# A STUDY OF STRIP TRANSMISSION LINES

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Diploma, Taipei Institute of Technology, 1963



A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1970

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# Introduction

## 1. The aims of the report

The last two decades have seen the large scale introduction of various forms of the strip transmission line, both shielded and unshielded, in an attempt to simplify the construction of microwave components. The high cost, bulk and ever-increasing complexity of equipment fabricated with standard waveguides has stimulated interest in several alternative types of transmission lines. Counterparts of most coaxial or waveguide components, such as hybrid junctions, directional couplers, etc., can be realized in planar form, and this has suggested the possibility of fabricating quite complicated component assemblies which would, moreover, have the advantage of relatively small bulk and weight.

Various forms of shielded strip transmission lines have been devised and discussed in the literature. Fig. 1 shows some configurations. Some have inner conductors consisting of a single strip or a solid inner bar, and with or without a filling of solid dielectric, Fig. 1a and 1c. Others have an inner conductor made up of a dielectric card with a thin conducting strip on one or both sides (with the dielectric occasionally not extending beyond the edges of the strips), and air spaced from the ground planes, Fig. 1b and 1d. In some applications, striplines are made of two microstrip lines (a microstrip line is made of a single strip and a ground plane on opposite

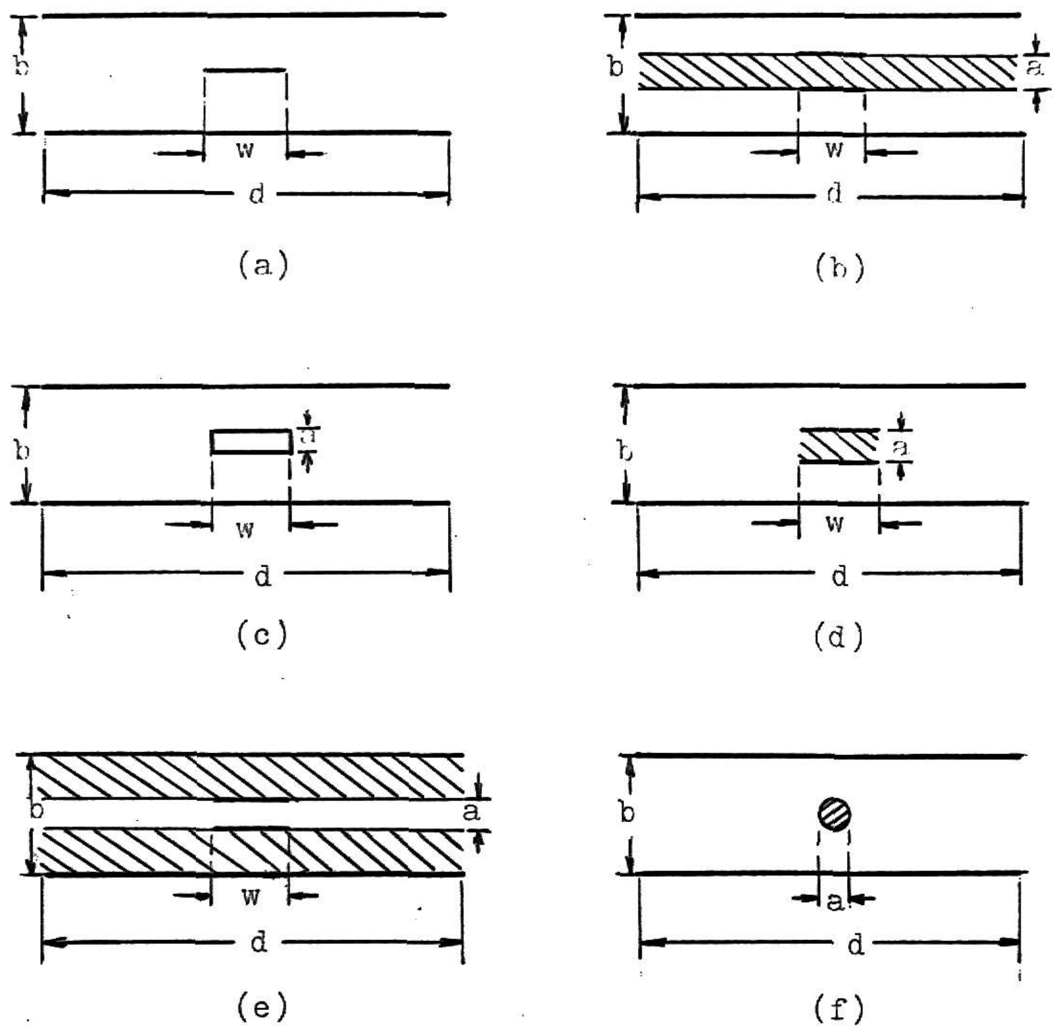



Fig. 1. Various forms of shielded strip line

(a) single strip; (b) double strip, dielectric supported; (c) solid inner bar; (d) double strip, partial dielectric supported; (e) two microstrip lines with the strips face to face; (f) circular inner rod.

faces of a sheet of dielectric material) with the ground planes on the top and bottom, Fig. 1e. Replacement of the rectangular bar by a circular rod has also been discussed, Fig. 1f, as this has obvious advantages in construction. These ground planes are sometimes closed at the sides to form a closed rectangular conducting tube.

As we know from transmission line theory, the transfer of energy from one point to another is only one use of a transmission line. At ultrahigh frequencies an equally important application is the use of sections of lines as circuit elements, because for frequencies higher than 150 MHz ordinary lumped circuit elements of high quality become difficult to construct. Here, the required transmission line sections become small enough to be used as circuit elements. They can be used in this manner up to about 3 GHz where their physical size then becomes too small and wave-guide techniques begin to take over. For both applications, an extensive use is made of the characteristic impedance of the line. For this reason, this important class of problems of computing the characteristic impedance in strip transmission lines will be considered in this report.

This report attempts

1) To outline methods applicable to this class of problems
2) To present the mathematical theory for their numerical analysis
3) To summarize the numerical analysis procedures and their use in constructing a computer program and
4) To give some important engineering results for practical considerations.

In chapter 2, formulas for the characteristic impedance of a strip line are developed and the numerical technique is introduced to obtain solutions. Chapter 3 gives the flow chart of a computer program based on the procedures discussed in Chapter 2. Some curves of the results for the specific case of Fig. 1e are shown in chapter 4. Also in chapter 4, the quality of computed results are discussed. Finally, conclusions are given in Chapter 5.

## 2. TEM-mode in a transmission line

When the strip conductors are enclosed between ground planes or embedded in a homogeneous isotropic dielectric medium of infinite extent, it is possible to assume uniform propagation in a TEM-mode, provided that certain critical dimensions do not exceed an appreciable fraction of a wavelength.

Before starting our analysis of the problem, we would like to review some characteristics of the TEM-mode in a transmission line. We shall begin by writing Maxwell's equations for the dielectric region in rectangular coordinates, with the propagation factor $e^{(j\omega t-\gamma z)}$ substituted. Hence we assume that the wave is propagating along the z-direction with a variation of $e^{-\gamma z}$. The curl equations are written below for fields in the dielectric of the system.

$$\nabla x\overline{E} = -j\omega\mu\overline{H}$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \qquad (1)$$

$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \qquad (2)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \qquad (3)$$

$$\nabla \times \overline{H} = j\omega\varepsilon\overline{E}$$

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x \tag{4}$$

$$-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \tag{5}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \tag{6}$$

It must be noted that the components $E_x$, $H_x$, $E_y$, etc., are functions of x and y only, by our agreement to take care of the z and time functions in the assumed $e^{(j\omega t-\gamma z)}$ factor.

From the above equations, it is possible to solve for $E_x$, $E_y$, $H_x$, or $H_y$ in terms of $E_z$ and $H_z$. We have

$$H_x = \frac{1}{\gamma^2 + K^2} \left(j\omega\varepsilon\frac{\partial E_z}{\partial y} - \gamma\frac{\partial H_z}{\partial x}\right) \tag{7}$$

$$H_y = \frac{1}{\gamma^2 + K^2} \left(j\omega\varepsilon\frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y}\right) \tag{8}$$

$$E_x = \frac{1}{\gamma^2 + K^2} \left(-\gamma\frac{\partial E_z}{\partial x} + j\omega\mu\frac{\partial H_z}{\partial y}\right) \tag{9}$$

$$E_y = \frac{1}{\gamma^2 + K^2} \left(-\gamma\frac{\partial E_z}{\partial y} + j\omega\mu\frac{\partial H_z}{\partial x}\right) \tag{10}$$

where,

$$K^2 = \omega^2\mu\varepsilon.$$

The total electric and magnetic intensities in the charge-free region between the conducting boundaries must also satisfy the wave equation

$$\nabla^2\overline{E} = -K^2\overline{E} \quad , \quad \nabla^2\overline{H} = -K^2\overline{H}$$

The three-dimensional $\nabla^2$ may be broken into two parts:

$$\nabla^2\overline{E} = \nabla^2_{xy}\overline{E} + \frac{\partial 2\overline{E}}{\partial z^2}$$

The first term is the two-dimensional Laplacian in the transverse plane, representing contributions to $\nabla^2$ from derivatives in this plane. The last term is the contribution to $\nabla^2$ from derivatives in the axial direction. By the assumption that the propagation function is $e^{-\gamma z}$ in the z direction,

$$\frac{\partial^2\overline{E}}{\partial z^2} = \gamma^2\overline{E}$$

Hence, the foregoing wave equations may be written as

$$\nabla^2_{xy}\overline{E} = -(\gamma^2 + K^2)\overline{E} \tag{11}$$

$$\nabla^2_{xy}\overline{H} = -(\gamma^2 + K^2)\overline{H} \tag{12}$$

Equations (11) and (12) are the differential wave equations that must be satisfied in the dielectric regions of transmission lines.

One of the solutions to the wave equations has neither electric nor magnetic fields in the direction of propogation, and is called TEM. The general relations between wave components as expressed by equations (7) to (10) show that, if $E_z$ and $H_z$ are zero, all other components must be zero also, unless $\gamma^2 + K^2 = 0$ at the same time. Thus, a TEM wave must satisfy

$$\gamma^2 + K^2 = 0$$

or,

$$\gamma = \pm jK = \pm j\omega\sqrt{\mu\varepsilon} \tag{13}$$

With Equation (13) satisfied, the wave equations (11) and (12) reduce to

$$\nabla^2_{xy} \bar{E} = 0 \qquad , \qquad \nabla^2_{xy} \bar{H} = 0 \tag{14}$$

These are in the form of the two-dimensional Laplace's equation in the transverse plane. Since $E_z$ and $H_z$ are zero, $\bar{E}$ and $\bar{H}$ lie entirely in the transverse plane. We recall that the electric field satisfies Laplace's equation under static conditions. The same is true for the magnetic field. Consequently it may be concluded that the field distribution in the transverse plane is exactly a static distribution, if it can be shown that the boundary conditions to be applied to the differential equations (14) are the same as those for a static field distribution. In a transmission line, the boundary condition for the TEM wave on a perfect conducting guide is that the electric field at the surface of the conductor can have a normal component only but not a tangential component, which is the same as the boundary condition at the conducting surface in the static case. Hence, the problem of concern is reduced to the two-dimensional electrostatic problem with the imposed boundary conditions in the transverse plane.

3. Validity of the TEM approach for composite lines

It is questionable to what extent the TEM approach is valid when a line has a discontinuous medium over its cross section. This question follows from the application of wave equations (11) and (12). For TEM propagation, the propagation constant is given by

$$\beta = I_m(\gamma) = \omega \sqrt{\mu\varepsilon}$$

If the dielectric permittivity $\varepsilon$ has two different values, one value in one medium and another value in the other medium, $\beta$ will have two values, one for each medium. This is contradictory since one wave can only possess one propagation constant. However, it is likely that a TEM wave will exist and its propagation constant can be expressed as

$$\beta = \omega \sqrt{\mu\varepsilon_e}$$

where $\varepsilon_e$ is an effective dielectric constant.

This is true because most of the power is concentrated in the region between the strip conductors and their closest ground planes, and in this region the field should be substantially TEM [1]. This suggests that the dominant mode is TEM and that the solutions based on the TEM approach give a reasonable approximation provided that the physical dimensions are much smaller than half a wavelength, which means that the operating frequency is far below cutoff for all higher order modes. Using a TEM approach to find the characteristic impedance or the propagation constant usually will cause a minute error, not amounting to more than a fraction of a per cent at frequencies of several GHz [2].

# Chapter 2

## Mathematical Analysis

In this chapter, we shall derive the fundamental equations and formulas for determining the parameters of the strip transmission line, i.e., the capacitance and characteristic impedance. We shall begin with Gauss's law, based on the assumption that the problem is TEM, and then derive Laplace's equation for the two-dimensional system. After that, we shall discuss a numerical technique to solve this two-dimensional Laplace equation, which will be used for the computer-aided calculation of some configurations of stripline in the later chapters.

Also, at the end of this chapter, we give some derived formulas, using the Schwarz-Christoffel transformation technique. In addition, there is another analytic solution which can be found in the literature, using the so called variational method. Formulas given by these two methods are usually tedious for practical use and are restricted to very simple configurations of the strip line. Because both analytic methods are so complicated, they will not be proved in this report. For a more detailed discussion, see [3], [4], and [5].

### 1. Gauss's law and Laplace's equation

We start with Gauss's law in order to formulate the equations for the numerical analysis. If the charge is continuously distributed throughout the volume with a charge density $\rho$, Gauss's law has the form

$$\oint_S \overline{D} \cdot d\overline{a} = \int_V \rho dv \tag{1}$$

By using the divergence theorem, Equation (1) can be changed into a differential form. From

$$\oint_S \overline{D} \cdot d\overline{a} = \int_V \nabla \cdot \overline{D}\, dv = \int_V \rho dv$$

it follows that

$$\nabla \cdot \overline{D} = \rho \tag{2}$$

which is the alternative statement of Gauss's law. In a region in which there are no charges ($\rho = 0$), it becomes

$$\nabla \cdot \overline{D} = 0 \tag{3}$$

As we mentioned before, our problem is concerned exclusively with a two-dimensional system. Moreover, Cartesian coordinates are suitable because of the configurations of the transmission line of interest. Thus, Equation (3) is written as, in the x-y plane

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0$$

or

$$\frac{\partial}{\partial x} \varepsilon E_x + \frac{\partial}{\partial y} \varepsilon E_y = 0 \tag{4}$$

where E is the electric field density and $\varepsilon$ is the permittivity of the medium at that point.

Recall that E is related to the potential V by

$$\overline{E} = -\nabla V \tag{5}$$

or

$$\overline{a}_x E_x + \overline{a}_y E_y = -\overline{a}_x \frac{\partial V}{\partial x} - \overline{a}_y \frac{\partial V}{\partial y} \tag{6}$$

in the x-y plane. This gives

$$E_x = -\frac{\partial V}{\partial x}\,, \qquad E_y = -\frac{\partial V}{\partial y} \tag{7}$$

Substituting Equation (7) into Equation (4) we have

$$\frac{\partial}{\partial x}\,\varepsilon\frac{\partial V}{\partial x} + \frac{\partial}{\partial y}\,\varepsilon\frac{\partial V}{\partial y} = 0 \tag{8}$$

Note that in a homogeneous medium, $\varepsilon$ is independent of position, and Equation (8) becomes

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \tag{9}$$

which is the usual form of the two-dimensional Laplace's equation.

2. Numerical solution and the finite-difference technique

We want to develop a numerical method to find the solution to Equation (9) by the so-called finite-difference method, which is an iterative technique and is suitable for the high-speed computer. To do so, consider a typical point P, not on a conductor, and eight adjacent points as shown in Fig. 2. Let the separation of each point be h/2 in each coordinate direction. The derivative of V with respect to x at point b is (with h small)

$$\left.\frac{\partial V}{\partial x}\right|_b \approx \frac{V_E - V_P}{h} \tag{10}$$

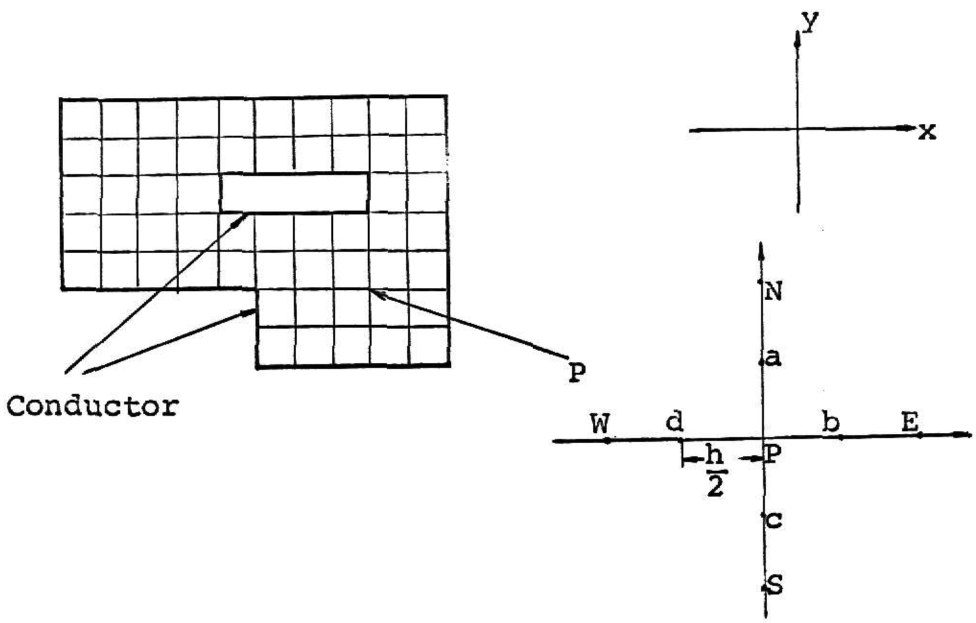



Fig. 2  Typical voltage node in a transmission line.

Similarly,

$$\left.\frac{\partial V}{\partial x}\right|_d \approx \frac{V_P - V_W}{h} \tag{11}$$

$$\left.\frac{\partial V}{\partial y}\right|_a \approx \frac{V_N - V_P}{h} \tag{12}$$

$$\left.\frac{\partial V}{\partial y}\right|_c \approx \frac{V_P - V_S}{h} \tag{13}$$

Continuing in the same manner, we have

$$\left.\frac{\partial}{\partial x}\varepsilon\frac{\partial V}{\partial x}\right|_P \approx \frac{\varepsilon_b \left.\frac{\partial V}{\partial x}\right|_b - \varepsilon_d \left.\frac{\partial V}{\partial x}\right|_d}{h} \tag{14}$$

Substituting equations (10), and (11) into (14), we have

$$\frac{\partial}{\partial x}\varepsilon \left.\frac{\partial V}{\partial x}\right|_P \simeq \frac{\varepsilon_b(V_E - V_P) - \varepsilon_d(V_P - V_W)}{h^2} \tag{15}$$

Similarly,

$$\frac{\partial}{\partial y}\ \varepsilon \left.\frac{\partial V}{\partial y}\right|_P \simeq \frac{\varepsilon_a \left.\frac{\partial V}{\partial y}\right|_a - \varepsilon_c \left.\frac{\partial V}{\partial y}\right|_c}{h}$$

$$\simeq \frac{\varepsilon_a(V_N - V_P) - \varepsilon_c(V_P - V_S)}{h^2} \tag{16}$$

Finally, we substitute equations (15) and (16) into (8), and after some manipulation we obtain

$$\frac{\varepsilon_a V_N + \varepsilon_b V_E + \varepsilon_c V_S + \varepsilon_d V_W}{\varepsilon_a + \varepsilon_b + \varepsilon_c + \varepsilon_d} - V_P = R \tag{17}$$

where R, the residual, is the difference between $V_P$ and the weighted average of the four adjacent voltages.

Note the case where the dielectric medium is discontinuous in the y-direction only (which is the only case we shall be concerned with), that is, for instance, the dielectric is discontinuous at point P. Then, $\varepsilon_b$ and $\varepsilon_d$ are assumed to be given by the average of the permittivities above and below the interface. That is,

$$\varepsilon_b = \varepsilon_d = \frac{\varepsilon_a + \varepsilon_c}{2} \tag{18}$$

This can be obtained from the requirement that the normal components of the electric flux density be continuous at the interfact [6].

Having formulated Equation (17), we are in position to employ a numerical relaxation method, the so called finite-difference method to solve our problem. A simple example will serve to illustrate this method. Consider a square duct containing a centrally located round wire, as shown in Fig. 3. Let it be required to find the potential distribution in it with +100 volts potential on the inner wire and zero volts on the duct. The space between conductors where a solution is desired is ruled off in meshes to establish points at which the potential is to be calculated, and a guess value is assigned to each point. From symmetry, only one-eighth of the whole cross section needs to be considered for this problem. At first, we guess the potential at all nonboundary points to be +50 volts. After calculating the potentials of the four points in the immediate neighborhood of each point between conductors, we substitute the potential values of these four adjacent points and the potential of the center point into Equation (17). It will be found that Equation (17) is not satisfied for some points with a nonzero residual. Each residual is written above the potential and is enclosed in parentheses as shown in Fig. 3. The potential of the point with the largest residual (-25 in Fig. 3b) is increased by the value of the residual. Once the potential of a point is changed, all other residuals have to be recalculated and will be changed also. This is shown in Fig. 3c. The procedure of finding the largest residual and adding it to the potential of that point is repeated until no smaller residual values can be reached. It can be seen that the mesh can be subdivided so as to make a more accurate solution as shown in

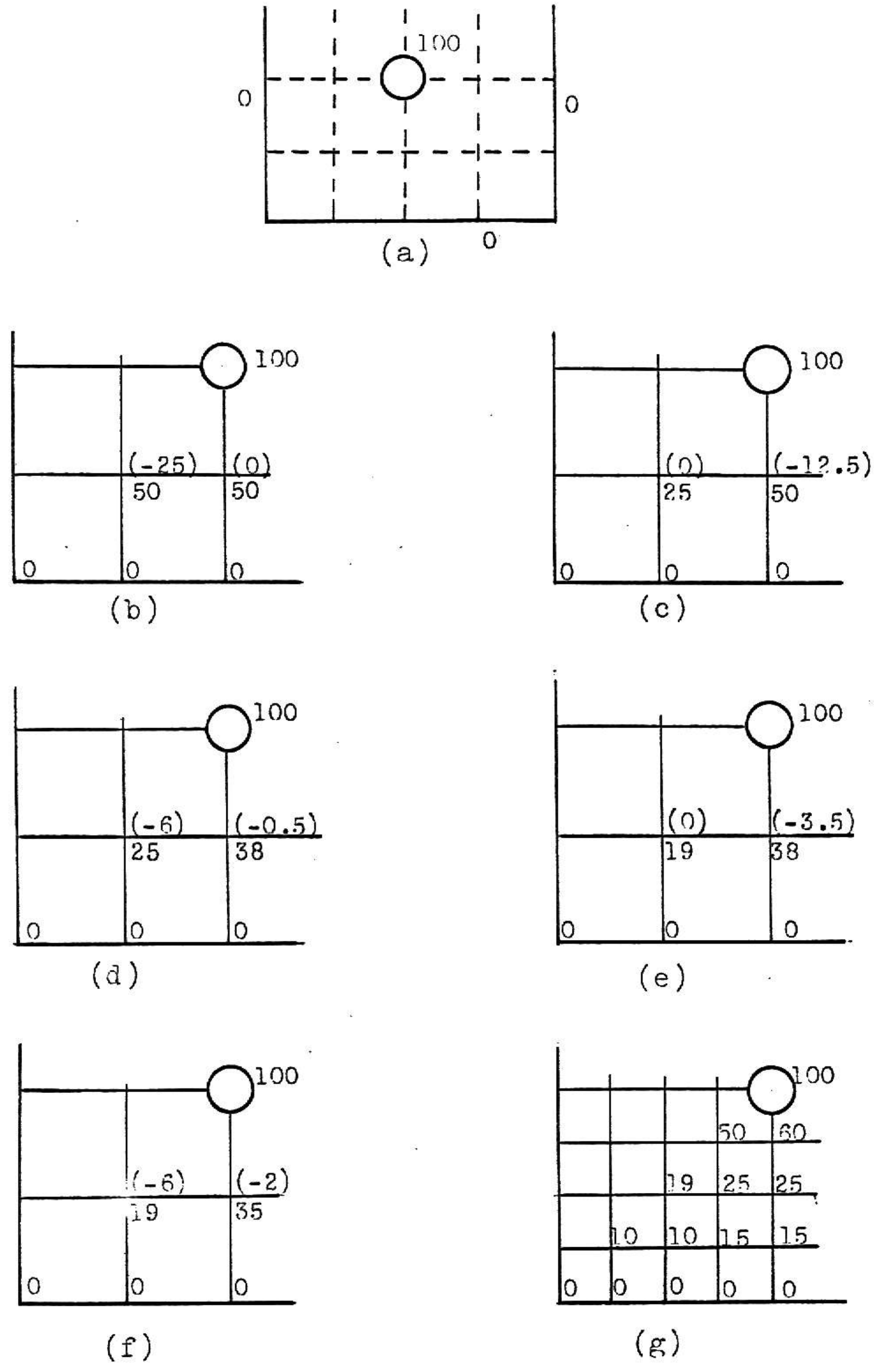



Fig.3. An example of the finite difference method.

Fig. 3g. Rather than use hand calculation, the computer might be employed to do this tedious work for more mesh nodes.

The relaxation process we have discussed is not fast because it involves a great deal of computation. A modified method, first proposed by Liebmann, has improved the speed of solution. [7] Liebmann's method converges fast and requires substantially less storage in the memory space of the computer. With this method, only one residual is calculated, instead of the entire array of residuals. Whenever the residual is found, the potential at that point is readjusted immediately. The advantages of this process are that it

1) involves only the constant repetition of a small group of machine orders,

2) allows the data relating to substantially large problems to be retained entirely in core, and

3) is terminable when any desired degree of residual tolerance has been attained.

Liebmann's method may be defined as the relaxation cycle,

$$V^{new}(I,J) = V^{old}(I,J) + \alpha\, R(I,J) \tag{19}$$

where $\alpha$ is a number called the overrelaxation factor, ranging in value from 1 to 2, $R(I,J)$ is the residual calculated from the potentials currently existing in the array of potential values and $(I,J)$ defines a mesh point. With each calculation, the old value of the potential $V(I,J)$ is replaced by an adjusted new value. With the aid of Equation (17), it is possible to write Equation (19) in the form

$$V^{n+1}(I,J)=V^{n}(I,J)+\alpha\left\{\frac{\varepsilon_a V^{n+1}(I-1,J)+\varepsilon_b V^{n}(I,J+1)+\varepsilon_c V^{n}(I+1,J)+}{\varepsilon_a+\varepsilon_b+\varepsilon_c+\varepsilon_d}\right.$$

$$\left.\frac{\varepsilon_d V^{n+1}(I,J-1)}{\varepsilon_a+\varepsilon_b+\varepsilon_c+\varepsilon_d} - V^{n}(I,J)\right\}$$

$$= (1-\alpha)V^n(I,J)+\alpha\left\{\frac{\varepsilon_a V^{n+1}(I-1,J)+\varepsilon_b V^n(I,J+1)+\varepsilon_c V^n(I+1,J)+\varepsilon_d V^{n+1}(I,J-1}{\varepsilon_a+\varepsilon_b+\varepsilon_c+\varepsilon_d}\right. \tag{20}$$

where superscripts indicate the order number of iteration cycle. The iteration cycles are successively carried out, with each cycle scanning all the points of the field. This calculation may be done for a considerable number of cycles until none of the residuals in the field has an absolute value greater than the tolerance desired. The speed of convergence depends on $\alpha$, the overrelaxation factor. The function of the overrelaxation factor, or so-called acceleration factor is explained as follows. As illustrated in the above example, throughout the relaxation process, potentials are increased by the corresponding residuals ($\alpha=1$) to depress residuals to zero. After the potential at a given point is changed, the residuals at all the other points will not remain zero. This knowledge gives an idea for the improvement of the process. That is, instead of relaxing with $\alpha=1$, each potential is overrelaxed ($\alpha>1$) such that the rise in residual that occurs on recalculation of the neighboring potential will bring the residual back to zero or some smaller value than would be the case for $\alpha=1$. It can be shown that this method is always convergent for $\alpha=1$, and always divergent for $\alpha=2$. Best convergence is obtained for some value between these two limits.

3. The characteristic impedance of the transmission line

Once the potential distribution of a transmission line is known the capacitance can be found without difficulty. To obtain capacitance, it is necessary to find the charge Q on the conductors. From Gauss's law, the enclosed charge in volume V is obtained from

$$Q = \int_V \rho dV = \oint_s \bar{D} \cdot d\bar{a} = \oint_s D_n \, da$$

where $D_n$ is the normal component of electric flux density and s is the surface enclosing V. $D_n$ is related to the electric intensity and potential by

$$D_n = \varepsilon E_n = -\varepsilon \frac{\partial V}{\partial n} \qquad (21)$$

where n denotes the normal coordinate.

We now form the surface enclosing the conductor by lines cutting between mesh points and drawn parallel to the coordinates, as shown in Fig. 4.

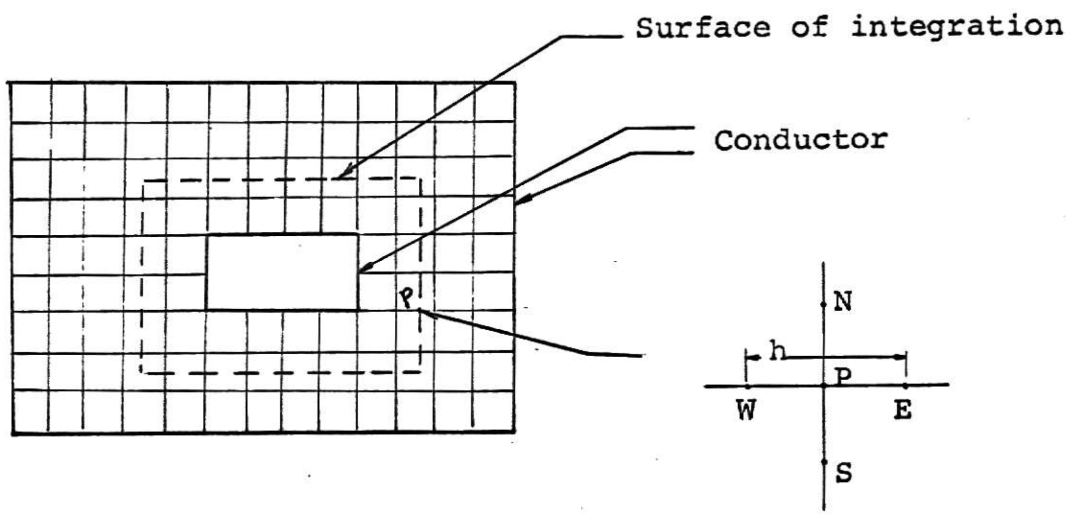



Fig. 4 Integration to determine charge.

As an illustration, if we choose a typical point P on the integration surface, then we have

$$- \frac{\partial V}{\partial n}\bigg|_P \simeq \frac{V_W - V_E}{h} \tag{22}$$

The electric flux density at that point is

$$D_n = -\varepsilon \frac{\partial V}{\partial n}\bigg|_P \simeq \varepsilon \frac{V_W - V_E}{h}$$

To obtain the total flux through the surface per meter of length we sum up all the flux densities at the potential nodes on the surface of integration. A special treatment is needed for the rectangular corners of this surface. Suppose it is desired to sum up the flux through two elemental areas adjacent to a typical corner as shown in Fig. 5. Let $\Delta\Psi_1$ and $\Delta\Psi_2$ be the amount of flux through these two areas. Referring to Fig. 5, we have

$$\Delta\Psi_1 = D_{np}\, h = \varepsilon(V_A' - V_p'')$$
$$\Delta\Psi_2 = D_{nQ}\, h = \varepsilon(V_A' - V_Q'')$$

and,

$$\Delta\Psi_1 + \Delta\Psi_2 = \varepsilon(2V_A' - V_p'' - V_Q'')$$

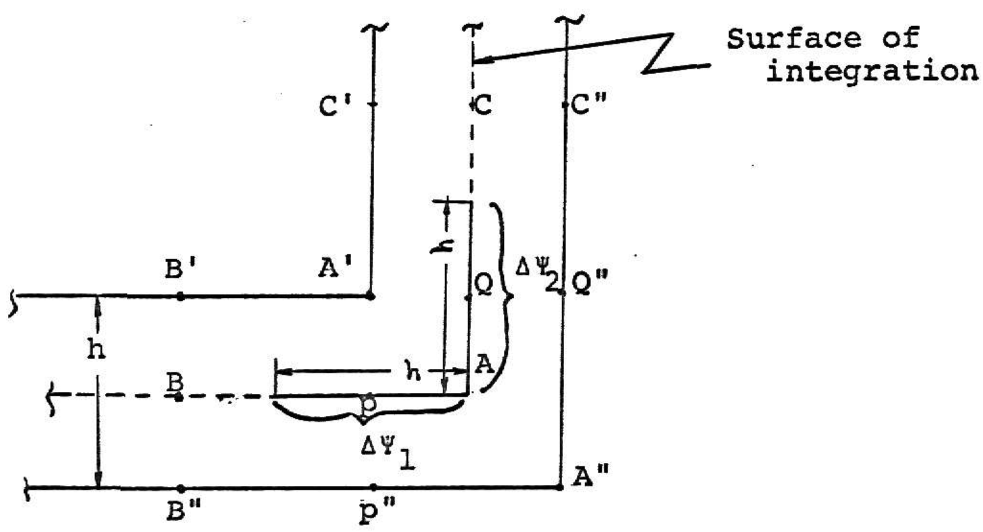



Fig. 5 A note on the flux calculation for corners.

Summing up other fluxes through the surface, the expression for the total flux becomes

$$Q = \Psi = \sum_i \Delta\Psi_i = \varepsilon\ (V'_B + 2V'_A + V'_C + \ldots\ldots)$$

$$- \varepsilon\ (V''_B + V''_p + V''_Q + V''_C + \ldots\ldots) \qquad (23)$$

We observe from the above expression that the potential of the inner vertex is used two times while that of the outer vertex is not used. Once the charge is computed, the capacitance can be found from

$$C = Q/V_c$$

where $V_c$ is the voltage difference between the conductors. Given capacitance, the characteristic impedance follows straightforwardly,

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

where, R, L, C, and G are the resistance, inductance, capacitance, and conductance per unit length of the transmission line. [8] At higher frequencies, the low-loss transmission line is of special interest, in which case we can assume that $R \ll \omega L$, $G \ll \omega C$ and therefore

$$Z_o = \sqrt{L/C} \qquad (24)$$

Since the inductance per unit length is not affected by the introduction of a pure dielectric within the line, L may be derived from the phase velocity of the TEM wave. Thus,

$$\nu = \omega/\beta = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon_e}} \qquad (25A)$$

and since L is independent of the dielectric within the line

$$\nu_\circ = 1/\sqrt{LC_\circ} \tag{25B}$$

or

$$L = \frac{1}{\nu_\circ^{2} C_\circ} \tag{25}$$

where $\nu_\circ$ is the velocity of light in air and $C_\circ$ is the capacitance of the transmission line with only air dielectrics.

Two cases of dielectric configuration need to be distinguished. When the medium is inhomogeneous, and we substitute Equation (25) into (24), we have

$$Z_\circ = \frac{1}{\nu_\circ \sqrt{C_\circ C}} \tag{26}$$

Hence, for the inhomogeneous case, two capacitances need to be determined. One is without dielectric, and the other is with dielectric.

However, when the medium is homogeneous, $C=\varepsilon_r C_\circ$ for the dielectric constant $\varepsilon_r$, and Equation (26) becomes

$$Z_\circ = \frac{1}{\nu_\circ C_\circ \sqrt{\varepsilon_r}} \tag{27}$$

Only $C_\circ$, the capacitance without the dielectric, is needed.

4. Analytic solutions

Some simple configurations of strip transmission lines have been solved analytically for the case of a homogeneous medium. Two methods may be found in the literature. One uses the Schwarz-Christoffel conformal transformation, while the other uses the variational approach.

An exact solution to the configuration shown in Fig. 1a is given by [3]

$$Z_o = 30\pi \frac{K(k')}{K(k)} \tag{28}$$

where

$$k = \frac{1}{\mathrm{Cosh}(\frac{\pi}{2}\frac{w}{b})}$$

$$k' = \sqrt{1-k^2}$$

$$K(k) = \int_o^{\pi/2} \frac{d\Psi}{\sqrt{1-k^2\ \sin^2\Psi}} \tag{29}$$

$$K(k') = \int_o^{\pi/2} \frac{d\Psi}{\sqrt{1-k'^2 \sin^2\Psi}}$$

K(k) and K(k') are elliptical integrals of the first kind. This solution is obtained by using the Schwarz-Christoffel comformal transformation. This technique also leads to the solution for the configuration shown in Fig. 1b with the dielectric slab removed [4]. The formula for finding that impedance is also given by Equation (28), where k is now related to the line geometry by

$$\frac{w}{b} = \frac{2}{\pi}\left\{\tan^{-1}h\left[\frac{k\frac{b}{a}-1}{\frac{1}{k}\frac{b}{a}-1}\right]^{\frac{1}{2}} - \frac{a}{b}\tan^{-1}\left[\frac{1}{k}\left(\frac{k\frac{b}{a}-1}{\frac{1}{k}\frac{b}{a}-1}\right)\right]^{\frac{1}{2}}\right\} \tag{30}$$

which is valid for $w/a>0.35$. The remainder of Equations (29) are formally the same. The quantity $K(k)/K(k')$ has been tabulated vs. k by Oberhettinger and Magnus [3].

Another analytic approach is the variation solution, which gives an upper bound of the exact impedance of the line. The variational solution relies upon finding a sufficiently accurate approximation for the charge distribution on the strip conductors. Then the Green's function technique is used to solve the resulting boundary value problem for an upper bound value. Unfortunately, the variation formula is very complicated as it involves several infinite series, and therefore will not be reproduced here.

# Chapter 3

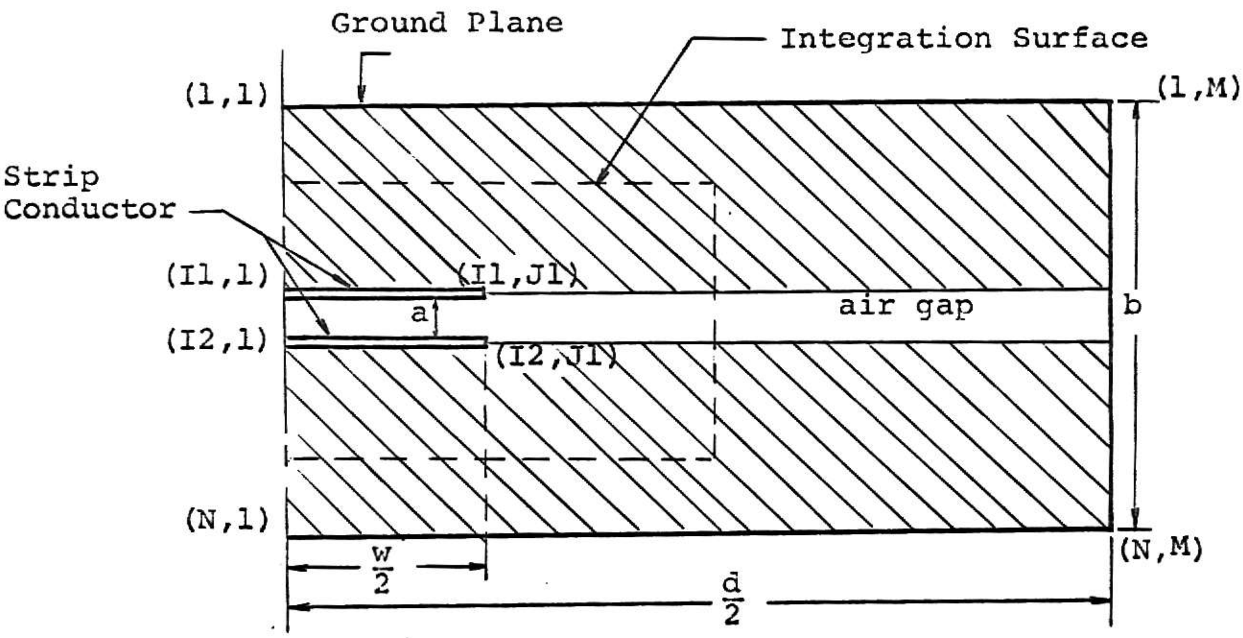
# Computer Program

## 1. Geometry of problem

Here, we want to employ the computer to find the characteristic impedance of the strip transmission line with the configuration shown in Fig. 1e. In construction, this kind of strip transmission line is made of two microstrip lines placed together with their strip conductors face to face. Because of construction difficulties (e.g. machine tolerances, etc.), there may be an air gap between the two faces as shown in Fig. 6. By symmetry, only one half of the whole cross section needs to be considered, as shown in Fig. 6, which also shows labels on the boundary locations (and hence the dielectric distribution). Whenever the ratios of line dimensions need to be changed we need only to change data of these locations for the computer program. Also note that in the calculation, we treat ground planes as closed at the sides, though they are not necessarily so in the practical construction. The error due to this treatment is small, if the length d is not too short. This is justified by the rapid exponential decay sideways of the field.

Essentially, there are two kinds of TEM modes which can exist in a multiconductor-transmission line. One of them is called the even mode, for which the respective voltages and currents on two inner conductors, for the three-conductor case, are equal and of the same sign. The other is the odd mode, for which voltages and currents are equal but of the opposite sign.

Fig. 6 A half cross-section detail of a strip transmission line.



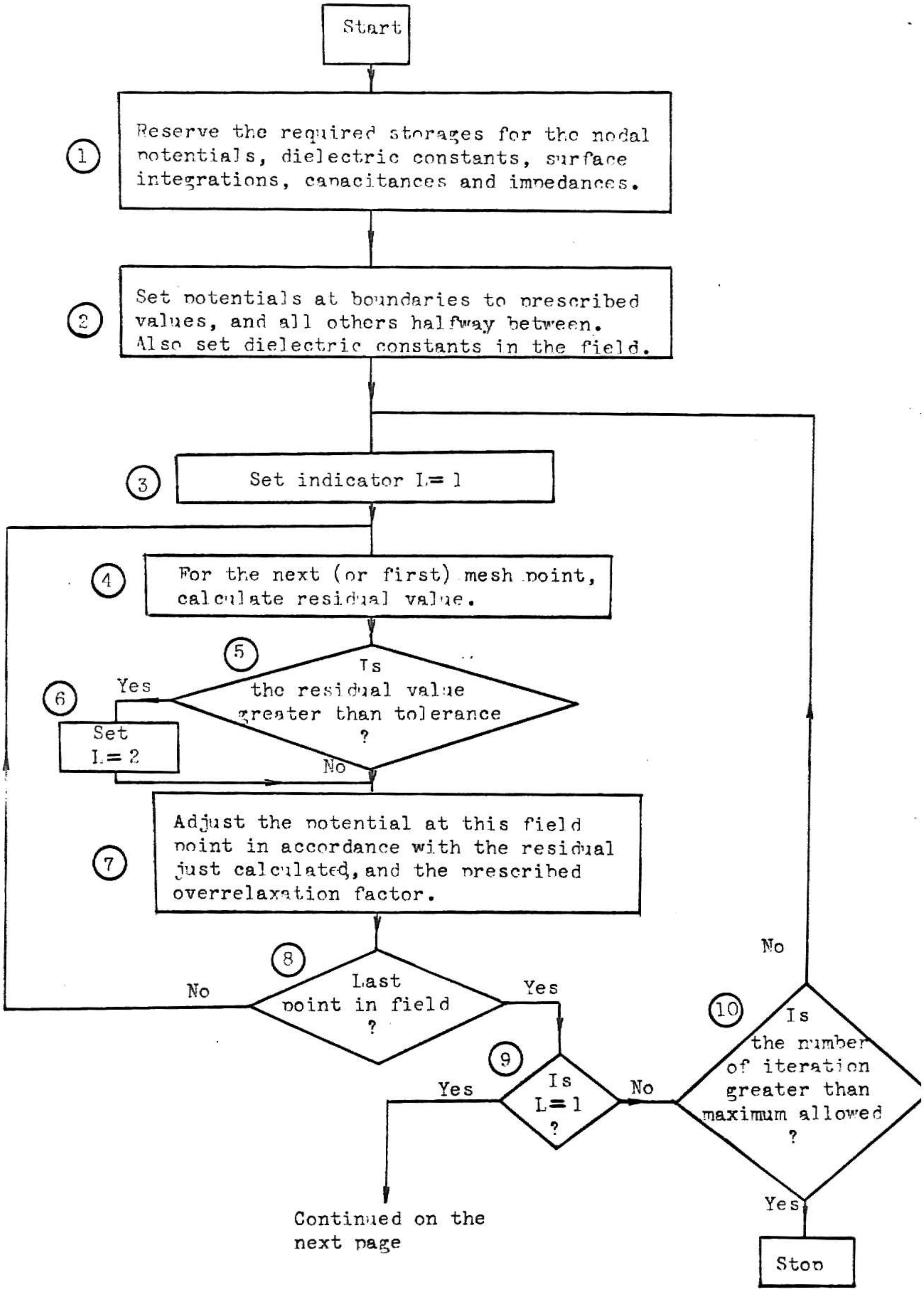
Any TEM mode in the line can be expressed as a linear combination of these two orthogonal TEM modes. For our particular case, the odd mode is not important because the two strips are electrically connected at the ends. Hence, let us now consider only the even mode existing in the line. That is, we can set equal potential values, say +100 volts, on both strips and zero potential values on the outer walls. These boundary conditions required in solving Laplace's equation for our problem.

2. Programming and the flow chart

It is now profitable to consider a few of the programming aspects of the problem. In the work that forms the subject of this report, it proved possible to solve problems in a reasonable time by using an IBM 360/50 computer.

A programming flow chart is shown in Fig. 7 to illustrate the finite-difference method as applied to solve our problem. The flow chart contains three main parts: (1) initial-condition setting; blocks 1 and 2, (2) iteration process; blocks 3 to 10 and (3) computation of impedance and output of results; blocks 11 and 12. In the first part, we set proper values of boundary conditions, guessed potential values at nonboundary points and the dielectric distributions as shown in Fig. 6 according to the configuration of the problem. The second part is to find the finite difference potential distributions, using the iteration technique discussed in Chapter 2 (refer to Equation (20) chapter 2). Once the potential distributions have been found, the capacitance and impedance can be obtained from Equations (23) and (26) or (27) of chapter 2 with the integral contour as shown in Fig. 6.

Finally, the results are printed out. In addition to the print-out of capacitance and impedance, the facility for the print-out of nodal potentials is also available on demand. This is useful whenever it is necessary to check whether the problem has been adequately represented by the finite-difference model or where a knowledge of the field distribution itself is desirable. A computer program using FORTRAN IV which is used to solve our problem and written according to the flow chart is listed in appendix I.

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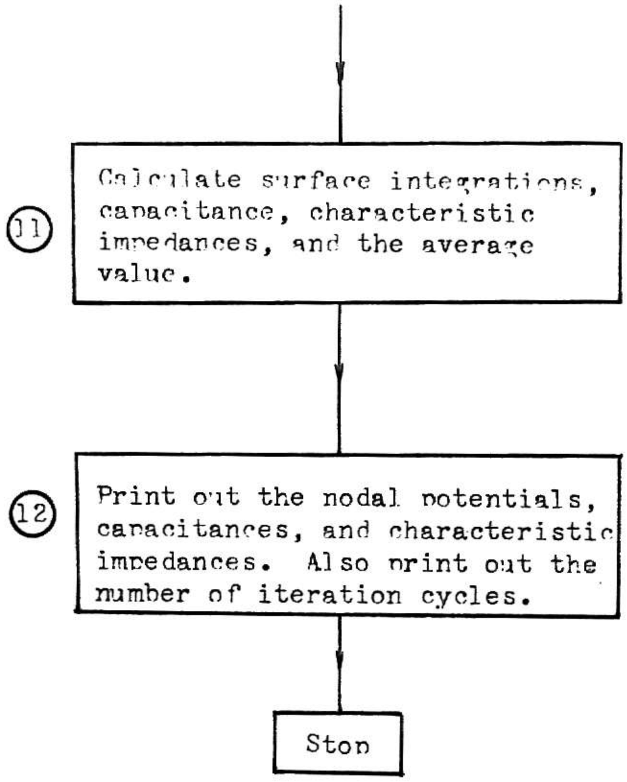


Fig. 7. The flow chart of the computer program.

# Chapter 4

# Computer Results

## 1. Numerical Results

Figs. 8 and 9 show the computer computation of impedance vs different strip spacings with a fixed line width (refer to Fig. 6). Two curves corresponding to an air gap between strips with dielectric elsewhere and solid dielectric everywhere are shown in each figure. The dielectric material used in Fig. 8 has $\varepsilon_r=10$, while in Fig. 9 $\varepsilon_r=2.56$. As observed from the figures, the impedance decreases monotonically as the spacing between strips increases. A physical explanation can be obtained from the fact that increasing the spacing decreases the distance between the strip and its ground plane and hence increases the inherent capacitance. It is also observed that a gap of $a/b=0.05$ causes a 6% change in $Z_o$ for $\varepsilon_r=10$ and 7% for $\varepsilon_r=2.56$ from the characteristic impedance for the zero gap case. This is probably not a sufficiently large change to seriously affect operation in most applications, but might be critical in some cases. The numerical results are given for a range of practical interest as this range encompasses the impedance neer 50 ohms.

A problem arises from the calculated result of capacitance (and hence that of characteristic impedance) found with such a numerical solution in that the result is only approximately independent of the surface of the application of Gauss's law, though it is completely independent of the surface for an exact solution. This is so because the finite difference equations (refer to

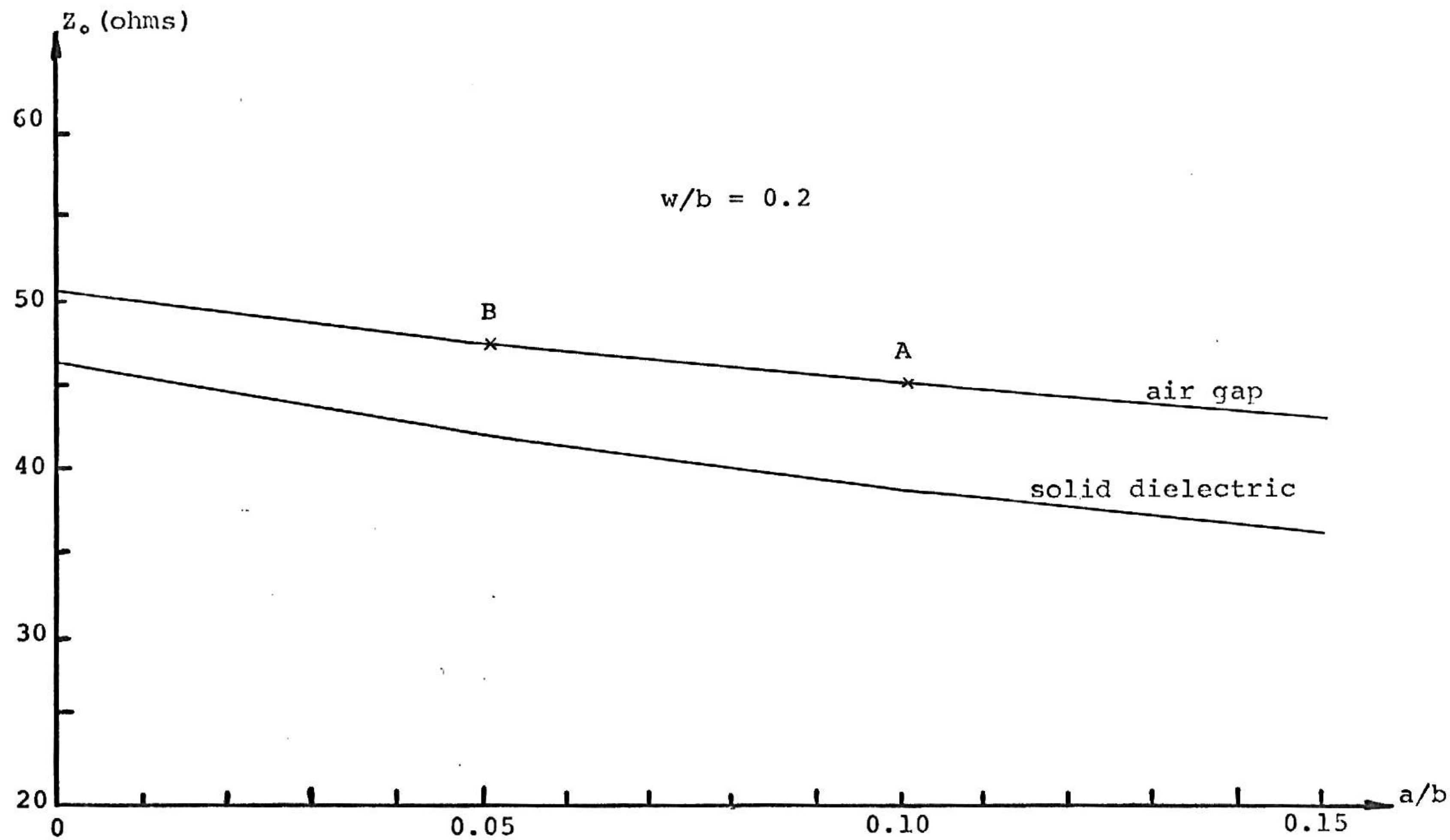


Fig. 8 The characteristic impedance of a strip transmission line (referred to Fig. 6) with $\varepsilon_r$=10, N=40, M=40, J1=4.

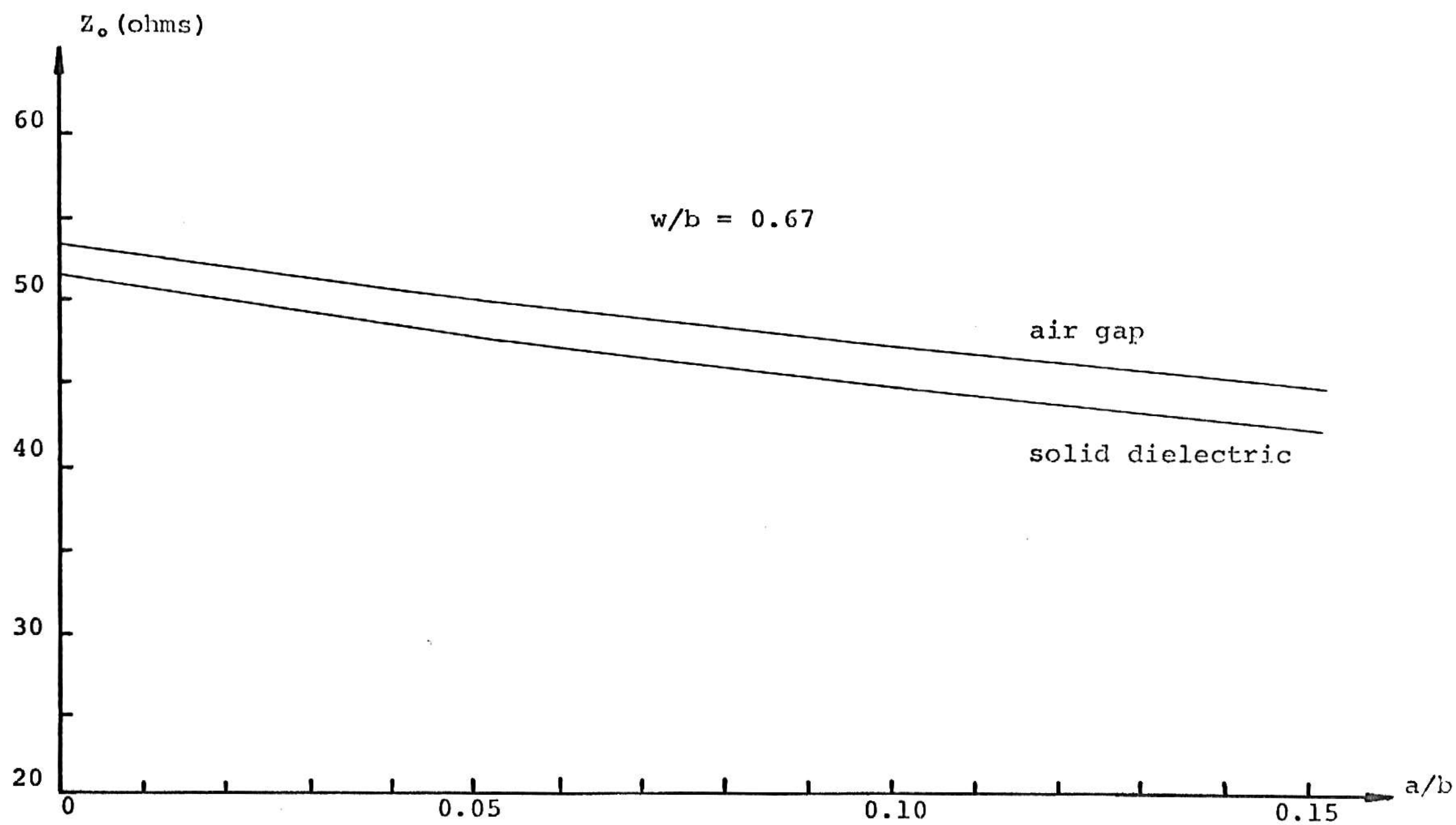


Fig. 9 The characteristic impedance of a strip transmission line (referred to Fig. 6) with $\varepsilon_r$=2.56, N=40, M=40, J1=13.

Equation (17), chapter 2) are not solved exactly (with R=0 for that equation). For example, the computer program for the point A in Fig. 8 yielded the capacitance range of values from 205.5 to 207.7 pf/meter. The limits of the range are within 0.5% of the average value. This gives us an indication that the problem was solved within a reasonable degree of exactness by the application of Gauss's law. In this report, the resulting impedance is defined as the arithmetical average of impedances calculated from the different integral contours. However, Sinnott [9] has pointed out that no theoretical reason has been found for one contour integration giving a more accurate value than any other. When the numerical solutions are reasonably precise, the contour about midway between boundaries gives a better result. Before leaving this subject, it is worth mentioning that there is an alternative method to find the capacitance by fitting a continuous function through the potential values and calculating the energy associated with this potential distribution [10].

2. Quality of computer results and number of iterations

From the approximation we made in equations (10) to (16) chapter 2, it is obvious that the accuracy of the impedance calculation depends on the grid size or the so called mesh size. In addition to the mesh error which is a function of h, there is also an iteration error due to the tolerance we allow. This error arose because the potentials are not solved with identically zero residuals. Milne [11] has shown that if the maximum absolute value of residuals does not exceed a positive quantity m, the maximum error in the iteration solution will not exceed

$m\rho^2/4h$. In this formula, $\rho$ is the radius of a circle with center at a node which just encloses the entire region R of the mesh point. Thus, the accuracy can be arbitrarily improved by decreasing the grid size and the tolerance. However, this results in increasing the number of iterations. As an illustration, we choose a point B in Fig. 8, which corresponds to $a/b = 2/39$, and calculate its values of impedance corresponding to two different tolerance values. When the tolerance is 0.05 volts, $Z_o$ is equal to 47.63 ohms and the number of iterations is found to be 123. When the tolerance is changed to 0.01, $Z_o$ is 48.22 ohms and the number of iterations is 182. We observe that the impedance for the case of the 0.05 tolerance differs only 1% from that of the 0.01 tolerance case, but the latter case has increased the number of iterations about 50% over that of the former. Note that Fig. 8 and 9 were obtained with 0.05 tolerance. As to the grid size, Fig. 8 and 9 were obtained with a 40x40 grid. Such a grid size gave a reasonable computer time and a reasonable accuracy (it is believed to be within 1% of the correct result). This can be justified by Johnson's work [12] for a similar problem. In that work it was shown that halving the grid size resulted in only about 0.5% change in the corresponding result. It is also believed that the convergence rate can be improved by using an optimum value of over-relaxation factor. Unfortunately, the optimum value of over-relaxation factor is not known theoretically and has to be found by trial and error [6].

## 3. Methods of improving the solution

In general, the over all accuracy of the finite difference technique can be improved by

1) Richardson's method of extrapolation
2) progressive mesh refinement, and
3) graded mesh

In Richardson's method, solutions are worked out for capacitance, using three mesh sizes in increasing order of refinement. These capacitance values are then substituted in Richardson's formula to provide an improved accuracy [2].

In the second method, the computation is started with a fairly coarse mesh size and the values obtained for the node potentials are used to interpolate starting potentials on a finer size mesh which is then further relaxed. Theoretically, this method gives the solution with any desired accuracy if this procedure is continued to any number of times. However, this is limited by the computer storage.

The third method employs a relatively coarse mesh everywhere except in the region where the potential changes rapidly. That is, in this region a finer mesh is used.

# Chapter 5

# Conclusions

The computation of characteristic impedances of various transmission lines supporting TEM modes is a problem of considerable importance for the design of microwave circuits. Despite the physical simplicity of strip transmission lines the rigorous mathematical analysis of their properties presents considerable difficulties. Solutions by analytic methods have been obtained for certain problems of restricted interest, but in many practical instances the solutions are rather tedious for actual use. Instead of using the rigorous analysis, this report has shown that finite difference techniques are particularly suited for the evaluation of the characteristic impedance of transmission lines by machine computation. The finite difference technique is a simple and accurate technique. This technique consists basically of a method for solving the field equations by replacing the domain between conductors by a finite set of mesh points and by solving Laplace's equation in finite difference form by digital computation. It is particularly useful for the special case of a shielded strip transmission line because exact solutions are not known and good approximations are only available for limited ranges of the line parameters.

Hence, in this report, this technique was outlined and its application shown in the development of a computer program for the numerical analysis of some problems. Considerable stress was placed on generality in devising the computer program, making it possible to solve an extensive range of large problems. Because of its generality the program becomes an important laboratory tool which can be used as an aid in solving particular design problems. Its use has been illustrated, in this report, in solving some practical problems. In fact, problems solved in this report are only simple examples of the multi-conductor-transmission line system. It can be seen that the finite difference technique and the computer program developed in this report can be applied without difficulty to solve many other multiconductor-transmission line problems. For transmission line concepts for multiconductor lines, a more detailed discussion can be found in [13].

# Appendix I

```
      C     ITERATION PROGRAM-STRIP TRANSMISSION LINE IMPEDANCE
            DIMENSION V(100,100),V1(20),V2(20),C1(20),C2(20),
           1 C(20),E(100),Z(20)
        300 READ (1,105,END=1) I1,I2,J1,N,M,ALP,EPS,VMAX,ER1
        105 FORMAT (5I3,4F5.3)
      C     I1,I2,J1,ARE TOP AND SIDE EDGES OF STRIP
      C     ER1 IS RELATIVE PERMITIVITY OF DIELECTRIC
      C     VMAX IS MAXIMUM POTENTIAL
      C     ALP IS OVERRELAXATION FACTOR
      C     EPS IS ACCURACY DESIRED
      C
      C     SET INITIAL CONDITIONS
      C
            DO 130 KKK=1,2
            NN = N-1
            MM = M-1
            VV=VMAX/2.
            DO 4 I=2,NN
            DO 4 J=1,MM
          4 V(I,J)=VV
            DO 5 J=1,J1
            V(I1,J)=VMAX
          5 V(I2,J)=VMAX
            DO 8 I=1,N
          8 V(I,M)=0.
            DO 58 J=1,M
         58 V(1,J)=0.
            DO 59 J=1,M
         59 V(N,J)=0.
         40 K=0
            WRITE (3,104)
        104 FORMAT('1')
      C
      C     SET PERMITTIVITY VALUES
      C
            DO 2 I=1,N
          2 E(I)=1.
            II=I1-1
            IK=I2+1
            JJ=J1+1
            IF (KKK-1) 61,61,62
         62 DO 71 I=1,II
         71 E(I)=ER1
            DO 72 I=IK,N
         72 E(I)=ER1
            E(I1)=(1.+ER1)/2.
            E(I2)=(1.+ER1)/2.
            WRITE (3,200) I1,I2,J1,N,M,ALP,EPS,VMAX,ER1
        200 FORMAT(1X,' I1 = ',I3,' I2 = ',I3,' J1 = ',I3,' N = ',I3,
           1 ' M = ',I3,' ALP = ',F5.3,' EPS = ',F5.3,
           1 ' VMAX = ',F5.1,' ER1 = ',F6.3)
            WRITE (3,201) (E(I),I=1,N)
        201 FORMAT(20F6.2)
      C
      C     START ITERATION PROCESS
      C
         61 AA=1.-ALP
          9 L=1
```

```
0041             K=K+1
          C      DO SYMMETRY AXIS
0042             DO 10 I=2,II
0043             EE=2.*E(I)+E(I+1)+E(I-1)
0044             A=(E(I-1)*V(I-1,1)+E(I+1)*V(I+1,1)+2.*E(I)*V(I,2))/EE
0045          10 V(I,1)=AA*V(I,1)+ALP*A
0046             DO 11 I=IK,NN
0047             EE=2.*E(I)+E(I+1)+E(I-1)
0048             A=(E(I-1)*V(I-1,1)+E(I+1)*V(I+1,1)+2.*E(I)*V(I,2))/EE
0049          11 V(I,1)=AA*V(I,1)+ALP*A
0050             IP=I1+1
0051             IQ=I2-1
0052             IF (IQ-IP-1) 74,75,75
0053          75 DO 90 I=IP,IQ
0054             EE=2.*E(I)+E(I+1)+E(I-1)
0055             A=(E(I-1)*V(I-1,1)+E(I+1)*V(I+1,1)+2.*E(I)*V(I,2))/EE
0056          90 V(I,1)=AA*V(I,1)+ALP*A
          C      DO INTERIOR POINTS
0057          74 DO 13 I = 2,II
0058             EE=2.*E(I)+E(I+1)+E(I-1)
0059             DO 13 J = 2,MM
0060             A=(E(I)*(V(I,J+1)+V(I,J-1))+E(I+1)*V(I+1,J)
               1 +E(I-1)*V(I-1,J))/EE
0061             IF (ABS(A-V(I,J))-EPS) 13,13,12
0062          12 L=2
0063          13 V(I,J)=AA*V(I,J)+ALP*A
0064             DO 15 I=I1,I2
0065             EE=2.*E(I)+E(I+1)+E(I-1)
0066             DO 15 J=JJ,MM
0067             A = (E(I)*(V(I,J+1)+V(I,J-1))+E(I+1)*V(I+1,J)
               1 +E(I-1)*V(I-1,J))/EE
0068             IF (ABS(A-V(I,J))-EPS) 15,15,14
0069          14 L=2
0070          15 V(I,J) = AA*V(I,J)+ALP*A
0071             IF (IQ-IP-1) 84,85,85
0072          85 DO 91 I=IP,IQ
0073             EE=2.*E(I)+E(I+1)+E(I-1)
0074             DO 91 J=2,J1
0075             A = (E(I)*(V(I,J+1)+V(I,J-1))+E(I+1)*V(I+1,J)
               1 +E(I-1)*V(I-1,J))/EE
0076             IF (ABS(A-V(I,J))-EPS) 91,91,92
0077          92 L=2
0078          91 V(I,J)=AA*V(I,J)+ALP*A
0079          84 DO 17 I=IK,NN
0080             EE=2.*E(I)+E(I+1)+E(I-1)
0081             DO 17 J=2,MM
0082             A=(E(I)*(V(I,J+1)+V(I,J-1))+E(I+1)*V(I+1,J)
               1  +E(I-1)*V(I-1,J))/EE
0083             IF (ABS(A-V(I,J))-EPS) 17,17,16
0084          16 L=2
0085          17 V(I,J)=AA*V(I,J)+ALP*A
0086             IF (K-150) 19,19,20
0087          19 GO TO (20,9),L
0088          20 WRITE (3,101) K,EPS
0089         101 FORMAT(1X,' NUMBER OF ITERATIONS ',I4,' TOLERANCE ',F5.3)
0090             WRITE(3,102) ALP
0091         102 FORMAT(1X,' OVERRELAXATION FACTOR IS ',F5.3)
          C
```

```
              C      PRINT VOLTAGE ARRAY
              C
0092                 DO 21 I = 1,N
0093              21 WRITE(3,103) (V(I,J),J = 1,25)
0094             103 FORMAT('0',26F5.1)
              C
              C      COMPUTE CAPACITANCE AND IMPEDANCE
              C
0095                 KK = I1-2
0096                 DO 27 K=1,KK
0097                 V1(K)=V(I1-K,1)*E(I1-K)+V(I2+K,1)*E(I2+K)
0098                 JJ=J1+K
0099                 DO 24 J=2,JJ
0100                 V1(K)=V1(K)+2.*V(I2+K,J)*E(I2+K)
0101              24 V1(K)=V1(K)+2.*V(I1-K,J)*E(I1-K)
0102                 IJ=I1-K
0103                 IK=I2+K
0104                 DO 25 I=IJ,IK
0105              25 V1(K)=V1(K)+2.*V(I,J1+K)*E(I)
0106              27 V2(K)=V1(K)-4.*V(I1-K,J1+K)*E(I1-K)
                   1 -4.*V(I2+K,J1+K)*E(I2+K)
0107                 KJ = I1-3
0108                 IF (KKK-1) 65,65,66
0109              65 DO 28 K=1,KJ
0110                 C1(K)=8.854*(V1(K)-V2(K+1))/VMAX
0111              28 Z(K)=1.0E+04/(C1(K)*SQRT(ER1)*2.997925)
0112                 AZ=0.
0113                 DO 30 K=1,KJ
0114              30 AZ=AZ+Z(K)
0115                 Y=AZ/KJ
0116                 GO TO 99
0117              66 DO 38 K=1,KJ
0118                 C2(K)=8.854*(V1(K)-V2(K+1))/VMAX
0119                 C(K)=SQRT(C1(K)*C2(K))
0120              38 Z(K)=1.0E+04/(C(K)*2.997925)
0121                 AZ=0.
0122                 DO 29 K=1,KJ
0123              29 AZ=AZ+Z(K)
0124                 Y=AZ/KJ
              C
              C      PRINT THE RESULT
              C
0125             106 FORMAT(1X,' I1 = ',I3,' I2 = ',I3,' J1 = ',I3,' N = ',I3,
                   1 ' M = ',I3,' ER1 = ',F6.3)
0126              99 IF (KKK-1) 81,81,82
0127              81 ER2=1.
0128                 WRITE(3,106) I1,I2,J1,N,M,ER2
0129                 GO TO 39
0130              82 WRITE(3,106) I1,I2,J1,N,M,ER1
0131              39 WRITE(3,108)
0132             108 FORMAT('0',' V1(K) = ')
0133                 WRITE(3,107)(V1(K),K=1,KK)
0134             107 FORMAT(13F10.2)
0135                 WRITE(3,109)
0136             109 FORMAT('0',' V2(K) = ')
0137                 WRITE(3,107)(V2(K),K=1,KK)
0138                 WRITE(3,110)
0139             110 FORMAT('0',' CAPACITANCE IN PF PER METER = ')
```

```
  111 FORMAT(13F9.3)
      IF (KKK-1) 120,120,122
  120 WRITE(3,111) (C1(K),K=1,KJ)
      GO TO 123
  122 WRITE(3,111) (C2(K),K=1,KJ)
  123 WRITE(3,112)
  112 FORMAT('0',' CHARACTERISTIC IMPEDANCE = ')
      WRITE(3,113) (Z(K),K=1,KJ)
  113 FORMAT(13F9.3)
      WRITE(3,114)
  114 FORMAT('0',' AVERAGE OF CHARACTERISTIC IMPEDANCE = ')
      WRITE(3,115) Y
  115 FORMAT(1F9.3)
  130 CONTINUE
      GO TO 300
    1 STOP
      END
```

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ACKNOWLEDGMENT

Words cannot express the writer's thanks to his advisor, Dr. Gary L. Johnson. Still, grateful acknowledgment is expressed to him for helpful counsel and correction. The writer feels also indebted to him for his guidance in forming the computer program.

A STUDY OF STRIP TRANSMISSION LINES

by

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Diploma, Taipei Institute of Technology, 1963

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

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1970

ABSTRACT

The purpose of this report is to study the variation of the characteristic impedance of shielded strip transmission lines as a function of line geometry by using the finite difference method for solving Laplace's equation.

The finite difference technique, which is particularly suited for machine computation, is reviewed. This technique is illustrated with a special kind of strip transmission line which is made of two microstrip lines placed together with their strip conductors face to face. It proved possible to solve the problem in a reasonable time by using an IBM 360/50 computer. A programming flow chart is shown and the developed program for the problem is listed.

The numerical results have been given in the form of curves. The impedance encompasses the range near 50 ohms, corresponding to a ratio of the strip width to the total thickness of the line of 0.2 with the dielectric material having a relative permittivity of $\varepsilon_r$=10, and to a ratio of 0.67 with $\varepsilon_r$=2.56. The ratio of the air gap between the two strip conductors to the total thickness is varied from 0 to 0.15. The quality of computer results is also discussed.