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## NOTATIONS

$D_{s}, D_{s}^{2} \quad$ First and second derivatives with respect to $s$
$D_{x}, D_{x}^{2} \quad$ First and second derivatives with respect to $x$
$D_{z}, D_{z}^{2} \quad$ First and second derivatives with respect to $z$
$E \quad$ Young's modulus of the arch material
£
8
H
$\mathrm{H}_{\mathrm{a}}$

I
L Span length
$M \quad$ Bending moment
Mo Bending moment under the load in a simply supported beam having the same span as that of the arch
$M_{a}, M_{b}$
n
Fixed end moment for ends $A$ and $B$ respectively
Non-dimensional parameter, $n=\frac{f}{L}$
Influence line for thrust force
A loading function characterizing the intensity of a distributed load
$\mathrm{R} \quad$ Initial radius of curvature of the arch axis
$\rho \quad$ Radius of curvature of the arch axis after deformation
s
$\theta$
${ }^{\prime}{ }_{q}, u_{h}$
$v$
$v_{h}$
$v_{q}$
Rise of the arch
A parameter, $\mathrm{g}^{2}=\frac{\mathrm{H}^{2}}{\mathrm{E}} \mathrm{L}^{2}$
True horizontal reaction
Assumed horizoncal reaction
Moment of inertia of the cross section

N
$q(x)$
F


Arc length along the arch axis
Angle between the horizontal and the tangent to the arch axis Non-dimensional functions

Total vertical displacement (+ve downward)
$v$ due to unit horizontal reaction $H$, with respect to the equilibrium position attained after the application of $\mathrm{H}_{\mathrm{a}}$
$v$ due to external load, with respect to the equilibrium position attained after the application of $H_{a}$

| $W$ | Radial displacement |
| :--- | :--- |
| $X, Y$ | Rectangular coordinates |
| $y(x)$ | The function describing the shape of the arch axis |
| $z$ | Non-dimensional parameter, $z=\frac{x}{L}$ |

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Elastic theory ${ }^{[1]}$ has been in use for many years in the design of structures. In general, this theory neglects any change in geometry of the structure due to strain. When the elastic displacements and axial forces are small, the error involved is usually small. But, when the elastic displacements are appreciable, or when the axial force is not a small fraction of its buckling value, the error introduced may be an important factor in the design. This awareness calls for a more refined method that could reduce the error in the analysis. Deflection theory ${ }^{[2]}$, the subject matter, is one such method. Since the coefficients of equilibrium equations in this theory depend on the displacements, the governing equations are not inear and hence principle of superposition is not applicable without special treatment.

Deflection theory, applied to a number of structures in the past, has shown that the behaviour of a structure can be predicted with much higher accuracy as compared to the elastic theory ${ }^{[3]}$. The theory was used in the design of the Rainbow arch bridge across the Niagara. Analysis of the arch rib showed that the quarter point deflection of 12 inches as computed by the elastic theory, increased to 21 inches by deflection theory. At the same time, it was observed that the unit stresses hiked up by $28 \%$.

In long spanned arches, the thrust due to dead load becomes an important consideration when determining the moments with the use of deflection theory. The present work is an attempt to test the applicability of a linearized deflection theory for a fixed-fixed
parabolic flexible arch. The stresses induced in a long span arch due to the deflection of the arch axis are very much significant. This fact solidifies our belief that the change in geonetry cannot be neglected. Wang $[4]$, in his Masters report, has put forward the deflection theory as applied to a parabolic arch of variable moment of inertia. It should be mentioned that Wang's work has been taken as the basis of this thesis.

If one uses iterative method of analysis, the internal forces of the arch determined by the elastic theory is used as the first approximation and are applied to calculate an approximate resulting deflected shape. Additional bending moments due to the deflection is calculated and used for the determination of the second approximate deflected position of the arch axis. If the loaded arch is stable, the deflection is finite and repetition of this procedure will yield the equilibrium position of the arch at which the forces and deflections are consistent ${ }^{〔 3]}$.

The derived governing differential equation is nonlinear, because one of the coefficients is the horizontal reaction which is a functional of deflection. To remove the difficulty, a linearization technique is employed. This linearization enabled the applicability of the principle of superposition under a preassigned horizontal thrust $H$. The resultant internal forces were expressed as a combination of effects due to transverse load on a beam and that due to the horizontal reaction and curvature of the arch. Shooting method was used to determine the influence lines of sectional forces for a set of carefully selected flexibility parameters. Using these influence lines, relationship between the variation of a stress at any section of
the arch to the variation of assumed horizontal reaction can be obtained. At the same time, the relationship between the corresponding computed horizontal reaction and the assumed horizontal reaction forces are obtained. By the use of the second relationship, the correct horizontal reaction force component can be calculated and used to find the correct value of the force of interest from the first relationship.


In the derivation of the equation, the following assumptions are being made.

1. Stresses and strains are within proportional limits.
2. The influence of the horizontal component of the deflection is small and can be neglected.
3. The change in slope of the axis of the arch due to the applied load, at any point, is so small that the differential length can be assumed as $d s=d x \sec \theta$.
4. The load is assumed to act directly on the arch axis.
5. The radius of curvature is large compared to the thickness of the areh rib so that the straight beam formula is applicable.
6. The arch axis is assumed to be inextensible, thus aeglecting the axial effects due to deformation.
7. The effects due to shearing strain is small.


Fig. 2 Reaction forces due to applied load

Based on the assumptions made, the bending moment at any point $m$, distant $x$ from the left end of the fixed-fixed arch can be written as,

$$
\begin{equation*}
M_{x}=M_{o}-H(y-v)+\left(M_{b}-M_{a}\right) \frac{x}{L}+M_{a} \tag{1}
\end{equation*}
$$

Equation (1) can be rewritten as,

$$
\begin{equation*}
M_{x}=M_{o}-H(y-v)+M_{a}\left(1-\frac{x}{L}\right)+M_{b} \frac{x}{L} \tag{2}
\end{equation*}
$$

We can express the relation between change in curvature and magnitude of bending moment $M$ by the equation,

EI $\left(\frac{1}{P}-\frac{1}{R}\right)=-M$
$M$ is taken positive when it produces a decrease in the initial curvature of the arch axis. Note chat, in the above case the moment is increasing the curvature.

(a)

(b)


Eig. 3 Change in curvature of arch axis
The change in curvature of the arch axis during bending will be found from a consideration of the deformation of a small element mn included between two radii with sn angle d $\theta$ between them.

The initial length and curvature of the small element mare, es $=R d \theta$ and $\frac{1}{R}=D_{s}(\theta)$

The curvature of the arch axis after bending is,
where, $d \theta+\Delta d \theta=$ Angle between normal cross sections at $m_{1}$ and $n_{1}$.
$d s+\Delta d s=$ Length of element $m_{1} n_{1}$

The angle between the tangent to the centerline at $m_{2}$ and the perpendicular to the radius mo is $D_{s}(w)$. The corresponding angle at cross section $n_{1}$ is $D_{s}(w)+D_{s}^{2}(w) d s$

Hence, $\quad d \theta=D_{s}(w)+D_{s}^{2}(w) d s-D_{s}(w)$

$$
\begin{equation*}
=D_{s}^{2}(w) d s \tag{6}
\end{equation*}
$$

As the angle $D_{S}(w)$ is very small, we could oeglect the same
while calculating the length of $m_{1} n_{1}$.

Length of mn $=R d \theta$

Length of $m_{1} n_{1}=(R-w) d e$

Hence $d s=(R-w) d \theta-R d \theta$
$d s=-w d \theta$

But from ( 4 ),

$$
d \theta=\frac{1}{R} d s
$$

Therefore,

$$
d s=-w \frac{1}{R} d s
$$

Hence (5) can be rewritten as,

$$
\frac{1}{\bar{\rho}}=\frac{\left[D_{s}(\theta)+D_{s}^{2}(w) d s\right]}{\left[\mathrm{ds}-\frac{w^{W}}{R} d s\right]}
$$

$$
\begin{align*}
& =\frac{d s\left[D_{s}(\theta)+\left\langle D_{s}^{2}(w)\right\}\right]}{=} \begin{array}{l}
d s\left[1-\frac{W}{R}\right] \\
= \\
\quad(1 / R)+D^{2}(w) \\
\quad 1-\frac{W}{R}
\end{array},
\end{align*}
$$

For $\mathbf{r}<1$,

$$
\frac{1}{\bar{l}=\bar{r}}=1+r+r^{2}+\ldots \ldots \ldots
$$

Thus for $w \ll R$, one has,

$$
\frac{1}{--(w / R)}=1+(w / R)+(w / R)^{2}+\cdots \cdots \cdot
$$

Neglecting small terms of higher order,

$$
\frac{1}{\rho}=\left(1+\left(\frac{w}{R}\right)\right) \frac{1}{R}+D_{s}^{2}(w)
$$

Substituting for $\frac{1}{\rho}$ in (3),

EL $\left(\frac{1}{R}\left(1+\left(\frac{W}{R}\right)\right)+D_{s}{ }^{2}(w)-\frac{1}{R} d=-M\right.$
$\left.\operatorname{EI}\left\{\frac{\underline{w}}{\mathbb{R}^{2}}\right\}+\left\langle D_{s}{ }^{2}(w)\right\}\right]=-M$
Equation ( 2 ) will therefore become,
$-E I\left[\frac{W}{R^{2}}+D_{s}^{2}(w)\right]=M_{0}-H(y-v)+M_{a}\left(1-\frac{x}{L}\right)+M_{b} \frac{x}{L}$

As the radius of curvature is large compared to the radial displacement $W$,
$-E I D_{s}{ }^{2}(w)=M_{0}-H(y-v)+M_{a}\left(1-\frac{X}{L}\right)+M_{b} \frac{x}{L}$

The vertical displacement of the arch axis is,

$$
\begin{aligned}
v & =w \cos \theta \\
\text { or }, \quad v & =v \sec \theta
\end{aligned}
$$



$$
D_{s}(w) \simeq D_{x}(v)
$$

$$
\text { and, } \begin{aligned}
D_{s}^{2}(w) & =D_{s}\left\{D_{x}(v)\right\} \\
& =\cos \theta D_{x}\left\{D_{x}(v)\right\} \\
& =\cos \theta D_{x}^{2}(v)
\end{aligned}
$$

As moment of inertia of the arch is constant, ( 8) can therefore be written as,
$-E I \cos \theta D_{x}^{2}(v)=M_{0}-H y+H v+M_{a}\left(1-\frac{x}{L}\right)+M_{b}\left(\frac{x}{L}\right)$

$$
\text { Defining } G(x)=\cos \theta
$$

$$
G(x) D_{x}^{2}(v)=-\frac{H}{E I} v+\frac{H}{E I} y+f(x)
$$

$$
\text { where, } \left.f(x)=-\frac{1}{E} I M_{0}(x)+M_{a}\left(1-\frac{x}{L}\right)+M_{b}\left(\frac{x}{L}\right)\right]
$$

or,

$$
\begin{equation*}
\left.G(x) D_{x}^{2}(v)=-\frac{1}{L^{2}} \frac{(H L}{E I}\right) v+\frac{H}{E I} y+f(x) \tag{9}
\end{equation*}
$$

The first term on the right hand side of equation (9) is the moment due to deflection and horizontal thrust ; the second term is the moment due to the action of the arch profile ; and last term is the end moment.
(9) can be rewritten as,

$$
G(x) D_{x}^{2}(v)+\frac{1}{L} 2\left(\frac{H L}{E L}\right) v=\frac{H}{E L} y+f(x)
$$

Differentiating twice and rearranging,
$\left.G(x) D_{x}^{4}(v)+2 G^{\prime}(x) D_{x}^{3}(v)+\left[G^{\prime \prime}(x)+\frac{1}{L^{2}} \frac{\left(H L^{2}\right.}{E L}\right)\right] D_{x}^{2}(v)$

$$
\begin{equation*}
=\frac{H}{E I} y^{\prime \prime}+\frac{q(x)}{E I} \tag{10}
\end{equation*}
$$

For a fixed ended arch, the vertical deflection and slope at the two ends are zero. Hence,

$$
\begin{align*}
& v(0)=v(L)=0 \\
& v^{\prime}(0)=v^{\prime}(L)=0 \tag{10a}
\end{align*}
$$

To solve the fourth order differential equation ( 10 ), one more constraint condition is required, as $H$ is also an unknown. The condition that the sum of horizontal displacements, through the span length $L$, is zero will give us this additional condition.

$$
\begin{equation*}
\int v^{\prime} y^{\prime} d x=0 \tag{10b}
\end{equation*}
$$

This consists of horizontal displacements due to both
external load and horizontal force

$$
\text { i.e. } \int v_{q}^{\prime} y^{\prime} d x+\int H v_{h}^{\prime} y^{\prime} d x=0
$$

## Linearization of the differential equation

The fourth order differential equation (10) has unknown coefficients. Also as $H$ is a functional of $v$, this poses to be a nonlinear problem. Solving this nonlinear problem directly is going to be a tedious job. In order to simplify this, a linearization technique is adopted, thus avoiding the multiplication of H with the functional of $v$, by assuming a parameter,

$$
\begin{equation*}
g^{2}=\frac{H_{a} L^{2}}{E I} \tag{10c}
\end{equation*}
$$

Substitution of this into eqn.( 10 ) yields a fourth order linear differential equation, with one unknown value $g$.

Since, for any proper value of $g$ assigned, the equation is transformed into a linear form, the superposition principle is applicable. Therefore, the vertical deflection $v$ can be split up as,

$$
\begin{array}{r}
v=v_{q}+H v_{h} \quad(\operatorname{lod}) \\
v_{\text {where }}, v_{q}=\text { total vert. disp. due to ext. load } \\
v_{h}^{q}=\text { total vert. disp. due to unit hor. } \\
\text { reaction } H
\end{array}
$$

Equation ( 10 ) therefore becomes,
$G(x) D_{x}^{4}\left(v_{q}+v_{h} H\right)+2 G^{\prime}(x) D_{x}^{3}\left(v_{q}+v_{h} H\right)$
$+\left[G^{\prime \prime}(x)+\frac{g_{2}^{2}}{L^{2}}\right] D_{x}^{2}\left(v_{q}+v_{h} H\right)$

$$
\begin{equation*}
=\frac{H}{E I} y^{\prime \prime}+\frac{q(x)}{E I} \tag{10e}
\end{equation*}
$$

This allows to find the solution in two simpler cases :
$G(x) D_{x}^{4}\left(v_{q}\right)+2 G^{\prime}(x) D_{x}^{3}\left(v_{q}\right)+\left[G^{\prime \prime}(x)+\frac{g_{L}^{2}}{2}\right] D_{x}^{2}\left(v_{q}\right)=\frac{q(x)}{E I}$
and
$G(x) D_{x}^{4}\left(v_{h}\right)+2 G^{\prime}(x) D_{x}^{3}\left(v_{h}\right)+\left[G^{\prime \prime}(x)+\frac{g_{2}^{2}}{2}\right] D_{x}^{2}\left(v_{h}\right)=\frac{y^{\prime \prime}}{E I}$
The bcundary conditions are,

$$
\begin{array}{rlrl} 
& v_{q}(0) & =v_{q}^{\prime}(0) & =v_{q}(L)=v_{q}^{\prime}(L)=0 \\
\text { and } & v_{h}(0) & =v_{h}^{\prime}(0)=v_{h}(L)=v_{h}^{\prime}(L)=0
\end{array}
$$

Differential equation for a parabolic arch

> In case of a parabolic arch, profile is characterized by, $y=\frac{4 f}{2}\left(x-\frac{x^{2}}{2}\right)$

Its first and second derivatives with respect to $x$ are, $y^{\prime}=\frac{4 f}{L}\left(1-\frac{2 x}{L}\right)=\tan \theta$
and $\quad y^{\prime \prime}=-\frac{8 f}{L^{2}} \quad, \quad$ respectively.
As defined earlier,

$$
\begin{equation*}
G(x)=\cos \theta=\frac{1}{\left(1+-y^{\prime 2}\right)} \tag{12d}
\end{equation*}
$$

Differentiating this successively, we can get $G^{\prime}(x)$ and $G^{\prime \prime}(x)$ For simplicity, some of the nondimensional terms are defined as follows.

$$
\begin{align*}
z & =\frac{x}{L} \\
n & =\frac{f}{L} \\
u_{q} & =-\frac{v_{q}}{L}  \tag{13}\\
Q(z) & =-\frac{q(x) L^{3}}{E I} \\
u_{h} & =-\frac{v_{h}}{L}
\end{align*}
$$

The differential equations ( 11 ) and ( 12 ) can therefore be written as,
$G(z) D_{z}^{4}\left(u_{q}\right)+2 G^{\prime}(z) D_{z}^{3}\left(u_{q}\right)+\left(G^{\prime \prime}(z)+g^{2}\right) D_{z}^{2}\left(u_{q}\right)$
$=Q(z)$
( 14 )
and its boundary conditions are,

$$
u_{q}(0)=u_{q}(L)=u_{q}(0)=u_{q}^{\prime}(L)=0
$$

$$
\begin{align*}
G(z) D_{z}^{4}\left(u_{h}\right)+2 G^{\prime}(z) & D_{z}^{3}\left(u_{h}\right)+\left(G^{\prime \prime}(z)+g^{2}\right) D_{z}^{2}\left(u_{h}\right) \\
& =-8 n r \tag{15}
\end{align*}
$$

and the corresponding boundary conditions are,

$$
\begin{equation*}
u_{h}(0)=u_{h}(L)=u_{h}(0)=u_{h}^{\prime}(L)=0 \tag{15a}
\end{equation*}
$$

where,

$$
r=\frac{L^{2}}{E I}
$$

Knowing the boundary values for the solution of a differential equation, an approximate solution at any section can be determined using the Runge-Kutta method of integration.

$$
\begin{aligned}
& G(z)=\frac{1}{\left\{1+\{4 n(1-2 z)\}^{2}\right\}^{0.5}} \\
& G^{\prime}(z)=\frac{32 n^{2}(1-2 z)}{\left[1+\{4 n(1-2 z)\}^{2}\right]^{0.5}} \\
& G^{\prime \prime}(z)=\frac{64 n^{2}}{\left[1+\{4 n(1-2 z)\}^{2}\right]^{1} \cdot 5}\left[\begin{array}{c}
3(4 n(1-2 z))^{2} \\
{\left[1+(4 n(1-2 z)\}^{2}\right]^{2}}
\end{array}\right.
\end{aligned}
$$

## Numerical solution of the differential equation

The solution for the deflection of a flexible fixed ended arch under the application of anit force is discussed in this section.

An arch with a preassigned horizontal reaction force $H_{a}$ can be visualized as a prestressed elastic arch as shown below. Our interest is in finding the deflection under a unit vertical force applied at $z=z_{0}$. The decomposition of the deflection functions can
be expressed as follows.

(a) Arch subjected to unit moving load

(c) Vertical displacement of the arch axis due to unit load The mathematical medel, presented above, can also be seen as a berce $H$ with varying moment of can also be seen as a bean with varying moment of inertia $I_{b}=I_{a} \operatorname{Cos} \theta$, where $I_{a}$ is the MI of
the arch.

(b) Arch subjected to horizontal force H alone

(d) Vertical displacement of the


5 (cl)

$5(d 1)$

The value of $H$ can be obtained by making the relative displacement between $A$ and $B$ equal to zero, as given by eqn ( $10 b$ ).

In order to use Runge-Kutta method of integration the fourth order differential equations of (14) and (15) can be transformed into two equivalent systems of four first order differential equations as follows.

1. Effects due to a unit external load

$$
\begin{aligned}
& \text { Letting }\left[u_{q}, u_{q}^{\prime}, u_{q}^{\prime \prime}, u_{q}^{\prime \prime \prime}\right]=\left[\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}\right]=\theta \\
& \text { Equation (14) yields the following system. } \\
& D_{z}\left(\theta_{1}\right)=\theta_{2} \\
& D_{z}\left(\theta_{2}\right)=\theta_{3} \\
& D_{z}\left(\theta_{3}\right)=\theta_{4} \\
& G\left\{D_{z}\left(\theta_{4}\right)\right\}=-\left[\left(G^{\prime \prime}+g^{2}\right) \theta_{3}+2 G^{\prime} \theta_{4}-Q\right] \\
& \quad \underline{B C} \theta_{1}(0)=\theta_{2}(0)=\theta_{1}(1)=\theta_{2}(1)=0
\end{aligned}
$$

Choosing the loading location $z=z_{0}$ as the shooting point, the following sets of initial values of $\theta$ for two homogeneous solutions are assumed.
$\theta^{1}(0)=\left\{\begin{array}{l}0 \\ 0 \\ 1 \\ 0\end{array}\right\}, \theta^{2}(0)=\left\{\begin{array}{l}0 \\ 0 \\ 0 \\ 1\end{array}\right\}, \theta^{3}(1)=\left\{\begin{array}{l}0 \\ 0 \\ 1 \\ 0\end{array}\right\}, \theta^{4}(1)=\left\{\begin{array}{l}0 \\ 0 \\ 0 \\ 1\end{array}\right\}\left(\begin{array}{l}16 a)\end{array}\right.$

By integrating from the two ends, the solutions for equation ( 14 ) can be expressed as the superposition of the two homogeneous solutions.

For $z<z_{0}, \quad \theta(z)=C_{1} \theta^{1 L+C} \theta^{2 L} \quad(17)$
For $z>z_{0} \quad \theta(z)=C_{3} \theta^{3 R}+C_{4} \theta^{4 R} \quad(18)$
L and $R$ in the superscript indicate left or right of the section under consideration.

The constants $C_{1}, C_{2}, C_{3}, C_{4}$ are determined from the condition that, at the loading point, deflection, slope, moment and shear difference obtained by integration from the two ends should match with the load.

Hence at the loading point $z=z_{0}$, each component of the solution vectors should satisfy the following conditions of continuity.

$$
\begin{align*}
& \theta_{1}^{L}=\theta_{1}^{R} \\
& \theta_{2}^{L}=\theta_{2}^{R} \\
& \theta_{3}^{L}=\theta_{3}^{R} \tag{18a}
\end{align*}
$$

The fourth condition for determination of the solution is to use the equilibrium condition $E_{y}=0$,
$-\left(G^{\prime} \theta_{3}^{L}+G \theta_{4}^{L}\right)+\left(G^{\prime} \theta_{3}^{R}+G \theta_{4}^{R}\right)=1$
at the loading point.
Substituting the obtained coefficients into equations (17) and (18), we can get the solution for equation ( 14 ).
2. Effects due to unit horizontal arch reaction

In order to find the required horizontal reaction force $H$, the deflection of the structure under the application of a unit horizontal force is used for calculating the deflection of the structure.

Letting [ $\left.u_{h}, u_{h}^{\prime}, u_{h}^{\prime \prime}, u_{h}^{\prime \prime} \cdot\right]=\left[k_{1}, k_{2}, k_{3}, k_{4}\right]=k$

Equation (15) yields the following system of first order differential equations.

$$
\begin{align*}
D_{z}\left(k_{1}\right) & =k_{2} \\
D_{z}\left(k_{2}\right) & =k_{3} \\
D_{z}\left(k_{3}\right) & =k_{4} \\
G\left\{D_{z}\left(k_{4}\right)\right\} & =-\left[\left(G^{\prime \prime}+g\right) k_{3}+2 G^{\prime} k_{4}+8 n r\right] \tag{19}
\end{align*}
$$

and the boundary conditions at the ends are,

$$
k_{1}(0)=k_{2}(0)=k_{2}(1)=k_{4}(1)=0
$$

Using the conditions of deformation for a symmetrical structure under the application of symmetric loading, the boundary conditions can be written as,

$$
k_{1}(0)=k_{2}(0)=k_{3}(1 / 2)=k_{4}(1 / 2)=0
$$

The solution can now be obtained as a superposition of two homogeneous solutions and a particular integral.

The initial values of $k^{(i)}$ are assumed as,

$$
k^{(1)}(0)=\begin{aligned}
& 0 \\
& 0 \\
& 1 \\
& 0
\end{aligned}, \quad k^{(2)}(0)=\begin{aligned}
& 0 \\
& 0 \\
& 0 \\
& 1
\end{aligned} \quad, \quad P(0)=\begin{aligned}
& 0 \\
& 0 \\
& 0 \\
& 0
\end{aligned}
$$

$P(z)$ is for the particular integral of the solution using homogeneous boundary conditions and, $k^{(1)}(z)$ and $k^{(2)}(z)$ are solutions of the homogeneous differential equation. Integrating from left end to the crown of the arch, the solution for equation ( 15 ) can be expressed as,

$$
\begin{equation*}
k(z)=P(z)+D_{1} k^{(1)}(z)+D_{2} k^{(2)}(z) \tag{19a}
\end{equation*}
$$

Slope and shear at the center of the arch are,

$$
\begin{align*}
& k_{2}(1 / 2)=P_{2}(1 / 2)+D_{1} k_{2}^{(1)}(1 / 2)+D_{2} k_{2}^{(2)}(1 / 2) \\
& k_{4}(1 / 2)=P_{4}(1 / 2)+D_{1} k_{4}^{(1)}(1 / 2)+D_{2} k_{4}^{(2)}(1 / 2) \tag{19b}
\end{align*}
$$

Solving for $D_{1}$ and $D_{2}$ equations ( 19 b ) and substituting them into (19a) yields the solution of the deflection of arch under the application of a unit horizontal force.

The solutions for equations (16) and (19) are obtained by assuming the value of $g$, which is given by,

$$
g^{2}=\frac{H_{a} L^{2}}{E I}
$$

The unknown horizontal arch reaction $H$ can be determined by making use of the additional constraint condition,

$$
\begin{aligned}
\int_{0}^{v^{\prime} y^{\prime} d x} & =0 \\
\text { As defined } & \text { earlier, } \\
v & =v_{q}+H v_{h} \\
v^{\prime} & =v_{q}^{\prime}+H v_{h}^{\prime} \\
\text { But, } \quad v_{q} & =u_{q} L \\
v_{h} & =u_{h} L
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
v^{\prime} & =\left(u_{q}^{\prime}+H u_{h}^{\prime}\right) L \\
y^{\prime} & =-\frac{4}{L}-\left[1-\frac{2 x}{L}\right] \\
& =[4 f(1-2 z)] \frac{1}{L} \\
& =\frac{1}{L} y^{\prime}(z)
\end{aligned}
$$

Hence,

$$
\int_{0}^{L}\left(u_{q}^{\prime}+H u_{h}^{\prime}\right) y^{\prime}(z) d z=0
$$

The horizontal force can be obtained by,

$$
H=\frac{\int_{0}^{L} u_{q^{\prime}}^{\prime} y^{\prime} d z}{u_{h}^{\prime} y^{\prime} d z}
$$

The non-dimensional function for $v$ and its derivatives can be expressed as,

$$
\begin{aligned}
u_{i}=\theta_{i}+ & H k_{i} \\
& \text { for, } \quad i=1,2,3,4
\end{aligned}
$$

The influence lines for the moment $M$, vertical shearing force $V$ and thrust $T H$ induced in the arch due to moving unit load can therefore be written as,

```
\(M(z, \bar{z}, g)=-G(z) u_{3}(z, \bar{z}, g)\)
\(V(z, \tilde{z}, g)=D_{z}(v)=-\left[G(z) u_{4}(z, \bar{z}, g)\right.\)
    \(\left.+G^{\prime}(z) u_{3}(z, \tilde{z}, g)\right\}\)
\(N(z, \bar{z}, g)=H(z, g) \operatorname{Cos}(\theta(z))+V(z, \bar{z}, g) \operatorname{SIN}\{\theta(z)\}\)
```

Thus, for any assumed value of $g$, the influence line for internal force component at the section of interest can be obtained.

## Procedure for stress analysis

The stresses at any section of the arch due to the given set of loading can be determined as follows.

Using a set of influence line for $M(z, \bar{z}, g)$, a set of $M$ for a given loading condition corresponding to the selected values of gare determined.

Under the same loading conditions, the values for horizontal reaction and the thrust force at the section are calculated corresponding to the $g$ values selected.

Therefore a set of points for functions $M(g), N(g)$ and $H(g)$ over a selected $g_{1}, g_{2}, \ldots, g_{n}$ are obtsined.

Let $g^{*}=-\mathrm{Ha}_{\mathrm{a}} \mathrm{L}^{2}$

The correct vglue of $g$, say $g_{0}$, is obtained such that $H_{\text {calculated }}$ and $H_{\text {assumed }}$ agree with each other. The sectional moment of interest $M$, under any given loading condition, can be calculated by using $H(g)$ such that,

$$
g^{*}\left(g_{0}\right)=g_{0}
$$

and is shown in Fig (1) of Appendix 1.

The computer program written in FORTRAN, by Wang ${ }^{[4]}$, was converted to BASIC language and made to work on the mini-computer TRS 80 Model II. The program was suitably modified to find the numerical solution for the loaded model shown below.

A dead load of 5 lbs at each of the loading points, as in the experiment, is used. The ratio of rise to span is 0.1333 and values of $0,0.318,0.636$ for $g$ were assumed. Three types of moving loads 1 lb, 5 lbs and 10 lbs were considered. The computer program, in BASIC language, is listed in Appendix 2. The results obtained for the various loading cases are tabulated. The sketch given below illustrates the method of determining the stresses.


Fig. 6 Loading points on the arch
W = Dead load
$P=$ Moving load

To verify the accuracy of the deflection theory, an experimental setup was designed. An arch model, made of steel, with fixed ends was built in the laboratory. The investigation was carried out for a number of loading conditions. Automatic data acquisition system, with an Apple - minicomputer, was used to record the strains registered by the strain gages. The strains were stored on a floppy disk.

It should be mentioned at this stage that, initially a rise of 12 inches and span of 60 inches was taken. But the strain induced in the middle third of the arch due to its profile was found to have crossed the yield strain of the arch material. This was also observed through the permanent strain set in the steel arch. Hence, another arch with a reduced rise of 8 inches and with the same span of 60 inches was built.

A series of experiments were conducted. Moving load was moved from one point to another and at each position the strains were read. There were totally ten loading points. Thus for each moving load there existed ten cases. On the outset, the experiment consisted of two phases. One phase was carried out with dead load and the other without the dead load. Application of dead load was achived by hanging equal load at all the ten loading points.

Moving loads used, for the case without dead load, were 0.5 , $1,2,4,6,8$ and 10 lbs . It was observed that there was negligibly small response from the arch when the loads were below 2 ibs. Therefore, only five types of moving loads were considered for the
second case when the dead load was applied. The total dead loads considered in the experiment were 10 lbs through 80 lbs at intervals of 10 lbs. For each type of dead load, five tests were carried out for the five moving loads. The data got from all these experiments were stored on a floppy diskette to be retrieved at a later stage. But later on it was discovered that, not all the data had been saved on the disk. Eailure of the efforts to retrieve all the data led to the repetition of the experiments.

This time the tests were repeated for dead loads ranging from 10 lbs to 90 lbs at intervals of 10 lbs . The five types of moving laads used were the same as those used previously. The computer was directed to print out the data as and when it received. A computer program was written to calculate the stresses, making use of the strains obtained from the experiment. These stresses were then compared with the stresses calculated from theory. This study showed a discrepancy of as high as $80 \%$ at some sections. In an attempt to understand the reason for this, all the stresses were normalized to a unit load. The normalized values were then compared with the coresponding calculated values. The comparison proved to be very much satisfactory, with an average accuracy of $90 \%$, except for the cases when the moving loads were applied at the extreme two points.

This outcome called for a recheck of the experimental setup. Closer inspections indicated the possibility of frictional forces, offered by the pulleys, being the main reason for the discrepancy in the final stresses obtained for the extreme points. Several trial experiments confirmed that the moving loads being used were too small to overcome the frictional resistance coming from the pulleys. This fact led to another set of experiments with moving loads of higher
magnitude applied at the two outermost loading points.
A constant dead load of 50 lbs was maintained while the moving loads were varied from 21 bs to 20 lbs at an interval of 2 lbs . Many sections of the arch were found to respond better when the moving load was above 10 lbs . Another BASIC program was written to make use of this new set of data and to compute the stresses for $1 \mathrm{lb}, \mathrm{l}$, l and 10 lbs , by interpolation. The calculation of the stresses indicated an improvement with an accuracy of as high as $75 \%$. The comparison between the experimental and theoretical stresses at the central span section, and the percentage accuracy achieved are given in the tables in Appendix 1. The stresses calculated for the other sections are given in Appendix 2.

The experiment was conducted on a fixed-fixed parabolic arch of span 60 inches. A wooden formwork for bending the arch into the desired parabolic profile was prepared and bolted on to the steel baseplate. The model was built horizontally on the baseplate which in turn was placed on a table.

To achieve the fixity at the ends, the movement in the three directions was considered and taken care of. The horizontal motion was controlled by two channel sections placed on either side of the arch model. The two sections were securely bolted to the baseplate. In addition a hat section was designed to resist the rotational movement. To allow for free movement of the arch, when loaded, and also to restrict its movement in the transverse direction, rollers were provided underneath the model at five locations.

The loads were applied symmetrically on the arch model at ten points. Five sections were chosen for analysing the stresses induced due to the applied loading. Aluminium wires were made use of for transferring the loads onto the arch. The wire ran parallel to the table-top and over a pulley to be connected to cylindrical loading bowl, prepared for the purpose. Six strain gages, 3 on the topface and 3 on the bottom face, were mounted at each of the five locations.

Material properties of the arch
Material $\quad$ : Cold rolled nild steel
Dimensions $\quad 1$ inch wide and 0.125 inch thick
Span of the arch : 50 inches
Rise of the arch : 8 inches

Steel was selected as the material for making the arch. Three specimens were prepared for tension testing. Riehlers testing machine was made use for the purpose. Two specimens were tested using the extensometer and the automatic graphical ability available in the Riehlers machine. The other specimen was tested for tension on the Riehlers machine, with the help of a dial gage. The two tests yielded comparable results. Fig (2) in Appendix 1 shows the stress-strain curve for the arch material.

The following results were obtained.

| Maximum load taken by the specimen | $=3670 \mathrm{lbs}$ |
| ---: | :--- |
| Ultimate load |  |
|  | $=2800 \mathrm{lbs}$ |
| Young's modulus | $=30 \times 10^{6} \mathrm{psi}$ |

Achieving fixity at the ends of the arch was one of the most important part in the experiment. As it can be visualized, the ends are said to be fixed if the movement of the end section is prevented in the three directions. The movement in the $X$ and $Y$ directions were locked by bolting the arch to two channel sections. To achieve proper lockage, one of the channels was first bolted firmly to the baseplate. The arch was then bolted to the two channel sections. Note that, at this stage the second channel was still not bolted to the baseplate. Only after the arch was bolted tightly to the two sections, was the second channel section secured to the baseplate. To achieve this, slot holes were drilled for the second channel in the baseplate. This procedure was followed to ensure proper fixture of the arch with the channel section.

In order to lock the rotational movement, and also to increase the moment of inertia at the end section, a hat section was designed. Five flat plates were welded togeather, as shown in the sketch, to form the hat section. The welded section was bolted to the channels and the baseplate.

The section thus built was satisfactory. In fact, after the wooden formwork was removed, the steel arch remained in its intended parabolic form. This was tested by putting the formwork back into its place, and the bolts in the formwork went right into the tapped holes made for them in the baseplate.

Roller supports

The main idea of testing the arch by placing it horizontally on the table was to reduce the possibility of the arch's movement in the rransverse direction. The arch will move in the transverse direction if the loading points do not pass exactly through the central section of the arch. By providing the rollers underneath the arch would reduce this problem. In the experiment, rollers were provided at five places.

Rollers also helped the arch in its free movement, caused due to the application of loads. The rollers were bolted to wooden blocks which in turn were made to sit under the arch. Care was taken to see that the roller-seat did not cause too much friction to the arch.

As mentioned earlier, five sections (symmetrical about the center line) were chosen for analysing the stresses. With six gages at each section, totally thirty gages were mounted. The surface where the gage was being mounted was carefully prepared by sanding away the dirt and rust to obtain a smooth, but not a highly polished surface. Solvents such as the degreaser and the neutralizer were then employed to remove all traces of oil or grease and to give the surface a proper chemicall affinity for the adhesive. The location on the arch was accurately marked and the gage positioned with the help of a rigid transparent tape. As the bonded type of strain gage is a high quality precision resistor, it must be attached to the specimen with a suitable adhesive.

M-Bond 200 was used as the adhesive in the present work. Maintaining the position and orientation of the gage by the tape, the adhesive was carefully applied. Since the adhesive is sufficiently strong to control the deformation of the strain-sensitive element in the gage, any residual stresses developed in the adhesive will influence the output of the strain gage. The adhesive was, therefore, subjected to gentle pressures over a suitable length of period to ensure a complete cure. After the gages were properly bonded to the structure, three lead wires were attached to the gage through anchor terminals. Since the strain gages are relatively fragile, care must be taken in attaching the lead wires to the soldering tabs. The properties of the strain gages, classified as Student gages by the Measurement Group Inc, are as follows.

| Gage type | $:$ EA - 06-240LZ-120 |
| :--- | :--- |
| Resistance in ohms | $: 120.0 \pm 0.3 \%$ |
| Gage factor at $75^{\circ} \mathrm{F}$ | $: 2.045 \pm 0.5 \%$ |
| Option | $: E$ |

Option
: E

As described by the suppliers, student strain gages are EA-series gages and are constructed with a 0.001 inch tough, flexible polyimide film backing. All student gages include option $E$, a polyi mide encapsulation of the grid face, with exposed solder tabs. Normal use temperature range for static strain measurements is $-100^{\circ} \mathrm{F}$ to $+350^{\circ} \mathrm{F}$.

The five sections chosen for analysing the stresses along with the positioning of the gages are shown in the sketch below.


Fig. 7 Strain gage sections
Loading applied on the structure was symuetrical. Loads were applied at ten points. Aluminium wires were used to transfer the load on the arch. Calculation of the bucking load was made by making use of the tables given in references ! 1 ] and [6].

From reference [1],

$$
q_{c r}=\frac{g_{4} E I}{L^{3}}
$$

where $g_{4}$ is obtained from the table below.

| f/L | 0.1 | 0.2 |
| :---: | :---: | :---: |
| $\mathrm{B}_{4}$ | 60.7 | 101.0 |

By interpolation, for $\mathrm{f} / \mathrm{L}=0.1333, \mathrm{~g}_{4}=74.133$

$$
\begin{aligned}
q_{c r} & =\frac{\left[74.133 \times 30 \times 10^{6} \times\left\{1 \times(1 / 8)^{3}\right\}\right]}{60^{3} \times 12} \\
& =1.67582 \mathrm{Lbs} / \mathrm{in}
\end{aligned}
$$

Calculation from reference [6] yielded,

$$
q_{c r}=1.69 \quad 1 \mathrm{bs} / \mathrm{in}
$$

Taking $q_{c r}=1.67582 \mathrm{lbs} / \mathrm{in}$,

$$
\begin{aligned}
& M_{c r}=\frac{{ }^{\text {cher }}}{} \mathrm{L}^{2}=\frac{1.67582 \times 60^{2}}{8}=\frac{-0-0}{8} \\
& =754.119 \mathrm{lb}-i \pi \\
& \mathrm{H}_{\mathrm{cr}}=\frac{\mathrm{M}_{\mathrm{cr}}}{\mathrm{f}}=\frac{754.119}{--2+0} \\
& =94.265 \mathrm{lbs}
\end{aligned}
$$

Sand and lead weights were used as loads on the structure. Special cylindrical loading bowls were prepared for the
purpose. Automatic data acquisition system, coupled with the apple minicomputer, was used to determine the strain. The free end of the lead wires coming from the strain gages were soldered to a pin connector. This pin is inserted into the sockets provided at the back of the data acquisition system.

The controller is essentially the brain of the data acquisition system and contains a microprocessor with several memory devices. The controller activates scanner and controls the time sequence of the switching from one channel to the next. Also, it stores the output, the channel number and the time when the reading is made in its operating memory. The final form of the data is then transmitted to the mini-computer.

Calculation of the stresses from the strains obtained

Let, Et = Strain obtained from the top surface of the arch.

Eb = Strain obtained from the bottom surface of the arch.

Aa $\quad$ Cross sectional area of the arch.
$M$ = Moment at the section under consideration.
$N \quad=$ Thrust at the section under consideration.

Both the moment and thrust contribute to the strain at any section. As the arch is basically a compression member, the contribution from the thrust is negative.

$$
\begin{aligned}
E t & =-\frac{N}{A_{a} E}-\frac{M c}{E I} \\
E b & =-\frac{N}{A_{a} E}+\frac{M c}{E I}
\end{aligned}
$$

Adding the two equations,

$$
N=-\frac{A E}{2}(E t+E b)
$$

Subtracting the two equations,

$$
M=\frac{E I}{2 c}(E b-E t)
$$

Shear (V) can also be calculated if the moments at two sections, a small distance apart, are known.

$$
\begin{aligned}
\mathrm{V}=\frac{M_{1}-M_{2}}{d s} \quad \text { where, } M_{1} \text { and } M_{2} \text { are the moments at } \\
\text { two sections distance dos apart. }
\end{aligned}
$$

Thus, the stresses are determined from the strains obtained from the experiment. A computer program, in BASIC language, was generated to calculate the stresses as explained above.

The following types of loadings have been considered and discussed here.

1 A moving load of 10 lbs with no dead load on the structure.
2 A moving load of 1 Ib with a dead load of 50 lbs (which is about $53 \%$ of the buckling load) on the structure.

3 A moving load of 5 lbs with a dead load of 50 lbs on the structure.

4 A moving load of 10 lbs with a dead load of 50 lbs on the structure.

5 Static loads of 5 lbs, 10 lbs and 101 bs at fourth, fifth and sixth loading points respectively, in addition to a dead load of 501 bs on the structure.

A computer program was generated to calculate the moment and thrust from the strain data obtained and has been made to run on the Zenith - 100 minimcomputer. A listing of the program is given in Appendix 2. The results obtained are tabulated and compared with the theoretical results. This comparison has been made at the center span. Influence lines for all the above loadings at three locations are drawn and shown through figures in Appendix 1.

As shown in Appendix 1 for the central span section, the moment values are in close agreement with the experimental values. For 10 lbs moving load, Table 2 shows that the accuracy is as high as $97 \%$. The accuracy is found to be $73 \%$ when the load is acting at 1 , indicating the failure of the load to influence the far off sections. The influence lines, at three sections are shown in Appendix 1 and the variation from the calculated values are marked.

However, it should be noted that at some sections the experimental moment values were normalized before being compared. The influence line diagram for the arch when a dead load of 50 bs was present, failed to agree with the corresponding theoretical results. The reasons for the disagreement are due to a number of factors, the most significant one being the frictional force.

As it has been explained in an earlier chapter, aluminium wires used for applying the loads on the structure, were carried over pulleys. Careful inspection indicated that the loads applied on the structure failed to overcome the friction exerted by the pulleys. The response from the arch due to some of the loads was observed to be negligibly small. In order to overcome this limitation, a series of experiments were conducted, by increasing the magnitude of the moving load each time, with a constant dead load of 501 ba . The magnitude of the moving loads ranged from 2 lb to 20 lbs at an interval of 2 lbs . In several of the cases, especially when the moving loads were near the ends, the arch was seen to respond only when the magnitude of the moving load was more than 101 bs .

The results obtained from this aeries of experimenta were then utilised in interpolating the strains for three moving load cases
of $\mathrm{l} \mathrm{lb}, 5 \mathrm{lbs}$ and $10 \mathrm{lbs}$. A computer program was generated for this interpolation and for the calculation of the stresses. It was observed that at some sections, the stresses were still not in agreement with the theoretical values. The experimental values were therefore normalized for a unit load. This treatment immediately yielded satisfactory results. The percentage variation of the values from the corresponding theoretical values are given in Appendix 1 , for stresses as obtained due to moving load of 10 lbs . The table indicatea very close agreement as the load approaches the section of interest. From this it could be clearly seen that, the influence of a load at any section of the arch is inversely proportional to its distance from the loaded point. The obvious reason is the frictional resistance from the pulleys.

Though utmost care was practiced in the mounting of strain gages, the possibility of human errors cannot be neglected. The characteristics of the adhesive used in mounting the gage are auch that it can influence apparent gage factor, hysteresis characteristics, resistance to stress relaxation, gage resistance, temperature induced zero drift and insulation resistance ${ }^{[5]}$. Such sensitiveness could blow up very small errora and might influence the final results. In fact, six of the thirty gages mounted, failed to respond eventhough a check on their mounting and resistance indicated satisfactory results.

From table 1 in the Appendix, it could be seen that the variation in the moments is not linear as the load is incresed from 1 lb to 10 lbs . This proves that the change in geometry of the structure has a considerable effect on the results.

However, further work in the area is required before using the method as a useful tool in the practical field.

Placing the arch in a vertical position and applying the loads directly, without any pulleys, would yield betcer results. Such a setup will remove the frictional forces, which influenced the final results considerably, in the present work.

It is also recommended that the wires carrying the load, should be made to sit directly on the arch. Connection of the wire to an eyebolt will generate local moments which are considerably large near the ends. Eye bolts were initially used in the present work, and later removed, at all but middle two loading points of the arch, after the above mentioned phenomena was observed. In the middle section, the alignment of the eye bolts was seen to be along the loading line. The influence line diagram, shown in Fig (3) of Appendix 1, indicates an offset in the values obtained for the middle two points. Thus, it is observed that the local moments due to the eyebolts, however small they were, have affected the final results.

Also, the horizontal component of the displacement was not considered in the present work. Hence, in order to predict the behaviour of the structure with a much higher accuracy, it is recommended that the horizontal component be taken into consideration while formulating the equilibrium conditions in all future works.

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## APPENDIX 1

Tables and figures
table 1
TABLE 2

| Loading at point no. | $\underset{(\%)}{ }=1 \mathrm{lb}$ | $P={\underset{(\%)}{5} \mathrm{lbs}}^{2}$ | $\mathrm{P}=\underset{(\%)}{10} \mathrm{lbs}$ |
| :---: | :---: | :---: | :---: |
| 1 | 76.76 | 78.63 | 72.31 |
| 2 | 90.80 | 93.82 | 88.55 |
| 3 | 99.55 | 98.04 | 93.13 |
| 4 | 83.33 | 94.90 | 92.32 |
| 5 | 83.34 | 76.54 | 96.72 |

Variation of Influence line coefficients for moment
at the central section of the arch
(in percentage)
$\mathrm{g}^{*} \mathrm{M}(\mathrm{g}) \times 10^{-3}$

* +0

Fig Al Graphical method to find the actual forces
Moving load $=10$ lbs at loading point 1.


亚

| Legend | Calculated |
| :---: | :--- |
| + | Experimental |


$\left[\begin{array}{lll}2\end{array}\right.$


P1 Fixing the formwork


P2 Placing the arch


P3 The hat section


P4 Sectional view of the fixed end


P5 General view of the end section



P7 Roller supports

APPENDIX 2
Computer programs and results

20 REM This program Galrulstas the influence line eoeftigients fo re fixen endeg erch with constant monent of inertia.
30 REM
40 REM
50 REM Some of the variabies used 1 n the prosram are explaned be 18
60 REM AN $=, \quad n=\uparrow / 1$
70 REM $G=\cos$ (Theta)
80 REM GP $=\Rightarrow \quad G^{2}=d G / d x$
90 REM YP $=\% \quad ध^{2}=d Y / d \%$
100 REM WI(I,J), TH(I;J), SU(I,J) $\Rightarrow$ Moment, Thrust and Shear at section I gt the arch due to unit loadet J.

120 DIM $A(4,5), \operatorname{AA}(4,21), \operatorname{AGN}(4), \operatorname{ALM}(8), \operatorname{AN(4)}, \operatorname{AP}(4), \operatorname{AUN}(4)$
$130 \operatorname{DIM}$ ET $(8,6,5), C(4), C N(21), \operatorname{DRIV}(21), D X(21), D Y(4), E(2)$
140 DIM F(21), FA(3,21), G(21), GP(21), HF(4,21), H(6,5), HX(21)
150 DIM IX(21), PA(3), PSA $(8,6,5), \operatorname{PVA}(4)$, (4)
160 DIM $S(4,10,21), S 1 P(21), \operatorname{SAN}(8,6: 5), \operatorname{SMNY}(4), \operatorname{SN}(21), \operatorname{SPS}(4,2$
1)

170 DIM SS(6), ST(6), SV(6,21), SYMX(4)
180 DIM T(21), T1(21): TA(4), TH(6,21), TIG(4, 21), TJP(4), TWH(4,
21)

190 DIM U( $8,6,5), W 1(6,21): W 2(11,21), W 3(11,21), W H(4,4,21)$
200 DIM XI( $5: Y(4), Y F(21), Y Y(5), Z(5)$
$210 \mathrm{FI}=3.14159265358$
$220 \mathrm{EI}=6000300$
230 cls
z40 FRINT "In the present works the rise to span retio is 8/60= $0.1333^{\prime \prime}$
250 INPUT "Rise to sPan retie ="; AN(3)
260 PRINT : PRINT "InPut the assumed values of H/HEr"
270 PRINT "In the present work, Hir was retoulated as 94.34 [Ref Eleztir Stability bir Timostramal. Hmax expected wes ob its. H/H $E r=50 / 84.34=0.636{ }^{\prime \prime}$
zed FRINT "For plotting ue reed a minimum of three points"
290 FRINT "Hence $0,(0+0.636) / 2$ and 0.636 were fed in as the inpu t"
3OD PRINT : PRINT : PRINT
310 FOR I=1 TO 3
320 INPUT "H/HIT=";ALM(I)
330 NEXT I
340 II $=3$
356 Fn JJ=1 T0 3
$3 \in 0 \quad x=0$
$370 H=1 / 20$
3 BO FOR $J=1$ TO 21
$390 G(J)=1 / E G R(1+(4 * A N(I I) *(1-2 * X)) * 2)$
$400 \mathrm{CN}(\mathrm{J})=\mathrm{G}(\mathrm{J})$
$410 \mathrm{YF}(J)=4 * A N(I I) *(1-2 * X)$
$420 \operatorname{SN}(J)=Y P(J) * C N(J)$
430 GP(J) $=32 * A N(I I) 42 *(1-2 * K) /((1+(4 * A N(I I) *(1-2 * X)) \uparrow 2) \uparrow(3 / 2)$
$440 \quad X=X+H$
450 NEXT J
$460 \mathrm{ALM}(\mathrm{JJ})=P I * A L M(J J)$
$470 \mathrm{NE}=4$
480 FOR J= 1 TO 2
$4901 \mathrm{MN}=\mathrm{J}+2$

```
500 FOR I = 1 TO 4
510 O(I)=2
520 Y(I)=0
530 NEXT I
540 Y(mN)=1
550 FOR M=1 T0 4
560 WH(M,J,1)=Y(M)
5 7 0 ~ N E X T ~ M ~
580 X=0
590 KK゙=2
GOD FOR K=z TO 22
610 GOSUE 2360
E2 FOR M=1 TO 4
630 WH(M:J,F()=Y(M)
640 NEXT M
650 NEXT K
600 NEXT J
670 ドK=>
680 FOR J=1 TO 4
690WH(J,3,1)=0
700 Y(J)=0
710 (J)
7% NEXT J
730 x=0
740 FOR J= 2 T0 21
750 GOSUS 2360
760 FOR K=1 T0 4
770 WH(k),\Xi:J)=Y(K)
780 NEXT K
79O NEXT J
8000 D=WH(2,1,11)*WH(4,2,11)-WH(2,2,11)*WH(4,1,11)
810 DI=-WH(2,3,11)*斯(4,2,11)+WH(2,2,11)*WH(4,3,11)
E20 DC=-WH(2,1,11)%WH(4,3,11)+WH(2,3,11)*WH(4,1,11)
830 C1=01/D
840 C2=D2/D
850 FOR J= 1 TO 4
E60 J1=J+1
870 FOF M= 1 T0 11
Eg0 JK=22-M
E70 TWH(J,M)=C1*WH(J,1,M)+CZ*WH(J,2,M)+WH(J,3,M)
900 TWH{J,JK)=TWH(J,M)*((-1)*J1)
910 NEXT M
720 NEXT J
930 FOR J=1 T0 21
940 T1(J)=TWH(2,I)*YP(J)
950 NEXT J
960 N=21
970 GOSUR 2840
980 DTH=AR
900 FOR J=: T0 21
1000 HF (1,J)=TWH(1;J)/DTH
1010 NEXT J
1020 FOR M=1 T0 21
1050 HF (3,M)=-G(M)*TNH(3,M)
1040 HF (4:M)=-GP(M)*TWH(3,M)-G(M)*TWH(4,M)
1050 NEXT M
1060 FOR J=1 T0 2
1070 MN=J+2
1080 CI=-1
1090 FOR M=1 T0 4
```

```
1100CI=CI*(-1)
11:0 FOR K=1 T0 21
1120 KM=22-K
1130 WH(M,MN:K)=WH(M,J:KM)*CI
$140 NEXT K゙gM, 」
1150 FOF I=2 T0 20
1160 FOF J=1 T0 3
1170 FOP K=1 T0 4
1180 A (J,K)=WH(J,隹,I)
1150 FOR L=1 TO 4
1200 A (4,L)=-GP(I)*WH(3,L,I)-G(I)*WH (4,L,I)
1210 A(L,5)=0
#20}NEXT 
1230 NEXT K゙,J
1240 FOR K゙=1 T0 4
1250 FOR J=3 TO 4
1260 A(kR,J)=-A(k,J)
1270 NEXT J.K゙
1280 A(4,5)=1
1290 GOSUE 2140
300 S1F(I)=[
ジロ FOR J=1 TO ご
I20 SIP(I)=SIP(I)-A(J,5)*(GP(I)*WH(3,J,I)+G(I)*WH(4,J,I))
1330 NEXT T
1340 FOR J=1 TO 11 ETEPE
1350 II=(j+1)/2
13605V(I1*I)=0
1570W3(I!,I)=Q
13E0 IF J y= I THEN 1460
1390 FOR K゙=1 T02
1400 EV(II,I)=SV(I&,I)-(GP(J)*WH(ぶ,N゙,J)+G(J)*WH(4,K゙,J))*A(K゙,5)
1410 W゙こ(II,I)=HF(J,I)*HF(I,I)
1420 WJ(II,I)=W3(II,I)-G(J)*A(K,5)*WH(3,N゙,J)
1430 W:(II,I)=W3(II,I)+W2(II,I)
1440 NEXT M
1450 GOTO 1520
1460 FOR k<=3 TO 4
1470 SV(II,I)=SV(II,I)-(GF(J)*WH(J,R゙,J)+G(J)*WH(4,G゙,J))*A(K゙,5)
1480 W2(I1,I)=HF(3,J)HHF(I;I)
1470W3(I1,I)=W3(I1,I)-G(U)*A(K゙,S)*WH(J,K,J)
1500 W1(II,I)=W3(II,I)+W2(II,I)
1510 NEXT ド
1520 NEXT
1530 NEXT I
1540 FOR, I=1 T0 6
1550W1(I,1)=0: SV(I,1)=0:SV(I,21)=0:WI(I,21)=0
1560 NEXT I
1570 51P(1)=1
1580 5V(1,1)=1
1590 FOR I=1 T0 6
1600 NP=2*I-1
1610 FOR J=1 T0 %1
1620 TH(I,J)=HF(I,J)*CN(NP)+EV(I,J)*SN(NP)
1630 SV(I,J)=(SU(I,J)+HF(4,NP)*HF(I,J))*CN(NP)
1640 NEXT J
1G50 ST(1)=HF(1,NP)*CN(NP)+S1P(I)*SN(NF)
1B&DSS(I)=(SIP(I)+HF(4,NP)*HF(I,NP))*CN(NP)
1670 NEXT I
1680 FOR XI=1 TO b
1690 FOR J=1 T0 21
```

```
    1700 TWH(1:J)=W1(XI:J)
    1710 TWH{2,J)=HE(1,J)
    1720 TWH(J,J)=TH(XI,J)
    1730 TWN'(4,5)=SV(XI:J)
    1740 NEXT J
    1750 NP=2%XI-1
    1760 AJJ=JJ-1
```



```
    17ED LFRINT
    177% LPRINT "LAMEDA= ", #WJ
    1800 LPRINT
    1Q10 LPRINT"Influermer limes for segtion #";NP
    1g2U LPRINT
    IESO LPRINT "Mament": : LPRINT "ThएuSt", : LPRINT "SHEAR"
    1840 FOF J=1 T0 21
    1850 LPRINT WI(XI,J), : LPRINT TH(XI:J), LPRINT SU(XI:J)
    106% NEXT S
    1E% SYSTEM "T"
    18SD IF XI < % THEN 192O
    1890 LPRINT
    1900 60T0 1990
    $10 REM CHER9E STH to HTS & SEV EO VSS
    1720 HTS=ST(X1)
    1750 vSS=5S(XI)
    1%40 LPRINT
    1950 REM LFRINT "(SPE,jel .jumP FG% Ht)"
    1CGR EEN LPRINT HTSE REM LFRINT VSS
    1970 TJP(3)=4TS
    1980 TJP(4)=V5S
    15%0 G05US 2%40
    2000 IF NP=11 THEN GOSUE.4100
    ZQ10 FOR J=1 TO J
    2D20 PGA(JJ:XI,J)=PA(J)
    2030 5AN(JJ,XI,J)=AGN(J)
    2040 NEXT J
2050 FOR K゙=1 TO 3
2060 ET(J:JyXI:KY)=T{隹)
2070 NEXT N゙
COBO NEXT XI
20%D NEXT JU
2100 PP=605
211D D=1.3*PI
Z2D END
```



```
2140 REM Subrnutire GJR(A)
```



```
216D N=4
2170N1=N+1
2180 DET=1.0
2190 FOF J=1 T0 N
2200 DIV=A(J.J)
210 S=1/DIV
2220 DET=DET*DIV
2230 FOR K゙=J TO N1
2240 A(J%ド)=A(J,ド)*S
2250 NEXT K
2260 FOR L=1 TO N
2270 IF (L-J)=0 THEN 2320
2280 AlJ=ーA(土,J)
2.%D FOR N゙=J TO N1
ころ00 A(L,K)=A(L,R゙)+AIJ*A(J,N゙)
23士0 NEXT K
23OD NEXT
2300 NEXT
2340 RETURN
```


2360 REM Subrouturie FHGG
2370 FE游 \#**%***************

```

```

\PiI=Y(I) \& DY(I)
2390 REM are the dependent variable \& its derivativis.All the G(I)
muzt ber initially
2400 REM Eet to zerrg ln the manत pragram.

```

```

2420 REM H = interval size
24305(1)=0.2528532188134524
2440 E (2)=1.707106781186547%
2450 H2=0.5*H
2460 GOSUR 2650
24706 FOR I=1 TO NEG!
2480 S=H2*DY(I)-G<br>)
2490 Y(I)=Y(I)+5
2500 g(I)=6{I}+す*E-H2*DY(I)
2510 NEXT !
2520 X = X+H2
2530 FOR JH=1 TO 2
2540 605UP 2650
2550 FOR I=1 TO NEO
256D B=E(JK゙)*(H*DY(I)-G(I))
2570 Y(I)=Y(I)+E

```

```

2SOK゙ NEXT \Psi,JK゙
2600 x = x+-2
2610 GOSUE 2\&70
IG20 FOR I=1 TO NEO
2630 S=0. 16066666666606606660*(H*DY(I)-2*6(I))
2640 Y (I)=Y(I)+B
250 6(I) =G(I)+3*S-H2*DY(I)
2660 NEXT I
2670 RETURN

```
```

2680 REM
********************
2670 REM
Subroutane aeriv
2700 REMi *********************
2710 L=NEG-1
2720 FOR I=1 TO L
2730 DY(I)=Y(I+1)
2740 NEXT I
2750 G=1/50R(1+(4*AN(II)*(1-2*X))\uparrow2)
2750 GP=32*AN(II)+2*(士-2*X)/((1+(4*AN(II)*(1-2*X))^2)^(3/2))
2770 S3=(-ALM(JJ)*2)*Y(3)-64*(AN(II)42)*(4B*AN(II)*2*(1-2*X)*2/(1
+16*AN(II)*2*(1-2*X)*2)-2)/({1+16*AN(II)*2*(1-2*X)*2)*(3/2))*Y(3)
-2*GP*Y(4)
2780 IF KKK=1 THEN 2810
2790 DY(4)=SE/G
2000 G0T0 2820
2810 DY(4)={-8*AN(II)+SE)/E
2E20 RETUNN

```
```

2BJO REM
284Q REM GuOroutinE INTGL

```

```

28S0 AR=T1(1)-T2(N)
2870 M=N-1
2800 F0F = = T0 M STEP 2
2870 J=I+1
2gDD AR=AR+4*TI(I)+2*T1(J)
2GIE NEXT I
2020 AR=AR*H/3
2930 RETURN

```

```

    2950 REM Subroutire TINTGO
    ```

```

    2%7d REM CHANGE ANV TO AVN ;FAV TO PVA; A TO AA; ANG TO AGN ; S
    YMN TO SNYN ; Y TO YY
    2700 NN=N+1
    2980 FOR I=1 TQ 3
    30DO YY(I)=O
    3010 Z(I)=0
    3020 NEXT I
    3030 FOR J=2 T0 20 STEP 2
    5040 J1=J-1
    3050 J2=J+1
    3060 IF J=2 THEN 3100
    ```

```

    3080 IF B % THEN 3100
    3070 60T0 52.0
3100 B=TWH(1,J)*TWH(1,J2)
311D IF B % THEN 3150
3120 60T0 3290
3130 FOR I=1 TO 3
3140 IF N=1 THEN 3190
3150 TF I < = 2 THEN 3170
3160 IF J=WN THEN 3180
3170 60T0 317%
3180 TWH(I:, W)=TJP(I)
3190 YY(I)=YY(I)+(TWH(I,JI)+4%TWH(I,J)+TWH(I,J2))/6D
3200 NEXT I.J
3220 M=51
32%D X=ABS(TWH(I,N))/(ABS(TWH(1,M))+AES(TWH(1,M+1)))
323D FOR I=1 TO J
324D YY(I)=YY(I)+TWH(I,M)+X/40
3250Z(I)=I(I)+(TWH(I,M)+4*TWH(I,M+1)+TWH(I,M+2))/60-TWH(I;M)*X/4
O
3200 NEXT I
3270 M=N
3280 6070 3360
3290 M=J
33DD X=ABS(TWH(1,M):/(ABS(TWH(1,M))+ABS(TWH(1,M+1)))
3\0 FOF I=1 TO 3
3320 Z(I)=Z(I)+(I-X)*TWH(I,M+I)/4O
33J0 YY(I)=YY(I)+(TWH(I,M-1)+4*TWH(I,M)+TWH(I,M+1))/60-Z(I)
3340 NEXT I
3350 M=f+N+2
360 FOR J=M TO 20 STEP 2
3370 J1=J-1
33ED J2=5+1
3370 IF J = 20 THEN 3460
3400Z E=TWH(1,J1)*TWH(1;J)
3410 IF D % OHEN 3430
3420 GOT0 3500
3430 E=TWH(1,J)*TWH(1,J2)
3440 IF B > D THEN 3460
3450 GOTO 3640
3400 FOR I=1 TO 3
3470 IF N=1 THEN 3520
3480 IF I < = 2 THEN 3520
3490 IF J = NAN THEN 3510
3500 60T0 3520
3510 TWH(I,J1)=TJP(I)

```
```

3520 Z(I)=Z(I)+(TWH(I,JI)+4*TWH(I,J)+TWH(I;J2))/60
35JD NEXT I
3540 NEXT J
3550 60T0 3770
7562 M=\1
3570 X=ABS(TWH(1,M))/(AES(TWH(1,M))+AES(TWH(1,M+1)))
3580 FOR I=1 TO 3
3500 I(I)=Z(I)+TWH(I四)*X/40
3600 YY(I)=YY(I)+(TWH(I,N)+4*TWH(I,M+I)+TWH(I,M+z))/G(D-TWH(I,M)*X
140
3610 NEXT 1
3620}M=M+
3630 GOT0 3710
3640 M=J
365(0 %=AES(TWH{1;M))/(ADS(TWH(1;M))+ABS(TWH(:,M+1)))
3660 F0% I=1 TO 3
3670 YY(1)=YY(I)+(1-X)*TWH1(I*M+1)/40
368(Z Z(I)=Z(I)+(TWH(InM-1)+4NTWH(I;M)+TWH(I;M+I))/60-(I-X)*TWH(I:
m+1)/40
3E4O NEXT I
3700 M=M+2
3710 FOR J=N TO 20 STEO
3720 J1=J-1
3730 J2=5+1
3740 FOR I=1 TO 3
J750 YY(I)=YY(I)+(TWH(I,J1)+4*TWH(I;J)+TWH(I;J%))/60
3760 NEXT ISJ
3770 IF YY(1) > 0 THEN 3540
37BD FOR I=1 TO 3
379D AGN(I)=YY(I)
3800 T(I)=YY(I)+Z(I)
3810 PA(I)=Z(I)
3E20 NEXT I
3630 GOT0 30%C
3640 FOR I=1 TO 3
3850 PA(I)=YY(I)
3860 AGN(I)=Z(I)
3870}T(I)=YY(I)+Z(I
3080 NEXT I
3890 60T0 4040
370D LPRINT "INTEGRATION OF INFLUENCE LINES"
3910 LPRINT " "* : LPRINT "MOMENT"* : LPRINT "HOR. REACTION",
: LPRINT "THRUST"
372D LPRINT "T.A.",
3930 FOR I=1 T0 3
3940 LPFINT T(J),
3950 NEXT I : LPRINT
3550 LPRINT "+ ",
3970 FOR I=1 TO 3
396D LFRINT PA(I):
3990 NEXT I : LPRINT
4OOD LPRINT "- M":
4010 FOR I=1 TO 3
4020 LPRINT AGN(I):
4030 NEXT I : LPRINT
4040 LPRINT : LPRINT : LPRINT : LPRINT : LPRINT
4050 FETURN
4060 END

```
```

4070 REM
40ED REM Stress Suturoutane

```


```

1b. LO(J) => Movang }rad ot IO lb.
4110LO(2)=1:LO(%)=5:LO(3)=10
4120) MS=0: HS=0
4130 FOF TT=1 TO =
4140 5TOF
4250 EOR DE=2 T0 20 STEP 2
4I60 ME=MS+W1(XI,BE)
4170 -SS=HS+TH(XI,EE)
4180 NEXT EE
4150 LPRINT "LAMEDA= ";AJJ
4200 LFPINT "LOEGJMS TYPG "% : LFFINT "H", : LPRINT "M", : LF
RINT "M-m
4%10 LPRINT
420 FON CO\#% TO 20 ETEP 2
423B LPRINT

```

```

4250 H=5*H5+LO(TT)*TH(XI,CC)
4260 M=(5*mS-LO(TT)*W%(XI,CC))*SD
4こ70 LPRL|T HsM%
42BO HN= (H-5*HS)/LO(TT)
429Q1 MN= (M/GD - 5*MS)/LO(TT)
4300 LPRINT MN
4310 NEXT CO
4320 LPRINT : LPRINT : LPRINT
4330 SVSTEN "T"
4340 NEXT TT
4350 RETUFN

```

Influence ines for section \(\#\)
\begin{tabular}{|c|c|c|}
\hline Monient 0 & Thrust
\[
.470497
\] & SHEAR
\[
622402
\] \\
\hline -. 0391584 & . 524804 & . 84.4854 \\
\hline -. 05779 & . 662932 & . 746975 \\
\hline -. 065213 & . 851708 & . 60841 \\
\hline -.0621867 & 1.053 & . 4.46069 \\
\hline -.0507659 & 1.27332 & . 274332 \\
\hline -. 0353365 & 1. 4.4352 & . 105186 \\
\hline -. 0176485 & 1.61846 & -. 051593 \\
\hline 1. 51565-04 & 1.72678 & -.1882\%2 \\
\hline . 0164971 & 1.78069 & -. 297144 \\
\hline . 0301808 & 1.77583 & -. 380238 \\
\hline . 04603394 & 1.7112 & -. 429461 \\
\hline . 0464462 & 1.58512 & -. 446473 \\
\hline . 04830065 & 1.41525 & -. 432711 \\
\hline .0460628 & 1.17673 & -. 391426 \\
\hline .0401782 & . 752287 & -. 327756 \\
\hline . 0015612 & . 692483 & -.248819 \\
\hline . 0213743 & . 43994 & -. 133846 \\
\hline .011267E & . 21767 & -.0843285 \\
\hline 3.30014E-03 & . 0614073 & -.0241931 \\
\hline - & 0 & 0 \\
\hline
\end{tabular}

Influence lanes for sertion \# 3
Moment
0
\(6.94316 E-03\)
.0261098
\(5.21727 E-03\)
\(-7.75265 E-05\)
-.014574
-.0167941
-.0157511
-8.275402
\(-3.60137 E-03\)
\(7.7842 E-04\)
\(4.4455 E-63\)
\(7.01286 E-03\)
\(8.32457 E-03\)
\(8.35735 E-03\)
\(7.24549 E-03\)
\(5.28754 E-03\)
\(2.95585 E-03\)
\(7.05022 E-04\)

Thrust
0
. 0.07551
. 203453
.795851
1.0204
1.2448
1.44892
1.61635
1.7366
1.7999
1.8020 e
1.74195
4.6218
1.44739
1. 22811
.977082
.71142
.452471
- 22645
.0632708
. 0

SHEAR
0
\(-.0326653\)
-. 118242 .679846 . 536402 .383513 . 231465 .0886846
-. 038126
-. 143896
-. 225106
\(-.279738\)
\(-.307254\)
\(-.308592\)
-. 286203
\(-.244102\)
\(-.187949\)
\(-.125153\)
\(-.0649987\)
-. 0187873
0


0
3. \(744585-23\)
. 0144999
. 03 56073
.0544456
-0324234
.014782
1. \(63467 E-03\)
\(-8.13843 E-23\)
- . 0147 万ワ7
-.0187t53
-. 0204955
- . 0200646
- 0187823
-. 0161688
-.0128781
\(-9.395535-23\)
-5.83787E-05
-2.86553E-03
\(-7.84873 E-94\) (D)

Thruzt
0
- D601519
. 21357
- 42415
. 661116
1. 20352
1. 42076
1.6015
1.73256
1. 80547
1. E152
1.76047
1. 64341
1.48987
\(2.2494 E\)
. 795614
.725968
.462184
.23128
.0647493
( 1

SHEAR
V
-. 0271644
\(-.2987191\)
- 20122
\(-.323061\)
.4981 Ј1
.365817
. 23935
.124281
- 0248881
-. 0557634
\(-.115774\)
\(-.154374\)
\(-.17202\)
\(-.170216\)
\(-.151751\)
-. 120663
-. 0823338
-.0435E7
- 012754

Intlagnee lines tor section 7
```

Moment
0
1.23903E-03
5.3801105-05
.0129634
.0244008
.0400261
.06002%4
.034509
.013507
-3.04731E-0.3
-.0153107
-.0235124
-.0279751
-.029109
-.0274183
-.0235081
-.0180519
-.0120014
-6.17676E-03

```

```

D

```
Throst
Thrist
    - 0225528
    \(.22234-.0770135\)
    \(.44204-.158074\)
    \(.689938-.255791\)
    \(.939501-.362785\)
    \(1.16501-.472844\)
    1.5698
    1.71165
    1.79407
    1.81179
    1.76326
    1.65062
    1.47971
    \(1.2602 \quad-.0452577\)
    \(1.00576-.0521011\)
    \(.734264 \quad-.0479619\)
    \(.468077-.0360157\)
    \(.234414-.0204025\)
    .0657
    \(\square\)
SHEAR
    D
-. 0210549
    .397251
    .27577
    .204111
    . 224827
    .0596744
    \(9.61698=-23\)
    -. 0251585
    \(2.2602-.0452577\)
\(-6.29892=-03\)
    0

\begin{tabular}{|c|c|c|}
\hline Moment 0 & Thrust 0 & \begin{tabular}{l}
SHEAR \\
©
\end{tabular} \\
\hline －5．6417日E－94 & ． 064379 & －．0144542 \\
\hline －1．24933E－23 & －25128 & －． 2555406 \\
\hline －7．20626E－04 & ． 456052 & －． 111359 \\
\hline 2． \(13068 E-25\) & ． 712745 & －．18－861 \\
\hline \(8.234395-83\) & ． 772058 & －26－3396 \\
\hline －0182905 & 1．21174 & －． 347061 \\
\hline ． \(032871 \%\) & 1．4141 & －． 436455 \\
\hline －05こ3461 & 1.56571 & －．52－678 \\
\hline － 02.67494 & 1．762ご & ． 387066 \\
\hline \(6.76175 E-25\) & 1．78707 & ． 312048 \\
\hline  & 1.74757 & ． 24224 \\
\hline －． 0183549 & 1． 64272 & ． 180745 \\
\hline －． 2238371 & 1． 47434 & ． 128472 \\
\hline －0254279 & 1．2581 & ． 0856773 \\
\hline －－205327 & 1．00574 & ． 0524838 \\
\hline －．01．71136 & ． 735278 & ． 2284445 \\
\hline －．013203 & ． 469288 & ．012721 \\
\hline －7．6378E－ 73 & －23526t & 4．D1773E－DJ \\
\hline －－n 07\％95－63 & ． 0659989 & 5． \(4854 E-D 4\) \\
\hline 0 & （1） & D \\
\hline
\end{tabular}
```

influence lines for sectign * 11

```

Momert Z
－1．6682
-5.3 B 67E－03
\(-7.44277 E-23\)
－． 0123708
\(-.0120515\)
－．0101857
－3．27688E－63
8． \(37877 E-23\)
－025205
\(.047452 t\)
－0．55205
8．3788E－02
－3． 2767 E－23
－．0101857
－0109518
\(-.0223708\)
－9．44576E－RU
－5． \(58866=-03\)
\(-1.66821 E-03\)
0

Thロリ」を
0
－ 0655687
． 23513
.465294
.720117
.994500
1.24152

1． 45241
1.6423
1.71203
1.7455
1.71 .202

1．6． 2
1．4 4241
1． 24172
.99408
． 728127
.45524
． 233515
.0255657
（2）

SHEAR
0
\(-7.54374=-23\)
－． 68424
－．0624119
－． 106252
\(-.158825\)
－．218602
\(-.284045\)
\(-.353705\)
\(-.46158\)
\(-.5\)
.42658
.353705
.284945
－ 2186 20
．150536
－106こ5こ
－062412
0287424
\(7.54389=-03\)
（a）
\begin{tabular}{|c|}
\hline 988991. \\
\hline 「6ごカワが－ \\
\hline \(61560^{\circ} 1-\) \\
\hline 二89125－ \\
\hline \(2506{ }^{\circ}\) \\
\hline \(25025^{\circ}\) \\
\hline LL9LZE．－ \\
\hline ごら\％＊I－ \\
\hline  \\
\hline \(598991^{\circ}\) \\
\hline 4， \(1-1.1\) \\
\hline
\end{tabular}

46.9531
47.3527
47.0918
46.3396
48.4991
48.5991
48.3396
47.8818
47.3527
46.9531
        14
46.9531
47.3527
47.0918
48.3396


\[
\begin{array}{ll}
H & 11 \\
94.1028 & -3.06214 \\
76.101 & -5.39454 \\
98.7464 & -6.44725 \\
101.035 & -3.54469 \\
102.333 & 4.9999 \\
102.333 & -3.54471 \\
101.035 & -6.44724 \\
98.7464 & -5.39454 \\
96.101 & -3.06214 \\
94.1028 &
\end{array}
\]
\[
\begin{array}{llllllllll}
-1 & M & \cdots & n & \infty & N & \infty & 0 & -4 \\
\cdots & + & + & + & + & + & + & + & +\cdots & + \\
\pi & \pi & \pi & \pi & \pi & \pi & \pi & \pi & \pi
\end{array}
\]
\[
\begin{aligned}
& \text { LAMEDA= } Q \\
& \text { LoAding } T \text { rae } \\
& W=5 \# \text { Arad } P=5 \\
& W=5 \# \text { Ard } P=5 \\
& W=5 \# \text { And } P=5 \\
& W=5 \# \text { And } P=5 \\
& W=5 \# \text { And } P=5 \\
& W=5 \# \text { And } P=5 \\
& W=5 \# \text { And } P=5 \\
& W=54 \text { And } P=5 \\
& W=5 \# \text { And } P=5 \\
& W=5 \# \text { And } P=5
\end{aligned}
\]
\[
I
\]
\[
\begin{aligned}
& 141.318 \\
& 145.314 \\
& 150.805 \\
& 155.183 \\
& 157.779 \\
& 157.779 \\
& 155.193 \\
& 150.605 \\
& 145.314 \\
& 141.318
\end{aligned}
\]
\[
\begin{aligned}
& 1-11 \\
& -.166825 \\
& -.744294 \\
& -1.2752 \\
& -.327677 \\
& 2.52652 \\
& -.32652 \\
& -.327682 \\
& -1.27519 \\
& -.944293 \\
& -.146826
\end{aligned}
\]

```

Intruence iines for sectagn \# I

```
\begin{tabular}{|c|c|c|}
\hline Mometot
\[
\theta
\] & Thrust
\[
.470497
\] & \begin{tabular}{l}
SHEAR \\
. 88.402
\end{tabular} \\
\hline -.0352444 & . 524524 & . 845171 \\
\hline -.06000514 & . 662217 & . 747767 \\
\hline -. 0666355 & . 85078 & - 6045856 \\
\hline -. 0627004 & 1.0622 & . 447422 \\
\hline -.0515005 & 1.27297 & . 275553 \\
\hline -. 0358054 & 1. 4638 & . 106066 \\
\hline -. 8179874 & 1.61742 & -.0512013 \\
\hline -1.7166:E-05 & 1.72 3 & -. 188434 \\
\hline . 0165121 & 1.78253 & -. 297772 \\
\hline . 030363 & 1.77769 & -. 3 B1227 \\
\hline . 0406605 & 1.71275 & -. 430639 \\
\hline . 0468657 & 1.59008 & -. 447654 \\
\hline - 0487635 & 1.41546 & -. 433724 \\
\hline . 0465017 & 1.19815 & -. 372151 \\
\hline . 0405753 & . 951075 & -. 328143 \\
\hline . 0318443 & . 690827 & - 248905 \\
\hline - 0 21553: & . 438447 & -. 163731 \\
\hline . 0113519 & .21863 & -. 0841658 \\
\hline 3.321335-63 & . 0610223 & -.024112 \\
\hline 0 & D & \(\square\) \\
\hline
\end{tabular}

Inftuence ianes for section \＃ 3
\begin{tabular}{|c|c|c|}
\hline Montent & Theust & SHEAR \\
\hline 0 & 0 & C \\
\hline 6．94835E－63 & ． 057306 & －．0320222 \\
\hline ，02610E & .204062 & －． 115752 \\
\hline \(5.162215-03\) & ． 797482 & ． 684138 \\
\hline －7．67624E－03 & 1．02308 & ． 541344 \\
\hline －． 0147505 & 1.24859 & ． 388177 \\
\hline －． 016993 & 1.45379 & ． 235177 \\
\hline －． 0159383 & 1.62261 & ． 0710195 \\
\hline －． 0.2730 & 1．74295 & －． 0373442 \\
\hline －8． \(35906 E-003\) & 1．80644 & －． 144631 \\
\hline －3．613065－03 & 1．80836 & －． 227148 \\
\hline B．57249E－04 & 1．74752 & －． 282754 \\
\hline 4.5633 E －23 & 1． 62629 & －． 310836 \\
\hline 7．17047Eー馬 & \(\pm .45056\) & －． 312317 \\
\hline E．49849E－03 & 1.2297 & －．23067\％ \\
\hline 8．52357E－03 & ． 97764 & －． \(24701 t\) \\
\hline 7． 3833 ごこ－03 & ． 711074 & －． 190107 \\
\hline \(5.3835 E-23\) & ． 451695 & －． 126504 \\
\hline 3．00666E－03 & ． 225438 & －． 0656404 \\
\hline 7．22608E－04 & ． 0629717 & －．018951 \\
\hline － & \(\square\) & 0 \\
\hline
\end{tabular}
```

LANBDA= ..28315
Intiuence lines for sestion 45

```
```

Moment

```
Moment
    ©
    ©
    3.78323E-03
    3.78323E-03
    . 0146447
    . 0146447
    .032874
    .032874
    .0547868
    .0547868
    . 0327487
    . 0327487
    . 0152356
    . 0152356
    \(1.76575 \mathrm{E}-2.3\)
    \(1.76575 \mathrm{E}-2.3\)
-8.123935-23
-8.123935-23
\(-.2148797\)
\(-.2148797\)
-. 018932 e
-. 018932 e
-. 0207205
-. 0207205
- 02025979
- 02025979
-.0180212
-.0180212
-. 0163691
-. 0163691
-.0150361
-.0150361
-7.41573E-23
-7.41573E-23
\(-5.96265 \mathrm{E}-03\)
\(-5.96265 \mathrm{E}-03\)
\(-2.89435=-03\)
\(-2.89435=-03\)
-7.9175-64
-7.9175-64
    0
```

Thrust
[
.2578679
.213341
.424480
. 662636
1.20666
1.4256
1.60738
1.7373
1.9125 ?
1.82215

1. 76677
2. 64862
1.47372
1.25183
.876063
.72582
.461587
.230603
. 0644704
$\square$

SHEAR
0
-. 0270771
-. 0982947
-. 200019
-. 320575
.502061
. 367855
. 242691
.12644
-0256547
$-.0563658$
-. 117545
$-.157009$
$-.175086$
$-.173327$
$-.154546$
$-.122869$
-. 083807
-. 0443389
-. 0130023
0

Intiuence iines for sertinn 7


Theust
0
. 062233
. 221571
.441107
.687506
.940059
1.17101

1. 57329
1.71627
1.77534
2. 81715
1.76826
$1=65470$
3. 48.271
1.26173
1.00631
.733 2
.467322
. 233704
. 0653778
0

SHEAR
Q
$-.021255$
$-.0776821$
$-.15525$
$-.257,38$
-. 3゙も3718
$-.472427$
.399032
.297998
.206158

- 126324
- 0604591

5. $677635-0.0$
-. 0256532

- . 0441251
-. 0531106
-. 0488776
$-0367153$
-. 0207936
$-6.410315-27$
2

Influerae lines for sertion 4 g

```
Momerit
    0
\(-5.55884=-04\)
\(-1.21348 E-03\)
\(-6.34805 E-64\)
    2. 29385E-03
    8. \(477075-93\)
    - 018623
    .03325
    . 0527135
    - 0272074
    \(6.855250-03\)
-8. \(36135=-63\)
\(-.0180201\)
- . 242264
-. 0257675
-. 0238804
-. 0173995
-. 01 उJ.
\(-7.139 \Xi 2 E-23\)
- 2. 29272Eーだ
    (a)
```

    0
    . 064023
    -2 2136
    .454677
    .711456
    .77128
    1.21181
    1.41518
    1. 56782
    1.7661:
    1.772.
    1.75062
    1.64321
    1.47576
    1. 25574
    1.00548
    .734383
    .468183
    .23479
    - 065655
    -
    SHEAR
(D)

```
TMTHE%
```

    \(-.0147285\)
    \(-.0545214\)
    -. 113308
    \(-.185673\)
    - . 66821
    \(-.35253\)
    \(-.439133\)
    \(-.523459\)
    .39028
    .314383
    .244988
    -183485
    .130837
    . 0.075974
    - 2538547
    - 029317
    .0131918
    4. \(21096 E-03\)
    5. 713 C5E-24
    0
    Infiuence innes for section ${ }^{\#} 11$

| momert $\square$ | Thr・リミさ 0 | SHEAR |
| :---: | :---: | :---: |
| －1．68451E－03 | ． 065190 c | －7．74203E－03 |
| －5．44258E－03 | ． 23252 | －．0296811 |
| －9．53616E－03 | ． 463722 | －． 0639283 |
| －．012497\％ | ． 726754 | －．10864E |
| －．0130604 | ． 983021 | －．162057 |
| －． 0102493 | 1.24176 | －． 22242 |
| －3．26172E－03 | 1.45307 | －． 288041 |
| 8．49195E－03 | 1.61571 | －． 357263 |
| ． 0254082 | 1.71395 | －． 428453 |
| ． 0476775 | 1.743 | －． 5 |
| ．0554003 | 1.71395 | ． 428453 |
| E．4519E－83 | 1.61371 | .357262 |
| －3．26182E－03 | 1．45307 | ． 288041 |
| －．0102493 | 1．24176 | －2242 |
| －．0130604 | ． 973621 | ． 162057 |
| －．012487 | ． 726784 | ． 108648 |
| －9．536185－03 | ． 44382 | ． 0639282 |
| －5．44258E－03 | ． 2325 | ．0296811 |
| －1．6E4535－03 | ． 0451908 | 7．74．196E－03 |
| 6 | 0 | ． |


| $\mathrm{H}-\mathrm{C}$ |
| :---: |
| - 48016 |
| -. 95.5643 |
| -1. 3 2ix ${ }^{2} 5$ |
| -.326172 |
| 2.54086 |
| 2.54 ¢ 6 |
| -. $3: 617$ |
| $-1.30265$ |
| -. 93.3635 |
| $-.168457$ |


|  | 8 | م | $n$ | - | 0 | N | 17 | N | M | +1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | 0 | r | $\pm$ | 5 | $\cdots$ | F | $\cdots$ | - | $\infty$ |
|  | -4 | $\cdots$ | $\checkmark$ | - | $\bigcirc$ | N | N | $\stackrel{1}{4}$ | \% |  |
|  | 5 | n | 0 | r. | $1{ }^{1}$ | $\cdots$ | r | $\bigcirc$ | 9 | (\%) |
|  | $\cdots$ | C | $\pm$ | + | $\bar{\square}$ | $\pm$ | $\pm$ | 8 | 10 | ल |
|  | - | - | sis | - | 8 | $\cdots$ | $\cdots$ | $\because$ | $\sim$ | - |
| $\Sigma$ | T | 1 | ; | 1 |  | 。 |  | 1 | 1 | 1 |

I
47.9521
47.3588
47.8803
48.3377
48.60055
48.6005
48.3397
47.8803
$47 .-1508$
46.9521

$$
\begin{aligned}
& \text { LAMBDA }=6283+7 \\
& \text { Lodding TYPe }
\end{aligned}
$$

I

$$
\begin{aligned}
& 54.647 \\
& 94.1473 \\
& 98.7499 \\
& 101.638 \\
& 102.342 \\
& 142.342 \\
& 101.039 \\
& 98.7409 \\
& 96.093 \\
& 94.0998
\end{aligned}
$$

-3.42685
-5.4282
94.0795
94.075
96.7499
101.038
102.342
142.342
101.039
99.7409
96.093
94.0978

$$
-6 n 4796
$$

$$
-3.5 / 4
$$

$$
5.06108
$$

$$
\begin{aligned}
& 5.05147 \\
& -3.54
\end{aligned}
$$

$$
-5.47964
$$

$$
-5.42230
$$

$$
-3.06686
$$

| L5サ8\% ${ }^{\text {\% }}$ |
| :---: |
| ¢29596 - |
| covere $7-$ |
| 17925*- |
|  |
| 9E4ty |
| c.192, - |
| E696\% ${ }^{\text {a }}$ - |
|  |
| 95y.5\% ${ }^{\text {a }}$ |
| 1.1-4. |

LAMEDA = -68519
Loadjng Trac

$$
\begin{aligned}
& 141.313 \\
& 145.299 \\
& 150.595 \\
& 155.180 \\
& 157.796 \\
& 157.796 \\
& 155.186 \\
& 150.595 \\
& 145.279 \\
& 141.313
\end{aligned}
$$

$$
\begin{aligned}
& W=5 \text { 戠 } A \Gamma D P=10 \\
& W=5 \# \text { Ary } P=10
\end{aligned}
$$

$$
\begin{aligned}
& W=54 \text { And } P=10 \\
& W=54 \text { And } P=10
\end{aligned}
$$

$$
\begin{aligned}
& 14 \\
& -4.85977 \\
& -9.56407 \\
& -11.6785
\end{aligned}
$$




| Manert 2 | Thrust .47647 | SHEAR <br> . 88.422 |
| :---: | :---: | :---: |
| -. 0395172 | 520004 | . 846017 |
| - B6086- |  | . 750153 |
| -. 067952 | . 547935 | . 613237 |
| -. 0243354 | 2.05575 | . 45157 |
| -.0531629 | 1.27189 | . 279380 |
| -. 0372877 | 1.46459 | 108848 |
| -.0190662 | 1. 5224 | -. 0479325 |
| -5.627575-74 | 1.73301 | -.188831 |
| . 2165402 | 1.75924 | -.301692 |
| . 23029477 | 1.7E343 | -.384288 |
| . 0416908 | 1.71752 | -. 434308 |
| . 0481851 | 1.59303 | -. 451342 |
| . 0502045 | 1.41606 | -. 436902 |
| .047892 | 1. 15634 | -. 374437 |
| . 04176 E | . 947291 | - 329303 |
| . 0327403 | . 686107 | -.249151 |
| . 0221182 | . 433531 | -.163402 |
| - $011617 \%$ | . 215429 | -.0836814 |
| す. 3888 E - | .0598410 | - . 0238684 |
| E | 0 | $\square$ |

Influence ianes for section \＃ 3

Momert
$\square$
$6.76214 E-23$
． 026076
4．98413E－05
－8．2687ミーか3
－． 0153084
$-.0176208$
$-.0165296$
－． 0131967
－E．©22E1E－03
－5．64882E－03
1．04489シーロ3
$4.93765 E-83$
7．67089E－0．
9．0504E－0．3
7．05075E－83
7． 02025 －－03
5．68733E－25
3． 26725 E － 3
9．686725－04 0

Thrust
D
.0575783
.206137
.802785
1.03182

1． 26074
1.46763
1.64135

1．76366
1.8277

1．8289\％
1.7657
1.64108

1．46124
－． 23605
.979809
.710311
． 445485
.223351
.0626775
Q

EHEAR
©
$-.0300172$
$-.100027$
.697473
.556789
.402844
.24692
.0784902
$-.0347407$
－． 146812
$-.233476$
$-29217$
$-.3221$
$-.324062$
$-.300658$
$-.256232$
－． 196937
－． 130781
$-.0576733$
－． 0184704
$\square$


| Monerit | Theust | SHEAR |
| :---: | :---: | :---: |
| 0 | D |  |
| 2.71315E-123 | .0584543 | -.0207763 |
| . 015095 | . 212758 | -.0963766 |
| . 0582978 | . 425567 | -. 196114 |
| . 0558512 | . 667882 | -. 31286 |
| . 033769 | 1.21712 | . 514433 |
| . 0160048 | 1.4407 | . 38257 |
| 2.18631E-03 | 1.6267 | . 253248 |
| -8.06804E-03 | 1.76142 | . 133295 |
| -.0151486 | 1.83595 | . 0281245 |
| -.0194521 | 1.8451 | -.0582332 |
| -.0213805 | 1.78356 | -. 12313 |
| -. 0213407 | i. 6657 | -. 165287 |
| -.0197434 | 1.45655 | -. 184812 |
| -. 0170052 | \%. 25959 | -. 183214 |
| -.0135375 | 1.0005t | -. 103435 |
| -9.766E-03 | . 726097 | -. 129886 |
| -6.10915E-06 | . 45959 | -.0884714 |
| -2.78636E-03 | . 228e0\% | -. 0467285 |
| -8.1357E-04 | . 0836475 | -.0126678 |
| 0 | $\square$ |  |

InfluenGe | anes fro sevtlan 掓?


Thrust
0

- D6:2805
.217308
.438685
. 688512
.942257
1.1778
1.58475
1.73136
2.E1553

1. 83477
1.76461
1.66844
1.4527
2. 26734
3. 00842
.733155
. 4651104
. 23:614

- D64457\%
(7)


0
－5ープー・•
$-.1153 \pm 4$
$-.194537$
－． 277591
$-.32327$
$-.447542$
－5． 25973
.393786
.322597
．25すヒ53
.17225
． 130465
－ 07369
－ 0502664
．0．321454
． 0147202
$4.8478=-83$
만

Influence ines for sertion \# 11

```
Mowert
    0
-1.73529E-83
-5.610245-05
-5.82654E-0J
-.0128479
-.0.0133765
-.0104454
-3.21328E-03
    8.84377E-03
    .0202379
    .0484544
    .02.50375
    8.84385E-7J
-3.21355-85
-.0164455
-.0135465
-.022848
-9.824475-08
-5.&1023*-03
-1.77527E-03
    Q
```

TMr.ust
0
. 064034
. 229467
.459676
. 722666
.950865
1. 24124
1.4551
1. 61805
1.71987
1. 75451
1.71989
i. 61805
1.4551
1. 24124
.990865
.722606
. 457575
-205467
. 0640343
0
BHEAP
0
$-8.37784 E-03$
-.032051
-. 068784
$-1.16324$
$-.172317$
-. 234534
- 300663
$-.368441$
$-.435624$
$-.5$
.435624
.36544
.300663
- 234354
. 172315
.216304
. 0657542
- 03)
8. $37805=-8.3$
0
E
$-1.3241 t$
-1.34465
ヶ859ロ＂た－
 41828.动 -1.47284
-2.88384
-1.86964 $24582^{\circ} 1-$
$177698^{\circ} 1-$
$I$

| ¢Sise |
| :---: |
|  |
|  |

45－817
27 $77^{\circ}-8$
$11,07 \cdot 8 t$

47.3449
ジずす。の方
$\begin{array}{ll}\text { LAMEHA＝} & \text { I．} 25664 \\ \text { Lgetinns Trpe }\end{array}$
（1） $\mathrm{AFOH}^{-1} \mathrm{P}$
$-1 \quad-1$

W上5办 Arid $P=1$



|  | tion | $\pm$ | N | \% | 07 | 0 | $\pm$ | N | $\stackrel{\square}{4}$ | $1{ }^{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | S | T10 | \% | - | t | $\infty$ | 5 | O | $\infty$ | $\cdots$ |
|  | E | 0 | 1 | S | $\cdots$ | $\cdots$ | * | 9, | 0 | a |
|  | - | S | N | - | \% | \% | - | 0 | $3$ | 2 |
|  | - | 0 | $\infty$ | S | $\varepsilon$ | S | Q | $\infty$ | 0 | * |
|  | g- | 0 | 0 | - | $\cdots$ | $\square$ | $\cdots$ | 5- | 0 | 0 |

$$
\begin{aligned}
& \text { LAMDIA }=1.20564 \\
& \text { L'Anirg TrPe }
\end{aligned}
$$



|  | $\cdots$ | $\pm$ | $\stackrel{+}{4}$ | $m$ | 0 | 01 | 5 |  | 15 | $\cdots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 547 | － | 7 | （－ | 0 | T1 | E | 10 | ＋ | $\cdots$ |
|  | － | 5 | $\cdots$ | $\cdots$ | ［10 | 0 | （1） | N | 4 | －- |
|  | $\square$ | 0 | ［1 | St | ？－ | N． | $\infty$ | 2 | ＋1 | $\infty$ |
|  | ［1） | $\cdots$ | － | 5＊ | － | ． | $r$ | ． | 1 | 10 |
|  | ＊ | － | $\rightarrow$ | － | － | － | － | $\cdots$ | ＊ | ， |
|  | $\pm$ | \％ | －1 | IT | －1 | T－ | i？ | $\cdots$ | $\square$ | 4 |
| $\underline{5}$ | I | 1 | ！ | ， |  |  |  | 1 | 1 | ， |

$141=286$
145.292
154.562
155.203
157.951
157.851
155.243
154.562
145.252
141.296

| $1.1=5$ | ATt | $\mathrm{P}=$ | 17 | 2t | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\dagger \mathrm{f}=5$ 折 | And | $P=$ | 10 | At | 2 |
| $12=5$ | Arid | $5=$ | 16 | H1 | 3 |
| $\vec{H}=5$ | And | $P=$ | 10 | ヨt | 4 |
| $\omega=54$ | Anti | $\mathrm{P}=$ | 138 | 日t | 5 |
| $W=54$ | And | $P=$ | 10 | $2 \pm$ | A |
| W＝5\＃ | And | $\mathrm{P}=$ | 1． 6 | \＃ | 7 |
| $t=54$ | Arit | $F=$ | 10 | at | E |
|  | And | $p=$ | 10 | 3t | 7 |
| $\ldots{ }_{5}$ | Afid | $P=$ | 10 | Et | 10 |

Experimental results

10 Res
Co Rem This program reads the strain values obtained froe the exxperiment conousted on the fixed ended arch, and calculates the stresses at the recuired sections.
30 REM The program also interpolates the strain values for any loading and determines the comresponding stresses.
40 REM
58 CLS
$66 \mathrm{Cl}=38 / 16$
78 C2=30/768
88 INPUT "Input the increwent of the load ":DP
$98 \mathrm{DIM} A(5,3), B(3,3), 6(3,3), E B(6), E T(6), \mathrm{PT}(3), \mathrm{PB}(3), \mathrm{CT}(3), \mathrm{CB}(3), \mathrm{AI}(4)$
180 FOR $I=1$ TO 6
110 $A(1,1)=1$
$120 \mathrm{~A}\left(\mathrm{I}, \mathrm{E}^{2}\right)=(\mathrm{I}-1) * \mathrm{BP}$
130 A $(1,3)=A(1,2)^{\wedge} 2$

158 FOR I=1 TO 3
160 FOR $J=1$ TO 3
$170 \mathrm{~B}(\mathrm{I}, \mathrm{J})=8$
180 FOR $K=1$ TO 5
$199 \mathrm{~B}(\mathrm{~L}, \mathrm{~J})=\mathrm{B}(\mathrm{l}, \mathrm{J})+A(\mathrm{~K}, \mathrm{I}) * \mathrm{~A}(\mathrm{~K}, \mathrm{~J})$
290 NEXTK
210 NEXT J
228 NEXT I
$2388(1,1)=B(2,2)+B(3,3)-B(2,3) * B(3,2)$
$2406(2,1)=-(B(2,1) \div B(3,3)-B(2,3)+B(3,1))$
2Se $6(3,1)=B(2,1)+B(3,2)-B(2,2) * B(3,1)$
$2606(1,2)=6(2,1)$
$2706(2,2)=B(1,1) * B(3,3)-E(1,3) * B(3,1)$
$2806(5,2)=-(B(1,1) * B(3,2)-B(1,2) \div B(3,1)$
C90 $6(1,3)=6(3,1)$
$3606(2,3)=6(3,2)$
$3106(3,3)=B(1,1) * B(2,2)-B(1,2) * B(2,1)$
$320 \mathrm{D}=\mathrm{B}(1,1) \times 6(1,1)+B(1,2) \times 5(2,1)+B(1,3)+G(3,1)$
330 FOR $I=1$ TO 3
340 FOR $J=1$ TO 3
$350 \mathrm{E}(\mathrm{I}, \mathrm{J})=6(\mathrm{I}, \mathrm{J}) / D$
360 NEXT J, I
370 FOR I=1 TO 3
368 FOR $\mathrm{J}=1$ TO 3
$39 \mathrm{~GB}(\mathrm{I}, \mathrm{J})=$ ?
$42 \mathrm{FOR} K=1$ TO 3
$410 \mathrm{~EB}(I, J)=5 B(I, J)+B(J, K) \div 5(K, J)$
420 NEXT K, J, I
438 ReN $N T=$ No. of tests.
448 REM NE $=$ NC. of experiments.
458 訾 $E T=5$ trains recorded by the gages on top of the arch.
460 RPM EB $=$ Strains recorded by the gages on bottom of the arch.
478 NT=10
488 REM N6 $=$ No. of pairs of gages
490 N6 = 8
500 FOR $\mathrm{BE}=1$ TO NG
510 LPRINT *****************H+*H*****
520 IF $\mathrm{BE}=1$ THEN LPRINT "i Strain gage nos. 3 and $4^{*}$ : 60 T0 620
530 IF BB=2 THEN LPRINT ": Strain gage nos. 7 and B" : G0T0 $6 \ell 0$
542 IF BB=3 THEN LPRINT "; Strain gage nos. 13 and $14^{*}$ : GOTO 600
550 If BR" $=4$ THEN LPRINT ** Strain gage nos. 17 and $18^{\circ}$ : 6070609
560 IF BE=5 THEN LPRINT " Strain gage nos. 19 and $29: 69 T 0600$

570 IF BE=6 THEN LPRINT " Strain gape nos. 21 and $22^{n}$ : $60 T 0600$
580 IF $B E=7$ THEN LPRINT "* Strain gase nos. 23 and 24 ": $60 T 0620$
590 IF B8-8 TheN LPRINT " Strain gage nos. 35 and 26 "

610 LPRIRT
629 FOR NE = 1 T0 NT
638 FOR $I=1$ TO 6
640 PERD EB(I)
650 IF NE ( 6 THEN 740
668 IF BE=1 THEN EB(I)=EB(I)-3.9: 60T0 838 678 IF $\mathrm{BE}=2$ TTEN $\mathrm{EB}(\mathrm{I})=E B(I)-.9$ : $60 T 0 \mathrm{~B} 28$ 688 IF Bis $=3$ THEN $E B(I)=E B(I)-.9$ : $60 T 0888$
 780 IF BR=5 THEN EB(I)=ER(I)-. 9 : GOTO 820
 720 IF $B E=7$ TKEN $E B(I)=E B(1)-1.9: 60 T 0828$ 730 IF $\mathrm{BB}=8 \mathrm{THEN} \mathrm{EB}(\mathrm{I})=E B(\mathrm{I})-3.9$; 60 TO 820 740 IF $B E=I$ ThEN $E B(I)=E B(1)-4 . B$; $60 T 0820$ 750 IF $\mathrm{BE}=2$ THEN EB(I) $=\mathrm{FB}(\mathrm{I})-3.9$ : $69 T \mathrm{COR}$ 760 IF $\mathrm{Bg}=3$ THEN EB(I) $=\mathrm{EB}(\mathrm{I})-1.9$ : EOTO 820 770 IF BB=4 THEN EE(I)=E8(I) : 0070 B20 780 IF BR=5 THEN EB(I) $=E 8(\mathrm{I})-1.9$ : GOTO 828 790 IF $\mathrm{BE}=6$ THEN $E B(\mathrm{I})=E \mathrm{E}(\mathrm{I})-.9$ : ECTO BCO B40 IF $\mathrm{BE}=7 \mathrm{TH}[\mathrm{N} \operatorname{EP}(\mathrm{I})=[\mathrm{EB}(\mathrm{I})-1.9: 60 T 0$ B2
 820 NEXT I
838 FDR I=I TO 6
840 READ ET (I)
850 IF NE ( 6 THEN 940
860 IF $\mathrm{BE}=1$ THEN ET(I) $=\mathrm{ET}(\mathrm{I})-2.9$ : 6070 IQEO
B70 IF $\mathrm{BE}=2$ THEN EI (I)=T(I)-.9: 6OTO 1020
889 IF $\mathrm{BB}=3$ THEN $E(\mathrm{I})=\underline{T}(\mathrm{I})-1.9$ : GOTO 1820
898 IF BB=4 THEN EI(I)=ET(I)-3.9: $60 T 0$ : 1828
900 IF BB=5 THEN ET(I)=ET(I)-3.9: GOTO 1ROC
910 IF BB=5 THEN Ei(I)=ET (I)-. 9 ; 60T0 1028
920 IF $\mathrm{BP}=7$ THON EI(I) $=\mathrm{ET}(\mathrm{I})-1.9$; GOTO 1028
932 IF B8=8 THEN EI(I)=ET (I) : GOTO IDEX
940 IF $\mathrm{BE}=1$ THEN ET(I) $=\mathrm{ET}(\mathrm{I})-3.9$ : 6070 IRC8
950 IF $\mathrm{BE}=2$ THEN $\operatorname{EI}(\mathrm{I})=\mathrm{ET}(\mathrm{I})+9.7$; GOTO IVE
960 IF $\mathrm{BB}=3$ THEN ET(I)=T(I)-1.9: 60TO I230
978 IF BE=4 THEN EI (I) $=\mathrm{ET}(\mathrm{I})-4 . \mathrm{B}$ : EOTO 1028
988 IF B8-5 THEN EI (I) $=E T(I)-.9$; $60 T 01020$
990 IF BB=6 THEN EI (I)=TT(I) -1.9 ; 80701820
1000 IF BE=7 THEN ET(I)=ET (I)-1.9: GUTO 1000
I010 IF B8=8 THEN ET(I)=ET(I)-. 9 : $60 \% 1800$
IROP NEXT I
1030 FOR $I=1$ TO 3
1040 PT (I) $=8:$ PB $(I)=8$
1850 FOR $K=1$ TO 6
1860 PT ( I$)=\mathrm{PT}(\mathrm{I})+\mathrm{A}(\mathrm{K}, \mathrm{I})$ सET $(\mathrm{K})$
1078 PB(I) $=P B(I)+R(K, I) \in E B(K)$
1688 NEXT K, I
I090 FOR I=1 TD 3
I180 CT (I) $=0$
$1110 \mathrm{CB}(\mathrm{I})=0$
1120 FOR $K=1$ TO 3
$1132 \mathrm{CT}(\mathrm{I})=\mathrm{CT}(\mathrm{I})+E(\mathrm{I}, \mathrm{K})+\mathrm{T} T(\mathrm{~K})$
$1140 \mathrm{CB}(\mathrm{I})=\mathrm{CE}(\mathrm{I})+G(\mathrm{I}, \mathrm{K}) * P E(\mathrm{~K})$
1158 NEXT K, I
IIS PRINT

1170 LPR1NT
1189 LPRINT "Loads at loading point f":NE
119 LPRINT
1200 LPRINT "Coefficients C1, C2 and C3 for equations of curve fitting"
1212 LPRINT
12ce LPRiNT " ", : LPRINT "Top gages", : LPRINT "Eot, gages"
1238 LPRINT
1240 FOR $1=1703$
1250 LPRINT "C";1,
1260 LPRINT CT(I),C5(1)
1278 NEXT I
1280 PI $(0)=0$
$1290 \mathrm{P}_{1}(1)=1$
1300 Pi (2) $=5$
$1310 P_{1}(3)=10$
1320 FOR $J=0$ TO 3
1330 RT(J) $=\left(\mathrm{CT}(3)+2{ }^{+}(\mathrm{J})+\mathrm{CT}(2)\right)+P 1(\mathrm{~J})+C T(1)$
$1340 \mathrm{RB}(\mathrm{J})=(\mathrm{CB}(3)+\mathrm{O}(\mathrm{J})+\mathrm{CE}(2)) * \mathrm{PI}(\mathrm{J})+C B(\mathrm{I})$
$1358 \mathrm{~N}=\mathrm{Ci} \div(\mathrm{RB}(\mathrm{J})+\mathrm{RT}(\mathrm{J}))$
$1368 \mathrm{~m}=\mathrm{CL} \pm(\mathrm{RB}(\mathrm{J})-\mathrm{RT}(\mathrm{J}))$
1370 IF N1 THEN NEN
1380 LPRINT
1398 LPRINT "
1400 LPRINT " Strain in bottom gage=";RB(J)

1420 NEXT J
1430 FOR $J=1$ TO 3
$1440 \mathrm{RT}(\mathrm{J})=R T(\mathrm{~J})-R T(0)$

1458 IF NII THEN N=N
 =" ${ }^{\text {N }}$
1480 NEXT J
1498 LPRINT CURF (12)
1500 LPRINT : LPRINT : PRINT
1510 NEXT RE
1538 NEXT BB
1538 PRINT
1548 END

1560 REX
DATA STATEKONTS

 U5P18. All data given belom are in the same order.
1590 DATA $-92.8,-83,-69.3,-63.5,-43.5,-41$ : REM Data for gage ${ }^{4} 4$ at 1 .
1689 DCTA 91.8,35.1,-25.4,-15.6,-71.3,-101.6: ReM Data for 13 at 2.
1518 DATA -91.8,-34.2,28.3,13.6,78.3,1e0.6: REM Data for at 2.
1620 DATA $45.9,40,11.7,4.8,-16.6,-52.7$ : Rem Data for 33 at 3.
1630 DATA $-43.9,-43.9,-14.5,-9.7,9.7,43.9:$ RED Data for 44 at 3. 1648 DATA 23.4,27.3,33.2,35.1,39,42.9 : RER Data for 43 at 4.
1650 DATA -28. $3,-30.2,-48,-43.9,-58.8,-57.6$ : REM Data for th at 4. 1660 DATA $22.4,27.3,57.5,58.6,88.9,92.8$ : RED Data for 13 at 5. 1570 DATA -22.4,-35. 1, $72.3,-72.3,-102,6,-109.4$ : REM Data for 44 at 5.
1688 DATA 53.7,70.3,97.7,130.9,164.1,198.3 : REM Data for 33 at 5.
1590 DATA -59.3,-85,-109.4,-142.5,-175. 8, -212 : REM Data for \#4 at 5.
1700 DATA 82, 88.9,189.4,144.5,168,175.8 ; REM Data for 33 at 7.
1710 DATA -85.9,-93.8,-113.3,-152.4,-175.8,-186.6 : RED Data for 44 at 7.
1728 DATA 74. $2,75.2,91.8,187.4,145,6,183.7$ : REM Data for $\$ 3$ at 8 .
1738 DATA -76. 2, -78.1, -94. 7, -167.4, -148.5,-187.6 : PEM Data for at 6.
1740 DATA 65, $85.9,88.9,185.5,116.2,115.2$; REM Data for 33 at 9.
1750 DATA -80. 1, -82, -84, -185.5, $-113.3,-115.3$ : REM Data for 4 at 9 .
1760 DATA 65.4,73.2,66.4,78.3,71.3,66.4 : REM Data for 3 at 18.
1778 DATA $-68.5,-68.4,-68.5,-54.4,-64.4,-61.5$ : REM Data for at 18.

1798 REM Data for chanmels 7 and 8

1810 DATA -88. 9, -90. 8, -77.1, $-85,-79.1,-78.1$ : REY. Data for 77 at 1.

103 DATA -87.9,-35. 1,28.3,17.5,70.3,101.5 : REM Data for 17 at 2.
1840 DATA -788.6, $-1899.3,-1279.1,-1451.1,-1145.2,-1184.2$ : REM Data for $\$ 8$ at 2 1850 DATA -37.1,-28.3,-4. $8,3.9,25.4,57.6$ : REM Data for 37 at 3.

1870 DATA $-34.2,-43.9,-67.4,-83,-165.5,-126:$ P9M Data for 77 at 4.
1889 DATA $-1479.4,-1468.7,-1514.6,-1544,-1538.1,-1471.6$ : REM Data for 8 at 4. 1890 DATA $-38.1,-50.8,-97.7,-183.5,-148.5,-163.1$ : REM Data for 77 at 5.
1980 DATR $-1668,-1671,-1635.8,-1635.8,-1589.9,-1425.7$ : Re4 Data for 18 at 5.
1918 DATA $-53.7,-76.2,-64,-116.2,-148.5,-184.6$ : REN Data for 77 at 6.
1520 DRTR $-11.7,5.8,12.7,-4,8,-15.6,189.4$ : REQ Data for 45 at 6.
1930 DRTA $-66.4,-72.3,-83,-182.6,-111.5,-112.3$ : RE5 Data for $\$ 7$ at 7.

1550 DATA -68.5, $58.8,-59.6,-59.6,-78.1,-56.7$ : FEM Data for 77 et 8 .
 1978 DATA $-58.8,-53.7,-49.8,-68.5,-59.6,-55.7$ : REM Data for 77 at 9.
1588 DATA $-182.7,-190.5,-197.3,-185.6,-157.3,-160.2$ : REM Data for at 9.
1998 DATA $-37.1,-42.9,-37.1,-38.1,-38.1,-33.2$ : REM Data for 77 at 10.
2008 DATA -202. $2,-194.4,-204.2,-289.1,-204.2,-214.9$ : REP Data for 18 at 10.

2020 PEY Data for chanmels 13 and 14

2040 DATA $-16.6,-17.5,-19.5,-16.6,-18.5,-28.5$ ；险 Data for 13 at 1. 2250 DATA 23．4，25．4，30．2，26．3，28．3，28．3 ：PEDM Data for $\$ 14$ at 1. 2068 DATA $-14.6,-24.4,-51.7,-44.9,-59.6,-57.4$ ：REM Data for $\$ 13$ at 2 ． 2078 DATA 22．4，33．2，58．6，50．8，63．5，69．3 ：Re＂Data for \＄i4 at 2. 20BD DATA $-30.2,-50.8,-68,4,-78.1,-98.8,-117.2$ ：REM Data for $\$ 13 a \ddagger 3$ ． 2890 DATA $39,57.6,72,3,79.1,86,9,111.4$ ：Re9 Data for $\$ 14$ at 3. 2100 DATA $-48,-58.6,-58.4,-78.1,-85.9,-95.7:$ REN Data for 13 at 4. 2118 DATA $42,58.6,67.4,74.2,82,85.9$ ：PEM Data for 14 it 4. 2120 DATA－36． $1,-36.1,34.2,42.9,96.7,131.9$ ：REM Data for 113 at 5. 2138 DATA $42,32.2,-35.1,-47.8,-93.8,-128.9$ ；REM Dat for 14 at 5. 2140 DATA $-36.1,-9.7,9.7,49.8,93.8,123.1$ ：距 Data for 113 at 6. 2150 DATA 23．4，－4．8，－22．4，$-55.7,-99.6,-127$ ：PREM Data for 14 at 6. 2168 DATM－23．4，$-31.2,-39,-36.1,-57.6,-77.1$ ：REM Data for $\$ 13$ at 7. 2170 DATA $26.3,33.2,39,32,2,55.7,72.3$ ：REM Data for 14 at 7. $21 B 8$ DATA $-3.9,-32.2,-42,-66.4,-87.9,-136.8$ ：REM Data for $\$ 13$ at 8. 2190 DATA $4.8,36.1,46.9,72.3,92.8,137.7$ ：REM Data for 14 at 8. 29 DATA－27． $3,-28,3,-36.1,-39,-65.4,-77.1$ ；REN Data for 13 at 9. 2210 DATA 34． $2,35.1,47.8,46,9,75.2,63$ ：REM Data for 14 at 9. 22ट0 DATA $-21.4,-14.6,-24.4,-31.2,-27.3,-31.2$ ：踇戍 Data for 113 at 18. 2230 DATA $35.1,28.3,39,44.9,40,44.9$ ：REM Data for 14 at 10

2270 DATA $-19.5,-21.4,-23.4,-22.4,-22.4,-25.4$ : REM Data for 117 at 1. 2388 DATh 19.5,17.5,23.4,20.5,19.5,19.5: Ras Data for $\$ 18$ at 1.
2798 DRTA $-19.5,-38.2,-51.7,-45.9,-61.5,-68.4$ : Re7 Data for $\$ 17$ at 2. 2380 DनTA $16.6,24.4,47.8,41,52.7,61.5$ : ReM Data for 118 at 2. 2318 DATA -33.2,-52.7,-72. $3,-82,-94.7,-121.1$; REM Data for 177 at 3. 2320 DATA 30.2,47.8,62.5,71.3,80.1,185.5 : REM Data for 18 at 3.
 2348 DRTA 33. $2,51.7,63.5,71.3,82,91.8$ : RPM Data for 18 at 4. 2350 DATA $-45.9,-49.8,2,9,6.8,45.9,76.2$ : REM Data for 17 at 5. 2350 DATA 40,34. 2, $-20.5,-29.3,-67.4,-94.7$ : REM Data for $\$ 18$ at 5. 2372 DRTA $-26.3,3.9,42.9,86.9,142.6,176.8$ : REN Data for 17 at 6. 2380 DATA $15.6,-15.6,-47.8,-49.9,-142.6,-177.8$ : REM Data for 118 at 6. 2396 DRTA $-11.7,-28.5,-27.3,-12.7,-30.2,-51.7$ : REM Data for $\$ 17$ at 7. 2400 DATA 19.5,26.3,30.2,11.7,31. $2,55.7$ : REM Data for $\$ 18$ at 7. 3410 DATA 2.5, $-27.3,-39,-64.4,-80.1,-129.9:$ KEM Data for 17 at 8. 2420 DATA 3.9,34.2,44.9,71.3,87.9,131.9: Ren Data for \#18 at 8 . 2430 DATA $-22.4,-20.5,-33,2,-33.2,-64.4,-74.2$ : REM Data for $\$ 17$ at 9. 2440 DRTA 29.3,31.2,43.9,42,72.3,81.1 : REM Data for \#18 at 9. 2450 DATA $-21.4,-13.6,-24.4,-30.2,-25.3,-31.2$ : REN Data for $\$ 17$ at 18. 2458 DATA 31. $2,24,4,37.1,42,9,39,43.9$ : REM Data for 18 at 10.

2480 Ren Data for channels 19 and 28

2500 DATR 39,33.2,37.1,35.1,31.2,33.2: REM Data for $\# 19$ at 1. 2510 DATA $-40,-37.1,-37.1,-34.2,-34.2,-35.1$ : fer Data for wed at 1. 2520 DATR 35.1,31.2, 39, 28. 3, 32.2, 32.2 : REM Data for $\$ 19$ at 2. С530 DATR -34. $2,-35.1,-42,-31.2,-37.1,-36.1$ : REM Data for 120 at 2 2548 DATA 37. 1, 32. $2,34.2,24.4,19.5,15.6$ : REN Data for $\$ 19$ at 3. 2550 DATR -38. 1, $-35.1,-39,-34.2,-29,3,-27.3$ : REM Data for $\ddagger 28$ at 3. 2560 DATR 14. 6, 11.7,7.8,2.9,-2.9, 6.8 : REM Data for 19 at 4. 2570 DATA $-24.4,-19.5,-17.5,-13.6,-9.7,-8.7$ : RCM Data for \#C2 at 4. 2589 DATR 4.8,2.9,-22. 4, -27.3, -47.8, -76.2 : REM Data for $\$ 15$ at 5. 2590 DATA $-7.8,-16.6,2,9,9.7,33.2,68.5$ : BeM Data for at 5. 2600 [ATR 4B. B, 18.5,17.5,-5.8, $-36.1,-56.6$ : REN Data for $\$ 19$ at 6 . 2610 DATA -63.5,-32. $2,-29.3,-2.9,30.2,50.8$; Rem Data for 520 at 6. 2620 DATR 53.7,49.8,46.9,45.9,54. 7,81.1 : REM Data for $\$ 19$ at 7. 2533 DATA $-53.7,-47.8,-52.7,-51.7,-57.6,-05$ : Rem Data fon $=20$ at 7. 3640 DATA 67.4,96.7,131.9,194.4, 252.1,316.6: REM data for 19 at 8 . 2650 DATA -67.4, -94. $7,-128,-188,6,-248.2,-316.6$ : REM Data for te at 6. C650 DATA 5. 7, 110. 4, 148.7,179.8,230.6,267.7 : REM Data for $\$ 19$ at 9. 2570 DATA $-85,-108.6,-132.8,-171.9,-224.7,-263.8$ : ROY Data for 720 at 9. 2588 DATR 88.1,89. 1, 85.9,95. 7, 87.9,99.6 : REM Data for 19 at 10. 2599 DATA $-8.4,-69,3,-72,3,-85,-77.1,-85.9:$ pem Data for tee at 10.

2718 REM Data for channels 21 and 22

2730 IATA 31.2,29. 3, 33.2,28.3,28.3,27.3: REF Data for 叛1 at 1.
2748 IATA $-33.2,-32.2,-31.2,-29,3,-29,3,-38.2$ : REX Data for 122 at 1. 2750 IATA $30.2,29.3,32.2,21,4,25.3,25.4$ : REN Data for $22!$ at 2.
2760 DATA $-30.2,-30.2,-35,1,-24,4,-30.2,-31.2$ : RE4 Data for te2 at 2 2778 IATA $33.2,25.4,30.2,22.4,15.6,11.7$ : FEM Data for wel at 3.
2780 IATA $-32.2,-27.3,-35.1,-29,3,-24,4,-22.4$ : RED Data for 122 at 3.
 286 DATA $-18.5,-16.6,-13.6,-9.7,-6.8,-7.8$ : FBM Data for 722 at 4. 2810 DATA 3.9, $9,-21.4,-27.3,-47.8,-75.2$ : REM Data for ${ }^{2} 21$ at 5. 2828 DATA $-5.8,-18.7,7.8,11.7,36.1,61.5$ : PREM Data for 122 at 5.
 2848 DATR $-59.6,-26.3,-26.3,1.9,35.1,53.7$ : RDM Data for $\mathbf{1 2 0}$ at 6. 2850 DATA $43.9,39,39,34.2,36,1,60.5$; Rem Data for 221 at 7. 2860 DPTR $-49.8,-42.9,-42,9,-48,-42,-68.4$ : REX Data for $\$ 22$ at 7. 2870 DATA 62.5,87.9,116. $2,171,221.8,276.5$ : REM Data for \#21 at 8 . 2880 DATR $-625,-87.9,-116.2,-171,-322.8,-280.4$ : KE? Data for ${ }^{2} 22$ at 8. 2898 DATA B5.9,101.6,132.7,182.7,234.5,283.4; REM Data for ${ }^{2} 21$ at 9. 29\% DATA - $89.1,-98,6,-133.8,-175.8,-227.6,-277.5$; R9\% Data for 120 a\% 9. 2910 DATA 72.3,73.2,77.1,92.8,84,94.7 : RER Data for el at 10. 2920 IATA $-63.5,-67.4,-69.3,-84,-74.2,-86.9$ : REM Data for 322 at 10.

2940 Rem Data for channels 23 and 24

2982 DATA 33. $2,15.5,29.3,14,6,12.7,13.6$ : REM Data for 123 at 1 .
2976 DATA $-29.3,-27.3,-25.4,-23.4,-21.4,-25.4$ : Rem Data for $\mathbf{2 5 4}$ at.
2980 DATA 14.6,14.6,17.5,8.7,12.7,13.6: REM Data for ${ }^{2} 23$ at 2.
2996 DATA $-24.4,-23.4,-29.3,-20.5,-24.4,-25.4$ : REF Data for 24 at 2
306 DATA $17.5,8.7,18.5,5,8,6,-3.9$ : REM Data fom 23 at 3.
3810 DATA $-25.4,-22.4,-30.2,-24.4,-21.4,-17.5$
328 DATA $-5.8,-6.8,-9.7,-13.6,-19.5,-21.4$
3630 DATA $-14.6,-8.7,-9.7,-5,8,-2,9,-5.8$
3048 DATA $3.9,-13.6,-31.2,-36.1,-53.7,-83$
3858 DATA $-1.9,-6.8,8.7,11.7,33.2,59.6$
3658 DATA $38.1,6.8,8.7,-18.5,-48.8,-70.3$
3070 DATA $-57.6,-25.4,-24.4,3.9,37.1,53.7$
3889 DATA $39,31.2,30.2,19.5,22.5,42.9$
3898 DATA -45.9, $-48,-38.1,-31.2,-29.3,-52.7$
3182 DATA 59.6,87.9,111.4, 159.2,204. $2,258,1$
3110 DFTA $-68.5,-82,-103.5,-151.4,-196.4,-243.3$
3128 DATA $87.9,185.5,147.5,192.5,247.2,390.9$
3132 DRTA -75. 2, -95. 7, -134. 8, -179.8,-232.5, -288.2
3148 DATA $75.2,86.1,82,97.7,50.8,102.6$
3158 DATA $-59.6,-60.5,-64,4,-77.1,-74,3,-82$

3170 REM Data for channels 25 and 26

319 DATA $-53.7,-50.8,-48.8,-47.8,-43.9,-45.9$
3200 DATA $88.9,85,88.9,85.9,84,82$
3.18 DATA $-49.8,-44,9,-43.9,-42,-37.1,-33.2$

3220 DATA 87.9, 84, 81. 1,76.2,74.2,68. 4
3230 ІАТА $-45.9,-49,-36.1,-29.3,-17.5,-3.9$
3240 DATA 85.9,75.2,74.2,63.5,50. $0,35.1$
3258 DATA $-24.4,-22,4,-14,6,-6,8, .9,6,8$
3258 DATA $59.6,55.7,47.0,41,31.2,20.5$
3278 DATA -4.8,-15.6,.9,1.9,29.3,56.0
3289 DATA 44.9,46.9,27.3,24.4,2.9,-16.5
3290 DATA $-89.9,-67.4,-83,-57.4,-51.7,-47.8$
3380 DATA 142.6, 128.1, 138.7,127, 114.3,118.4
3318 DATA $-81.1,-85,-95.7,-116.2,-140.7,-171.9$
3320 DATA 146.5,148.5, 163. 1, 181. 7,284. 2, 233.5
333 DATÂ $-82,-184.5,-126,-172,9,-231.5,-320.9$
3340 DATA 153.4, 177.8, 201.3,244. 3, 360, 365.4
3358 DATA -97.7, -117.2,-153.4,-197.3, -252. 1, -334.8
3368 DATA 188.7,1\%.4, 229.6,272.6,319.5, 37e. 3
3370 DATA $-76.2,-86.1,-77.1,-85.9,-74.2,-82$
3380 DATM $160.2,162.2,159.2,168,155.3,163.1$

## 

+ Strain gage nos. 3 and 4


Loads at loading point $* 1$
Coefficients C1, C2 and $\mathrm{C3}$ for equations of curve fittimg


Loads at loading point 12

Coefficients Cl , C 2 and CJ for equations of curve fitting


Loads at loading point $\$ 3$
Coefficients $\mathrm{Cl}, \mathrm{CL}$ and $\mathrm{C3}$ for equations of eurve fitting
Top gages Bot. gages
01 -49.20143 42.02857
C $2 \quad 2.772499-4.35286$
C3 .5866075 -. 52857 1
$W=51$ and $P=8$ Strain in top gage $=-49.82143$
Strain in bottom gage= 42.8285
Wosent $M=3.556641$
Thrust $\mathrm{N}=-13.11163$
$1=5$ and $P=1$ Strain in top gage-45.65033
Strain in bottom gage $=37.14713$
Monent $M=3.234744$
Thrust $\mathrm{N}=-15.96598$
$4=5$ and $P=5$ Strain in top gage-20. 49375
Strain in botton gage $=7.049965$
Howent $M=1.875926$
Thrust $\mathrm{N}=$-25.297!
$W=51$ and $P=18$ Strain in top gage $=37.3643$
Strain in botton gage=-54. 37724
Hosent $m=-3.582873$
Thrust $\mathrm{N}=-31.86175$
$u=0$ and $P=1$
Normalized Moment $=-.321851$
Normalized Thrust $=-2.854357$
$\omega=8$ and $p=5$
Normalized Fonent $=-.4 \% 1428$
Normalized Thrust $=-2.419894$
$\psi=9$ and $\mathrm{P}=10$
Normalized Moment $=-.7139514$
Normalized Thrust $=-1.875013$

Loads at loading point 4
Coefficients $\mathrm{Cl}, \mathrm{C} 2$ and $\mathrm{C3}$ for equations of curve fitting
Top gages Bot. gages
C $1 \quad-31.31672 \quad 18.60981$
C $2 \quad-2.455551 \quad 2.27858$
C $3 \quad-5.758858 \mathrm{E}-8 \mathrm{C}-.8357135$
$4=5$ and $\mathrm{P}=0$ Strain in top gage -31.31872
Strain in bottom gage $=18.60601$
Howent $=1.94963$
Thrust $N=-23.83250$
$W=5$ and $P=1$ Strain in top gage $=-33.82385$
Strain in bottom gage $=20.84287$
Moment 采 2.135419
Thrust $\mathrm{N}=-24.33934$
$\mathrm{N}=54$ and $\mathrm{p}=5$ Strain in top gage $=-45.22819$
Strain in bottom gage= 29.12008
Homent $=2.695635$
Thrust $N=-29.86521$

Whyi and $P=10$ Strain in top gage $=-61.62509$
Strain in bottom gage $=37.81449$
Mowent $1=3.884358$
Thrust $N=-44.64488$
0 and $P=1$
Nornalized Moment $=.1857814$
Nornalized Thrust $=-.5267587$
$u=0$ and $P=5$
Normalized Mowent $=.1891995$
Mornalized Thrust $=-1.206525$
$1=0$ and $P=18$
Normalized Moment $=.193472$
Normalized Thrust $=-2.68123$

Loads at loading point $\$ 5$
Coefficients Cl, ©2 and C3 for equations of curve fitting


Loads at loading point 6
Coefficients $\mathrm{Cl}, \mathbb{C}$ and $\mathbf{C 3}$ for equatiors of curve fitting


Loads at loading point 17
Coefficients Cl, C2 and C3 for equations of curve fitting

|  | Top gages | Bot. gages |
| :---: | :---: | :---: |
| C 1 | -82. 9964 | 72.21876 |
| $C 2$ | -9.921935 | 9.874884 |
| C 3 | -. 1343842 | 7.188416E-02 |
| $\\|=5 *$ and $P=0$Moment | Strain Strain | op gage=-82.9964 tottom gage $=72.21876$ |
|  | Homent $\mathrm{F}=6.86278$ |  |
| Thrust $\mathrm{N}=-20.20368$ |  |  |
| $4=5 \%$ and $P=1$ | Strain Strain | op gage $=-93.05272$ ton gage $=82.15673$ |
| Nowent $M=6.844119$ |  |  |
| Thrust $\mathrm{N}=-28.42999$ |  |  |
| $\underline{W}=5{ }^{\text {a }}$ and $P=5$ | Strain Strain | $0 \mathrm{page}=-135.9657$ thos gage 123.3783 |
| Howent $1=10.13062$ |  |  |
| Thrust N=-23.68:38 |  |  |
| $y=53$ and $P=10$ | Strain | 10p gage=-195.6542 |
|  | Strain i | tot gage |
| Howent ${ }^{1}=14.60134$ |  |  |
| Thrust $\mathrm{N}=32.8395$ |  |  |
| 10 and $\mathrm{P}=1$ |  |  |
| Mormalized Moment $=.7813395$ |  |  |
| Normelized Thrust $=-.2059092$ |  |  |
| $W=0$ and $P=5$ |  |  |
| Normalized Moment $=.8135689$ |  |  |
| Normalized Thrust $=-.675659$ |  |  |
| 108 and $p=18$ |  |  |
| Nornalized Mowent $x .8538555$ |  |  |
| Mormalized Thrust $=-1.261597$ |  |  |

Loads at loading point : 8
Coefficients $\mathrm{Cl}, \mathrm{CD}_{2}$ and CJ for equations of curve fittirg
Top gages Bot. gages
C $1 \quad-79.4435969$
C $2 \quad 1.522888 \quad-1.876965$
C 31.2678581 .213841
$\|=5$ and $p=0 \quad$ Strain in top gage $=-79.443$
Strain in bottom gage $=69.9607$
Moeent 在 5.836892
Thrust $N=-17.77931$
$4=5$ and $P=1$ Strain in top gage=-79.18798
Strain in bottom gage $=78.23756$
Howent $\|=5.831466$
Thrust No-17.04452
$W=5$ and $P=5$ Strain in top gage $=-103.5253$
Strain in bottoe gage $=94.92188$
Mowent $M=7.751841$
Thrust $\mathrm{N}=-16.13133$
$4=51$ and $p=18$ Strain in top gage=-191.0625
Strain in botton gage= 188.575 !
Moment $=14.51469$
Thrust $\mathrm{N}=-19.54843$
$1=0$ and $P=1$
Hormalized Monent $=-4.615188 E-23$
Normalized Thrust $=.7347965$
$4=0$ and $p=5$
Normalized Hoeent $=.3831519$
Norwalized Thrust $=.3295956$
10 and $P=18$
Normalized Moment $=.8578666$
Normalized Thrust $=-, 1769114$

Loads at loading point $\$ 9$
Coefficients C1, t2 and C3 for equations of curve fitting


Loads at loading point 10

Coefficients $\mathrm{Cl}, \mathbf{C 2}$ and $\mathbf{C 3}$ for equations of curve fitting


*     * 
+ Strain gage nos. 7 and 8


Loads at loading point $\leqslant 1$
Coefficients $\mathrm{Cl}, ~ C 2$ and $[3$ for equations of curve fitting

|  | Top gages | Bot. gaqes |
| :--- | :---: | :---: |
|  |  |  |
| C 1 | -826.3672 | -93.86059 |
| C 2 | -8.690918 | 1.965518 |
| C 3 | -.1398289 | $-7.4556355-82$ |

$U=5$ and $D=8 \quad$ Strain in top gage=-826.3572 Strain in bottom gage $=-93.86069$
Koment $M=28.61353$
Thrust $N=-1725.427$
$H=5$ and $P=1$ Strain in top quge=-635.189 Strain in bottom gage=-92. 8C972
Hoeent $M=29.0296 \mathrm{E}$
Thrust $N=-1738.535$
$W=5$ and $P=5$ Strain in top gage=-873.0925 Strain in bottom gage $=-85.197 e 1$
Howent 俗 30.73811
Thrust N $=-1798.668$
H=5l and $p=10$ Strain in top gage $=-926.3592$
Strain in botton gage=- 82,26115
Mowent $M=32.97258$
Thrust $N=-1891.163$
$W=0$ and $p=1$
Nermalized Mowent $=.4161297$
Normalized Thrust $=-13.18778$
$W=8$ and $P=5$
Normalized Howent $=.4249143$
Normalized Thrust $=-14.6461$ ?
$H=0$ and $P=10$
Normalized Mowent $=.4359047$
Normalized Thrust $=-16.5735$

Loads at loading point ?
Coefficients C1, C2 and [3 for equations of curve fitaing

|  | Top gages Bot. gages |
| :---: | :---: |
| C. 1 | -779.7315 -85.53216 |
| C 2 | -192.2941 24.59855 |
| C 3 | 16.52385 -.6691952 |
| $4=54$ and $P=0$ | Strain in top page=-779.7315 Strain in bottom gage=-86.53216 |
| Homent $\mathrm{H}=27.8781$ |  |
| Thrust $\mathrm{N}=-1624,244$ |  |
| $105 \%$ and $P=1$ | Strain in top gage=-955. 4119 Strain in botton gage=-6さ, 61082 |
| Mowent $=34.87584$ |  |
| Thrust $N=-1998.733$ |  |
|  | Strain in top gagew-1327.661 Strain in bottom gage $=19.69858$ |
| Howent $n=52.63892$ |  |
| Thrust $\mathrm{N}=-2452.444$ |  |
| $\\|=5 *$ and $P=18$ | Strain in top gage=-1849.487 |
|  | Strain in bottom gage= 92,45377 |
| Mowent $M=44.68395$ |  |
| Thrust $\mathrm{N}=-1794.288$ |  |
| $W=0$ and $P=1$ |  |
| Normalized Moment $=7.796945$ |  |
| Normalized Thrust $=-284.5403$ |  |
| $\\|=0$ and $\mathrm{P}=5$ |  |
| Normalized Moment $=5.118563$ |  |
| Normalized Thrust $=-165.6399$ |  |
| $4=0$ and $P=10$ |  |
| Normalized Moment $=1.752585$ |  |
| Normalized Thrust $=-17.90437$ |  |

Loads at loading point $\$ 3$
Coefficients $\mathrm{Cl}, \mathrm{CR}$ and $[3$ for equations of curve fitting

|  | Top gages | Bot. gages |
| :---: | :---: | :---: |
| C1 - | -1317.172 | -40. 25714 |
| C 2 - | -48.82031 | 4.06144 |
| C 3 | 2.862427 | . 5 |
| $4-5 *$ and $P=?$ | Strain Strain | op gage=-1317.172 tot 1001 gaģe $=-48.25714$ |
| Howent | nt $h=43.87$ |  |
| Thrust | 5. $\mathrm{N}=-2545$. |  |
| Win and $P=1$ | Strajn Strain | op gage=-1355. 13 tot tou gage=-35.6957 |
| Moment | at 10 51.54 |  |
| Tmrust | t $\mathrm{N}=-2587$. |  |
| $N=51$ and $P=5$ | Strain Strain | op gage=-1449.713 <br> thou gage=-7.449944 |
| Moment | t $h=56.33$ |  |
| Thrust | t $\mathrm{N}=-2733$. |  |
| $y=5$ and $P=10$ | Strain Strain in | top gage-1439.132 <br> ot tow gage= 58.35726 |
| Nowent | t $m=50.18$ |  |
| Thrust | \% $\mathrm{N}=-2623$. |  |
| $4=0$ and $P=1$ |  |  |
| Mormalized Moment $=1.660911$ |  |  |
| Nornalized Thrust $=-62.61834$ |  |  |
| $4=8$ and $P=5$ |  |  |
| Normalized Moment $=1.291782$ |  |  |
| Norralized Thrust $=-37.40014$ |  |  |
| 10 and $\mathrm{P}=10$ |  |  |
| Norualized Mowent $=.8383785$ |  |  |
| Normadized Threst $=-5.877365$ |  |  |

Loats at loading point $\$ 4$
Coefficients $\mathrm{Cl}_{1}$ C2 and $\mathrm{C3}$ for equations of curve fitting

|  | Top gages | Bot. gages |
| :--- | :--- | :--- |
|  |  |  |
| C 1 | -1450.478 | -36.44284 |
| C 2 | -23.771 | -7.197927 |
| C 3 | 2.8998 | -.223217 |

$W=51$ and $\rho=0 \quad$ Strain in top gagre $=-1450.470$ Strain in botton gage $=-36.44284$
Noment: $M=55.23575$
Thrust $N=-2767.977$
$W=5$ and $P=1$ Strain in top gage=-1472.15
Strain in botton gage=-43.85399
Noment $1=55.79281$
Thrust $N=-2842.507$

Strain in botton gage=-77.9629!
Moment $M=56.28678$
Thrust $N=-2990.286$
$*=5$ and $p=10 \quad$ Strain in top gage=-1478.282
Strain in bottom gage $=-138.6438$
Howent $4=52.64212$
Thrust $N=-3816.736$
$H=0$ and $D=1$
Normalized moment $=.5570598$
Nornalized Thrust $=-54.53867$
40 and $9=5$
Normalized Moment $=.1942059$
Normalized Thrust $=-48.45153$
$4=0$ and $p=10$
Normalized Moment $=-.2593634$
Mormalized Thrust $=-22.87593$

Loads at loading point :5
Coefficients Cl , 2 and C for equations of curve fitting


## Loads at loading point 6

Coefficients $C: C 2$ and $C 3$ for equations of curve fitting
Top gages Bot. gages
C1 5.3e5012 $\quad-56.77161$
C $2 \quad-13.2802 \quad-5.497803$
C $3 \quad 2.661162 \quad-.7410851$
$W=5$ and $\rho=0 \quad$ Strain in top gage= 5.325012
Strain in bottom gage $=-56.77161$
Moment $M=-2.425649$
Thrust $N=-98.46236$
$W=51$ and $P=1$ Strain in top gage $=-5.81482!$
Strain in bottom gage $=-63.8185$
Howent $\mathrm{H}=-2.234237$
Thrust $N=-129.64 E$
*-5\# and $p=5$ Strain in top gagpe=-9. 146914
Strain in bottom gage=-182. 7878
Homent $1=3.657845$
Thrust $\mathrm{N}=-299.0775$
$W=51$ and $p=10$ Strain in top gage $=79.43926$
Strain in botton gage $=-185,858$ !
Mowent $\mathrm{N}=-10.36318$
Thrust $\mathrm{N}=-199.5354$
$U=0$ and $P=1$
Normalized Moment $=.1914119$
Nornalized Thrust $=-32.5836$
$H \Rightarrow 8$ and $p=5$
Normalized Koment $=-.2464392$
Normalized Thrust $=-22.65303$
$W=0$ and $P=10$
Normalized Moment $=-.793753$
Normalized Thrust $=-10.3873$

Loads at loading point $\geqslant 7$
Coefficients $\mathrm{Cl}, \mathrm{CL}$ and Cl for equations of eurve fitting


Loads at loading point ©
Coefficients $\mathrm{Cl}_{1}$ C2 and $\mathrm{CB}_{3}$ for equations of curve fitting

|  | Top gages | Bot. gages |
| ---: | ---: | ---: |
| C 1 | -166.5249 | -60.40363 |
| C 2 | 5.470093 | 4.293427 |
| C 3 | -.3361626 | -.8949164 |

$H=5 t^{2}$ and $P=0$ Strain in top gage $=-166.5249$ Strain in bottom gage=-60.40363
Homent $k=4.145363$
Thrust $\mathrm{N}=-425.491$
$U=5$ and $P=1$ Strain in top gages-161.391
Strain in botton gage $=-56.91512$
Moment $\#=4.681888$
Thrust $\mathrm{N}=-409.3235$
$W=5$ and $P=5$ Strain in top gage $=-147.5785$
Strain in bottom gage $=-59.0594$
Moment $H=3.451778$
Thrust $N=-387.4461$
$4=5$ and $P=10$ Strain in top gage $=-145.4482$ Strain in bottom gage $=-97.961$ Homent $H=1.854658$
Thrust $N=-456.3773$
$\psi=0$ and $p=1$
Nortal ized Moment $=-6.427489 E-22$
Normalized Thrust $=-16.16787$
10 and $P=5$
Normalized Moment $=-.137517$
Hormalized Thrust $=-7.688982$
$u=0$ and $P=10$
Normal ized Mowent $=-.2298785$
Normalized Thrust $=-3,088632$

Loads at loading point $\%$
Coefficients Cl , $\mathrm{C2}$ and $\mathrm{C3}$ for equations of curve fitting

|  | Top gages | Bot. gages |
| :--- | :--- | :--- |
|  |  |  |
| C 1 | -186.0235 | -50.82855 |
| C 2 | -4.164551 | -1.737854 |
| C 3 | .7351683 | $9.821319 E-82$ |

$W=5$ and $P=0$ Strein in top gagex-185. 8035
Strain in bottom gage=-50. 82855
Moment $=5.239271$
Thrust $N=-444.8601$
$\|=5$ and $P=1$ Strain in top gage $=-189.4319$
Strain in bottom gages-52. 4682
Howent $=5.35144$
Thrust N $=-453.5626$
$H=51$ and $P=5$ Strain in top gage- 188.4222
Strain in bottoo gage $=-57.06249$
Mowent $\mathrm{F}=5.13124$
Thrust $N=-462.2839$
W-5* and $P=10$ Strain in top gagex-154.033
Strain in bottom gage $=-58.38578$
Howent $=3.736218$
Thrust N=-398. 285 :
$0=0$ and $p=1$
Hormalized Mowent $=6.987384 E-02$
Hormalized Thrust $=-9.502559$
$H=0$ and $p=5$
Hormalized Mowent $=-.8298962$
Norwalized Thrust $=-3.244758$
$H=8$ and Pr 18
Hormalized Moment $=-.1544852$
Normalized Thrust $=-4.57744$

Loacs at loading point 10
Coefficients C1, C2 and $\mathbf{C 3}$ for equations of curve fitting

|  | Top gages | Bot. gages |
| :---: | :---: | :---: |
| C 1 | -208. 7534 | -39.19646 |
| C 2 | . 1872998 | -. 8886752 |
| C 3 | -. 158444 | . 135264 |
| H 5 and $D=0$ | Strain in <br> Strain $h=6.318$ | Op gage-209. 7534 bot toa gage $=-39.19646$ |
| Thrust $\mathrm{N}=-449.9859$ |  |  |
| $1=51$ and $P=1$ | Strain in Strain i $M=6.283$ | ep gagee-290.7965 bottom gage $=-39.94387$ |
| Thrust N=-451.3882 |  |  |
| W=53 and $p=5$ | Strain i Strain i | Op gage-203.97e botton gage=-48. 28822 |
| Mowent $n=6.396475$ |  |  |
| Thrust $N=-457.8866$ |  |  |
| 105 and $P=18$ | - Strain Strain i | top gage $=-214.7248$ bottom gage=-34.48677 |
| Mreent $n=7.840157$ |  |  |
| Thrust $\mathrm{N}=-467.2984$ |  |  |
| $4=0$ and $P=1$ |  |  |
| Normalized Moment $=-2.7516095-22$ |  |  |
| Normalized Thrust $=-1.482303$ |  |  |
| 40 and $\mathrm{p}=5$ |  |  |
| Normalized Moment $=.0171316$ |  |  |
| Morwalized Thrust $=-1.59614$ |  |  |
| $4=8$ and $P=10$ |  |  |
| Nornalized Poment $=7.2933962-82$ |  |  |
| Normalized Thrust $=-1.738447$ |  |  |

## 

* Strain page nos. 13 and 14


Loads at loading point \# 1
Coefficients $\mathrm{Cl}, \mathbb{C 2}$ and $\mathrm{C3}$ for equations of curve fitting
Top gages Bot. gages
C 1 21.72856 -19.20359
C $2 \quad 1.365066 \quad-5.230713 E-02$
[ 3 -9.464 $645-02-2.276892 E-02$

What and Pe 0 Strain in top gage 21.72856 Strain in bottoa gage=-19.00359
Moment Mo-1. 5911
Thrust $N=-5,169329$

W=51 and $P=1 \quad$ Strain in top pages 25.99993 Strain in bottom gage=-19.87865
moment $\%=-1.643656$
Thrust $N=-7.356494$
$\|=5$ and $P=5$ Strain in top gage= 26. 18752 Strain in bottom gapes-19, 83432
Moment $\%-1.797728$
Thrust N=-11.91285
$W=54$ and Px 18 Strain in top page= 25.914
Strain in botton gage-21.80346

Thrust $N=-7.767925$
0 and $P=1$
Norwalized Moment $=-5.255617 E-92$
Normalized Thrust $=-2,241165$
$4=0$ and $P=5$
Normalized Moment $=-4.132576 E-92$
Normalized Thrust $=-1,360585$
$\omega=0$ and Pe 18
Normalized Mowent $=-2.729715 E-92$
Normalized Thrust $=.25985 \%$

Loads at loading point ic
Coefficients $\mathrm{Cl}, \mathrm{C}$ and $\mathrm{C3}$ for equations of curve fitting
Top gages Bot. gages
C $1 \quad 20.53571 \quad-16.15717$
C 2 7.921888 -7.879273
C 3 -. 3393923 . 2696419
$\|=51$ and $P=0$ Strain in top gage 20.53571
Strain in bottom gage $=-16.15717$
moment $h=-1.433315$
Thrust $\mathrm{N}=-\mathbf{6 . 2 6 9 7 6 3}$
$4=54$ and $P=1$ Strain in top gages 28.1184
Strain in bottom gagee-23.7668
Howent $f=-2.826765$
Thrust $N=-8.159248$
$1=51$ and $p=5$ Strain in top gage $=51.68131$
Strain in bottoa gage $=-48.81248$
Moment $1=-3.925539$
Thrust $N=-5.379845$
$W=5$ and $P=10$ Strain in top gage= 65.9873
Strain in bottom gage $=-67.985$
Noment $\mathrm{H}=-5.2381 \%$
Thrust No-3. 897014
$H=8$ and $P=1$
Normalized Wament $=-.59345$
Normalized Thrust $=-5.851494 \mathrm{E}-82$
$H=0$ and $P=5$
Normalized Mowent $=-.4984447$
Normalized Thrust $=-.5661435$
$\psi=8$ and $P=18$
Norwalized Moment $=-.379688$
Normalized Thrust $=-1.212678$

Loads at loading point 3
Coefficients $\mathrm{Cl}, \mathrm{CD}$ and $\mathrm{C3}$ for equations of curve fitting


Loads at loading point 4
Coefficients $C 1, C 2$ and $\mathbb{C 3}$ for equations of curve fitting
Top gages Bot. gages

| C 1 | 41.36869 | -43.48569 |
| ---: | ---: | ---: |
| C 2 | 7.097351 | -7.608612 |
| C 3 | -.2792165 | .2321434 |

$W=5$ and $P=0$ Strain in top gage= 41.36969
Strain in bottom gaga=-43.48569
Mowent $=-3.314312$
Thrust $\mathrm{N}=-3.984375$
$4=53$ and $P=1$ Strain in top gage= 48.17903 Strain in botten gaga $=-52.86216$
Mowent $=-3.86879$
Thrust $=-5.038873$

W-5* and $P=5$ Strain in top gage $=69.87283$
Strain in botton gage=-75.72516
Homent $=-5.697391$
Thrust $N=-10.97461$
$\psi=5 *$ and $P=10$ Strain in top gage $=84.43255$
Strain in bottom gage $=-96.35747$
Moment $=-7.06211$
Thrust N=-22. 359 羿
$\mathrm{V}=8$ and $\mathrm{P}=1$
Normalized Moment $=-.544846$
Normalized Thrust $=-1.846498$
$4=0$ and $P=5$
Normalized Homent $=-.4746158$
Normalized Thrust $=-1.398848$
$\$ 28$ and $P=10$
Normalized Moment $=-.3747793$
Normalized Thrust $=-1.837465$

## Loads at loading point $\$ 5$

Coefficients C1, C 2 and C 3 for equations of curve fitting
Top gages Bot. gages

| $C 1$ | 45.81779 | -45.51432 |
| ---: | ---: | ---: |
| $C 2$ | -15.94482 | 12.98593 |
| $C 3$ | -.1843796 | .4910690 |

W54 and $\mathrm{P}=0$ Strain in top gage 46. 01779 Strain in bottom gage=-45.51432
inwent -3.515473
Thrust $N=.9448 e 31$
$W=5$ and $P=1$ Strain in top page $=29.88859$
Strain in bottom gagg=-32,1182:
Moment $M=-2.422141$
Thrust $\mathrm{N}=-4.188548$

V=5" and $p=5$ Strain in top gagex-38.31582
Strain in bottor gage $=31.28758$
Howent $M=2.718893$
Thrust $\mathrm{N}=-13.17794$

Hz5l and $p=10$ Strain in top gagez-131. 8684
Strain in bottom gage $=132.643$
Homent $M=10.33248$
Thrust $k=-1.452284$
$W=0$ and $P=1$
Norwalized Moment $=1.153332$
Nornalized Thrust $=-5.124572$
$\psi=0$ and $P=5$
Normalized Moment $=1.258871$
Normalized Thrust $=-2.824393$
$k=0$ and $p=16$
Normalized Moment $=1.390795$
Normalized Thrust $=5.082669 \mathrm{E}-82$

Leads at loading point * 6

Coefficients $\mathrm{Cl}, \mathrm{C}$ and Cl for equations of curve fitting

|  | Top gages | Bot. gages |
| :--- | ---: | ---: |
| C 1 | 20.79999 | -37.64646 |
| C 2 | -10.76358 | 11.33981 |
| C 3 | -.4517899 | .5840169 |

$W=5$ and $P=0$ Strain in top gage 20.79999
Strain in bottom gage=-37. 64646
Mowent $\mathrm{n}=-2.253864$
Thrust $\mathrm{N}=-31.58713$
$W=51$ and $P=1$ Strain in top gage 9.584618
Strain in bottom gage=-25.88262
Moment $M=-1.382314$
Thrust $\mathrm{N}=-30.48876$
$4=51$ and $P=5$ Strain in top page $=-44.31256$
Strain in bottom gage $=31.65308$
Mowent $h=2.96741$
Thrust $N=-23.73681$
$W=5$ and $P=18$ Strain in top gage $=-132.0148$
Strain in bottow gage $=126.1534$
mowent $x=10.68469$
Thrust $N=-10.99019$
$k=0$ and $p=1$
Normalized Moment $=.9087501$
Normalized Thrust $=-1.178352$
$v=0$ and $\mathrm{P}=5$
Normalized Moment $=1.050995$
Normalized Thrust $=-1.57896$
$H=0$ and $P=10$
Normelized Howent $=1.236776$
Mormalized Thrust $=-2.65963$

## Loads at loading point :7

Coefficients $\mathrm{Cl}_{1} \mathrm{CL}$ and CO for equations of eurve fitting
Top gages Bot. gages

C $1 \quad 27.55356 \quad-26.26968$
C 2 -1.173019 -. 1623383
C 3 . $532599-.4634824$
$N=5 F^{2}$ and $p=0$ strain in top gage= 27.55356
Strain in bottom gage=-26. 25968
Mowent
Thrust $N=-2.424145$

Strain in bottom gage $=-26.9065$
Mowent $H=-2.182399$
Thrust $\mathrm{N}=1.2423995-82$
$W=5 t^{2}$ and $P=5$ Strain in top gage= 35.00319
Strain in botton gage=-39.15943
Howent $H=-2.896977$
Thrust $\mathrm{N}=-7.792962$
$W=51$ and $P=10$ Strain in top gage= 69.28ex6
Strain in botton gage $=-76.2323$
moeent $N=-5.67635$
Thrust $N=-13.4853$
$H=0$ and $P=1$
Mormalized Mowent $=-2.185534 \mathrm{E}$ - 84
Normalized Thrust $=-2.411721$
$\Rightarrow$ and $p=5$
Normalized Mowent $=-.1589717$
Normalized Thrust $=-2.843421$
$4=8$ and $p=18$
Normalized Moment $=-.3574231$
Normalized Thrust $=-1.583047$

Loads at loading point $\& 8$
Coefficients $\mathrm{Cl}_{1} \mathbb{C 2}$ and C for equations of curve fitting


Loads at loading point $\mid 9$
Coefficients $\mathrm{Cl}, \mathrm{CL}_{2}$ and $\mathrm{C3}$ for equations of curve fitting


Loads at loading point * 18
Coefficients $\mathrm{Cl}, \mathrm{CZ}$ and C 3 for equations of curve fitting

|  | Top gages | Bot. gages |
| :--- | :--- | :--- |
|  |  |  |
| C 1 | 30.13927 | -19.13214 |
| C 2 | 1.459839 | -1.399468 |
| C 3 | $-1.741828 E-92$ | $5.803189 E-03$ |

$N=5$ and $\mathrm{P}=8$ Strain in top gage= 38.13927
Strain in botton gage=-19.13214
Mowent $1=-1.924654$
Thrust $N=-20.63836$
$W=5$ and $P=1$ Strain in top gage $=31.5817$
Strain in bottom gage-e-28.52581
Mrment $M=-2.835449$
Thrust $N=-26.72979$
105 and $P=5$ Strain in top gage $=37.00321$
Strain in bottom gage- 25.9844
Moment $M=-2.469453$
Thrust $\mathrm{N}=-28.65826$
$W=5$ and $P=18$ Strain in top gage $=42.99563$
Strain in bottod gage=-32.54651
Howent $y=-2.950394$
Thrust $N=-19.59399$
$H=0$ and $P=1$
Normalized Moment $=-1107849$
Normalized Thrust $=9.143114 E-22$
$W=0$ and $P=5$
Norwalized Moment $\approx-.1071578$
Norealized Thrust $=4.378796 E-83$
$W=8$ and $P=18$
Mormalized Moment $=-, 1866239$
Normalized Thrust $=-.184381$

## 

* Sirain gage nos. 17 and 18

H

Loads at loading point \# 1
Coefficients Cl , $\mathrm{CL}_{2}$ and $\mathrm{C3}$ for equations of curve fitting


Loads at loading point 12
Coefficients $\mathrm{Cl}_{4} \mathrm{C2}$ and C 3 for equations of curve fitting

|  | Top gages | Bot. gages |
| :---: | :---: | :---: |
| C1 | 11.76427 | -19.99714 |
| C 2 | 6.188935 | 6.653214 |
| C3 - | -. 1866874 | . 1981779 |
| K=5 and $\mathrm{P}=0$ | Strain Strain in | op gage $=11.76427$ <br> botton gages-19.99714 |
| Mosent | t $m=-1.237$ |  |
| Thrust | t $\mathrm{N}=-15.26$ |  |
| $\\|=5 \%$ and $p=1$ | Strain in Strain i | op gage $=17.76659$ <br> otton gage=-26. 37017 |
| Moment | the-1.724 |  |
| Thrust | $t \mathrm{~N}=-16.13$ |  |
| $\\|=5 *$ and $P=5$ | Strain i <br> Strain i | pp gage $=38.84376$ tton gage $=-48.41876$ |
| Moment | \% $=-3.377$ |  |
| Thrust | $t \mathrm{~N}=-19.45$ |  |
| $4=54$ and $P=10$ | Strain Strain in | 00p gage= 54. 99288 <br> th om gage $=-67.42148$ |
| Moment | the-4.781 |  |
| Thrust | t $\mathrm{N}=-23.30$ |  |
| $\\|=0$ and $P=1$ |  |  |
| Morualized Moment $=-.4869282$ |  |  |
| Normitized Thrust $=-.8636287$ |  |  |
| $\psi=0$ and $p=5$ |  |  |
| Norwalized Mowent $=-.4288555$ |  |  |
| Nornalized Thrust $=-8.870495$ |  |  |
| $V=8$ and $P=10$ |  |  |
| Noremalized Moment $=-$, 3544647 |  |  |
| Nornalized Thrust $=-.0035 / 55$ |  |  |

Loads at loeding point \# 3
Coefficients $\mathrm{Cl}, \mathrm{C} 2$ and $\mathrm{C3}$ for equations of curve fitting

|  | Top gages Bot. gages |
| :---: | :---: |
| C1 | $27.90711-35.30504$ |
| C 2 | 6.201111 -7.58956\% |
| C 3 | 6.8748485-82 -5.652451E-82 |
| $4=54$ and $P=8$ | Strain in top gage= 27.99711 <br> Strain in botton gage $=-35.32564$ |
| Mowent | ( $\mathrm{m}-2.470026$ |
| Thrust | $t=-13.98863$ |
| $N-5 *$ and $P=1$ | Strain in top gage $\mathbf{3 4 . 1 7 6 5 7}$ <br> Strain in bottox gape=-42.87288 |
| nowent | 爯-3.813634 |
| Thrust | ( $\mathrm{N}=-16.49883$ |
| $12=51$ and $p=5$ | Strain in top gage 66.63137 |
|  | Strain in bottoes gage=-74.89869 |
| Mowent | M=-5.293831 |
| Thrust | k=-26.73622 |
| W=54 and $P=18$ | Strain in top gage $=9.78386$ |
|  | Strain in bottom gage $=-117.7826$ |
| Moment | t $m=-8.38186$ |
| Thrust | $\mathrm{N}=-39.35529$ |
| $W=0$ and $P=1$ |  |
| Norma | alized Mowent $=-.5436285$ |
| Norma | alized Thrust $=-2.58215$ |
| $4=0$ and $p=5$ |  |
| Norma | alized Monent $=-.564765$ |
| Norme | alized Thrust $=-2.565517$ |
| $4=8$ and $P=18$ |  |
| Nornalized Moment $=-.5311854$ |  |
| Normatized Thrust $=-2.544565$ |  |

Loads at loading point is
Coefficients C1, C2 and C3 for equations of curve fitting


## Loads at loading point 5

Coefficients $\mathrm{Cl}, \mathrm{C} 2$ and C 3 for equations of curve fitting


Loads at loading point $\$ 6$
Coefficients Cl , $\mathbf{C 2}$ and C 3 for equations of curve fitting


Loads at loading point 4
Coefficients $\mathrm{Cl}, \mathrm{C2}$ and $\mathrm{C3}$ for equations of curve fitting


Loads at loading point \$ 8
Coefficients $\mathrm{Cl}, \mathbf{C 2}$ and $\mathbf{C 3}$ for equations of curve fitting


Loads at loading point $\geqslant 9$
Coefficients Cl; $C 2$ and $C 3$ for equations of curve fitting
Top zages Bot. gages
C $1 \quad 25.1392 B \quad-24.19641$
C 2.7512818 . 3337482
C $3 \quad .4683237-.5915184$

Strain in bottom gage-24.19641
Mowent $1=-1.527176$
Thrust $N=-1.767883$
$U=5 \#$ and $P=1$ Strain in top gager 26.35887
Strain in bottow gage=-24.45419
Howent $M=-1.984885$
Thrust $N=-3.571272$
$1=54$ and $P=5$ Strain in top gage 40.60328
Strain in bottoo gage=-37.31567
Moeent $\mathrm{M}=-3.843799$
Thrust $1=-6.164274$
$W=53$ and $P=10$ Strain in top gage= 79.48247
Strain in botto gage=-B9.01805
Howent $M=5.238288$
Thrust \% $\%$-. 998715
$H=0$ and $P=1$
Norsalized Monent $=-5.778952-Q 2$
Normalized Thrust $=-1.883389$
$4=1$ and $P=5$
Mormalized Mowent $=-.2233067$
Hormalized Thrust $=.8799782$
$\|=0$ and $p=10$
Norealized Moment $=-.4383832$
Normalized Thrust $=-$.2758598

Loads at loading point 10

Coefficients $\mathrm{Cl}, \mathrm{C} 2$ and C 3 for equations of eurve fitting
Top gages Bot. gages
C $1 \quad 23.95785 \quad-21.0687$
C $2 \quad 1.968399-1.117333$
C 3 -3.5266885-02 -2.098274E-02
$N=5$ and $P=$ Strain in top gaga= 23.95785
Strain in botten gace-21.0007
Homent $\mu=-1.758928$
Thrust $\mathrm{N}=-5.450907$

When $\mathrm{F}=1$ Strain in top gage= 25.90998
Strain in botton gagez-22. 19962
Moment $\mathrm{M}=-1.878986$
Thrust $N=-6.941189$
$W=54$ and $p=5$ Strain in top gage 32.92817
Strain in botton gage-27.17193
Homent $\mathrm{F}=-2.34766$
Thrust $N=-10.792 \%$

What and $\rho=10$ Strain in top gage 40.12515
Strain in bottom gages-34. 3323
Homent 欮一2.909494
Thrust $\mathrm{N}=-16.8616$
$1=0$ and $P=1$
Norwalized Moment $=-1199784$
Normalized Thrust $=-1.49868$
$\|=0$ and $P=5$
Morwalized Moment $=-.1177455$
Norsalized Thrust $=-1.06841$
40 and $p=10$
Normalized Mowent $=-.1149566$
Normalized Thrust $=.54106 \mathrm{~g}$

Loads at loading point : :
Coefficients $\mathrm{Cl}_{1} \mathbb{C}$ and $\mathbf{C 3}$ for equations of curve fitting


Loads at loading point $\leqslant 2$
Coefficients $\mathrm{Cl}_{1}$ C2 and CO for equations of curve fitting
Top gages Bot, gages
C $1 \quad-35.71073 \quad 32.78356$
C $2 \quad-.6698151 \quad-.1876755$
C3 6.026554E-82-2.7231275-42
$W=5$ and $P x$ Strain in top gager -35.71673 Strain in bottom gage $=32.7035$ B
Homent $m=2.672434$
Thrust $N=-5.63839$
1451 and $p=1$ Strain in top gage=-36. 32928 Strain in bottom gagee $=32,48858$
Howent $=2.68785$
Thrust $N=-7.184243$
$U-51$ and $P=5$ Strain in top gage $=-37.55316$
Strain in bottom gage= 31.09443
Homent $1=2.651156$
Thrust $N=-12.12888$
$W=54$ and $p=10$ Strain in top gage $=-36.38232$
Strain in bottom gage $=28.10371$
Howent $M=2.518985$
Thrust $N=-15.5224$
$1 k=0$ and $p=1$
Nornalized Homent $=1.541585 \mathrm{E}-\mathrm{Q} 2$
Noralized Thrust $=-1.545854$
$4=0$ and $P=5$
Horwalized Mceent $=1.744375 \mathrm{E}-03$
Mormalized Thrust $=-1,298898$
$n=6$ and $p=10$
Normalized Moment $=-1.534484 E-92$
Normalized Thrust $=-.988401 ?$

Laads at loading point : 3
Coefficients $\mathrm{Cl}_{1} \mathrm{CP}_{2}$ and Cl for equations of curve fitting


Loads at loading point : 4
Coefficients $\mathrm{Cl}, \underset{\sim}{C}$ and $\mathbf{C 3}$ for equations of curve fitting
Top gages Bot. gages
C 1 -25.16872 13.82143
C 2 2.128391 -1.651785
C $3 \quad-5.312538 E-82-5.6249985-62$
$N=5$ and $P=8$ Strain in top gage=-25. 16972
Strain in bottom gages 13.82143
Howent 乍 1.49149
Thrust $N=-22.76117$
$W=51$ and $P=1$ Strain in top gager- 23.00545 Strain in botton gage $=11.32339$
Mowent $1=1.343314$
Thrust $N=-22.89135$
U-5il and $P=5$ Strain in top gage=-15. 8469
Strain in bottom gage= 3.30625
Howent $\mid=.7481699$
Thrust $N=-23.5137$
$W=5$ and $P=10$ Strain in top gage $=-9.189342$
Strain in bottom gage=-9.22142
\#cment $\mathrm{M}=-1.253839 \mathrm{E}-03$
Thrust $\mathrm{N}=-34.52018$
$1 * 0$ and $P=1$
Normelized Monent $=-.1481758$
Aormalized Thrust $=.6698984$
100 and $P=5$
Normalized Moment $=-.148554$
Nornalized Thrust $=-.1585073$
$H=8$ and $P=18$
Nornalized Howent $=-.1492743$
Hormalized Thrust $=-1.175922$

Loads at loading point
Confficients $\mathrm{Cl}_{3}, \mathbb{C}$ and C 3 for equations of curve fitting


Loads at loading point \# 6
Coefficients $\mathrm{Cl}, \mathrm{CO}$ and C 3 for equations of curve fitting
Top gages Bot. gages
C $1 \quad 503.14644 \quad 43.76428$
C $2 \quad 8.214829 \quad-7.156784$
C 3 . $2915182-.384465$
$1-54$ and $p=8 \quad$ Strain in top gage-63. 14644
Strain in bottom gages 43.75428
Howent $=4.1762$
Thrust $\mathrm{N}-35.341 \mathrm{~m}$
$W=54$ and $p=1$ 5train in top gage=-54. 54209
Strain in bottom gage 36.38393
Moment $k=3.552466$
Thrust $N=-34.38199$
$\|=54$ and $p=5$ Strain in top gagem-14.78434
Strain in bottom gage= . 3687287
Mowent $f=.5919168$
Thrust $\mathrm{F}=-27.82927$
$W=5{ }^{2}$ and $\rho=10 \quad$ Strain in top gage $=46.15367$ Strain in botton gage--58. 25009
Moment $1=-4.156397$
Thrust 18.93979
$N=8$ and $p=1$
Normalized Moment $=-.6237343$
Norsalized Thrust $=-1.959558$
$\|=0$ and $p=5$
Norsalized Moment $=-.7168567$
Hormalized Thrust $=-1.862454$
$W=8$ and $p=18$
Norwitilized moment $=-.8332597$
Nornalized Thrust $=-1.741876$

Loads at loading point 7
Coefficients $\mathrm{Cl}_{1} \mathbb{C 2}$ and C 3 for equations of curve fitting


Loads at loading point * 6
Coefficients $\mathrm{Cl}, \mathrm{C} 2$ and C 3 for equations of curve fitting


Loads at loading point if
Coefficients $\mathrm{C1}, \mathrm{C} 2$ and $\mathrm{C3}$ for equations of curve fitting

|  | Top gages | Bot. gages |
| :--- | :--- | :--- |
| C 1 | -85.68933 |  |
| C 2 | -9.38713 |  |
| C 3 | -.724426 | 9.840405 |
|  | -.8924256 | .888389 |

$\|=5 *$ and $P=0 \quad$ Strain in top gage $=-85.68933$ Strain in bottom gage $=92.30713$
moment $M=6.952987$
Thrust $\mathrm{N}=-12.4887$
$W=5$ and $P=1$ Strain in top gage=-96. 38618 Strain in bottom gage $=182.2359$
Moment $1=7.755553$
Thrust $\mathrm{N}=-11.11826$
$W$-5* and $P=5$ Strain in top gage=-156.6221 Strain in bottom gaje $=159.7189$
Mowent $N=12.35707$ Thrust $N=-5.89654$
$U=5 *$ and $P=18 \quad$ Strain in top gages-272. 1762 Strain in botto gage $=271.5502$ Moment $1=21.23931$ Thrust $\mathrm{H}=-1.173763$
$1=0$ and $P=1$
Normalized Moment $=.8025643$
Normailzed Thrust $=-1.290107$
$\|=0$ and $p=5$
Normalized Nowent $=1.080817$
Normslized Thrust $=-1.322374$
$\omega=0$ and $P=10$
Normalized Moment $=1.428532$
Normalized Thrust $=-1.358213$

## Loads at loading point $\geqslant 10$

Coefficients $\mathrm{Cl}, \mathrm{C} 2$ and $\mathrm{C3}$ for equations of curve fitting


* Strain qage nos. 21 and 22


Loads at loading point \# 1
Coefficients Cl , $\mathbb{C 2}$ and C 3 for equations of curve fitting

|  | Tap gages | Bot. pages |
| :---: | :---: | :---: |
| C 1 | -35.43215 | 29.99643 |
| C 2 | . 9683914 | . 1841187 |
| C 3 | -.0682664 | -4, 955938\%-82 |

$W=5$ and $p=8 \quad 5$ train in top qage $=-35.43215$ Strain in bottom gage 29.99643
Moment $=2.555884$
Thrust $N=-10.19897$
145 and Pa 1 Strain in top gage=-34. 52482 Strain in bottom gagz= 30.05899
moment $h=2.523462$
Thrust $N=-3.306938$
$W=51$ and $p=5 \quad 5 t r a i n$ in top gages-33.0969
Strain in bottom gage 29.27816
Moment $=2.397463$
Thrust $=-5.285132$
$1=5 \#^{\prime \prime}$ and $P=10$ Strain in top gage=-31. 77507
Strain in bottom gages 26.0835
Movent $=2.268251$
Thrust $=-10.67425$
$4=9$ and $p=1$
Morsalized Mowent $=-3.3342395-$ Q2
Normalized Thrust $=-1,885827$
$H=8$ and $p=5$
Mormalized Mowent $=-.8316681$
Normalized Thrust $=.9813666$
$\Rightarrow$ and $p=10$
Noral ized Homent $=-2.5575235-0.2$ Normalized Thrust $=-4,888834 E-22^{2}$

Laads at laading point 2
Coefficients $\mathrm{Cl}_{1}$ [2 and C 3 for equations of curve fitting


Loads at loading point 3
Coefficients C1, [2 and [3 for equations of curve fitting


Loads at loading point 4
Coefficients $\mathrm{Cl}, \mathbf{C 2}$ and $\mathrm{C3}$ for equations of curve fitting


## Loads at loading point :5

Coefficients Cl , 22 and C for equations of curve fitting

coads at loasing point : 6
Coeffisients CI, C2 and C3 for equations of curve fitting

| Top gages Bot. gages |  |  |
| :---: | :---: | :---: |
| C1 - | -56.5893 | 34.9536 |
| C2 | B. 479828 | -7.245178 |
| C3 | , 2647324 | -. 3334847 |
| $W=5 \#$ and $P=0$ | Strain in top gagee-56,5893 |  |
|  | Strain in bottom gages 34.9536 |  |
| Moment M=3.575894 |  |  |
| Thrust $\mathrm{N}=-40.56693$ |  |  |
| $W=5 \#$ and $P=1$ | Strain in top gagex-47.84474 |  |
|  | Strain in bottea gage 27.37484 |  |
| Wrant $\mu=2.938268$ |  |  |
| Thrust $\mathrm{N}=-38.38088$ |  |  |
| $n=5{ }^{2}$ and $P=5$ | Strain in top gage=-7.571846 |  |
|  | Strain in bottor gage=-9.689409 |  |
|  | M $=-7.959332 \mathrm{E}-{ }^{2}$ |  |
| Thrust $N=-32.21486$ |  |  |
| $\cdots=5 x^{2}$ and $P=10$ | Strain in top gage $=54.68282$ |  |
|  | Strain in bottom gapme-70.04665 |  |
| Macent | \% $=-4.903$ |  |
| Thrust | ( $\mathrm{N}=-30.38$ |  |
| $W=0$ axd $F=1$ |  |  |
| Normalized Monent $=-.637625$ |  |  |
| Norma'ized Thrust $=-2.186858$ |  |  |
| $h=9$ and $p=5$ |  |  |
| Normalized Roment $=-.7318973$ |  |  |
| Norcalized Thrust $=-1.670416$ |  |  |
| $\omega=0$ and $P=10$ |  |  |
| Nornalized Moment $=-$, 8479365 |  |  |
| Normalized Thrust $=-1.825863$ |  |  |

Loads at loading point \# 7
Coefficients $\mathrm{Cl}, \mathrm{C} 2$ and C 3 for equations of curve fitting


Lcats at loading point B

Copfficients $\mathrm{Cl}, \mathrm{Cl}$ and $\mathrm{C3}$ for equations of curve fitting


Loacs at loading point 5
Coefficients $\mathrm{Cl}, \mathrm{CD}$ and $\mathrm{C3}$ for equations of curve fitting
Too gages Bot. gages
C! $\quad-78.62146 \quad 78.34998$
C 2 - $20.25537 \quad$ 10.5476!
C3 -.997329: . 3812545
$H=5$ and $P=$ Strain in top gage $=-78.62146$
Strain in bottow gage $=78.34998$
Moment $1=6.131697$
Thrust $\mathrm{N}=-.5858332$
$k=5$ and $P=1$ Strain in top gage $=-89.87416$
Strain in bottos gage $=88.87894$
Hosent $M=7.821621$
Thrust $N=8.783341 E-23$
\#54 and $2=5$ Strain in top gage $=-154.8314$
Strain in bottem page $155.619 \%$
Fowent $t=12.12698$
Thrust $N=-1.477547$
$h=5 \%$ and $P=10$ Strain in top gage $=-280.9074$
Strain in bottom gage $=281.9515$
Kosent. $1=24.98558$
Thrust $\mathrm{N}=-1.95774!$
$h=8$ and $p=$ :
Norkalized Moment $=.8899045$
Normalized Thrust $=.5178166$
$\mathrm{N}=0$ and $\mathrm{F}=5$
Norwal:zed Moment $=1.199057$
Normalized Thrust $=.397316$
$W=8$ and $F=18$
Norwalized Moment $=1.585498$
Normalized Thrust $=.2466774$

Loads at loading poift: 10
Copfficients Cl , C 2 and CJ for equations of curve fitting


## **

* Strair gage nos. 23 and 24


Loads at loading point ! !

Cosfficients $\mathrm{Cl}, \mathrm{C} 2$ and $\mathrm{C3}$ for equations of curve fitting


Loas5 at loading point \# 2
Coefficients Cl , $\mathrm{C2}$ and $\mathrm{C3}$ for equations of curve fitting


Load5 at Loading point 13
Coefficients C1, C 2 and C for equations of curve fitting
Top gages Elot. gages
C1 $-26.05072 \quad 14.825$
C 2 -1.439461 -. 390893
C 3 . $212946-.1691965$
$W=5 \frac{1}{*}$ and $P=0$ Strain in top gage=-26. 86872
Strain in bottor gage $=14.825$
Movent M 1.1 .565849
Thrust $N=-22.56693$
$h=5 \#$ and $F=1$ Strain in top gage $=-27.28724$ Strain in botton page= 13.46491
Worent $\%=1.591881$
Thru5: No-25.9168\%
$h=5{ }^{\prime}$ and $p=5$ Strain in top gages- 27.93438 Strain in bettom gaga= 7.848623
Worent $W=1.357461$
Thrust $\mathrm{N}=-37.67579$
$W=5$ and $p=18 \quad$ Strain in top gage $=-19.16074$
Strein in bettom gapa=-6.603576
Voment $N=.4827816$
Thrust $\mathrm{N}=-48.68309$
$W=\varnothing$ and $P=1$
Nomalized Homent $=2.603308 E-42$
Normalized Thrust $=-3.349882$
$W=2$ and $p=5$
Aorwa!ized Mowent $=-3.36775 \mathrm{SE}-82$
Normalized Thrust $=-3.821762$
$h=2$ and $P=18$
Normelized Mowent $=-.1683147$
Norralized Thrust $=-2.611611$

Loads at loading point |
Coefficients Cl , C 2 and $\mathrm{C3}$ for equations of curve fitting

$W=5$ Hid $^{1} p=1$ Strain in top gage=-14.2975 Strain in bottom gage=-6. 162235
Fonent $k=.2318532$
Thrust $\mathrm{N}=-41.73666$
$W=5$ \# and $p=5$ Strain in top gage=-8. 337513
Strain in bottoe gage=-13.84065
Norent $M=-2149564$
Thrust $N=-41.5848$
insent and $p=: \ell \quad$ Strain in top gage=-6. 842894
Strain in bottoe gage=-24.25364
Monent $K=-.6801071$
Thrust $N=-58.38599$
$W=0$ and $P=1$
Nornalized Moment $=-.1228889$ Normalized Thrust $=-1.917338$
$\omega=0$ and $p=5$ Normalizec Moment $=-.1137801$ Normalized Thrust $=.4139876$
$\omega=0$ and $P=18$
Normalized Moment $=-.1834041$
kormalized Thrust $=-1.4652$

Loas at loading point 5
Coefficients Cl , CL and $\mathrm{C3}$ for equations of curve fitting


Loads at loading point \# 6
Coofficients C1, $\mathbf{C 2}$ and C ] for equations of curve fitting


Loads at loading point $\$ 7$
Coefficients $\mathrm{Cl}, \mathrm{C} 2$ and $\mathrm{C3}$ for equations of curve fitting

|  | Top gages | Bot. papes |
| ---: | ---: | ---: |
| C 1 | -50.51874 | 39.77857 |
| C 2 | 6.611603 | -7.431059 |
| C 3 | -.6548263 | .76982 ch |

$U-51$ and $0=8$ Strain in top gage=-59.51874 Strain in bottom gage $=39.77857$
Koment $M=3.526926$ Thrust $\mathrm{N}=-22.12283$
$W=5$ and $p=1$ Strain in top gage=-44.55316
Strain in bottom gage= 33.05732
Nonent Ma 3.031659
Thrust $N=-21.5547$
$W=5{ }^{*}$ and $p=5$ Strain in top gape $=-33.80324$
Strain in botto gage $=28.35879$
Fowent $k=2.116835$
Thrust $N=-25.18959$
$W=5$ and $p=10 \quad$ Strain in top gage $=-49.79675$
Strain in botto gape= 36.45815
Moernt $10=3.369019$
Thrust $\mathrm{N}=-25.82487$
$W=0$ and $p=1$
Normalized Moment $=-.4952685$
Normalized Thrust $=-1.431871$
$h=0$ and $P=5$
Normalized Moment $=-.2821663$
Normalized Tnrust $=-1.01335 \mathrm{t}$
$\omega=8$ and $p=10$
Normalized Moment $=-1.575069 \mathrm{E}-42$ Normalized Thrust $=-.4982041$

Loads at loading point \& 8
Coefficients C1, C2 and C3 for equations of curve fitting

|  | Top gages | Bot. gages |
| :--- | :--- | :--- |
| C 1 | -61.3501 | 57.48578 |
| C 2 | -8.778198 | 11.58641 |
| C 3 | -.9856181 | .7767945 |

$W=5$ and $P=0 \quad 5 t r a i n$ in top gage -51.3501
Strain in bottom gage $=57.48578$
Homent $=4.642027$
Thrust $\mathrm{N}=-7.24559$
$W=5$ and $\mathrm{P}=1$ Strain in top gage-71.11491 Strain in botton gage $=69.76898$
Monent $M=5.583278$
Thrust $N=-2.523623$
$W=5$ and $P=5$ Strain in top gage=-129, 9065 Strain in bottom gaye $=134.4377$ *ovent $h=10.32595$ Thrust $N=-8.495894$
$W=54$ and $P=10$ Strain in top gage=-247.7939
Strain in bottom gage $=250.2293$
Howent $M=19.45483$
Thrust $N=-4.566422$
$H=0$ and $P=1$
Normalized Moment $=.8612598$
Normalized Thrust $=\mathbf{- 4 . 7 2 1 9 7 6}$
$W=0$ and $P=5$
Nommalized Moment $=1.136784$
Normalized Thrust $=\mathbf{- 3 . 1 4 8 2 9 8}$
$H=0$ and $P=10$ Norwalized Monent $=1.481281$ Nomalized Ttrust $=-1.18128$

Loats at loading point $\# 9$
Coefficients Cl, Ce and C for equations of curve fitting

|  | Top gages | Bot. gages |
| :--- | :--- | :--- |
| C 1 |  |  |
| C 2 | -74.71423 | 62.78931 |
| C 3 | -11.43427 | 11.68596 |
|  | -1.02858 | 1.82813 |

$W=5$ and $p=8$ Strain in top gagez-74. 71423 Strain in bottom gage $=82.78931$
Honent
Thrust $N=-15.14076$

W-54 and $\mathrm{P}=1$ Strain in top gage=-87.17708 Strain in bottom gage $=95.42339$
Homert $1=7.132831$
Thrust $N=-15.46184$
$W=5 \%$ and $P=5$ Strain in top gage-157.6001 Strein in bottom gages 166,5223
Moment $A=12.56103$
Thrust $N=-16.72929$
$W=5 \#$ and $P=18$ Strain in top gage=-291. 9149
Strain in bottom gage $=301.6618$
Moment $=23.18659$
Thrust $\mathrm{N}=-18.27559$
$H=0$ and $P=1$
Normalized Moment $=.9893489$
Niomalized Thrust $=.3218783$
$K=8$ and $P=5$
Nonslized Monent $=1.38171$ Normalized Thrust $=.3177852$
$W=Q$ and $P=18$
Normalized Moment $=1.783411$ Normalized Thrust $=.3134823$

Loads at loading point 10
Coefficients $\mathrm{C1}$, 22 and $\mathrm{C3}$ for equations of curve fitting
Top gages Bot. gages

C $1 \quad-60.54285 \quad 72.92582$
C2 -1.701447 2.671265
C 3 -5.060115E-02-3.120422E-03
$N=54$ and $P=0$ Strain in top gage $=-60.54285$ Strain in bottom gage= 72.92582
Monent $M=5.213589$
Thrust $\mathrm{N}=-23.21657$
$W=5 \frac{1}{2}$ and $P=1$ Strain in top gage -62.2943 Strain in botton gage= $\mathbf{7 5 . 5 9 3 1 5}$
Homent $=5.3962 \mathrm{c} 9$
Thrust $\mathrm{N}=-24.93538$
$W=5 A^{2}$ and $P=5 \quad$ Strain in top gape-70.30011 Strain in bottom gage $=86.20333$
Horent $=\mathrm{E} .113416$
Thrust $\mathrm{N}=-29.81854$
$U=54$ and $P=18$ Strain in top gage $=-82.55743$
Strain in botion gage $=99.32562$
howent $=7.104887$
Thrust $\mathrm{N}=-31.44837$
$\mathrm{H}=0$ and $\mathrm{P}=1$
Normalized Moment $=.1726404$
Normalized Thrust $=-1.718883$
$W=0$ and $\mathrm{P}=5$
Normalized Moment $=.1799654$
Normalized Thrust $=-1,320394$
$\mathrm{H}=8$ and $\mathrm{P}=1 \mathrm{C}$
Normalized Nomant $=.1891218$
Normalized Thrust $=.82237 \%$

## $\$$ Strain gage nos. 25 and 26



Loads at loading point 1
Coefficients Cl, C2 and $\mathbf{C 3}$ for equations of curve fitting

|  | Top gages | Bot. gages |
| :--- | :--- | ---: |
|  |  |  |
| $C 1$ | 86.96879 | -59.73218 |
| $C 2$ | $3.3928825-92$ | 1.773428 |
| $C 3$ | $-6.115723 E-02$ | -.0977726 |

$W=5$ and $P=0 \quad$ Strain in top pape $=85,96079$ Strain in bottom gager-59.73218
Froent $M=-5.738194$
Thrust $N=-51.65354$
$W=5$ and $P=1$ Strain in top qage= B6.93265 Strain in bottom gagpe-56.85653
Foment $M=-5.663539$
Thrust $N=-54.14273$
$W=5$ and $P=5$ Strain in top gage= 85.5969
Strain in bottom gage $=-53.38938$
Mowent $=-5.425829$
Thrust $N=-68.53921$
$W=5{ }^{*}$ and $P=18 \quad$ Strain in top gage $=81.17526$
Strain in botton gang $=-51.77522$
Moment $M=-5.193378$
Ihrust $N=-55.12509$
$h=0$ and $p=1$
Norwalized Moment $=6.655425 E-80$
Norealized Thrust $=-3.68909$
$H=0$ and $p=5$
Nornalized Mowent $=6.883304 \mathrm{E}-92$
Noralized Thrust $=-1.897113$
$W=8$ and $p=10$
Normalized Mowent $=5.368158 E-82$ Normalized Thrust $=.487145$

Loads at loating point : ?

Coefficients $\mathrm{Cl}, \mathrm{CZ}$ and CJ for equations of curve fitting


Loads at loading point 3
Coefficients $\mathrm{Cl}, \mathrm{C2}$ and $\mathrm{C3}$ for equations of eurve fitting


Loads at loasing point \$4
Coefficients $\mathrm{C1}, \mathrm{C}$ and $\mathrm{C3}$ for equations of curve fitting
Top gages Bot. gages
C $1 \quad 56.95716 \quad-31.43929$
C $2 \quad$-2. $882855 \quad 2.485893$
$\begin{array}{lll}C .3 & -1857147 & \text { B. } 525754 E-62\end{array}$
$W=5$ and $p=$ Strain in top gager 58.95716 Strain in bottor gage=-31.43929
Moment $\mathrm{M}=-\mathbf{3} .531111$
Thrust $N=-51.596$
$W=54$ and $P=$ : Strain in top gage= 56,68859
Strain in bottor gage $=-28.85813$
Mopent $\quad 1=-3.342059$
Thrust $\mathrm{N}=-52.16336$
$W=5 *$ and $P=5$ Strain in top gager 43.98001 Strain in bottom gager-16. 87813
Morent M $=-2.374146$
Thrust $k=-52.66680^{\circ}$
$W=5$ and $D=10$ Strain in top gage= 19.55713
Strain in botto gage= 1.9464
Moment $1=-.6879191$
Thrust $N=-40.31912$
$H=0$ and $P=1$
Normalized Moment $=.189952$
Norabiized Thrust $=.5673588$
$w=0$ and $p=5$
Normalized Mowent $=.231393$
Mormalized Thrust $=-1859958$
$\mathrm{W}=0$ and $\mathrm{p}=10$
Normalized Movent $=.2843192$ Normalized Thrust $=-1.127688$

## Loads at loading point 5

Coefficients $\mathrm{Cl}, \mathrm{CZ}$ and $\mathrm{C3}$ for equations of curve fitting
Top gages Bot. gages
C1 45.27857 -12.72501
C $2 \quad-.8932147 \quad-3.246245$
C $3-.55625 \quad .9156256$
$N=5$ and $p=0 \quad$ Strain in top gage 45.27857
Strain in bottom gage-12.72561
Howent: $1=-2.265765$
Thrust $N=-61.03794$
$U=5 w^{2}$ and $D=1$ Strain in top gage 43.82911
Strain in botton gage $=-15.85563$
Honent $N=-2.388185$
Thrust $\mathrm{N}=-53.95028$
$W=54$ and $p=5$ Strain in top gage 25.90525
Strain in botto gage-6.965599
Howent $N=-1.287963$
Thrust $N=-39.87623$
$W=5$ and $P=18$ Strain in top gagam-19.27857
Strain in bottox gage= 46.37569
Homent $N=2.5645 \%$
Thrust $N=-52.885 \%$
$\omega=0$ and $p=1$
Normalized Moment $=-3.442823 E-82$
Normalized Thrust $=-7.887657$
$W=8$ and $p=5$
Normalized Mowent $=.1955604$
Normalized Thrust $=-4.392343$
$W=2$ and $p=18$
Normalized Moment $=.4838361$
Normalized Thrust $=-1.823196$

## Loats at loading point $\mid 6$

Coefficients Cl , CL and $\mathrm{C3}$ for equations of surve fitting


Loads at loading point $\# 7$
Coefficients $\mathrm{Cl}, \mathrm{C2}$ and $\mathrm{C3}$ for equations of curve fitting


Loads at loading point 8
Coefficients $\mathrm{Cl}_{1}$ [2 and C 3 for equations of curve fitting

|  | Top gaģes | Bot. gages |
| :---: | :---: | :---: |
| C 1 | [55. 2643 | -67.58203 |
| C 2 | 6.692407 | -4.654907 |
| C3 | 1.490189 | -1.769374 |
| $\begin{array}{ll} k=5 \% \text { and } p=0 & \begin{array}{l} \text { Strain in top gagee } 15.2643 \\ \text { Strain in bottom gage=-87.58263 } \end{array} \end{array}$ |  |  |
| Moment M=-9.485184 |  |  |
| Thrust $N=-126.9842$ |  |  |
| W=5i and $\mathrm{P}=$ : | Strain in Strain in | 70page 162.8459 <br> tt on gage=-93.94631 |
| Nowent $\mathrm{H}=-16.03656$ |  |  |
| Thrust $\mathrm{N}=-129.1885$ |  |  |
| $W=51$ |  |  |
| Moment $\mathrm{H}=-14.76984$ |  |  |
| Thrust $\mathrm{N}=-130.1055$ |  |  |
| $4=54$ and $\rho=18$ Strain in top gages $=365,2972$ | Strain in | 20p gages 365,2872 |
|  | Strain in | tion gage=-385. 8685 |
| Monent | H=-26.18 |  |
| Thrust | $\mathrm{N}=-112.76$ |  |
| $\\|=0$ and $P=1$ |  |  |
| Normalized Moment $=\mathbf{-} .5447999$ |  |  |
| Norealized Thrust $=-2.284341$ |  |  |
| $W=0$ and $P=5$ |  |  |
| Normalized Moment $=-1.644732$ |  |  |
| Normalized Thrust $=.6464515$ |  |  |
| $\mathrm{H}=0$ and $\mathrm{P}=10$ |  |  |
| Normalized Koment $=-1.669846$ |  |  |
| Normalized Thrust $=-1.414494$ |  |  |

Loads at loading point 9
Coefficients C1, C2 and C3 for equations of curve fitting


Loads at loading point * 10

Coefficients $\mathrm{Cl}, \mathrm{C}$ and Cz for equations of curve fitting


# EXPERIMENTAL AND THEORETICAL INVESTIGATION 

OF A SHALLOW FLEXIBLE ARCH

## By

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B.E. (CIVIL), Bangalore University, Bangalore, India, 1980 M.Tech. (Engg. Mech), Indian Institute of Technology, Madras, India, 1982 AN ABSTRACT OF A MASTER'S THESIS

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This thesis presents a study of the experimental and theoretical investigation of a fixed ended flexible parabolic arch. The study is an attempt to test the applicability of a linearized deflection theory. The governing differential equation, based on the consideration of deflection of the arch, is derived and linearized for the construction of influence lines. A computer program in BASIC language, using the RungeKutta numerical integration method to construct the influence lines, through the technique of shooting method, is presented. Influence lines for the unit vertical force acting on the arch, for a given set of flexibility parameters, are constructed. The actual forces at the section of interest are then determined numerically, with the knowledge that the assumed and computed flexibility parameters must be equal. A series of experiments were conducted to verify the theoretical prediction. A model of steel arch, of span 60 inches and rise 8 inches, was built in the laboratory for this purpose. A comparison of the calculated stresses with the values obtained experimentally has been made at the end section as well as the central span section.

