METHODS OF TEACHING ANGULAR MOMENTUM

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B. S., Concordia Teachers College, 1966

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE
PHYSICAL SCIENCE TEACHING

Department of Physics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

Approved by:

Major Professor

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CHAPTER I

INTRODUCTION

People have observed the motion of planets for thousands of years. For hundreds of years they have studied motion of other objects. Because of the increasing application of technology in everyday life, education in the sciences receives greater emphasis. Students are taught to wonder about observations of nature. How can winds in a tornado develop such high speeds? Why is the length of a day on earth constant? Why does the rate of rotation of a skater increase as she pulls in her arms? Such questions may be answered by an understanding of angular momentum.

Angular momentum is an aspect of motion. Rotating bodies are usually involved. Teaching the concepts of angular momentum at the introductory level can be done in various ways. Many illustrations and demonstrations are possible.

DEFINITION OF TERMS

The reader is presumed to be acquainted with much of the terminology in this report. For completeness, however, a brief collection of definitions of the basic terms which will be used often is given below.

A scalar quantity is a quantity which has magnitude only. Kinetic energy is one example.

A vector has both a magnitude and a direction. Examples of vectors are force, $\vec{F}$, linear momentum, $\vec{p}$, and angular momentum, $\vec{L}$. An arrow above a symbol indicates that it is a vector.

Velocity, $\vec{v}$, is defined as the rate of change of displacement, $\vec{s}$, with time, t,

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t},$$

$$\vec{v} = \frac{d\vec{s}}{dt}.$$

Speed is the magnitude of the velocity vector. Speed is a scalar.

Angular velocity, $\vec{\omega}$, is the rate of change of the positive angle theta, θ, with time and is directed along the axis around which θ changes. The direction of $\vec{\omega}$ is the direction in which a right hand screw would advance if turned in the direction of the changing angle as in the figure below.

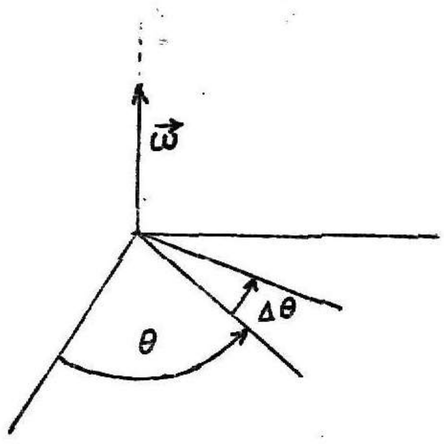

Figure showing direction of angular velocity vector.

The magnitude of the angular velocity, or angular speed, ω, is defined as,

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t};$$

$$\omega = \frac{d\theta}{dt}.$$

The moment of inertia, I, of a point mass, m, rotating about a fixed axis is defined as the product of the mass times the square of the distance, r, to the axis of rotation,

$$I = mr^2.$$

The moment of inertia, I, of a body is the sum of the moments of inertia of each individual particle making up the body,

$$I = \sum_i m_i r_i^2.$$

Linear momentum, $\vec{p}$, is defined as the product of the mass of a particle times its velocity,

$$\vec{p} = m\vec{v}.$$

Force, $\vec{F}$, is related to the time rate of change of linear momentum according to Newton's second law as,

$$\vec{F} = \frac{d\vec{p}}{dt}.$$

The moment of force is defined as the vector cross product of the radius vector, $\vec{r}$, from a point and the force vector,

$$\text{moment of force} = \vec{r} \times \vec{F}.$$

The moment of force is called torque, $\vec{T}$,

$$\vec{T} = \vec{r} \times \vec{F}.$$

The moment of momentum is the vector cross product of the radius vector and the linear momentum vector,

$$\text{moment of momentum} = \vec{r} \times \vec{p}.$$

Angular momentum, $\vec{L}$, is defined as the product of the moment of inertia and the angular velocity, $\vec{\omega}$,

$$\vec{L} = I\vec{\omega}.$$

Angular momentum is also defined as the cross product of the radius vector and linear momentum vector,

$$\vec{L} = \vec{r} \times \vec{p}.$$

Torque may be shown to be equal to the time rate of change of angular momentum by use of the above definitions and Newton's second law,

$$\vec{T} = \frac{d\vec{L}}{dt}.$$

Precession is the gradual movement of the angular momentum vector resulting from an applied torque. If the torque is perpendicular to the angular momentum, the change in angular momentum is also at right angles to the angular momentum. This causes the magnitude of the angular momentum to remain unchanged but causes the direction of the angular momentum vector to change in space.

Rotational kinetic energy, E_{rot}, is defined according to the following equation,

$$E_{rot} = \tfrac{1}{2}I\omega^2.$$

THE PROBLEM

The purpose of this study is to answer the following questions:

1. What methods of instruction, illustrations, and demonstration materials have been used in the teaching of the concept of angular momentum at the introductory level?

2. What changes over the years have resulted in the methods and materials used?

TIMELINESS OF THIS PAPER

This paper is pertinent because of the variation of methods available in teaching angular momentum. The author has undertaken this study to determine what is being and has been used by educators and to summarize the instruction techniques in one paper.

PROCEDURE EMPLOYED IN THE STUDY

The author collected his information for this report from available literature. The materials are organized in such a way as to be able to answer the questions stated in the problem.

CHAPTER II

METHODS OF INSTRUCTION, ILLUSTRATIONS, AND DEMONSTRATIONS USED IN TEACHING ANGULAR MOMENTUM

Various methods of instruction are used in teaching angular momentum. Numerous illustrations are available. A variety of demonstrations have been found useful. Those presented in this paper are such that have been found to be of value in use at the introductory level for the student's first exposure to angular momentum. Methods and materials used in advanced dynamics, atomic structure, or quantum mechanics have not been included.

The theoretical treatment of angular momentum at this level is fairly straight forward. A great deal of variation in teaching of the theory does not exist. The following pages are devoted especially to the illustrations and demonstrations used.

During the late 1800's angular momentum was defined as the moment of momentum. The moment of momentum was defined as the product of mass and velocity times the perpendicular distance from a point to the line of travel. The principle of the conservation of moment of momentum was taught in first year college physics classes.[1,2] In more recent times, however, angular momentum is usually defined differently. A rigid body rotating with an angular velocity about an axis has an angular momentum given by the product of the moment of inertia of the body about the axis and the angular velocity.[3] This definition seems to be more suitable for further mathematical application.

Some of the most important and most general laws used in physics are conservation laws. In particular, the study of conservation of angular momentum might be worthwhile. The total angular momentum of a system is conserved.[4] Winans defines angular momentum as stored angular impulse. He states that since for every angular impulse there is another angular impulse equal in magnitude and opposite in direction, the total angular impulse stored in the universe must be zero. Therefore, the total angular momentum in the universe is zero.[5]

Angular momentum is a vector.[6] Angular momentum is defined as the cross product of the radius vector and the linear momentum vector.[7,8] A change of angular momentum is given by the product of torque and time.[9] If there is no external torque the angular momentum must remain constant. Thinking about the situation another way, angular momentum is distance times perpendicular component of momentum. Since rate of change of momentum is force, the rate of change of angular momentum is distance times the perpendicular component of force. The rate of change of angular momentum equals torque. Thus for a stationary object to keep from rotating the sum of torques about a point must be zero.[10]

Forces may be central. A central force is one which is directed along a line joining the centers of mass of two bodies and thus produces no torque on the system. When a force is central the angular momentum relative to the center of force is a constant. As an example, since the gravitational force of attraction of the earth to the sun is central, the angular momentum of the earth about the sun is constant.[11] When a force is axial, the component of the angular momentum along the axis is constant.[12]

Whenever the moment of inertia of a rotating system changes, one must examine whether there is conservation of angular momentum or conservation of kinetic energy. If no external torques are present, angular momentum is conserved. Angular momentum is proportional to the angular velocity. Kinetic energy is proportional to the square of the angular velocity. Thus both cannot, in general, remain constant when the moment of inertia changes.[13] In general, if the radius of a rotating object decreases the angular velocity increases.[14]

The angular momentum of a rotating body is not necessarily parallel to the angular velocity.[15,16] Resnick cites an example of the angular momentum vector of an unsymmetrical rotating dumbbell whose bar makes an angle with the fixed axis of rotation passing through its center of mass.[17]

In an article by Armstrong the suggestion is made that changes in angular momentum should be defined in terms of ordinary Newtonian principles of forces and reactions.[18] When writing the analogous expression for Newton's second law he says authors state torque is equal to the time rate of change of angular momentum. However, he says, this is not always so. He states:

> "Suppose that a wheel was made by packing sand together (say) and it was set rotating, no torques acting. Suppose that the sand started to flake off, so that I was decreasing. Would ω increase? Surely it would remain constant. For the fact that pieces have come away is not going to affect the motion of the remaining parts."[19]

It should be remembered, however, that if the total system is considered, the angular momentum remains constant. In the above example the pieces that have come away from the wheel could still be considered as part of the system.

An early illustration of the use of angular momentum, although not stated as such, was given by Arnott in 1833. He cites the example of a flywheel used to equalize the effect of an irregular force.[20] In a recent physics text such a use of conservation of angular momentum was illustrated. A flywheel is used to maintain constant speed of rotation of a motor.[21]

In 1916 Arnold said that angular momentum is the principle which is taken advavtage of to hold the Brennan motor-rail car in an upright position. He said it is also the principle of the Sperry stabilizer used in boats and aeroplanes.[22]

A demonstration often used is that of a student on a turntable. He stands with arms extended and a weight in each hand. He is given a small angular speed. As he lowers his arms or pulls them in his speed increases greatly. Since angular momentum is conserved, the decrease of moment of inertia necessitates higher angular velocity.[23,24]

In a situation similar to the previous one a mechanical apparatus is used. It consists of two weights free to slide on a horizontal rod that can rotate about a vertical axis. The system is set into rotation with the weights at the ends of the rod. The weights are then pulled toward the center by strings that pass over pulleys to a handle held at the center above the rod. As the radial distance of the weights decreases, the angular speed of the system increases.[25,26] In another similar apparatus two metal spheres are mounted on the ends of thin metal rods. The rods are pin-hinged at the lower ends, enabling them to move in a vertical plane. The rods are supported by two thin strings which are held on the main axis by a clamp stand. The spheres are set at a particular distance apart by adjusting the support. The system is set into rotation and the period of rotation is timed. The moment of inertia is then changed by raising or lowering the support. The rotation rate is again measured. Calculations show that within an error of a few per cent the angular momentum remains constant. The apparatus is suitable for use in a quantitative experiment.[27]

The student on a turntable with a bicycle wheel offers a good demonstration. The student stands on a turntable and holds before him the front wheel of a bicycle. The wheel has handles on both ends of its axle. The wheel is held in a vertical plane passing through the axis of the turntable. If the student spins the wheel no rotation of the turntable results. If he then turns the spinning wheel into a horizontal plane, he will rotate in the sense opposite to the spin of the wheel. Turning the wheel over through 180 degrees reverses his direction of motion. Bringing it back to the vertical stops him. If he starts the wheel while it is held in a horizontal plane, he acquires equal and opposite angular momentum. In this case the two momenta can be shown to be equal by the fact that the student stops rotating if he turns the wheel to a vertical plane while it continues to spin or if he stops it while it is in a horizontal plane.[28-30] An electric drill may be used instead of a bicycle wheel but less spectacularly.[31]

It is interesting to demonstrate that angular momentum is a vector quantity that may be moved from place to place. The student stands at rest on a turntable. An assistant hands him a bicycle wheel spinning rapidly about a vertical axis. When the student turns the wheel over he gains an angular velocity. He then hands the wheel back to his assistant. The assistant rotates the axis through 180 degrees and returns it to the student on the turntable. The angular momentum of the student is doubled when he turns the wheel over a second time. Additional angular momentum may be given to him by repeating the process. If the assistant does not turn over the wheel one time but turns it over each time thereafter the process becomes subtractive. The angular velocity of the student decreases to zero. He then starts in the other direction. Similarly, six or more transfers of angular momentum may take place with a single spin of the wheel.[32]

In another demonstration the student stands on the turntable and swings a baseball bat. As the bat is given angular momentum in one direction the student and turntable acquire equal angular momentum in the opposite direction.[33]

Conservation of angular momentum may be shown by mounting a circular track on a bicycle wheel free to rotate about a vertical axis. A toy locomotive and a car with weights are run along the track. The track moves in the opposite direction. By changing the loading of the train, it is possible to control the velocities of both train and track relative to a given fixed point.[34]

The principle of conservation of angular momentum may be used effectively by some without their knowledge of the principle. A diver can control his angular velocity by controlling the curvature of his body.[35] A circus performer makes similar use of the principle.[36] A gymnast swinging around on a horizontal bar can change his angular velocity by drawing up his knees during motion.[37] In a manner similar to a student on a turntable, a ballet dancer[38] or figure skater[39] may start a spin with arms extended and then bring them in close to her body to increase angular speed. A cat, through the use of angular momentum conservation may execute a turn in midair.[40,41] When a moving skater catches the hand of a stationary skater, the first skater will have an angular momentum equal to the product of his momentum times the shortest distance between the two skaters if the second skater remains stationary.[42] Because a torque is needed to cause a change of angular momentum, the rotation of the wheels helps maintain the balance of a bicycle.[43] It is easier to learn to ride a bicycle with large wheels rather than small ones because of the greater angular momentum.[44]

Because the angular momentum of a rotating body remains constant in both magnitude and direction in the absence of external torque, conservation of angular momentum preserves the constant tilt of the polar axis of the earth.[45,46] The rotation rate of the earth is also constant. Even though tides produce frictional forces, the change in the length of an average day produced by tides is only a small fraction of a second each century.[47]

The constancy of direction of angular momentum may be shown by a spinning gyroscope. The gyroscope is supported in gimbals providing freedom of motion about three mutually perpendicular axes. If the axis of the spinning gyroscope is set in any direction, it will maintain that direction while the stand on which it is supported is moved along a devious path.[48]

Under proper conditions, gyroscopes may be shown to precess. Most of the phenomena of precession may be shown with a bicycle wheel equipped with handles on both ends of its axle. The wheel is given a spin by hand. One handle is then slipped into a loop of string for support. When the other handle is released, the wheel precesses about a vertical axis. At the same time its horizontal spin axis descends to become somewhat vertical. If a force is applied to the unsupported end of the axle in an attempt to accelerate the precession, the center of gravity rises. As the wheel spins more slowly, the wheel precesses more rapidly. The precession may be made more rapid by adding a weight to the unsupported end of the axle. If the opposite end of the axle is supported in the loop of string while the wheel spins in the same direction, the direction of precession will reverse. From these phenomena, a few of the rules of gyroscopic motion may be deduced. Included are such rules as the relation between directions of spin, torque, and precession and the relation between the magnitudes of spin, torque, and precession.[49] A discussion of precession and vectors involved is given by such authors as Black,[50] McCormick,[51] Benham,[52] and others.[53-56] The magnitude of precession is given by the magnitude of the torque divided by the magnitude of the angular momentum.[57,58]

Others have published articles dealing with gyroscopes and their motions. Sheldon uses the example of the gyrocompass.[59] Hufford describes the use of a Ford speedometer attached to a cylinder to measure its angular speed.[60] Eaton describes a method of construction of a large demonstration gyroscope. The rim of a bicycle wheel is weighted and used. Such an apparatus may be used to illustrate a turn indicator and ship stabilizer.[61] A similar large scale gyroscope has been described by Dosso and Vidal.[62] A demonstration used to stimulate interest involves first securing in a suitcase a large massive wheel driven by a battery powered motor. A volunteer is then asked to turn the suitcase as rapidly as he can.[63]

The earth is not a perfect sphere. Therefore torques provided by the moon and sun cause precession of the earth's axis of rotation. The precession period is about 26 000 years.[64]

Nutation of a gyroscope is discussed by Feynman. With a small gyroscope, however, this form of motion is difficult to observe of demonstrate because it damps out quickly due to friction in the gimbal bearings. Nutation may more easily be observed using a large massive sheel spinning more slowly.[65]

Some examples of effects produced by rotating bodies are included in introductory physics texts. Smith states that an aircraft with a single propeller acts like a gyroscope when trying to turn.[66,67] One form of helicopter has two propellers that rotate in opposite directions to keep the sum of angular momentum equal to zero.[68] Some helicopters use small propellers at their tails to keep from rotating about a vertical axis during flight.[69,70] This last example is not a case of gyroscopic effect. The frictional torque of air on the large propeller is made equal and opposite to the torque produced by the small propeller.

Angular momentum finds a place in planetary motion. Conservation of angular momentum requires a planet to move in an elliptical orbit about the sun.[71] For a planet's orbit a certain velocity is required for a given distance from the sun. A planet therefore cannot fall into the sun. To do so would require a greatly increased speed. Because gravitational energy is not great enough to produce the necessary kinetic energy increase, this process is forbidden by conservation laws.[72] Because the direction of the angular momentum vector is constant, a planet's orbit must lie in a plane.[73]

A low orbit earth satellite experiences some air resistance. The friction produces torque and the angular momentum decreases. Thus the radius of the orbit also decreases. Since the satellite moves closer to the earth, the potential energy decreases and the kinetic energy increases. The satellite's speed, therefore, also increases.[74]

Freier introduces an interesting application of angular momentum. The solar system has a tremendous quantity of angular momentum about the sun. Many theories of its creation met their downfall in failing to satisfy the law of conservation of angular momentum.[75]

Kepler's law of equal areas is a special case of the law of conservation of angular momentum. The area of a triangle swept out by a planet in a given time is approximately half the radius of the orbit times the perpendicular component of velocity. Kepler observed equal areas were swept out in equal time intervals. Multiplying by twice the mass of a planet results in a term equal to the angular momentum of the planet. Thus the angular momentum during one time interval is equal to that during any other. The total angular momentum of the planet and sun system and, thus, of the solar system is conserved.[76,77]

Angular momentum is conserved in a collision. Freier describes a useful demonstration apparatus. Ball A swings as a bifilar pendulum. At the lowest point of the swing it strikes an identical ball B. A stops while B turns freely about a vertical axis and strikes A again on the back side. B then stops and A completes its swing.[78] A two particle collision must take place in such a way that the plane of the trajectories after collision is the same as the plane of the trajectories before collision.[79]

Some other examples illustrating the conservation of angular momentum are given by various authors. Kolin says a man walking on the earth produces a slight change in the earth's rotation.[80] When fired, a rifle must acquire an angular momentum equal and opposite to that imparted to the bullet inside the muzzle.[81] Gamow suggests a possible method of turning a space ship. Mount a wheel on an axis perpendicular to the axis of the ship. If the wheel is set to rotating, the ship must rotate in the opposite direction. When the ship has rotated far enough the wheel is stopped. The ship stops rotating.[82]

A long projectile is given a rapid angular velocity about the long axis so that it will strike head on.[83] Likewise, a football is given angular momentum when thrown. Some rockets are fired in a similar manner.[84] A torpedo maintains its direction of motion through the water because of the angular momentum of the small engine which propels the torpedo through the water.[85]

Jones cites the principles used in driving a skidding automobile. The principal external forces that act on the car are frictional forces exerted on the wheels by the road. With the brake released, these forces are nearly perpendicular to the planes of the wheels. Suppose the rear wheels are skidding toward the right. The forces on them are toward the left. If the front wheels are now turned toward the right the external forces on the front wheels are directed toward the right and backward. The resultant torque is such as to decrease the angular momentum involved in the skid. Suppose the front wheels are turned toward the left. The forces acting on the front and rear wheels are more nearly parallel. Any torque they may exert is much smaller. Steering into the skid best uses the external forces available for reducing angular momentum.[86]

Angular momentum finds its use on the race track. The flywheel in most American racing cars rotates counterclockwise. Race tracks are usually built so that only left turns are made. As a car turns left, the torque on the motor acts in such a way as to force the front of the car downward. The front wheels dig into the road making the turns easier to execute.[87]

An apparatus has been described to demonstrate the rotation of the earth. A beam, curved slightly downward in the shape of a bowl and containing a small groove, is hung up horizontally on a long wire. Two equal spherical masses may be allowed to roll simultaneously from the ends of the beam to the middle. Since originally the beam is at rest relative to its surroundings, it has a specific angular velocity for a given latitude. When the masses are displaced the beam suddenly assumes a new angular velocity due to its change in moment of inertia. The new angular velocity will produce an easily detectable movement relative to the surroundings.[88,89]

Variation of the moment of inertia of the earth may produce changes in the earth's angular velocity.[90] During the twenty year period from 1910 to 1930 the length of the day changed by 0.004 7 seconds.[91] If the interior of the earth cools and shrinks the rate of rotation will increase slightly.[92] Building skyscrapers slows the earth's rotation. Mountain erosion increases the rotation.[93]

Conservation of angular momentum helps a tornado develop high speed winds. As the rotating air mass is drawn closer to the center because of the low pressure, the radius decreases rapidly. Conservation of angular momentum requires a corresponding increase in speed.[94]

If while a mass on a string is rotating in a horizontal circle the string is pulled down through a hole, angular momentum is conserved and the angular speed must increase.[95-98] In this case angular momentum is constant and kinetic energy increases.[99] If the mass swings around a peg so as to wind around it, kinetic energy remains constant and the angular momentum decreases.[100,101]

In one demonstration a wooden rod about two feet long grooved spirally along its length like the threading on a shaft is used. The pitch of the thread is not constant. Several wooden rings of different size and color are placed on the shaft. The rod is held vertically with the rings at the top. When the rings are released they gravitate down the rod. Since they have different moments of inertia they move at different rates.[102]

Another demonstration apparatus uses a pie plate suspended by three threads. The three threads are attached to a single long thread supported overhead. A weight rests on a ball-bearing screw arrangement about fifteen centimeters above the center of the pie plate. The weight may gravitate spirally downward, following grooves in the vertical screw. As it descends, the weight accelerates with a clockwise rotation as seen from above. At the bottom of its descent, friction quickly brings the weight to rest. The angular momenta of the weight and the pie plate should be equal and opposite. Therefore, when the descending weight stops rotating, the whole system should stop.[103]

The tippe top is a small toy top. It can be used in an effective demonstration. If set down while spinning, the top turns itself over. If an arrow is painted on the top and the top is spun on a flat surface, the top spins opposite the direction of the arrow after inversion. Angular momentum is conserved.[104]

In another demonstration two croquet balls are fixed to strings and hung from a support. The balls are given a circular motion about each other so that the string twists up. As the radius decreases the speed increases. The converse takes place when the string untwists.[105] In order to extend this demonstration, a string is prepared with loops at each end. When the system is completely wound up, it is stopped, and a loop is slipped over each ball. When released the angular velocity decreases as previously, but when the connecting string becomes taut, the moment of inertia becomes constant and the torque from the supporting strings causes an increase in angular velocity.[106]

Elliptic motion with conservation of angular momentum may be demonstrated. Under certain conditions a steel ball rolled in a glass funnel will describe an elliptic motion. Except for frictional losses, energy is also conserved.[107]

An apparatus consisting of a six inch diameter wooden disk supported by a loop of string passing through two holes drilled on either side of the center of the disk may be used as a demonstration apparatus. The string is twisted a few turns, and then by properly timed pulls the disk is set revolving at high speed about a horizontal axis. It revolves first in one direction and then in another. While the disk is spinning the string may be swung in almost any manner without changing the direction of the disk's axis.[108]

A pocket watch may be suspended by its ring from a nail filed to a sharp edge. The periodic changes of angular momentum of the balance wheel necessitate equal and opposite changes in some other body. If the period of the watch swinging as a pendulum from the nail is approximately equal to the period of the balance wheel, a large amplitude of swing of the whole watch results. If the watch is placed face down on a very smooth surface it also oscillates through a small angle. The motion may be made more visible by reflecting a beam of light from a small mirror mounted on the back of the watch.[109]

An apparatus using two circular masses with their axes connected by a spring from a window curtain roller may be used in a demonstration illustrating rotary action and reaction. The spring is wound by turning one mass. Both masses are then released simultaneously. The masses acquire opposite rotations. Each gains angular momentum opposite that of the other.[110]

A demonstrator may turn himself completely around on a turntable while always maintaining zero net angular momentum. To do so he extends his arms, executes a turn to the left, retracts his arms, executes a turn to the right, and repeats the process a number of times.[111]

Baseballs may be thrown or caught at arm's length by a student as he stands on a rotating turntable while his assistant stands on the floor. Each throw or each catch by the same hand increases the angular momentum of the student. He may decrease it or start himself in the other direction by catching the balls or throwing them with the other hand.[112]

Sutton describes a demonstration apparatus used to illustrate the method of how to work up in a swing. A similar apparatus uses an electronically operated swing.[113]

An early type of steam engine may be represented by a cylindrical boiler pivoted on a vertical axle. Four tubes emerge from the boiler. Each has a nozzle directed tangentially. Steam is generated in the boiler and escapes through the nozzles. The boiler is set into rotation. The angular momentum of the steam equals that of the boiler.[114]

A lawn sprinkler illustrates conservation of angular momentum. In a similar manner, a tall tin can may be set on a float or supported by a thread. Two tubes emerge from the can near the bottom. The tubes are placed at opposite ends of a diameter and bent in the same sense at their ends. When the can is filled with water, the water leaving through the jets imparts angular momentum to the can.[115]

One demonstration may be used to show that angular momentum may be transferred by friction. A gyroscope is pivoted on a vertical axis in a balanced framework that is supported by a thread. The gyroscope is given a spin and the framework released. As the wheel gradually slows down because of friction, the framework as a whole takes up rotation with conservation of angular momentum.[116]

Several new techniques for demonstrations and experiments have been suggested. Hecht has suggested using shadow projections for collision experiments.[117] A simple spring gun described by Rice[118] has been used to shoot steel balls into a plastic bottle attached to the end of a plate which is free to rotate about a fixed vertical shaft. The object of the experiment is to compare the angular momentum of the ball relative to the fixed shaft before collision with that of the rotating system after the ball is caught in the bottle.[119] The rotational speed of the system may be measured by use of an impact timer which strikes pressure sensitive tape applied to the rim of the plate.[120] Multiple exposure photography has been used. In one example a photograph shows that a rotating freely moving wrench rotates uniformly about its center of mass.[121] Air pucks on tables provide dynamics experiments with very low friction. Such equipment should prove useful in developing future angular momentum experiments.[122,123]

CHAPTER III

CHANGES IN INSTRUCTION METHODS

Until recent decades angular momentum received little emphasis in introductory college physics courses. The book authorized by Prince Leopold of Tuscany first published in 1684 contains four experiments with projectiles but contains nothing else involving mechanics.[124] Of those college texts studied by the author, none published prior to 1892 contained a definition or mention of angular momentum. The early 1900's saw angular momentum being defined in some college physics texts. Sheldon in 1926 stated the law of conservation of angular momentum and introduced vectors to discuss precession of a top.[125] Later texts used more examples and illustrations of angular momentum. Few used vectors to any extent. The use of vectors to define and introduce angular momentum in introductory college physics texts came during the late 1950's. Since that time and especially during the late 1960's vector definitions and vector calculations have become much more established.

High school physics texts such as Modern Physics,[126] Physics by the Physical Science Study Committee,[127] and Project Physics by Harvard Project Physics[128,129] do not include angular momentum as a specific topic in the regular texts.

The illustrations and demonstrations used in the teaching of angular momentum have not changed a great deal. An early demonstration was that of a man with outstretched arms on a rotating turntable. As the man pulls in his arms his angular velocity increases. This demonstration is still one of the most popular. As new and better equipment becomes available, demonstrations and experiments have been developed using the new types of apparatus. Some examples are the use of electronic timers on pressure sensitive tape, air pucks and air tables, and multiple exposure photographic techniques.

Vectors and mathematics increasingly dominate introductory physics courses. Rather than rely on only qualitative discussions as was often done in early college physics texts, specific numerical examples are used. More quantitative interpretation is possible.

A resource letter published in the American Journal of Physics in 1965 makes several suggestions. The major portion of the article is devoted to treatment of rigid body mechanics. The author says an understanding of angular momentum is a prerequisite to such study. However, he points out that "there is no necessity to attempt teaching rigid body motion in an introductory course."[130] He says a thorough understanding of vectors, the formulas used to describe angular momentum, and their limitations should be understood in order to study dynamics of rotational motion at the advanced level.[131]

CHAPTER IV

SUMMARY AND CONCLUSIONS

The teaching of angular momentum at the introductory level has progressed during the past century from a mere definition of the term to a mathematical introduction using various illustrations. The basic illustrations and demonstrations used have not varied or changed greatly. However, as new electronic and other materials for use in experiments and demonstrations become available there is a trend toward a greater use of them. The trend seems to be that as students gain better understanding of and ability in mathematical abstractions in the high school and elementary school, the methods used in teaching of angular momentum are also becoming increasingly more quantitative and mathematical.

Some demonstrations for use in the classroom are worthy of special recommendation. The student on a turntable with weights in his hands may be used for an easily observable qualitative study of the change of angular speed with a change of moment of inertia.

The apparatus with a variable moment of inertia is suitable for use in a quantitative study of angular momentum that might be done in a laboratory period. A gun of the type described previously which is used to shoot steel balls into a plastic bottle attached to the end of a plate which is free to rotate about a fixed vertical shaft may be used as another quantitative laboratory experiment. In this experiment the angular momentum before and after a collision may be compared.

The student holding a weighted bicycle wheel while standing on a turntable offers a good qualitative investigation of the vector nature of angular momentum. The tippe top may be used in a very simple demonstration of the conservation of direction of the angular momentum vector.

Studies in precession lend themselves quite well to the use of the weighted bicycle wheel. Such a wheel may also be used to demonstrate gyroscopic effects.

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METHODS OF TEACHING ANGULAR MOMENTUM

by

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B. S., Concordia Teachers College, 1966

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE
PHYSICAL SCIENCE TEACHING

Department of Physics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

METHODS OF TEACHING ANGULAR MOMENTUM

The purpose of the report was to answer the following questions:

1. What methods of instruction, illustrations, and demonstration materials have been used in the teaching of the concept of angular momentum at the introductory level?

2. What changes over the years have resulted in the methods and materials used?

The report contains brief descriptions of the various instruction methods, illustrations, and demonstrations used in teaching angular momentum. Demonstrations and illustrations used in teaching angular momentum have not changed significantly during past years. There is, however, a tendency toward a more quantitative mathematical treatment of the subject.