description
705	Phone Cords/Acc.
1224	PC mouse
3406	GPS Acc.
4810	Speaker Acc.
4811	TV Mount
4815	Coax Acc.
4820	Surge Prot.
4835	Remote
4840	MP3 Charge/Acc.
4841	MP3 Charge/Dock
4842	MP3 Acc.
4860	Cd/Dvd/VHS cleaner
8034	PS2 Acc.
8045	PSP Acc.
8118	Wii Acc.

Table 5: Ink Sub-Population

SKU	Description
1259	Gloss Paper
1260	BlK Printer Ink
1262	Color Printer Ink
1264	Combo Ink

Table 6: Computer Sub-Population

SKU	Description
1101	Desktop PC
1115	PC des equip.
1130	Laptop
1131	Laptop 2
1162	Printer
1168	Printer 2
1207	Webcam
1209	Router
1216	Hard drive
1267	Hard drive

Table 7: TV/DVD Sub-Population

SKU	Description
3639	Combo TV/DVD
3641	19"-22" TV
3643	32" TV
3646	40"/42" TV
3647	46"/47" TV
4010	DVD Player

3.2 Experimentwise Error Rate

It is important to note that the experimentwise error rate is likely very large for our pragmatic approach within this chapter. The α values for all the experiments considered is greater than the individual experiment. Steel and Torrie (1980) state that a true experimentwise error rate must clearly allow any and all possible hypotheses to be tested and that it is desired that each treatment has a meaningful set of contrasts. The experimentwise error rate approximation can be seen in equation (64), below:

$$\alpha_{EW} = 1 - (1 - \alpha_{PC})^{C} \tag{64}$$

α_{EW} is the experimentwise error rate, α_{PC} is the per comparison error rate, and C is the number of comparisons. In the case of the home electronics scenario the per comparison error rate was set at .05 and there were a total of 28 comparisons made. In this case the experimentwise error rate would be .7622. This is high and it is likely that there is at least one type I error, but we believe there is overwhelming evidence of similar mean characteristics. With our research it is understood that there is a good chance of inflated type I errors due to the procedures we used to make sub-populations. It is important to be conscious of this as results are examined later in the research.

Chapter 4: Computer Program

Small values of n have been examined for the SPC approach to cycle counting in Huschka (2009) using complete enumeration. In this chapter, we present a recursive approach to consider larger sample sizes. This approach is embedded in a computer program so that larger sample sizes can be considered. The data that was provided by the large retail environment was utilized to compare the results of the program with the results of complete enumeration. We will show that this program can achieve the same type I error rates as complete enumeration for n=1, n=2, and n=3. After n=3 complete enumeration becomes quite computationally burdensome.

4.1 Defining the Recursive Relationship

As stated previously, complete enumeration becomes computationally burdensome and is not effective to utilize as n grows larger. Miller (2008) develops an approach called conditional probabilities that is further used by Huschka (2009). Although this method is somewhat effective for estimating type I error rates, it is not effective for large sample sizes. With a recursive relationship, loops can easily be set to calculate n for much larger sizes, to provide precise estimates of type I error rates. Equations (65), (66), and (67) show the relationship that was developed:

$$Q_j^1 = \alpha * \frac{e^{-\lambda} * \lambda^j}{j!} \quad \forall\, j = 0, 1, 2, \ldots \tag{65}$$

$$Q_i^n = \sum_{j=0}^{i} Q_j^1 * Q_{i-j}^{n-1} \tag{66}$$

$$\alpha_n{}' = 1 - \left(\sum_{i=\lceil LCL \rceil}^{\lfloor UCL \rfloor} Q_i^n\right) \tag{67}$$

From equation (65), (66), and (67) the nomenclature follows as *n* is the total number of SKUs, *i* is the total number of errors, and *j* is the current iteration of the errors. Equation (65) initializes the recursive function. *Q* gives the final probability at *n*=1 for each error, *j*. Using this initialization it is possible to utilize this equation to solve for any value of n as seen in equation (66). The final equation (67) determines the final type I error. With this recursive equation in place it is easy to examine larger sizes of n. The next step was developing a program that could read data and take the necessary steps to find error rates for the data.

4.2 Computer Program

A computer program was created to calculate larger values of n and can be found in Appendix A. There are two main parts to the program. The first part begins reading the number of product types in each sub-population and the corresponding α and λ values. Once these values are calculated, the program utilizes equations (54) and (55) to find the expected value and variance for the sub-population. With the expected value and variance the LCL and UCL can be calculated with equations (56) and (57). The second part of the program applies the recursive relationship to calculate each of the probabilities and ultimately find the type I error rate. The final program takes the α and λ values that are specified by the user and give a print out from n=1 to n=150. This can be edited to be larger or smaller depending on preference of the user. Figure 5, below, is an example of the output screen from the program.

```
n= 138 ex val: 2.750 var: 7.813 LCL: 281.0 UCL: 478.0 Type I: 0.002220
n= 139 ex val: 2.750 var: 7.813 LCL: 283.4 UCL: 481.1 Type I: 0.002292
n= 140 ex val: 2.750 var: 7.813 LCL: 285.8 UCL: 484.2 Type I: 0.002270
n= 141 ex val: 2.750 var: 7.813 LCL: 288.2 UCL: 487.3 Type I: 0.002342
n= 142 ex val: 2.750 var: 7.813 LCL: 290.6 UCL: 490.4 Type I: 0.002319
n= 143 ex val: 2.750 var: 7.813 LCL: 293.0 UCL: 493.5 Type I: 0.002297
n= 144 ex val: 2.750 var: 7.813 LCL: 295.4 UCL: 496.6 Type I: 0.002367
n= 145 ex val: 2.750 var: 7.813 LCL: 297.8 UCL: 499.7 Type I: 0.002343
n= 146 ex val: 2.750 var: 7.813 LCL: 300.2 UCL: 502.8 Type I: 0.002413
n= 147 ex val: 2.750 var: 7.813 LCL: 302.6 UCL: 505.9 Type I: 0.002389
n= 148 ex val: 2.750 var: 7.813 LCL: 305.0 UCL: 509.0 Type I: 0.002210
n= 149 ex val: 2.750 var: 7.813 LCL: 307.4 UCL: 512.1 Type I: 0.002277
```

Figure 5: Computer Program Output

Comparisons of the program are made to complete enumeration to ensure that the correct expected value, variance, LCL, UCL, and Type I error are calculated. To do this a sub-population was arbitrarily picked with size of m=10 and fixed alpha and lambda equal to .1 and 2.5 respectively. Table 8, below, provides the results of complete enumeration vs. the computer program.

Table 8: Complete Enumeration vs. Computer Program

	n=	Exp. Value	Variance	LCL	UCL	Type I Error
Complete Enumeration	1	2.5	2.5	0	7.243	0.004245
	2	2.5	2.5	0	11.708	0.00545
	3	2.5	2.5	0	15.716	0.004603
Computer Program	1	2.5	2.5	0	7.243	0.004245
	2	2.5	2.5	0	11.708	0.00545
	3	2.5	2.5	0	15.716	0.004603

Table 8 shows that the computer program is validated with complete enumeration and has the ability to give accurate results. In the type I error rate column, it can be seen that the computer program provides the same precision as complete enumeration up to 6 decimal places.

Chapter 5: Type I Error Rates

Based upon observations made analyzing real world data, an experiment of numerous different type I error rates are constructed to evaluate the type I error rate in a balanced fashion. There are numerous different type I error rates that could be examined, but ultimately the real world retail environment data is used to derive the type I error rates examined. In this section larger values of n and differing sub-populations are examined. Specifically, n=5, n=75, and n=150 are explored. Additionally m=5, m=10, and m=15 are examined. Defining the $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}$ values that are utilized is a little more complex and will be explained in the following section.

5.1 Defining α and λ Values

In this section we will explore $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}$ values when m=5, but any of the scenarios can be replicated for differing values of m. Five different methods are used for $\boldsymbol{\alpha}$ which are fixed, delta increase, delta decrease, skewed top, and skewed bottom. When observing $\boldsymbol{\alpha}$ values it is important to keep in mind that each value is a proportion of the population and thus the sum of all the $\boldsymbol{\alpha}$'s within a sub-population must add up to 1. When the $\boldsymbol{\alpha}$ values are fixed the total proportion of the sub-population, 1, is divided by the size of the sub-population, m. This can be seen in equation (68):

$$\boldsymbol{\alpha_{fixed} = \frac{1}{m}} \tag{68}$$

In the fixed scenario when observing m=5, $\boldsymbol{\alpha}$=.2 for all $\boldsymbol{\alpha}$'s within the sub-population. The next scenario for $\boldsymbol{\alpha}$ is delta increase and delta decrease. These two states are calculated in the same manner the only difference is increase starts at the minimum and goes to the maximum, while decrease starts at the maximum and goes to the minimum. The initialization of this state can be seen, below, in equation (69):

$$\boldsymbol{\Delta_{\mathrm{total}} = 1 + 2 + 3 + \cdots + m} \tag{69}$$

When using the delta approach the first step is to calculate the delta total which can be done by using equation (69). In the case of m=5 the delta total would be equal to 15. Once this is calculated the next step is to calculate the individual $\boldsymbol{\alpha}$ values for the delta approach with equation (70):

$$\boldsymbol{\alpha_{delta}} = \frac{\boldsymbol{\Delta_i}}{\boldsymbol{\Delta_{total}}} \tag{70}$$

Equation (70) allows for each individual $\boldsymbol{\alpha}$ value to be calculated for the delta state. In the case of m=5 and delta increase state $\boldsymbol{\alpha_1}$=.0666, $\boldsymbol{\alpha_2}$=.1333, $\boldsymbol{\alpha_3}$=.2000, $\boldsymbol{\alpha_4}$=.2666, and $\boldsymbol{\alpha_5}$=.3333. To utilize the delta decrease state these values are reversed starting at .3333.

The final scenario that is observed is the skewed top and skewed bottom. This scenario is observed because it is important to understand how a single product could potentially skew results if it is a high proportion of the population. In the case of skewed top $\boldsymbol{\alpha_1}$ was arbitrarily set to equal .5, while the remaining $\boldsymbol{\alpha}$'s proportions are evenly distributed utilizing equation (68). In the case of skewed top $\boldsymbol{\alpha_1}$=.5, $\boldsymbol{\alpha_2}$=.125, $\boldsymbol{\alpha_3}$=.125, $\boldsymbol{\alpha_4}$=.125, and $\boldsymbol{\alpha_5}$=.125. Skewed bottom is obtained by swapping $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_5}$.

Now that the process to find each of the $\boldsymbol{\alpha}$ values has been defined the next step is to define the process to find each of the $\boldsymbol{\lambda}$ values. Similarly to $\boldsymbol{\alpha}$ these values will differ as there are changes in *m* and the observations below will be for *m*=5. There are four different methods utilized for $\boldsymbol{\lambda}$ which are: flat minimum, flat maximum, linear, and weighted.

The real world data helped drive the $\boldsymbol{\lambda}$ values that are used. From the data it was found that all of the $\boldsymbol{\lambda}$ values were greater than .5 and less than 5. These values are then set as the maximum and minimum values that are used for testing. Flat minimum is sets all of the $\boldsymbol{\lambda}$ values equal to .5, while flat maximum sets all of the $\boldsymbol{\lambda}$ values equal to 5. The linear approach is an increase in a linear manner from a starting value of .5 to an ending value of 5. The equation to find $\boldsymbol{\lambda_i}$ can be seen, below, in equation (71) and (72):

$$\boldsymbol{\delta} = \frac{\mathbf{4.5}}{\boldsymbol{m-1}} \quad (71)$$

$$\boldsymbol{\lambda_i} = \mathbf{.5} + (\boldsymbol{i} * \boldsymbol{\delta}) \quad (72)$$

In equation (71) 4.5 is used because it is the difference between 5 and .5. When looking at m=5 the $\boldsymbol{\lambda}$ values are as follows $\boldsymbol{\lambda_1}$= .5, $\boldsymbol{\lambda_2}$= 1.625, $\boldsymbol{\lambda_3}$=2 .75, $\boldsymbol{\lambda_4}$= 3.875, and $\boldsymbol{\lambda_5}$= 5. The final approach for $\boldsymbol{\lambda}$ values is the weighted scenario. This is similar to the idea of skewed $\boldsymbol{\alpha}$'s. All of the $\boldsymbol{\lambda}$ values are at the minimum of .5 while the last $\boldsymbol{\lambda}$ value in the sub-population is heavily weighted with a value of 5. Each of the approaches outlined in this section is chose to test numerous different possible scenarios that could occur in the real world. It is not an exhaustive list of possible scenarios that can happen in the real world. The exact values that are used in each of these different scenarios are found in Appendix B.

5.2 Type I Error Rate Results

Using every scenario of $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}$ that have been outlined nine detailed tables can be found in Appendix C that show the type I error rate for every scenario. From these tables it was determined that *m* has no effect on the type I error rate when $\boldsymbol{\lambda}$ is set to flat minimum or flat maximum. A separate table for flat minimum and flat maximum for each value of n are examined. In Table 9, below, a list of the type I error rates for the flat minimum and flat maximum are presented:

Table 9: Flat Min/Max Type I Error Rates

	n=5	n=75	n=150
Flat Minimum	0.004245	0.003124	0.002952
Flat Maximum	0.002240	0.002123	0.002045

Table 9 shows that the type I error rates are all relatively low, no matter the scenario, but a Flat maximum gives a little bit lower type I error rate. In all cases the type I errors are similar to the classic

$\overline{x}$ chart with normality assumed, but it does appear that the SPC approach may have a bit better type I error for larger $\lambda's$ or average inventory inaccuracy.

It is also important to note that as *n* increases the type I error rate continues to decrease. We now extend our type I error rates to λ conditions of linear and weighted. Table 10, below, has the type I error rates for the linear and weighted scenarios:

Table 10: Linear/ Weighted Type I Error Rates

Lambda		Linear			Weighted		
Alpha	n=	m=5	m=10	m=15	m=5	m=10	m=15
Fixed	5	0.003287	0.003811	0.003451	0.008133	0.014378	0.014846
	75	0.002607	0.002436	0.002523	0.003238	0.003674	0.003928
	150	0.002371	0.002428	0.002408	0.002821	0.003233	0.003499
Delta Increase	5	0.002612	0.003149	0.002854	0.004641	0.008996	0.010831
	75	0.002490	0.002431	0.002528	0.002555	0.003280	0.003566
	150	0.002392	0.002176	0.002146	0.002383	0.002680	0.002936
Delta Decrease	5	0.004992	0.005308	0.004642	0.014846	0.016851	0.012562
	75	0.002761	0.002669	0.002694	0.003928	0.005110	0.003829
	150	0.002641	0.002490	0.002370	0.003499	0.003646	0.003255
Skewed Top	5	0.004669	0.005867	0.007527	0.010831	0.016788	0.014115
	75	0.002532	0.002617	0.002835	0.003566	0.004038	0.004887
	150	0.002529	0.002524	0.002598	0.002936	0.003375	0.003834
Skewed Bottom	5	0.003221	0.002531	0.002697	0.003309	0.003309	0.003309
	75	0.002194	0.002345	0.002401	0.002517	0.002517	0.002517
	150	0.002130	0.002216	0.002232	0.002343	0.002343	0.002345

Table 10 was designed to display type I error rates in a readable format that allows for easy comparison between scenarios. The first observation made from Table 10 is that some of the type I error rates are reaching above 1%. A type I error rate above 1% is not uncommon, but it is a value that is much higher than the classic value of .0027 for an $\overline{x}$ chart assuming normality. Looking at Table 10 it is noted that all of the values above 1% occurred when $\boldsymbol{\lambda}$ is weighted and n=5. There is some evidence that increasing *n* for the weighted conditions of $\boldsymbol{\lambda}$ may lead to excessive type I error rates.

In the weighted scenario, all of the values are set to .5 except for a single value that is set to 5. This causes a high variance for the number of errors and pushes the control limits further out. When looking at lower values of *n*, the calculation of the LCL will return a negative value that is outside the realm of feasible errors. In the case of UCL, it does not get out far enough to make up for the large variance in probabilities, and in turn, it causes the type I error rate to increase. From looking at Table 10 it is seen that the type I error rate is well below 1% when n=75 or n=150. This should not be a cause for a concern.

The second finding from Table 10 is that type I error rates decrease as n increases. This is no surprise, as such behavior is expected due to the central limit theorem. Table 10 confirms the hypothesis and matches the findings from Table 9. When *n*=150 it is evident that the highest type I error rate is .003834. As with Table 9 this is not below the .0027 that is necessary to meet the normality assumption, but is still an acceptable type I error rate.

5.3 Type I Error Rate Conclusions

Several general observations are made from Table 9 and Table 10. The first was that no matter what value of m, if the $\lambda's$ are equal, the type I error rate is the same for all conditions. This makes logical sense because the expected value and variance are the same no matter how many sub-populations exist.

The second observation made is that as *n* increases the type I error rate dwindles. As this is examined further it is found that this is what should be expected. The central limit theorem states that as sample sizes increase, with a mean and variance, the sampling distribution of the mean approaches a normal distribution. In our scenarios, the sampling distribution is tending to a normal distribution as more SKUs are pulled from the mixture Poisson process. This is the cause for slight decrease in the the type I error rates as *n* increases.

The final observation made is that the majority of our type I error rates were below 1% which is an acceptable level. However, there are a few cases in which the type I error rate is seen to inflate higher than 1%. The only cases of this happening were when n=5 with λ set to weighted**.** It is determined that this is due to the large variance that is caused when λ is weighted. From these findings it can be determined that this would not be a serious issue for any real world environment because it is unlikely that a real world retail environment would ever be looking at such small values of n. It is however important to note this and keep it in mind when looking at scenarios that would come close to fitting a weighted state.

Chapter 6: Type II Error Rates

In this section type II error rates are discussed. Shifts in $\boldsymbol{\lambda}$ at 10%, 25%, and 35% will be observed. Since type I error rates are calculated for nine different tables above with three different shifts in $\boldsymbol{\lambda}$ this would require a total of 27 tables. The same table format will be utilized as was used for type I error rates. Appendix D presents all 27 tables and these may be referenced as necessary. Before moving forward it is important to mention that this is not an exhaustive list of all type II error rates. In fact there are an infinite number of scenarios for which type II error rates could be determined. This section provides a balanced set of scenarios and examines how different shifts in $\boldsymbol{\lambda}$ affect the type II error rates.

6.1 Type II Error Rates $\boldsymbol{\lambda = 10\%}$ Shift

A smaller shift of $\lambda = 10\%$ begin the examination of type II error rates. This is determined to be small by running numerous tests and finding that the type II error rates were unlikely to be detected in most scenarios. In Table 11, below, the type II error rates for flat minimum and flat maximum can be seen at different levels of *n*:

Table 11: Flat Min/Max, $\lambda = 10\%$, Type II Error Rates

	n=5	n=75	n=150
Flat Minimum	0.999411	0.997919	0.993811
Flat Maximum	0.998703	0.908302	0.727124

When n=5, both flat minimum and flat maximum have fairly high type II error rates which means it is unlikely that this shift would be detected. As *n* starts to increase it is observed that the type II error rate decreases for both flat minimum and flat maximum. When looking at flat maximum, it is seen that it decreases at a much faster rate as n increases, since the flat maximum has much higher values of λ than flat minimum. If a higher λ is shifted by a percentage it causes the shifted λ's to increase at a faster rate. As the values of λ increase, the expected value will also increase. Thus a shift with higher λ values will cause its expected value to move away from the assumed expected value of the control limits at a much faster rate. This allows for quicker detection of a type II error rate, because it more quickly moves away from the mean as the shift increases. In Table 12, below, we present the type II errors for linear and weighted λ's with a 10% increase:

Table 12: Linear/ Weighted, $\lambda = 10\%$, Type II Error Rates

Lambda		Linear			Weighted		
Alpha	n=	m=5	m=10	m=15	m=5	m=10	m=15
Fixed	5	0.998952	0.998764	0.998900	0.995902	0.994410	0.994102
	75	0.988917	0.988462	0.987505	0.998757	0.999220	0.999060
	150	0.968217	0.961694	0.963686	0.996467	0.997809	0.998351
Delta Increase	5	0.999246	0.993420	0.999100	0.998124	0.996924	0.993901
	75	0.977045	0.975535	0.973590	0.997302	0.998816	0.999081
	150	0.931200	0.917313	0.913742	0.992634	0.996698	0.997459
Delta Decrease	5	0.997988	0.998796	0.998175	0.994102	0.988564	0.991189
	75	0.994695	0.993649	0.993119	0.998757	0.998964	0.998233
	150	0.983399	0.980218	0.976875	0.998351	0.997903	0.996157
Skewed Top	5	0.998079	0.997430	0.997910	0.993901	0.993295	0.998599
	75	0.996231	0.996960	0.996786	0.999081	0.999104	0.990800
	150	0.989766	0.990549	0.990625	0.997495	0.998354	0.990109
Skewed Bottom	5	0.999497	0.999329	0.999589	0.999365	0.999365	0.999365
	75	0.981053	0.976383	0.973948	0.993985	0.993985	0.993984
	150	0.942092	0.929416	0.924577	0.983733	0.983733	0.983732

Similar to Table 11, we see that in Table 12 the type II error rates are relatively high. This should not be of any concern because it is a small shift in $\lambda = 10\%$. Similar to the type I error rate, the type II error rate decreases as *n* increases in almost every scenario examined in Table 12.

It should be noted there are some cases that when n increases there is not a decrease in the type II error rate. These cases were isolated only to the scenario when λ is in the weighted state. These few rare cases are examined and it is found that it only occurred when moving from n=5 to n=75. It is observed that there are ten cases where it didn't decrease and of these cases the largest increase was by .010004.

This is a similar situation to the problems observed with the weighted state for type I error rates. Due to the high variance of the λ values, the distribution becomes quite dispersed. In the case of n=5, the type II error rate is lower because the LCL is truncated to zero, and the UCL is unable to detect the dispersion. This is not a huge concern, but is important to note when looking at data similar to the weighted state at low levels of n. There could also be some round off error due to the utilization of a program, but the values are small enough it is not of any concern. A larger shift in λ is explored in the next section.

6.2 Type II Error Rates $\lambda = 25\%$ Shift

Following the same format as above, the type II error rates for a shift in $\lambda = 25\%$ are observed when the base state is set to flat minimum and flat maximum. Below, Table 13 presents type II error rates at different levels of n for each state:

Table 13: Flat Min/Max, $\lambda = 25\%$, Type II Error Rates

	n=5	n=75	n=150
Flat Minimum	0.998905	0.987006	0.938342
Flat Maximum	0.994993	0.170157	0.004697

Table 13 meets all the conditions that were observed in Table 11. It can be seen that the type II error rate decrease as n increases and that flat maximum is decreasing at a much faster rate than flat minimum. It is somewhat alarming to see n=150 return one type II error rate equal to .938342 and another equal to .004697. After further investigation, it can be attributed to the larger λ values compared to such small λ values. Keep in mind that two extremes for λ values are used when looking at flat minimum and flat maximum. Table 14, below, provides the type II error rates for the linear and weighted states:

Table 14: Linear/ Weighted, $\lambda = 25\%$, Type II Error Rates

Lambda		Linear			Weighted		
Alpha	n=	m=5	m=10	m=15	m=5	m=10	m=15
Fixed	5	0.999817	0.999777	0.999806	0.998539	0.997555	0.997226
	75	0.863697	0.845308	0.832716	0.994180	0.997976	0.998283
	150	0.554144	0.486133	0.486053	0.966540	0.981999	0.986121
Delta Increase	5	0.999705	0.999366	0.999408	0.999531	0.998926	0.997401
	75	0.683259	0.646398	0.625272	0.981731	0.994822	0.996991
	150	0.246691	0.189437	0.175521	0.915006	0.969900	0.979121
Delta Decrease	5	0.999483	0.999720	0.999544	0.997226	0.993543	0.994776
	75	0.942325	0.924152	0.916258	0.998283	0.997650	0.992211
	150	0.759122	0.703063	0.668434	0.986120	0.979259	0.961661
Skewed Top	5	0.999496	0.999254	0.999407	0.997401	0.996716	0.995321
	75	0.967231	0.973700	0.972843	0.996991	0.998750	0.998575
	150	0.860963	0.874276	0.876791	0.979121	0.986179	0.982998
Skewed Bottom	5	0.999887	0.999757	0.999832	0.999908	0.999908	0.999908
	75	0.739638	0.674209	0.648259	0.943876	0.943876	0.943876
	150	0.311082	0.234131	0.212416	0.776429	0.776429	0.776428

The first thing observed when looking at Table 14 is that it met all of the findings that we previously discovered in Table 12. When looking at the linear state, it is seen that as n increases, the type II error rate decreases. When looking at the weighted state, it is seen that there are a few cases that the type II error rates are not decreasing as n is increasing. In Table 14 there were only six cases of the type II error rate not decreasing as n increased. Like Table 12, it only happened from n=5 to n=75. It was also found that the largest discrepancy between the two values was an increase of .004017 which is much better than .010004 we saw in Table 12. It is seen that potential round off errors have lessened in table 8.

When λ is shifted it moves the entire distribution to the right which allows for improved detection. This is important with a highly variable state with a low level of n because the control limits are now detecting more probabilities within the realm of possible errors. As shifts in λ continue to grow this issue will dwindle until reaching a point that the type II error rates will all decrease as any value of n increases.

The final observation that was made from Table 14 is that Type II error rates drop in many of the scenarios. It was observed that the cases that type II error rates reached prevalent levels were when n=150, and it was set to a linear state. There was also somewhat of a trend when α was equal to skewed bottom and delta increase. In these cases there were larger drops in the type II error rate. These findings will continue to be monitored as larger shifts in λ are explored.

6.3 Type II Error Rates $\lambda = 35\%$ Shift

The final shift in a type II Error rate that will be observed will be a shift in $\lambda = 35\%$. Below, Table 15 shows the type II error rates with a base condition set to flat minimum and flat maximum:

Table 15: Flat Min/Max, $\lambda = 35\%$, Type II Error Rates

	n=5	n=75	n=150
Flat Minimum	0.999328	0.967226	0.837217
Flat Maximum	0.988425	0.011245	0.000004

From Table 15 it is seen that the type II error rates continue to decrease as n increases. It is also seen that at the flat maximum condition, it decreases at a much faster rate than the flat minimum condition. From Table 11, Table 13, and Table 15 it is noted that as the shift in λ increases there is a decrease in the type II error rate. This makes sense as the shift becomes more pronounced. Although this isn't a surprise, it should be noted that type II error rates behaved as expected for this circumstance. Table 16, below, shows the type II error rates for the linear and weighted scenarios:

Table 16: Linear/ Weighted, $\lambda = 35\%$, Type II Error Rates

Lambda		Linear			Weighted		
Alpha	n=	m=5	m=10	m=15	m=5	m=10	m=15
Fixed	5	0.999885	0.999931	0.999942	0.999263	0.998577	0.998305
	75	0.643636	0.602687	0.578296	0.985186	0.994754	0.995438
	150	0.182622	0.130891	0.128160	0.903518	0.948496	0.958904
Delta Increase	5	0.998330	0.998934	0.999050	0.999816	0.999465	0.998516
	75	0.331281	0.282605	0.260015	0.951121	0.986946	0.992413
	150	0.023377	0.012786	0.010724	0.762204	0.913643	0.940445
Delta Decrease	5	0.999790	0.999893	0.999817	0.998305	0.995543	0.996284
	75	0.836859	0.787829	0.768067	0.995438	0.993325	0.979684
	150	0.437658	0.351804	0.310054	0.958904	0.937276	0.892541
Skewed Top	5	0.999793	0.999671	0.999742	0.998516	0.997936	0.996920
	75	0.908467	0.925504	0.924025	0.992413	0.996560	0.995976
	150	0.630679	0.661848	0.668583	0.940445	0.958754	0.949448
Skewed Bottom	5	0.999892	0.999690	0.999763	0.999979	0.999979	0.999978
	75	0.409233	0.317150	0.286780	0.844875	0.844875	0.844874
	150	0.040273	0.020018	0.015894	0.462642	0.462642	0.462642

When studying Table 16 the first finding noted is that there is evidence that supported a previous theory. From Table 14 it was hypothesized that as shifts in λ increased there would be a point at which all type II error rates would decrease as any value of n increases. When looking at Table 16 every case that n increases results in a decrease in λ.

The second observation is that there are definitive trends that are causing type II error rates to drop. The first of these trends that is observed is that in almost every case when comparing a type II error rate with constant α, m, and n the linear state will result in a lower type II error rate than the weighted state. The only time this did not happen was in a few cases when n=5. This is due to the high variability with a low value of n that was previously discussed.

There are two causes for the lower type II error rates for the linear state. The first is because it has lower variability like the weighted state. This means that the control limits are much tighter and as shifts occur they will move out of the control limits faster and allow them to be detected more quickly. The second reason is that the linear state has much higher values of λ than in the weighted state. These values are higher λ's causing the expected value to change much quicker thus moving away from the assumed mean quicker.

The next trend that was seen was that when α was set to delta increase and skewed bottom, detection of type II error rates happened quicker. This relates back to having λ's that are at a higher level thus causing a higher expected value. In the case of the delta increase and skewed bottom the highest proportioned α value lines up with the highest value of λ. This causes the expected value to be much higher than other scenarios and causes shifts to move away from the mean at a much quicker rate. The conclusion section ties all of these findings together.

6.4 Type II Error Rate Conclusions

After observing three different types of shifts in λ, a few very important discoveries were made and a few questions that need to be explored further. The first area that needs to be explored is why flat maximum with a percentage shift is able to detect quicker than flat minimum. Below, Table 17 and Table 18 show the control limits for each value of n and the corresponding control limits with each shift of λ that is observed.

Table 17: Flat Min CL vs. Shift CL

	Base Min	10% Shift	25% Shift	35% Shift
n=5	[0,7]	[0,8]	[0,8]	[0,9]
n=75	[19,56]	[22,61]	[26,67]	[29,72]
n=150	[47,98]	[53,105]	[59,117]	[61,121]

Table 18: Flat Max CL vs. Shift CL

	Base Max	10% Shift	25% Shift	35% Shift
n=5	[10,40]	[12,43]	[14,48]	[16,51]
n=75	[317,433]	[352,473]	[404,534]	[429,564]
n=150	[644,806]	[713,882]	[816,997]	[885,1073]

When observing Table 17 and Table 18 we see that as the shift is increased the control limits move further away from the base condition as should be expected. As n increases we also see that the distribution is shifting away from the base condition quicker as we previously discovered. When comparing Table 17 and Table 18 it is observed that the distribution for flat maximum is shifting away from the base condition much quicker than flat minimum. Looking at the 35% shift when n=150 for flat minimum we see that the shifted control limits take into consideration a good portion of the base conditions control limits. When compared to a 35% shift when n=150 for flat maximum it is seen that the shifted control limits are completely outside of the base control limits.

This is the reason that in this scenario we see a type II error rate of .837217 for flat minimum while flat maximum returns a type II error rate of .000004. When observing percentage shifts the flat maximum will move quicker out of the base conditions control limits due to a higher expected value. This makes it much easier to detect shifts when λ values are higher or there is significant weight put on an individual λ value.

Another important take away from the type II error rate analysis was that m has minimal effect on the resulting type II error rate. Through all of the tables when everything else was held constant comparing type II error rates for different values of m gave very similar results. There was only one instance when there was a discrepancy larger than .06 in all of the tables. The majority of inconsistencies happened when using α values that had repeating decimals which caused numerical instability.

The third finding from the type II error rate examination was that when there is a single λ value that is highly weighted it causes a high variance. Due to this high variance the probabilities stretch out and this causes an issue for small values of n. In the case of n=5 this causes the type II error rate to be lower because the LCL pushes negative and the UCL is unable to detect the large variance of probabilities.

The final conclusion from the type II error rate analysis is that as n increases a decrease in the type II error rate can be expected. Although this can't be assumed for every state it is definitely the case in a large majority of the scenarios. The only time this conclusion was not valid was when λ was set to the weighted state for the reasons listed in the previous paragraph. It is important that when performing analysis on data to keep this in mind and understand how to find a scenario that would resemble the weighted state.

Chapter 7: Conclusion and Future Work

7.1 Conclusions

The work by Huschka (2009) set the background for this thesis and brought up the need to examine larger sample sizes. Huschka proposed simulation, but we developed a recursive approach that eliminated the need for simulation. The development of this thesis has successfully advanced the belief that SPC can be utilized as an efficient method to improve inventory control systems in a real world environment as Huschka (2009) believed to be the case. Although complete enumeration was an effective method to find type I error rates for small values of n, it was impossible to apply this to real world environments for large sample sizes. The development of a recursive function and a program that can calculate large values of *n* proved that SPC is utilized in a resourceful and practical manner. There are a few important lessons and findings from this research that will now briefly be discussed.

As n increases a decrease in type I and type II error rates should be expected in most scenarios. It was found that the only time this did not occur is when looking at smaller values of n and when λ is set to the weighted state. To avoid these issues it would be recommended to observe your λ values closely to ensure that there is not a single value that is an outlier from the rest of the sub-population. It is also suggested that when utilizing the SPC approach to use sample sizes greater than 50.

This research was able to determine that m has minimal to no affect on type I and type II error rates. It was found that there was some variance due to round off error and repeating decimals, but it was negligible in most scenarios. This is an important find because it allows the practitioner to make comparisons between sub-populations of different sizes of m.

The final finding from this research was the effect larger expected values have on the detection rate of type II errors. When observing percentage shifts, the flat maximum distribution will move quicker out of the base conditions control limits due to a higher expected value. This will assist the practitioner when observing the expected values of differing populations and making connections to the type II error rates. In the future work section we will look at a method that allows for equal detection of type II error rates and other methods to advance this research.

7.1 Future Work

The research completed in this thesis takes the previous work of Huschka (2009) and proves the usefulness of SPC in real world retail environments. With this progression, the next steps of this work should be to look at methods to further improve upon the current model and computer program work that was created in this thesis.

The modeling techniques used in this thesis only scratched the surface of possible situations that could come up in the real world. Type II errors could be extended beyond just the shifts in λ that are observed in this research. Model shifts in α should be examined first. Additionally, since an analytical approach is now available for large sample sizes the type I error rates should be fixed at .0027. Then from there finding the resulting type II error rates could be analyzed. This would allow for an equal detection process for all type II error rates.

Aside from the modeling techniques, there are also significant improvements that need to be made to the computer program. In its current state, it is effective for finding type I and type II error rates, but it is not efficient. For each value in the type II error rate table λ, α, m, n, and control limits had to be adjusted manually. After entering each of these values, the program calculates the type II error rate for that specific location in the table. Adding on to the program to allow it to calculate type II error rates for different values of m and n for a certain set of α and λ values would allow it to be much more efficient. There are also memory allocation issues. The current program can only calculate up to values of n=150. After that point issues start to occur and the program crashes. It would be a huge improvement to rework resource allocations within the program to allow it to calculate up to n=200.

The final area to progress the work in this thesis is to utilize an updated and efficient computer program to observe a retail environment over a period of time. Although real world data was used in this thesis, it was a snapshot of data. Being able to observe data over a six month to one year period would allow for the observation of the complexities of a real world environment over time. This could open up new ideas on how to improve the current work and show that the current work is effective in all situations instead of isolated scenarios.

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Appendix A – Computer Program

```
#include "stdafx.h"
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
#include "math.h"

typedef struct Data_S Data;
typedef struct Families_S Families;
typedef struct Product_S Product;

#define MAXFAMILY 1
#define MAXITER 151
#define MAXSKUS 16
#define MAXHISTORY 0
#define MAXERRORS 822


struct Data_S {
	Families *family;
	int numfamilies;
	int totaldatapts;
	double *totalerrorprob;
};

struct Families_S {
	Product *prod;
	int numprod;
	double ucl[MAXITER];
	double lcl[MAXITER];
	double expectval;
	double var;
	double prob[MAXITER][MAXERRORS];
};


struct Product_S {
	double alpha;
	double lamda;
	double *probabilities;
	int probabilitylength;
	int *errorhistory;
	int historylength;
	int familynum;
```

```
        int productnum;
};

void    alphalambda (Data *data);

void    calcprob (Data *data, int numberit);


int _tmain(int argc, _TCHAR* argv[])
{
        int errors, i, j, k, numberit;
        Data *data=NULL;

        data =(Data*)calloc( 1, sizeof( Data ));
        data->totalerrorprob =(double*)calloc(MAXERRORS, sizeof(double));
        data->family =(Families*)calloc(MAXFAMILY, sizeof(Families));
        for(i=0;i<MAXFAMILY;i++){
                data->family[i].prod =  (Product*)calloc(MAXSKUS, sizeof(Product));
                for (j=0; j<MAXSKUS; j++) {

                }
        }

        alphalambda (data);

        numberit=1;

        calcprob(data, numberit);

        return 0;
}


void    alphalambda (Data *data) {

        int cnt,i, j, k, l, sum, totpicks;
        double sumd, sumstdev, var, UCL, LCL;

        data->numfamilies=1;
        for (i=0; i<data->numfamilies; i++) {

            data->family[i].prod[1].alpha=.2;
                    data->family[i].prod[2].alpha=.2;
                    data->family[i].prod[3].alpha=.2;
                    data->family[i].prod[4].alpha=.2;
                    data->family[i].prod[5].alpha=.2;

                    data->family[i].prod[1].lamda=.5;
```

```
				data->family[i].prod[2].lamda=.5;
				data->family[i].prod[3].lamda=.5;
				data->family[i].prod[4].lamda=.5;
				data->family[i].prod[5].lamda=.5;

				data->family[i].numprod=5;

	}

	getchar();

	for (i=0; i<data->numfamilies; i++) {
		sumd=0;
		for (j=1; j<=data->family[i].numprod; j++) {

		}
	}

   for (i=0; i<data->numfamilies; i++) {
		sumd=0;
		sumstdev=0;
		for (j=1; j<=data->family[i].numprod; j++) {
			sumd+=data->family[i].prod[j].alpha*data->family[i].prod[j].lamda;
			sumstdev+=data->family[i].prod[j].alpha*(((data->family[i].prod[j].lamda*data-
>family[i].prod[j].lamda)+data->family[i].prod[j].lamda));
			var=sumstdev-(sumd*sumd);
		}

		data->family[i].var=var;
		data->family[i].expectval=sumd;

		for (j=1; j<MAXITER; j++) {
			data->family[i].lcl[j]=(double)j*data->family[i].expectval-3*sqrt(j*data-
>family[i].var);
			data->family[i].ucl[j]=(double)j*data->family[i].expectval+3*sqrt(j*data-
>family[i].var);
			if ( data->family[i].lcl[j]<0) data->family[i].lcl[j]=0;
		}
	}
}

void	calcprob (Data *data, int numberit)
{

	int cnt,i, j, k, l, m,  totpicks;
```

```
	double sumd, pr, finalprob
[MAXFAMILY][MAXERRORS],prob[MAXFAMILY][MAXERRORS],origprob[MAXERRORS],totalprob[MAXERR
ORS],fact ;
	double answerprob [MAXFAMILY][MAXITER][MAXERRORS];
	double type1error;

	for (i=0; i<data->numfamilies; i++) {
		for (l=0; l<MAXERRORS; l++) {
			finalprob[i][l]=0;
			prob[i][l]=0;
		}
	}

	for (i=0; i<data->numfamilies; i++) {
		for (l=0; l<MAXERRORS; l++) {
			prob[i][l]=0;
			finalprob[i][l]=0;
		}
		for (j=1; j<=data->family[i].numprod; j++) {
			for (l=0; l<MAXERRORS; l++) {
				pr= pow(2.71828,(-data->family[i].prod[j].lamda));

				for (m=1;m<=l; m++) {
					pr=pr*data->family[i].prod[j].lamda/(double)m;
				}
				pr=pr*data->family[i].prod[j].alpha;
				prob[i][l]+=pr;
			}
		}
		for (l=0; l<MAXERRORS; l++) {

		}
	}

  numberit=145;
	for (i=0; i<data->numfamilies; i++) {
		for (j=0; j<=numberit; j++) {
			for (k=0; k<MAXERRORS; k++) {
				if (j==1) {
					answerprob[i][1][k]	=prob[i][k];
				} else	answerprob[i][j][k]=0;
			}
		}
	}
	for (i=0; i<data->numfamilies; i++) {
		for (j=2; j<=numberit; j++) {
			sumd=0;
			for (k=0; k<MAXERRORS; k++) {
```

```
				for (l=0; l<=k; l++){
					answerprob[i][j][k]+=answerprob[i][1][l]*answerprob[i][j-1][k-l];

				}
				sumd+=answerprob[i][j][k];
			}
		}
	}

	for (i=0; i<data->numfamilies; i++) {

		for (j=1; j<=numberit; j++) {

			for (k=0; k<MAXERRORS; k++) {
				data->family[i].prob[j][k]=answerprob[i][j][k];

			}
		}
	}

	for (i=0; i<data->numfamilies; i++) {

		for (j=1; j<=numberit; j++) {
			sumd=0;
			for (k=288; k<=411; k++) {
				sumd+=data->family[i].prob[j][k];
			}
		type1error=1-sumd;
		printf("n= %d ex val: %.3f var: %.2f LCL: %.1f UCL: %.3f ", j, data->family[i].expectval,
data->family[i].var, data->family[i].lcl[j], data->family[i].ucl[j]);
		printf("Type II: %f\n", sumd);
		}
	}
}
```

Appendix B – λ, α, and m Values for Tables

Appendix C-Table 1: m=15 Values

m=15	Fixed	Delta Increase	Delta Decrease	Skewed Top	Skewed Bottom
Alpha 1	0.067	0.008	0.125	0.500	0.036
Alpha 2	0.067	0.017	0.117	0.036	0.036
Alpha 3	0.067	0.025	0.108	0.036	0.036
Alpha 4	0.067	0.033	0.100	0.036	0.036
Alpha 5	0.067	0.042	0.092	0.036	0.036
Alpha 6	0.067	0.050	0.083	0.036	0.036
Alpha 7	0.067	0.058	0.075	0.036	0.036
Alpha 8	0.067	0.067	0.067	0.036	0.036
Alpha 9	0.067	0.075	0.058	0.036	0.036
Alpha 10	0.067	0.083	0.050	0.036	0.036
Alpha 11	0.067	0.092	0.042	0.036	0.036
Alpha 12	0.067	0.100	0.033	0.036	0.036
Alpha 13	0.067	0.108	0.025	0.036	0.036
Alpha 14	0.067	0.117	0.017	0.036	0.036
Alpha 15	0.067	0.125	0.008	0.036	0.500
m=15	**Flat Min.**	**Flat Max**	**Linear**	**Weighted**	
Lambda 1	0.50	5.00	0.50	0.50	
Lambda 2	0.50	5.00	0.82	0.50	
Lambda 3	0.50	5.00	1.14	0.50	
Lambda 4	0.50	5.00	1.46	0.50	
Lambda 5	0.50	5.00	1.79	0.50	
Lambda 6	0.50	5.00	2.11	0.50	
Lambda 7	0.50	5.00	2.43	0.50	
Lambda 8	0.50	5.00	2.75	0.50	
Lambda 9	0.50	5.00	3.07	0.50	
Lambda 10	0.50	5.00	3.39	0.50	
Lambda 11	0.50	5.00	3.71	0.50	
Lambda 12	0.50	5.00	4.04	0.50	
Lambda 13	0.50	5.00	4.36	0.50	
Lambda 14	0.50	5.00	4.68	0.50	
Lambda 15	0.50	5.00	5.00	5.00	

Appendix C-Table 2: m=10 Values

m=10	Fixed	Delta Increase	Delta Decrease	Skewed Top	Skewed Bottom
Alpha 1	0.100	0.018	0.182	0.500	0.056
Alpha 2	0.100	0.036	0.164	0.056	0.056
Alpha 3	0.100	0.055	0.145	0.056	0.056
Alpha 4	0.100	0.073	0.127	0.056	0.056
Alpha 5	0.100	0.091	0.109	0.056	0.056
Alpha 6	0.100	0.109	0.091	0.056	0.056
Alpha 7	0.100	0.127	0.073	0.056	0.056
Alpha 8	0.100	0.145	0.055	0.056	0.056
Alpha 9	0.100	0.164	0.036	0.056	0.056
Alpha 10	0.100	0.182	0.018	0.056	0.500
m=10	**Flat Min.**	**Flat Max**	**Linear**	**Weighted**	
Lambda 1	0.50	5.00	0.50	0.50	
Lambda 2	0.50	5.00	1.00	0.50	
Lambda 3	0.50	5.00	1.50	0.50	
Lambda 4	0.50	5.00	2.00	0.50	
Lambda 5	0.50	5.00	2.50	0.50	
Lambda 6	0.50	5.00	3.00	0.50	
Lambda 7	0.50	5.00	3.50	0.50	
Lambda 8	0.50	5.00	4.00	0.50	
Lambda 9	0.50	5.00	4.50	0.50	
Lambda 10	0.50	5.00	5.00	5.00	

Appendix C-Table 3: m=5 Values

m=5	Fixed	Delta Increase	Delta Decrease	Skewed Top	Skewed Bottom
Alpha 1	0.200	0.067	0.333	0.500	0.125
Alpha 2	0.200	0.133	0.267	0.125	0.125
Alpha 3	0.200	0.200	0.200	0.125	0.125
Alpha 4	0.200	0.267	0.133	0.125	0.125
Alpha 5	0.200	0.333	0.067	0.125	0.500
m=5	**Flat Min.**	**Flat Max**	**Linear**	**Weighted**	
Lambda 1	0.50	5.00	0.50	0.50	
Lambda 2	0.50	5.00	1.63	0.50	
Lambda 3	0.50	5.00	2.75	0.50	
Lambda 4	0.50	5.00	3.88	0.50	
Lambda 5	0.50	5.00	5.00	5.00	

Appendix C – All Type I Error Rate Tables

Appendix D-Table 1: n=5, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.004245	0.002240	0.003287	0.008133
	Delta Increase	0.004245	0.002240	0.002612	0.004641
	Delta Decrease	0.004245	0.002400	0.004992	0.014846
	Skewed Top	0.004245	0.002240	0.004669	0.010831
	Skewed Bottom	0.004245	0.002240	0.003221	0.003309

Appendix D-Table 2: n=75, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.003124	0.002123	0.002607	0.003238
	Delta Increase	0.003124	0.002123	0.002490	0.002555
	Delta Decrease	0.003124	0.002123	0.002761	0.003928
	Skewed Top	0.003124	0.002123	0.002532	0.003566
	Skewed Bottom	0.003124	0.002123	0.002194	0.002517

Appendix D-Table 3: n=150, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.002953	0.005444	0.002371	0.002821
	Delta Increase	0.002953	0.005444	0.002392	0.002383
	Delta Decrease	0.002953	0.005444	0.002641	0.003499
	Skewed Top	0.002953	0.005444	0.002529	0.002936
	Skewed Bottom	0.002953	0.005444	0.002130	0.002343

Appendix D-Table 4: n=5, m=10

		Lambda			
		Flat Min.	Flat Max.	Linear	Weighted
Alpha	Fixed	0.004245	0.002240	0.003811	0.014378
	Delta Increase	0.004245	0.002240	0.003149	0.008996
	Delta Decrease	0.004245	0.002240	0.005308	0.016851
	Skewed Top	0.004270	0.002240	0.005867	0.016788
	Skewed Bottom	0.004270	0.002240	0.002531	0.003309

Appendix D-Table 5: n=75, m=10

		Lambda			
		Flat Min.	Flat Max.	Linear	Weighted
Alpha	Fixed	0.003124	0.002123	0.002436	0.003674
	Delta Increase	0.003124	0.002123	0.002431	0.003280
	Delta Decrease	0.003124	0.002123	0.002669	0.005110
	Skewed Top	0.003124	0.002123	0.002617	0.004038
	Skewed Bottom	0.003124	0.002123	0.002345	0.002517

Appendix D-Table 6: n=150, m=10

		Lambda			
		Flat Min.	Flat Max.	Linear	Weighted
Alpha	Fixed	0.002953	0.005444	0.002445	0.003233
	Delta Increase	0.002953	0.005444	0.002176	0.002680
	Delta Decrease	0.002953	0.005444	0.002490	0.003646
	Skewed Top	0.002953	0.005444	0.002524	0.003375
	Skewed Bottom	0.002953	0.005444	0.002216	0.002343

Appendix D-Table 7: n=5, m=15

		Lambda			
		Flat Min.	Flat Max.	Linear	Weighted
Alpha	Fixed	0.004245	0.002240	0.003451	0.014846
	Delta Increase	0.004245	0.002240	0.002854	0.010831
	Delta Decrease	0.004245	0.002240	0.004642	0.012562
	Skewed Top	0.004245	0.002240	0.007527	0.014115
	Skewed Bottom	0.004245	0.002240	0.002697	0.003309

Appendix D-Table 8: n=75, m=15

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.003124	0.002123	0.002523	0.003928
	Delta Increase	0.003124	0.002123	0.002528	0.003566
	Delta Decrease	0.003124	0.002123	0.002694	0.003829
	Skewed Top	0.003124	0.002123	0.002835	0.004887
	Skewed Bottom	0.003124	0.002123	0.002401	0.002517

Appendix D-Table 9: n=150, m=15

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.002952	0.005444	0.002408	0.003499
	Delta Increase	0.002953	0.005444	0.002146	0.002936
	Delta Decrease	0.002953	0.005444	0.002370	0.003255
	Skewed Top	0.002953	0.005444	0.002598	0.003834
	Skewed Bottom	0.002953	0.005444	0.002232	0.002345

Appendix D – All Type II Error Rate Tables

Appendix E-Table 1: y=10%, n=5, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.999411	0.998703	0.998952	0.995902
	Delta Increase	0.999411	0.998703	0.999246	0.998124
	Delta Decrease	0.999411	0.998703	0.997988	0.994102
	Skewed Top	0.999411	0.998703	0.998079	0.993901
	Skewed Bottom	0.999411	0.998703	0.999497	0.999365

Appendix E-Table 2: y=10%, n=75, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.997919	0.908302	0.988917	0.998757
	Delta Increase	0.997919	0.908302	0.977045	0.997302
	Delta Decrease	0.997919	0.908302	0.994695	0.999066
	Skewed Top	0.997919	0.908302	0.996231	0.999081
	Skewed Bottom	0.997919	0.908302	0.981053	0.993985

Appendix E-Table 3: y=10%, n=150, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.993811	0.727124	0.968217	0.996467
	Delta Increase	0.993811	0.727124	0.931200	0.992634
	Delta Decrease	0.993811	0.727124	0.983399	0.998351
	Skewed Top	0.993811	0.727124	0.989766	0.997495
	Skewed Bottom	0.993811	0.727124	0.942092	0.983733

Appendix E-Table 4: y=10%, n=5, m=10

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.999411	0.998703	0.998764	0.994410
	Delta Increase	0.999411	0.998703	0.993420	0.996924
	Delta Decrease	0.999411	0.998703	0.998796	0.988564
	Skewed Top	0.999411	0.998703	0.997430	0.993295
	Skewed Bottom	0.999411	0.998703	0.999329	0.999365

Appendix E-Table 5: y=10%, n=75, m=10

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.997919	0.908302	0.988462	0.999220
	Delta Increase	0.997919	0.908302	0.975535	0.998816
	Delta Decrease	0.997919	0.908302	0.993649	0.998964
	Skewed Top	0.997919	0.908302	0.996960	0.999104
	Skewed Bottom	0.997919	0.908302	0.976383	0.993985

Appendix E-Table 6: y=10%, n=150, m=10

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.993811	0.727124	0.961694	0.997809
	Delta Increase	0.993811	0.727124	0.917313	0.996698
	Delta Decrease	0.993811	0.727124	0.980218	0.997903
	Skewed Top	0.993811	0.727124	0.990549	0.998354
	Skewed Bottom	0.993811	0.727124	0.929416	0.983733

Appendix E-Table 7: y=10%, n=5, m=15

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.999411	0.998703	0.998900	0.994102
	Delta Increase	0.999411	0.998703	0.999100	0.993901
	Delta Decrease	0.999411	0.998703	0.998175	0.991189
	Skewed Top	0.999411	0.998703	0.997910	0.998599
	Skewed Bottom	0.999411	0.998703	0.999589	0.999365

Appendix E-Table 8: y=10%, n=75, m=15

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.997919	0.908302	0.987505	0.999060
	Delta Increase	0.997919	0.908302	0.973590	0.999081
	Delta Decrease	0.997919	0.908302	0.993119	0.998233
	Skewed Top	0.997919	0.908302	0.996786	0.990800
	Skewed Bottom	0.997919	0.908302	0.973948	0.993984

Appendix E-Table 9: y=10%, n=150, m=15

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.993811	0.727124	0.963686	0.998351
	Delta Increase	0.993811	0.727124	0.913742	0.997459
	Delta Decrease	0.993811	0.727124	0.976875	0.996157
	Skewed Top	0.993811	0.727124	0.990625	0.998109
	Skewed Bottom	0.993811	0.727124	0.924577	0.983732

Appendix E-Table 10: y=25%, n=5, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.998905	0.994993	0.999817	0.998539
	Delta Increase	0.998905	0.994993	0.999705	0.999531
	Delta Decrease	0.998905	0.994993	0.999483	0.997226
	Skewed Top	0.998905	0.994993	0.999496	0.997401
	Skewed Bottom	0.998905	0.994993	0.999887	0.999908

Appendix E-Table 11: y=25%, n=75, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.987006	0.170157	0.863697	0.994180
	Delta Increase	0.987006	0.170157	0.683259	0.981731
	Delta Decrease	0.987006	0.170157	0.942325	0.998283
	Skewed Top	0.987006	0.170157	0.967231	0.996991
	Skewed Bottom	0.987006	0.170157	0.739638	0.943876

Appendix E-Table 12: y=25%, n=150, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.938342	0.004697	0.554144	0.966540
	Delta Increase	0.936983	0.004697	0.246691	0.915006
	Delta Decrease	0.936983	0.004697	0.759122	0.986120
	Skewed Top	0.936983	0.004697	0.860963	0.979121
	Skewed Bottom	0.937940	0.004697	0.311082	0.776429

Appendix E-Table 13: y=25%, n=5, m=10

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.998900	0.994988	0.999777	0.997555
	Delta Increase	0.998900	0.994988	0.999366	0.998926
	Delta Decrease	0.998900	0.994988	0.999720	0.993543
	Skewed Top	0.998900	0.994988	0.999254	0.996716
	Skewed Bottom	0.998900	0.994988	0.999757	0.999908

Appendix E-Table 14: y=25%, n=75, m=10

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.987006	0.170157	0.845308	0.997976
	Delta Increase	0.987006	0.170157	0.646398	0.994822
	Delta Decrease	0.987006	0.170157	0.924152	0.997650
	Skewed Top	0.987006	0.170157	0.973700	0.998750
	Skewed Bottom	0.987006	0.170157	0.674209	0.943876

Appendix E-Table 15: y=25%, n=150, m=10

		Lambda			
		Flat Min.	Flat Max.	Linear	Weighted
Alpha	Fixed	0.938342	0.004697	0.486133	0.981999
	Delta Increase	0.938342	0.004697	0.189437	0.969900
	Delta Decrease	0.938342	0.004697	0.703063	0.979259
	Skewed Top	0.938342	0.004697	0.874276	0.986179
	Skewed Bottom	0.938342	0.004697	0.234131	0.776429

Appendix E-Table 16: y=25%, n=5, m=15

		Lambda			
		Flat Min.	Flat Max.	Linear	Weighted
Alpha	Fixed	0.998900	0.994988	0.999806	0.997226
	Delta Increase	0.998900	0.994988	0.999408	0.997401
	Delta Decrease	0.998900	0.994988	0.999544	0.994776
	Skewed Top	0.998900	0.994988	0.999407	0.995321
	Skewed Bottom	0.998900	0.994988	0.999832	0.999908

Appendix E-Table 17: y=25%, n=75, m=15

		Lambda			
		Flat Min.	Flat Max.	Linear	Weighted
Alpha	Fixed	0.987006	0.170157	0.832716	0.998283
	Delta Increase	0.987006	0.170157	0.625272	0.996991
	Delta Decrease	0.987006	0.170157	0.916258	0.992211
	Skewed Top	0.987006	0.170157	0.972843	0.998575
	Skewed Bottom	0.987006	0.170157	0.648259	0.943876

Appendix E-Table 18: y=25%, n=75, m=15

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.938342	0.004697	0.486053	0.986121
	Delta Increase	0.936983	0.004697	0.175521	0.979121
	Delta Decrease	0.936983	0.004697	0.668434	0.961661
	Skewed Top	0.936983	0.004697	0.876791	0.982998
	Skewed Bottom	0.937940	0.004697	0.212416	0.776428

Appendix E-Table 19: y=35%, n=5, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.999328	0.988425	0.999885	0.999263
	Delta Increase	0.999328	0.988425	0.998330	0.999816
	Delta Decrease	0.999328	0.988425	0.999790	0.998305
	Skewed Top	0.999328	0.988425	0.999793	0.998516
	Skewed Bottom	0.999328	0.988425	0.999892	0.999979

Appendix E-Table 20: y=35%, n=75, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.967226	0.011245	0.643636	0.985186
	Delta Increase	0.967226	0.011245	0.331281	0.951121
	Delta Decrease	0.967226	0.011245	0.836859	0.995438
	Skewed Top	0.967226	0.011245	0.908467	0.992413
	Skewed Bottom	0.967226	0.011245	0.409233	0.844875

Appendix E-Table 21: y=35%, n=150, m=5

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.837217	0.000004	0.182622	0.903518
	Delta Increase	0.837217	0.000004	0.023377	0.762204
	Delta Decrease	0.837217	0.000004	0.437658	0.958904
	Skewed Top	0.837217	0.000004	0.630679	0.940445
	Skewed Bottom	0.837217	0.000004	0.040273	0.462642

Appendix E-Table 22: y=35%, n=5, m=10

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.999328	0.988425	0.999931	0.998577
	Delta Increase	0.999328	0.988425	0.998934	0.999465
	Delta Decrease	0.999328	0.988425	0.999893	0.995543
	Skewed Top	0.999328	0.988425	0.999671	0.997936
	Skewed Bottom	0.999328	0.988425	0.999690	0.999979

Appendix E-Table 23: y=35%, n=75, m=10

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.967226	0.011245	0.602687	0.994754
	Delta Increase	0.967226	0.011245	0.282605	0.986946
	Delta Decrease	0.967226	0.011245	0.787829	0.993325
	Skewed Top	0.967226	0.011245	0.925504	0.996560
	Skewed Bottom	0.967226	0.011245	0.317150	0.844875

Appendix E-Table 24: y=35%, n=150, m=10

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.837217	0.000004	0.130891	0.948496
	Delta Increase	0.837217	0.000004	0.012786	0.913643
	Delta Decrease	0.837217	0.000004	0.351804	0.937276
	Skewed Top	0.837217	0.000004	0.661848	0.958754
	Skewed Bottom	0.837217	0.000004	0.020018	0.462642

Appendix E-Table 25: y=35%, n=5, m=15

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.999328	0.988425	0.999942	0.998305
	Delta Increase	0.999328	0.988425	0.999050	0.998516
	Delta Decrease	0.999328	0.988425	0.999817	0.996284
	Skewed Top	0.999328	0.988425	0.999742	0.996920
	Skewed Bottom	0.999328	0.988425	0.999763	0.999978

Appendix E-Table 26: y=35%, n=75, m=15

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.967226	0.011245	0.578296	0.995438
	Delta Increase	0.967226	0.011245	0.260015	0.992413
	Delta Decrease	0.967226	0.011245	0.768067	0.979684
	Skewed Top	0.967226	0.011245	0.924025	0.995976
	Skewed Bottom	0.967226	0.011245	0.286780	0.844874

Appendix E-Table 27: y=35%, n=150, m=15

		Lambda			
Alpha		Flat Min.	Flat Max.	Linear	Weighted
	Fixed	0.837217	0.000004	0.128160	0.958904
	Delta Increase	0.837217	0.000004	0.010724	0.940445
	Delta Decrease	0.837217	0.000004	0.310054	0.892541
	Skewed Top	0.837217	0.000004	0.668583	0.949448
	Skewed Bottom	0.837217	0.000004	0.015894	0.462642