DESIGN CONSIDERATIONS OF A SAVONIUS WIND ROTOR SUPPORTED AT THE BOTTOM

by

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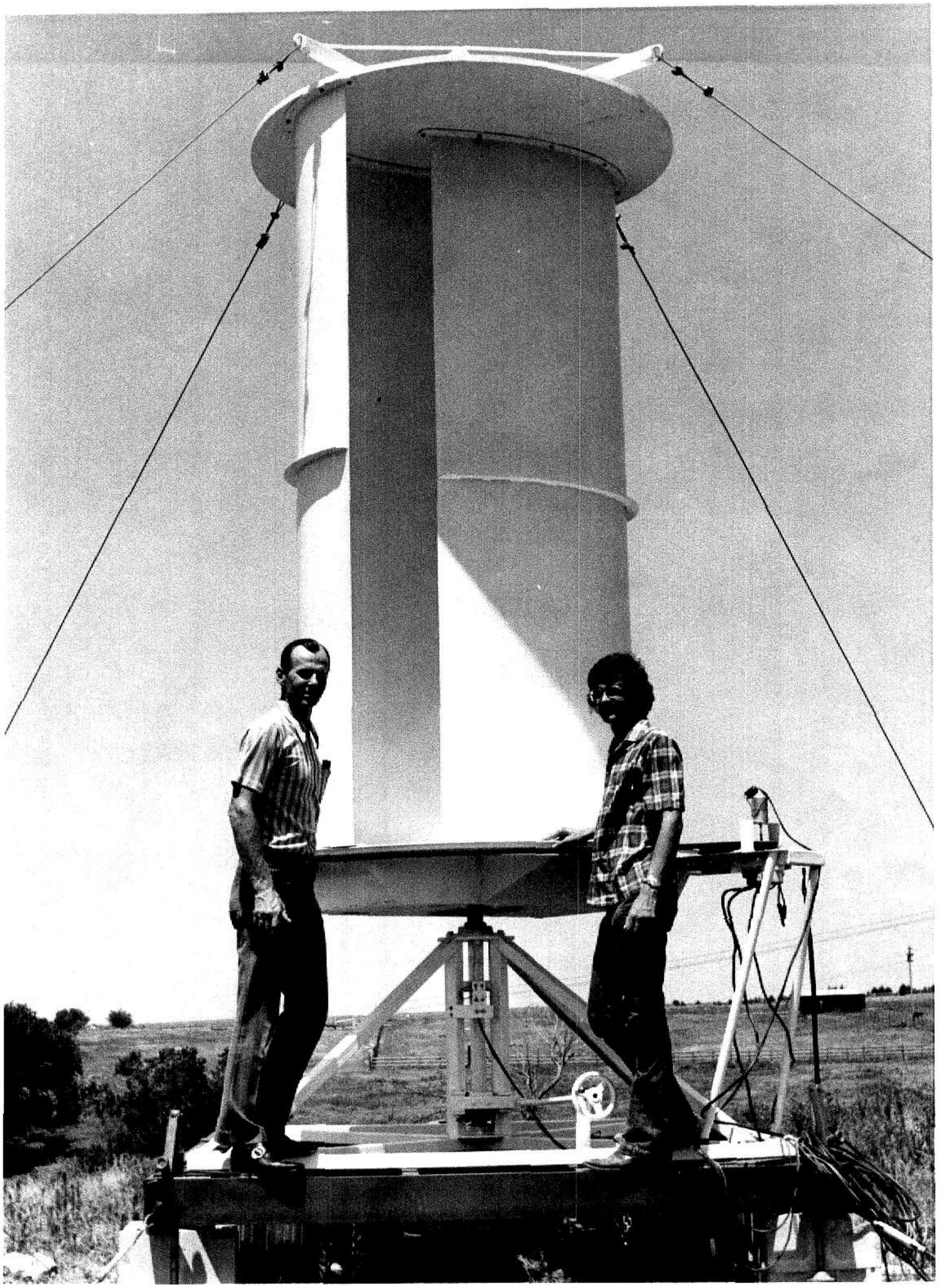
CHAPTER I

INTRODUCTION

In an effort to find solutions to the growing energy shortage, a team of professors and students at Kansas State University began studying the possibility of building some kind of wind rotor to utilize wind for the generation of electrical energy. The study began in the fall of 1974 and culminated in the summer of 1975 with the building of a prototype Savonius [1] wind rotor. In this design the wind vanes turn about a vertical axis, unlike a conventional windmill which turns about a horizontal axis (see Fig. 1).

When the rotor was being designed the decision was made to build the prototype with bearing supports only at the bottom as opposed to bearing supports at both the top and bottom. One reason for making it this way is that the latter method needs an external framework which may cause undesirable flow disturbances. Secondly, the curved shape of the rotor vanes, plus the fact they are fastened to end plates results in inherent stiffness. Thus, the support at the top may not be needed at all.

After the prototype was assembled it was discovered that the shaft used was too small. First of all, small horizontal forces applied at the top of the rotor were found to cause noticeable deflections. These deflections were virtually caused by bending of the shaft. Also, the fundamental transverse frequency of the assembly (about 125 cpm)



SAVONIUS WIND ROTOR

Figure 1

was much lower than anticipated. Therefore, the prototype had to be supported at the top. However, a machine with support only at the bottom was still considered feasible since a stiffer shaft would remedy the above problems.

The purpose of this work is to arrive at a proper shaft diameter for the prototype supported at the bottom only. It is also desired to develop a computer program of such flexibility that shaft diameters for rotors of various sizes and shapes could be determined.

The design criteria considered here are stress due to high winds, and vibration problems. First, shaft stress will be estimated, then a minimum size shaft will be determined so as to ensure no permanent deformation. Next, vibration, which turns out to be a more serious problem, is discussed in much more detail.

CHAPTER II

DESCRIPTION OF THE WIND ROTOR

Before proceeding with the analysis of the shaft design, the overall dimensions and thicknesses of the materials used in the rotor assembly will be presented. The various components of the machine are identified in Fig. 2.

The two rotor vanes are made of 14 gauge steel sheet and have the cross-sectional shape shown in Fig. 2a and 3. The rotor height is 10 feet. For additional stiffness a ¼ inch thick external rib is welded to the center of each vane. End flanges are also welded to the vanes. These flanges are made of ¼ inch steel plate and are bolted to the lower support assembly and upper end plate.

The upper end plate is a disc 6 feet in diameter made of 16 gauge steel sheet. Attached to it is a circular rim made of ¼ x 1 inch flat bar stock.

The main part of the lower support is a central box type structure of 11 gauge steel sheet. This structure supports the vanes and is attached to the shaft. The prototype shaft has a diameter of 2.4 inches.

The base is a framework 7 feet square and 32 inches high. A standard 2 - 7/16 inch diameter self aligning flange type ball bearing is mounted at the top of the base and a 1 - ½ inch diameter bearing of the same type is located at the bottom of the base.

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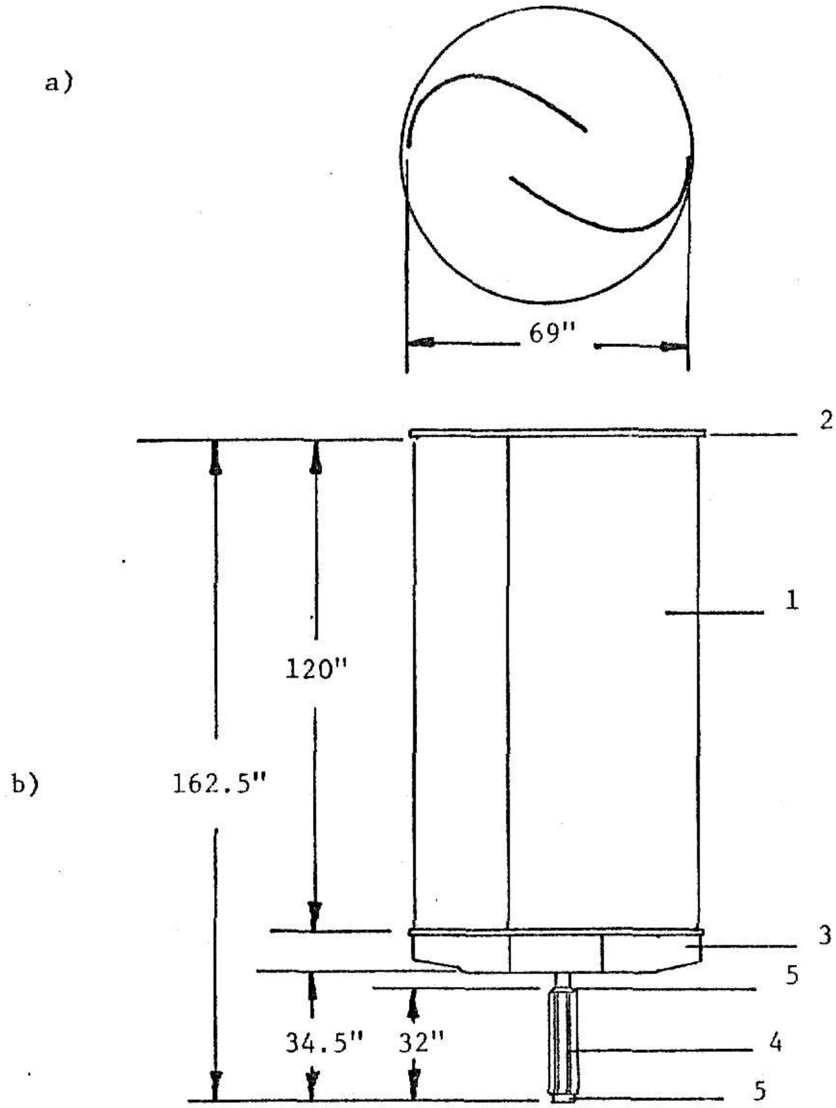
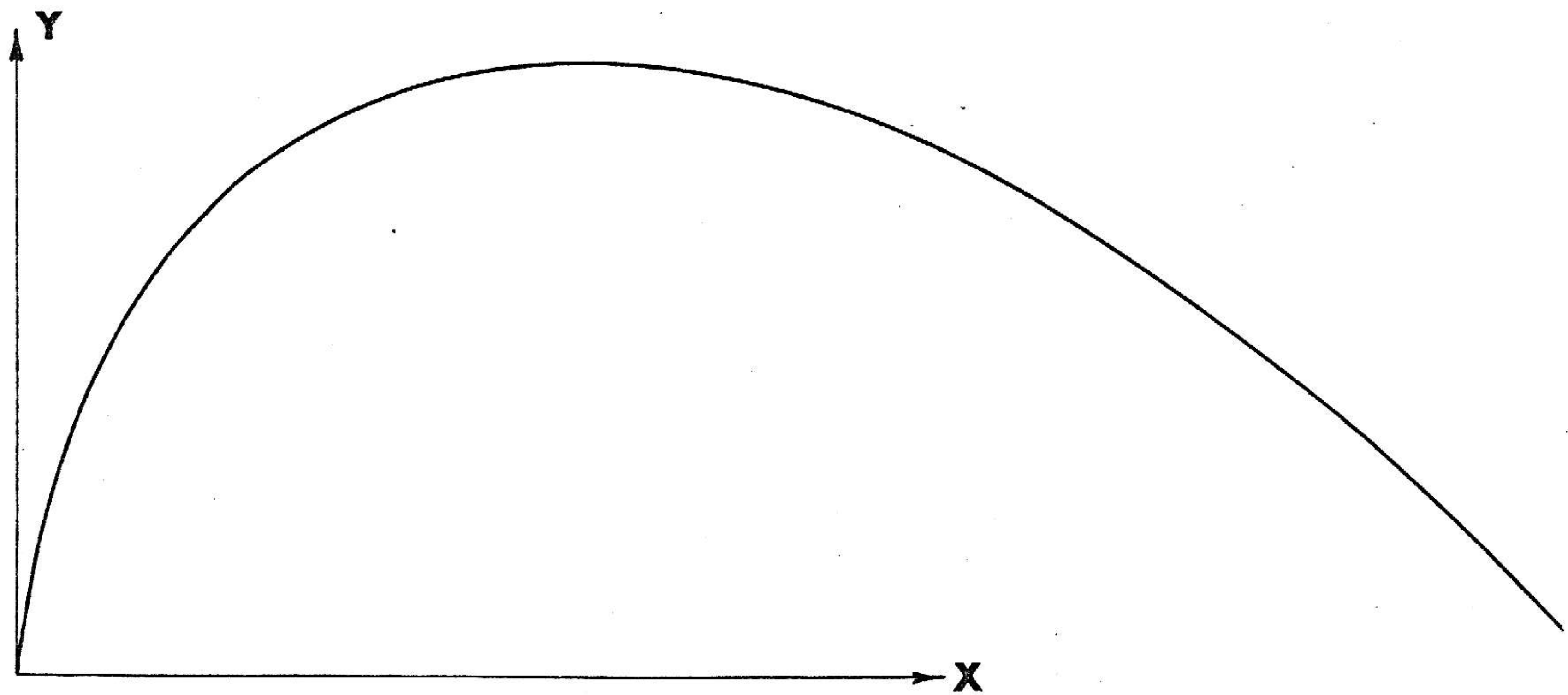


1 - Rotor Vanes
2 - Upper End Plate
3 - Lower Support
4 - Shaft
5 - Bearing Support

Rotor Components

Figure 2



VANE PROFILE

Figure 3

X	Y
0.0	0.0
1.0	5.2
2.0	8.1
4.0	11.8
6.0	14.2
8.0	15.8
10.0	17.0
12.0	17.8
14.0	18.2
16.0	18.5
18.0	18.5
20.0	18.4
22.0	18.0
24.0	17.4
26.0	16.7
28.0	16.0
30.0	14.9
32.0	13.7
34.0	12.3
36.0	11.0
38.0	9.2
40.0	7.5
42.0	5.7
44.0	3.6
46.0	1.4

Vane Profile

Table 1

CHAPTER III

THE SHAFT STRESS DUE TO HIGH WINDS

In this section the maximum shaft stress due to high winds is approximated. The analysis is thought to be somewhat conservative.

To begin with, it is customary to express the total drag D on an object as

$$D = C_D \rho \frac{v^2}{2} A \qquad (1)$$

where C_D is the drag coefficient, ρ the fluid density, v the fluid velocity, and A the projected area to the fluid flow [2].

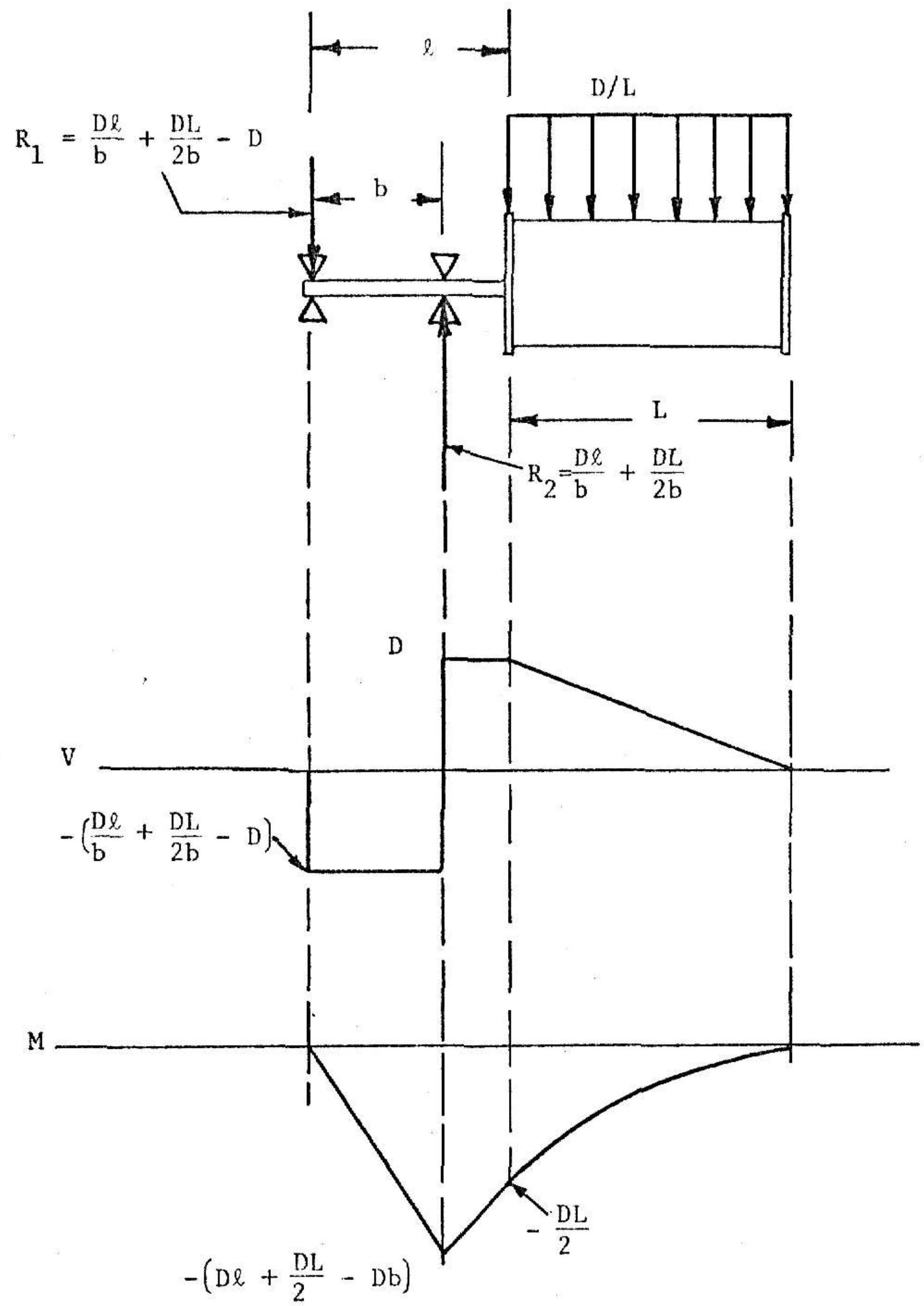
To get some idea of the total force on the prototype rotor the largest possible projected area of the vanes (see Fig. 2a) is used in the above equation. Also, the value of C_D is estimated to be 2.0 which is the drag coefficient for flat plates [2].

The maximum stresses in the shaft can now be found using the value of D for the load on the rotor vanes. The associated shear and bending moment diagrams are shown in Fig. 4 where D/L is the distributed load on the vanes. It follows from the figure that the maximum principal shear and bending stresses in the shaft are respectfully

$$\tau_1 = \left(\frac{4}{3}\right) \frac{V_{max.}}{A} = \left(\frac{4}{3}\right) \left(\frac{D\ell}{b} + \frac{DL}{2b} - D\right) / \pi r^2 \qquad (2)$$

and

$$\sigma_1 = \frac{M_{max.} C}{I} = \left(D\ell + \frac{DL}{2} - Db\right) r / [(\tfrac{1}{4}) \pi r^4] \qquad (3)$$



Shear and Moment Diagram of the Rotor

Figure 4

where r is the shaft radius. It turns out that τ_1 is small when compared to σ_1, so therefore, the maximum stress in the shaft is essentially σ_1 the maximum principle bending stress.

Considering an arbitrary wind speed of 100 m.p.h., the stress in the 2.4 inch prototype shaft turns out to be

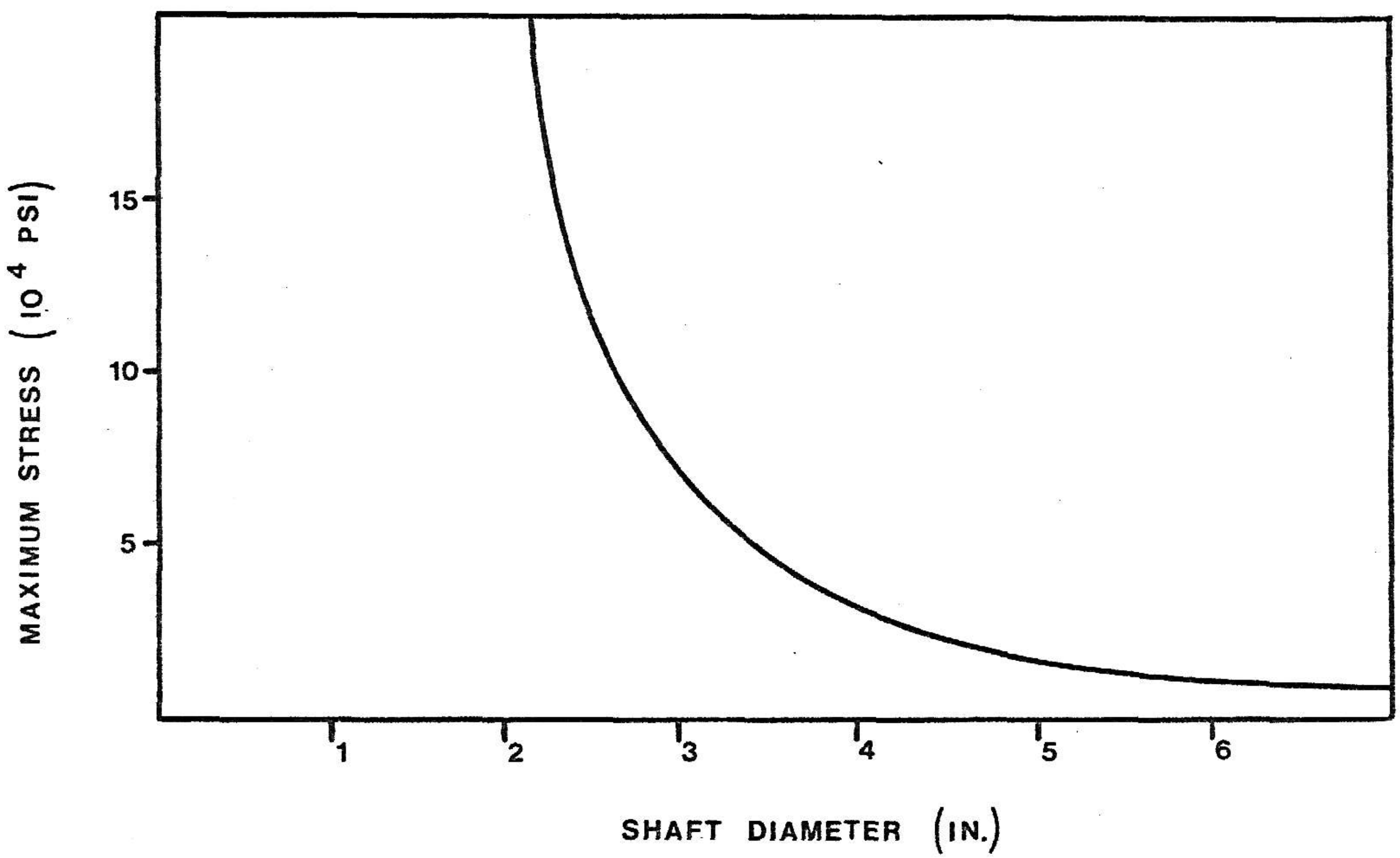
$$\sigma max = 135{,}453 \text{ lb./in.}^2$$

which is clearly above the elastic limit of 50,000 lb./in.^{2} for the shaft. In determining the proper shaft size needed it is wise to introduce a safety factor, especially since it is not known how good the approximated value of stress is. If a factor 1.5 is used the design criterion becomes

$$\sigma max \leq \frac{\sigma yield}{1.5} = 33{,}333 \text{ lb./in.}^2 \qquad (4)$$

which requires that the rotor shaft be about 3.85 inches in diameter. The relationship between shaft diameter and approximate maximum stress is shown in Fig. 5.

By the same analysis as above the stress in the rotor vanes is approximated to be 2034 lb./in.^{2} which is well below the elastic limit. However, a special note should be made that buckling of the rotor vanes has not been considered in this thesis. This should also be considered in the design of the vanes.



Max. Shaft Stress Vs. Shaft Diameter

Figure 5

Shaft Diameter (in.)	Max. Stress (lb./in.^{2})
1.00	1,872,501
2.00	234,063
2.40	135,453
3.00	69,352
3.75	35,508
3.80	34,125
3.85	32,813
4.00	29,258
5.00	14,980
6.00	8,669
7.00	5,459

Approximate Shaft Stress

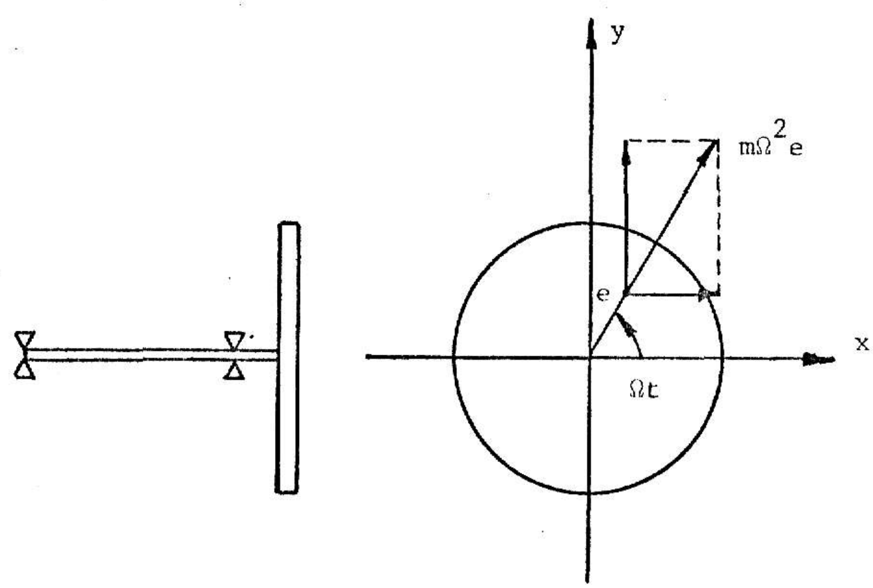
Table 2

CHAPTER IV

THE PROBLEM OF VIBRATIONS

Vibration is of great concern in the design of any rotating machinery. If the RPM of a machine reaches what is called a critical speed a condition of resonance will exist. That is the amplitude of vibration will greatly increase and a machine such as the wind rotor will tend to shake itself apart.

In rotating machinery the effect of unbalance acts like a variable forcing function. Consider a disk of mass m on a shaft running at constant angular speed Ω as shown in Fig. 6. Let the center



Rotating Disk

Figure 6

of gravity of the disk be at a radial distance e (eccentricity) from the center of the shaft. Obviously, there will be a centrifugal force on the shaft of

$$F = m\Omega^2 e = m\Omega^2 e \cos\Omega t\ \hat{i} + m\Omega^2 e \sin\Omega t\ \hat{j}. \qquad (5)$$

This force will normally tend to cause a whirling type of motion with the same frequency as that of the shaft rotation. However, according to Den Hartog [3] bearing effects may cause shafts to have slightly different stiffnesses in one direction than another. For example, if the rotating shaft in Fig. 6 has one value of stiffness in the x-direction and another in the y-direction, the system will "resonate" in the x and y-directions at different shaft speeds. The path of the vibrations will be nearly transverse and the frequency will be the same as the shaft RPM [3]. In the case of the wind rotor the unbalance will act as a forcing function with the same frequency as the shaft rotation.

Another "exciting" force on the rotor is the wind itself. To understand this, imagine the rotor vanes at some position with respect to the wind load. As the rotor turns, the configuration it presents to the wind changes and thus the force of the wind changes correspondingly. Because there are two symmetric vanes it can be seen that for every revolution of the shaft the same vane configuration will be presented to the wind twice. Thus, the variable force by the wind will have a frequency of twice the rotor RPM. Therefore, if the rotor is spinning at one-half the value of a natural whirl or natural transverse frequency the system will begin resonating.

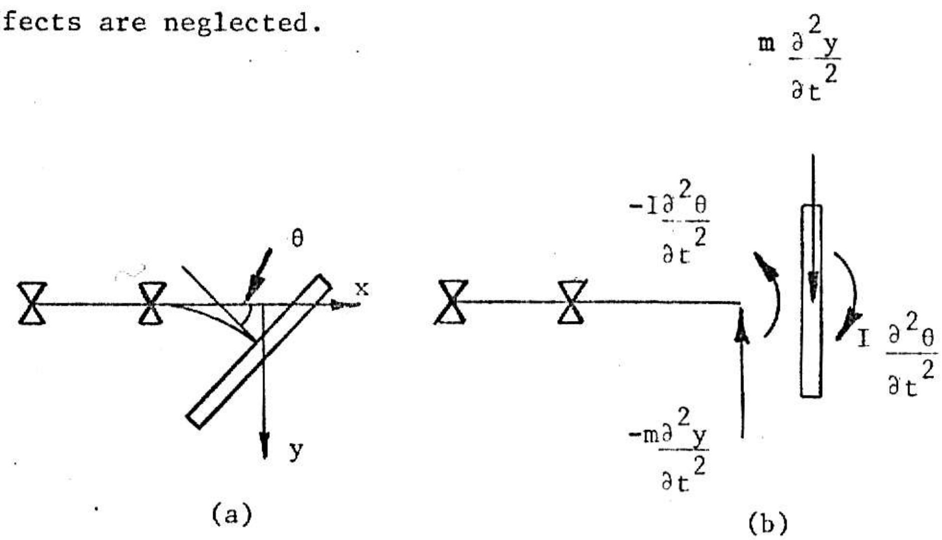
The logical procedure is to determine the proper shaft size that will ensure that the natural whirl and transverse frequencies are out of the range (above) of the frequencies of the forcing functions.

To determine what the maximum "exciting" frequencies are the maximum rotor RPM must be determined. According to Savonius [1] the maximum possible ratio of vane-tip speed to wind velocity is approximately 1.7. If the rotor is allowed to run at winds of up to, say, 50 mph the resulting maximum rotor or shaft speed for the prototype (vane-tip diameter of 69 inches) is 414 cpm. The forcing function due to unbalance will, therefore, have a maximum frequency of 414 cpm, while the one due to wind will be 828 cpm.

CHAPTER V

THE TRANSVERSE FUNDAMENTAL FREQUENCY EQUATION

To make the development of the frequency equation more easily understood, first consider the simple case of a thin disk connected to a shaft. This system, shown in Fig. 7, is assumed to be in simple harmonic motion. Before beginning the analysis it should be noted that the mass of the shaft as well as rotational and gravitational effects are neglected.



Thin Disk in Harmonic Motion

Figure 7

When the disk is accelerating in the positive y and θ directions, as in the case in Fig. 7b, a force and moment of

$$F = m\left(\partial^2 y/\partial t^2\right) \tag{6}$$

$$M = I\left(\partial^2\theta/\partial t^2\right) \tag{7}$$

acting at the center of gravity are required to account for the acceleration. Since there are no outside forces, the force and moment on the disk are exerted by the shaft at the connecting point. The force and moment on the disk are of course acting in the same direction as the accelerations. The force and moment by the disc on the shaft will be equal and opposite.

Now consider the case of the wind rotor in simple harmonic motion shown in Fig. 8. Again, the mass of the shaft, and rotational, and gravitational effects are neglected. Also, the rotor is considered rigid. For further simplicity the rotor is assumed to consist of three parts. They are:

1. The lower support with mass Mb and rotational inertia about its diameter of Ib (The lower support is considered to be a thin disk.);
2. The rotor vanes with mass Mv, rotational inertia Iv about its center of gravity, and length L;
3. The upper end plate with mass Mt and rotational inertia about its diameter of It.

First consider the lower support separately from the rest of the rotor as shown in Fig. 8b. From the previous discussion the force on it alone would be

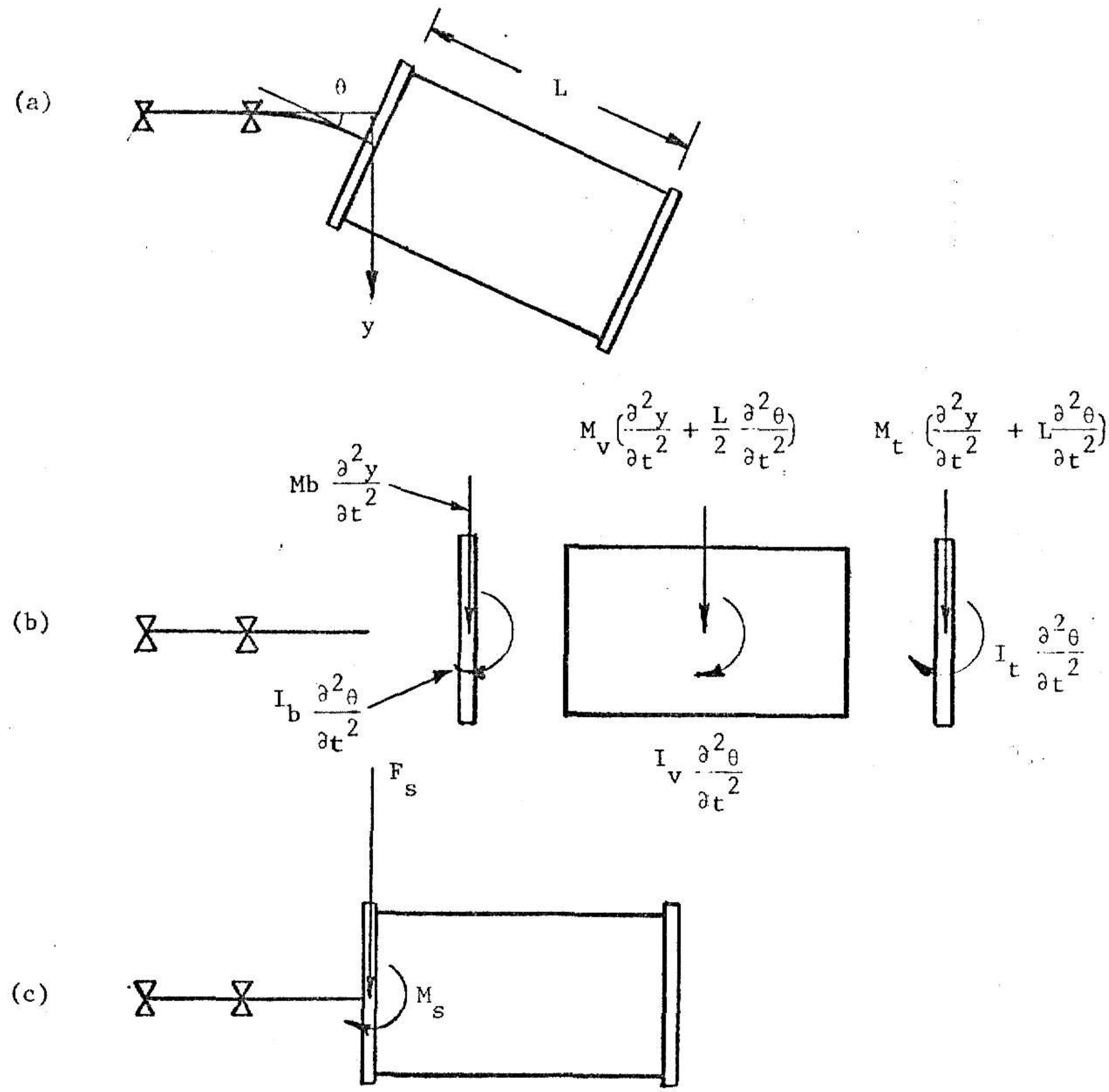
$$Mb\left(\partial^2 y/\partial t^2\right)$$

and the moment

$$Ib\left(\partial^2 \theta/\partial t^2\right).$$

Now the center of gravity of the rotor vanes is undergoing a linear acceleration of

$$\partial^2 y/\partial t^2 + (L/2)\, \partial^2 \theta/\partial t^2.$$



Rotor in Harmonic Motion

Figure 8

So a force of

$$Mv\left(\partial^2 y/\partial t^2 + (L/2)\partial^2\theta/\partial t^2\right)$$

and a moment of

$$Iv(\partial^2\theta/\partial t^2)$$

applied at the center of gravity would account for its acceleration. Since the vanes are assumed to be rigid the force at the center of gravity can be replaced by a force and moment at the shaft connection point. It follows that the force and moment exerted by the shaft (at the point of connection) required for the vane acceleration is, respectfully

$$Mv\left(\partial^2 y/\partial t^2 + (L/2)\partial^2\theta/\partial t^2\right)$$

and

$$Iv(\partial^2\theta/\partial t^2) + Mv(L/2)\left(\partial^2 y/\partial t^2 + (L/2)\partial^2\theta/\partial t^2\right).$$

For the upper end plate the force and moment required at its center of gravity would be, respectfully

$$Mt\left(\partial^2 y/\partial t^2 + (L)\ \partial^2\theta/\partial t^2\right)$$

and

$$It(\partial^2\theta/\partial t^2).$$

The force and moment by the shaft for the upper end plate will be, respectfully

$$Mt\left(\partial^2 y/\partial t^2 + (L)\ \partial^2\theta/\partial t^2\right)$$

and

$$It\ (\partial^2\theta/\partial t^2) + Mt\ (L)\ \left(\partial^2 y/\partial t^2 + (L)\ \partial^2\theta/\partial t^2\right).$$

By superposition the total force Fs and moment Ms by the shaft is

$$Fs = Mb(\partial^2 y/\partial t^2) + Mv\left(\partial^2 y/\partial t^2 + (L/2)\partial^2\theta/\partial t^2\right) + Mt\left(\partial^2 y/\partial t^2 + (L)\partial^2\theta/\partial t^2\right) \quad (8)$$

and

$$Ms = (Ib + Iv + It)\partial^2\theta/\partial t^2 + Mv(L/2)\left(\partial^2 y/\partial t^2 + (L/2)\frac{\partial^2\theta}{\partial t^2}\right) + Mt(L)\left(\partial^2 y/\partial t^2 + (L)\partial^2\theta/\partial t^2\right). \quad (9)$$

These are shown in Fig. 8c. The force and moment on the shaft will, of course, be equal and opposite.

We are now ready to discuss the elastic properties of the shaft at the connecting point to the rotor. These are described by three influence coefficients:

δ_{11} is the deflection y at the rotor connection point from a 1 lb. force.

δ_{12} is the angle θ at the rotor connection point from a 1 lb. force.

δ_{12} is also the deflection y at the connecting point from a 1 in.-lb. moment.

δ_{22} is the angle θ at the connecting point from a 1 in.-lb. moment.

These influence coefficients, determined in Appendix A, are

$$\delta_{11} = (\ell^3 - 2b\ell^2 + b^2\ell)/(3EI), \quad (10)$$

$$\delta_{12} = (3\ell^2 - 4\ell b + b^2)/(6EI), \quad (11)$$

$$\delta_{22} = (3\ell - 2b)/(3EI). \quad (12)$$

Here, ℓ is the length of the shaft, b the distance between the bearing supports, E the modulus of elasticity for the shaft material, and I the area moment of inertia of the shaft.

The shaft equations can now be written as

$$y = \delta_{11}(-F_s) + \delta_{12}(-M_s) \tag{13}$$

and

$$\theta = \delta_{12}(-F_s) + \delta_{22}(-M_s) \tag{14}$$

where $-F_s$ and $-M_s$ are the force and moment on the shaft by the rotor which were just determined. By rearranging terms the equations can be written

$$y = -\delta_{11}(Mb + Mv + Mt)\partial^2 y/\partial t^2 - \delta_{11}L(Mv/2 + Mt)\partial^2\theta/\partial t^2 \tag{15}$$
$$-\delta_{12}L(Mv/2 + Mt)\partial^2 y/\partial t^2 - \delta_{12}\left[Ib + Iv + It + (Mv/2 + Mt)L^2\right]\frac{\partial^2\theta}{\partial t^2}$$

and

$$\theta = -\delta_{12}(Mb + Mv + Mt)\partial^2 y/\partial t^2 - \delta_{12}L(Mv/2 + Mt)\partial^2\theta/\partial t^2 \tag{16}$$
$$-\delta_{22}\left[Ib + Iv + It + (Mv/2 + Mt)L^2\right]\frac{\partial^2\theta}{\partial t^2} - \delta_{22}\, L(Mv/2 + Mt)\ \frac{\partial^2 y}{\partial t^2}$$

Next assume solutions for the shaft equation to be of the following form:

$$y\ (x,t) = y(x)F(t) = y\left[A \sin(\omega t-\emptyset)\right] \tag{17}$$

$$\partial y\ (x,t)/\partial x \approx \theta(x,t) = \theta(x)F(t) = \theta\left[A \sin(\omega t-\emptyset)\right]. \tag{18}$$

Small angles only are considered above while A and the phase angle $\emptyset$ are unknown constants and ω is the natural frequency. By substitution of the above expressions into equations (15) and (16) and by letting

$$B = Mb + Mv + Mt$$

$$C = L(Mv/2 + Mt)$$

$$D = Ib + Iv + It + (Mv/2 + Mt)L^2$$

the equations become

$$y = (\delta_{11} B + \delta_{12}C)\omega^2 y + (\delta_{11}C + \delta_{12}D)\omega^2 \theta \tag{19}$$

and

$$\theta = (\delta_{12}B + \delta_{22}C)\omega^2 y + (\delta_{12}C + \delta_{22}D)\omega^2 \theta. \tag{20}$$

Next for each of the two above equations let

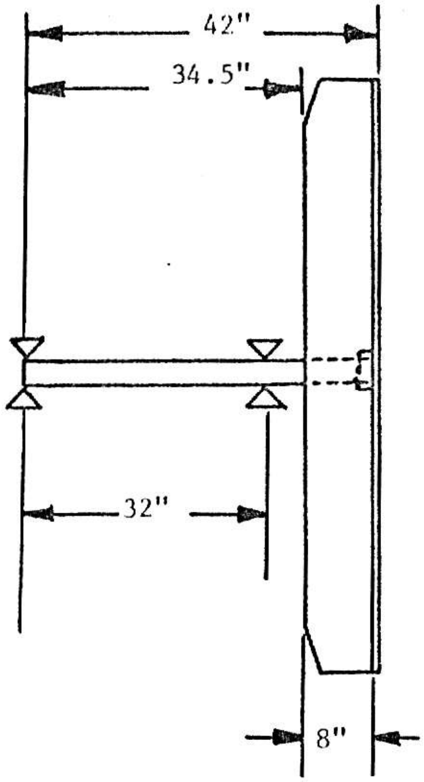
$$E = \delta_{11}B + \delta_{12}C$$
$$F = \delta_{11}C + \delta_{12}D$$
$$G = \delta_{12}B + \delta_{22}C$$
$$H = \delta_{12}C + \delta_{22}D.$$

Now find the ratio y/θ for each of equations (19) and (20) then equate the results. This can be done because the two ratios are equal since they are written for the same instant in time and point in space. So the final equation becomes

$$(FG - EH)\omega^4 + (E + H)\omega^2 - 1 = 0. \tag{21}$$

Solving this equation for the case of the prototype the natural frequency is calculated to be 135 cpm. In view of the assumptions made this agrees quite well with the experimentally determined value of 125 cpm. Values for the masses and moments of inertia are given in Appendix B (see Figs. 2 and 9 for lengths).

A special note should be made of the fact that when the prototype was built, the shaft was intended to be securely fastened at the bottom of the lower support. However, there turned out to be quite a bit of "play" at this point. So, the effective length of the shaft is actually 42 inches rather than 34.5 inches (see Fig. 9).

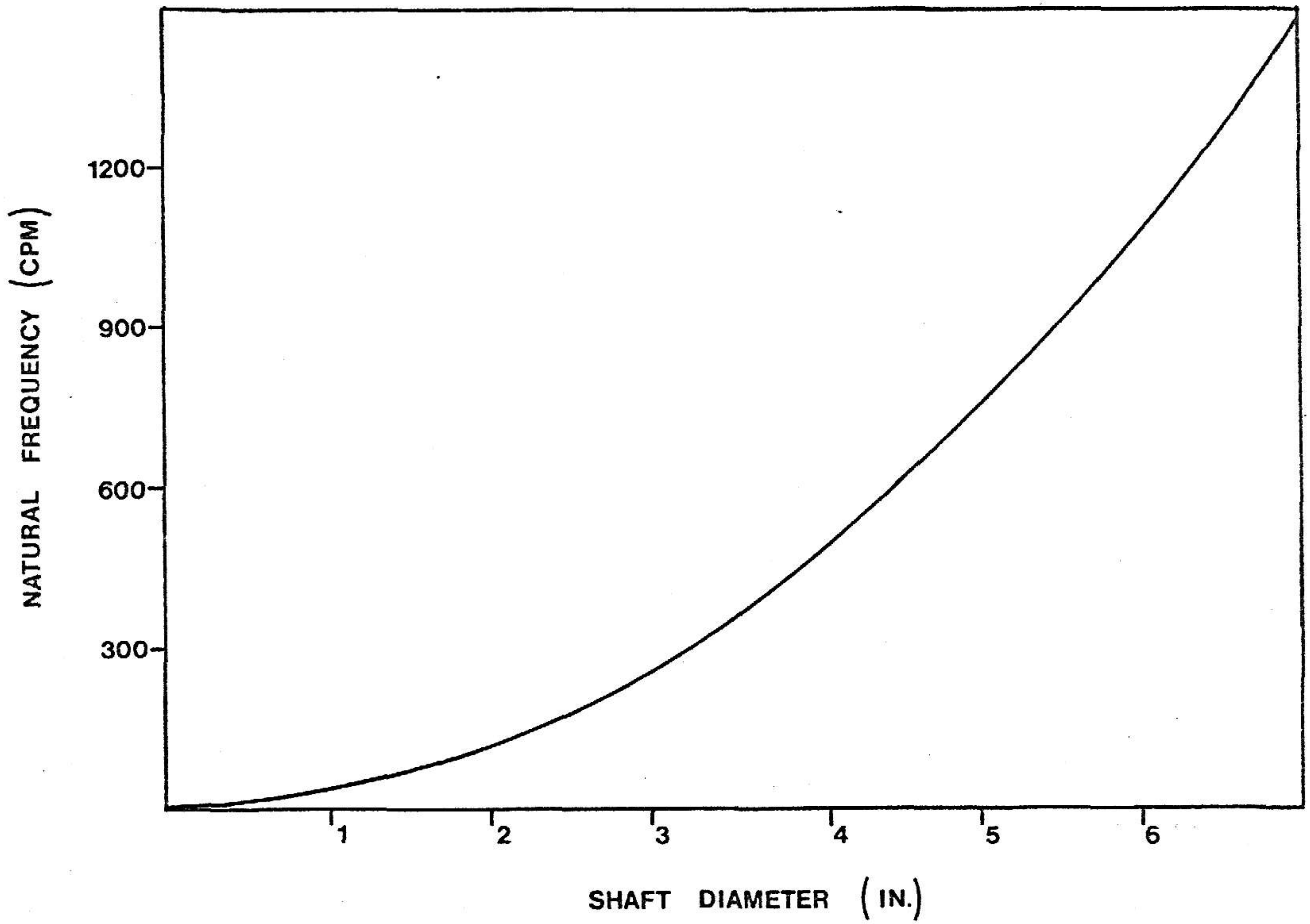


Lower Support And Shaft

Figure 9

Fig. 10 shows the effect that varying the shaft diameter has on the natural frequency. It is assumed that any future wind rotors will be securely fastened at the bottom of the lower support, so only the case of the 34.5 inch shaft is considered here. (note that although the associated computer program treats the lower support by itself as a thin disk, the height of the lower support is not neglected when the positions of the vanes and upper end plate relative to the shaft are considered.)

Now, remembering that the wind will tend to excite the rotor with a frequency of twice the shaft speed; it follows that the desired natural frequency of the system should be at least twice that of the maximum shaft RPM. This will ensure that a condition of "transverse resonance" will never be reached. If the maximum shaft speed is 414 RPM (corresponding to a 50 mph wind) the natural frequency of the rotor should be



Natural Frequency vs. Shaft Diameter

Figure 10

Shaft Diameter (in.)	Natural Transverse Frequency (cpm)
0.50	8
1.00	30
2.00	120
2.40	172
3.00	270
3.75	421
4.00	480
5.00	748
6.00	1079

Natural Frequencies

Table 3

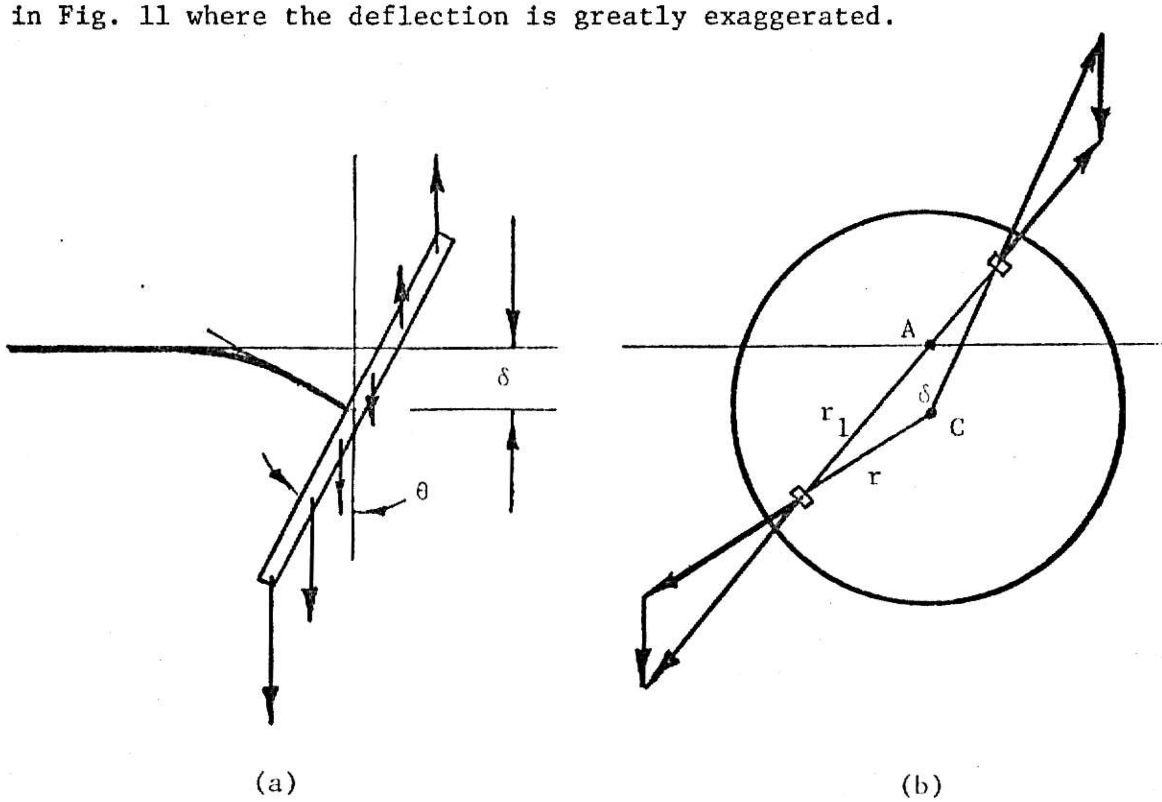
828 cpm. From Fig. 10 it can be seen that a shaft diameter of 5.25 inches is needed. This value for the shaft size is the minimum requirement for the transverse frequency criterion. (It should be pointed out that the effect of damping on the natural frequency has been neglected.)

CHAPTER VI

THE WHIRL EQUATION

The equation relating possible whirl frequencies to shaft speed will first be determined then its significance will be discussed. The same basic type of approach used in finding the natural transverse frequency will be employed in developing the equation for whirl. In this analysis the mass of the shaft and gravitational effects are neglected. Also, the rotor is assumed to be rigid.

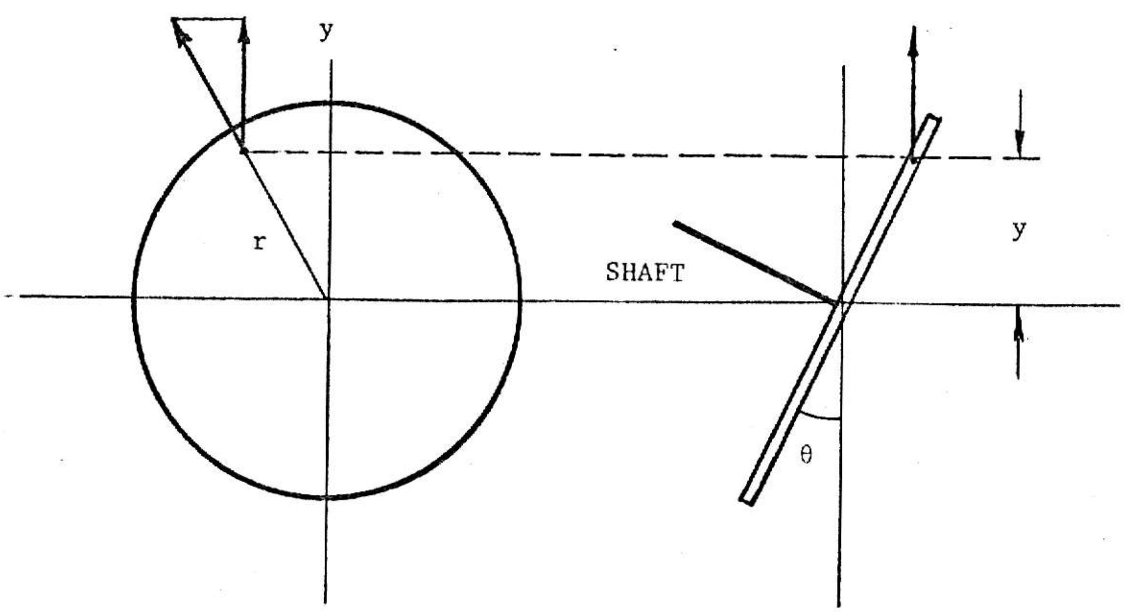
First of all, consider the centrifugal forces on a thin disk in whirl with whirl speed ω and shaft deflection δ. This case is shown in Fig. 11 where the deflection is greatly exaggerated.



Centrifugal Forces On A Disk In Whirl

Figure 11

In Fig. 11b we see that the centrifugal force of a mass element dm is $\omega^2 r_1 dm$ directed away from point A. This force can be resolved into two components, $\omega^2 \delta dm$ vertically down and $\omega^2 r dm$ directed away from the disk or shaft center C. The forces $\omega^2 \delta dm$ for the various mass elements add together to a single force $m\omega^2 \delta$ (where m is the total mass of the disk) acting downward at point C. The forces $\omega^2 r dm$ all radiate from the center of the disk C and their influence becomes clear from Fig. 12, as follows.



Centrifugal Moment Effect

Figure 12

The y-component of the force $\omega^2 r dm$ is $\omega^2 y dm$. The moment arm of this elemental force is $y\theta$, where θ is the (small) angle of the disk with respect to the vertical. Thus, the moment of a small particle dm being $\omega^2 y^2 \theta dm$, the total moment M of the centrifugal forces is

$$M = \omega^2\theta\int y^2 dm = \omega^2\theta Id \tag{22}$$

where Id is the mass moment of inertia of the disk about one of its diameters [3]. The moment acts in the counterclockwise direction for the disk shown. That is, it tends to straighten the shaft.

Now consider the case of the rotor in whirl shown in Fig. 13. Noting the deflection configuration in Fig. 13b and that the downward centrifugal force is directly proportional to the deflection; the centrifugal forces on the rotor can be reduced to the point forces shown in Fig. 13c. The force

$$Mv\omega^2 y$$

acting at L/2, is written in compliance with the rectangular area in 13b while the force

$$(\tfrac{1}{2})\ Mv\omega^2 L\theta$$

acting at (2/3)L is for the triangular area. The centrifugal moments shown in 13d are

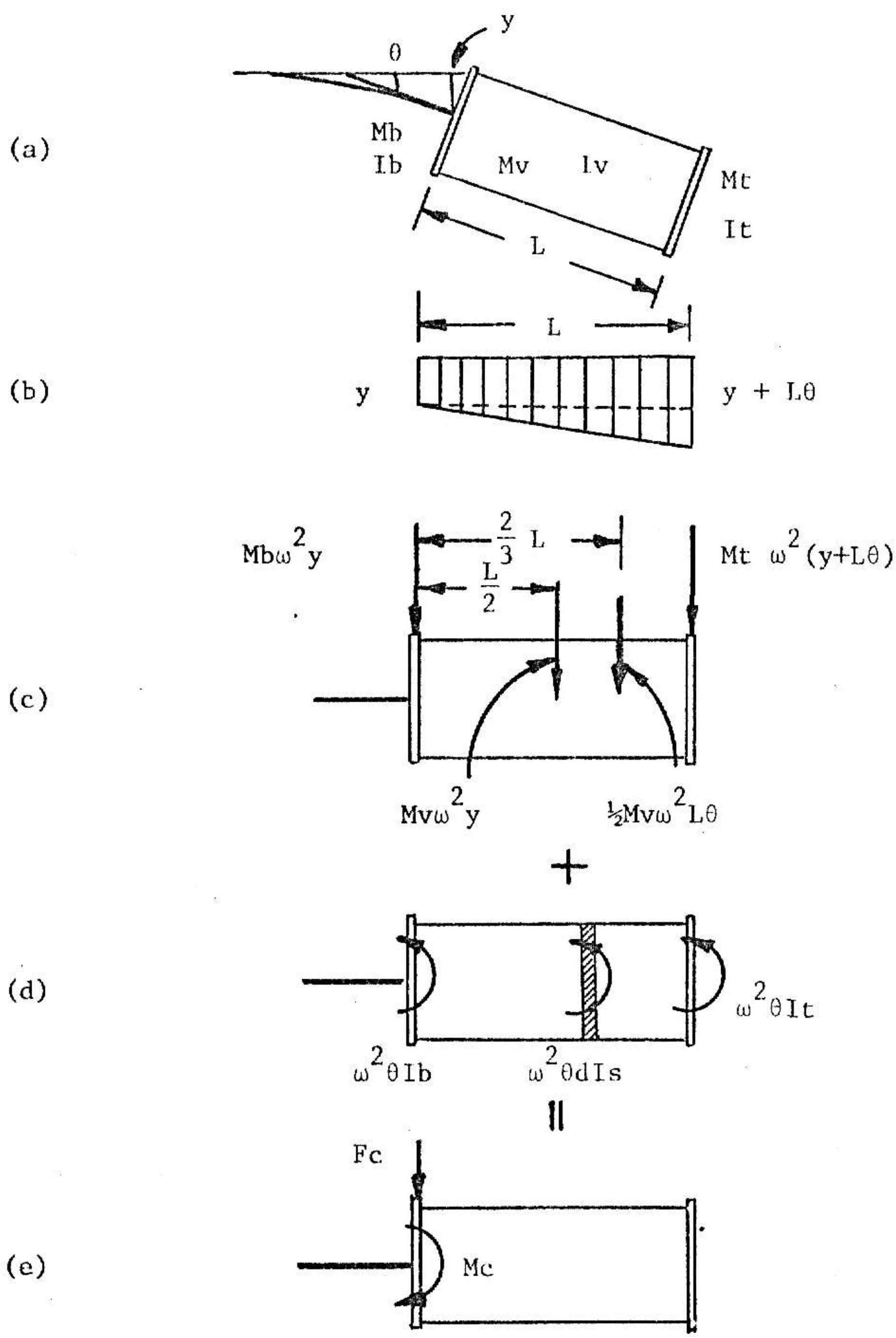
$$(Ib + Is + It)\theta\omega^2$$

where $\omega^2\theta Is$ is the sum of the moments of the infinitesimal vane elements. The value of Is is

$$Is = \rho L I_{AA} \tag{23}$$

where L is the vane height, ρ their density, and I_{AA} the area moment of inertia of the vanes.

Since the rotor is assumed to be rigid an equivalent force and moment can be written at the point of the shaft connection. This is the force and moment on the shaft from the centrifugal effects (see Fig. 13e). The expressions for these are



Centrifugal Effect On The Rotor In Whirl

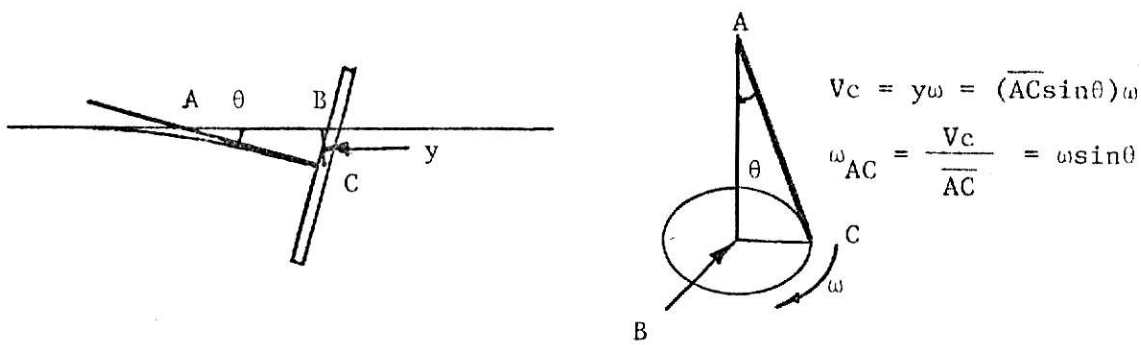
Figure 13

$$Fc = (Mb + Mv + Mt)\omega^2 y + (Mv/2 + Mt)L\omega^2\theta \tag{24}$$

and

$$Mc = \left(Mv(L/2) + MtL\right)\omega^2 y + \left((1/3)MvL^2 + MtL^2 - Ib - Iv - It\right)\omega^2\theta. \tag{25}$$

Next, consider the gyroscopic effects of a thin disk on the end of a rotating and whirling shaft. Assume the deflected shaft and disk in Fig. 14 to be spinning and whirling with angular speeds Ω and ω respectfully. Not only does the disk spin about its perpendicular axis,

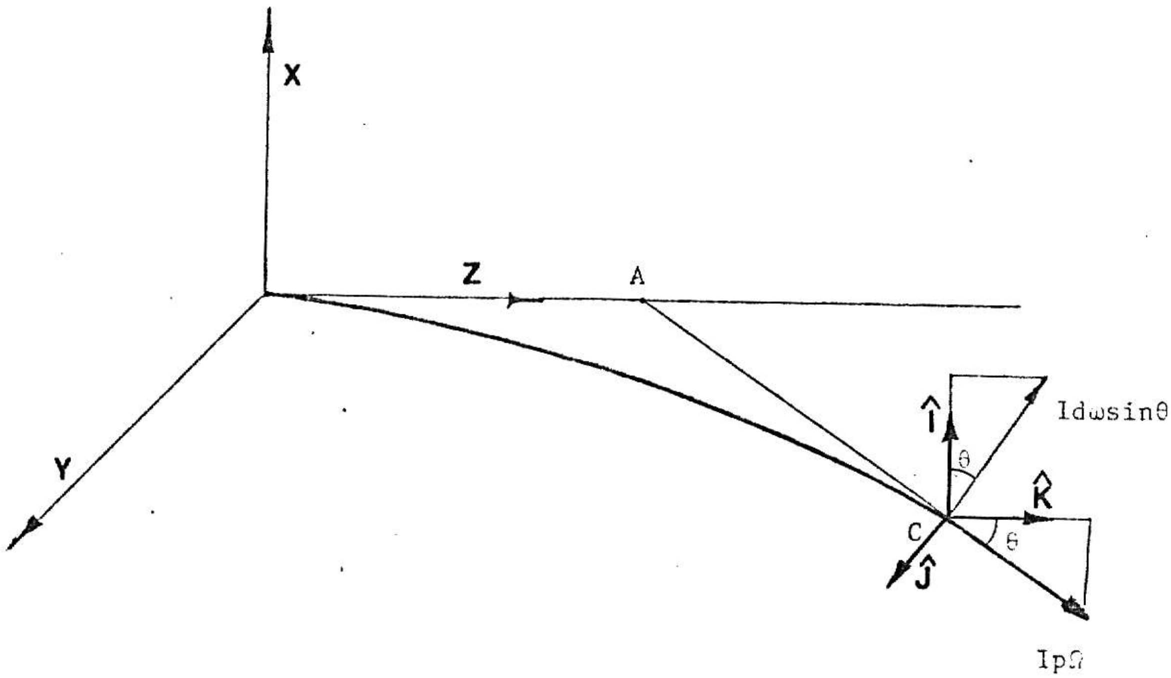


Whirling And Rotating Disk

Figure 14

but it also rotates about its diameter with angular speed ω_{AC} which is equal to $\omega\sin\theta$. The angular momentum vectors for the disk are shown in Fig. 15 where Ip is the mass polar moment of inertia of the disk, and Id is the mass moment of inertia of the disk about one of its diameters. In the position of the disk indicated (directly below the $\hat{k}$-axis), the vectors will be in the $\hat{i} - \hat{k}$ plane. Both Ω and ω

are assumed to be in the counterclockwise sense, so the



Angular Momentum Vectors

Figure 15

angular momentum vector $\overline{H}$ is

$$\overline{H} = (Id\omega\sin\theta\cos\theta - Ip\Omega\sin\theta)\hat{i} + (Id\omega\sin^2\theta + Ip\Omega\cos\theta)\hat{k}. \qquad (26)$$

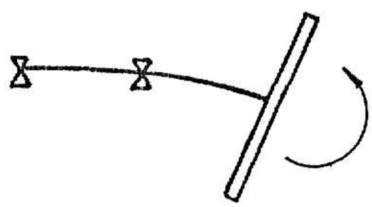
The time rate of change of the angular momentum is

$$d\overline{H}/dt = (Id\omega^2\sin\theta\cos\theta - Ip\Omega\omega\sin\theta)\,\hat{j} \qquad (27)$$

where $di/dt = \omega j$ and $dk/dt = 0$. For small angles the expression becomes

$$d\overline{H}/dt = (Id\omega^2 - Ip\Omega\omega)\theta j. \qquad (28)$$

Now, a moment having the same magnitude and sense as the change in angular momentum is required to produce it [4]. This moment comes from the shaft, and by action and reaction the moment by the disk on the shaft is equal and opposite. The moment on the shaft by the disk is, thus,



Gyroscopic Moment On The Shaft

Figure 16

$$M_G = (Ip\Omega\omega - Id\omega^2)\theta, \tag{29}$$

as shown in Figure 16. In extending this analysis to the wind rotor the values of Ip and Id will be different, but the expression for the moment on the shaft will remain the same. Ip simply becomes the sum of the polar moments of the various parts (Ipb + Ipv + Ipt) and Id is evaluated at the shaft connection by use of the parallel axis theorem ($Ib + Iv + It + MvL^2/4 + MtL^2$).

We are now ready to write the shaft equations taking into account the centrigugal and gyroscopic effects. The influence coefficients, determined in Appendix A, are again used. Letting

$$A = Mb + Mv + Mt$$

$$B = (Mv/2 + Mt)L$$

$$C = Mv\ (L/2) + MtL$$

$$D = (1/3)\ MvL^2 + MtL - Ib - Is - It$$

the equations are

$$y = \delta_{11}(A\omega^2 y + B\omega^2\theta) + \delta_{12}(C\omega^2 y + D\omega^2\theta + Ip\Omega\omega\theta - Id\theta\omega^2), \quad (30)$$

and

$$\theta = \delta_{12}(A\omega^2 y + B\omega^2\theta) + \delta_{22}(C\omega^2 y + D\omega^2\theta + Ip\Omega\omega\theta - Id\theta\omega^2), \quad (31)$$

where Ip is the polar moment of inertia for the rotor and Id is the moment of inertia of the rotor about the shaft connection. The same procedure of finding the ratio y/θ for each relation and equating the two is again used, as it was in the previous chapter. The two ratios are equal, of course, since they represent the same point in space at the same instant in time. The resulting equation is

$$(F\cdot H - E\cdot J)\omega^4 + (G\cdot H - E\cdot K)\omega^3 + (J + E)\omega^2 + K\omega - 1 = 0, \quad (32)$$

where

$$E = \delta_{11}A + \delta_{12}C$$

$$F = \delta_{11}B + \delta_{12}D - \delta_{12}Id$$

$$G = \delta_{12}Ip\Omega$$

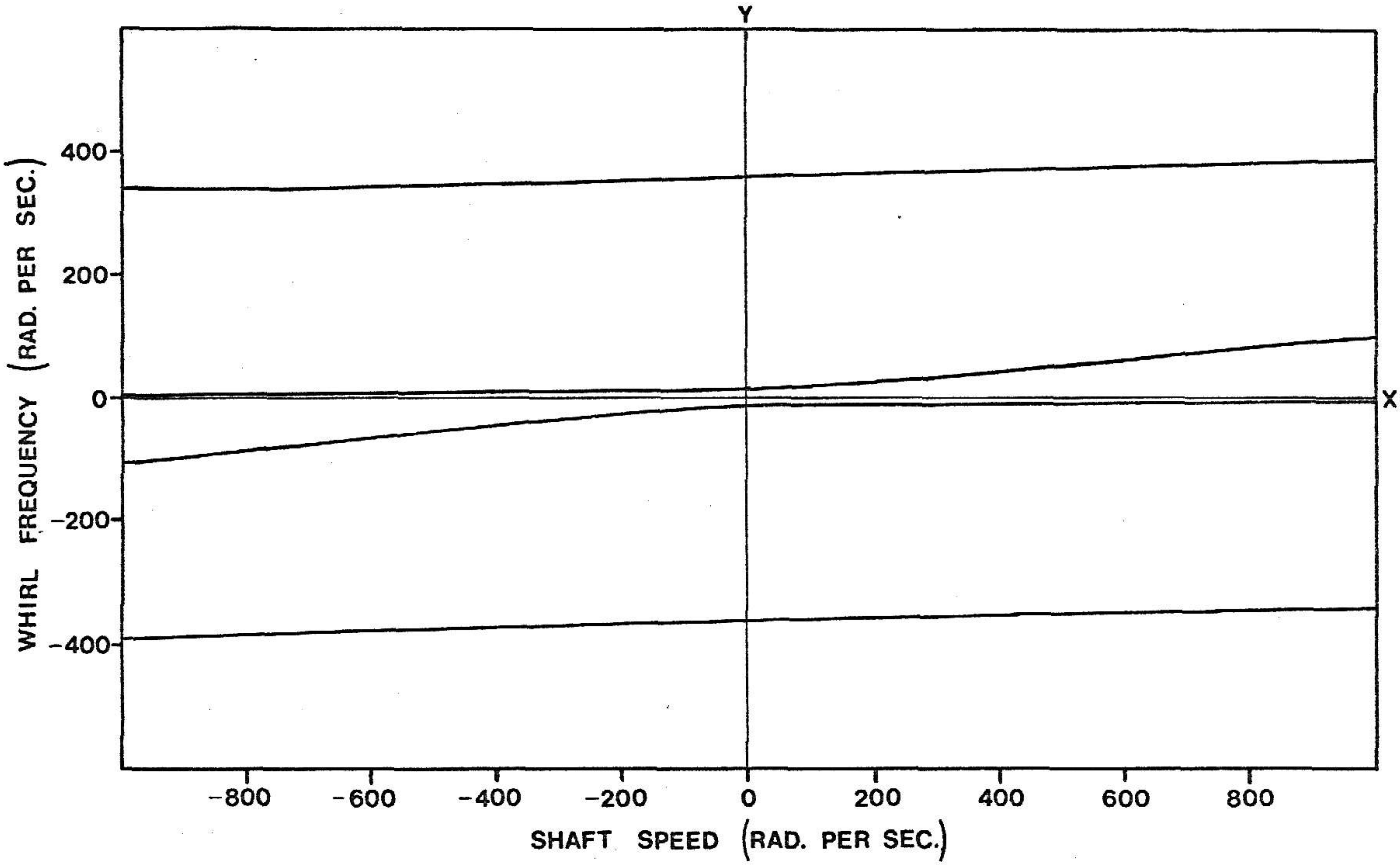
$$H = \delta_{12}A + \delta_{22}C$$

$$J = \delta_{12}B + \delta_{22}D - \delta_{22}Id$$

$$K = \delta_{22}Ip\Omega.$$

The equation is then solved by an iterative type procedure.

In Fig. 17 the whirl frequency is plotted against the shaft speed. From this figure it follows that there are four distinct possible whirl frequencies for each shaft speed. Equation (32) and Fig. 17 do not indicate that the rotor must whirl at every shaft speed, but if it does whirl it must do so with one of four frequencies corresponding to that shaft speed. According to Den Hartog [3] various frequency ratios ω/Ω

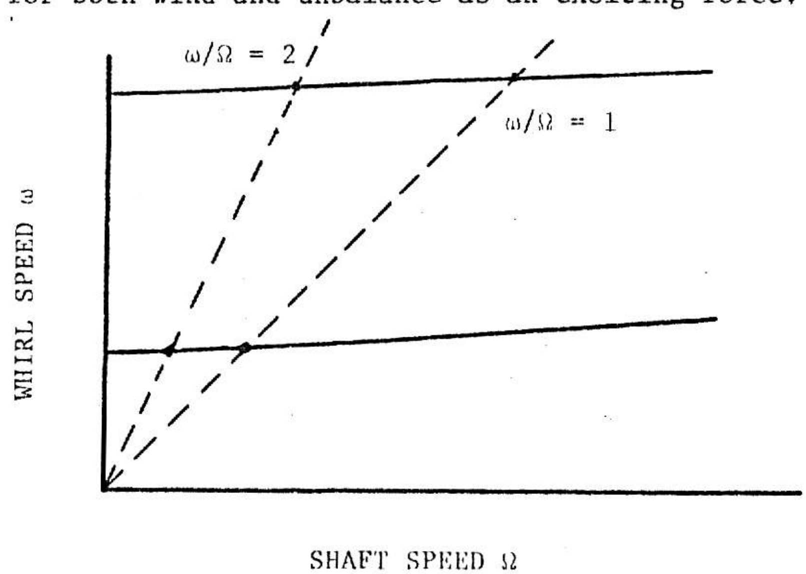


Whirl Frequency vs. Shaft Speed

Figure 17

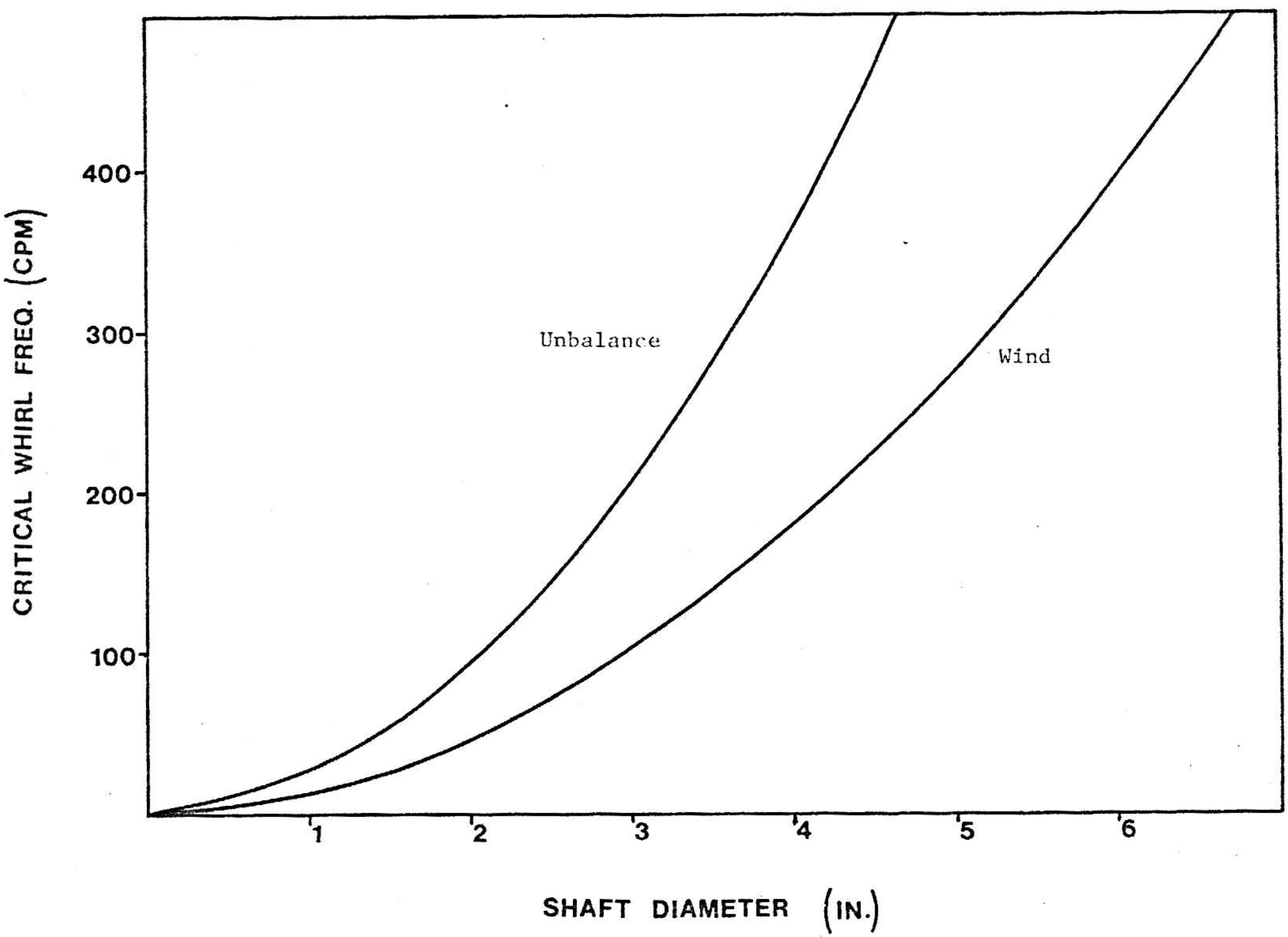
for thin disks have been observed. It is not fully understood why some machines whirl at certain ω/Ω ratios while others whirl at different values of ω/Ω. However, it appears that an "exciting" force of some kind is required to start the machine whirling. For instance, unbalance will act as a forcing function with a frequency of the shaft RPM. From this it seems logical that most machines will exhibit (at a certain critical RPM) a whirl to shaft speed ratio of $\omega/\Omega = 1$. This is exactly the case for almost all machines [3].

For the wind rotor the wind itself acts as a forcing function with twice the frequency of the rotation. It is not conclusive, but it appears quite probable that this will cause the rotor to whirl with the ratio $\omega/\Omega = 2$. This turns out to be a more critical design factor than unbalance ($\omega/\Omega = 1$) since the shaft should be designed such that the ω/Ω ratio must be always larger than 2 rather than 1 (see Fig. 18). The shaft size needed is just over 6 inches. This insures that the ratio $\omega/\Omega = 2$ is not attained within the range of the operating speeds. The relationship between shaft size and critical whirl frequency is shown in Fig. 19 for both wind and unbalance as an exciting force.



Whirl Ratios

Figure 18



Critical Whirl Frequency vs. Shaft Diameter

Figure 19

Shaft Diameter (in.)	Critical Whirl Frequency (c.p.m.) $\omega/\Omega = 2$	$\omega/\Omega = 1$
0.50	4	7
1.00	12	24
2.00	45	93
2.40	65	133
3.00	102	208
4.00	181	371
5.00	282	577
6.00	406	829
7.00	553	1130

Critical Whirl Frequencies

Table 4

CHAPTER VII

SUMMARY

In designing the shaft for the prototype Savonius wind rotor supported at the bottom, three design problems were considered. They were: shaft stress, the natural transverse frequency, and the critical whirl frequency. Of the three, the critical whirl frequency turns out to be the most severe problem, requiring a minimum shaft diameter of just over [illegible] inches. This is a much larger shaft size than was anticipated when this work was begun. For rotors of the size discussed it appears that it may be best to support them at both top and bottom. For smaller rotors this analysis, together with the computer program in Appendix D, should be helpful in deciding on the appropriate shaft size.

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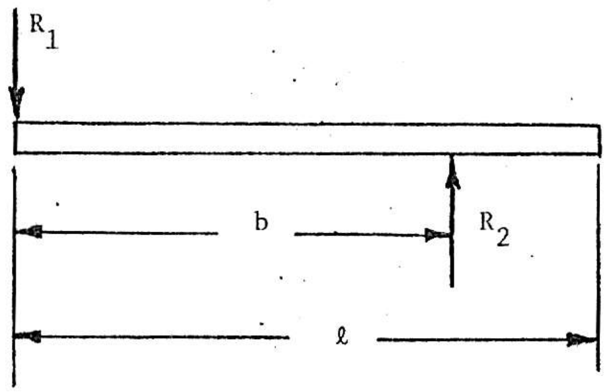
APPENDIX A

INFLUENCE COEFFICIENTS



Clebsch's method for beam deflection is used in calculating the influence coefficients for the shaft [5]. The equation of moment is written using unit step functions then integrated to find the deflection and angle equations.

$$EIY''(x) = M(x) = R_1 x\ u(x-0) - R_2(x-b)u(x-b)$$

$$EIY'(x) = (R_1 x^2/2)u(x-0) - \left(R_2(x-b)^2/2\right)u(x-b) + c_1$$

$$EIY(x) = (R_1 x^3/6)u(x-0) - \left(R_2(x-b)^3/6\right)u(x-b) + c_1 x + c_2$$

The boundary conditions are

$$EIY(0) = 0, \text{ and } EIY(b) = 0.$$

From this it follows that c_2 equals 0, and c_1 equals $-R_1 b^2/6$. The equations for deflection and slope become

$$Y(x) = \left((R_1 x^3/6)u(x-0) - \left(R_2(x-b)^3/6\right)u(x-b) - R_1 b^2 x/6\right)/EI,$$

and

$$\theta \approx Y'(x) = \left((R_1 x^2/2)u(x-0) - \left(R_2(x-b)^2/2\right)u(x-b) - R_1 b^2/6\right)/EI.$$

Now applying a 1 lb.-force downward at the end of the shaft and evaluating the deflection and slope also at the end of the shaft will give us two influence coefficients. For the 1 lb.-force, R_1 and R_2 become $(\ell-b)/b$ and ℓ/b respectfully. So two of the coefficients are

$$\overset{1\text{-}1b}{y(\ell)} = (\ell^3 - 2b\ell^2 + b^2\ell)/3EI = \delta_{11},$$

and

$$\overset{1\text{-}1b}{\theta(\ell)} = (3\ell^2 - 4\ell b + b^2)/6EI = \delta_{12}.$$

Now apply a 1 in.-lb. moment clockwise at the end of the shaft. For the deflection and slope equations R_1 and R_2 are both equal to 1/b. Thus the coefficients are

$$\overset{1\ \text{in-}1b}{y(\ell)} = (3\ell^2 - 4\ell b + b^2)/6EI,$$

which is the same as δ_{12}, and

$$\overset{1\ \text{in-}1b}{\theta(\ell)} = (3\ell - 2b)/3EI = \delta_{22}.$$

APPENDIX B

MASS AND INERTIA VALUES

The masses of the various wind rotor components are determined by use of the equation

$$m = \rho V.$$

where ρ and V stand for density and volume, respectfully. The value of ρ used for all components is 0.284 lb/in^3.

The calculations of the masses are shown below. When appropriate the moment of inertia calculations are also shown.

Upper End Plate

$$m = (0.284)(\pi)(36)^2(0.0598)$$

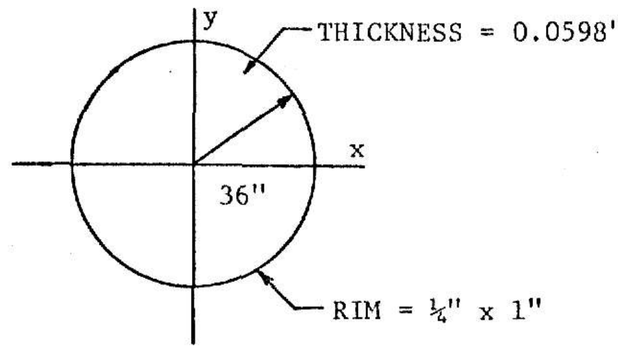
$$m = 69.1 \text{ lb}$$

$$Ix = Iy = \tfrac{1}{4}mr^2$$

$$= \tfrac{1}{4}(69.1)(36)^2$$

$$= 22{,}404 \text{ lb-in}^2$$

$$Iz = (2)(22{,}404) = 44{,}807 \text{ lb-in}^2$$



Circular Rim (Upper End Plate)

$$m = (0.284)(\pi)(1)\ \left((36.25)^2 - (36)^2\right) = 16.1 \text{ lb}$$

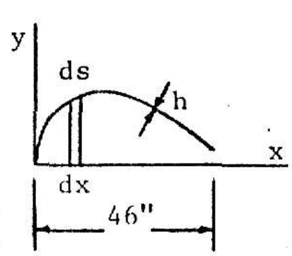
$$Ix = Iy = \tfrac{1}{4}m\,\frac{(r_2^4 - r_1^4)}{(r_2^4 - r_1^2)} = (\tfrac{1}{4})(16.1)\ \frac{\left((36.25)^4 - (36)^4\right)}{\left((36.25)^2 - (36)^2\right)}$$

$$= 10{,}506 \text{ lb-in}^2$$

$$Iz = (2)(10{,}506) = 21{,}012 \text{ lb-in}^2$$

Wind Vanes and Vane Flanges

These values are determined by the computer program by numerical integration. The equations used are shown on the following page.

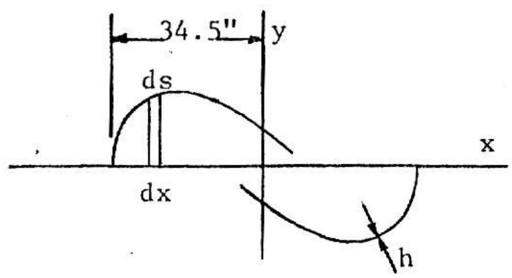


$$m = \int dm = 2\rho hz\int ds = 2\rho hz\int (dx^2 + dy^2)^{\frac{1}{2}} = 2\rho hz\int_0^{46} \left(1 + (\frac{dy}{dx})^2\right)^{\frac{1}{2}} dx$$

$$Ix(\text{area}) = \int y^2 dA = 2h\int_0^{46} y(x)^2 \left(1 + (\frac{dy}{dx})^2\right)^{\frac{1}{2}} dx$$

$$Ix(\text{mass}) = \int (y^2 + z^2) dm = 2\rho h \int_{-\frac{z}{2}}^{\frac{z}{2}} \int_0^{46} (y(x)^2 + z^2)\left(1 + (\frac{dy}{dx})^2\right)^{\frac{1}{2}} dxdz$$

$$= 2\rho z Ix(\text{area}) + \frac{1}{6}\rho hz^3 s$$



$$Iy(\text{area}) = \int x^2 dA = 2h \int_{-34.5}^{11.5} x^2 \left(1 + (\frac{dy}{dx})^2\right)^{\frac{1}{2}} dx$$

$$\text{or } Iy(\text{area}) = 2h \int_0^{46} (x - 34.5)^2 \left(1 + (\frac{dy}{dx})^2\right)^{\frac{1}{2}} dx$$

$$Iy(\text{mass}) = \int (x^2 + z^2) dm = 2\rho h \int_{-\frac{z}{2}}^{\frac{z}{2}} \int_0^{46} \left((x-34.5)^2 + z^2\right)\left(1 + (\frac{dy}{dx})^2\right)^{\frac{1}{2}} dxdz$$

$$= 2\rho z\ Iy(\text{area}) + \frac{1}{6}\rho hz^3 s$$

$$Iz(\text{mass}) = \int (x^2 + y^2)\ dm = 2\rho h \int_{-\frac{z}{2}}^{\frac{z}{2}} \int_0^{46} \left((x-34.5)^2 + y(x)^2\right)\left(1 + (\frac{dy}{dx})^2\right)^{\frac{1}{2}} dxdz$$

$$= 2\rho z\left[Ix(\text{area}) + Iy(\text{area})\right]$$

Values for the vanes and flanges are listed below.

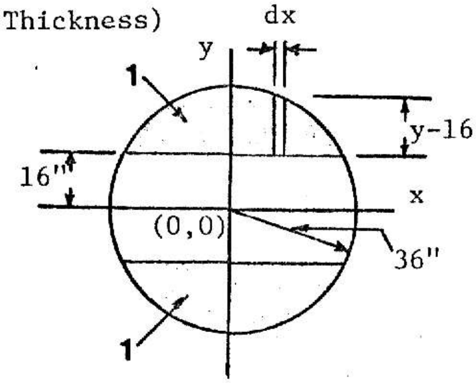
	2 Vanes	2 End Plate Flanges	2 External Ribs (center of vanes)
h (in.)	0.0747	4.0	2.0
z (in.)	120.0	0.25	0.25
Ix(area) (in.^{4})	1,605	85,944	42,972
Iy(area) (in.^{4})	3,746	200,589	100,295
Ix(mass) (lb-in^2)	435,570	6,103	3,052
Iy(mass) (lb-in^2)	508,536	14,243	7,122
mass (lb.)	317.4	35.6	17.7
s (in.)	62.3	62.3	62.3

Note that the prototype rotor was constructed with two end plate flanges at the upper end plate and four at the top of the lower support.

Lower Support

The calculation of mass of each component is shown below.

1 - Two Filler Plates (0.0598" Thickness)



$$x^2 + y^2 = r^2$$

$$x^2 = 16^2 = 36^2$$

$$x = \pm 32.25$$

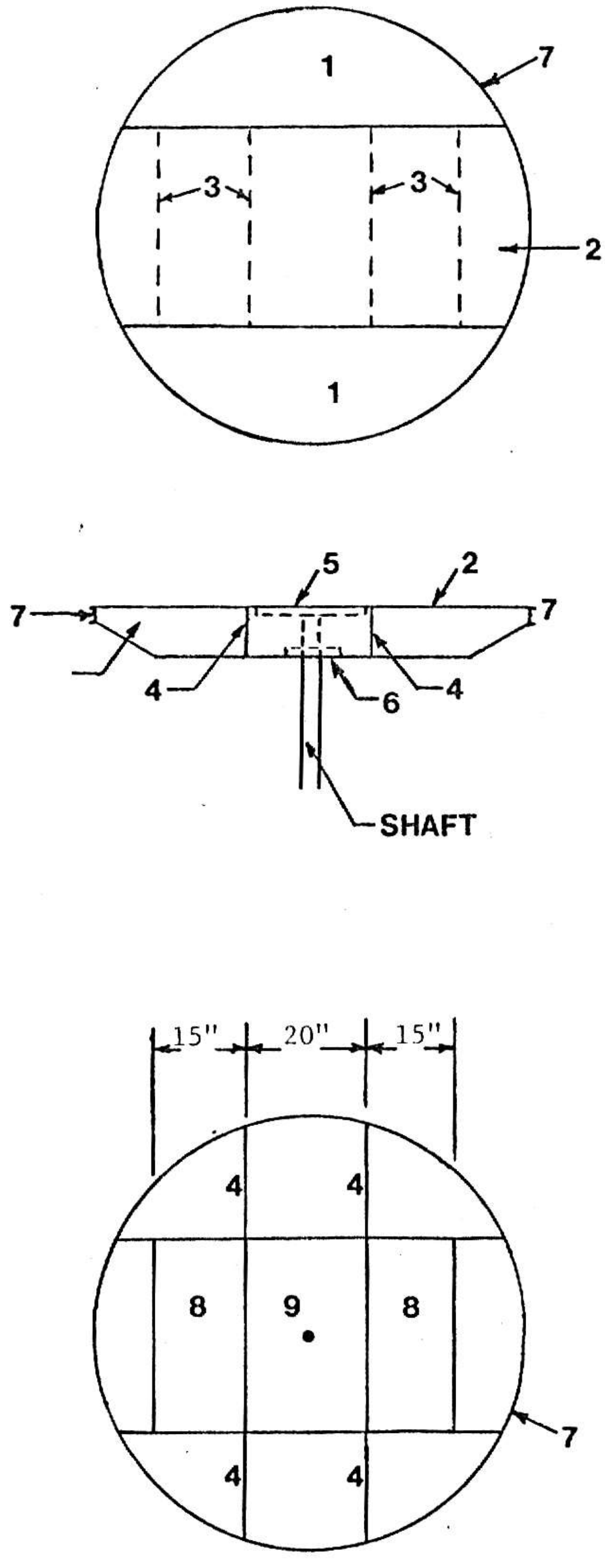
$$\text{area} = 2\int_{-32.25}^{32.25} (y-16)dx$$

$$= 4\int_{0}^{32.25} \left((36^2 - x^2)^{\frac{1}{2}} - 16\right)dx$$

$$= r\left(\frac{x}{2}(36^2 - x^2)^{\frac{1}{2}} + \frac{36^2}{2}\sin^{-1}\frac{x}{36} - 16x\right)_{0}^{32.25}$$

$$= 1846 \text{ in}^2$$

m = (.284)(.0598)(1846) = 31.3 lb

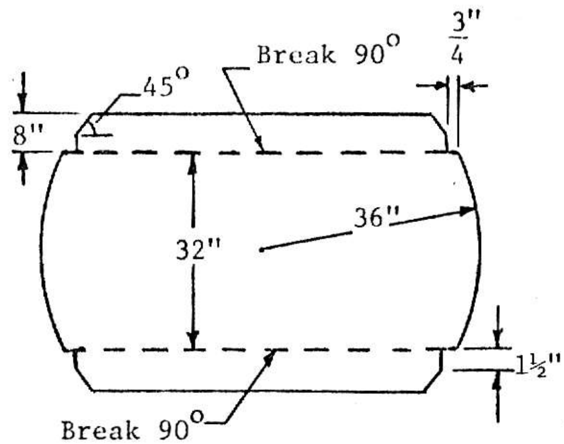


TOP VIEW

SIDE VIEW

BOTTOM VIEW

2 - Formed Steel Plate (0.1196" Thickness)



$$\text{area} = \pi(36)^2 - 1846 + (2)(63)(1\tfrac{1}{2}) + (4)(\tfrac{1}{2})(6\tfrac{1}{2})^2 + (2)(6.5)(50)$$

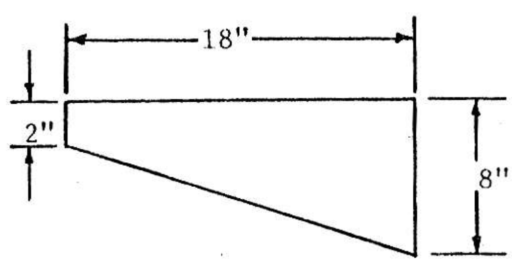
$$= 3149 \text{ in}^2$$

$$m = (0.284)(0.1196)(3149) = 100.1 \text{ lb}$$

3 - Four $32\frac{3}{32}$" x $7\frac{3}{4}$" Steel Sheets (0.1196" Thickness)

$$m = (4)(0.284)(0.1196)(32.09375)(7.75) = 33.8 \text{ lb}$$

4 - Four Slanted Supports (0.1196" Thickness)



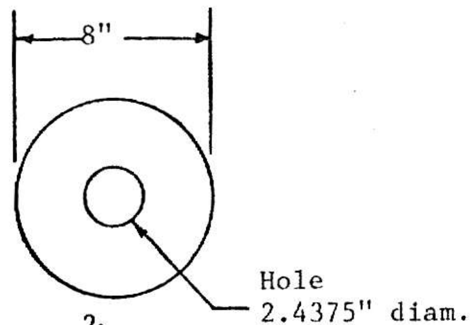
$$\text{area} = (4)\left[(2)(18) + (\tfrac{1}{2})(6)(18)\right] = 360 \text{ in}^2$$

$$m = (0.284)(0.1196)(360) = 12.2 \text{ lb}$$

5 - <u>18" Diameter Circular Plate</u> (½" Thickness)

$$m = (0.284)(\tfrac{1}{2})\left[(\pi)(9)^2\right] = 36.1 \text{ lb}$$

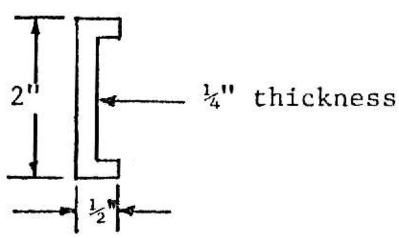
6 - <u>Bottom Circular Plate</u> (½" Thickness)



$$m = (0.284)(\tfrac{1}{2})(\pi)\left[(8)^2 - (2.4375)^2\right](\tfrac{1}{4})$$

$$= 6.5 \text{ lb}$$

7 - <u>Channel Iron</u>



$$m = (0.284)(2\pi)(36)\left[(2)(\tfrac{1}{4}) + (2)(\tfrac{1}{4})^2\right]$$

$$= 40.2 \text{ lb}$$

8 - <u>Two $32\frac{3}{32}$" x 15" Steel Sheets</u> (0.1196" Thickness)

$$m = (2)(0.284)(0.1196)(32.09375)(15) = 32.7 \text{ lb}$$

9 - <u>$32\frac{3}{32}$" x 20" Steel Sheet</u> (0.1196" Thickness)

$$m = (0.284)(0.1196)\left[(32.09375)(20) - \tfrac{1}{4}\pi(2.4)^2\right] = 21.7 \text{ lb}$$

7.5" of Shaft (2.4" diameter)

$$m = (0.284)(7.5)(\tfrac{1}{4})(\pi)(2.4)^2 = 9.6 \text{ lb}$$

Lower Support $\Sigma m = 324.2$ lb

The values of the moments of inertia of the lower support are roughly estimated to be 1½ times that of a thin disk with the same mass. That is,

$$Ix = Iy = (1.5)(\tfrac{1}{4}mr^2)$$

and

$$Iz = 2Ix = 2Iy$$

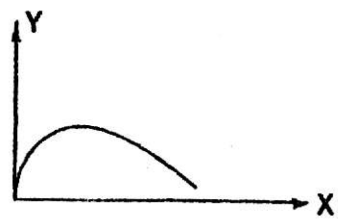
where r is 36 inches. Justification for approximating these values is that the final results are not very dependent on their accuracy. For instance, it was found that varying these values by as much as 50% resulted in less than 2% change in the critical shaft speeds.

APPENDIX C

DESCRIPTION OF THE COMPUTER PROGRAM

It is assumed that the user has a basic knowledge of Fortran programming techniques.

First of all, the steps within the program will be briefly presented, then the required input and corresponding output will be discussed. The program itself can be divided into three steps.



The first step fits the vane profile to a 3rd, 4th, or 5th order polynomial equation. This is done by using the method of least squares in which the degree of the polynomial is determined by comparison of the rms error. The purpose of this step is to allow easy analysis of any reasonable vane profile.

Next, using the polynomial equation just determined, the computer performs numerical integrations to determine the masses, lengths, and moments of inertia of the rotor vanes and flanges (see Appendix B).

The third part consists of the equations for stress, transverse natural frequency, and critical whirl frequency. Iteration is used to find the critical whirl frequency.

The input symbols are as follows:

1. NDATA - The number of vane (x,y) coordinate points to be considered fitting the vane profile to an equation. It can be any integer number from 5 through 99.
2. XC - The x-coordinates used in finding the vane profile equation (in inches).

3. YC - The y-coordinates used in finding the vane equation (in inches).

4. CASE - Enter 1 to specify that the rotor is securely fastened at the bottom of the lower support. A 2 entered indicates that it is not.

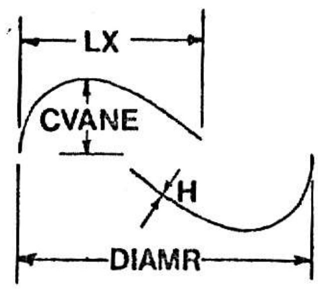
5. DIAMS - Shaft Diameter in inches

6. E - Shaft Modulus of Elasticity in psi

7. L - Length of the Shaft in inches from the bottom bearing to the bottom of the lower support

8. B - Distance between the two bearings in inches

9. HLSUPP - Thickness of the lower support in inches



10. LX - in inches (see drawing)

11. CVANE - The largest y-coordinate of the vane in inches (see drawing)

12. DIAMR - in inches (see drawing)

13. H - vane thickness in inches

14. DENS - vane density in lb/in^3

15. LV - vane height in inches

16. NFT - number of end flanges on the upper end plate

17. NFB - number of end flanges on the lower support

18. DENSF - flange density in lb/in^3

19. FW - flange width in inches

20. FTH - flange thickness in inches

21. NFVS - number of flanges on the vanes

22. DENSFV - flange density in lb/in^3

23. FWVS - flange width in inches

24. FTHVS - flange thickness in inches

25. MT - Mass of the upper end plate (excluding flanges) in lb

26. MB - Mass of the lower support (excluding flanges) in lb

27. IDT - mass moment of inertia of the upper end plate (excluding flanges) in $lb\text{-}in^2$

28. IPT - polar mass moment of inertia of the upper end plate (excluding flanges) in $lb\text{-}in^2$

29. IDB - mass moment of inertia of the lower support (excluding flanges) in $lb\text{-}in^2$

30. IPB - polar mass moment of inertia of the lower support (excluding flanges) in $lb\text{-}in^2$

31. VELW - maximum wind velocity (in mph) considered in determining stress

32. USEWV - maximum wind velocity (in mph) at which the rotor is allowed to operate

33. WL - the desired lower bound (in rad/sec) for a listing of possible whirl speeds due to unbalance

34. WF - the desired upper bound (in rad/sec)

35. DW - the desired increments in going from WL to WF

OUTPUT SYMBOLS

LENGTH - vane length

C - largest y-coordinate determined from the vane equation

XX - the x-coordinate corresponding to C

MVS - Vane mass plus the mass of the flanges on the middle of the vanes

IYM - moment of inertia of the vanes only

IXM - moment of inertia of the vanes only

IPTOT - Total polar moment of inertia of the rotor

FMT - mass of the flanges on the upper end plate

FMB - mass of the flanges on the lower support

FMVS - mass of the flanges on the vanes

TFIY - sum of the flange mass moment of inertia about the y-axis

TFIX - sum of the flange mass moment of inertia about the x-axis

The rest of the output is self-explanatory.

APPENDIX D

COMPUTER PROGRAM AND SAMPLE OUTPUT

ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE

P-474

```
     $JOB              ,TIME=(,10)
1          SUBROUTINE SPCTOR(NDATA,XC,YC,CASE,DIAMS,E,L,B,FLSUPP,LX,CVANE,
          XDIAMR,H,DENS,LV,NFT,NFB,DENSF,FW,FTF,NFVS,DENSFV,FWVS,FTHVS,MT,MB,
          XIDT,IPT,IDP,IPB,VELW,LSCWV,WL,WF,DW)
2          DOUBLE PRECISION A(9,9),XC(99),YC(99),X,Y,A1,A2,A3,A4,A5,A6,SUM,IX
          XA,IYA,IR,IL,IAVE,LENGTH,LL,LR,LAVE,IXM,IYM,ID,DENS,LV,B,EI,MVS,MT,
          XMB,LX,IP,WL,DW,WF,DEL,A11,A12,A22,NU1,NU2,W,WS,AM,BM,CM,DIAMS,L,ID
          XB,IDT,IPB,IPT,IDV,IPV,WDIFF,WDIFF1,MTOT,IDVSUM,IDSUM
3          INTEGER CASE
4          RMS=10.E 09
5          N=2
6      151 N=N+1
7          IF(N.EQ.6)GO TO 747
8          RMSI=RMS
9          DO 800 I2=1,N
10         DO 169 I=1,N
11         SUM=0.0
12         DO 170 I1=1,NDATA
13     170 SUM=XC(I1)**(I+I2)+SUM
14     169 A(I2,I)=SUM
15     800 CONTINUE
16         N1=N+1
17         DO 400 I=1,N
18         SUM=0.0
19         DO 401 I1=1,NDATA
20     401 SUM=YC(I1)*XC(I1)**(I)+SUM
21     400 A(I,N1)=SUM
22         DO 200 J=1,N
23         DIV=A(J,J)
24         S=1.0/DIV
25         DO 201 K=J,N1
26     201 A(J,K)=A(J,K)*S
27         DO 202 I=1,N
28         IF(I-J) 203,202,203
29     203 AIJ=-A(I,J)
30         DO 204 K=J,N1
31     204 A(I,K)=A(I,K)+AIJ*A(J,K)
32     202 CONTINUE
33     200 CONTINUE
34         A4=0.0
35         A5=0.0
36         A1=A(1,N1)
37         A2=A(2,N1)
38         A3=A(3,N1)
39         IF(N.EQ.3) GO TO 900
40         A4=A(4,N1)
41         IF(N.EQ.4) GO TO 900
42         A5=A(5,N1)
43     900 CONTINUE
44         RMS=0.0
45         DO 850 I1=1,NDATA
46         X=XC(I1)
47         Y=A1*X+A2*(X**2)+A3*(X**3)+A4*(X**4)+A5*(X**5)
48     850 RMS=RMS+((Y-YC(I1))**2)/NDATA
49         IF(RMS.LE.RMSI)GO TO 151
50     747 N=N-1
51         DO 410 I2=1,N
52         DO 469 I=1,N
53         SUM=0.0
54         DO 470 I1=1,NDATA
```

```
  470 SUM=XC(I1)**(I+I2)+SUM
  469 A(I2,I)=SUM
  410 CONTINUE
      N1=N+1
      DO 420 I=1,N
      SUM=0.0
      DO 411 I1=1,NDATA
  411 SUM=YC(I1)*XC(I1)**(I)+SUM
  420 A(I,N1)=SUM
      DO 440 J=1,N
      DIV=A(J,J)
      S=1.0/DIV
      DO 441 K=J,N1
  441 A(J,K)=A(J,K)*S
      DO 442 I=1,N
      IF(I-J) 443,442,443
  443 AIJ=-A(I,J)
      DO 444 K=J,N1
  444 A(I,K)=A(I,K)+AIJ*A(J,K)
  442 CONTINUE
  440 CONTINUE
      A4=0.0
      A5=0.0
      A1=A(1,N1)
      A2=A(2,N1)
      A3=A(3,N1)
      IF(N.EQ.3) GO TO 911
      A4=A(4,N1)
      IF(N.EQ.4) GO TO 911
      A5=A(5,N1)
  911 CONTINUE
      RMS=0.0
      DO 450 I1=1,NDATA
      X=XC(I1)
      Y=A1*X+A2*(X**2)+A3*(X**3)+A4*(X**4)+A5*(X**5)
  450 RMS=RMS+((Y-YC(I1))**2)/NDATA
      LENGTH=0.
      LL=0.
      IXA=0.
      IL=0.
      ILY=0
      IYA=0.
      I=0
      NDIV=1000
      DX=LX/FLOAT(NDIV)
   11 CONTINUE
      I=I+1
      X=I*DX
      Y=A1*X+A2*(X**2)+A3*(X**3)+A4*(X**4)+A5*(X**5)
      DYODX=A1+2*A2*X+3*A3*(X**2)+4*A4*(X**3)+5*A5*(X**4)
      LR=SQRT(1.+(DYODX**2))
      IRY=(((X-DIAMR/2.)**2)*H)*(SQRT(1.+(DYODX**2)))
      IR=((Y**2)*H)*(SQRT(1.+(DYODX**2)))
      IYAVE=(ILY+IRY)/2.0
      LAVE=(LL+LR)/2.
      IAVE=(IL+IR)/2.0
      LENGTH=LENGTH+LAVE*DX
      IYA=IYA+IYAVE*DX
      IXA=IXA+IAVE*DX
      ILY=IRY
```

```
      LL=LR
      IL=IR
      IF(I.LE.NDIV)GO TO 11
      PRINT305,VELW
  305 FORMAT('1','MAX. WIND VELOCITY CONSIDERED (STATIC CASE) =',F6.1,'
     XMPH')
      PRINT21,DIAMS
   21 FORMAT('0','SHAFT DIAMETER =',F6.3,' INCHES')
      Q=.002378*2.*(VELW*5280./3600.)**2/2.*DIAMR/12.*LV/12.
      SMS=(Q*L/B+Q*LV/(2.*B)-Q)/(3.141592654*DIAMS**2/4.)*4./3.
      SMBS=(Q*L+Q*LV/2.-Q*B)*DIAMS/2./(3.141592654*DIAMS**4/64.)
      VMS=Q/(2.*LENGTH*H)
      VMBS=Q*LV/2.*CVANE/(IXA*2.)
      PRINT301,SMBS
      PRINT302,VMBS
      PRINT303,SMS
      PRINT304,VMS
  301 FORMAT(' ','MAX. PRINCIPAL BENDING STRESS (SHAFT) =',E16.7,' PSI')
  302 FORMAT(' ','MAX. PRINCIPAL BENDING STRESS (VANES) =',E16.7,' PSI')
  303 FORMAT(' ','MAX. PRINCIPAL SHEAR STRESS (SHAFT) =',E16.7,' PSI')
  304 FORMAT(' ','MAX. PRINCIPAL SHEAR STRESS (VANES) =',E16.7,' PSI')
      X=0.
      J=0
      DX=2.*DX
   33 CONTINUE
      X=J*DX+X
      DYDDX=A1+2*A2*X+3*A3*(X**2)+4*A4*(X**3)+5*A5*(X**4)
      J=J+1
      IF(DYDDX.GE.0)GO TO 33
      XX=X-DX/2
      Y=A1*XX+A2*(XX**2)+A3*(XX**3)+A4*(XX**4)+A5*(XX**5)
      C=Y
      IXM=2.*IXA/12.**4*DENS/32.2*1728.*LV/12.+DENS*1728./32.2*H/12.*(LV
     X/12.)**3/6.*LENGTH/12.
      IYM=2.*IYA/12.**4*DENS/32.2*1728.*LV/12.+DENS*1728./32.2*H/12.*(LV
     X/12.)**3/6.*LENGTH/12.
      FI=FW*FTH*DENSF
      FMT=FI*NFT*(LENGTH+FTH)
      FMB=FI*NFB*(LENGTH+FTH)
      FI=FI/H
      FIDT=NFT*FI*IYA
      FIDTMI=NFT*FI*IXA
      FIDB=NFB*FI*IYA
      FIDBMI=NFB*FI*IXA
      FIV=FWVS*FTHVS*DENSFV
      FMVS=FIV*NFVS*LENGTH
      FIV=FIV/H
      FIDV=NFVS*FIV*IYA
      FIDVMI=NFVS*FIV*IXA
      IDV=IYM
      IDVSUM=2.*IXA/12.*DENS/32.2*LV/12.
      IF(IYM.GE.IXM)GO TO 41
      FIDT=NFT*FI*IXA
      FIDTMI=NFT*FI*IYA
      FIDB=NFB*FI*IXA
      FIDBMI=NFB*FI*IYA
      FIDV=NFVS*FIV*IXA
      FIDVMI=NFVS*FIV*IYA
      IDV=IXM
      IDVSUM=2.*IYA/12.*DENS/32.2*LV/12.
```

```
   41 CONTINUE
      ID=IDV+(IDB+IDT+FIDT+FIDB+FIDV)/(144.*32.2)
      IDSUM=IDVSUM+(IDB+IDT+FIDTMI+FIDBMI+FIDVMI)/(144.*32.2)
      IPV=2.*(IXA+IYA)/(12.**4)*DENS/32.2*1728.*LV/12.+(NFT+NFB)*FW/H*(I
     XXA+IYA)*DENSF/32.2/144.*FTH+NFVS*FWVS/H*(IXA+IYA)*DENSFV/32.2/144.
     X*FTHVS
      IP=IPV+(IPB+IPT)/(12.**2*32.2)
      EI=144.*E*3.141592654/64.*(DIAMS/12.)**4
      B=B/12.
      MVS=2.*LV*LENGTH*H*DENS/32.2+FMVS/32.2
      MB=(MB+FMB)/32.2
      MT=(MT+FMT)/32.2
      AM=MB+MVS+MT
      BM=MVS/2.+MT
      CM=MVS/3.+MT
      IF(CASE.EQ.2)GO TO 378
      IF(CASE.EQ.1)GO TO 377
  377 CONTINUE
      LV=LV+HLSUPP
  378 CONTINUE
      LV=LV/12.
      IF(CASE.EQ.1)GO TO 477
      L=L+HLSUPP
  477 CONTINUE
      L=L/12.
  547 CONTINUE
      A11=(((L-B)*(L**3)/(6.*B))-((L*(L-B)**3)/(6.*B))-(L-B)*B*L/6.)/EI
      A12=((L**2)/2.-2.*L*B/3.+(B**2)/6.)/EI
      A22=((L**2)/(2.*B)-((L-B)**2)/(2.*B)-B/6.)/EI
      ALPH11=A11*12.
      ALPH22=A22/12.
      PRINT551,ALPH11
      PRINT552,A12
      PRINT553,ALPH22
  551 FORMAT('0','ALPHA11 =',E16.7,' IN./LB.')
  552 FORMAT(' ','ALPHA12 =',E16.7,' 1/LB.')
  553 FORMAT(' ','ALPHA22 =',E16.7,' 1/LB.-IN.')
      PRINT379,CASE
  379 FORMAT('0','CASE ',I1)
      PRINT310,USEWV
  310 FORMAT('0','MAX. WIND VELOCITY CONSIDERED (OPERATING CASE) =',F5.1
     X,' MPH')
      VV=1.7*USEWV*5280./3600./(DIAMR/(2.*12.))
      WCPM=VV*60./(2.*3.141592654)
      PRINT351,VV,WCPM
  351 FORMAT('0','MAX. SHAFT SPEED =',F6.2,' RADIANS PER SECOND =',F7.2,
     X' CYCLES PER MINUTE')
      MTOT=AM
      T1=A11*MVS*LV/2.+A11*MT*LV+A12*MVS/4.*LV**2+A12*MT*LV**2+A12*ID
      T2=A12*MVS*LV/2.+A12*MT*LV+A22*ID+A22*MVS/4.*LV**2+A22*MT*LV**2
      TO1=A11*MTOT+A12*MVS*LV/2.+A12*MT*LV
      TO2=A12*MTOT+A22*MVS*LV/2.+A22*MT*LV
      TA=T1*TO2-T2*TO1
      TB=T2+TO1
      TSR=TB**2+4.*TA
      WT=(SQRT(TSR)-TB)/(2.*TA)
      WT=SQRT(WT)
      WTCPM=WT*60./(2.*3.141592654)
      PRINT657,WTCPM
  657 FORMAT('0','CRITICAL SHAFT SPEED (TRANSVERSE) DUE TO UNBALANCE =',
```

```
     XF7.2,' CPM')
      WT=WTCPM/2.
      WDIFF1=10.E 30
      DEL=10.E-09
      I=0
      DO 555 I3=1,10000
      I=I+1
      W=I*.3
      D1=A11*AM*W**2+A12*BM*LV*W**2-1.
      D2=A12*AM*W**2+A22*BM*LV*W**2
      IF(ABS(D1).LT.DEL) GO TO 555
      IF(ABS(D2).LT.DEL) GO TO 555
      D3=A22*IP*W/D2-A12*IP*W/D1
      IF(ABS(D3).LT.DEL) GO TO 555
      RO=IDSUM
      NU1=-A11*BM*LV*W**2-A12*CM*(LV**2)*W**2+A12*RO*W**2-A12*(ID+MVS/4.
     X*LV**2+MT*LV**2)*W**2
      NU2=-A12*BM*LV*W**2-A22*CM*(LV**2)*W**2+A22*RO*W**2-A22*(ID+MVS/4.
     X*LV**2+MT*LV**2)*W**2+1.
      WS=(NU1/D1-NU2/D2)/D3
      WDIFF=(W-WS)**2
      IF(WDIFF.GT.WDIFF1) GO TO 556
  555 WDIFF1=WDIFF
  556 CONTINUE
      W=W-.15
      WWCPM=W*60./(2.*3.141592654)
  557 FORMAT('0','CRITICAL SHAFT SPEED (WHIRL) DUE TO UNBALANCE =',F7.2,
     X' CPM')
      PRINT557,WWCPM
  790 FORMAT('-','CRITICAL SHAFT SPEED (TRANSVERSE) DUE TO WIND =',F7.2,
     X' CPM')
      PRINT790,WT
      WDIFF2=10.E 30
      I=0
      DO 700 I1=1,1000
      I=I+1
      W=I*.3
      D1=A11*AM*W**2+A12*BM*LV*W**2-1.
      D2=A12*AM*W**2+A22*BM*LV*W**2
      IF(ABS(D1).LT.DEL) GO TO 700
      IF(ABS(D2).LT.DEL) GO TO 700
      D3=A22*IP*W/D2-A12*IP*W/D1
      IF(ABS(D3).LT.DEL) GO TO 700
      RO=IDSUM
      NU1=-A11*BM*LV*W**2-A12*CM*(LV**2)*W**2+A12*RO*W**2-A12*(ID+MVS/4.
     X*LV**2+MT*LV**2)*W**2
      NU2=-A12*BM*LV*W**2-A22*CM*(LV**2)*W**2+A22*RO*W**2-A22*(ID+MVS/4.
     X*LV**2+MT*LV**2)*W**2+1.
      WS=(NU1/D1-NU2/D2)/D3
      WDIFF3=(W-2.*WS)**2
      IF(WDIFF3.GT.WDIFF2) GO TO 701
  700 WDIFF2=WDIFF3
  701 CONTINUE
      W=(W-.15)/2.
      W=W*60./(2.*3.141592654)
  702 FORMAT('0','CRITICAL SHAFT SPEED (WHIRL) DUE TO WIND =',F7.2,' CPM
     X')
      PRINT702,W
      PRINT558
  558 FORMAT('-','       WHIRL SPEED (RAD./SEC.)        SHAFT SPEED (RAD./S
```

```
      XEC.)')
      I=0
  111 CONTINUE
      W=WL+I*DW
      I=I+1
      D1=A11+AM*W**2+A12*BM*LV*W**2-1.
      D2=A12*AM*W**2+A22*BM*LV*W**2
      IF(ABS(D1).LT.DEL) GO TO 199
      IF(ABS(D2).LT.DEL) GO TO 199
      D3=A22*IP*W/D2-A12*IP*W/D1
      IF(ABS(D3).LT.DEL) GO TO 199
      NU1=-A11*BM*LV*W**2-A12*CM*(LV**2)*W**2+A12*RC*W**2-A12*(ID+MVS/4.
     X*LV**2+MT*LV**2)*W**2
      NU2=-A12*BM*LV*W**2-A22*CM*(LV**2)*W**2+A22*RC*W**2-A22*(ID+MVS/4.
     X*LV**2+MT*LV**2)*W**2+1.
      WS=(NU1/D1-NU2/D2)/D3
      PRINT,W,WS
  199 CONTINUE
      IF(W.LT.WF) GO TO 111
  733 FORMAT('1','THE COEFFICIENTS DEFINING THE VANE EQUATION ARE')
      PRINT733
  102 FORMAT('0',6E16.8)
      WRITE(6,102)  (A(I,N1),I=1,N)
  860 FORMAT('0','RMS =',E16.7)
      PRINT860,RMS
      PRINT99,LENGTH
      PRINT55,C
      PRINT44,XX
      PRINT107,IYA
      PRINT22,IXA
  107 FORMAT(' ','IYA =',E16.7,' IN.**4')
   22 FORMAT(' ','IXA =',E16.7,' IN.**4')
      TFIY=(NFT+NFB)*FI*IYA+NFVS*FIV*IYA
      TFIX=(NFT+NFB)*FI*IXA+NFVS*FIV*IXA
      IYM=IYM*32.2*144.
      IXM=IXM*32.2*144.
      IPV=IPV*32.2*144.
      MVS=MVS*32.2
      PRINT713,MVS
      PRINT710,IYM
      PRINT711,IXM
      PRINT712,IPV
      PRINT714,FMT
      PRINT715,FMB
      PRINT716,FMVS
      PRINT717,TFIY
      PRINT718,TFIX
  710 FORMAT(' ','IYM = ',F10.2,' LB.-IN**2')
  711 FORMAT(' ','IXM = ',F10.2,' LB.-IN**2')
  712 FORMAT(' ','IPTOT = ',F10.2,' LB.-IN**2')
  713 FORMAT(' ','MVS = ',F7.2,' LB.')
  714 FORMAT(' ','FMT = ',F6.2,' LB.')
  715 FORMAT(' ','FMB = ',F6.2,' LB.')
  716 FORMAT(' ','FMVS = ',F6.2,' LB.')
  717 FORMAT(' ','TFIY = ',F9.2,' LB.-IN.**2')
  718 FORMAT(' ','TFIX = ',F9.2,' LB.-IN.**2')
   99 FORMAT(' ','LENGTH = ',F6.3,' INCHES')
   44 FORMAT(' ','XX = ',F6.3,' INCHES')
   55 FORMAT(' ','C = ',F6.3,' INCHES')
      PRINT554
```

```
  554 FORMAT('0','        VANE X-COORDINATE                VANE Y-COORDINATE')
      I=0
  500 CONTINUE
      X=2.*I
      I=I+1
      Y=A1*X+A2*(X**2)+A3*(X**3)+A4*(X**4)+A5*(X**5)
      PRINT,X,Y
      IF(X.LT.46.)GO TO 500
  513 FORMAT('1')
      PRINT513
      RETURN
      END

      DOUBLE PRECISION A(9,9),XC(99),YC(99),X,Y,A1,A2,A3,A4,A5,A6,SUM,IX
     XA,IYA,IR,IL,IAVE,LENGTH,LL,LR,LAVE,IXM,IYM,ID,DENS,LV,B,EI,MVS,MT,
     XMB,LX,IP,WL,DW,WF,DEL,A11,A12,A22,NU1,NU2,W,WS,AM,BM,CM,DIAMS,L,ID
     XB,IDT,IPB,IFT,ICV,IPV,WDIFF,WDIFF1,MTCT,IDVSUM,IDSUM
      INTEGER CASE
      READ,NDATA
      READ,(XC(I),I=1,NDATA)
      READ,(YC(I),I=1,NDATA)
      READ,CASE
      READ,DIAMS,E,L,B,FLSUPP
      READ,LX,CVANE,DIAMR,H,DENS,LV,NFT,NFB,DENSF,FW,FTH,NFVS,DENSFV,
     XFWVS,FTHVS,MT,MB,IDT,IPT,ICB,IPB,VELW,USEWV,WL,WF,DW
      CALL SECTOR(NDATA,XC,YC,CASE,DIAMS,E,L,B,FLSUPP,LX,CVANE,
     XDIAMR,H,DENS,LV,NFT,NFB,DENSF,FW,FTH,NFVS,DENSFV,FWVS,FTHVS,MT,MB,
     XIDT,IPT,ICB,IPB,VELW,USEWV,WL,WF,DW)
      STOP
      END

$ENTRY
```

MAX. WIND VELOCITY CONSIDERED (STATIC CASE) = 100.0 MPH

SHAFT DIAMETER = 2.400 INCHES
MAX. PRINCIPAL BENDING STRESS (SHAFT) = 0.1354529E 06 PSI
MAX. PRINCIPAL BENDING STRESS (VANES) = 0.2023684E 04 PSI
MAX. PRINCIPAL SHEAR STRESS (SHAFT) = 0.1693162E 04 PSI
MAX. PRINCIPAL SHEAR STRESS (VANES) = 0.3158347E 03 PSI

ALPHA11 = 0.3235959E-04 IN./LB.
ALPHA12 = 0.3466848E-05 1/LB.
ALPHA22 = 0.4390822E-06 1/LB.-IN.

CASE 2

MAX. WIND VELOCITY CONSIDERED (OPERATING CASE) = 50.0 MPH

MAX. SHAFT SPEED = 43.36 RADIANS PER SECOND = 414.08 CYCLES PER MINUTE

CRITICAL SHAFT SPEED (TRANSVERSE) DUE TO UNBALANCE = 135.37 CPM

CRITICAL SHAFT SPEED (WHIRL) DUE TO UNBALANCE = 110.29 CPM

CRITICAL SHAFT SPEED (TRANSVERSE) DUE TO WIND = 67.69 CPM

CRITICAL SHAFT SPEED (WHIRL) DUE TO WIND = 53.71 CPM

WHIRL SPEED (RAD./SEC.)	SHAFT SPEED (RAD./SEC.)
-0.25000000000000000D 02	-0.23909141386461920D 03
-0.23000000000000000D 02	-0.21070942067250440D 03
-0.21000000000000000D 02	-0.18143762666489820D 03
-0.19000000000000000D 02	-0.15098685750007120D 03
-0.17000000000000000D 02	-0.11693240651088690D 03
-0.15000000000000000D 02	-0.84624527862900240D 02
-0.13000000000000000D 02	-0.47014537728558110D 02
-0.11000000000000000D 02	-0.42924069268436460D 01
-0.90000000000000003D 01	0.46959265410133880D 02
-0.70000000000000004D 01	0.11406146089300200D 03
-0.50000000000000004D 01	0.21604585818898760D 03
-0.30000000000000004D 01	0.42269074625678800D 03
-0.10000000000000004D 01	0.13619869215493570D 04
0.99999999999999963D 00	-0.13619869215493970D 04
0.29999999999999997D 01	-0.42269074625678810D 03
0.49999999999999997D 01	-0.21604585818898770D 03
0.69999999999999997D 01	-0.11406146089300210D 03
0.89999999999999995D 01	-0.46959265410134040D 02
0.11000000000000000D 02	0.42924069268434900D 01
0.13000000000000000D 02	0.47014537728557950D 02
0.15000000000000000D 02	0.84624527862900080D 02
0.17000000000000000D 02	0.11693240651088560D 03
0.19000000000000000D 02	0.15098685750007100D 03
0.21000000000000000D 02	0.18143762666489800D 03
0.23000000000000000D 02	0.21070942067250420D 03
0.25000000000000000D 02	0.23909141386461910D 03

E COEFFICIENTS DEFINING THE VANE EQUATION ARE

0.40082279D 01 -0.33758497D 00 0.13847665D-01 -0.27988247D-03 0.21221062D-05

S = 0.2688645E 00
NGTH = 62.335 INCHES
= 18.327 INCHES
= 15.696 INCHES
A = 0.1875212D 04 IN.**4
A = 0.8026968D 03 IN.**4
S = 335.08 LB.
M = 508535.86 LB.-IN**2
M = 435569.56 LB.-IN**2
TOT = 253604.14 LB.-IN**2
T = 35.55 LB.
B = 71.10 LB.
VS = 17.70 LB.
IY = 49851.98 LB.-IN.**2
IX = 21362.25 LB.-IN.**2

VANE X-COORDINATE	VANE Y-COORDINATE
0.0000000000000000D 00	0.0000000000000000D 00
0.2000000000000000D 01	0.6772486550286286D 01
0.4000000000000000D 01	0.1144832559378423D 02
0.6000000000000000D 01	0.1454117764437372D 02
0.8000000000000000D 01	0.1647352772298625D 02
0.1000000000000000D 02	0.1758483224526578D 02
0.1200000000000000D 02	0.1813966830922942D 02
0.1400000000000000D 02	0.1833589256292826D 02
0.1600000000000000D 02	0.1831274019210813D 02
0.1800000000000000D 02	0.1815907360787034D 02
0.2000000000000000D 02	0.1792143153433250D 02
0.2200000000000000D 02	0.1761222779628922D 02
0.2400000000000000D 02	0.1721789022687298D 02
0.2600000000000000D 02	0.1670700955521482D 02
0.2800000000000000D 02	0.1603848825410519D 02
0.3000000000000000D 02	0.1516968962765452D 02
0.3200000000000000D 02	0.1406458625895436D 02
0.3400000000000000D 02	0.1270190545773770D 02
0.3600000000000000D 02	0.1108329774804029D 02
0.3800000000000000D 02	0.9241445795860747D 01
0.4000000000000000D 02	0.7248253496321734D 01
0.4200000000000000D 02	0.5222974703830857D 01
0.4400000000000000D 02	0.3340366154741275D 01
0.4600000000000000D 02	0.1838836360012238D 01

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DESIGN CONSIDERATIONS OF A SAVONIUS WIND ROTOR SUPPORTED AT THE BOTTOM

by

CARL LAWRENCE JACOBS

B. S., Kansas State University, 1974

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
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1976

ABSTRACT

The energy shortage facing this nation has caused a great deal of concern to all portions of society. Tapping unused sources of power such as the wind has become a major goal of many scientists and engineers. At Kansas State University a vertical axis wind rotor was built to produce energy from the wind. The shaft design of this rotor, supported at the bottom only, was studied in this thesis.

In designing the shaft three problems were considered. The first was stress due to high winds. To begin with, the maximum possible wind velocity was arbitrarily picked to be 100 mph. Next, the drag force on the rotor vanes was approximated to be the same as the force on a flat plate with the same projected area. From this the stress in the shaft was calculated, and, hence, the minimum shaft size ensuring no permanent deformation could be chosen.

The second problem considered was the possibility of the rotor exhibiting "transverse resonance" within the range of the operating speed. First, the maximum possible shaft speed was found, then the equation for the natural frequency was derived. Considering the wind as the crucial "forcing function," the minimum shaft size could then be specified to ensure that resonance would not be reached.

The third problem considered was the possibility that the rotor might reach a "critical whirl" speed. To arrive at a proper shaft size to ensure that this would not happen, the equation for possible whirl frequencies was determined. Again the wind was assumed to be

the crucial "forcing function."

Of the three design problems whirl turned out to be the most critical, requiring a shaft diameter of just over 6 inches for the K-State wind rotor. This was much larger than was anticipated and so, it appears that it may be best to support rotors of this size at the top as well as the bottom. However, for smaller vertical-axis wind machines this analysis and the computer program developed should be helpful in deciding on the proper shaft size.