AN INVESTIGATION OF THE GOAL PROGRAMMING METHOD

by

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THIS BOOK CONTAINS NUMEROUS PAGES WITH THE ORIGINAL PRINTING BEING SKEWED DIFFERENTLY FROM THE TOP OF THE PAGE TO THE BOTTOM.

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The writer is indebted to Dr. Lynn E. Bussey for suggesting this research topic, proffering unlimited amounts of his time, demanding performance beyond that the writer conceived as possible, and most of all for his unlimited patience.

EDWIN OLIVER BAUMGARTEN

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER I

RESEARCH OBJECTIVES AND INTRODUCTION

Research Objectives

One of the objectives of this research is to investigate the method of goal programming as it is applied to the allocation of resources within the firm. A problem typically faced by the firm is one in which a limited amount of resources is to be allocated within the firm, which requires that certain decision parameters be established and selected in a manner that will optimally allocate the resources of the firm subject to a series of constraints. This kind of problem is typified by the so-called "optimal mix problem", the solution to which specifies certain operating parameters that will accomplish an optimal allocation of resources. An "optimal mix problem" will be used in this report to illustrate goal programming and the operating policy it provides the firm.

A second objective of this thesis is to incorporate an extension of the goal programming method, in which consequential actions for goal underachievement or overachievement are incorporated directly into the model.

Introduction

Goal programming is an extension of linear programming originally formulated by A. Charnes, W. W. Cooper [2], and Yuji Ijiri [7], which incorporates the concept of goals into the linear programming format. The concept of a "goal" must be distinguished from a "constraint" to differentiate goal programming from traditional linear programming. A constraint describes environmental conditions imposed upon the decision maker, which he cannot alter. A goal, on the other hand, is the result of a management desire. Management establishes and quantifies its goals with the idea that if these goals are attained, or even closely approached, the firm will operate successfully.

The traditional linear programming model seeks to optimize its objective function within a set of environmental constraints which bound the set of possible solutions. The only place a management desire can be expressed in the model is in the objective function and the optimal attainment of this desire is a result of the optimization process. Any subsidiary goals which management wishes to attain must be formulated as environmental constraints and as such are no longer goals. These constraints cannot be violated and, as a result, specify the feasible, attainable solutions.

The formulation of the allocation problem as a goal programming problem, on the other hand,allows the manager to establish multiple goals and place these goals, unaltered, into the model. These goals may be subjectively ranked and weighted in order of importance to the firm. The weights placed on goals typify the relative importance of each goal to management and force the optimization method to satisfy critical goals before attempting to satisfy less important goals. Only true environmental restrictions are treated as constraints to the solution.

A goal should be thought of as a target set by management. It is possible for such a target to be set at a level higher than that which can actually be attained. When confronted with such a situation, goal programming does not abandon the target but seeks to satisfy it as well as possible, even though it cannot be fully met. This factor becomes important when the firm is faced with a decision in which there is a conflict of incompatible goals. Using goal programming, it is possible to force the computational procedure to satisfy a goal, even though the solution that results lies outside what would be the convex set of feasible solutions allowed in a linear programming formulation of the same problem. Thus, the conflict of incompatible goals can be resolved in a logical and efficient manner.

Goal programming can be adapted to include information from the books of account (accounting records), so that full use can be made of the present and past internal business states in the final selection of proposed future operating plans. The use of the "goal" approach makes it possible for management to establish subgoals for various operating departments. The process of establishment of these subgoals requires dissemination of information between departments, which in itself results in an improved communication system. Multiperiod operating policies can be developed for all suborganizations, in line with total organization goals, thus giving management an integrated and time-ordered plan for the direction of the firm.

Goal programming is extremely flexible in that it allows for the incorporation of multiple targets into a proven computational procedure (linear programming). These goals may be compatible or incompatible, and weighted in order of relative importance to indicate superordinate and subordinate targets, to be approached or even attained in the final solution. The ease of incorporating accounting information makes data gathering a small part of problem formulation and creates a powerful decision tool which management can use to develop operating policies which are in line with the present internal financial status of the firm. (Barnard [1] says, "Decisions must be made in light of reality"). The establishment of explicit goals for use in the model provides improved coordination between departments and managers at all levels of the organization.

An Illustration of an "Optimal Mix" Linear Programming Problem

Linear programming, and the simplex method, can best be illustrated by a simple mix problem. In the mix problem the firm seeks to determine an operating policy which will result in the optimal use of certain resources of the firm.

To motivate this exposition, consider an example. The Chop-N-Block Corporation produces two products, a knife and a wooden cutting board. Each product requires the following inputs:

	Knife	Board
Direct Material Cost ($)	.50	1.00
Direct Labor Cost ($)	.50	1.00
Sales Price ($)	3.00	5.00
Machine Time (Hr.)	.50	.25
Assemble Time (Hr.)	1.00	1.00

In order to keep this example uncomplicated, we will make the following assumptions. Consumer demand for the products is strong, which means all knives and cutting boards that can be produced can be sold for the prices indicated. Production is limited by three restrictions. First, the firm has $28.00 in liquid resources (cash + bank credit + collections from prior sales) at the beginning of Period 1, the beginning of the firm's planning horizon. Second, we assume no cash sales (i.e. all goods are sold on credit) and no cash-on-hand required minimum balance, therefore this \$28.00 is the amount available to meet material and labor costs in Period 1. Third, the firm employs one machinist and the equivalent of two and one-half assemblers (each working 8 hours per period). This means that machine capacity and assembly capacity are 8 and 20 hours per period respectively. The problem to be solved then is, how many knives and cutting boards should the firm produce in Period 1 in order to maximize its profits, given the financial and capacity constraints?

The solution is straightforward. Let X_1 represent the number of knives to be produced and X_2 represent the number of cutting boards to be produced. The objective function and constraints are developed as follows. The profit on each product is the difference between the sales price and production costs for labor and material:

$$\begin{aligned} \text{PROFIT} &= (\text{Price} - \underset{\text{Cost}}{\text{Material}} - \underset{\text{Cost}}{\text{Labor}})_K \cdot X_1 \\ &\quad + (\text{Price} - \underset{\text{Cost}}{\text{Material}} - \underset{\text{Cost}}{\text{Labor}})_B \cdot X_2 \\ &= (3.00 - .50 - .50)X_1 + (5.00 - 1.00 - 1.00)X_2 \\ &= 2.00X_1 + 3.00X_2 \end{aligned}$$

Total material and labor costs for a knife and a cutting board are \$1.00 and \$2.00 respectively and must be paid from available cash resources. This leads to the financial constraint that total cash production costs must be equal to or less than available cash resources, or $1X_1 + 2X_2 \leq 28$. The total problem formulation thus becomes

Maximize PROFIT = $2X_1 + 3X_2$

subject to

(1)	$.5X_1 + .25X_2 \leq 8$ hours	(Machine capacity constraint)
(2)	$1X_1 + 1X_2 \leq 20$ hours	(Assembly capacity constraint)
(3)	$1X_1 + 2X_2 \leq 28$ dollars	(Financial resource constraint)
(4)	$X_1, X_2 \geq 0$	(Nonnegativity conditions)

The procedure for the solution of this linear programming problem is the simplex method (for an exposition of the simplex method see Hadley [5]). Constraints are converted to equalities by the addition of slack variables, S_j, where necessary:

Max PROFIT = $2X_1 + 3X_2 + 0S_1 + 0S_2 + 0S_3$

subject to

(1)	$.5X_1 + .25X_2 + S_1 + 0 + 0 = 8$	(Machine capacity constraint)
(2)	$1X_1 + 1X_2 + 0 + S_2 + 0 = 20$	(Assembly capacity constraint)
(3)	$1X_1 + 2X_2 + 0 + 0 + S_3 = 28$	(Financial resources constraint)
(4)	$X_1, X_2 \geq 0$	(Nonnegativity conditions)

The graphical solution, Figure 1, and the optimal solution, Figure 7 in Appendix A indicate that the optimal production policy is to make 12 knives and 8 cutting boards, resulting in a profit of \$48.00.[1] The cash resources (\$28.00), machine capacity (8 hours), and assembly capacity (20 hours) are fully utilized and restrict the simplex method from finding a more optimal solution. This total utilization of resources is evidenced by the fact that the slack variables for each of these resources (S_1, S_2, and S_3) are equal to zero.

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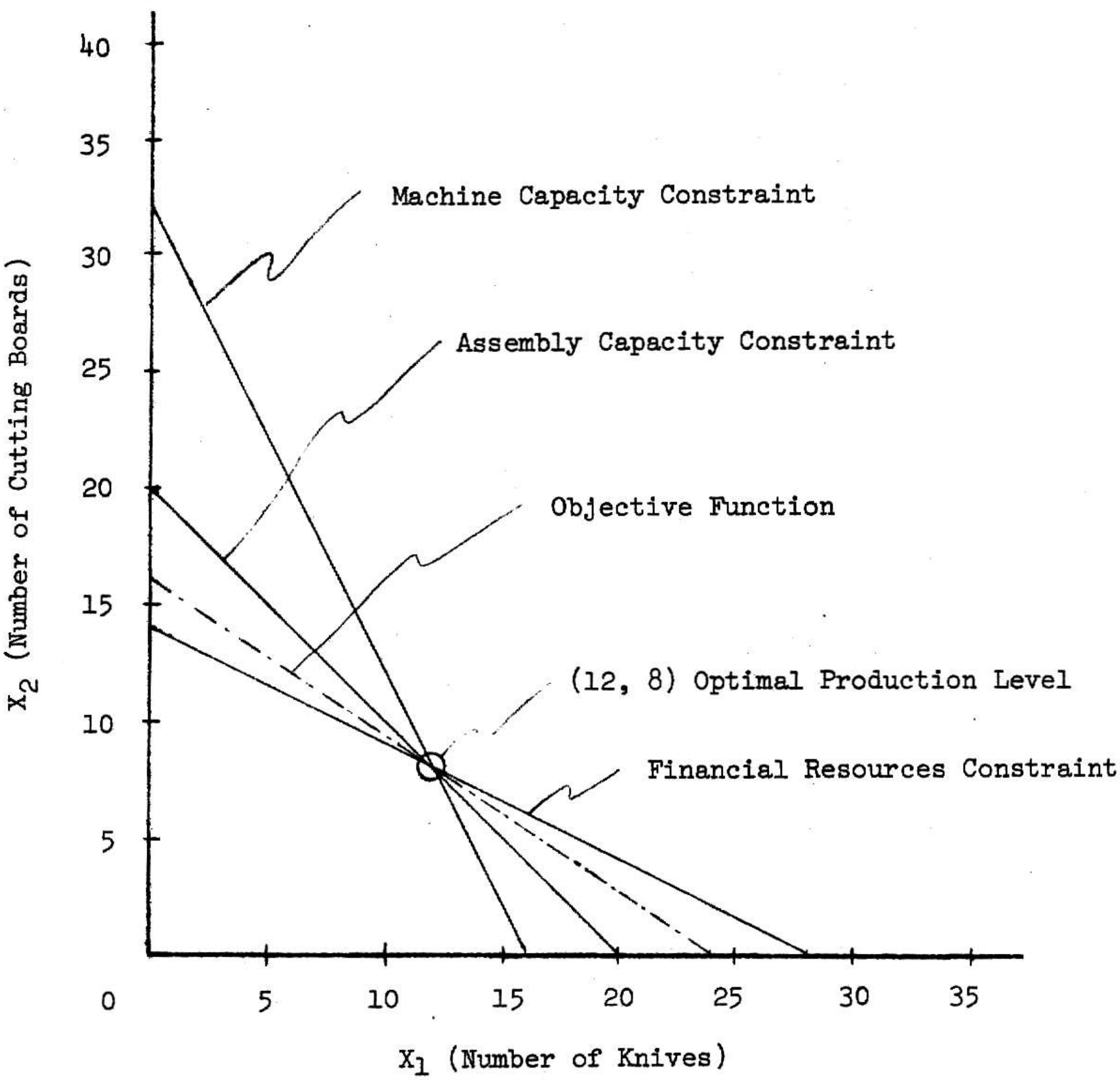


Figure 1. Graphical Solution to the "Optimal Mix" Linear Programming Problem

FOOTNOTES

[1] $2(12) + 3(8) = 48$.

CHAPTER II

THE GOAL PROGRAMMING METHOD

The entity being examined here is a firm. A real problem faced by all firms is the necessity to translate the overall purpose of the firm into subordinate purposes which can be made operational at subordinate levels of the organization. In the end, the accomplishment of subordinate purposes must result in the accomplishment of the firm's overall purpose. This implies control and direction of operating departments by establishing subordinate purposes which will result in attainment of the overall objective of the firm.

An optimization model of the firm must allow for the incorporation of both overall and subordinate goals. It must, through the optimization procedure, ferret out the operating policy or policies that best satisfy the overall goal of the organization through the satisfactory accomplishment of subordinate objectives. It is possible that subgoals may be in conflict with each other and as such parts of subgoals may have to be sacrificed to attain satisfactory accomplishment of the overall goal. The model should incorporate a scheme for ranking subgoals in order of importance, so as to permit attainment of some subgoals in preference to others in case of subgoal conflict.

This kind of "satisficing" behavior is not atypical to that evidenced by the modern day firm. Goal programming is a method by which subordinate goals can be successfully incorporated into a mathematical decision model so that the overall objective will be approached through the accomplishment of subobjectives.

The term "goals" should be distinguished from "constraints". Constraints are environmental conditions which are imposed on the firm and as such are inviolable. Goals are the announced desires of management expressed in mathmatical terminology. Constraints in optimization terminology are considered to be inviolate limits, whereas goals may be viewed as behaving like "constraints" but with the capability of being intentionally violated, with the added feature that the amount and direction of violation is explicitly determined by the optimization procedure.

The Concept of a Goal and a "Target"

In conventional linear programming formulations, feasibility constraints act as absolute bounds on the convex set of feasible solutions. No constraint is permitted to be violated in any basic solution, optimal or otherwise. It is possible to convert what would otherwise be a bounding constraint into a goal, so that the goal incorporates a defined right-hand-side "target" as an approachable, but perhaps unattainable, objective. The specific intent in goal construction is to permit intentional deviation away from a right-hand side target value without causing infeasibility in the basic solutions.

The concept of goal construction and target values (objectives) can be developed as follows. Let Y be a deviation from an announced target value, T, resulting from some actual performance, A, being less than, greater than, or equal to the target. From this reasoning, then $A + Y = T$, given $A, T > 0$; $A, T = 0$; $A, T < 0$. Thus, Y may take on values that are either positive, negative, or zero with respect to the target and the actual performance, irrespective of the signs or values of A and T.

One recognizes the deviation, Y, as being an unsigned variable, whose value and sign are determined by the value and sign of the actual performance, A, in relation to the value and sign of the target, T. To use linear programming computational techniques to solve goal programming problems, it is necessary that all decision variables, including the deviation variable, Y, be nonnegative. One technique for handling a variable that is unrestricted as to sign is to replace the unsigned variable with the difference between two nonnegative variables. Let the unsigned variable, Y, be the difference between two nonnegative variables, Y^- and Y^+, or $Y = Y^- - Y^+$. We define Y^- to be the amount that a target is undershot, that is, a "deficiency" deviation from goal attainment. Y^+ is accordingly defined as the amount that a target is overshot, that is, an "excess" deviation from goal attainment.

Let $Y = Y^- - Y^+$, given that $Y^-, Y^+ \geq 0$ and $Y^- \cdot Y^+ = 0$. The characteristic that the product of Y^- and Y^+ equals zero is a result of the simplex method which guarantees that at most one of the deviation variables, Y^- and Y^+, will be in solution (have a positive value) at any iteration.

There are three possible sign conditions for Y with respect to the target, T, and actual performance, A. Since $Y = Y^- - Y^+$, $(Y^-, Y^+ \geq 0)$, and since at most one of the deviation variables, Y^- and Y^+, can be in solution in any given basis, there are three combinations of Y^- and Y^+ which result in conditions identical to those for the unsigned deviation variable, Y (see Table 1).

TABLE I

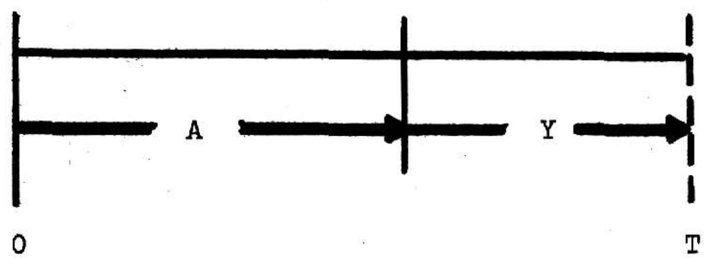
CONDITIONS FOR $Y = Y^- - Y^+$, $(Y^-, Y^+ \geq 0)$

	When:	Then:
(1)	$Y > 0$	$Y^- > 0, Y^+ = 0$
(2)	$Y < 0$	$Y^- = 0, Y^+ > 0$
(3)	$Y = 0$	$Y^- = 0, Y^+ = 0$

Condition 1 corresponds to goal underattainment, or a "deficiency" deviation from the target as earlier defined. Condition 2 corresponds to a goal overattainment, or an "excess" deviation from the target as earlier defined. Condition 3 corresponds to zero deviation, or "exact" attainment of the target.

To illustrate this concept, consider a cash <u>in</u>flow equation. Let the target, T, represent the amount of cash which one wishes to accumulate; let A represent the actual cash accumulation; and let Y be a value which must be added to A in order to equal T. A cash inflow is implied when A is positive.

(1) If cash is <u>under</u>accumulated, then $T > A$ (given $T, A \geq 0$), and Condition 1 in Table 1 holds true:



Since by definition

$$A + Y = T \quad , \qquad (Y > 0) \; ;$$

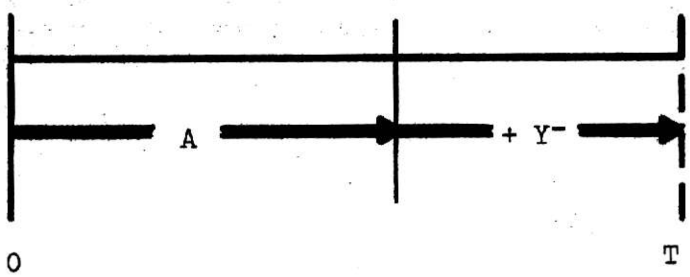
then on substituting $Y = Y^- - Y^+$ and the requirement from Table 1 that $Y^- > 0$, $Y^+ = 0$, one obtains

$$A + Y^- - Y^+ = T \quad , \qquad (Y^- > 0, \; Y^+ = 0) \; ;$$

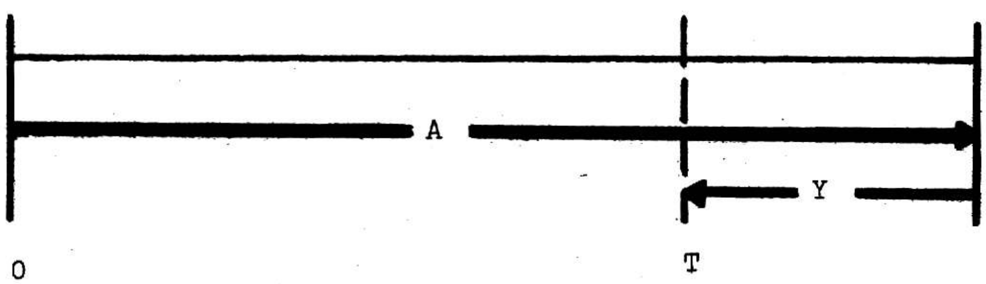
or

$$A + Y^- = T \qquad [1]$$

which is illustrated as:



(2) If cash is <u>over</u>accumulated, then $T < A$ (given $T, A \geq 0$), and Condition 2 in Table 1 holds true:



Since by definition

$$A + Y = T \quad , \qquad (Y < 0) ;$$

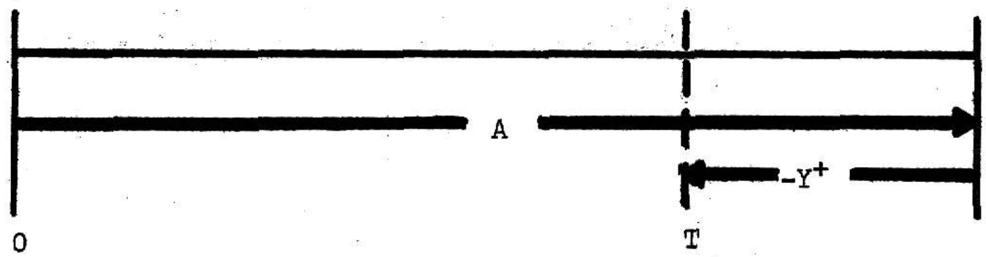
then on substituting $Y = Y^- - Y^+$ and the requirement that $Y^- = 0$, $Y^+ > 0$, one obtains

$$A + Y^- - Y^+ = T \quad , \qquad (Y^- = 0, Y^+ > 0);$$

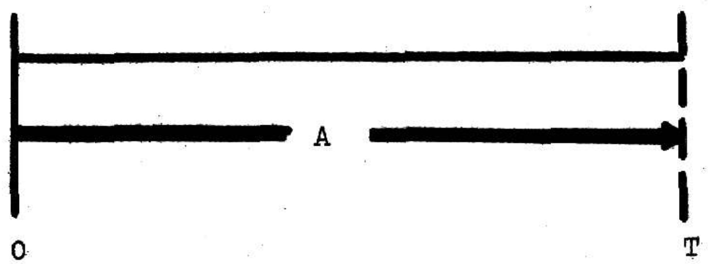
or

$$A - Y^+ = T \tag{2}$$

which is illustrated as:



(3) If cash is <u>neither</u> <u>under</u>accumulated <u>nor</u> <u>over</u>accumulated, then $A = T$ (given T, $A \geq 0$), so that Condition 3 in Table 1 holds:



Since by definition

$$A + Y = T \quad , \qquad (Y = 0) \; ;$$

then on substituting $Y = Y^- - Y^+$ and the requirement that $Y^- = 0$, $Y^+ = 0$, one obtains

$$A + Y^- - Y^+ = T \quad , \qquad (Y^- = 0, \; Y^+ = 0) \; ;$$

or

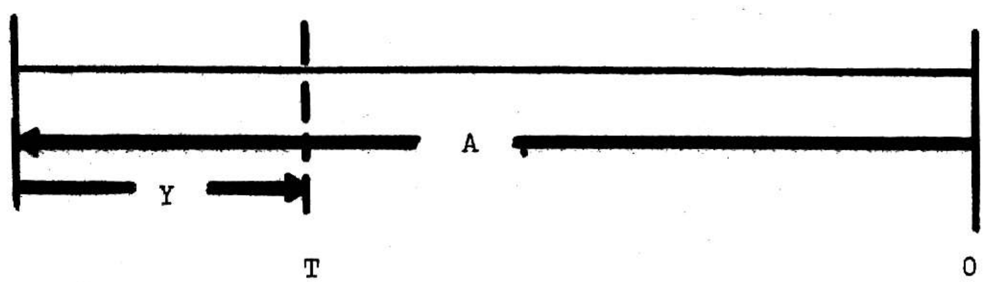
$$A = T \qquad [3]$$

Since it is known that at most one of the deviation variables, Y^- or Y^+, can be in solution in any given basis, these three Equations, [1], [2], and [3], can be combined to form Equation [4], representing the general cash <u>in</u>flow goal situation:

$$A + (Y^- - Y^+) = T \quad , \qquad (A, T \geq 0 \; ; \; Y^-, Y^+ \geq 0) \qquad [4]$$

In a similar manner, a general cash <u>out</u>flow goal equation can be developed. Let the target, T, represent a limit to the amount of cash one wishes to spend; let A represent the actual cash outflow; and let Y be a value which must be added to A in order to equal T. A cash outflow is implied when A is negative. Hereafter, we define $A, T \leq 0$.

(1) If the cash outflow <u>exceeds</u> the limit then $T > A$ (given A, $T \leq 0$), and Condition 1 in Table 1 holds true:



Since by definition

$$A + Y = T \quad , \quad Y > 0 \; ;$$

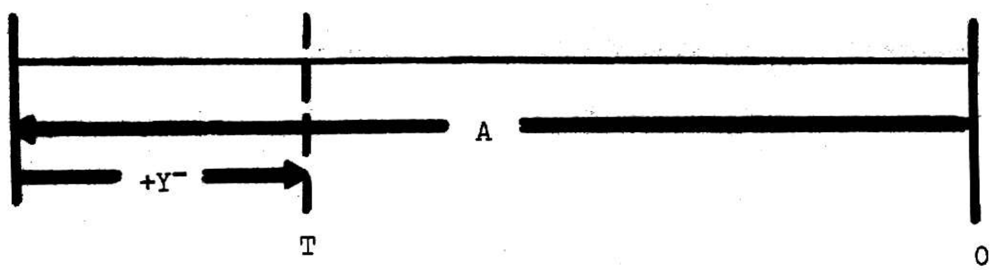
then on substituting $Y = Y^- - Y^+$ and the requirement from Table 1 that $Y^- > 0$, $Y^+ = 0$, one obtains

$$A + Y^- - Y^+ = T \quad , \quad (Y^- > 0,\ Y^+ = 0) \; ;$$

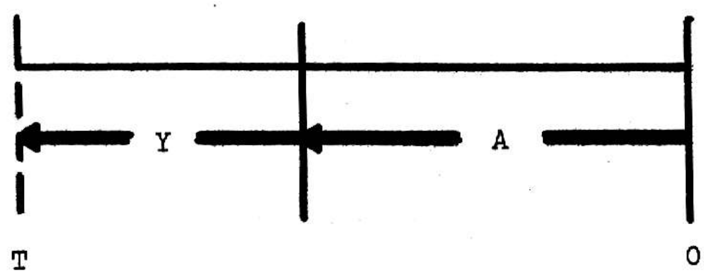
or

$$A + Y^- = T \tag{5}$$

which is illustrated as:



(2) If the cash outflow <u>does not exceed</u> the limit then $T < A$ (given $A, T \leq 0$), and Condition 2 in Table 1 holds true:



Since by definition

$$A + Y = T \quad , \quad (Y < 0) \; ;$$

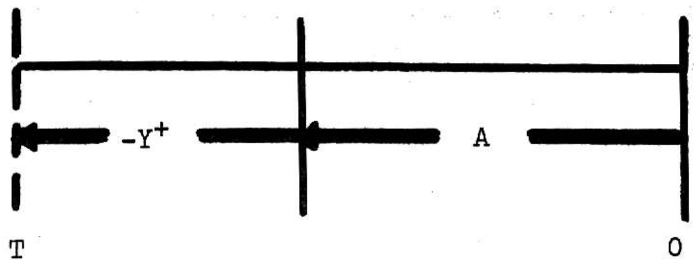
then on substituting $Y = Y^- - Y^+$ and the requirement from Table 1 that $Y^- = 0$, $Y^+ > 0$, one obtains

$$A + Y^- - Y^+ = T \quad , \quad (Y^- = 0,\ Y^+ > 0) \; ;$$

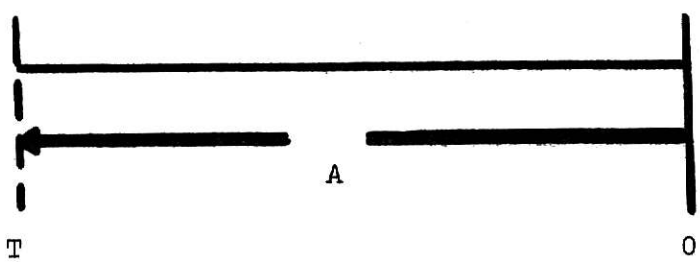
or

$$A - Y^+ = T \qquad [6]$$

which is illustrated as:



(3) If the cash outflow <u>exactly</u> <u>equals</u> the limit, then $T = A$ (given $A, T \leq 0$), so that Condition 3 in Table 1 holds true:



Since by definition

$$A + Y = T \quad , \qquad (Y = 0) ;$$

then on substituting $Y = Y^- - Y^+$ and the requirement from Table 1 that $Y^- = 0$, $Y^+ = 0$, one obtains

$$A + Y^- - Y^+ = T \quad , \qquad (Y^- = 0, Y^+ = 0) ;$$

or

$$A = T \tag{7}$$

Again since we know that at most one of the deviation variables, Y^- or Y^+, can be in solution in any given basis, these three Equations, [5], [6], and [7] can be combined to form Equation [8] representing the general cash <u>out</u>flow goal situation:

$$A + Y^- - Y^+ = T \quad , \qquad (A, T \le 0 ; Y^-, Y^+ \ge 0)$$

Recalling that earlier we defined A, $T \le 0$, then on multiplying both sides by (-1), one obtains

$$A - (Y^- - Y^+) = T \quad , \qquad (A, T \ge 0 ; Y^-, Y^+ \ge 0) \tag{8}$$

To summarize the foregoing briefly, note that both cash <u>in</u>flow and cash <u>out</u>flow goals are formulated in the form $A \pm Y = T$, with the sign on the deviation variable, Y, or its conjugate set, $Y^- - Y^+$, being the distinguishing difference. Thus, from Equation [4] a cash inflow goal is formulated as

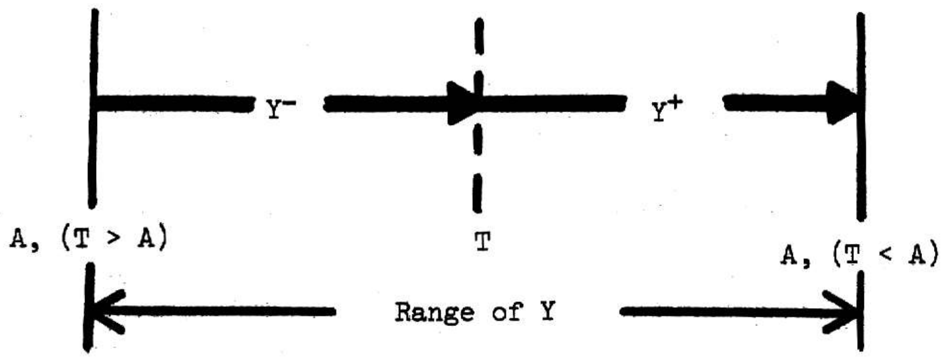
$$A + (Y^- - Y^+) = T$$

whereas from Equation [8] a cash outflow goal is formulated as

$$A - (Y^- - Y^+) = T \quad ,$$

with the understanding that Y^- and Y^+ are defined to be nonnegative deviation variables in the linear programming model.

Now, consider the formulation of the objective function. Observe that Y^- and Y^+ are defined as nonnegative variables, so that the sum of these two variables measures the range of the deviation variable Y about the target, T (recall that at most one of the nonnegative deviation variables Y^- and Y^+ will take on a positive value in any solution basis).



The simplex method should attempt to minimize the range of Y so as to approach the target as closely as possible. Therefore the objective function can be expressed in the form

$$\text{Min} \quad Z = Y^- + Y^+ \ .$$

The concepts developed in this section will be used to formulate problems in the goal programming format in the sections that follow.

An Illustration of a Goal Programming Problem

The simple product-mix example problem, which was illustrated in a prior section, will now be converted into a goal programming problem to illustrate the formulation of this type of problem. Let X_1 again represent the number of knives to be produced and X_2 be the number of cutting boards to be produced in a particular decision period. Profit will now be treated as a consequential variable resulting from the optimization procedure. Although the previous simplex linear programming problem formulation yielded an optimal solution in which the maximum profit attainable is $48.00, we now assume a profit target of $60.00, which is obviously in excess of the attainable profit.

Using the general form for a cash inflow goal, Equation [8], the profit goal is formulated as follows:

$$2X_1 + 3X_2 + (Y^- - Y^+) = 60$$

in which one can identify the contribution to profit and overhead as being $2X_1 + 3X_2$ and the profit target as 60. The question now is, what manufacturing policy should be undertaken to best satisfy environmental (production) constraints and the firm's profit goal? The goal programming formulation of the problem is given on the following page.

$$\text{Min} \quad Z = Y^- + Y^+$$

subject to

(1)	$.5X_1 + .25X_2$	≤ 8	(Machine capacity constraint)
(2)	$1X_1 + 1X_2$	≤ 20	(Assembly capacity constraint)
(3)	$1X_1 + 2X_2$	≤ 28	(Financial constraint)
(4)	$2X_1 + 3X_2 + Y^- - Y^+$	$= 60$	(Profit goal)
(5)	$X_1, X_2, Y^-, Y^+ \geq 0$		(Nonnegativity condition)

The environmental constraints, (1), (2), (3), are unchanged from the prior linear programming example. The profit goal, (4), is constructed such that the actual profit attainment, $2X_1 + 3X_2$: can fall short of the target, in which case $Y^- > 0$, $Y^+ = 0$; can exceed the target, in which case $Y^- = 0$, $Y^+ > 0$; or can equal the profit target of \$60.00 exactly, in which case $Y^- = 0$, $Y^+ = 0$. In order that the actual profit attainment be as close to the profit target as possible, the range of deviation from the profit goal must be as small as the environmental constraints will allow, that is, the range of deviation from the target, $Y^- + Y^+$, must be minimized. Thus, the object function becomes

$$\text{Min} \quad Z = Y^- + Y^+$$

All constraints are converted to equalities through the addition of slack variables and the simplex method is used to find an optimal solution. The simplex solution tableaus are included in Figure 8 of Appendix A. The goal programming formulation yields the following optimal solution:

$X_1 = 12.0$ $S_1 = 0.0$ $Y^- = 12.0$

$X_2 = 8.0$ $S_2 = 0.0$ $Y^+ = 0.0$

$S_3 = 0.0$

$$Z = Y^- + Y^+ = 12.0 + 0.0 = 12.0$$

If the firm undertakes this solution, it will produce and sell 12 knives and 8 cutting boards in Period 1. Machine capacity, assembly capacity, and available cash resources will be fully utilized, as is indicated by the zero value of the slack variables in the corresponding constraints (S_1, S_2, $S_3 = 0$). One should note that the profit goal of \$60.00 is not attained (as the simple linear programming formulation indeed demonstrated it could not be), but that the deficiency variable, Y^-, has a value of 12.0. This indicates an <u>under</u>attainment of the profit goal by \$12.00. Substituting $Y^- = 12.0$ and $Y^+ = 0.0$ into the profit goal equation,

$$2X_1 + 3X_2 + 12.0 = 60$$

one realizes that the actual profit is therefore 60 - 12 or \$48.00. The goal programming solution, in this case, is exactly equivalent to that obtained using traditional linear programming problem formulation.

<u>Expanded Goal Programming Problem</u>

Consider a slightly modified situation for the Chop-N-Block Corporation as summarized in Table 2 and Table 3. Changes from the prior problem are:

1. The company incurs fixed expenses in the amount of \$5.00 each period, which must be paid in cash.

2. Cash dividends amounting to $2.50 are to be paid at the end of each period and $2.50 is to be spent each period for equipment replacement.

3. Instead of selling on a cash basis, the firm now sells all finished products on a one period credit basis. It is assumed that all receivables are collected when due. Labor and material expenses must be paid in cash in the same period as when incurred. The firm again can sell all that it produces so there is no change in inventory from period to period.

Table 2 is a summary of cost and physical data for this altered situation.

TABLE 2

COST AND PHYSICAL DATA

	Knife	Cutting Board
Machine time/unit	1/2	1/4
Assembly time/unit	1	1
Selling price/unit	$3.00	$5.00
Variable expenses/unit (Labor, material, etc.)	$1.00	$2.00
Contribution to profit and overhead/unit	$2.00	$3.00

Total machine time available	8 hours/period
Total assembly time available	20 hours/period
Fixed cash expenses	$5.00/period
Expenditures for equipment	$2.50/period
Cash dividends	$2.50/period

Table 3 illustrates the balance sheet of the company at the beginning of Period 1 (end of Period 0).

TABLE 3

BALANCE SHEET, BEGINNING OF PERIOD 1

Assets		Liabilities and Net Worth	
Cash	$20.00	Bank Loan	$10.00
Accounts Receivable	30.00	Long Term Bonds Payable	30.00
Inventory	- 0 -		
Plant & Equipment	24.00	Stockholder's Equity	34.00
	$74.00		$74.00

The sum of cash and accounts receivable are the firm's "quick" assets which can be used to pay cash expenditures in Period 1, such as fixed and variable expenses, dividends, and equipment replacement. Taxes are assumed to be nonexistent and the inventory balance is assumed to be zero because the firm is able to sell all goods that it produces. These simplifying assumptions are made in order to concentrate attention on the goal programming solution method without burying it in extraneous details.

It is assumed also that the board of directors of the firm has approved a plan for plant expansion in Period 3. Part of the funds for this project will be obtained through debt financing and part through the sale of capital stock in Period 2. In order to command a good market price for its stock in Period 2, management judges that the firm must (1) continue to pay its dividends and (2) make a satisfactory profit in

Period 1. The board feels that potential investors would deem a profit of \$35.00 in Period 1 to be a "satisfactory" profit and one which the organization can accomplish in the coming period. Since the total of fixed expenses, dividends, and expenditures for equipment is \$10.00 per period, the <u>net</u> profit objective is equivalent to a contribution to profit and overhead of \$45.00 from the production and sale of knives and cutting boards.

It is further assumed that the president of the firm has established a policy of maintaining a cash balance that is at least \$2.00 greater than the sum of short term liabilities (in this case \$10.00 + \$2.00 = \$12.00). Also, it is assumed that the contractual conditions of the long term bonds require that the firm's net working capital (cash + accounts receivable + inventory - short term liabilities) at the end of the period be twice the face value of the bond debt (in this case 2 · 30 = \$60.00). The question now is, what level of production optimally satisfies these requirements?

As before, letting X_1 represent the number of knives to be produced and X_2 represent the number of cutting boards to be produced, the goal programming model will be formulated as follows. The two technical constraints remain unchanged from prior formulations:

$$(1) \quad .5X_1 + .25X_2 \leq 8 \quad \text{(Machine capacity constraint)}$$

$$(2) \quad 1X_1 + 1X_2 \leq 20 \quad \text{(Assembly capacity constraint)}$$

The profit goal is now formulated as:

$$(3) \quad 2X_1 + 3X_2 + Y^- - Y^+ = 45 \quad \text{(Contributions to profit and overhead)}$$

The end of period cash balance requirement modifies the financial constraint from the original equation to:

20	+	30	-	10	-	$(1X_1 + 2X_2)$	$\geq$	12
Beginning cash balance		Accounts receivable		Cash fixed expenses, equipment expenditures, dividend payment		Cash variable expenses		Required end of period balance

which reduces to

(4) $1X_1 + 2X_2 \leq 28$ (Minimum cash balance requirement)

The minimum net working capital balance constraint is formulated in the following manner:

20	+	30	-	10	-	10	+	$(2X_1 + 3X_2)$	$\geq$	60
Beginning cash balance		Accounts receivable		Bank loan		Cash fixed expenses, equipment expenditures, dividend payment		Net contribution to profit and overhead		Required end of period balance

which reduces to

(5) $2X_1 + 3X_2 \geq 30$ (Working capital requirement)

Since the overall aim is to meet the profit goal of $45.00, the objective function will again be to minimize the range of deviation of profit contribution. Thus, a formal statement of the revised goal programming problem is to

$$\text{Min} \quad Z = Y^- + Y^+$$

subject to

$$
\begin{array}{llr}
(1) & .5X_1 + .25X_2 \leq 8 & \text{(Machine capacity constraint)} \\
(2) & 1X_1 + 1X_2 \leq 20 & \text{(Assembly capacity constraint)} \\
(3) & 2X_1 + 3X_2 + Y^- - Y^+ = 45 & \text{(Contribution to profit and overhead)} \\
(4) & 1X_1 + 2X_2 \leq 28 & \text{(Cash balance requirement)} \\
(5) & 2X_1 + 3X_2 \geq 30 & \text{(Working capital requirement)} \\
(6) & X_1, X_2, Y^-, Y^+ \geq 0 & \text{(Nonnegativity condition)}
\end{array}
$$

The graphical solution to this problem is illustrated in Figure 2, and the simplex iterations are included as Figure 9 in Appendix A. In examining this figure, it should be noted that the profit goal of $45.00 can be satisfied by more than one optimal basic feasible solution; in this case, two.

The two optimal basic feasible solutions are illustrated and discussed below. The first optimal basic feasible solution is

$$
\begin{array}{lll}
X_1 = 6.0 & S_2 = 3.0 & Y^- = 0.0 \\
X_2 = 11.0 & S_3 = 0.0 & Y^+ = 0.0 \\
S_1 = 2.25 & S_4 = 15.0 &
\end{array}
$$

and if it is implemented as a production policy, the firm will produce 6 knives and 11 cutting boards. The contribution to profit and overhead goal of $45.00 is exactly attained, indicated by the fact that the deviation variables, Y^- and Y^+, both take on values of zero. This can be further verified by substituting the solution values for X_1 and X_2 into the marginal profit contribution expression, $2X_1 + 3X_2$. There is an excess of 2.25 hours of machine capacity, an excess of 3.0 hours of

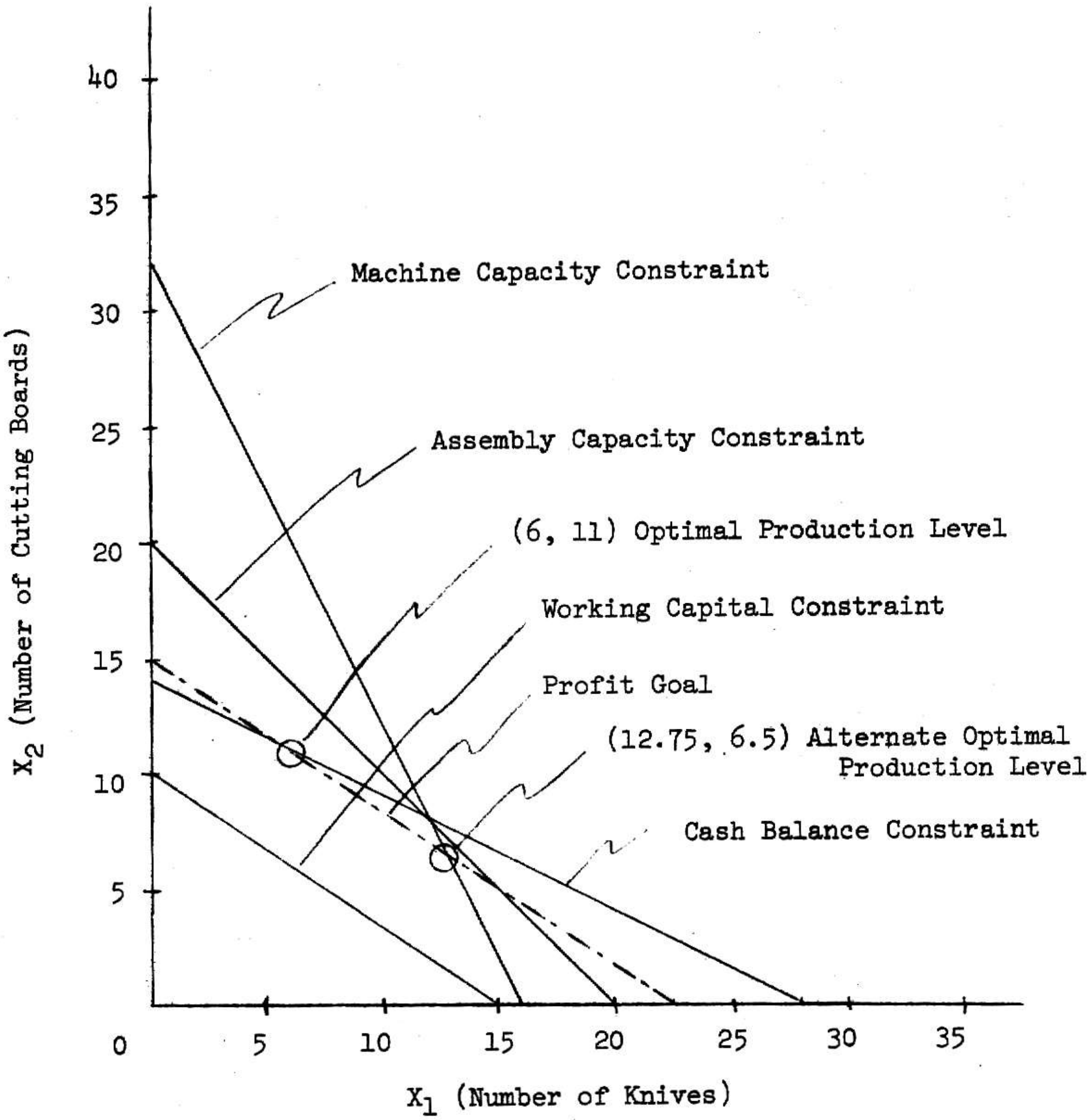


Figure 2. Graphical Solution to Expanded Goal Programming Problem with a Profit Goal of $45.00

assembly capacity, and the required working capital balance of \$60.00 is exceeded by \$15.00 as is evidenced by the positive values of the slack variables (S_1, S_2, S_4) which are associated with these constraints. All available cash resources are utilized ($S_3 = 0$).

The second optimal basic feasible solution

$X_1 = 12.75$	$S_2 = 0.75$	$Y^- = 0.0$
$X_2 = 6.5$	$S_3 = 2.25$	$Y^+ = 0.0$
$S_1 = 0.0$	$S_4 = 15.0$	

requires that the firm produce 12.75 knives and 6.5 cutting boards per period. Again the deviation variables, Y^- and Y^+, are equal to zero, denoting that the profit and overhead contribution goal of \$45.00 was attained exactly. Machine capacity is fully utilized ($S_1 = 0$), but the positive values of the slack variables in the assembly capacity, cash balance, and working capital balance constraints show that there will be 0.75 hours of assembly capacity, \$2.25 cash, and \$15.00 working capital which are not employed by this production plan.

In addition to the two optimal basic feasible solutions to the foregoing problem which satisfy the profit goal, there are other optimal solutions which will accomplish the same result. A theorem in linear programming states that if a problem has more than one optimal basic feasible solution, then any convex combination of these solutions is also an optimal solution (but not necessarily basic) (for proof see Hadley [5], p. 99). Given p optimal basic feasible solutions to a problem, a convex combination of these solutions is defined as a weighted average of the p solutions, where the weights are nonnegative and sum to unity. That is, if a problem has p different optimal basic feasible solutions, $\underline{X}_j$ (where $\underline{X}_j$ is an n-component solution vector and j = 2, 3, ..., p), then a convex combination, $\underline{X}$, of these solutions results from assigning weights U_j to the solution vectors $\underline{X}_j$, or

$$\underline{X} = U_j\underline{X}_j$$

if the conditions $U_j \geq 0$ and $\Sigma U_j = 1$ are met. $\underline{X}$ is an optimal solution but not necessarily a basic one.

Thus, in the foregoing example problem, the two optimal basic feasible solutions define the two extremal points of a straight line (Figure 2) connecting the two solutions. The other optimal nonbasic solutions lie along the straight line. For example, by assigning weights of 0.2 and 0.8, respectively, to the two optimal basic feasible solutions obtained previously, a third optimal nonbasic solution can be obtained. Consider the decision variables X_{1a} and X_{1b}, which are the values of X_1 obtained from Optimum a and Optimum b, respectively, in the foregoing example. The convex combination of X_{1a} and X_{1b}, resulting from assigning weights $U_a = 0.2$ and $U_b = 0.8$, is thus

$$\begin{aligned} X_{1c} &= U_aX_{1a} + U_bX_{1b} \\ &= 0.2(6.0) + 0.8(12.75) \\ &= 11.40. \end{aligned}$$

The same procedure is applied to the other basis and non-basis variables in the two basic solution vectors $\underline{X}_a$ and $\underline{X}_b$, so that one obtains

$X_{1c} = 11.40$ $S_{2c} = 1.20$ $Y_c^- = 0$

$X_{2c} = 7.40$ $S_{3c} = 1.80$ $Y_c^+ = 0$

$S_{1c} = 0.45$ $S_{4c} = 15.00$

which is a third optimal solution, but nonbasic. This solution satisfies the environmental constraints and the profit target of $45.00. In a similar fashion, an infinity of optimal nonbasis solutions can be found by using weights of different magnitude, all of which will be optimal and satisfy the environmental constraints and the same profit target.

An examination of the graphical solution (Figure 2) reveals that the profit and overhead contribution goal of $45.00 not only can be met, but in fact can be exceeded[1]. Had management set the profit target at $60.00, for example, the optimal solution would be

$X_1 = 12.0$ $S_2 = 0.0$ $Y^- = 12.0$

$X_2 = 8.0$ $S_3 = 0.0$ $Y^+ = 0.0$

$S_1 = 0.0$ $S_4 = 18.0$

This solution indicates that the optimal employment of resources will result when the firm produces 12 knives and 8 cutting boards. In this case the profit and overhead contribution goal of $60.00 is underattained by $12.00 which is evidenced by the deficiency deviation variable, Y^-, having a value of 12.0. Thus, the actual contribution to profit and overhead for this solution is $48.00.[2] There is no idle machine capacity, assembly capacity, or liquid resources (S_1, S_2, S_3 = 0), but the required minimum working capital balance is exceeded by $18.00 ($S_4$ = 18).

The Incorporation of Multiple Goals

In all of the foregoing examples, the profit target has been treated as a management goal with all of the other requirements treated as environmental constraints. To illustrate the flexibility of goal programming, it is desirable to treat not only profit but also cash and working capital balances as goals rather than as constraints. Previously, when the liquidity requirements (cash and working capital) were formulated as constraints, these requirements were required to be satisfied absolutely in order to retain feasibility in the optimal solutions. When these requirements are formulated as goals, intentional deviation from the goals can be permitted.

In the following multiple-goal formulation of the example problem, the technical constraints due to machine and assembly capacity remain unchanged:

(1) $.5X_1 + .25X_2 \leq 8$ (Machine capacity constraint)

(2) $1X_1 + 1X_2 \leq 20$ (Assembly capacity constraint)

Also, the contribution to profit and overhead goal remains unchanged; except now the deviation variables for this goal are denoted as Y_3^- and Y_3^+, to distinguish them from other new deviation variables to be defined below. Thus, the contribution to profit and overhead goal becomes

(3) $2X_1 + 3X_2 + Y_3^- - Y_3^+ = 45$ (Contribution to profit and overhead)

where Y_3^- and Y_3^+ represent an underattainment and overattainment, respectively, of the $45.00 goal established for the contribution to profit and overhead.

Additionally, let Y_4^- and Y_4^+ be the end-of-period under- and over-accumulation of cash. Since the end-of-period cash balance goal is stated in terms of cash outflow decision variables, the deviation variable $(Y^- - Y^+)$ must be subtracted from the left-hand side in order to yield the end-of-period goal. Thus

(4) $1X_1 + 2X_2 - (Y_4^- - Y_4^+) = 28$ (Cash balance goal)

now expresses the end-of-period cash balance as a goal, rather than as a constraint.

Also, let Y_5^- and Y_5^+ be the end-of-period deficiency and excess, respectively, in the working capital balance at the end of the decision period. Since the working capital requirement is formulated in terms of cash inflow decision variables, the deviation variable $(Y_5^- - Y_5^+)$ must be added to the left side in order to yield the end of period goal. Thus

(5) $2X_1 + 3X_2 + (Y_5^- - Y_5^+) = 30$ (Working capital requirement)

expresses the end-of-period working capital balance as a goal, rather than as a constraint.

Now consider the objective function, into which multiple goals are to be introduced. Suppose management desires first and foremost to attain a profit contribution of $45.00, and only secondarily to reach the cash and working capital end-of-period balances if possible. Specifically, suppose management considers satisfying the working capital balance requirement to be three times as important as satisfying the minimum cash balance goal, with both of these goals subordinated to the primary goal of profit attainment. Suppose also that the working capital goal must not be underattained, but that working capital goal overattainment is equally satisfactory as meeting the minimum end of period working capital balance exactly. Also, suppose that cash should be used as fully as possible, therefore neither an excess nor a deficiency in the required \$12.00 end-of-period cash balance is desireable. Suppose also that management considers that the overriding profit goal must not be underattained.

To formulate these goals into the objective function, let M and N be very large numbers ($M,N > 0$) and let $M >>> N$, such that M is so much larger than N that no number k ($k > 0$) can make $kN \geq M$. Using the weights M and N as defined, the objective function can be written as

$$\text{Min} \quad Z = MY_3^- + NY_4^- + NY_4^+ + 3NY_5^-.$$

Thus, MY_3^- expresses the heavily weighted potential underattainment of profit, $3NY_5^-$ expresses the less weighted potential underattainment of the working capital goals, and $N(Y_4^- + Y_4^+)$ expresses the lowest weighted potential deficiency and excess in the cash balance, all of which are to be minimized.

This formulation of the objective function demonstrates the incorporation of multiple management goals which are selectively and arbitrarily weighted in order of importance, so the simplex method will selectively optimize amoung the competing goals. The objective function can be interpreted more fully as follows:

1. Management has set profit target attainment as its primary and overriding goal. Exceeding the target is considered as acceptable as exactly meeting the goal, as evidenced by the absence of the excess variable Y_3^+ in the objective function. Since Y_3^- carries the largest

relative weighting factor (M), it will be driven from the basis before the other deviation variables, thus satisfying management's primary profit goal before attempting to satisfy the other goals.

2. After the profit target has been satisfied or exceeded, management wants to maintain or exceed the working capital minimum balance. The deficiency variable Y_5^- is placed in the objective function with a relative weight of N = 3 (N<<<M), to insure minimization of this deviation after the profit goal but before the cash balance goal.

3. After both of the above goals have been approached or satisfied, management then wants cash to be used as fully as possible with no excess or deficiency in the required cash balance. Thus, the deviation $(Y_4^- + Y_4^+)$ carries the least relative weight, N = 1.

The multiple goal problem can thus be stated formally as follows:

$$\text{Min} \quad Z = MY_3^- + NY_4^- + NY_4^+ + 3NY_5^- ,$$

subject to

$$
\begin{array}{lll}
(1) & .5X_1 + .25X_2 \leq 8 & \text{(Machine capacity constraint)} \\
(2) & 1X_1 + 1X_2 \leq 20 & \text{(Assembly capacity constraint)} \\
(3) & 2X_1 + 3X_2 + Y_3^- - Y_3^+ = 45 & \text{(Contribution to profit and overhead goal)} \\
(4) & 1X_1 + 2X_2 - Y_4^- + Y_4^+ = 28 & \text{(Cash balance requirement goal)} \\
(5) & 2X_1 + 3X_2 + Y_5^- - Y_5^+ = 30 & \text{(Working capital requirement goal)} \\
(6) & X_i, Y_j^-, Y_j^+ \geq 0 \quad (i = 1, 2;\ j = 3, 4, 5) & \text{(Nonnegativity Constraints)}
\end{array}
$$

The IBM MPS/360 programming system [12,13,14] was used to determine the optimal solution to this problem. In the machine formulation, the relative weights used in the objective function were $M = 99999(10^{30})$, $N = 100$, and $3N = 300$. The optimal solution is as follows:

$X_1 = 6.0$	$Y_3^- = 0$	$Y_5^- = 0.0$
$X_2 = 11.0$	$Y_3^+ = 0$	$Y_5^+ = 15.0$
$S_1 = 2.25$	$Y_4^- = 0$	
$S_2 = 3.0$	$Y_4^+ = 0$	

Contribution to Profit & Overhead = $2X_1 + 3X_2$ = \$45.00.

This solution indicates that the profit goal of \$45.00 is attained exactly (Y_3^-, $Y_3^+ = 0$), and that there is an excess of 2.25 hours of machine capacity and 3.0 hours of assembly capacity. The end of period cash balance is \$12.00, the required minimum as evidenced by the zero valued deficiency variables Y_4^- and Y_4^+. There is an excess of \$15.00 of working capital ($Y_5^+ = 15.0$) above the required end of period minimum balance of \$60.00.

Referring to the optimal tableau in Figure 11 (Appendix A), it is concluded that an alternate optimal basic feasible solution exists. The non-basis variable Y_3^+ contains a zero value in the current objective function row, Z', which indicates that Y_3^+ can enter the basis without altering the value of the objective function. This is the requirement for the existence of an alternate optimum. The alternate optimal basic feasible solution is:

$X_1 = 12.0$	$Y_3^- = 0.0$	$Y_5^- = 0.0$
$X_2 = 8.0$	$Y_3^+ = 3.0$	$Y_5^+ = 18.0$
$S_1 = 0.0$	$Y_4^- = 0.0$	
$S_2 = 0.0$	$Y_4^+ = 0.0$	

In this solution, the profit goal of \$45.00 is not only attained but exceeded by \$3.00 (as indicated by $Y_3^- = 0$ and $Y_3^+ = 3.0$) when the firm produces $X_1 = 12$ knives and $X_2 = 8$ cutting boards. There is no idle machine capacity or assembly capacity as the slack variables, S_1 and S_2, associated with these constraints take on values of zero. The end of period cash balance exactly equals the minimum balance goal of \$12.00 (since Y_4^-, $Y_4^+ = 0$). The working capital goal of \$60.00 is exceeded by \$18.00 ($Y_5^- = 0$, $Y_5^+ = 18.0$). The graphical solution (Figure 3) depicts the two optimal basic feasible solutions.

The question arises as to why the profit goal of \$45.00 was attained exactly in the first optimal solution and then, in the alternate optimum, this goal was exceeded by $Y_3^+ = \$3.00$? The answer lies simply in the fact that the objective function was constructed so as to minimize <u>under</u>attainment of the profit goal (Y_3^-), but not to prohibit overattainment; that is, Y_3^+ was not included in the minimization objective. In seeking the alternate optimum, the variable Y_3^+ was the non-basis variable whose adjusted objective row value was zero, and therefore the candidate variable to be brought into solution. In bringing Y_3^+ into solution, feasibility is retained by removing S_2 (assembly time slack), and optimality is retained because the incoming variable, Y_3^+, does not make any additional contribution to the objective function <u>as it is defined</u>. Simply, overattainment of the profit goal is permitted because there is no optimization function term which requires the variable Y_3^+ to be minimized.

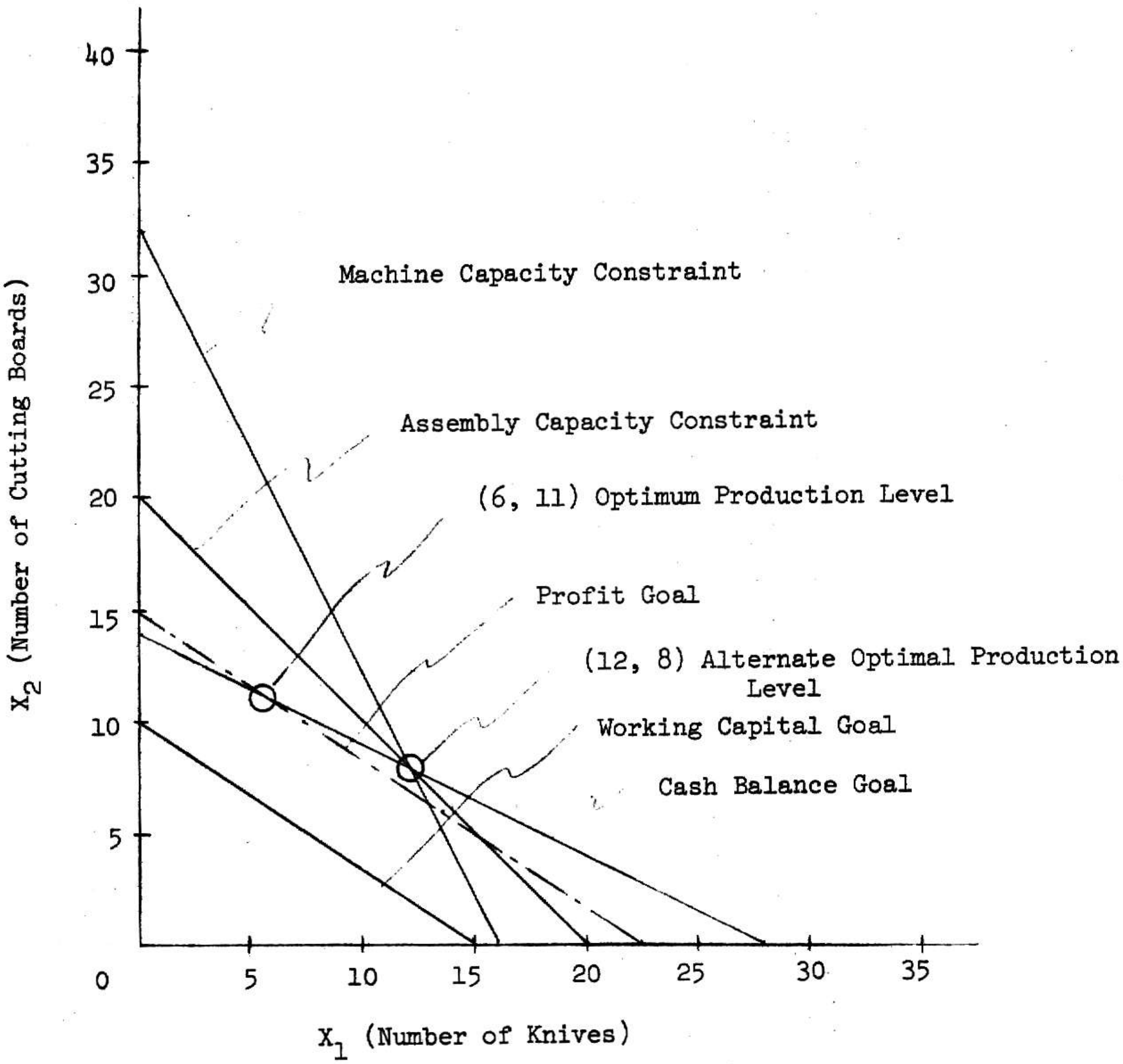


Figure 3. Solution to Multiple Goal Example

Incompatible Multiple Goals

In the last example, all goals and constraints were satisfied by both the original and alternate optimal solutions. Note that the maximum profit that can be attained without violating any constraints is $48.00 (see Figure 3), corresponding to the optimal solution point (X_1 = 12.0, X_2 = 8.0). Now, consider a situation in which one or more of the announced goals could not be attained exactly. For example, what if management had specified a target of $60.00 for contribution to profit and overhead, instead of setting the profit goal at the original $45.00? This new target lies beyond the feasible area in which all goals and constraints can be satisfied. If the new target of $60.00 is met, which goal will be violated?

Although the answer is not obvious, a prediction might be made from the weighting factors in the objective function. Since underattainment of the profit goal is most heavily weighted, one would expect the deficiency variable Y_3^- to be driven out of solution (Y_3^- goes to zero) first, thus attaining the profit target. One might also predict that the working capital goal will be satisfied because its deviation variable (Y_5^-) carries the second largest weighting factor (3N), and therefore Y_5^- should also be driven from solution. Because the technical constraints (machine and assembly capacities) are feasibility constraints which cannot be violated, one would expect then that the cash balance requirement goal would "yield" to the optimization pressure, as its deviation variables (Y_4^- and Y_4^+) carry the lowest weighting factor in the objective function.

To motivate an illustration of the behavior of incompatible multiple goals in the goal programming format, consider an extension of the previous example. Suppose now that, the profit goal equation is altered to include the new profit target:

$$(3) \quad 2X_1 + 3X_2 + Y_3^- - Y_3^+ = 60$$

To insure <u>exact</u> attainment of the $60.00 profit target, the objective function is also strengthened by the inclusion of a Y_3^+ term, so that the objective becomes:

$$\text{Min} \quad Z = MY_3^- + MY_3^+ + NY_4^- + NY_4^+ + 3NY_5^- \ .$$

The other constraints and goals remain unchanged from the previous multiple goal formulation.

Under this formulation, the optimal solution is:

$X_1 = 0.0$	$Y_3^- = 0.0$	$Y_5^- = 0.0$
$X_2 = 20.0$	$Y_3^+ = 0.0$	$Y_5^+ = 30.0$
$S_1 = 3.0$	$Y_4^- = 12.0$	
$S_2 = 0.0$	$Y_4^+ = 0.0$	

The profit goal of $60.00 is exactly attained (Y_3^+, $Y_3^- = 0$); machine and assembly constraints are not violated (although there are 3.0 hours of idle machine capacity); the working capital minimum is exceeded ($Y_5^+ = 30.0$), but the cash balance goal is sacrificed ($Y_4^- = 12.0$), just as predicted. The large weighting factors attached to the profit deviation variables and the working capital deviation variables have forced the violation of the weakest goal (cash balance requirement).

This example demonstrates the applicability of goal programming to a situation involving incompatible goals. A hierarchy denoting relative goal importance can be established by management and in turn translated mathematically into the objective function of the model. As the goal programming objective function is composed of deviation variables (each representing a deviation from a specific management goal) which can be weighted, it is possible to force the computational procedure to attempt the selective minimization of these deviations from goals by weighting each deviation variable in the objective function with a numerical coefficient representing the relative importance of the goal in management's hierarchy of goals. Traditional linear programming methods not only cannot express multiple goals, but have no built-in mechanism to force selective satisfaction of multiple goals - a necessary characteristic for the logical resolution of conflict between incompatible multiple goals.

Consequential Actions Associated with Incompatible Goals

When one or more of management's announced goals fails of attainment, it is possible that the firm must undertake a consequential or remedial action in order to guarantee that the unattained goal will be met. For example, in the previous incompatible goal example, the end-of-period cash balance goal failed of attainment (the goal was underattained as evidenced by $Y_4^- = 12.0$). A subsequent (or contemporaneous) action required of the firm as a consequence of this result might be, for example, the short-term borrowing of funds in order to assure end-of-period liquidity maintenance (attainment of the cash balance goal). In this example then, the cash balance underattainment, $Y_4^- = 12.0$, would signal not only the necessity for making the loan but also the size of it.

The goal programming format is more powerful than the conventional linear programming format for handling what can be called consequential or dependent decisions. For example, in the prior example, had the end-of-period cash balance amount been formulated as a constraint, the optimization would not have proceeded to the point where the firm's profit goal could be attained, even though this was the firm's announced overriding objective. Optimization simply would have stopped when the cash balance constraint became "tight". On the other hand, the goal programming format in fact permitted <u>intentional</u> underattainment of the low-ranked cash balance goal so that the overriding profit goal could be attained exactly, while at the same time explicitly signalling cash balance underattainment in the optimal solution.

Now, if one were to devise a consequential decision procedure (to borrow money, for example) that would be invoked only when a goal is adversely violated, so that the goal is <u>consequentially</u> attained as a result of the signalled underattainment, then one would have, in effect, the same end result that a constraint formulation would provide but without the disadvantages of the inviolable constraint formulation. Such a decision procedure is obviously more powerful than a straight constraint formulation, since it provides for secondary or remedial action to be taken that is conditioned upon a prior result. That is, if the prior result is favorable, no remedial action need be taken and the consequential decision procedure is not invoked, but if the prior result is unfavorable then the remedial decision procedure is invoked. The method by which such a decision procedure can be incorporated into the goal programming model will be exemplified on the following page.

In the incompatible goal example presented earlier, attainment of the end-of-period cash balance goal of \$12.00 was sacrificed in order to obtain a maximum profit operating level. The question, then, is what consequential or remedial action should be taken by the firm as a result of underattaining the cash balance goal? One straightforward remedial action would be for the firm to borrow \$12.00 in Period 1 to make up the end-of-period cash deficiency ($Y_4^- = 12.00$). This would insure on-target attainment of the cash balance goal, which would in turn provide the level of funds at the beginning of Period 2 which management deems to be necessary for the operational level contemplated.[3] Associated with the short-term borrowing of funds, however, is an interest charge which acts to reduce the attainable profit. In writing the loan procedure into the goal programming model, therefore, one needs to reflect not only the effect of making the loan but also the effect of the interest penalty associated with the loan.

Assume that the firm has access to a free money market (i.e., it can borrow or lend all of the money it desires at a current established interest rate). Assume also that the current established interest rate on short-term loans is 10% per period. If a cash deficiency, Y_4^-, will exist at the end of the period, the firm must borrow Y_4^- dollars from the free money market to satisfy the minimum cash balance requirement. The interest charge on this short-term loan will be 10% of Y_4^- dollars and will decrease the contributions to profit and overhead in Period 1 by $0.1Y_4^-$ dollars.

Since it is, in general, possible for the firm to overattain a cash balance goal as well as underattain it, the question arises as to what consequential action should be undertaken in the event of overattainment. If the firm overaccumulates cash during the period, a cash excess (Y_4^+) will result, which can be lent on the assumed free money market at an interest rate of 10%. The revenue from such a loan acts to increase the contribution to profit and overhead by $0.1Y_4^+$ dollars. (The functional equivalent of "lending" excess funds is the purchase by the firm of short-term securities, such as certificates of deposit and marketable bonds). Thus, a consequential decision procedure, in this case, can be incorporated for both underaccumulation and overaccumulation of cash.

To implement this decision procedure, we assume that the firm uses an accrual method of accounting, so that if funds are borrowed in Period 1, for example, to provide cash balance goal attainment at the end of Period 1, then the interest penalty on the loan is also properly chargeable against profit in Period 1. Likewise, we assume that if excess funds are invested in Period 1, then the accrued interest income is credited to profit in Period 1. Using this assumption, then, the goal for contribution to profit and overhead becomes

$$(3) \quad 2X_1 + 3X_2 + Y_3^- - Y_3^+ - 0.1(Y_4^- - Y_4^+) = 60$$

when the potential accrued interest penalty (or credit) is reflected.

Note also that no change is necessary in the cash balance goal (Equation (4), p. 35). The reason is that the cash balance deviation variables, Y_4^- and Y_4^+, are now viewed as being amounts that are to be borrowed or invested in the free money market, rather than as simple deficiency or excess deviations from the goal.

Similarly, the working capital goal formulation (Equation (5), p. 35) requires only slight modification as a result of borrowing or lending funds. Working capital is defined as the net of cash, receivables, inventory, and short term liabilities. When borrowing is effected, both cash and short-term liabilities are increased by the same increment, so no change in working capital results. Similarly, when lending is effected, cash is decreased but receivables are increased by the same increment, so no net change in working capital results.

However, as a result of borrowing, an interest penalty (charge) is incurred which, in an accrual accounting system, results in an increase in short term liabilities (accounts payable). This increase in short-term liabilities reduces working capital, so the result in the working capital goal of borrowing Y_4^- dollars is to decrease end-of-period working capital by $0.1Y_4^-$ dollars. Conversely, if excess funds (Y_4^+ dollars) is "lent" then the net effect is to increase accounts receivable (interest income receivable) by $0.1\ Y_4^+$ dollars, which is a net increase in working capital. Thus, the revised working capital goal is

$$2X_1 + 3X_2 + (Y_5^- - Y_5^+) - 0.1(Y_4^- - Y_4^+) = 30$$

When the consequential "borrowing - lending" decision procedure is incorporated into the incompatible goal example, then, the revised problem can be formally stated as

$$\text{Min} \quad Z = MY_3^- + MY_3^+ + NY_4^- + NY_4^+ + 3NY_5^-$$

subject to:

$$(1) \quad .5X_1 + .25X_2 \leq 8 \text{ (Machine capacity constraint)}$$

$$(2) \quad 1X_1 + 1X_2 \leq 20 \text{ (Assembly capacity constraint)}$$

$$(3) \quad 2X_1 + 3X_2 + Y_3^- - Y_3^+ - 0.1Y_4^- + 0.1Y_4^+ = 60 \text{ (Contribution to profit and overhead)}$$

$$(4) \quad 1X_1 + 2X_2 - Y_4^- + Y_4^+ = 28 \text{ (Cash balance goal)}$$

$$(5) \quad 2X_1 + 3X_2 + Y_5^- - Y_5^+ - 0.1Y_4^- + 0.1Y_4^+ = 30 \text{ (Working capital goal)}$$

$$(6) \quad X_1, X_2, Y_i^-, Y_i^+ \geq 0 \; , \quad (i = 3,4,5) \text{ (Nonnegativity conditions)}$$

A unique optimal solution to this problem exists which is as follows:

$X_1 = 0$	$Y_3^- = 1.2$	$Y_5^- = 0$
$X_2 = 20.0$	$Y_3^+ = 0$	$Y_5^+ = 28.8$
$S_1 = 3.0$	$Y_4^- = 12.0$	
$S_2 = 0$	$Y_4^+ = 0$	

This solution requires that the firm produce no knives and 20 cutting boards. There will be 3 hours of idle machine capacity but assembly capacity will be fully utilized ($S_1 = 3$, $S_2 = 0$). Whereas, in the previous example the profit goal of $60.00 was attained exactly (Y_3^-, $Y_3^+ = 0$), the profit goal is now underattained by $1.20 ($Y_3^- = 1.2$). Similarly, the working capital goal was previously overattained by $30.00, whereas now it is overattained by only $28.80 ($Y_5^+ = 28.8$). These changes in the profit goal and working capital goal deviations result from the interest penalty (charge) on the short-term loan of \$12.00 ($Y_4^- = 12.0$), which was the consequential action of violating the end-of-period cash balance goal ($Y_4^- = 12.0$). In effect, the gross contribution to profit and overhead by producing 20 cutting boards is (3.00) (20) = \$60.00, but the interest expense reduces this to a net value of \$58.80 for the period. Likewise, the accrual of \$1.20 in incurred interest expense reduces the end-of-period cash balance overattainment from \$30.00 to \$28.80.

To further illustrate the flexibility of goal programming, consider a comparison of two formulations of an example, the first formulation with no consideration of consequential actions resulting from deviation from goals and the second formulation incorporating these consequences. In this example assume that there are no changes from the previous example except that now management desires a contribution to profit and overhead of \$57.00.

Formulating the problem with no consideration of the consequences of goal violation, the formal statement of the problem is as follows:

$$\text{Min} \quad Z = MY_3^- + MY_3^+ + NY_4^- + NY_4^+ + 3NY_5^-$$

subject to

(1) $.5X_1 + .25X_2 \leq 8$ (Machine capacity constraint)

(2) $1X_1 + 1X_2 \leq 20$ (Assembly capacity constraint)

(3) $2X_1 + 3X_2 + Y_3^- - Y_3^+ = 57$ (Contribution to profit and overhead)

(4) $1X_1 + 2X_2 - Y_4^- + Y_4^+ = 28$ (Cash balance goal)

(5) $2X_1 + 3X_2 + Y_5^- - Y_5^+ = 30$ (Working capital goal)

(6) $X_1, X_2, Y_i^-, Y_i^+ \geq 0$, $(i = 3,4,5)$ (Nonnegativity conditions)

which results in the solution

$X_1 = 3.0$	$Y_3^- = 0.0$	$Y_5^- = 0.0$
$X_2 = 17.0$	$Y_3^+ = 0.0$	$Y_5^+ = 27.0$
$S_1 = 2.25$	$Y_4^- = 9.0$	
$S_2 = 0.0$	$Y_4^+ = 0.0$	

The formulation of the same problem, this time considering penalties, is formally stated as:

$$\text{Min} \quad Z = MY_3^- + MY_3^+ + NY_4^- + NY_4^+ + 3NY_5^-$$

subject to

(1) $.5X_1 + .25X_2 \leq 8$ (Machine capacity constraint)

(2) $1X_1 + 1X_2 \leq 20$ (Assembly capacity constraint)

(3) $2X_1 + 3X_2 + Y_3^- - Y_3^+ - 0.1Y_4^- + 0.1Y_4^+ = 57$ (Contribution to profit and overhead)

(4) $1X_1 + 2X_2 - Y_4^- + Y_4^+ = 28$ (Cash balance goal)

(5) $2X_1 + 3X_2 + Y_5^- - Y_5^+ - 0.1Y_4^- + 0.1Y_4^+ = 30$ (Working capital goal)

(6) $X_1, X_2, Y_i^- Y_i^+ \geq 0$, $(i = 3,4,5)$ (Nonnegativity conditions)

which results in the solution

$X_1 = 2.0$	$Y_3^- = 0.0$	$Y_5^- = 0.0$
$X_2 = 18.0$	$Y_3^+ = 0.0$	$Y_5^+ = 27.0$
$S_1 = 2.5$	$Y_4^- = 10.0$	
$S_2 = 0.0$	$Y_4^+ = 0.0$	

One recognizes that although the contribution to profit and overhead goal of $57.00 is met in both cases, the production policies of the two solutions differ. In the second case, the level of production is increased by 1 knife and 1 cutting board so that the net contribution to profit and overhead from the products and the interest expense will equal the goal of $57.00. Thus when the consequences of goal violation are incorporated, goal programming not only tells management what goals will be violated and how much they will be violated but what revisions are necessary to production levels in order to satisfy the hierarchy of management desires. A comparison of these two cases is shown graphically in Figure 4.

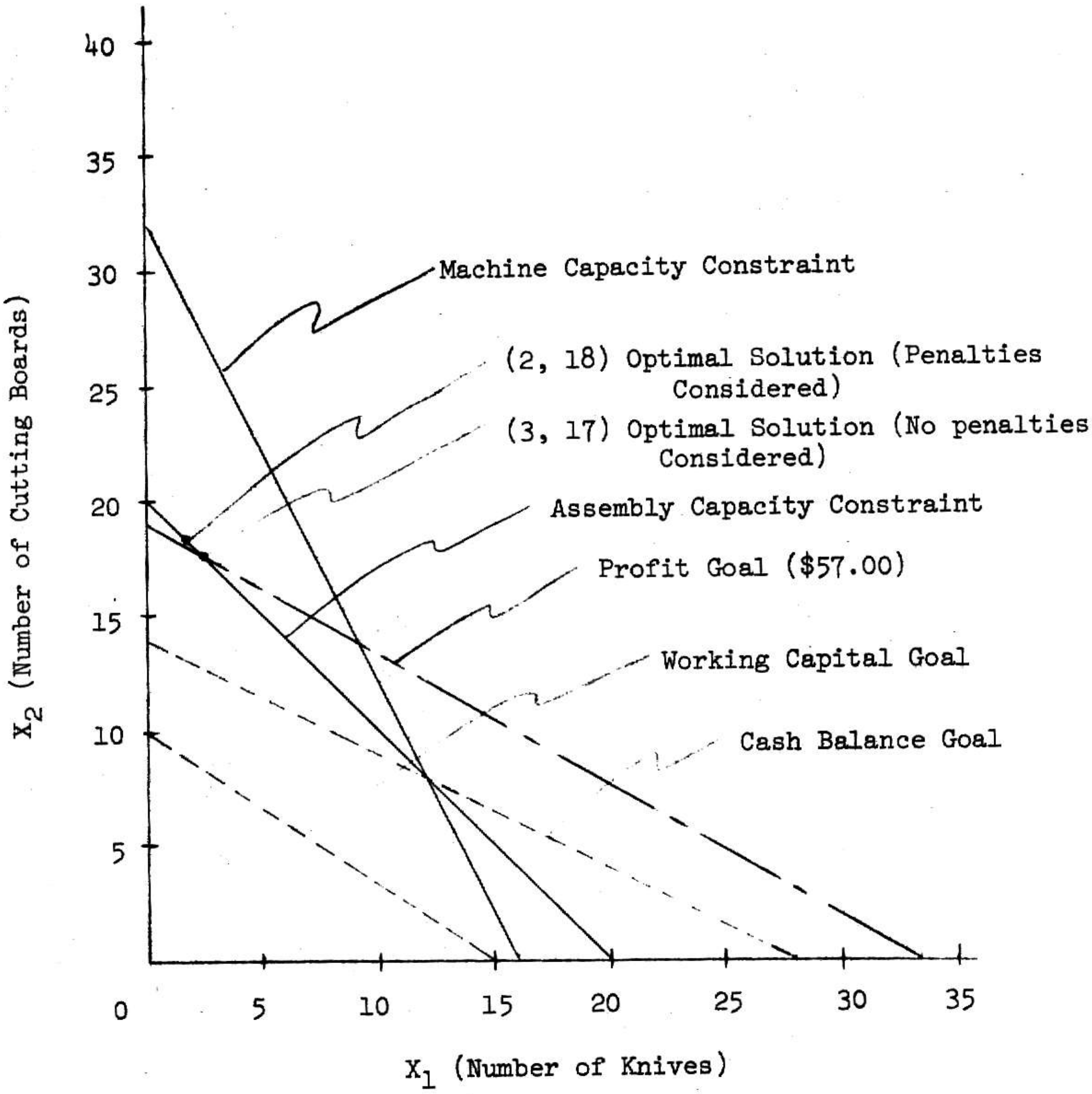


Figure 4. Comparison of Incompatible Goal Solutions With and Without Penalties in the Model Formulation

Incorporation of Books of Account

One of the major advantages of goal programming devolves from the fact that the model can be constructed in terms of financial variables and financial relationships, rather than in terms of other decision variables such as production quantities, for example. This has an obvious appeal from the standpoint of management of the firm, since most operating variables in the firm can be expressed in monetary values. The data base in which monetary amounts are periodically recorded so as to reflect changes in the firm's operating position is the firm's accounting system, or specifically, the "books of account" of the firm. It is not only possible but desirable to use this data base as a fundamental input in the formulation of the goal programming problem. Accounting data comprise a primary source of information in the formulation of goals and constraints affecting the operating policies of the firm. Additionally, the operating decisions chosen will directly affect future transactions which are to be recorded in the books of account. From a financial prediction (budget) standpoint, it is easier to define operating and decision variables in terms of financial units (dollars) as opposed to production units. Thus, the integration of accounting data and the goal programming model is a reasonable and logical undertaking.

From the accounting viewpoint, a firm is composed of assets, liabilities, and owner equity, with the following relationship:

ASSETS = LIABILITIES + OWNER EQUITY

Each of these is a major classification of several accounts, with each account in the firm's "books of account" denoting a particular type of asset, liability, or equity. These accounts, when viewed in total, comprise the "books of account".

An accounting convention, "double entry bookkeeping", is based on the fundamental equation above. The occurence of a business transaction causes entries to at least two accounts in the books of account. One of these entries will be a debit (left hand) entry and the other will be a credit (right hand) entry. Increasing the value of an asset account requires a debit or left-hand entry. Decreasing an asset account value requires a credit or right-hand entry. For liability and equity accounts the opposite holds true; that is, a debit entry decreases the value of the account and a credit entry increases the net balance of a liability or equity account. Two simple examples will illustrate this point. A firm purchases $1,000.00 of materials on credit. The accountant will record this transaction as a $1,000.00 debit to materials (an increase in an asset) and a $1,000.00 credit to accounts payable (increase in a liability). When the actual payment of the $1,000.00 liability is transacted, a $1,000.00 debit will be made to accounts payable and a $1,000.00 credit will be recorded in the cash account. The net effect of the two transactions is to convert one $1,000.00 asset (cash) into another $1,000.00 asset (materials). In a second example, 100 shares of company stock is sold at a price of $10.00 per share. This transaction increases both the firm's cash balance and stockholders equity. The accountant debits the cash account for $1,000.00 and credits the stockholder's equity account for $1,000.00.

For the purposes of the goal programming model, transactions such as these can be illustrated very simply through the use of an n x n account matrix and a standard variable notation to describe each entry in the matrix. The n x n matrix in Table 4 has a column <u>and</u> a row for <u>each</u> of the n accounts in the books of account. In the matrix, credit entries are recorded in the <u>column</u> under the account to be credited and debit entries are recorded in the <u>row</u> corresponding to the account to be debited. Thus, only one entry to the matrix is required for any one transaction that can be represented by a simple debit-credit entry. In Table 4, each entry in the spread-sheet matrix has a numerical value. In formulating the goal programming model, some of these entries will be unknown variables; thus, it is convenient to represent an entry in general in the spread-sheet matrix as a variable, X_{ij}, where X is the unknown dollar amount of the transaction, i is the account to be debited and j is the account to be credited (k, j = 1, 2, ..., n), and n is the total number of accounts in the matrix.

In the formulation of a goal programming model, changes in account balances are also needed. Let the change in the balance of a particular account, t, (t = 1, 2, ..., n) during a given period, be Δ_t. Then Δ_t is given by

$$\Delta_t = \sum_{j=1}^{n} X_{tj} - \sum_{i=1}^{n} X_{it} \qquad [9]$$

The first term in Equation [9] is the total of the debit entries to account t during the period and the second term is the total of the credit entries to the same account t during the period. The following conditions describe the net change (Δ_t) in the balance of account t during the period:

TABLE 4

AN ILLUSTRATION OF A SPREAD SHEET ACCOUNT MATRIX

CREDITS

DEBITS	Cash (C)	Receiv-ables (R)	Inven-tory (I)	Plant & Equip-ment (P)	Interest Payable (D)	Loans (L)	Bonds (B)	Owner Equity (E)	Total Debits
Cash (C)		(b)30.00				(g)10.00			40.00
Receivables (R)	(a)38.00							(c)58.00	96.00
Inventory (I)									- 0 -
Plant and Equipment (P)	(e)2.50								2.50
Interest Payable (D)									- 0 -
Loans (L)									- 0 -
Bonds (B)									- 0 -
Owner Equity (E)	(d)5.00 (f)2.50				(h)1.00				8.50
Total Credits	48.00	30.00	- 0 -	- 0 -	1.00	10.00	- 0 -	58.00	147.00

TABLE 4 "Continued"

(a) variable production costs; $X_{RC(K)} = \$2.00$, $X_{RC(B)} = \$36.00$

(b) collection of receivables; X_{CR} - \$30.00

(c) net profit on goods sold; $X_{RE} = \$58.00$

(d) payment of fixed expenses; $X_{EC(1)} = \$5.00$

(e) equipment replacement; $X_{PC} = \$2.50$

(f) payment of cash dividend; $X_{EC(2)} = \$2.50$

(g) short term loan to meet liquidity goal; $X_{CL} = \$10.00$

(h) interest expense payable; $X_{ED} = \$1.00$

TABLE 5

RESULTING EFFECTS OF Δ_t ON ACCOUNT BALANCES

If:	Then:
$\Delta_t > 0$	An asset account balance has increased. or A liability or net worth account balance has decreased.
$\Delta_t < 0$	An asset account balance has decreased. or A liability or net worth account balance has increased.

In formulating a problem in terms of financial units as opposed to production units it is more convenient to express subscripts in alphabetic notation rather than in numerical notation as used above. The notation used for a financial variable is X_{ij}, where X is the dollar amount of the transaction, i is the account to be debited in the spreadsheet matrix, and j is the account to be credited. The following alphabetic abbreviations are used for the subscripts i and j, and represent the account titles in Table 4:

C = Cash R = Accounts Receivable I = Inventory P = Plant and Equipment	Asset Accounts
D = Interest Expense Payable L = Loans Payable B = Bonds Payable	Liability Accounts
E = Owner Equity	Net Worth Account

Net entries, X_{ij}, into the account matrix may arise from several accounting transactions which give rise to the final net entry. To describe how a typical net entry is expressed in general notation, consider the variable $X_{RC(K)}$. This entry is considered in the subsequent sections of this report to be the "total variable cost" of producing knives. In its variable form, it represents a net amount of money that is to be debited to accounts receivable and credited to cash as a result of producing the entire lot of knives. The question is, why does it represent the net effect (in the books of account) of a total variable cost? The explanation can be made in terms of a sequence of accounting transactions, as follows. To facilitate the explanation, abbreviated T-accounts will be used to illustrate the elementary transactions (as in Figure 5).

To produce one knife, variable production inputs (labor and material) are purchased for cash (entries "a" and "b" to "Cash", "Direct Labor Expense", and "Materials" in Figure 5). The materials are then transformed by the production process and labor into a finished knife which is carried to "Inventory" at the cost value of the production inputs (entries "C"). When the knife is sold on credit, the sale is recorded in "Sales" and "Accounts Receivable" at the sales value (entries "d"); and the cost of the sale is recorded to "Cost of Goods Sold" (entries "e"). The net revenue from the sale is reflected in the "Profit and Loss" summary account by the transfer entries "f" and "g". On closing the books of account at the end of the period, the net profit is transferred as a net increase to "Owner Equity" (entries "h").

The $2.00 balance in "Owner Equity" represents an increase in equity from the beginning of the period. This is offset by an increase

Cash

	.50 (a)
	.50 (b)

Accounts Receivable

(d) 3.00	

Owner Equity

	2.00 (h)

Materials

(a) .50	.50 (c)

Labor Expense

(b) .50	.50 (e)

Inventory

(c) 1.00	1.00 (e)

Cost of Goods Sold

(e) 1.00	1.00 (g)

Sales

(f) 3.00	3.00 (d)

Profit & Loss

(g) 1.00	3.00 (f)
(b) 2.00	

a = Purchase of materials

b = Purchase of labor

c = Transfer to finished goods inventory at cost

d = Sales of knife for $3.00

e = Record <u>cost</u> of the sale

f = Reflect revenue from sale

g = Reflect cost of generating revenue from sale

h = Transfer of net profit to owner equity

Figure 5. Illustrative Example Using T-Accounts

in "Accounts Receivable" of \$3.00 and a decrease in "Cash" of \$1.00. The question then is, how much of the accounts receivable increase is not owner equity? Thus, the difference between the \$3.00 increase in "Accounts Receivable" and the \$2.00 increase in "Owner Equity" is a <u>cost</u>, which in this case is the equivalent of the total variable cost (or, the original value of the labor and materials purchased). Thus, a debit to "Accounts Receivable" (R) and a credit to cash (C) in <u>like amount</u>, X_{RC}, represents the net effect of incurring a total variable cost when X is stated in terms of cost. Differentiating now between the total variable cost of producing knives and that of producing cutting boards, $X_{RC(K)}$ is defined as the total variable cost of producing knives and the $X_{RC(B)}$ is defined as the total variable cost of producing cutting boards, per period.

To illustrate now how the goal programming model can be formulated in terms of financial variables, consider again the incompatible goal example formulated on page 48, which incorporated goal unattainment consequences. This previous example, formulated in terms of <u>production</u> variables, was stated as

$$\text{Min} \quad Z = MY_3^- + MY_3^+ + NY_4^- + NY_4^+ + 3NY_5^-$$

subject to

$$(1) \quad .5X_1 + .25X_2 \leq 8 \text{ (Machine capacity constraint)}$$

$$(2) \quad 1X_1 + 1X_2 \leq 20 \text{ (Assembly capacity constraint)}$$

$$(3) \quad 2X_1 + 3X_2 + Y_3^- - Y_3^+ - 0.1Y_4^- + 0.1Y_4^+ = 57 \text{ (Contribution to profit and overhead)}$$

$$(4) \quad 1X_1 + 2X_2 - Y_4^- + Y_4^+ = 28 \text{ (Cash balance goal)}$$

$$(5) \quad 2X_1 + 3X_2 + Y_5^- - Y_5^+ - 0.1Y_4^- + 0.1Y_4^+ = 30 \text{ (Working capital goal)}$$

$$(6) \quad X_1, X_2, Y_i^-, Y_i^+ \geq 0 \quad , \quad (i = 3,4,5) \text{ (Nonnegativity conditions)}$$

Note that in the cash balance goal

$$(4) \quad 1X_1 + 2X_2 - Y_4^- + Y_4^+ = 57$$

the expression $(1X_1 + 2X_2)$ is the total cash outflow (and therefore, the total variable cost) required to purchase the inputs (labor and material) for the production of both knives and cutting boards during the decision period. Note also that $1X_1$ is the total variable cost of producing knives only and $2X_2$ is the total variable cost of producing cutting boards only, per period. Recall now that $X_{RC(K)}$ and $X_{RC(B)}$ were defined earlier as the total variable costs of producing knives and cutting boards, respectively. Therefore, the relationships which equate total variable cost in one system of variables to total variable cost in the other system are

$$1X_1 = X_{RC(K)}$$

and $$2X_2 = X_{RC(B)}$$

which can be solved in terms of the production variables, X_1 and X_2. Thus:

$$X_1 = X_{RC(K)} \qquad [10]$$

and $$X_2 = X_{RC(B)} / 2 \qquad [11]$$

By substituting Equations [10] and [11] into the goal and constraint equations of the model as constructed with production variables, one obtains the equivalent model in terms of financial variables, which can be formally stated as follows:

$$\text{Min} \quad Z = MY_3^- + MY_3^+ + NY4^- + NY_4^+ + 3NY_5^-$$

subject to

(1) $.5X_{RC(K)} + .125X_{RC(B)} \leq 8$ (Machine capacity constraint)

(2) $1X_{RC(K)} + .5X_{RC(B)} \leq 20$ (Assembly capacity constraint)

(3) $2X_{RC(K)} + 1.5X_{RC(B)} + Y_3^- - Y_3^+ - 0.1Y_4^- + 0.1Y_4^+ = 57$ (Contribution to profit and overhead)

(4) $1X_{RC(K)} + 1X_{RC(B)} - Y_4^- + Y_4^+ = 28$ (Cash balance goal)

(5) $2X_{RC(K)} + 1.5X_{RC(B)} + Y_5^- - Y_5^+ - 0.1Y_4^- + 0.1Y_4^+ = 30$ (Working capital goal)

(6) $X_{RC(K)}, X_{RC(B)}, Y_i^-, Y_i^+ \geq 0$, $(i = 3,4,5)$ (Nonnegativity conditions)

which results in the solution

$$
\begin{array}{lll}
X_{RC(K)} = 2.0 & Y_3^- = 0.0 & Y_5^- = 0.0 \\
X_{RC(B)} = 36.0 & Y_3^+ = 0.0 & Y_5^+ = 27.0 \\
S_1 = 2.5 & Y_4^- = 10.0 & \\
S_2 = 0.0 & Y_4^+ = 0.0 &
\end{array}
$$

The implementation of this solution will result in a total variable production cost of \$38.00, \$2.00 of which is the variable cost of producing knives ($X_{RC(K)} = 2.0$) and \$36.00 of which is the variable cost of producing cutting boards ($X_{RC(B)} = 36.0$) in Period 1. There will be 2.5 hours of idle machine capacity ($S_1 = 2.5$) and assembly capacity will be fully utilized ($S_2 = 0$). The contribution to profit and overhead goal of \$57.00 is attained exactly ($Y_3^-, Y_3^+ = 0$), but only at the expense of the cash balance goal which is undershot by \$10.00 ($Y_4^- = 10.0$) and in turn causes the firm to borrow \$10.00. The working capital goal is exceeded by \$27.00 ($Y_5^+ = 27.0$). The graphical solution to this example is illustrated in Figure 6.

In order that the production equivalents of this optimal financial operating policy can be communicated to the production department in the form of a production quota, one only has to solve the relationships $X_1 = X_{RC(K)}$ and $X_2 = X_{RC(B)} / 2$ to set performance objectives of $X_1 = 2$ knives and $X_2 = 18$ cutting boards[4] for the production department in Period 1.

It should be noted that this solution yields the same production policy as that given by the goal programming formulation in terms of production variables. By defining the variables in terms of financial units (dollars), management can immediately discern the effects of a

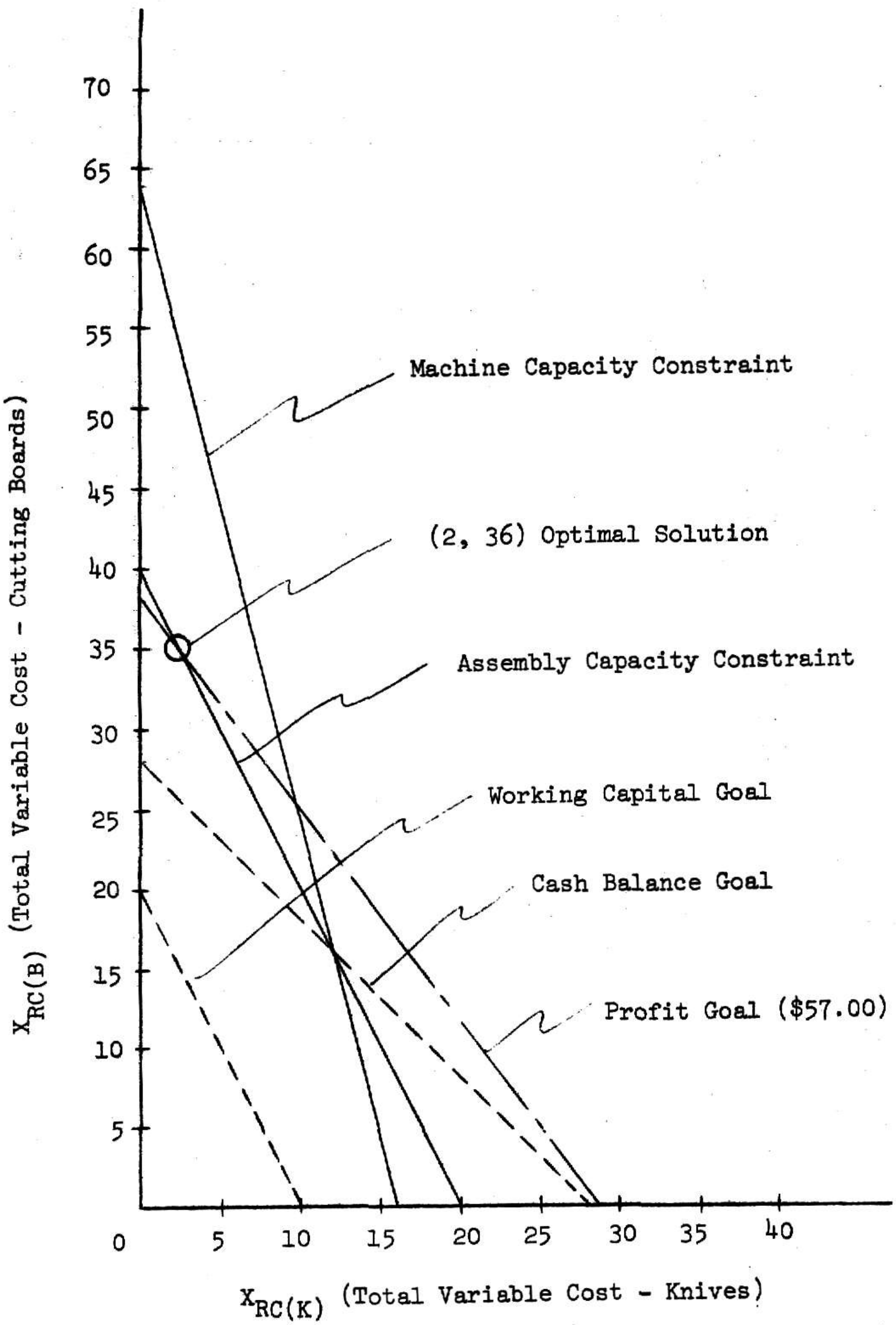


Figure 6. Goal Programming Solution Using Accounting Variables

proposed operating policy on the future financial state of the firm and thus can give it direction in a more informed manner.

Development of an Account Matrix and Projected Balance Sheet

To show the financial effects of a proposed operating policy on the firm, the optimal solution to the goal programming model is incorporated into the account matrix. That is, the optimal decision values and the values of the resulting deviation variables are incorporated into the account matrix, in which the X_{ij} entries are those resulting from the goal program. The following variables are used to develop the exposition in this section.

$X_{RC(K)}$ = Total variable cost per period of producing knives

$X_{RC(B)}$ = Total variable cost per period of producing cutting boards

$X_{EC(1)}$ = Fixed cash expenses

$X_{EC(2)}$ = Cash Dividend payments

X_{PC} = Expenditures for equipment replacement

X_{CR} = Cash receipt due to collection of receivables

X_{RE} = Net profit contributions from the sale of finished goods

X_{CL} = Amount of the short term loan liability undertaken

X_{ED} = Interest expense accrued

These variables represent the net effects of the entries made to record the accounting transactions associated with these variables. The variables $X_{RC(K)}$ and $X_{RC(B)}$ were explained in a previous section of this report. They are the total variable costs of producing knives and cutting boards per period, respectively. The variable $X_{EC(1)}$ describes the net result of incurring and paying \$5.00 in fixed expenses for the period. The detailed accounting transactions are illustrated in the T-accounts below:

Cash		Fixed Expenses		Profit & Loss		Owner Equity	
	5.00(a)	(a)5.00	5.00(b)	(b)5.00	5.00(c)	(c)5.00	

Fixed expenses are paid in cash and recorded as entries "a" in the books of account. At the end of the period, the "Fixed Expenses" balance is transferred to the "Profit and Loss" summary account (entries "b") and then the balance of "Profit and Loss" is transferred to "Owner Equity" (entries "c"). The net of these transactions ("a", "b", "c") is a credit to cash (C) and a debit to equity (E), or in variable notation $X_{EC(1)}$.

The variable $X_{EC(2)}$ represents the net of the accounting transactions recording the payment of cash dividends. The net profits generated by the firm during the period can either be distributed to stockholders as cash dividends or be retained in the firm to finance growth and other needs. If all net profits are retained in the firm, owner equity is increased by the amount of the net profit. When cash dividends are paid, net profit is decreased and the increase in owner equity from net profit is smaller than it would be if all profits were retained for use by the firm. The payment of cash dividends, then, decreases both "Owner Equity" (E) and "Cash" (C) as represented by the variable $X_{EC(2)}$.

The variable X_{PC} represents the credit entry to "Cash" and the debit entry to "Equipment" (P) which are made when equipment is purchased for cash. The collection of receivables requires a debit entry to "Cash" (C) and a credit entry to "Accounts Receivable" (R) as described by the variable X_{CR}.

In the development of the variable X_{RC} on page 59 of the previous section it was noted that the production and sale of one knife resulted in a \$3.00 increase in "Accounts Receivable", a \$1.00 decrease in "Cash" and a \$2.00 increase in "Owner Equity". It was noted that \$1.00 of the \$3.00 increase in "Accounts Receivable" was the recovery of the variable cost of producing a knife. The remaining \$2.00 of the increase to "Accounts Receivable" then is net profit, which increases the balance of "Owner Equity" by \$2.00. The recording of net profit and increase in owner equity is thus reflected by the net variable X_{RE}.

If a loan is undertaken, "Cash" (C) will be debited and "Loans Payable" (L) will be credited, resulting in an entry, X_{CL}, to represent the corresponding debit and credit entries. As accrual accounting methods are assumed to be employed, the interest charge will be recorded in the period the loan is undertaken. The net effect of entries recording the interest charge is a credit to "Interest Payable" (D) and a debit to "Owner Equity" (E), represented by the variable X_{ED}. The rationale for this net entry is similar to that for the variable $X_{EC(1)}$ on page 64, except that the expense is not paid in cash. That is, "Interest Expense" decreases net profit which in turn results in a decrease in "Owner Equity", while "Interest Payable" increases liability, thus preserving the integrity of the double-entry account system.

The account matrix illustrated in Table 4 depicts the net accounting transactions which are caused by the optimal operating policy determined on page 62. In that example problem the profit goal is \$57.00 and the consequential actions resulting from goal under- or overattainment are incorporated in the model. Using the data illustrated in Table 4 and Equation [9], the projected end-of-period account balances can be calculated. The relationship

$$\text{ENDING BALANCE} = \text{BEGINNING BALANCE} + \Delta_t \qquad [12]$$

is used to calculate the end-of-period balance for an asset account, recalling from Table 5 that $\Delta_t > 0$ signals an increase in an asset account balance and $\Delta_t < 0$ signals a decrease in an asset account balance over the period. The relationship

$$\text{ENDING BALANCE} = \text{BEGINNING BALANCE} - \Delta_t$$

is used to calculate the end-of-period balance for a liability or net worth account, recalling from Table 4 that $\Delta_t > 0$ signals a decrease and $\Delta_t < 0$ signals an increase in a liability or net worth account balance over the period.

As an example, we use Equation [9] to calculate the change in the cash account balance over Period 1. Thus:

$$\begin{aligned}\Delta_C &= \sum_{j=1}^{8} X_{Cj} - \sum_{i=1}^{8} X_{iC} \\ &= 40.00 - 48.00 \\ &= -8.00\end{aligned}$$

on substituting $\Delta_C = 8.00$ into Equation [12],

$$(\text{ENDING BALANCE})_{\text{CASH}} = 20.00 + (-8.00)$$
$$= 12.00$$

In the same manner, the end-of-period balances for the other asset accounts are found. The results are:

Accounts Receivable = \$96.00

Inventory = \$0.00

Plant and Equipment = \$26.50

Using Equation [9], the change in the Loans Payable account balance over Period 1 is calculated as

$$\Delta_L = \sum_{j=1}^{8} X_{Lj} - \sum_{i=1}^{8} X_{iL}$$
$$= 0.0 - 10.00$$
$$= -10.00$$

Substituting $\Delta_L = -10.00$ into Equation [13],

$$(\text{ENDING BALANCE})_{\text{LOAN PAYABLE}} = 10.00 - (-10.00)$$
$$= 20.00$$

In the same manner, the end-of-period balances for the remaining liability and net worth accounts are found. They are:

Interest Payable = \$1.00

Bonds Payable = \$30.00

Owner Equity = \$83.50

Table 6 is the projected end-of-period Balance Sheet for the firm based on the assumption that the optimal policy derived from the goal programming solution is implemented.

TABLE 6

BALANCE SHEET, END OF PERIOD I

Assets		Liabilities and Net Worth	
Cash	$ 12.00	Interest Expense Payable	$ 1.00
Accounts Receivable	96.00	Bank Loans	20.00
Inventory	- 0 -	Long Term Bonds Payable	30.00
Plant & Equipment	26.50	Stockholder's Equity	83.50
	$134.50		$134.50

One should realize from the foregoing that the formulation of the goal programming model in terms of financial variables not only yields optimal operating policies (in terms of goals) for the firm, but also provides the basis for estimating the effect of those policies on the financial position of the firm at a future date. Since actual operating policies will directly affect the transactions to be recorded in the books of account, it is desirable to estimate the effect of these policies in advance. Projected balance sheets are the means of estimating the net effects of operating policies, and it has been demonstrated that balance sheets can be obtained easily from the matrix of accounts and the optimal values of operating variables obtained from the goal programming solution.

An obvious extension of the method would be, therefore to incorporate the matrix of accounts and a balance sheet calculating routine, along with the goal programming solution routine, into a computer program so that the output would provide not only the goal programming problem solution, but also the projected balance sheet(s) resulting from the solution. Such a procedure would give management both the "means" (operating policies), and the "end" (projected balance sheets).

Further, by making incremental changes in the values of the inputs (goals), the analyst thus could develop several optimal alternate operating policies - in effect, providing a sensitivity analysis on goal values - thus providing management with an enlarged set of alternate actions, together with the projected consequences (in the balance sheet) of these proposed action.

FOOTNOTES

[1]Simplex iterations are shown in Figure 10, Appendix A.

[2]$2X_1 + 3X_2 + Y^- - Y^+ = 60$

$2(12) + 3(8) + 0 - 12 = 60$

therefore, $2(12) + 3(8) = 48$

[3]Whether or not the end-of-period cash balance of $12.00 is reasonable or adequate is not at issue here. It is to be recognized, of course, that cash balances in reality fluctuate from period to period, and that the absolute level to be maintained at the end of a given period is not only a function of prior levels of cash usage and generation, but also of future cash needs and potential generation. In reality, the right-hand side goal value in a cash balance goal formulation probably is not a constant, as assumed here, but rather a function of the firm's production and operating levels, which can and do change from period to period. For simplicity, however, we assume the goal value to be a constant which has _a priori_ been determined by management to be satisfactory.

[4]$X_1 = X_{RC(K)} = 2.0;$

$X_2 = X_{RC(B)} / 2 = 36.0/2 = 18.0$

CHAPTER 3

SUMMARY

In this thesis the goal programming method is investigated as an economic model of the firm. The process of managing at all levels of the firm requires a continual definition and subdivision of goals and objectives that in turn give direction to the detailed activities of the firm. For the firm to be successful, it is necessary that the attainment of the subgoals formulated for each operating segment of the firm contribute to the accomplishment of the overall objective of the firm. Therefore, the method used to model the firm should possess a format which is capable of incorporating multiple goals and subgoals in an ordered hierarchy. The model should also be capable of selectively optimizing the goals and subgoals of the firm.

The formats of traditional linear programming models provide for the optimization of one overriding goal, formulated as the objective function, and require that all goals other than the superordinate goal be formulated as environmental constraints. The constraints act as absolute bounds to the optimization procedure.

The goal programming format, on the other hand, is capable of incorporating quantified multiple management goals which can be subjectively weighted to signify the relative importance of each goal in the hierarchy of management desires. Using linear programming computational techniques, the goal programming method optimizes the allocation of resources so as to obtain the most satisfactory solution relative to the hierarchy of goals established by management.

Why should management entertain a "satisficing" behavior when the possibility of a global optimum allocation of resources may be possible? The answer to this question comes from considering the nature and characteristics of the firm itself. The firm is a complex organization of what might be described as "men, money, machines, material, and methods." When management undertakes the formulation of an overall operating policy for the firm, the total integration of these components of the firm should be considered. Management must be able realistically to project levels of performance for each operating period and segment of the firm which are attainable (or believed to be attainable) relative to the present mode of operation and standards of performance of the firm. Even if a global optimal operating policy could be found for the firm, this does not mean that the steps to attain that optimum can be implemented and the result achieved in the immediate period, or even in a few foreseeable periods in the future. A change in policy may require movement toward the new objective over several operating periods before this change becomes an operational reality. Management gives direction to the firm through the establishment of a realistic hierarchy of goals and thus the method used to model the firm should develop operating policies which are consistent with these realistic objectives. Herein lies the applicability and superiority of goal programming as a model of the firm.

The goal programming method will obtain a satisfactory solution to the optimal allocation of resources even though the goals established by management are incompatible. It will also signal goal incompatibility. These results are accomplished by the hierarchical weighting of the deviation variables which compose the objective function, so that the computational procedure will attempt to minimize these deviations logically in an ordered sequence. The goal programming solution obtained for the incompatible goal situation may require the underattainment of particular subgoals but the goals that are underattained will be those which have a lower "status" in the hierarchy of management desires. From the viewpoint of the firm as a whole, the solution to this incompatible multiple goal problem will indicate an optimal allocation of resources so that the major objectives of the firm are attained. Because multiple goals must be formulated as inviolable environmental constraints, traditional linear programming models cannot resolve the conflict of incompatible constraints through selective satisfaction of particular environmental constraints at the expense of violating other constraints and retain feasibility in the solution. The incompatible goal programming formulation can be extended to yield a solution that not only depicts the amount that particular goals are under- or overattained but the consequences of such goal unattainment also.

The goal programming formulation can be easily adapted to incorporate data from the books of account. When the variables in the goal programming model are defined in terms of financial units, the solution values for the operating variables can be used directly to construct a projected balance sheet of the firm. This provides the financial <u>effect</u> of the operating variables. Given both an operating policy and the results of that policy, management possesses better knowledge on which to base decisions that will, more probably, lead to successful operation of the firm <u>in the direction of the goals</u> specified by it.

APPENDIX A

SIMPLEX ITERATIONS TO ILLUSTRATIVE EXAMPLES

Min $Z = 2X_1 + 3X_2$

First Tableau

		2	3	0	0	0	0	
		X_1	X_2	S_1	S_2	S_3	b	θ
0	S_1	1/2	1/4	1	0	0	8	32
0	S_2	1	1	0	1	0	20	20
0	S_3	1	[2]	0	0	1	28	14
	Z'	2	3	0	0	0	0	

Second Tableau

		2	3	0	0	0	0	
		X_1	X_2	S_1	S_2	S_3	b	θ
0	S_1	3/8	0	1	0	-1/8	9/2	12
0	S_2	[1/2]	0	0	1	-1/2	6	12
3	X_2	1/2	1	0	0	1/2	14	28
	Z'	1/2	0	0	0	-3/2	42	

Optimal Tableau

		2	3	0	0	0	0	
		X_1	X_2	S_1	S_2	S_3	b	θ
0	S_1	0	0	1	-3/4	1/4	0	
2	X_1	1	0	0	2	-1	12	
3	X_2	0	1	0	-1	1	8	
	Z'	0	0	0	-1	-1	48	

Figure 7. Simplex Iterations to the "Optimal Mix" Linear Programming Problem

$$\text{Min} \quad Z = 2X_1 + 3X_2$$

First Tableau

		X_1	X_2	Y^+	S_1	S_2	S_3	Y^-	b	Θ
		0	0	1	0	0	0	1	0	
0	S_1	1/2	1/4	0	1	0	0	0	8	32
0	S_2	1	1	0	0	1	0	0	20	20
0	S_3	1	[2]	0	0	0	1	0	28	14
1	Y^-	2	3	-1	0	0	0	1	60	20
	Z'	-2	-3	2	0	0	0	0	60	

Second Tableau

		X_1	X_2	Y^+	S_1	S_2	S_3	Y^-	b	Θ
		0	0	1	0	0	0	1	0	
0	S_1	3/8	0	0	1	0	-1/8	0	9/2	12
0	S_2	[1/2]	0	0	0	1	-1/2	0	6	12
0	X_2	1/2	1	0	0	0	1/2	0	14	28
1	Y^-	1/2	0	-1	0	0	-3/2	1	18	36
	Z'	-1/2	0	2	0	0	3/2	0	18	

Optimal Tableau

		X_1	X_2	Y^+	S_1	S_2	S_3	Y^-	b	Θ
0	S_1	0	0	0	1	-3/4	1/4	0	0	
0	X_1	1	0	0	0	2	-1	0	12	
0	X_2	0	1	0	0	-1	1	0	8	
1	Y^-	0	0	-1	0	-1	-1	1	12	
	Z'	0	0	2	0	1	1	0	12	

Figure 8. Simplex Method Solution to the "Optimal Mix" Goal Programming Problem

Two Phase Method

Phase I : Min $Z_1 = A_5$

First Tableau

		0	0	0	0	0	0	0	0	1	0	
		X_1	X_2	Y^+	S_4	S_1	S_2	Y^-	S_3	A_5	b	θ
0	S_1	1/2	1/4	0	0	1	0	0	0	0	8	32
0	S_2	1	1	0	0	0	1	0	0	0	20	20
0	Y^-	2	3	-1	0	0	0	1	0	0	45	15
0	S_3	1	2	0	0	0	0	0	1	0	28	14
1	A_5	2	[3]	0	-1	0	0	0	0	1	30	10
	Z'	-2	-3	0	1	0	0	0	0	0		

Optimal Tableau

		0	0	0	0	0	0	0	0	1	0	
		X_1	X_2	Y^+	S_4	S_1	S_2	Y^-	S_3	A_5	b	θ
0	S_1	1/3	0	0	1/12	1	0	0	0	-1/12	11/2	
0	S_2	1/3	0	0	1/3	0	1	0	0	-1/3	10	
0	Y^-	0	0	-1	1	0	0	1	0	-1	15	
0	S_3	-1/3	0	0	2/3	0	0	0	1	-2/3	8	
0	X_2	2/3	1	0	-1/3	0	0	0	0	1/3	10	
	Z'	0	0	0	0	0	0	0	0	1	0	

Figure 9. Solution to Expanded Goal Programming Problem with a Profit Goal of $45.00

Phase II : Min $Z_2 = Y^- + Y^+$

First Tableau

		0	0	1	0	0	0	1	0	0	
		X_1	X_2	Y^+	S_4	S_1	S_2	Y^-	S_3	b	Θ
0	S_1	1/3	0	0	1/12	1	0	0	0	11/2	66
0	S_2	1/3	0	0	1/3	0	1	0	0	10	30
1	Y^-	0	0	-1	1	0	0	1	0	15	15
0	S_3	-1/3	0	0	[2/3]	0	0	0	1	8	12
0	X_2	2/3	1	0	-1/3	0	0	0	0	10	-30
	Z'	0	0	2	-1	0	0	0	0	15	

Second Tableau

		0	0	1	0	0	0	1	0	0	
		X_1	X_2	Y^+	S_4	S_1	S_2	Y^-	S_3	b	Θ
0	S_1	3/8	0	0	0	1	0	0	-1/8	9/2	12
0	S_2	1/2	0	0	0	0	1	0	-1/2	6	12
1	Y^-	[1/2]	0	-1	0	0	0	1	-3/2	3	6
0	S_4	-1/2	0	0	1	0	0	0	3/2	12	-24
0	X_2	1/2	1	0	0	0	0	0	1/2	14	28
	Z'	-1/2	0	2	0	0	0	0	3/2	3	

Figure 9. "Continued"

<u>Optimal</u> <u>Tableau</u>

		0	0	1	0	0	0	1	0	0	
		X_1	X_2	Y^+	S_4	S_1	S_2	Y^-	S_3	b	Θ
0	S_1	0	0	3/4	0	1	0	-3/4	[1]	9/4	9/4
0	S_2	0	0	1	0	0	1	-1	1	3	3
0	X_1	1	0	-2	0	0	0	2	-3	6	-2
0	S_4	0	0	-1	1	0	0	1	0	15	15
0	X_2	0	1	1	0	0	0	-1	2	11	11/2
	Z'	0	0	1	0	0	0	1	0	0	

<u>Alternate</u> <u>Optimal</u> <u>Tableau</u>

		0	0	1	0	0	0	1	0	0	
		X_1	X_2	Y^+	S_4	S_1	S_2	Y^-	S_3	b	Θ
0	S_3	0	0	3/4	0	[1]	0	-3/4	1	9/4	9/4
0	S_2	0	0	1/4	0	-1	1	-1/4	0	3/4	-3/4
0	X_1	1	0	1/4	0	3	0	-1/4	0	51/4	17/4
0	S_4	0	0	-1	1	0	0	1	0	15	∞
0	X_2	0	1	-1/2	0	-2	0	1/2	0	13/2	-13/4
	Z'	0	0	1	0	0	0	1	0	0	

Figure 9. "Continued"

Two Phase Method

Phase I : Min $Z_1 = A_5$

First Tableau

		0	0	0	0	0	0	0	0	1	0	
		X_1	X_2	Y^+	S_4	S_1	S_2	Y^-	S_3	A_5	b	θ
0	S_1	1/2	1/4	0	0	1	0	0	0	0	8	32
0	S_2	1	1	0	0	0	1	0	0	0	20	20
0	Y^-	2	3	-1	0	0	0	1	0	0	60	20
0	S_3	1	2	0	0	0	0	0	1	0	28	14
1	A_5	2	[3]	0	-1	0	0	0	0	1	30	10
	Z'	-2	-3	0	1	0	0	0	0	0	30	

Optimal Tableau

		0	0	0	0	0	0	0	0	1	0	
		X_1	X_2	Y^+	S_4	S_1	S_2	Y^-	S_3	A_5	b	θ
0	S_1	1/3	0	0	1/12	1	0	0	0	-1/12	11/2	
0	S_2	1/3	0	0	1/3	0	1	0	0	-1/3	10	
0	Y^-	0	0	-1	1	0	0	1	0	-1	30	
0	S_3	-1/3	0	0	2/3	0	0	0	1	-2/3	8	
0	X_2	2/3	1	0	-1/3	0	0	0	0	1/3	10	
	Z'	0	0	0	0	0	0	0	0	1	0	

Figure 10. Solution to Expanded Goal Programming Problem with a Profit Goal of $60.00

Phase II ; Min $Z_2 = Y^- + Y^+$

First Tableau

		0	0	1	0	0	0	1	0	0	
		X_1	X_2	Y^+	S_4	S_1	S_2	Y^-	S_3	b	θ
0	S_1	1/3	0	0	1/12	1	0	0	0	11/2	66
0	S_2	1/3	0	0	1/3	0	1	0	0	10	30
1	Y^-	0	0	-1	1	0	0	1	0	30	30
0	S_3	-1/3	0	0	[2/3]	0	0	0	1	8	12
0	X_2	2/3	1	0	-1/3	0	0	0	0	10	-30
		0	0	2	-1	0	0	0	0		

Second Tableau

		0	0	1	0	0	0	1	0	0	
		X_1	X_2	Y^+	S_4	S_1	S_2	Y^-	S_3	b	θ
0	S_1	3/8	0	0	0	1	0	0	-1/8	9/2	12
0	S_2	[1/2]	0	0	0	0	1	0	-1/2	6	12
1	Y^-	1/2	0	-1	0	0	0	1	-3/2	18	36
0	S_4	-1/2	0	0	1	0	0	0	3/2	12	-24
0	X_2	1/2	1	0	0	0	0	0	1/2	14	28
	Z'	-1/2	0	2	0	0	0	0	3/2	18	

Figure 10. "Continued"

Optimal Tableau

		0	0	1	0	0	0	1	0	0	
		X_1	X_2	Y^+	S_4	S_1	S_2	Y^-	S_3	b	Θ
0	S_1	0	0	0	0	1	-3/4	0	1/4	0	
0	X_1	1	0	0	0	0	2	0	-1	12	
1	Y^-	0	0	-1	0	0	-1	1	-1	12	
0	S_4	0	0	0	1	0	1	0	1	18	
0	X_2	0	1	0	0	0	-1	0	1	8	
	Z'	0	0	2	0	0	1	0	1	12	

Figure 10. "Continued"

$$\text{Min} \quad Z = MY_3^- + NY_4^- + NY_4^+ \; 3NY_5^-$$

First Tableau

		0	0	0	N	0	0	0	M	N	3N	0	
		X_1	X_2	Y_3^+	Y_4^-	Y_5^+	S_1	S_2	Y_3^-	Y_4^+	Y_4^-	b	Θ
0	S_1	1/2	1/4	0	0	0	1	0	0	0	0	8	32
0	S_2	1	1	0	0	0	0	1	0	0	0	20	20
M	Y_3^-	2	3	-1	0	0	0	0	1	0	0	45	15
N	Y_4^+	1	2	0	-1	0	0	0	0	1	0	28	14
3N	Y_5^-	2	[3]	0	0	-1	0	0	0	0	1	30	10
	Z'	-2M-7N	-3M-11N	M	2N	3N	0	0	0	0	0	45M+118N	

Second Tableau

		0	0	0	N	0	0	0	M	N	3N	0	
		X_1	X_2	Y_3^+	Y_4^-	Y_5^+	S_1	S_2	Y_3^-	Y_4^+	Y_5^-	b	Θ
0	S_1	1/3	0	0	0	1/12	1	0	0	0	-1/12	11/2	66
0	S_2	1/3	0	0	0	1/3	0	1	0	0	-1/3	10	30
M	Y_3^-	0	0	-1	0	1	0	0	1	0	-1	15	15
N	Y_4^+	-1/3	0	0	-1	[2/3]	0	0	0	1	-2/3	8	12
0	X_2	2/3	1	0	0	-1/3	0	0	0	0	1/3	10	-30
	Z'	1/3N	0	M	2N	-M-2/3N	0	0	0	0	3N	15M + 8N	

Figure 11. Multiple Goal Example Simplex Solution With a Profit Goal of $45.00

Third Tableau

		0	0	0	N	0	0	0	M	N	3N	0	
		X_1	X_2	Y_3^+	Y_4^-	Y_5^+	S_1	S_2	Y_3^-	Y_4^+	Y_5^+	b	θ
0	S_1	3/8	0	0	1/8	0	1	0	0	-1/8	0	9/2	36
0	S_2	1/2	0	0	1/2	0	0	1	0	-1/2	0	6	12
M	Y_3^-	1/2	0	-1	[3/2]	0	0	0	1	-3/2	0	3	2
0	Y_5^+	-1/2	0	0	-3/2	1	0	0	0	3/2	-1	12	-8
0	X_2	1/2	1	0	-1/2	0	0	0	0	1/2	0	14	-28
	Z'	-1/2M	0	M	N-3/2M	0	0	0	0	N+3/2M	3N	3M	

Fourth Tableau

		0	0	0	N	0	0	0	M	N	3N	0	
		X_1	X_2	Y_3^+	Y_4^-	Y_5-	S_1	S_2	Y_3^-	Y_4^+	Y_5^-	b	θ
0	S_1	1/3	0	1/12	0	0	1	0	-1/12	0	0	17/4	51/4
0	S_2	1/3	0	1/3	0	0	0	1	-1/3	0	0	5	15
N	Y_4^-	[1/3]	0	-2/3	1	0	0	0	2/3	-1	0	2	6
0	Y_5^+	0	0	-1	0	1	0	0	1	0	-1	15	∞
0	X_2	2/3	1	-1/3	0	0	0	0	1/3	0	0	15	45/2
	Z'	-1/3N	0	2/3N	0	0	0	0	M-2/3N	2N	3N	2N	

Figure 11. "Continued"

Optimal Tableau

	X_1	X_2	Y_3^+	Y_4^-	Y_5^+	S_1	S_2	Y_3^-	Y_4^+	Y_5^-	b	θ
	0	0	0	N	0	0	0	M	N	3N	0	
S_1	0	0	3/4	-1	0	1	0	-3/4	1	0	9/4	3
S_2	0	0	[1]	-1	0	0	1	-1	1	0	3	3
X_1	1	0	-2	3	0	0	0	2	-3	0	6	-3
Y_5^+	0	0	-1	0	1	0	0	1	0	-1	15	-15
X_2	0	1	1	-2	0	0	0	-1	2	0	11	11
Z'	0	0	0	N	0	0	0	M	N	3N	0	

Alternate Optimal Tableau

	X_1	X_2	Y_3^+	Y_4^-	Y_5^+	S_1	S_2	Y_3^-	Y_4^+	Y_5^-	b	θ
	0	0	0	N	0	0	0	M	N	3N	0	
S_1	0	0	0	-1/4	0	1	-3/4	0	1/4	0	0	
Y_3^+	0	0	1	-1	0	0	1	-1	1	0	3	
X_1	1	0	0	1	0	0	2	0	-1	0	12	
Y_5^+	0	0	0	-1	1	0	1	0	1	-1	18	
X_2	0	1	0	-1	0	0	-1	0	1	0	8	
Z'	0	0	0	N	0	0	0	M	N	3N	0	

Figure 11. "Continued"

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AN INVESTIGATION OF THE GOAL PROGRAMMING METHOD

by

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AN ABSTRACT OF A MASTER'S THESIS

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This thesis summarizes some of the developments of other investigators in the method of "Goal Programming". In addition, the prior work reported herein has been extended briefly to include a new concept called "consequential action".

Goal programming is a method of optimally allocating resources within the firm when the firm has an ordered, weighted hierarchy of goals. A simple economic conceptualization of the firm is used to illustrate the application of goal programming when the firm is assumed to have both compatible and incompatible multiple goals or objectives. Using this same concept, an example was formulated to demonstrate the integration of the goal programming method and the accounting system of the firm.

The goal programming solution to the incompatible goal problem requires that the exact attainment of particular subgoals be sacrificed so that major goals can be met. The goal programming method also was extended from prior work to define a new concept, that of "consequential action". A consequential action is an action that the firm must undertake as a result of failure to attain a particular goal. An example of a consequential action is the forced borrowing of funds if a cash balance goal is underattained. This concept is formulated quantitatively and exemplified.