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/A COMPARISON BETWEEN MATHEMATICS PLACEMENT EXAMINATION
AND ACT MATHEMATICS ON CERTAIN CLASSES OF STUDENTS
AT KANSAS STATE UNIVERSITY/

by

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A MASTER'S REPORT

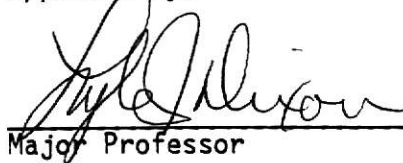
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CHAPTER 1

INTRODUCTION

In the United States, precollege data have been useful and helpful in selecting students for admissions to colleges and universities. Numerous predictive studies of academic achievement have been done in almost every major field of study (Eysenck, 1974). Prediction of a certain criteria such as final course grades, first semester grade point average, or other measures of overall academic achievement made in order to facilitate a decision with regard to an individual student have been widely used, both for counseling and admission.

Another use of precollege data is the placement of entering freshmen in appropriate course levels according to their ability. It is difficult when one tries to predict another's academic achievement using precollege data for there may be other factors affecting success which one can never really foresee. However, within certain limits of error, academic success in terms of grades can be predicted on a large group of students. It is wished that this prediction of success be on a recommended basis. Freshmen were enrolled in a certain mathematics course in which they might have a small probability of success due to a lack of certain mathematical ability. These students could be recommended to take other math courses compatible with their ability.

Khan (1974) proposed that study of predictions at higher institutions should not be done every year, but on an as-needed basis

to check the validity of the prediction system; for it will become obsolete with time.

Therefore, this correlational study investigated the comparison between Mathematics Placement Examination scores and ACT Mathematics scores as precollege data to predict academic success in Intermediate Algebra, College Algebra, and Calculus I. The scores were not intended to be used exclusively, but as an additional asset for the advisor and the student to select a math course which would be more compatible with his ability.

Statement of the Problem

To advise the students enrolled in the beginning mathematics courses on which courses they should take, depending on their math ability, is one of the important things that counselors and faculty advisors at Kansas State University must do. Such beginning mathematics courses are Intermediate Algebra, College Algebra, and Calculus I. Evidence for this need could be seen by the number of students enrolled in the above three courses either failing or withdrawing from their respective courses. For example, approximately 359 enrolled in Intermediate Algebra, and from this population only 339 received a grade (A, B, C, D, F). There were 1049 students enrolled in College Algebra, with only 781 receiving a grade (A, B, C, D, F). And 491 enrolled in Calculus I, with only 405 receiving a grade (A, B, C, D, F). All subjects enrolled in the fall semester 1985-86.

Even though there are many studies related to predicting the first semester grade, there are relatively few studies that are concerned

about course placement exams or success in individual courses, especially in Intermediate Algebra, College Algebra, and Calculus I.

A predictive relationship between placement exams and success level in Intermediate Algebra, College Algebra, and Calculus I could help counselors and faculty advisors place students in beginning college mathematics courses in order for the students to have a better chance of success. In this study, the students' success in Intermediate Algebra, College Algebra, and Calculus I was measured by their final course grades in respective mathematics courses.

Purpose

In this study a set of certain data was collected on 359 Intermediate Algebra students, 1049 College Algebra students, and 491 Calculus I students enrolled in the fall semester of 1985-86.

The data collected included two predictor variables--ACT Mathematics score and Mathematics Placement Exam scores, and one criterion variable--the final course grade received. The primary purpose of this study was to determine the correlation between each of the predictor variables and the criterion variable.

The second purpose was to determine the correlation between the two predictor variables.

The third purpose was to determine which is the best and most easily usable predictor of academic performance in each of the courses.

Significance of the Study

The use of the results from this study is intended as an additional tool for the advisors and the student to select a mathematics course which would be more compatible with his ability, interests, and major field so as to insure maximum success.

In addition, the score of the Mathematics Placement Examination would show certain deficiencies in the student's mathematical knowledge so that the instructors could put more emphasis on certain areas of mathematics.

Limitations

This study was limited to 359 Intermediate Algebra students, 1049 College Algebra students, and 491 Calculus I students enrolled in the fall semester of 1985-86. Also, this study was limited to the number of factors such as the absence of ACT Mathematics scores and Mathematics Placement scores. Furthermore, the number of students who received a W and who dropped the course in the beginning of the fall semester of 1985-86 are not reflected in this study.

Definition of Terms

1. American College Test (ACT): Before applying for an admission to college, the high school students usually take one of the three major tests of academic ability: the American College Test (ACT), the Scholastic Aptitude Test (SAT), or the National Merit Scholarship Qualifying Test (NMQST).

The ACT academic test covers four subjects: Mathematics, English, Natural Sciences, and Social Studies. However, in this study, the ACT Mathematics score is used as one of the predictor variables to predict academic success.

2. **Mathematics Placement Exam:** At Kansas State University, the use of Mathematics Placement Exam has only been used recently for students enrolled in Intermediate Algebra, College Algebra, and Calculus I. The Mathematics Placement Exam was taken by all the subjects in the first week of school, during fall semester 1985-86.

In this study, the Mathematics Placement Exam is used as the second predictor variable to predict academic performance in each course.

Mathematics Placement Examination Form A/4A (1981) was used in this study.

3. **Grades:** At Kansas State University, the grade policy used is by letter grades, i.e., A, B, C, D, F. For this study, the letter grades were converted to a numerical grade based on the four point system: $A = 4$, $B = 3$, $C = 2$, $D = 1$, $F = 0$.
4. **Level of Significance:** Level of significance is a probability value that is considered rare in the sampling distribution specified under the null hypothesis that are willing to assert the operation of nonchance factors.

Common significance levels are 0.01 and 0.05.

5. **Degrees of Freedom:** The number of degrees of freedom is equal to the number of observations minus the number of algebraically independent linear restrictions placed upon them.
6. **Mean:** The mean of a collection of measure is the sum of the measures divided by the number of measures, and the mean can be calculated by the following formula:

$$M = \frac{\sum X}{N}$$

M = Mean of Measures

N = Number of Measures

$\sum X$ = Sum of Measures

7. **Variance:** The variance denoted by σ^2 is defined as the mean of squared deviations from the mean:

$$\sigma^2 = \frac{\sum (X_i - \mu)^2}{N}$$

μ = Sample Mean

8. **Standard Deviation:** The standard deviation, denoted by σ , is the square root of the variance:

$$\sigma = \sqrt{\sigma^2} = \left(\frac{\sum (X_i - \mu)^2}{N} \right)^{1/2}$$

9. **Correlation Coefficient:** A correlation coefficient is a measure that expresses the extent or relation between two variables. The value of the correlation coefficient varies between +1.00 and -1.00. Both extremes represent perfect relationship between the variables and 0.00 represents the absence of a relationship.

A positive relationship means that individuals obtaining high scores on one variable tend to be paired with a high score on another variable.

Similarly, two variables are said to be negatively correlated when a high score on one is accompanied by a low score on the other. Thus, the two variables vary in the opposite direction. In this study, Pearson correlation coefficient was used, and can be calculated by:

$$r = \frac{SP}{\sqrt{SS_X \cdot SS_Y}}$$

$$\text{where } SP = \text{Sum of Product} = \sum XY - \frac{(\sum X)(\sum Y)}{N}$$

$$SS_X = \text{Sum of Squares of } X = \sum X^2 - \frac{(\sum X)^2}{N}$$

$$SS_Y = \text{Sum of Squares of } Y = \sum Y^2 - \frac{(\sum Y)^2}{N}$$

Assumptions of Pearson r:

- (1) Pearson r assumes that the relationship between the two variables is linear. If the relationship is not linear, the correlation will be greatly underestimated by Pearson r formula.
- (2) The significance of correlation is a function of sample size. A small correlation with a large sample may be statistically significant.
- (3) Pearson r helps in making predictions, but is not a statement of causality.

10. Correlation Matrix: The correlation matrix measures several variables available. The correlation matrix also represents correlation between each pair in matrix form.

Rows: variables from top to bottom.

Columns: variables from left to right.

Correlations are entered in cells of matrix, and the matrix is symmetric.

11. Null Hypothesis: A statement that specifies hypothesized values for one or more of the population parameters.¹ It commonly involves the hypothesis of "no difference."

¹Runyon, Richard P. and Audrey Haber, Fundamentals of Behavioral Statistics (Addison-Wesley Publishing Company, June 1984).

CHAPTER 2

THE REVIEW OF RELATED LITERATURE

In recent years, many predictive studies have been done to determine what single variable, or what combination of predictor variables, were the best predictors for predicting academic success in beginning mathematics courses.

This chapter has been divided into three sections as follows:

- I. The first section is related to general studies of Mathematics Placement Exam as a predictor for predicting success in introductory courses.
- II. The second section is related to general studies of ACT Mathematics as a predictor for predicting success in mathematics courses.
- III. The third section is related to general studies of Mathematics Placement Exam and ACT Mathematics as predictors for predicting success in beginning mathematics courses.

I. General Studies of Mathematics Placement Exam as Predictor of Academic Achievement in Introductory Mathematics Courses.

The Mathematical Association of America (MAA) has introduced the Mathematics Placement Exam for guidance and placement in introductory Mathematics courses offered by colleges and universities throughout the nation. In this section, we will investigate how far the studies have gone using the placement exam as a predictor for success.

Sims² conducted a study that fulfilled the following objectives:

1. To determine if there is a significant correlation at the 0.05 level between success in college algebra and prior mathematics achievement as measured by the MAA Placement test BA/1.
2. To determine students' attitude towards Mathematics.
3. To determine predictive ability of certain cognitive, affective, and demographic factors.

The Placement Exam, the attitude scale, and a student data sheet were administered to all students enrolled in College Algebra at Richland College, Dallas, Texas, in the spring semester of 1979. 253 subjects have completed ten independent variables; however, only 135 completed the course.

The results indicated that there is a significant Pearson r correlation at .05 level between placement test scores and course grades for all students and between placement test scores and scores on the standardized final examinations. The Mathematics Placement Test, together with age and attitude subscale emerged as the best three predictors for students who completed the course.

Even though Clark³ didn't use the MAA Placement Exam, he conducted a study at Kings River Community College to determine whether there were student characteristics that could help to predict success in

²Sims, Georgia Lee, "Prediction of Success in College Algebra at Richland College in Dallas, Texas," Doctoral Dissertation, Florida State University, 1979.

³Clark, Robert M., "Math Courses Survey: Math 5A Math Analysis I.," Research Report (143), statistical material (110), RIE, Kings River Community College, Reedley, California, December 1981.

college-level Calculus course (Math Analysis I). The subjects are all the students that enrolled in Math Analysis I in Fall 1979 or Fall 1980. The predictor variables are: year of high school graduation, high school attended, Mathematics Placement test, high school GPA, high school math courses and grades, college units taken before Math Analysis I, GPA of college courses before and Math Analysis I. A C grade or better in Math Analysis I is the criterion variable.

The results revealed that students had less than 50% chance of success in Math Analysis I if they scored less than the 94th percentile on the placement test, less than a 3.0 GPA in high school, received a grade less than a B in any other college math course, and had less than a 3.0 college GPA.

Placing entering freshmen into appropriate math courses to insure maximum success is an ever-continuing process.

Ahrens⁴ investigated which predictor variables could be used to place entering freshmen mathematic students into suitable math courses. The predictor variables used were departmental math placement exam scores, high school math backgrounds, and the recommendations of the students' advisor. The subjects consisted of incoming freshmen in August 1977 who enrolled in West Virginia University.

At the end of the semester, grades for the students in the various mathematics courses were obtained and matched to the earlier individual test scores of the departmental Math Placement exam. The study

⁴Ahrens, Steve, "Analysis and Classification of Entering Freshmen Mathematics Students Using Multiple Discriminate Function Analysis," Research Report (143), Conference Paper (150), RIE, December 1981.

revealed that the two best predictors were high school math background and the grade on the departmental placement exam.

Characteristics of students also play a role in determining success in introductory math courses. Clark⁵ reported that analyses were done on the grades and the characteristics of students in Math A (Elementary Algebra), Math B (Plane Trigonometry), Math C (Trigonometry), and Math D (Intermediate Algebra) to determine which variables were significantly related to student success. The predictor variables are placement test scores, high school GPA, prior college units taken, prior college GPA, high school math grades, and prior college math grades.

The findings of the study were 1) students had less than 50% chance of success in Math A if they scored below the 49th percentile on any mathematics placement test; 2) no significance between grade in high school and Math A; 3) the only significant success factor in Math B was college GPA; 4) no significant predictor of success in Math C; and, 5) students in Math D had less than 50% chance of success if they scored below the 56th percentile on the placement test.

At Iowa State University, the placement exam score is being used as an advisory tool to place entering freshmen into appropriate math

⁵Clark, Robert M., "Summary Analysis of Students and Grades: Mathematics A, Elementary Algebra; Mathematics B, Plane Geometry; Mathematics C, Trigonometry; Mathematics D, Intermediate Algebra," Research Report (143), Statistical Material (110), RIE, Kings River Community College, Reedley, California, September 1982.

classes according to their math ability as is done by Parker.⁶ High scores on ACT exams showed that it is insufficient to predict success in Calculus, thus the placement exams are given to all incoming freshmen but not to transfer students. The subjects were allowed to choose two exams which they believed appropriate from among five exams: Arithmetic, Algebra (2), Trigonometry, and Calculus.

The study revealed that even though the placement exam is advertised, most students did not take the exam seriously. The placement exam is an easily usable predictor of student's success in math courses; in fact, it is a better predictor than the student's grade in the prerequisite course.

II. General Studies of ACT Mathematics Usage Score As Predictor of Academic Achievement in Introductory Mathematic Courses.

The American College Testing began providing the ACT Assessment Program for students planning to take up post-secondary institutions in 1959. It is primarily concerned with the purpose, content, and use of ACT Mathematics Usage Test.

Ferguson and Schmeiser⁷ both investigated just how far the ACT Mathematics Usage Test is related to future college academic performance as indicated in the Technical Report for the ACT Assessment Program (1973). Tests can only measure directly what a person can do

⁶Parker, Bill, "The Placement Exam at Iowa State University," Unpublished Report to Kansas State University Faculty, 1985.

⁷Ferguson, Richard L. and Cynthia B. Schmeiser, "The Mathematics Usage Test of the ACT Assessment Program: An Overview of its Purpose, Content, and Use." The Mathematics Teacher, March 1978, pp. 182-191.

at the time of testing. The uses of Mathematics Usage scores are guiding and placing students in beginning Mathematics courses in colleges and universities. Table 1 shows the median multiple correlations for predicting first semester college grades in various mathematics courses given by colleges that participate in ACT Standard Research Services. Column I provides correlations when four ACT test scores were predictors. Column II shows the correlations with student's self-reported high school grades as predictors, and Column III when ACT scores and high school grades were used together to predict college grades in mathematics.

Table I indicates that the ACT tests are valid predictors for academic success in first year college mathematics.

In the study by Williams,⁸ the effects of achievement in the pre-calculus mathematics courses taken by transfer students and by native students at the University of Southern Mississippi was assessed and used to obtain equations for predicting success in Calculus I and Calculus II. There were 88 natives and 41 junior college transfers. The predictor variables were GPA in College Algebra, Trigonometry, Analytic Geometry, ACT Mathematics Usage score, ACT Composite score, sex, and type of student (native or transfer). The criteria was GPA in Calculus I, Calculus II, and Calculus III.

⁸Williams, Raymond, "A Study of Differences in Achievement in Pre-Calculus and Calculus courses by Junior College Transfer and Non-Transfer Students at University of Southern Mississippi," Doctoral Dissertation, 1980.

TABLE 1

Median Multiple Correlations for Colleges Using High School Self-Reported
Grade Average and ACT Composite Test Scores to Predict
First Semester College Grades in Mathematics

Years of Participation in Standard Research Services	Number of Colleges	(1) ACT Composite Test Scores	(2) High School Grade Average	(3) Combined ACT and HS GPA
1968 - 1970	281	0.418	0.428	0.510
1970 - 1973	262	0.440	0.437	0.521
1971 - 1974	245	0.443	0.428	0.519
1972 - 1975	207	0.457	0.432	0.535
1973 - 1976	203	0.458	0.428	0.532

Multiple regression analysis of data showed that ACT Mathematics and ACT Composite scores were significantly related to achievement in Calculus I ($p = 0.0002$, $F(2,88) = 11.069$, $R^2 = 0.2$).

Research by Dykes⁹ also indicated that ACT Mathematics has a significant correlation with GPA in College Algebra. The sample included 188 freshmen enrolled in College Algebra as their first college math course during the fall semesters of 1976-77, 1977-78, and 1978-79 at Copiah-Lincoln Junior College. The predictors were ACT Mathematics score, ACT Composite score, GPA's in high school mathematics, cumulative GPA in high school, and scores on the Cooperative Mathematics Tests, Algebra II section.

The purpose of the study is to determine correlations between each of the five predictors and the criteria of GPA in College Algebra, and to determine which combination of predictors are most significant in predicting the GPA in College Algebra.

Pearson product-moment correlation was conducted between each of the predictors and GPA in College Algebra. All correlations were significant at 0.05 level. The results revealed that ACT Mathematics was third with correlation coefficient $r = 0.535$, which indicates that ACT Mathematics is a valid predictor variable to predict GPA in College Algebra.

⁹Dykes, Isaac J., "Prediction of Success in College Algebra at Copiah-Lincoln Junior College," Doctoral Dissertation, University of Mississippi, 1980.

In 1972, Burnside¹⁰ recommended that predictive systems should be obtained at individual colleges and universities to predict academic success in mathematics.

O'Neal¹¹ conducted a research to examine the relationship of certain measurable characteristics of the students enrolled in initial Calculus courses at the University of Mississippi and Mississippi Gulf Coast Junior College. ACT Composite scores, ACT Mathematics scores, the number of units of high school mathematics, high school GPA, and the age in months beyond the age of seventeen were the predictor variables. The criterion was the final course grade.

The result emerged that the best equation for predicting success in Calculus I at the university is:

$$I_u = 0.153D + 1.39G + 0.580E - 0.930F - 5.93$$

The equation for the junior college is:

$$I_j = 0.064D + 0.295G + 0.283$$

where:

I_u is the grade in Calculus I at the university

I_j is the grade in Calculus I at the junior college

D is the ACT Mathematics score

G is the high school math GPA

¹⁰Burnside, Lucy H., "Prediction of Success in Mathematics as a Major Field of Study at the Public Universities in Mississippi," Doctoral Dissertation, University of Mississippi, 1972, p. 118.

¹¹O'Neal, Larry D., "A Comparison of the Predictors of Success of University and Junior College Students in the Initial Calculus Course," Doctoral Dissertation, University of Mississippi, 1980.

E is the number of units of high school math

F is the high school GPA

ACT Mathematics has the highest correlation with the course grade in Calculus I than the other predictors on both the university and the junior college. Also, this study concluded that different equations are required for predicting grades in Calculus I in individual institutions.

III. General Studies of Mathematics Placement Exam and ACT Mathematics Scores As Predictors of Academic Achievement in Introductory Mathematics Courses.

In this section, both predictors, Mathematics Placement Exam and ACT Mathematics score were used to predict academic achievement in beginning mathematics courses. The use of precollege data, namely the placement of beginning students in appropriate course levels, will become increasingly important as college population becomes more heterogeneous.¹² Horst (1957) found that specified predictors based on specific subject material predicted better than general predictors.

In the fall of 1968, the American College Testing Program (ACT) provided a new test, the Mathematics Placement Exam, for use in college placement. Shevel and Whitney¹³ conducted a study to determine whether

¹²Suddarth, Betty and Edgar S. Wirt, "Predicting Course Placement Using Precollege Information," College and University, Winter 1974, p. 186.

¹³Shevel, Linda R. and Douglas R. Whitney, "Predicting Validity of the Mathematics Placement Examination," Educational and Psychological Measurement, 1969, vol. 29, pp. 895-901.

the Mathematics Placement Exam offered sufficient improvement in the prediction of college mathematics grade to warrant its addition to a college's testing program. Another purpose was to compare the differential predictive validity of Mathematics Placement Exam with the addition of ACT scores and high school grades for classes that differed in average math ability and for classes that covered different types of material.

From Table 2, we can conclude that the two correlations are similar for lower level courses, but that the correlation is markedly larger for upper level courses. Mathematics Placement Exam adds more to the prediction of success for upper level courses.

From Table 3, the Mathematics Placement Exam has greater predictive ability for higher level courses than for remedial courses. Hence, the Mathematics Placement Examination is the most useful, easily usable predictor, relative to other predictors, for recommended use in placement into higher level courses.

To summarize the above research, the addition of the Mathematics Placement Examination improves college mathematics grade prediction and the increase is noticeable for higher level courses. The exam had differential predictive ability for both content and ability groupings. Prediction of grades by ACT scores and high school grades in low level courses is not much improved by the use of Mathematics Placement Examination.

TABLE 2
Comparison of Standard ACT Prediction with
Standard ACT Prediction
Plus Mathematics Placement Tests

Class Categories	Std. ACT Prediction Median	Std. ACT Prediction Plus Math Placement Median	Median Increase in R
Business Math (3 Classes)	.561	.585	.051
Intermediate Algebra (4 Classes)	.564	.630	.074
College Algebra (6 Classes)	.526	.639	.086
Trigonometry (6 Classes)	.632	.714	.105
Calculus (4 Classes)	.532	.742	.206
Honors (1 Class)	.375	.771	.396

TABLE 3

Means, SDs and Median Correlations of Variables with First Semester
College Mathematics Grades for Four Classes

Variable	<u>Intermediate Algebra</u>			<u>Trigonometry</u>			<u>College Algebra</u>			<u>Calculus</u>		
	Mean	SD	Med. r	Mean	SD	Med. r	Mean	SD	Med. r	Mean	SD	Med. r
College Grade	1.80	1.27	1.00	1.72	1.32	1.00	2.05	1.24	1.00	1.81	1.35	1.00
Math Placement Scores	15.04	5.45	.38	19.97	6.94	.46	23.19	7.95	.48	31.58	8.30	.50
ACT Math	19.42	4.22	.36	23.77	4.53	.30	25.03	4.18	.43	28.85	3.70	.15

Supporting the above study, Frankel¹⁴ evaluated the validity of placement tests and other predictor variables to predict first quarter mathematics grades. The sample included students of the University of Southern Illinois. There were 214 students who took Mathematics Placement 076, 132 students took Mathematics Placement 107, and 317 students took Advanced Mathematics Placement. The predictor variables were Mathematics Placement test scores, high school rank, high school math grade, and the ACT English, Math, Natural Science, and Composite score. The criteria was the final course grade.

The results showed that the prediction of course grades yielded multiple R's ranging from .375 to .654. The highest multiple R is 0.654 between Math 107 Placement test and Math 108.

The order of the best predictors for all beginning math courses indicated that the Math Placement score appeared in the first or second place five times in the six major regressions. ACT Math scores were in third place three times. A conclusion could be made that the Math Placement Exam score is the best single predictor to predict academic achievement in first-year college mathematics.

A study conducted by Helmick¹⁵ seems to agree with the above two studies. The purpose is to develop a possible methodology that could

¹⁴Frankel, Charlene M., "The Validity of Placement Tests and Other Predictor Variables in the Placement of Students in Beginning Mathematics Courses at Southern Illinois University at Carbondale," Doctoral Dissertation, SIU, 1976.

¹⁵Helmick, Faith E., "Evaluation of the Placement Test for First-Year Mathematics at the University of Akron," Doctoral Dissertation, 1983.

assist institutions of higher education to evaluate the effectiveness of mathematics placement programs. The subjects were 795 students who took the Mathematics Placement tests and completed the course "3450: III Algebra" at the University of Akron, Ohio. The final predictors were Mathematics Placement Test score, the overall high school GPA and the ACT Mathematics score. The study concluded that institutionally designed Mathematics Placement Test is a good predictor of success in college algebra.

Kohler¹⁶ conducted a study to report the findings of the ability of the Cooperative Mathematics Test, Algebra III, Form B (COOP) as the placement test, the ACT Mathematics Usage Test, and the ACT Composite (ACTC) to predict College Algebra GPA. The subjects were 161 students enrolled in college Algebra in the fall semester 1972 at Mississippi State University. Pearson product-moment correlations between all pairs of the four variables were computed.

Table 4 is shown on the next page. Even though the correlation between GPA in College Algebra and COOP ($r = 0.53$) is the highest, the correlation between GPA and ACTM is $r = 0.52$. Thus, the COOP is a moderately reliable test for placement purposes.

¹⁶Kohler, Emmett T., "The Relationship Between the Cooperative Mathematics Test, Algebra III, ACT Mathematics Usage Test, ACT Composite, and Grade Point Average in College Algebra," Educational and Psychological Measurement, 1973, vol. 33, pp. 929-931.

TABLE 4
Pearson Correlation Coefficient Between
Predictor Variables and Criterion Variables

Test	1	2	3	4
1. COOP	-			
2. ACTM	0.63	-		
3. ACTC	0.53	0.74	-	
4. GPA	0.53	0.52	0.4	-

Summary of Studies

Review of related studies indicates that a variety of predictors were used to predict academic achievement in introductory mathematics courses. Most of the studies concluded that ACT Mathematics Usage score and Mathematics Placement Examination score were among the best predictors for predicting success in beginning college mathematics. For lower level courses, the Mathematics Placement Exam seemed to be a moderately reliable predictor compared to ACT Mathematics, but for higher level math courses, such as Calculus I, the Mathematics Placement Examination is the best single predictor for academic achievement.

In this study, the reliability of using the Mathematics Placement Examination, Form 4/4A by the Mathematical Association of America (1981) and ACT Mathematics Usage scores to predict success in Intermediate Algebra, College Algebra, and Calculus I will be investigated.

CHAPTER 3

DESIGN OF STUDY

The primary purpose of this study was to find out which predictor variable, the ACT Mathematics scores or the Mathematics Placement scores, best predict the course grade for Intermediate Algebra, College Algebra, and Calculus I, respectively. In order to do this, correlations will be computed between the two predictor variables and the grades in the three courses. In addition, the correlation between the predictor tests will be computed.

The Variables Used in This Study

The variables used in this study are those which Kansas State University had on file for the subjects under study. These variables are Kansas State University identity numbers of the subjects, ACT Mathematics scores for each subject, Mathematics Placement scores for each subject, and the final course grade received, (including letter grade A, B, C, D, F, and numerical grades A = 4, B = 3, C = 2, D = 1, F = 0).

Subjects

The subjects used for this study include 359 students who enrolled in Intermediate Algebra, 1049 students who enrolled in College Algebra, and 491 students who enrolled in Calculus I. All subjects were enrolled in the fall semester of 1985-86.

Data

A request was made in the Student Academic Office for the information required for this study. There were not any names printed on the computer print-out for any subjects in order to preserve confidentiality. The subjects were observed by their observation numbers. For instance, the subjects in Intermediate Algebra were numbered 1 through 359, the subjects in College Algebra were numbered 1 through 1049, and the subjects in Calculus I were numbered 1 through 491.

From the information listed for each subject, only the following pieces of data were used:

1. Social Security Number (SSN) of Kansas State University for each subject.
2. Letter grades that were received by each subject at the end of the fall semester 1985-86 (A, B, C, D, F).
3. Numerical grades that were converted from the letter grades received (A = 4, B = 3, C = 2, D = 1, F = 0).
4. The ACT Mathematics score for each subject. A minimum score of 0 and a maximum score of 36.
5. The Mathematics Placement score for each subject. A minimum score of 0 and a maximum score of 32.

6. The class identification for each subject enrolled in each separate course:

MATH 010 for Intermediate Algebra

MATH 100 for College Algebra

MATH 220 for Calculus I

7. Observation numbers for each subject:

1-359 for Intermediate Algebra

1-1049 for College Algebra

1-491 for Calculus I

Statistical Method for Processing the Data

All the data was analyzed by using Statistical Analysis System (SAS), which is available to all at Kansas State University.

Pearson Product-Moment Correlation Coefficient and Related Hypotheses

Pearson r correlation is an index of the relationship between two variables.

In this study, Pearson r correlation was calculated between final course grade and the two predictor variables—ACT Mathematics score and Mathematics Placement score. Also, Pearson r correlation was computed between ACT Mathematics score and Mathematics Placement score.

There were 359 subjects who enrolled in Intermediate Algebra; only 339 had received a grade, only 781 subjects in College Algebra had received a grade, and only 405 subjects had received a grade in Calculus I.

TABLE 5

Summary and Abbreviation for the Variables Used for This Study

Abbreviation	Description
SSN	Social Security Number of each subject.
L-GRADE	Letter grades received by each subject. A, B, C, D, F.
N_GRADE	Numerical grades converted from the letter grades received by each subject. A = 4, B = 3, C = 2, D = 1, F = 0.
PLACEMENT	The Mathematics Placement score for each subject, 0 through 32.
ACT_MATH	The ACT Mathematics Score of each subject from 0 through 36.
CLASS ID	Class identification number of each subject. MATH 010 - Intermediate Algebra MATH 100 - College Algebra MATH 220 - Calculus I
CARD ID	Observation numbers of each subject. 1 - 359 Intermediate Algebra 1 - 1049 College Algebra 1 - 491 Calculus I

Accordingly, the subjects that received grades in each individual course are the ones who contributed to the calculation of Pearson product-moment coefficient correlation.

Listed below are two hypotheses that pertain to the relationship between independent variables and the criterion variable, the course grade in Intermediate Algebra.

Hypothesis IA 1

The correlation coefficient between the ACT Mathematics score and final course grade will not be different from zero.

Hypothesis IA 2

The correlation coefficient between the Mathematics Placement Examination score and final course grade will not be different from zero.

There are two hypotheses that pertain to the relationship between predictor variables and final course grade in College Algebra listed below.

Hypothesis CA 1

The correlation coefficient between ACT Mathematics and final course grade will not be different from zero.

Hypothesis CA 2

The correlation coefficient between the Mathematics Placement Exam score and final course grade will not be different from zero.

The following hypotheses are those that pertain to the

relationship between the two predictor variables and final course grade in Calculus I as the criterion variable.

Hypothesis CALC 1

The correlation coefficient between the ACT Mathematics score and the course grade will not be different from zero.

Hypothesis CALC 2

The correlation coefficient between the Mathematics Placement Exam score and course grade will not be different from zero.

Listed below are hypotheses that pertain to the relationship between each of the predictor variables, ACT Mathematics score and Mathematics Placement score in each course.

Hypothesis IA 3

The correlation coefficient between the ACT Mathematics score and the Mathematics Placement Exam score for Intermediate Algebra will not be different from zero.

Hypothesis CA 3

The correlation coefficient between the ACT Mathematics score and the Mathematics Placement score for College Algebra will not be different from zero.

Hypothesis CALC 3

The correlation coefficient between the ACT Mathematics score and the mathematics Placement score for Calculus I will not be different from zero.

CHAPTER 4

ANALYSIS OF DATA

Correlation Analysis

Pearson product-moment correlation coefficients were computed between each of the predictor variables and the criterion variable, and between the two predictor variables.

Table 6 shows the Pearson correlation coefficients among the predictor variables and criterion variable in Intermediate Algebra as presented in a correlation matrix.

Table 7 shows the Pearson correlation coefficients among the predictor variables and the criteria in College Algebra as presented in a correlation matrix.

Table 8 shows the Pearson correlation coefficients among the predictor variables and the criteria in Calculus I as presented in a correlation matrix.

Nine hypotheses concerning the correlation coefficients between the variables were analyzed, and the results were as follow.

Hypothesis IA 1

The null hypothesis was rejected, ACT Mathematics score and final course grade in Intermediate Algebra had a correlation coefficient of 0.163 which is significant at 0.015 level.

TABLE 6
MATH 010 - Intermediate Algebra

Correlation Coefficients/Prob under $H_0: r = 0$ /Number of Observations

	N_GRADE	PLACEMENT	ACT_MATH
N_GRADE	1.000	0.285	0.163
	0.000	0.001	0.0150
	339	233	221
PLACEMENT	0.285	1.000	0.245
	0.0001	0.000	0.0017
	233	247	162
ACT_MATH	0.163	0.245	1.000
	0.0150	0.0017	0.000
	221	162	227

TABLE 7
MATH 100 – College Algebra

Correlation Coefficients/Prob under $H_0: r = 0$ /Number of Observations

	N_GRADE	PLACEMENT	ACT_MATH
N_GRADE	1.000	0.497	0.468
	0.000	0.001	0.0001
	781	489	614
PLACEMENT	0.497	1.0000	0.6002
	0.0001	0.0000	0.0001
	489	693	552
ACT_MATH	0.468	0.6002	1.0000
	0.0001	0.0001	0.0000
	614	552	881

TABLE 8
MATH 220 - Calculus 1

Correlation Coefficients/Prob under $H_0: r = 0$ /Number of Observations

	N_GRADE	PLACEMENT	ACT_MATH
N_GRADE	1.000	0.5034	0.3704
	0.000	0.0001	0.0001
	405	328	331
PLACEMENT	0.5034	1.0000	0.5427
	0.0001	0.000	0.0001
	328	405	333
ACT_MATH	0.3705	0.5427	1.0000
	0.0001	0.0001	0.0000
	331	333	395

Hypothesis IA 2

The null hypothesis was rejected, Mathematics Placement score and final course grade in Intermediate Algebra had a correlation coefficient of 0.285 which is significant at 0.0001 level.

Hypothesis IA 3

The null hypothesis was rejected, ACT Mathematics score and Mathematics Placement score in Intermediate Algebra had a correlation coefficient of 0.245 which is significant at 0.0001 level.

Hypothesis CA 1

The null hypothesis was rejected, ACT Mathematics score and final course grade in College Algebra had a correlation coefficient of 0.468 which is significant at 0.001 level.

Hypothesis CA 2

The null hypothesis was rejected, Mathematics Placement score and final course grade in College Algebra had a correlation coefficient of 0.497 which is significant at 0.0001 level.

Hypothesis CA 3

The null hypothesis was rejected, ACT Mathematics score and Mathematics Placement score in College Algebra had a correlation of 0.6002 which is significant at 0.0001 level.

Hypothesis CALC 1

The null hypothesis was rejected, ACT Mathematics score and final course grade in Calculus I had a correlation coefficient of 0.3705 which is significant at 0.0001 level.

Hypothesis CALC 2

The null hypothesis was rejected, Mathematics Placement score and final course in Calculus I had a correlation coefficient of 0.5034 which is significant at 0.0001 level.

Hypothesis CALC 3

The null hypothesis was rejected, ACT Mathematics Placement score in Calculus I had a correlation coefficient of 0.5427 which is significant at 0.0001 level.

CHAPTER 5

FINDINGS AND CONCLUSIONS

Findings

From Chapter 4, a correlational analysis of data was done and the findings are reported below.

I. Intermediate Algebra

All three null hypotheses concerning Intermediate Algebra were rejected. This implies that each correlation between the predictors and the criteria is significant at $p < .01$ level. ACT Mathematics score has a correlation coefficient with $r = 0.163$ at $p = 0.015$ when the final course grade in Intermediate Algebra is the criterion variable. The correlation coefficient for Mathematics Placement Examination with final course grade is $r = 0.245$ which is significant at $p = 0.0001$. Thus, the Mathematics Placement Examination is the better predictor for success in Intermediate Algebra than the ACT Mathematics.

II. College Algebra

Again, the three null hypotheses concerning College Algebra were rejected. Each correlation between the two predictors and the criteria is significant at $p < .01$ level. The Mathematics Placement Examination has the highest correlation with the final course grade in College Algebra ($r = 0.497$, $p = 0.0001$). ACT Mathematics has a correlation of $r = 0.468$ at $p = 0.0001$ level. Also, the correlation between Mathematics Placement Examination and ACT Mathematics is $r = 0.6002$

which is significant at $p = 0.0001$ level. Thus, for College Algebra, we can conclude that the Mathematics Placement Examination is the better single predictor for academic performance.

III. Calculus I

The three null hypotheses concerning Calculus I were rejected. Therefore, each correlation between the two predictors and the final course grade is significant at $p < 0.01$ level.

The Mathematics Placement Examination has the highest correlation with the course grade in Calculus I with $r = 0.5034$ which is significant at $p = 0.0001$ level, while ACT mathematics has a correlation $r = 0.3075$, $p = 0.0001$ with the course grade in Calculus I. There is a substantial increase of 0.1329 correlation between Mathematics Placement Examination and ACT Mathematics in predicting academic success in Calculus I.

In addition, the correlation between Math Placement Exam and ACT Mathematics is $r = 0.5427$ at $p = 0.0001$.

A conclusion could be made that Mathematics Placement Examination improved the prediction of success in Calculus I; hence, it is the best single predictor for Calculus I.

IV. Item Analysis

As a further analysis of the test data, not related to this study, but of interest to those wanting to use the Mathematics Placement Examination, an item analysis was made of the correct responses to the Mathematics Placement Examination taken by freshmen enrolled in either Intermediate Algebra, College Algebra, or Calculus I at Kansas State

University. There were 32 questions in the Mathematics Placement Examination, Form A/4A by MAA (1981).

From Table 9, there were 5 items that failed to produce less than 20% difference in scores from College Algebra and Calculus I (questions 2, 6, 7, 8, 9). This indicates that Calculus I students have a better math ability than College Algebra students on nearly all the questions.

There were 6 items that failed to produce less than 20% difference in scores from College Algebra and Intermediate Algebra (questions 1, 8, 10, 11, 20, 31). Therefore, College Algebra students are better than Intermediate Algebra students.

Thus, the Mathematics Placement Examination had differential predictive ability for both content and ability groupings.

Conclusions

From the results of the present study at Kansas State University, we can conclude that both predictors, the Mathematics Placement Examination (Form A/4A) scores and ACT Mathematics scores did relate significantly ($p < .01$) to the final course grade in each course of Intermediate Algebra, College Algebra, and Calculus I.

Furthermore, the two tests correlate quite highly with each other. This indicates that either tests could be used for prediction. However, since the coefficients for correlation are higher for the Mathematics Placement Examination when tested against grades, it appears to be the better of the predictors to use in advising students.

TABLE 9
Percentage of Correct Responses Per Question on the Mathematics
Placement Exam. Form A/4A (1981).

Item	Calc. I (CI)	Col. Alg. (CA)	Int. Alg. (IA)	Diff. CI to CA	Diff. CA to IA
1	78	55	41	23	15
2	92	81	56	11	25
3	88	67	39	21	28
4	83	62	26	21	36
5	86	49	59	37	10
6	79	66	42	13	24
7	92	78	50	14	28
8	50	32	25	18	7
9	85	78	46	7	32
10	82	44	25	42	19
11	70	50	47	20	3
12	89	66	30	23	36
13	86	39	14	47	25
14	70	31	10	39	21
15	88	66	32	22	34
16	63	41	17	22	24
17	90	55	29	35	26
18	78	46	19	32	27
19	75	48	7	27	41
20	82	36	17	46	19
21	70	29	6	41	23
22	66	37	9	29	28
23	63	29	6	34	23
24	69	39	7	30	32
25	94	63	15	31	48
26	71	46	10	25	36
27	73	32	9	41	23
28	66	31	6	35	25
29	65	36	8	29	28
30	58	31	5	27	26
31	41	19	3	22	16
32	66	32	7	34	25

COLLEGE ALG.

$\bar{X} = 15.30$
 $N = 1049$
 $\sigma = 5.65$

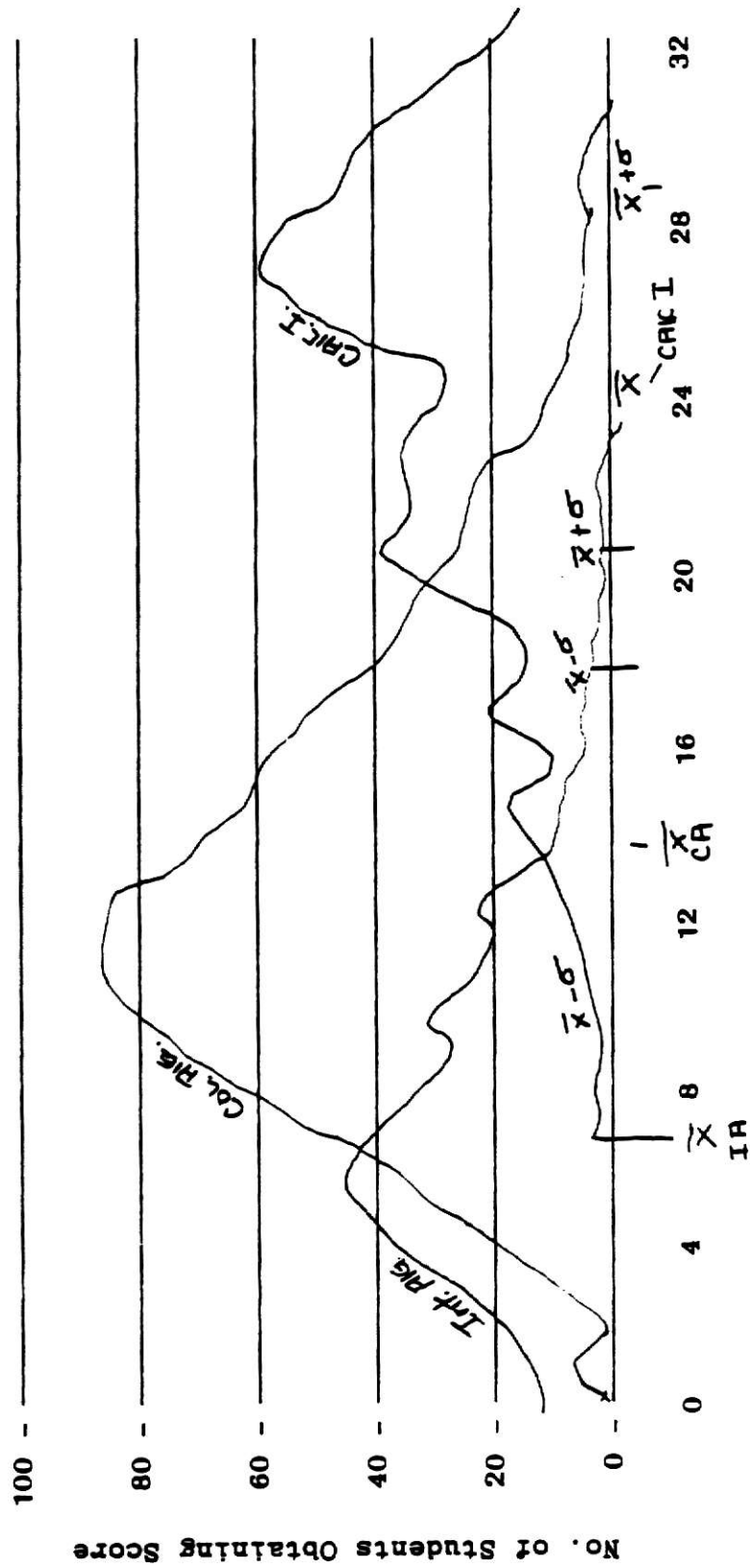
CALC I.

$\bar{X} = 24.23$
 $N = 491$
 $\sigma = 5.0$

INT. ALGE.

$\bar{X} = 7.36$
 $N = 359$
 $\sigma = 3.98$

Distribution of Scores
 On The Mathematics Placement
 Exam, Form A/4A (1981) By Type
 Of Enrollment



No. of Correct Responses

Figure 1.

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APPENDIX

TABLE A
Class ID = MATH 010

Variable	N	Mean	Std_Dev	Sum	Min	Max
N_GRADE	339	1.8761	1.3063	636.0	0.0	4.0
PLACEMENT	247	7.9231	4.3652	1957.0	0.0	25.0
ACT_MATH	227	13.8414	5.6162	3142.0	1.0	28.0

Table A shows a copy of computer print-out of statistics in Intermediate Algebra.

Sum of Squares and Crossproducts

	N_GRADE	PLACEMENT	ACT_MATH
N_GRADE	1770	4125	6227
PLACEMENT	4125	20193	20023
ACT_MATH	6227	20023	50618

TABLE B
Class ID = MATH 100

Variable	N	Mean	Std_Dev	Sum	Min	Max
N_GRADE	781	1.6927	1.2931	1322.0	0.0	4.0
PLACEMENT	693	16.0592	5.6602	11129.0	0.0	30.0
ACT_MATH	811	19.7632	5.7470	16028.0	1.0	35.0

Table B shows a copy of computer print-out of statistics in College Algebra.

Sum of Squares and Crossproducts

	N_GRADE	PLACEMENT	ACT_MATH
N_GRADE	3542	17106	24452
PLACEMENT	17106	200893	188242
ACT_MATH	24452	188242	343518

TABLE C
Class ID = MATH 220

Variable	N	Mean	Std_Dev	Sum	Min	Max
N_GRADE	405	1.9975	1.3116	809.0	0.0	4.0
PLACEMENT	405	25.2296	4.6750	10218.0	8.0	32.0
ACT_MATH	395	26.4911	4.2804	10464.0	8.0	36.0

Table C shows a copy of computer print-out of statistics in Calculus I.

Sum of Squares and Crossproducts

	N_GRADE	PLACEMENT	ACT_MATH
N_GRADE	2311	18623	18738
PLACEMENT	18623	266626	227840
ACT_MATH	18738	227840	284422

A COMPARISON BETWEEN MATHEMATICS PLACEMENT EXAMINATION
AND ACT MATHEMATICS ON CERTAIN CLASSES OF STUDENTS
AT KANSAS STATE UNIVERSITY

by

SAKIRAH ZAKARIA

B.S., Kansas State University, 1984

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Mathematics

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1986

The objectives of this study were 1) to determine whether there is a significant correlation at the 0.01 level between academic success in Intermediate Algebra, College Algebra, and Calculus I and prior mathematics achievement as measured by the Mathematical Association of America Mathematics Placement Examination, Form A/4A (1981) and ACT Mathematics Usage Scores; and, 2) to determine which predictor variables best predict success in each mathematics course.

The predictor variables were Mathematics Placement Examination scores and ACT Mathematics scores, and the criterion variable was the final course grades received.

At the beginning of the fall semester 1985, the placement tests were administered to all students who enrolled in Intermediate Algebra, College Algebra, and Calculus I at Kansas State University.

Pearson Product-Moment Correlations were computed between each pair of predictor variables, and between each predictor variable and the criterion variable.

A significant correlation at the 0.01 level was found between Mathematics Placement Exam scores and course grades, ACT Mathematics scores and course grades, and between Mathematics Placement Exam scores and ACT Mathematics scores in each mathematics course.

The highest correlation, with grades in Intermediate Algebra, College Algebra, and Calculus I, is the Mathematics Placement Examination with $r = 0.285$, $p = 0.0001$, $r = 0.497$, $p = 0.0001$ and $r = 0.5034$, $p = 0.0001$, respectively.