

AN EXPERIMENTAL STUDY OF FLOW PATTERNS  
AND  
HEAT TRANSFER BY NATURAL CONVECTION  
INSIDE CUBICAL ENCLOSURES

by

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## NOMENCLATURE

A	inside surface area of cube
$C_p$	heat capacity of fluid inside cube
g	local acceleration of gravity
Gr	Grashof number, $\frac{\beta g \Delta T \rho^2 W^3}{\mu^2}$
h	average heat transfer coefficient
k	thermal conductivity of fluid inside cube
$k_g$	thermal conductivity of cube wall (plexiglas)
Nu	Nusselt number, $\frac{hW}{k}$
Pr	Prandtl number, $\frac{C_p \mu}{k}$
Q	total heat transfer rate
Ra	Rayleigh number, $Gr \cdot Pr$
S	thickness of the cube wall
T	fluid temperature
$T_{av}$	average fluid temperature
$T_c$	fluid temperature at center of cube
$T_o$	outside wall temperature or bath temperature
$T_s$	inside wall temperature
V	volume of cube
W	inside width of the cube
$\beta$	coefficient of volume expansion of fluid

$\Delta T$	fluid temperature difference between inside wall and center of cube
$\mu$	viscosity of fluid
$\rho$	fluid density
$\frac{dT}{dt}$	rate of change of fluid temperature inside cube

## INTRODUCTION

In technology and in the natural world around us, we constantly meet transport processes in fluids in which the motion is driven simply by the interaction of a difference in density with a gravitational field. The density difference may be due to a varying composition or, more often may be the result of temperature differences. Atmospheric and oceanic circulations, cooling oil in a power transformer, sterilization of canned foods, crystal growth from the melt, and fluid flow next to a heated or cooled object are all examples of fluid motions due to temperature differences.

All such occurrences are very similar and are called "natural convection" or "free convection". They are quite different from transport processes driven by "forced" convection from a fan or pump. The principal differences are that in natural convection little is known a priori about the resulting flow; the flow and temperature fields are invariably coupled and must be considered together; and the flows are relatively weak. The velocities are usually so small that the momentum and viscous effects are of the same order.

Among all the possible causes of natural convection the ones that have been subjected to the greatest engineering and technical investigation are those where the heater (or cooler) used to

produce the convection has much smaller dimensions than those of the vessel used to contain the fluid. The study of these cases has been primarily conditioned by the practical demands of steam boiler plants. The combination of these conditions is included under the general concept of the "external problem" of heat convection.

In contrast, when the dimensions of the heater or the cooler are comparable with the dimensions of the vessel containing the fluid, the general concept is considered as an "internal problem". Of these cases, the ones investigated in the most detail are those of heat transfer from one solid body to another through a thin layer of fluid, and from the wall of a pipe to the fluid moving within it. These phenomena are encountered in geophysics in drilled wells and problems of the geothermics of underground reservoirs, double glazing of windows, gas filled cavities surrounding the core of nuclear reactors, heat losses from pipes carrying steam or other heated fluids, furnace design, cryogenic storage containers, etc. They are of considerable practical importance as well as theoretically interesting.

The studies of internal convection problem in rectangular enclosures have focused mainly on numerical solutions. Due to the complexity of the governing equations, and the very large amount of calculation, the agreement among numerical results often is not good. Therefore, the understanding of the heat transfer process in a cube has to depend largely on experimental work.

In this research, the internal fluids, distilled water, ethylene glycol, glycerin, and their aqueous solutions were

observed by small particles suspended in the fluid. Several flow patterns were observed when a cube was subjected to a step change in temperature of outside wall. The analytic solutions of this kind of internal problems are still impossible. Although several numerical studies of the similar rectangular system were available, the consistency of these solutions was not good. Therefore, experiment results showing streamlines and heat transfer data are needed to provide the basis for evaluating the numerical techniques. The center temperature and the temperature difference between the center of the cube and outside wall were measured to calculate the heat transfer coefficients. Nusselt number was correlated as function of Rayleigh number.

## LITERATURE SURVEY

Due to the limited amount of literature about natural convection inside cubes, either experimental or numerical, this survey will cover natural convection within other major geometries such as rectangles, cylinders, and spheres.

Because of multidimensional calculations, natural convection problems require tremendous amounts of computer time and computer storage, therefore most investigations are based on experimental work. Recently more and more numerical results are becoming available. In order to carry out these calculations, authors have to make some assumptions and simplifications which limit the system parameters, the Grashof and Rayleigh numbers, to small values. Even so, the agreement between computational results is not very good. This survey will include some numerical results especially in the rectangular coordinate system.

### I. Rectangular Enclosures

Owing to the wide applications and geometric simplicity, natural convection inside rectangular enclosures has been widely investigated. The most important applications are heat transfer in the space between bricks of a cavity wall and in the region between the panes of glass in double glazed windows. In these examples, the air gap is used for insulation and it is important

to be able to determine the rate of heat transfer across the gap resulting from a temperature difference between two opposing sides.

#### A. Horizontal Layers In Rectangular Enclosures

One of the earliest descriptions of heat transfer by natural convection was written in the 1790's by Benjamin Thompson, Count Rumford. He introduced the idea to account for the transport of heat in an apple pie. There had been earlier proposals of a convective mechanism for atmospheric circulation, and a number of anecdotal reports were added to the literature throughout the 19th century. It was not until about 1900, however, that systematic investigations were undertaken. The most influential experimental work in that period was done by the French investigator Henri Bénard(11)\* in 1901. The true nature of the convective system he studied has only recently been recognized. He found that a thin layer of spermaceti, when heated from below, performed motions of distinct regular patterns (Figure 1). These cellular patterns are now called Bénard cells. Experiments have revealed that the fluid flow directions of Bénard cell, the ascent or decent in the middle of the cells, change from fluid to fluid. Usually liquids and gases have opposite circulation directions; liquids ascending and gases decending in the middle of the cell. Graham(49) suggested that the difference in circulation directions may be a result of

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\* Number in parenthesis refers to literature listed in the bibliography.



the fact that the variations in kinematic viscosity with temperature are opposite in liquids and gases. This suggestion was confirmed by Tippelskirch in 1956(113). He investigated the direction of circulation for liquid sulphur for different temperature regions. The kinematic viscosity of sulphur decreases or increases with temperature accordingly as the temperature is less than or greater than  $153^{\circ}\text{C}$ . The motion is upward in the middle of the cell in the lower temperature region and downward in the higher region.

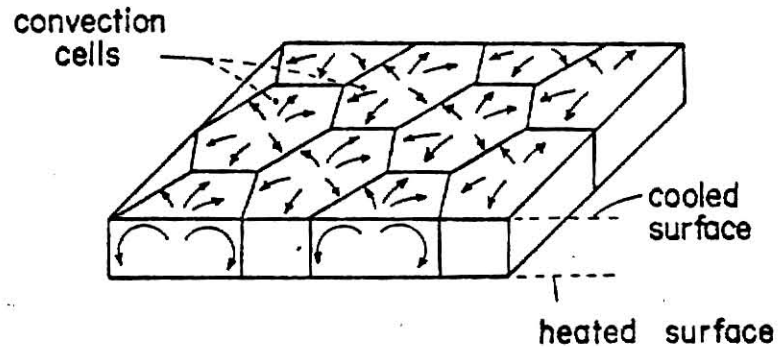


Figure. 1. Bénard, or Cellular Convection within an Enclosed Horizontal Liquid Layer.

Chandra(19) made use of smoke to visualize the flow and found that the width of the hexagonal cells increased with increasing Rayleigh number. Schmidt and Saunders(104) applied an optical method to the same subject and determined the critical Rayleigh number to be 45.000. from which turbulent motion started.

Recently Ahlers and Behringer(1) studied the onset of turbulence in a Rayleigh-Benard cell. The aspect ratio,  $A$ , is defined as the diameter of the cell divided by twice of the cell height. They suggested that,

$$Ra_t = Ra_c \quad \text{for } A = 57$$

$$Ra_t = 2 \times Ra_c \quad \text{for } A = 4.72$$

$$Ra_t = 11 \times Ra_c \quad \text{for } A = 2.08$$

( $Ra_t$  is the Rayleigh number when turbulence starts and  $Ra_c$  is the critical Rayleigh number for  $A = \infty$ ).

Although there are two parameters for natural convection flow, Gollub and Benson(48) studied the dynamics of a convecting fluid by setting the Prandtl number equal to 2.5, and classifying the phenomena into four regions: (1) for  $1 < Ra/Ra_c < 20$ , it was time independent; (2) for  $20 < Ra/Ra_c < 39.8$ , it was strictly periodic; (3) for  $31.3 < Ra/Ra_c < 41.9$ , it was a quasiperiodic state; (4) finally, a chaotic state occurred for  $Ra/Ra_c > 38.2$ . In the domains of overlap between these regions, the state was determined by initial conditions. Motivated by these experiments, Curry(27) used a Prandtl number of 10 and obtained similar results.

The heat transferred by free convection across horizontal air layers was measured by Mull and Reiher(82). Their results were correlated by Jacob(57), who presented the empirical equations as,

$$Nu = 0.295 Gr^{0.25} \quad 10^4 < Gr < 3.7 \times 10^5$$

$$Nu = 0.068 Gr^{0.33} \quad 3.7 \times 10^5 < Gr$$

de Graaf and van der Heid(28) measured heat transfer across horizontal layers by an optical technique. They observed that air

remained stationary until the Rayleigh number reached 2,000, and that turbulence was well defined at  $Ra = 60,000$ . The heat transfer results were given as

$Nu = 1$	$Gr < 2.0 \times 10^3$
$Nu = 0.0507 Gr^{0.40}$	$2.0 \times 10^3 < Gr < 5.0 \times 10^4$
$Nu = 3.8$	$5.0 \times 10^4 < Gr < 2.0 \times 10^5$
$Nu = 0.0426 Gr^{0.37}$	$2.0 \times 10^5 < Gr$

O'toole and Silverston(85) correlated heat transfer data (supplied by previous investigators) and obtained the following equations:

(a) For initial or creeping region :

$$Nu = 0.00238 Ra^{0.816} \quad \text{when } 1700 < Ra < 3500$$

(b) For the laminar region :

$$Nu = 0.229 Ra^{0.252} \quad \text{when } 3500 < Ra < 10^5$$

(c) For the turbulent region :

$$Nu = 0.104 Ra^{0.305} Pr^{0.084} \quad \text{when } 10^5 < Ra < 10^9$$

Koschmieder(66) found that the shapes of convective flow patterns were determined by their boundaries. With a circular lateral wall he obtained a system of concentric round rolls, while in a rectangular frame he observed rectangular patterns which transformed into rolls and combinations of rolls and rectangles.

The most important contribution to the theory of natural convection was given by Rayleigh in 1916(98). He showed that the onset of convective flow is determined by a dimensionless parameter which is now called the Rayleigh number. Although Rayleigh's theory was not applied to the system examined by Benard; nevertheless his work was the starting point of almost

all modern theories of convection.

Jeffreys(60,61), Low(73), Pellow and Southwell(90), and Reid and Harris(99,100) analyzed convective systems mathematically and concluded that the critical Rayleigh number was around 1709.

#### B. Vertical Layers In Rectangular Enclosures

In 1930, Mull and Reiher(82) suggested that an empirical equation for free convection of air between vertical surfaces was of the form,  $Nu = C(Gr)^m(H/W)^n$ , where  $H/W$  is the height-to-width ratio. In 1946 Jacob(58) analyzed the experimental results of Mull and Reiher and proposed the following correlations,

(A) Conduction region :

$$Nu = 1 \qquad \qquad \qquad Gr < 2000$$

(B) Laminar region :

$$Nu = 0.18 Gr^{1/4}(H/W)^{-1/9} \qquad \qquad 2 \times 10^4 < Gr < 2 \times 10^5$$

(C) Turbulent region :

$$Nu = 0.065 Gr^{1/3}(H/W)^{-1/9} \qquad \qquad 2 \times 10^5 < Gr < 1.1 \times 10^7$$

He also suggested that a correction for fluids other than air could be made by multiplying by the term,  $(Pr/0.72)^m$ , where  $m = 1/4$  for laminar flow and  $1/3$  for turbulent flow. The above equations were applicable only when  $3 < H/W < 40$ .

de Graaf and van der Held(28) combined their measurements with those of Mull and gave a smooth curve irrespective of  $H/W$  value with a maximum deviation of 10%. Their measurements gave,

$$Nu = 1 \qquad \qquad \qquad Gr < 7,000$$

$$Nu = 0.0384 Gr^{0.37} \qquad \qquad 10^4 < Gr < 8 \times 10^4$$

$$Nu = 0.0317 Gr^{0.37} \qquad \qquad 2 \times 10^5 < Gr$$

In 1961 Eckert and Carlson(37) used an interferometer to examine the temperature distribution in enclosures with several aspect ratios. They found that the heat transfer mechanism could be divided into three principle regions. First, for  $Ra < 3,200$  and  $L/W=20$ , there was an almost linear temperature gradient across the cavity over most of the central portion, corresponding to predominantly conduction heat transfer. Convection became more significant at the higher Rayleigh number and the flow eventually shifted from a "transition region" to a boundary layer type convection pattern, at  $Ra > 58,000$ .

In 1965 Elder(39,40) used a photographic technique of suspended particles to study the flow patterns. Using paraffin and silicone oil he observed stable secondary cellular flows at  $Ra > 10^5$  and tertiary flows at  $Ra > 10^6$

The first attempt to solve the related governing partial differential equations of natural convection of vertical rectangular layers was made by Batchelor(6) in 1954. He obtained an approximate solution through a power series expansion in the Rayleigh number. By his approximation, the solution was limited to Rayleigh numbers less than 1,000. The next significant analytical solution was presented by Poots(92) in 1958. He assumed that the dimensionless temperature and stream function could be represented by double infinite series of orthogonal functions whose coefficients were functions of the Prandtl and Rayleigh number, and the length-to-width ratio. Fourier transforms and iterative methods were used to determine the coefficients. The results were verified within an error of only

3%, by de vahl Davis and Kettleborough(31,32) who used finite difference techniques.

Similar solutions for free convection from a vertical plate were presented by Sparrow and Gregg(109) in 1958. By defining the wall temperature as either (i)  $T_w - T_o = N X^n$  or (ii)  $T_w - T_o = M e^{mx}$  and neglecting the axial component of the equation of motion, they were able to reduce the coupled partial differential equations to nonlinear ordinary differential equations. In Sparrow and Gregg's solution, it was assumed that there was a thin boundary layer. For liquid metals, however, the thermal boundary layer can no longer be considered to be thin. Chang, Akins and Bankoff(20) solved the same governing equations of Sparrow's system by a perturbation method and extended the applicability to liquid metals.

van Leeuwen, Looman and Schenk(115) investigated laminar free convection heat transfer in a vertical rectangular corner experimentally and presented results in a graphical form. Riley and Poots(101) divided the three dimensional corner into four regions (two flat plate boundaries, one corner boundary, and a potential flow region) and assumed a boundary flow and a potential flow for each region. The numerical results presented for air were in good agreement with Leeuwen's experimental results. Recently, Ramakrishna(96) analyzed the same system by the similar classification method and gave calculation results for both a similar and a non-similar method. The latter method exhibited better agreement.

### C. Inclined Layers In Rectangular Enclosures

Mull and Reiher(82) performed a test with an air layer inclined at 45 degrees. On the basis of their result, they suggested a linear interpolation between the vertical layer and the horizontal layer for Nusselt number.

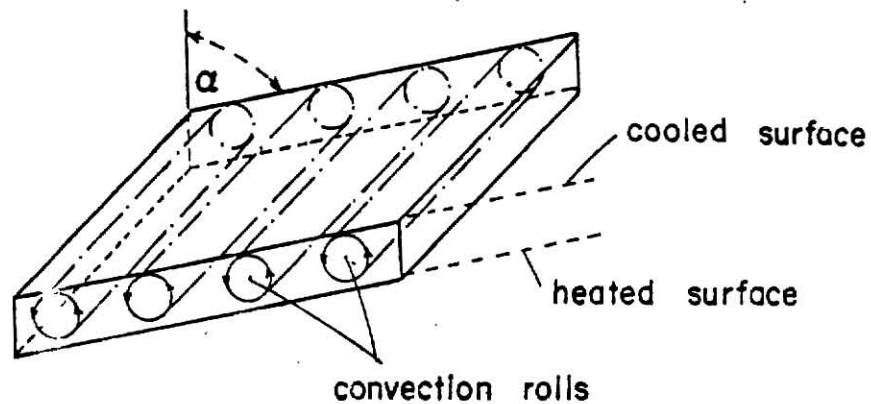


Figure 2. Schematic Diagram of Roll-Type Convection within Inclined Air Layers.

By means of an optical technique and smoke tests de Graff and van der Held(28) found, for angles greater than 10 degrees to the horizontal, that the flow of air was in the form of rolls(Figure 2), provided the Rayleigh number was between 2,000 and 40,000. It was found to be turbulent for Rayleigh number greater than 40,000. A linear interpolation was again proposed (when  $5,000 < Gr < 60,000$ ) for Nusselt number with angles of inclination between 20 and 70 degrees to the horizontal. The horizontal and vertical Nusselt number, respectively, are given as follows,

$$Nu_h = 0.0426 Gr^{0.37}$$

$$Nu_v = 0.0384 Gr^{0.37}$$

The convective heat transfer through liquids contained within inclined rectangular layers has been investigated by Dropkin and Somerscales(36). For  $Ra > 1.5 \times 10^5$ , the empirical relationship was given as,

$$Nu = C Ra^{0.33} Pr^{0.074}$$

where C depends upon the inclination. Catton, Ayyaswamy, and Clever(16) used the Galerkin method to investigate, theoretically, natural convection in an inclined rectangular region, and their results indicate a pronounced aspect ratio dependency. Their solutions were two dimensional and limited up to 120 degrees. For this range they presented predictions for aspect ratios from 0.2 to 20 and Rayleigh numbers up to  $10^6$ . Ayyaswamy and Catton(4) investigated the boundary layer region and showed that, when the Rayleigh number is sufficiently high, a simple rescaling of the results for 90 degrees can be accomplished. They found that for an angle, A, up to 110 degrees

$$Nu(A) = Nu(90^\circ)(\sin A)^{1/4}$$

Ozoe, et al.(86) have reported theoretical work on aspect ratios from 1 to 4 and Rayleigh number up to  $10^4$  with some experimental confirmation. Arnold, et al.(3) measured steady natural convection heat transfer for Rayleigh numbers between  $10^3$  and  $10^6$ . The angle of inclination varied from  $0^\circ$  (heated from bottom) to  $180^\circ$  (heated from top), with aspect ratios of 1, 3, 6, and 12. They presented results in graphical form, made comparisons with previous work, and gave the result for angles between  $0^\circ$  and  $90^\circ$ .

$$Nu(Ra, A) = 1 + (Nu(Ra, 90^\circ) - 1) \times \sin A$$



In his dissertation, Arnold(2) extended the previous analytical Galerkin method into two and three dimensions. He presented Nusselt number as function of Rayleigh number, D/L, W/L and tilt angle.

Randall, et al.(97) used interferometric techniques to study the effects of Grashof number, tilt angle and aspect ratio on both the local and average heat transfer coefficients. They determined, for the laminar boundary layer region, that

$$Nu = 0.118(Gr Pr \cos(A-45^\circ)^2)^{0.29}$$

#### D. Long Rectangular Cavity - Two Dimensional Problems

The analysis of multidimensional natural convection is important in many industrial and geophysical problems; for example, the study of natural convection cooling of nuclear fuel in shipping flasks and in water filled storage bays. Cooling has to be analyzed not only during routine operation of these facilities but also in postulated accidents. This leads to a need for time dependent multidimensional analysis.

Regardless of the practical applications, with its coupled physical phenomena, the natural convective heat transfer has also become one of the most challenging and important typical problems currently under investigation in computational mechanics.

The motion is described by the equation of continuity, equation of energy, and equations of motion, which incorporate the Boussinesq assumption, i.e., all physical properties of the fluid are assumed to be constant except for the density in the buoyancy force terms that drive the motion. (There is an exception by

Gartling(47), who considered viscosity as function of temperature.) The governing equations may be written. in dimensionless form. as.

Equation of Continuity:

$$\nabla u = 0$$

Equations of Motion :

$$\frac{du}{dt}^* = -\nabla P + \nabla^2 u + Gr \theta$$

Equation of Energy :

$$\frac{d\theta}{dt} = \frac{1}{Pr} (\nabla^2 \theta)$$

where Gr equals zero except in the vertical direction.

The dependent variables of the above system are pressure. temperature. and velocity components. usually called "primitive variables". The use of primitive variables in incompressible flow problems is reported by Harlow and Welch(52) and Hirt. et al.(55). This technique is called the "Marker And Cell" (MAC) method. Hirt and Cook(54) first suggested the use of the MAC method in natural convection problems. Alternately. the equations may be formulated in terms of vorticity. stream function. and temperature (23,45,68,76,77,83,117) by eliminating the pressure terms in the equations of motion and introducing vorticity and stream function relations as.

$$\begin{aligned}\omega &= \frac{\partial U_y}{\partial x} - \frac{\partial U_x}{\partial y} \\ U_x &= \frac{\partial \psi}{\partial y} \\ U_y &= - \frac{\partial \psi}{\partial x}\end{aligned}$$

---

\* Total differential.

The equation of continuity will be satisfied automatically after introducing the stream function. The resulting governing equations become.

$$\begin{aligned}\frac{d\theta}{dt} &= \frac{1}{Pr} \nabla^2 \theta \\ \frac{d\omega}{dt} &= \nabla^2 \omega + Gr \frac{\partial \theta}{\partial x} \\ \omega &= -\nabla^2 \psi\end{aligned}$$

These governing equations are more attractive than those written in terms of primitive variables, especially for two dimensional problems, because it reduces the number of equations by one and it ensures that the equation of continuity is automatically satisfied. As discussed by Piacsek and Williams(91), failure to satisfy the continuity equation using the primitive variable equations can lead to convective instability. More over, the difficulties associated with pressure boundary conditions are avoided, the corresponding conditions for vorticity being much simpler and the procedure based on the primitive variable equations was less satisfactory in its convergence behavior. The third alternative is the elimination of the vorticity by the use of its definition (31,63,76). The advantage of this method is that the number of the dependent variables is again decreased by one. However, the reduction in the number of variables is gained at the expense of admitting higher order derivatives in the system equations.

In the above systems, the steady state solution can be found by directly solving the steady state equations(31,88) or by solving the unsteady state equations. In the latter case, we may proceed through time until the solutions cease to change

significantly. When the steady state equations are solved, a double iterative scheme is necessary since they are elliptic and coupled. When the unsteady state equations are solved, the outer iteration is replaced by a progression through time. Both methods require very lengthy calculations.

The most popular boundary conditions used are adiabatic horizontal walls with isothermal hot and cold vertical walls. Kublbeck, et al.(68) used three adiabatic walls and an isothermal side wall to study the transient problem. In a recent analysis by Sparrow, et al.(110), one vertical wall was cooled by an external natural convection boundary-layer flow, while another vertical wall was isothermal.

In calculation procedures, most authors (23,30,31,68,117) have suggested the use of the Alternating Directional Implicit method(ADI) to solve the time dependent, two dimensional, coupled, parabolic-elliptic equations. Alternating direction methods were first introduced by Peaceman, Rachford and Douglas(33,89) as a tool to solve the heat equation. Later, Douglas (34,35) showed the value of ADI method in the approximation of the solutions of parabolic and elliptic differential equations in multidimensional problems by finite difference method. Other methods, such as the Crank and Nicholson method(83), the time splitting method(45), the false transient method(76,77), the Galerkin method(30), the Chorin method(88), and, most recently, the Pantankar method(110) also have been used.

From the practical point of view, the most important purpose of studying heat transfer problems is to determine the heat

transfer coefficient, or Nusselt number. Most papers have given temperature, stream function and vorticity in the form of contour maps as well. Velocity profiles are also available in some papers(30,31).

Constrained by nonlinear stability, most of the computation techniques were limited by  $Gr < 10^5$ . Fromm(45) made the high Grashof number (up to  $10^{12}$ ) problems calculable through the design of a mixed system of nonlinear difference equations which had purely leading-phase-error properties.

Although transient solutions of natural convective heat transfer are important in many cases, sometimes only the steady solution is of interest. Mallinson and de Vahl Davis(77) found if the steady solution exists and is unique, it may be obtained more efficiently by introducing false transient terms into the time independent governing equations. The false transient terms lead to a set of parabolic equations which are solved by marching through a distorted time; no inner iterations are involved and the rate of transient convergence can be enhanced by the use of different time steps for the different equations. The true transient solution is lost, but at large time the transient terms decay and the true steady state solution is recovered. By this method, the vorticity equation and stream function equation can be converted to,

$$F \frac{d\omega}{dt} = \nabla^2 \omega + Gr \frac{\partial \theta}{\partial x}$$

$$G \frac{d\psi}{dt} = \omega + \nabla^2 \psi$$

where F and G are functions of vorticity and stream function,

respectively, for false time steps. After extensive tests on accuracy and speed of the method, they found this method was at least one, and perhaps two, orders of magnitude faster than a conventional, double iterative procedure.

Denny and Clever(30) used ADI method and Galerkin(43,44) method to deal with high Prandtl number Boussinesq fluid in a square two dimensional cavity. They concluded that conditionally stable ADI methods of solution for elliptic partial differential equations may successfully be applied to highly nonlinear, thermal driven flows. However, as the Rayleigh number becomes large the time steps must be decreased to keep the solutions stable. The Galerkin method of solution is an attractive alternative to finite difference methods. The computational time required to advance solutions sequentially through a series of Rayleigh numbers is essentially independent of the magnitudes of Rayleigh numbers. This method proved to be increasingly more efficient than finite difference methods, requiring about 1/2 to 1/3 as much time at the expense of errors of the order of 10%, but the advantage in computational time is to some extent negated by a more tedious preparation of the problem and somewhat larger storage requirements. These disadvantages, of course, become more severe with increasing complexity of a given problem. All things considered, they could not judge which method was superior.

Elder(41) observed two secondary rolls at  $Ra = 9.6 \times 10^6$ ,  $Pr = 7.14$ . Similar rolls were observed by de vahl Davis(31) at  $Ra = 1.5 \times 10^5$ ,  $Pr = 1000$  and were also evident in solutions published by Fromm(45) for  $Ra = 10^5$ ,  $Pr = 1$ , by Quon(95) for  $Ra = 8.0 \times 10^5$ ,  $Pr =$

7.14, and by Cormack, et al.(24) for  $Ra = 1.4 \times 10^5$  and  $Pr = 6.983$ . Mallinson and de Vahl Davis(76) presented the solution of natural convection heat transfer in a square cavity by false transient method(77) and discussed the phenomena of secondary flow. He believed that the secondary rolls did not result from an instability of the base flow, but were a direct consequence of the convective distortion of the temperature field. The flow may be regarded as being driven by the generation of vorticity, which leads to a clockwise rotation, and is generated by positive  $\partial\theta/\partial x$ . For  $Ra = 10,000$ ,  $\partial\theta/\partial x$  is positive. Negative vorticity is produced within the viscous boundary layers and the ultimate form of the steady flow is determined by vorticity equation. As Rayleigh number increases, the development of thermal boundary layers intensifies  $\partial\theta/\partial x$  in the vicinity of the walls, and the convection within each layer lead to negative  $\partial\theta/\partial x$  in the center. A vorticity sink thus separates the regions of concentrated vorticity generation, and, provided viscous diffusion does not completely smear the distribution of vorticity, two secondary flows are formed. In their investigation, reversed temperature gradients appeared if  $Ra > 3.0 \times 10^4$ , for  $0.1 < Pr < 100$ . However, viscous diffusion retarded the development of secondary rolls until  $Ra > 6.0 \times 10^4$ .

Kublbeck et al.(68) pointed out that the equal stepsize finite difference methods had over-emphasized the changes in the center part. In order to have denser grids in the regions close to vertical boundaries, they introduced transformation equations, which concentrate the grid points in the boundary layer regions.

The suggested relation was,

$$P(x, \epsilon) = \frac{s}{2} \left\{ 1 + \tan \left[ \frac{\pi}{2} \left( \frac{2x}{s} - 1 \right) \epsilon \right] / \tan \left( \frac{\pi}{2} \epsilon \right) \right\}$$

They also gave estimated CPU time as function of grid points for the CDC CYBER 175.

Patterson and Imberger(88) used a modified version of the finite difference method proposed by Chorin(21) to study the unsteady state heat transfer phenomena within a rectangular cavity. They observed pure conduction dominated flow at  $Ra = 0.1$ . At  $Ra = 21$ , a conduction dominated flow with some convective effects is expected. When  $Ra = 1,000$ , it is expected to be dominated by convection. They also confirmed the independency between the Prandtl number and the Nusselt number for large Rayleigh number and steady state condition. The Nusselt number, in this case, is proportional to  $Ra^{0.25}$ .

Sparrow and Prakash(110) dealt with Navier-Stokes equation for steady state natural convection in a square enclosure, of which, one side wall was cooled by an external natural convection boundary layer flow. The calculation scheme they employed for the internal part was that of Patankar's(87). It began with guessed values for primitive variables and refining them using pre-calculated boundary condition until convergence. They correlated the average Nusselt number to be

$$Nu = 0.0907 Gr^{0.285} \quad \text{for } Gr > 10,000$$

There was a 1.8% difference when compared with the results of standard enclosure problems, which were (though not necessarily



precisely) :

$$Nu = 0.141 \text{ Gr}^{0.29}$$

#### E. Cubical Enclosures -- Three Dimension Problems

It has been known for a long time that methods for two dimensional Navier-Stokes equation and related equations can be extended to three dimensional problems. However, there had been no successful results until Aziz and Hellums' in 1967(5). They pointed out the difficulties of three dimension problems as follows. First, there have been considerable difficulties with stability and with computational requirements even in two dimensions -- these problems become more critical in three dimensions. Second, the widely used ADI method in two dimensions does not go over to three dimensions in a straight forward way. Third, the formulation of the equations of motion in terms of the vector potential is not well known, but has been found to be essential. It is interesting to note that the only two references(81,112) on the use of the vector potential in solving the Navier-Stokes equations available in 1967 contradicted one another. One stated that the vector potential must vanish on all solid boundaries the other indicated that on solid walls the normal derivatives of the normal components must vanish. Fourth, the magnitude of the computations in three dimensions is about  $10N$  times the magnitude in two dimensions, where  $N$  is grid number in each dimension. Problems requiring a few minutes of computer time for two dimensions require hours of time for three dimensions. Fifth, the stream function in three dimensions is not a scalar as

in two dimensions. The velocity components defined by stream function vector are

$$\vec{v} = \nabla \times \vec{\psi}$$

The major difference between Aziz and Hellums' calculation method and conventional two dimensional calculation methods was that the parabolic portion of the problem was solved by the ADI method, while the elliptic portion is solved by the Successive Over Relaxation (SOR) method. The computer process time for one time step of their method in two dimensions was about 20 seconds, but 4 minutes in three dimensions, while estimated conventional method time was 30 minutes.

In 1973, Mallinson and de vahl Davis(77) extended the false transient method to solve steady state three dimension problems and used a particle track method to present their results. They observed differences among the results of Aziz and Hellums(5), Catton, Edward(17), and their own, but failed to explain the reasons.

Four years later, Mallinson and de vahl Davis(76) solved the problem again. They used the false transient term parameter  $F(w) = 0.05$  (it was 1 in their first paper). The solution was obtained after 100 iterations, which required about 30 minutes of central processor time on the PDP-10 or approximately 50 minutes on the IBM 360/50. The authors discussed the effect of aspect ratios and governing parameters on three dimension flows, and concluded that there existed a three-dimensional end effect, the form of which depends highly on the governing parameters. The effect arises from two mechanisms: an initial mechanism dominates

when Prandtl number is small, whereas, a weaker thermal effect dominates at large Prandtl number.

Chan and Banerjee(18) discussed the advantages of the primitive variables(MAC) method in three dimensional problems. First, boundary conditions were much easier to handle for complex flow geometries, such as nuclear fuel shipping flasks and storage bays. Second, the MAC method required about the same amount of CPU time per grid point per time step for two or three dimensions, while ADI method and time splitting(45) required progressively more CPU time per grid point per time step as the dimension increases. In this paper, heat transfer coefficients were calculated for both two and three dimensional problems. The same order of calculation requirements were reported for both two and three dimensions. They ranged from 0.01 to 0.02 second per grid point per time step. The higher value was required during rapid changes.

Experimental results for three dimensional heat transfer were given by Arnold(2). He measured the Nusselt number using tilt angle, aspect ratios, and Rayleigh number as parameters and presented his data in graphical form. For the case of non-tilt cube, the Nusselt number is proportional to the Rayleigh number to the power of about 0.32. In order to give comparison to experimental work with which high Prandtl number silicone oils were used, numerical solutions were also calculated by Galerkin method by assuming infinite Prandtl number and neglecting the nonlinear terms in the equation of motion.

## II. Cylindrical Enclosures

From a practical standpoint, heat transfer by natural convection within cylindrical enclosures might arise in connection with cylindrical storage tanks, in pipelines under condition of no flow, or in many other situations. But, perhaps more important, knowledge of natural convection phenomena is needed as a limiting condition in the study of combined forced and free convection in tube flow.

### A. Horizontal Cylindrical Enclosures

Martini and Churchill(78) investigated the natural convection in a long horizontal cylinder with vertical halves of the wall at different uniform temperatures. They found that the local heat transfer coefficient varied considerably with angle and increased only slightly with increasing wall temperature difference, while the overall Nusselt number had a constant value of approximately 7.0 over a range of wall temperature differences from 35 to 365°F.

Deaver and Eckert(29) investigated the transient natural convection inside a long horizontal cylinder. An interferometer was used to obtain temperature profiles, from which the Nusselt number was given as,

$$\begin{array}{ll} \text{Nu} = 8 & \text{for Ra} < 1,000 \\ \text{Nu} = 1.181 \text{ Ra}^{0.214} & \text{for } 5.0 \times 10^5 < \text{Ra} < 10^7 \end{array}$$

Recently Bejan and Rossie(9) used thymol blue PH indicator to visualize the flow. Velocity and temperature profiles were presented and they correlated

$$Nu = 9.1 (Ra \cdot H/L)^{(1/4)}$$

Martini and Churchill(78) applied the finite difference technique to solve the governing equations for long horizontal cylinder. They found discrepancies between the experimental and computed temperature profiles. Later, Hellums and Churchill(53) again solved this system, and obtained the following correlation:

$$Nu = 0.365 Ra^{(1/4)}.$$

Ostrach(84) solved the stream function governing equation using a nonuniform temperature distribution around the cylinder circumference. The distribution was,

$$T(R_o, \theta) = T_o + \Delta T \cos(\theta + \phi)$$

Several streamline contour maps were presented in this paper.

Leong and de vahl Davis (69) applied the ADI method to a long horizontal cylinder. They approximated the Nusselt number by,

$$Nu = 1.375 Ra^{(0.74-0.1075(\log Ra)+0.027(\log Ra)^2)}$$

#### B. Horizontal Annuli

Some decades ago the first experimental results on the steady state heat transfer by free convection between isothermal horizontal concentric cylinders were published by Beckmann(8). Kraussold(67) extended Beckmann's work by using several fluids to account for Prandtl number variations. He proposed the results as a ratio of equivalent conductivity to the molecular conductivity,

$K_c/K = 1.0$	$\log(Ra) < 3.0$
$K_c/K = 0.11 Ra^{0.29}$	$3.8 < \log(Ra) < 6.0$
$K_c/K = 0.4 Ra^{0.2}$	$6.0 < \log(Ra)$

Liu et al.(72) reported the results correlated differently.  
His results were,

$$K_c/K = 1.0 \quad \log(R) < 3.0$$

$$K_c/K = 0.135 R^{0.278} \quad \log(R) > 3.5$$

where  $R = Pr^2 Gr / (1.36 + Pr)$

Lis(71) fitted his experimental heat transfer data as,

$$\log(K_c/K) = 0.0794 + 0.0625 \log(X) + 0.0154(\log(X))^2$$

where  $X = Ra_{Di} \times (1 - Di/Do)^{6.5}$ , and the ranges of the parameters are,

$$4.0 \times 10^4 < Ra_{Di} < 4.7 \times 10^{10}$$

$$2 < Do/Di < 4$$

$$0.645 < Pr < 1.32$$

Fujita et al.(46) rearranged previous investigators' data and concluded that the Nusselt number was only dependent on a modified Rayleigh number.

Bishop and Carley(12) studied the flow patterns of convection in an annulus by photographs and motion pictures of smoke. They reported the existence of a "crescent eddy" pattern and a "kidney shaped eddy" pattern.

About the same time, Grigull and Hanf(52) presented numerous photographs for flows within cylinder annuli. They showed three different regions,

- (1) A two-dimensional pseudo-conductive region for Gr less than 2,400.
- (2) A three-dimensional transition region for Gr between 2,400 and 30,000.
- (3) A two-dimensional, fully developed, laminar convection region for  $30,000 < Gr < 716,000$ .

Experimental work for various changes in wall temperature has been reported by Evans and Stefany(42). van de Sande and Hamer(114) measured the heat transfer rate for both concentric and eccentric annulus under constant heat flux for steady and transient natural convection.

A theoretical solution was reported by Crawford and Lemlich(25). They solved the stream function and temperature equations by the Gauss-Seidel iteration. Convergence was very slow in their method. Mack and Bishop(74) used power series expansions of the Rayleigh number. They found that it was only feasible to use three terms. This restricted their solutions to low Rayleigh numbers. Bejan and Tien(10) solved the governing equations, in primitive variables form, for a concentric horizontal annulus with a different end wall temperature by the perturbation method. Singh and Elliott(108) solved the system in stream function form for a vertically stratified fluid in the annulus. Stream function, temperature distribution and velocity components were reported for several cases, with the Grashof number limited to the order of one.

### C. Vertical Cylinders

There are two types of natural convection heat transfer inside vertical cylinders: "open" and "closed" thermosyphons. A general literature survey has been given by Japikse(59). Fundmentally, the open system is more attractive since it is capable of producing higher heat transfer rates.

### (1) Open Thermosyphon

Martin and Cohen(79) described an experimental investigation of heat transfer by free convection in an heated vertical tube sealed at its lower end, while the upper end opened into a mounted large reservoir. They found that turbulent flow in a closed end tube was less effective than laminar boundary layer flow so long as turbulence interfered with the movement of the heated fluid filling the tube. Only if the conditions permitted the existence of a turbulent boundary layer was it likely that turbulent flow would show superior rates of heat transfer.

The first and most fundamental theoretical analysis of natural convection heat transfer in open thermosyphons was given by Lighthill(70), who divided the flow into three regions according to a parameter, the ratio of Rayleigh number and length-to-radius.

### (2) Closed Thermosyphons

In the closed thermosyphons, in addition to the effects created by the internal fluid boundary in both heated and cooled sections, there is an additional amplification at the junction of the two regions. Lighthill(70) considered that the closed system could be treated as a synthesis of open systems coupled together in some manner by a region of limited extent. He suggested that complete mixing of the opposing streams in the coupling region could probably be a reasonable approximation to the truth.

Bayley and Lock(7) also studied the heat transfer



characteristics of a closed thermosyphon for various Prandtl number and aspect ratios. They found that conduction was associated with high Prandtl numbers. In general, heat was transferred by conduction near the walls of the tube and a combination of conduction and convection near the center. The boundary position was strongly influenced by the Prandtl number.

Japikse(59) observed the flow regions and proposed analytical models for predicting the heat transfer efficiency of the various exchange mechanisms. For high viscosity and low temperature differences, evidence of conduction was found while for low viscosity and high temperature differences, mixing was observed. He concluded that turbulence existed for  $Ra/(R L)$  values above approximately  $1.6 \times 10^7$ .

#### D. Vertical Annuli

In 1962, MacPherson and Stuart(75) conducted an experimental study of natural convection heat transfer of helium and carbon dioxide in vertical annuli with different, but constant, wall temperatures. It was found that for Rayleigh number less than 1,000 the Nusselt number was equal to one. Sheriff(107) carried out experiments with high pressure carbon dioxide. The hot wall was controlled at constant heat flux while the cold wall was kept at a constant temperature. The correlation equation was given in the form of,

$$Nu = 0.25 Ra^{0.3} \left( \frac{H}{t} \right)^{-0.25}$$

Schwab and de Witt(105) solved the governing partial differential equations by ADI method. They found that when the Rayleigh number approached 1,000, large temperature gradients grew near the vertical walls giving rise to the formation of thermal boundary layers and fluid velocities sufficient to form hydrodynamic boundary layers. A unicellular flow pattern was generated in the enclosure. When  $Ra > 5,000$ , a fully developed boundary-layer flow existed in the cavity. The interaction of two boundary-layers on the vertical walls led to a nearly uniform vertical temperature gradient in the core of the annulus. Also the streamlines in the central portion of the cavity were found to be nearly horizontal.

Thomas and de vahl Davis(111) solved the system by primitive variables. Heat transfer correlations were reported as,

(1) Conduction Region :  $F < 400$

$$Nu = 0.595 Ra^{0.101} Pr^{0.024} \left( \frac{L}{r_o - r_i} \right)^{-0.052} \left( \frac{r_o}{r_i} \right)^{0.525}$$

(2) Transition Region :  $400 < F < 3,000$

$$Nu = 0.202 Ra^{0.294} Pr^{0.029} \left( \frac{L}{r_o - r_i} \right)^{-0.246} \left( \frac{r_o}{r_i} \right)^{0.243}$$

(3) Boundaary Layer Region :  $F > 3,000$

$$Nu = 0.286 Ra^{0.258} Pr^{0.006} \left( \frac{L}{r_o - r_i} \right)^{-0.238} \left( \frac{r_o}{r_i} \right)^{0.442}$$

where  $F = Ra / ((r_o - r_i) L)$

### III. Spherical Enclosures

It is well known that a sphere has the characteristic of minimum surface-to-volume ratio. Therefore many storage tanks are spherical, especially when low heat transfer rates are desirable. The investigation of natural heat transfer in spherical enclosures can be classified into hollow spheres and spherical annuli.

#### A. Hollow Spheres

In 1956, Schmidt(103) investigated the transient natural convection heat transfer inside a sphere when it was subjected to a step change in uniform wall temperature. The Rayleigh numbers of his experimental data were in the range of  $10^8$  to  $10^{12}$ . From the experimental data a heat transfer correlation of the follow form was obtained,

$$Nu = 0.098 Ra^{0.345}$$

Chow(22) used flow visualization method to observe the flow patterns of pseudosteady-state natural convection inside spheres. In his experiments, outside wall temperature was increased steadily to maintain a constant temperature difference between center of the sphere and outside wall. For Rayleigh number smaller than  $10^7$ , the flows were laminar, and the streamlines were found to be crescent-shaped. The location of the circulation center was found to move toward the wall and downward with increasing Rayleigh number. Velocity profiles were determined and the overall heat transfer in the laminar

region was correlated as,

$$Nu = 0.80 Ra^{0.3}$$

Pustovoit(94) presented the numerical solution for transient natural convection heat transfer inside a sphere when its wall was subjected to a step temperature change. Primitive variables were expanded in terms of the Grashof number while only the first order approximate solution was solved. Kee, et al.(65) solved the system by modifying the ADI method through a stream function-vorticity approach. Their modification increased the computational efficiency by treating the convective and source terms in the transport equations in an explicit way(64). The computation time ranged from hundreds to thousands of seconds as the Grashof number varied from  $10^5$  to  $10^{10}$ . The Nusselt number, as a function of modified Grashof number was reported in graphical form. Recently Hwang and Lee(56) combined the ADI method and SOR method to present the transient solution of natural convection inside a sphere. They reported streamlines, temperature profiles, the mean temperature, and the overall heat transfer coefficient as functions of time.

#### B. Spherical Annuli

Bishop et al.(13) investigated the flow patterns and temperature distributions occurring between two concentric isothermal spheres of various diameter ratios with air in the intervening space. Three convective patterns, namely the "crescent-eddy" type, the "kidney-shape-eddy" and

"falling-vortices" were observed. Later, Bishop et al.(14) presented temperature distributions and a heat transfer correlation for air,

$$Nu = 0.106 Gr^{0.276}$$

with  $0.25 < L/R_i < 1.5$  and  $2.0 \times 10^4 < Gr < 3.6 \times 10^6$

Scanlan et al.(102) used water and silicone oil to cover a wider range of Prandtl and Rayleigh numbers. They recommended an overall heat transfer correlation,

$$K_{eff}/K = 0.228 Ra^{0.226} \quad \text{for } 0.09 < L/R_i < 1.81$$

Weber et al.(116) investigated the heat transfer between vertical eccentric spheres. He found the above relation still applied as long as the absolute value of  $e/L$  was smaller than 0.75. Where  $e$  is eccentricity and  $L$  is the difference of the diameters of the two spheres.

More data were made available by Yin et al.(118). They also described how to make use of a particular detergent to visualize the flow.

Sevruk(106) solved the governing equations by an approximate method, but no numerical data were reported; probably because of the difficulties in making use of the solution, which was unwieldy.

In Hardee's(51) perturbation solution, three terms were considered in a power series expansion of the Rayleigh number, with the Rayleigh number limited to 1,600, which is only one tenth of the available experimental value. Therefore no close

correlation could be made.

Recently Caltagirone et al.(15) applied the ADI method to the problem. Radius ratios between 1.15 and 3 and Rayleigh numbers varying from 100 to  $10^5$  were considered. Two kinds of solutions were observed for the same parameters depending on the initial radial velocity at  $r=(R_i+R_o)/2$ ,  $\Phi=0$ . For positive initial radial velocity, convergence was quickly obtained. For negative initial radial velocity, convergence was very slow and a secondary flow occurred.

## APPARATUS

The specific system chosen for this investigation was a fluid filled cube with heat entering through all six walls. For  $t < 0$  the fluid was to be uniform in temperature and stagnant. At  $t \geq 0$  the wall temperature was maintained at a higher temperature. The objectives of this research were to study the average heat transfer coefficient, to observe the flow patterns, and to measure the velocity profiles inside the cube. By dimensional analysis, it is expected that natural convection heat transfer data can be correlated by an equation of the form

$$Nu = f(Gr, Pr)$$

Thus, in order to determine the unknown function, all the variables involved in the Nusselt, Grashof, and Prandtl numbers are required. The necessary measurements were rate of temperature change, temperature difference, and time.

The design and construction of the equipment and instruments are described in three major groups: those associated with the internal fluid, those associated with the temperature control, and the photographic equipment.

### I. INTERNAL FLUID AND TEMPERATURE MEASURING DEVICES

The fluid inside the cube was either distilled water, ethylene glycol, glycerin, or aqueous solutions of glycerin and

ethylene glycol. These provided fluids with viscosities ranging from 1 to 1,500 centipoise. Two methods were tested to make the flow visible: "AJAX" detergent(93) and small glass spheres. The detergent produced particles which were so fine that the fluid looked milky. In addition, the brightness of the particles was not constant, probably due to the asymmetry of the particles. Better results were found from the use of spherical particles -- industrial grade microballoons (IG 101), manufactured by Emerson and Cuming, Inc. Therefore throughout the experiments, working fluids with dispersed microballoons at concentrations ranging from one-tenth to three-tenths of a gram per liter were used for studying the flow patterns.

Three sizes of cubes, 2", 3", and 4" (inside dimension), and made of 1/8" plexiglas were used. Three plexiglas tubes, one-eighth inch inside diameter and one-fourth inch outside diameter, were attached on top cover of each cube for filling with fluid, inserting the thermocouple, and supporting the cube. Figure 3 shows a typical cube.

Two copper-constantan thermocouples each with 1/16" stainless steel sheath were employed for temperature measurements. One was located at the center of the cube and the other in the outside bath. The time constant of these thermocouples were about fifty milliseconds each. A Keithley 180 Digital Nanovoltmeter and a Honeywell Electronik 19 Potentiometric recorder were used for displaying and recording the output from the thermocouples. The maximum sensitivity of these instruments was one microvolt ( $0.02^{\circ}\text{C}$ ).



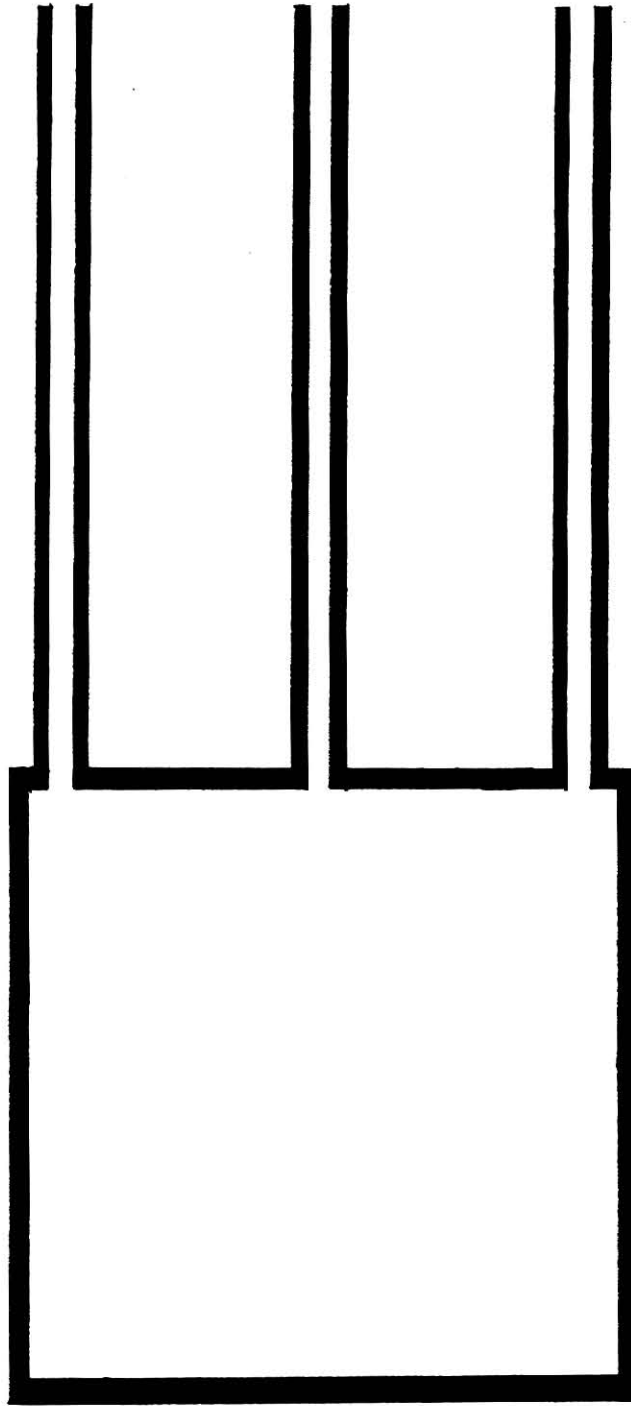


Figure 3. A typical Cube Used in This Study.

## II. TEMPERATURE CONTROL EQUIPMENT

The main purpose of the equipment in this group was to supply and control heat to maintain an uniform and constant temperature around the cube. Tap water was used as the heating fluid. Because the angles between walls and incident rays or refracted rays were at right angles, the index of refraction differences between internal and external fluids was not a problem in this experiment.

A rectangular tank 18" long, 12" wide, and 15" deep, and made of 1/4" plate glass was used as the external fluid container. A Tecam TU-14 Tempunit thermostat was used to keep constant bath temperature which gave a stability of  $0.02^{\circ}\text{C}$ . The back wall of the bath was sheeted with rough, black board to absorb unwanted reflected light.

## III. LIGHT SOURCE AND CAMERA ARRANGEMENT

The objectives of this group of equipment were to illuminate the microballoons and to take pictures showing the flow patterns inside the cube. A sketch showing the principal features of the light source and camera is given in Figure 4. A Viewlex slide projector, model V-25C, with 500 watt lamp was used for the light source. A slide with a small slit in the center was used to produce a thin (about 1/8"), vertical slit of light through the center of the cube. The wall perpendicular to the light source was blocked with a black plate which had an about 1/8" vertical slit down the center, which could be adjusted to change the

thickness of the light plane entering the bath to ensure good illumination of the microballoons. The position of the cube was adjusted so that the light plane passed through the neighborhood of the center. (The exact center was avoided to prevent direct reflection of light from the thermocouple which would ruin the pictures.)

A Miranda FV, 35 mm, single-lens reflex camera was located perpendicular to the direction of the plane of light and therefore could photograph a major square through the cube. An appropriate combinations of lenses and extension rings were used to provide close-up pictures. Kodak Plus-X and Tri-X pan film were used. Both gave good results.

Figure 5 is a photograph of the experimental set-up.

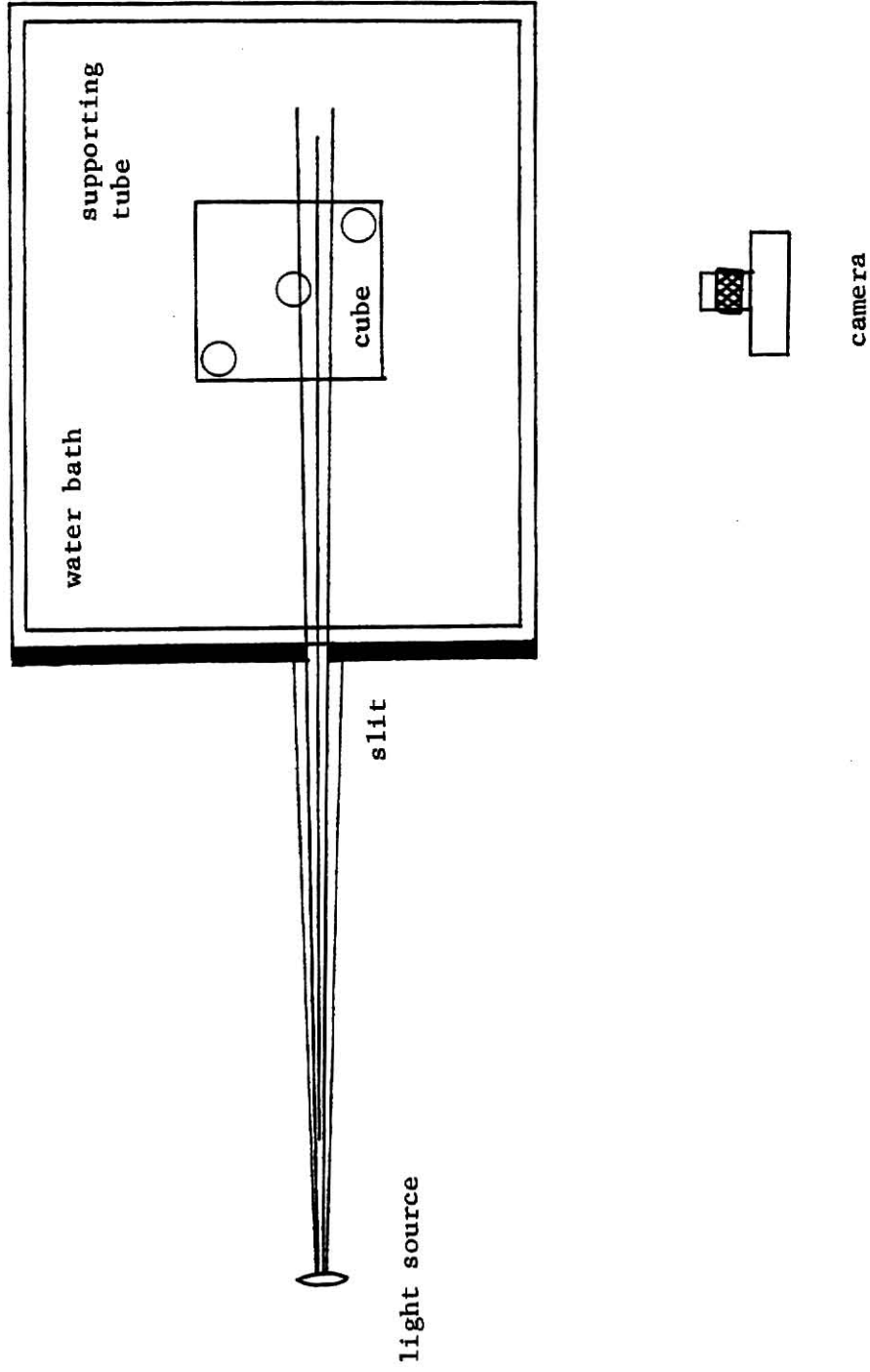


Figure 4. Top View of Location of Photographic Equipment.

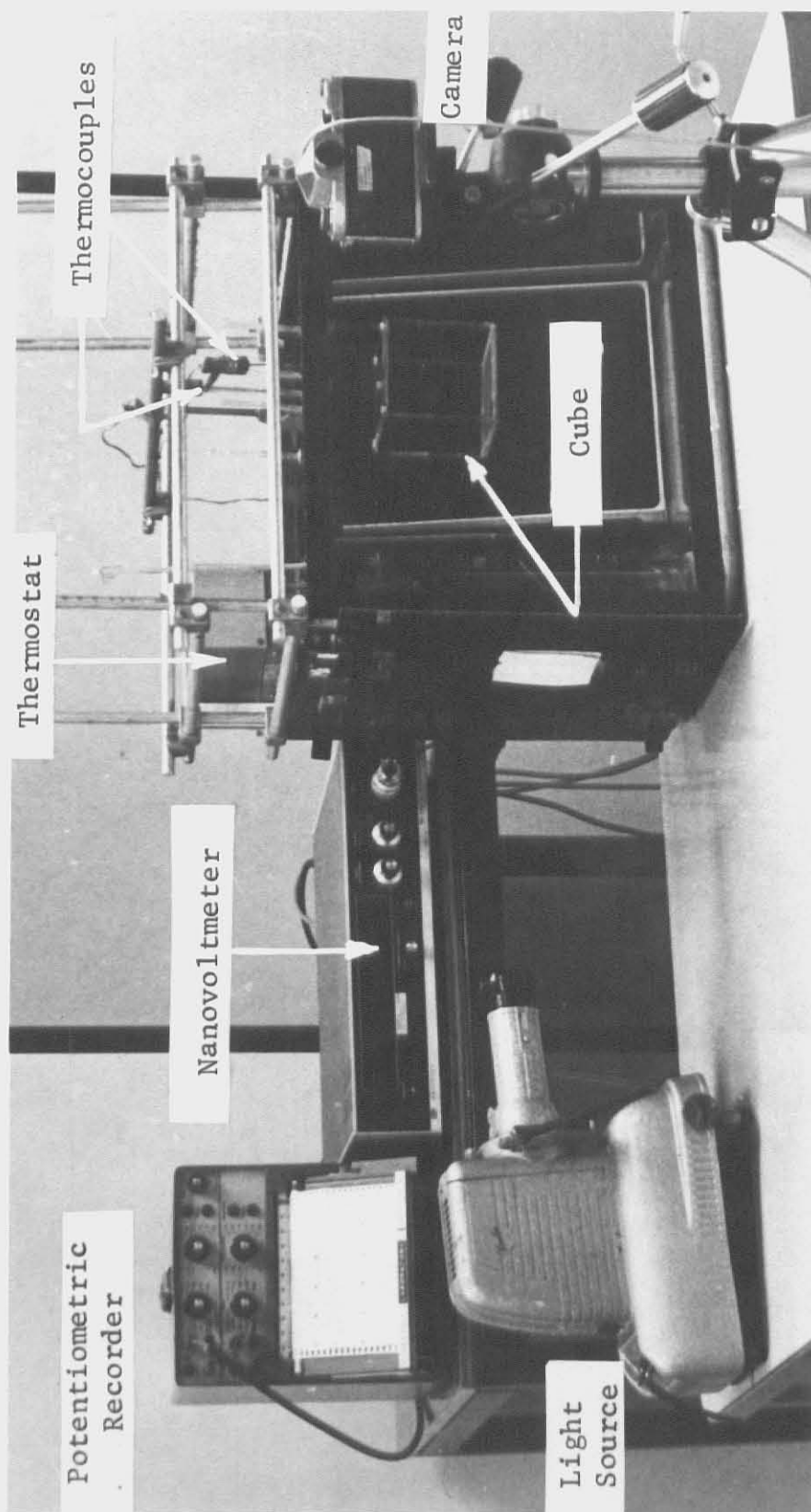


Figure 5. Assembled Apparatus.

## EXPERIMENTAL PROCEDURE

The distilled water used in this experiment was boiled first for deaeration purposes in order to minimize undesirable bubble collection on the cube.

Industrial grade glass microballoons (IG- 101), manufactured by Emerson and Cuming, Inc., were poured into the boiled water to make a solution of about 0.3 gram per liter and were stirred well. These hollow glass spheres, ranging in size from 10 to 300 microns in diameter, have a density  $0.34 \text{ gm/cm}^3$ . The water and suspended particles were drawn from the lower section of the container after allowing 20 minutes for the particles larger than about 20 microns to float to the top. An estimate of floating rate is available in Appendix A. The distilled water with particles was then poured into the cube. A thermocouple was inserted into the center of the cube through center tube. The floating velocities of microballoons suspended in glycerin and ethylene glycol were much slower than those suspended in distilled water. For example, the particles stayed suspended in glycerin uniformly for more than one week. The amount of microballoons needed could be decreased, therefore, according to the viscosity. For pure glycerin, only one-tenth gram (or less) of the microballoons per liter was enough to give good lumination. Moreover, the slot on the outside wall of the bath could also be changed slightly to ensure a proper lumination.

The whole system was set to a temperature about  $3^{\circ}\text{C}$  above room temperature, where it remained for several hours, or over night, to allow temperature equilibrium.

The camera; with appropriate lenses, extension rings, and/or close-up lenses; was set at the proper position and was focused on the particles suspended in the fluid in the plane of light. The f-stop depended on the exposure time of each picture to be taken. To start an experiment, a pre-calculated amount of hot water was added quickly to the bath and the thermostat was adjusted to a higher temperature. (Step changes of temperature between  $1^{\circ}\text{C}$  and  $4^{\circ}\text{C}$  were used.) At the same time, a timer and the temperature recorder were turned on and run throughout the experiment. Figure 6 shows a typical temperature difference history. Multiple exposure pictures were made periodically by opening the camera shutter and covering the camera lens. After a number of experiments and comparisons, the following multi-exposure method was found to give the best information about the flow patterns and showed the best "readability". Three exposures were made on each picture; the exposure time of the first exposure was twice as long as the exposure time of the second and the third exposures. Therefore the speed and the direction of particles could be calculated. The total exposure time per picture was from 30 seconds to 12 minutes according to the velocity (or viscosity). The total time of an experiment was about one and a half hours for water and 6 hours for pure glycerin for  $1^{\circ}\text{C}$  step change. Increases in the temperature step change did not increase the required time significantly, since the

temperature difference dropped very fast during the initial range of the experiment.

Heat transfer coefficients were calculated assuming quasi-steady state (discussed in next section). Several constant temperature difference experiments were also made to check this assumption. After the step change, instead of maintaining a constant bath temperature, the temperature difference was kept constant by manually controlling the heater and by adding ice cubes. Although this controlling method was crude, it was not too difficult to keep the deviation within  $0.1^{\circ}\text{C}$ . The increasing of bath temperature was also recorded by reading a thermometer accurate to  $0.05^{\circ}\text{C}$ .



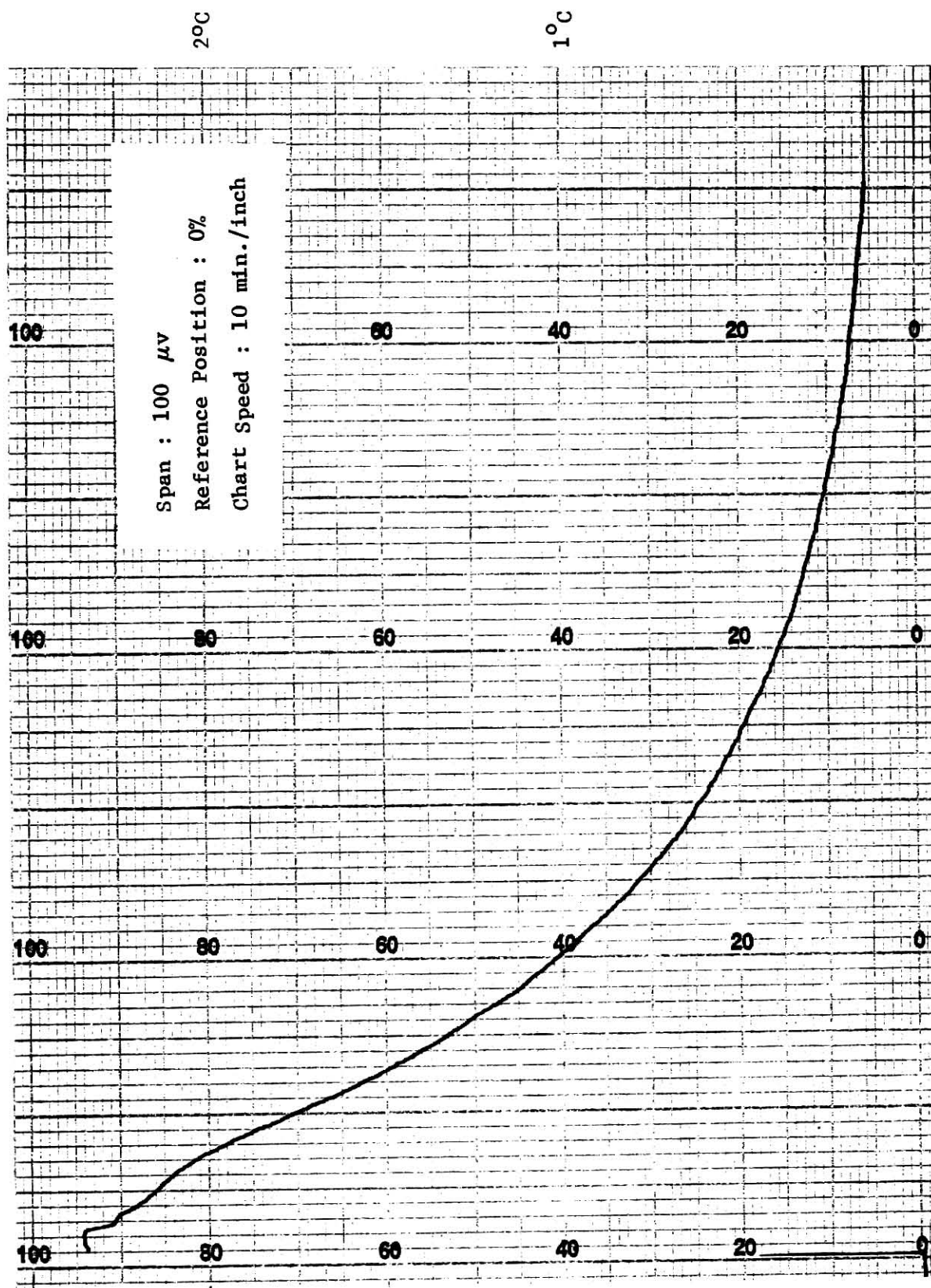


Figure 6. Typical Temperature Difference Curve for Step Temperature Change.

## HEAT TRANSFER DATA ANALYSIS

The average heat transfer coefficient is defined here as

$$Q = h A \Delta T \quad (1)$$

where

$Q$  = total heat transfer rate,

$h$  = average heat transfer coefficient,

$A$  = inside surface area of the cube,

$\Delta T$  = fluid temperature difference between the inside wall and the center of the cube.

The total heat transfer rate  $Q$  can be estimated from the equation by assuming quasi-steady state condition

$$Q = C_p \rho V \frac{dT}{dt} \quad (2)$$

where

$C_p$  = heat capacity of fluid inside the cube,

$\rho$  = density of the fluid inside the cube,

$V$  = volume of the cube

$\frac{dT}{dt}$  = rate of change of fluid temperature at the center of the cube.

The properties of the fluid were taken at the temperature,  $T_{av}$ , defined by

$$T_{av} = 0.75T_s + 0.25T_c \quad (3)$$

where

$T_s$  = temperature of fluid at the inner surface of the cube,

$T_c$  = temperature at the center of the cube.

The total heat transfer rate through the wall by conduction can be approximately evaluated by

$$Q = \frac{k_g A (T_o - T_s)}{S} \quad (4)$$

where

$k_g$  = thermoconductivity of the wall,

$T_o$  = outside wall temperature,

$T_s$  = inside wall temperature,

$S$  = thickness of the wall.

Thus, from equation (2) and (4) the inside wall temperature reduces to

$$T_s = T_o - \frac{C_p \rho S W}{6k_g} \frac{dT}{dt} \quad (5)$$

The temperature difference was recorded continuously which allowed the calculation of the rate of the center temperature change at any time. Assuming that the rate of the center temperature change can represent the rate of the average temperature change, the above quasi-steady state analysis can be applied to calculate heat transfer coefficient. And, the Nusselt number is defined by

$$Nu = \frac{hW}{k} \quad (6)$$

where

$W$  = inside length of the cube,

$k$  = thermal conductivity of fluid inside the cube at  $T_{av}$ .

Substituting equation (1) and (2) into equation (6) gives

$$Nu = \frac{C_p \rho W^2}{6k\Delta T} \frac{dT}{dt}$$

The dimensionless groups, Grashof number, Prandtl number, and

Rayleigh number, are defined as

$$Gr = \frac{\beta g \Delta T \rho^2 W^3}{\mu^3}$$

$$Pr = \frac{C_p \mu}{k}$$

$$Ra = Pr \cdot Gr$$

The physical properties of water were calculated by curve fitting equations available in reference (62). The physical properties of ethylene glycol, glycerin and their aqueous solutions were available in reference (26,80). In Appendix B were computer programs used to calculate these dimensionless groups. The index of refraction is almost linear with respect to concentration for ethylene glycol and glycerin, it was used to measure the concentration in this experiment. In these computer programs, inside wall temperature was calculated first by trial and error method. The term,  $dT/dt$ , was determined by the slope of the temperature response curve (Figure 6). For constant temperature difference experiments,  $dT/dt$  was calculated by the bath temperature increasing rate. For aqueous solution of ethylene glycol and glycerin, all physical properties, except viscosity, were found from tabulated data and treated as constant in the calculation. The logarithm of viscosity for a particular concentration were correlated by a quadratic equation with respect to the inverse of absolute temperature in the temperature range of interest.

## DISCUSSION OF RESULTS

### I. FLOW PATTERNS

Before starting a run, the fluid inside the cube was at a uniform temperature, and at rest. To begin a run, hot water was added into the external bath, and the thermostat was adjusted to a new higher temperature and kept constant. This produced a rapid temperature change which closely approximated a step change. The temperature difference between outside and inside wall of the cube caused the transfer of heat by conduction through the wall. Therefore, the fluid close to the inside wall of the cube became hotter than that near the center. At the place near the top wall of the cube, the hotter fluid (with lower density), was above the colder fluid (with higher density) and produced a stable situation. But at the place near the bottom wall, the situation was just the reverse; the hotter and lighter fluid was in the lower layer near the wall, causing an unstable condition to exist. Hence, the fluid remained motionless until the critical Rayleigh number was reached and then, as more heat penetrated into the cube, natural convection began. Although the fluid heated by bottom of the cube would tend to move upward, it must descend somewhere to form closed streamlines. Because the center part is far away from side wall, the flow, driven by the heating of the bottom of the cube, ascended in the center core and descended

where close to the side walls. At the same time, heated by the side walls, the fluid along the inside vertical walls started to flow upward along the wall until it reached the upper part of the cube; then it tended to bend in and flowed downward until it met the upward flowing streams at the lower part of the cube to form closed streamlines. These two factors tended to drive the fluid in different directions. The flow patterns were determined by the relative intensities of these two factors. The heating area of side walls was much larger than that of bottom wall, therefore, the flow driven by heating of side walls, upward along side wall and downward along center core, was the most important flow pattern and covered most of the time. It is called the "main flow", and shown in Figure 7. Since the flow direction caused by heating of the bottom wall was the reverse of the main flow, it is called the "reverse flow". Usually the driving force of the reverse flow was too small, and, there was no reverse flow. As the relative driving force of the reverse flow increased due to the decrease of the driving force of main flow, small reverse flow was observed at the place close to the bottom center of the cube, as shown in Figure 8. When the relative driving force of the reverse flow was even larger, there were two flows in the cube of similar size. This is shown in Figure 9. Sometimes the driving force of the reverse flow was so large that it dominated the whole cube, and only a reverse flow could be observed (Figure 10). Later, the effect of the heating from side walls would become dominant again, reestablishing the main flow. Figure 11 is a sequence of pictures showing the formation of the reverse flow.

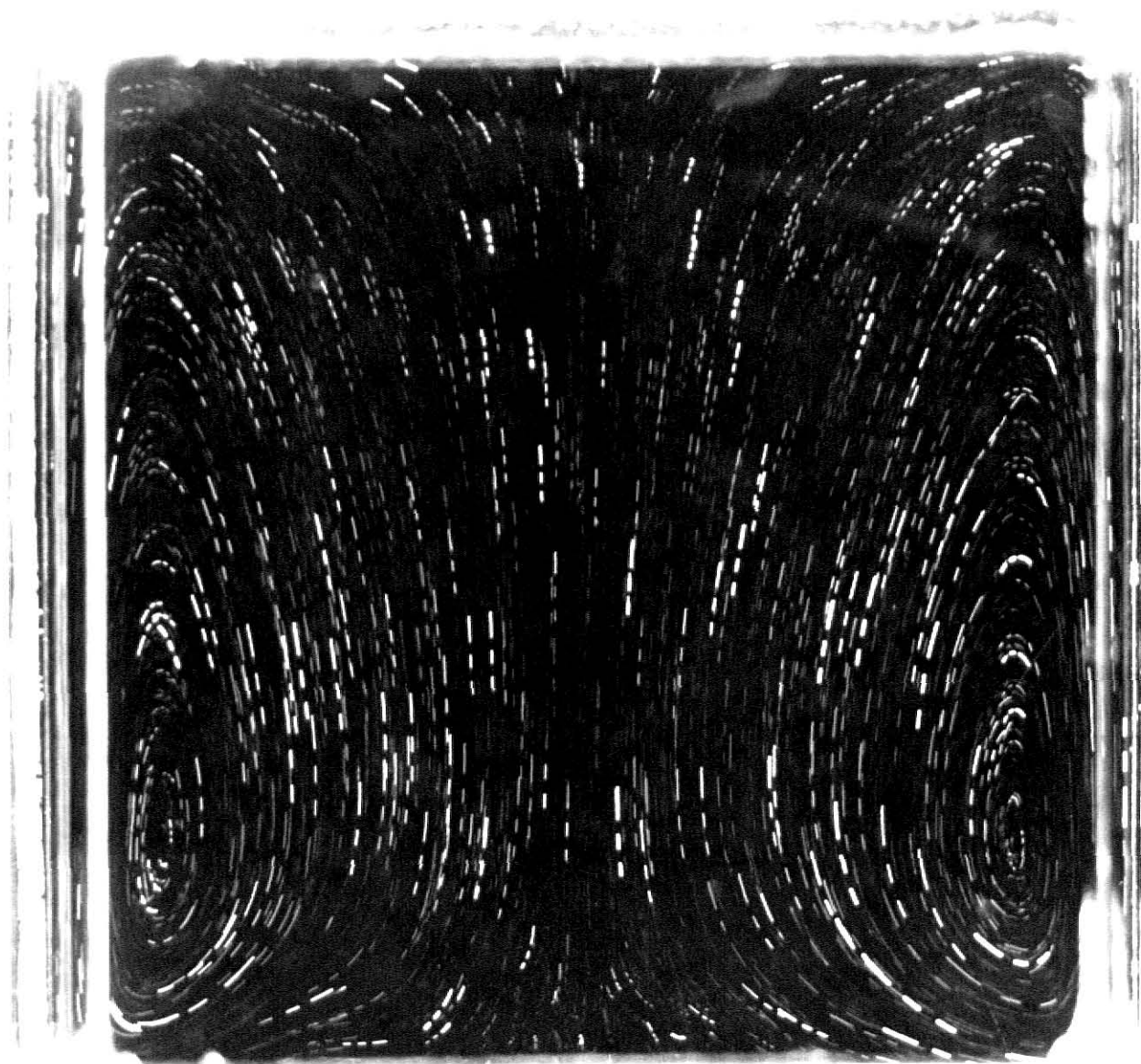


Figure 7. Photograph of Flow Pattern --- Main Flow.



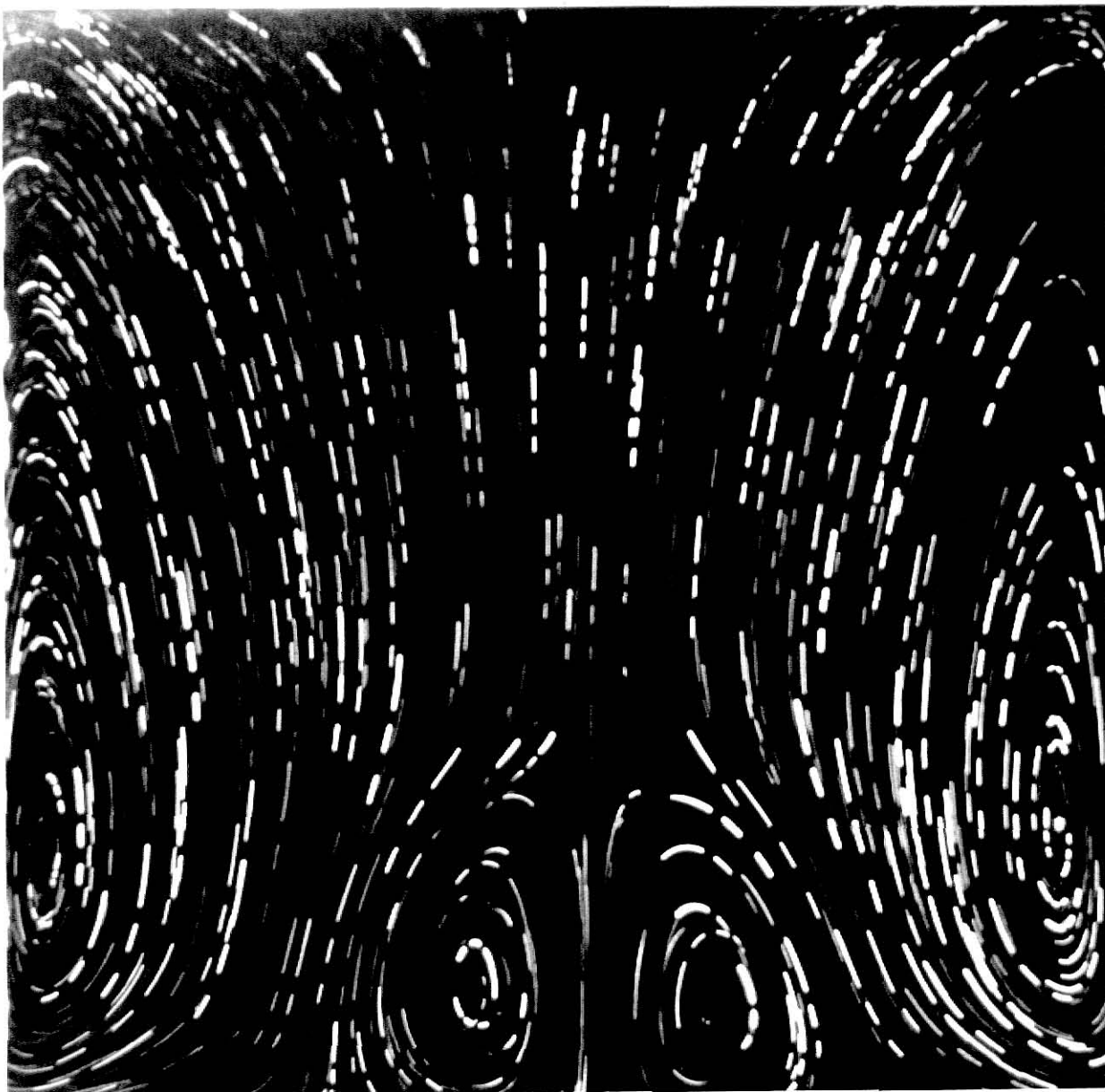


Figure 8. Photograph of Flow Pattern --- Small Reverse Flow  
Coexisting with the Main Flow.



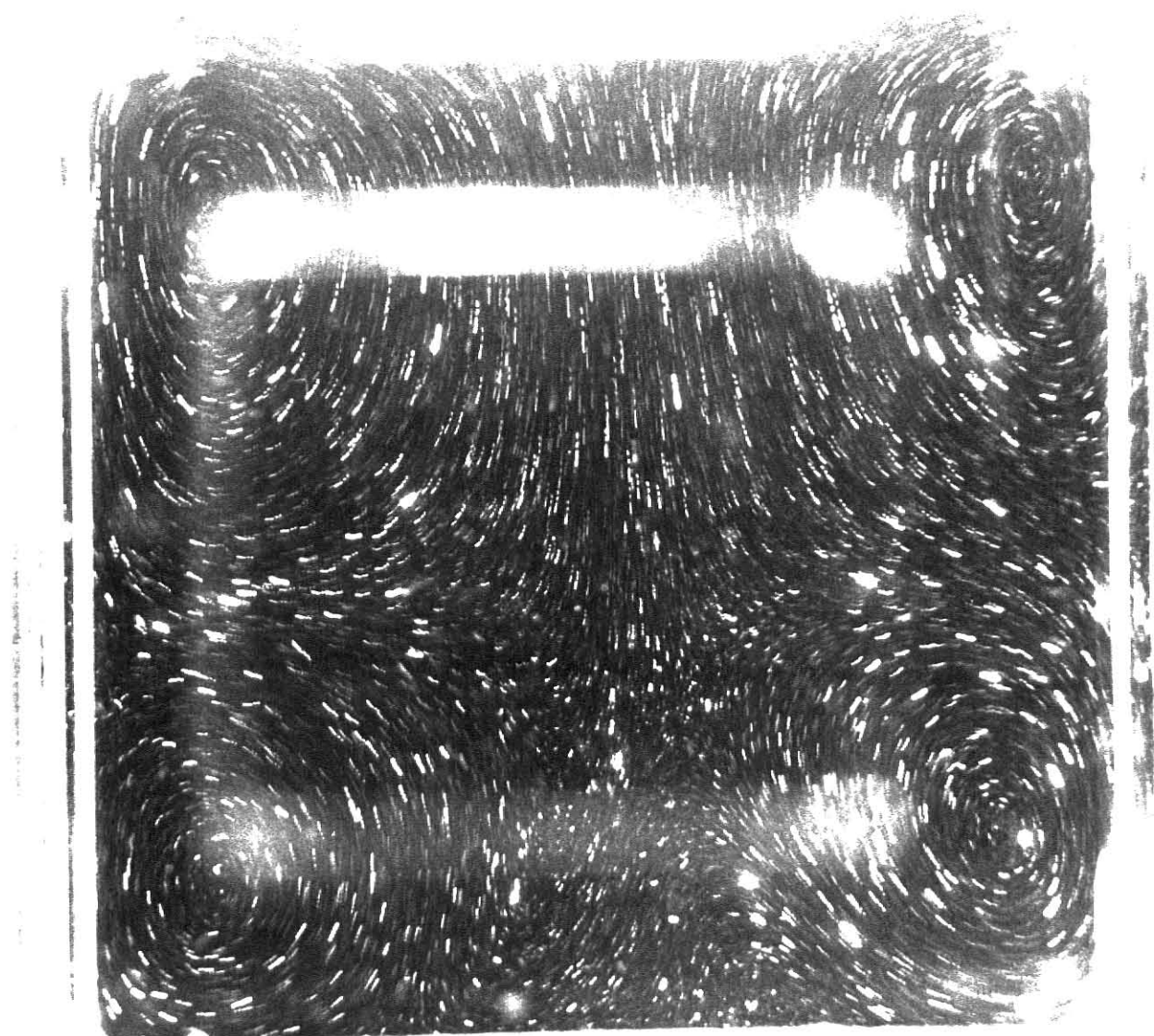


Figure 9. Photograph of Flow Pattern --- Main Flow and Reverse Flow of Similar Size.

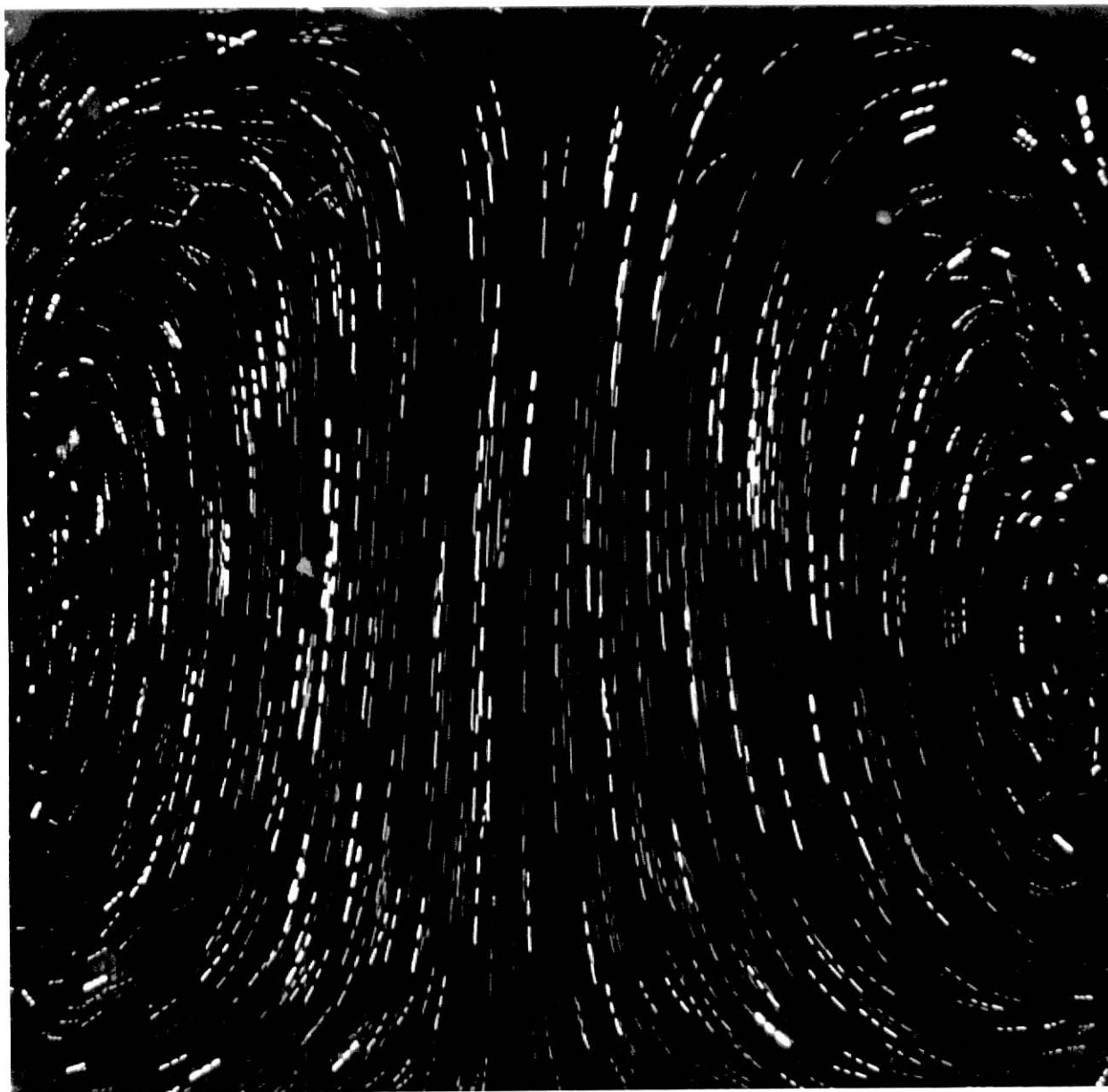
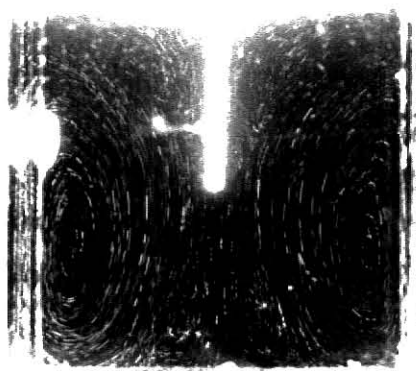
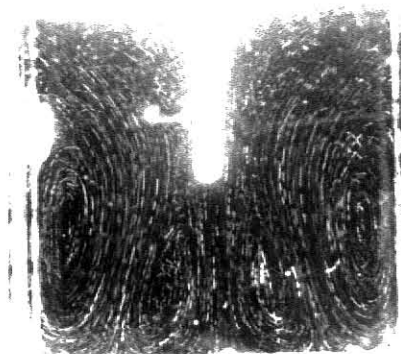


Figure 10. Photograph of Flow Pattern --- Single Reverse Flow.



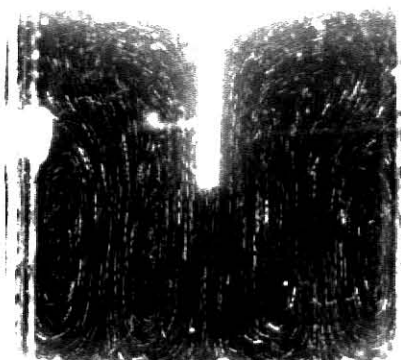
20 min.



25 min



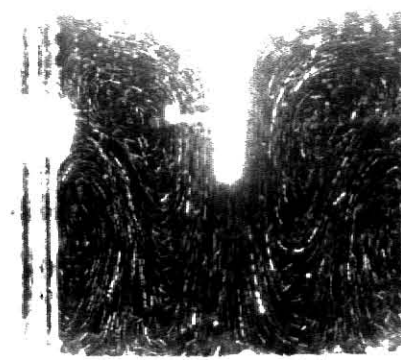
30 min.



35 min

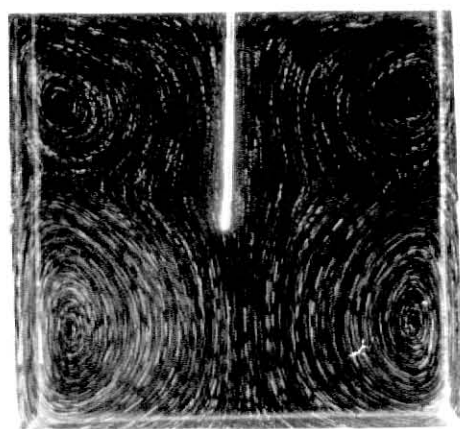


40 min.

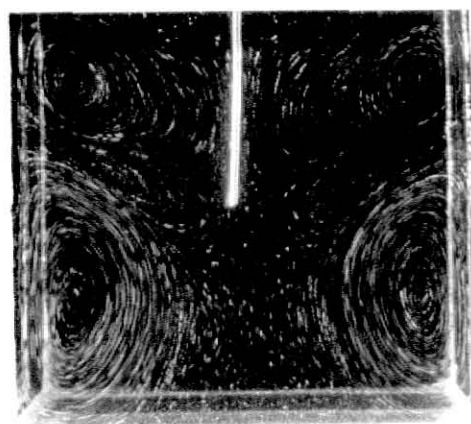


50 min.

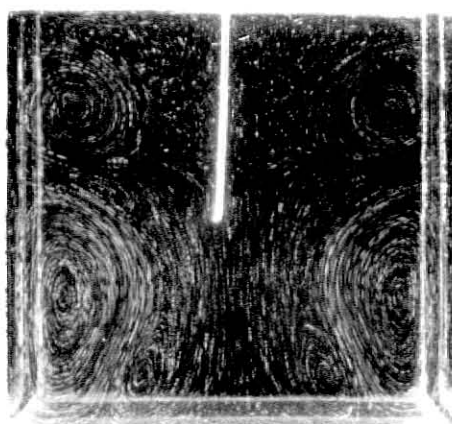
Figure 11. The Formation of Reverse Flow in 2" Cube.



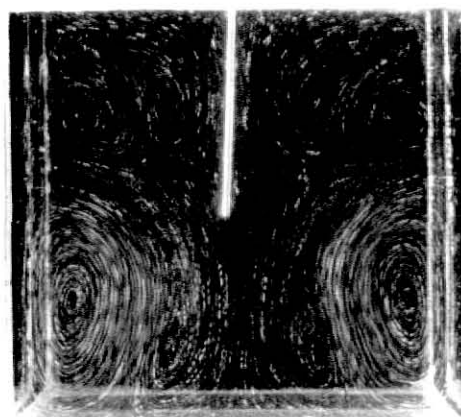
10 min.



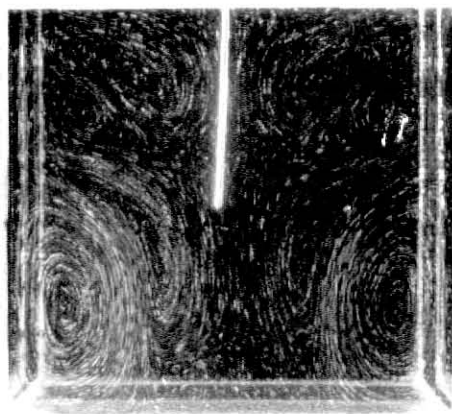
30 min.



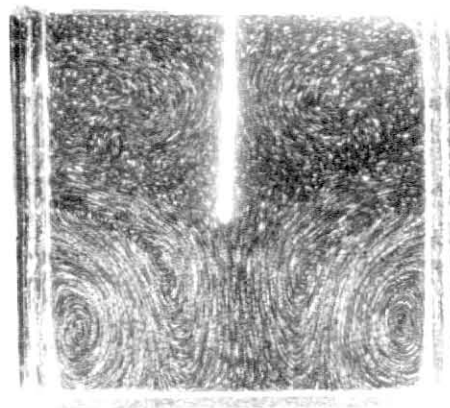
50 min.



80 min.

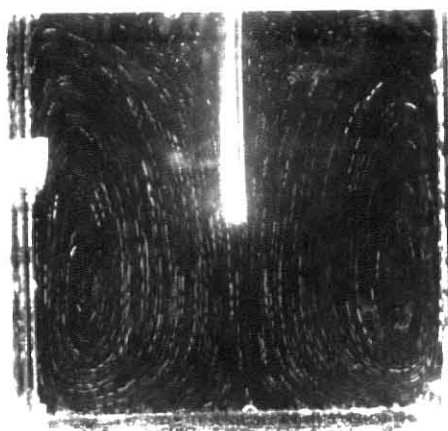


110 min.

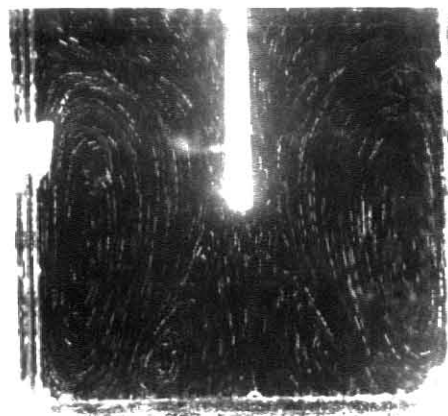


160 min.

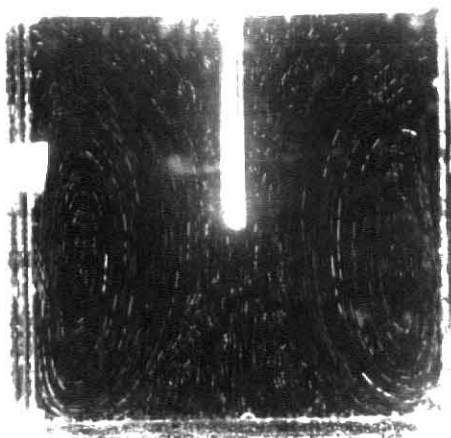
Figure 12. A Sequence of Complicate Flow Patterns in 4" Cube.



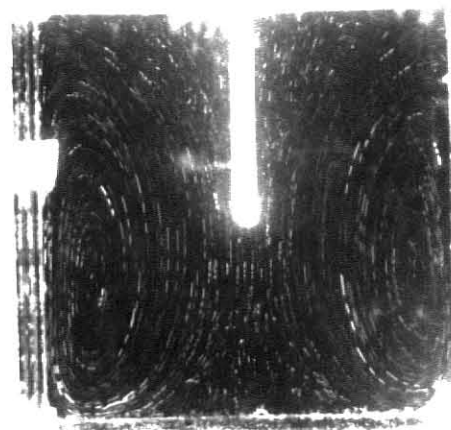
5 min.



8 min.



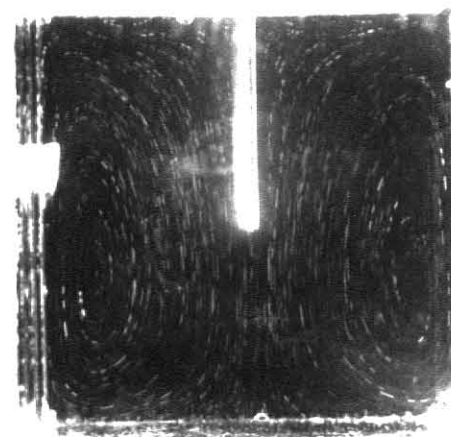
14 min.



17 min.

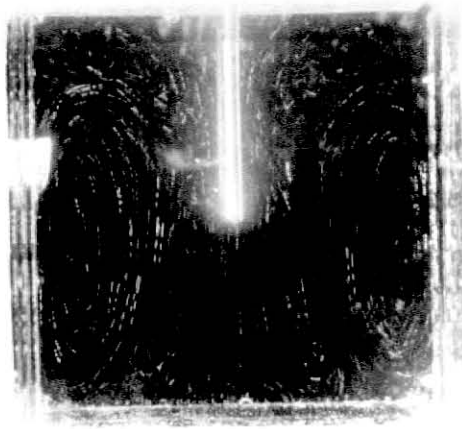


25 min.

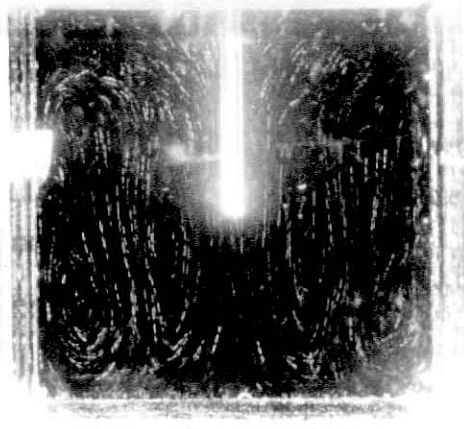


30 min.

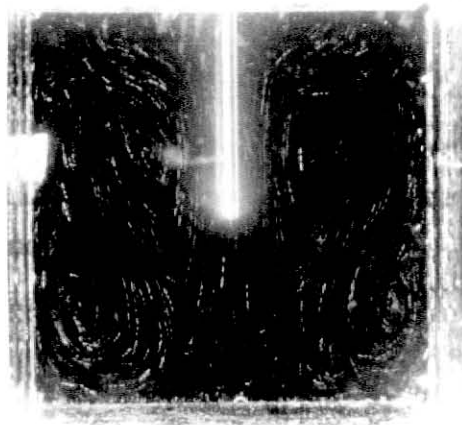
Figure 13. A Sequence of Typical Flow Pattern Changes.



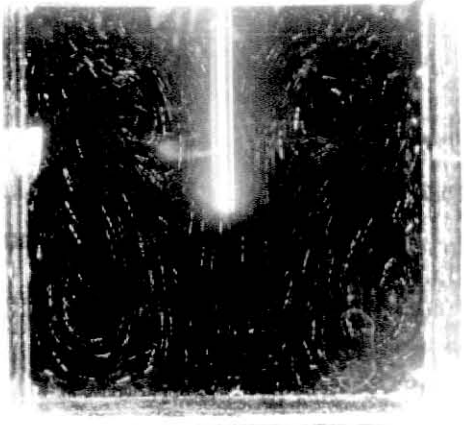
35 min.



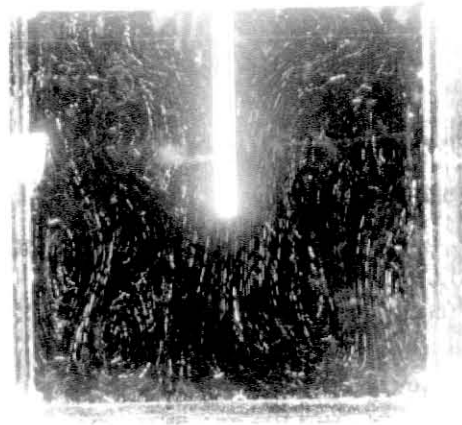
42 min.



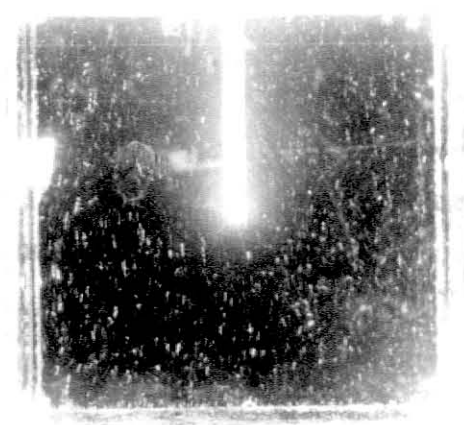
50 min.



55 min.



70 min.



90 min.

Figure 13. A Sequence of Typical Flow Pattern Changes (continued).



For main flow pattern, if the average velocity flux for upward and downward flows were equivalent, the thickness of the fluid layer flowing upward would be about twenty percent of the cube dimension. The actual thickness was between twenty and thirty percent, most likely due to wall effects. The velocity decreased to zero at the wall and retarded the average velocity of the entire upward stream.

The height of the center of circulation was about 25% from bottom wall for main flow and about 50% from bottom wall if the pattern contained a single reverse flow as shown in Figure 10.

In larger cubes (for example, 4"), or for a fluid with higher viscosity, the side walls did not have as much influence on the center fluid, therefore more than two circulation patterns could exist. Figure 12 shows a sequence of these complicated flow patterns. The flow patterns could not be correlated by Rayleigh number, by Rayleigh number and Prandtl number together, or by time, but they were surprisingly reproducible indicating something other than random changes. More study on these flow patterns is necessary to find the exact cause and correlation for the changes from one flow pattern to another.

In these photographs, the exposure time for first streak was twice as long as the second and third streaks, therefore this kind of photographs gave information of flow direction and velocity as well. The direction of flow was from long streak to short streak. Figure 13 is a sequence of photographs showing typical flow pattern changes in a 2" cube with 88% glycerin solution. They cover most of the flow patterns that were observed. For low

viscosity fluids, several of the patterns could not be observed, perhaps due to rapid changes between patterns which were missed by the photographic technique. For clarity, the streamlines were traced (Figures 14 to 19) from photographs to show the general patterns of the flow. Figure 20 shows five flow pattern change sequences on time scales. Cube size, inside fluid, viscosity, temperature step change, and initial temperature were given for each sequence.

## II. VELOCITY PROFILES

The local velocity inside the cube was estimated from the length of the streaks in the photograph and the time of exposure. The velocity profiles at a horizontal plane passing through the centers of circulation were determined and plotted in Figure 21 through 26. This choice of presentation was so that the figures would also show the location of the center of circulation (position of zero velocity). In the figures, positive velocities indicate upward flow and negative velocities downward flow.

Figures 21, 22, and 23 were produced from one run in which ethylene glycol solution in a 3" cube and was subjected to a step temperature change of one degree centigrade. The figures show results at 37 minutes, 50 minutes, and 90 minutes after the temperature change. The velocity decreased from 37 minutes to 50 minutes, as expected, but increased from 50 minutes to 90 minutes. In observing the sequence of photographs taken in this experiment, there appeared a special double circulation pattern (showed in Figure 9) between 50 minutes and 90 minutes. The flow accelerated



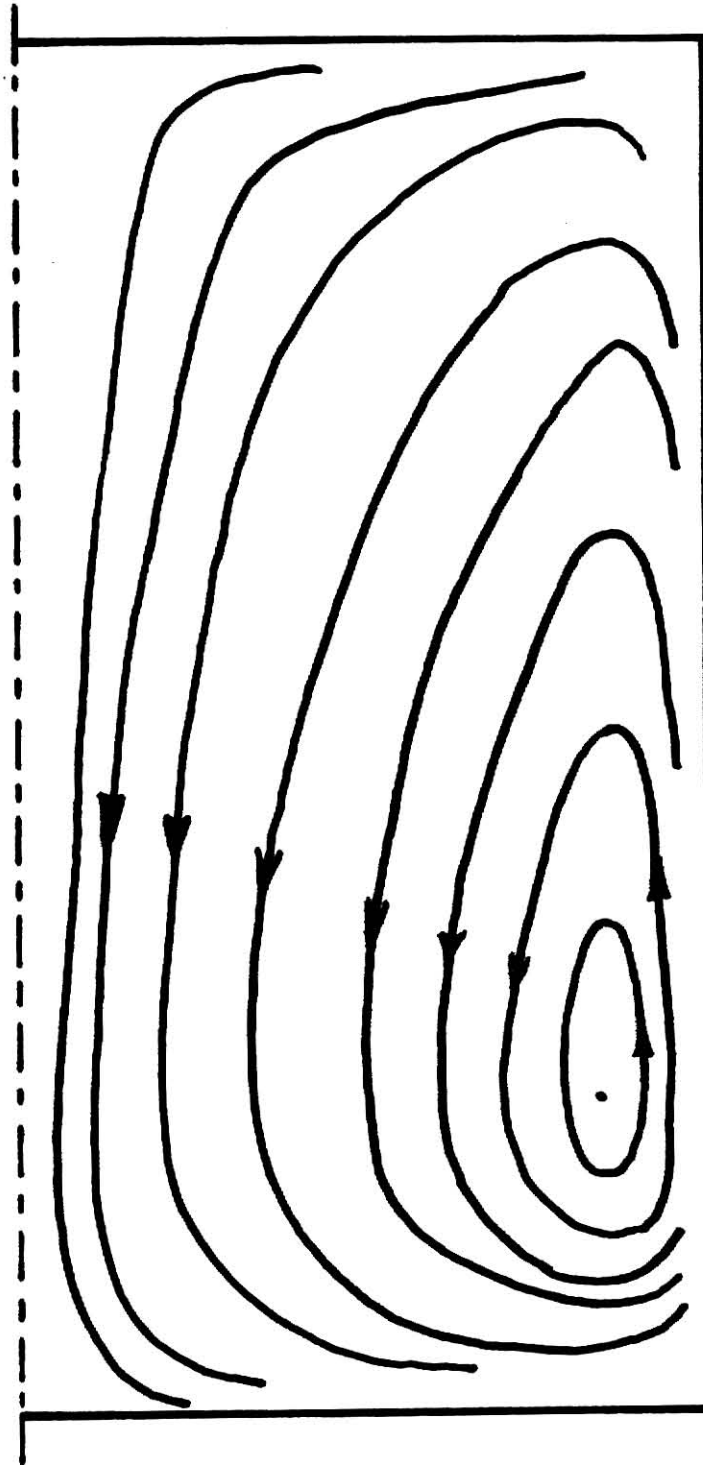


Figure 14. Streamlines of the Main Flow with Large Driving Force.

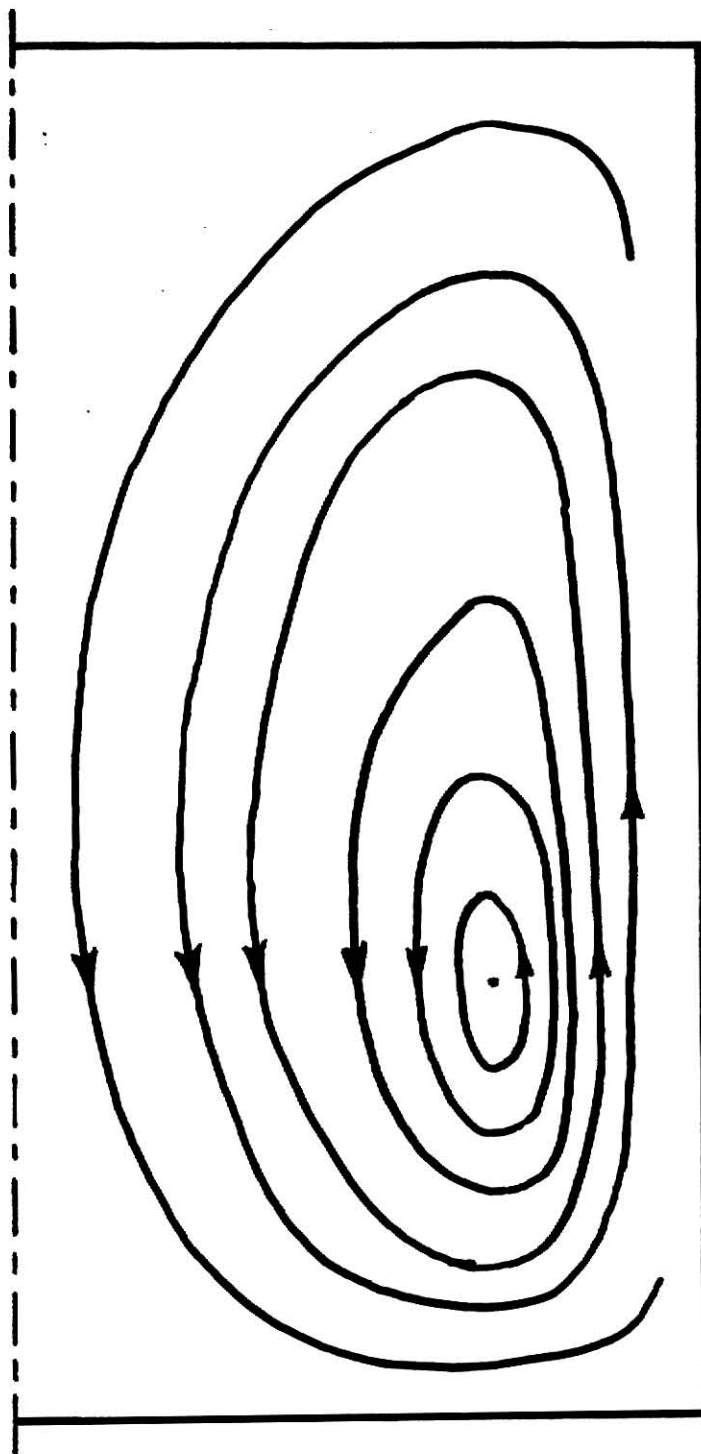


Figure 15. Streamlines of the Main Flow with Small Driving Force.

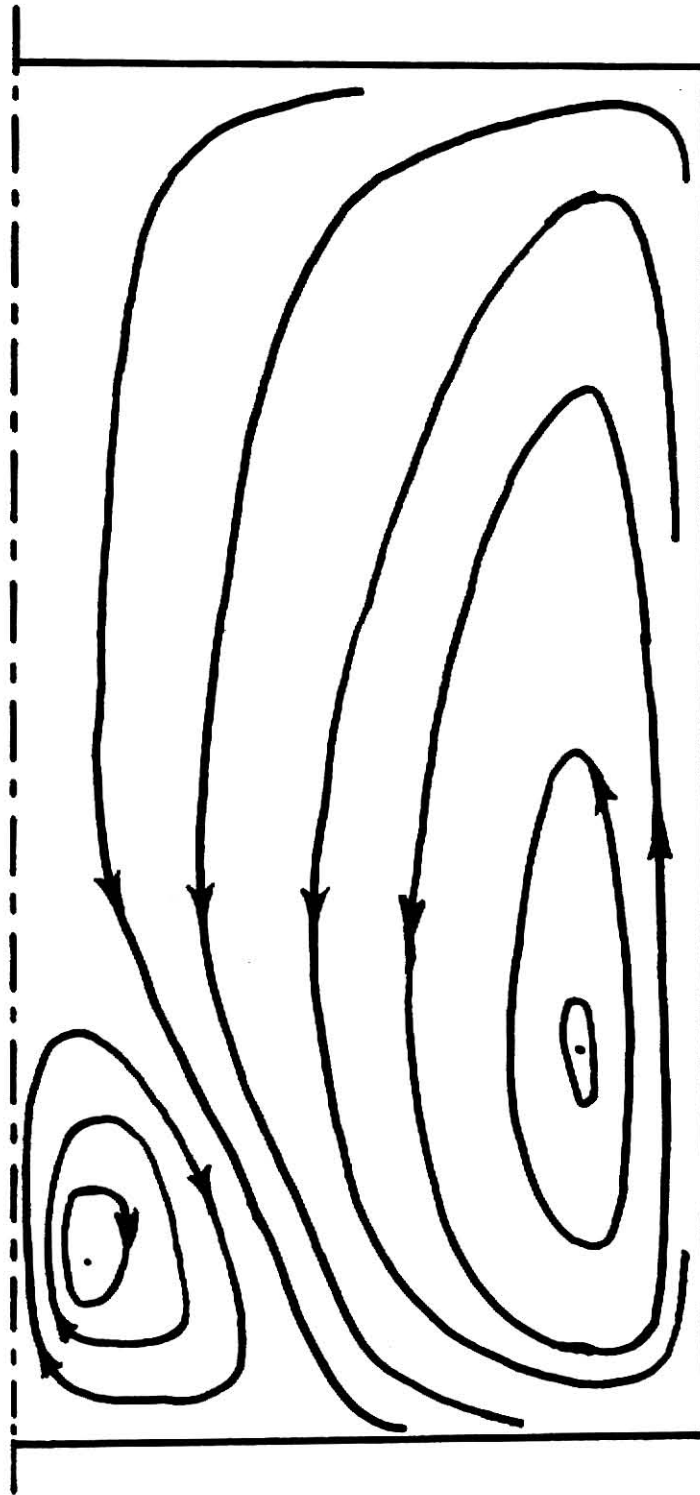


Figure 16. Streamlines of the Main Flow Coexisting with a Small Reverse Flow.

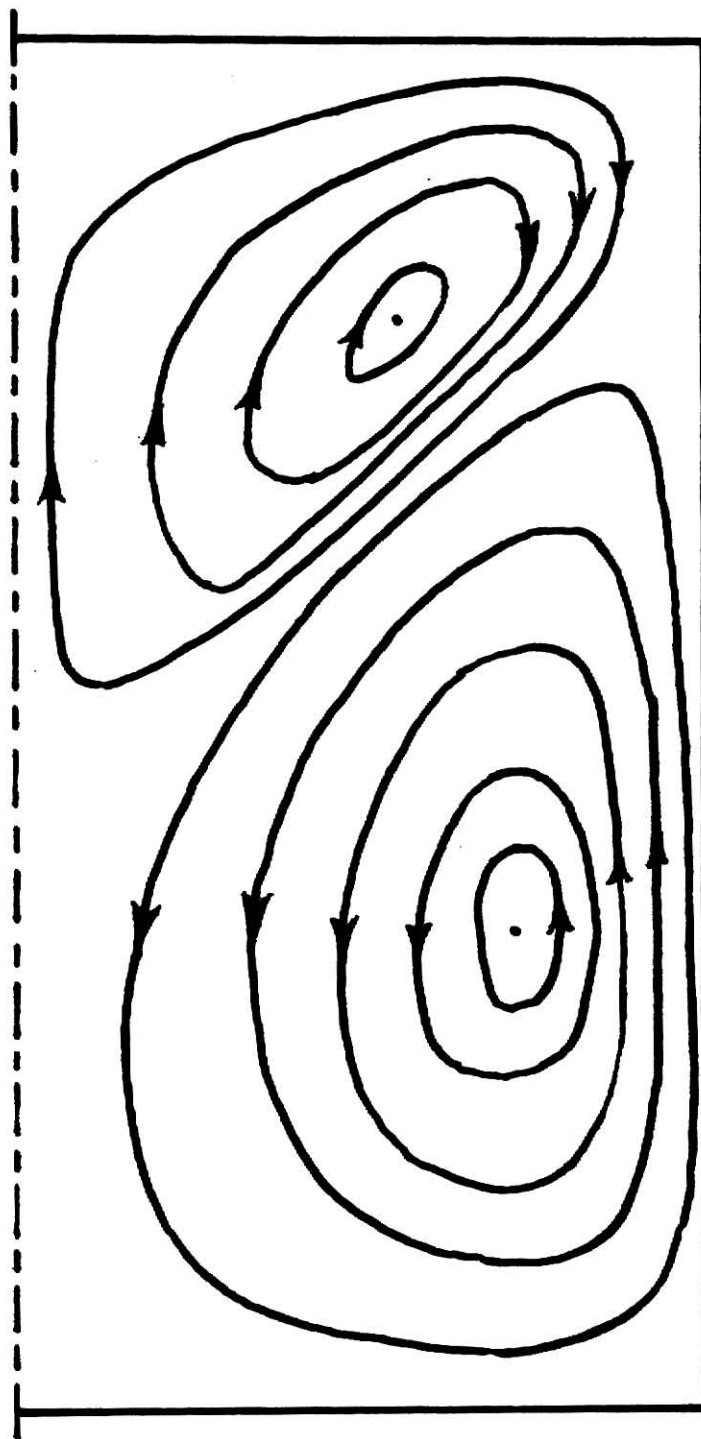


Figure 17. Streamlines of the Flow Pattern with Main Flow and Reverse Flow in Similar Size.

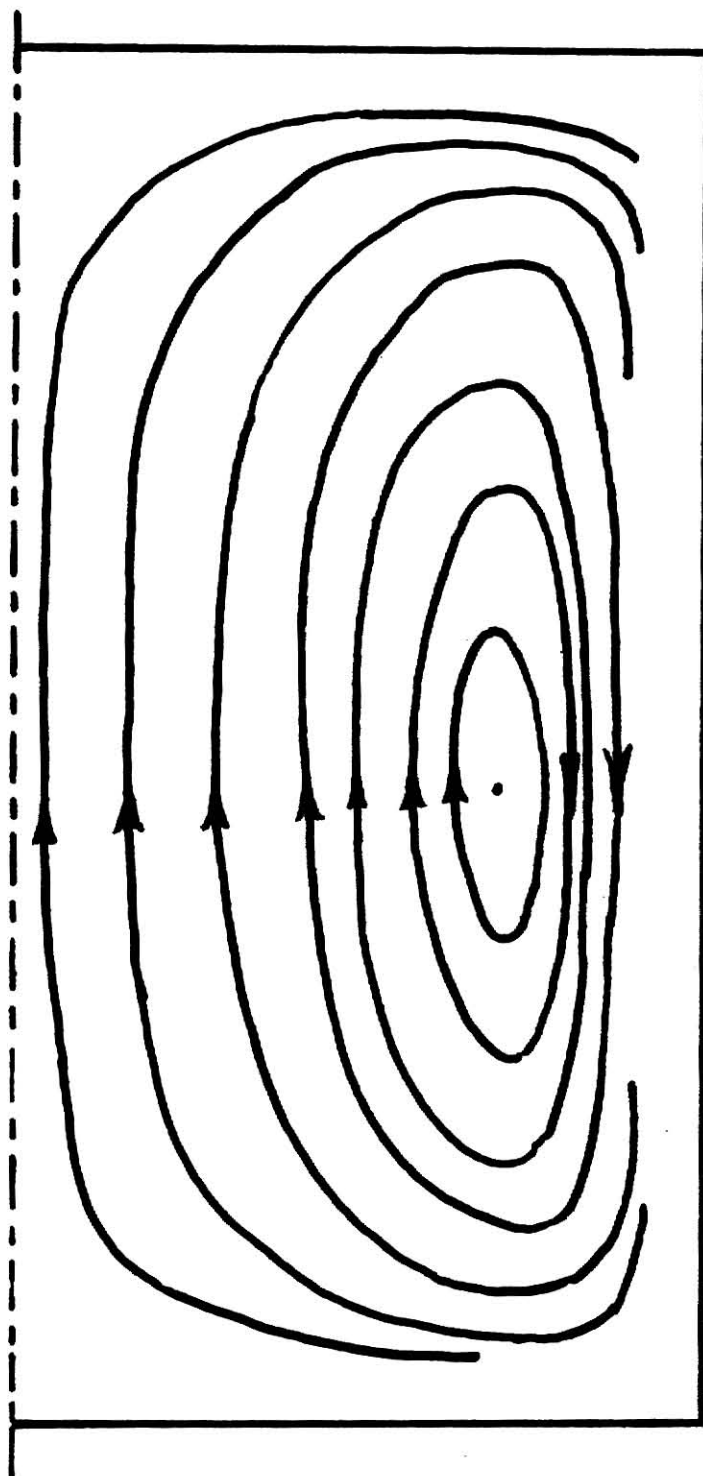


Figure 18. Streamlines of the Reverse Flow Pattern.

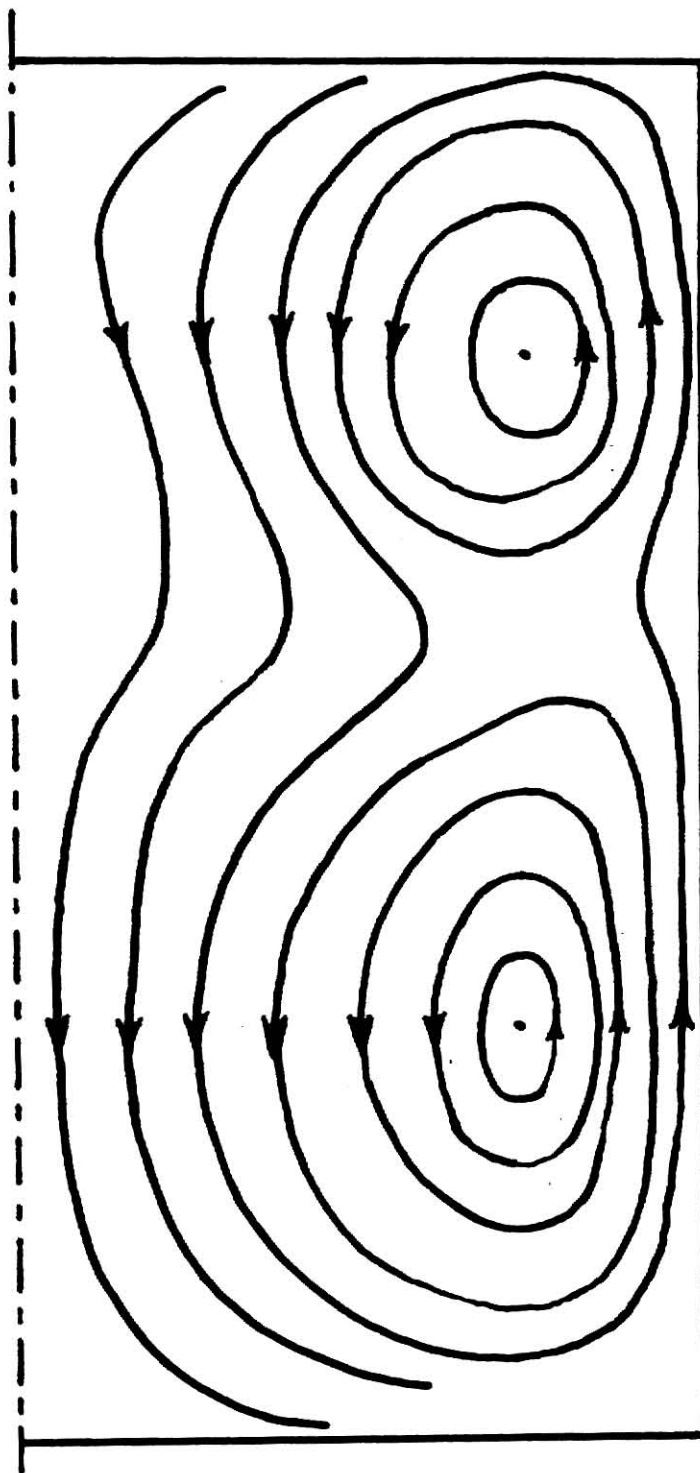


Figure 19. Streamlines of Two Circulation Patterns Moving in the Same Direction.

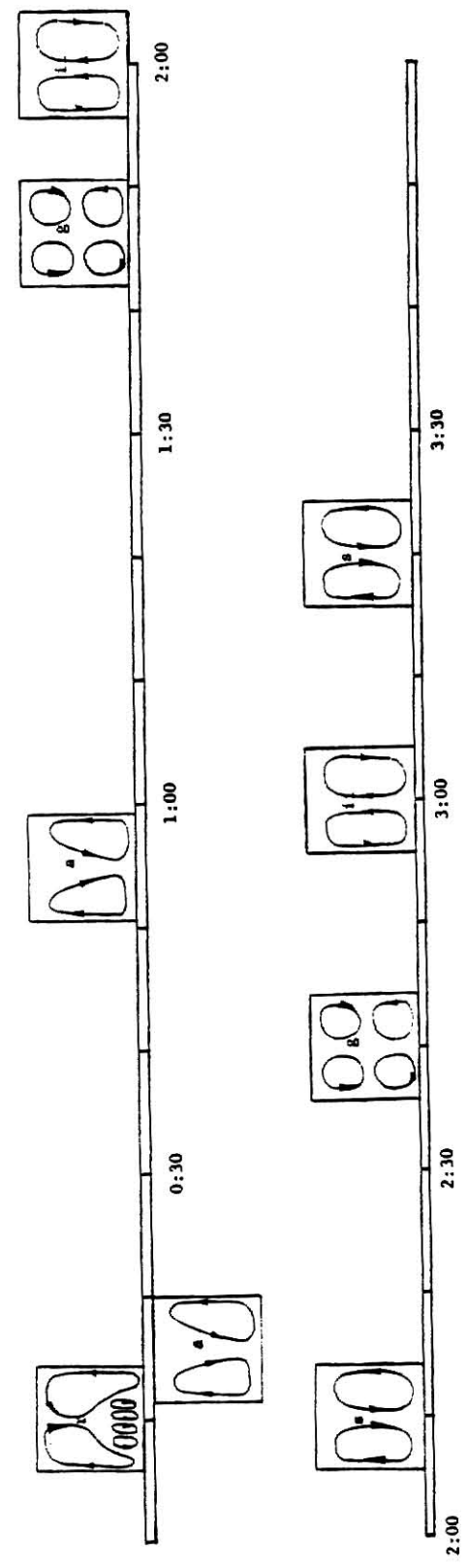
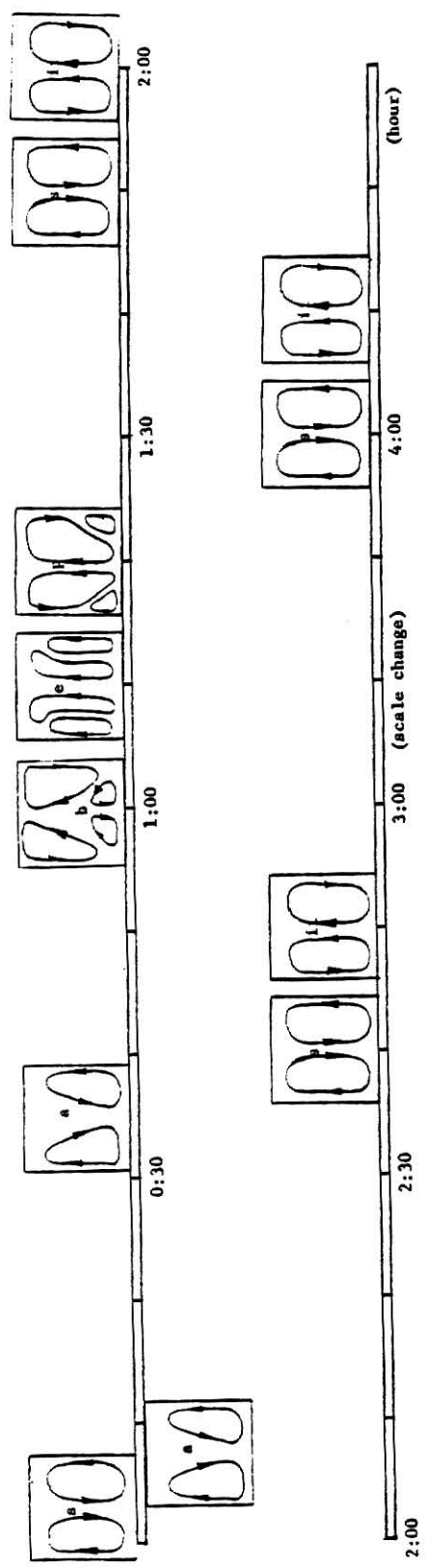
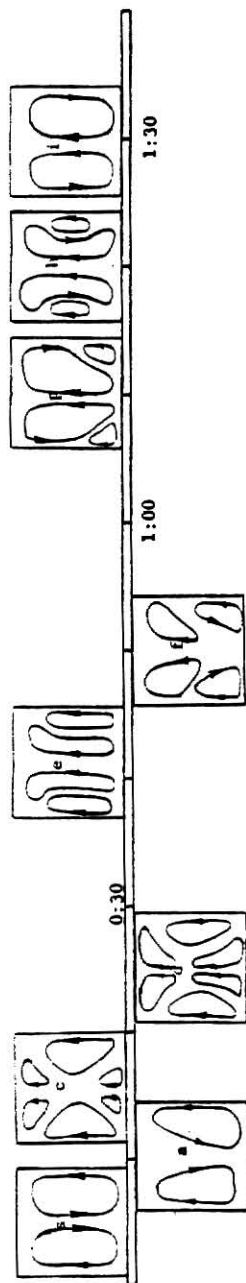
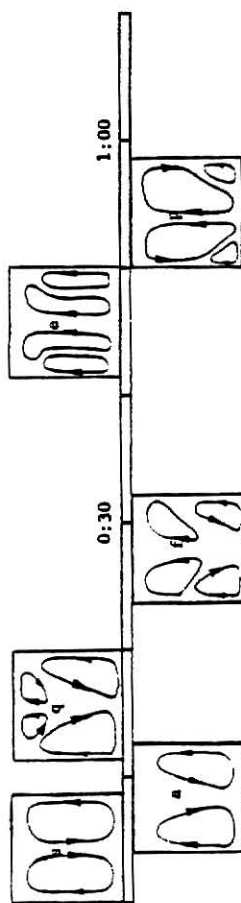


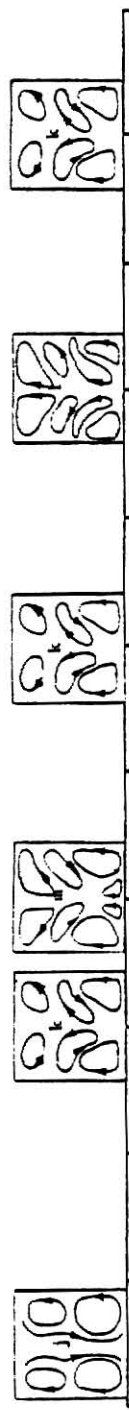
Figure 20. Flow Pattern Change on Time Scales.



(c) 2" cube, 88% glycerin ( $\mu = 150$  cp),  $\Delta T = 2^\circ\text{C}$ ,  $T_0 = 22^\circ\text{C}$



(d) 2" cube, glycerin ( $\mu = 1400$  cp),  $\Delta T = 3.4^\circ\text{C}$ ,  $T_0 = 21^\circ\text{C}$



(e) 4" cube, glycerin ( $\mu = 1400$  cp),  $\Delta T = 3^\circ\text{C}$ ,  $T_0 = 21^\circ\text{C}$

Figure 20. Flow Pattern Change on Time Scales. (continue)



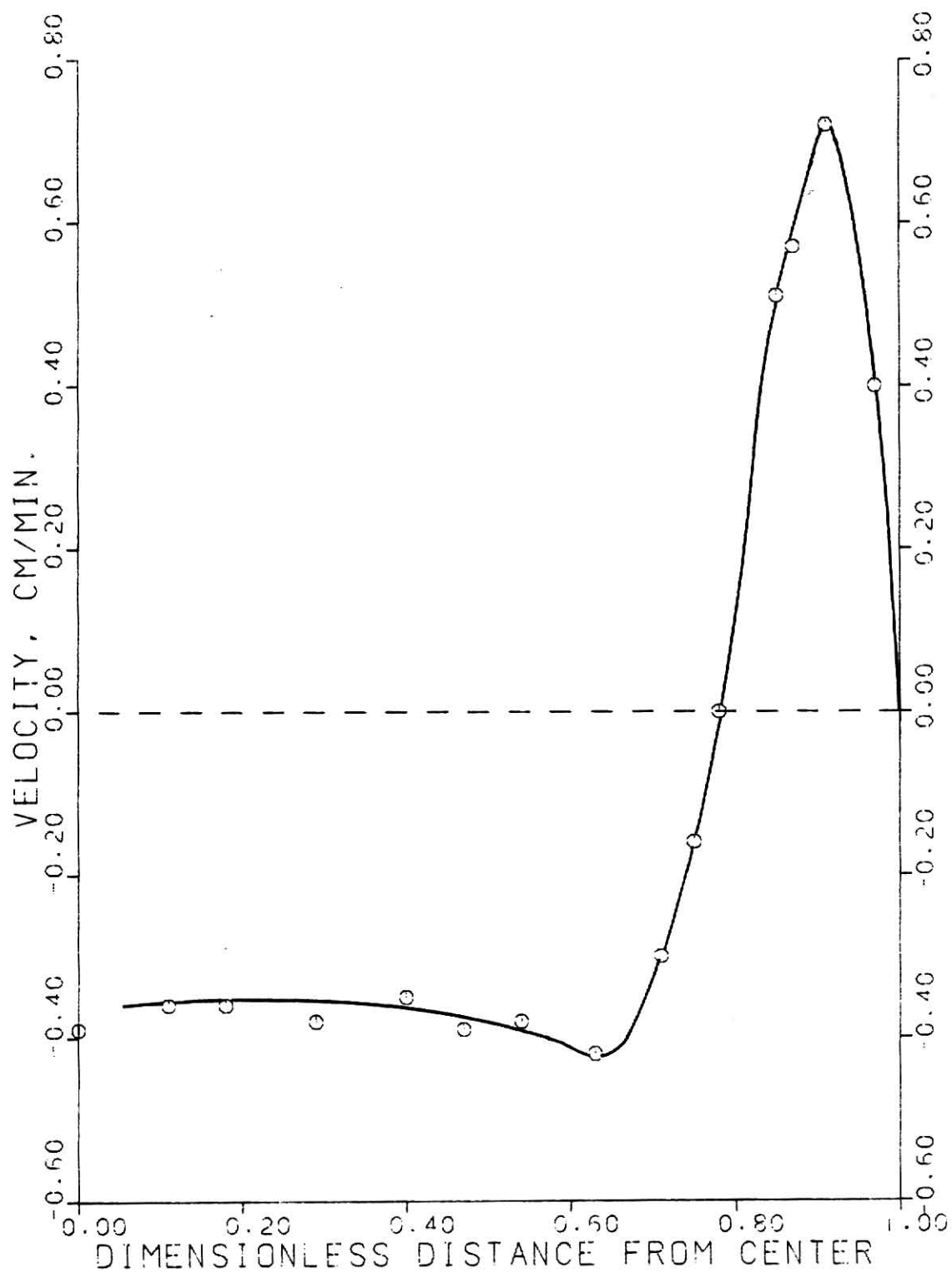


Figure 21. Velocity Profile of Ethylene Glycol in 3" Cube, 37 min. after 1°C Step Change.

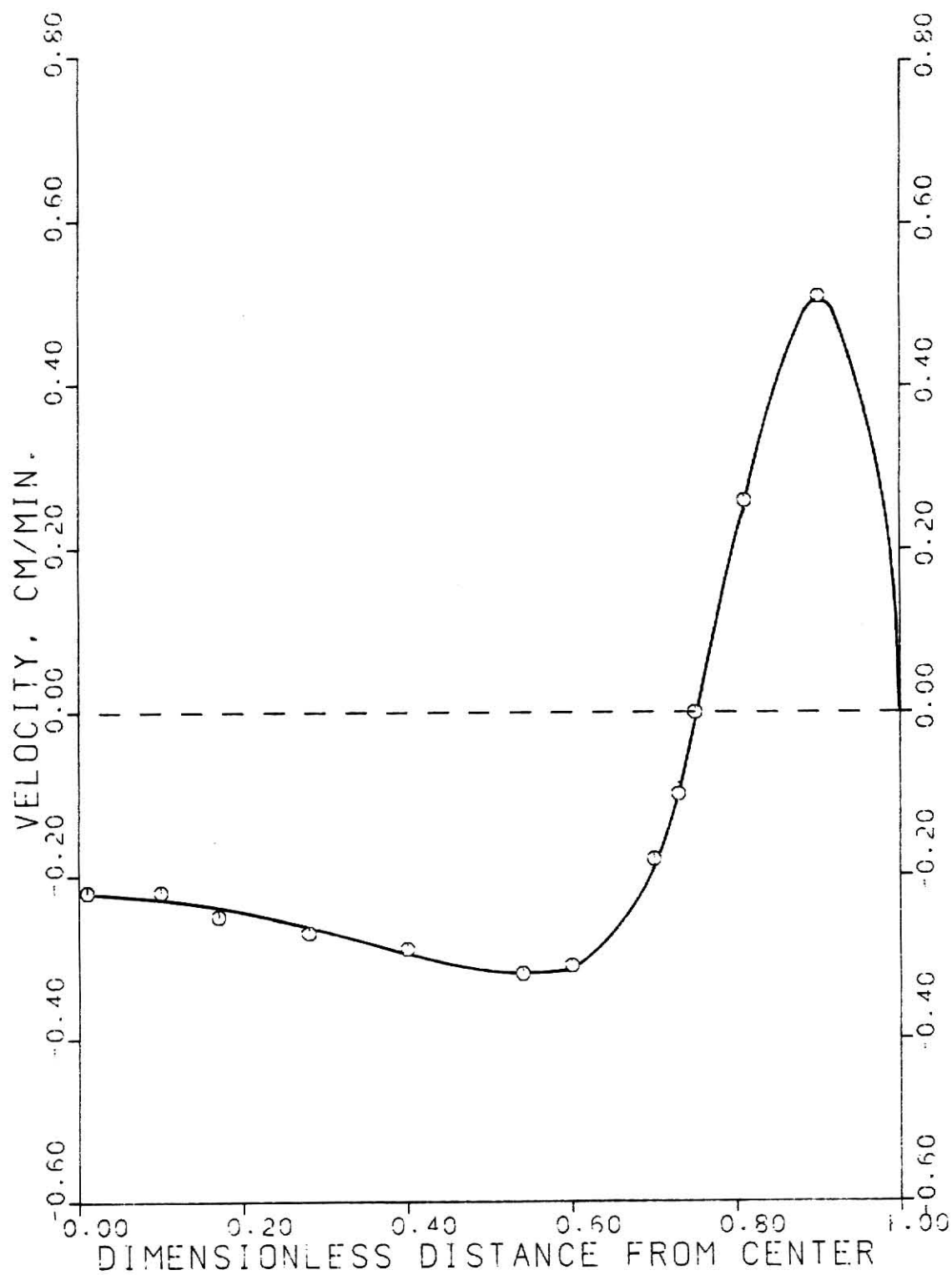


Figure 22. Velocity Profile of Ethylene Glycol in 3" Cube, 50 min.  
after 1°C Step Change.

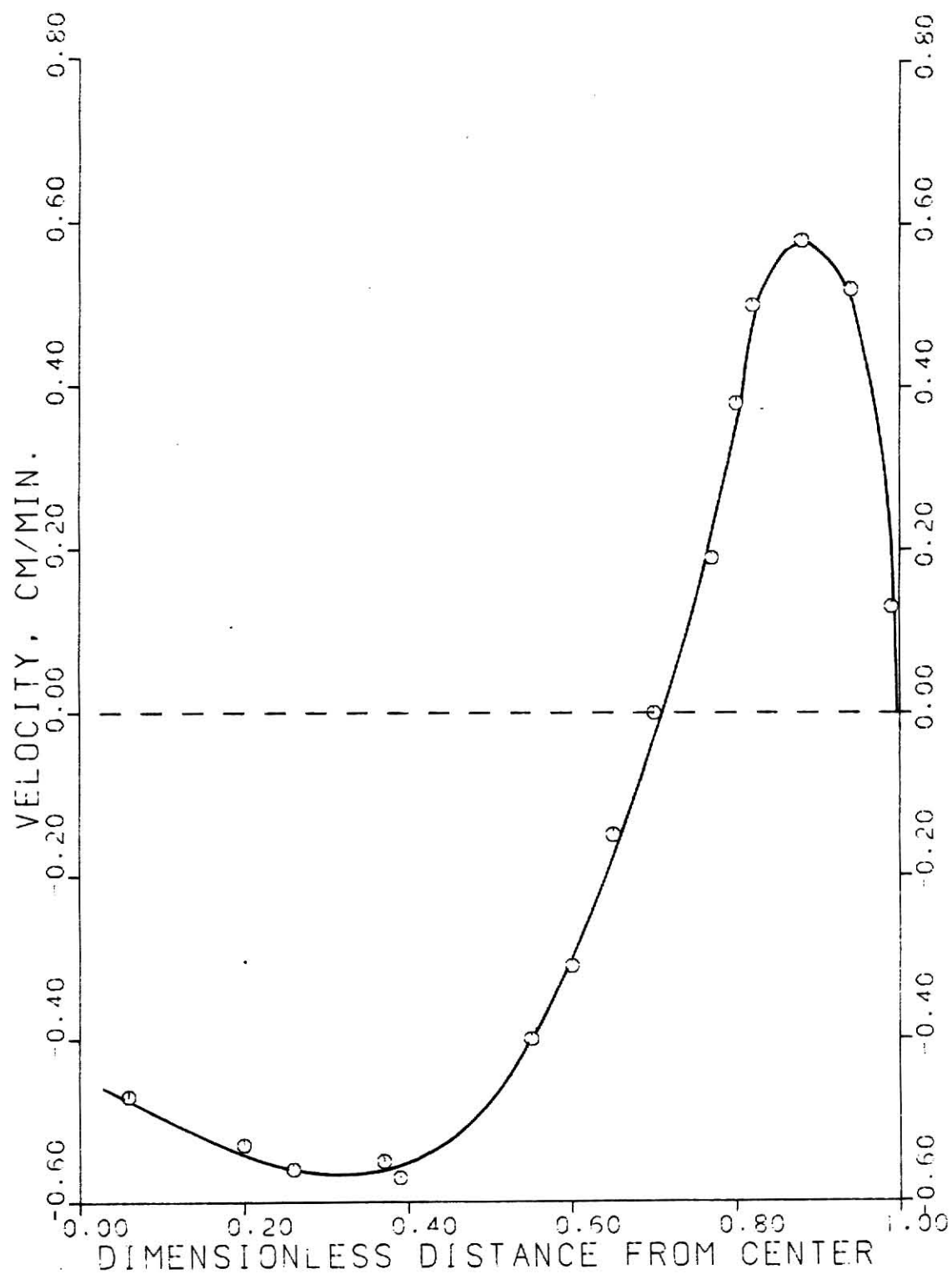


Figure 23. Velocity Profile of Ethylene Glycol in 3" Cube, 90 min.  
after 1°C Step Change.

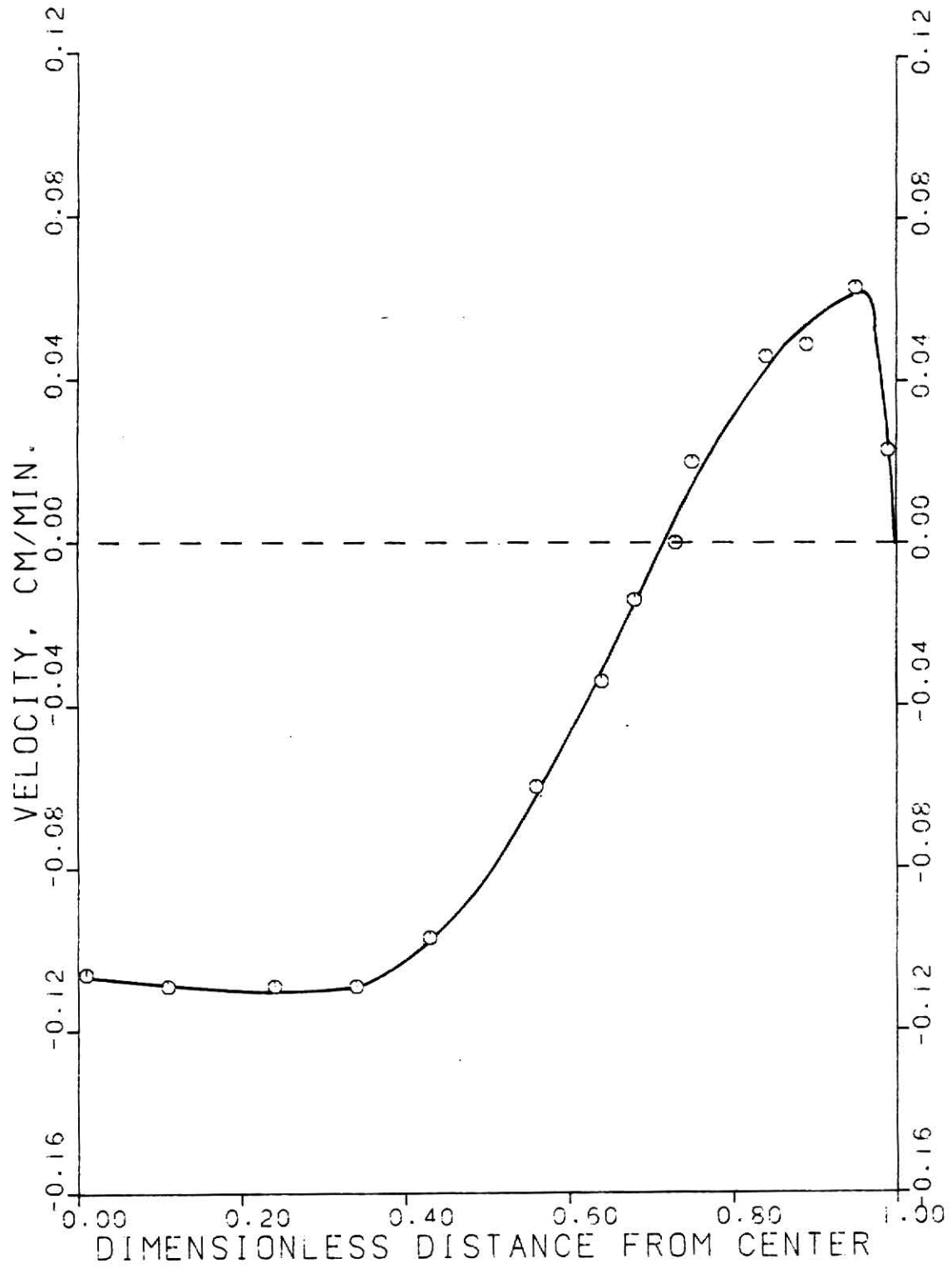


Figure 24. Velocity Profile of Glycerin in 2" Cube ( $Ra = 10^4$ ).

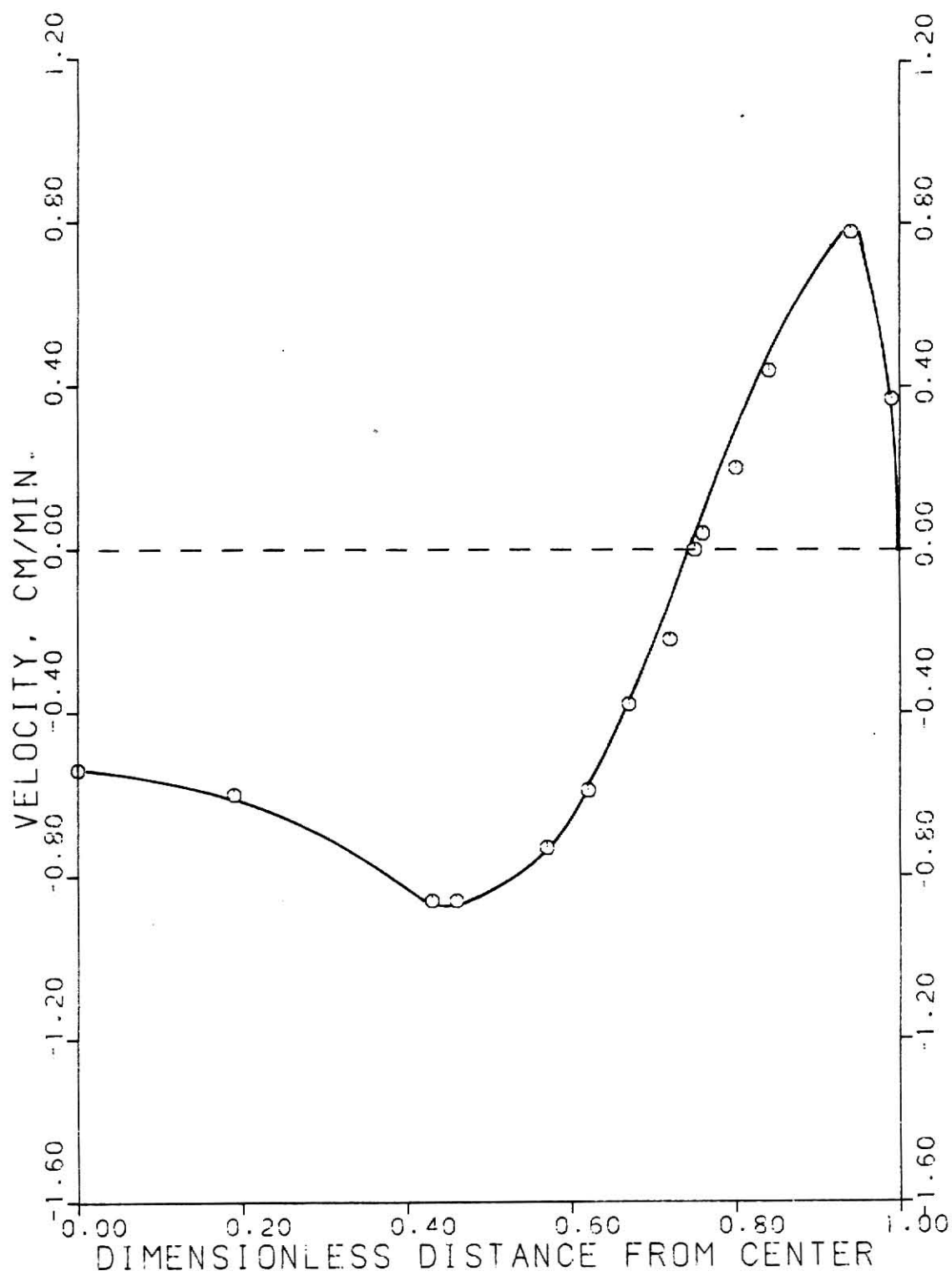


Figure 25. Velocity Profile of Glycerin in 2" Cube ( $Ra = 7 \times 10^4$ ).

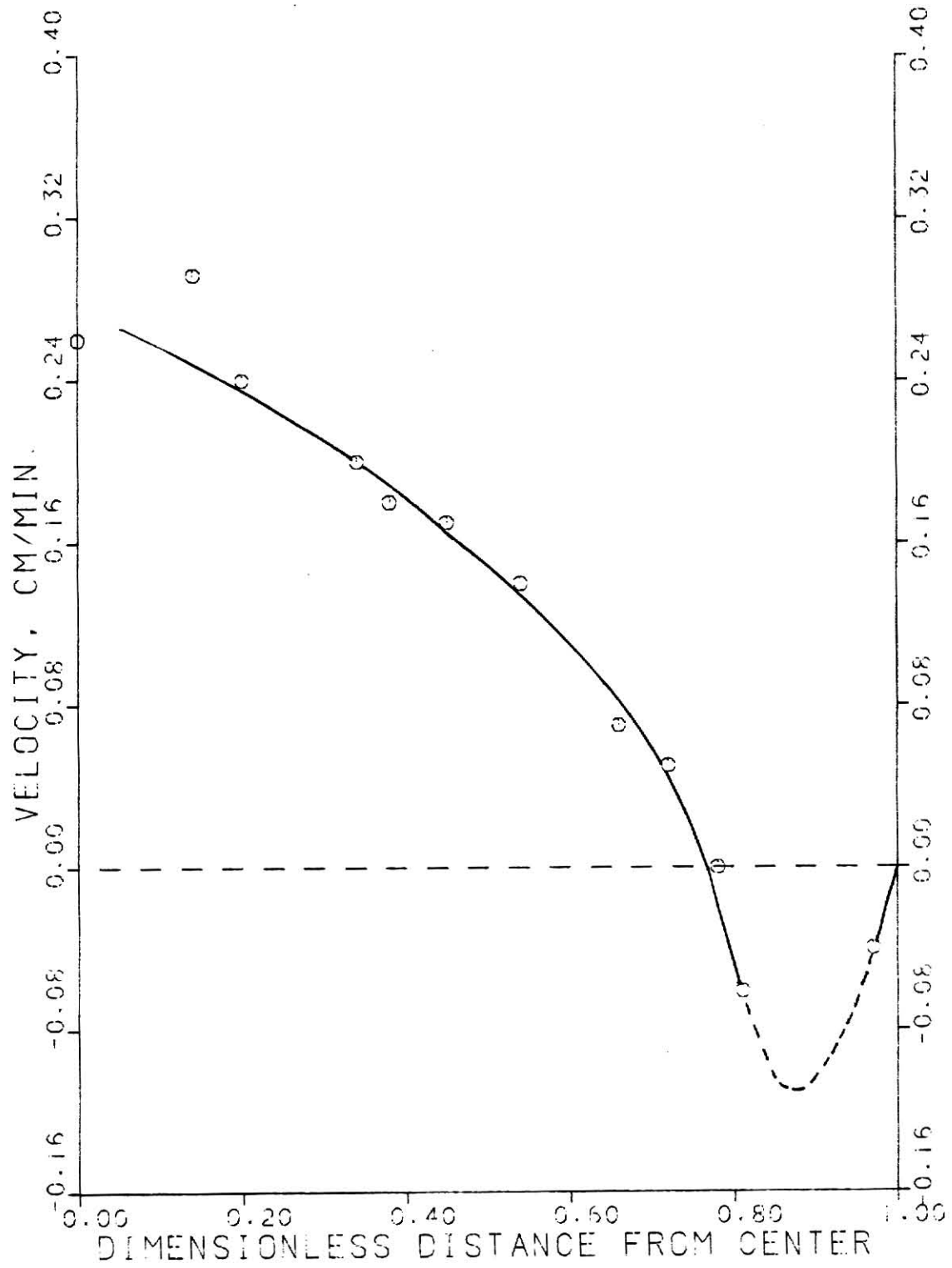


Figure 26. Velocity Profile of a Typical Reverse Flow for Ethylene Glycol in 3" Cube.

after combination of two circulations into one. The discontinuity in velocity change, or flow pattern change, made the correlation of fluid flow by Rayleigh number impossible. Figure 24 and 25 are velocity profiles for 2" cube, the working fluid was glycerin solution with viscosity about 240 centipoise. These profiles show that the velocity dropped rapidly and changed sign, then, when approaching center core, stayed constant for about 50% of the range around the center. Figure 26 shows a reverse flow velocity profile.

### III. HEAT TRANSFER CORRELATION

By dimensional analysis for natural convection, the heat transfer correlation may be expected to be of the form

$$Nu = f(Gr, Pr)$$

Actually, if inertial forces are assumed to be negligible as compared to buoyancy and viscous forces, or the Prandtl number is larger than 5, similarity considerations predict that the Nusselt number can be expressed as a function of the Rayleigh number alone(38). That is,

$$Nu = f(Ra)$$

O'toole and Silverston(85) were successful in correlating previous investigators' experimental data for the laminar region by an equation of the form

$$Nu = c(Ra)^a$$

Therefore, the heat transfer data obtained in this experiment were correlated using Nusselt number and Rayleigh number alone.

The relationship between Nusselt number and Rayleigh number

was fitted with linear and quadratic correlations by means of the least square method. In the range  $5 \times 10^3 < Ra < 10^7$ , the best fit of the data were:

Linear correlation

$$\text{Log Nu} = -0.222 + 0.235 (\text{Log Ra}) \quad \text{or}$$

$$\text{Nu} = 0.600 \text{ Ra}^{0.235}$$

$$(\text{correlation coefficient} = 0.947)$$

Quadratic correlation

$$\text{Log Nu} = -0.606 + 0.380 (\text{Log Ra}) - 0.013 (\text{Log Ra})^2$$

$$(\text{correlation coefficient} = 0.948)$$

Since the quadratic fit is not better than the linear one, the linear relation is suggested. The correlation and data are shown in Figure 27. Several psuedo steady state (constant temperature difference) runs were made and the results compared with step temperature change data. Both types of data are included in Figure 27. The assumption of psuedo steady state in heat transfer calculation was thereby confirmed.

If we consider the situation as the Rayleigh number decreases toward zero, the heat transfer approaches that for pure conduction. Assuming quasi-steady state and symmetry in three coordinates, a rough approximation of the heat transfer equation is

$$3k \frac{d^2 T}{dx^2} = \frac{Q}{V}$$

The boundary conditions are

$$\frac{dT}{dx} = 0$$

$$\text{at } x = 0$$

$$T = T_s$$

$$\text{at } x = W/2$$



The solution of the above equation is

$$T = T_s + \frac{Q}{24kW} \left[ \left( \frac{2x}{W} \right)^2 - 1 \right]$$

The temperature at the center of the cube is

$$T_c = T_s - \frac{Q}{24kW}$$

The convective heat transfer coefficient is defined as

$$h = \frac{Q}{A(T_s - T_c)}$$

Therefore

$$h = \frac{4k}{W}, \text{ or}$$

$$Nu = 4$$

The intercept of the linear correlation obtained from this research for Nusselt number of 4 is about 3,000. Thus, it is reasonable to say that when Rayleigh number is less than 3,000, the heat transfer is essentially by conduction.

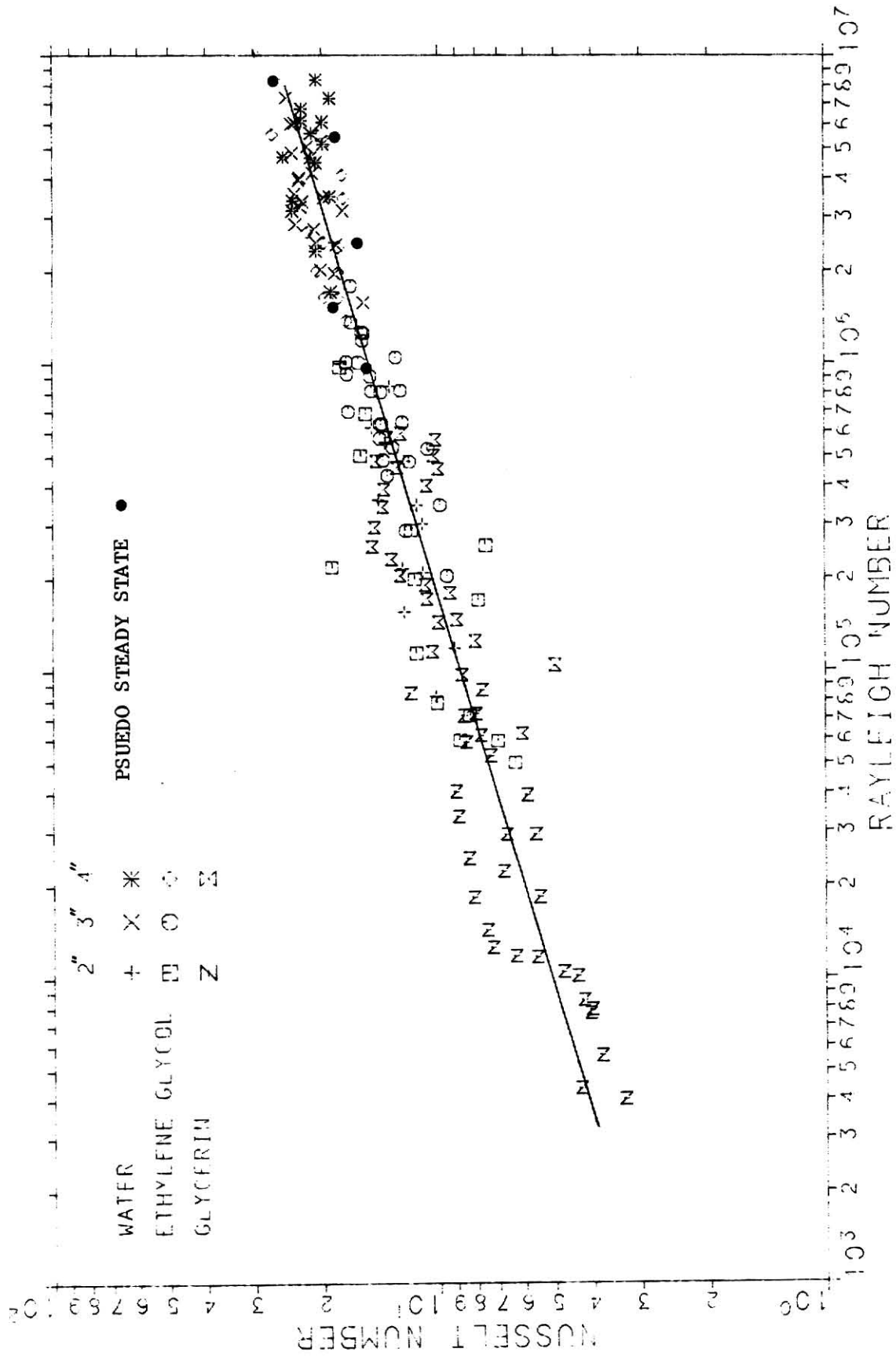


FIGURE 27. NATURAL CONVECTION HEAT TRANSFER CORRELATION

## CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

To the author's knowledge, this is the only experimental investigation of the flow patterns for natural convection heat transfer inside a cube for any wall conditions. The observed transient flow patterns were too complex to be correlated at this time. In order to explain and analyze the complicity of these flow patterns, it is recommended that pseudosteady state and steady state experiments be carried out. The former has been made in this study to support the assumption of similarity between transient state and pseudosteady state in heat transfer rate, but no flow pattern observations were made. The steady state condition could be realized by designing a heat source and heat sink and insulating all other walls. Heating from one side wall and cooling from opposite side wall, heating from bottom and cooling from top are examples of steady state designs. Simpler transient information could also be found by insulating several walls, for example, heating only the bottom, or only the top, etc. These methods would allow study of the effects of the various single driving forces. Comparison with published numerical results would also be possible for some of these simple cases. Additional studies with different sized cubes and a wider range of viscosity would also be helpful in arriving at a predictive method for the flow patterns.

For the correlation of natural convection heat transfer the suggested relationship between the Nusselt number and the Rayleigh number is  $Nu = c Ra^a$ . The present study succeeded in correlating data in the above form, and gave

$$Nu = 0.6 Ra^{0.235}$$

O'toole and Silverston(85) gave  $a = 0.252$  for horizontal layer and Jacob(58) gave  $a = 0.25$  for vertical layer. Compared with their results, the low of  $a$  value in the present study was expected since the heat transfer rate was based on all six walls and the heating effect of the top wall was much smaller than other walls. Measurement of the temperature profiles within the cube would be valuable in giving more information about the mechanism of heat transfer.

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## APPENDIX A

### FLOATING VELOCITY OF PARTICLES

Usually for particles of diameter greater than 10 microns the Brownian effect, which is a stochastic phenomenon and is very difficult to determine, can be neglected. The settling or floating effect of particles in a fluid in the presence of a gravitational field can be estimated by Stoke's law

$$F = 3\pi\mu vd \quad (1)$$

where  $F$  is the force on the particle,  $\mu$  the fluid viscosity,  $d$  the equivalent sphere diameter,  $v$  the velocity. The force is equal to the weight difference of particle and the displaced fluid,

$$F = (\rho_p - \rho_f) g \left(\frac{\pi}{6} d^3\right) \quad (2)$$

where  $\rho_p$  is the particle density,  $\rho_f$  the fluid density,  $g$  the gravitational acceleration.

Using these two equations, an expression for estimating the settling or floating velocity can be obtained.

$$u = \frac{(\rho_p - \rho_f)gd^2}{18\mu} \quad (3)$$

The sizes of microballoons used in this research ranged in size from 10 microns to 300 microns, therefore the floating velocity was between 0.2 cm/min. and 200 cm/min. for water, 0.01 cm/min. and 10 cm/min. for ethylene glycol, and 0.0002cm/min. and 0.2 cm/min. for glycerin. This is the reason to allow several hours or even overnight setting in preparing the working fluid. Most of the photographs presented in this thesis were from ethylene glycol and glycerin to avoid the errors generated from floating velocity. To have better photographs for water, or to decrease the floating velocity of the particles in water, heavier particles would appropriate (the average density of the



microballooms used was only about one third of the water). From the above calculation, the particles showed on photographs can be assumed to follow the streamlines with negligible error.

## APPENDIX B

### COMPUTER PROGRAMS FOR CALCULATING DIMENSIONLESS GROUPS

```

C
C   THIS PROGRAM IS TO CALCULATE PR, GR, RA, AND NU FOR NATURE
C   HEAT CONVECTION WITHIN A CUBE, WHICH IS FILLED WITH WATER.
C
C   DEFINITION OF VARIABLES:
C   ALL VARIABLES ARE IN C.G.S. UNITS.
C
C   BETA : THERMAL EXPENSION COEFFICIENT OF THE FLUID.
C   CP   : HEAT CAPACITY.
C   DT   : TEMPERATURE DIFFERENCE BETWEEN TOW AND TC, INPUT.
C   F****: FUNCTION STATEMENT OF ****. ( T IN DEGREE C)
C   GR   : GRASHOF NUMBER.
C   K    : THERMAL CONDUCTIVITY OF THE FLUID.
C   N    : NUMBER OF DATA POINTS TO BE CALCULATED,
C   NE   : NUMBER IN EXPERIMENT.
C   NU   : NUSSELT NUMBER.
C   PR   : PRANDTL NUMBER.
C   RA   : RAYLEIGH NUMBER.
C   RHO  : DENSITY.
C   S    : THICKNESS OF THE WALL.
C   TAV  : AVERAGE TEMPERATURE OF FLUID,WEIGHTED 70% TIW, 30%TC
C         AND USED TO DETERMINED PROPERTIES.
C   TC   : CENTER TEMPERATURE.
C   TCH  : RATE OF TEMPERATURE CHANGE IN CENTER POINT,INPUT.
C   TIW  : INSIDE WALL TEMPERATURE,CALCULATE.
C   TOW  : TEMPERATURE OF THE OUTSIDE WALL.
C   TTD  : TEMPERATURE DIFFERENCE BETWEEN TIW AND TC, USED TO
C         CALCULATE GR AND DEFINE NU.
C   V    : VOLUME PER UNIT MASS.
C   VIS  : VISCOSITY OF THE FLUID.
C   W    : WIDTH OF THE CUBE.
C
C   DATA ARRANGEMENT :
C
C   0. (FREE),NSET(NUMBER OF EXPERIMENT)
C   1. (A40),RUN NUMBER
C   2. (A40),FLUID NAME
C   3. (FREE),TOW,DT,W,S,N(NUMBER OF DATA)
C   4. (FREE),NE,DT,TCH(ONE CARD FOR EACH DATA SET)
C
C   REAL K,NU
C   FVIS(T)=
C   $ 1.0/(2.1482*((T-8.435)+SQRT(8078.4+(T-8.435)**2))-120.)
C   FBETA(T)=-0.5874E-4+0.16983E-4*T-0.2349E-6*T*T+0.26253E-8
C   $ *T*(T*T)-0.10974E-10*(T*T)*(T*T)
C   FRHO(T)=1.0002+0.22345E-4*T-0.66394E-5*(T*T)+0.31178E-7
C   $ *T*(T*T)-0.8877E-10*(T*T)*(T*T)
C   FPR(T)=0.1222E02-0.3523*T+0.53521E-2*(T*T)-0.41941E-4
C   $ *T*(T*T)+0.13217E-6*(T*T)*(T*T)
C   FK(T)=0.13271E-2+0.59344E-5*T-0.39312E-7*(T*T)+0.10154E-9
C   $ *T*(T*T)
C   READ(5,*)NSET
C   DO 200 J=1,NSET
C   READ(5,1) RUNO
C   READ(5,1) FLUID

```

```

      READ(5,*)TOW,DT,W,S,N
      TAV=TOW-0.6*DT
      RHO=FRHO(TAV)
      PR=FPR(TAV)
      K=FK(TAV)
      BETA=FBETA(TAV)
      CP=PR*K/FVIS(TAV)
      WRITE(6,10) RUNO
      WRITE(6,20) FLUID
      WRITE(6,30) W,S
      WRITE(6,40) RHO,CP,K,BETA
      WRITE(6,50) TOW,DT
      WRITE(6,60)
C     THERMAL CONDUCTIVITY OF PLEXIGLASS IS 4.96E-4 CAL/SEC CM C
      DO 100 I=1,N
      READ(5,*)NE,DT,TCH
      TC=TOW-DT
      TAV=TOW-0.6*DT
90    CONTINUE
      RHO=FRHO(TAV)
      PR=FPR(TAV)
      K=FK(TAV)
      BETA=FBETA(TAV)
      CP=PR*K/FVIS(TAV)
      CC =980.*RHO*RHO*BETA*W*W*W
      CCC=CP*RHO*W*W/6.0/K
      C4=CP*RHO*W*S/6./4.96E-4
      TIW=TOW-C4*TCH
C
C     VISCOSITY IS EXPRESSED AS FUNCTION OF TEMPERATURE.
C
      TAVT=TAV
      TAV=0.7*TIW+0.3*TC
      IF (ABS(TAVT-TAV).GT.0.001) GC TO 90
      TTD=TIW-TC
      VIS=FVIS(TAV)
      PR=FPR(TAV)
      GR=CC*TTD/VIS/VIS
      RA=PR*GR
      NU=CCC*TCH/TTD
      WRITE(6,70)NE,TC,TIW,TTD,VIS,PR,GR,RA,NU
100   CONTINUE
200   CONTINUE
      1  FORMAT(A40)
      10 FORMAT(1H1,'EXPERIMENT NUMBER',9X,A40/)
      20 FORMAT(1X,'FLUID',21X,A40/)
      30 FORMAT( 1X,'WIDTH OF THE CUBE =', 2X,F10.2//1X,
        $ 'THICKNESS OF THE WALL =',F10.5/)
      40 FORMAT(/1X,'PHYSICAL PROPERTIES OF FLUID'//10X,'DENSITY',
        $ 21X,F6.4/10X,'HEAT CAPACITY',15X,F6.4/10X,
        $ 'THERMAL CONDUCTIVITY COEF.',E12.4/10X,
        $ 'THERMAL EXPANSION COEF.',3X,E12.4/)
      50 FORMAT(/1X,'OUTSIDE WALL TEMPERATURE',F16.2//1X,
        & 'INITIAL TEMPERATURE DIFFERENCE',F10.2)
      60 FORMAT(/1X,'NO. TC TIW TTD',8X,'VISCOSITY',
        $ 11X,'PR',14X,'GR',14X,'RA',14X,'NU'/1X,110(1H-)/)
      70 FORMAT(1X,I3,3F8.3,5E16.4)
      STOP
      END

```

THIS PROGRAM IS TO CALCULATE PR, GR, RA, AND NU FOR NATURE HEAT CONVECTION WITHIN A CUBE  
 PROPERTIES OF THE FLUID ARE ASSUMED CONSTANT OVER THE TEMPERATURE RANGE EXCEPT VISCOSITY, WHICH IS FUNCTION OF TEMPERATURE.

DEFINITION OF VARIABLES:  
 ALL VARIABLES ARE IN C.G.S. UNITS

BETA : THERMAL EXPANSION COEFFICIENT OF THE FLUID.  
 CP : HEAT CAPACITY.  
 DT : TEMPERATURE DIFFERENCE BETWEEN TOW AND TC, INPUT.  
 GR : GRASHOF NUMBER.  
 K : THERMAL CONDUCTIVITY OF THE FLUID.  
 N : NUMBER OF DATA POINTS TO BE CALCULATED,  
 NE : NUMBER IN EXPERIMENT.  
 NU : NUSSELT NUMBER.  
 PR : PRANDTL NUMBER.  
 RA : RAYLEIGH NUMBER.  
 RHO : DENSITY.  
 S : THICKNESS OF THE WALL.  
 TAV : AVERAGE TEMPERATURE OF FLUID, WEIGHTED 70% TIW, 30% TC AND USED TO DETERMINE PROPERTIES.  
 TC : CENTER TEMPERATURE.  
 TCH : RATE OF TEMPERATURE CHANGE IN CENTER POINT, INPUT.  
 TIW : INSIDE WALL TEMPERATURE, CALCULATE.  
 TOW : TEMPERATURE OF THE OUTSIDE WALL.  
 TTD : TEMPERATURE DIFFERENCE BETWEEN TIW AND TC, USED TO CALCULATE GR AND DEFINE NU.  
 V : VOLUME PER UNIT MASS.  
 VIS : VISCOSITY OF THE FLUID.  
 W : WIDTH OF THE CUBE.

DATA ARRANGEMENT :

0. (FREE), NSET (NUMBER OF EXPERIMENT)
1. (A40), RUN NUMBER
2. (A40), FLUID NAME
3. (FREE), THREE COEFFICIENTS FOR VISCOSITY
4. (FREE), RHO, CP, K, BETA
5. (FREE), TOW, DT, W, S, N (NUMBER OF DATA)
6. (FREE), NE, DT, TCH (ONE CARD FOR EACH DATA SET)

```
REAL K,NU
DIMENSION A(3)
READ(5,*)NSET
DO 200 J=1,NSET
  READ(5,1) RUNO
  READ(5,1) FLUID
  READ(5,*)A
  READ(5,*)RHO,CP,K,BETA
  READ(5,*)TOW,DT,W,S,N
  WRITE(6,10) RUNO
  WRITE(6,20) FLUID
  WRITE(6,30) W,S
```

```

WRITE(6,40)RHO,CP,K,BETA
WRITE(6,50) TOW,DT
WRITE(6,60)
C   =CP/K
CC =980.*RHO*RHO*BETA*W*W*W
CCC=CP*RHO*W*W/6.0/K
C4=CP*RHO*W*S/6./4.96E-4
C   THERMAL CONDUCTIVITY OF PLEXIGLASS IS 4.96E-4 CAL/SEC CM C
DO 100 I=1,N
READ(5,*)NE,DT,TCH
TC=TOW-DT
TIW=TOW-C4*TCH

C   VISCOSITY IS EXPRESSED AS FUNCTION OF TEMPERATURE.
C
C   TAV=0.7*TIW+0.3*TC
T=TAV+273.15
VIS=EXP(A(1)+A(2)/T+A(3)/T/T)
TTD=TIW-TC
PR=VIS*C
GR=CC*TTD/VIS/VIS
RA=PR*GR
NU=CCC*TCH/TTD
WRITE(6,70)NE,TC,TIW,TTD,VIS,PR,GR,RA,NU
100 CONTINUE
200 CONTINUE
1   FORMAT(A40)
10  FORMAT(1H1,'EXPERIMENT NUMBER',9X,A40/)
20  FORMAT(1X,'FLUID',21X,A40/)
30  FORMAT(1X,'WIDTH OF THE CUBE =',2X,F10.2//1X,
$ 'THICKNESS OF THE WALL =',F10.5/)
40  FORMAT(//1X,'PHYSICAL PROPERTIES OF FLUID'//10X,'DENSITY',
$ 21X,F6.4/10X,'HEAT CAPACITY',15X,F6.4/10X,
$ 'THERMAL CONDUCTIVITY COEF.',E12.4/10X,
$ 'THERMAL EXPANSION COEF.',3X,E12.4/)
50  FORMAT(//1X,'OUTSIDE WALL TEMPERATURE',F16.2//1X,
& 'INITIAL TEMPERATURE DIFFERENCE',F10.2)
60  FORMAT(///2X,'NO.    TC      TIW      TTD',8X,'VISCOSITY',
$ 11X,'PR',14X,'GR',14X,'RA',14X,'NU'/1X,110(1H-)/)
70  FORMAT(1X,I3,3F8.3,5E16.4)
STOP
END

```

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AN EXPERIMENTAL STUDY OF FLOW PATTERNS  
AND  
HEAT TRANSFER BY NATURAL CONVECTION  
INSIDE CUBICAL ENCLOSURES

by

YUE-SHYANG LIN  
B. S., National Taiwan University, 1978

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AN ABSTRACT OF A MASTER'S THESIS

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1982



This thesis presents the results of an experimental investigation of natural convection heat transfer to fluid enclosed in a cube, of which all six walls were subjected to a temperature step change. Three sizes of cubes, 2", 3", and 4" (inside dimension) were made. The fluid inside the cube was either distilled water, ethylene glycol, glycerin, or aqueous solutions of glycerin and ethylene glycol. These provided fluids with viscosities ranging from 0.8 to 1500 centipoise. Small hollow glass particles about 10 microns or more in diameter were suspended in water, and photographs of the particles were taken to show the flow patterns. Temperature at cube center was recorded to calculate heat transfer coefficient.

Several flow patterns were observed which have not been reported previously and were quite different from general numerical prediction. Velocity profiles of the main flow pattern at a horizontal plane passing through the center of circulation were determined. The overall heat transfer in laminar range was found to be correlated by

$$Nu = 0.6 Ra^{0.235}$$