THERMOELASTIC STRESS AND DISPLACEMENT IN A THIN ROD DUE TO AN INSTANTANEOUS HEAT SOURCE

by

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NOMENCLATURE

An	Cofficient of equation (12)
Bn	Cofficient of equation (13)
Ce	Cofficient of thermal expansion
E	Young's modulus
$G_1(n,0)$, $G_2(n,0)$	Define in section 3.3
$G_1'(n,0)$	
$F_1(n,m)$, $F_2(n,m)$	Define in section 3.3
$F_3(n,m)$, $F_4(n,m)$	
$F_1'(n,m)$	
$F_3'(n,m)$	
$H(x)$, $H(x-x_0)$	Heaviside step function
$H(x-x_1)$	
k	thermal conductivity
L	span length of rod
L{ }	Laplace transformation operator
Q_0	Dimensionless heat quantity, $q_0\alpha/kT_0$
q_0	Heat quantity per unit time and volume
$q(x,t)$	Heat generation
s	dummy variable used in Laplace transformation
t	time variable
$T(x,t)$	Temperature
T_0	Reference temperature

$u(x,t)$	Displacement
$\bar{U}(\xi,\tau)$	Dimensionless displacement
$U(\xi,\tau)$	Laplacement transformation of $U(\xi,\tau)$ with respect to τ.
x_0, x_1	Partical point on bar
x	Axial coordinate
x'	Relative axial coordinate
V	Velocity of elastic wave propagation $(E/\rho)^{1/2}$
α	Thermal diffusivity
β	Difine in equation (41)
$\psi_1(s)$	Cofficient of equation (35), defined by equation (36)
$\psi_2(s)$	Cofficient of equation (35), defined by -quation (37)
$\varepsilon(x,t)$	Strain
$\lambda_1, \lambda_2, \lambda_3, \lambda_4$	Defined by equation (39)
η	Dummy variable
ρ	Density
$\Theta(\xi,\tau)$	Dimensionless temperature, T/T_0
$\sigma(x,t)$	Stress
τ	Dimensionless time variable, $\alpha t/L^2$
ξ	Dimensionless length, x/L
ξ'	Relative dimensionless length, x'/L
ξ_0 ξ_1	Dimensionless form of x_0, x_1
$\delta(x-x'), \delta(\tau)$	Dirac's Delta function
$\delta(t), \delta(\xi-\xi')$	

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CHAPTER 1

INTRODUCTION

The determination either of the temperature distribution or of the subsequent mechanical responses exposed to the action of heat source in solids has been paid considerable attention both in research and engineering [1,2]. Research on this topic and the development of appropriate methods of analysis are some of the most interesting areas of current technical activity, not only in connection with nuclear reactors, but also in fields such as those of welding, metal cutting, metallurgical processes, high-speed flight, power plant design and so forth.

Apparently, the theory of heat flow due to a heat source originated in connection with arc welding. Rosenthal, P. [4] in 1935 was the first to apply the exact theory of heat flow due to a heat source to arc welding, by using the experimentally established principle of a quasi-stationary state [3,5]. A particular case of the general solution was treated independently by Boulton and Lance Martin [3,6] in 1936. Bruce [7] applied the method of instantaneous sources to another particular case of welding and, in 1941, Mahla [8] extended this method to a three dimensional case. More recently, More complete investigations of thermal stresses in an infinite cylinder due to steady-state or transient temperature variation have since been made by a number of authors [9.10,11,12,13,14,15,16,17]. In reviewing the literature, attention was centered on the case of temperature variation and thermal stress in rods resulting in many efforts which have been made by different investigators [18,19,20,21,22].

In the present research, a simple problem of thermal stress and displacement in a thin finite rod has been considered. The heat source is instantaneously generated over a finite portion of the rod, one end of the rod is fixed with the other free, and both ends are kept at zero temperature. This researsh is of intrinsic interest itself because the rod problem reprents the simplest of all engineering structures.

The problem is approached from the standpoint of classical linear, uncoupled, thermoelastic theory. The analysis is composed of two distinct problem; i.e., heat conduction neglecting the mechanical coupling effect and elasticity regarding the inertia effect. Furthermore, the material is assumed to be homogeneous and isotropic with respect to both its thermal and mechanical responses, and its physical properties are independent of temperature.

At first, this research is concerned with the derivation of the temperature distribution field. Assuming the temperature gradients in the cross section of the rod to be negligible and also that heat losses through the surface to the sorroundings medium is not considered, one obtains one uncoupled heat conduction equation. The partial differential equation is solved for the finite long rod by the technique of Laplace transformation method. A " long-time" solution [23] is obtained. Associated with the given temperature variation, an elementary thermoelastic theory was applied to derive the governing differential equations under the thermal load. Laplace transformation has been found convenient for the solution of this problem. In addition, the effect of this temperature induced the thermal deformation and the propagating stress wave in the rod will be studied by using the finite difference approximation methods.

CHAPTER 2

TEMPERATURE DISTRIBUTION IN A THIN ROD INDUCED BY A LINE HEAT SOURCE

2.1 Derivation of the Heat Conduction Equation

The coordinate system to be considered is described in Fig. 1.

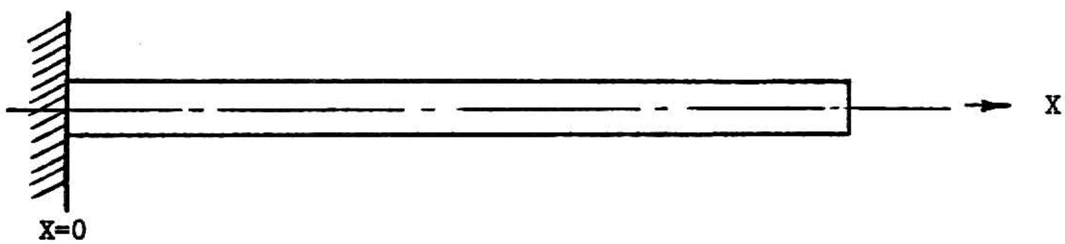


Fig. 1 Coordinate system

For analysis of the above described thin rod, the linear, uncoupled heat conduction theory will be used subjected to the following assumptions:

1. Any conversion of mechanical energy into heat is neglected.
2. The material is assummed to be homogeneous and isotropic with respect to its thermal response , and all physical properties are regarded to be independent of temperature.
3. The temperature gradient in the cross section of the rod

is neglected and also the loss through the surface to the sorrounding medium is not considered in the present investigation.

Under these assumptions the energy balance equation for an element of the rod leads to the Fourier Heat Conduction Equation [24].

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{\alpha}\frac{\partial T}{\partial x} = -\frac{q(x,t)}{k} \qquad (1)$$

where q(x,t) is the heat generation. For a point impulsive heat source at zero time

$$q(x,t) = q_0\delta(t)\delta(x-x') \qquad (2)$$

and for an instantaneous line heat source at zero time

$$q(x,t) = q_0\delta(t)\{H(x-x_0)-H(x-x_1)\} \qquad (3)$$

where α is the thermal diffusivity, k is the thermal conductivity, q_0 is the quantity of heat generated by the heat source per unit time and volume, t is the time variable, x is the position variable, H(x) is the Heaviside step function and $\delta(t)$ is the well-known Dirac's Delta function.

Equation (1) and equation (3) constitute the equations of heat conduction to be solved in the present investigation.

2.2 Method of solution

The equation of heat conduction for an isotropic rod of finite length due to a point impulsive heat source at zero time is first solved by using the Laplace transformation method. With the aid of integration with respect to the space coordinate, the solution for a thin rod of finite length due to a constant instantaneous line heat source can be obtained.

2.2.1 Solution of Heat Conduction Equation for a point impulsive heat source

Consider a thin rod of finite length as shown in Fig. 2.

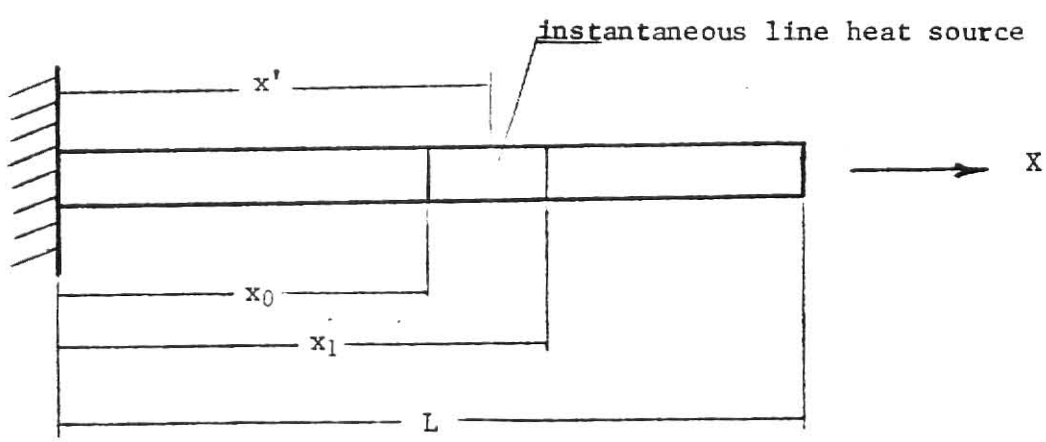


Fig. 2 An instantaneous heat source distributed on the thin rod

The rod is instantly heated by a point heat source at x'. Immediately remove the point heat source and let the rod cool naturally.

The differential equation governing the heat conduction in this investigation as derived in section 2.1 is

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{\alpha}\frac{\partial T}{\partial t} = - \frac{q_0}{k}\,\delta(t)\delta(x-x') \qquad (4)$$

where $x \leq x' \leq x$. Introduction of dimensionless quantities $\xi = x/L$, $\tau = \alpha t/L^2$, $\Theta = T/T_0$, reduces equation (4) to

$$\frac{\partial^2 \Theta}{\partial \xi^2} - \frac{\partial \Theta}{\partial \tau} = \frac{q_0 \alpha}{kT_0}\,\delta(\tau)\delta(\xi-\xi')$$

$$= - Q_0\delta(\tau)\delta(\xi-\xi') \qquad (5)$$

where $Q_0 = q_0\alpha/kT_0$, $\xi' = x'/L$ and T_0 is a reference temperature.

For the rod with a finite length L, the boundary conditions and initial conditions must be provided in order to describe the problem completely. In the present investigation we suppose that the system is at rest initially. Thus the initial conditions are

$$T = 0 \qquad \text{for} \quad 0 \leq x \leq L \quad \text{at } t = 0 \qquad (6)$$

or in terms of dimensionless quantities

$$\Theta = 0 \qquad \text{for} \quad 0 \leq \xi \leq 1 \quad \text{at } \tau = 0 \qquad (7)$$

Prescribed temperature at both ends are

$$T = 0 \qquad \text{for} \quad x = 0,\ L \tag{8}$$

or

$$\Theta = 0 \qquad \text{for} \quad \xi = 0,\ 1 \tag{9}$$

Taking the Laplace transform of equation (5) and introducing the initial condition, i.e., Eq.(7), we obtain

$$\frac{d^2\overline{\Theta}}{d\xi^2} - s\overline{\Theta} = -\,Q_0\delta(\xi-\xi') \tag{10}$$

with the boundary condition

$$\overline{\Theta}(0,s) = \overline{\Theta}(1,s) = 0 \tag{11}$$

where the notation

$$\overline{\Theta}(\xi,s) = L\{\Theta(\xi,\tau)\} = \int_0^\infty e^{-s\tau}\Theta(\xi,\tau)\,d\tau$$

is introduced and $\overline{\Theta}(\xi,s)$ is referred to as the Laplace transform of $\Theta(\xi,s)$ with respect to τ. As a solution of Eq. (10) satisfying Eq. (11), we assume

$$\overline{\Theta}(\xi,s) = \sum_{n=1}^{\infty} A_n \sin(n\pi\xi) \tag{12}$$

where A_n is independent of ξ. We also assume

$$\delta(\xi-\xi') = \sum_{n=1}^{\infty} B_n \sin(n\pi\xi) \tag{13}$$

Since the set of functions $\sin(n\pi\xi)$ is orthogonal over (0,1), both sides of equation (13) are multiplied by $\sin(n\pi\xi)$ and integrated to obtain

$$Bn = \int_0^1 \delta(\xi-\xi')\sin(n\pi\xi)$$

$$= 2\ \sin(n\pi\xi') \tag{14}$$

where n = 1,2,3,4,5 .... ∞

and also

$$\delta(\xi-\xi') = 2 \sum_{n=1}^{\infty} \sin(n\pi\xi')\sin(n\pi\xi) \tag{15}$$

where n = 1,2,3,4,5 .... ∞

Hence, substituting equation (12) and equation (15) into equation (10), we have

$$A_n = \frac{2\ Q_0\ \sin(n\pi\xi')}{s + n^2\pi^2}$$

where n = 1,2,3,4,5 .... ∞

and

$$\overline{\theta}(\xi,s) = 2\ Q_0 \sum_{n=1}^{\infty} \frac{\sin(n\pi\xi')\sin(n\pi\xi)}{s + n^2\pi^2} \tag{16}$$

By inversion, the temperature distribution for an impulsive point heat source is

$$\Theta(\xi,\tau) = 2\ Q_0 \sum_{n=1}^{\infty} \sin(n\pi\xi')\sin(n\pi\xi)\ e^{-n^2\pi^2\tau} \tag{17}$$

2.2.2 Solution of Heat Conduction Equation for a thin rod of finite length due to an instantaneous line heat source

For an instantaneous line heat source distributed over $\xi_0 \leq \xi' \leq \xi$ we integrate the preceding result of equation (17) with respect to ξ' between the limits $\xi' = \xi_0$, $\xi = \xi_1$ and get

$$\Theta(\xi,\tau) = \int_{\xi_0}^{\xi_1} [2\ Q_0 \sum_{n=1}^{\infty} \sin(n\pi\xi)\ e^{-n^2\pi^2\tau} \sin(n\pi\xi')]\ d\xi'$$

$$= 2\ Q_0 \sum_{n=1}^{\infty} \frac{\sin(n\pi\xi)}{n\pi} [\cos(n\pi\xi_0) - \cos(n\pi\xi_1)]\ e^{-n^2\pi^2\tau} \tag{18}$$

Equation (18) is the solution for temperature variation in terms of nondimensional quantities.

2.3 Evaluation of analytical solution of temperature variation

2.3.1 Evaluation of an infinite series

Since the temperature variation $\Theta(\xi,\tau)$ is expressed in terms of exponential and trigonometric functions, the numerical calculation and convergence of the infinite series must be studied. For convenience, let us repeat equation (18)

$$\Theta(\xi,\tau) = 2Q_0 \sum_{n=1}^{\infty} \frac{\sin(n\pi\xi)}{n\pi} e^{-n^2\pi^2\tau}[\cos(n\pi\xi_0)-\cos(n\pi\xi_1)]$$

Let

$$\text{sum} = \sum_{n=1}^{\infty} \frac{\sin(n\pi\xi)}{n\pi} e^{-n^2\pi^2\tau}[\cos(n\pi\xi_0)-\cos(n\pi\xi_1)] \tag{19}$$

From the properties of trigonometric functions, the absolute values of $\sin(n\pi\xi)$ and $\cos(n\pi\xi_0)-\cos(n\pi\xi_1)$ should be less than or equal to 1 and 2 respectively, i.e., $|\sin(n\pi\xi)| \leq 1$ and $|\cos(n\pi\xi_0)-\cos(n\pi\xi_1)| \leq 2$. Therefore, equation (19) can be expressed as

$$\text{sum} \leq 2\sum_{n=1}^{\infty} \frac{e^{-n^2\pi^2\tau}}{n\pi} \tag{20}$$

Upon examining the above equation, the right hand side of the inquality converges very rapidly if τ is very large, but, obviously, it is inconvenient for calculations pertaining to small times.

Expanding the exponential function $e^{-n^2\pi^2\tau}$ in series form, we find that

$$e^{-n^2\pi^2\tau} = \frac{1}{1+(n^2\pi^2\tau)+(n^2\pi^2\tau)^2/2!+(n^2\pi^2\tau)^4/4!+\cdots\cdots\cdots\cdots}$$

From the above equation, we obtain

$$e^{-n^2\pi^2\tau} < \frac{1}{n^2\pi^2\tau} \tag{21}$$

Inequalities given by equation (20) and equation (21) can be combined in the following form

$$\text{sum} < 2 \sum_{n=1}^{\infty} \frac{1}{n^3\pi^3\tau} \tag{22}$$

Therefore, the series sum and the temperature variation $\Theta(\xi,\tau)$ are convergent, as can be easily proved by combining equation (20) and (22) and applying a comparison test [25].

For the purpose of computation, some particular values, i.e., $\xi = 0.5$, $\xi_0 = 1.0$, $\xi_1 = 1.1$ were substituted in equation (18). Table (1) reveals that the larger the value τ, the less terms one needs to calculate of the series for a required convergent value.

n =	time = 1.00 sec. τ = 0.000546	time = 5.00 sec. τ = 0.002728	time = 10.00sec. τ = 0.005457	time = 50.00 sec. τ = 0.027284
1	0.463740E-01	0.453858E-01	0.441759E-01	0.356181E-01
2	-0.697310E-02	-0.355711E-02	0.234663E-03	0.170531E-01
3	-0.697185E-02	-0.355608E-02	0.235468E-03	0.170532E-01
4	-0.372094E-01	-0.249781E-01	-0.136881E-01	0.166097E-01
5	0.109819E-01	0.314627E-02	0.657693E-03	* 0.166755E-01
6	0.109779E-01	0.314445E-02	0.657002E-03	0.166755E-01
7	0.234975E-01	0.750134E-02	0.182155E-02	0.166755E-01
8	-0.110178E-01	-0.119356E-02	0.269767E-03	0.166755E-01
9	-0.110120E-01	-0.119255E-02	0.269881E-03	0.166755E-01
10	-0.110247E-01	-0.119403E-02	0.269781E-03	
11	0.829180E-02	0.231327E-03	0.324598E-03	
12	0.828616E-02	0.231073E-03	0.324593E-03	
13	0.313685E-02	0.959647E-04	0.323166E-03	
14	-0.488285E-02	-0.216592E-04	0.322566E-03	
15	-0.487877E-02	-0.216272E-04	0.322566E-03	
16	0.189591E-03	-0.121557E-05	0.322586E-03	
17	0.226229E-02	0.288477E-05	* 0.322588E-03	
18	0.226012E-02	0.288275E-05	0.322588E-03	
19	-0.806473E-03	0.159643E-05	0.322588E-03	
20	-0.808938E-03	0.159598E-05	0.322588E-03	
21	-0.808095E-03	0.159604E-05	0.322588E-03	
22	0.493160E-03	0.163477E-05		
23	0.211947E-03	0.163155E-05		
24	0.211731E-03	0.163155E-05		
25	-0.163950E-03	0.163101E-05		
26	-0.360885E-04	* 0.163107E-05		
27	-0.360621E-04	0.163107E-05		
28	0.210599E-04	0.163107E-05		
29	0.371366E-05	0.163107E-05		
30	0.371366E-05	0.163107E-05		
31	0.122185E-04			
32	-0.149779E-05			
33	-0.149410E-05			
34	-0.917515E-05			
35	0.157397E-05			
36	0.156955E-05			
37	0.351390E-05			
38	-0.791600E-06			
39	-0.789100E-06			
40	-0.792929E-06			
41	0.372507E-06			
42	0.371586E-06			
43	0.189205E-06			
44	-0.286520E-07			
45	-0.284014E-07			
46	0.501237E-07			
47	0.744581E-07			
48	0.744074E-07			
49	0.542989E-07			
50	0.542687E-07			

* denoting a required convergent value

Table 1 τ influences the convergence of the series

2.3.2 Numerical example and result

For numerical calculations a finite rod of span length L = 1.5 ft., made of copper, has been assumed. The temperature has been calculated at points along the longitudinal axis, with k = 224.0 Btu/hr-ft°F, ρ = 558.0 Ib/ft^3, Ce = 0.091 Btu/Ib°F, α = 4.42 ft^2/hr, and E = 1.872 × 10^9 Lbf/ft^2.

The numerical calculations of the dimensionless quantities shown in equation (18) were carried out using an IBM 370/158 computer at the Computing Center of Kansas State University.

The results were found by summing terms for evaluation; for instance n = 26, at ξ = 0.5, τ = 0.002728, gave very good convergence. Throughout the computations, the dimensionless quantity Q_0 has been taken as .019731, this choice was assumed so that a unit quantity of heat is introduced.

Part of the numerical values of the dimensionless temperature have been illustrated in Fig. (3), Fig. (4), table (2) and table (3). Thus table (2) and Fig.(3) show the variation of the dimensionless temperature v.s. the dimensionless space variable ξ along the longitudinal axis of the rod as a line heat source is induced between ξ_0 = 0.133 and ξ_1 = 0.120, while table (3) and Fig. (4) illustrate the temperature distribution as a line heat source is induced between ξ_0 = 0.667 and ξ_1 = 0.733 at various instants after the thermal shock.

CHAPTER 3

THERMAL STRESS AND DEFORMATION DUE TO THE GIVEN TEMPERATURE VARIATION

3.1 Derivation of Governing Differential Equation of thermoelasticity for a thin rod

We shall confine ourselves to an elastic, isotropic homogeneous thin rod , with respect to both its mechanical and thermal properties, and assume that plane cross sections remain plane, and that only axial stress is present, being uniformly distributed over the cross section. Let $u = u(x,t)$ be the longitudinal component of the displacement at a point x and at any time t. As in the linear theory of elasticity the strain-displacement relation can be derived directly from purely geometrical considerations.

For small displacements, the strain $\varepsilon(x,t)$ at any point x and at any time t is connected with the displacement vector by the relation

$$\varepsilon(x,t) = \frac{\partial u(x,t)}{\partial x} \tag{23}$$

If only a one dimensional problem is considered, then $\sigma(y,t)$ and $\sigma(z,t)$ can be neglected, i.e., $\sigma(y,t) = \sigma(z,t) = 0$

For the present investigation, only thermal loading is considered. If we use the assumptions of section 2.1 and further assume that there is no coupling of the temperature and strain fields, i.e., no mechanical energy due to the strain is converted into heat, the governing equations of thermoelasticity for an isotropic homogeneous thin rod are derived.

Since the thermal expansion contributes to part of the direct strain, the temperature will appear explicitly in the stress-strain relation, which now has the form

$$\sigma(x,t) = E[\varepsilon(x,t) - Ce\ T(x,t)] \tag{24}$$

with E denoting Young's modulus, Ce denoting the coefficient of linear thermal expansion and $\sigma(x,t)$ denoting the stress at a point and at time t.

When there exist sudden changes of temperature in this rod, the influence of inertia can not be neglected; we have then to investigate the equations of motion.

The basic differential equations governing the extensional motion of a thin, homogeneous rod are the equation of motion [23].

$$\frac{\partial \sigma}{\partial x} = \rho\left(\frac{\partial^2 u}{\partial t^2}\right) \tag{25}$$

and the equation of compatibility

$$\frac{\partial^2 u}{\partial x \partial t} = \frac{\partial^2 u}{\partial t \partial x} \tag{26}$$

Substitution of equations (23) and (24) into equation (25) to eliminate σ leads to the displacement equation, where ρ is the density of the material of the rod.

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 u}{\partial t^2} + Ce(\frac{\partial T}{\partial x}) \tag{27}$$

where $V = (E/\rho)^{1/2}$ is the velocity of elastic wave propagation. Successive substitutions from equation (23) into equation (24) and from equation (26) into equation (25) to eliminate u lead to the stress equation.

$$\frac{\partial^2 \sigma}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 \sigma}{\partial t^2} + \rho Ce\frac{\partial^2 T}{\partial t^2} \tag{28}$$

3.2 Solution of the thermal stress and deformation

We suppose that the system was initially at rest and in the stress free state thus the initial conditions are

$$u = \frac{\partial u}{\partial t} = 0 \qquad \text{for } 0 \leq x \leq L,\ t = 0 \tag{29}$$

$$\sigma = \frac{\partial \sigma}{\partial t} = 0 \qquad \text{for } 0 \leq x \leq L,\ t = 0 \tag{30}$$

Since one end of the rod is kept fixed and the other free and there are no tractions on the lateral surface, the boundary conditions are

$$u = 0 \qquad \text{at } x = 0 \tag{31}$$

$$\frac{\partial u}{\partial x} = 0 \qquad \text{at } x = L \tag{32}$$

Introducing dimensionless quantities $\xi = \frac{x}{L}$, $U = \frac{u}{L}$, $\Theta = \frac{T}{T_0}$, $\tau = \frac{\alpha t}{L^2}$, displacement equation (27) becomes

$$\frac{\partial^2 U}{\partial \xi^2} = \frac{\alpha^2}{V^2 L^2} \frac{\partial^2 U}{\partial \tau^2} + Ce\ T_0 \frac{\partial \Theta}{\partial \xi} \tag{33}$$

with

$$U(\xi,0) = \frac{\partial U(\xi,0)}{\partial \tau} = 0 \tag{34}$$

Taking the Laplace transformation of equation (33) and using equation (34), we obtain

$$\frac{d^2 \overline{U}}{d\xi^2} - \frac{\alpha^2}{V^2 L^2} s^2 \overline{U} = Ce\ T_0 \left(\frac{\partial \overline{\Theta}}{\partial \xi} \right) \tag{35}$$

where $\overline{U}(\xi,s) = \int_0^\infty e^{-s\tau} U(\xi,\tau) d\tau$ is the Laplace transformation of $U(\xi,\tau)$ with respect to τ.

Differentiating equation (22) with respect to ξ and substituting in the above equation we rewrite the equation as

$$\frac{d^2 \overline{U}}{d\xi^2} - \frac{\alpha^2}{V^2 L^2} s^2 \overline{U}$$

$$= 2\ Q_0 Ce\ T_0 \sum_{n=1}^{\infty} \frac{\cos(n\pi\xi)}{s + n^2\pi^2} [\cos(n\pi\xi_0) - \cos(n\pi\xi_1)] \tag{36}$$

with the boundary conditions

$$\bar{U} = 0 \qquad \text{at } \xi = 0 \tag{37}$$

$$\frac{d\bar{U}}{d\xi} = 0 \qquad \text{at } \xi = 1 \tag{38}$$

General solution of equation (36) is

$$\bar{U}(\xi,s) = C_1 e^{\psi_2(s)\xi} + C_2 e^{-\psi_2(s)\xi} - \sum_{n=1}^{\infty} \frac{\psi_1(s)}{n^2\pi^2+\psi_2^2} \cos(n\pi\xi) \tag{39}$$

where

$$\psi_1(s) = \frac{2\ Q_0 CeT_0}{s + n^2\pi^2} [\cos(n\pi\xi_0) - \cos(n\pi\xi_1)] \tag{40}$$

$$\psi_2(s) = \frac{\alpha s}{VL} \tag{41}$$

Solution of equation (39) satisfying equation (37) and (38) is

$$\bar{U}(\xi,s) = [\frac{1}{1 + e^{2\psi_2}} \sum_{n=1}^{\infty} \frac{\psi_1}{n^2\pi^2+\psi_2^2}] e^{\psi_2\xi} + [\frac{e^{2\psi_2}}{1 + e^{2\psi_2}} \sum_{n=1}^{\infty} \frac{\psi_1}{n^2\pi^2+\psi_2^2}] e^{-\psi_2\xi}$$

$$- \sum_{n=1}^{\infty} \frac{\psi_1}{n^2\pi^2+\psi_2^2} \cos(n\pi\xi)$$

$$= [\frac{e^{\psi_2\xi} + e^{(2-\xi)\psi_2}}{1 + e^{2\psi_2}}] \sum_{n=1}^{\infty} \frac{\psi_1}{n^2\pi^2+\psi_2^2} - [\sum_{n=1}^{\infty} \frac{\psi_1}{n^2\pi^2+\psi_2^2}] \cos(n\pi\xi)$$

$$= [\frac{\cosh(1-\xi)\psi_2}{\cosh(\psi_2)}] \sum_{n=1}^{\infty} \frac{\psi_1}{n^2\pi^2+\psi_2^2} - [\sum_{n=1}^{\infty} \frac{\psi_1}{n^2\pi^2+\psi_2^2}] \cos(n\pi\xi)$$

Substituting Eq. (40) and Eq. (41) into the above equation, we obtain

$$\overline{U}(\xi,s) = \frac{\mathrm{Cosh}[(1-\xi)\alpha s/VL]}{\mathrm{Cosh}(\alpha s/VL)} \sum_{n=1}^{\infty} \frac{2Q_0CeT_0[\mathrm{Cos}(n\pi\xi_0)-\mathrm{Cos}(n\pi\xi_1)]}{(s+n^2\pi^2)(n^2\pi^2+\alpha^2s^2/V^2L^2)}$$

$$-\sum_{n=1}^{\infty} \frac{2Q_0CeT_0[\mathrm{Cos}(n\pi\xi_0)-\mathrm{Cos}(n\pi\xi_1)]\mathrm{Cos}(n\pi\xi)}{(n^2\pi^2+\alpha^2s^2/V^2L^2)(s+n^2\pi^2)} \quad (42)$$

Letting

$$\lambda_1 = \frac{\alpha}{VL}$$

$$\lambda_2 = \lambda_1(1-\xi)$$

$$\lambda_3 = \frac{2Q_0CeT_0}{\lambda_1^2} \quad (43)$$

$$\lambda_4 = \frac{n^2\pi^2}{\lambda_1^2}$$

and rearranging Eq.(43) we obtain the simple form as

$$\overline{U}(\xi,s) = \lambda_3 \sum_{n=1}^{\infty} \frac{\mathrm{Cosh}(\lambda_2 s)}{\mathrm{Cosh}(\lambda_1 s)} \cdot \frac{[\mathrm{Cos}(n\pi\xi_0)-\mathrm{Cos}(n\pi\xi_1)]}{(s^2+\lambda_4)(s+n^2\pi^2)}$$

$$-\lambda_3 \sum_{n=1}^{\infty} \frac{[\cos(n\pi\xi_0) - \cos(n\pi\xi_1)]\cos(n\pi\xi)}{(s^2+\lambda_4)(s+n^2\pi^2)} \quad (44)$$

Inversion

$$L^{-1}\left[\frac{1}{(s^2+\lambda_4)(s+n^2\pi^2)}\right]$$

$$= -\frac{\mathrm{Cos}(\sqrt{\lambda_4}\tau)}{n^4\pi^4+\lambda_4} + \frac{n^2\pi^2\sin(\sqrt{\lambda_4}\tau)}{(n^4\pi^4+\lambda_4)(\sqrt{\lambda_4})} + \frac{e^{-n^2\pi^2\tau}}{n^4\pi^4+\lambda_4}$$

$$= \frac{\beta\mathrm{Sin}(\sqrt{\lambda_4}\tau-\phi)}{n^4\pi^4+\lambda_4} + \frac{e^{-n^2\pi^2\tau}}{n^4\pi^4+\lambda_4} \tag{45}$$

Where

$$\beta\cos(\phi) = n^2\pi^2/\sqrt{\lambda_4},\quad \beta\sin(\phi) = 1,\quad \phi = \tan^{-1}(\sqrt{\lambda_4}/n^2\pi^2)$$

Also

$$L^{-1}\left[\frac{\mathrm{Cosh}(\lambda_2 s)}{s^2\mathrm{Cosh}(\lambda_1 s)}\right]$$

$$= \tau - \frac{8\lambda_1}{\pi^2}\sum_{m=1}^{\infty}\frac{\mathrm{Sin}[(2m-1)\pi\xi/2]\mathrm{Sin}[(2m-1)\pi\tau/2\lambda_1]}{(2m-1)^2} \tag{46}$$

And

$$L^{-1}\left[\frac{s^2}{(s^2+\lambda_4)(s+n^2\pi^2)}\right]$$

$$=\left(\frac{\lambda_4}{n^4\pi^4+\lambda_4}\right)\left[\cos(\sqrt{\lambda_4}\tau) - \frac{n^2\pi^2}{\sqrt{\lambda_4}}\sin(\sqrt{\lambda_4}\tau)\right] + \left(\frac{n^4\pi^4}{n^4\pi^4+\lambda_4}\right)e^{-n^2\pi^2\tau}$$

$$= \frac{-\lambda_4\beta}{(n^4\pi^4+\lambda_4)}\left[\sin(\sqrt{\lambda_4}\tau-\phi)\right] + \left(\frac{n^4\pi^4}{n^4\pi^4+\lambda_4}\right) e^{-n^2\pi^2\tau} \tag{47}$$

Hence, by the application of Eq. (46) and Eq. (47) and the convolution theorem of the Laplace transform [26], we obtain

$$\chi(\xi,\tau)$$

$$=L^{-1}\left[\frac{\mathrm{Cosh}(\lambda_2 s)}{\mathrm{Cosh}(\lambda_1 s)}\ \frac{1}{(s^2+\lambda_4)(s+n^2\pi^2)}\right]$$

$$=L^{-1}\left[\frac{\mathrm{Cosh}(\lambda_2 s)}{s^2\mathrm{Cosh}(\lambda_1 s)}\ \frac{s^2}{(s^2+\lambda_4)(s+n^2\pi^2)}\right]$$

$$=\int_0^{\tau}\left[\frac{-\lambda_4\beta \mathrm{Sin}(\sqrt{\lambda_4}\,\eta-\phi)}{n^4\pi^4+\lambda_4}+\frac{n^4\pi^4 e^{-n^2\pi^2\eta}}{n^4\pi^4+\lambda_4}\right] X$$

$$X\cdot\left\{(\tau-\eta)-\frac{8\lambda_1}{\pi^2}\sum_{m=1}^{\infty}\frac{\mathrm{Sin}[(2m-1)\pi\xi/2]\mathrm{Sin}[(2m-1)\pi(\tau-\eta)/2\lambda_1]}{(2m-1)^2}\right\}d\eta \quad (48)$$

where η is a dummy variable

After integrating Eq. (48) from zero to τ we obtain

$$\chi(\xi,\tau)$$

$$=-\frac{\sqrt{\lambda_4}\beta}{n^4\pi^4+\lambda_4}\left\{-\frac{1}{\sqrt{\lambda_4}}[\sin(\sqrt{\lambda_4}\tau-\phi)+\mathrm{Sin}\phi]+\tau\cos(\phi)\right\}$$

$$+\frac{n^2\pi^2}{n^4\pi^4+\lambda_4}\left\{\tau-\frac{1}{n^2\pi^2}[1-e^{-n^2\pi^2\tau}]\right\} -$$

$$- \frac{8n^4\pi^4\lambda_1}{(n^4\pi^4+\lambda_4)\pi^2} \sum_{m=1}^{\infty} \frac{\sin[(2m-1)\pi\xi/2]}{\{n^4\pi^4+[(2m-1)\pi/2\lambda_1]^2\}^{1/2}(2m-1)^2} \quad X$$

$$X \quad \{ \sin[\frac{(2m-1)\pi\tau}{2\lambda_1} - \tan^{-1}(\frac{2m-1}{2\lambda_1 n^2\pi})] + e^{-n^2\pi^2\tau}\sin[\tan^{-1}(\frac{2m-1}{2\lambda_1 n^2\pi})]\}$$

$$+ \frac{4\lambda_4\lambda_1^2\beta}{(n^4\pi^4+\lambda_4)\pi^3} \sum_{m=1}^{\infty} \frac{\sin[(2m-1)\pi\xi/2]}{(2m-1)^2} \quad X$$

$$X \ \{\frac{\sin(\sqrt{\lambda_4}\tau-\phi) + \sin[\phi+(2m-1)\pi\tau/2\lambda_1]}{n+(2m-1)/2} -$$

$$- \frac{\sin(\sqrt{\lambda_4}\tau-\phi) + \sin[\phi-(2m-1)\pi\tau/2\lambda_1]}{n-(2m-1)/2}\}$$

Therefore , the inverse transform of Eq. (44) is the solution of the displacement equation , i.e.,

$$U(\xi,\tau)$$

$$= \lambda_3 \sum_{n=1}^{\infty} \chi(\xi,\tau)[\cos(n\pi\xi_0)-\cos(n\pi\xi_1)]$$

$$-\lambda_3 \sum_{n=1}^{\infty} \frac{1}{n^4\pi^4 + \lambda_4} \{\beta \sin(\sqrt{\lambda_4} -\phi) + e^{-n^2\pi^2\tau}\} \ X$$

$$X \ [\cos(n\pi\xi_0)-\cos(n\pi\xi_1)] \ \cos(n\pi\xi) \tag{50}$$

Where $\chi(\xi,\tau)$ is shown in Eq. (49) rearranging Eq. (50) to combine the summation of single and double forms we obtain

$U(\xi,\tau)$

$$= \lambda_3 \sum_{n=1}^{\infty} \frac{[\beta \mathrm{Sin}(\sqrt{\lambda_4}\tau-\phi)+e^{-n^2\pi^2\tau}][1-\mathrm{Cos}(n\pi\xi)][\mathrm{Cos}(n\pi\xi_0)-\mathrm{Cos}(n\pi\xi_1)]}{n^4\pi^4+\lambda_4} -$$

$$- \lambda_3 \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \{\frac{8n^4\pi^4\lambda_1}{\pi^2(n^4\pi^4+\lambda_4)(2m-1)^2\{n^4\pi^4+[(2m-1)\pi/2\lambda_1]^2\}^{1/2}} \times$$

$$\times \{\mathrm{Sin}[\frac{(2m-1)\pi\tau}{2\lambda_1} - \mathrm{Tan}^{-1}(\frac{2m-1}{2\lambda_1 n^2\pi})] + e^{-n^2\pi^2\tau}\mathrm{Sin}[\mathrm{Tan}^{-1}(\frac{2m-1}{2\lambda_1 n^2\pi})]\}$$

$$- \frac{4\lambda_4\lambda_1^2\beta}{\pi^3(n^4\pi^4+\lambda_4)(2m-1)^2} \times$$

$$\times \{\frac{\sin(\sqrt{\lambda_4}\tau-\phi)+\sin[\phi+(2m-1)\pi\tau/2\lambda_1]}{n+(2m-1)/2} - \frac{\sin(\sqrt{\lambda_4}\tau-\phi)+\sin[\phi-(2m-1)\pi\tau/2\lambda_1]}{n-(2m-1)/2}\}\} \times$$

$$\times \sin[(2m-1)\pi\xi/2)][\cos(n\pi\xi_0)-\cos(n\pi\xi_1)] \qquad (51)$$

From equation (51) it is evident that $\frac{\partial U}{\partial \xi} = 0$ for $\xi = 1$. Also, it can be shown that $U(0,\tau) = 0$ and $U(\xi,0) = \frac{\partial U(\xi,0)}{\partial \tau} = 0$, for $0 \le \xi \le 1$. Thus the solution satisfies the boundary and initial conditions. The normal stress can be easily verified from equation (18), equation (24) and equation (51) to be

$$\frac{\sigma(\xi,\tau)}{E}$$

$$= \frac{\partial U(\xi,\tau)}{\partial \xi} - Ce\ T_0\Theta(\xi,\tau)$$

$$= \lambda_3 \sum_{n=1}^{\infty} \left\{ \frac{n\pi[\beta \sin(\sqrt{\lambda_4}\tau-\phi) + e^{-n^2\pi^2\tau}]}{n^4\pi^4 + \lambda_4} - \frac{2Q_0CeT_0e^{-n^2\pi^2\tau}}{n\pi\lambda_3} \right\} \times$$

$$\times \; \{ \sin(n\pi\xi)[\cos(n\pi\xi_0)-\cos(n\pi\xi_1)]\} -$$

$$- \lambda_3 \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \left\{ \frac{8n^4\pi^4\lambda_1\{\sin[\frac{(2m-1)\pi\tau}{2\lambda_1} - \tan^{-1}(\frac{2m-1}{2\lambda_1 n^2\pi})] + e^{-n^2\pi^2\tau}\sin[\tan^{-1}(\frac{2m-1}{2\lambda_1 n^2\pi})\}}{\pi^2(n^4\pi^4+\lambda_4)(2m-1)^2\{n^4\pi^4+[(2m-1)^2/2\lambda_1]\}^{1/2}} \right. -$$

$$- \frac{4\lambda_4\lambda_1^2\beta \left\{ \frac{\sin(\sqrt{\lambda_4}\tau-\phi)+\sin[\phi+(2m-1)\pi\tau/2\lambda_1]}{n + (2m+1)/2} - \frac{\sin(\sqrt{\lambda_4}-\phi)+\sin[\phi-(2m-1)\pi\tau/2\lambda_1}{n - (2m-1)/2} \right.}{\pi^3(n^4\pi^4+\lambda_4)(2m-1)^2}$$

$$\times \left\{ \frac{(2m-1)\pi}{2} \cos[\frac{(2m-1)\pi\xi}{2}][\cos(n\pi\xi_0)-\cos(n\pi\xi_1)]\right\} \qquad (52)$$

By equation (52), $\sigma = 0$ at $\xi = 1$. Hence the condition of end $\xi = 1$ being stress-free is satisfied. From the expression of U and σ, it is evident that both U and σ remain bounded as $\tau \to \infty$ which is expected.

3.3 Examining the convergence of the analytical solutions of thermal stress and deformation.

An examination of solution of the displacement and stress obtained in section 3.2 shows that all solutions are expressed in the forms of combinations of single and double infinite series. Therefore, a study of the convergence of the single and double infinite series becomes necessary. For instance, the expression of the displacement solution is

$$U(\xi,\tau) = \sum_{n=1}^{\infty}\frac{G_1(n,0)}{G_2(n,0)} - \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{F_1(n,m)}{F_2(n,m)} - \frac{F_3(n,m)}{F_4(n,m)}\right]$$

where $G_1(n,0)$, $G_2(n,0)$, $F_1(n,m)$, $F_2(n,m)$, $F_3(n,m)$ and $F_4(n,m)$ are polynomials in the variable m and n and

$$G_1(n,0) = \lambda_3[\beta\sin(\sqrt{\lambda_4}\tau-\phi)+e^{-n^2\pi^2\tau}][1-\cos(n\pi\xi)][\cos(n\pi\xi_0)-\cos(n\pi\xi_1)]$$

$$G_2(n,0) = n^4\pi^4+\lambda_4$$

$$F_1(n,m) = 8\lambda_3\lambda_1 n^4\pi^4\{\sin[\frac{(2m-1)\pi\tau}{2\lambda_1}-\tan^{-1}\frac{(2m-1)}{2\lambda_1 n^2\pi}] + e^{-n^2\pi^2\tau}\sin[\tan^{-1}(\frac{2m-1}{2\lambda_1 n^2\pi})]\}X$$

$$X\ \sin[(2m-1)\pi\xi/2][\cos(n\pi\xi_0)-\cos(n\pi\xi_1)]$$

$$F_2(n,m) = \pi^2(n^4\pi^4+\lambda_4)(2m-1)^2\{n^4\pi^4+[(2m-1)\pi/2\lambda_1]\}^{1/2}$$

$$F_3(n,m) = 4\lambda_4\lambda_1^2\beta\{\frac{\sin(\sqrt{\lambda_4}\tau-\phi)+\sin[\phi+(2m-1)\pi\tau/2\lambda_1]}{n+(2m-1)/2} -$$

$$-\frac{\sin(\sqrt{\lambda_4}\tau-\phi)+\sin[\phi-(2m-1)\pi\tau/2\lambda_1]}{n-(2m-1)/2}\}\sin[(2m-1)\pi\xi/2]\ X$$

$$X\ [\cos(n\pi\xi_0)-\cos(n\pi\xi_1)]$$

$$F_4(n,m) = \pi^3(n^4\pi^4+\lambda_4)(2m-1)^2$$

Since the degrees of $G_1(n,0)$, $F_1(n,m)$ and $F_3(n,m)$ are lower than that of $G_2(n,0)$, $F_2(n,m)$ and $F_4(n,m)$ respectively, series $\sum_{n=1}^{\infty}\frac{G_1(n,0)}{G_2(n,0)}$, $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{F_1(n,m)}{F_2(n,m)}$ and $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{F_3(n,m)}{F_4(n,m)}$ are convergent, as can be easily proved by applying a comparision test [26].

Examing the solution of stress, we can express the equation (52) as the following form

$$\frac{\sigma(\xi,\tau)}{E} = \sum_{n=1}^{\infty}\frac{G_1'(n,0)}{G_2(n,0)} - \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{F_1'(n,m)}{F_2(n,m)} - \frac{F_3'(n,m)}{F_4(n,m)}\right] - CeT_0\Theta(\xi,\tau)$$

where $G_1'(n,0)$, $F_3'(n,m)$ and $F_3'(n,m)$ are the derivatives of $G_1(n,0)$, $F_1(n,m)$ and $F_3(n,m)$ with respect to the independent variable, respectively.

$$G_1'(n,0) = \lambda_3[\beta\sin(\sqrt{\lambda_4}\tau-\phi)+e^{-n^2\pi^2\tau}][\cos(n\pi\xi_0)-\cos(n\pi\xi_1)][n\pi\sin(n\pi\xi)]$$

$$F_1'(n,m) = 8\lambda_3\lambda_1 n^4\pi^4\{\sin[\frac{(2m-1)}{2\lambda_1}-\tan\frac{-1(2m-1)}{2\lambda_1 n^2\pi}]+ e^{-n^2\pi^2\tau}\sin[\tan^{-1}(\frac{2m-1}{2\lambda_1 n^2\pi})]\}X$$

$$X\ [\cos(n\pi\xi_0)-\cos(n\pi\xi_1)][(2m-1)\pi\xi/2]\cos[(2m-1)\pi\xi/2]$$

$$F_3'(n,m) = 4\lambda_4\lambda_1^2\beta\{\frac{\sin(\sqrt{\lambda_4}\tau-\phi)+\sin[\phi+(2m-1)\pi\xi/2\lambda_1]}{n+(2m-1)/2} -$$

$$-\frac{\sin(\sqrt{\lambda_4}\tau-\phi)+\sin[\phi-(2m-1)\pi\xi/2\lambda_1]}{n+(2m-1)/2}\}[\cos(n\pi\xi_0)-\cos(n\pi\xi_1)]\ X$$

$$X\ \ [(2m-1)\pi\xi/2]\cos[(2m-1)\pi\xi/2]$$

Comparing the degrees of $G_1'(n,0)$, $F_1'(n,m)$ and $F_3'(n,m)$ to that of $G_1(n,0)$, $F_1(n,m)$ and $F_3(n,m)$ respectively, we find that the degrees of $G_1'(n,0)$, $F_1'(n,m)$ and $F_3'(n,m)$ are greater than that of $G_1(n,0)$

$F_1(n,m)$ and $F_3(n,m)$. Therefore, the convergence of the series $\sum_{n=1}^{\infty}\frac{G_1'(n,0)}{G_2(n,m)}$, $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{F_1'(n,m)}{F_2(n,m)}$ and $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{F_3'(n,m)}{F_4(n,m)}$ should be slower than that of the series $\sum_{n=1}^{\infty}\frac{G_1(n,0)}{G_2(n,0)}$, $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{F_1(n,m)}{F_2(n,m)}$ and $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{F_3(n,m)}{F_4(n.m)}$ respectively. By repeated trials of numerical evaluation, even when n and m up to 300, the required convergent result of $\sum_{n=1}^{\infty} G_1'(n,0)/G_2(n,0)$, $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} F_1'(n,m)/F_2(n,m)$ and $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} F_3'(n,m)/F_4(n,m)$ is not reached, which indicated that the convergence is very slow.

For our purpose of obtaining the results of thermal stress and deformation, a finite difference approximate method based on the result of the given temperature variation is introduced instead of directly evaluating the analytical solutions of stress and deformation,i.e., equation (46) and equation (47) respectively.

CHAPTER 4

SOLUTIONS OF THE DIFFUSION, DISPLACEMENT AND STRESS WAVE EQUATION BY FINITE-DIFFERENCE METHODS

In the present investigation, finite difference approximations to the solution of the diffusion equation are so chosen as to give an explicit computational program for the unknown function. The solution then proceeds step by step, i.e., it matches in a direction normal to the boundary along which the initial condition is specified, guided by boundary conditions along the transverse boundaries of the open region. Then based on the given temperature field, implict computational programs are used for solving the displacement equation and stress wave equation. An implicit finite difference approximation procedure which is unconditionally stable for the solution of the diffusion equation is also presented. Results obtained by this method are compared with that evaluated analytical solution and with that obtained by an explicit finite difference method.

4.1 On the solution of the Diffusion Equation

In section 2.1 the liberation of source as just described means that a quantity of energy is instantaneously liberated at $t = 0$; this may be taken to imply that at $t = 0$ an instantanous rise of temperature of an amount $\alpha q_0/k$ will take place [23] and therefore an equivalent problem is considered instead of solving the equation (22), equation (51) and equation (52) directly [see Appendix I], i.e.,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{52}$$

$T = 0$ for $x = 0, L$ at $t \geq 0$

$T = 0$ for $x < x_0$, $x > x_1$ at $t = 0$

$= \alpha q_0/k$ for $x_0 \leq x \leq x_1$ at $t = 0$ (53)

$\frac{\partial T}{\partial t} = 0$ for $0 \leq x \leq L$ at $t = 0$

4.1.1 The explicit form of the Diffusion Equation

In order to approximate the solution of equation (52) and equation (53), a network of grid points is first established through out the region $0 \leq x \leq L$, $0 \leq t$, with grid spacings Δx, Δt. In this problem, it is easy to ensure that grid points lie on the boundaries of x and t. For any grid point (i,j) that does not have i=0, or j=0. The derivatives of equation (52) are now replaced by the finite-difference forms suggested by using the second central difference in the x direction and first forward difference in the t direction.

$$\frac{T_{i,j+1} - T_{i,j}}{\Delta t} = \alpha \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2} \tag{53}$$

Letting $r = \alpha\Delta t/(\Delta x^2)$, this may be rewriten as

$$T_{i,j+1} = rT_{i-1,j} + (1-2r)T_{i,j} + rT_{i+1,j} \tag{54}$$

In Fig. (5) the crosses and circles indicate those grid points involed in the time and space differences respectively.

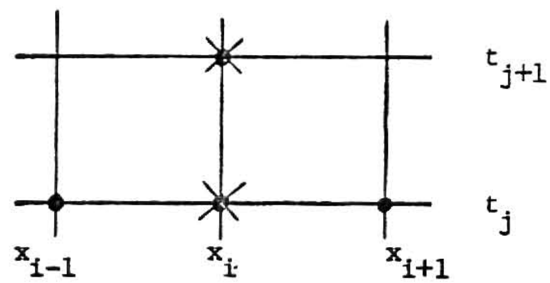


Fig.5 the explicit form

It has been established that the calculations will be stable [27] and the solution of equation (54) will closely approximate that of equation (52) provided that $r \leq 1/2$. Furthermore it was proven [27] that the solution of equation (54) will converge to that of equation (52) as both the time and space increments Δt and Δx approch zero assuming that inequality $r \leq 1/2$ is satisfied. This method has the advantages of being simple and easy to program.

4.1.2 The implicit form of the Diffusion equation

The stability restrictions inherent in explicit methods require very small steps in the t direction. Therefore, an implicit method, in which stability for all $r > 0$ is ensured, is applied to obtain a solution to compare with the " long time " analytical solution.

Using the second central difference in the x direction and the first backward difference (suggested by Crank and Nicolso [1947] and Laasonen [1949]) in the t direction at the point (i,j+1), the difference equation is

$$\frac{T_{i,j+1} - T_{i,j}}{\Delta t} = \frac{T_{i+1,j+1} - 2T_{i,j+1} + T_{i-1.j+1}}{(\Delta x)^2} \cdot \alpha$$

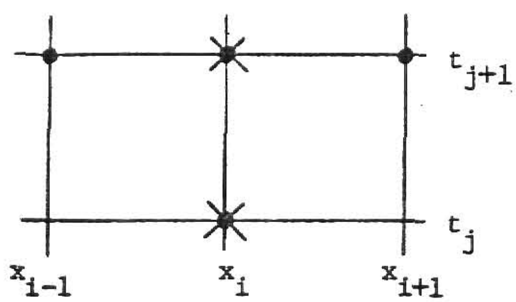
or

$$(1+2r)T_{i,j+1} = T_{i,j} + r(T_{i+1,j+1} + T_{i-1,j+1}) \tag{55}$$

where $r = \alpha\Delta t/(\Delta x)^2$

That is, the above relation exists between the values of T at the four points shown in the space-time grid of Fig.(6)

Fig. 6 The implicit form



This method is unconditionally stable, However, the use of this method requires the solution of a large number of simultaneous, linear, algebraic equations at each time step. Iterative methods are usually utilized to accomplish this solution.

4.2 On the solution of the Displacement Equation and Stress Wave Equation

Equation (27) is a hyperbolic partial differential equation. Following our usual approach, we select a network of points (i,j) with spacing Δx and Δt and approximate the governing equation as follows:

$$\frac{U_{i-1,j} - 2U_{i,j} + U_{i+1,j}}{(\ x)^2} = \frac{1}{V^2}\ \frac{U_{i,j-1} - 2U_{i,j} + U_{i,j+1}}{(\Delta t)^2} +$$

$$+ \ Ce\ \frac{T_{i+1,j} - T_{i-1,j}}{2(\Delta x)}$$

or

$$U_{i,j+1} = R_1^2 U_{i-1,j} + 2(1-R_1^2)U_{i,j} + R_1^2 U_{i+1,j} - U_{i,j-1}$$

$$+ R_2(T_{i+1,j} - T_{i-1,j})$$

where $R_1 = V\Delta t/\Delta x$, $R_2 = CeR_1\Delta x/2$, This is an explicit recurrence formula.

Basing on this explicit recurrence formula, an improved implicit method [28] is introduced. The method is carried on an approximate difference for $\partial^2 u/\partial x^2$ at the time steps j-1 and j+1 respectively. Then, an average recurrence formula between time steps j-1 and j+1 are shown as

$$\frac{\partial^2 u}{\partial x^2}\ \frac{\partial^2 u}{\partial t^2} = \frac{1}{2(\Delta x)^2}(U_{i-1,j-1} - 2U_{i,j-1} + U_{i+1,j-1} +$$

$$+ U_{i-1,j+1} - 2U_{i,j+1} + U_{i,j+1}) -$$

$$- \frac{1}{(\Delta t)^2} (U_{i,j-1} - 2U_{i,j} + U_{i,j+1}) \qquad (56)$$

Introducing,

$$Ce \frac{\partial T}{\partial x} = Ce \frac{T_{i+1,j} - T_{i-1,j}}{2(\Delta x)}$$

equation (27) can be approximated as

$$\frac{R_1^2}{2} U_{i-1,j-1} - (1+R_1^2)U_{i,j-1} + \frac{R_1^2}{2} U_{i+1,j-1} +$$

$$+ \frac{R_1^2}{2} U_{i-1,j+1} - (1+R_1^2)U_{i,j+1} + \frac{R_1^2}{2} U_{i,j+1} + 2U_{i,j}$$

$$= R_2 (T_{i+1,j} - T_{i-1,j}) \qquad (57)$$

This is an implicit recurrence formula. Fig. (7) is the typical relaxation pattern for equation (57). The use of this method requires the solution of a large number of simultaneous, linear, algebraic equations at each time step. Iterative methods are utilized to accomplish this solution.

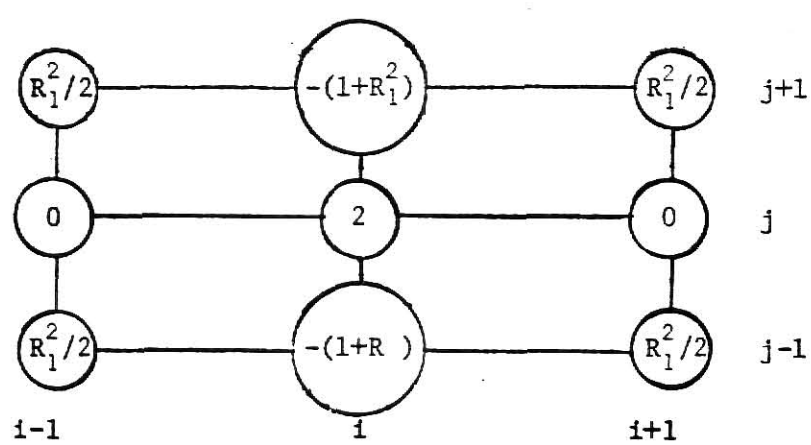


Fig. 7 Typical relaxation pattern

Having established the values of U at any time step, we can apply the following approximate formula to study the solution of the stress σ.

$$\frac{\sigma}{E} = \frac{\partial u}{\partial x} - CeT$$

$$= \frac{U_{i+1,j} - U_{i-1,j}}{2\Delta x} - T_{i,j}$$

4.3 Numerical example and results

For numerical calculations, the same example presented in section 2.3.2 is carried out. All the computer runs were performed on the IBM 360 machine.

Part of the numerical results of the temperature, stress and displacement in the thin rod have been illustrated in Fig. 8 through 10 and table 4 through table 6. Fig. 8 and table 4 show the variation of temperature, which are carried out by explicit and implicit finite difference methods respectively. The result of the analytical solution is also compared with the results carried out by finite difference methods in Fig. 8. Fig. 9 and Fig. 10 show the results for an elastic wave traveling through a bar in the direction of continuously decreasing temperature. A typical time and space increments that lead to excellent result was $\Delta x = 0.05$, $\Delta t = 0.00005$. Fig. 10 shows two compression stress waves traveling in two opposite direction after a thermal shock. The wave velocity measured from Fig. 10 is about 1800 ft/sec. Which is very close to the value $(E/\rho)^{1/2}$.

CONCLUSIONS

The mathematical model with its solutions and the popular finite difference methods for an isotropic finite rod due to an instantaneous heat source have been studied. Obviously, Laplace transformation method is convenient for the solution of this problem. Although this solution is formally exact in that it is able to satisfy all the boundary and initial conditions as well as the governing differential equations, it will give rise to considerable difficulties in the numerical evaluation, for the convergence of the analytical solution with single and double series forms is very slow.

For our purpose, the finite difference method with carefully selected time and space increments has the advantage of investigating the stress wave pattern in the thin rod after a thermal shock is introduced.

Based on the study of this research, the following research is recommended and suggested.

1. In the present analysis, the edges of the rod are assumed to be fixed and freely supported. Analyses involving boundary conditions, such as clamped and simple edges have not been considered and have been left open for future investigations.

2. Although the idea of an instantaneous heat source is an idealization, experimental approximations are still of value. An

application of the laser beam technique has been done successfully by many investigators [29]. Therefore it is significant to pursue such techniques so that an experiment which closely approximate the present problem can be carried out in the near future.

The present analysis is concerned with the isotropic, homogeneous thin rod. Therefore, the study of the propagation of a stress wave in a nonhomogeneous thin rod and the effect of temperature gradients on the propagation of a stress wave due to an instantaneous heat source provides a new task for future investigation.

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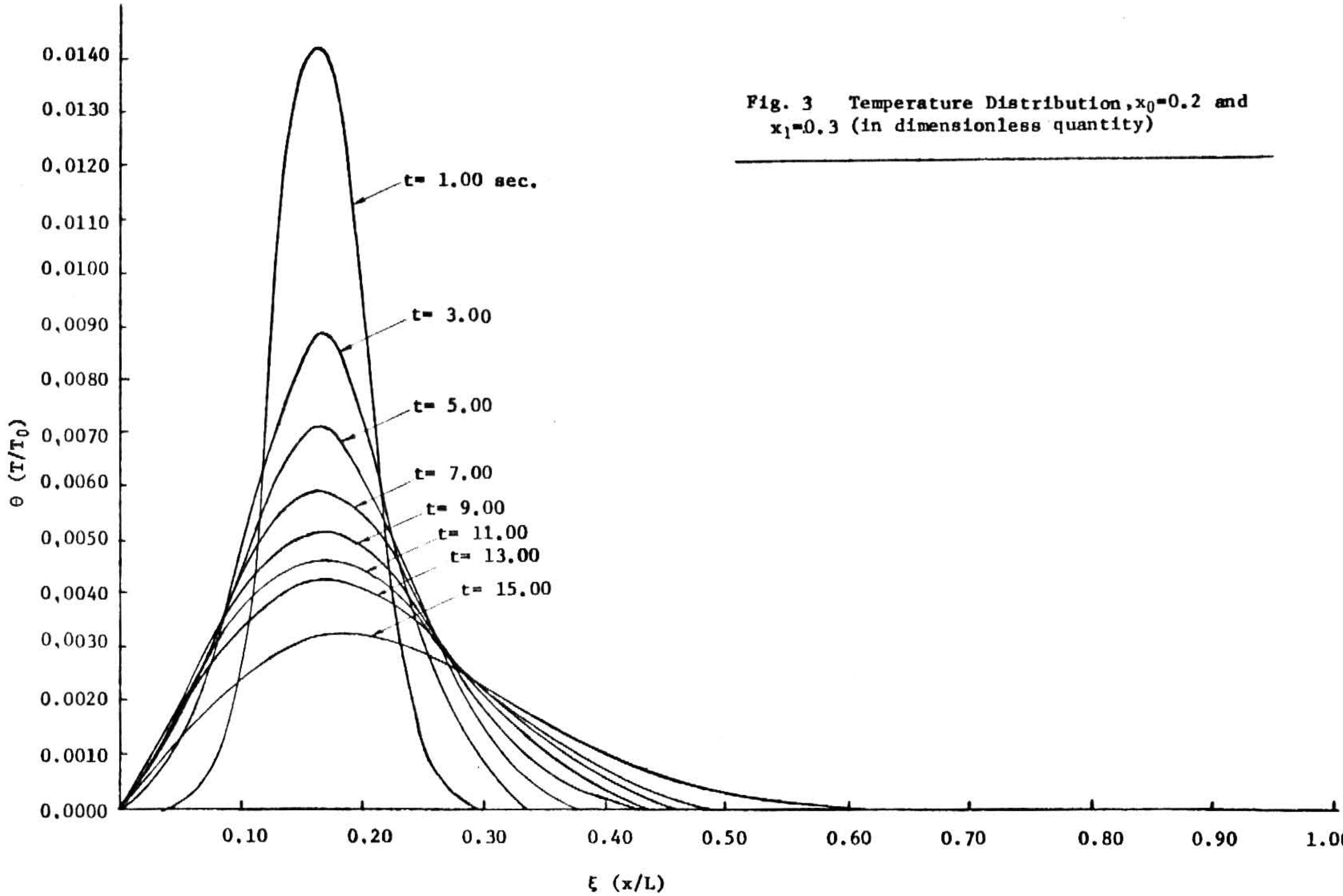

Fig. 3 Temperature Distribution, x_0=0.2 and x_1=0.3 (in dimensionless quantity)

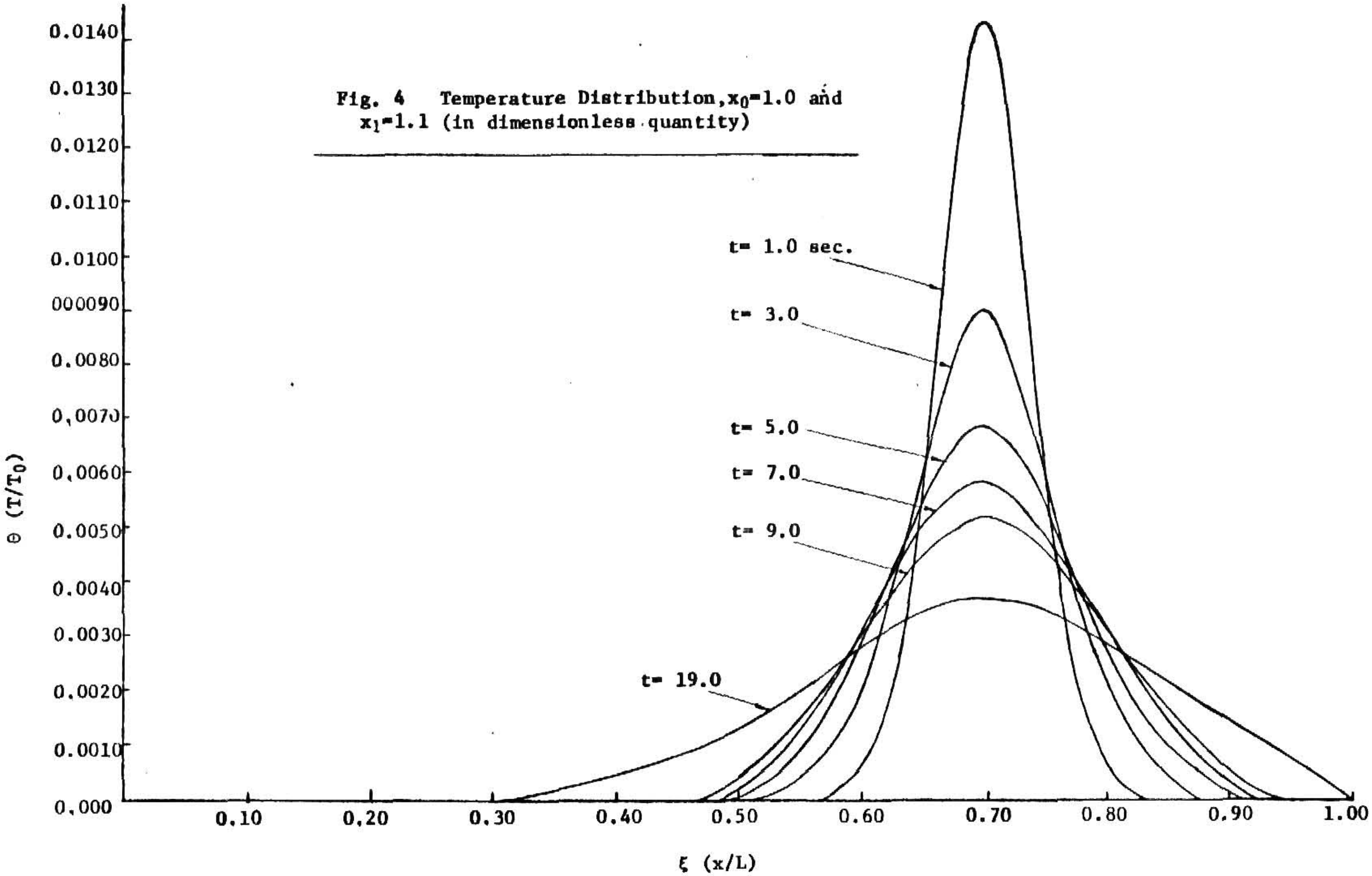

Fig. 4 Temperature Distribution, x_0=1.0 and x_1=1.1 (in dimensionless quantity)

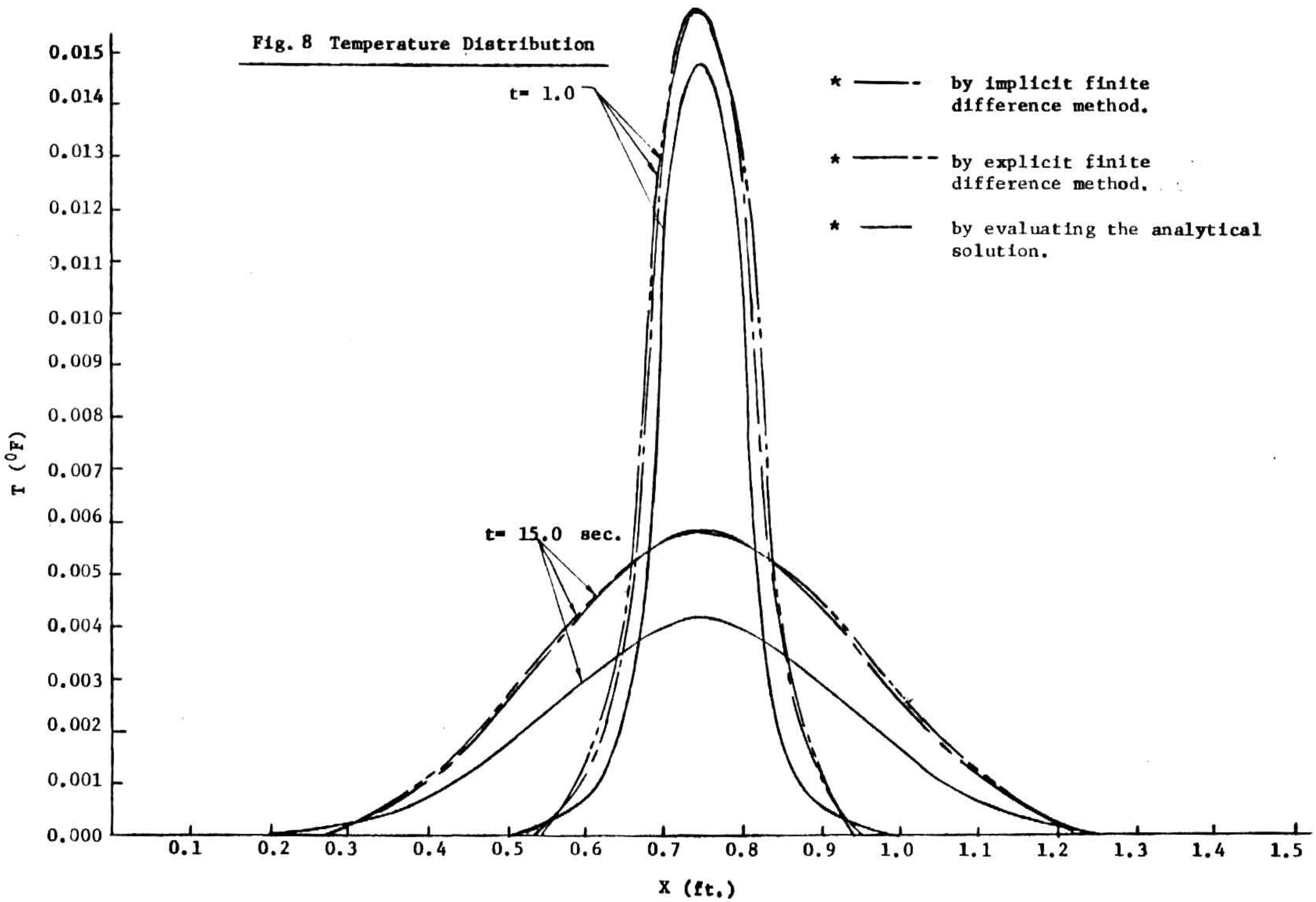

Fig. 8 Temperature Distribution

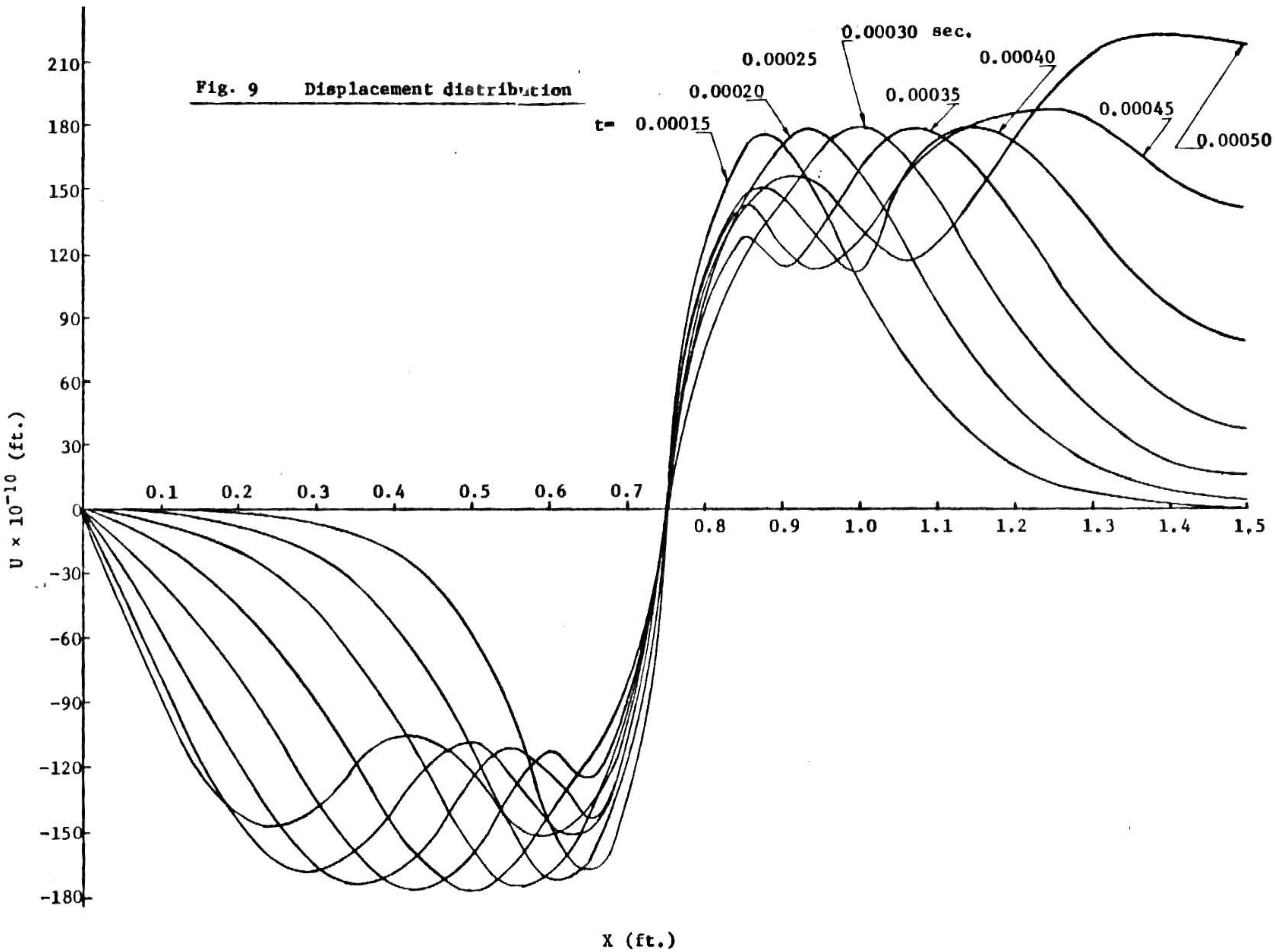

Fig. 9 Displacement distribution

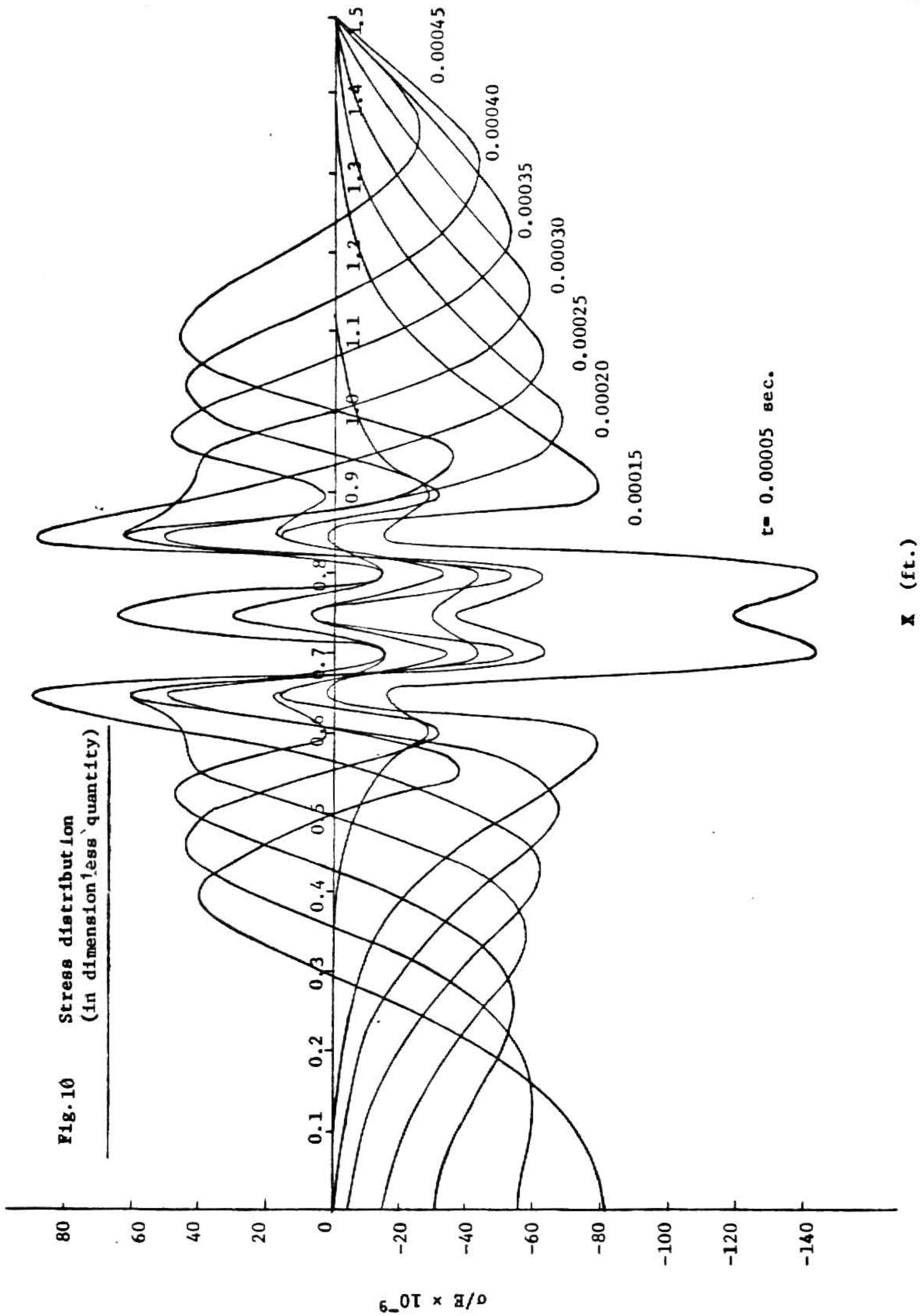

Fig.10 Stress distribution
(in dimensionless quantity)

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Table 2 Temperatures solved by evaluating analytical solution, $x_0=0.2$ and $x_1=0.3$

```
                    TEMPERATURE DISTRIBUTION (F)

        *  THE SECTION LENGTH IS 0.1 FT.                        *
        *  X0 = 0.20      AND      X1 = 0.30                    *
        *  XX0= 0.1333    AND      XX1= 0.2000                  *

AT TIME=     1.0000     SECOND

  0.000000   0.000430   0.009436   0.009436   0.000430   0.000001   0.000000   0.000000
  0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000
AT TIME=     3.0000     SECOND

  0.000000   0.002207   0.007459   0.007459   0.002212   0.000191   0.000005   0.000000
  0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000
AT TIME=     5.0000     SECOND

  0.000000   0.002854   0.006244   0.006247   0.002918   0.000634   0.000064   0.000003
  0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000
AT TIME=     7.0000     SECOND

  0.000000   0.002946   0.005448   0.005468   0.003142   0.001036   0.000196   0.000021
  0.000001   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000
AT TIME=     9.0000     SECOND

  0.000000   0.002824   0.004859   0.004915   0.003183   0.001331   0.000360   0.000063
  0.000007   0.000001   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000
AT TIME=    11.0000     SECOND

  0.000000   0.002627   0.004386   0.004489   0.003147   0.001536   0.000523   0.000124
  0.000021   0.000002   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000
AT TIME=    13.0000     SECOND

  0.000000   0.002415   0.003989   0.004143   0.003078   0.001675   0.000670   0.000198
  0.000043   0.000007   0.000001   0.000000   0.000000   0.000000   0.000000   0.000000
AT TIME=    15.0000     SECOND

  0.000000   0.002211   0.003648   0.003850   0.002994   0.001767   0.000798   0.000276
  0.000073   0.000015   0.000002   0.000000   0.000000   0.000000   0.000000   0.000000
```

AT TIME= 17.0000 SECOND

0.000000	0.002024	0.003351	0.003595	0.002903	0.001826	0.000905	0.000354
0.000109	0.000027	0.000005	0.000001	0.000000	0.000000	0.000000	0.000000

AT TIME= 19.0000 SECOND

0.000000	0.001856	0.003090	0.003369	0.002808	0.001861	0.000993	0.000428
0.000149	0.000042	0.000010	0.000002	0.000000	0.000000	0.000000	0.000000

Table 3 Temperatures solved by evaluating analytical solution, x_0=1.0 and x_1=1.1

TEMPERATURE DISTRIBUTION (F)

```
*  THE SECTION LENGTH IS 0.1 FT.                    *
*  X0 = 1.00     AND     X1 = 1.10                  *
*  XX0= 0.6667   AND     XX1= 0.7333                *
```

AT TIME= 1.0000 SECOND

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000001	0.000430	0.009436	0.009436	0.000430	0.000001	0.000000	0.000000

AT TIME= 3.0000 SECOND

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000005
0.000191	0.002212	0.007459	0.007459	0.002212	0.000191	0.000005	0.000000

AT TIME= 5.0000 SECOND

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000003	0.000064
0.000634	0.002918	0.006247	0.006247	0.002918	0.000634	0.000064	0.000000

AT TIME= 7.0000 SECOND

0.000000	0.000000	0.000000	0.000000	0.000000	0.000001	0.000021	0.000196
0.001036	0.003142	0.005470	0.005470	0.003142	0.001036	0.000194	0.000000

AT TIME= 9.0000 SECOND

0.000000	0.000000	0.000000	0.000000	0.000001	0.000007	0.000063	0.000360
0.001331	0.003183	0.004922	0.004922	0.003183	0.001331	0.000353	0.000000

AT TIME= 11.0000 SECOND

0.000000	0.000000	0.000000	0.000000	0.000002	0.000021	0.000124	0.000523
0.001536	0.003150	0.004510	0.004510	0.003150	0.001534	0.000502	0.000000

AT TIME= 13.0000 SECOND

0.000000	0.000000	0.000000	0.000001	0.000007	0.000043	0.000198	0.000671
0.001676	0.003085	0.004186	0.004186	0.003085	0.001669	0.000628	0.000000

AT TIME= 15.0000 SECOND

0.000000	0.000000	0.000000	0.000002	0.000015	0.000073	0.000276	0.000798
0.001770	0.003009	0.003923	0.003923	0.003007	0.001755	0.000725	0.000000

AT TIME= 17.0000 SECOND

0.000000	0.000000	0.000001	0.000005	0.000027	0.000109	0.000354	0.000905
0.001831	0.002929	0.003704	0.003704	0.002924	0.001805	0.000796	0.000000

AT TIME= 19.0000 SECOND

0.000000	0.000000	0.000002	0.000010	0.000042	0.000149	0.000428	0.000995
0.001871	0.002850	0.003518	0.003517	0.002841	0.001829	0.000845	0.000000

Table 4 **Temperatures solved by evaluating analytical solution, x_0=0.7 and x_1=0.8**

TEMPERATURE DISTRIBUTION (F)

* THE SECTION LENGTH IS 0.1 FT. *
* X0 = 0.70 AND X1 = 0.80 *
* XX0= 0.4667 AND XX1= 0.5333 *

AT TIME=	1.0000	SECOND					
0.000000	0.000000	0.000000	0.000000	0.000000	0.000001	0.000430	0.009436
0.009436	0.000430	0.000001	0.000000	0.000000	0.000000	0.000000	0.000000
AT TIME=	3.0000	SECOND					
0.000000	0.000000	0.000000	0.000000	0.000005	0.000191	0.002212	0.007459
0.007459	0.002212	0.000191	0.000005	0.000000	0.000000	0.000000	0.000000
AT TIME=	5.0000	SECOND					
0.000000	0.000000	0.000000	0.000003	0.000064	0.000634	0.002918	0.006247
0.006247	0.002918	0.000634	0.000064	0.000003	0.000000	0.000000	0.000000
AT TIME=	7.0000	SECOND					
0.000000	0.000000	0.000001	0.000021	0.000196	0.001036	0.003142	0.005470
0.005470	0.003142	0.001036	0.000196	0.000021	0.000001	0.000000	0.000000
AT TIME=	9.0000	SECOND					
0.000000	0.000001	0.000007	0.000063	0.000360	0.001331	0.003183	0.004922
0.004922	0.003183	0.001331	0.000360	0.000063	0.000007	0.000001	0.000000
AT TIME=	11.0000	SECOND					
0.000000	0.000002	0.000021	0.000124	0.000523	0.001536	0.003150	0.004510
0.004510	0.003150	0.001536	0.000523	0.000124	0.000021	0.000002	0.000000
AT TIME=	13.0000	SECOND					
0.000000	0.000007	0.000043	0.000198	0.000671	0.001676	0.003085	0.004186
0.004186	0.003085	0.001676	0.000671	0.000198	0.000043	0.000007	0.000000
AT TIME=	15.0000	SECOND					
0.000000	0.000015	0.000073	0.000276	0.000798	0.001769	0.003009	0.003923
0.003923	0.003009	0.001769	0.000798	0.000276	0.000073	0.000015	0.000000

AT TIME= 17.0000 SECOND

0.000000	0.000026	0.000109	0.000354	0.000905	0.001831	0.002929	0.003704
0.003704	0.002929	0.001831	0.000905	0.000354	0.000109	0.000026	0.000000

AT TIME= 19.0000 SECOND

0.000000	0.000040	0.000149	0.000428	0.000995	0.001871	0.002850	0.003518
0.003518	0.002850	0.001871	0.000995	0.000428	0.000149	0.000040	0.000000

Table 5 TEMPERATURE DISTRIBUTION (F)
SOLVED BY IMPLICIT FINITE DIFFERENCE METHOD
* THE SEGMENT LENGTH IS 0.05 FT.*

TIME=1.0000

0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000003	0.000024	0.000185	0.001137	0.005127	0.014396	0.017454
0.014396	0.005127	0.001137	0.000185	0.000024	0.000003	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.0	

TIME=3.0000

0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000001	0.000006
0.000027	0.000113	0.000419	0.001315	0.003404	0.006999	0.010967	0.012596
0.010967	0.006999	0.003404	0.001315	0.000419	0.000113	0.000027	0.000006
0.000001	0.000000	0.000000	0.000000	0.000000	0.000000	0.0	

TIME=4.9999

0.0	0.000000	0.000000	0.000000	0.000001	0.000003	0.000013	0.000045
0.000144	0.000412	0.001041	0.002293	0.004339	0.006943	0.009262	0.010204
0.009262	0.006943	0.004339	0.002293	0.001041	0.000412	0.000144	0.000045
0.000013	0.000003	0.000001	0.000000	0.000000	0.000000	0.0	

TIME=******

0.0	0.000011	0.000028	0.000062	0.000125	0.000240	0.000436	0.000748
0.001210	0.001844	0.002643	0.003556	0.004486	0.005300	0.005860	0.006060
0.005860	0.005300	0.004486	0.003556	0.002643	0.001844	0.001210	0.000748
0.000436	0.000240	0.000125	0.000062	0.000028	0.000011	0.0	

Table 6 Temperature (F), displacement and stress solved by finite difference method

* segment length is 0.05 feet *

```
.....................................

AT TIME=     0.0        SECOND
  TEMPERATURE
     0.0         0.0         0.0         0.0         0.0         0.0         0.0
     0.0         0.0         0.0         0.0         0.0         0.0         0.0
     0.01973     0.01973     0.01973     0.0         0.0         0.0         0.0
     0.0         0.0         0.0         0.0         0.0         0.0         0.0
     0.0         0.0         0.0
  DISPLEMENT
  0.0         0.0         0.0         0.0         0.0         0.0         0.0
  0.0         0.0         0.0         0.0         0.0         0.0         0.0
  0.0         0.0         0.0         0.0         0.0         0.0         0.0
  0.0         0.0         0.0         0.0         0.0         0.0         0.0
  0.0         0.0         0.0
  STRESS
  0.0         0.0         0.0         0.0         0.0         0.0         0.0
  0.0         0.0         0.0         0.0         0.0         0.0         0.0
  0.0         0.0         0.0         0.0         0.0         0.0         0.0
  0.0         0.0         0.0         0.0         0.0         0.0         0.0
  0.0         0.0         0.0

.....................................

AT TIME=     0.000050 SECOND
  TEMPERATURE
     0.0         0.0         0.0         0.0         0.0         0.0         0.0
     0.0         0.0         0.0         0.0         0.0         0.0         0.00000
     0.01973     0.01973     0.01973     0.00000     0.0         0.0         0.0
     0.0         0.0         0.0         0.0         0.0         0.0         0.0
     0.0         0.0         0.0
  DISPLEMENT
 0.0          -0.33396E-12-0.86701E-12-0.19169E-11-0.41097E-11-0.87524E-11-0.18613E-10
-0.39569E-10-0.84116E-10-0.17881E-09-0.38010E-09-0.80798E-09-0.17176E-08-0.36511E-08
-0.31735E-08-0.33770E-15 0.31735E-08 0.36511E-08 0.17176E-08 0.80798E-09 0.38010E-09
 0.17881E-09 0.84117E-10 0.39571E-10 0.18617E-10 0.87617E-11 0.41294E-11 0.19589E-11
 0.95630E-12 0.52378E-12 0.40350E-12
  STRESS
-0.65792E-11-0.85701E-11-0.15830E-10-0.32426E-10-0.68354E-10-0.14503E-09-0.30817E-09
-0.65503E-09-0.13924E-08-0.29598E-08-0.62918E-08-0.13375E-07-0.28431E-07-0.14563E-07
-0.14699E-06-0.12004E-06-0.14699E-06-0.14563E-07-0.28431E-07-0.13375E-07-0.62918E-08
-0.29598E-08-0.13924E-08-0.65499E-09-0.30810E-09-0.14488E-09-0.68027E-10-0.31731E-10
-0.14352E-10-0.55280E-11 0.0
```

```
AT TIME=     0.000100 SECOND
  TEMPERATURE
     0.0            0.0            0.0            0.0            0.0            0.0            0.0
     0.0            0.0            0.0            0.0            0.0            0.00000        0.00000
     0.01973        0.01973        0.01973        0.00000        0.00000        0.0            0.0
     0.0            0.0            0.0            0.0            0.0            0.0            0.0
     0.0            0.0            0.0
  DISPLEMENT
 0.0          -0.37882E-11-0.94365E-11-0.19677E-10-0.39362E-10-0.77613E-10-0.15170E-09
-0.29402E-09-0.56446E-09-0.10711E-08-0.20031E-08-0.36761E-08-0.65771E-08-0.11351E-07
-0.93639E-08 0.90452E-15 0.93639E-08 0.11351E-07 0.65771E-08 0.36761E-08 0.20031E-08
 0.10711E-08 0.56447E-09 0.29406E-09 0.15177E-09 0.77762E-10 0.39665E-10 0.20291E-10
 0.10679E-10 0.62925E-11 0.50329E-11
  STRESS
-0.75763E-10-0.94365E-10-0.15888E-09-0.29925E-09-0.57936E-09-0.11234E-08-0.21641E-08
-0.41276E-08-0.77708E-08-0.14386E-07-0.26050E-07-0.45740E-07-0.76751E-07-0.27876E-07
-0.69988E-07 0.37688E-08-0.69988E-07-0.27876E-07-0.76751E-07-0.45740E-07-0.26050E-07
-0.14386E-07-0.77705E-08-0.41270E-08-0.21630E-08-0.11211E-08-0.57471E-09-0.28986E-09
-0.13999E-09-0.56461E-10 0.0

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AT TIME=     0.000150 SECOND
  TEMPERATURE
     0.0            0.0            0.0            0.0            0.0            0.0            0.0
     0.0            0.0            0.0            0.0            0.00000        0.00000        0.00000
     0.01973        0.01973        0.01973        0.00000        0.00000        0.00000        0.0
     0.0            0.0            0.0            0.0            0.0            0.0            0.0
     0.0            0.0            0.0
  DISPLEMENT
 0.0          -0.22831E-10-0.54757E-10-0.10807E-09-0.20236E-09-0.37035E-09-0.66660E-09
-0.11794E-08-0.20447E-08-0.34559E-08-0.56503E-08-0.88249E-08-0.12878E-07-0.16765E-07
-0.12525E-07 0.66771E-14 0.12525E-07 0.16765E-07 0.12878E-07 0.88249E-08 0.56504E-08
 0.34560E-08 0.20448E-08 0.11797E-08 0.66725E-09 0.37162E-09 0.20482E-09 0.11284E-09
 0.63923E-10 0.40387E-10 0.33424E-10
  STRESS
-0.45662E-09-0.54757E-09-0.85244E-09-0.14760E-08-0.26228E-08-0.46424E-08-0.80902E-08
-0.13781E-07-0.22765E-07-0.36056E-07-0.53690E-07-0.72272E-07-0.79398E-07 0.35126E-08
-0.15848E-07 0.66989E-07-0.15848E-07 0.35127E-08-0.79398E-07-0.72272E-07-0.53689E-07
-0.36055E-07-0.22763E-07-0.13776E-07-0.80809E-08-0.46243E-08-0.25879E-08-0.14090E-08
-0.72449E-09-0.30499E-09 0.0
```

```
AT TIME=     0.000200 SECOND
   TEMPERATURE
      0.0         0.0         0.0         0.0         0.0         0.0         0.0
      0.0         0.0         0.0         0.00000     0.00000     0.00000     0.00000
      0.01973     0.01973     0.01973     0.00000     0.00000     0.00000     0.00000
      0.0         0.0         0.0         0.0         0.0         0.0         0.0
      0.0         0.0         0.0
   DISPLEMENT
  0.0         -0.95149E-10-0.22020E-09-0.41227E-09-0.72354E-09-0.12298E-08-0.20380E-08
 -0.32886E-08-0.51408E-08-0.77200E-08-0.10994E-07-0.14539E-07-0.17193E-07-0.16790E-07
 -0.10762E-07 0.20935E-13 0.10762E-07 0.16790E-07 0.17193E-07 0.14539E-07 0.10995E-07
  0.77206E-08 0.51419E-08 0.32907E-08 0.20420E-08 0.12372E-08 0.73739E-09 0.43786E-09
  0.25716E-09 0.18067E-09 0.15435E-09
   STRESS
 -0.19030E-08-0.22020E-08-0.31712E-08-0.50334E-08-0.81751E-08-0.13145E-07-0.20589E-07
 -0.31027E-07-0.44314E-07-0.58536E-07-0.68192E-07-0.61988E-07-0.22510E-07 0.54294E-07
 -0.15589E-07 0.31730E-07-0.15588E-07 0.64295E-07-0.22509E-07-0.61986E-07-0.68188E-07
 -0.58528E-07-0.44299E-07-0.30999E-07-0.20535E-07-0.13046E-07-0.79936E-08-0.47022E-08
 -0.25719E-08-0.11282E-08 0.0

.......................................

AT TIME=     0.000250 SECOND
   TEMPERATURE
      0.0         0.0         0.0         0.0         0.0         0.0         0.0
      0.0         0.0         0.00000     0.00000     0.00000     0.00000     0.00000
      0.01973     0.01973     0.01973     0.00000     0.00000     0.00000     0.00000
      0.00000     0.0         0.0         0.0         0.0         0.0         0.0
      0.0         0.0         0.0
   DISPLEMENT
  0.0         -0.30289E-09-0.67742E-09-0.12045E-08-0.19816E-08-0.31242E-08-0.47556E-08
 -0.69730E-08-0.97769E-08-0.12953E-07-0.15923E-07-0.17666E-07-0.16989E-07-0.13781E-07
 -0.77174E-08 0.83307E-13 0.77175E-08 0.13782E-07 0.16989E-07 0.17667E-07 0.15925E-07
  0.12956E-07 0.97826E-08 0.69834E-08 0.47744E-08 0.31579E-08 0.20416E-08 0.13106E-08
  0.86302E-09 0.62411E-09 0.54937E-09
   STRESS
 -0.60578E-08-0.67742E-08-0.90161E-08-0.13041E-07-0.19197E-07-0.27740E-07-0.38488E-07
 -0.50213E-07-0.59799E-07-0.61465E-07-0.47134E-07-0.10655E-07 0.38849E-07 0.92692E-07
 -0.45672E-07-0.29160E-07-0.45670E-07 0.92696E-07 0.38856E-07-0.10643E-07-0.47112E-07
 -0.61425E-07-0.59727E-07-0.50082E-07-0.38255E-07-0.27327E-07-0.18473E-07-0.11786E-07
 -0.68649E-08-0.31365E-08 0.0
```

```
.......................................

AT TIME=     0.000300 SECOND
    TEMPERATURE
       0.0          0.0          0.0          0.0          0.0          0.0          0.0
       0.0          0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
       0.01973      0.01973      0.01973      0.00000      0.00000      0.00000      0.00000
       0.00000      0.00000      0.0          0.0          0.0          0.0          0.0
       0.0          0.0          0.0
    DISPLEMENT
   0.0         -0.77313E-09-0.16729E-08-0.28265E-08-0.43554E-08-0.63547E-08-0.88486E-08
  -0.11720E-07-0.14624E-07-0.16926E-07-0.17782E-07-0.16538E-07-0.13647E-07-0.11941E-07
  -0.73828E-08 0.39854E-12 0.73837E-08 0.11942E-07 0.13649E-07 0.16542E-07 0.17790E-07
   0.16940E-07 0.14648E-07 0.11762E-07 0.89211E-08 0.64795E-08 0.45681E-08 0.31850E-08
   0.22702E-08 0.17548E-08 0.15890E-08
    STRESS
  -0.15463E-07-0.16729E-07-0.20533E-07-0.26826E-07-0.35283E-07-0.44932E-07-0.53654E-07
  -0.57752E-07-0.52059E-07-0.31585E-07 0.38848E-08 0.41356E-07 0.45967E-07 0.62612E-07
  -0.64069E-07-0.35844E-07-0.64063E-07 0.62627E-07 0.45995E-07 0.41407E-07 0.39771E-08
  -0.31423E-07-0.51777E-07-0.57266E-07-0.52824E-07-0.43530E-07-0.32945E-07-0.22979E-07
  -0.14302E-07-0.68117E-08 0.0

.......................................

AT TIME=     0.000350 SECOND
    TEMPERATURE
       0.0          0.0          0.0          0.0          0.0          0.0          0.0
       0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
       0.01973      0.01973      0.01973      0.00000      0.00000      0.00000      0.00000
       0.00000      0.00000      0.00000      0.0          0.0          0.0          0.0
       0.0          0.0          0.0
    DISPLEMENT
   0.0         -0.16217E-08-0.33973E-08-0.54550E-08-0.78627E-08-0.10581E-07-0.13403E-07
  -0.15915E-07-0.17511E-07-0.17566E-07-0.15840E-07-0.13076E-07-0.11341E-07-0.12752E-07
  -0.94265E-08 0.17475E-11 0.94306E-08 0.12758E-07 0.11350E-07 0.13092E-07 0.15869E-07
   0.17615E-07 0.17595E-07 0.16056E-07 0.13639E-07 0.10970E-07 0.84969E-08 0.64750E-08
   0.50136E-08 0.41402E-08 0.38507E-08
    STRESS
```

```
-0.32434E-07-0.33973E-07-0.38333E-07-0.44654E-07-0.51257E-07-0.55405E-07-0.53343E-07
-0.41076E-07-0.16512E-07 0.16707E-07 0.44905E-07 0.44995E-07 0.32367E-08 0.19109E-07
-0.55940E-07 0.50621E-08-0.55915E-07 0.19167E-07 0.33447E-08 0.45185E-07 0.45231E-07
 0.17258E-07-0.15593E-07-0.39557E-07-0.50866E-07-0.51420E-07-0.44946E-07-0.34833E-07
-0.23348E-07-0.11629E-07 0.0


.......................................

AT TIME=    0.000400 SECOND
  TEMPERATURE
    0.0          0.0          0.0          0.0          0.0          0.0          0.00000
    0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
    0.01973      0.01973      0.01973      0.00000      0.00000      0.00000      0.00000
    0.00000      0.00000      0.00000      0.00000      0.0          0.0          0.0
    0.0          0.0          0.0
  DISPLEMENT
 0.0          -0.28206E-08-0.57234E-08-0.87310E-08-0.11749E-07-0.14523E-07-0.16634E-07
-0.17579E-07-0.16982E-07-0.14951E-07-0.12425E-07-0.11122E-07-0.12364E-07-0.14405E-07
-0.10625E-07 0.67065E-11 0.10641E-07 0.14426E-07 0.12399E-07 0.11179E-07 0.12520E-07
 0.15107E-07 0.17238E-07 0.17993E-07 0.17297E-07 0.15570E-07 0.13378E-07 0.11226E-07
 0.94757E-08 0.83508E-08 0.79646E-08
  STRESS
-0.55412E-07-0.57234E-07-0.59104E-07-0.60259E-07-0.57925E-07-0.48851E-07-0.30553E-07
-0.34782E-08 0.26278E-07 0.45572E-07 0.38290E-07 0.60933E-09-0.32827E-07 0.17353E-07
-0.39360E-07 0.29149E-07-0.39275E-07 0.17546E-07-0.32475E-07 0.12082E-08 0.39280E-07
 0.47181E-07 0.28855E-07 0.58964E-09-0.24229E-07-0.39187E-07-0.43441E-07-0.39023E-07
-0.28748E-07-0.15111E-07 0.0


.......................................

AT TIME=    0.000450 SECOND
  TEMPERATURE
    0.0          0.0          0.0          0.0          0.0          0.00000      0.00000
    0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
    0.01973      0.01973      0.01973      0.00000      0.00000      0.00000      0.00000
    0.00000      0.00000      0.00000      0.00000      0.00000      0.0          0.0
    0.0          0.0          0.0
  DISPLEMENT
 0.0          -0.40445E-08-0.79499E-08-0.11514E-07-0.14434E-07-0.16328E-07-0.16843E-07
-0.15866E-07-0.13793E-07-0.11673E-07-0.10941E-07-0.12397E-07-0.14750E-07-0.14767E-07
-0.95770E-08 0.22758E-10 0.96286E-08 0.14838E-07 0.14858E-07 0.12569E-07 0.11214E-07
 0.12108E-07 0.14477E-07 0.16929E-07 0.18473E-07 0.18789E-07 0.18036E-07 0.16828E-07
 0.15518E-07 0.14562E-07 0.14215E-07
  STRESS
-0.80839E-07-0.79499E-07-0.74697E-07-0.64845E-07-0.48141E-07-0.24084E-07 0.46198E-08
 0.30501E-07 0.41928E-07 0.28520E-07-0.72403E-08-0.38093E-07-0.23691E-07 0.51689E-07
-0.35575E-07 0.85473E-08-0.35320E-07 0.52256E-07-0.22687E-07-0.36440E-07-0.45094E-08
 0.32623E-07 0.48215E-07 0.39964E-07 0.18597E-07-0.38723E-08-0.19610E-07-0.25677E-07
-0.22662E-07-0.13033E-07 0.0
```

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APPENDIX I

AN EQUIVALENT SOLUTION OF TEMPERATURE VARIATION

For the purpose of our numerical work we consider an equivalent problem to the problem which was presented in section 2.2.2 instead of solving this problem directly.

The linear partial differential equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

subject to the boundary and initial conditions

$$T = 0 \qquad \text{at } x = 0, \text{ for } t \geq 0$$

$$T = 0 \qquad \text{at } x = L, \text{ for } t \geq 0$$

$$T = \frac{q_0 \alpha}{k}\{H(x-x_0)-H(x-x_1)\} \qquad \text{at } t = 0, \text{ for } 0 \leq x \leq L$$

where q_0 α k are known constants.

Let us note that only the x-direction yields a characteristic value problem; then, with the proper choice of separation constant, the product solution $T(x,t) = X(x)\tau(t)$ gives

$$\frac{d^2X}{dx^2} + \lambda^2 X = 0; \quad X(0) = X(L) = 0 \tag{A1}$$

$$\frac{d\tau}{dt} + \alpha\lambda^2\tau = 0 \tag{A2}$$

The solution of equation (A1) is

$X_n(x) = A_n\psi_n(x)$, $\psi_n(x) = \sin(\lambda_n x)$, characteristic functions,

$\lambda_n L = n\pi$ $\qquad n = 1,2,3,\cdots\cdots\cdots$characteristic values,

and the solution of equation (A2) is

$$\tau_n(t) = C_n e^{-\alpha\lambda_n^2 t} \tag{A3}$$

Hence the product solution becomes

$$T(x,t) = \sum_{n=1}^{\infty} a_n\, e^{-\alpha\lambda_n^2 t} \sin(\lambda_n x) \tag{A4}$$

where $a_n = A_n C_n$

Finally, introducing the initial condition, $T(x,0) = \alpha q_0/k\{H(x-x_0)-H(x-x_1)\}$, into equation (A4) gives

$$T(x,0) = \alpha q_0/k\{H(x-x_0)-H(x-x_1)\}$$

$$= \sum_{n=1}^{\infty} a_n \sin(\lambda_n x) \tag{A5}$$

Equation (A5) is the Fourier sine series expansion of $T(x,0)$ over the interval $(0,L)$. Multiplying both sides of equation (A5) by $\sin(\lambda_m x)$ and integrating the result over the interval $(0,L)$, where $\sin(\lambda_m x)$ is the mth term in the set, we have

$$\int_0^L T(x,0)\ \sin(\lambda_m x)dx = \sum_{n=1}^{\infty} a_n \int_0^L \sin(\lambda_n x)\sin(\lambda_m x)dx \tag{A6}$$

where $\sin(\lambda_m x)$ is the mth term in the set. Using the orthogonality of the set, we find that all terms in the sum on the right of equation (A6) are zero except the term corresponding to $n = m$. Hence we obtain

$$a_n = \frac{2}{L}\int_0^L T(x,0)\sin(\lambda_n x)dx$$

$$= \frac{2\alpha q_0}{kn\pi}[\cos(\lambda_n x_0)-\cos(\lambda_n x_1)] \tag{A7}$$

Substituting equation (A7) into equation (A4), we have

$$T(x,t) = \frac{2\alpha q_0}{k}\sum_{n=1}^{\infty} \frac{\sin(\lambda_n x)}{n\pi}\ e^{-\alpha\lambda_n^2 t}[\cos(\lambda_n x_0)-\cos(\lambda_n x_1)]$$

where $\lambda_n = n\pi/L$. Hence some solution can be derived in this manner by this method.

```
C
C     ....THIS PROGRAMM USED TO EVALUATE THE ANALYSIS
C          SOLUTION OF TEMPERATURE VARIATION
C
C     SPL...THE SPAN LENGTH OF ROD                   (FT)
C     CON...THERMAL CONDUCTIVITY                     (BTU/HR,FT,F)
C     CAP...SPECIAL HEAT COEFFICIENT                 (BTU/LB.F)
C     DENS..DENSITY OF THE ROD                       (IB/CUBIC FT)
C     ALPH..THERMAL DIFFUSIVITY                      (SQUARE FT/HR.)
C     TIMEF...TIME FOR CONTROLLING THE
C            COMPUTATION OF THIS PROGRAM             (SECOND)
C     U0... HEAT QUATITY DUE TO A CONSTANT
C            INSTANTANEOUS HEAT SOURCE               (BTU/CUBIC FT,SEC.)
C     X0,X1...PARTICULAR POINTS, BETWEEN THAT
C            A CONSTANT HEAT QUANTITY U0 IS INDUCED
C     E...  YOUNHS ELASTIC MUDULUS                   (IBF/SQUARE FT)
C     COEF...COEFFICIENT OF LINEAR THERMAL
C             EXPANSION
C
C
      DIMENSION TEMP(16)
   10 FORMAT(F3.1,F5.1,F5.3,F5.1,F4.2,F4.1,F7.1,E9.3,E7.1)
   11 FORMAT(5X,'N= ',I4,'   THE SOLUTION CAN NOT CONVERGE',
     C'AT EXACT VALUE')
   12 FORMAT(//,42X,'TEMPERATURE DISTRIBUTION (F)',//,25X,
     C'  *  THE SECTION LENGTH IS 0.1 FT.                        *')
   15 FORMAT(1H1)
   14 FORMAT(//)
   17 FORMAT(2F5.2)
   18 FORMAT(25X,'  *  X0 =',F5.2,'   AND    X1 =',F5.2,22X,'*')
   22 FORMAT(25X,'  *  XX0=',F7.4,' AND    XX1=',F7.4,20X,'*',/)
   40 FORMAT(1H0,10X,'AT TIME=',F11.4,'     SECOND ',//,
     C2(/,10X,8F12.6))
      MM=15
      READ (5,10) SPL,CON,CAP,DENS,ALPH,TIMEF,U0,E,COEF
      WRITE(6,15)
      WRITE(6,14)
      WRITE(6,12)
C
C     ....A UNIT HEAT QUANTITY IS INTRODUCED
C
      U0=U0/U0
C
C     ....CHANGE TIME UNIT (HOUR) TO TIME UNIT (SECOND)
C
      CON=CON/3600.0
      ALPH=ALPH/3600.0
      PI=3.1415296
      Q0=U0*ALPH/CON
C
C     ....KDUMYA USED FOR CONTROLLING THE READ CARD
C
```

```
      KDUMYA=1
 4500 CONTINUE
      READ(5,17) X0,X1
      WRITE(6,18) X0,X1
      XX0=X0/SPL
      XX1=X1/SPL
      EPI=1.0E-12
      WRITE(6,22) XX0,XX1
      TIME=1.0
 2000 CONTINUE
      TD=ALPH*TIME/(SPL**2)
      X=0.0
      MK=1
 3000 CONTINUE
      XX=X/SPL
C
C     ....SUMMING THE TERMS
C
      SUM=0.0
      N=1
      S1=0.0
  200 CONTINUE
      PIN=N*PI
      PIN2=PIN**2
      COSN=COS(PIN*XX0)-COS(PIN*XX1)
      SUM=SIN(PIN*XX)/PIN*EXP(-PIN2*TD)*COSN+SUM
C
C     ....COMPARE THE RESULTS OF SUMMING TERMS (FIVE TERMS)
C
      IF (N.GT.1) GO TO 110
      S1=SUM
      N=N+1
      GO TO 200
  110 IF(N.GT.2) GO TO 210
      S2=SUM
      N=N+1
      GO TO 200
  210 IF(N.GT.3) GO TO 300
      S3=SUM
      N=N+1
      GO TO 200
  300 IF(N.GT.4) GO TO 400
      S4=SUM
      N=N+1
      GO TO 200
  400 IF(N.GT.5) GO TO 500
      S5=SUM
      N=N+1
      GO TO 200
  500 CONTINUE
      D1=ABS(S2-S1)
      D2=ABS(S3-S1)
```

```
      D3=ABS(S4-S1)
      D4=ABS(S5-S1)
      S1=S2
      S2=S3
      S3=S4
      S4=S5
      S5=SUM
      IF((D1.LE.EPI).AND.(D2.LE.EPI).AND.(D3.LE.EPI).AND.
     C(D4.LE.EPI)) GO TO 1000
      IF(N.EQ.100) GO TO 800
      N=N+1
      GO TO 200
  800 WRITE(6,11) N
      GO TO 810
 1000 CONTINUE
      IF(MK.EQ.MM) GO TO 780
      GO TO 790
  780 S1=0.0
  790 CONTINUE
      TEMP(MK)=2.0*Q0*ABS(S1)
      X=X+0.1
      MK=MK+1
      IF(MK-MM) 3000,3000,4000
 4000 CONTINUE
      WRITE(6,40) TIME,(TEMP(KK),KK=1,MM)
      TIME=TIME+2.0
      IF(TIME-TIMEF) 2000,2000,810
  810 CONTINUE
      WRITE(6,15)
      KDUMYA=KDUMYA+1
C
C     ....USING L TO CONTROL READ CARD
C
      L=3
      IF(KDUMYA-L) 4500,4500,4600
 4600 CONTINUE
      WRITE(6,15)
      STOP
      END
```

SAMPLE DATA IS AS FOLLOWS

```
SPL....1.5
CON....224.0
CAP....0.091
DENS....553.0
ALPH....4.42
TIMEF....11.0
U0....35000.0
E....1.872E 09
COEF....9.3E-06
X0....0.2, X1....0.3
X0....1.0, X1....1.1
X0....0.7, X1....0.3
```

APPENDIX III

```
C       ....THIS PROGRAM USED THE IMPLICIT FINITE DIFFERENCE
C           TO SOLVE THE TEMPERATURE VARIATION
        DIMENSION T(31,2),A(29,30)
        COMMON/W2/A
     10 FORMAT(F5.3,F5.1)
     12 FORMAT(2X,'TIME=',F6.4,/,3X,4(8F9.6,/,3X))
     15 FORMAT(//,'.......................................')
C
C
C       ALPH..THERMAL DIFFUSIVITY                  (SQUARE FT/HR.)
C       CON...THERMAL CONDUCTIVITY                 (BTU/HR,FT,F)
C
C
        READ(5,10) ALPH,CON
        NN=31
        NN1=NN-1
        NN2=NN-2
        ALPH=ALPH/3600
        CON=CON/3600
        DTAU=0.05
        TIMEF=3.0
        DX=0.050
        R=ALPH*DTAU/(DX*DX)
C
C       .....AN UNIT HEAT QUANTIFY BE INTRODUCED.....
C
        Q0=1.0
        TO=ALPH/CON*Q0
        TAU=0.0
C
C       .....SET AND PRINT INITIAL TEMPERATURES......
C
        DO 100 I=1,31
    100 T(I,1)=0.0
        T(15,1)=TO
        T(16,1)=TO
        T(17,1)=TO
        WRITE(6,15)
        WRITE(6,12) TAU,(T(I,1),I=1,NN)
C
C       .....SET BOUNDARY VALUES......
C
        T(1,1)=0.0
        T(NN,1)=0.0
        T(1,2)=0.0
        T(NN,2)=0.0
C
C       .....REFORM CALCULATIOUS OVER SUCESSIVE TIME STEPS....
C
```

```
  500 CONTINUE
      TAU=TAU+DTAU
C
C     .....COMPUTE NEW TEMPERATURE.......
C
      DO 200 I=1,NN2
      DO 200 J=1,NN1
      A(I,J)=0.0
  200 CONTINUE
      DO 300 I=1,NN2
      A(I,I)=-(1.0+2.0*R)
      IF(I.EQ.1) GO TO 350
      IF(I.EQ.NN2) GO TO 360
      A(I,I+1)=R
      A(I,I-1)=R
      A(I,NN1)=-T(I+1,1)
      GO TO 300
  350 A(I,I+1)=R
      A(I,NN1)=-T(I+1,1)
      GO TO 300
  360 A(I,I-1)=R
      A(I,NN1)=-T(I+1,1)
  300 CONTINUE
      CALL GAUS2 (NN2)
      DO 320 I=1,NN2
      T(I+1,2)=A(I,NN1)
  320 CONTINUE
C
C     .....PRINT TEMPERATURES WHEN APPROPRIATE......
C
      WRITE(6,15)
      WRITE(6,12) TAU,(T(I,2),I=1,NN)
C
C     .....CHANGE NEW TEMPERATURES TO OLD TEMPERATURES AND STORE
C
      DO 400 I=2,30
      T(I,1)=T(I,2)
  400 CONTINUE
      IF(TAU-TIMEF) 500,500,600
  600 STOP
      END
C
C
      SUBROUTINE GAUS2 (N)
      DIMENSION  A(29,30)
      COMMON/W2/A
      N1=N+1
      DO 200 J=1,N
      DIV=A(J,J)
      S=1.0/DIV
      DO 201 K=J,N1
  201 A(J,K)=A(J,K)*S
```

```
      DO 202 I=1,N
      IF(I-J) 203,202,203
  203 AIJ=-A(I,J)
      DO 204 K=J,N1
  204 A(I,K)=A(I,K)+AIJ*A(J,K)
  202 CONTINUE
  200 CONTINUE
      RETURN
      END
```

SAMPLE DATA IS AS FOLLOWS

```
ALPH....4.42
CON....224.0
```

APPENDIX IV

```
C
C     .....THIS PROGRAM USED THE FINITE DIFFERENCE METHODS
C          TO  SOLVE THERMOELASTICITY PROBLEM
C
C     .....A UNIT HEAT QUANTY OF HEAT IS INTRODUCED
C
      DIMENSION T(31,3),DISPL(31,3),STRESS(31,3),RIGHT(29),
     CLEFT(29),ERROR(29)
      COMMON/Z1/ T
      COMMON/Z2/ DISPL
      COMMON/Z4/ STRESS
      COMMON/Z5/ DSPLX,TIMEO,XO,X1
      COMMON/Z6/ DENS,E,ALPH
      COMMON/Z7/COEF
      COMMON/Z8/LEFT
      COMMON/Z9/ RIGHT
      COMMON/Z10/ ERROR
   10 FORMAT(F3.1,F5.1,F5.3,F5.1,F4.2,F4.1,F7.1,E9.3,E7.1)
   17 FORMAT(2F5.2)
   30 FORMAT(1H1,25X,' TEMPERATURE (F)  STRAIN AND STRESS
     C(PSF) ARE GIVE IN AT SPACING OF',F10.6,' FT   APART',/)
   40 FORMAT(1HO,' AT TIME= ',F11.6,' SECOND ',/,6X,
     C'TEMPERATURE',/,3(4X,8E16.9,/),4X,7E16.9)
   42 FORMAT(6X,'DISPLEMENT',/,3(4X,8E16.9,/),4X,7E16.9)
   29 FORMAT(//,'.......................................')
   50 FORMAT(6X,'STRESS',/,3(4X,8E16.9,/),4X,7E16.9)
C
C
C     CON...THERMAL CONDUCTIVITY                    (BTU/HR,FT,F)
C     SPL...THE SPAN LENGTH OF ROD                  (FT)
C     CAP...SPECIAL HEAT                            (BTU/IB,F)
C     DENS..DENSITY OF THE ROD                      (IB/CUBIC FT)
C     ALPH..THERMAL DIFFUSIVITY                     (SQUARE FT/HR.)
C     TIMEF...TIME FOR CONTROLLING THE
C             COMPUTATION OF THIS PROGRAM           (SECOND)
C     UO... HEAT QUATITY DUE TO A CONSTANT
C             INSTANTANEOUS                         (BTU/CUBIC FT,SEC.)
C     XO,X1...PARTICULAR POINTS, BETWEEN THAT
C             A CONSTANT HEAT QUANTITY UO IS INDUCED
C     E...  YOUNHS ELASTIC MODULUS                  (IBF/SQUARE FT)
C     COEF...COEFFICIENT OF LINEAR THERMAL
C              EXPANSION                            (FT/FT,F)
C
C
      READ (5,10) SPL,CON,CAP,DENS,ALPH,TIMEF,UO,E,COEF
      NUMB=31
      NM1=NUMB-1
      NM2=NUMB-2
      UO=UO/UO
```

```
      CON=CON/3600.0
      ALPH=ALPH/3600.0
      DTIME=0.00005
      DSPLX=0.050
      XX=0.050
      READ(5,17) X0,X1
      TIME=0.0
      K=1
      KK=1
      X=0.0
      F=U0*ALPH/CON
C
C     .....SET INITIAL TEMPERATURES(AN EQUIVALENT
C          PROBLEM WAS REPLACED)
C
      DO 62 I=1,NUMB
      T(I,1)=0.0
   62 CONTINUE
      T(15,1)=F
      T(16,1)=F
      T(17,1)=F
      DO 100 I=1,NUMB
      DISPL(I,K)=0.0
      STRESS(I,K)=0.0
  100 CONTINUE
C
C     SET BOUNDARY VALUES
C
      T(1,1)=0.0
      T(1,2)=0.0
      T(1,3)=0.0
      T(NUMB,1)=0.0
      T(NUMB,2)=0.0
      T(NUMB,3)=0.0
      DISPL(1,1)=0.0
      DISPL(1,2)=0.0
      DISPL(1,3)=0.0
      WRITE(6,30) XX
      WRITE(6,29)
      WRITE (6,40) TIME,(T(I,K),I=1,NUMB)
      WRITE(6,42) (DISPL(I,K),I=1,NUMB)
      WRITE(6,50) (STRESS(I,K),I=1,NUMB)
  999 CONTINUE
      K=K+1
      KK=KK+1
      TIME=TIME+DTIME
      CALL TEMP (NUMB,K,DTIME)
      CALL DISP (DTIME,DSPLX,K,NUMB)
      CALL ELAST (K,NUMB,COEF,F,DSPLX)
      IF(KK.GT.2) GO TO 250
      WRITE(6,29)
      WRITE (6,40) TIME,(T(I,2),I=1,NUMB)
```

```
      WRITE(6,42) (DISPL(I,2),I=1,NUMB)
      WRITE(6,50) (STRESS(I,2),I=1,NUMB)
      WRITE(6,29)
      GO TO 260
  250 CONTINUE
      WRITE (6,40) TIME,(T(I,3),I=1,NUMB)
      WRITE(6,42) (DISPL(I,3),I=1,NUMB)
      WRITE(6,50) (STRESS(I,3),I=1,NUMB)
      WRITE(6,29)
  260 CONTINUE
      IF(K-2) 270,270,280
  280 CONTINUE
      DO 271 I=1,NUMB
      T(I,1)=T(I,2)
      T(I,2)=T(I,3)
      DISPL(I,1)=DISPL(I,2)
      DISPL(I,2)=DISPL(I,3)
      STRESS(I,1)=STRESS(I,2)
      STRESS(I,2)=DISPL(I,3)
  271 CONTINUE
      K=2
  270 CONTINUE
      IF(KK-50) 290,290,300
  290 CONTINUE
      GO TO 999
  300 CONTINUE
      STOP
      END
C
C     .....USING EXPLICIT METHOD TO SOLVE TEMPERATURE
C
      SUBROUTINE TEMP (NM,K,DTIME)
      DIMENSION T(31,3),A(29,30)
      COMMON/Z1/ T
      COMMON/Z5/ DSPLX,TIME0,X0,X1
      COMMON/Z6/ DENS,E,ALPH
      NM1=NM-1
      T(1,K)=0.0
      T(NM,K)=0.0
      B=ALPH*DTIME/(DSPLX*DSPLX)
      DO 100 I=2,NM1
      T(I,K)=B*T(I-1,K-1)+(1.0-2.0*B)*T(I,K-1)+B*T(I+1,K-1)
  100 CONTINUE
      RETURN
      END
C
C     .....USING IMPROVED IMPLICIT METHOD TO SOLVE DISPLACEMENT
C
      SUBROUTINE DISP (DTIME,DSPLX,K,M)
      DIMENSION A(30,31),DISPL(31,3),T(31,3)
      COMMON/Z1/ T
      COMMON/Z2/ DISPL
```

```
      COMMON/Z6/ DENS,E,ALPH
      COMMON/Z7/COEF
      COMMON/W2/A
      M1=M-1
      M2=M-2
      VELO=SQRT(E/DENS)
      RATO=VELO*DTIME/DSPLX
      RRATO=RATO**2
      RA=(1+RRATO)
      COR=COEF*DSPLX*RRATO
      DO 100 I=1,M1
      DO 100 J=1,M1
      A(I,J)=0.0
  100 CONTINUE
      IF(K-2) 300,300,200
  300 CONTINUE
      DO 800 I=1,M1
      A(I,I)=-2.0*RA
      IF(I.EQ.1) GO TO 500
      IF(I.EQ.M1) GO TO 550
      A(I,I+1)=RRATO
      A(I,I-1)=RRATO
      A(I,M)=COR*(T(I+2,K-1)-T(I,K-1))/2.0-2.0*DISPL(I+1,K-1)
      GO TO 700
  500 CONTINUE
      A(I,I+1)=RRATO
      A(I,M)=COR/2.0*T(I+2,K-1)-2.0*DISPL(I+1,K-1)
      GO TO 700
  550 CONTINUE
      A(I,I-1)=RRATO*2.0
      A(I,M)=-COR*T(I,K-1)-2.0*DISPL(I+1,K-1)
  700 CONTINUE
  800 CONTINUE
      GO TO 810
  200 CONTINUE
      DO 840 I=1,M1
      A(I,I)=-1.0*RA
      IF(I.EQ.1) GO TO 505
      IF(I.EQ.M1) GO TO 515
      A(I,I+1)=RRATO/2.0
      A(I,I-1)=RRATO/2.0
      A(I,M)=COR*(T(I+2,K-1)-T(I,K-1))/2.0-2.0*DISPL(I+1,K-1)
     C-(RRATO/2.0*DISPL(I,K-2)-RA*DISPL(I+1,K-2)
     C+RRATO/2.0*DISPL(I+2,K-2))
      GO TO 710
  505 CONTINUE
      A(I,I+1)=RRATO/2.0
      A(I,M)=COR*(T(I+2,K-1)-T(I,K-1))/2.0-
     C(-RA*DISPL(I+1,K-2)+RRATO/2.0*DISPL(I+2,K-2))
     C-2.0*DISPL(I+1,K-1)
      GO TO 710
  515 A(I,I-1)=RRATO
```

```
      A(I,M)=-COR*T(I,K-1)-(RRATO*DISPL(I,K-2)-RA*DISPL(I+1,
     CK-2))-2.0*DISPL(I+1,K-1)
 710  CONTINUE
 840  CONTINUE
 810  CONTINUE
      CALL GAUS2 (M1)
      DO 900 I=1,M1
 900  DISPL(I+1,K)=A(I,M)
      RETURN
      END
      SUBROUTINE ELAST (K,NUMB,COEF,E,DSPLX)
      DIMENSION T(31,3),DISPL(31,3), STRESS(31,3)
      COMMON/Z1/ T
      COMMON/Z2/ DISPL
      COMMON/Z4/ STRESS
      IF(K-2) 100,100,300
 100  CONTINUE
      DO 150 I=1,NUMB
      IF(I.EQ.1) GO TO 190
      IF(I.EQ.NUMB) GO TO 200
      STRESS(I,2)=(DISPL(I+1,2)-DISPL(I-1,2))/(2.0*DSPLX)
     C-COEF*T(I,2)
      GO TO 150
 190  STRESS(I,2)=DISPL(I+1,2)/DSPLX-COEF*T(I,2)
      GO TO 150
 200  STRESS(I,2)=-COEF*T(I,2)
 150  CONTINUE
      GO TO 500
 300  CONTINUE
      DO 390 I=1,NUMB
      IF(I.EQ.1) GO TO 310
      IF(I.EQ.NUMB) GO TO 320
      STRESS(I,3)=(DISPL(I+1,3)-DISPL(I-1,3))/(2.0*DSPLX)
     C-COEF*T(I,3)
      GO TO 390
 310  STRESS(I,3)=DISPL(I+1,3)/DSPLX-COEF*T(I,3)
      GO TO 390
 320  STRESS(I,3)=-COEF*T(I,3)
 390  CONTINUE
 500  CONTINUE
      RETURN
      END
      SUBROUTINE GAUS2 (N)
      DIMENSION  A(30,31)
      COMMON/W2/A
      N1=N+1
      DO 200 J=1,N
      DIV=A(J,J)
      S=1.0/DIV
      DO 201 K=J,N1
 201  A(J,K)=A(J,K)*S
      DO 202 I=1,N
```

```
      DO 204 K=J,M1
204   A(I,K)=A(I,K)+AIJ*A(J,K)
202   CONTINUE
200   CONTINUE
      RETURN
      END
```

SAMPLE DATA IS AS FOLLOWS

```
SPL....1.5
CON....224.0
CAP....0.091
DENS....558.0
ALPH....4.42
TIMEF....11.0
U0....35000.0
E....1.372E 09
COEF....9.3E-06
```

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THERMOELASTIC STRESS AND DEFORMATION IN A THIN ROD DUE TO AN INSTANTANEOUS HEAT SOURCE

by

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AN ABSTRACT OF A MASTER'S THESIS

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1976

This research presents an analytical and numerical solution for the transient temperature and the associated thermal stress and deformation which arise in an istropic thin rod of finite length due to an instantaneous heat source distributed over a finite portion of the rod. The problem is approached from the standpoint of classical linear, uncoupled, thermoelastic theory. The material of the rod is assumed to be homogeneous and isotropic with respect to both its thermal and mechanical response, and its physical properties to be independent of temperature.

Assuming the temperature gradients in the cross section of the rod to be negligible, and also that heat loses through the surface to the sorroundings medium is not considered. The diffusion equation is solved by the technique of Laplace transformation. A " long-time " solution for the simple boundary conditions (zero temperature boundary) is obtained. Associated with the given temperature variation, an elementary thermoelastic theory was applied to derive the governing differential equations under the thermal load. For the fixed end and free end boundary conditions, the Laplace transformation method leads directly to to the solutions in terms of a double infinite series. For the poor convergence of the double infinite series, it gives rise to considerable difficulties in the numerical evaluation. For this research, the finite difference approximate method with carefully selected time and space increments has the advantage of investigate the stress wave and deformation patterns in the thin rod . A graphical presentation is made of predicted temperature distributions, stress and deformation.