A MODEL OF CORN RESPONSE TO AVAILABLE MOISTURE AND AN ECONOMIC MODEL TO SCHEDULE IRRIGATIONS

by

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CHAPTER I

INTRODUCTION

Because of increased interest in the scheduling of irrigations, which has been largely due to the dwindling supplies of irrigation water and the increasing costs of pumping it, there have been numerous attempts to develop irrigation scheduling models. The degree of usefulness of those models vary widely, but generally they can be found to be lacking either in economic considerations or in their representation of the biological and physical processes involved in plant-water relationships.

Since there was a need for a model that would integrate the economic, biological, and physical aspects of irrigation scheduling in a realistic fashion, two models were developed and are presented here.

The first model, a crop response model, was developed to simulate corn grain yield response to soil moisture availability. It integrated the biological and physical aspects of irrigation scheduling. In developing that model pertinent literature was reviewed regarding corn growth both in relation to water availability and crop modeling. Data were collected and several agronomic functions were estimated. Those functions included two growth functions and a plant-soil moisture stress function. The model was then simulated using the test data.

The second model was an economic irrigation scheduling model. It combined the biological and physical relationships in the crop response model with economic aspects of irrigation scheduling. The economic criterion used was to maximize total revenue net of irrigation costs. The crop response model was combined with a water balance equation and the criterion function to make the model. Those components were cast into a dynamic programming framework which resulted in a multiperiod decision model. That model was then solved for two different levels of irrigation costs.

CHAPTER II

CORN DEVELOPMENT AND ITS RESPONSE TO SOIL MOISTURE STRESS

Growth and Development of the Corn Plant

The construction of the corn crop response model involved an understanding of the growth and development of the corn plant in relation to the whole range of water availabilities. The following subsections contain a brief review of the present understanding of corn growth as it relates to grain yield and water availability.

Two Phases of Corn Growth

The corn plant's "development can be divided into two phases, vegetative and ear development [43]." The corn plant can be divided into those two phases because most of the earlier growth occurs in the vegetative parts while most of the later growth takes place in the ear parts. For example, the leaves and tassel are fully developed two to three days before silking and the stalk has ceased to elongate by silking; therefore just prior to silking vegetative growth has practically ceased. Thereafter, most accumulations of dry matter in the vegetative parts will be in the form of labile carbohydrates which will later be translocated to the grain. In addition to vegetative growth ceasing just prior to silking, the ear development phase begins. The ear development phase commenses with the silks elongating, the cob growing and the ovules enlarging. From that point on, most of the dry matter accumulation in the plant occurs in the ear. The interval during which vegetative growth ceases and ear development commences is between tasseling and silking.

Vegetative Growth Phase

The vegetative growth phase begins with plant emergence and ends a few days after tasseling but a few days prior to silking at about stage 4.5.[1] During that time the plant produces practically all of its leaves, leaf sheaths, stalk, tassel, husks and ear shanks. The length of that period is variable [22] and dependent on fertility [22], temperature [43], [13], and available moisture [43].

From emergence (stage 0) to 35 days after emergence, DAE, (stage 2.5) most of the growth is in the leaves and leaf sheaths. During that period the leaf growth results in a growing number of leaves being exposed to sunlight, causing the rate of photosynthesis to increase. Therefore, during the first 35 DAE dry matter accumulates at an increasing rate.

The first two weeks after emergence are important because by the end of the first week (stage 0.5) the plant is feeding itself by photosynthesis and by the end of the second week (stage 1) the tassel has been initiated. Kiesselback [30] found that initiation of leaf primordia ceases with the initiation of the tassel. Hanway [22] also has observed that all the leaves and ear shoots are initiated by stage 1. "Thus, the number of leaves that will develop on the corn plant has been determined by this time (stage 1)." Therefore, environmental conditions need to be favorable for sufficient number of leaf initials and

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[1]The use of stages in this paper refer to Hanway's stages as reported in [22] and [23].

tassel initiation. The number of leaves initiated at that time is the first step in determining the plant's photosynthetic capacity.

During the third week the stalk elongates and by stage 1.5 (21 DAE) the growing point is at or above the soil surface. By the end of the fourth week (stage 2) the stalk has started rapid elongation. The stalk will continue to elongate until tasseling. Conditions favorable enough to allow sufficient development in the stalk are needed during that time. The diameter of the stalk will largely determine its strength, so as to avoid lodging, and its volumnmetric capacity for labile carbohydrates [13], which will later be stored and then transferred to the grain.

At stage 2 (28 DAE) the plant is in the middle of rapid leaf formation and by stage 3 (42 DAE) leaf enlargement is complete and maximum leaf area is attained.

One can not expect to increase the size of the leaves after that time. At stage 3 only about half of the leaves are fully exposed to sunlight and therefore only half are functional at that time. After stage 3 the leaves will continue to emerge from the whorl and become functional. As the leaves emerge from the whorl they develop green color and continue to increase in weight. Although by stage 3 a potential leaf area is determined, functional leaf area continues to develop. Claassen and Shaw [9] found that severe soil moisture stress can significantly affect the leaf area at stages 2, 2.5, and 3.

"By stage 3, enough leaves are exposed to sunlight so the dry matter accumulation is rapid [23]." That rapid rate continues nearly constant until near maturity [23], [22], [14]. Claassen and Shaw [9] reported reductions in total dry matter production as high as 17 per cent at stage 2.5 and 15 per cent at stage 3 for short severe periods of stress. Those same stages are also important to the development of the tassel and ear. At stage 2 the tassel begins to develop rapidly and by stage 2.5 it is developing rapidly. That rapid development continues through stage 3. At stage 3.5 the tassel is nearly full size. Severe stress st stages 2.5 or 3 "appear to have the most effect on tassel emergence [9]." Silking was found to be delayed from 2 to 5 days by a severe stress at 2.5, 3 or 3.5 [9].

The ear shoots are developing at stage 2.5. The ear shoots first develop the ear shank and husks. After all the husk initials are formed, the growing point of the ear shoot elongates to form the beginning of the ear [13]. At stage 3 the uppermost ears were beginning rapid development and "the potential number of ovules on the top (major) ear is determined at that time [23]." That rapid development continues at stage 3.5 when "the number of ovules which develop silks and thus the number of kernels is being determined [23]."

Claassen and Shaw [9] found that the first stress period for the cob coincided with stages 2.5 and 3. They also reported that water deficits had an adverse effect on husk weight at stages 2.5, 3, and 3.5. During that same period, stages 2.5 and 3, "the earliest significant stress effects on yield" were reported by those researchers [10]. They reported as much as a 12 to 15 per cent reduction in yields due to a short period of severe stress at either of those stages. They noted severe stress at stage 2.5 and 3 reduced the percentage of developed kernels. Stress at stage 2.5 affected the middle and lower sections of the ear while stress at stage 3 affected the top section of the ear. Hanway [23] notes that a moisture deficiency at stage 3.5 "may seriously reduce the number of kernels that develop."

At stage 3, the stalk is in the middle of the period of rapid growth and elongation which continues into stage 3.5 and 4. The stalk grows by elongation at each successively higher internode until the last internode below the tassel has been fully elongated. Claassen and Shaw [9] observed "that short stress periods had their greatest effect on stalk height late in the vegetative period, i.e. during elongation of the tassel and (or) upper internodes of the stalk." They found that stress reduced the height by as much as 6.4 cm at stage 3.5 and 15.2 cm at stage 4 [9].

The tassel emerges from the leaf whorl at stage 4 (56 DAE) and is almost fully developed [23], [13]. From tassel emergence to its full development and shedding of pollen, it will take approximately ten days at which time vegetative growth is completed [13]. During that time the silks are developing, starting at the base of the top ear at stage 3.5 and at tassel emergence they are elongating rapidly from the base of the ear. Hanway observed that moisture stress at that time "will delay silking more than tassel emergence and pollen shedding [23]."

Also at tassel emergence the ear shoot begins to develop rapidly. The husks and the ear within the husks are the parts that are developing at that point. The husks are visible at the uppermost ear and the ear is about one inch long. Both are beginning rapid growth at that time.

With tassel emergence complete the plant's vegetative growth has practically ceased. Except for the accumulation of labile carbohydrates that will later be translocated to the grain, dry matter accumulation in the leaves, leaf sheathes and stalk has ceased. The plant is ready for grain production which will be discussed in terms of ear development.

Ear Development Phase

The ear development phase begins approximately halfway between tasseling and silking (about 61 DAE or about stage 4.5) and ends at physiological maturity (stage 10). Researchers disagree as to whether the length of that phase is relatively constant or varies. Some researchers [22], [12], [44] have observed relatively constant periods from silking to maturity under different environmental conditions and different planting rates. Yet, Hanway and Russell [24] and Duncan [13] report the length of the ear development phase varying as much as 17 days, and the former reported that the length of that phase had marked effect on yields. Hanway and Russell, and Duncan found that the type of hybrid, different planting dates, and different geographical location affect the length of the ear development phase.

At the beginning of the ear development phase, vegetative growth has practically ceased. The tassel is almost fully emerged and the silks are elongating rapidly and will soon emerge. Also the stalk and leaf sheaths are beginning to accumulate labile carbohydrates that will later be transported to the grain [13], [23]. The accumulation of labile carbohydrates usually occurs after vegetative growth and before the rapid near linear rate of dry matter accumulation in the grain. That period is approximately 17 days long and ends by stage 6. Under normal conditions approximately 20 per cent to 50 per cent of the dry matter in the grain will come from accumulations elsewhere in the plant [13], [8].

By stage 5 (66 DAE) most of the plants are silking and pollen is shedding. An individual plant will shed pollen for about a week [13] while the silks will continue to elongate until they are fertilized [23]. The silks are receptive from 10 days to two weeks or more [13], but most of the pollination occurs in a short period of time. "Extreme heat or drought may damage a high percentage of tassels or environmental stresses may delay silking until after all pollen is shed in some cases, particularly in single cross hybrids or inbreds with high uniformity [13]."

From stage 4.5 through 5 is the most critical period of corn plant development in terms of grain yield. At that time the number of ovules that will be fertilized is determined. Moisture stress at that time may result in poor pollination and seed set. Claassen and Shaw [10] observed a 53 per cent grain yield reduction and significant reductions in kernel numbers from moisture stress at that time. They reported significant reductions in kernel weights from stress during or after silking. These same authors report that others have observed grain yield reduction from 40 per cent to 73 per cent as a result of 4 to 8 days of wilt at silking. Wormley [49] reports research done at Davis, California, where "yields dropped 43 per cent to 88 per cent as the result of water deficits during tasseling and silking. Claassen and Shaw [10] reported a yield reduction of 29 per cent due to a short severe moisture stress when silking was almost completed (97 per cent silking).

The cob is also developing rapidly through that period. During late silking the cob is sensitive to stress [9] and by stage 6 the cob is fully developed.

At stage 6 (78 DAE) the kernels are in the blister stage. They have enlarged considerably but contain very little dry matter. Stage 6 is the beginning of the period of rapid, nearly constant rate of dry matter accumulation in the kernel, which will continue until near maturity, stage 9. Starting with stage 6, almost all the dry matter accumulation is in the grain.

Moisture stress at stage 6 may cause significant reductions in grain yield. Claassen and Shaw [10] observed reduced yields of approximately 30 per cent as a result of a short severe moisture stress at about that time. They also reported that other researchers had found yield reductions of 48 per cent and 22 per cent in single-eared and two-eared varieties for a 4 to 5 day stress during the blister stage.

Moisture stress at stage 6 has several effects that may result in reduced grain yields. The first is that during moisture stress the rate of photosynthesis will be lowered which decreases the rate of dry matter accumulation in the grain. Also moisture stress at that stage and later stages may result in firing or hastened senescence of the lower leaves [9] which may reduce the rate of photosynthesis in later stages. Moisture stress at that time may also reduce translocation of labile carbohydrates to the grain from other plant parts.

Hanway [22], [23] reports the loss of nitrogen and phosphorus from other plant parts to the developing grain begins at stage 6 and continues until physiological maturity. The translocation of labile carbohydrates probably begins at the same time. Therefore "factors that limit translocation (during that period) will also limit grain yield [8]." Boyer found that translocation was less sensitive to water stress than photosynthesis. He reported that dry matter continued to accumulate in the grain from other plant parts when leaf photosynthesis had virtually ceased.

Through both stages 7 (90 DAE) and 8 (102 DAE) dry matter accumulation in the grain is rapid. "Unfavorable conditions (at either stage) will result in unfilled kernels and 'chaffy' ears [23]." At stage 7 the kernels are in the dough stage. Moisture stress at that stage resulted in approximately 30 per cent reduction in yield in tests conducted by Claassen and Shaw [10].

Denting begins with a few kernels at stage 8. By stage 9 (114 DAE) all kernels are fully dented. This "denting is associated with drying of the ear. . .[44]."

At stage 9 the corn plant has almost reached maturity. The embryo in the kernels is morphologically mature. The rate of dry matter accumulation has begun to decline and there will be relatively little increase in grain weight after stage 9.

Physiological maturity occurs at stage 10 (126 DAE). Physiological maturity is understood as *ceasation of dry matter* accumulation [22], [23], [44], [13]; therefore growth is completed. The grain will no longer increase in dry weight, but it will continue to lose moisture. Senescence of the husks and leaves has begun.

Wormley [49] reports that California researchers could not improve yields significantly with irrigations after the blister stage. He also reported that sometimes irrigations after the blister stage (stage 6) actually cut yields. Claasses and Shaw [10] stressed the corn plant after the dough stage (stage 7) in only one year. Their stress treatments after stage 7 either increased the yield over the control (when stressed at stage 8) or had no effect (when stressed at stage 9). Those limited data indicate that sometime after stage 7 water deficits have little or no adverse effect on grain yields and may actually increase yields.

Plant-Soil Moisture Stress

Soil moisture stress takes place when the plant can not withdraw enough water from the soil to meet the demands placed on it by the atmosphere.

The plant obtains its water needs from thin films and wedges that surround the soil particles. It is by these films and wedges that the soil holds water against gravitational forces. In other words, the soil attracts water with a potential energy known as soil-water potential.[2] The maximum amount of water a soil will hold against gravitational forces after it has been saturated is known as field capacity. The soil-water potential at field capacity is about -1/5 to -1/3 bar [29].

The plant can withdraw water from the soil until the soil-water potential reaches approximately -15 bars [29]. When the soil-water potential reaches this level the plant can no longer meet its evaporative needs and looses turgor, or wilts. The soil moisture content at which the plant undergoes complete wilting is known as the permanent wilting point.

The amount of water held in the soil between permanent wilting point and field capacity is the amount of water available for plant uptake, and is called available soil moisture. The amount of available soil moisture present expressed as a percentage of the total possible amount for a particular soil is known as available soil moisture percentage.

The atmosphere places demands on the plant for water described as the evaporative potential of the atmosphere. Some of the factors affecting evaporative potential are solar radiation, temperature, humidity, and wind speed.

The plant meets those demands through transpiration. The plant withdraws water from the soil through the roots. The water travels up the stem and is transpired mostly from the leaves. As the water is withdrawn from the soil, the soil holds the remaining water more tightly, thus

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[2] Soil-water potential has a negative value because its direction is opposite that of the plant. A unit of soil-water potential is commonly called the bar and is equal to 1,000,000 ergs per gram of water [29].

the plant must work harder to extract remaining water and the soil-water potential decreases.

When the soil-water potential obtains approximately -5 bars (available soil moisture percentage is about 35) the leaf-water potential may be approximately -15 to -20 bars. At this point, the plant loses turgor and begins to close its stomata which reduces transpiration. The closing stomata also reduces photosynthesis, thereby reducing the rate of dry matter accumulation and the growth rate. Reduction in the growth rate might seriously affect grain yield depending on its severity and its timing.

CHAPTER III

THE CROP RESPONSE MODEL

The crop response model was developed to simulate corn grain yield response to soil moisture availability throughout the corn plant's development. The model integrated the biological and physical relationships discussed in the previous chapter. To represent the growth and development of the corn plant a growth function was used, which also represented the plant's response to water as the plant developed. To represent the plant's response to various levels of available soil moisture a plant-soil moisture stress function was used.

The crop response model was represented by

$$X_d = [\Gamma(d)]^{\sigma_d} X_{d-1} \qquad d = 1,2,3,\ldots,D'' \qquad (1)$$

where X_d was the state of corn development in period d, X_{d-1} was the state of corn development in the previous period (d-1), $\Gamma(d)$ was the appropriate relative growth rate, σ_d was the plant-soil moisture stress function, and D'' was maturity.

The model was represented by recursive equation because plant growth in any period is a function of accumulated growth in the previous period and the relative growth rate.

The plant-soil moisture stress function was assumed to affect only the relative growth rate (potential for growth in this period) and not previous growth. It was a non-decreasing function being equal to zero at permanent wilting point and one at field capacity. Therefore when the soil was at field capacity there was full growth and when the soil was at permanent wilting point there was no growth.

The final result of the crop response model was a yield index. It indicated the percentage of optimum yield calculated by the model given the soil moisture conditions during the growing season.

Estimation of the Growth Function

The crop response model consisted of two phases that corresponded with the two phases of corn growth. In the development of the model I was particularly interested in the direct effect of water availability on grain yield. I was interested in the effect of water on plant growth only to the extent that it affected grain yield. The latter situation existed in the vegetative growth phase, since it was during that phase that the plant developed its "factory" from which it would produce the grain. During the vegetative period I was primarily interested in those aspects of vegetative growth that affected grain yield. A more direct relationship exists between water availability and grain yield during ear development because nearly all of the dry matter produced then accumulates in the grain and other ear parts. Therefore the crop response model consisted of two phases -- vegetative growth and ear development.

Vegetative Growth

Vegetative growth is a complicated process. It involves the development of a number of different plant parts. Growth of most of those plant parts is initiated at different times. While several may be developing at the same time, they are usually at different stages of development and are developing at different rates. In addition, Claassen and Shaw's [9] observation that "the effects of moisture stress on final-dry-matter yield of each vegetative component was closely related to the coincidence of water deficit with initial and (or) rapid growth phase of the respective period" makes it difficult to estimate the corn plant's response to water deficits during vegetative growth, especially in relation to grain yield.

Fortunately, grain yield is greatly affected by and highly correlated with two measures of vegetative growth -- leaf area and stalk size. Both components provide much of the photosynthetic capacity of the plant. A number of researchers including Eik and Hanway [15], Hanway [18], and Duncan [13] have noted significant relationship between leaf area and grain yield. The size of the stalk is also thought to affect grain yield. Stalk size determines the amount of labile carbohydrates that can be stored in the stalk for later transport to the grain [13].

The development of leaf area and stalk size, as well as the development of the tassel and ear shoots during vegetative growth has led to the conclusion that "grain yield potential at anthesis (pollination) is a function of the previous growth of the plant [13]." To represent previous (vegetative) growth, a vegetative growth function was estimated using a dry matter accumulation curve reported by Hanway [23] (Figure 1).

The portion of the dry matter accumulation curve in Figure 1 from emergence to 60 DAE (just prior to silking) was used in estimating the growth function for the vegetative phase. It was estimated using least squares regression and an exponential equation. The exponential equation used was

$$G_t = e^{\gamma+\delta t} \qquad t = 1,2,3,\ldots,T \qquad (2a)$$

where G_t was dry matter accumulation at day t, t corresponded with the days in Hanway's data, T was the last day of vegetative growth which was 60, and γ and δ were unknown constants to be estimated. By taking a log transformation of (2a), I was able to estimate γ and δ using least squares regression. The least squares estimate for γ and δ was -1.7 and 0.094, respectively, with an R^2 of .947. The growth curve obtained from the estimated function is shown with Hanway's curve in Figure 2.

The vegetative growth function expressed in (2a) is only applicable when the vegetative period is exactly 60 days long. To generalize this function for vegetative growth phases of various lengths, I substituted (T/D)d for t in equation (2a) where d is the day of the vegetative phase, D is the total number of days for a particular vegetative growth phase, and T is the total number of days for the vegetative phase in Hanway's data. This substitution implies that the same growth curve is applicable for all lengths of vegetative phases, only the time scale is changed. The more generalized expression of (2a) is

$$G_d = e^{\gamma+\delta(T/D)d} = e^{\gamma+\delta Td/D} \qquad d = 1,2,3,\ldots,D \qquad (2b)$$

where T = 60.

No attempt was made to model the factors that affect the length of the vegetative growth phase because little information is available on that aspect of corn growth.

Equation (2b) was expressed in recursive form because the recursive form more accurately describes the daily process stated earlier. The recursive form of (2b) is

$$G_d = e^{\gamma+\delta Td/D} = e^{\gamma T/D}\, e^{\gamma+\delta T(d-1)/D} = e^{\delta T/D}\, G_{d-1} \qquad (3)$$

where G_{d-1} is accumulated growth in the previous day and $e^{\delta T/D}$ is relative growth rate.

Equation (3) represents the vegetative growth function when water availability is sufficient so as not to limit growth.

Since it was assumed that soil moisture stress affected only the relative growth rate which in this case is $e^{\delta T/D}$, the relative importance of water to yield as the plant grows from emergence to just prior to silking as indicated by that model can be determined by analyzing $e^{\delta T/D}$.

Since $e^{\delta T/D}$ is constant for a particular vegetative growth phase, the same degree of soil moisture stress will have the same percentage effect on plant growth anywhere in the vegetative phase. In other words, all plants having the same levels of stress at different periods in the vegetative phase would have the same total growth. Presently with the available information, one can not assess the appropriateness of that simplification.

Physiologically the entire period is important. Throughout that time, the plant is initiating the development of different parts of the plant. For example, the leaves are continually developing, but different aspects of leaf development are initiating, rapidly growing and ceasing at different times. The first aspect of leaf development is leaf initialization which is completed by the end of the second week after emergence. Yet potential leaf area is not determined until the sixth week after emergence and the final dry weight of the leaves is not complete until the end of vegetative growth.

The understanding of grain yield response to conditions during the vegetative phase is not well integrated or very complete. Our present understanding is based on test plot experiments that have provided useful but fragmented information. Those experiments provide information that usually deal with only one level of soil moisture deficit or with only a few stages of vegetative growth. Nor is there much information available on the interactions of different periods of water stress during vegetative growth.

Based on the information available the relationship expressed in (3) was considered a reasonable representation of corn growth under favorable soil moisture conditions. To represent that relationship when water availability is less favorable, a plant-soil moisture stress function, σ_d, was added to (3)

$$G_d = [e^{\delta T/D}]^{\sigma_d} G_{d-1} \qquad d = 1,2,3,\ldots,D \qquad (4)$$

The reason for expressing the growth relationship in this manner is the same as it was for equation (1).

The vegetative growth relationship expressed in (4) was the one used in the crop response model as stated in equation (1). At the end of the vegetative period it gives an indication of the state of plant growth at the beginning of ear development.

Ear Development

The ear development phase of the corn crop response model is concerned with the effect of water deficits on grain yield after vegetative growth. During ear development, grain yield is determined by prior vegetative growth, kernel capacity established at pollination and photosynthate available for filling the kernels. Since prior vegetative growth was determined by the vegetative growth phase of the model, it is considered fixed during the ear development phase. Therefore, the ear development phase of the model was concerned with how water availability affects both pollination of the kernels and their subsequent development.

The effects of water stress on grain yields during ear development is much better understood than during vegetative growth. The effects of stress are most intense at the beginning of that period and decrease as the ear develops. With that in mind I developed the following ear development function.

As before, dry matter accumulation was used as an indication of growth. The combined dry matter curves for the cob, silks and grain parts by Hanway [23] were used in estimating the ear development function (See Figure 3).

Dry matter accumulation in those combined parts first increases at an increasing rate, then increases at a decreasing rate as the ear matures, and ceases at maturity. That process can best be represented by a modified logistic equation. While the growth rate declines as growth approaches some maximum for a logistic curve, the situation here is one where the growth rate declines as the ear nears maturity date. That type of growth is shown in the following equation:

$$\frac{dH_{t'}}{dt'} = \delta' H_{t'} \left(\frac{T' - t'}{T'}\right) \qquad t' = 1,2,3,\ldots,66 \qquad (5a)$$

where t' is days into the ear development phase, $H_{t'}$ is accumulated ear development at day t', T' is total number of days in ear development phase and δ' *is a constant.*

To obtain my modified logistic equation I rewrote (5a) in the form of partial fractions

$$\frac{dH_{t'}}{H_{t'}} = \delta'\left(\frac{T' - t'}{T'}\right) dt' = \delta' dt' - \delta'\frac{t'}{T'} dt' \tag{5b}$$

and integrated both sides to obtain

$$\log H_{t'} = \delta' t' - \frac{\delta' t'^2}{2T'} + C \tag{6a}$$

where C is the constant of integration. Then I rewrote (6a) as

$$\log H_{t'} = C + \delta' \left(t' - \frac{t'^2}{2T'}\right) \tag{6b}$$

and took the exponential to obtain

$$H_{t'} = e^{C+\delta'(t'-t'^2/2T')} \tag{7}$$

where C and δ' are constants to be estimated.

Equation (6b) was estimated by least squares to obtain -3.573 for C and 0.109 for δ'. The coefficient of determination for the estimate was .994. The estimated curve and Hanway's curve are shown in Figure 3.

That estimate as expressed in (7) represents ear development when that phase is exactly 66 days long. To generalize (7) for ear development phases of various lengths I substituted $(T'/D')d'$ for t' in (7) to obtain

$$H_{d'} = e^{C + \delta'(T'd'/D'-(T'd'/D')^2/2T')} \tag{8a}$$

where $T' = 66$, D' is the length of ear development phase under study and d' represents the day in that ear development phase.

Since there were indications that the length of ear development period and grain yield were related I added another term to represent that relationship. That term was the ratio of D' over T'. The product of that ratio and the ear development function at D' gave the level of ear development at maturity.

Again as with vegetative growth I used the recursive form of (8a) since the recursive form more accurately describes the daily process. The basic steps I followed in the development of the recursive form of (8a) were

$$H_{d'} = e^{C}\, e^{\delta' T' d'/D' \;-\; \delta' T' d'^{2}/2D'^{2}} \qquad (8b)$$

$$H_{d'} = e^{C}\, e^{\delta' T'((d'-1)+1)/D' \;-\; \delta' T'((d'-1)+1)^{2}/2D'^{2}}$$

$$H_{d'} = e^{C}\, e^{\delta' T'(d'-1)/D'+\delta' T'/D'-\delta' T'(d'-1)^{2}/2D'^{2}}$$

$$e^{-2\delta' T' d'/2D'^{2}+\delta' T'/2D'^{2}}$$

$$H_{d'} = e^{C}\, e^{\delta' T'(d'-1)/D'-\delta' T'(d'-1)^{2}/2D'^{2}}$$

$$e^{\delta' T'/D'-2\delta' T' d'/2D'^{2}+\delta' T'/2D'^{2}}$$

$$H_{d'} = e^{(\delta' T'/D')(1+1/(2D'))}\; e^{-\delta' T' d'/D'^{2}}\; H_{d'-1} \qquad (9)$$

Equation (9) is the ear development function under favorable conditions. When soil water conditions were less than favorable, the resulting stress decreased the relative growth rate, $e^{(\delta' T'/D')(1+1/2D')}e^{-\delta' T' d'/D'^{2}}$.

Unlike the vegetative growth function, relative growth in the ear development function was not constant, but changed over time. The relative growth rate decreased as the ear matured because the second term of the relative growth rate, $e^{-\delta' T' d'/D'^2}$, decreased as d' increased while the first term, $e^{(\delta' T'/D')(1+1/2D')}$, remained constant.

The decreasing rate of relative growth indicates that the availability of water to the plant was most critical to grain yield at the beginning of that period. It also indicates that the importance of water decreased throughout that period until maturity when its availability is no longer crucial to grain yield.

Those properties closely agree with the previous discussion of the ear development period and the present understanding of grain yield response to water during that period.

To represent the effect of soil moisture availability on relative growth I used the plant-soil moisture stress function, $\sigma_{d'}$, similarly as I did in the vegetative growth stage. The ear development stage of the model was represented by

$$H_{d'} = \left[e^{(\delta' T'/D')(1+1/2D')} \quad e^{-\delta' T' d'/D'^2} \right]^{\sigma_{d'}} H_{d'-1} \qquad (10)$$

where $d' = 1,2,3,\ldots,D'$.

Equations (4) and (10) represent vegetative growth and ear development, respectively. Interfacing those two equations essentially produced the corn crop response model as stated in equation (1).

The vegetative growth relationship at day D represented the yield potential of the corn plant at that time, just prior to anthesis. To represent that mathematically I used the ratio of (4) at day D over (3) at day D. In other words the ratio of actual vegetative growth over potential growth under favorable conditions.

That ratio was then multiplied by the intercept of the ear development function which was then used for $H_{d'-1}$ when d'=1.

Estimation of the Plant-Soil Moisture Stress Function

The corn crop response model in the previous section was complete except for knowing the correct functional form of the plant-soil moisture stress function, σ_d. In reviewing the pertinent literature little information concerning the form of σ_d was found. Several possible forms were assumed and tested including one proposed by Jensen [26]. None of those assumed forms were satisfactory over the range of my test data, therefore I estimated an equation for σ_d.

Since there was no observable daily data from which to estimate the relationship of available soil moisture percentage and plant-soil moisture stress, I estimated the plant-soil moisture stress function implicitly through yield response. The crop response model gives yield response to available soil moisture by the interaction of the growth function and the stress function. Since the estimated forms in the previous section were assumed to be the appropriate growth functions, then the plant-soil moisture stress function is the only unknown and can be estimated if its functional form is assumed or known and data containing daily available soil moistures and corresponding grain yields are available.

The appropriate functional form for σ_d was not known, but it was assumed to be a nondecreasing function with a range of zero (at permanent wilting point) and one (at field capacity). Therefore I assumed that σ_d was a piece wise linear function of available soil moisture percentage, AM_d, which would allow the estimation of the unknown functional form by approximating it with linear estimates of each of a number of segments over the whole range. By forcing the adjacent segments to intersect, I obtained what is known as a spline function.

I assumed that three intervals or "pieces" would be sufficient to indicate the shape of the σ_d. Therefore σ_d would be the following:

$$\sigma_d = \begin{cases} a_1+b_1AM_d & 0.000 \leq AM_d \leq 0.333 \\ a_2+b_2AM_d & 0.333 \leq AM_d \leq 0.667 \\ a_3+b_3AM_d & 0.667 \leq AM_d \leq 1.000 \end{cases} \qquad (11a)$$

where the a's and b's were unknown coefficients. To generalize the notation i was used for the subscript of a and b. The value of i would then correspond to the interval that a and b represented.

$$\sigma_d = a_i+b_iAM_d \qquad \text{for } i = 1,2,3 \qquad (11b)$$

In estimating (11b), AM_d, the available soil moisture, was the independent variable. The dependent variable was grain yield. Therefore the crop response model in (1) was expressed relative to final grain yield, which was

$$X'_{D''} = (D'/T')[\Gamma(D'')]^{\sigma_{D''}} X_{D''-1} \qquad (12a)$$

where $X'_{D''}$ was final grain yield.

Since linear least squares regression was used to estimate that relationship, (12a) needed to be expressed in linear form. In order to get the model in linear form I first expressed the recursive equation (12a) in product form,

$$X'_{D''} = (D'/T') \prod_{d=1}^{D''} [\Gamma(d)]^{\sigma_d} X_0 \qquad (12b)$$

where X_0 was plant development at the beginning of the period, i.e. plant development at emergence. Then I divided both sides by (D'/T') and took the log transformation of it to obtain

$$\text{Log}(X'_{D''}T'/D') = \sum_{d=1}^{D''} \sigma_d \text{Log}\Gamma(d) + \text{Log}X_0 \tag{12c}$$

Even though (12c) was a linear equation it was not in a form that could have been estimated using least squares regression. In order to get (12c) into the needed form I first substituted the assumed form of σ_d, (11b), into (12c) to get

$$\text{Log}(X'_{D''}T'/D') = \sum_{d=1}^{D''} (a_i+b_i AM_d)\text{Log}\Gamma(d) + \text{Log}X_0 \tag{13a}$$

Then I expanded (13a),

$$\text{Log}(X'_{D''}T'/D') = \sum_{d=1}^{D''} a_i \text{Log}\Gamma(d) + \sum_{d=1}^{D''} b_i AM_d \text{Log}\Gamma(d) + \text{Log}X_0 \tag{13b}$$

and rearranged constants and terms to obtain

$$\text{Log}(X'_{D''}T'/D') = \text{Log}X_0 + a_i \sum_{d=1}^{D''} \text{Log}\Gamma(d) + b_i \sum_{d=1}^{D''} AM_d \text{Log}\Gamma(d) \tag{13c}$$

Equation (13c) expressed the model in a form so that σ_d could be estimated using linear regression. The regression equation that represented (13c) was

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 \tag{13d}$$

where $Y = \text{Log}(X'_{D''}T'/D')$; each odd numbered X_j corresponded to a $\sum_{d=1}^{D''} \text{Log}\Gamma(d)$ for each segment, each even numbered X_j corresponded to a $\sum_{d-1}^{D''} AM_d \text{Log}\Gamma(d)$ for each segment, $\beta_0 = \text{Log}X_0$, each odd numbered β_j

corresponded with each a_i and each even numbered β_j corresponded to each b_i.

If equation (13d) was estimated without restrictions on the coefficients, the end points of each of the segments would not necessarily join the end points of the segment on either side of it. If those end points did not join then it would be a discontinuous function, but I felt that a continuous function would be more appropriate.

Therefore to estimate σ_d I estimated the regression equation (13d) subject to linear side conditions that forced the end points of adjacent segments to intersect and that restricted σ_d to zero at permanent wilting point and to equal one at field capacity. Those linear side conditions follow:

$$\beta_1 = 0 \tag{14a}$$

$$\beta_1 + 0.333\ \beta_2 = \beta_3 + 0.333\ \beta_4 \tag{14b}$$

$$\beta_3 + 0.667\ \beta_4 = \beta_5 + .0667\ \beta_6 \tag{14c}$$

$$\beta_5 + 1.000\ \beta_6 = 1.000 \tag{14d}$$

Those side conditions were constraints on the β coefficient. The first side condition (14a) constrains β_1 which is the y-intercept of the first segment. The slope of that segment was given by β_2. Setting β_1 equal to zero forces σ_d to equal zero at zero available soil moisture.

The next two side conditions, (14b) and (14c), force the join points to equal. To get the join points to equal, I set the segments equal to each other at the points where both segments cover the same available soil moisture level. Since there were three segments, there were two points of intersection. Those points of intersection were where AM_d equaled 0.333

and 0.667. Those points of intersection correspond to the intersection of segments one and two, two and three, respectively.

Likewise to get the end point of the last segment to equal one, I set that segment equal to one when AM_d equals one.

In order to estimate (13d) subject to (14a) through (14d), I used those constraints in expressing three of the β coefficients as linear combinations of the remaining three β coefficients which were then estimated using least squares regression.

The first step in expressing three of the coefficients as linear combinations of the other three was to drop out β_1 since it was zero and to add β_0. I then wrote (14b) through (14c) as a system of linear equations,

$$\begin{aligned} 0.333\beta_2 - \beta_3 - 0.333\beta_4 \qquad\qquad\qquad &= 0 \\ \beta_3 + 0.667\beta_4 - \beta_5 - 0.667\beta_6 &= 0 \\ \beta_5 + \qquad \beta_6 &= 1 \end{aligned} \tag{15}$$

I then expressed the system of equations in (15) in matrix form, so that I could use matrix algebra in the following transformations. The matrix equation of (15) was

$$\left[\begin{array}{ccc|ccc} 0 & .333 & -1 & -.333 & 0 & 0 \\ 0 & 0 & 1 & .667 & -1 & -.667 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right] \left[\begin{array}{c} \beta_0 \\ \beta_2 \\ \beta_3 \\ \hline \beta_4 \\ \beta_5 \\ \beta_6 \end{array}\right] = \left[\begin{array}{c} 0 \\ 0 \\ 1 \end{array}\right] \tag{16a}$$

or

$$\overline{B}\ \overline{\beta} = \overline{V} \tag{16b}$$

To express three of the coefficients as linear combinations of the other three, I partitioned both $\overline{B}$ and $\overline{\beta}$ as shown in (16a) into $\overline{B}_1$, $\overline{B}_2$, $\overline{\beta}_1$, and $\overline{\beta}_2$ respectively. After partitioning $\overline{B}$ and $\overline{\beta}$, (16b) was also represented by

$$\left[\overline{B}_1 \;\vdots\; \overline{B}_2\right] \begin{bmatrix} \overline{\beta}_1 \\ \hline \overline{\beta}_2 \end{bmatrix} = \overline{V} \tag{16c}$$

I then expressed $\overline{\beta}_2$ as linear combination of $\overline{\beta}_1$, $\overline{B}_1$, $\overline{B}_2$, and $\overline{V}$, by solving (16c) for $\overline{\beta}$ to obtain

$$\overline{\beta}_2 = \overline{B}_2^{-1}\,\overline{V} - \overline{B}_2^{-1}\,\overline{B}_1\,\overline{\beta}_1 \tag{17}$$

Next I substituted (17) into $\overline{\beta}$ for $\overline{\beta}_2$ to get

$$\overline{\beta} = \begin{bmatrix} \overline{\beta}_1 \\ \hline \overline{\beta}_2 \end{bmatrix} = \begin{bmatrix} \overline{\beta}_1 \\ \hline \overline{B}_2^{-1}\,\overline{V} - \overline{B}_2^{-1}\,\overline{B}_1\,\overline{\beta}_1 \end{bmatrix} = \begin{bmatrix} \overline{0} \\ \hline \overline{B}_2^{-1}\,\overline{V} \end{bmatrix} + \begin{bmatrix} \overline{I} \\ \hline -\overline{B}_2^{-1}\,\overline{B}_1 \end{bmatrix} \overline{\beta}_1 \tag{18}$$

By so doing I had expressed the coefficients represented by $\overline{\beta}_2$ as a linear combination of the coefficients represented by $\overline{\beta}_1$.

With $\overline{\beta}_2$ expressed as a linear combination of $\overline{\beta}_1$ in equation (18) I substituted (18) for the coefficients in the regression equation. The regression equation that I substituted (18) into was a simplified expression of (13d). It was obtained by droping out $\beta_1 X_1$, since β_1 was set equal to zero in (14a). That regression equation was

$$Y = \overline{\beta}\,\overline{X} \tag{19a}$$

where $\overline{X} = \begin{bmatrix} X_0 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix}$

I then substituted (18) into (19a) to obtain

$$\overline{Y} = \overline{X}\left(\begin{bmatrix} \overline{0} \\ \hline \overline{B}_2^{-1}\ \overline{V} \end{bmatrix} + \begin{bmatrix} \overline{I} \\ \hline -\overline{B}_2^{-1}\ \overline{B}_1 \end{bmatrix} \overline{\beta}_1 \right) \qquad (19b)$$

In review, equation (19b) was a modification of the original regression equation (13d) where one coefficient was dropped and some of the coefficients had been expressed as linear combinations of the others using linear side conditions that were placed on the coefficient to restrict their values. The equation in (20b) was restricted so that it would start at zero and end at one with all segments intersecting at the join points.

Even though the coefficients in (19b) were expressed as linear combinations of $\overline{\beta}_1$, the equation in (19b) was no longer in the form of a linear regression equation. To get (19b) into the form of a linear regression equation again I transformed (20b) into the following equivalent expressions:

$$\overline{Y}-\overline{X} \begin{bmatrix} \overline{0} \\ \hline \overline{B}_2^{-1}\ \overline{V} \end{bmatrix} = \overline{X} \begin{bmatrix} \overline{I} \\ \hline -B_2^{-1}-B_1 \end{bmatrix} \overline{\beta}_1 \qquad (20a)$$

or

$$\overline{W} = \overline{\beta}_1\ \overline{R} \qquad (20b)$$

where $\bar{W} = \bar{Y} - \bar{X} \begin{bmatrix} \bar{0} \\ \hline \bar{B}_2^{-1}\,\bar{V} \end{bmatrix}$ represented the dependent variable, $\bar{\beta}_1$ represented the coefficients to be estimated, and $\bar{R} = \bar{X} \begin{bmatrix} \bar{I} \\ \hline -\bar{B}_2^{-1}\,\bar{B}_1 \end{bmatrix}$ represented the independent variables.

The regression equation (20b) had only three coefficients to be estimated compared to seven in (13d). That was accomplished by setting one of them equal to zero and expressing three of them as linear combinations of the other three. Reducing the number of coefficients needed to be estimated to three resulted in increasing the degrees of freedom of the estimate by four.

To estimate σ_d using the spline function in (11a) and the crop response model in (12a) I used the regression equation in (20b) and the data described in the following section.

Data

The crop data were collected from test plots at two different locations, Manhattan, and Scandia Agricultural Experiment Fields over three years, 1974, 1975, and 1976. The Manhattan Experiment Field is located south of Manhattan, Kansas in the southwestern corner of northeast Kansas. The Scandia Experiment Field is located in the northcentral part of Kansas. There are considerable differences in the soil types and environmental conditions between these locations.

There were five treatments at Manhattan in 1974 and 1975. The first treatment received no irrigation. The next three treatments received one irrigation each. Each of these irrigations were at different times, tasseling, silking, and dough stage. The last treatment consisted of three irrigations, one at each of the above irrigation times.

For the Manhattan 1976 tests three different planting dates with no irrigation were used.

The Scandia data were for the years 1974 and 1975. There the plots were irrigated when soil moisture fell to a certain level. For each year there were three different treatments one at each of three different levels of soil moisture.

The data from those test plots needed to estimate σ_d were daily available soil moisture and grain yields. The data from those test plots included the amounts and dates of irrigations and precipitation as well as planting dates, soil moisture at the beginning of the season, soil type information, and grain yields. The dates of each of the different stages of development and leaf area at each day were available for most of the plots. Temperature and solar radiation information were obtained from the closest weather station. With that information I was able to generate daily available soil moisture percentage, AM_d, for each of these plots by using a model developed for that purpose by Edward T. Kanemasu at the Evapotranspiration Laboratory at Kansas State University. The daily soil moisture series generated for each test plot are presented in Figures 4 through 8.

The soil moisture was at or near field capacity during vegetative growth in most of the plots, which was due to environmental conditions present during those years. That is, enough rain was received to keep the soil profile near field capacity during vegetative growth. As a result those data lack available soil moisture levels over most of the possible range during vegetative growth. That would have made estimation of σ_d for that period over the whole range difficult. The accuracy of such an estimate would be dubious at best.

In contrast, the data for the ear development phase contained soil moisture levels over most of the range.

Since there was not a sufficient range of soil moisture values during vegetative growth but there were during ear development I estimated σ_d for the latter period and assumed that that same relationship would hold true during vegetative growth. The available soil moisture series for the ear development phase for each test plot are presented in Figures 9 through 13.

Results

The results I obtained from the regression equation in (20b) using the data presented in Figures 9 through 13 were the estimated values for the coefficients in $\overline{\beta}_2$. To obtain the estimated value for all the coefficients in $\overline{\beta}$, I solved for $\overline{\beta}_1$ using equation (17).

The estimated values I obtained for $\overline{\beta}$ were the following:

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{bmatrix} = \begin{bmatrix} 1.6494 \\ 2.4641 \\ 0.7551 \\ 0.1989 \\ 0.66299 \\ 0.337 \end{bmatrix} \tag{21}$$

The coefficient of determination for the estimate was 0.81.

To obtain the spline function estimated to represent σ_d from (21), I recalled the description of each of the β coefficients from (13d) and (11a). The interpretation of the results in (21) follow:

$\text{Log } X_0 = 1.6494$

$$\sigma_d = \begin{cases} 2.4641\ AM_d & 0.000 \leq AM_d \leq 0.333 \\ 0.7551 + 0.1989\ AM_d & 0.333 \leq AM_d \leq 0.667 \\ 0.66299 + 0.337\ AM_d & 0.667 \leq AM_d \leq 1.999 \end{cases} \qquad (22)$$

The plot of (22) is presented in Figure 14.

The estimate of σ_d in (21) had all the expected properties discussed earlier. The estimate was a nondecreasing function with a range between zero and one. In other words, as available soil moisture increased the value of σ_d also increased. Biologically that meant that as water became more available for plant uptake, the growth rate increased, because more of the potential growth rate was realized.

To illustrate the appropriateness of that estimate I substituted (22) into (1) and simulated the crop response model. I made two different simulations of the model. The first simulation was just for the ear development phase and the second was for the whole crop response model. The results of those two simulations are presented in Tables 1 and 2 and Figures 15 and 16.

Since the final result of the crop response model is a yield index instead of grain yield in bushels per acre, I multiplied the yield index for each plot by the quotient of the sum of the yields for all the test plots and the sum of the yield indices for all the test plots. That product was the yield estimate by the crop response model for each test plot[3].

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[3] In other words,

$$\text{ESTIMATED YIELD} = X_{D''} \sum_{j=1}^{J} \text{ACTUAL YIELD}_j \; \sum_{j=1}^{J} X_{jD''}$$

where j = 1,2,3,...J and j refers to a particular test plot in a particular year.

To determine the statistical relationship between the actual yield and the yield estimated by the crop response model I used regression analysis. I used actual yield as the independent variable and the estimated yield as the dependent variable. For each simulation I determined two different regressions. The first was for the Manhattan data alone and the second was using both the Manhattan and the Scandia data. The results for the first simulation in which I used only the ear development portion of the model were

$$y = 35.18 + 0.76X$$

with an R^2 of 0.85 for the Manhattan data and

$$y = 45.79 + 0.45X$$

with an R^2 of 0.48 for the Manhattan and Scandia data. The results of the regression analysis of the second simulation were

$$y = 22.85 + 0.89X$$

with an R^2 of 0.83 using the data from Manhattan and

$$y = 39.62 + 0.60X$$

with an R^2 of 0.46 for the data from Manhattan and Scandia.

The coefficients of determination indicate that the model does a pretty good job of estimating the yield for the Manhattan data. They also indicate that when the Scandia data is added, it does not perform nearly as well.

The slope coefficient indicates whether the model was doing an equally good job of estimating yield over the whole range of actual yields. If the model was doing equally well over the whole range then the slope coefficient would have been one. The best results in that respect were obtained for the Manhattan data using the whole crop response model. All the slope coefficients indicate that the model was overestimating the low yields and underestimating the high ones.

The other coefficient does not mean much unless the slope is close to one. In that case, if it was different from zero it would then indicate that on the average the model was over- or underestimating by the amount of the coefficient.

It appears that either there are problems with the Scandia data or that there are differences in the locations that the model did not account for. That indicates that closer analysis of the data collected at the Scandia site is needed to determine its accuracy. More good data is also needed to help in fuller testing the model's accuracy.

In spite of the problems with the Scandia data the model did do a good job of estimating yields over three years of data from the Manhattan location.

I think that the good estimates obtained for the Manhattan location indicate considerable promise for the methodology represented in the crop response model. Therefore I proceeded to cast it into a dynamic programming framework in order to obtain a decision model for optimizing irrigation schedules.

CHAPTER IV

ECONOMIC IRRIGATION SCHEDULING

In developing the following economic irrigation scheduling model, I used the crop response model developed in the previous chapter. To it I added a water balance equation and a criterion function. I then cast these components into a dynamic programming framework, which resulted in a multiperiod decision model.

Dynamic Programming

Since economic irrigation scheduling involves the optimization of many interrelated decision periods, dynamic programming was chosen as the method of optimization. Dynamic programming is an approach to problem solving that is characterized by the task of finding a sequence of decisions that optimizes an appropriately defined criterion function. That approach to problem solving is based on The Principle of Optimality as set forth by Bellman [4].

> An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

As as result of that principle, I was able to set my model up in a series of 18 periods, each a week long and to solve it by starting with the terminal period and moving backwards to the initial period. For each period there were two state variables, one decision variable, one return variable, and two transformation functions. The state variables represent the condition or state of the process. In the scheduling model, the two state variables were the state of crop development and the state of the available soil moisture. The decision variable controls the state of the process for a particular period. The decision variable for the irrigation scheduling model was the amount of irrigation water delivered to the soil. The return variable gives the utility or return associated with a given decision. The return for a particular period is obtained from the criterion function. The transformation functions are used to express the state of the process as a function of the state of the process in the previous period and the decisions in this period. There is a transformation function for each state variable. In the scheduling model the crop response model and a water balance equation were used as the transformation functions for the state of crop development and the state of available soil moisture respectively.

Criterion Function

Since my chief concern in this model was the scheduling of irrigations, I chose a criterion function that maximizes total revenue net of irrigation costs. In that criterion function I assumed that decisions concerning seeding, fertilizing, and other cultural practices have been decided in advance, and therefore not included. The criterion function was expressed as

$$\text{Maximize } V = P_c X'_{D''} - \sum_{d=1}^{D''} h(I_d) \tag{23}$$

where P_c was the price of corn per bushel, $X'_{D''}$ was the harvestable grain yield in bushels per acre, and $h(I_d)$ was the cost of irrigation per acre in period d. The cost of irrigation was calculated as follows:

$$h(I_d) = C_I I_d + S \tag{24a}$$

or

$$h(I_d) = 0 \tag{24b}$$

where (24a) was the cost of irrigation when the decision was to irrigate and where (24b) was the cost associated with the decision not to irrigate. C_I was the cost of applying one acre inch of water and S was the set up charge for each irrigation. Harvestable grain yield, $X'_{D''}$, was obtained by multiplying the yield index obtained from the crop response model by the optimum harvestable grain yield.

Water Balance Equation

A water balance equation was used to calculate available soil moisture percentage, AM_d, which was the independent variable in the crop response model. In the scheduling model, AM_d was no longer the independent variable, but it was still necessary for the calculation of plant development. AM_d was calculated using the independent variable in the scheduling model which was the amount of irrigation water applied to the soil, I_d.

The water balance equation used to calculate AM_d was

$$AM_d = \frac{W_d \times 100}{Z_d} = \frac{(W_{d-1} - E_{d-1} - D_{d-1} - RO_{d-1} - R_{d-1} - I_{d-1}) \times 100}{Z_d} \tag{25a}$$

where Z_d was the maximum amount of available water a soil was capable of holding in the root zone, and W_d was the amount of available water actually held in period d, which was the amount that was held in period d-1 minus the evapotranspiration, E_{d-1}, minus drainage, D_{d-1}, minus run off, RO_{d-1}, plus the amount entering through rainfall, R_{d-1}, and the amount entering through irrigation, I_{d-1} [6].

I simplified (25a) by considering only the water that entered the root zone and I assumed that deep percolation did not occur. I therefore dropped D_{d-1} and RO_{d-1} from (25a). I further simplified (25a) by assuming that no rain would occur during the whole growing season, $R_d = 0$ for all d. In the more arid parts of western United States that is a reasonable assumption.

After those simplifications the water balance equation used to calculate AM_d was

$$AM_d = \frac{W_d \times 100}{Z_d} = \frac{(W_{d-1} - E_{d-1} + I_{d-1}) \times 100}{Z_d} \qquad (25b)$$

To calculate evapotranspiration, E_d, the following equation, proposed by Jensen [26], was used:

$$E_d = (\alpha_d \beta_d + \gamma_d) E_d^* \qquad (26)$$

where E_d^*, the evaporative potential of the atmosphere, was estimated from meteorological data, β_d was the crop coefficient, γ_d was surface evaporation coefficient, and α_d indicated the plant's ability to absorb water as soil moisture stress varies. α_d was one at field capacity and zero at permanent wilting point. For α_d I used the following functional form, as proposed by Jensen:

$$\alpha_d = Ln(AM_d + 1)/Ln(101) \qquad (27)$$

The crop coefficient, β_d, indicated the relationship between plant size and evapotranspiration. Empirical data from 1970 tests at Scandia, Kansas were used for β_d,

The amount of surface evaporation which occurs following the wetting of the soil surface upon receipt of new soil moisture was indicated by γ_d. Jensen proposed that γ_d for the first, second, and third day after a rain or irrigation is $(0.9-\alpha_d\beta_d)0.8$, $(0.9-\alpha_d\beta_d)0.5$, and $(0.9-\alpha_d\beta_d)0.3$, respectively. Starting with the fourth day after a rain or irrigation, γ_d was assumed to be zero. It was also constrained to be less than or equal to the added soil moisture.

Scheduling Model

Using the optimality principle of dynamic programming, I decomposed the crop response model into a series of recursive equations starting at the end of the process. I let $g_n(X_n,AM_n)$ be the return associated with only the last N-n+1 periods and $f_n(X_n,AM_n)$ be the return associated with the current period where n refered to a particular period, N was the maximum number of periods, X_n was the state of crop development at period n and AM_n was the state of the available soil moisture. For the last period the return was

$$g_N(X_N,AM_N) = \max_{I_n} (P_c X_N' - f_n(X_n,AM_n)) \tag{28a}$$

where I_n was the decision maximized, P_c was the price of corn per bushel, and X_N' was the harvestable grain yield in bushels per acre.

For any other period the recursive equation of the model was

$$g_n(X_n,AM_n) = \max_{I_n} (g_{n-1}(X_{n+1},AM_{n+1}) - f_n(X_n,AM_n)) \tag{28b}$$

The equations in (28a) and (28b) represented the crop response model, the criterion function and the water balance equation in the dynamic programming framework. The economic irrigation scheduling model consisted of (28a) and (28b) subject to (1), (12a), (22), (23), (24a), (24b), (25b), (26), and (27).

Results

The model was simulated for two different irrigation costs. The costs used represented two different extremes in irrigation costs. The low price chosen was $1.24 per acre inch at the well head or $2.06 per acre inch delivered to the soil, assuming 60 per cent efficiency. The high price chosen was $7.14 per acre inch at the well head or $11.90 per acre inch delivered to the soil.

The other two parameters were held constant. The optimum harvestable grain yield was specified at 140 bushels per acre and the price received per bushel was $2.00.

In determining both schedules, it was assumed that the soil was at field capacity at the beginning of the growing season. To account for the cost of applying the water that would be needed to bring the field from permanent wilting point to field capacity, one would have to add $20.60 to the cost of pumping for the lower pumping cost schedule and $119.00 to the higher pumping cost schedule. Those schedules are presented in Table 3.

The amount of irrigation water applied during the different periods was affected by two factors that should be considered when analyzing the schedules. The first was that the water requirements of the plant were not as great during the early periods when the plant was small than at later stages when the plant was larger. Therefore its demands could be met with a smaller amount of water. The second factor was that during vegetative growth the roots were occupying an ever increasing volume of soil. Therefore as the plant would use up the water in the volume of soil occupied by its roots in the previous stage, it would help meet its own water needs by occupying additional soil volume. In contrast, that did not occur during ear development since the roots had ceased all major growth by silking.

The schedules in Table 3 indicate that the early ear development and vegetative growth were given the highest priority, followed by late ear development.

Increasing the cost of irrigation had a significant impact on the economic schedules. The costs per acre inch delivered to the soil was increased from $2.06 to $11.90. That resulted in decreasing the amount of water applied by six inches. The periods that were affected by that decrease were 13, 14, 15, and 16. The yield was reduced by 23.8 bushels per acre and the total revenue net of irrigation costs was reduced by $165.13.

Those results indicated that the economic irrigation scheduling model not only responded to a significant change in the cost of irrigation but also responded in a realistic way. As the cost of the input increased, the amount used was decreased, but more significantly, the amount of water applied was decreased where it would contribute the least amount to yield or gross revenue. As reported in chapter two, grain yield is least sensitive to lower levels of soil moisture during the latter part of ear development.

CHAPTER V

CONCLUSION

This study resulted in the development of both a crop response model and an economic irrigation scheduling model for corn.

The crop response model simulated grain yield response to the availability of soil moisture. The model consisted of two major components. The first component was a growth function and the second was a plant-soil moisture stress function.

The growth function represented the effect of the interrelationship of both the stage of plant development and plant-soil moisture stress on grain yield potential. The growth function was divided into two phases corresponding to the two phases of corn development -- vegetative growth and ear development. They were estimated using dry matter accumulation data, least squares regression, and an exponential equation for the vegetative growth phase and a modified logistic equation for the ear development phase.

The plant-soil moisture stress function indicated the relationship between available soil moisture percentage and plant-soil moisture stress. Since there was no observable daily data on that relationship, it was estimated implicitly through grain yield response using the estimated growth functions, least squares regression, a piece wise linear or spline function, and data containing daily available soil moisture and corresponding grain yields from Manhattan and Scandia, Kansas test fields during 1974, 1975, and 1976.

The crop response model was simulated using the test data. It did particularly well when considering only the data from the Manhattan location, but did not perform nearly as well on the data from the Scandia location. I concluded that either there were some inaccuracies in the Scandia data or there were some factors at the Scandia location that the model did not account for that were relatively important.

In comparing this model with similar models, I found it worthy of further consideration. I would suggest several avenues for future investigation of the crop response model. The first would be a check on the Scandia data as to its accuracy. The next would be to obtain more data for further testing, especially data from other regions. I would also like to see the model expanded to take into account factors that affect maturity and the length of the various stages. More testing and research on the effect of the length of the growth phases on yield is recommended. Another useful direction would be to develop crop response models for other crops, using the same basic methodology. Crop response models for wheat and grain sorghum would be particularly useful since they are the other major cultivated crops grown on irrigated land in the Great Plains.

The economic irrigation scheduling model determined the optimum schedule for the irrigation of corn based on the criterion of the maximization of total revenue net of irrigation costs. The scheduling model used the crop response model, a water balance equation, and a criterion function. They were cast into a dynamic programming framework which resulted in a multiperiod decision model.

The scheduling model was solved for two different irrigation cost levels. The two costs were representative of the two extremes in irrigation costs. The resulting schedules were significantly different. The amount of water applied in the higher irrigation cost simulation was lower during the later stages of ear development. The total revenue net of irrigation costs was also lower for the higher irrigation cost simulation.

The accuracy of the scheduling model was hard to determine. To adequately test the model, field tests need to be conducted. That was not possible during this study, but could be a possibility for further investigation.

Since intensive empirical testing was not yet possible, I compared the schedules produced by the model to the present understanding of corn's response to water availability. It was apparent from that type of analysis that the model had done reasonably well in both scheduling irrigations and responding to increased prices.

Another test as to the model's accuracy would be to analyze the accuracy of its major components, since the scheduling model's accuracy will be largely dependent on the components' accuracy. That would involve the testing of the crop response model and the water balance equation. The accuracy of the crop response model has already been discussed, therefore leaving the water balance equation to be tested.

The water balance equation used in the scheduling model was based on the work of Biere [6] and Jensen [26]. I assumed that the equation would be appropriate for my own model. Further research into its appropriateness for my model would therefore be useful. I suggest that the evapotranspiration function, the crop coefficient, and root capacity be given particular attention.

The two models presented here offer considerable potential in the area of water resource allocation. They can be used in analyzing both micro and macro aspects of the problem. They can also be used in planning policies as well as analyzing policies that already exist.

The models' major strength, particularly the crop response model, is that they are based on actual data. The functions used in the models were estimated from actual data rather than synthesized or merely assumed.

APPENDIX

Table 1. Actual Grain Yield Versus Grain Yield Estimated by the Ear Development Phase of the Crop Response Model.

Location	Year	Plot	Actual[1] Yield	Estimated[1] Yield	State of[2] Growth at Maturity	Average[2] Available Moisture
Manhattan	1974	1	45.90	75.514	44.933	33.6
		2	106.30	114.415	68.080	56.2
		3	112.80	114.365	68.050	58.1
		4	70.20	107.612	64.031	54.0
		5	146.10	157.132	93.497	96.2
	1975	1	95.80	96.645	58.498	46.5
		2	105.20	121.685	73.654	62.8
		3	134.90	129.749	78.535	72.9
		4	103.00	116.215	70.343	65.7
		5	153.80	153.030	92.626	92.6
	1976	1	45.20	76.441	70.620	57.0
		2	56.60	80.629	65.828	49.6
		3	62.80	54.265	37.354	27.5
Scandia	1974	1	143.80	87.106	67.955	58.1
		2	131.50	82.557	64.407	48.8
		3	40.80	22.897	17.863	21.4
	1975	1	164.63	107.891	71.465	67.9
		2	122.28	100.970	66.881	60.2
		3	48.37	90.668	60.057	58.3

[1]In bushels per acre

[2]In per cent

Table 2. Actual Grain Yield Versus Grain Yield Estimated by the Crop Response Model.

Location	Year	Plot	Actual[1] Yield	Estimated[1] Yield	State of[2] Growth at Anthesis	State of[2] Growth at Maturity	Average[2] Available Moisture
Manhattan	1974	1	45.90	69.404	81.503	36.622	60.0
		2	106.30	114.194	88.508	60.256	74.1
		3	112.80	105.110	81.503	55.463	72.8
		4	70.20	98.903	81.503	52.188	70.6
		5	146.10	156.827	82.508	82.752	94.9
	1975	1	95.80	103.292	94.779	55.443	71.8
		2	105.20	132.454	96.528	71.096	80.5
		3	134.90	138.673	94.779	74.434	85.0
		4	103.00	124.208	94.779	66.670	81.4
		5	153.80	166.573	96.528	89.410	95.4
	1976	1	45.20	78.639	87.748	61.968	78.8
		2	56.60	75.158	82.662	54.415	71.1
		3	62.80	40.873	66.794	24.950	53.1
Scandia	1974	1	143.80	78.924	80.349	54.601	75.1
		2	131.50	74.595	80.127	51.607	71.2
		3	40.80	20.413	79.060	14.122	59.1
	1975	1	164.63	113.027	92.900	66.392	81.0
		2	122.28	105.776	92.900	62.132	76.9
		3	48.37	94.984	92.900	55.793	75.9

[1]In bushels per acre

[2]In per cent

Table 3. Two Irrigation Schedules that Maximize Total Revenue Net of Irrigation Costs.

Period	Lower Pumping Costs	Higher Pumping Costs
1	0.0	0.0
2	0.3	0.3
3	0.3	0.3
4	0.5	0.5
5	0.6	0.6
6	0.6	0.6
7	0.7	0.7
8	1.3	1.3
9	1.4	1.4
10	2.0	2.0
11	2.0	2.0
12	1.5	1.5
13	2.0	1.5
14	2.0	0.0
15	2.0	0.0
16	1.5	0.0
17	0.5	0.5
18	0.0	0.0
Total Acre Inches Applied Per Acre	19.20	13.20
Harvestable Grain Yield in Bushels Per Acre	135.80	112.00
Total Revenue in Dollars Per Acre	271.60	224.00
Irrigation Costs in Dollars Per Acre	39.55	157.08
Total Revenue Net of Irrigation Costs Per Acre	232.05	66.92

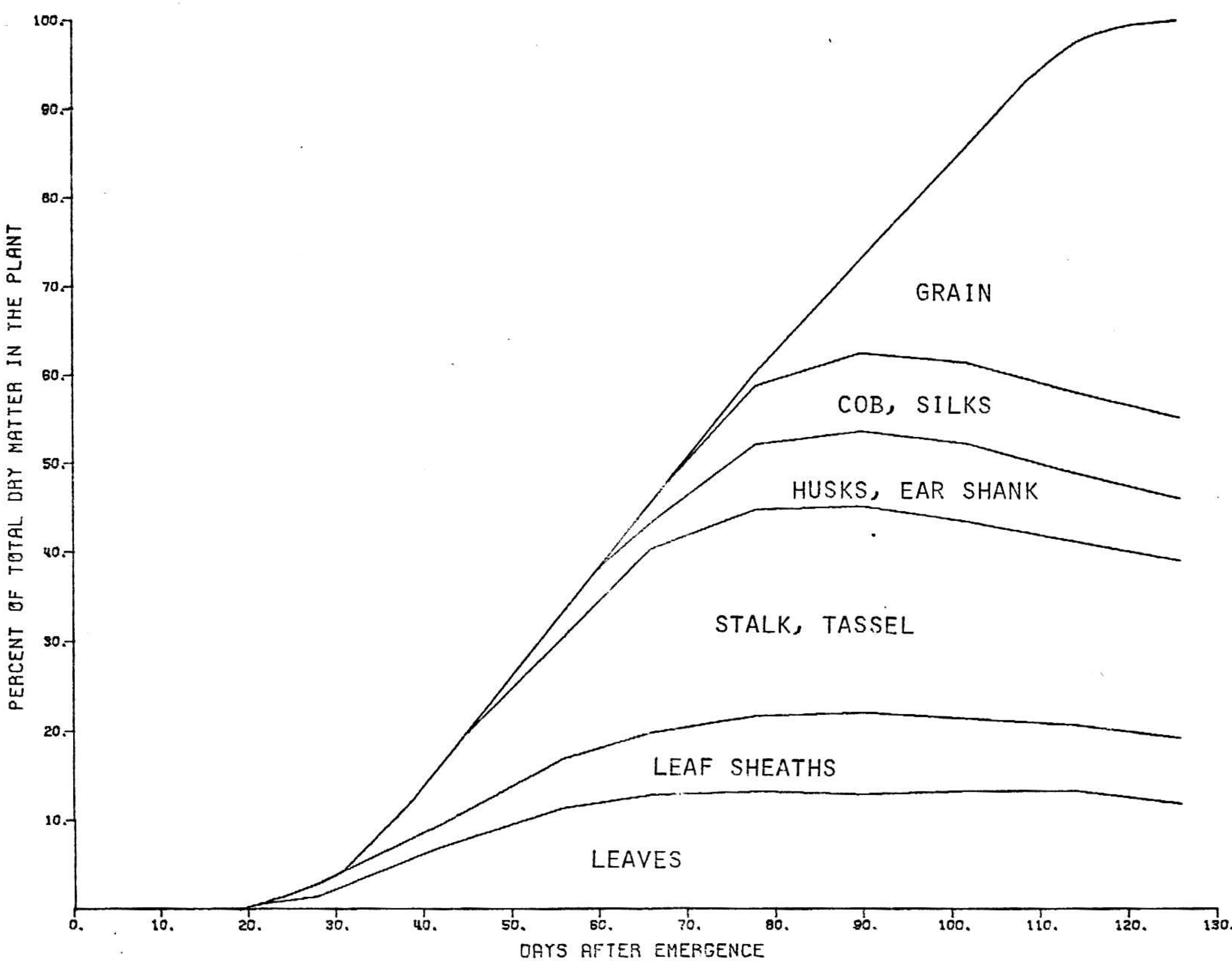

FIGURE 1. DRY MATTER ACCUMLATION IN THE CORN PLANT REPRODUCED USING DATA FROM HANWAY

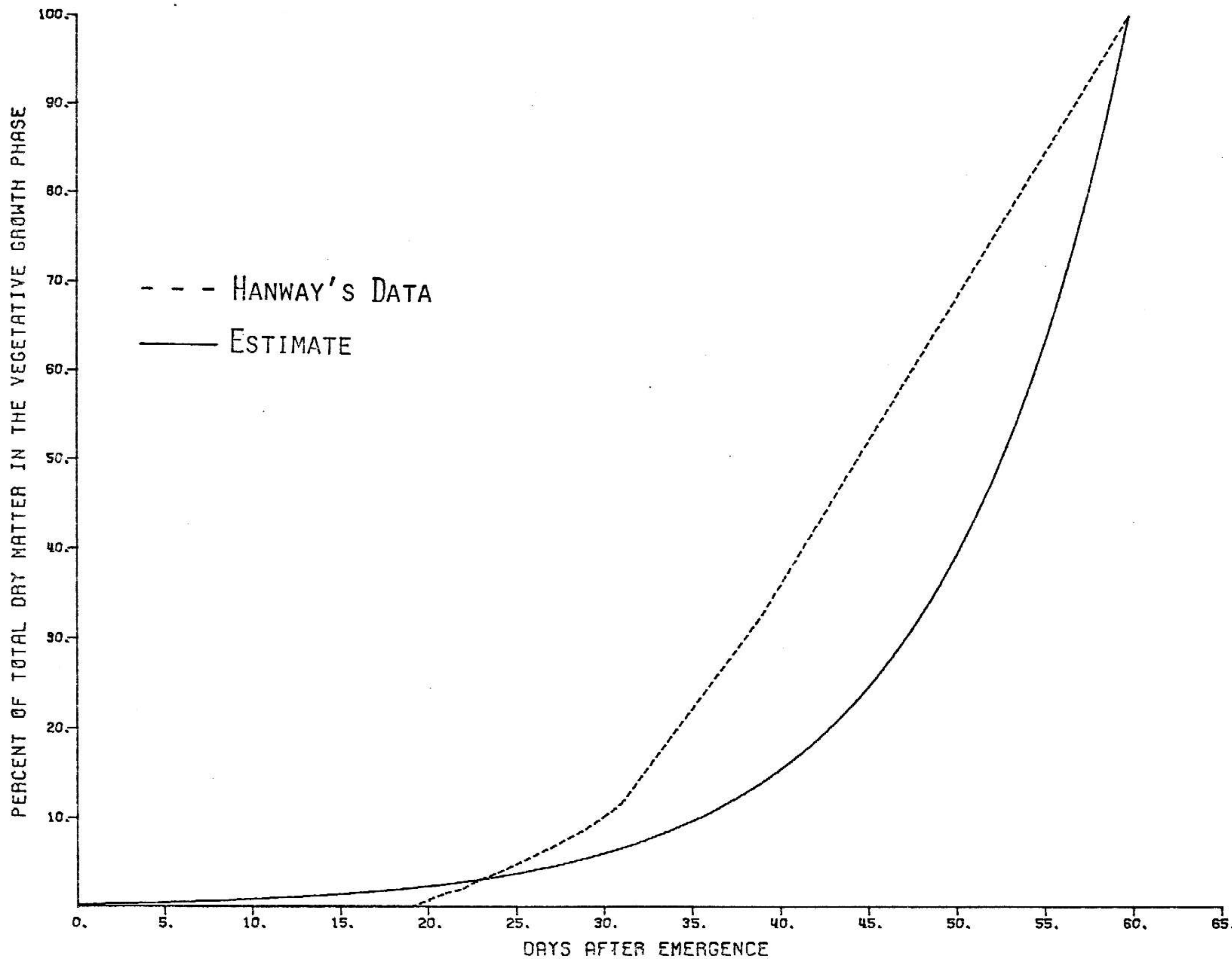

FIGURE 2. ESTIMATED EXPONENTIAL VEGETATIVE GROWTH FUNCTION FOR CORN AND DATA FROM HANWAY

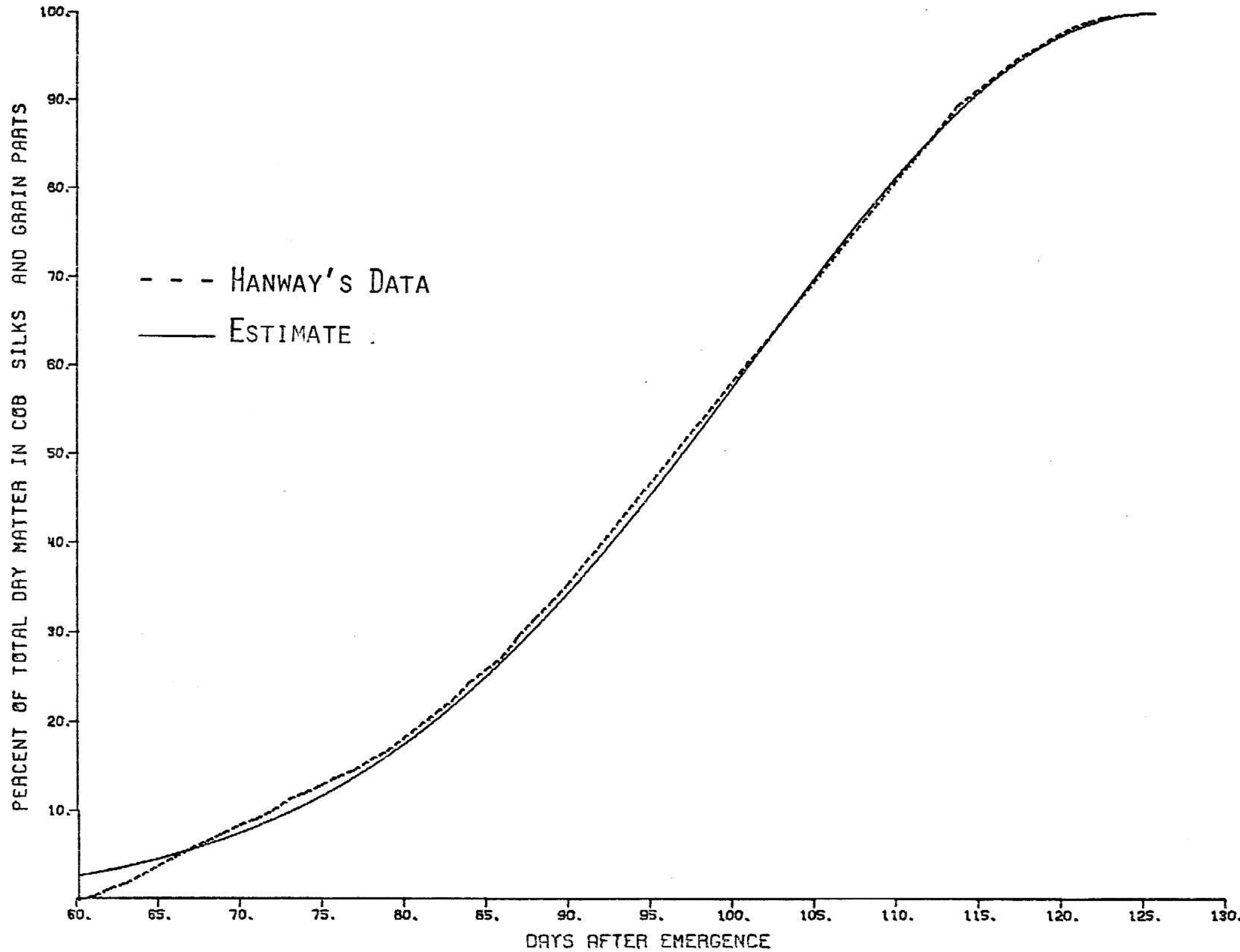

FIGURE 3. ESTIMATED MODIFIED LOGISTIC EAR DEVELOPMENT FUNCTION FOR CORN AND DATA FROM HANWAY

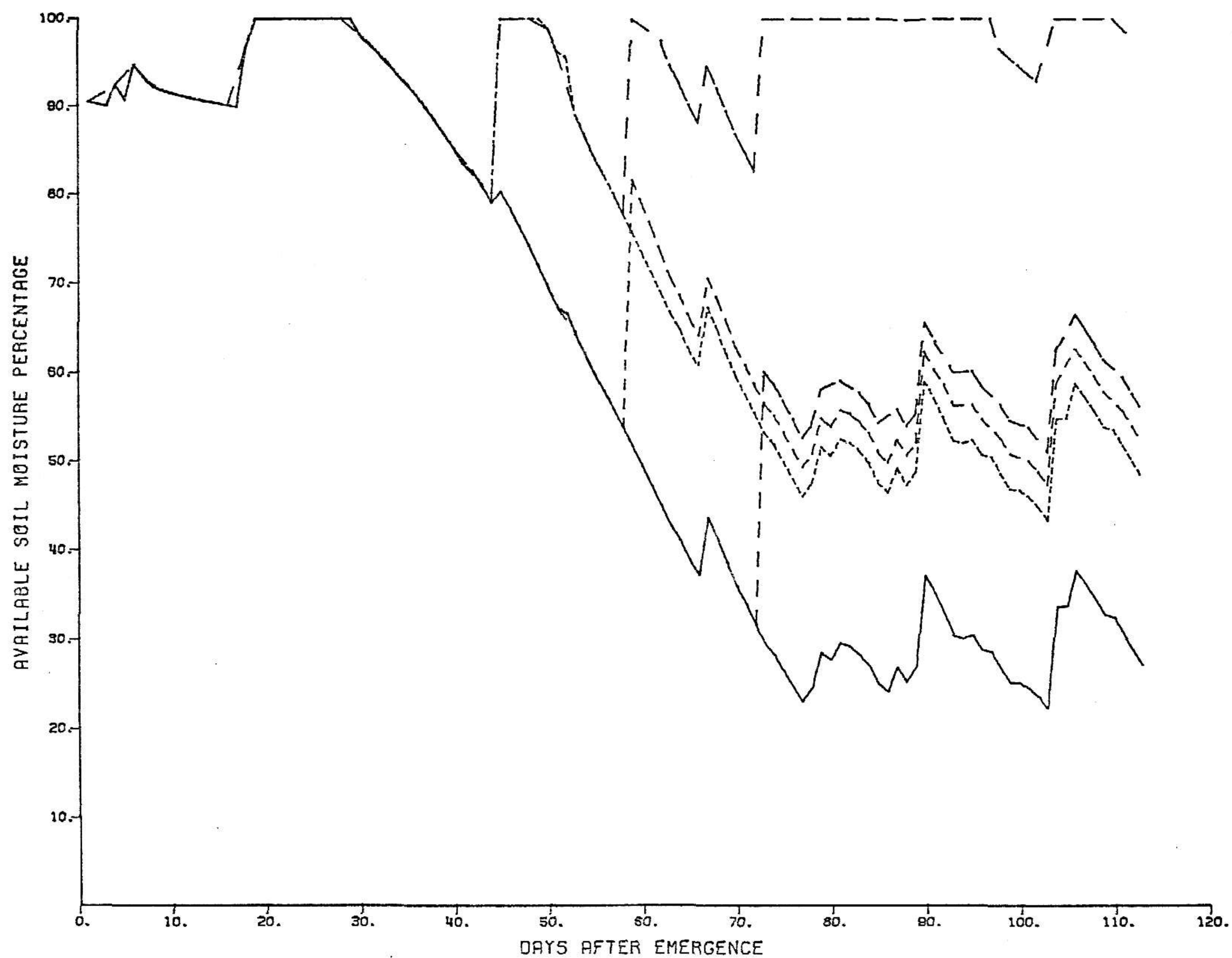

FIGURE 4. AVAILABLE SOIL MOISTURE PERCENTAGE FOR FIVE CORN IRRIGATION TREATMENTS AT MANHATTAN, KANSAS IN 1974

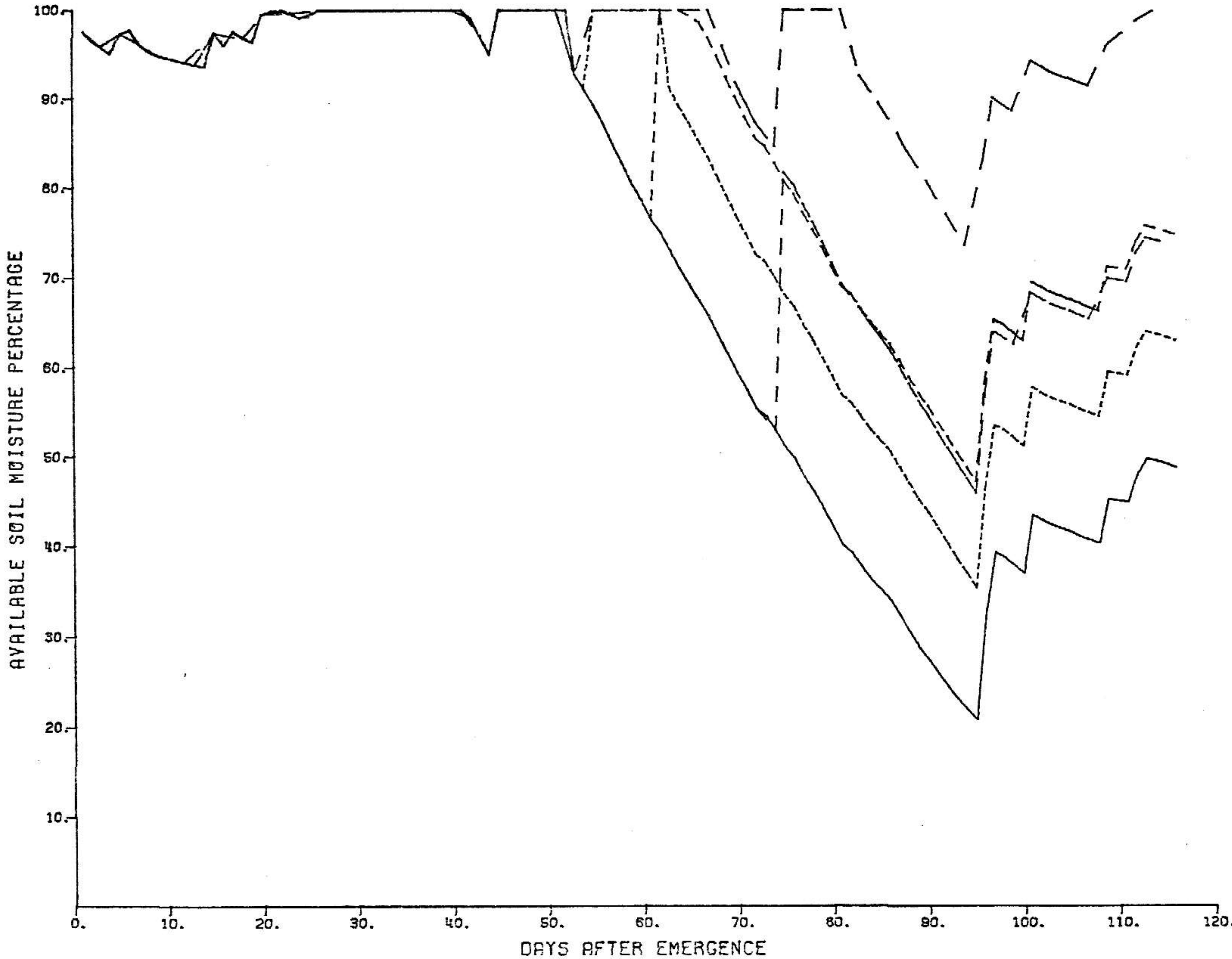

FIGURE 5. AVAILABLE SØIL MØISTURE PERCENTAGE FØR FIVE CØRN IRRIGATIØN TREATMENTS AT MANHATTAN, KANSAS IN 1975

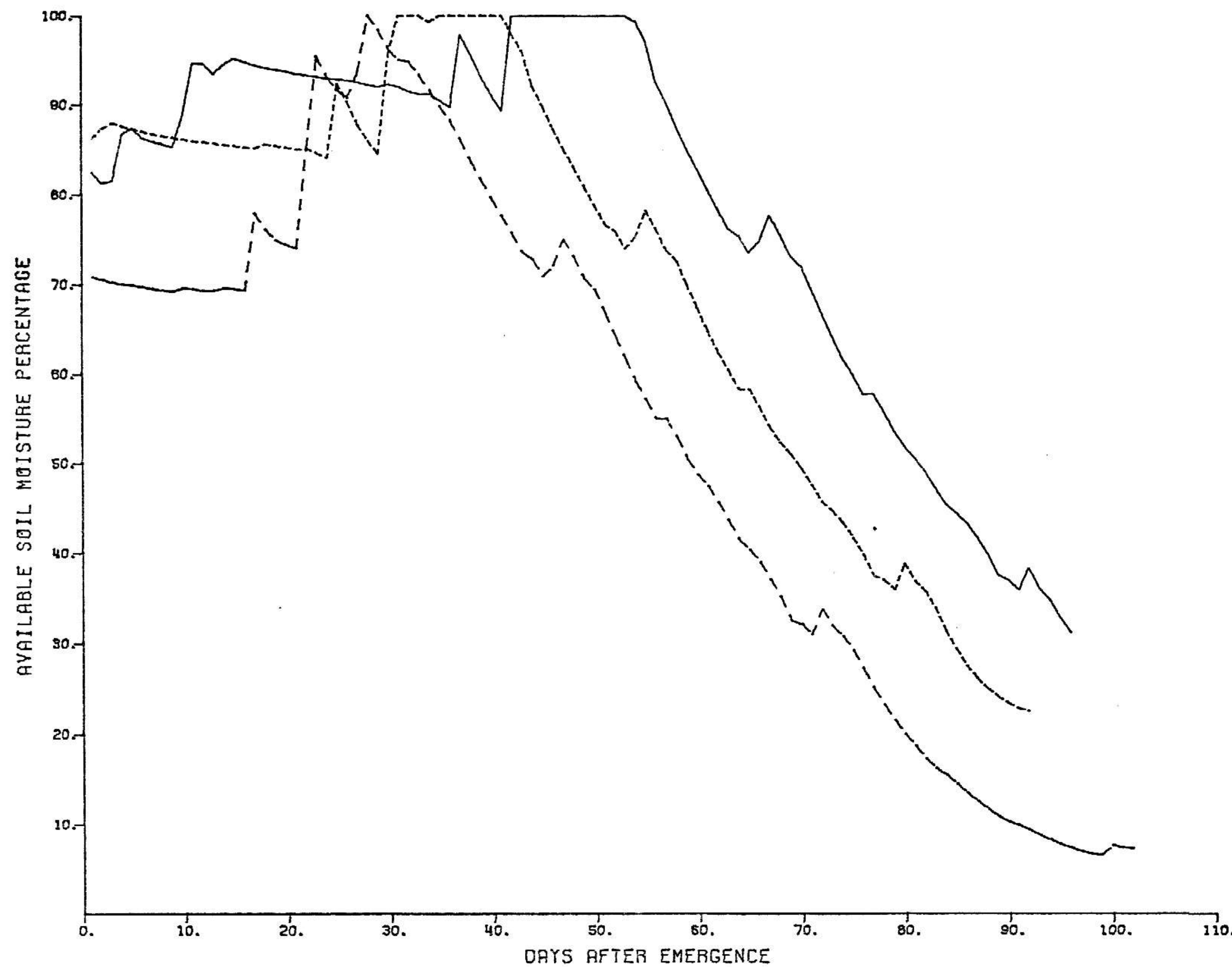

FIGURE 6. AVAILABLE SØIL MØISTURE PERCENTAGE FØR THREE CØRN PLANTING DATES AT MANHATTAN, KANSAS IN 1976

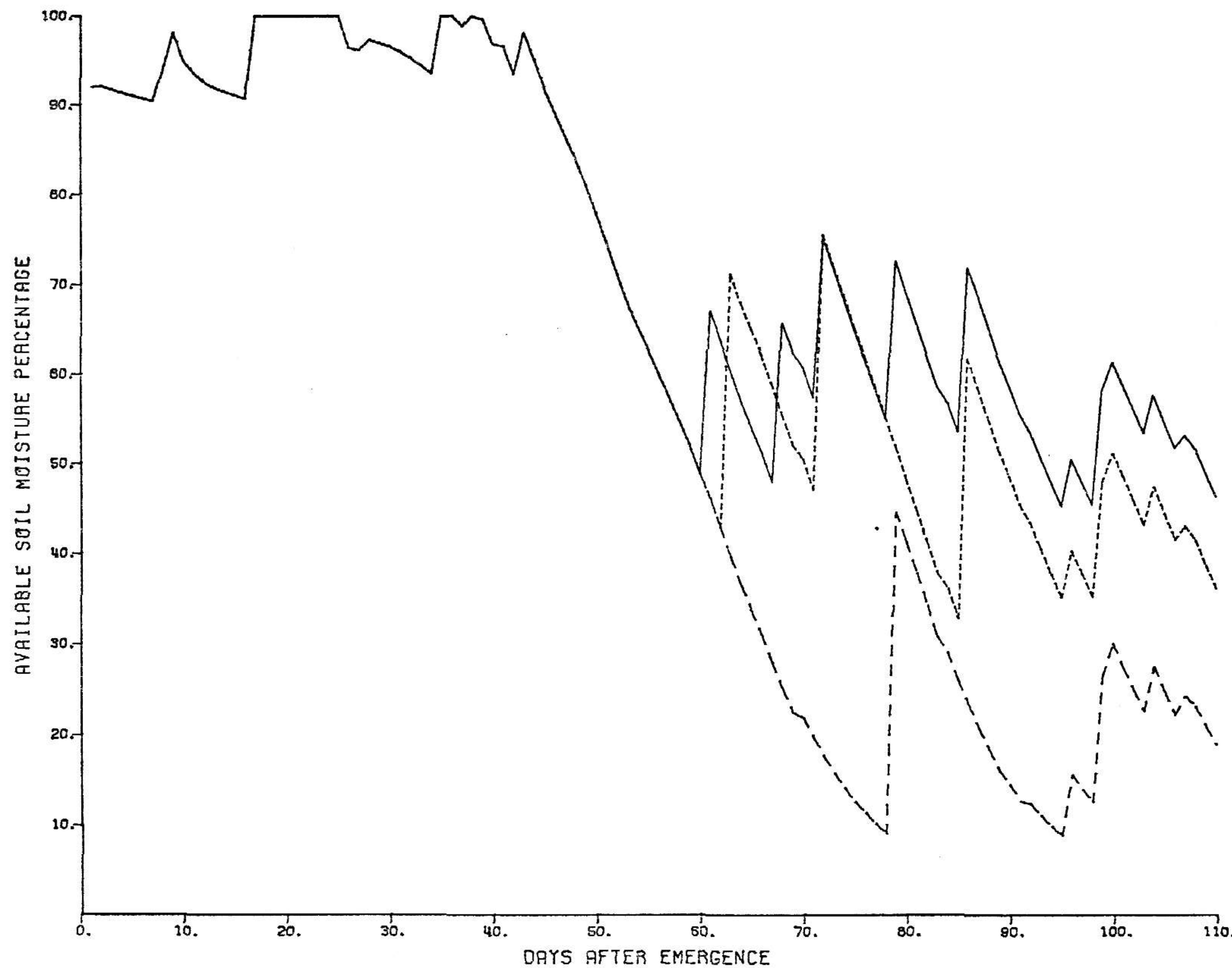

FIGURE 7. AVAILABLE SØIL MØISTURE PERCENTAGE FØR THREE CØRN IRRIGATIØN TREATMENTS AT SCANDIA, KANSAS IN 1974

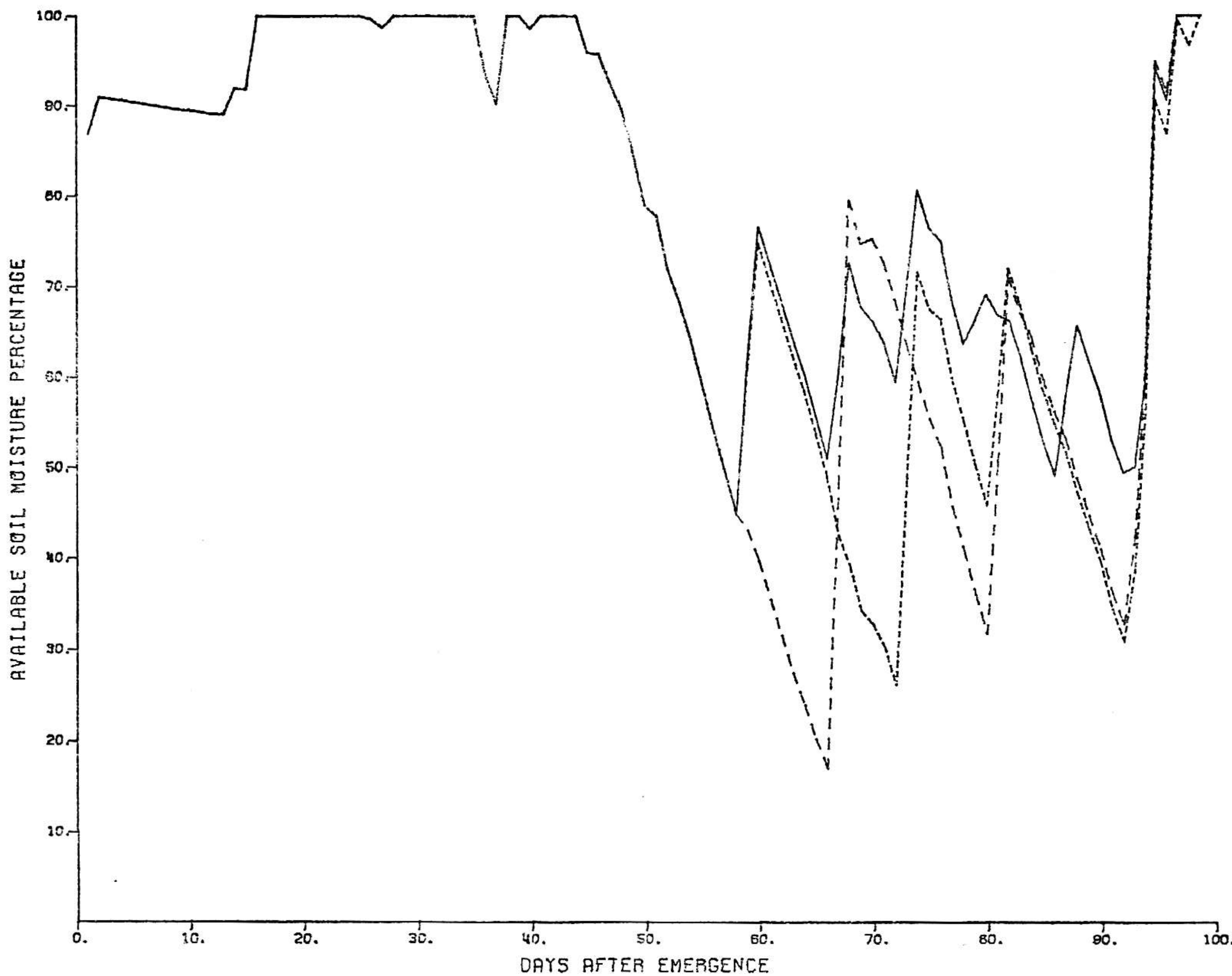

FIGURE 8. AVAILABLE SOIL MOISTURE PERCENTAGE FOR THREE CORN IRRIGATION TREATMENTS AT SCANDIA, KANSAS IN 1975

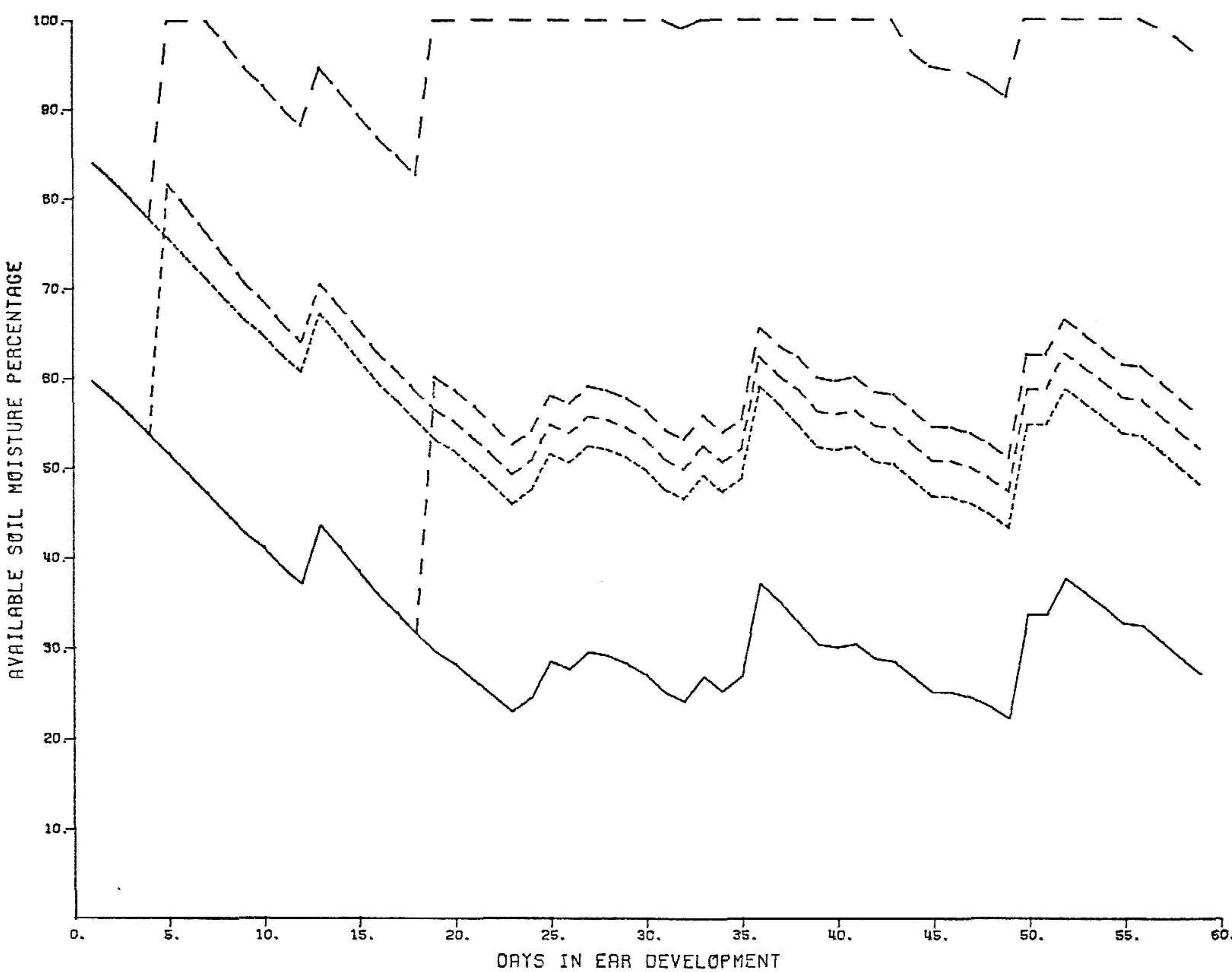

FIGURE 9. AVAILABLE SØIL MØISTURE PERCENTAGE FØR FIVE CØRN IRRIGATIØN TREATMENTS AT MANHATTAN, KANSAS IN 1974

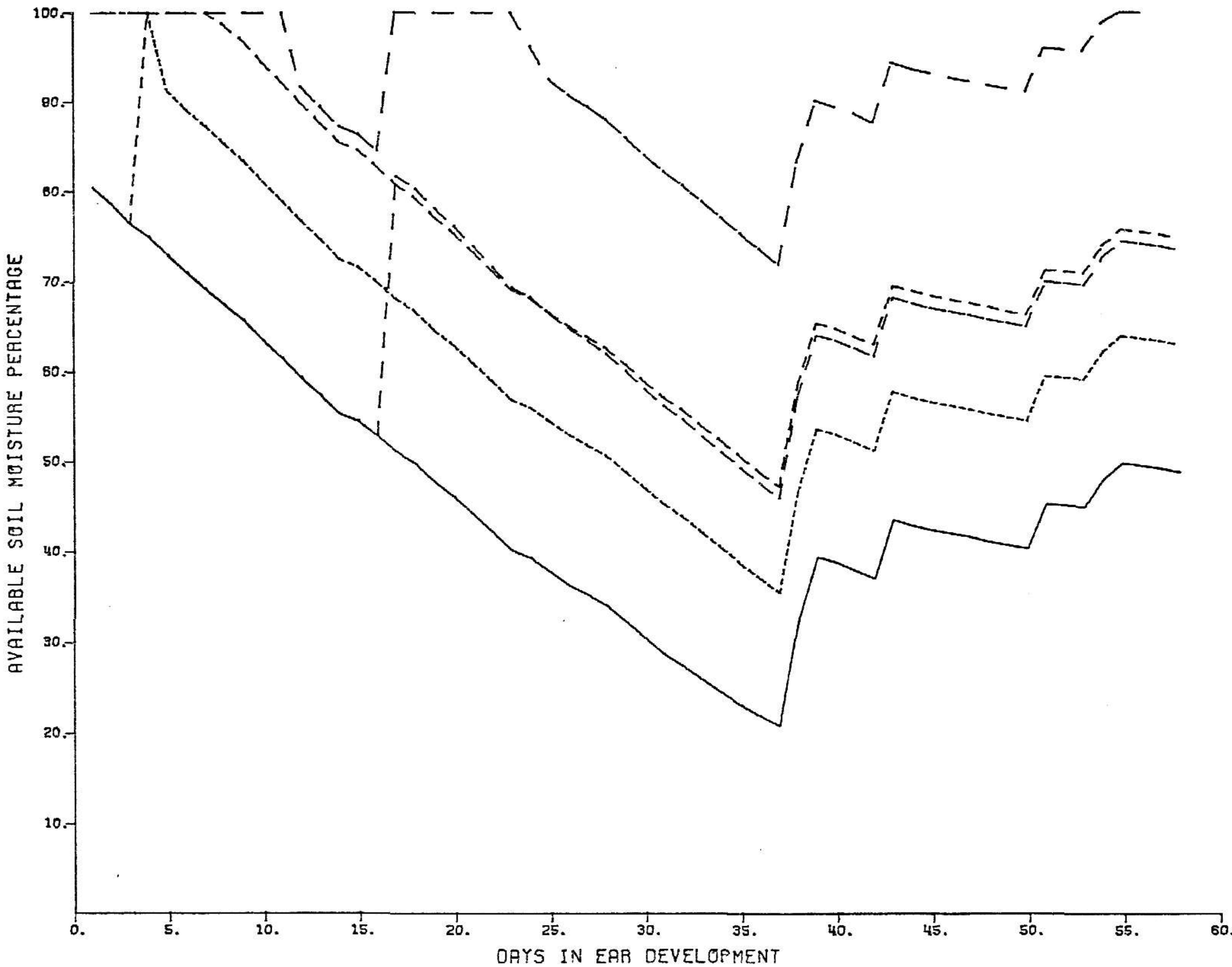

FIGURE 10. AVAILABLE SØIL MØISTURE PERCENTAGE FØR FIVE CØRN IRRIGATIØN TREATMENTS AT MANHATTAN, KANSAS IN 1975

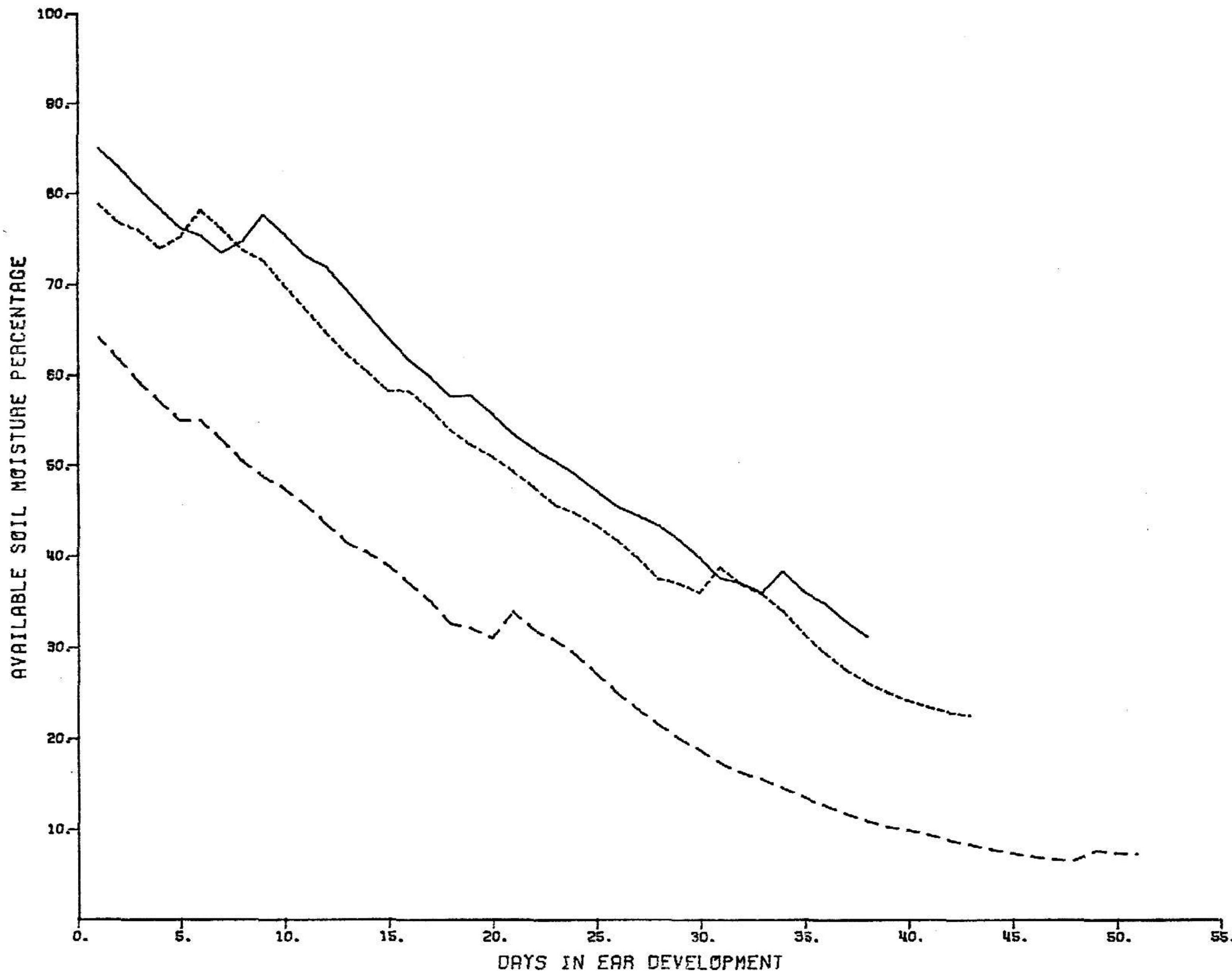

FIGURE 11. AVAILABLE SØIL MØISTURE PERCENTAGE FØR THREE CØRN PLANTING DATES AT MANHATTAN, KANSAS IN 1976

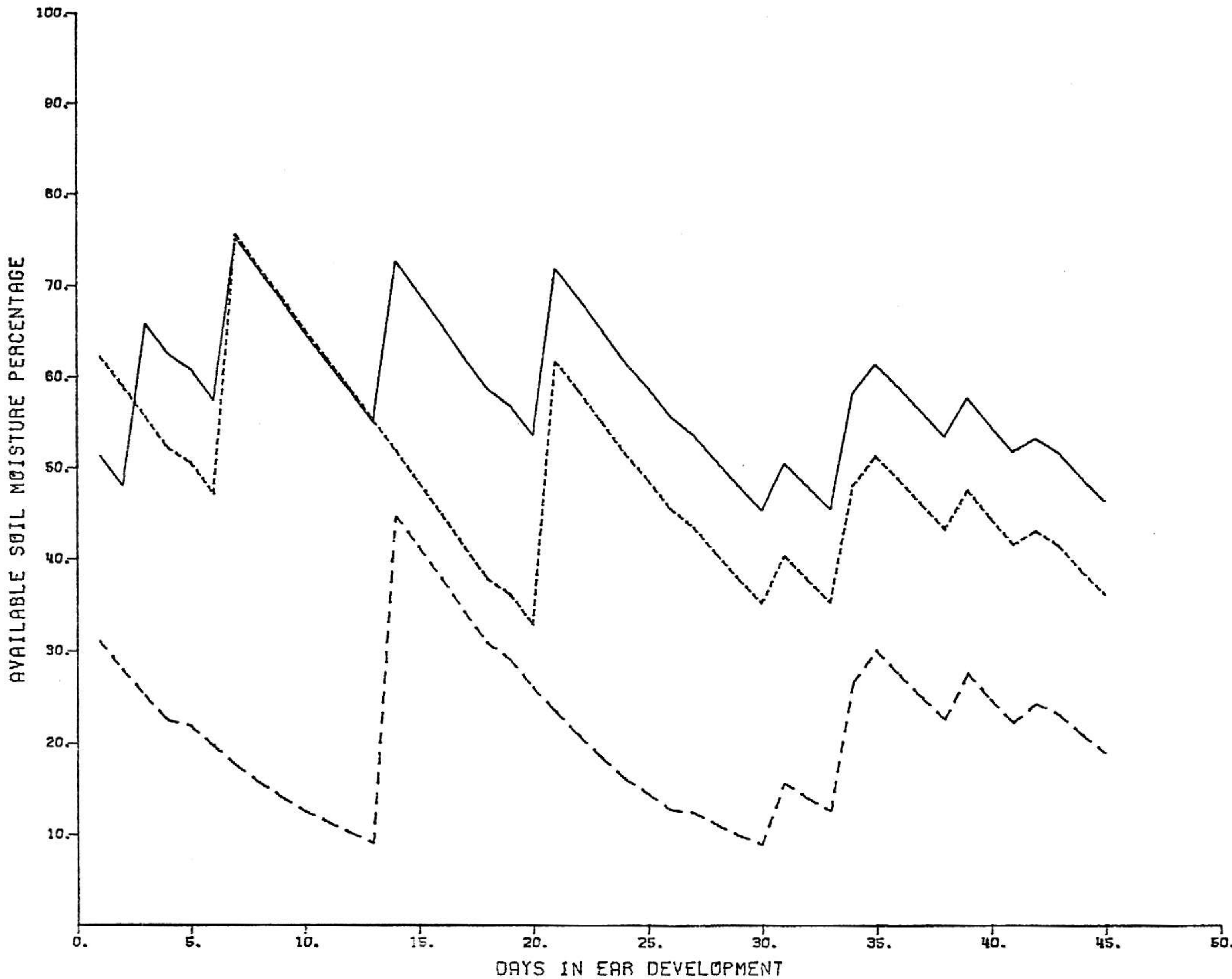

FIGURE 12. AVAILABLE SØIL MØISTURE PERCENTAGE FØR THREE CØRN IRRIGATIØN TREATMENTS AT SCANDIA, KANSAS IN 1974

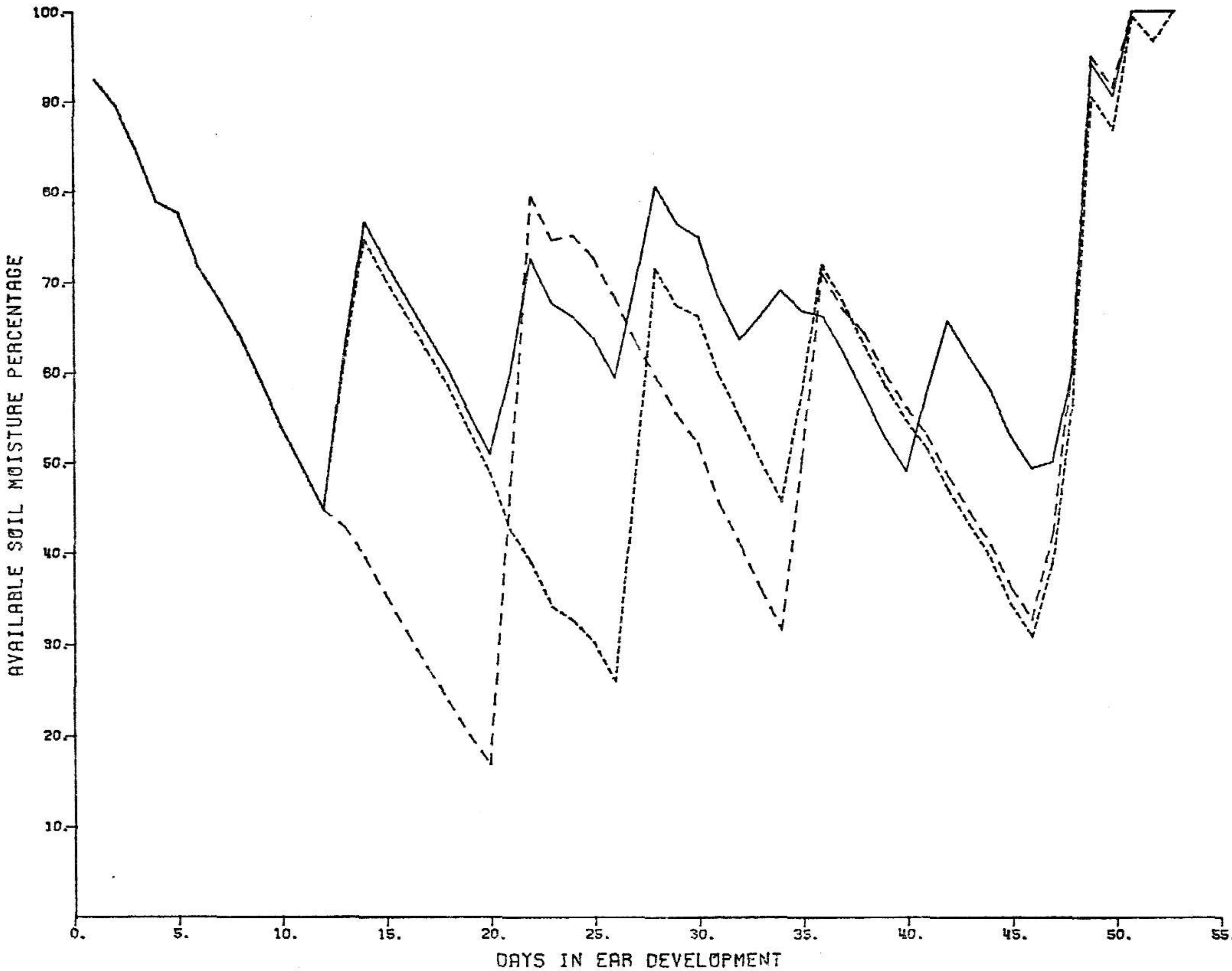

FIGURE 13. AVAILABLE SØIL MØISTURE PERCENTAGE FØR THREE CØRN IRRIGATIØN TREATMENTS AT SCANDIA, KANSAS IN 1975

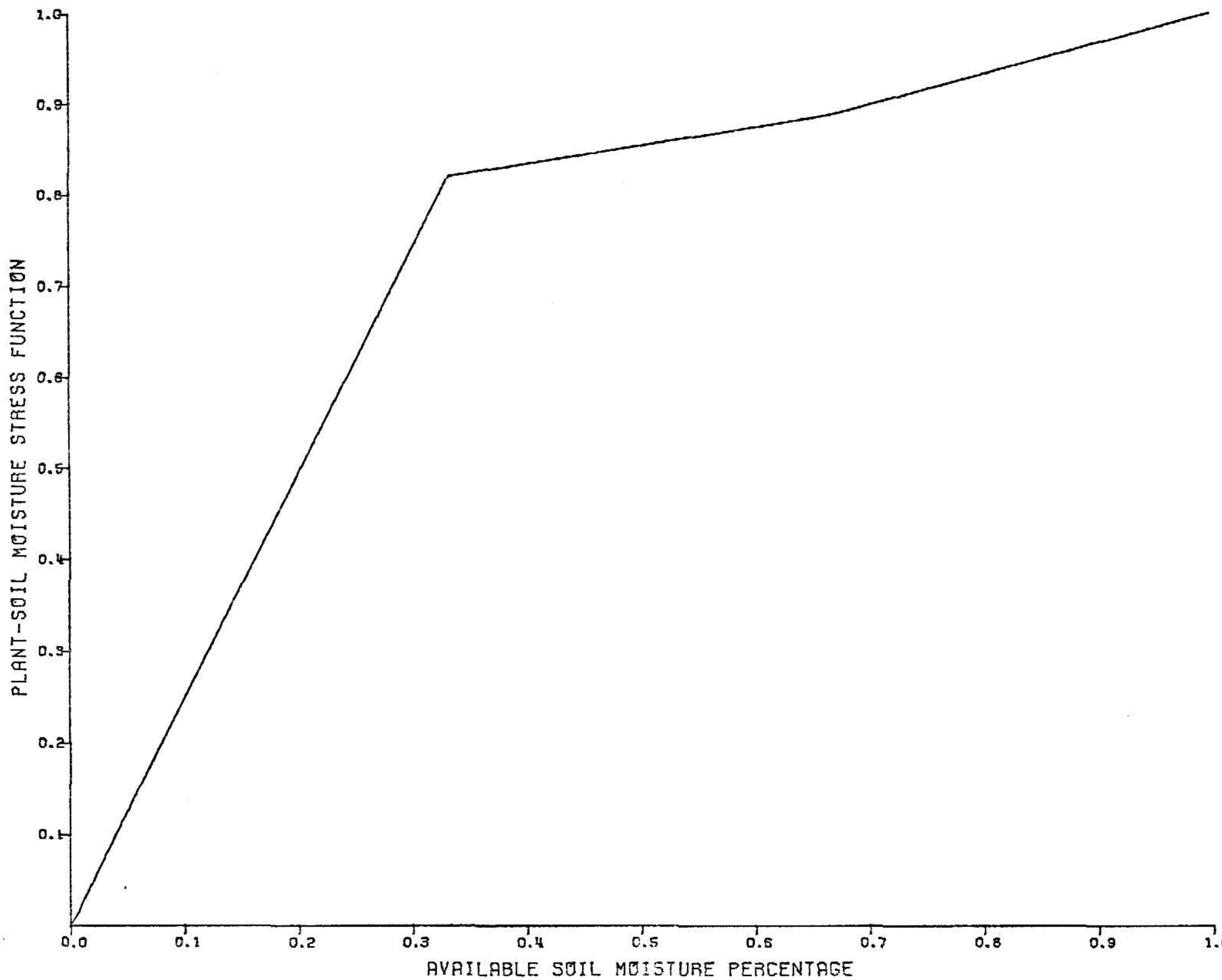

FIGURE 14. PLANT-SØIL MØISTURE STRESS FUNCTIØN ESTIMATED USING A THREE SEGMENT SPLINE EQUATIØN

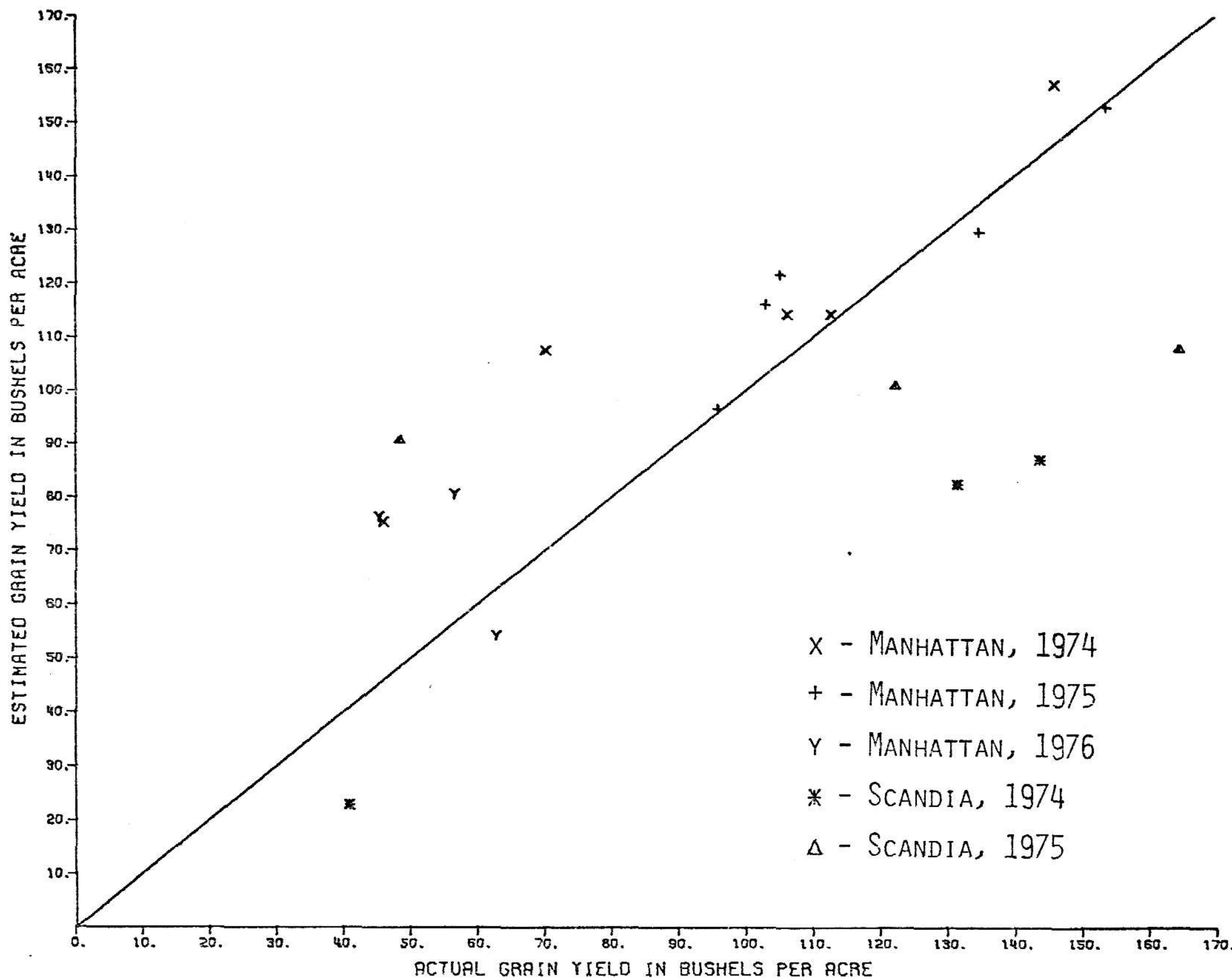

FIGURE 15. ACTUAL GRAIN YIELD VERSUS GRAIN YIELD ESTIMATED BY EAR DEVELOPMENT PHASE OF THE CROP RESPONSE MODEL

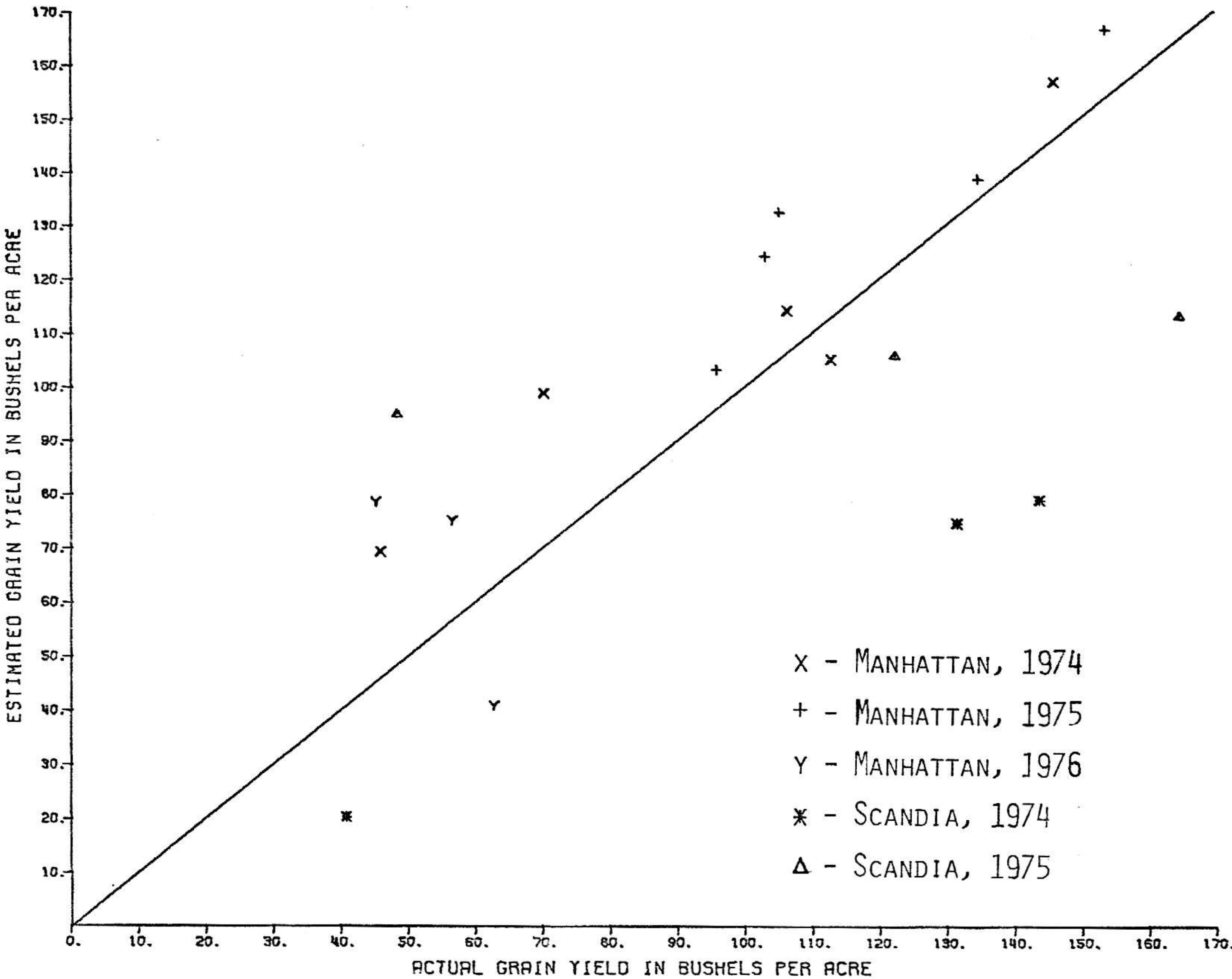

FIGURE 16. ACTUAL GRAIN YIELD VERSUS GRAIN YIELD ESTIMATED BY THE CRØP RESPØNSE MØDEL

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A MODEL OF CORN RESPONSE TO AVAILABLE MOISTURE AND AN ECONOMIC MODEL TO SCHEDULE IRRIGATIONS

by

THOMAS HENRY MORGAN

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1977

This study was concerned with the modeling of the economic, biological and physical aspects of corn irrigation scheduling. Two models were developed and simulated -- a crop response model and an economic irrigation scheduling model.

The crop response model was developed to simulate corn grain yield response to soil moisture availability by integrating the biological and physical aspects of irrigation scheduling. After reviewing pertinent literature regarding corn growth in relation to water availability and crop modeling, two growth functions and a plant-soil moisture stress function were estimated. The two growth functions represented two phases of corn development -- vegetative growth and ear development. They indicated the relationship of the stage of plant development and plant-soil moisture stress on grain yield potential. They were estimated using dry matter accumulation data, least squares regression and an exponential equation for vegetative growth and a modified logistic curve for ear development. The plant-soil moisture stress function indicated the relationship between available soil moisture percentage and plant-soil moisture stress. Since there was no observable daily data on that relationship it was estimated implicitly through yield response using the estimated growth functions, least squares regression, a piece wise linear or spline function and data containing daily available soil moisture and corresponding grain yields from Manhattan and Scandia, Kansas test fields during 1974, 1975, and 1976.

The crop response model was simulated over the test data. It did particularly well when considering only the data from the Manhattan location, but did not perform as well when the data from the Scandia location was considered. The poorer performance on the Scandia data was attributed to either inaccuracies in the data or some factor important to the Scandia location but not to the Manhattan location that the model did not take into account.

The economic irrigation scheduling model combined the crop response model, a water balance equation and the criterion of the maximization of total revenue net of irrigation costs in the dynamic programming framework to obtain a multiperiod decision model. The scheduling model was solved for two levels of irrigation costs, which were representative of the two extremes in irrigation costs. As the cost of irrigation increased, the amount of water applied decreased, as well as the total revenue net of irrigation costs. The most significant result was that the amount of water applied was decreased during the periods where agronomic research has shown it would contribute the least to yield or gross revenue.

The two models offer considerable potential in the area of water resource allocation, where they can be used in analyzing both micro and macro aspects of the problem. The models' major strength, particularly the crop response model, is that they are based on actual data. The functions used in the models were estimated from actual data rather than synthesized or merely assumed.