COMPUTER INTEGRATED MANUFACTURE OF
OPTIMAL PLATE CAMS

by

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TO MY PARENTS

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Chapter I

INTRODUCTION

The rapid growth of the computer industry has provided the engineering world with a whole array of powerful computing machines that have completely altered most aspects of engineering practice . Computer Aided Design (CAD), Computer Aided Manufacture (CAM), Computer Integrated Engineering (CIE) and Computer Integrated Manufacture (CIM) are some of the technologies that are a direct result of the computer revolution.

CAD and CAM are probably the fastest expanding areas in industry today. These two fields initially emerged as related but independent technologies . A large volume of current research is focussed on efforts to link these two technologies together for maximum production efficiency. Traditionally, designers have worked in isolation, taking only design considerations and performance requirements into account while creating a design; manufacturing limitations are generally not given much consideration.

Computer Integrated Manufacturing (CIM) is a technology developed to incorporate CAD and CAM into a united framework. CIM can be divided into two categories based on the function it performs. The first category deals with the creation and manipulation of a common database that can be used by the designer as well as the production planner. This is a very significant aspect of CIM and plays a leading role in streamlining communications between the design and manufacturing departments within a company. Most of the current research work in the field of CIM falls into this category. The second category is generally known as designing for producibility. The motivation here is the existence of a basic weakness in conventional design: even though the design process may create an efficient design, it does not always ensure that the design is one that can be easily produced, since the designer is operating essentially independently of manufacturing considerations. In conventional design it is quite common to sacrifice performance to obtain better producibility. However, it is often possible to enhance producibility even without sacrificing performance if we tackle the design and manufacturing problems simultaneously. This approach, therefore, exploits the idea of introducing manufacturing considerations at the design level itself.

The research described in this thesis is an effort to implement the "design for producibility" aspect of Computer Integrated Manufacturing. The objective set for this work was to devise an analytical framework for the integration of CAD and CAM and to develop software which allows the designer to implement not only performance considerations, but also manufacturability at the stage of designing. In this environment, the designer includes manufacturing considerations such as manufacturing time, manufacturing cost, production sequences, cutting speed, cutter diameter, feed, etc ., while evaluating and modifying candidate designs. This effort will broaden the scope of the design problem by establishing tighter links between the design and manufacture of a product; it will also enable designers to bridge the long-standing gap between performance and manufacturing considerations.

In this research, the specific problem of designing and manufacturing a plate cam is considered . It is assumed that the cam will be produced by end milling on a Numerical Control Machine. The specifications of a Pratt & Whitney Tapemate Series C NC machine were taken into consideration. It is further assumed that the cam will be cut from a rectangular blank, where the size and orientation of the blank will be provided by the program. Efforts have been made to reduce the cost of manufacturing the plate cam while keeping stringent constraints on the performance requirements.

A constrained nonlinear optimization approach has been adopted in this research to achieve design/manufacturing integration. This approach provides a convenient unified framework within which design and manufacturing considerations can be dealt with simultaneously. It also provides a systematic, iterative method for choosing the values of the design/manufacturing parameters in order to create a product that is optimal with respect to design as well as manufacture.

To formulate the optimization problem, the design engineer is required to specify the following:

1) A set of design variables whose values must be determined to obtain the optimum solution. The engineer is also required to specify initial

values for these variables.

2) An objective function to be minimized or maximized.

3) Constraint functions that represent feasibility conditions that the final design must satisfy.

The type of cams considered in this research are polynomial cams, i.e., cams whose profiles are specified by a polynomial of suitable degree . In conventional design, the number of coefficients of the polynomials are set equal to the number of boundary conditions specified. The evaluation of these coefficients is then just a matter of solving a set of linear equations, as explained in Chapter 2 . The values of the coefficients are therefore uniquely determined[1]; however, the design obtained may not be a feasible design from either the manufacturing or the performance point of view . The approach taken in this research is to choose the order of the polynomial to be one higher than that required for satisfying the specified boundary conditions. This extra coefficient can then be treated as a design variable in the optimization problem.

The base diameter of the cam plays an important role in cam-follower mechanisms . The minimum possible base diameter is usually determined by the size of the roller follower, the pressure angle requirement and the required lift . On the other hand, large cams are undesirable because of the bulkiness of the mechanism and the presence of heavy rotating parts . The base diameter is also treated as a design variable; however the upper and lower bounds on it need to be specified . As an option, the base diameter can be treated as constant during the optimization procedure, if the user wishes to do so.

After establishing the design variables, the second specification in a nonlinear optimization technique is to define a cost function that is to be minimized. Here, the manufacturing cost is considered to be directly related to manufacturing time; hence the objective function chosen here is the cutting time . This cost function, however, can be altered as desired without significantly changing the general method that is developed in this thesis.

The third issue in the optimization process is the set of constraints. Here, both the designer and manufacturer can impose their respective constraints on the design . The designer may constrain the design by putting constraints on undercutting, pressure angle, velocity, acceleration, jerk, contact stresses, etc, while the manufacturer can impose constraints on cutting speed, feed, etc.

Once an initial set of values is given for the design parameters, the coefficients of the polynomials are computed using the boundary conditions and the initial values assigned to the design variables . Now, using these polynomials, the constraints are evaluated at specified points around the cam contour. The tool path is then generated automatically using an algorithm that ensures that the path generated will be the most efficient one that satisfies the manufacturing tolerance requirements . After the path has been generated, the cutting time associated with the path can be estimated. As mentioned earlier, this cutting time is the cost function . If all constraints are satisfied then the estimated cost is taken directly as the cost function; in case there are violated constraints then the highest constraint violation is multiplied by a penalty factor (usually of the order of 10^5 or higher), and added to the estimated cost to obtain the cost function. In essence, if any constraint is found to be violated in any region, the design is forced to move away from that region.

Each design is passed through a grid search non-linear optimization algorithm in order to compute a new, updated design. By cycling through a suitable number of iterations, the design variables are made to converge to a final design which minimizes the cutting time and satisfies the constraints that are imposed. The final design is than utilized to generate Numerical Control machine codes which can be used to produce the cam.

The design aspects of polynomial plate cams are presented in Chapter 2. Manufacturing aspects and the generation of the tool path are discussed in Chapter 3. Chapter 4 deals with the formulation of the nonlinear optimization problem with emphasis on the choice of design variables and the formulation of cost/constraint functions. Some of the numerical examples that were used to test the proposed model are presented in Chapter 5, and some conclusions and recommendations for future research are discussed in Chapter 6.

Chapter II

PLATE CAM DESIGN

Cams are used to lead a follower element by direct contact through desired motion patterns for the performance of a specific task. Cams and followers may be classified in different ways. Based on physical structure, cams can be classified as wedge cams, barrel cams or plate cams[1]. Similarly, followers can be classified as roller followers, knife-edge followers or flat face followers. This work focuses on plate cams with roller followers having pure translational motions. However, with proper adjustments, other types of cams and/or followers may also be incorporated.

CAM AND FOLLOWER MECHANISM

Cam and follower mechanisms are simple and inexpensive; they have few moving parts and, compared to linkages, they occupy less space. Cams are used to make the follower undergo various specified motion patterns, defined in the form of rise, return and dwell phases, with control on position, velocity, acceleration, jerk,

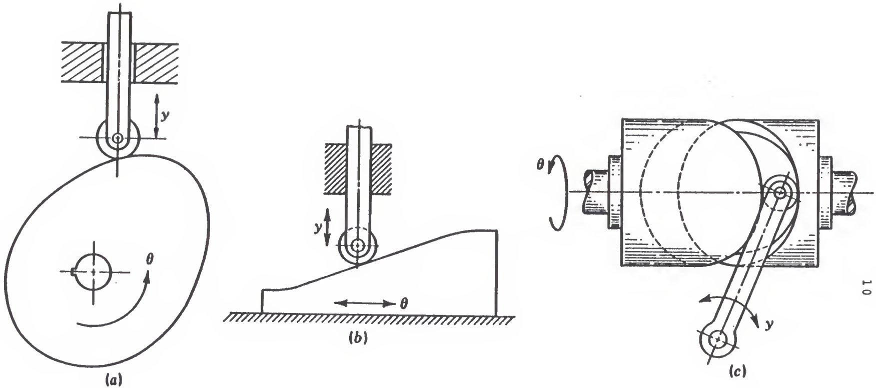
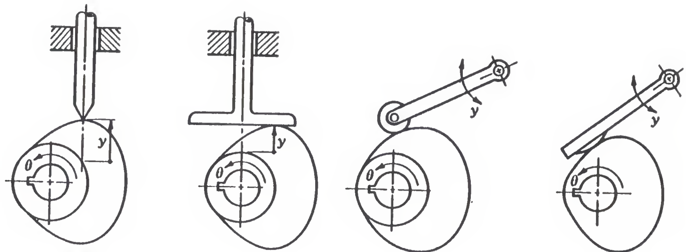


Plate Cam   Wedge Cam   Barrel Cam

Figure 1(a): Types of Cams
(Extracted from Reference {1})



(a) Knife-edge follower

(b) Flat-face follower

(c) Roller follower

(d) Curved-shoe follower

Figure 1(b): Types of Followers
(Extracted from Reference {1})

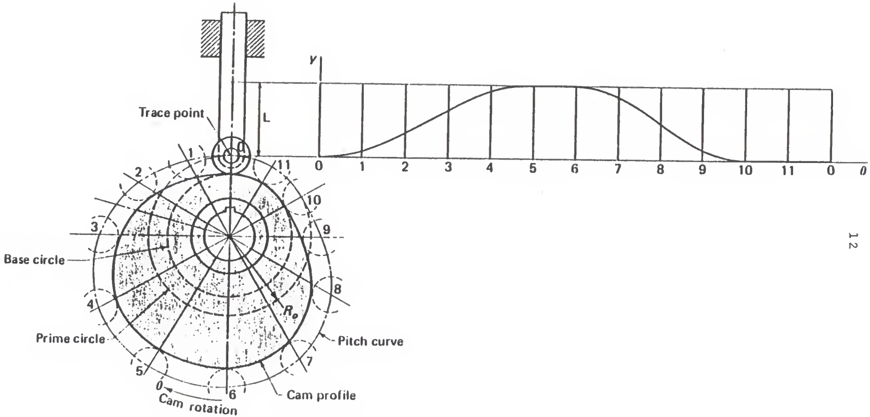


Figure 2: Geometrical description of Cam and Follower mechanism (Extracted from Reference {1})

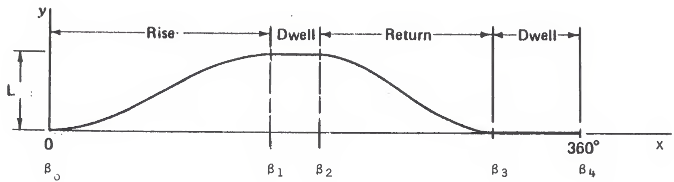


Figure 3: Follower displacement diagram
(Extracted from Reference {1})

etc. A reciprocating roller follower is illustrated in Figure 2. The follower maintains direct contact with the cam and, as the cam rotates (generally with constant angular velocity), the follower moves to and fro along its axis.

The displacement diagram (Figure 3) provides a clearer understanding of the performance of the cam-follower mechanism. Here, the abscissa represents one cycle of input motion (x). The ordinate represents the motion of the follower (y). If a cam is rotating with constant angular velocity "w", when the cam rotates by "x" the follower will move a distance "y" that is dependent on the cam rotation, i.e.,

$$y=f(x) \tag{1}$$

The overall follower motion is divided into four zones, as shown in Figure 3. The region where the follower is moving away from the cam center of rotation is called the rise zone. The maximum rise is the total lift "L". The region where the follower is approaching towards the cam center of rotation is termed the return zone. All other regions are dwell zones. The dwell following the rise is called the top dwell and the one following the return is called the bottom dwell. In the dwell zones, the follower precisely maintains constant position.

The end points of these zones are defined by angles " β_j", j=0,1,...,4, as shown in the figure. It should be noted that β_1 is the starting point of the cycle, and β_4 is the final point of the cycle; hence, the difference between them should always be equal to 360 degrees.

With these β_j´s known " $\Delta\beta_i$ ", the specific range of any zone i can be computed by:

$$\Delta\beta_i = \beta_j - \beta_{j-1} \tag{2}$$

The procedure by which a relationship is developed between the follower motion and cam rotation is given below:

The displacement of the follower can be described by an equation of the form.

$$y=f(z) \tag{3}$$

Here, "z" is the dimensionless ratio of the angle of rotation of the cam from the start of the current zone to the total range of the zone; mathematically,

$$z = \theta / \Delta\beta_i \qquad (4)$$

where

θ is the angle of cam rotation measured from the start of zone i.

The ratio "z" is generally termed the "time ratio" and varies from 0 to 1.

It is assumed that the follower position is normalized by dividing the true position by the total lift. Thus, the normalized position "y" in Equation (3) varies from 0 to 1 . Actual position can therefore be computed by multiplying the normalized position by the total lift. We can also define the reduced velocity as the derivative of position with respect to cam rotation within a zone " θ ", i.e.,

$$y' = L * dy/d\theta = L * f'(z) \qquad (5)$$

where

L is the total lift of the follower.

Similarly, the second derivative of position with respect to cam rotation within a zone is called the reduced acceleration and is given by Equation (6); the third derivative of position with respect to cam rotation within a zone is called the reduced jerk and is given by Equation (7).

$$y'' = L * d^2y/d^2\theta = L * f''(z) \qquad (6)$$

$$y''' = L * d^3y/d^3\theta = L * f'''(z) \qquad (7)$$

The true position, velocity, acceleration and jerk of the follower are given by Equations (8), (9), (10) and (11) respectively:

$$y^* = y * L \qquad (8)$$

where

y^* is the true position

L is the total lift of the follower

$$y = y' * w \qquad (9)$$

$$y = y'' * w^2 \tag{10}$$

$$y = y''' * w^3 \tag{11}$$

The radial distance from the cam center to a point on the cam profile is given by:

$$r(z) = L * y + r_b \tag{12}$$

where

$r(z)$ is the radial distance to the cam profile

r_b is the base circle radius of the cam

The actual follower motion depends on the particular application that the mechanism is designed for and on the boundary conditions specified. It is important, in making the transition from one zone to next , that we maintain continuity of position and its derivatives up to the degree required for the application in view. In order to ensure this continuity, boundary conditions are enforced at the transition points between adjoining zones. The boundary conditions are generally expressed in terms of normalized position, reduced velocity, reduced acceleration, etc., and are specified at each terminal point of each rise and each return zone. For example, for the follower motion shown in Figure 4, the boundary

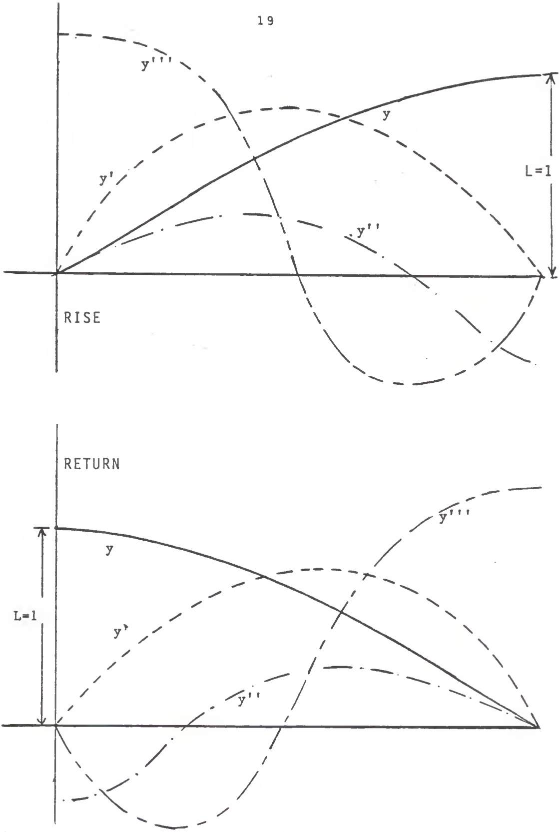


Figure 4: Illustration of boundary conditions

conditions are specified as follows;

RISE ZONE

$z = 0$

$y = 0, \quad y' = 0, \quad y'' = 0$

$z = 1$

$y = 1, \quad y' = 0, \quad y''' = 0$

RETURN ZONE

$z = 0$

$y = 1, \quad y' = 0, \quad y''' = 0$

$z = 1$

$y = 0, \quad y' = 0 \quad y'' = 0$

There are various techniques, both graphical and analytical, to establish a relationship between the input motion of the cam and the output motion of the follower. In conventional design, standard curves are adopted to generate the required cam contour for specified follower motions and boundary conditions[2]. Some of the standard curves that are commonly used are cycloidal curves, simple harmonic curves, modified harmonic curves, parabolic curves, etc . The criteria for selecting the appropriate curves to be used are based almost entirely

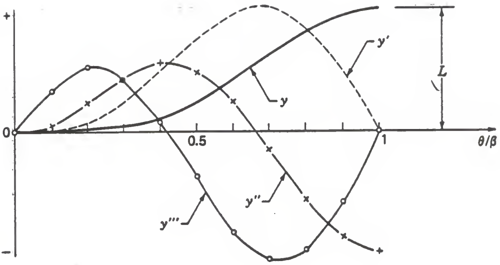


Figure 5(a): Displacement diagram and derivatives for full-rise modified harmonic motion
(Extracted from Reference {1})

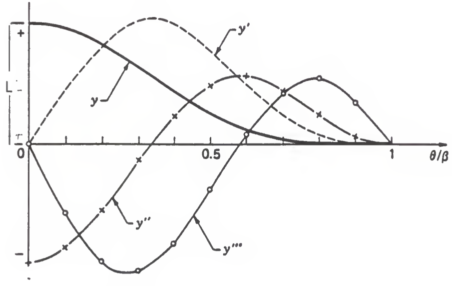


Figure 5(b): Displacement diagram and derivatives for full-return modified harmonic motion (Extracted from Reference {1})

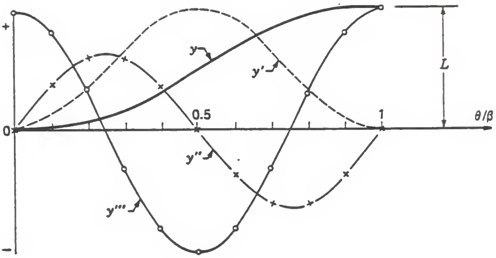


Figure 5(c): Displacement diagram and derivatives for full-rise cycloidal motion (Extracted from Reference {1})

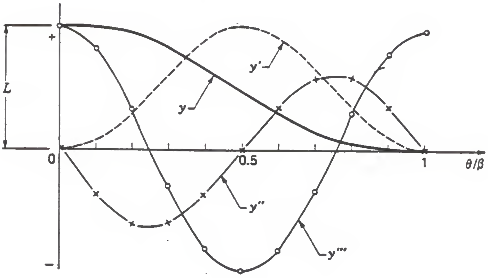


Figure 5(d): Displacement diagram and derivatives for full-return cycloidal motion (Extracted from Reference {1})

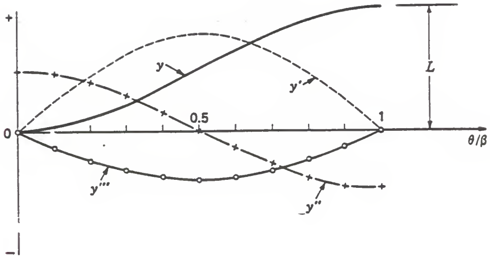


Figure 5(e): Displacement diagram and derivatives for full-rise simple harmonic motion
(Extracted from Reference {1})

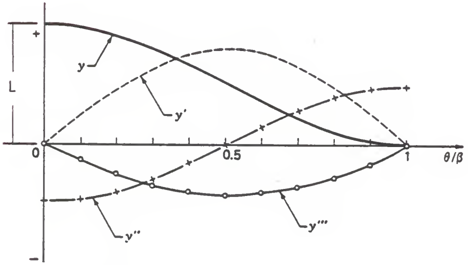


Figure 5(f): Displacement diagram and derivatives for full-return simple harmonic motion
(Extracted from Reference {1})

on the boundary conditions to be satisfied. Figures 5a-5f illustrate the displacement diagrams and derivatives for full-rise and full-return for some of these standard motions[1]. If the follower is required to have zero acceleration at each terminal point of each zone then cycloidal motion will be the obvious choice, whereas if the jerk is required to be zero at each terminal, modified harmonic motion will be selected. This is a tedious procedure since each different case must be implemented by a different curve. Moreover, apart from being cumbersome, this procedure restricts the range of possible solutions since standard curves are only a small subset of the set of possible solutions which satisfy the specified boundary conditions.

POLYNOMIAL CAMS

The approach adopted in this research is to use polynomials to define the cam profile. Polynomials provide a convenient means by which a wide variety of motion patterns can be generated[3]. Even complicated cam profiles which would be difficult to generate using standard curves can be conveniently described using polynomials. By merely adding successive higher order terms to the polynomial, different curves can be generated. These polynomials can be employed as single continuous functions, or may comprise parts of a piecewise defined function[3]. In the current work, it is assumed that two different polynomials are used to represent the rise and return respectively.

The polynomials considered in this work are functions of the time ratio "z". Any number of boundary conditions, specified with respect to z, can be satisfied by using an appropriate order of polynomial. The order of the polynomial is generally taken to be n-1, where n is the total number of boundary conditions specified.

A typical rise or return polynomial can be written in the form:

$$y(z)=c_1+c_2*z+c_3*z^2+c_4*z^3+ --- +c_n*z^{(n-1)} \qquad (13)$$

The derivative of this polynomial with respect to " θ " will give the reduced velocity:

$$y'(z)=L/\Delta\beta *(c_2+2*c_3*z+3*c_4*z^2+--+(n-1)*c_n*z^{(n-2)}) \qquad (14)$$

where

L is the total lift of the follower.

Similarly, the second derivative of the polynomial will yield the reduced acceleration, and the third derivative will give us the reduced jerk:

$$y''(z)=L/\Delta\beta*(2*c_3+6*c_4*z+--+(n-1)*(n-2)*c_n*z^{(n-3)}) \quad (15)$$

$$y'''(z)=L/\Delta\beta*(6*c_4+---+(n-1)*(n-2)*(n-3)*c_n*z^{(n-4)}) \quad (16)$$

Equations (13), (14), (15) and (16) can be used to compute the coefficients of the polynomial once the boundary conditions are known.

The radial distance to the cam profile "r" can now be expressed by:

$$r(z)=L*(c_1+c_2*z+c_3*z^2+c_4*z^3+ --- +c_n*z^{(n-1)})+r_b \quad (17)$$

where

r_b is the base circle radius of the cam

Note that the first, second, and third derivative this radial distance are equal to y', y'', y''' respectively.

EVALUATION OF POLYNOMIAL COEFFICIENTS

In conventional cam design the number of free coefficients in the polynomial is taken to be equal to the number of boundary conditions specified. In this case the coefficients are uniquely determined through the solution of a set of simultaneous linear equations[1]. A modified version of this approach is utilized in this work and is explained below.

Instead of setting the order of the polynomial to be equal to be one less than the number of boundary conditions it must satisfy, we choose it to be of the same order as the number of boundary conditions . The leading coefficient can then be chosen arbitrarily and can therefore be treated as a design variable . The solution scheme is setup in such a way that any change in this design variable will automatically effect a change in the remaining coefficients of the polynomial so that the boundary conditions are enforced . The process by which this is achieved is the following:

1) Define the boundary conditions.

   For example, let us consider the boundary conditions for a rise cycle only:

At z = 0:

$$y = 0, \quad y' = 0, \quad y''=0.$$

At z = 1:

$$y = 1, \quad y' = 0, \quad y'''= 0.$$

2) Select a polynomial of order one less than the number of boundary conditions:

$$y=c_1+c_2*z+c_3*z^2+c_4*z^3+c_5*z^4+c_6*z^5 \qquad (18)$$

3) Raise the order of this polynomial by one:

$$y=c_1+c_2*z+c_3*z^2+c_4*z^3+c_5*z^4+c_6*z^5+c_7*z^6 \qquad (19)$$

4) Compute y', y'' & y''':

$$y'=c_2+2*c_3*z+3*c_4*z^2+4*c_5*z^3+5*c_6*z^4+6*c_7*z^5 \qquad (20)$$

$$y''=2*c_3+6*c_4*z+12*c_5*z^2+20*c_6*z^3+30*c_7*z^4 \qquad (21)$$

$$y'''=6*c_4+24*c_5*z+60*c_6*z^2+120*c_7*z^3 \qquad (22)$$

5) Write the boundary conditions in matrix form, using the above Equations to substitute for y, y', y'' and y''':

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 0 & 0 & 6 & 24 & 60 & 120 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad (23)$$

In general, this matrix equation can be simply written as:

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}_{n,n+1} \begin{bmatrix} c_1 \\ \vdots \\ c_{n+1} \end{bmatrix}_{n+1} = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix}_{n} \qquad (24)$$

The matrix A can be partitioned as:

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}_{n,n+1} = \left[\begin{array}{c:c} & \\ \bar{A} & V \\ & \end{array}\right]_{n,n+1} \qquad (25)$$

where

[V] is an (n x 1) column vector that is identical to the (n+1)th column of [A]

[$\bar{A}$] is the (n x n) submatrix consisting of the first n columns of [A]

6) Now, Equation (24) can be manipulated into the following form:

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}_{n,n} \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}_{n} = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix}_{n} + c_{n+1} \begin{bmatrix} \\ V \\ \\ \end{bmatrix}_{n,1} \qquad (26)$$

7) The last step is to solve this system of linear equations to obtain c_1 through c_6.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 6 & 24 & 60 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_7 \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ -6 \\ -120 \end{bmatrix}$$

For example, let the design variable be equal to 1 i.e., $c_7 = 1$

Then, after solving the above system of equations, the final polynomial coefficients obtained are:

$c_1 = c_2 = c_3 = 0.$

$c_4 = 5.3326$

$c_5 = -4.666$

$c_6 = -.6666$

DESIGN CONSIDERATIONS

Before setting up the design process, we must first establish a set of important design considerations. In the current work, these design considerations are enforced through constraints in the optimization process. Design considerations are user dependent and may vary from case to case. Some basic design considerations are explained below; however, the user may add performance criteria as desired.

Pressure Angle

One of the most critical parameters in cam design is the pressure angle. The pressure angle is defined as the angle between the axis of the follower stem and the line of action of the force exerted by the cam on the roller follower. High values of pressure angle should be avoided in order to reduce friction, minimize chattering of the

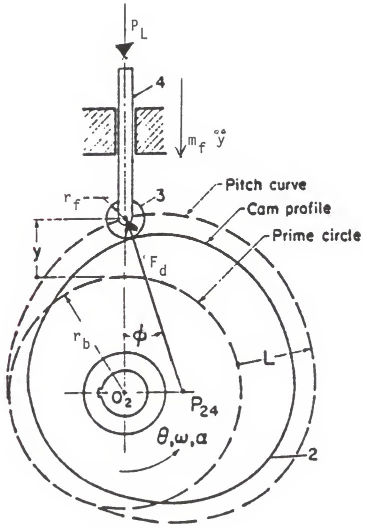


Figure 6: Plate Cam with reciprocating roller follower
(Extracted from Reference {17})

follower, and to obtain better force transmission[2]. As can be seen in Figure 6, only the force component along the axis of the follower stem is used to overcome output load. By keeping the value of Φ as low as possible the tangential component (unused component) of force is reduced, thus reducing the sliding friction between the follower and its guideway . The distance from the cam center of rotation to the trace point, "Ho" is given by[4]:

$$H_o = sqrt((r_b + r_f)^2 - e^2) \quad (27)$$

where

r_b is the base circle radius of the cam

r_f is the roller follower radius

e is the follower offset

The instantaneous distance between the trace point and the cam center of rotation, "H" is given by,[4]

$$H = H_o + y * L \quad (28)$$

where

y is the normalized position of the follower

L is the total lift of the follower

Now, the pressure angle is given by:

$$\Phi = \tan^{-1}[(y'-e)/H] \qquad (29)$$

where

y' is the reduced follower velocity at any instant

The maximum pressure angle should generally be less than 30 degrees for most applications.

Undercutting

The curvature of the cam is another important consideration in cam design. If the curvature of the cam is too high, the follower may not be able to track the surface precisely[1]; there will also be an increase in the contact stresses developed. This phenomenon is called undercutting and must be avoided if the cam is to perform satisfactorily. With a roller follower, the undercutting occurs in a convex curve when the radius of the roller follower is greater than the radius of curvature of the cam profile[2]. With a concave curve, it occurs either when the radius of curvature of the pitch curve is less than zero or when it is less than the radius of the roller follower. For a safe design, the radius of

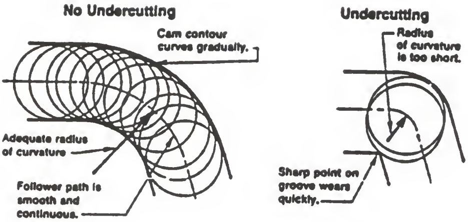


Figure 7: Illustration of Undercutting (Extracted from Reference {16})

curvature of the profile must be greater than one and a half times the radius of the roller follower[4].

The radius of curvature of the cam is given by:

$$\nabla = \frac{[H^2 + (y' - e)^2]^{3/2}}{H^2 + (y' - e)(2*y' - e) - H_o} \qquad (30)$$

There are two different ways to avoid undercutting[2]:

1) Use a roller follower with small radius. In this case, however there will be high stresses in the follower.

2) Use a large base radius for the cam. However, if the cam is too large it will create vibration problems and increase space requirements.

In this work the roller follower radius is kept constant and therefore undercutting effectively imposes limits on the base radius.

Separation

The above mentioned pressure angle and undercutting considerations are very important in the design process as they impose limits on cam size . Another essential consideration in the design of a cam-follower mechanism is the assurance of continuous contact between cam and follower surfaces at all times during the cycle. Here, we are required to take a look at the dynamics of the system (Figure 6). The driving force "F_d" is given by:

$$F_d = (P_l + m_f * \ddot{y})/\cos \phi \qquad (31)$$

where

P_l is the preload specified by the user (including friction, spring force,etc.).

m_f is the mass of the follower mechanism.

$\ddot{y}$ is the true acceleration of the follower.

Physically, the force F_d can only act upwards on the follower. If the follower acceleration required to track the cam profile on the return is too high, a negative value of F_d might be required to maintain contact. Since this is not possible, this situation will result in separation between the follower and cam surfaces, and this is generally unacceptable . In order to avoid separation the resultant driving force is required to be positive all through the cycle. One way to prevent the driving force from going negative is to increase the preload[6], but that will result in an increase in the contact stress developed . The best way to avoid separation is to keep the acceleration on the return as low as possible by controlling the profile. In this work, separation between the follower and cam surfaces is avoided by restricting the driving force to be positive throughout the motion.

Stress Considerations

If we consider an ideal case of two convex rigid bodies, we will have a case of straight line contact. However, in practice the bodies deform to give area contact. As studied by Hertz, stress distribution between two contacting bodies with cylindrical shape and with perfect alignment is such that the compressive stress is a maximum at the center of the contact area and decreases to zero at the ends; however, in actuality, deflection and misalignment may lead to extremely high local stress generation[2]. In this work, the contact stress is computed by using the Hertzian Equation[4]

$$\sigma_c^2 = \frac{F_d (\nabla + r_f)}{\pi * t_c * r_f * \nabla * [(1-v_c^2)/E_c+(1-v_f^2)/E_f]} \qquad (32)$$

Stress is a very critical factor because many problems of high wear and failure are caused by excessive stresses; consequently, the designer is required to take steps to minimize the stresses encountered.

Elimination of oscillation in the follower velocity

The use of high order polynomials tends to produce oscillations in the path. These oscillations are highly undesirable in cam follower mechanisms. In order to avoid these fluctuations, constraints are imposed on the velocity of the follower to ensure a steady rise and a steady return. During the rise cycle it is required that the follower velocity always be positive, whereas during the return cycle it should always be negative.

All the above-mentioned design considerations are enforced as design constraints which need to be satisfied by the final design. However, the user may include any other design considerations that he wishes to use as constraints in the optimization problem. The exact constraint equations that are used to implement the considerations discussed above are outlined in Chapter 4.

Chapter III

MANUFACTURING ASPECTS

The production of cams using NC machines was started in the mid 50's. Since then, digital computers have been utilized extensively to design cams and to generate data points for machining the cam profile. These data points are transferred onto a metallic tape or other data transfer medium, which is then fed to the NC machine. Here, except for the required machine setup, human involvement is almost completely eliminated[7]. The automation of cam production has not only reduced the manufacturing time drastically, but has also increased the quality of the product.

PRODUCTION PROCESS

For the purpose of this research, the NC machine considered is a Pratt & Whitney Tapemate Series C Numerical Control machine. This machine can perform operations like milling, drilling, boring and tapping economically and efficiently[8]. Table 1 provides the relevant specifications of this machine. The end milling

Table 1: Numerical Control Machine Specifications

		Specifications: Metric System	Specifications: Inch System
Capacity			
Drilling	FC25 Cast iron	Max. 20mm dia.	Max. 1/4" dia.
	S45C Mild steel	Max. 20mm dia.	Max. 1/4" dia.
Tapping	FC25 Cast iron	3 ~ 16mm dia.	1/8 ~ 5/8" dia.
	S45C Mild steel	3 ~ 16mm dia.	1/8 ~ 5/8" dia.
Milling	End mill	Max. 16mm dia.	Max. 5/8" dia.
Table			
Servo motor		FANUC DC servo motor model 5	
Size		610 x 460 mm	24.1/64" x 18.7/64"
Table travel		500 x 380 mm	20" x 15"
Least command increment		0.01 mm/pulse	0.0002"/pulse
Rapid traverse		6000 mm/min.	250 ipm
Feed rate		1 ~ 2000 mm/min.	0.05 ~ 100 ipm
Positioning accuracy		±0.05 mm/300 mm	±0.002"/12"
Repeatability		±0.01 mm	±0.0005"
Max. weight on table		150 kg	330 lbs
Spindle head (Z-axis)			
Servo motor		FANUC DC servo motor model 0	
Spindle travel		220 mm	8.5"
Distance from spindle nose to table surface		130 ~ 350	5 ~ 13.5"
Least command increment		0.01 mm/pulse	0.0002"/pulse
Rapid traverse		4000 mm/min.	150 ipm
Feed rate		1 ~ 2000 mm/min.	0.05 ~ 100 ipm
Positioning accuracy		±0.05 mm/300 mm	±0.002"/12"
Repeatability		±0.01 mm	±0.0005"
Number of tools		7	7
CNC			
Controlled axes		X, Y, Z	
Simultaneously controllable axes		2 axes of X-Y or X-Z	
Interpolation		Linear and Circular interpolation	
Fixed cycle		G81, G82, G85, G89/G84, G86 (with DC spindle motor)	
Tool length offset /tool radius offset		Max. 8 values stored in the memory	
Dwell		o	
Buffer storage		o	
Manual return to zero		o	
Automatic return to zero		o	
MDI and DPL		o	
Total weight (Approx.)		2,000 kg	4,500 lb
Power supply		AC200/220V or 380/415V ±10%, 3 phase, 50/60Hz, 5KVA	AC230V ±10%, 3 phase, 60Hz, 5KVA
Basic Option			
Spindle Motor	FANUC DC spindle motor model 2	Spindle speed: 50 ~ 4000 rpm (97 steps) Power: 1.7kW (Continuous rated value)	
	Pole change type induction motor	Spindle speed (see below) Power: 0.75/1.5/1.5KW(8/4/2P)	
Options			
Part program storage and editing		33/66ft (10/20m) of the tape information	
Coolant		o	
Additional tool length offset and tool radius offset		4/8/16 values in the memory	
Toolings		Many sorts of toolings are available. In detail, refer to the descriptions.	

Spindle speed (Pole change type induction motor):

	V belt position: Low			V belt position: High		
	Low	Medium	High	Low	Medium	High
50 Hz	230	460	920	700	1,400	2,800
60 Hz	280	560	1,120	840	1,680	3,360

Power: 0.75/1.5/1.5KW(8/4/2P)

slotting process is selected for manufacturing the cam. It is assumed that the cam will be cut from a rectangular blank, the dimensions of which will be provided by the optimization process.

In practice, the selection of milling cutters for a certain task is totally based on the engineer´s experience. For this work, the required machining data was acquired from [9]. This reference provides a table which prescribes preferred values of machining parameters for end mill slotting on alloy steel material. According to this table, a tool diameter of 3/4" is the most appropriate from the point of view of both tool life and productivity. This is also evident from table 2, as the 3/4" diameter tool provides a good feed rate and cutting speed for a wide range of materials. Some of the prescribed machining data obtained from [9] is given below:

Tool material	------	High Speed Steel
Tool diameter	------	3/4 inches
Number of teeth	------	4
Depth of cut	------	0.25 inches
Feed per tooth	------	0.002inches
Cutting speed	------	110 feet per minute

Table 2: Machining Data for End Milling Process

MATERIAL	HARDNESS BHN	CONDITION	DEPTH OF CUT inches	SPEED fpm	FEED – Inches per Tooth, WIDTH OF SLOT – Inches: 1/4	1/2	3/4	1 to 2	HSS TOOL MATERIAL except as noted
1. FREE MACHINING CARBON STEELS, WROUGHT Low Carbon Resulfurized 1212 1213 1215	100 to 150	Hot Rolled or Annealed	.250	125			.0025	.0035	M2, M7
			.050	135	.001	.0015	.002	.003	M2, M7
	150 to 200	Cold Drawn	.250	110			.0025	.0035	M2, M7
			.050	120	.001	.0015	.002	.003	M2, M7
Low Carbon Resulfurized 1108 1118 1109 1119 1110 1144 1116 1211 1117	100 to 150	Hot Rolled or Annealed	.250	110	-	-	.0025	.003	M2, M7
			.050	120	.001	.0015	.002	.0025	M2, M7
	150 to 200	Cold Drawn	.250	90	-	-	.0025	.003	M2, M7
			.050	100	.001	.0015	.002	.0025	M2, M7
Medium Carbon Resulfurized 1132 1141 1137 1145 1139 1146 1140 1151	175 to 225	Hot Rolled, Normalized, Annealed or Cold Drawn	.250	105	-	-	.0025	.003	M2, M7
			.050	115	.001	.0015	.002	.0025	M2, M7
	275 to 325	Quenched and Tempered	.250	75	-	-	.0025	.003	M2, M7
			.050	85	.001	.0015	.002	.0025	M2, M7
	325 to 375	Quenched and Tempered	.250	45			.002	.0025	M2, M7
			.050	50	.0005	.001	.0015	.002	M2, M7
	375 to 425	Quenched and Tempered	.250	25	-	-	.0015	.002	T15, M33, M41 Thru M47
			.050	30	.0005	.0007	.001	.0015	T15, M33, M41 Thru M47
	100 to 150	Hot Rolled, Normalized, Annealed or Cold Drawn	.250	135	-	-	.0025	.0035	M2, M7
			.050	145	.001	.0015	.002	.003	M2, M7
	150 to 200	Hot Rolled, Normalized, Annealed or Cold Drawn	.250	120	-	-	.0025	.0035	M2, M7
			.050	130	.001	.0015	.002	.003	M2, M7
	200 to 250	Hot Rolled, Normalized, Annealed or Cold Drawn	.250	105			.0025	.003	M2, M7
			.050	115	.001	.0015	.002	.0025	M2, M7

(Extracted from Reference {9})

Once the cutting speed is set and the tool diameter is known, the required spindle speed "V_s" in rpm can be calculated from[10]

$$V_s=(12*V_c)/(\pi * d_t) \tag{33}$$

where

V_c is the nominal cutting speed in fpm

d_t is the tool diameter in inches

There are only six fixed spindle speed available on the NC machine, so the closest lower speed (V_s') is automatically selected. Using this spindle speed actual cutting speed can be computed from

$$V_{ca}=12/(60* \pi *d_t) * V_s' \tag{34}$$

where

V_{ca} is the actual cutting speed in fpm

V_s' is the actual spindle speed in rpm

Also, the feed in inches per second can be computed from the known spindle speed[14]:

$$Feed = fpt * N_t * V_s'/60 \tag{35}$$

where

fpt is feed per tooth in inches

N_t is the number of teeth in the tool

TURNS IN TOOL PATH

In computing the cutting time it was anticipated that the number of turns in the tool path would be a significant factor. A simple test was conducted to verify this hypothesis and to obtain data regarding the time lost in these turns. The details of this test are given below:

The basic idea of the test was to measure the time required for the tool to travel a fixed distance through different paths. All other parameters such as the feed rate and speed are kept constant. The NC machine was first programmed so that the tool traversed the sides of a square of side 4 inches. The machine was then programmed to drive the tool from corner 1 to corner 3 and back to corner 1 along the staircase path shown in Figure 6. This path has 20 turns in the tool path. Similar staircase paths were generated with 40, 80 and 100 turns. Note that the total length of all these paths is equal.

In each case, the tool was driven along the generated path for 20 cycles. The results obtained are shown in Table 3. Based on these results, an average time

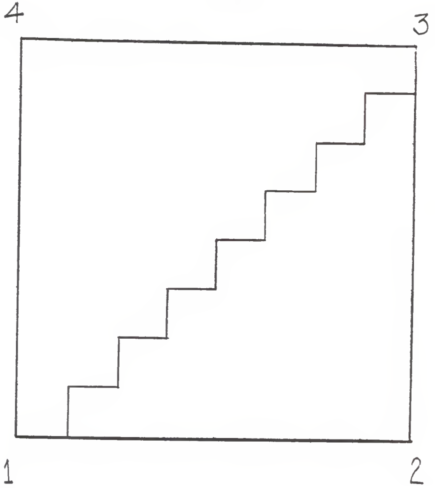


Figure 8: Estimation of time lapse per turn in tool path

Table 3: Estimation of Time Lapse per turn

Case 1

No. of cycles	No. of turns/cycl	Total time (Min)
20	4	14.133
20	4	14.155

Case 2

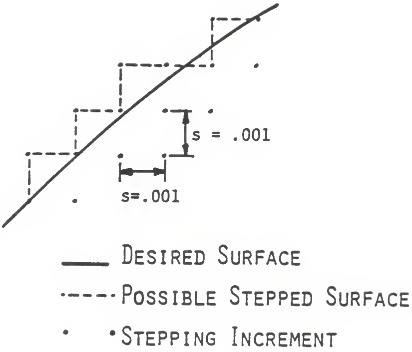
Number of cycles	Number of turns/cycl	Total time(min)	Total time lapse(min)	Time lapse/turn
20	20	16.35	2.216	.33seconds
20	40	19.35	5.216	.39seconds
20	80	23.947	9.81	.37seconds
20	100	39.633	25.58	.38seconds

lapse of 0 .4 seconds per turn was assumed in all subsequent calculations.

GNERATION OF DATA POINTS

The generation of data points for the tool path for manufacturing is a critical and time consuming step in process automation . Conventionally, a series of fixed steps is taken along both the x-axis and the y-axis to generate the cam contour. A detailed description of this method is presented in Reference [6] . The size of this fixed step is determined at the most critical section of the cam profile, where the step size is usually required to be very small in order to meet the desired accuracy. This approach has been modified in this work . It is assumed that it is not necessary to maintain the same step size throughout the cam contour because at some sections of the cam a larger step size may be permissible for the same accuracy . The advantage of this variable-step strategy is that it minimizes the number of turns in the tool path, thus reducing the manufacturing time significantly.

A Newton-Raphson iterative procedure is adopted to generate the variable steps along the x and y axes over



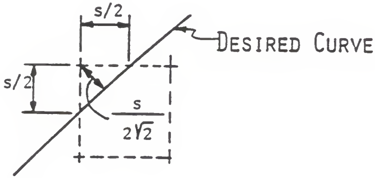


Figure 9: Fixed step path generation (Extracted from Reference {6})

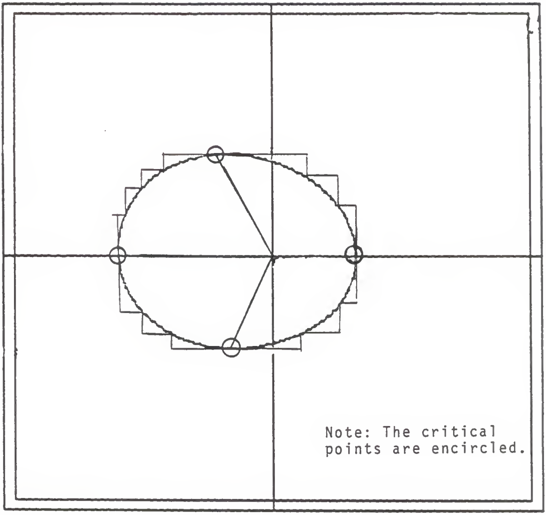


Figure 10: Variable step path generation

the cam contour. It is assumed that the movement of the tool is restricted to be on the outside of the cam profile at all points. In other words the cutter never cuts into the cam profile, but remain outside the actual profile within a pre-specified accuracy " ε ", as shown in Figures 11a and 11b. This strategy makes it possible to improve the accuracy further by later passing the cam through finishing processes, such as profile grinding.

The variable step generation technique developed is described below:

1) The entire profile of the cam is divided into zones based on the sign of the slope, as shown in Figure 10. This is done by computing the critical points with respect to the coordinate axis, i.e. points at which the slope is either zero or infinity. Between any two consecutive critical points, the slope "dy/dx" maintains the same sign.

2) When all the critical points are known, the process of data point generation begins at the first critical point and follows the cam contour until it comes around to the same starting point. As explained above, the tool path is required to lie outside of the cam contour. For

example, if slope is positive, the tool should first take a step away from the cam contour along the x-axis and then step towards the cam contour along the y-axis. The radial distance between the data point and the cam profile must be equal to the required accuracy "ε".

This approach is implemented as follows:

If dy/dx > 0

$$f(\theta) = (r(\theta) + \varepsilon) * \sin(\theta) - y_o = 0 \qquad (36)$$

$$g(\alpha) = r(\alpha) * \cos(\alpha) - x_1 = 0 \qquad (37)$$

Else if dy/dx < 0

$$f(\theta) = (r(\theta) + \varepsilon) * \cos(\theta) - x_o = 0 \qquad (38)$$

$$g(\alpha) = r(\alpha) * \sin(\alpha) - y_1 = 0 \qquad (39)$$

where

θ and α are the angle of cam rotation

$r(\theta)$ is the instantaneous radial distance to the cam profile

In order to generate the tool path, we only need to solve the above equation for θ and α at each step. Referring to Figure 11(a), the slope in section 1 is negative. Suppose, x_o and y_o are known coordinates of a point on the profile, and the angle is known. Starting from this position we want to move along the y-axis to

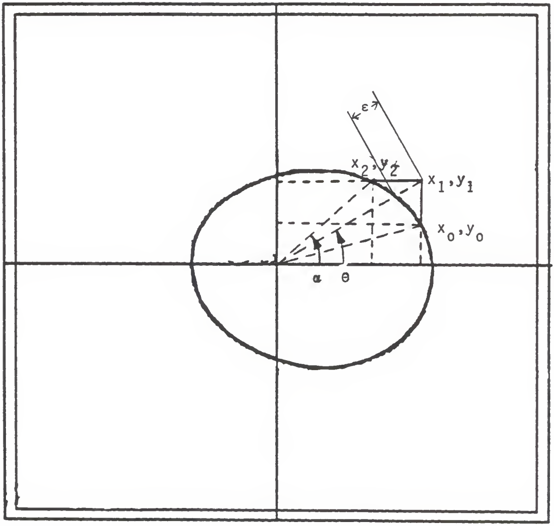


Figure 11(a): Generation of data points

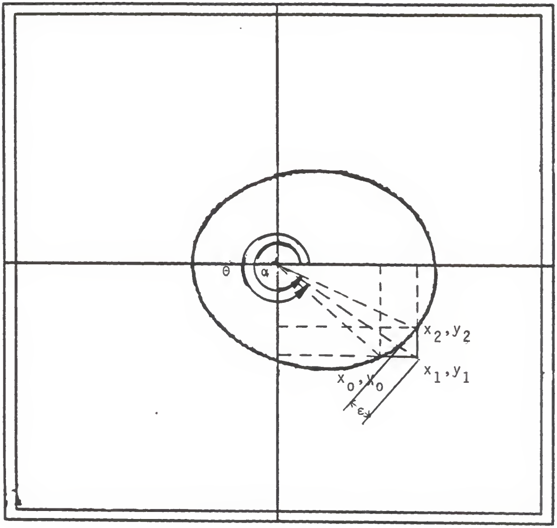


Figure 11(b): Generation of data points

(x_1, y_1) such that the maximum radial deviation from the true profile is " ε ". The new angle θ is computed by solving Equation 38 using an iterative Newton-Raphson procedure described by

$$\theta_{n+1} = \theta_n - f(\theta_n)/f'(\theta_n) \qquad (40)$$

where

n is the iteration number

and $f'(\theta)$ is given by;

$$f'(\theta_n)=r'(\theta_n) * \cos(\theta_n)-(r(\theta_n) + \varepsilon) * \sin(\theta_n) \qquad (41)$$

By using this iterative procedure the angle " θ " can be obtained. This will enable us to calculate the step along the y-axis. Next the tool must be moved to the position (x_2, y_2) shown in the figure. This is achieved by again using a Newton Raphson iteration to find the angle " α " in Eq. 39:

$$\alpha_{n+1} = \alpha_n - g(\alpha_n)/g'(\alpha_n) \qquad (42)$$

$$g'(\alpha_n) = r'(\alpha_n) * \sin(\alpha_n) + r(\alpha_n) * \cos(\alpha_n) \qquad (43)$$

In the same manner, data points can be generated in sections of the cam contour where the slope is positive by solving from the functions given in Equations 36 and 37.

The total number of steps generated is recorded along with the data points. The total number of turns in a tool path is considered to be equal to the total number of steps taken by the tool.

THE ESTIMATION OF CUTTING TIME

Once the data points for the tool path have been generated for the entire cam contour, the distance "S_{tp}" that the tool will travel in order to cut the cam, can be computed as follows:

$$S_{tp} = [(x_j - x_{j-1})^2 + (y_j - y_{j-1})^2]^{1/2} \qquad (44)$$

where

x_j is the x-coordinate of data point "j"

y_j is the y-coordinate of data point "j"

When the feed, total distance travelled and total number of turns are known, the cutting time can be estimated as follows:

$$F = S_{tp}/Feed + 0.4 * N_{stp} \tag{45}$$

where

N_{stp} is the total number of turns.

This estimate of cutting time is then used as the cost function in the optimization problem that will be discussed in detail in the next chapter.

Chapter IV

OPTIMIZATION

Nonlinear optimization techniques provide a convenient means through which design objectives can be achieved; by using suitable constraints, we can be assured that the design satisfies the required performance criteria . In order to establish an optimization problem, the optimality criteria need to be defined. In order to do this, the following are required to be identified:

1) A set of design variables

2) A cost function that is to be minimized

3) A set of constraints that must be satisfied

1) CHOICE OF DESIGN VARIABLES

The first and foremost problem is to pick out the right set of design variables to work with. A thorough investigation is needed to determine which parameters should be treated as design variables and which parameters should be kept constant. It may appear safe to treat all parameters in sight as design variables, but for diverse reasons this is not possible or practical[11]

Material selection, for instance, is not treated as design variable in practice; rather it is selected on the basis of experience by the engineer. The number of design variables must also be reduced as much as possible in order to avoid computational complexities and to cut down computing time. In this research, quite a few parameters could have been easily treated as design variables, for example, tool diameter, cutting speed, roller diameter, cam and roller thickness, etc . Doing so would have resulted in a much more complex problem, which would have been very expensive to optimize; in certain cases, reaching the optimum solution may not even have been possible.

The approach taken in this work was to treat the cam diameter and the orientation of the cam on the NC machine table as design variables in addition to the extra coefficients in the rise and return polynomials . This combination of design variables provides a convenient and practicable approach to optimization in the design/manufacture environment . These design variables are explained in detail below.

The leading polynomial coefficients:

The polynomial is a convenient tool to describe a set of points in a plane. The order of the polynomial depends on the number of conditions it is required to satisfy, i.e., the number of boundary conditions in our case. As explained in Chapter 2, the approach taken in this work is to choose the order of the rise and return polynomials to be one higher than the order required to satisfy all the boundary conditions. The leading coefficients of these polynomials, which are treated as design variables, can be chosen arbitrarily; the remaining coefficients can then be calculated in order to satisfy the boundary conditions. The optimization process will choose values of the leading coefficients in such a way that feasibility and cost minimization are achieved. These design variables usually have a strong effect when the design is in the infeasible region, i.e., when some of the constraints are not satisfied.

The cam base diameter:

In the cam-follower mechanism, it is generally always desirable to have the smallest cam[2]. Small cams allow us to build a compact cam-follower mechanism, thus minimizing problems of space allocation. Also, the follower path per cycle will be short if the cam size is reduced, which in turn will reduce the wear. Vibration of large parts is always undesirable, and the use of small cams also reduces the chances of unbalancing at high speed. Smaller cams are also cheaper to produce. On the other hand, if the cam becomes too small we may encounter large pressure angles, high contact forces and high stresses[2]. We may also run into problems of undercutting.

From these considerations, it is clear that the base diameter of the cam should be treated as a design variable in the optimization problem. The cam size, as a design variable, plays an important role in both satisfaction of constraints as well as in reduction of cost.

Orientation of the cam on the NC table:

In the previous chapter the variable step generation technique that has been incorporated into this work was explained. Since the steps generated for the cutting tool are either along the x-axis or the y-axis, these steps are very strongly dependent on the orientation of these axes. If the coordinate axis is rotated on the cam surface, a different set of steps is obtained and this has a direct effect on the manufacturing time. For this reason, the angle representing the orientation of the coordinate axes is treated as a design variable. This angle of orientation of the axes is the same as that of the orientation of the blank on the table of NC machine. If this angle is initially assumed to be zero, then the orientation of the coordinate axis will be such that the rise polynomial begins from the positive x-axis. Since the step size and number of turns are key factors in the computation of the cost function, this design variable plays a critical role in reducing the manufacturing cost. However, it does not affect the feasibility of the design.

2) COST

The minimization of a cost function is the primary objective of optimization. As mentioned earlier, the task of this research is to reduce the manufacturing time. This cost function utilized here can be broken up into two parts. The first part depends on the tool diameter, feed rate, total distance traversed and spindle speed, whereas the second part is comprised of the time lapse per turn multiplied by the total number of turns in the tool path that will be required to cut the cam . The details of the estimation procedure for the manufacturing time are covered in Chapter 3.

3) CONSTRAINTS

Defining a proper set of constraints is very critical in obtaining a usable design . A complete knowledge and understanding of the problem is required before the constraints are identified. Efforts should be made to review all performance criteria. Constraints are the only restrictions on the search procedure that prevent the search from converging at an undesirable or dangerous solution.

In this research, constraints are imposed to prevent high values of pressure angle and contact stress.
Also, undercutting and follower separation are avoided by imposing suitable constraints . Constraints are also imposed to keep the cam profile free of undesirable oscillations. As an option,the user is allowed to impose new constraints as desired, such as restricting the maximum values of jerk, acceleration and velocity.

The explicit defining inequalities for these constraints are the following:

1) Undercutting

$$r(\theta) - 1.5 * r_f <= 0. \qquad (46)$$

where

$r(\theta)$ is the instantaneous radial distance to the cam profile

θ is the angle of the cam rotation

r_f is the radius of the roller follower

2) Elimination of oscillation in follower motion

For rise

$$-1 * y' <= 0. \qquad (47)$$

For return

$$y' <= 0. \qquad (48)$$

where

y' is the reduced follower velocity

3) Pressure angle

$$\Phi_i - \Phi_d <= 0. \qquad (49)$$

where

Φ_i is the instantaneous pressure angle

Φ_d is the specified maximum pressure angle

4) Contact Stress

$$\sigma_i^2 / \sigma_d^2 - 1 \leq 0. \qquad (50)$$

where

σ_i is the instantaneous contact stress on the cam contour

σ_d is the specified design stress

5) Follower separation

$$-1 * F_d \leq 0. \qquad (51)$$

where

F_d is the driving force

The preceding constraints are required to be satisfied at all times during a cycle . Checking for constraint violations at very short intervals usually increases the computing time significantly; at the same time if these intervals are too long, then it is always possible that the optimization process might miss some constraint violations.

In this work each rise and return cycle is divided into 15 equally spaced nodes on which the constraints are evaluated . This number, however, can be changed for a more precise check on constraint violation. The constraint functions are constant all through the dwell zones. For this reason no nodes are specified in the dwell zone; however, the constraints are evaluated at each terminal of each zone in order to account for the behavior of constraints in the dwell zones.

THE OPTIMIZATION STRATEGY

The optimization strategy is based on the concept of an exterior penalty function method. A pseudo - objective function "F^*" is defined as follows:

$$F^* = F + R * g \qquad (52)$$

where

F is the true cost function, in this case the cutting time

R is a penalty multiplier, usually specified to be of the order of 10^5 or 10^6

g is the highest constraint violation

The minimum cost for the constrained problem is then found approximately by finding the unconstrained minimum of the pseudo-objective function[12]. Looking at the values of "g", it can be said that no penalty is imposed if all constraints are satisfied; however if one or more constraints are violated, than the highest violation is multiplied by "R" and added as a penalty to the cost function. The unconstrained minimum of F^* is therefore likely to be found inside the feasible region, since any constraint violation causes a sharp increase in the pseudo-objective function.

The cost function "F", which corresponds to the manufacturing time, is discontinuous with respect to the design variables. This is true because the manufacturing time depends on the number of turns in the tool path. Whenever a small design change causes a change in the number of turns of the tool path, the cost function will undergo a step change. Because of this fact, gradient based methods are not applicable in this case. The method that was finally used to perform the optimization was a directed grid search technique[13] which does not require gradients. Although it is a relatively crude method, it performed adequately well in the current work.

OUTPUT

Once the design is complete and manufacturing cost/time is optimized, the result can be displayed on the screen in the form of graphs and cam profile plots. The final task of the process is the generation of NC codes, which can then be transferred to metallic or paper tape. The tape is then fed to the NC machine. The output also provides the size of the rectangular blank and orientation of the cam blank on the table of the NC machine.

Chapter V

NUMERICAL EXAMPLES

The techniques developed in the preceding chapters was implemented in a computer program. In order to verify the approach and its implementation, several numerical examples were solved, some of which are presented in this chapter.

For these examples, machining data was obtained from Reference [9] and the physical properties were taken from Reference [15]. The machining data and the physical properties were kept unchanged in all the examples. A complete set of these constant parameters is given below.

Physical Properties:

Material:Hardened Steel SAE 1010(for both cam & follower)
Density of the material = 0.28 lb/in^2
Youngs modulus of elasticity = 29 x 10^6
Poison´s Ratio = 0.29

Machining Data:

Tool Material: High Speed Steel
Tool diameter = 0.75 inches
Number of teeth = 4
Cutting speed = 109.95 fpm
Feed per tooth = 0.0025 inch/tooth
Spindle speed = 560 rpm
Feed rate = 9.333 x 10^{-2} inch/sec

Example 1:

INPUT

Design Specifications:

Preload = 320.0 lbs
Maximum Allowable Stress = 250000 psi
Maximum Pressure Angle = 12.0 degrees
RPM = 1000
Mass of follower mechanism = .02 lbsm
Lift = 1.20
Cam Thickness = 0.25 inches
Follower diameter = 0.85
Follower thickness = 0.25
Required accuracy = 1.000E-02

Cycle Characteristics:

Rise	Dwell	Return	Dwell
0.0-180.0	180.0-185.0	185.0-350.0	350.0-360.0

Boundary Conditions:

RISE
At z=0.

$y=0$, $y'=0$, $y''=0$, $y'''=0$

At z=1.

$y=1$, $y'=0$, $y'''=0$,

RETURN
At z=0.

$y=1$, $y'=0$, $y'''=0$,

At z=1

$y=0$, $y'=0$, $y''=0$, $y'''=0$

Design Variables:

Number of design variables = 4

Physical significance of design variables:

B(1) is the leading coefficient of the rise polynomial.

B(2) is the leading coefficient of the return polynomial.

B(3) is the base radius of the cam.

B(4) is the orientation of the cam.

Design Variable	Initial Value	Upper Bound	Lower Bound
B(1)	4.0	180.	-180.
B(2)	-0.072	180.	-180.
B(3)	1.5	10.	1.
B(4)	12.7	180.	-180.

COMMENTS

The design procedure started from the infeasible region with maximum constraint violation of 5.9955; the pressure angle constraint was found to be the most violated. When the design stepped into the feasible region for the first time the cost function was found to be 927.2 seconds.

FINAL RESULTS

Design Variables

B(1) = 3.996

B(2) = 2.0160

B(3) = 2.7398

B(4) = 18.684

Coefficients of polynomials

	RISE	RETURN
1	0.00	1.00
2	0.00	0.00
3	0.00	-4.495
4	0.00	0.00
5	5.005	9.959
6	-1.400E-2	-5.919
7	-7.987	-2.560
8	3.996	2.016

Maximum Values	Magnitude	Location on cam profile (degrees)
Force	.486E+03	291.071
Velocity	.833E+02	128.570
Pressure Angle	.118E+02	115.714
Contact Stress Sqr	.316E+11	279.286
Positive Accel	.813E+04	291.071
Negative Accel	-.160E+05	180.000
Positive Jerk	.202E+07	232.143
Negative Jerk	-.228E+07	141.429

Highest Constraint = $2.223368 * 10^{-10}$

Cutting Time = 795.01 sec

Number of turns = 1298

Tool path distance = 25.7430 inch

% reduction in cost from
the first feasible design = 14.2587

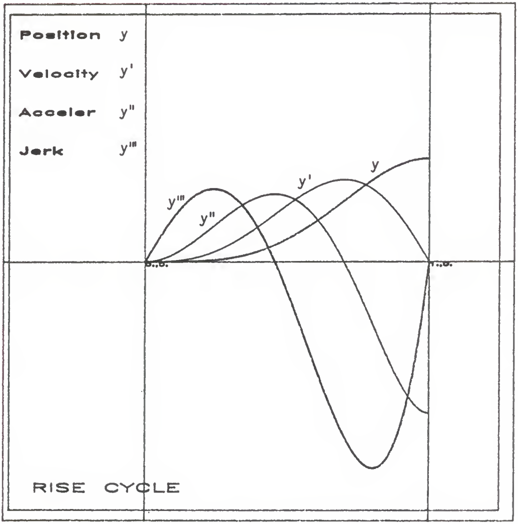


Figure 12(a): Example 1: Results

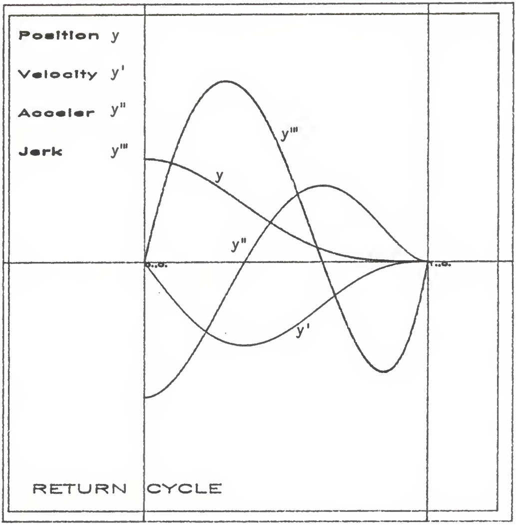


Figure 12(b): Example 1: Results

Example 2:

INPUT

Design Specifications:

Preload = 330.0 lbs
Maximum Allowable Stress = 250000 psi
Maximum Pressure Angle = 20.0 degrees
RPM = 1000
Mass of follower mechanism = .02 lbsm
Lift = 1.00 inch
Cam Thickness = 0.25 inches
Follower diameter = 0.75
Follower thickness = 0.25
Required accuracy = 1.000E-02

Cycle Characteristics:

Rise	Dwell	Return	Dwell
0.0-150.0	150.0-160.0	160.0-330.0	330.0-360.0

Boundary Conditions:

RISE
At z=0.

y=0,	y´=0,	y´´=0,	y´´´=0

At z=1.

y=1,	y´=0,	y´´=0,	y´´´=0

RETURN
At z=0.

y=1,	y´=0,	y´´=0,	y´´´=0

At z=1

y=0,	y´=0,	y´´=0,	y´´´=0

Design Variables:

Number of design variables = 4

Physical significance of design variables:

B(1) is the leading coefficient of the rise polynomial.

B(2) is the leading coefficient of the return polynomial.

B(3) is the base radius of the cam.

B(4) is the orientation of the cam.

Design Variable	Initial Value	Upper Bound	Lower Bound
B(1)	0.0	180.	-180.
B(2)	0.0	180.	-180.
B(3)	1.5	10.	1.
B(4)	0.0	180.	-180.

COMMENTS

The design procedure started within the feasible region with the initial cost of 533.1 seconds.

FINAL RESULTS

Design Variables

B(1) = 0.000

B(2) = 0.000

B(3) = 1.45689

B(4) = 12.168

Coefficients of polynomials

	RISE	RETURN
1	0.00	1.00
2	0.00	0.00
3	0.00	0.00
4	-2.168E-19	-4.3368E-19
5	35.000	-34.999
6	-84.000	83.999
7	70.000	-69.999
8	-20.000	19.999
9	0.00	0.00

Maximum Values	Magnitude	Location on cam profile (degrees)
Force	.585E+03	42.857
Velocity	.875E+02	75.00
Pressure Angle	.198E+02	64.286
Contact Stress Sqr.	.384E+11	42.857
Positive Accel	.120E+05	42.857
Negative Accel	-.120E+05	107.143
Positive Jerk	.255E+07	21.429
Negative Jerk	-.336E+07	75.000

Highest Constraint = $3.55739019 \times 10^{-09}$

Cutting Time = 488.8689 secs

Number of turns = 800

Tool path distance = 15.7610 inch

% reduction in cost = 8.45155

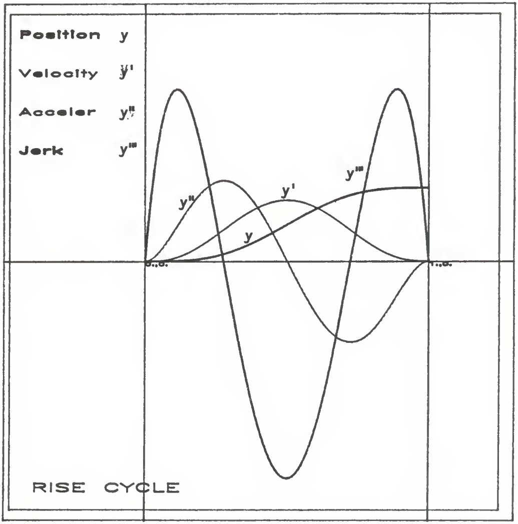


Figure 13(a): Example 2: Results

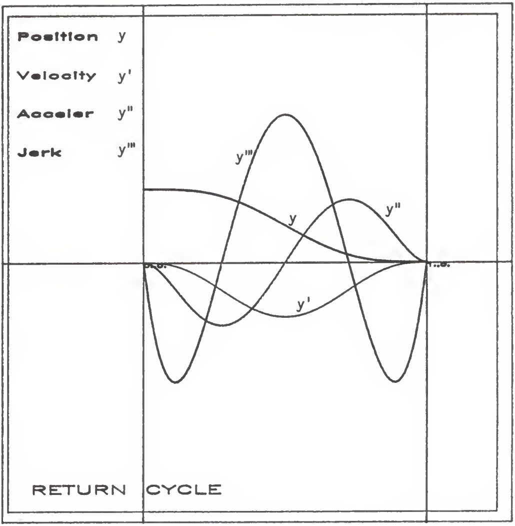


Figure 13(b): Example 2: Results

Example 3:

INPUT

Design Specifications:

Preload = 320.0 lbs
Maximum Allowable Stress = 250000 psi
Maximum Pressure Angle = 18.0 degrees
RPM = 1000
Mass of follower mechanism = .02 lbsm
Lift = 1.50
Cam Thickness = 0.25 inches
Follower diameter = 1.00
Follower thickness = 0.25
Required accuracy = 1.000E-02

Cycle Characteristics:

Rise	Dwell	Return	Dwell
0.0-190.0	190.0-190.0	190.0-360.0	360.0-360.0

Boundary Conditions:

RISE
At z=0.
y=0, y´=0, y´´=0, y´´´=0
At z=1.
y=1, y´=0, y´´´=0,

RETURN
At z=0.
y=1, y´=0, y´´´=0,
At z=1
y=0, y´=0, y´´=0, y´´´=0

Design Variables:

Number of design variables = 4

Physical significance of design variables:

B(1) is the leading coefficient of the rise polynomial.

B(2) is the leading coefficient of the return polynomial.

B(3) is the base radius of the cam.

B(4) is the orientation of the cam.

Design Variable	Initial Value	Upper Bound	Lower Bound
B(1)	0.0	180.	-180.
B(2)	0.0	180.	-180.
B(3)	1.5	10.	1.
B(4)	0.0	180.	-180.

COMMENTS

The design procedure started from the infeasible region with maximum constraint violation of 23.75; the force constraint was found to be the most violated. When the design stepped into the feasible region for the first time the cost function was found to be 1388 seconds.

FINAL RESULTS

Design Variables

B(1) = 1.368

B(2) = 3.204

B(3) = 3.6548

B(4) = 28.872

Coefficients of polynomials

	RISE	RETURN
1	0.00	1.00
2	0.00	0.00
3	0.00	-4.199
4	0.00	0.00
5	8.29	6.999
6	-9.212	1.99E-2
7	0.554	-7.015
8	7.368	3.204

Maximum Values	Magnitude	Location on cam profile (degrees)
Force	.651E+03	299.286
Velocity	.940E+02	135.715
Pressure Angle	.104E+02	122.143
Contact Stress Sqr	.292E+11	299.286
Positive Accel	.973E+04	299.286
Negative Accel	-.160E+05	192.0
Positive Jerk	.217E+07	238.571
Negative Jerk	-.204E+07	149.256

Highest Constraint = $2.63293682\ 10^{-10}$

Cutting Time = 1053.88 seconds
Number of turns = 1718
Tool path distance = 34.1495 inches
% reduction in cost from
the first feasible design = 24.129

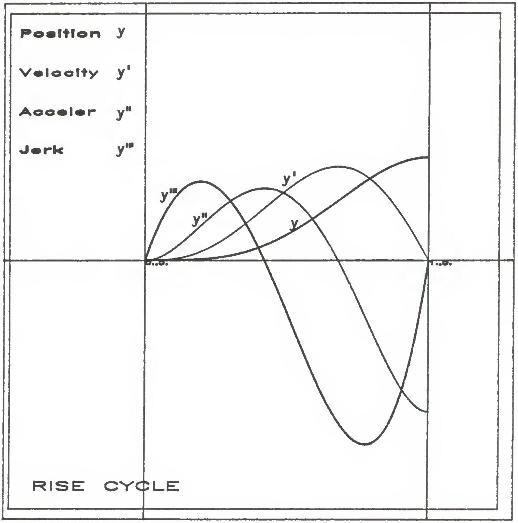


Figure 14(a): Example 3: Results

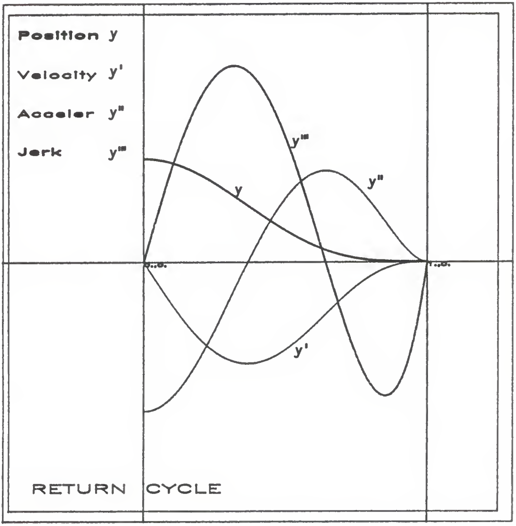


Figure 14(b): Example 3: Results

Chapter 6

CONCLUSION

The motivation for the work presented in this thesis was the perceived need for new techniques in the area of design for producibility. The concept of accounting for manufacturing considerations at the design level is a very significant aspect of this work and addresses a major weakness of Computer Integrated Manufacture as implemented in industry today.

The goal set for this work was to achieve integration of design and manufacture into a unified framework and to account for manufacturing considerations in a convenient, general manner. A mathematical model, based on nonlinear optimization concepts was developed; this model makes it possible to handle both design and manufacturing considerations simultaneously. Based on this model, a solution technique is also developed.

The use of nonlinear optimization techniques offers distinct advantages. A wide variety of problems can be easily accommodated . Both design and manufacturing parameters can be conveniently included in the problem statement and solution.

The solution technique for the nonlinear optimization problem is a systematic iterative process. Starting from any initial design, the technique produces a sequence of improved designs until the optimal solution is reached . Furthermore, the technique can be easily implemented in a computer code.

Another major advantage of this approach is that the design process becomes highly automated. Once the initial values of the design variables are specified and the cost and constraint functions are defined, the process finds the optimal design with no further user interaction. This not only cuts down the amount of designer time that is required but also reduces the level of skill required to produce the design.

The model also guarantees an improvement in the initial design . With any initial design the solution first proceeds in the direction of constraint correction and then in the direction of cost reduction. It provides at least as good a design as that which would be produced by a human designer using conventional design techniques.

The implementation of the proposed solution technique proved to be very reliable and computationaly viable as can be seen from the numerical examples. The examples also point out the generality of formulation. The model can handle any number of boundary conditions as well as any specified values for the lift, rise/return zone widths, maximum pressure angle, maximum stress, etc.

Although the results were very encouraging, there are some serious limitations to this work:

1) Manufacturing considerations have not been treated in great detail.
2) Only plate cams with roller followers are considered.
3) Only single rise and single return cam profiles are considered.
4) The numerical technique that has been used is somewhat crude, even though it has been effective.

Future work is required to address the above limitations. The model should be expanded so that manufacturing parameters such as cutting speed, tool diameter, feed, etc., can also be treated as design variables. Tolerances on the machined surface should be given more consideration, and studies should be done on how these tolerances relate to the manufacturing parameters. The use of piecewise polynomial definitions (e.g., B-Splines) instead of the single polynomial approximation used in this thesis, will make it possible to design cams that can satisfy more rigorous performance constraints and provide greater reduction in the cost.

The model can also be extended to include other type of cams/followers and to account for cases with multiple rises and multiple returns. Finally, there is room for improvement in the numerical methods that have been used. These extensions will make the model more comprehensive and greatly expand the range of applications for which it can be used.

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COMPUTER INTEGRATED MANUFACTURE OF OPTIMAL PLATE CAMS

by

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ABSTRACT

The objective of this work is to integrate the design and manufacture of Plate Cams through the use of non-linear optimization techniques . The underlying concept is design for producibility. A constrained non-linear minimization problem is set up in which the cost function is taken to be the manufacturing time required to produce the cam. Constraints are imposed to ensure that the design is acceptable with respect to criteria such as pressure angle, undercutting, stresses developed, etc . The design variables are the base radius of the cam, the orientation of the cam blank on the NC machine table, and the leading coefficients of the polynomials which describe the rise and return cycle of the cam. A grid search method is used to find the constraint optimum iteratively . This iterative scheme was successfully implemented in a FORTRAN computer program . At each iteration the tool path corresponding to the current cam design is automatically generated . The cost and constraint function are also evaluated automatically. After the optimum is found, the results are displayed graphically and in printed form, and NC code required for producing the cam is also generated. Examples are presented to demonstrate the effectiveness of the approach in automatically designing plate cams that are optimal with respect to design and manufacture.