ON DETERMINING THE POWER OF A TEST AFTER DATA COLLECTION

by

WILLIAM AVRAM CHERNOFF

B.S., Chapman University, 2007

A REPORT

Submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Statistics
College of Art and Sciences

KANSAS STATE UNIVERSITY
Manhattan, Kansas
2009

Approved by

____
Major Professor
Dr. Leigh Murray

ABSTRACT

The term retrospective power describes methods for estimating the true power of a test after data have been collected. These methods have been recommended by some authors when null hypothesis of a test cannot be rejected. This report uses simulations to study power as a construct of an observed effect, variance, sample size, and set level of significance under the balanced one-way analysis of variance model for normally distributed populations with constant variance.

Retrospective power, as a construct of sample data, is not recommended when the null hypothesis of a test cannot be rejected. When the p-value of the test is large, estimates for true power tend to fall below the 0.80 level and width-minimized confidence limits for true power tend to be wide.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGEMENTS

I would like to thank Dr. Leigh Murray, Dr. James Neill, and Dr. Weixing Song for contributing in the development of this report. I thank Dr. Murray for her role as major professor; for her guidance through retrospective power; and for her topical and gritty digressions on statistical consulting. I thank Dr. Neill for sitting on my committee; for always having a moment for questions; and for his ability to impart calm and fortitude during the rapidity and rigor of graduate studies. I thank Dr. Song for acting as a member of my committee; for helping me with SAS; and for his staunch encouragement and winning sense of humor.

I would also like to thank the faculty at Chapman University for preparing me for graduate studies. I owe much of my personal and academic growth to Dr. Atanas Radenski, Dr. Peter Jispen, Dr. Drew Moshier, Dr. Adrian Vajiac, and Dr. Mihaela Vajiac. For all the people I met at Chapman University, you are the most appreciated.

I would like to thank the friends I have made through the K-State Statistics department. I thank Seth Demel for all-night study sessions, Lee Goerl for 'talking stats' at social functions, and Aleksandra Banasik for discussing teaching strategies over early morning coffee. You made home feel larger than southern California.

Finally, I would like to thank my mother Valerie Chernoff, my sister Elyse Felder, and the friends I hold dear for putting up with all this 'Kansas/Graduate School' business. Your thoughts and affections are a constant source of inspiration.

1 Introduction

For many researchers, failing to reject the null hypothesis of statistical test is disconcerting. In some cases, the research design is questioned: was the study powerful enough to detect a meaningful effect? While valid statistical methods exist for calculating power before data have been collected, determining power retrospectively is seen in the literature as controversial. Here we explore power, as described in Thomas (1997), as a construct of sample data using the balanced one-way ANOVA model. Section 2 discusses methods for calculating prospective power; methods for estimating retrospective power; and concerns associated with estimating retrospective power. Section 3 outlines the balanced one-way ANOVA model; a method for estimating power using the observed effect, variance, sample size, and set significance level; and a method for constructing a minimal (1 - α)100% confidence interval for true power. In addition, Section 3 contains conditions for simulations and implementation methods used for simulations. Section 4 summarizes the simulation results, displaying the location of true power, median estimated power, average estimated power, average p-value of tests and average confidence limits produced from the simulations. Section 4 also discusses the effect of population conditions on power calculations. Flow charts, code, and complete output are given in appendices A-D. By examining power after data have been collected and statistical tests have been performed, the perspective shifts from calculating power to estimating power.

2 Literature Review

Calculating the power of a statistical test before data have been collected, denoted *prospective power*, is generally regarded by statisticians as a good practice for obtaining conclusive results (American Statistical Association 1999). Power calculations are useful in determining adequate sample sizes for an up coming study (Lenth 2001). The power of a statistical test can be computed using a set significance level, sample size, effect, and variance (Lenth 2001; Gerard et al. 1998). Lenth (2001) provides a list of discussion-techniques for eliciting meaningful effect and variance values for a potential study. Power for t-tests, multiple regression, one-way anova, and other common statistical methods can be computed using the SAS procedure POWER (SAS Institute Inc. 2004b). Power for linear models with fixed class effects and contrast statements can be calculated using the SAS procedure GLMPOWER (SAS Institute Inc. 2004a). For many researchers, prospective power calculations are within reach.

Calculating the power of a statistical test after data have been collected, i.e. *retrospective power*, is less accepted by many statisticians as compared to prospective power (Hoenig and Heisey 2001; Thomas 1997; Yuan and Maxwell 2005). Retrospective power has been proposed for interpreting results when the null hypothesis cannot be rejected (Taylor and Muller 1995; Hogarty and Kromrey 2001) and for determining the sample size needed for the observed data to show statistically significant results (Steidl et al. 1997; Thomas 1997). Some popular methods for calculating retrospective power included: an observed sample size and given significance level where effect and variance come from literature (Yuan and Maxwell 2005); an observed effect, observed sample size, and given significance level (Yuan and Maxwell 2005) where variance comes from literature; an observed variance, observed sample size, and given significance level (Thomas 1997; Yuan and Maxwell 2005; Steidl et al. 1997) where effect comes from literature; and an observed effect, an observed sample size, an observed variance, and given significance level (Gerard et al. 1998; Thomas 1997; Hoenig and Heisey 2001). Of these four methods only the last method is reviewed here.

Calculating retrospective power using an observed effect, an observed variance, an observed sample size, and given significance level has been shown to produce biased estimates when using an observed noncentrality parameter (Gerard et al. 1998). Thomas (1997) suggests adjusting the observed noncentrality parameter as shown in Wright and O'Brien (1988) and Johnson et al. (1995) to remove the bias in calculating power. The adjusted observed noncentrality parameter can produce negative values, which are typically set to zero (Gerard et al. 1998). A median estimator for the noncentrality parameter has been proposed by Taylor and Muller (1996) as a means for correcting the retrospective power calculation. The median estimator for the noncentrality parameter under-estimates half the time and over-estimates half the time. Here, the adjusted observed noncentrality parameter is explored.

Concerns about using retrospective power procedures to represent true power exist in the literature. Zumbo and Hubley (1998) provide a Bayesian argument against retrospective power as a logical representative for true power. Hoening and Heisey (2001) note that retrospective power obtain through observed values is redundant with the p-value of the test.

Due to the bias associated with retrospective power, confidence intervals for effect have been proposed in lieu of retrospective power analyses when the null hypothesis cannot be rejected (Gerard et al. 1998; Thomas 1997; Steidl et al. 1997).

In this paper, we examine the method presented in Thomas (1997) for estimating the power of a test in the context of the balanced one-way ANOVA model.

3 Methods

3.1 Model

To investigate post-hoc power, the balanced one-way analysis of variance model is considered. The following briefly reviews this model and the usual analysis as described in Kuehl (2000) and Milliken and Johnson (2009). The means model for this situation is

$$y_{ij} = \mu_i + \varepsilon_{ij} \text{, with } \varepsilon_{ij} \text{'s i.i.d } N\left(0,\sigma^2\right) \text{ for } i = 1,...,p, j = 1,...,n \text{ .} \tag{1.1}$$

Here y_{ij} represents the observed value of the jth observation from the ith treatment group and μ_i denotes the population mean of the ith treatment group. The term ε_{ij} represents the random error associated with the jth observation from the ith treatment group. The errors are assumed independent and identically distributed from a normal population with mean 0 and variance σ^2.

The null and alternative hypotheses are that of no difference and that of some difference among the *p* population means, respectively:

$$\begin{aligned} &H_0 : \mu_1 = ... = \mu_p \\ &H_a : \text{ at least two } \mu_i\text{'s not equal} \end{aligned} \tag{1.2}$$

When the sample means are not “too different” from each other we deem the data consistent with the null hypothesis of equal means. We do not conclude that the population means are all the same but acknowledge that sample means as too similar to be called different. When at least two sample means exist which “strongly” differ we favor the alternative hypothesis and conclude that not all of the population means are equal to one another.

Two estimates of variance are used to measure the amount of evidence against the null hypothesis. The first estimate of variance is based only on the assumptions of the model. The second estimate of variance is based on the assumptions of the model and the assumption that the null hypothesis is true. Of these two estimates of variance, the second estimate has a larger expected mean square than the first estimate when the null hypothesis is false.

The mean square for error is the first estimate of σ^2 and is calculated by taking the average of the sample variances s_1^2, s_2^2, ..., s_p^2:

$$\begin{aligned} MSE &= \frac{s_1^2 + s_2^2 + \ldots + s_p^2}{p} \\ &= \frac{1}{p(n-1)} \sum_{i=1}^{p} \sum_{j=1}^{n} \left(y_{ij} - \bar{y}_{i\bullet}\right)^2 \\ &= \frac{1}{p(n-1)} \left\{ \sum_{i=1}^{p} \sum_{j=1}^{n} y_{ij}^2 - n \sum_{i=1}^{p} \bar{y}_{i\bullet}^2 \right\} \\ &= \frac{1}{p(n-1)} \left\{ \sum_{i=1}^{p} \sum_{j=1}^{n} y_{ij}^2 - n \bar{Y}^T \bar{Y} \right\} \\ &= \frac{SSE}{DFE}, \end{aligned} \tag{1.3}$$

where $\bar{y}_{i\bullet} = \frac{1}{n} \sum_{j=1}^{n} y_{ij}$, $\bar{Y}^T = \left[\bar{y}_{1\bullet}, \bar{y}_{2\bullet}, \ldots, \bar{y}_{p\bullet}\right]$, SSE is the error sum of squares, and DFE is the error degrees of freedom. Under the assumptions of the model,

$$\frac{(DFE)(MSE)}{\sigma^2} \sim \chi^2_{(DFE)} \tag{1.4}$$

and

$$E(MSE) = \sigma^2 . \tag{1.5}$$

The mean square for treatments (MST) is the second estimate of σ^2 and is calculated as the sample variance of the sample means $\bar{y}_{1\bullet}$, $\bar{y}_{2\bullet}$, ..., $\bar{y}_{p\bullet}$. That is,

$$
\begin{aligned}
MST &= \frac{n}{p-1}\sum_{i=1}^{p}\left(\bar{y}_{i\bullet}-\bar{y}_{\bullet\bullet}\right)^2 \\
&= \frac{n}{p-1}\left\{\sum_{i=1}^{p}\bar{y}_{i\bullet}^2 - p\bar{y}_{\bullet\bullet}^2\right\} \\
&= \frac{1}{p-1}\left\{\sum_{i=1}^{p}\frac{y_{i\bullet}^2}{n} - \frac{y_{\bullet\bullet}^2}{np}\right\} \\
&= \frac{1}{p-1}\left\{n\bar{Y}^T\bar{Y} - \frac{n}{p}\left(\bar{Y}^T J\right)^2\right\} \\
&= \frac{SST}{DFT},
\end{aligned} \tag{1.6}
$$

where $y_{i\bullet} = \sum_{j=1}^{n} y_{ij}$, $y_{\bullet\bullet} = \sum_{i=1}^{p}\sum_{j=1}^{n} y_{ij}$, $J^T = [1,1,...,1]$, SST is the treatment sum of squares, and DFT is the treatment degrees of freedom. Under the assumptions of the model and the assumption that the null hypothesis is true

$$\frac{(DFT)(MST)}{\sigma^2} \sim \chi^2_{(DFT)} \tag{1.7}$$

and

$$E(MST) = \sigma^2 . \tag{1.8}$$

Under the assumptions of the model and the assumption that the null hypothesis is not true

$$\frac{(DFT)(MST)}{\sigma^2} \sim \chi^2_{(DFT,\lambda)} \tag{1.9}$$

and

$$E(MST) = \sigma^2 + \frac{n}{p-1}\sum_{i=1}^{p}\left(\mu_i - \bar{\mu}_{\bullet}\right)^2 , \tag{1.10}$$

where

$$\lambda = \frac{n}{\sigma^2}\sum_{i=1}^{p}\left(\mu_i - \bar{\mu}_{\bullet}\right)^2 \tag{1.11}$$

denotes the noncentrality parameter.

Under the assumptions of the model and assumption that the null hypothesis is true,

$$\frac{\left[(DFT)(MST)/\sigma^2\right]/DFT}{\left[(DFE)(MSE)/\sigma^2\right]/DFE} \sim F_{(DFT,DFE)}, \tag{1.12}$$

where (1.7) and (1.9) are independently distributed χ^2 random variables. Under the assumptions of the model and the assumption that the alternative hypothesis is true,

$$\frac{\left[(DFT)(MST)/\sigma^2\right]/DFT}{\left[(DFE)(MSE)/\sigma^2\right]/DFE} \sim F_{(DFT,DFE,\lambda)}. \tag{1.13}$$

The observed F-statistic for the test is calculated as the ratio of the mean square for treatments over the mean square for error. This statistic is used to measure the amount of evidence in the sample data against the null hypothesis,

$$F = \frac{MST}{MSE} = \frac{\left[(DFT)(MST)/\sigma^2\right]/DFT}{\left[(DFE)(MSE)/\sigma^2\right]/DFE} \tag{1.14}$$

Given the assumptions of the means model and the assumption that the null hypothesis is true, the mean square for error should be of similar size to the mean square for treatments, practically speaking. However, given the null hypothesis is false, the mean square for error should be smaller than the mean square for treatments. An uncommonly large *F*-statistic leads to the conclusion that not all population means are equal. The *p*-value of the test gives the probability of obtaining an *F*-statistic greater than or equal to the one observed. The more evidence in the sample data against the null hypothesis of equal means the smaller the *p*-value.

3.2 Estimated Power

Power is estimated using the observed effect and the observed sampling variance. Let the cumulative distribution function of the noncentral F-distribution be denoted as

$$F(DFT, DFE, \lambda) \, . \tag{2.1}$$

The power of the F-test is defined as

$$\text{power} = 1 - F\left(F_{crit} \mid DFT, DFE, \lambda\right) , \tag{2.2}$$

where F_{crit} is the $100 \cdot (1-\alpha)$ percentile from a central F-distribution with numerator degrees of freedom DFT, denominator degrees of freedom DFE, level of significance α, and noncentrality parameter λ (Thomas 1997). A positively-biased estimate for the power of an F-test is

$$\widehat{\text{power}} = 1 - F\left(F_{crit} \mid DFT, DFE, \hat{\lambda}\right) \tag{2.3}$$

where $\hat{\lambda} = SST / MSE$ (Thomas 1997). The estimated noncentrality parameter can be adjusted to remove the upward bias. Therefore, an unbiased estimate of power (Wright and O'Brien 1988, and Johnson et al. 1995, cited in Thomas 1997) is

$$\widehat{\text{power}}_{adj} = 1 - F\left(F_{crit} \mid DFT, DFE, \hat{\lambda}_{adj}\right) \tag{2.4}$$

where $\hat{\lambda}_{adj}$ is the adjusted estimated noncentrality parameter and is computed as

$$\hat{\lambda}_{adj} = [\hat{\lambda} \cdot (DFE - 2) / DFE] - DFT \, . \tag{2.5}$$

Note that for $\hat{\lambda} \cdot (DFE - 2) / DFE$ less than DFT, $\hat{\lambda}_{adj}$ can be negative.

Upper and lower confidence limits for the true power are therefore

$$\widehat{\text{power}}_U = 1 - F\left(F_{crit} \mid DFT, DFE, \hat{\lambda}_U\right) \tag{2.6}$$

and

$$\widehat{\text{power}}_L = 1 - F\left(F_{crit} \mid DFT, DFE, \hat{\lambda}_L\right), \tag{2.7}$$

where $\hat{\lambda}_U$ and $\hat{\lambda}_L$ satisfy $F\left(F_{obs} \mid DFT, DFE, \hat{\lambda}_U\right) = \alpha_U$ and $F\left(F_{obs} \mid DFT, DFE, \hat{\lambda}_L\right) = 1 - \alpha_L$ in which α_U is the upper tail probability and α_L lower tail probability (Thomas 1997). The upper and lower tail probabilities α_U and α_L are those which define the $100 \cdot (1 - \alpha_U - \alpha_L)$ percent confidence interval for true power.

3.3 Confidence Interval Optimization

Confidence limits for true power were minimized to explore retrospective power under a "best" case scenario. Confidence limits set under $\alpha_L = \alpha_U = \alpha / 2$ are not generally optimal (i.e. narrowest confidence interval) because of the skewed nature of the noncentral F-distribution. Setting upper and lower probabilities equal may under-represent, or over-represent, true power.

For a user-specified level of significance α, we consider the $100(1-\alpha)$% confidence interval for true power, where

$$\alpha = \alpha_L + \alpha_U \ . \tag{3.1}$$

We denote the inverse cumulative distribution function of the noncentral F-distribution by

$$F^{-1}\left(\alpha_c, DFT, DFE, \hat{\lambda}_{adj}\right), \tag{3.2}$$

where α_c is a lower tail probability, DFT the numerator degrees of freedom, DFE the denominator degrees of freedom, and $\hat{\lambda}_{adj}$ the noncentrality parameter. We denote the quantile associated with α_L as

$$F_L = F^{-1}\left(\alpha_L, DFT, DFE, \hat{\lambda}_{adj}\right), \tag{3.3}$$

and the quantile associated with α_U as

$$F_U = F^{-1}\left(1-\alpha_U, DFT, DFE, \hat{\lambda}_{adj}\right). \tag{3.4}$$

The width of the $100(1-\alpha)$% confidence interval for true power is minimized by finding the distance between the lower and upper quantiles such that

$$W = F_U - F_L \tag{3.5}$$

is minimized. Note, for fixed α we have the constraint that $\alpha = \alpha_L + \alpha_U$ (3.1) which gives

$$\alpha_U = \alpha - \alpha_L . \tag{3.6}$$

As such, we wish to find the value of α_L which minimize the distance

$$W(\alpha_L) = F^{-1}\left(1-\alpha+\alpha_L, DFT, DFE, \hat{\lambda}_{adj}\right) - F^{-1}\left(\alpha_L, DFT, DFE, \hat{\lambda}_{adj}\right), \tag{3.7}$$

for

$$\alpha_L \in (0,\alpha). \tag{3.8}$$

A method for finding α_L such that $W(x)$ is minimal is by the Golden Section Search (GSS) as described in Press et al. (2007). For a unimodal function $W(x)$ defined for $x \in [a,b]$, the Golden Section Search method seeks to bracket the value k in $[a,b]$ such that $W(k) < W(x)$ for all other $x \in [a,b]$. The value k is considered found when brackets are reached whose distance is within a user-specified tolerance level. The GSS procedure is named for its use of the mathematical constant

$$\varphi = \frac{-1+\sqrt{5}}{2} \approx 0.618034 \ . \tag{3.9}$$

The ends points a and b are used to construct two new possible bracket points. We calculate the first new point, which we call c, by

$$c = a + \varphi\left(b-a\right), \tag{3.10}$$

and the second new point, which we call d, by

$$d = a + \varphi^2 (b-a) \ . \tag{3.11}$$

These new points are then evaluated with the function $W(x)$ such that

$$u = W(c) \tag{3.12}$$

and

$$v = W(d) \ . \tag{3.13}$$

One of two cases may arise:

(1) $u > v$

(2) $u \leq v$

For the first case (Figure 3.1.a) the value k must be in $[a,c]$ since the function $W\left(x\right)$ is unimodal. The interval $[a,c]$ will then have width $c-a=\varphi\left(b-a\right)$ (3.10). For the second case (Figure 3.1.b) the value k must be in $\left[d,b\right]$, again since the function $W(x)$ is unimodal. Using the equation $d = a + \varphi^2(b-a)$ (3.11) and the fact that $\varphi^2 = 1-\varphi$, it can be shown that $b-d=\varphi(b-a)$. Therefore,

Figure 3.1.a: Update Bracket Interval for Minimand for v less than u

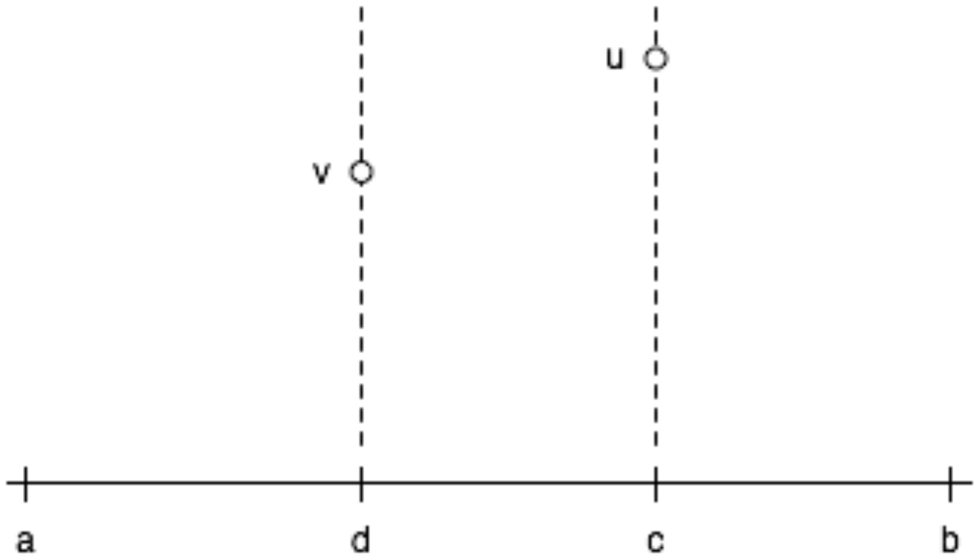
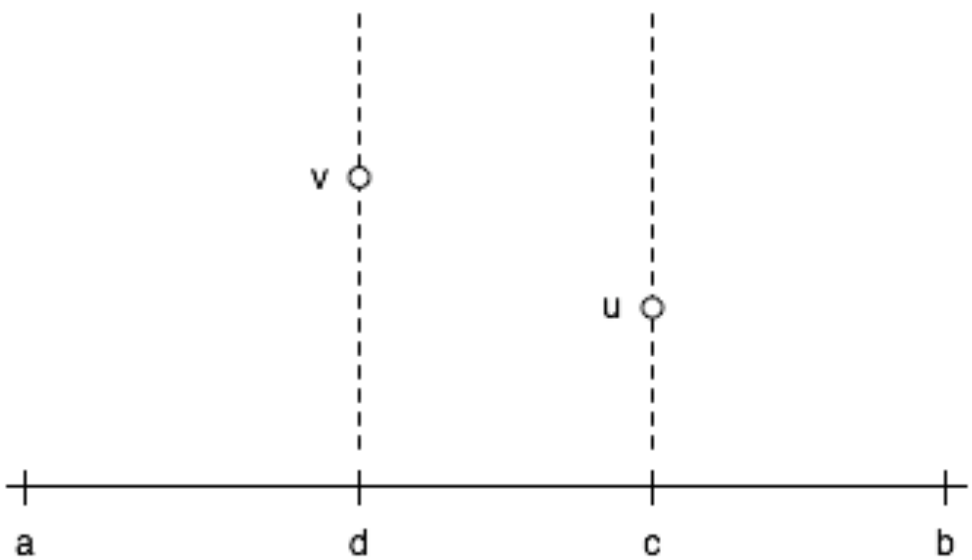


Figure 3.1.b: Update Bracket Interval for Minimand for v greater than u



$$c - a = b - d \ , \tag{3.14}$$

that is, the interval width for case (1) equals the interval width for case (2). That is, we get a consistent reduction factor. Additionally,

$$\varphi = \frac{\varphi^2(b-a)}{\varphi(b-a)} = \frac{d-a}{b-d} \tag{3.15}$$

and

$$\varphi = -1 + \frac{1}{\varphi} = -1 + \frac{\varphi(b-a)}{\varphi^2(b-a)} = -\frac{d-a}{d-a} + \frac{\varphi(b-a)}{d-a} = \frac{-(d-a)+\varphi(b-a)}{d-a} = \frac{c-d}{d-a} \tag{3.16}$$

so that

$$\frac{c-d}{d-a} = \frac{d-a}{b-d} \tag{3.16}$$

that is, using φ to pick c and d maintains the spacing among points as the brackets are updated. Therefore, we only need to calculate one new point and make one new function evaluation per interval reduction.

3.4 Parameter Settings

Based on the balanced one-way ANOVA model, true power and estimated power were obtained for a range of population conditions and simulation outcomes, respectively, for three sizes of number of treatments, $p = 2$, 3, and 4.

The two-treatment class has a single symmetric mean arrangement. The three-treatment class has one symmetric mean arrangement and one asymmetric mean arrangement. The four-treatment class has two symmetric mean arrangements and one asymmetric mean arrangement. For each mean arrangement a range of population standard deviations and sample sizes are considered. These population conditions are summarized in Table 3.1. Table 3.1 also includes under each mean arrangement the sum of squared differences between population means and their grand population mean which is the numerator of the noncentrality parameter λ (1.10). These values were chosen so that treatment populations should be discernibly different at a 95% empirical interval when the population standard deviation is "small" and/or sample size is "large," and indiscernibly different when the population standard deviation is "large" and/or sample size is "small."

Table 3.1: Summary of Parameter Settings for Simulations

Case	Mean = μ_i	$\sum(\mu_i - \bar{\mu}_{\bullet})^2$
Two-Sample Symmetric	30, 40	50.0000
Three-Sample Symmetric	30, 35, 40	50.0000
Three-Sample Asymmetric	30, 40, 40	66.6667
Four-Sample Symmetric A	30, 35, 35, 40	50.0000
Four-Sample Symmetric B	30, 30, 40, 40	100.0000
Four-Sample Asymmetric	30, 40, 40, 40	75.0000
	Standard Deviation = σ	2.5 to 10.0 by 0.5 and 20.0
	Sample Size = n	5 to 14 by 1 and 15 to 50 by 5
	Significance Level = α	0.05

3.5 Iteration

For each combination of mean arrangement, population standard deviation, and sample size, 100 simulations were produced. Each simulation produced *p* random samples where *p* denotes the number of treatment populations. The *p* samples were produced to have equal samples size n. Each random sample was produced from a random seed value, an integer between 1 and 1,000,000,000, obtained through the website www.random.org. Random samples were constructed to reflect a normally distributed population with mean μ_i and equal population standard deviation where $i = 1,...,p$. Note that random samples were initially generated in part by the SAS® software function RANNOR. RANNOR was seen to produce dependent samples when called multiple times within the IML procedure. Therefore, RANNOR was encapsulated in a macro which resulted in independent random number generation.

For each simulation, the following calculations were made from the generated data. An ANOVA table was calculated as outlined in 3.1.1. True power and estimated power were calculated as outlined in 3.1.2. Confidence limits for true power were calculated and optimized as outlined in 3.1.2 and 3.1.3, respectively. Tolerance for bracket intervals in 3.1.3 was set to 1E-4 as suggested by the procedure FMINBND in MATLAB (2009) and the package OPTIMIZE in R (R Development Core Team 2009).

Several problems were noted with the SAS software function PROBF which was recommended by Thomas (1997) for calculating power (SAS Institute Inc. 2006a). First, the SAS function PROBF reports an invalid argument when the supplied noncentrality parameter is large with respect to numerator degrees of freedom, denominator degrees of freedom, and *F* critical value. This problem is noted by O'Brien (1988). Taylor and Muller (1996) suggest setting the noncentrality large enough such that PROBF evaluates to zero or one where appropriate. The SAS software function CDF (SAS Institute Inc. 2006b) offers some relief to the PROBF issue though invalid arguments can still be had for extreme parameter values.

Second, PROBF reports an error when the supplied noncentrality parameter is negative. Gerard et al. (1998) suggest setting the noncentrality parameter to zero. For some random random seed values the data produced result with a negative adjusted estimated noncentrality parameter. The negative value occurs in some instances when adjusting the estimated noncentrality parameter (2.5), as noted previously.

Finally, FNONCT reports an error when either of the probabilities $1-\alpha_L$ or α_U are greater than the probability associated with the observed F statistic from a central F-distribution with degrees of freedom DFT and DFE. Thomas (1997) suggests setting the noncentrality parameter equal to zero. In this case the error associated with FNONCT is noted in the SAS software documentation.

For each combination of mean arrangement, population standard deviation, and sample size, calculations for the 100 simulations are averaged. Notable averages include average estimated power and average upper and lower confidence limits about true power. The median for estimated power was also determined. Missing values were deleted.

The SAS program (Version 9.1.3) used can be found in Appendix B. Flow charts for the SAS program can be found in Appendix A (arranged through OmniGraffle Pro® version 5). This document was processed using the iWork ’09® application Pages. Mathematical expression were set using MathType® version 6.

4 Results

Output for simulation summaries is plotted as power versus sample size for population standard deviations 2.5 to 10.0 by 0.5 and 20.0. Power ranges from 0.0 to 1.0 and sample size ranges from 5 to 14 by 1 and 15 to 50 by 5. The average p-value of the test is presented in each plot. The complete display of simulation plots can be found in Appendix C. In the Results, four plots are presented from each treatment class for each treatment mean arrangement, power plotted against sample size, where sample size ranges from 1 to 35 at population standard deviations of 2.5, 5.0, 10.0, and 20.0. A maximum sample size of 35 is presented since power approached 1.0 for sample sizes over 35 for the simulation configuration used here. Population standard deviations 2.5 and 20.0 represent two extreme cases for variability while population standard deviations 5.0 and 10.0 represent two medium cases for variability. Plots for each treatment arrangement appear as a moving-window like landscape from a moving vehicle.

4.1 Two-Sample Symmetric Case

Plots for population standard deviations 2.5, 5.0, 10.0, and 20.0 with a maximum sample size of 35 are seen in Figure 4.1.a, 4.1.b, 4.1.c, and 4.1.d, respectively. The p-value for the test decreases as the sample size increases, such that the null hypothesis can be rejected when $n \geq 5$, $n \geq 6$, $n \geq 14$, and $n > 35$ for respective values of $\sigma = 2.5$, $\sigma = 5.0$, $\sigma = 10.0$, and $\sigma = 20.0$. True power is always between the average lower and upper confidence limits for true power. Power and confidence limits are higher and closer together at larger sample sizes, as expected. For $\sigma = 2.5$, true power is very close to the average upper limit. For $\sigma = 5.0$, true power is closer to the average upper limit than the lower limit for $n < 30$; is half-way between the average upper and lower limits when both limits and true power approach 1.0 for $n \geq 30$. For $\sigma = 10.0$, true power starts (at $n = 5$) around the average lower limit, and is located half-way between average confidence limits at $n = 9$. For $n > 9$, true power is closer to the average upper limit than the average lower limit. When $\sigma = 20.0$, true power is located near the average lower limit (for $n = 5$), and obtains an approximate location half-way between average limits for $n = 30$. True power exhibits an increasing positive trend toward the average upper limit for each plot, though the incline of the curve decreases for larger values of σ.

Power curves appear generally ordered as,

$$power_{true} > \widehat{power}_{med} > \widehat{power}_{avg} . \qquad (4.1)$$

The general ordering of power curves is strictly seen for σ of 2.5 and 5.0. For $\sigma = 10.0$, average estimated power starts (at $n = 5$) above median estimated power but adopts the general ordering for sample sizes 10 and greater. When σ equals 20.0, average estimated power starts (at $n = 5$) above true power, and median estimated power starts below true power. Average estimated power appears (for $\sigma = 20.0$) below true power at sample sizes 13 and greater, and below median estimated power at sample sizes 35 and greater. For samples sizes greater than 35, the general ordering appears to hold for any value of σ presented here.

Two-Sample Symmetric Case: Power vs Sample size

Figure 4.1.a: Sigma 2.5

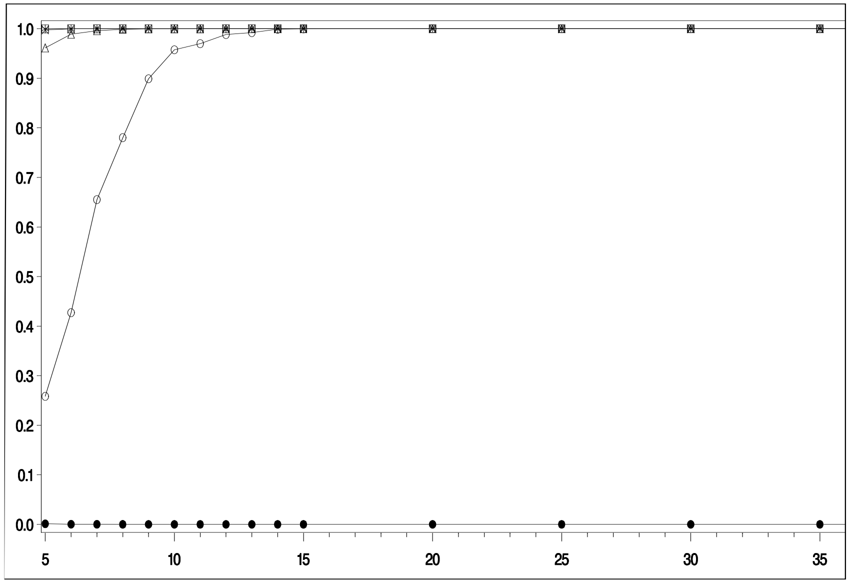


Figure 4.1.b: Sigma 5.0

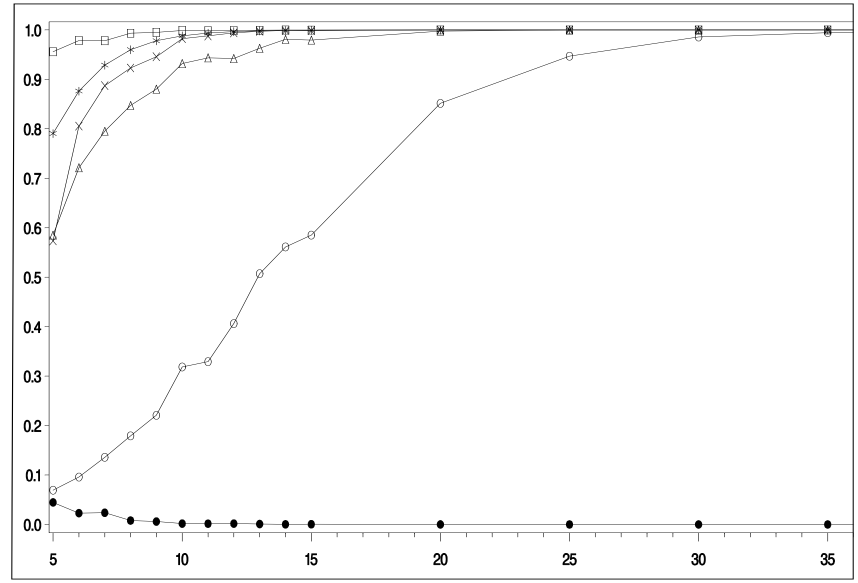


Figure 4.1.c: Sigma 10.0

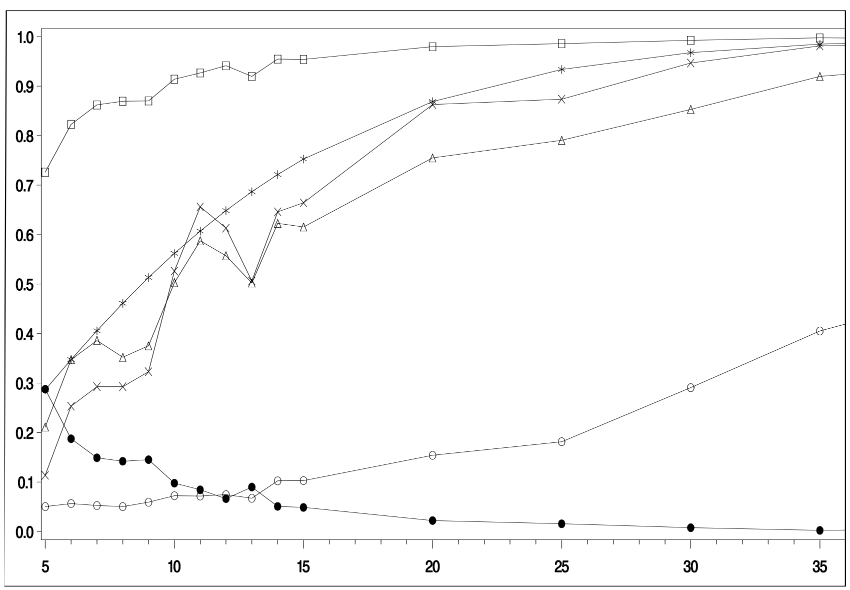
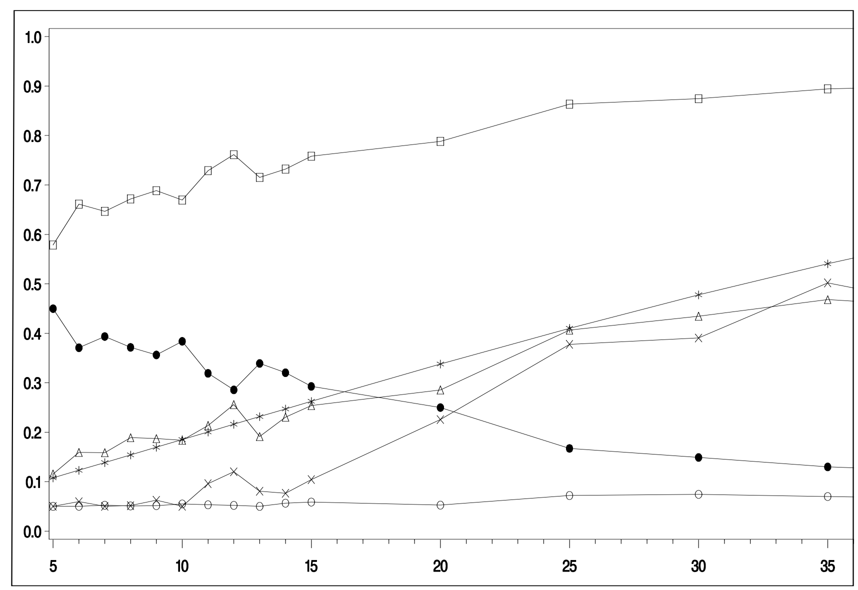


Figure 4.1.d: Sigma 20.0



4.2 Three-Sample Symmetric Case

Plots for population standard deviations 2.5, 5.0, 10.0, and 20.0 with a maximum sample size of 35 are seen in Figure 4.2.a, 4.2.b, 4.2.c, and 4.2.d, respectively. Similarly to the two-sample case, the p-value for the test decreases as the sample size increases, such that the null hypothesis can be rejected when $n \geq 5$, $n \geq 8$, $n \geq 16$, and $n > 35$ for values of $\sigma = 2.5$, $\sigma = 5.0$, $\sigma = 10.0$, and $\sigma = 20.0$, respectively. True power is always contained within the average limits. As with the two-sample case, power and confidence limits tend to be higher and closer together for larger sample sizes. True power is very close to the average upper limit for $\sigma = 2.5$. For $\sigma = 5.0$, true power is located near the upper average limit and is approximately half-way between the average limits for $n \geq 35$. When $\sigma = 10.0$, true power is close to the average lower limit (at $n = 5$), and is approximately half-way between the confidence limits at $n = 12$. For $\sigma = 20.0$, true power is nearer to the average lower limit than the average upper limit for sample sizes up to 30, but for $n = 35$ true power is approximately half-way between the confidence limits. At all values σ considered here, true power tend toward the average upper limit.

As with the two-sample case, the general ordering (4.1) tends to hold. The general ordering of power is seen for σ = 2.5 and 5.0. For $\sigma = 10.0$, average estimated power starts (at $n = 5$) above the median estimated power, but takes on the general ordering at sample sizes 13 and greater. For $\sigma = 20.0$, true power begins (at $n = 5$) below average estimated power, and above median estimated power. Average estimated power (for $\sigma = 20.0$) is below true power at sample sizes 30 and greater, and below median estimated power at sample sizes greater than 35. The general position of power curves (4.1) tends to hold when samples sizes are large.

Three-Sample Symmetric Case: Power vs Sample size

Figure 4.2.a: Sigma 2.5

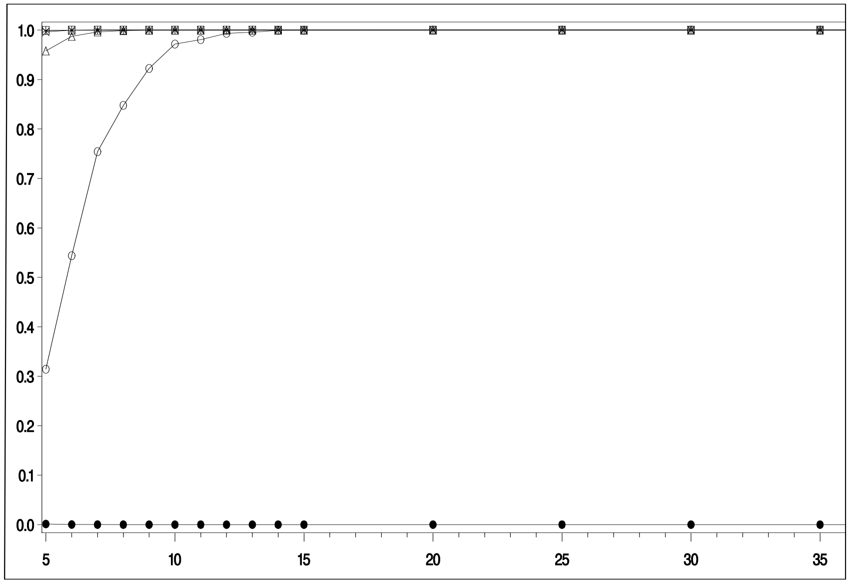


Figure 4.2.b: Sigma 5.0

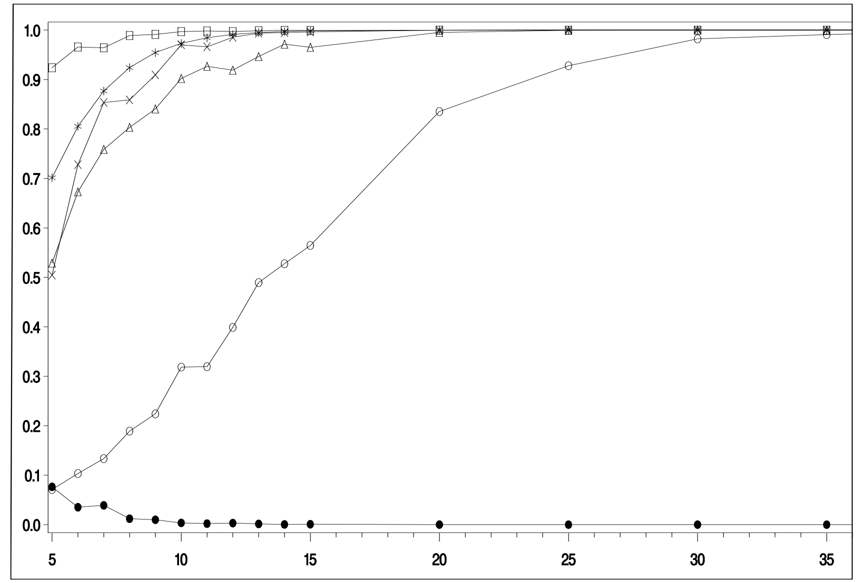


Figure 4.2.c: Sigma 10.0

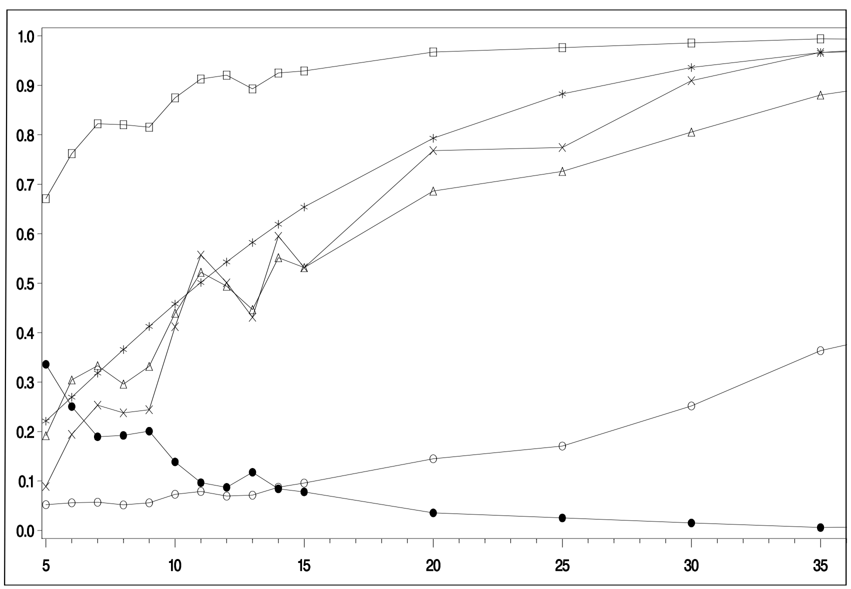
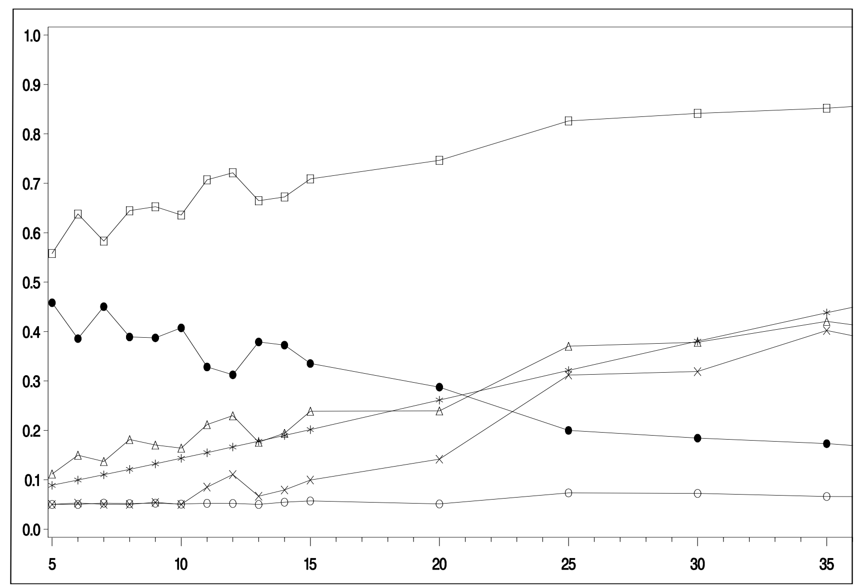


Figure 4.2.d: Sigma 20.0



4.3 Three-Sample Asymmetric Case

Plots for population standard deviations 2.5, 5.0, 10.0, and 20.0 with a maximum sample size of 35 are seen in Figure 4.3.a, 4.3.b, 4.3.c, and 4.3.d, respectively. Again, the p-value for the test decreases as the sample size increases, such that the null hypothesis can be rejected when $n \geq 5$, $n \geq 6$, $n \geq 14$, and $n > 35$ for respective values of $\sigma = 2.5$, $\sigma = 5.0$, $\sigma = 10.0$, and $\sigma = 20.0$. True power is between the average upper and lower limits for all values of the population standard deviation considered here. As with the previous cases, power and confidence limits are higher and closer together for larger sample sizes. For $\sigma = 2.5$, true power is indistinguishable from the average upper limit. For $\sigma = 5.0$, true power is closer to the average upper limit than the average lower limit, and true power is approximately half-way between the average upper and lower limits when both limits approach 1.0 ($n \geq 30$). For $\sigma = 10.0$, the average upper and lower limits are centered around true power for $n = 9$. When $\sigma = 20.0$, true power is located near the average lower limit (for $n = 5$), and is approximately half-way between the average upper and lower limits at $n = 30$. As with previous cases, true power shows an upward trend toward the average upper limit for larger sample sizes.

As with prior cases, the general ordering of true power, median estimated power, and average estimated power (4.1) tends to hold. For σ = 2.5 and 5.0, the general ordering of power curves can be seen in Figure 4.3.a. For $\sigma = 10.0$, average estimated power starts (at $n = 5$) above the median estimated power, but adopts the general ordering for sample sizes larger than 10. When $\sigma = 20.0$, average estimated power starts (at $n = 5$) above true power, and median estimated power starts (at $n = 5$) below true power. Average estimated power appears below true power for sample sizes 15 and greater, and below median estimated power for sample sizes greater than 35. As with the previous cases, the general order of the power curves (4.1) is maintained for large sample sizes.

Three-Sample Asymmetric Case: Power vs Sample size

Figure 4.3.a: Sigma 2.5

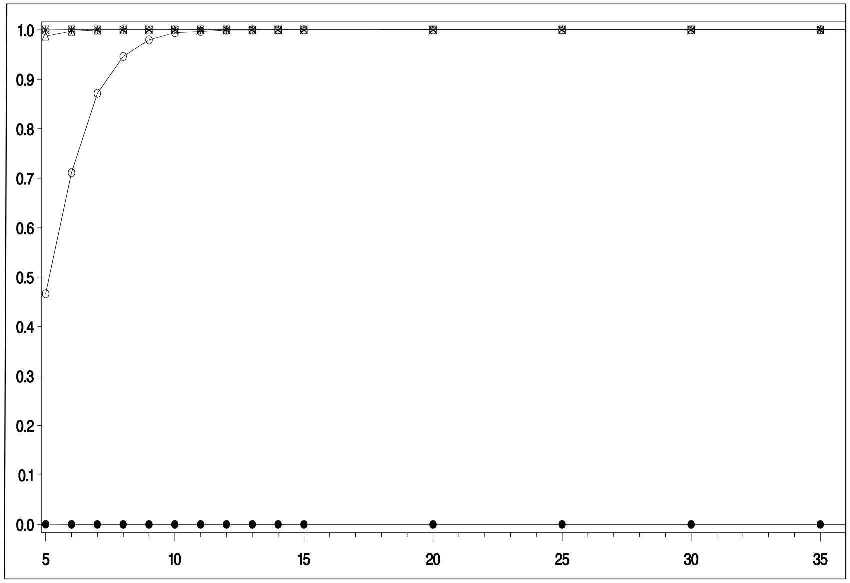


Figure 4.3.b: Sigma 5.0

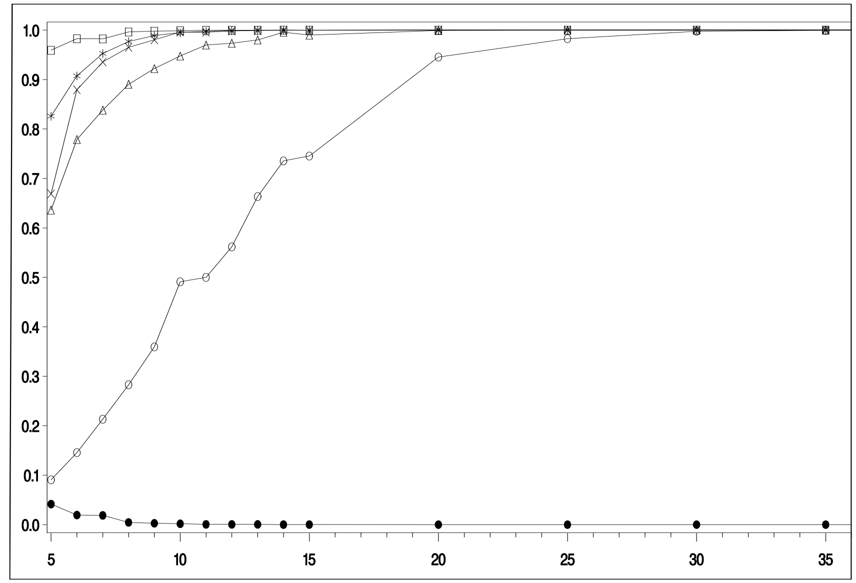


Figure 4.3.c: Sigma 10.0

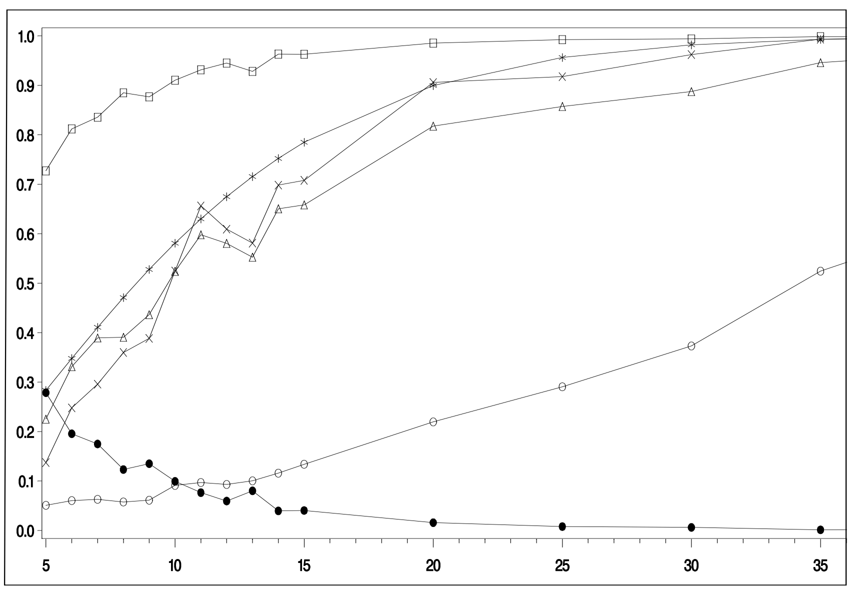
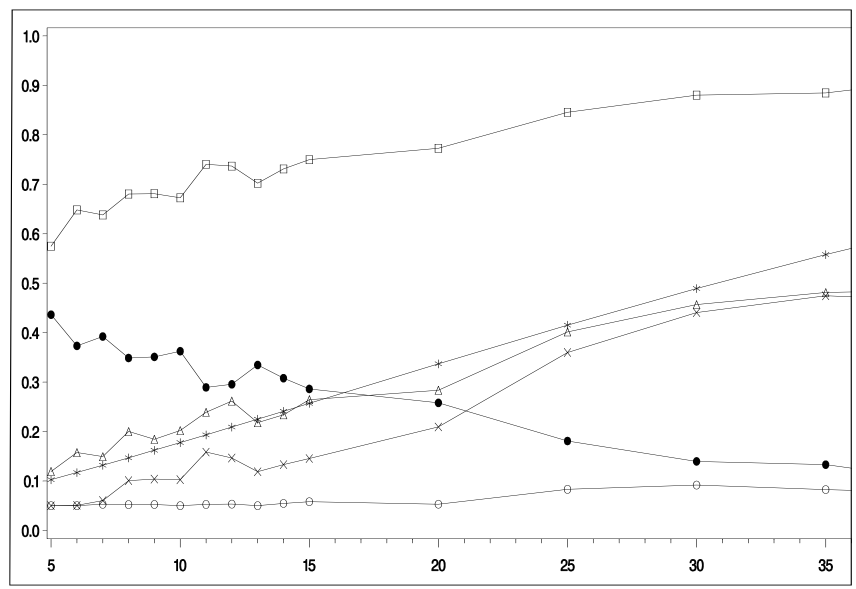
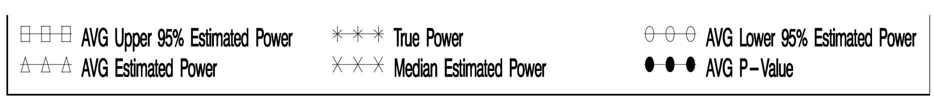


Figure 4.3.d: Sigma 20.0





4.4 Four-Sample Symmetric Case A

Plots for population standard deviations 2.5, 5.0, 10.0, and 20.0 with a maximum sample size of 35 are seen in Figure 4.4.a, 4.4.b, 4.4.c, and 4.4.d, respectively. For larger sample sizes, the p-value for the test decreases such that the null hypothesis can be rejected when $n \geq 5$, $n \geq 8$, $n \geq 17$, and $n > 35$ for respective values of $\sigma = 2.5$, $\sigma = 5.0$, $\sigma = 10.0$, and $\sigma = 20.0$. True power, as seen in previous cases, is always between the average upper and lower limits for true power. Further, true power and average confidence limits tend toward 1.0 for large sample sizes. For $\sigma = 2.5$, true power is identical with the average upper confidence limit. For $\sigma = 5.0$, average confidence limits center are symmetric about true power when both limits equal 1.0. For $\sigma = 10.0$, average upper and lower confidence limits are centered around true power when $n = 14$. For $\sigma = 20.0$, true power is close to the average lower limit. True power displays a positive increasing trend toward the average upper limit, where the steepness of the curve reduces for larger values of σ.

The general order of the power curves (4.1) again tends to occur. The typical ordering always occurs for σ = 2.5 and 5.0. For $\sigma = 10.0$, median estimated power starts (at $n = 5$) below the median estimated power, but maintains the typical ordering at sample sizes 14 and greater. For $\sigma = 20.0$ and $n = 5$, power curves are ordered as average estimated power, true power, and median estimated power. The average estimated power (for $\sigma = 20.0$) appears below true power at sample sizes 15 and greater, and below median estimated power at sample sizes larger than 35. As with previous cases, the typical order of power curves (4.1) is seen for larger sample sizes.

Four-Sample Symmetric Case A: Power vs Sample size

Figure 4.4.a: Sigma 2.5

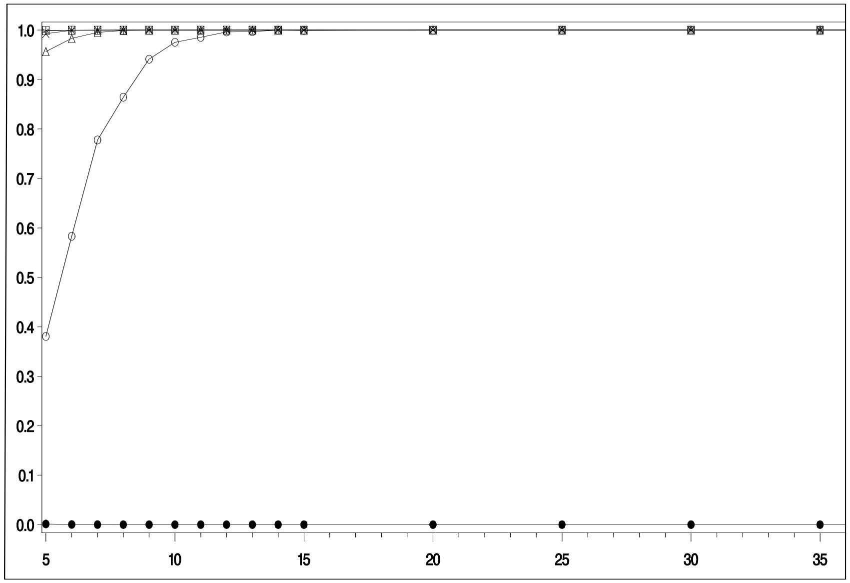


Figure 4.4.b: Sigma 5.0

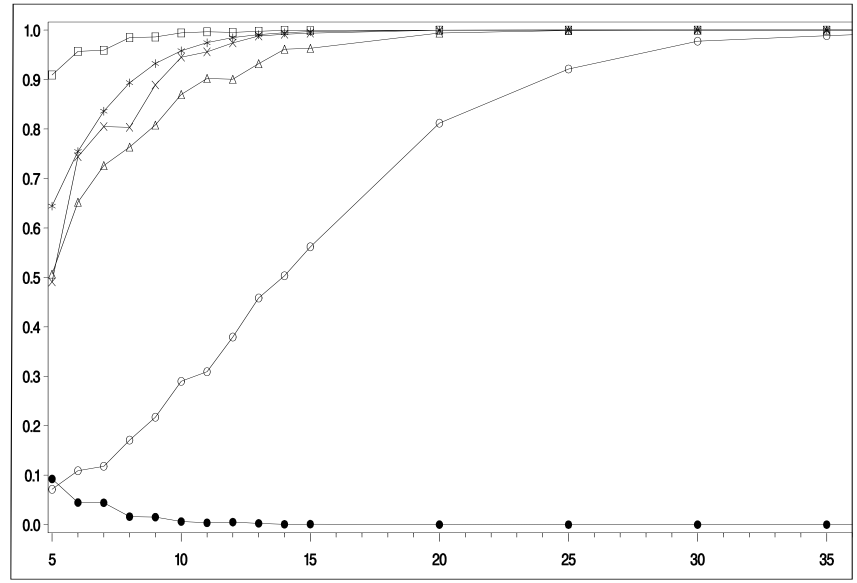


Figure 4.4.c: Sigma 10.0

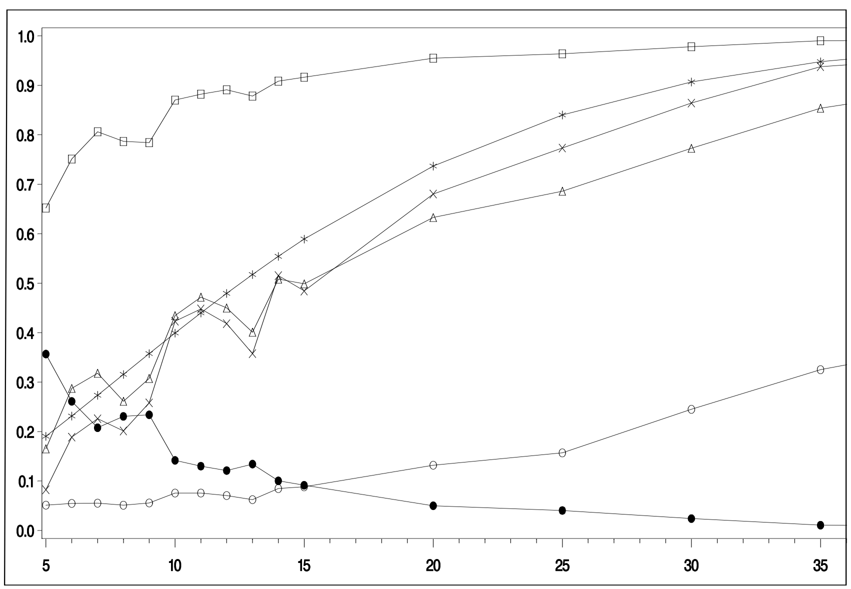
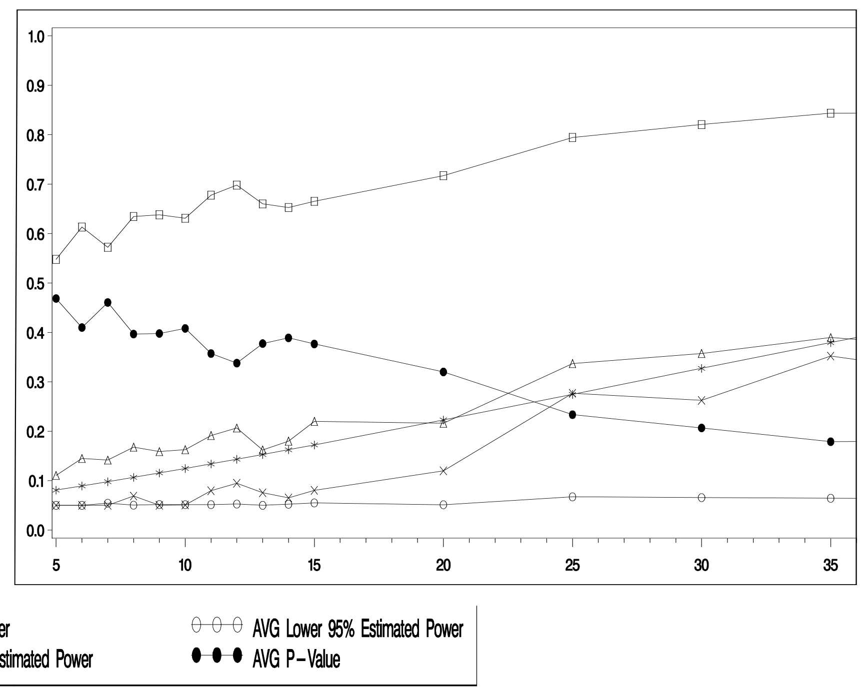


Figure 4.4.d: Sigma 20.0



4.5 Four-Sample Symmetric Case B

Plots for population standard deviations 2.5, 5.0, 10.0, and 20.0 with a maximum sample size of 35 are seen in Figure 4.5.a, 4.5.b, 4.5.c, and 4.5.d, respectively. As the sample size increases, the p-value for the test decreases such that the null hypothesis can be rejected when $n \geq 5$, $n \geq 5$, $n \geq 10$, and $n > 35$ for $\sigma = 2.5$, $\sigma = 5.0$, $\sigma = 10.0$, and $\sigma = 20.0$, respectively. As with previous cases, average lower and upper confidence limits always contain true power. True power and confidence limits become narrower and approach 1.0 for larger sample sizes. For $\sigma = 2.5$, true power is identical to the average upper limit. For $\sigma = 5.0$, true power is just below the average upper limit and is centered between average limits when both limits approach 1.0. For $\sigma = 10.0$, true power begins (at $n = 5$) around the average lower limit, and centers between the average confidence limits for $n = 8$. When $\sigma = 20.0$, average confidence limits are symmetric around true power for $n = 25$. As with prior cases, true power has an increasing positive trend toward the average upper limit for each plot. Notably, the rate of increase of the true power curve decreases for larger values of σ.

As seen previously, average estimated power, median estimated power, and true power are consistently between the lower and upper confidence limits for true power. Power curves have the typical order (4.1) as in previous cases. The usual ordering of power is seen for σ = 2.5 and 5.0. For $\sigma = 10.0$, average estimated power starts (at $n = 5$) above the median estimated power, yet takes on the general ordering at sample sizes 10 and greater. For $\sigma = 20.0$, true power is below average estimated power and above median estimated power for $n = 5$. Further, for $\sigma = 20.0$, average estimated power appears below true power at sample sizes 13 and greater, and below median estimated power at sample sizes 30 and greater. As seen earlier, the general ordering (4.1) holds for large sample sizes.

Four-Sample Symmetric Case B: Power vs Sample size

Figure 4.5.a: Sigma 2.5

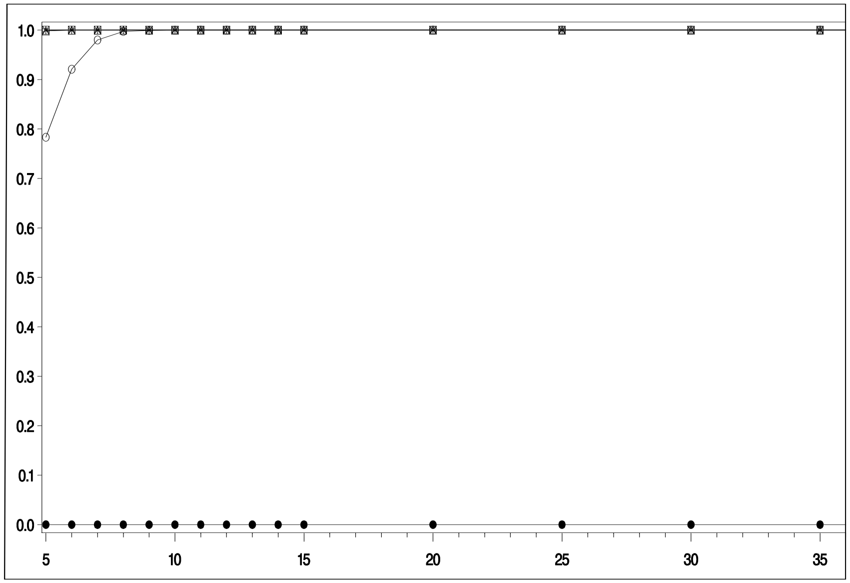


Figure 4.5.b: Sigma 5.0

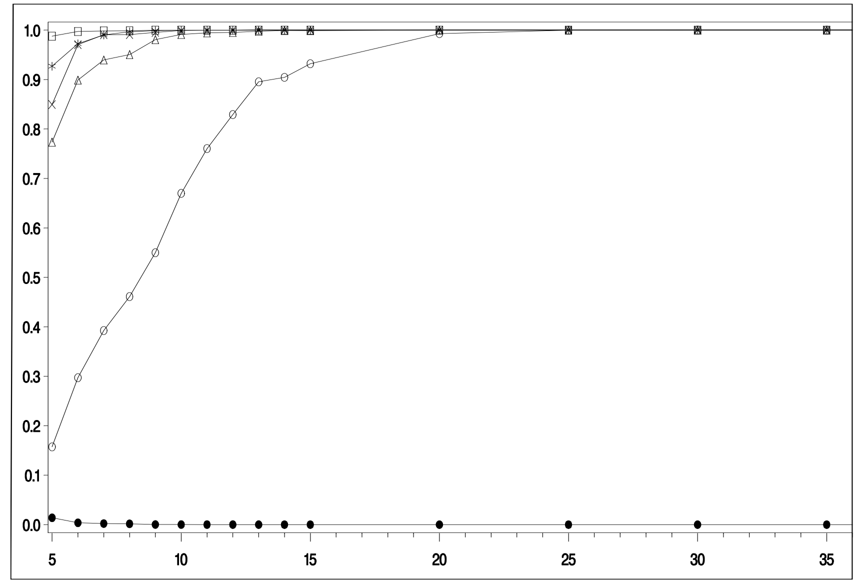


Figure 4.5.c: Sigma 10.0

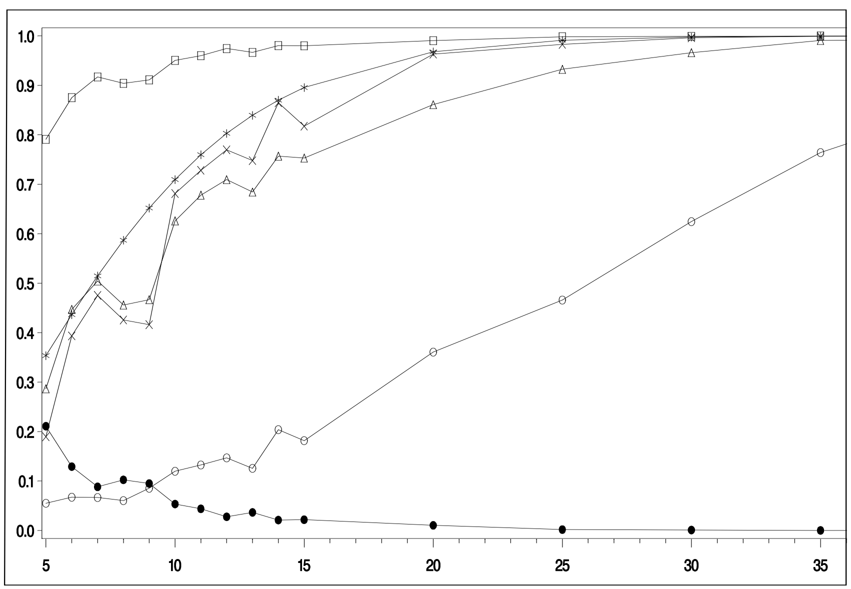
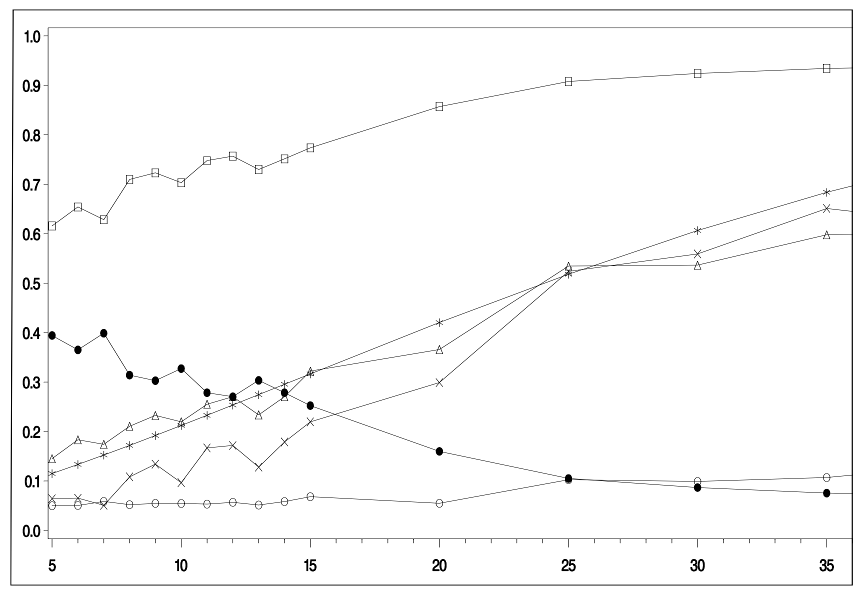
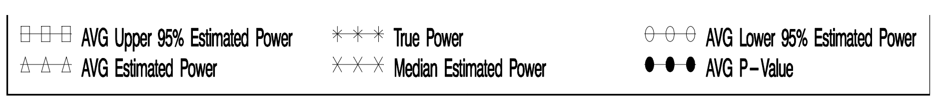


Figure 4.5.d: Sigma 20.0





4.6 Four-Sample Asymmetric Case

Plots for population standard deviations 2.5, 5.0, 10.0, and 20.0 with a maximum sample size of 35 are seen in Figure 4.6.a, 4.6.b, 4.6.c, and 4.6.d, respectively. The p-value for the test decreases as the sample size increases, as expected, such that the null hypothesis can be rejected when $n \geq 5$, $n \geq 6$, $n \geq 14$, and $n > 35$ for respective values of $\sigma = 2.5$, $\sigma = 5.0$, $\sigma = 10.0$, and $\sigma = 20.0$. True power lies between the average lower and upper limits for all values of σ. As seen in prior cases, true power and average confidence limits are higher and closer together for large sample sizes. For $\sigma = 2.5$, true power is indistinct from the average upper limit. For $\sigma = 5.0$, true power is close to the average upper limit and is approximately very close (at power approximately 1.0) with the average upper limit at $n = 10$. For $\sigma = 10.0$, average confidence limits are equally spaced about true power when $n = 9$. When $\sigma = 20.0$, average upper and lower limits are centered around true power for $n = 30$. As usual, true power shows an increasing upward trend toward the average upper confidence limit for each plot. Notably, the incline of the curve for true power climbs with less rapidity for larger values of σ.

The power curves are hold the usual ordering (4.1) as seen previously. The general ranking of power curves (4.1) is exemplified for $\sigma = 2.5$ and 5.0. For $\sigma = 10.0$, average estimated power starts (at $n = 5$) above the median estimated power but takes on the general ordering at sample sizes 9 and greater. For $\sigma = 20.0$, average estimated power starts above true power, while median estimated power starts below true power. Average estimated power (for $\sigma = 20.0$) appears below true power at sample sizes 20 and greater, and below median estimated power at sample sizes greater than 35. When sample sizes are large, the power curves follows the general ordering (4.1).

Four-Sample Asymmetric Case: Power vs Sample size

Figure 4.6.a: Sigma 2.5

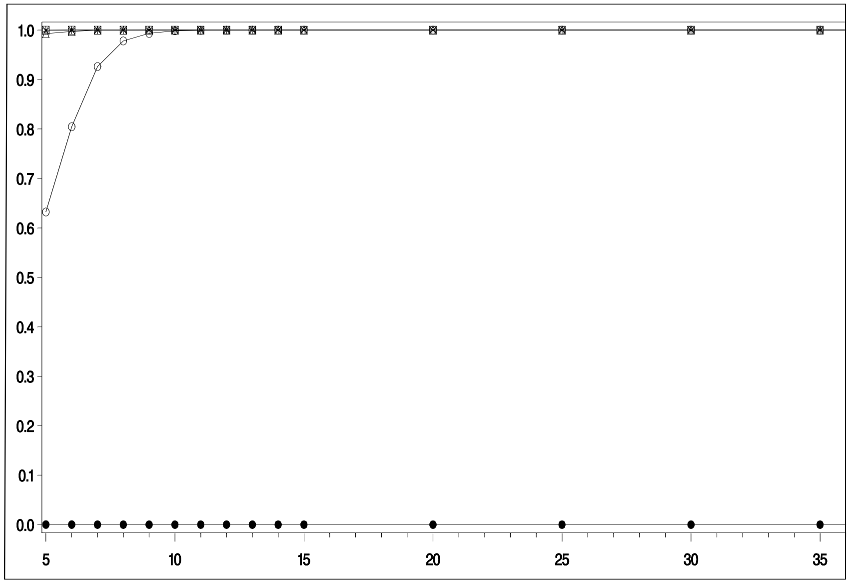


Figure 4.6.b: Sigma 5.0

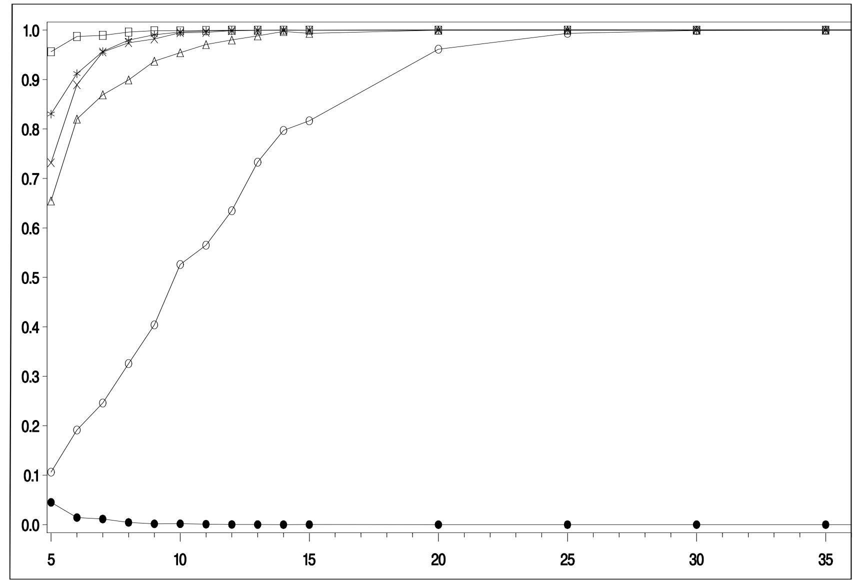


Figure 4.6.c: Sigma 10.0

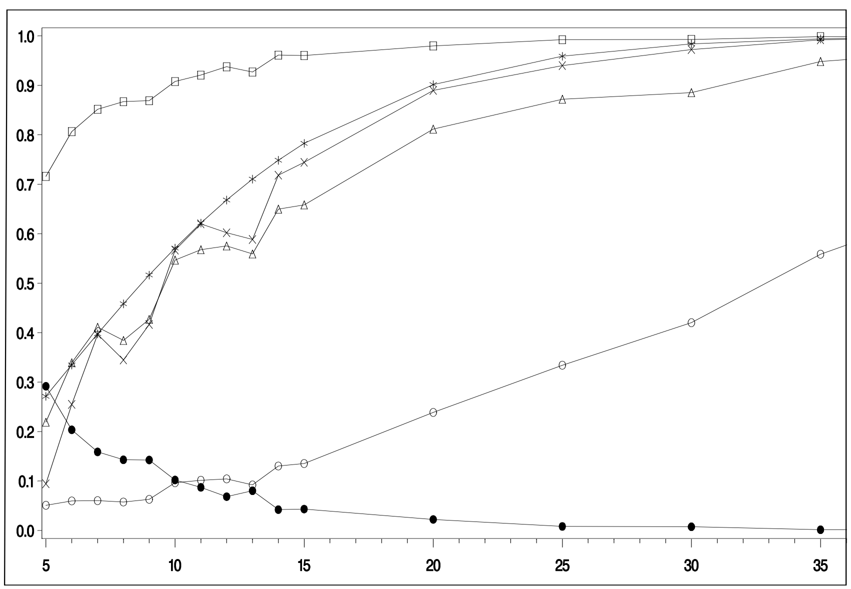
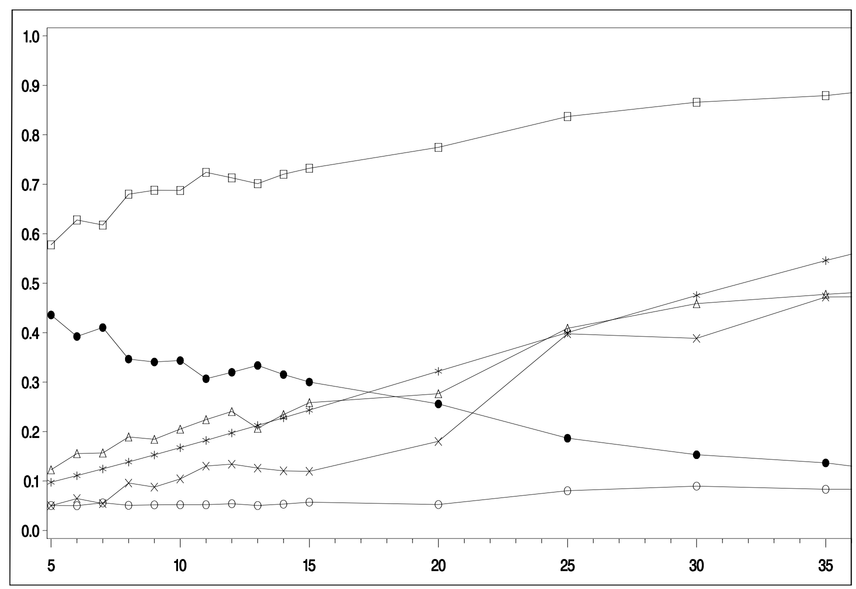
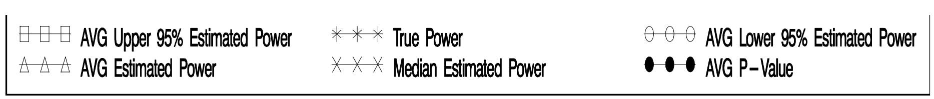


Figure 4.6.d: Sigma 20.0





4.7 Symmetric Cases

For symmetric treatment arrangements, confidence limits for the two-sample case, three-sample case, and four-sample case A do not appear substantially different. As an exception, confidence limits for the four-sample case B are narrower than the other symmetric cases. Average lower and upper confidence limit widths are ordered, from least to greatest, as four-sample case B, four-sample case A, three-sample case, and two-sample case. Average lower and upper confidence limits always contain true power.

For all symmetric cases, true power shows a positive increasing trend toward the average upper limit. For σ = 2.5 and 5.0, all symmetric cases show true power near the average upper limit. For $\sigma = 10.0$, true power starts (at $n = 5$) close to the average lower limit, reaches a half-way distance between average limits for $n = 5 \text{ to } 15$, and then proceeds toward the average upper limit. For $\sigma = 20.0$, true power is generally close to the average lower limit for all symmetric cases, though the two-sample case and four-sample case B reach a half-way distance between average limits at $n = 30$ and $n = 25$, respectively.

4.8 Asymmetric Cases

For the two asymmetric cases, confidence interval widths are noticeably different from one another. The four-sample case has narrower average lower and upper confidence limits for true power than the three-sample case. True power is always between the average lower and upper confidence limits.

For both asymmetric cases, true power exhibits an upward trend toward the average upper limit. For σ = 2.5 and 5.0, true power is never half-way between the average upper and lower limits. For $\sigma = 10.0$, true power starts (at $n = 5$) around the average lower limit, reaches the half-way distance between average limits around $n = 8 \text{ to } 9$, and then moves closer to the average upper limit. For $\sigma = 20.0$, true power is close to the average lower limit, but reaches the half-way distance between average limits around $n = 30 \text{ to } 35$.

4.9 Symmetric Cases versus Asymmetric Cases

In both symmetric and asymmetric cases, true power is always between the average lower and upper limits. Generally, symmetric cases appear to have wider average confidence limits than the asymmetric cases, with the exception of the four-sample case B, which has the narrowest confidence limits among all cases considered. Symmetric and asymmetric cases both show that true power is closer to the average upper limit than the lower limit for large sample sizes. In terms of average limit widths, cases are ordered from least to greatest as follows: four-sample case B, four-sample asymmetric case, three-sample asymmetric case, four-sample case A, three-sample symmetric case, then the two-sample case.

Both symmetric cases and asymmetric cases have power curves typically ordered, from highest to lowest, as true power, median estimated power, then average estimated power. The typically ordering of power curves is seen for σ = 2.5 and 5.0. For $\sigma = 10.0$, average estimated power starts (at $n = 5$) above median estimated power, but crosses below median estimated power for samples sizes between 10 to 15. For $\sigma = 20.0$, average estimated power is above true power, and median estimated power is below true power. Further (for $\sigma = 20.0$), average estimated power drops below true power, then proceeds below median estimated power as sample sizes increase. All power curves are within the average lower and upper limits.

5 Conclusion

Simulations were constructed for a range of population conditions under the balanced one-way ANOVA model. Each simulation case was iterated 100 times and plots for average estimated power, median estimated power, true power, average lower 95% limit, and average upper 95% limit were produced. An average p-value for *F*-tests was also plotted for each simulation setup.

When the population standard deviation was small (from $\sigma = 2.5 \text{ to } 5.0$), confidence limits for true power were wider for small sample sizes and narrower for medium-to-large sample sizes. Under all sample sizes considered here, average limits contained true power. Notably, true power was never half-way between the average upper and lower confidence limits except when sample sizes were large and both limits were very close to 1.0. Additionally, true power was always close to the average upper limit and above the average estimate for power. When significant differences exist among the population treatment means, the average p-value is small, true power is large, and the average retrospective estimate for power provides a higher lower limit for true power than the average lower limit.

When the population standard deviation was large (from $\sigma = 10.0 \text{ to } 20.0$), average confidence limits for true power were generally wide for small and large sample sizes. True power was contained within the average limits: close to the average lower limit for small sample sizes and progressive toward the average upper limit for large sample sizes. In all cases, the average estimate for power was present above true power (when sample sizes were small) and below true power (when sample sizes were large). When the null hypothesis of equal population means was difficult to reject, i.e. when the p-value of the test was large, true power was low and confidence limits for true power were wide.

When the null hypothesis cannot be rejected, the test performed is seen as under powered. The results here show that true power is hard to determine retrospectively as laid out in Thomas (1997) when the null hypothesis cannot be rejected, since the average 95% confidence limits were wide and the average estimate for power fails to act as a lower bound for true power under all values of σ.

The presence of nonsignificant results can lead researchers to question the power of their test. When power has not been calculated *a priori,* researchers may see *retrospective* methods as a viable technique for “figuring out what power was.” As an example of Thomas' technique to calculate power *post hoc*, two possible consulting situations are described below. Each uses an observed *F*-statistic, numerator and denominator degrees of freedom, and the desired level of significance. The p-value of the test is also provided.

For the first example, consider an observed *F*-statistic of 1.5 with 1 numerator degree of freedom and 40 denominator degrees of freedom with a p-value of 0.2278. An estimate for true power is

$$\widehat{\text{power}} = 1 - F\left(4.085 \mid 1,40,0.425\right) = 0.0975 ,$$

where $F_{0.05,1,40} = 4.085$ is the upper tail critical value from a central *F*-distribution with 1 numerator degree of freedom and 40 denominator degrees of freedom, and $\hat{\lambda}_{adj} = \left(\left(1.5\right)\left(1\right)\left(40-2\right)\right)/40-1 = 0.425$ is the adjusted estimated noncentrality parameter. The minimized 95% confidence limits for true power are

$$\widehat{\text{power}}_L = 1 - F\left(4.085 \mid 1,40,0.00000\right) = 0.05 \text{ and}$$

$$\widehat{\text{power}}_U = 1 - F\left(4.085 \mid 1,40,\ 8.2801704\right) = 0.8017 ,$$

where the estimated lower and upper noncentrality parameters $\hat{\lambda}_L = 0.000000$ and $\hat{\lambda}_U = 8.2801704$ are determined by the lower and upper tail probabilities $\alpha_L = 0.000048$ and

$\alpha_U = 0.049952$ through use of the Golden Section Search method (Section 3.3). Using the estimate for power, a researcher would conclude that the test was under powered since this estimated value is below 0.8 (Cohen 1988). Notably, using the point estimate to represent true power is uninformative since the confidence limits have true power anywhere from 0.05 to 0.8017. The confidence interval here is similar to the confidence limits in Figure 4.1.d for the symmetric two population mean arrangement with $\sigma = 20.0$ at a sample size of 22. Figure 4.1.d likens the clients' observations to an extreme case where discerning treatment groups as different is difficult.

For the second example, consider an observed *F*-statistic of 4.05 with 1 numerator degree of freedom and 40 denominator degrees of freedom with a p-value of 0.0509. Power is estimated as

$$\widehat{\text{power}} = 1 - F\left(4.085 \mid 1,40,2.8475\right) = 0.3773,$$

where $F_{0.05,1,40} = 4.085$ is the upper tail critical value from a central *F*-distribution with 1 numerator degree of freedom and 40 denominator degrees of freedom, and $\hat{\lambda}_{adj} = \left(\left(4.05\right)\left(1\right)\left(40-2\right)\right)/40 - 1 = 2.8475$ is the adjusted estimated noncentrality parameter. Through the Golden Section Search (Section 3.3), the minimized lower and upper tail probabilities are found to be $\alpha_L = 0.000048$ and $\alpha_U = 0.049952$ which yield lower and upper estimated noncentrality parameters $\hat{\lambda}_L = 0.000000$ and $\hat{\lambda}_U = 13.59003$. The estimated 95% lower and upper confidence limits for true power are given by

$$\widehat{\text{power}}_L = 1 - F\left(4.085 \mid 1,40,0.00000\right) = 0.05 \text{ and}$$

$$\widehat{\text{power}}_U = 1 - F\left(4.085 \mid 1,40,13.59003\right) = 0.9491.$$

Using the estimate for power, a researcher would conclude that the test was under-powered since this estimated value is below 0.8 (Cohen 1988). Notably, using the point estimate to represent true power is uninformative since the confidence limits have true power anywhere from 0.05 to 0.9491. The confidence interval here is similar to the confidence limits in Figure 4.1.d for the two-sample symmetric case with $\sigma = 20.0$ at a sample size of 8. Figure 4.5.c likens the clients observations to the a case where discerning treatment groups as different is difficult.

When determining the power of a test *post hoc*, power is not calculated but estimated. For researchers looking to resolve power after observations have been collected, confidence limits for true power can help quantify the uncertainty inherent in a *post hoc* power calculation. Examples 1 and 2, along with the plots in the results section, show a high degree of variability when the null hypothesis cannot be rejected. Hence, researchers are recommended to incorporate power analyses in the *planning* stages of their studies, as opposed to waiting until data have been collected.

References

American Statistical Association (1999), ‘Ethical Guidelines for Statistical Practice’.

Cohen, J. (1988), *Statistical Power Analysis for the Behavioral Sciences*, Lawrence Erlbaum Associates, Inc., 365 Broadway, Hillsdale, New Jersey 07642.

Gerard, P. D., Smith, D. R. and Weerakkody, G. (1998), 'Limits of Retrospective Power Analysis', *The Journal of Wildlife Management* **62**(2), 801-807.

Haahr, M. (2009), *Random Integer Generator*, Trinity College, Dublin, Ireland, http://www.random.org.

Hoenig, J. M. and Heisey, D. M. (2001), 'The Abuse of Power: The Pervasive Fallacy of Power Calculations for Data Analysis', *The American Statistician* **55**(1), 19-24.

Hogarty K. Y. and Kromrey, J. D. (2001), ‘Probabilistic Hindsight: A SAS Macro for Retrospective Statistical Power Analysis’, *SAS Users Group International Proceedings*, 202-26.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995), *Continuous Univariate Distributions*, Vol. 2 (2nd ed.), John Wiley & Sons, New York, New York.

Kuehl, R. O. (1999), *Design of Experiments: Statistical Principles of Research Design and Analysis*, (2nd e.), Duxbury Press.

Lenth, R. V. (2001), 'Some Practical Guidelines for Effective Sample Size Determination', *The American Statistician* **55**(3), 187-193.

MATLAB (2009), *fminbnd*, The MathWorks, http://www.mathworks.com/access/helpdesk/help/techdoc/ref/fminbnd.html.

Milliken, G. A. and Johnson, D. E (2009), Analysis of Messy Data, Volume I: Designed Experiments, (2nd ed.), Chapman & Hall/CRC.

O'Brien, R. G. (1988), 'Review: [untitled]', *The American Statistician* **42**(4), 266-270.

Press, W. H., Teukolsky, S. A., Vetterling, W. T. and Flannery, B. P. (2007), *Numerical Recipes: The Art of Scientific Computing*, (3rd ed.), Cambridge University Press.

R Development Core Team (2009), *optimize*, R Foundation for Statistical Computing, Vienna, Austria, http://www.R-project.org.

SAS Institute Inc. 2006a. SAS® 9.1.3 Language Reference: Dictionary, Fifth Edition, Volumes 1-4 page 574. Cary, NC: SAS Institute Inc.

SAS Institute Inc. 2006b. SAS® 9.1.3 Language Reference: Dictionary, Fifth Edition, Volumes 1-4 page 434-447 . Cary, NC: SAS Institute Inc.

SAS Institute Inc. 2004a. SAS® 9.1 User's Guide, page 1929. Cary, NC: SAS Institute Inc.

SAS Institute Inc. 2004b. SAS® 9.1 User's Guide, page 3411. Cary, NC: SAS Institute Inc.

Steidl, R. J., Hayes, J. P. and Schauber, E. (1997), 'Statistical Power Analysis in Wildlife Research', *The Journal of Wildlife Management* **61**(2), 270-279.

Taylor, D. J. and Muller, K. E. (1995), 'Computing Confidence Bounds for Power and Sample Size of the General Linear Univariate Model', *The American Statistician* **49**(1), 43-47.

Taylor, D. J. and Muller, K. E. (1996), 'Bias in linear model power and sample size calculation due to estimating noncentrality', *Communications in Statistics - Theory and Methods* **25**(7), 1595-1610.

Thomas, L. (1997), 'Retrospective Power Analysis', *Conservation Biology* **11**(1), 276-280.

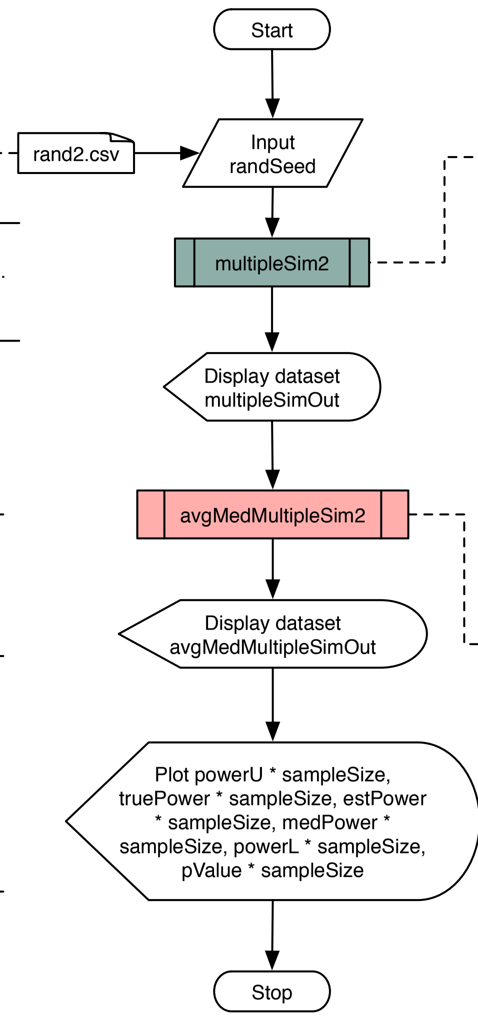
Wright, S. P. and O'Brien, R. G. (1988). 'Power analysis in an enhanced GLM procedures: what is might look like. *SAS Users Group International Proceedings 13*, 1097-1102.

Yuan, K.-H. and Maxwell, S. (2005), 'On the Post Hoc Power in Testing Mean Differences', *Journal of Educational and Behavioral Statistics* **30**(2), 141-167.

Zumbo, B. D. and Hubley, A. M. (1998), 'A Note on Misconceptions Concerning Prospective and Retrospective Power', *Journal of the Royal Statistical Society. Series D (The Statistician)* **47**(2), 385--388.

Appendix A: Two-Sample Flow Chart of Power Simulations

Program Name: twoSample.sas



* rand2.csv is a two column file of random numbers generated from http:/www.random.org. For each iteration two random values are used to seed two random samples.

* Datasets Defined

randSeed := dataset of random seeds

* Program Description

twoSample.sas acts as a controller interface which accepts user defined values for two population means, upper and lower bounds for a range of population standard deviations, a maximum sample size, a Type I error rate, and the number of iterations per sample size to population standard deviation combination.

* Flowchart page 2

* multipleSim2 calls the macro multipleSim2.sas.

* This macro manages the production of simulations. Calls are made to the macro simulation2.sas which performs calculations based on *Thomas 1997*.

* Calculations for each simulation are stored in a dataset called multipleSimOut.

* Flowchart page 14

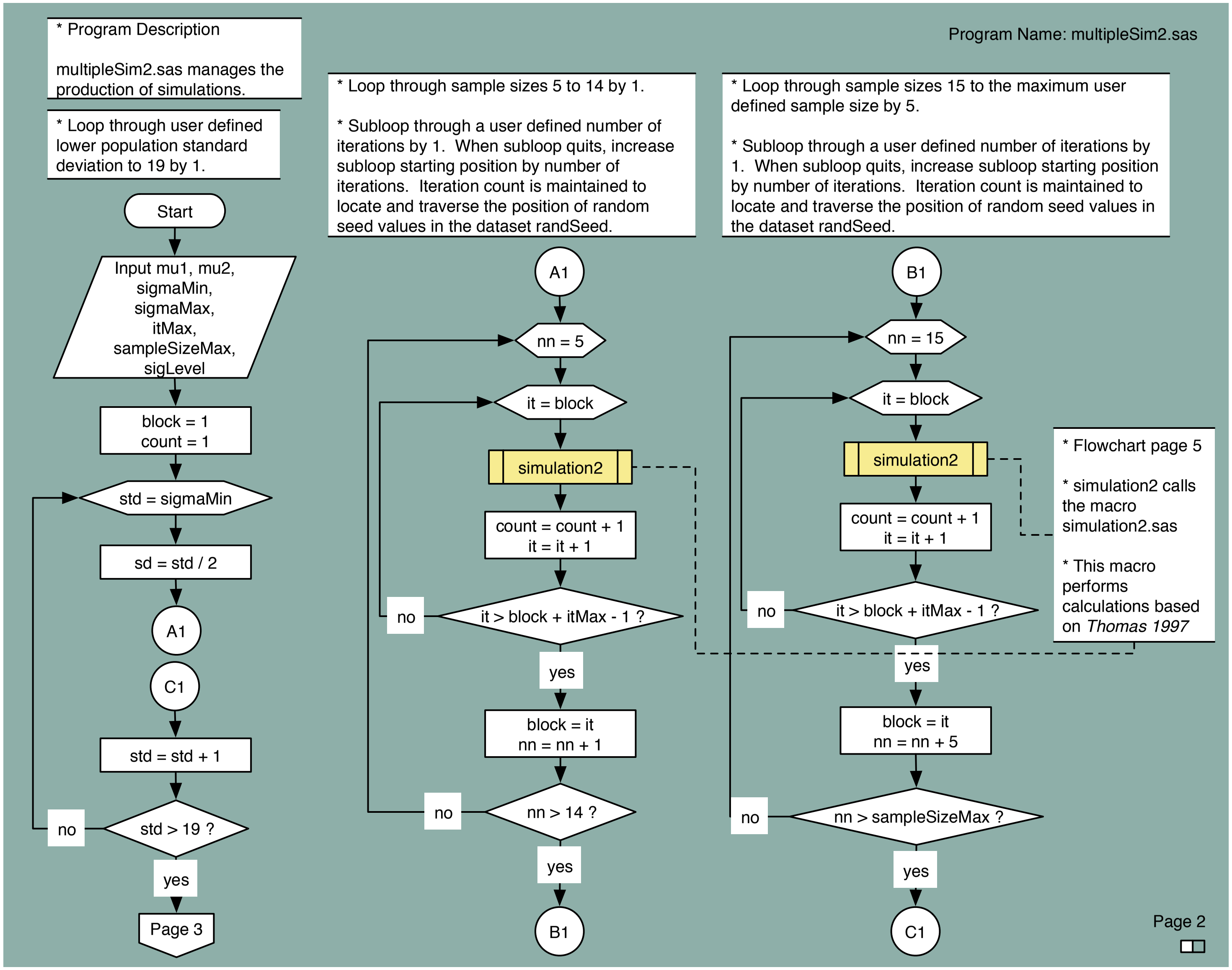
* avgMedMultipleSim2 calls the macro avgMedMultipleSim2.sas.

* This macro manages the consolidation of simulations. Calls to the macro avgMedSim2.sas average over the rows of multipleSimOut by the user defined number of iterations. avgMedSim2 also determines the median of the calculated power values of the rows.

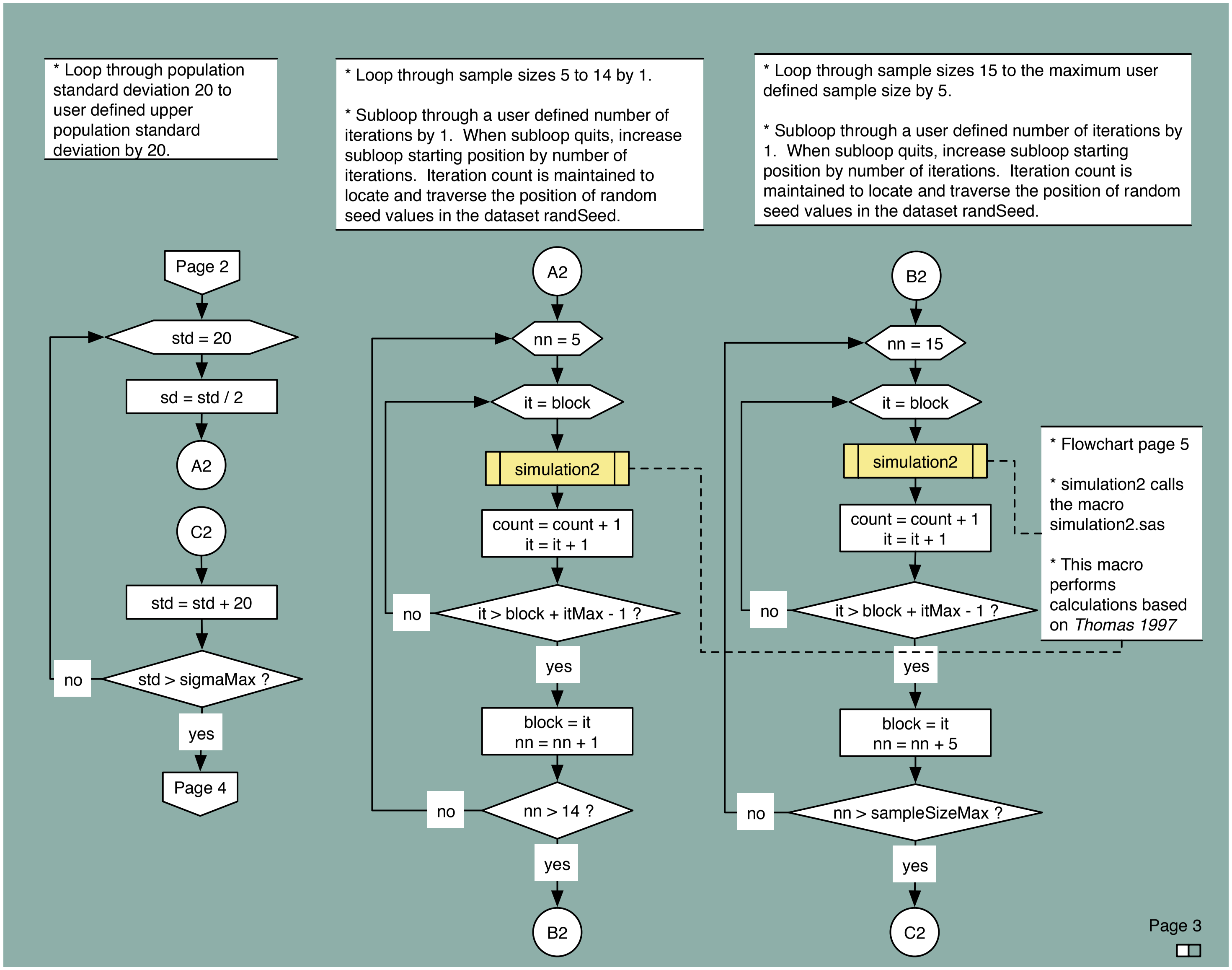
* Calculations for each iteration block are stored in a dataset called avgMedMultipleSimOut.

Page 1

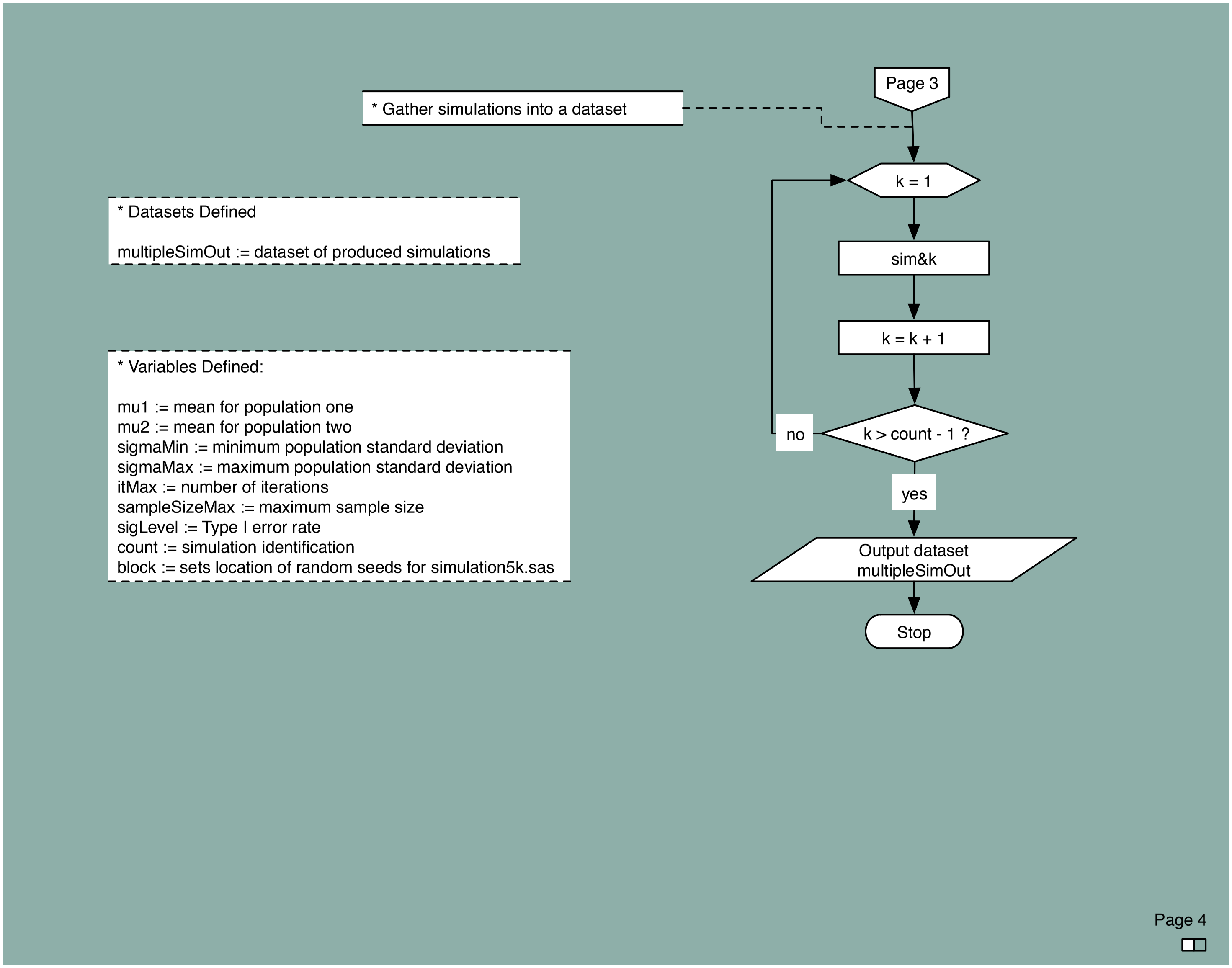
Appendix A: Two-Sample Flow Chart of Power Simulations



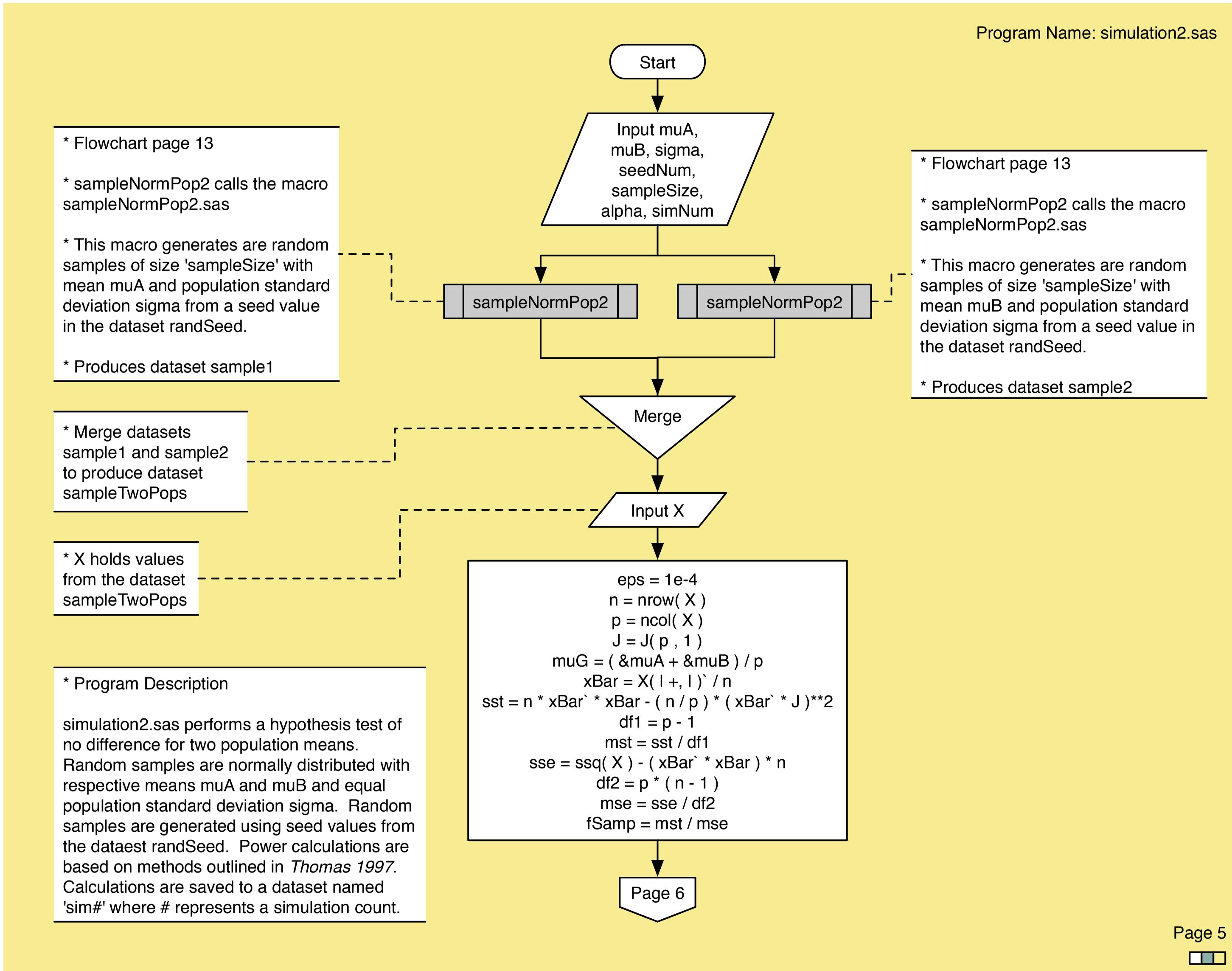
Appendix A: Two-Sample Flow Chart of Power Simulations



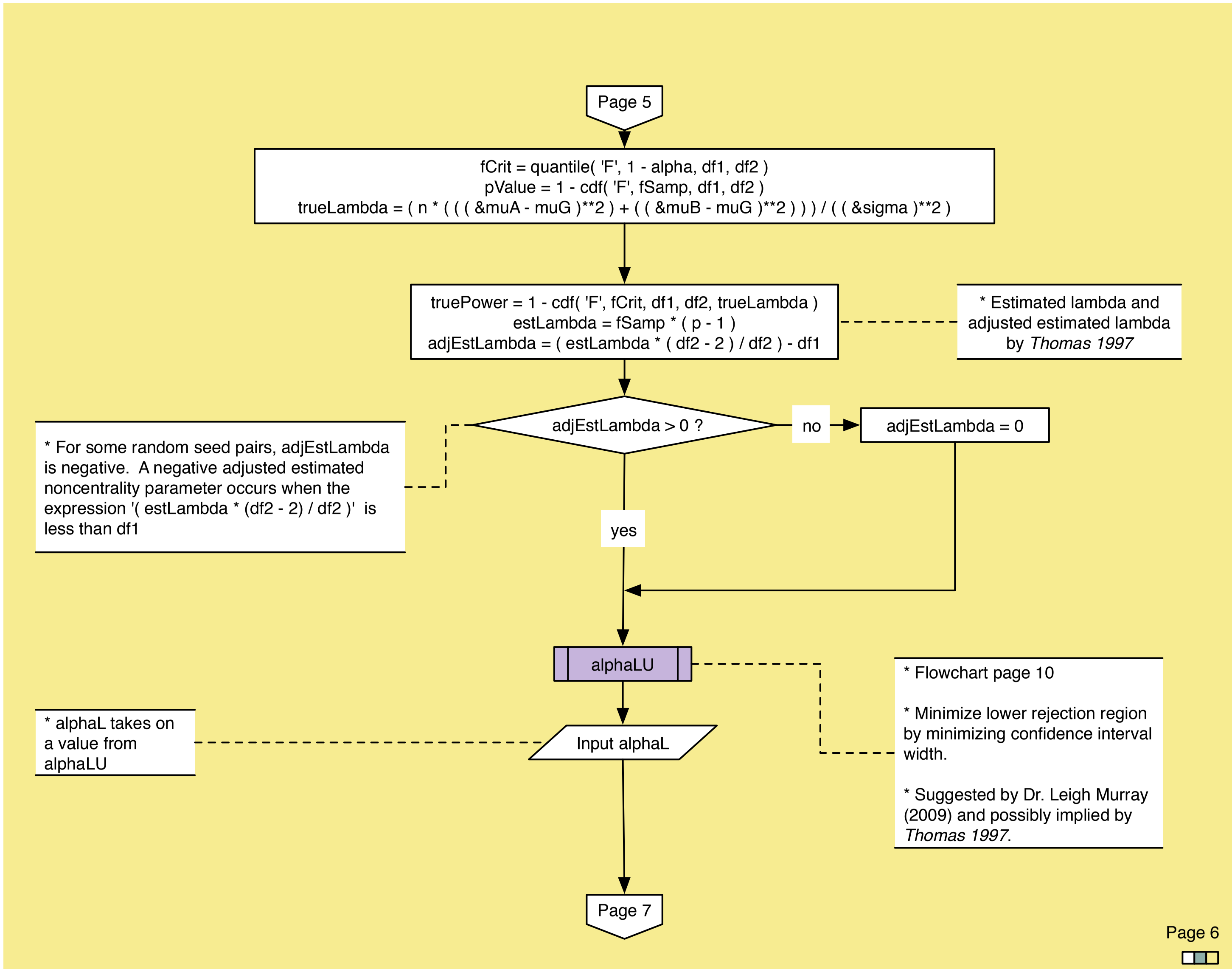
Appendix A: Two-Sample Flow Chart of Power Simulations



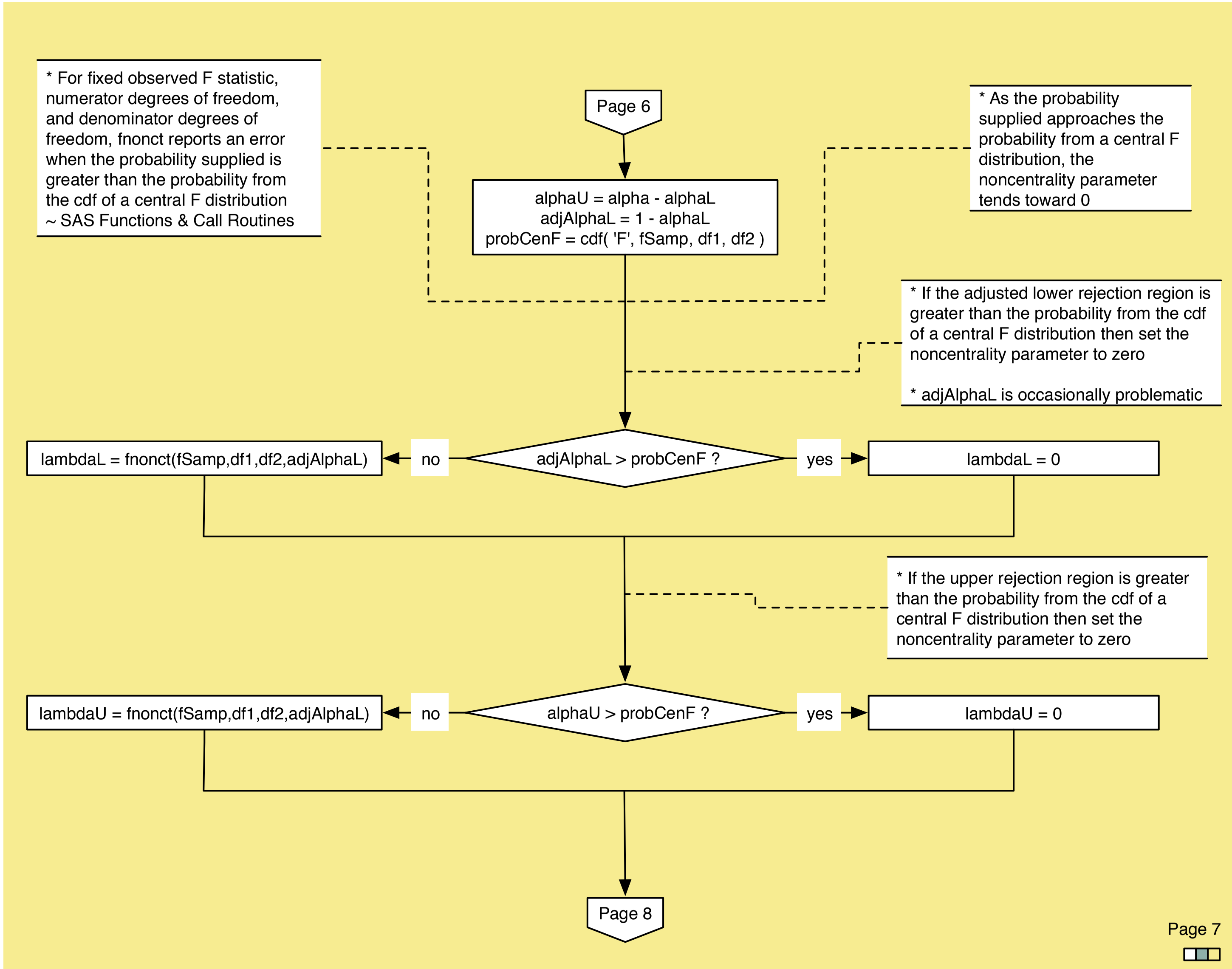
Appendix A: Two-Sample Flow Chart of Power Simulations



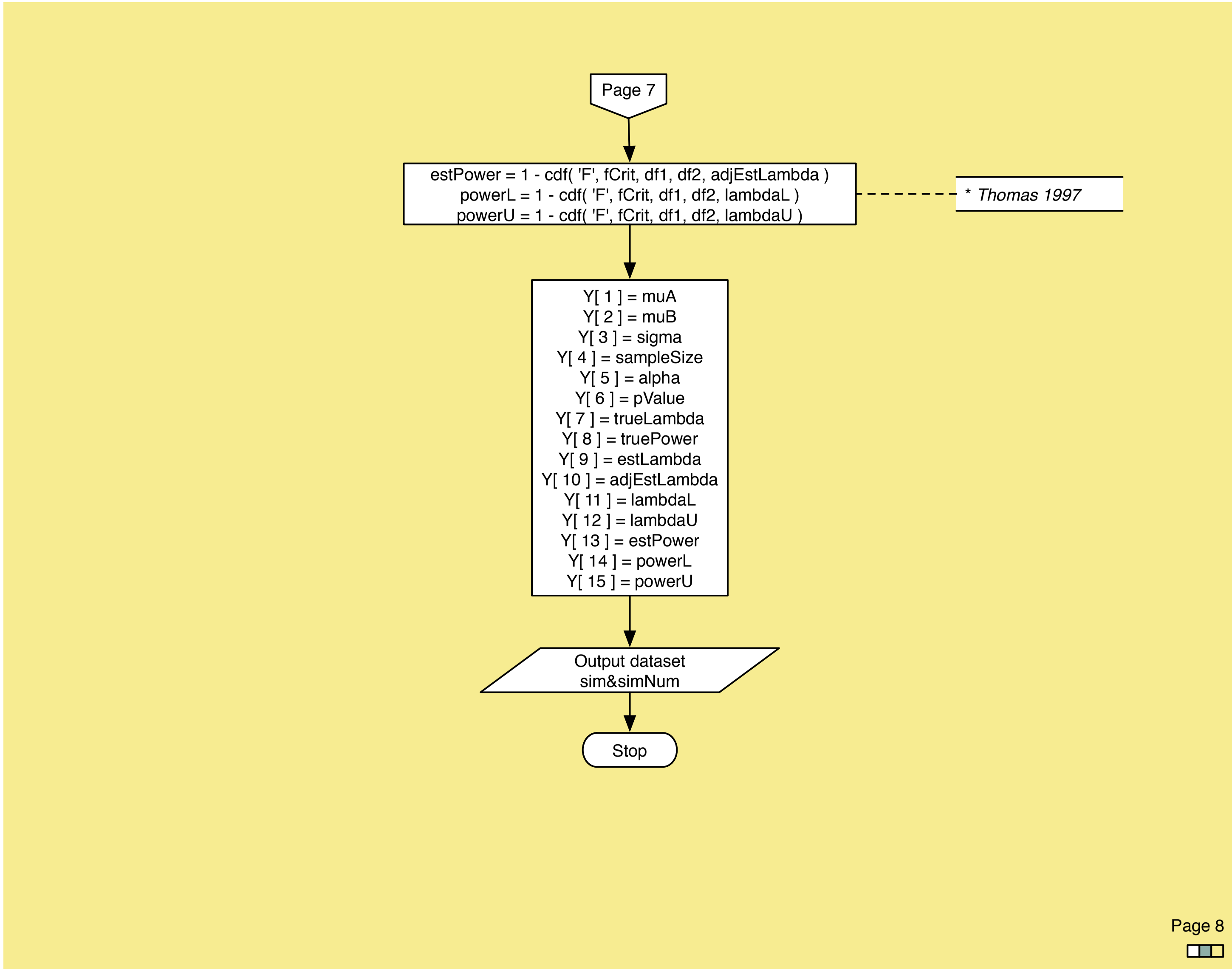
Appendix A: Two-Sample Flow Chart of Power Simulations



Appendix A: Two-Sample Flow Chart of Power Simulations



Appendix A: Two-Sample Flow Chart of Power Simulations



Appendix A: Two-Sample Flow Chart of Power Simulations

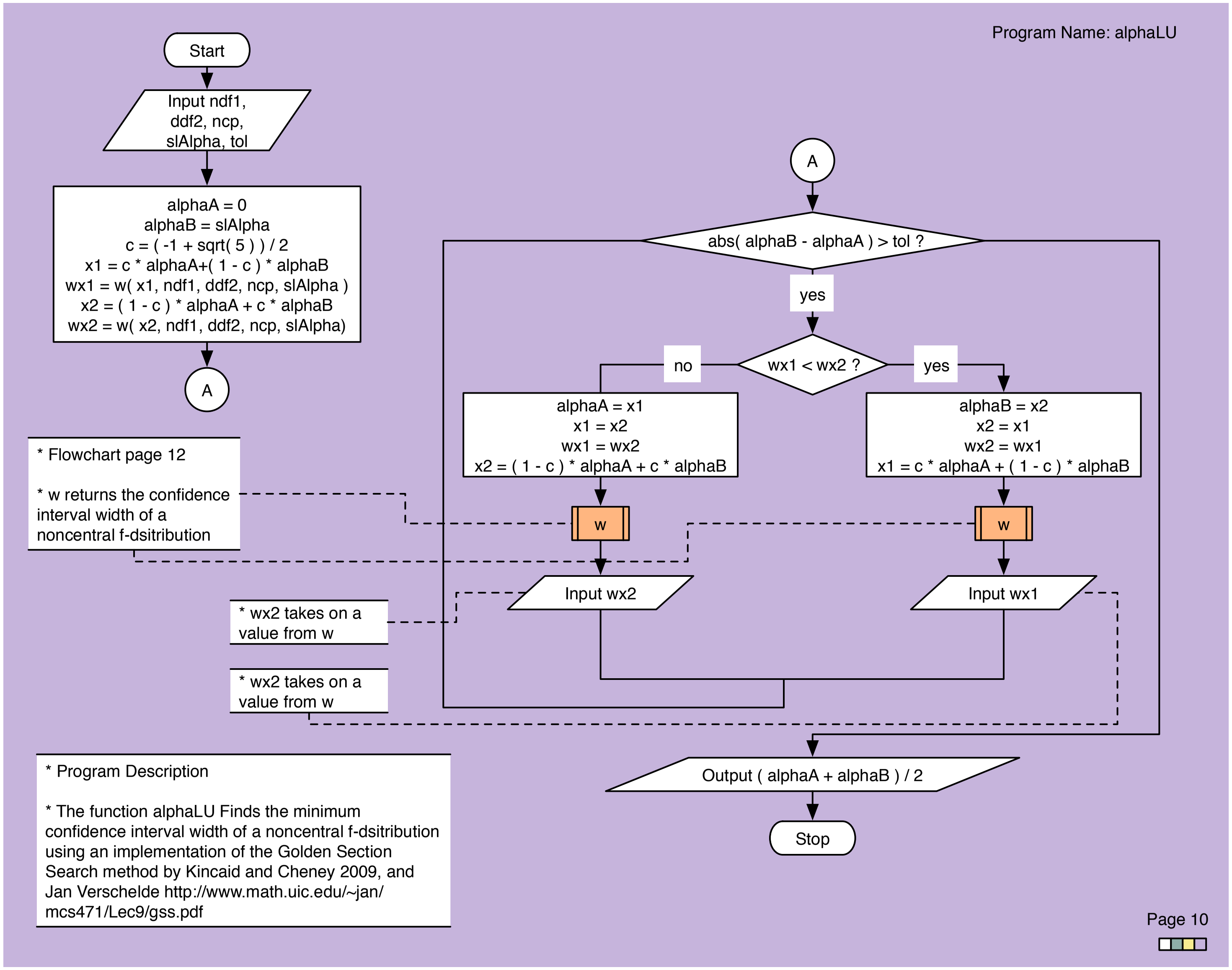
* Variables Defined

muA := mean for population one
muB := mean for population two
sigma := population standard deviation
seedNum := location of random seeds
sampleSize := sample size
alpha := level of significance
simNum := simulation identification
X := samples from normal populations
eps := tolerance for Golden Section Search methods
n := number rows
p := number columns
xBar := sample mean
sst := sum of squares for treatment
df1 := numerator degrees of freedom
mst := mean square for treatment
sse := sum of squares for error
df2 := denominator degrees of freedom
mse := mean square for error
fSamp := observed F statistic
fCrit := critical value from a central F distribution
pValue := p-value for the test
trueLambda := noncentrality parameter
truePower := power for the test
estLambda := calculated noncentrality parameter
adjEstLambda := adjusted calculated noncentrality parameter
alphaL := optimized lower critical region
alphaU := optimized upper critical region
probCenF := probability of observed F statistic of the central F distribution
estPower := calculated power
powerL := lower confidence interval for calculated power
powerU := upper confidence interval for calculated power

* Datasets Defined

sampleTwoPops := random samples dataset
sim&simNum := simulation dataset

Appendix A: Two-Sample Flow Chart of Power Simulations



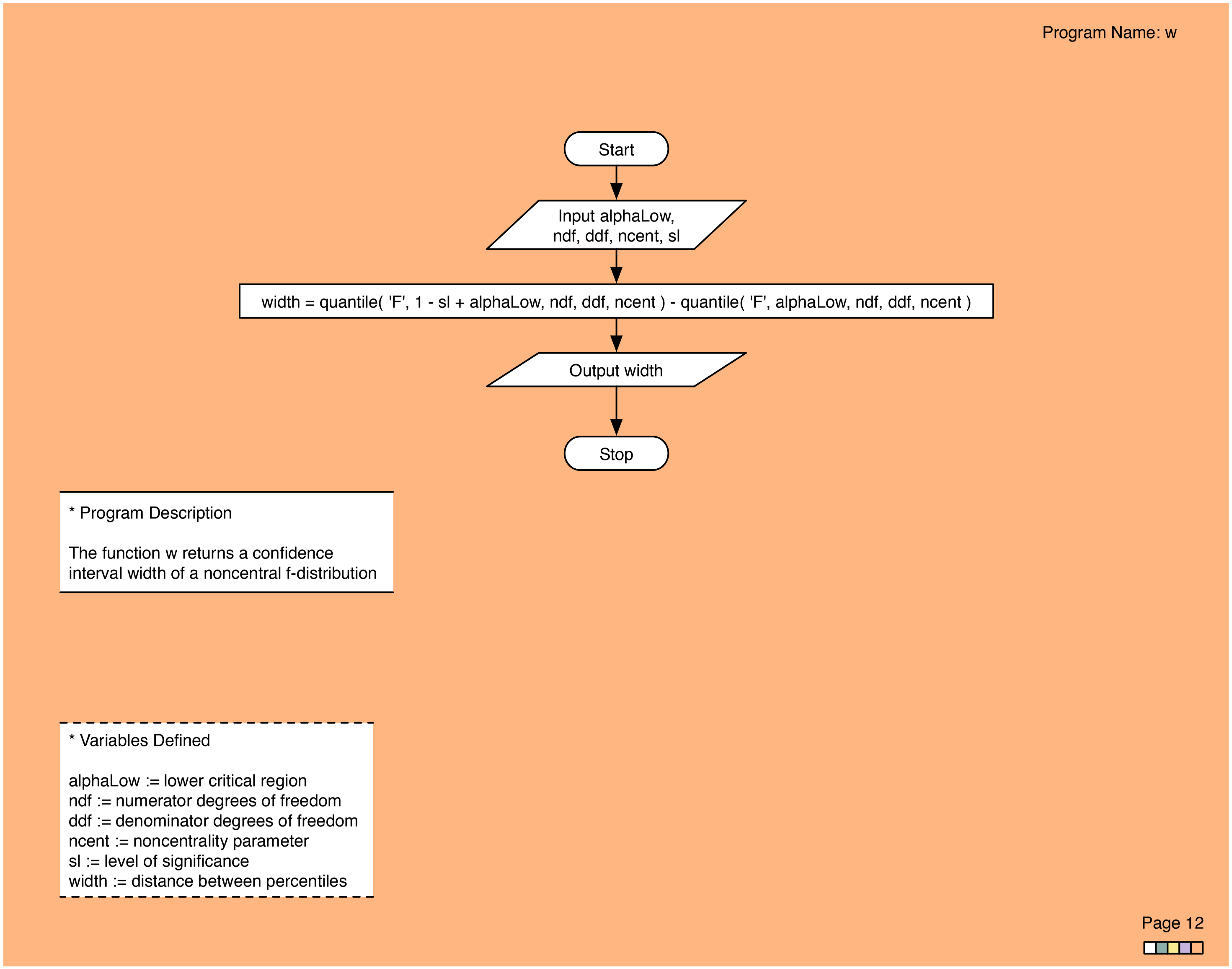
Appendix A: Two-Sample Flow Chart of Power Simulations

* Variables Defined

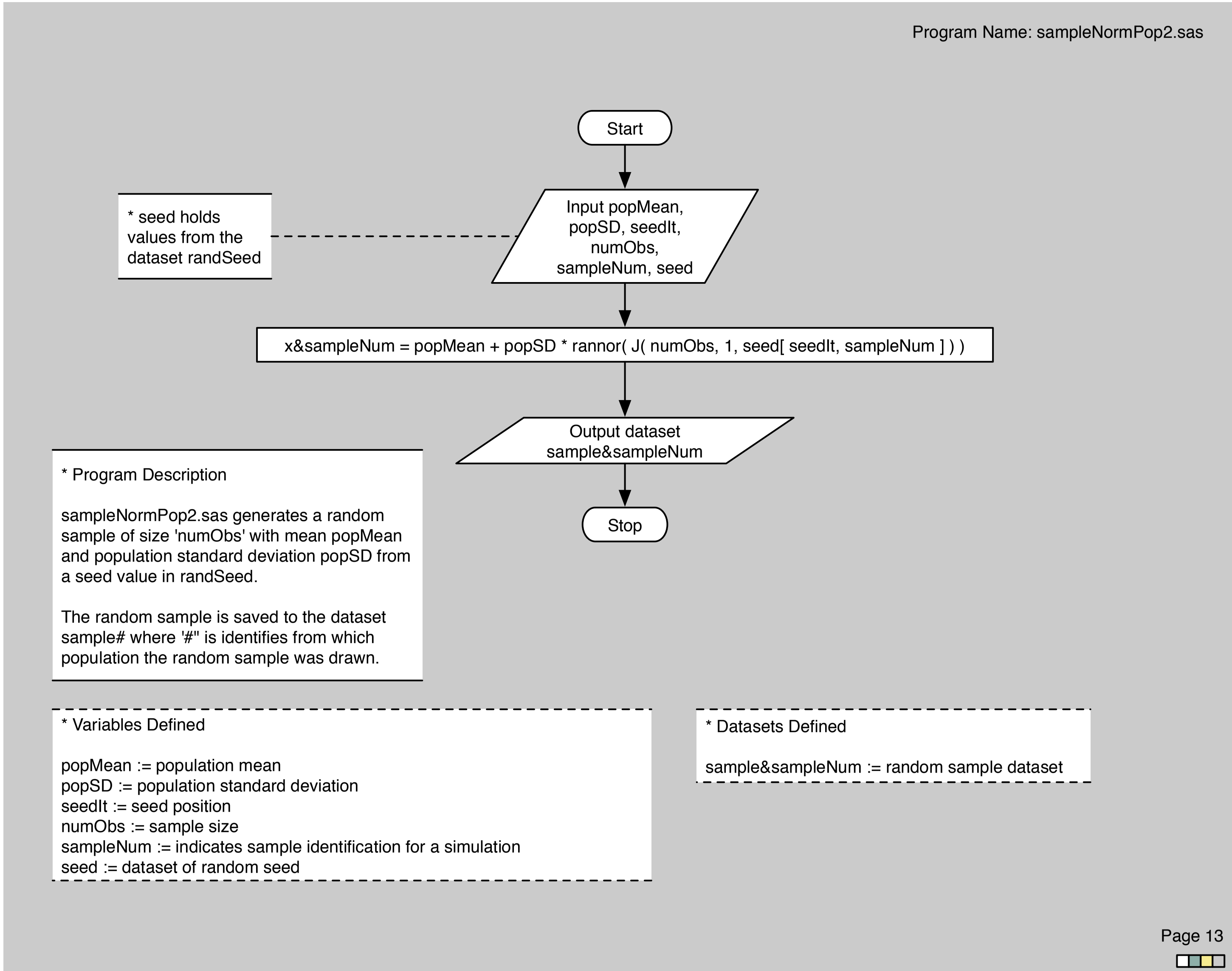
ndf1 := numerator degrees of freedom
ddf2 := denominator degrees of freedom
ncp := noncentrality parameter
slAlpha := level of significance
tol := tolerance for Golden Section Search method
alphaA := lower bound for optimized lower alpha
alphaB := upper bound for optimized lower alpha
c := Golden ratio constant reduction factor
x1 := percentile associated with alphaA
wx1 := height associated with x1
x2 := percentile associated with alphaB
wx2 := height associated with x2

Page 11

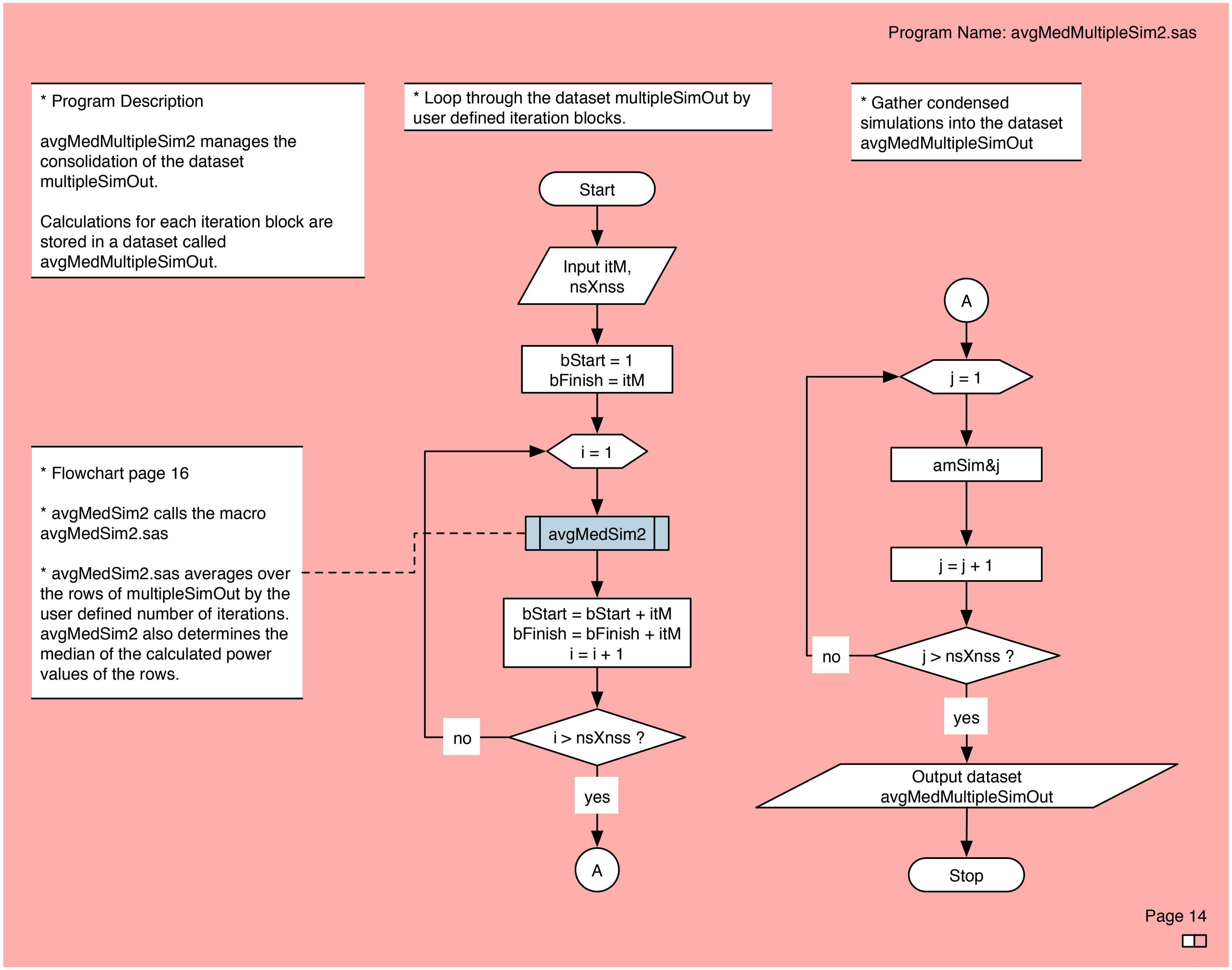
Appendix A: Two-Sample Flow Chart of Power Simulations



Appendix A: Two-Sample Flow Chart of Power Simulations



Appendix A: Two-Sample Flow Chart of Power Simulations



Appendix A: Two-Sample Flow Chart of Power Simulations

* Variables Defined

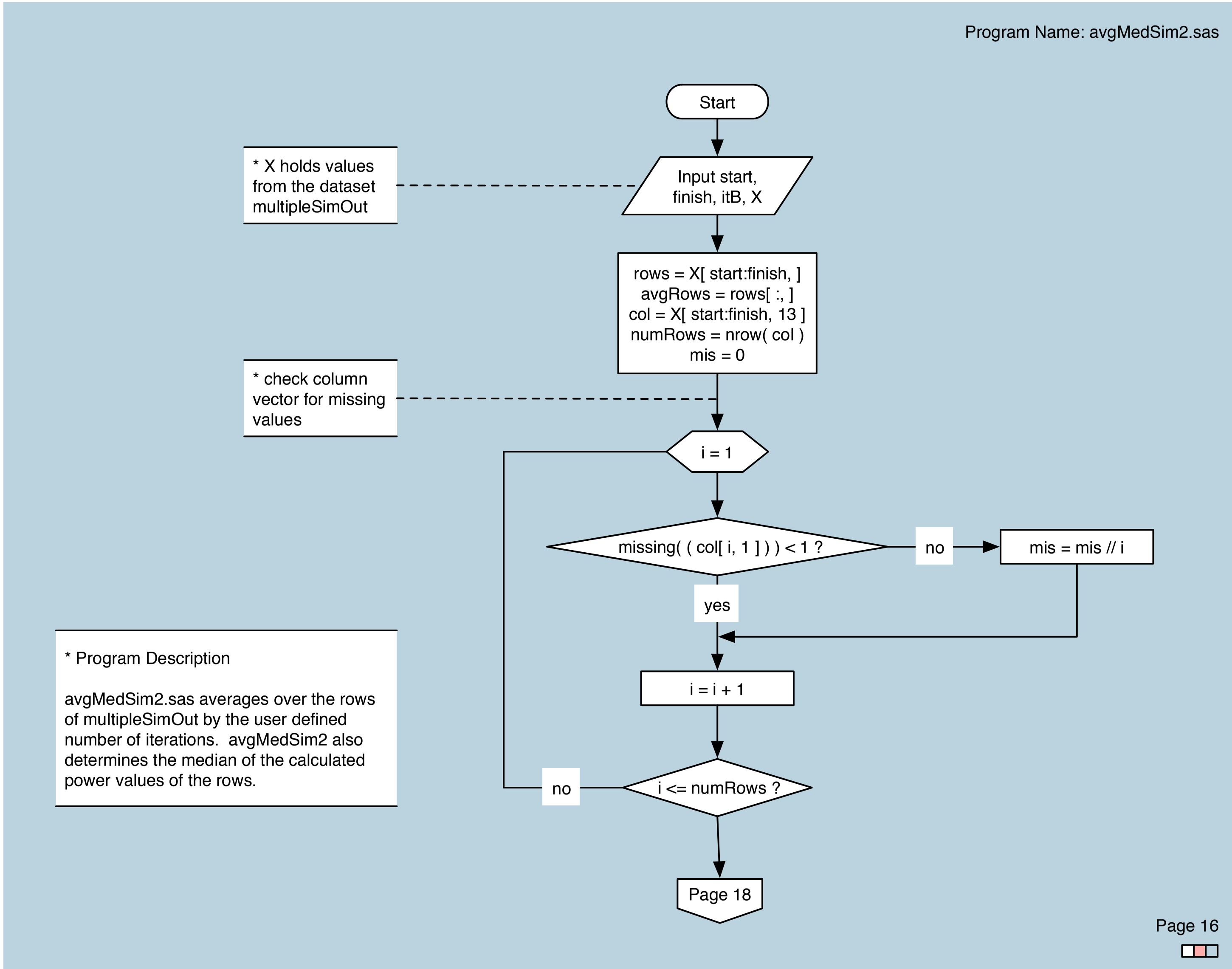
itM := maximum number of iterations
nsXnss := number of population standard deviations considered multiplied by the number of sample sizes considered
bStart := start of seed block
bFinish := end of seed block

* Datasets Defined

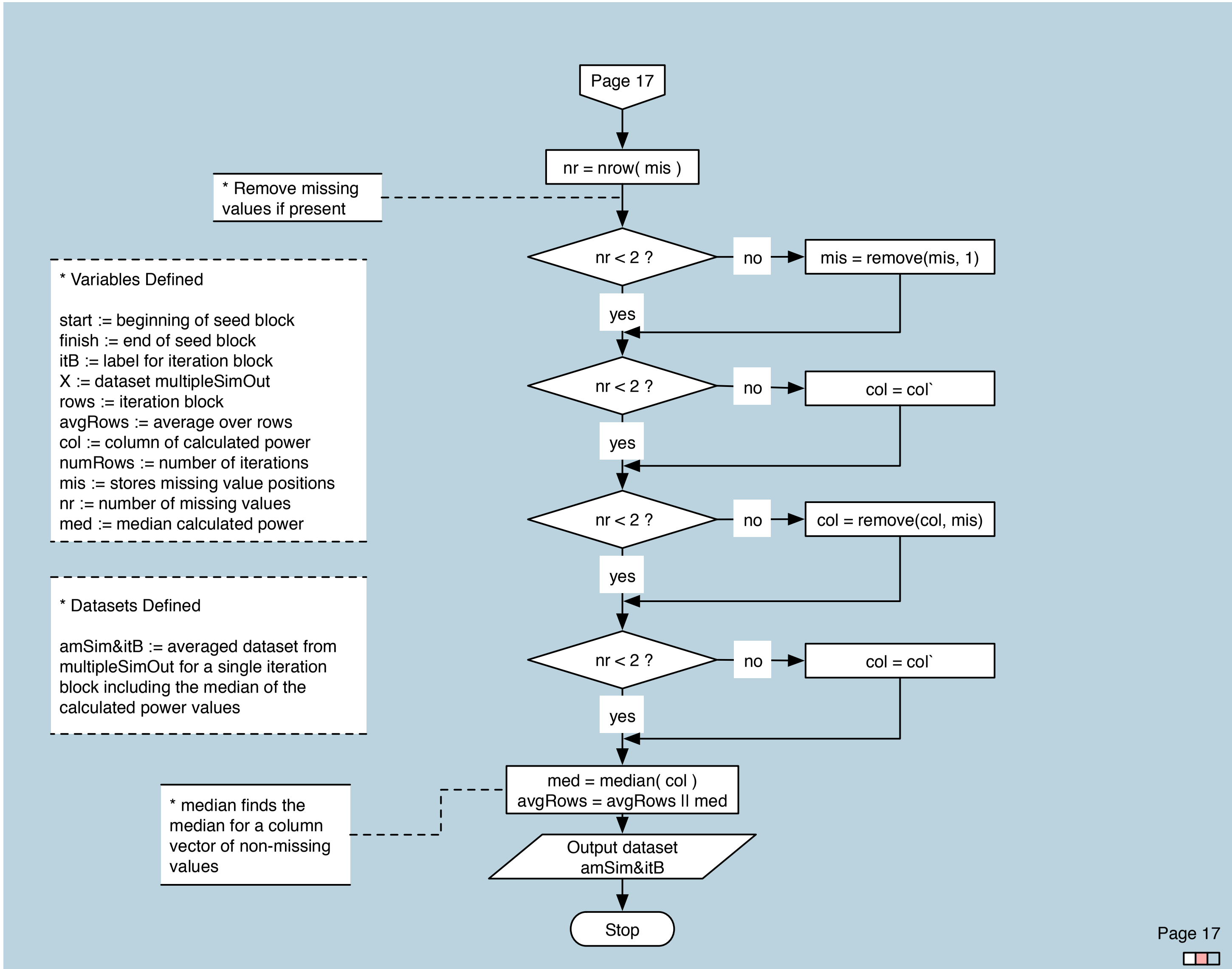
avgMedMultipleSimOut := averaged dataset of multipleSimOut by iteration blocks including the median for the calculated power values in each iteration block

Page 15

Appendix A: Two-Sample Flow Chart of Power Simulations



Appendix A: Two-Sample Flow Chart of Power Simulations



Appendix A: Three-Sample Flow Chart of Power Simulations

Program Name: threeSample.sas

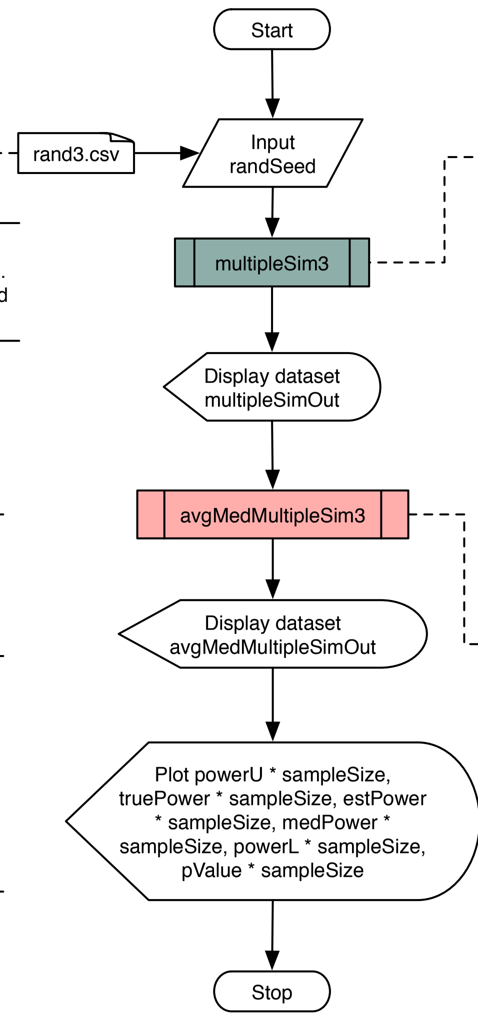
* rand3.csv is a three column file of random numbers generated from http:/www.random.org. For each iteration three random values are used to seed three random samples.

* Datasets Defined

randSeed := dataset of random seeds

* Program Description

threeSample.sas acts as a controller interface which accepts user defined values for three population means, upper and lower bounds for a range of population standard deviations, a maximum sample size, a Type I error rate, and the number of iterations per sample size to population standard deviation combination.



* Flowchart page 2

* multipleSim3 calls the macro multipleSim3.sas.

* This macro manages the production of simulations. Calls are made to the macro simulation3.sas which performs calculations based on *Thomas 1997*.

* Calculations for each simulation are stored in a dataset called multipleSimOut.

* Flowchart page 14

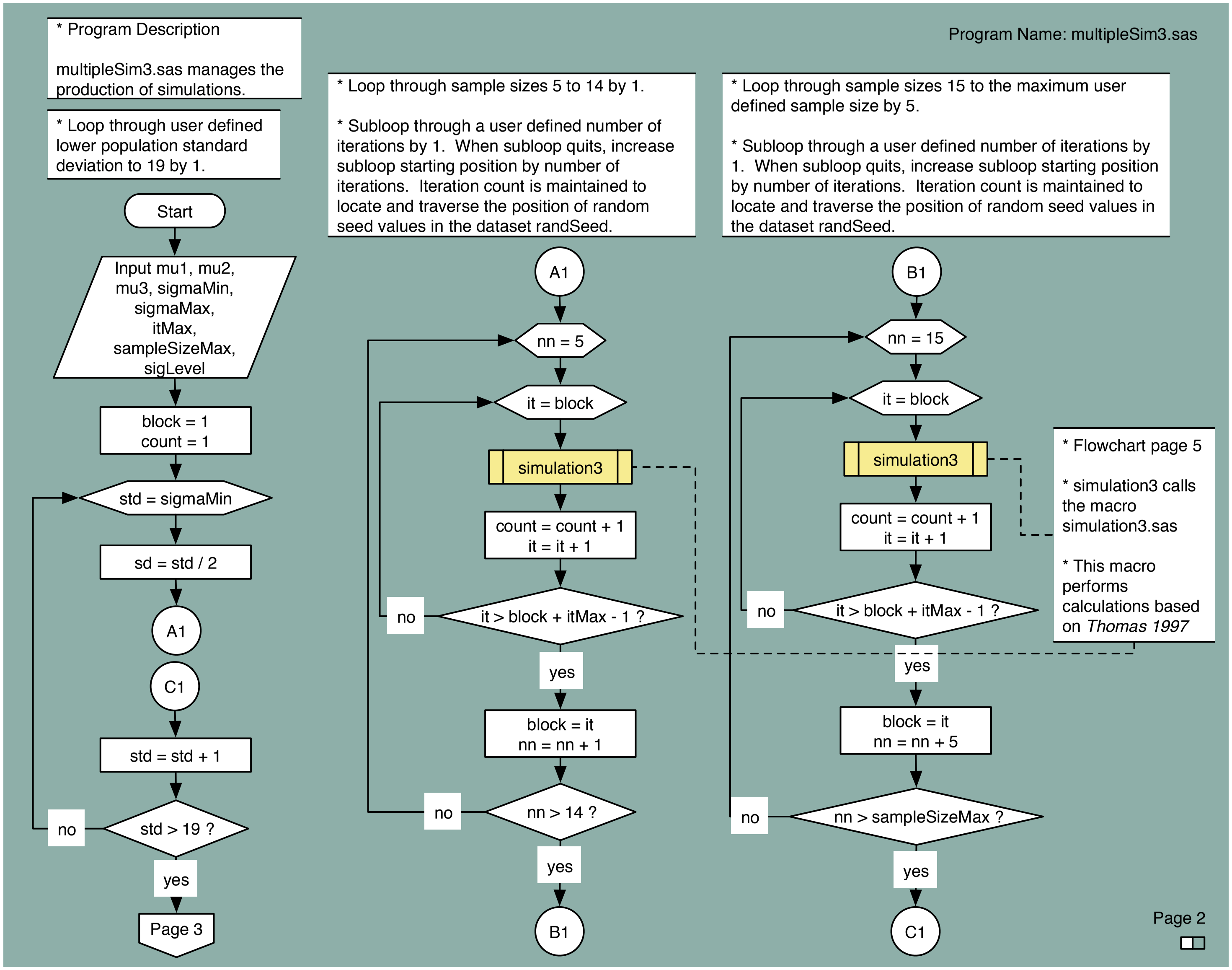
* avgMedMultipleSim3 calls the macro avgMedMultipleSim3.sas.

* This macro manages the consolidation of simulations. Calls to the macro avgMedSim3.sas average over the rows of multipleSimOut by the user defined number of iterations. avgMedSim3 also determines the median of the calculated power values of the rows.

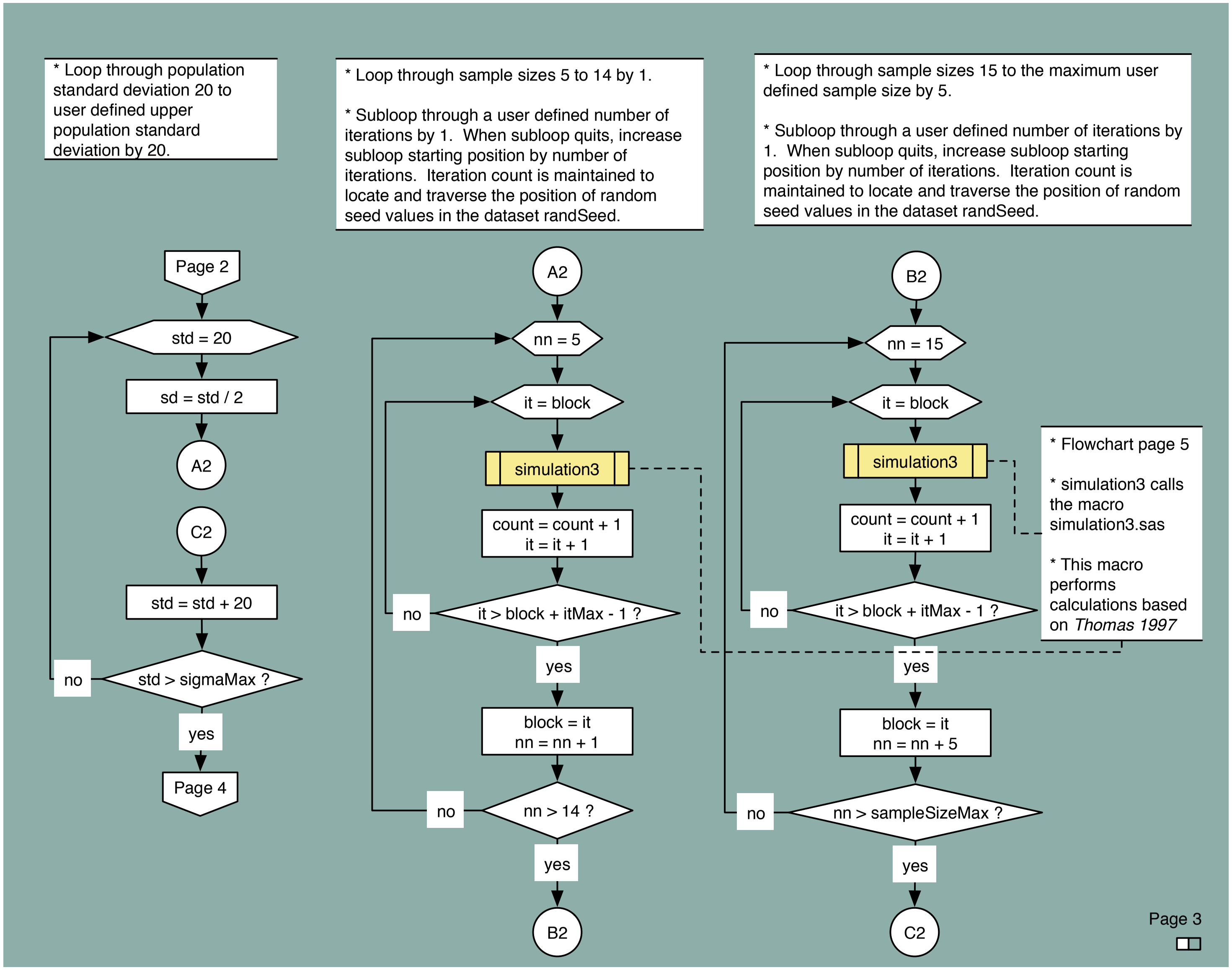
* Calculations for each iteration block are stored in a dataset called avgMedMultipleSimOut.

Page 1

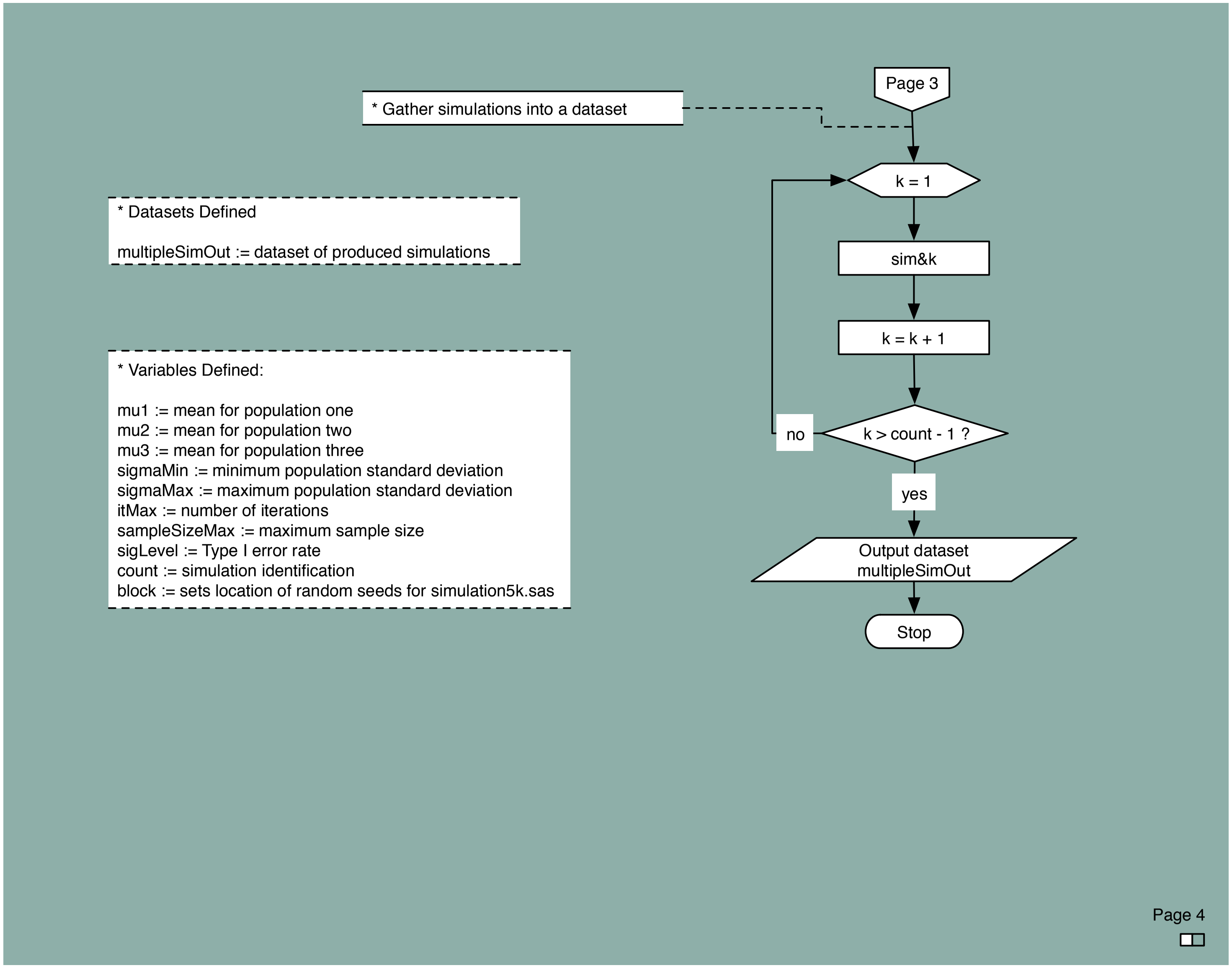
Appendix A: Three-Sample Flow Chart of Power Simulations



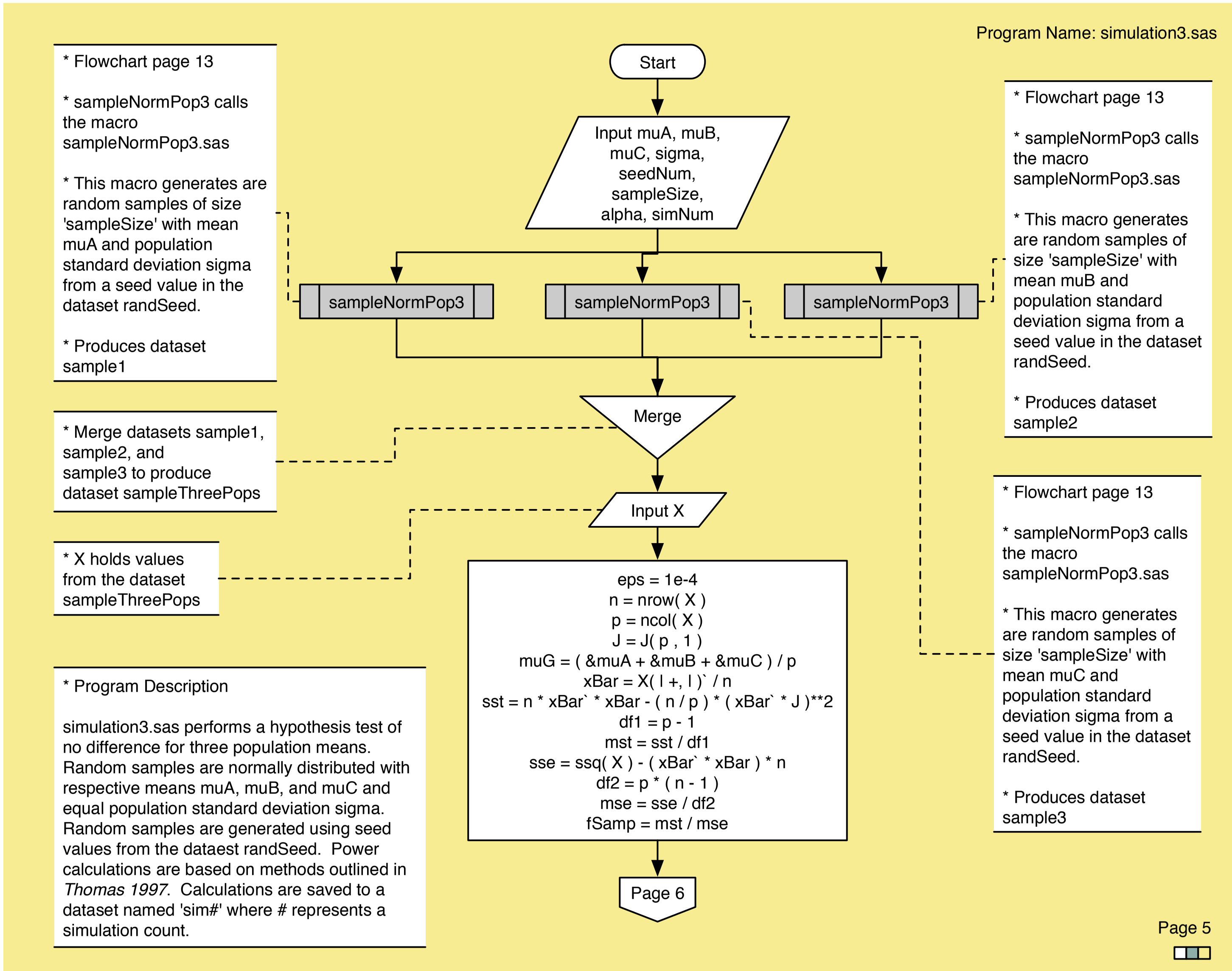
Appendix A: Three-Sample Flow Chart of Power Simulations



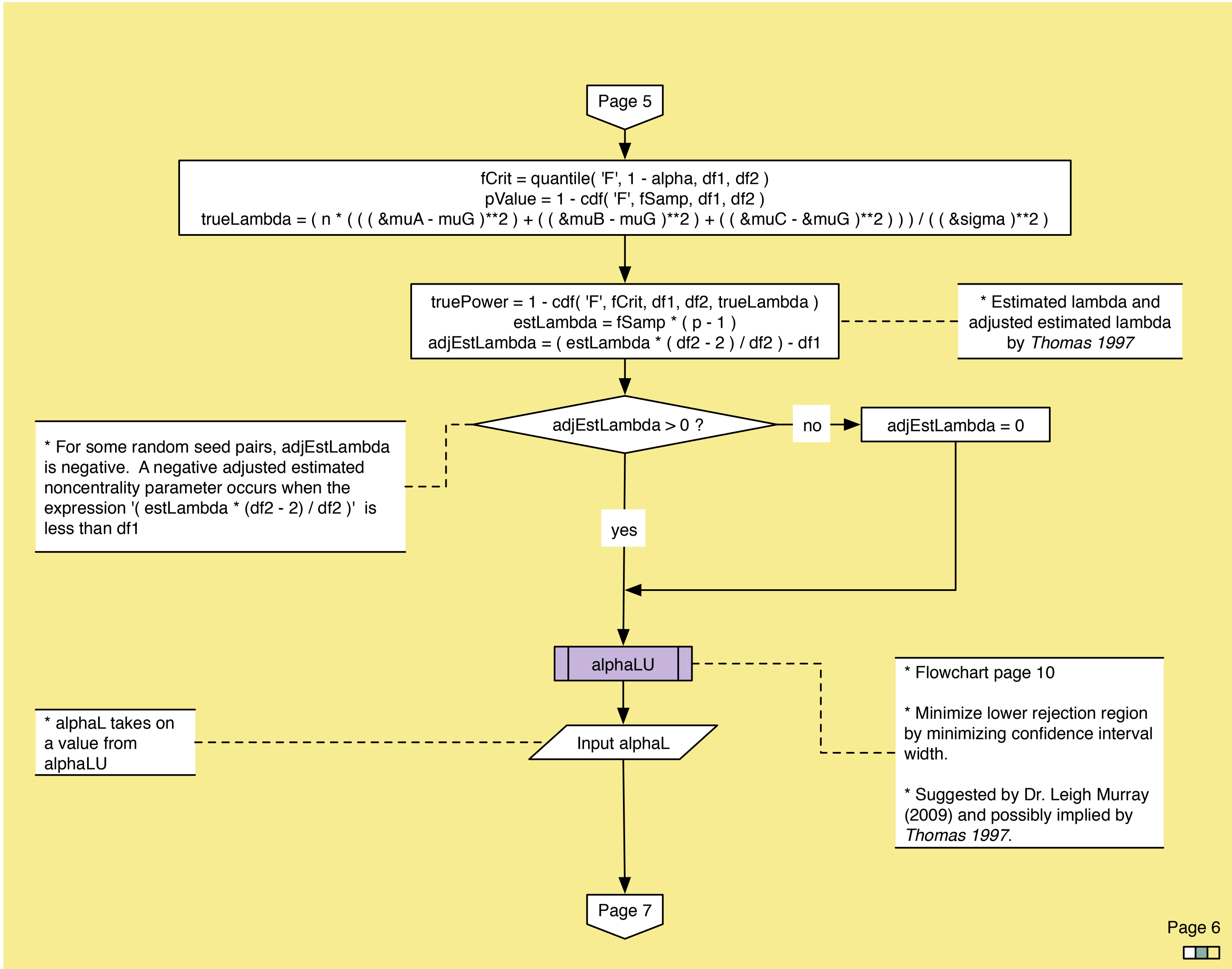
Appendix A: Three-Sample Flow Chart of Power Simulations



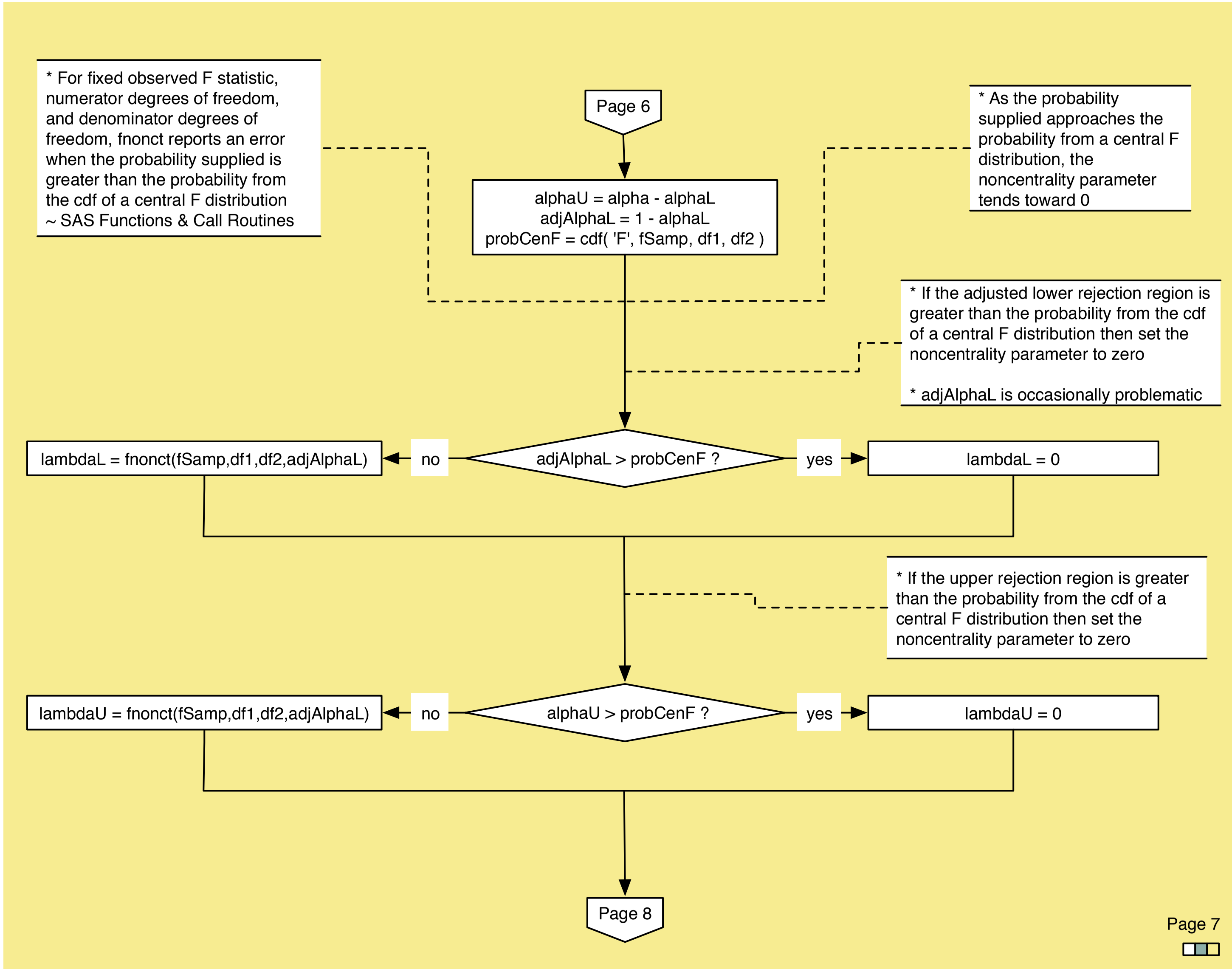
Appendix A: Three-Sample Flow Chart of Power Simulations



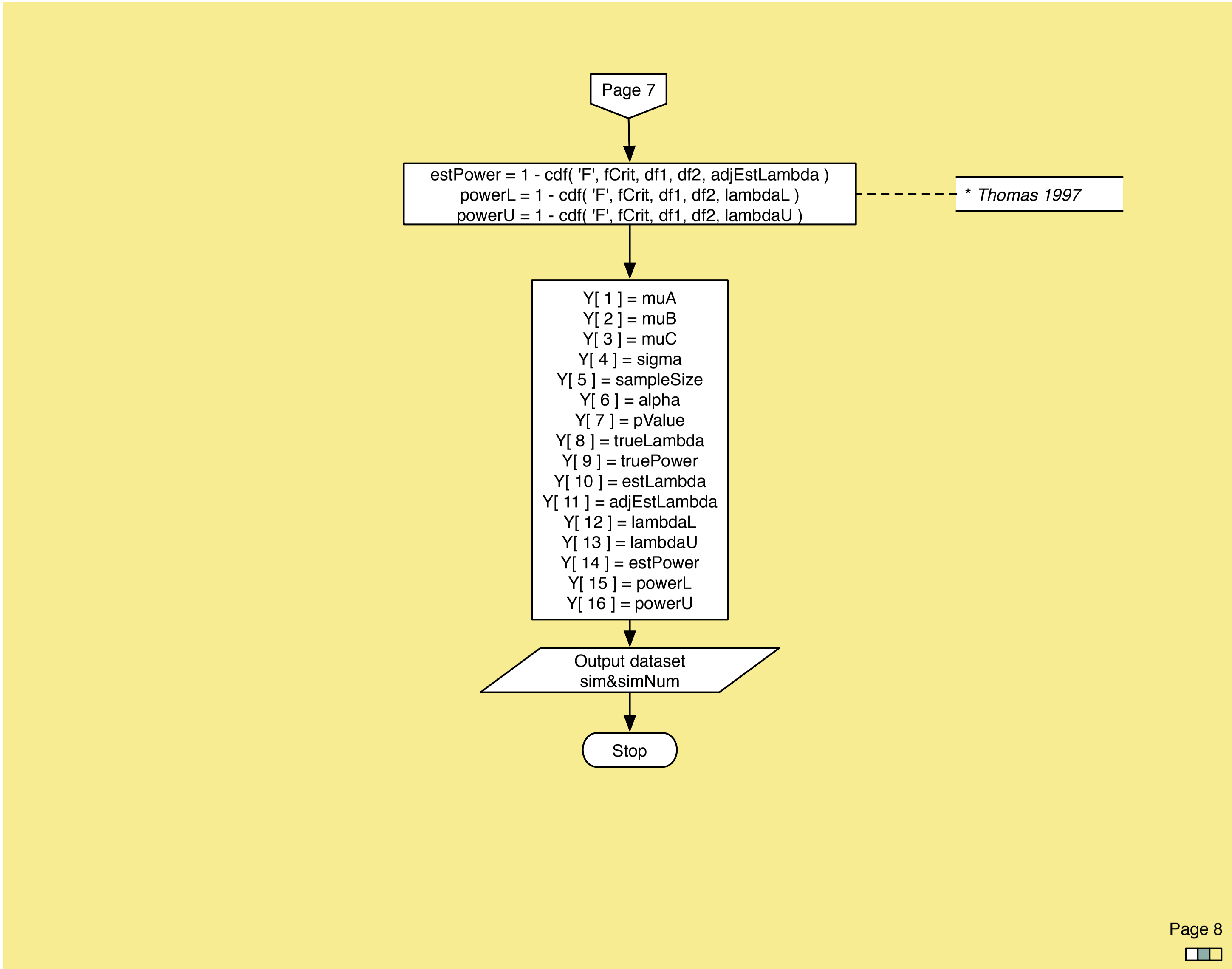
Appendix A: Three-Sample Flow Chart of Power Simulations



Appendix A: Three-Sample Flow Chart of Power Simulations



Appendix A: Three-Sample Flow Chart of Power Simulations



Appendix A: Three-Sample Flow Chart of Power Simulations

* Variables Defined

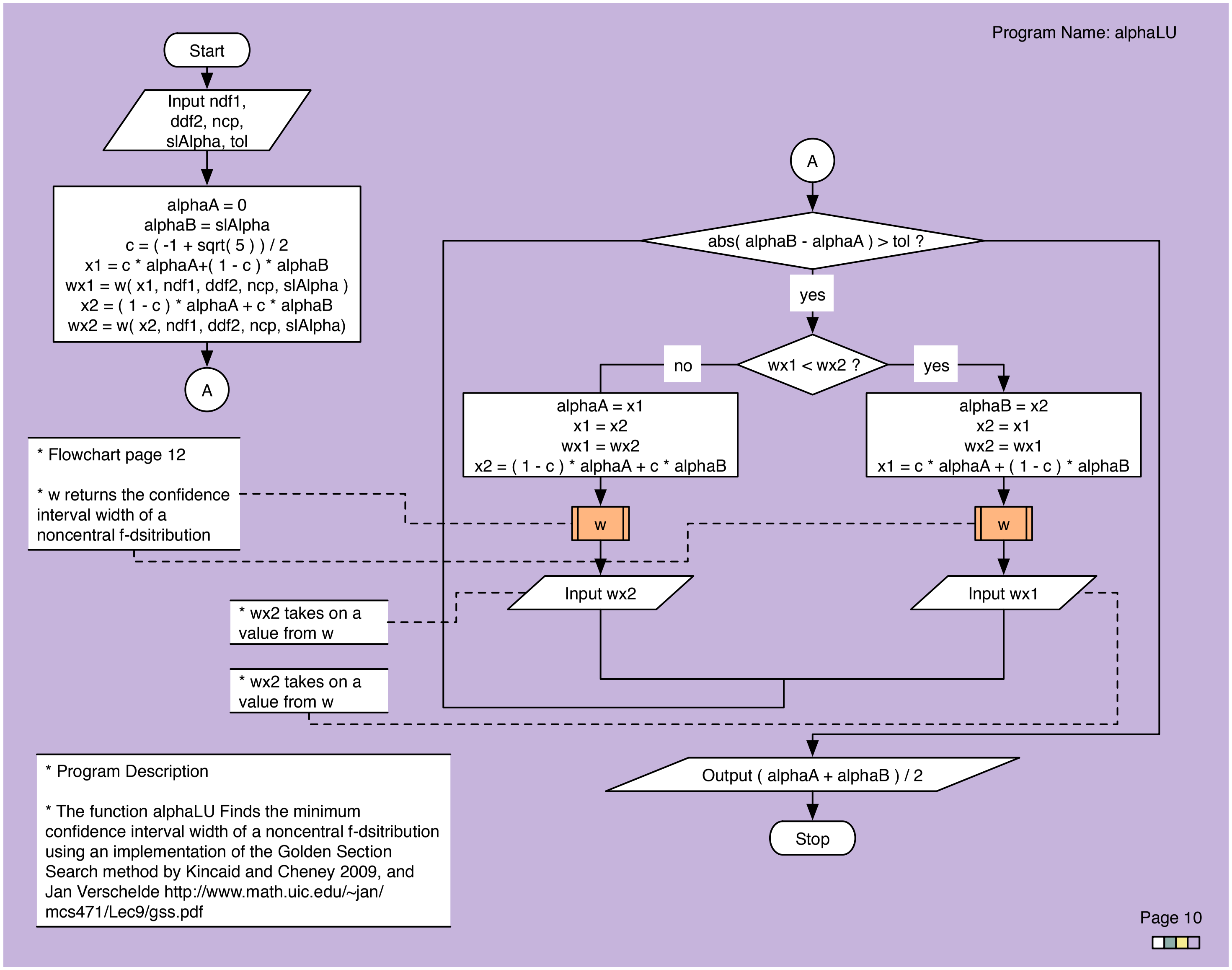
muA := mean for population one
muB := mean for population two
muC := mean for population three
muG := average of the tree population means
sigma := population standard deviation
seedNum := location of random seeds
sampleSize := sample size
alpha := level of significance
simNum := simulation identification
X := samples from normal populations
eps := tolerance for Golden Section Search methods
n := number rows
p := number columns
xBar := sample mean
sst := sum of squares for treatment
df1 := numerator degrees of freedom
mst := mean square for treatment
sse := sum of squares for error
df2 := denominator degrees of freedom
mse := mean square for error
fSamp := observed F statistic
fCrit := critical value from a central F distribution
pValue := p-value for the test
trueLambda := noncentrality parameter
truePower := power for the test
estLambda := calculated noncentrality parameter
adjEstLambda := adjusted calculated noncentrality parameter
alphaL := optimized lower critical region
alphaU := optimized upper critical region
probCenF := probability of observed F statistic of the central F distribution
estPower := calculated power
powerL := lower confidence interval for calculated power
powerU := upper confidence interval for calculated power

* Datasets Defined

sampleThreePops := random samples dataset
sim&simNum := simulation dataset

Page 9

Appendix A: Three-Sample Flow Chart of Power Simulations



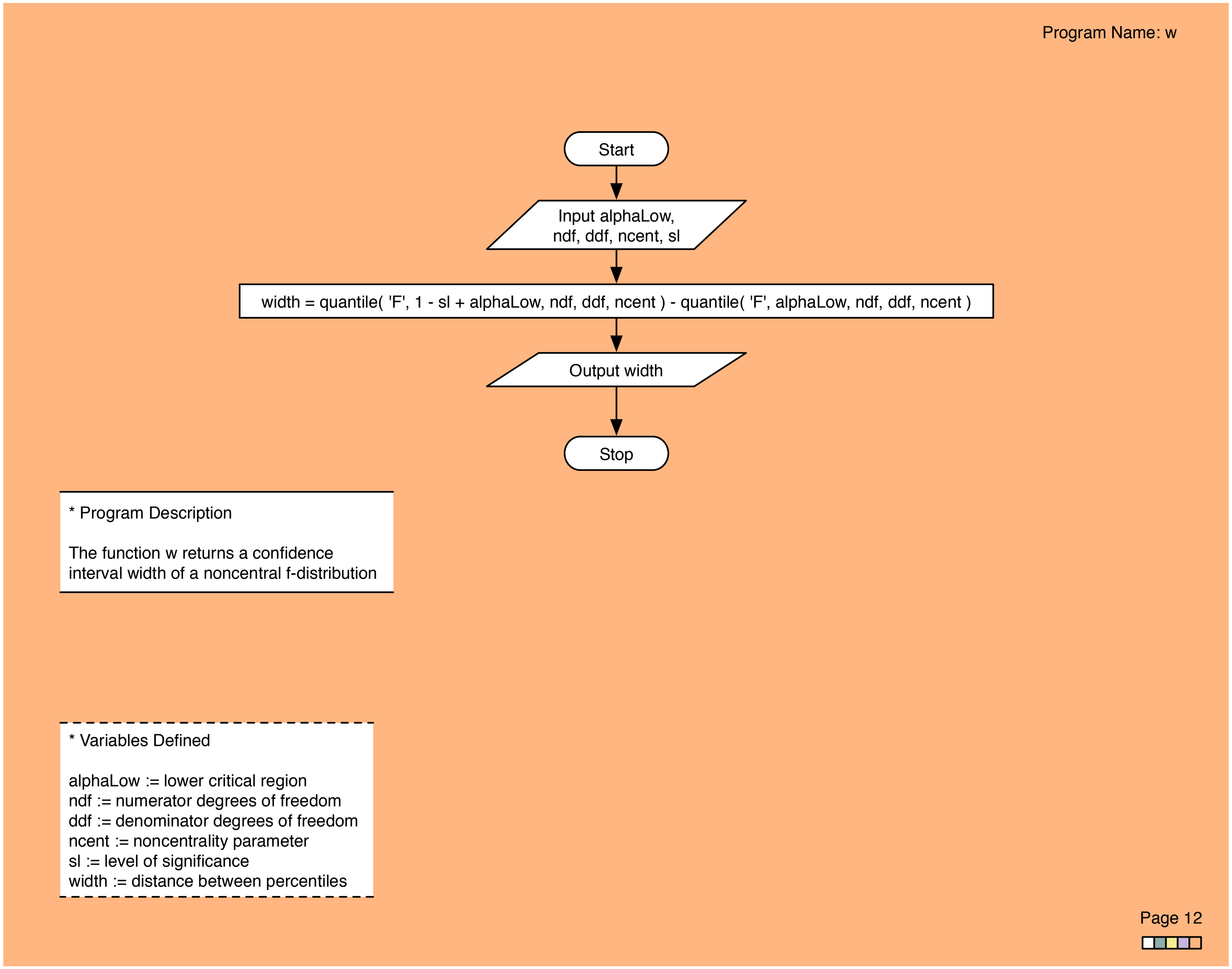
Appendix A: Three-Sample Flow Chart of Power Simulations

* Variables Defined

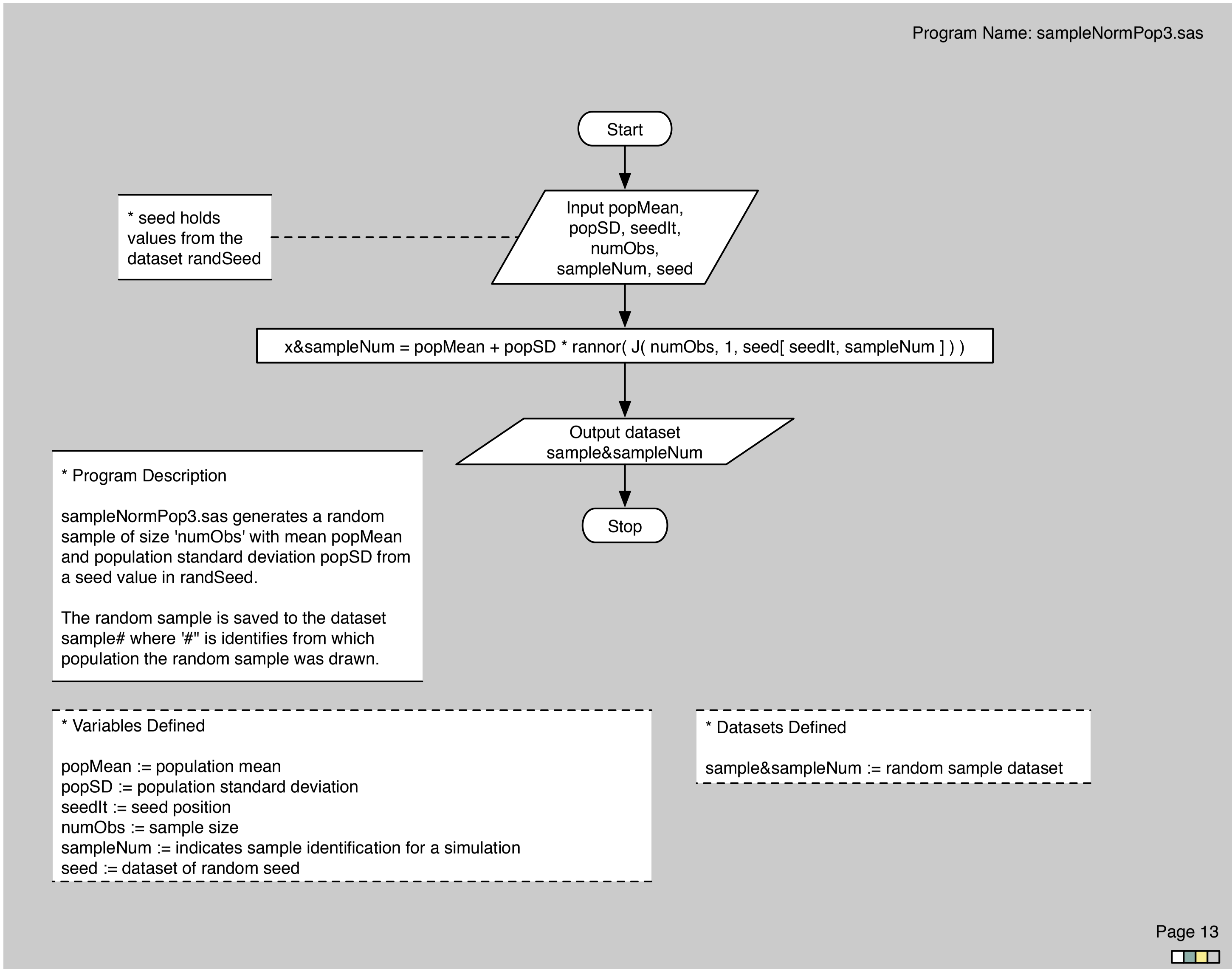
ndf1 := numerator degrees of freedom
ddf2 := denominator degrees of freedom
ncp := noncentrality parameter
slAlpha := level of significance
tol := tolerance for Golden Section Search method
alphaA := lower bound for optimized lower alpha
alphaB := upper bound for optimized lower alpha
c := Golden ratio constant reduction factor
x1 := percentile associated with alphaA
wx1 := height associated with x1
x2 := percentile associated with alphaB
wx2 := height associated with x2

Page 11

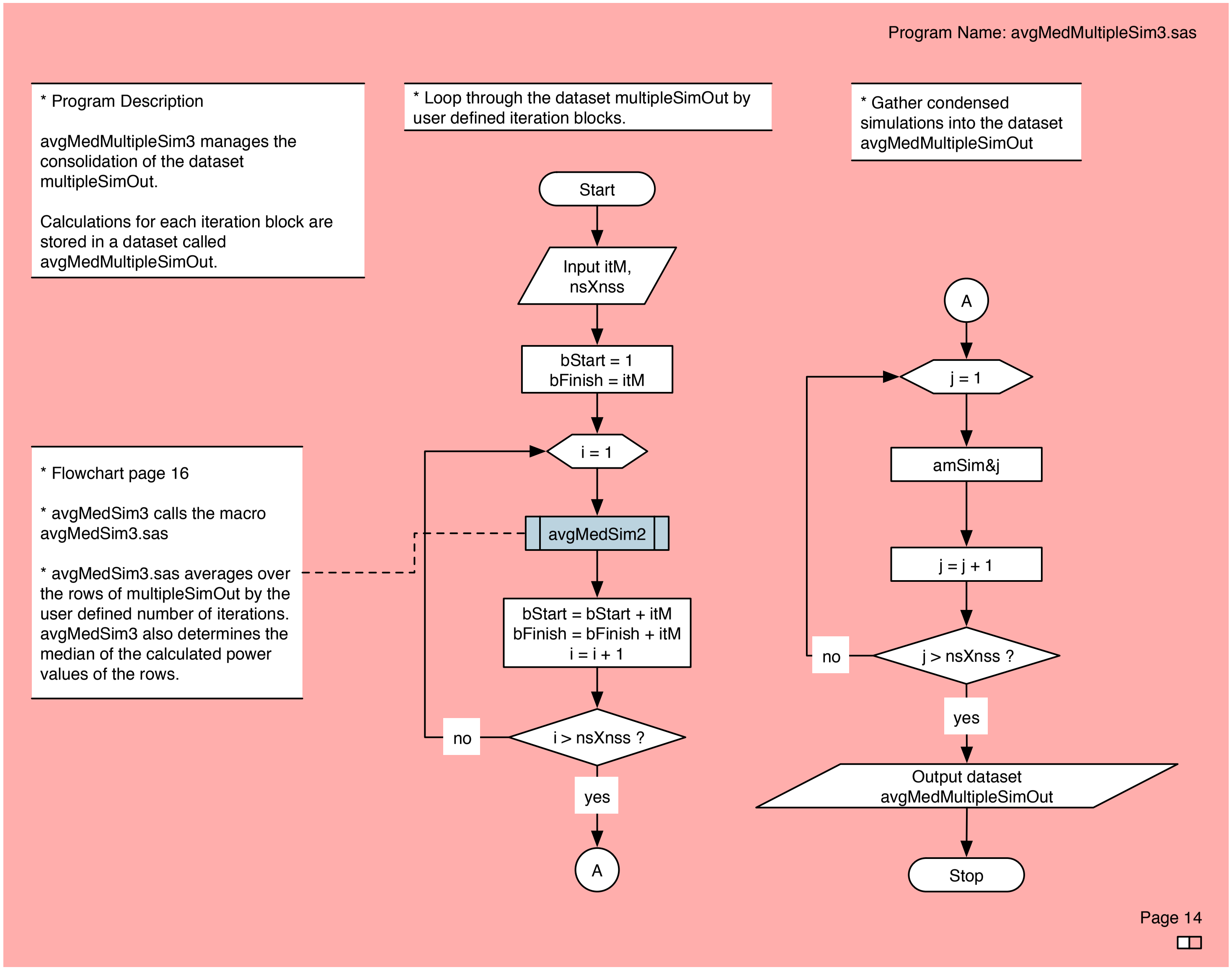
Appendix A: Three-Sample Flow Chart of Power Simulations



Appendix A: Three-Sample Flow Chart of Power Simulations



Appendix A: Three-Sample Flow Chart of Power Simulations



Appendix A: Three-Sample Flow Chart of Power Simulations

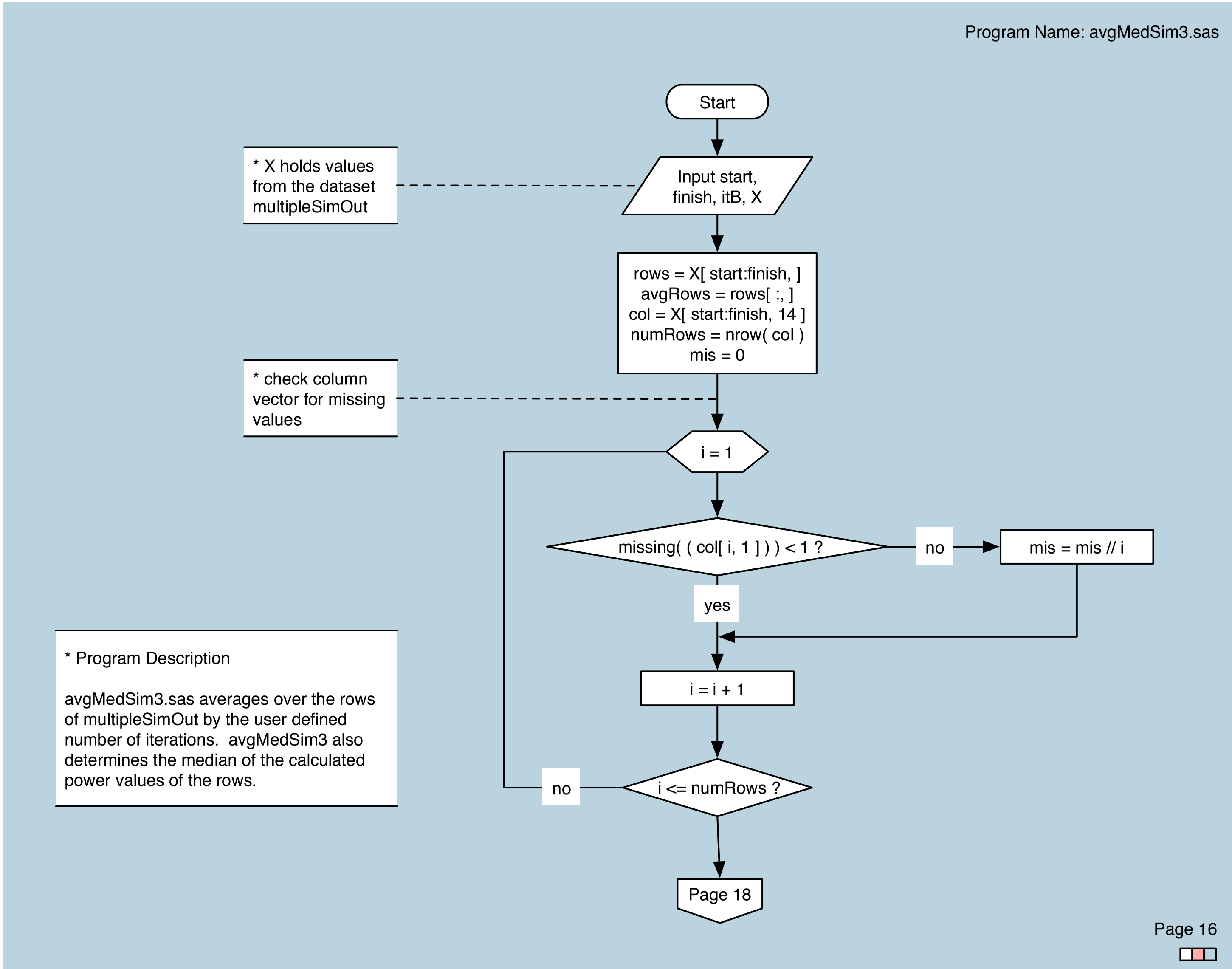
* Variables Defined

itM := maximum number of iterations
nsXnss := number of population standard deviations considered multiplied by the number of sample sizes considered
bStart := start of seed block
bFinish := end of seed block

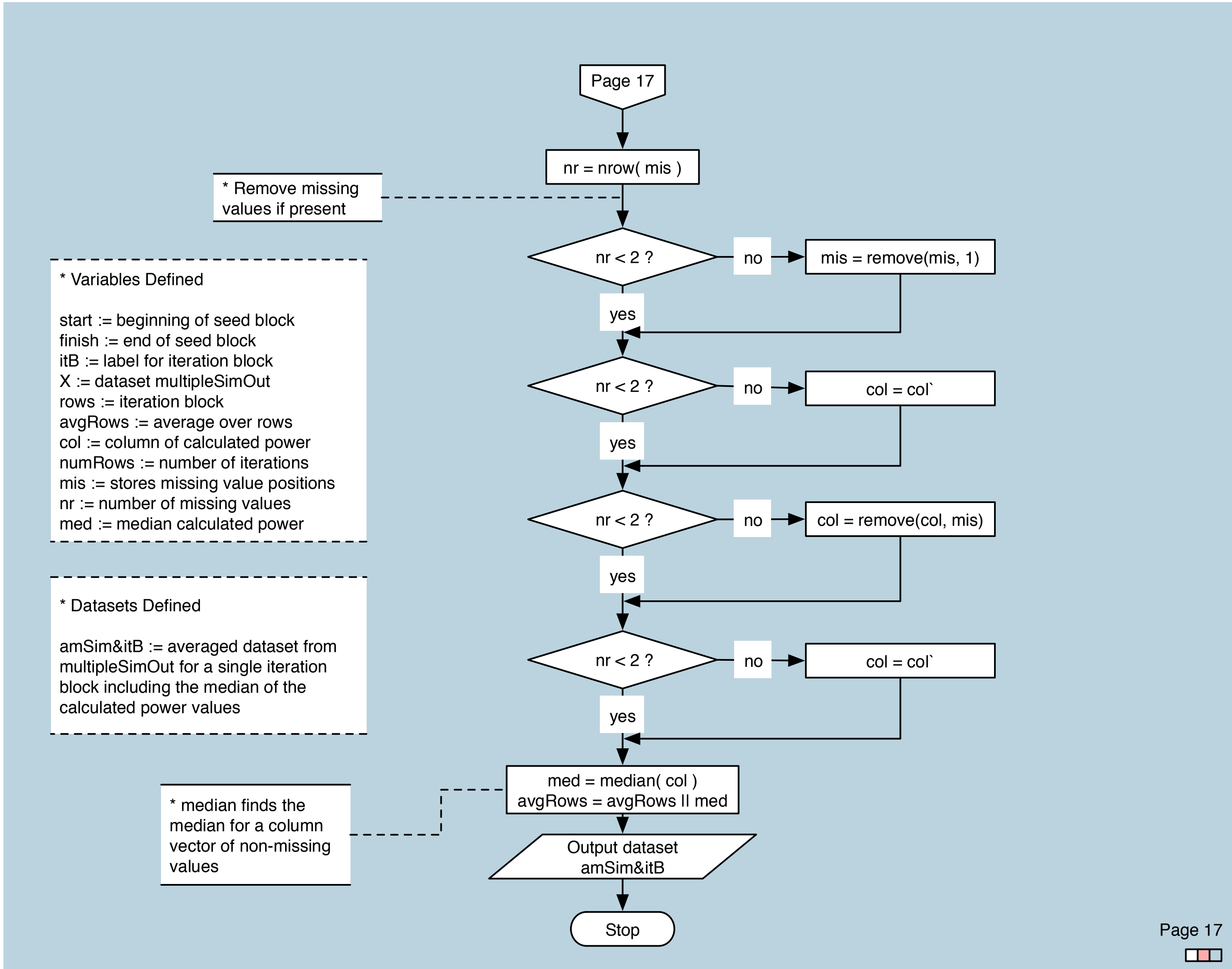
* Datasets Defined

avgMedMultipleSimOut := averaged dataset of multipleSimOut by iteration blocks including the median for the calculated power values in each iteration block

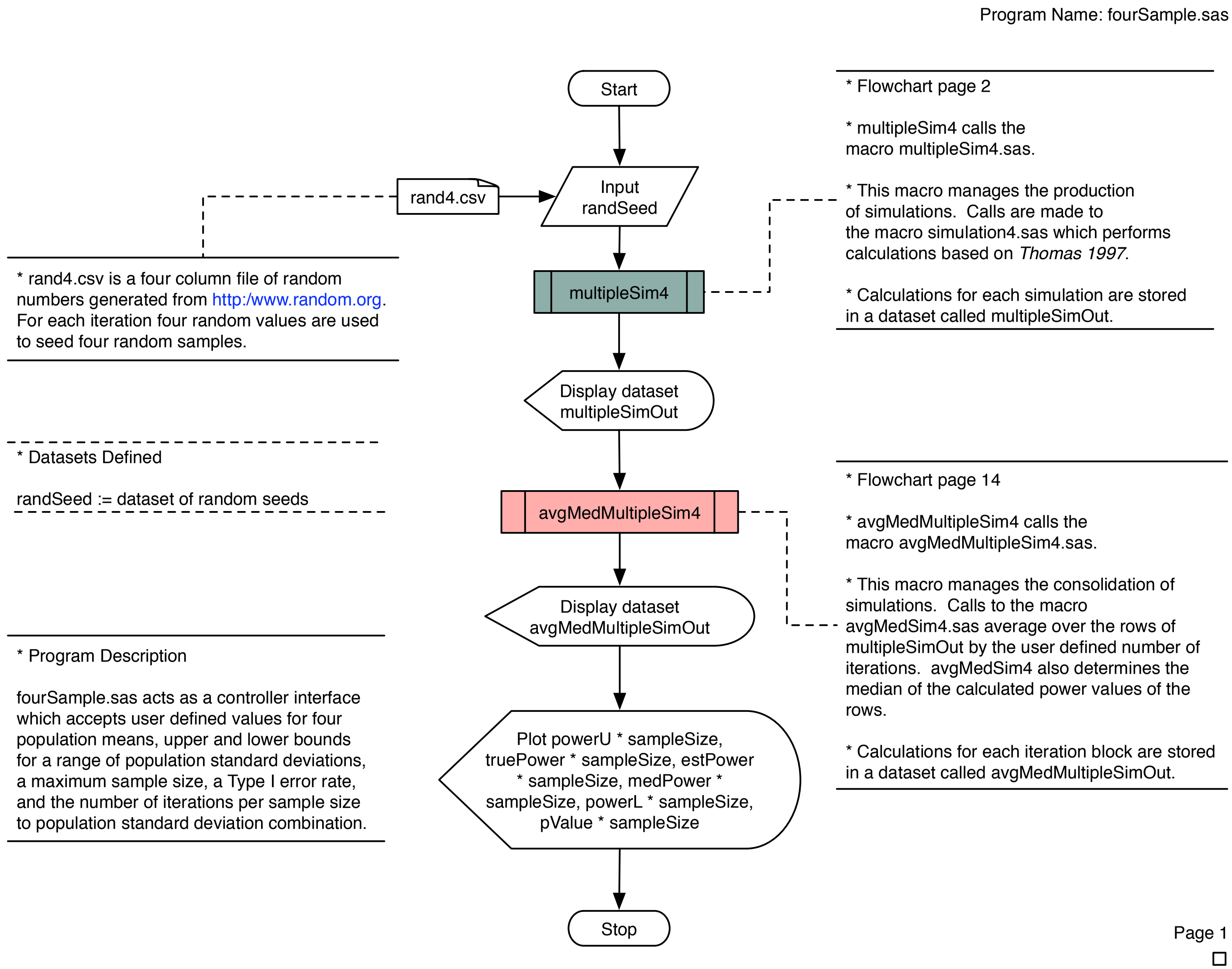
Appendix A: Three-Sample Flow Chart of Power Simulations



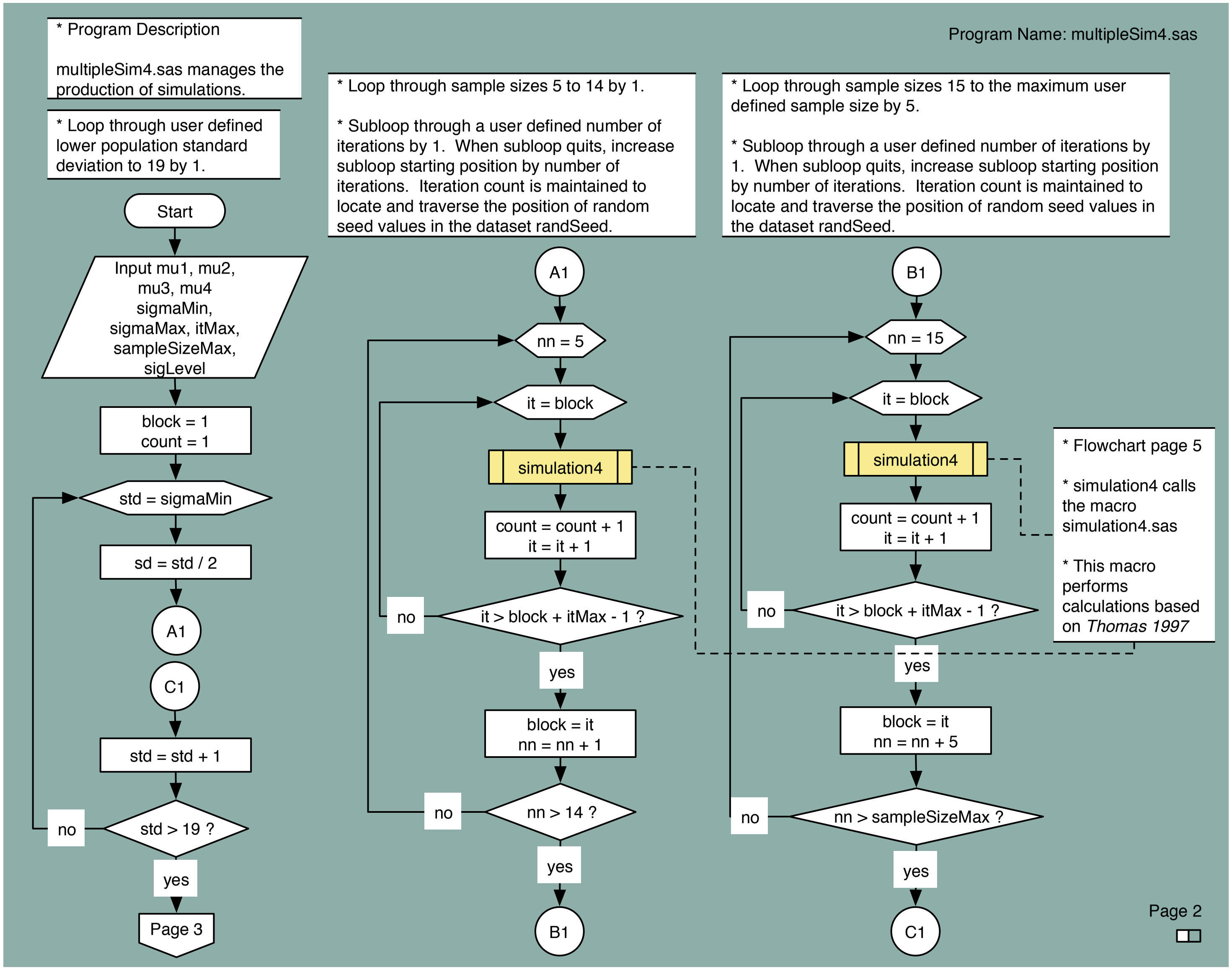
Appendix A: Three-Sample Flow Chart of Power Simulations



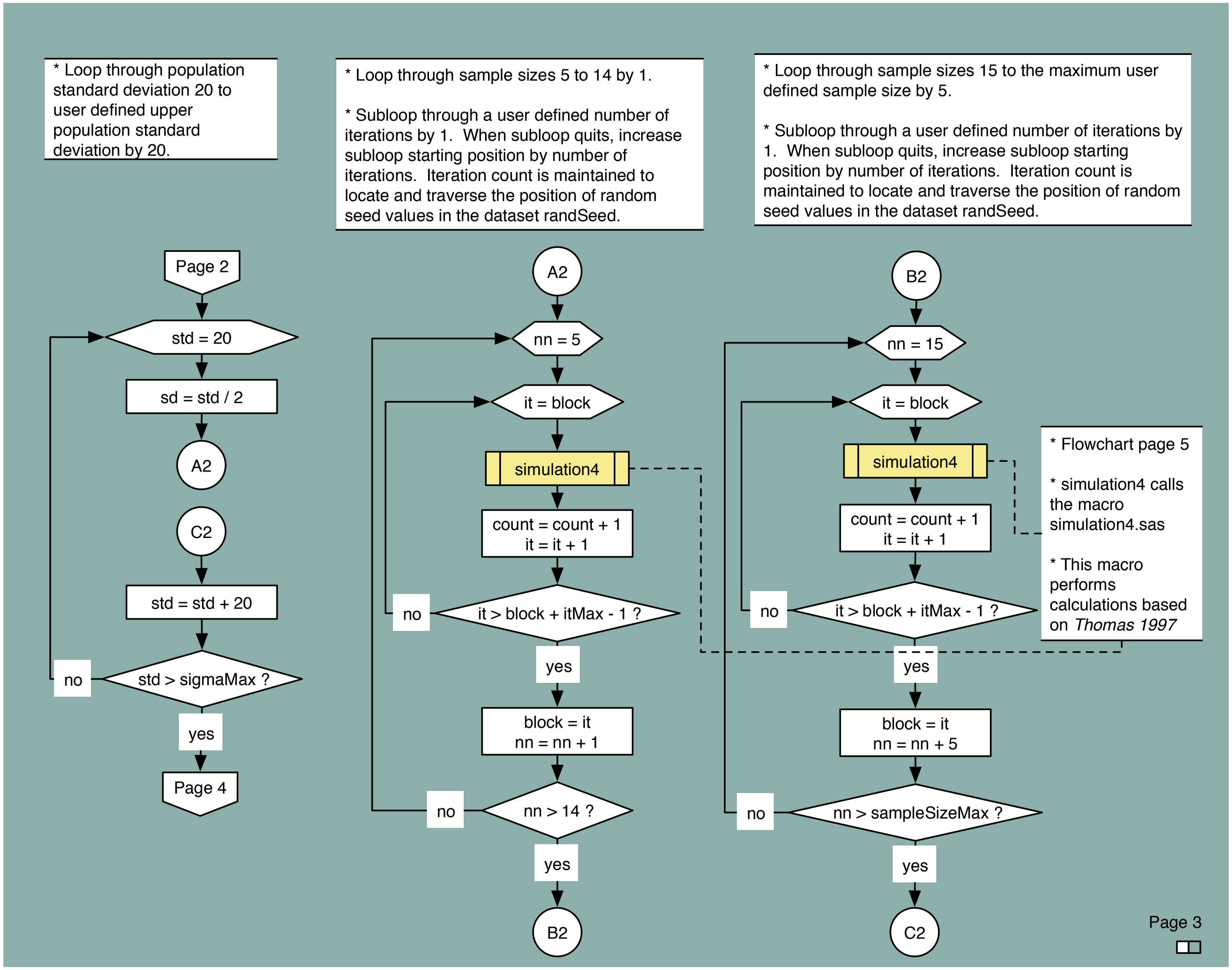
Appendix A: Four-Sample Flow Chart of Power Simulations



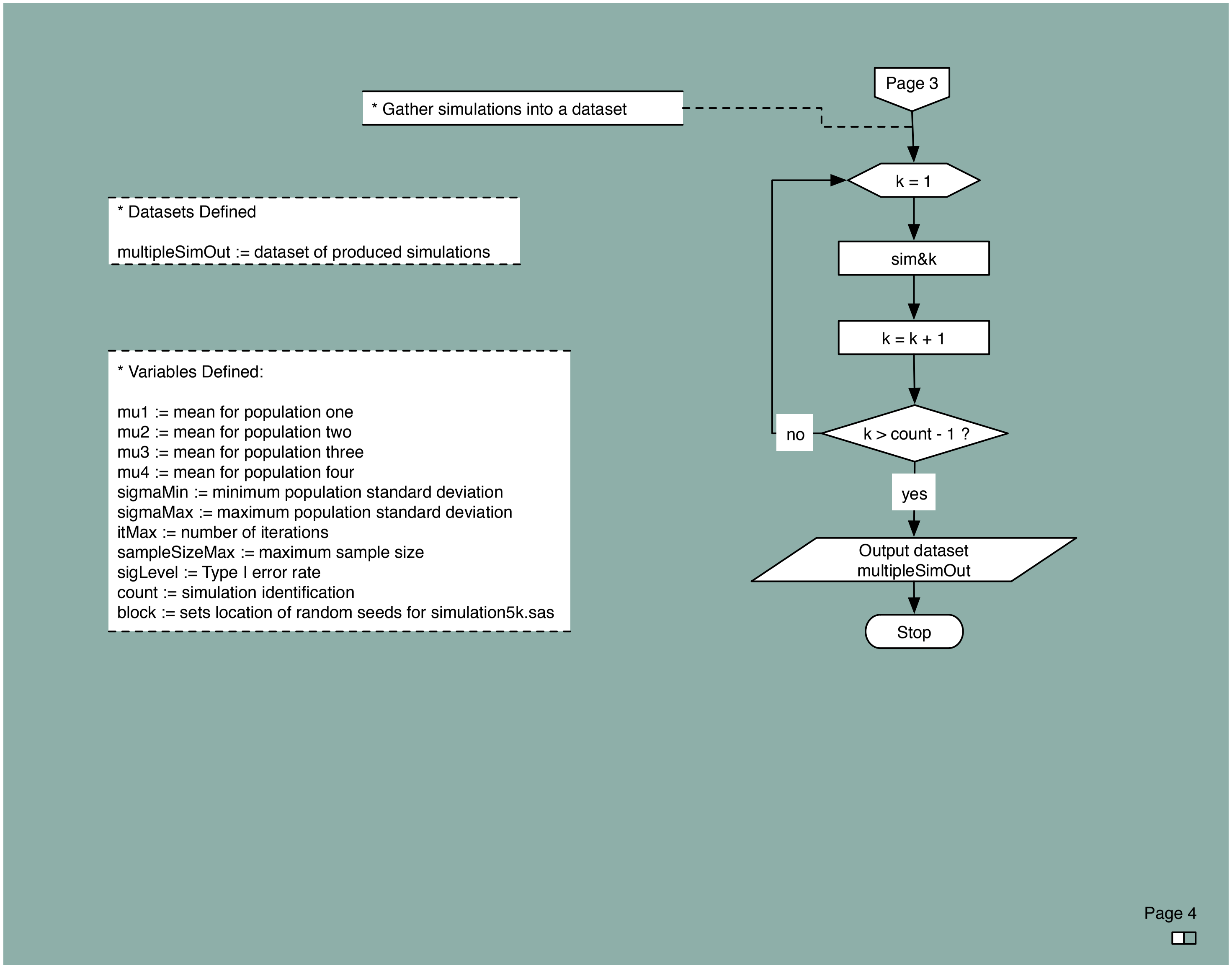
Appendix A: Four-Sample Flow Chart of Power Simulations



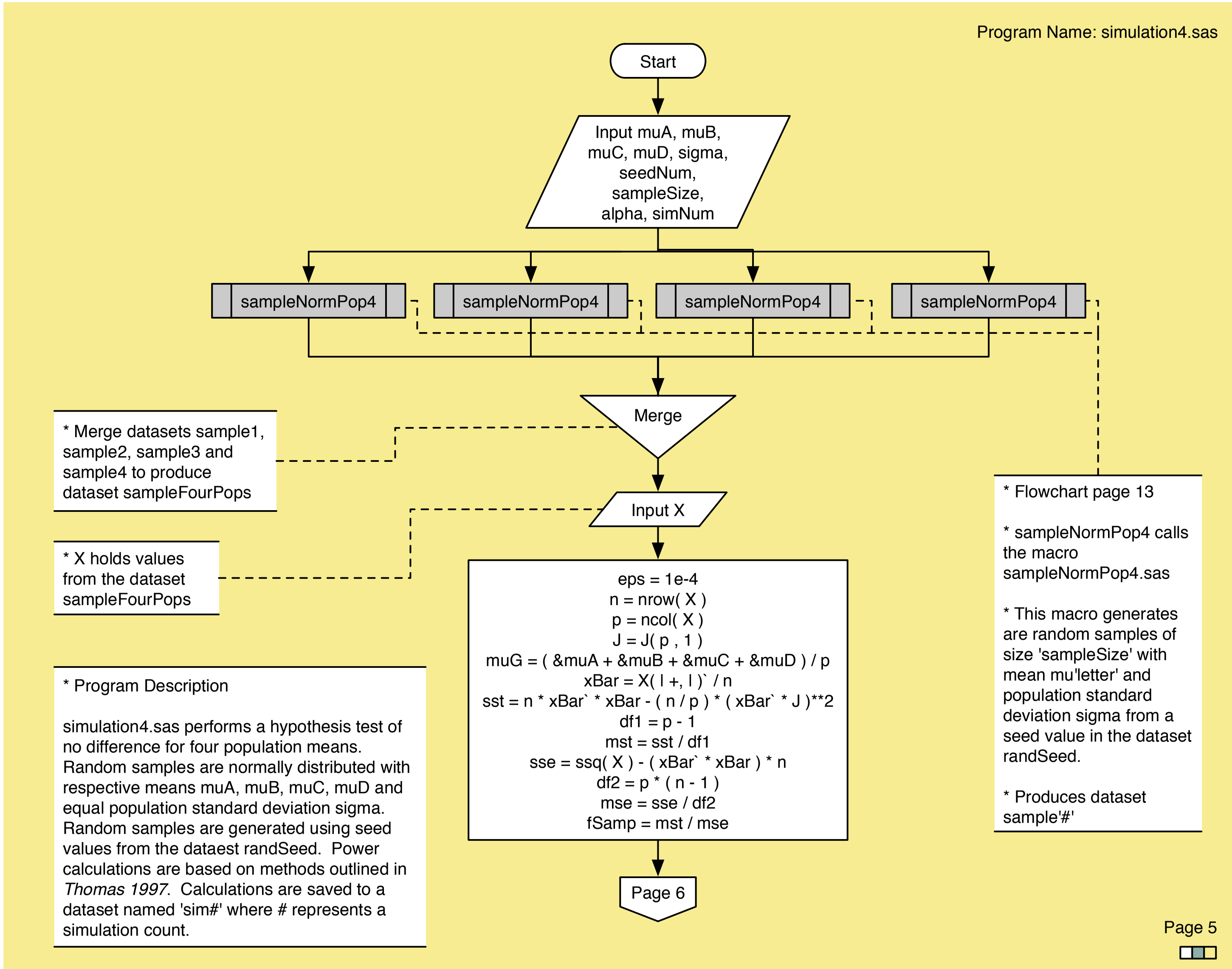
Appendix A: Four-Sample Flow Chart of Power Simulations



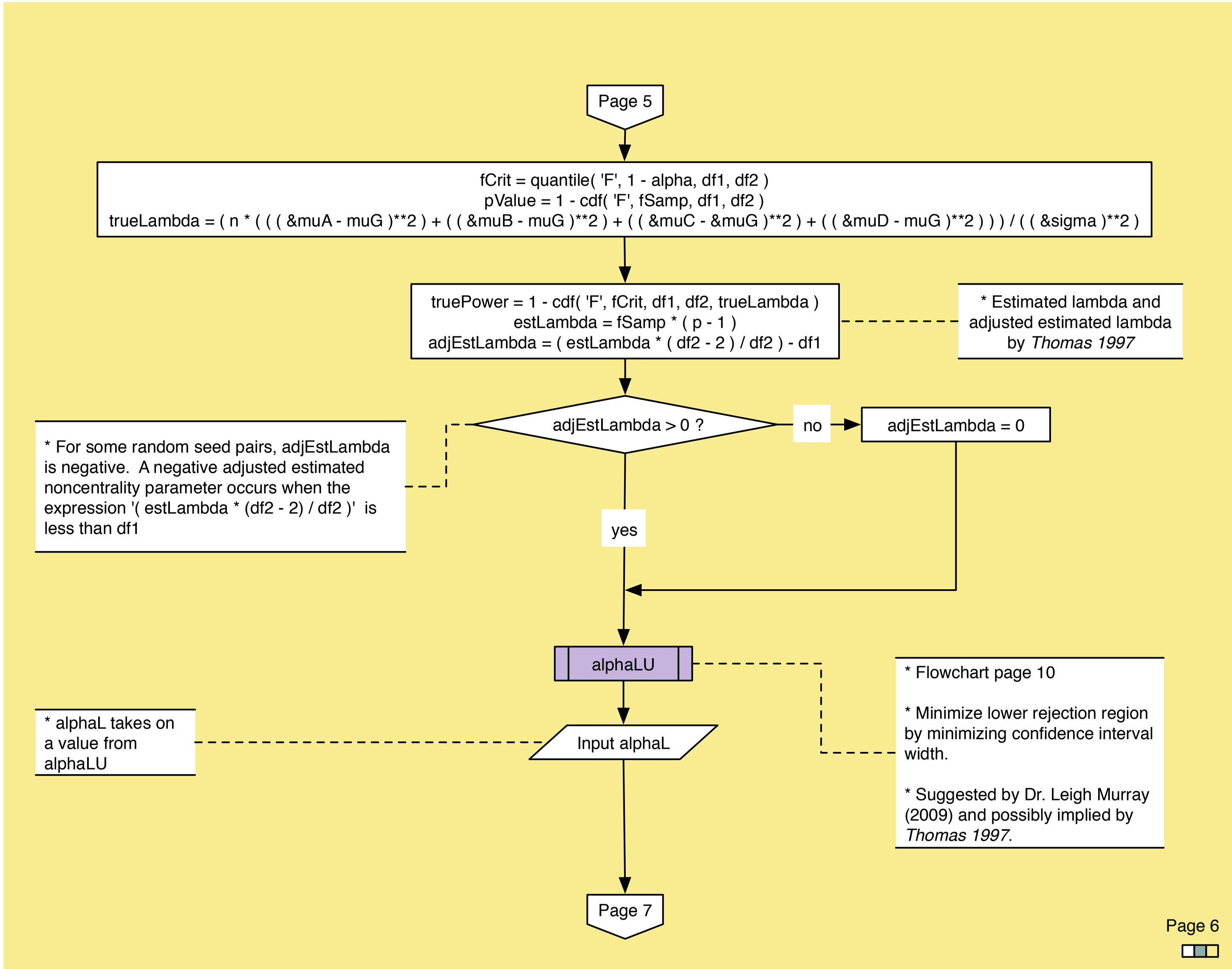
Appendix A: Four-Sample Flow Chart of Power Simulations



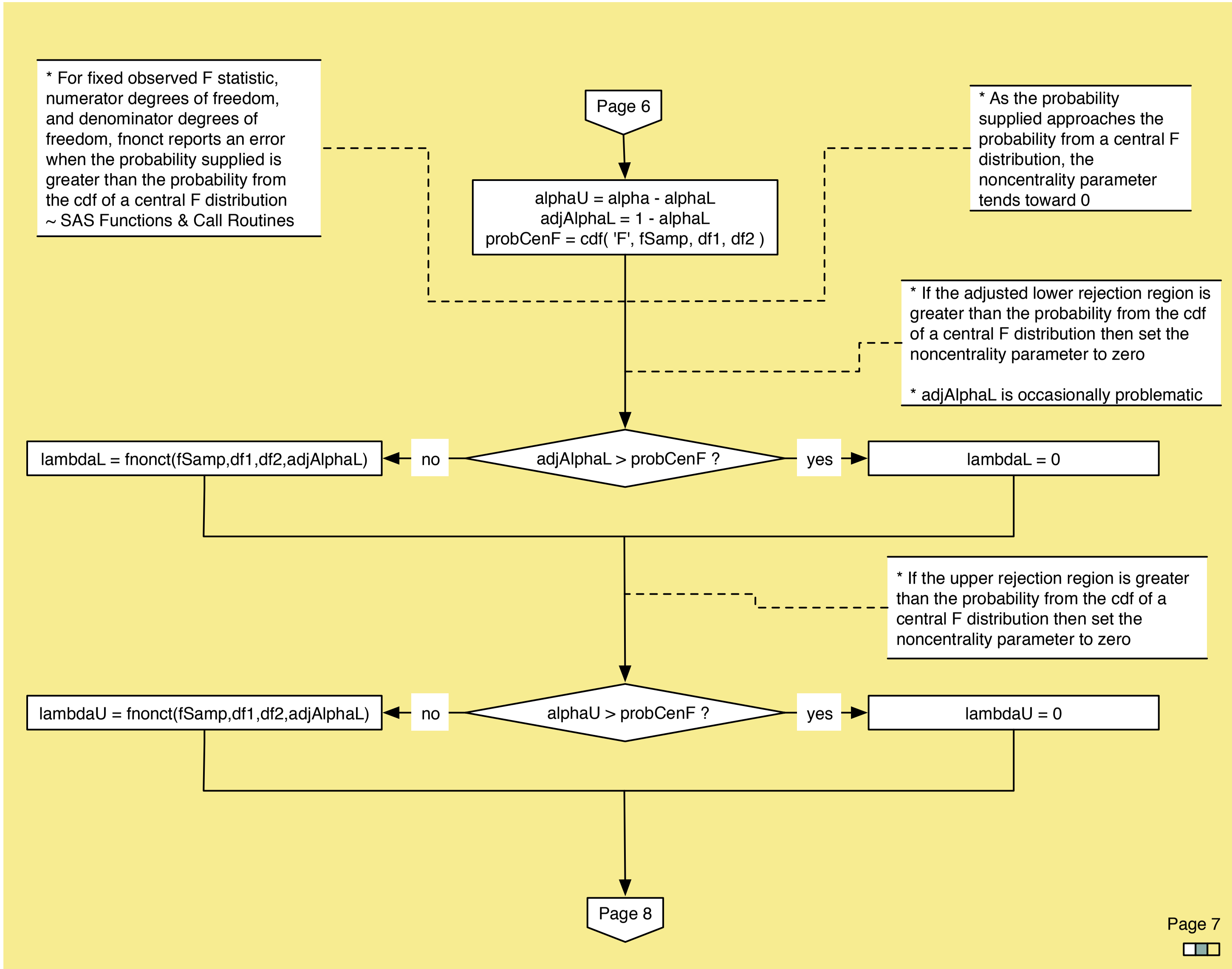
Appendix A: Four-Sample Flow Chart of Power Simulations



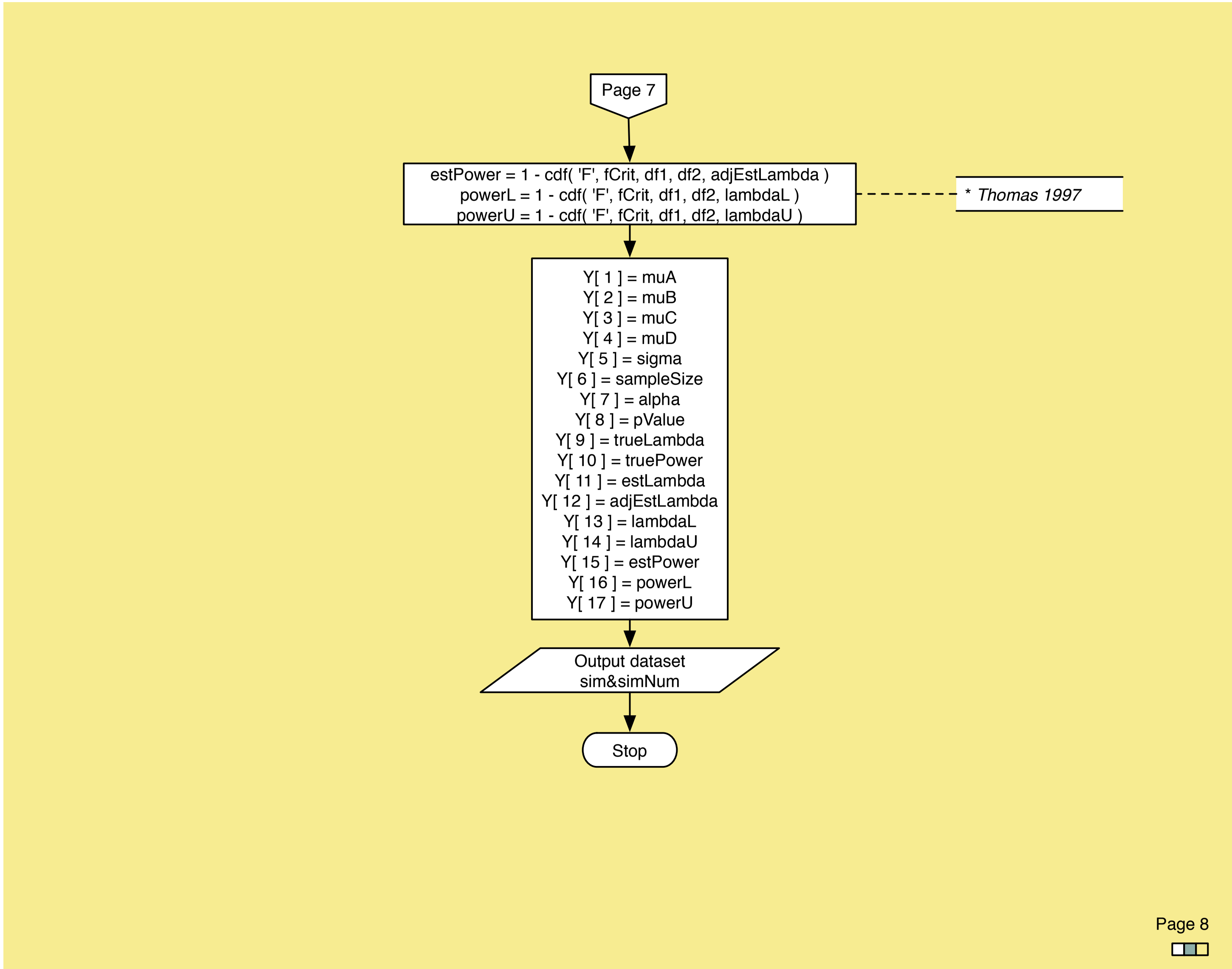
Appendix A: Four-Sample Flow Chart of Power Simulations



Appendix A: Four-Sample Flow Chart of Power Simulations



Appendix A: Four-Sample Flow Chart of Power Simulations



Appendix A: Four-Sample Flow Chart of Power Simulations

* Variables Defined

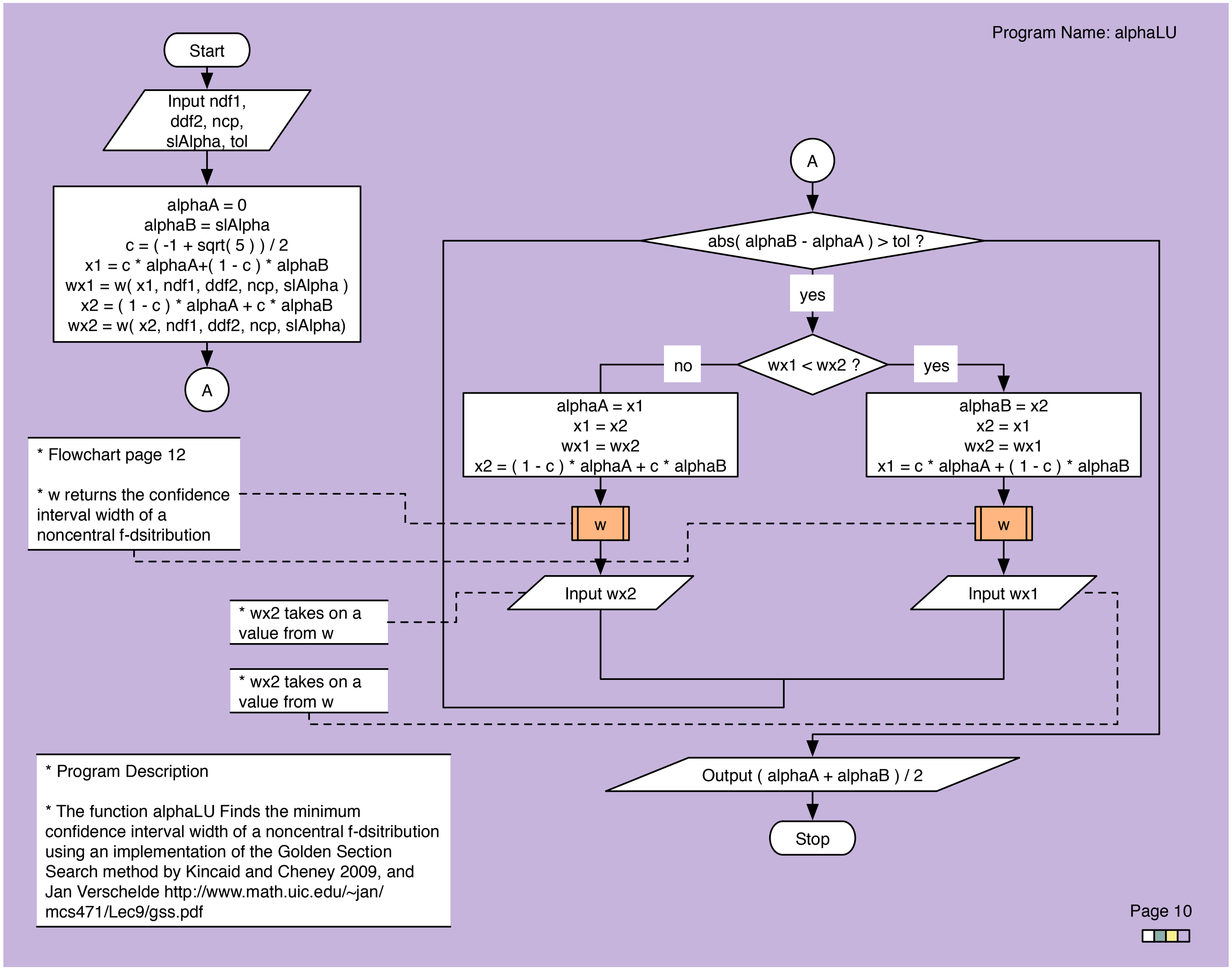
muA := mean for population one
muB := mean for population two
muC := mean for population three
muD := mean for population four
muG := average of the tree population means
sigma := population standard deviation
seedNum := location of random seeds
sampleSize := sample size
alpha := level of significance
simNum := simulation identification
X := samples from normal populations
eps := tolerance for Golden Section Search methods
n := number rows
p := number columns
xBar := sample mean
sst := sum of squares for treatment
df1 := numerator degrees of freedom
mst := mean square for treatment
sse := sum of squares for error
df2 := denominator degrees of freedom
mse := mean square for error
fSamp := observed F statistic
fCrit := critical value from a central F distribution
pValue := p-value for the test
trueLambda := noncentrality parameter
truePower := power for the test
estLambda := calculated noncentrality parameter
adjEstLambda := adjusted calculated noncentrality parameter
alphaL := optimized lower critical region
alphaU := optimized upper critical region
probCenF := probability of observed F statistic of the central F distribution
estPower := calculated power
powerL := lower confidence interval for calculated power
powerU := upper confidence interval for calculated power

* Datasets Defined

sampleFourPops := random samples dataset
sim&simNum := simulation dataset

Page 9

Appendix A: Four-Sample Flow Chart of Power Simulations



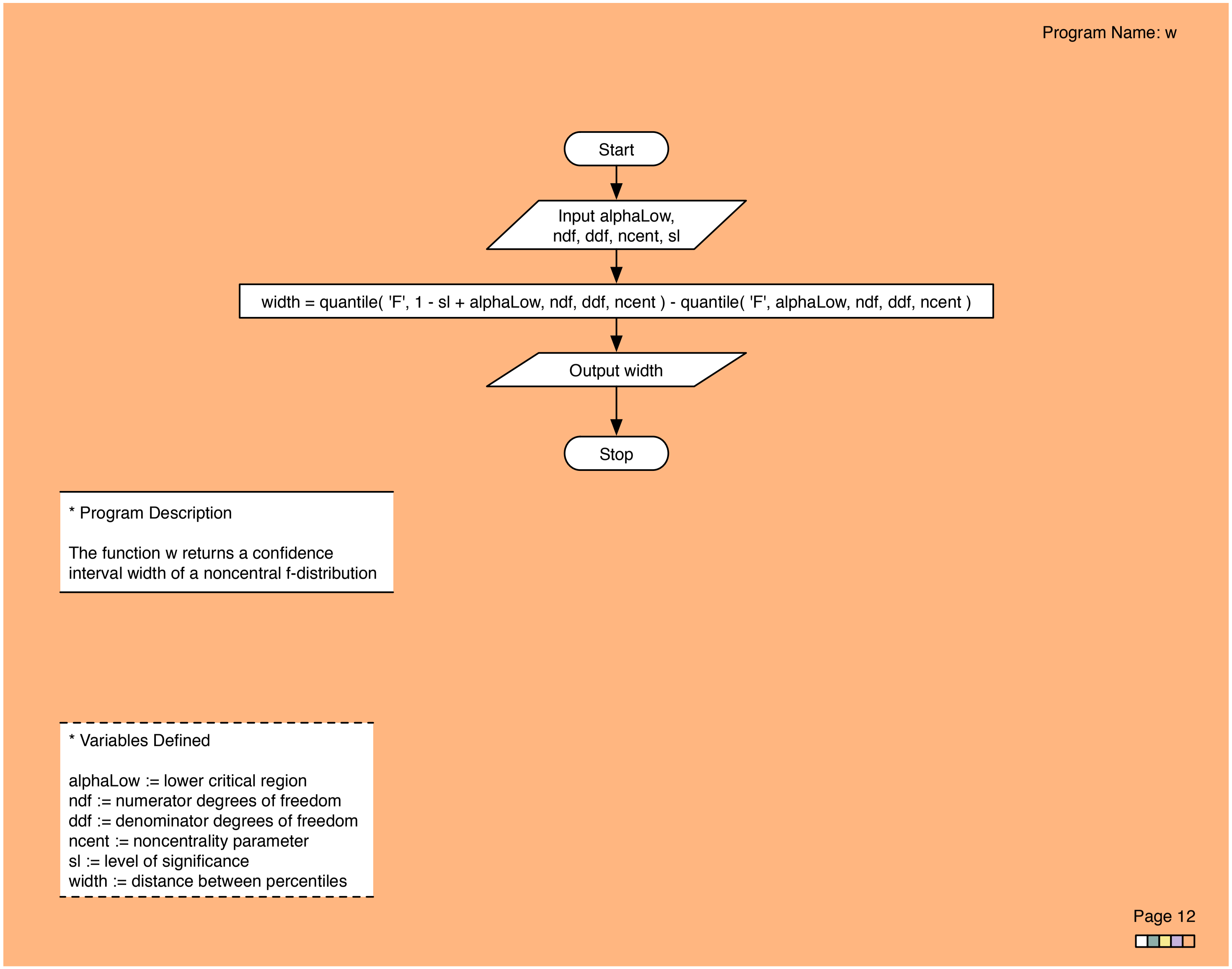
Appendix A: Four-Sample Flow Chart of Power Simulations

* Variables Defined

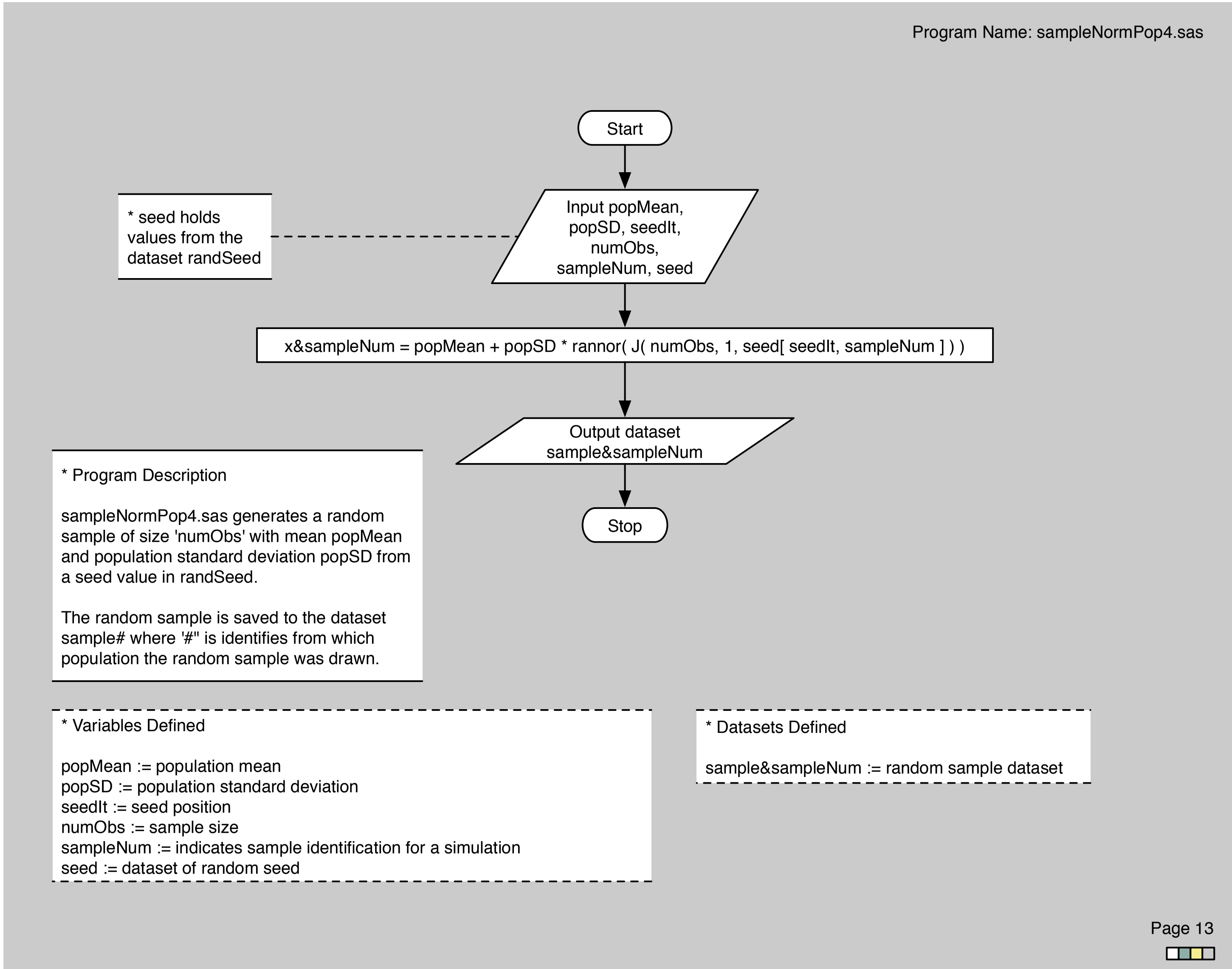
ndf1 := numerator degrees of freedom
ddf2 := denominator degrees of freedom
ncp := noncentrality parameter
slAlpha := level of significance
tol := tolerance for Golden Section Search method
alphaA := lower bound for optimized lower alpha
alphaB := upper bound for optimized lower alpha
c := Golden ratio constant reduction factor
x1 := percentile associated with alphaA
wx1 := height associated with x1
x2 := percentile associated with alphaB
wx2 := height associated with x2

Page 11

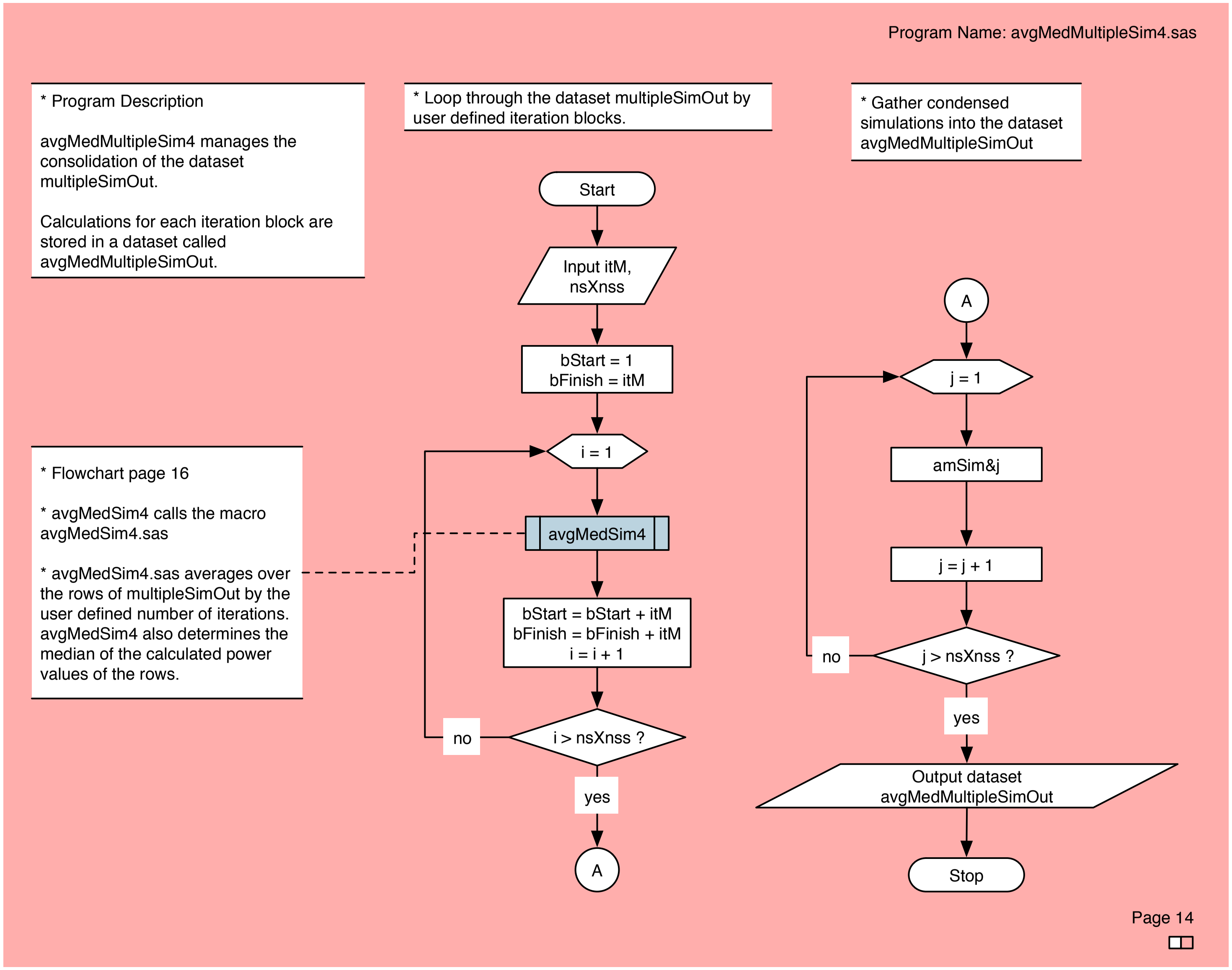
Appendix A: Four-Sample Flow Chart of Power Simulations



Appendix A: Four-Sample Flow Chart of Power Simulations



Appendix A: Four-Sample Flow Chart of Power Simulations



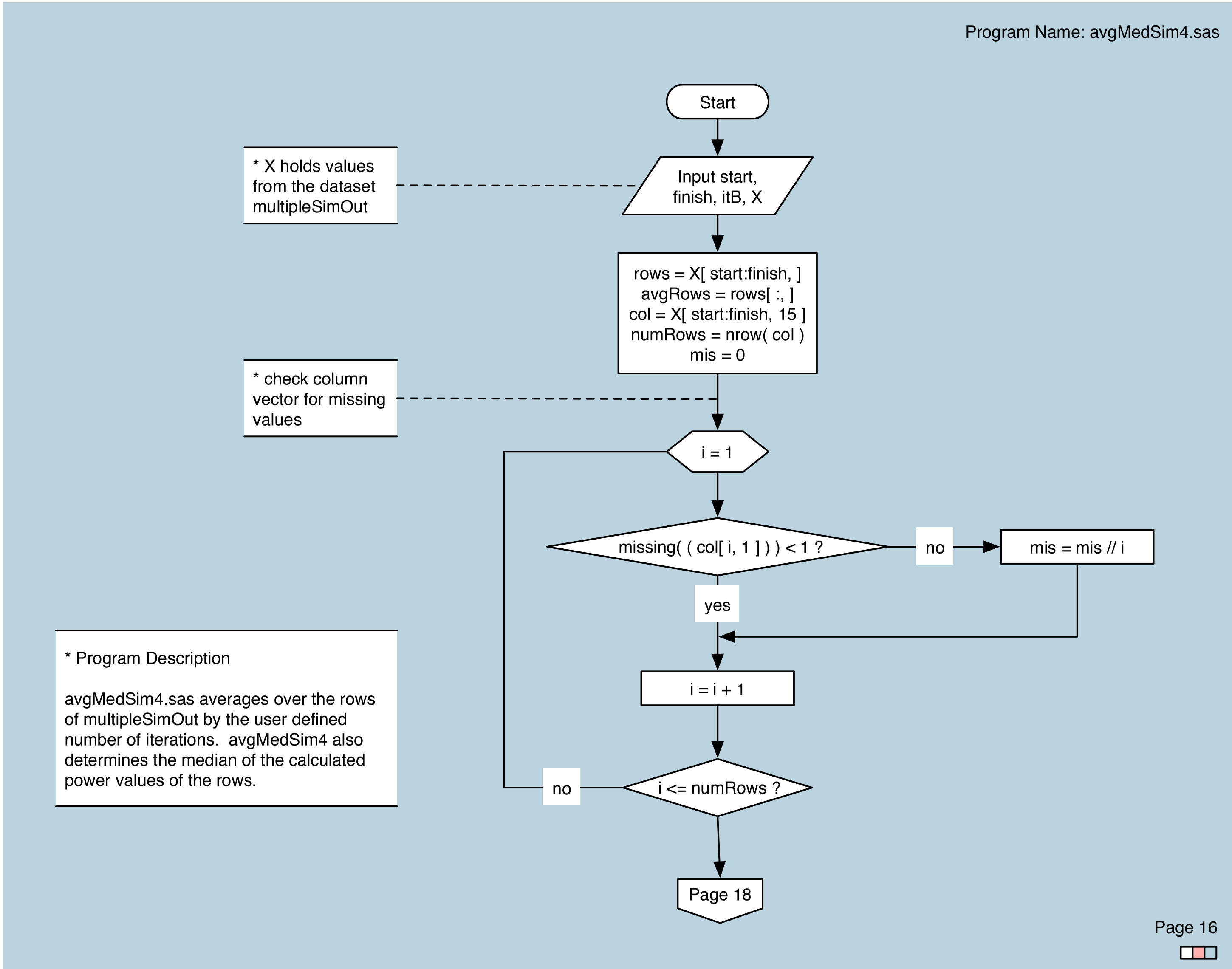
* Variables Defined

itM := maximum number of iterations
nsXnss := number of population standard deviations considered multiplied by the number of sample sizes considered
bStart := start of seed block
bFinish := end of seed block

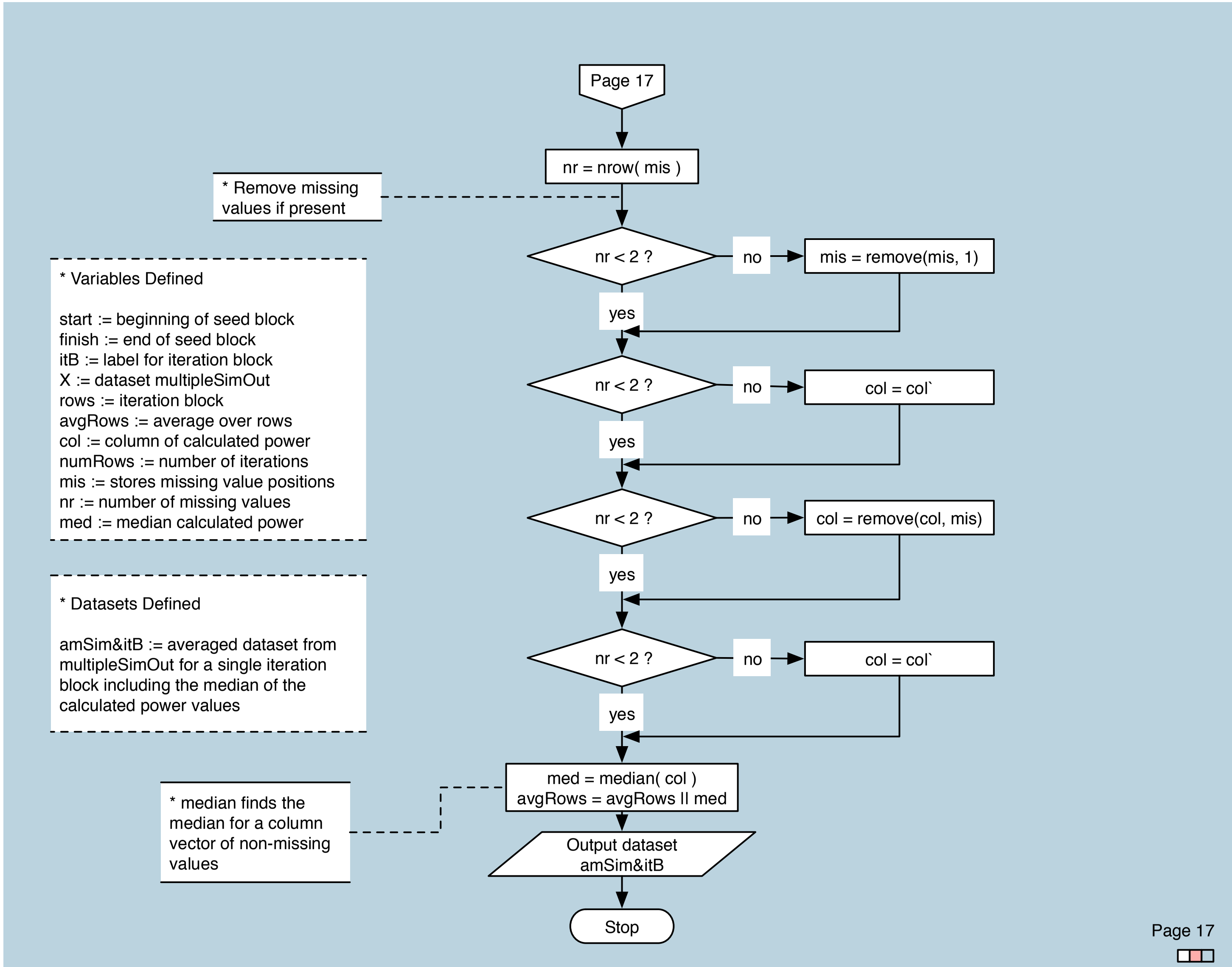
* Datasets Defined

avgMedMultipleSimOut := averaged dataset of multipleSimOut by iteration blocks including the median for the calculated power values in each iteration block

Appendix A: Four-Sample Flow Chart of Power Simulations



Appendix A: Four-Sample Flow Chart of Power Simulations



Appendix B: Two-Sample SAS Code for Power Simulations

```
/* twoSample **************************************************************/
/*                                                                        */
/* Requirement(s):                                                        */
/* rand2.csv, multipleSim2.sas, avgMedMultipleSim2.sas                    */
/*                                                                        */
/* Program Description:                                                   */
/* twoSample.sas acts as a controller interface which accepts user        */
/* defined values for two population means, upper and lower bounds        */
/* for a range of population standard deviations, a maximum sample size,  */
/* a type I error rate, and the number of iterations per sample size      */
/* to population standard deviation combination.                          */
/*                                                                        */
/**************************************************************************/


proc printto print='\Path\to\Output\Folder\twoSampleOut.lst' log='\Path\to\Output\Folder\twoSampleOut.log';
run;


/* Input random seed file to the dataset randseed ***********************/
/*                                                                       */
/* rand2.csv is a two column file of random numbers generated from       */
/* http:/www.random.org.  For each iteration of multipleSim2 two random  */
/* values are used to seed two random samples.                           */
/*                                                                       */
/*************************************************************************/

data randSeed;
    infile "\Path\to\File\rand2.csv" dlm=',';
    input seedCol1 seedCol2;
run;


/* Produce simulations **********************************************/
/*                                                                  */
/* The multipleSim2 manages the production of simulations.          */
/* Calls are made to the macro simulation2.sas which performs       */
/* calculations based on Thomas 1997.                               */
/*                                                                  */
/* Calculations for each simulation are stored in a dataset called  */
/* multipleSimOut.                                                  */
/*                                                                  */
/********************************************************************/

%multipleSim2(mu1=30,mu2=40,sigmaMin=5,sigmaMax=40,itMax=100,sampleSizeMax=50,sigLevel=0.05);
```

```
/* Print multipleSimOut */

proc print data=multipleSimOut;
run;


/* Produce averages and medians for simulations ***********************/
/*                                                                    */
/* The avgMedMultipleSim2 manages the consolidation of simulations.   */
/* Calls are made to avgMedSim2 for each sigma by sample size         */
/* combination.  avgMedSim2 averages over a user specified number     */
/* of rows from multipleSimOut.  avgMedSim2 also determines the       */
/* median of the calculated power values of the rows.                 */
/*                                                                    */
/* Calculations for each iteration block are stored in a dataset      */
/* called avgMedMultipleSimOut.                                       */
/*                                                                    */
/**********************************************************************/

%avgMedMultipleSim2(itM=100,nsXnss=306);


/* Print avgMedMultipleSimOut */

proc print data=avgMedMultipleSimOut;
run;


/* Plot avgMedMultipleSimOut */

goptions ftitle=swiss ftext=swiss;
symbol1 value=square interpol=join width=0.5 color=blue;
symbol2 value=star interpol=join width=0.5 color=red;
symbol3 value=triangle interpol=join width=0.5 color=yellow;
symbol4 value='x' interpol=join width=0.5 color=orange;
symbol5 value=circle interpol=join width=0.5 color=blue;
symbol6 value=dot interpol=join width=0.5 color=green;
title2 height=1.4 "Sample Size vs Power";
axis1 order=(0.0 to 1.0 by 0.1) label=(angle=90 'Power') minor=none;
axis2 order=(5 to 50 by 5) label=(angle=0 'Sample Size');
legend label=none value=(h=.8 'Upper AVG Estimated Power' 'True Power' 'AVG Estimated Power' 'Median Power' 'Lower AVG Estimated Power' 'AVG P-Value')
       position=center;
```

Appendix B: Two-Sample SAS Code for Power Simulations

```
proc gplot data=avgMedMultipleSimOut;
    plot powerU*sampleSize truePower*sampleSize estPower*sampleSize medPower*sampleSize powerL*sampleSize
pValue*sampleSize/overlay vaxis=axis1 haxis=axis2 hminor=4 legend=legend1;
    by sigma;
run;
```

Appendix B: Two-Sample SAS Code for Power Simulations

```
/* multipleSim2 ****************************************************************/
/*                                                                            */
/* Requirement(s):                                                            */
/* simulation2.sas                                                            */
/*                                                                            */
/* Program Description:                                                       */
/* multipleSim2 Iterates simulation2.sas and combines simulation output to a  */
/* dataset.                                                                   */
/*                                                                            */
/* Input:                                                                     */
/* mu1           := population one mean                                       */
/* mu2           := population two mean                                       */
/* simgaMin      := minimum population standard deviation controller          */
/* sigmaMax      := maximum population standard deviation controller          */
/* itMax         := number of iterations                                      */
/* sampleSizeMax := largest sample size considered                            */
/* sigLevel      := level of significance                                     */
/*                                                                            */
/* Note(s):                                                                   */
/* In the subsubloop iterations run in blocks.  This is done to               */
/* traverse the dataset randSeed, where by each call to the macro             */
/* simulation2 gets a different random seed position.                         */
/*                                                                            */
/* This macro contains two main loops.  The first main loop                   */
/* iterates from the minimum user define population standard deviation to 19  */
/* by 1.  The second main loop iterates from 20 to the maximum user defined   */
/* population standard deviaiton by 20.  Each main loop has two subloops.     */
/* The first subloop iterates from a system defined minimum sample size of 5  */
/* to 14 by 1.  The second subloop iterates from 15 to the maximum user       */
/* defined sample size by 5.  Each subloop has a subsubloop.                  */
/* Each subsubloop iterates by a use defined number of iterations.  After     */
/* a subsubloop executes, the starting position of an iteration 'block' is    */
/* increased by the the user defined number of iterations.                    */
/*                                                                            */
/******************************************************************************/


%macro multipleSim2(mu1=,mu2=,sigmaMin=,sigmaMax=,itMax=,sampleSizeMax=,sigLevel=);

%local std sd nn it block s n i b;
%let block = 1;
%let b = 1;
%let count = 1;
```

```
/* Produce simulations */

    %do std = &sigmaMin %to 19 %by 1;
    %let sd = %sysevalf(&std/2);
        %do nn = 5 %to 14 %by 1;
            %do it = &block %to &block+&itMax-1 %by 1;
                %simulation2(muA=&mu1,muB=&mu2,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count);
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
        %do nn = 15 %to &sampleSizeMax %by 5;
            %do it = &block %to &block+&itMax-1 %by 1;
                %simulation2(muA=&mu1,muB=&mu2,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count);
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
    %end;
    %do std = 20 %to &sigmaMax %by 20;
    %let sd = %sysevalf(&std/2);
        %do nn = 5 %to 14 %by 1;
            %do it = &block %to &block+&itMax-1 %by 1;
                %simulation2(muA=&mu1,muB=&mu2,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count);
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
        %do nn = 15 %to &sampleSizeMax %by 5;
            %do it = &block %to &block+&itMax-1 %by 1;
                %simulation2(muA=&mu1,muB=&mu2,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count);
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
    %end;
```

```
/* Combine simulations into a dataset and name columns */

    data multipleSimOut;
        set
            %do k = 1 %to &count-1 %by 1;
                sim&k
            %end;
            ;   /* Double do-loop to reference datasets.  ';' closes 'set' */

        rename col1=mu1 col2=mu2 col3=sigma col4=sampleSize col5=sigLevel col6=pValue col7=trueLambda col8=truePower
col9=estLambda col10=adjEstLambda col11=lambdaL col12=lambdaU col13=estPower col14=powerL col15=powerU;

    run;


%mend multipleSim2;
```

Appendix B: Two-Sample SAS Code for Power Simulations

```
/* simulation2 ***********************************************************/
/*                                                                       */
/* Requirement(s):                                                       */
/* sampleNormPop2.sas                                                    */
/*                                                                       */
/* Program Description:                                                  */
/* simulation2.sas performs a hypothesis                                 */
/* test of no difference for two population means.  Random samples       */
/* are normally distributed with respective means muA and muB with equal */
/* population standard deviation sigma.  Random samples are generated    */
/* using seed values from the dataest randSeed.  Power calculations      */
/* are based on methods outlined in Thomas 1997.  Calculations are       */
/* saved to a dataset named 'sim#' where # represents the                */
/* simulation count.                                                     */
/*                                                                       */
/* Input:                                                                */
/* muA          := mean of population one                                */
/* muB          := mean of population two                                */
/* sigma        := standard deviation of populations one and two         */
/* seedNum      := position for random seed                              */
/* sampleSize   := sample size taken for populations one and two         */
/* alpha        := level of significance                                 */
/* simNum       := simulation ID                                         */
/*                                                                       */
/* Note(s):                                                              */
/* Optimal lower significance level by Golden Section Search method      */
/*                                                                       */
/*************************************************************************/


%MACRO simulation2(muA=,muB=,sigma=,seedNum=,sampleSize=,alpha=,simNum=);

    /* Generate random samples from two normal populations */

        %sampleNormPop2(popMean=&muA,popSD=&sigma,seedIt=&seedNum,numObs=&sampleSize,sampleNum=1);
        %sampleNormPop2(popMean=&muB,popSD=&sigma,seedIt=&seedNum,numObs=&sampleSize,sampleNum=2);


    /* Merge datasets sample1 and sample2 to produce dataset sampleTwoPops */

    data sampleTwoPops;
        merge sample1 sample2;
    run;
```

```
/* Main */

proc iml;
    reset nolog;
    use sampleTwoPops;
    read all into X;


    /* Function ***********************************************************/
    /*                                                                    */
    /* Return the confidence interval width for a noncentral              */
    /* f-dsitribution                                                     */
    /*                                                                    */
    /**********************************************************************/

    start w(alphaLow,ndf,ddf,ncent,sl);

        width = quantile('F',1-sl+alphaLow,ndf,ddf,ncent)-quantile('F',alphaLow,ndf,ddf,ncent);
        return(width);

    finish w;


    /* Function: optimal lower critical region ****************************/
    /*                                                                    */
    /* Finds the minimum confidence interval width using an               */
    /* implementation of the Golden Section Search method                 */
    /* similiar to Press et al. (2007).                                   */
    /*                                                                    */
    /* Note(s):                                                           */
    /* The Golden Section Search method finds a value which minimizes     */
    /* a function.  The function must be one dimensional and unimodal.    */
    /*                                                                    */
    /* Imagine a 'typical' pdf of an F distribution.  Say we want to      */
    /* setup an interval such that the area covered by the sum of the two */
    /* tails is alpha.  For each alpha we can find the                    */
    /* associated quantile using the SAS function 'quantile'.             */
    /* Since alpha is specified, knowing the lower area easily leads to   */
    /* the upper area.  Here we consider only the lower                   */
    /* area.  As such we can relate interval width as a function of       */
    /* lower alpha size.  As we increase or decrease the size of the      */
    /* lower area we increase or decrease the width of the interval.      */
    /* Assuming a minimum interval width exists, we optimize over the     */
    /* values [0.00, 0.05].                                               */
    /*                                                                    */
    /**********************************************************************/

```

```
    start alphaLU(ndf1,ddf2,ncp,slAlpha,tol);

        alphaA = 0;                                 /* Initial lower bracket on optimal region */
        alphaB = slAlpha;                           /* Initial upper bracket on optimal region */

                                                    /* Note 1 **********************************/
                                                    /*                                        */
                                                    /* For each iteration an interval         */
                                                    /* [alphaA, alphaB] is retained           */
                                                    /*                                        */
                                                    /******************************************/

        phi = (-1+sqrt(5))/2;                       /* Golden ratio constant reduction factor */
        a1 = alphaA+phi*(alphaB-alphaA);            /* Set test bracket a1 */
        wa1 = w(a1,ndf1,ddf2,ncp,slAlpha);          /* Calculate Interval width for a1 */
        a2 = alphaA+phi*phi*(alphaB-alphaA);        /* Set test bracket a2 */
        wa2 = w(a2,ndf1,ddf2,ncp,slAlpha);          /* Calculate interval width for a2 */

                                                    /* Note 2 ************************************/
                                                    /*                                          */
                                                    /* The intial setup invokes two function    */
                                                    /* calls.  The following loop uses one      */
                                                    /* function call per iteration.             */
                                                    /*                                          */
                                                    /********************************************/

        do while (abs(alphaB-alphaA)>tol);          /* Note 3 ************************************/
                                                    /*                                          */
                                                    /* We continue until the absolute difference */
                                                    /* between an upper and a lower bracket is  */
                                                    /* negligable. Tolerance is set to1e-4.     */
                                                    /*                                          */
                                                    /********************************************/

                                                    /* Note 4 ************************************/
                                                    /*                                          */
                                                    /* Four brackets are maintained at any      */
                                                    /* given time.                              */
                                                    /*                                          */
                                                    /********************************************/

            if wa1>wa2 then do;                     /* If wa1 > wa2 then minimum width is between */
                                                    /* alphaA and a1                              */
```

```
                                                          /* Note 5 ***********************************/
                                                          /*                                          */
                                                          /* Within the interval [alphaA, a1] we have */
                                                          /* the width wa2.  a2 is situated by the    */
                                                          /* expression alphaA+phi*(a1-alphaA).       */
                                                          /*                                          */
                                                          /********************************************/

            alphaB = a1;                                  /* Updated alphaB */
            a1 = a2;                                      /* Save a2 */
            wa1 = wa2;                                    /* Save wa2 */
            a2 = alphaA+phi*phi*(alphaB-alphaA);          /* Set new test bracket a2 */
            wa2 = w(a2,ndf1,ddf2,ncp,slAlpha);            /* Calculate interval width for new a2 */
        end;
        else do;                                          /* If wa1 < wa2 then minimum width is between */
                                                          /* a2 and alphaB                              */

                                                          /* Note 6 ***********************************/
                                                          /*                                          */
                                                          /* Within the interval [a2, alphaB] we have */
                                                          /* the width wa1.  a1 is situated by the    */
                                                          /* expression alphaA+phi*phi*(alphaB-a2).   */
                                                          /*                                          */
                                                          /********************************************/

            alphaA = a2;                                  /* Updated alphaA */
            a2 = a1;                                      /* Save a1 */
            wa2 = wa1;                                    /* Save wa1 */
            a1 = alphaA+phi*(alphaB-alphaA);              /* Set new test bracket a1 */
            wa1 = w(a1,ndf1,ddf2,ncp,slAlpha);            /* Calculate interval width for new a1 */
        end;
    end;

    out = (alphaA+alphaB)/2;
    return(out);

finish alphaLU;


/* Calculations */

eps = 1e-4;                                               /* Stopping criteria for GSS method */
n = nrow(X);                                              /* Number of rows */
p = ncol(X);                                              /* Number of columns */
J = J(p,1);

xBar = X(|+,|)`/n;                                        /* Sample means */
```

```
        muG = (&muA+&muB)/p;                                  /* Mean of population means */
        sst = n*xBar`*xBar-(n/p)*(xBar`*J)**2;                /* Sum of squares for treatments */
        df1 = p-1;                                            /* Degrees of freedom for treatment */
        mst = sst/df1;                                        /* Mean square for treatment */
        sse = ssq(X)-(xBar`*xBar)*n;                          /* Sum of squares for error */
        df2 = p*(n-1);                                        /* Degrees of freedom for error */
        mse = sse/df2;                                        /* Mean square for error */
        fSamp = mst/mse;                                      /* Observed F */

        fCrit = quantile('F',1-&alpha,df1,df2);               /* F critical value */

        pValue = 1-cdf('F',fSamp,df1,df2);                    /* P-Value */

        trueLambda = (n*(((&muA-muG)**2)+((&muB-muG)**2)))/((&sigma)**2);       /* trueLambda */

        truePower = 1-cdf('F',fCrit,df1,df2,trueLambda);      /* truePower */

        estLambda = fSamp*(p-1);                              /* Estimated lambda */
        adjEstLambda = (estLambda*(df2-2)/df2)-df1;           /* Adjusted estimated lambda */

        if adjEstLambda < 0 then adjEstLambda = 0;            /* Note 7 **********************/
                                                              /*                              */
                                                              /* For some random seeds the   */
                                                              /* noncentrality parameter is  */
                                                              /* negative.                   */
                                                              /*                              */
                                                              /* A negative noncentrality    */
                                                              /* parameter occurs when       */
                                                              /* the expression              */
                                                              /* estLambda*(df2-2)/df2 is    */
                                                              /* less than df1.              */
                                                              /*                              */
                                                              /******************************/


        alphaL = alphaLU(df1,df2,adjEstLambda,&alpha,eps);    /* Lower significance */
        alphaU = &alpha-alphaL;                               /* Upper significance */
```

```

        adjAlphaL = 1-alphaL;                                    /* Note 8 ********************************/
        probCenF = cdf('F',fSamp,df1,df2);                       /*                                       */
                                                                 /* fnonct reports an error for a supplied */
                                                                 /* probability greater than the          */
                                                                 /* probability from the cdf of a central  */
                                                                 /* F distribution.                       */
                                                                 /*                                       */
                                                                 /* As the probability supplied gets large */
                                                                 /* fnonct tends toward zero.  probCenF    */
                                                                 /* with fnonct is small around zero.      */
                                                                 /* Using adjAlphaL would then be smaller  */
                                                                 /* about zero.  Set to zero.              */
                                                                 /*                                       */
                                                                 /****************************************/


        if adjAlphaL > probCenF then lambdaL = 0;                /* See note 8 */
            else lambdaL = fnonct(fSamp,df1,df2,adjAlphaL);
        if alphaU > probCenF then lambdaU = 0;                   /* See note 8 */
            else lambdaU = fnonct(fSamp,df1,df2,alphaU);


        estPower = 1-cdf('F',fCrit,df1,df2,adjEstLambda);        /* Estimated power */
        powerL = 1-cdf('F',fCrit,df1,df2,lambdaL);               /* Lower limit for power */
        powerU = 1-cdf('F',fCrit,df1,df2,lambdaU);               /* Upper limit for power */

        /* Output simulation to a data set */

        Y = {"&muA" "&muB" "&sigma" "&sampleSize" "&alpha" "." "." "." "." "." "." "." "." "." "."};
        Y = num(Y);
        Y[6] = pValue;
        Y[7] = trueLambda;
        Y[8] = truePower;
        Y[9] = estLambda;
        Y[10] = adjEstLambda;
        Y[11] = lambdaL;
        Y[12] = lambdaU;
        Y[13] = estPower;
        Y[14] = powerL;
        Y[15] = powerU;

        create sim&simNum from Y;
        append from Y;

    quit;

```

```
    proc datasets lib=work nolist;
        delete sample1 sample2 sampleTwoPops;
    quit;
    run;

%MEND simulation2;
```

```
/* sampleNormPop2 *************************************************************/
/*                                                                            */
/* Program Description:                                                       */
/* sampleNormPop2 generates a random sample from a normal population          */
/* and outputs the values to a dataset                                        */
/*                                                                            */
/* Input:                                                                     */
/* seed       := dataset of random seeds from randSeed                        */
/* popMean    := population mean                                              */
/* popSD      := population standard deviation                                */
/* seedIt     := seed value position for generating random observations      */
/* numObs     := sample size                                                  */
/* sampleNum  := identifies sample                                            */
/*                                                                            */
/******************************************************************************/


%macro sampleNormPop2(popMean=,popSD=,seedIt=,numObs=,sampleNum=);

proc iml;
    reset nolog;
    use randSeed;
    read all into seed;

    x&sampleNum = &popMean+&popSD*rannor(J(&numObs,1,seed[&seedIt,&sampleNum]));
    test = seed[&seedIt,&sampleNum];

    create sample&sampleNum from x&sampleNum[colname={obs&sampleNum}];
    append from x&sampleNum;

quit;

%mend sampleNormPop2;
```

Appendix B: Two-Sample SAS Code for Power Simulations

```
/* avgMedMultipleSim2 ***********************************************/
/*                                                                  */
/* Requirement(s):                                                  */
/* avgMedSim2.sas                                                   */
/*                                                                  */
/* Program Description:                                             */
/* avgMedMultipleSim2 calls avgMedSim2 for each sigma by sample size */
/* combination.                                                     */
/*                                                                  */
/* Input:                                                           */
/* itM          := number of iterations                             */
/* nsXnss       := number of iteration groups                       */
/*              (sigmas times number of sample sizes)               */
/*                                                                  */
/********************************************************************/


%macro avgMedMultipleSim2(itM=,nsXnss=);

    /* Produce average and median of simulations */

        %let bStart = 1;
        %let bFinish = &itM;

        %do i = 1 %to &nsXnss %by 1;
            %avgMedSim2(start=&bStart,finish=&bFinish,itB=&i);
            %let bStart = &bStart+&itM;
            %let bFinish = &bFinish+&itM;
        %end;


    /* Combine average simulations into a data set and name columns */

    data avgMedMultipleSimOut;
        set
            %do j = 1 %to &nsXnss %by 1;
                amSim&j
            %end;
            ;   /* Double do-loop to reference data sets.  ';' closes 'set' */
        rename col1=mu1 col2=mu2 col3=sigma col4=sampleSize col5=sigLevel col6=pValue col7=trueLambda col8=truePower col9=estLambda col10=adjEstLambda col11=lambdaL col12=lambdaU col13=estPower col14=powerL col15=powerU col16=medPower;

    run;

%mend avgMedMultipleSim2;
```

Appendix B: Two-Sample SAS Code for Power Simulations

```
/* avgMedSim2 *******************************************************/
/*                                                                  */
/* Program Description:                                             */
/* avgMedSim2 averages over a user specified number of rows from    */
/* multipleSimOut.  avgMedSim2 also determines the median           */
/* of the calculated power values of the rows.                      */
/*                                                                  */
/* Input:                                                           */
/* X            := gets dataset multipleSimOut                      */
/* start        := beginning of iteration block                     */
/* finish       := end of iteration block                           */
/* itB          := number of iterations                             */
/*                                                                  */
/********************************************************************/


%macro avgMedSim2(start=,finish=,itB=);

proc iml;

    reset nolog;
    use multipleSimOut;
    read all into X;

    /* Average rows for an iteration block */

    rows = X[&start:&finish,];
    avgRows = rows[:,];


    /* Median for column calculated power */

    col = X[&start:&finish,13];                         /* Column of calculated power */

    numRows = nrow(col);                                /* Number of rows */

    mis = 0;                                            /* Storage for positions of missing values */
    i = 1;

    do while (i <=numRows);                             /* Note 1 ***********************/
        if missing((col[i,1])) > 0 then mis=mis//i;     /*                              */
        i=i+1;                                          /* Loop through column of       */
    end;                                                /* of calculated power values.  */
                                                        /* If a missing value is found  */
                                                        /* record array position.       */
                                                        /*                              */
                                                        /********************************/
```

```

    nr = nrow(mis);                                      /* Number of missing values */

                                                         /* Note 2 *******************/
                                                         /*                          */
                                                         /* If 'nr' is greater than  */
                                                         /* one then we have missing */
                                                         /* values.  These values are */
                                                         /* removed from the column. */
                                                         /*                          */
                                                         /****************************/

    if nr>1 then mis = remove(mis,1);                    /* Remove initialize position */
    if nr>1 then col = col`;                             /* Transpose a column to row */
    if nr>1 then col = remove(col,mis);                  /* Remove missing values */
    if nr>1 then col = col`;                             /* Transpose row to a column */

    med = median(col);                                   /* Find median */


    /* Concatinate median to average row */

    avgRows=avgRows||med;


    create amSim&itB from avgRows;                       /* Output to a dataset */
    append from avgRows;

quit;
%mend avgMedSim2;
```

Appendix B: Three-Sample SAS Code for Power Simulations

```
/* threeSample ************************************************************/
/*                                                                        */
/* Requirement(s):                                                        */
/* rand3.csv, multipleSim3.sas, avgMedMultipleSim3.sas                    */
/*                                                                        */
/* Program Description:                                                   */
/* threeSample.sas acts as a controller interface which accepts user      */
/* defined values for three population means, upper and lower bounds      */
/* for a range of population standard deviations, a maximum sample size,  */
/* a type I error rate, and the number of iterations per sample size      */
/* to population standard deviation combination.                          */
/*                                                                        */
/**************************************************************************/


proc printto print='\Path\to\Output\Folder\threeSampleOut.lst' log='\Path\to\Output\Folder\threeSampleOut.log';
run;



/* Input random seed file to the dataset randseed **************************/
/*                                                                         */
/* rand3.csv is a three column file of random numbers generated from       */
/* http:/www.random.org.  For each iteration of multipleSim3 three random  */
/* values are used to seed three random samples.                           */
/*                                                                         */
/***************************************************************************/

data randSeed;
    infile "\Path\to\File\rand3.csv" dlm=',';
    input seedCol1 seedCol2 seedCol3;
run;


/* Produce simulations **********************************************/
/*                                                                  */
/* The multipleSim3 manages the production of simulations.          */
/* Calls are made to the macro simulation3.sas which performs       */
/* calculations based on Thomas 1997.                               */
/*                                                                  */
/* Calculations for each simulation are stored in a dataset called  */
/* multipleSimOut.                                                  */
/*                                                                  */
/********************************************************************/

%multipleSim3(mu1=30,mu2=40,mu3=35,sigmaMin=5,sigmaMax=40,itMax=5,sampleSizeMax=50,sigLevel=0.05);

```

```

/* Print multipleSimOut */

proc print data=multipleSimOut;
run;


/* Produce averages and medians for simulations ***********************/
/*                                                                    */
/* The avgMedMultipleSim3 manages the consolidation of simulations.   */
/* Calls are made to avgMedSim3 for each sigma by sample size         */
/* combination.  avgMedSim3 averages over a user specified number     */
/* of rows from multipleSimOut.  avgMedSim3 also determines the       */
/* median of the calculated power values of the rows.                 */
/*                                                                    */
/* Calculations for each iteration block are stored in a dataset      */
/* called avgMedMultipleSimOut.                                       */
/*                                                                    */
/**********************************************************************/

%avgMedMultipleSim3(itM=5,nsXnss=306);


/* Print avgMedMultipleSimOut */

proc print data=avgMedMultipleSimOut;
run;


/* Plot avgMedMultipleSimOut */

goptions ftitle=swiss ftext=swiss;
symbol1 value=square interpol=join width=0.5 color=blue;
symbol2 value=star interpol=join width=0.5 color=red;
symbol3 value=triangle interpol=join width=0.5 color=yellow;
symbol4 value='x' interpol=join width=0.5 color=orange;
symbol5 value=circle interpol=join width=0.5 color=blue;
symbol6 value=dot interpol=join width=0.5 color=green;
title2 height=1.4 "Sample Size vs Power";
axis1 order=(0.0 to 1.0 by 0.1) label=(angle=90 'Power') minor=none;
axis2 order=(5 to 50 by 5) label=(angle=0 'Sample Size');
legend label=none value=(h=.8 'Upper AVG Estimated Power' 'True Power' 'AVG Estimated Power' 'Median Power' 'Lower AVG Estimated Power' 'AVG P-Value')
       position=center;
```

```
proc gplot data=avgMedMultipleSimOut;
    plot powerU*sampleSize truePower*sampleSize estPower*sampleSize medPower*sampleSize powerL*sampleSize
pValue*sampleSize/overlay vaxis=axis1 haxis=axis2 hminor=4 legend=legend1;
    by sigma;
run;
```

Appendix B: Three-Sample SAS Code for Power Simulations

```
/* multipleSim3 ***************************************************************/
/*                                                                            */
/* Requirement(s):                                                            */
/* simulation3.sas                                                            */
/*                                                                            */
/* Program Description:                                                       */
/* multipleSim3 Iterates simulation3.sas and combines simulation output to a  */
/* dataset.                                                                   */
/*                                                                            */
/* Input:                                                                     */
/* mu1           := population one mean                                       */
/* mu2           := population two mean                                       */
/* mu3           := population three mean                                     */
/* simgaMin      := minimum population standard deviation controller          */
/* sigmaMax      := maximum population standard deviation controller          */
/* itMax         := number of iterations                                      */
/* sampleSizeMax := largest sample size considered                            */
/* sigLevel      := level of significance                                     */
/*                                                                            */
/* Note(s):                                                                   */
/* In the subsubloop iterations run in blocks.  This is done to               */
/* traverse the dataset randSeed, where by each call to the macro             */
/* simulation2 gets a different random seed position.                         */
/*                                                                            */
/* This macro contains two main loops.  The first main loop                   */
/* iterates from the minimum user define population standard deviation to 19  */
/* by 1.  The second main loop iterates from 20 to the maximum user defined   */
/* population standard deviaiton by 20.  Each main loop has two subloops.     */
/* The first subloop iterates from a system defined minimum sample size of 5  */
/* to 14 by 1.  The second subloop iterates from 15 to the maximum user       */
/* defined sample size by 5.  Each subloop has a subsubloop.                  */
/* Each subsubloop iterates by a use defined number of iterations.  After     */
/* a subsubloop executes, the starting position of an iteration 'block' is    */
/* increased by the the user defined number of iterations.                    */
/*                                                                            */
/******************************************************************************/


%macro multipleSim3(mu1=,mu2=,mu3=,sigmaMin=,sigmaMax=,itMax=,sampleSizeMax=,sigLevel=);

%local std sd nn it block s n i b;
%let block = 1;
%let b = 1;
%let count = 1;
```

```
/* Produce simulations */

    %do std = &sigmaMin %to 19 %by 1;
    %let sd = %sysevalf(&std/2);
        %do nn = 5 %to 14 %by 1;
            %do it = &block %to &block+&itMax-1 %by 1;

%simulation3(muA=&mu1,muB=&mu2,muC=&mu3,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count);
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
        %do nn = 15 %to &sampleSizeMax %by 5;
            %do it = &block %to &block+&itMax-1 %by 1;

%simulation3(muA=&mu1,muB=&mu2,muC=&mu3,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count);
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
    %end;
    %do std = 20 %to &sigmaMax %by 20;
    %let sd = %sysevalf(&std/2);
        %do nn = 5 %to 14 %by 1;
            %do it = &block %to &block+&itMax-1 %by 1;

%simulation3(muA=&mu1,muB=&mu2,muC=&mu3,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count);
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
        %do nn = 15 %to &sampleSizeMax %by 5;
            %do it = &block %to &block+&itMax-1 %by 1;

%simulation3(muA=&mu1,muB=&mu2,muC=&mu3,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count);
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
    %end;
```

```
/* Combine simulations into a dataset and name columns */

    data multipleSimOut;
        set
            %do k = 1 %to &count-1 %by 1;
                sim&k
            %end;
            ;   /* Double do-loop to reference datasets.  ';' closes 'set' */

        rename col1=mu1 col2=mu2 col3=mu3 col4=sigma col5=sampleSize col6=sigLevel col7=pValue col8=trueLambda col9=truePower col10=estLambda col11=adjEstLambda col12=lambdaL col13=lambdaU col14=estPower col15=powerL col16=powerU;

    run;


%mend multipleSim3;
```

Appendix B: Three-Sample SAS Code for Power Simulations

```
/* simulation3 ***************************************************************/
/*                                                                          */
/* Requirement(s):                                                          */
/* sampleNormPop3.sas                                                       */
/*                                                                          */
/* Program Description:                                                     */
/* simulation3.sas performs a hypothesis                                    */
/* test of no difference for three population means.  Random samples        */
/* are normally distributed with respective means muA, muB and muC with equal */
/* population standard deviation sigma.  Random samples are generated       */
/* using seed values from the dataest randSeed.  Power calculations         */
/* are based on methods outlined in Thomas 1997.  Calculations are          */
/* saved to a dataset named 'sim#' where # represents the                   */
/* simulation count.                                                        */
/*                                                                          */
/* Input:                                                                   */
/* muA          := mean of population one                                   */
/* muB          := mean of population two                                   */
/* muC          := mean of population three                                 */
/* sigma        := standard deviation of populations one, two, and three    */
/* seedNum      := position for random seed                                 */
/* sampleSize   := sample size taken for populations one, two, and three    */
/* alpha        := level of significance                                    */
/* simNum       := simulation ID                                            */
/*                                                                          */
/* Note(s):                                                                 */
/* Optimal lower significance level by Golden Section Search method         */
/*                                                                          */
/****************************************************************************/


%MACRO simulation3(muA=,muB=,muC=,sigma=,seedNum=,sampleSize=,alpha=,simNum=);

    /* Generate random samples from three normal populations */

        %sampleNormPop3(popMean=&muA,popSD=&sigma,seedIt=&seedNum,numObs=&sampleSize,sampleNum=1);
        %sampleNormPop3(popMean=&muB,popSD=&sigma,seedIt=&seedNum,numObs=&sampleSize,sampleNum=2);
        %sampleNormPop3(popMean=&muC,popSD=&sigma,seedIt=&seedNum,numObs=&sampleSize,sampleNum=3);


    /* Merge datasets sample1, sample2 and sample3 to produce dataset sampleThreePops */

    data sampleThreePops;
        merge sample1 sample2 sample3;
    run;
```

```
/* Main */

proc iml;
    reset nolog;
    use sampleThreePops;
    read all into X;


    /* Function ***********************************************************/
    /*                                                                    */
    /* Return the confidence interval width for a noncentral              */
    /* f-dsitribution                                                     */
    /*                                                                    */
    /**********************************************************************/

    start w(alphaLow,ndf,ddf,ncent,sl);

        width = quantile('F',1-sl+alphaLow,ndf,ddf,ncent)-quantile('F',alphaLow,ndf,ddf,ncent);
        return(width);

    finish w;


    /* Function: optimal lower critical region ****************************/
    /*                                                                    */
    /* Finds the minimum confidence interval width using an               */
    /* implementation of the Golden Section Search method                 */
    /* similar to Press et al. (2007).                                    */
    /*                                                                    */
    /* Note(s):                                                           */
    /* The Golden Section Search method finds a value which minimizes     */
    /* a function.  The function must be one dimensional and unimodal.    */
    /*                                                                    */
    /* Imagine a 'typical' pdf of an F distribution.  Say we want to      */
    /* setup an interval such that the area covered by the sum of the two */
    /* tails is alpha.  For each alpha we can find the                    */
    /* associated quantile using the SAS function 'quantile'.             */
    /* Since alpha is specified, knowing the lower area easily leads to   */
    /* the upper area.  Here we consider only the lower                   */
    /* area.  As such we can relate interval width as a function of       */
    /* lower alpha size.  As we increase or decrease the size of the      */
    /* lower area we increase or decrease the width of the interval.      */
    /* Assuming a minimum interval width exists, we optimize over the     */
    /* values [0.00, 0.05].                                               */
    /*                                                                    */
    /**********************************************************************/
```

```

start alphaLU(ndf1,ddf2,ncp,slAlpha,tol);

    alphaA = 0;                                         /* Initial lower bracket on optimal region */
    alphaB = slAlpha;                                   /* Initial upper bracket on optimal region */

                                                        /* Note 1 **********************************/
                                                        /*                                         */
                                                        /* For each iteration an interval          */
                                                        /* [alphaA, alphaB] is retained            */
                                                        /*                                         */
                                                        /*******************************************/

    phi = (-1+sqrt(5))/2;                               /* Golden ratio constant reduction factor */
    a1 = alphaA+phi*(alphaB-alphaA);                    /* Set test bracket a1 */
    wa1 = w(a1,ndf1,ddf2,ncp,slAlpha);                  /* Calculate Interval width for a1 */
    a2 = alphaA+phi*phi*(alphaB-alphaA);                /* Set test bracket a2 */
    wa2 = w(a2,ndf1,ddf2,ncp,slAlpha);                  /* Calculate interval width for a2 */

                                                        /* Note 2 ************************************/
                                                        /*                                           */
                                                        /* The intial setup invokes two function     */
                                                        /* calls.  The following loop uses one       */
                                                        /* function call per iteration.              */
                                                        /*                                           */
                                                        /*********************************************/

    do while (abs(alphaB-alphaA)>tol);                  /* Note 3 ************************************/
                                                        /*                                           */
                                                        /* We continue until the absolute difference */
                                                        /* between an upper and a lower bracket is   */
                                                        /* negligable. Tolerance is set to1e-4.      */
                                                        /*                                           */
                                                        /*********************************************/

                                                        /* Note 4 ************************************/
                                                        /*                                           */
                                                        /* Four brackets are maintained at any       */
                                                        /* given time.                               */
                                                        /*                                           */
                                                        /*********************************************/

        if wa1>wa2 then do;                             /* If wa1 > wa2 then minimum width is between */
                                                        /* alphaA and a1                              */




```

```

                                                /* Note 5 ***********************************/
                                                /*                                          */
                                                /* Within the interval [alphaA, a1] we have */
                                                /* the width wa2.  a2 is situated by the    */
                                                /* expression alphaA+phi*(a1-alphaA).       */
                                                /*                                          */
                                                /********************************************/

            alphaB = a1;                        /* Updated alphaB */
            a1 = a2;                            /* Save a2 */
            wa1 = wa2;                          /* Save wa2 */
            a2 = alphaA+phi*phi*(alphaB-alphaA);  /* Set new test bracket a2 */
            wa2 = w(a2,ndf1,ddf2,ncp,slAlpha);  /* Calculate interval width for new a2 */
        end;
        else do;                                /* If wa1 < wa2 then minimum width */
                                                /* is between a2 and alphaB        */

                                                /* Note 6 ***********************************/
                                                /*                                          */
                                                /* Within the interval [a2, alphaB] we have */
                                                /* the width wa1.  a1 is situated by the    */
                                                /* expression alphaA+phi*phi*(alphaB-a2).   */
                                                /*                                          */
                                                /********************************************/

            alphaA = a2;                        /* Updated alphaA */
            a2 = a1;                            /* Save a1 */
            wa2 = wa1;                          /* Save wa1 */
            a1 = alphaA+phi*(alphaB-alphaA);    /* Set new test bracket a1 */
            wa1 = w(a1,ndf1,ddf2,ncp,slAlpha);  /* Calculate interval width for new a1 */
        end;
    end;

    out = (alphaA+alphaB)/2;
    return(out);

finish alphaLU;


/* Calculations */

eps = 1e-4;                                     /* Stopping criteria for GSS method */
n = nrow(X);                                    /* Number of rows */
p = ncol(X);                                    /* Number of columns */
J = J(p,1);
muG = (&muA+&muB+&muC)/p;                       /* Mean of population means */
xBar = X(|+,|)`/n;                              /* Sample means */
```

```
        sst = n*xBar`*xBar-(n/p)*(xBar`*J)**2;                  /* Sum of squares for treatments */
        df1 = p-1;                                              /* Degrees of freedom for treatment */
        mst = sst/df1;                                          /* Mean square for treatment */
        sse = ssq(X)-(xBar`*xBar)*n;                            /* Sum of squares for error */
        df2 = p*(n-1);                                          /* Degrees of freedom for error */
        mse = sse/df2;                                          /* Mean square for error */
        fSamp = mst/mse;                                        /* Observed F */

        fCrit = quantile('F',1-&alpha,df1,df2);                 /* F critical value */

        pValue = 1-cdf('F',fSamp,df1,df2);                      /* P-Value */

        trueLambda = (n*(((&muA-muG)**2)+((&muB-muG)**2)+((&muC-muG)**2)))/((&sigma)**2);  /* trueLambda */

        truePower = 1-cdf('F',fCrit,df1,df2,trueLambda);        /* truePower */

        estLambda = fSamp*(p-1);                                /* Estimated lambda */
        adjEstLambda = (estLambda*(df2-2)/df2)-df1;             /* Adjusted estimated lambda */

        if adjEstLambda < 0 then adjEstLambda = 0;              /* Note 7 **********************/
                                                                /*                             */
                                                                /* For some random seeds the   */
                                                                /* noncentrality parameter is  */
                                                                /* negative.                   */
                                                                /*                             */
                                                                /* A negative noncentrality    */
                                                                /* parameter occurs when       */
                                                                /* the expression              */
                                                                /* estLambda*(df2-2)/df2 is    */
                                                                /* less than df1.              */
                                                                /*                             */
                                                                /******************************/


        alphaL = alphaLU(df1,df2,adjEstLambda,&alpha,eps);      /* Lower significance */
        alphaU = &alpha-alphaL;                                 /* Upper significance */
```

```
    adjAlphaL = 1-alphaL;                                /* Note 8 **********************************/
    probCenF = cdf('F',fSamp,df1,df2);                   /*                                         */
                                                         /* fnonct reports an error for a supplied */
                                                         /* probability greater than the           */
                                                         /* probability from the cdf of a central  */
                                                         /* F distribution.                        */
                                                         /*                                         */
                                                         /* As the probability supplied gets large */
                                                         /* fnonct tends toward zero.  probCenF    */
                                                         /* with fnonct is small around zero.      */
                                                         /* Using adjAlphaL would then be smaller  */
                                                         /* about zero.  Set to zero.              */
                                                         /*                                         */
                                                         /*******************************************/


    if adjAlphaL > probCenF then lambdaL = 0;            /* See note 8 */
        else lambdaL = fnonct(fSamp,df1,df2,adjAlphaL);
    if alphaU > probCenF then lambdaU = 0;               /* See note 8 */
        else lambdaU = fnonct(fSamp,df1,df2,alphaU);


    estPower = 1-cdf('F',fCrit,df1,df2,adjEstLambda);    /* Estimated power */
    powerL = 1-cdf('F',fCrit,df1,df2,lambdaL);           /* Lower limit for power */
    powerU = 1-cdf('F',fCrit,df1,df2,lambdaU);           /* Upper limit for power */

    /* Output simulation to a data set */

    Y = {"&muA" "&muB" "&muC" "&sigma" "&sampleSize" "&alpha" "." "." "." "." "." "." "." "." "." "."};
    Y = num(Y);
    Y[7] = pValue;
    Y[8] = trueLambda;
    Y[9] = truePower;
    Y[10] = estLambda;
    Y[11] = adjEstLambda;
    Y[12] = lambdaL;
    Y[13] = lambdaU;
    Y[14] = estPower;
    Y[15] = powerL;
    Y[16] = powerU;

    create sim&simNum from Y;
    append from Y;

quit;
```

```
    proc datasets lib=work nolist;
        delete sample1 sample2 sample3 sampleThreePops;
    quit;
    run;

%MEND simulation3;
```

Appendix B: Three-Sample SAS Code for Power Simulations

```
/* sampleNormPop3 *************************************************************/
/*                                                                            */
/* Program Description:                                                       */
/* sampleNormPop3 generates a random sample from a normal population          */
/* and outputs the values to a dataset                                        */
/*                                                                            */
/* Input:                                                                     */
/* seed       := dataset of random seeds from randSeed                        */
/* popMean    := population mean                                              */
/* popSD      := population standard deviation                                */
/* seedIt     := seed value position for generating random observations       */
/* numObs     := sample size                                                  */
/* sampleNum  := identifies sample                                            */
/*                                                                            */
/******************************************************************************/


%macro sampleNormPop3(popMean=,popSD=,seedIt=,numObs=,sampleNum=);

proc iml;
    reset nolog;
    use randSeed;
    read all into seed;

    x&sampleNum = &popMean+&popSD*rannor(J(&numObs,1,seed[&seedIt,&sampleNum]));
    test = seed[&seedIt,&sampleNum];

    create sample&sampleNum from x&sampleNum[colname={obs&sampleNum}];
    append from x&sampleNum;

quit;

%mend sampleNormPop3;
```

Appendix B: Three-Sample SAS Code for Power Simulations

```
/* avgMedMultipleSim3 ***********************************************/
/*                                                                  */
/* Requirement(s):                                                  */
/* avgMedSim3.sas                                                   */
/*                                                                  */
/* Program Description:                                             */
/* avgMedMultipleSim3 calls avgMedSim3 for each sigma by sample size */
/* combination.                                                     */
/*                                                                  */
/* Input:                                                           */
/* itM          := number of iterations                             */
/* nsXnss       := number of iteration groups                       */
/*              (sigmas times number of sample sizes)               */
/*                                                                  */
/********************************************************************/


%macro avgMedMultipleSim3(itM=,nsXnss=);

    /* Produce average and median of simulations */

        %let bStart = 1;
        %let bFinish = &itM;

        %do i = 1 %to &nsXnss %by 1;
            %avgMedSim3(start=&bStart,finish=&bFinish,itB=&i);
            %let bStart = &bStart+&itM;
            %let bFinish = &bFinish+&itM;
        %end;


    /* Combine average simulations into a data set and name columns */

    data avgMedMultipleSimOut;
        set
            %do j = 1 %to &nsXnss %by 1;
                amSim&j
            %end;
            ;   /* Double do-loop to reference data sets.  ';' closes 'set' */
        rename col1=mu1 col2=mu2 col3=mu3 col4=sigma col5=sampleSize col6=sigLevel col7=pValue col8=trueLambda
col9=truePower col10=estLambda col11=adjEstLambda col12=lambdaL col13=lambdaU col14=estPower col15=powerL
col16=powerU col17=medPower;

    run;

%mend avgMedMultipleSim3;
```

```
/* avgMedSim3 *******************************************************/
/*                                                                  */
/* Program Description:                                             */
/* avgMedSim3 averages over a user specified number of rows from    */
/* multipleSimOut.  avgMedSim3 also determines the median           */
/* of the calculated power values of the rows.                      */
/*                                                                  */
/* Input:                                                           */
/* X            := gets dataset multipleSimOut                      */
/* start        := beginning of iteration block                     */
/* finish       := end of iteration block                           */
/* itB          := number of iterations                             */
/*                                                                  */
/********************************************************************/


%macro avgMedSim3(start=,finish=,itB=);

proc iml;

    reset nolog;
    use multipleSimOut;
    read all into X;

    /* Average rows for an iteration block */

    rows = X[&start:&finish,];
    avgRows = rows[:,];


    /* Median for column calculated power */

    col = X[&start:&finish,14];                         /* Column of calculated power */

    numRows = nrow(col);                                /* Number of rows */

    mis = 0;                                            /* Storage for positions of missing values */
    i = 1;

    do while (i <=numRows);                             /* Note 1 ***********************/
        if missing((col[i,1])) > 0 then mis=mis//i;     /*                              */
        i=i+1;                                          /* Loop through column of       */
    end;                                                /* of calculated power values.  */
                                                        /* If a missing value is found  */
                                                        /* record array position.       */
                                                        /*                              */
                                                        /********************************/
```

```
    nr = nrow(mis);                                     /* Number of missing values */

                                                        /* Note 2 *******************/
                                                        /*                          */
                                                        /* If 'nr' is greater than  */
                                                        /* one then we have missing */
                                                        /* values.  These values are */
                                                        /* removed from the column. */
                                                        /*                          */
                                                        /***************************/

    if nr>1 then mis = remove(mis,1);                   /* Remove initialize position */
    if nr>1 then col = col`;                            /* Transpose a column to row */
    if nr>1 then col = remove(col,mis);                 /* Remove missing values */
    if nr>1 then col = col`;                            /* Transpose row to a column */

    med = median(col);                                  /* Find median */


    /* Concatinate median to average row */

    avgRows=avgRows||med;


    create amSim&itB from avgRows;                      /* Output to a dataset */
    append from avgRows;

quit;
%mend avgMedSim3;
```

Appendix B: Four-Sample SAS Code for Power Simulations

```
/* fourSample *************************************************************/
/*                                                                        */
/* Requirement(s):                                                        */
/* rand4.csv, multipleSim4.sas, avgMedMultipleSim4.sas                    */
/*                                                                        */
/* Program Description:                                                   */
/* fourSample.sas acts as a controller interface which accepts user       */
/* defined values for four population means, upper and lower bounds       */
/* for a range of population standard deviations, a maximum sample size,  */
/* a type I error rate, and the number of iterations per sample size      */
/* to population standard deviation combination.                          */
/*                                                                        */
/**************************************************************************/


proc printto print='\Path\to\Output\Folder\fourSampleOut.lst'
             log='\Path\to\Output\Folder\fourSampleOut.log';
run;



/* Input random seed file to the dataset randseed ************************/
/*                                                                        */
/* rand4.csv is a four column file of random numbers generated from       */
/* http:/www.random.org.  For each iteration of multipleSim4 four random  */
/* values are used to seed four random samples.                           */
/*                                                                        */
/**************************************************************************/

data randSeed;
    infile "\Path\to\File\rand4.csv" dlm=',';
    input seedCol1 seedCol2 seedCol3 seedCol4;
run;


/* Produce simulations ***********************************************/
/*                                                                  */
/* The multipleSim4 manages the production of simulations.          */
/* Calls are made to the macro simulation4.sas which performs       */
/* calculations based on Thomas 1997.                               */
/*                                                                  */
/* Calculations for each simulation are stored in a dataset called  */
/* multipleSimOut.                                                  */
/*                                                                  */
/********************************************************************/

%multipleSim4(mu1=30,mu2=40,mu3=35,mu4=35,sigmaMin=5,sigmaMax=40,itMax=100,sampleSizeMax=50,sigLevel=0.05);
```

```


/* Print multipleSimOut */

proc print data=multipleSimOut;
run;


/* Produce averages and medians for simulations ***********************/
/*                                                                    */
/* The avgMedMultipleSim4 manages the consolidation of simulations.   */
/* Calls are made to avgMedSim4 for each sigma by sample size         */
/* combination.  avgMedSim4 averages over a user specified number     */
/* of rows from multipleSimOut.  avgMedSim4 also determines the       */
/* median of the calculated power values of the rows.                 */
/*                                                                    */
/* Calculations for each iteration block are stored in a dataset      */
/* called avgMedMultipleSimOut.                                       */
/*                                                                    */
/**********************************************************************/

%avgMedMultipleSim4(itM=100,nsXnss=306);


/* Print avgMedMultipleSimOut */

proc print data=avgMedMultipleSimOut;
run;


/* Plot avgMedMultipleSimOut */

goptions ftitle=swiss ftext=swiss;
symbol1 value=square interpol=join width=0.5 color=blue;
symbol2 value=star interpol=join width=0.5 color=red;
symbol3 value=triangle interpol=join width=0.5 color=yellow;
symbol4 value='x' interpol=join width=0.5 color=orange;
symbol5 value=circle interpol=join width=0.5 color=blue;
symbol6 value=dot interpol=join width=0.5 color=green;
title2 height=1.4 "Sample Size vs Power";
axis1 order=(0.0 to 1.0 by 0.1) label=(angle=90 'Power') minor=none;
axis2 order=(5 to 50 by 5) label=(angle=0 'Sample Size');
legend label=none value=(h=.8 'Upper AVG Estimated Power' 'True Power' 'AVG Estimated Power' 'Median Power'
       'Lower AVG Estimated Power' 'AVG P-Value')
       position=center;
```

```
proc gplot data=avgMedMultipleSimOut;
    plot powerU*sampleSize truePower*sampleSize estPower*sampleSize medPower*sampleSize powerL*sampleSize
    pValue*sampleSize/overlay vaxis=axis1 haxis=axis2 hminor=4 legend=legend1;
    by sigma;
run;
```

Appendix B: Four-Sample SAS Code for Power Simulations

```
/* multipleSim4 ***************************************************************/
/*                                                                            */
/* Requirement(s):                                                            */
/* simulation4.sas                                                            */
/*                                                                            */
/* Program Description:                                                       */
/* multipleSim4 Iterates simulation4.sas and combines simulation output to a  */
/* dataset.                                                                   */
/*                                                                            */
/* Input:                                                                     */
/* mu1           := population one mean                                       */
/* mu2           := population two mean                                       */
/* mu3           := population three mean                                     */
/* mu4           := population four mean                                      */
/* simgaMin      := minimum population standard deviation controller          */
/* sigmaMax      := maximum population standard deviation controller          */
/* itMax         := number of iterations                                      */
/* sampleSizeMax := largest sample size considered                            */
/* sigLevel      := level of significance                                     */
/*                                                                            */
/* Note(s):                                                                   */
/* In the subsubloop iterations run in blocks.  This is done to               */
/* traverse the dataset randSeed, where by each call to the macro             */
/* simulation4 gets a different random seed position.                         */
/*                                                                            */
/* This macro contains two main loops.  The first main loop                   */
/* iterates from the minimum user define population standard deviation to 19  */
/* by 1.  The second main loop iterates from 20 to the maximum user defined   */
/* population standard deviaiton by 20.  Each main loop has two subloops.     */
/* The first subloop iterates from a system defined minimum sample size of 5  */
/* to 14 by 1.  The second subloop iterates from 15 to the maximum user       */
/* defined sample size by 5.  Each subloop has a subsubloop.                  */
/* Each subsubloop iterates by a use defined number of iterations.  After     */
/* a subsubloop executes, the starting position of an iteration 'block' is    */
/* increased by the the user defined number of iterations.                    */
/*                                                                            */
/******************************************************************************/


%macro multipleSim4(mu1=,mu2=,mu3=,mu4=,sigmaMin=,sigmaMax=,itMax=,sampleSizeMax=,sigLevel=);

%local std sd nn it block s n i b;
%let block = 1;
%let b = 1;
%let count = 1;
```

```
/* Produce simulations */

    %do std = &sigmaMin %to 19 %by 1;
    %let sd = %sysevalf(&std/2);
        %do nn = 5 %to 14 %by 1;
            %do it = &block %to &block+&itMax-1 %by 1;

%simulation4(muA=&mu1,muB=&mu2,muC=&mu3,muD=&mu4,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count)
;
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
        %do nn = 15 %to &sampleSizeMax %by 5;
            %do it = &block %to &block+&itMax-1 %by 1;

%simulation4(muA=&mu1,muB=&mu2,muC=&mu3,muD=&mu4,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count)
;
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
    %end;
    %do std = 20 %to &sigmaMax %by 20;
    %let sd = %sysevalf(&std/2);
        %do nn = 5 %to 14 %by 1;
            %do it = &block %to &block+&itMax-1 %by 1;

%simulation4(muA=&mu1,muB=&mu2,muC=&mu3,muD=&mu4,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count)
;
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
        %do nn = 15 %to &sampleSizeMax %by 5;
            %do it = &block %to &block+&itMax-1 %by 1;

%simulation4(muA=&mu1,muB=&mu2,muC=&mu3,muD=&mu4,sigma=&sd,seedNum=&it,sampleSize=&nn,alpha=&sigLevel,simNum=&count)
;
            %let count = %eval(&count+1);
            %end;
            %let block = &it;
        %end;
    %end;
```

```
/* Combine simulations into a dataset and name columns */

    data multipleSimOut;
        set
            %do k = 1 %to &count-1 %by 1;
                sim&k
            %end;
            ;   /* Double do-loop to reference datasets.  ';' closes 'set' */

        rename col1=mu1 col2=mu2 col3=mu3 col4=mu4 col5=sigma col6=sampleSize col7=sigLevel col8=pValue
col9=trueLambda col10=truePower col11=estLambda col12=adjEstLambda col13=lambdaL col14=lambdaU col15=estPower
col16=powerL col17=powerU;

    run;


%mend multipleSim4;

```

```
/* simulation4 ***********************************************************/
/*                                                                       */
/* Requirement(s):                                                       */
/* sampleNormPop4.sas                                                    */
/*                                                                       */
/* Program Description:                                                  */
/* simulation4.sas performs a hypothesis                                 */
/* test of no difference for four population means.  Random samples      */
/* are normally distributed with respective means muA, muB, muC and muD  */
/* with equal population standard deviation sigma.  Random samples are   */
/* generated using seed values from the dataest randSeed.  Power         */
/* calculations are based on methods outlined in Thomas 1997.            */
/* Calculations are saved to a dataset named 'sim#' where # represents   */
/* the simulation count.                                                 */
/*                                                                       */
/* Input:                                                                */
/* muA          := mean of population one                                */
/* muB          := mean of population two                                */
/* muC          := mean of population three                              */
/* muD          := mean of population four                               */
/* sigma        := standard deviation of populations one and two         */
/* seedNum      := position for random seed                              */
/* sampleSize   := sample size taken for populations one and two         */
/* alpha        := level of significance                                 */
/* simNum       := simulation ID                                         */
/*                                                                       */
/* Note(s):                                                              */
/* Optimal lower significance level by Golden Section Search method      */
/*                                                                       */
/*************************************************************************/


%MACRO simulation4(muA=,muB=,muC=,muD=,sigma=,seedNum=,sampleSize=,alpha=,simNum=);

    /* Generate random samples from four normal populations */

        %sampleNormPop4(popMean=&muA,popSD=&sigma,seedIt=&seedNum,numObs=&sampleSize,sampleNum=1);
        %sampleNormPop4(popMean=&muB,popSD=&sigma,seedIt=&seedNum,numObs=&sampleSize,sampleNum=2);
        %sampleNormPop4(popMean=&muC,popSD=&sigma,seedIt=&seedNum,numObs=&sampleSize,sampleNum=3);
        %sampleNormPop4(popMean=&muD,popSD=&sigma,seedIt=&seedNum,numObs=&sampleSize,sampleNum=4);


    /* Merge datasets sample1, sample2, sample3 and sample4 to produce dataset sampleFourPops */

    data sampleFourPops;
        merge sample1 sample2 sample3 sample4;
    run;
```

```


/* Main */

proc iml;
    reset nolog;
    use sampleFourPops;
    read all into X;

    /* Function ***********************************************************/
    /*                                                                    */
    /* Return the confidence interval width for a noncentral              */
    /* f-dsitribution                                                     */
    /*                                                                    */
    /**********************************************************************/

    start w(alphaLow,ndf,ddf,ncent,sl);

        width = quantile('F',1-sl+alphaLow,ndf,ddf,ncent)-quantile('F',alphaLow,ndf,ddf,ncent);
        return(width);

    finish w;

    /* Function: optimal lower critical region ****************************/
    /*                                                                    */
    /* Finds the minimum confidence interval width using an               */
    /* implementation of the Golden Section Search method                 */
    /* similar to Press et al. (2007).                                    */
    /*                                                                    */
    /* Note(s):                                                           */
    /* The Golden Section Search method finds a value which minimizes     */
    /* a function.  The function must be one dimensional and unimodal.    */
    /*                                                                    */
    /* Imagine a 'typical' pdf of an F distribution.  Say we want to      */
    /* setup an interval such that the area covered by the sum of the two */
    /* tails is alpha.  For each alpha we can find the                    */
    /* associated quantile using the SAS function 'quantile'.             */
    /* Since alpha is specified, knowing the lower area easily leads to   */
    /* the upper area.  Here we consider only the lower                   */
    /* area.  As such we can relate interval width as a function of       */
    /* lower alpha size.  As we increase or decrease the size of the      */
    /* lower area we increase or decrease the width of the interval.      */
    /* Assuming a minimum interval width exists, we optimize over the     */
    /* values [0.00, 0.05].                                               */
    /*                                                                    */
    /**********************************************************************/

```

```
      start alphaLU(ndf1,ddf2,ncp,slAlpha,tol);

          alphaA = 0;                                               /* Initial lower bracket on optimal region */
          alphaB = slAlpha;                                         /* Initial upper bracket on optimal region */

                                                                    /* Note 1 **********************************/
                                                                    /*                                         */
                                                                    /* For each iteration an interval          */
                                                                    /* [alphaA, alphaB] is retained            */
                                                                    /*                                         */
                                                                    /*******************************************/

          phi = (-1+sqrt(5))/2;                                     /* Golden ratio constant reduction factor */
          a1 = alphaA+phi*(alphaB-alphaA);                          /* Set test bracket a1 */
          wa1 = w(a1,ndf1,ddf2,ncp,slAlpha);                        /* Calculate Interval width for a1 */
          a2 = alphaA+phi*phi*(alphaB-alphaA);                      /* Set test bracket a2 */
          wa2 = w(a2,ndf1,ddf2,ncp,slAlpha);                        /* Calculate interval width for a2 */

                                                                    /* Note 2 ************************************/
                                                                    /*                                           */
                                                                    /* The intial setup invokes two function     */
                                                                    /* calls.  The following loop uses one       */
                                                                    /* function call per iteration.              */
                                                                    /*                                           */
                                                                    /*********************************************/

          do while (abs(alphaB-alphaA)>tol);                        /* Note 3 ************************************/
                                                                    /*                                           */
                                                                    /* We continue until the absolute difference */
                                                                    /* between an upper and a lower bracket is   */
                                                                    /* negligable. Tolerance is set to1e-4.      */
                                                                    /*                                           */
                                                                    /*********************************************/

                                                                    /* Note 4 ************************************/
                                                                    /*                                           */
                                                                    /* Four brackets are maintained at any       */
                                                                    /* given time.                               */
                                                                    /*                                           */
                                                                    /*********************************************/

              if wa1>wa2 then do;                                   /* If wa1 > wa2 then minimum width is between */
                                                                    /* alphaA and a1                              */
```

```
                                                        /* Note 5 ***********************************/
                                                        /*                                          */
                                                        /* Within the interval [alphaA, a1] we have  */
                                                        /* the width wa2.  a2 is situated by the     */
                                                        /* expression alphaA+phi*(a1-alphaA).        */
                                                        /*                                          */
                                                        /********************************************/

            alphaB = a1;                                /* Updated alphaB */
            a1 = a2;                                    /* Save a2 */
            wa1 = wa2;                                  /* Save wa2 */
            a2 = alphaA+phi*phi*(alphaB-alphaA);        /* Set new test bracket a2 */
            wa2 = w(a2,ndf1,ddf2,ncp,slAlpha);          /* Calculate interval width for new a2 */
        end;
        else do;                                        /* If wa1 < wa2 then minimum width is between */
                                                        /* a2 and alphaB                               */

                                                        /* Note 6 ***********************************/
                                                        /*                                          */
                                                        /* Within the interval [a2, alphaB] we have  */
                                                        /* the width wa1.  a1 is situated by the     */
                                                        /* expression alphaA+phi*phi*(alphaB-a2).    */
                                                        /*                                          */
                                                        /********************************************/

            alphaA = a2;                                /* Updated alphaA */
            a2 = a1;                                    /* Save a1 */
            wa2 = wa1;                                  /* Save wa1 */
            a1 = alphaA+phi*(alphaB-alphaA);            /* Set new test bracket a1 */
            wa1 = w(a1,ndf1,ddf2,ncp,slAlpha);          /* Calculate interval width for new a1 */
        end;
    end;

    out = (alphaA+alphaB)/2;
    return(out);

finish alphaLU;


/* Calculations */

eps = 1e-4;                                             /* Stopping criteria for GSS method */
n = nrow(X);                                            /* Number of rows */
p = ncol(X);                                            /* Number of columns */
J = J(p,1);
muG = (&muA+&muB+&muC+&muD)/p;                          /* Mean of population means */
xBar = X(|+,|)`/n;                                      /* Sample means */
```

```
sst = n*xBar`*xBar-(n/p)*(xBar`*J)**2;                  /* Sum of squares for treatments */
df1 = p-1;                                              /* Degrees of freedom for treatment */
mst = sst/df1;                                          /* Mean square for treatment */
sse = ssq(X)-(xBar`*xBar)*n;                            /* Sum of squares for error */
df2 = p*(n-1);                                          /* Degrees of freedom for error */
mse = sse/df2;                                          /* Mean square for error */
fSamp = mst/mse;                                        /* Observed F */

fCrit = quantile('F',1-&alpha,df1,df2);                 /* F critical value */

pValue = 1-cdf('F',fSamp,df1,df2);                      /* P-Value */

trueLambda = (n*(((&muA-muG)**2)+((&muB-muG)**2)+((&muC-muG)**2)+((&muD-muG)**2)))/((&sigma)**2);
                                                        /* trueLambda */

truePower = 1-cdf('F',fCrit,df1,df2,trueLambda);        /* truePower */

estLambda = fSamp*(p-1);                                /* Estimated lambda */
adjEstLambda = (estLambda*(df2-2)/df2)-df1;             /* Adjusted estimated lambda */

if adjEstLambda < 0 then adjEstLambda = 0;              /* Note 7 *********************/
                                                        /*                            */
                                                        /* For some random seeds the   */
                                                        /* noncentrality parameter is  */
                                                        /* negative.                   */
                                                        /*                             */
                                                        /* A negative noncentrality    */
                                                        /* parameter occurs when       */
                                                        /* the expression              */
                                                        /* estLambda*(df2-2)/df2 is    */
                                                        /* less than df1.              */
                                                        /*                             */
                                                        /*******************************/


alphaL = alphaLU(df1,df2,adjEstLambda,&alpha,eps);      /* Lower significance */
alphaU = &alpha-alphaL;                                 /* Upper significance */
```

```
        adjAlphaL = 1-alphaL;                                   /* Note 8 ********************************/
        probCenF = cdf('F',fSamp,df1,df2);                      /*                                       */
                                                                /* fnonct reports an error for a supplied */
                                                                /* probability greater than the          */
                                                                /* probability from the cdf of a central  */
                                                                /* F distribution.                       */
                                                                /*                                       */
                                                                /* As the probability supplied gets large */
                                                                /* fnonct tends toward zero.  probCenF    */
                                                                /* with fnonct is small around zero.      */
                                                                /* Using adjAlphaL would then be smaller  */
                                                                /* about zero.  Set to zero.              */
                                                                /*                                       */
                                                                /*****************************************/


        if adjAlphaL > probCenF then lambdaL = 0;               /* See note 8 */
            else lambdaL = fnonct(fSamp,df1,df2,adjAlphaL);
        if alphaU > probCenF then lambdaU = 0;                  /* See note 8 */
            else lambdaU = fnonct(fSamp,df1,df2,alphaU);


        estPower = 1-cdf('F',fCrit,df1,df2,adjEstLambda);       /* Estimated power */
        powerL = 1-cdf('F',fCrit,df1,df2,lambdaL);              /* Lower limit for power */
        powerU = 1-cdf('F',fCrit,df1,df2,lambdaU);              /* Upper limit for power */

        /* Output simulation to a data set */

        Y = {"&muA" "&muB" "&muC" "&muD" "&sigma" "&sampleSize" "&alpha" "." "." "." "." "." "." "." "." "." "."};
        Y = num(Y);
        Y[8] = pValue;
        Y[9] = trueLambda;
        Y[10] = truePower;
        Y[11] = estLambda;
        Y[12] = adjEstLambda;
        Y[13] = lambdaL;
        Y[14] = lambdaU;
        Y[15] = estPower;
        Y[16] = powerL;
        Y[17] = powerU;

        create sim&simNum from Y;
        append from Y;

    quit;
```

```
    proc datasets lib=work nolist;
        delete sample1 sample2 sample3 sample4 sampleFourPops;
    quit;
    run;

%MEND simulation4;

```

Appendix B: Four-Sample SAS Code for Power Simulations

```
/* sampleNormPop4 *************************************************************/
/*                                                                            */
/* Program Description:                                                       */
/* sampleNormPop4 generates a random sample from a normal population          */
/* and outputs the values to a dataset                                        */
/*                                                                            */
/* Input:                                                                     */
/* seed       := dataset of random seeds from randSeed                        */
/* popMean    := population mean                                              */
/* popSD      := population standard deviation                                */
/* seedIt     := seed value position for generating random observations      */
/* numObs     := sample size                                                  */
/* sampleNum  := identifies sample                                            */
/*                                                                            */
/******************************************************************************/


%macro sampleNormPop4(popMean=,popSD=,seedIt=,numObs=,sampleNum=);

proc iml;
    reset nolog;
    use randSeed;
    read all into seed;

    x&sampleNum = &popMean+&popSD*rannor(J(&numObs,1,seed[&seedIt,&sampleNum]));
    test = seed[&seedIt,&sampleNum];

    create sample&sampleNum from x&sampleNum[colname={obs&sampleNum}];
    append from x&sampleNum;

quit;

%mend sampleNormPop4;
```

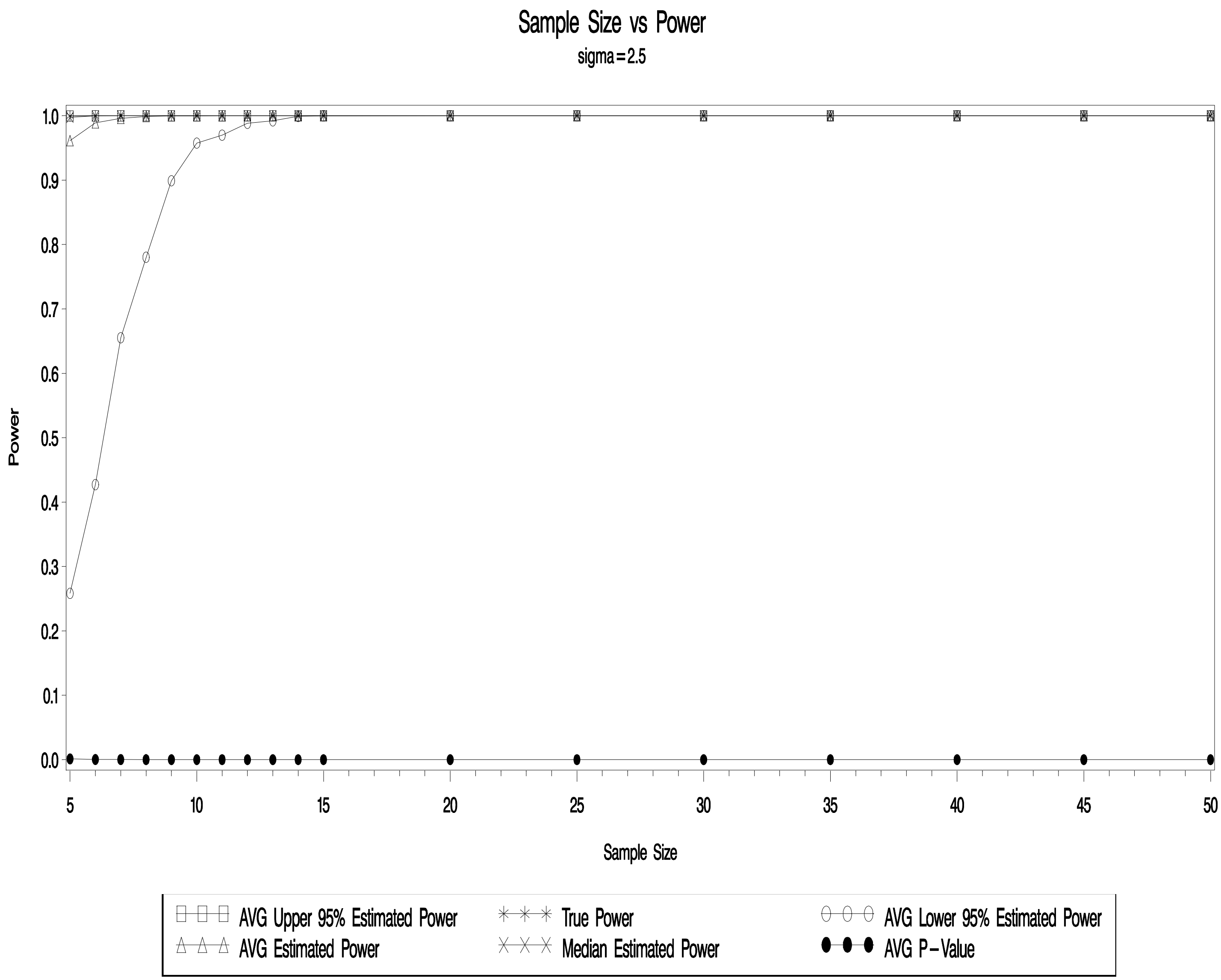
```
/* avgMedMultipleSim4 *************************************************/
/*                                                                    */
/* Requirement(s):                                                    */
/* avgMedSim4.sas                                                     */
/*                                                                    */
/* Program Description:                                               */
/* avgMedMultipleSim4 calls avgMedSim4 for each sigma by sample size */
/* combination.                                                       */
/*                                                                    */
/* Input:                                                             */
/* itM          := number of iterations                               */
/* nsXnss       := number of iteration groups                         */
/*                 (sigmas times number of sample sizes)              */
/*                                                                    */
/**********************************************************************/


%macro avgMedMultipleSim4(itM=,nsXnss=);

    /* Produce average and median of simulations */

        %let bStart = 1;
        %let bFinish = &itM;

        %do i = 1 %to &nsXnss %by 1;
            %avgMedSim4(start=&bStart,finish=&bFinish,itB=&i);
            %let bStart = &bStart+&itM;
            %let bFinish = &bFinish+&itM;
        %end;


    /* Combine average simulations into a data set and name columns */

    data avgMedMultipleSimOut;
        set
            %do j = 1 %to &nsXnss %by 1;
                amSim&j
            %end;
            ;   /* Double do-loop to reference data sets.  ';' closes 'set' */
        rename col1=mu1 col2=mu2 col3=mu3 col4=mu4 col5=sigma col6=sampleSize col7=sigLevel col8=pValue
col9=trueLambda col10=truePower col11=estLambda col12=adjEstLambda col13=lambdaL col14=lambdaU col15=estPower
col16=powerL col17=powerU col18=medPower;

    run;

%mend avgMedMultipleSim4;
```

Appendix B: Four-Sample SAS Code for Power Simulations

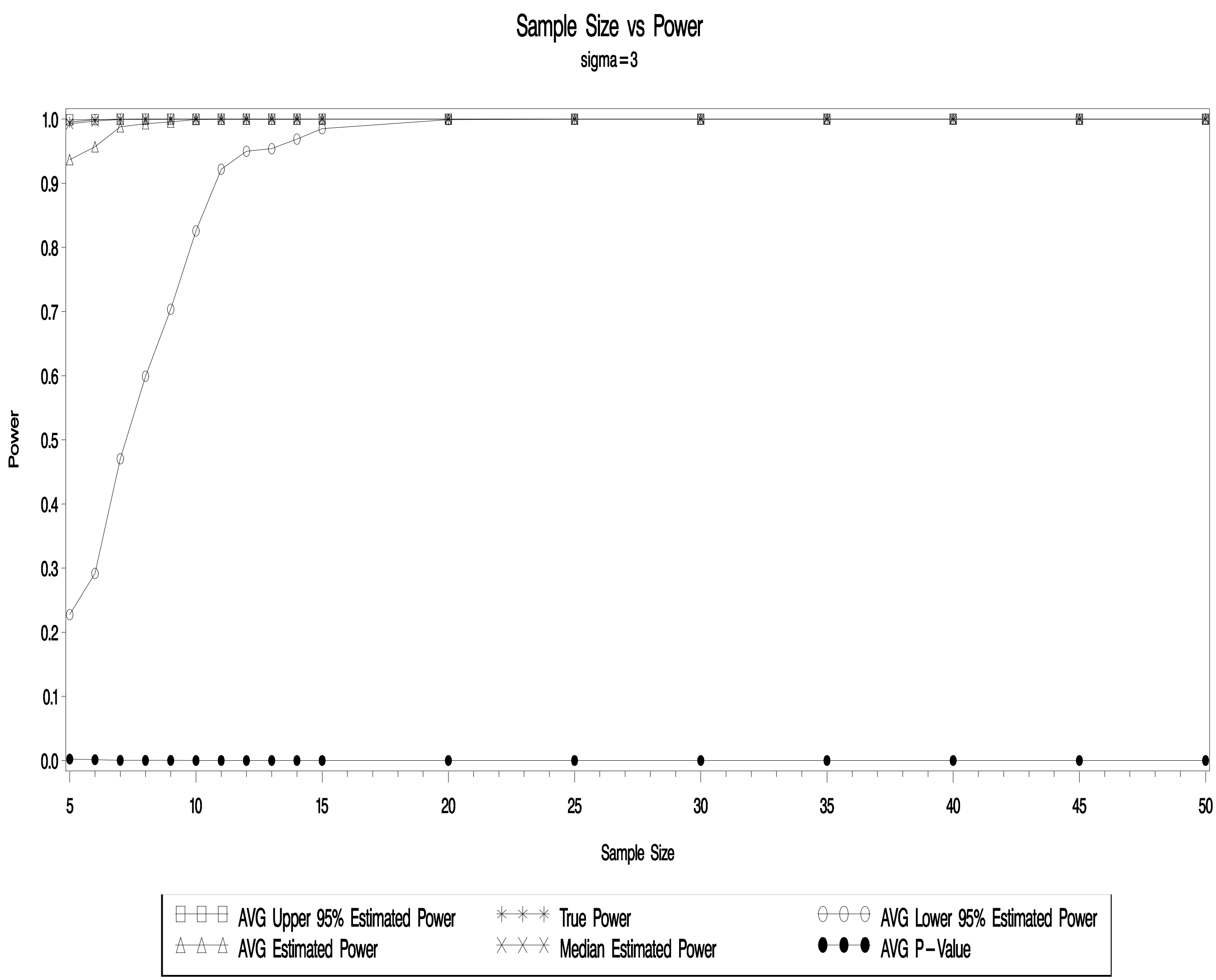
```
/* avgMedSim4 *******************************************************/
/*                                                                  */
/* Program Description:                                             */
/* avgMedSim4 averages over a user specified number of rows from    */
/* multipleSimOut.  avgMedSim4 also determines the median           */
/* of the calculated power values of the rows.                      */
/*                                                                  */
/* Input:                                                           */
/* X            := gets dataset multipleSimOut                      */
/* start        := beginning of iteration block                     */
/* finish       := end of iteration block                           */
/* itB          := number of iterations                             */
/*                                                                  */
/********************************************************************/


%macro avgMedSim4(start=,finish=,itB=);

proc iml;

    reset nolog;
    use multipleSimOut;
    read all into X;

    /* Average rows for an iteration block */

    rows = X[&start:&finish,];
    avgRows = rows[:,];


    /* Median for column calculated power */

    col = X[&start:&finish,15];                         /* Column of calculated power */

    numRows = nrow(col);                                /* Number of rows */

    mis = 0;                                            /* Storage for positions of missing values */
    i = 1;

    do while (i <=numRows);                             /* Note 1 ***********************/
        if missing((col[i,1])) > 0 then mis=mis//i;     /*                              */
        i=i+1;                                          /* Loop through column of       */
    end;                                                /* of calculated power values.  */
                                                        /* If a missing value is found  */
                                                        /* record array position.       */
                                                        /*                              */
                                                        /********************************/
```

```

    nr = nrow(mis);                                      /* Number of missing values */

                                                         /* Note 2 *******************/
                                                         /*                          */
                                                         /* If 'nr' is greater than  */
                                                         /* one then we have missing */
                                                         /* values.  These values are */
                                                         /* removed from the column. */
                                                         /*                          */
                                                         /****************************/

    if nr>1 then mis = remove(mis,1);                    /* Remove initialize position */
    if nr>1 then col = col`;                             /* Transpose a column to row */
    if nr>1 then col = remove(col,mis);                  /* Remove missing values */
    if nr>1 then col = col`;                             /* Transpose row to a column */

    med = median(col);                                   /* Find median */


    /* Concatinate median to average row */

    avgRows=avgRows||med;


    create amSim&itB from avgRows;                       /* Output to a dataset */
    append from avgRows;

quit;
%mend avgMedSim4;
```

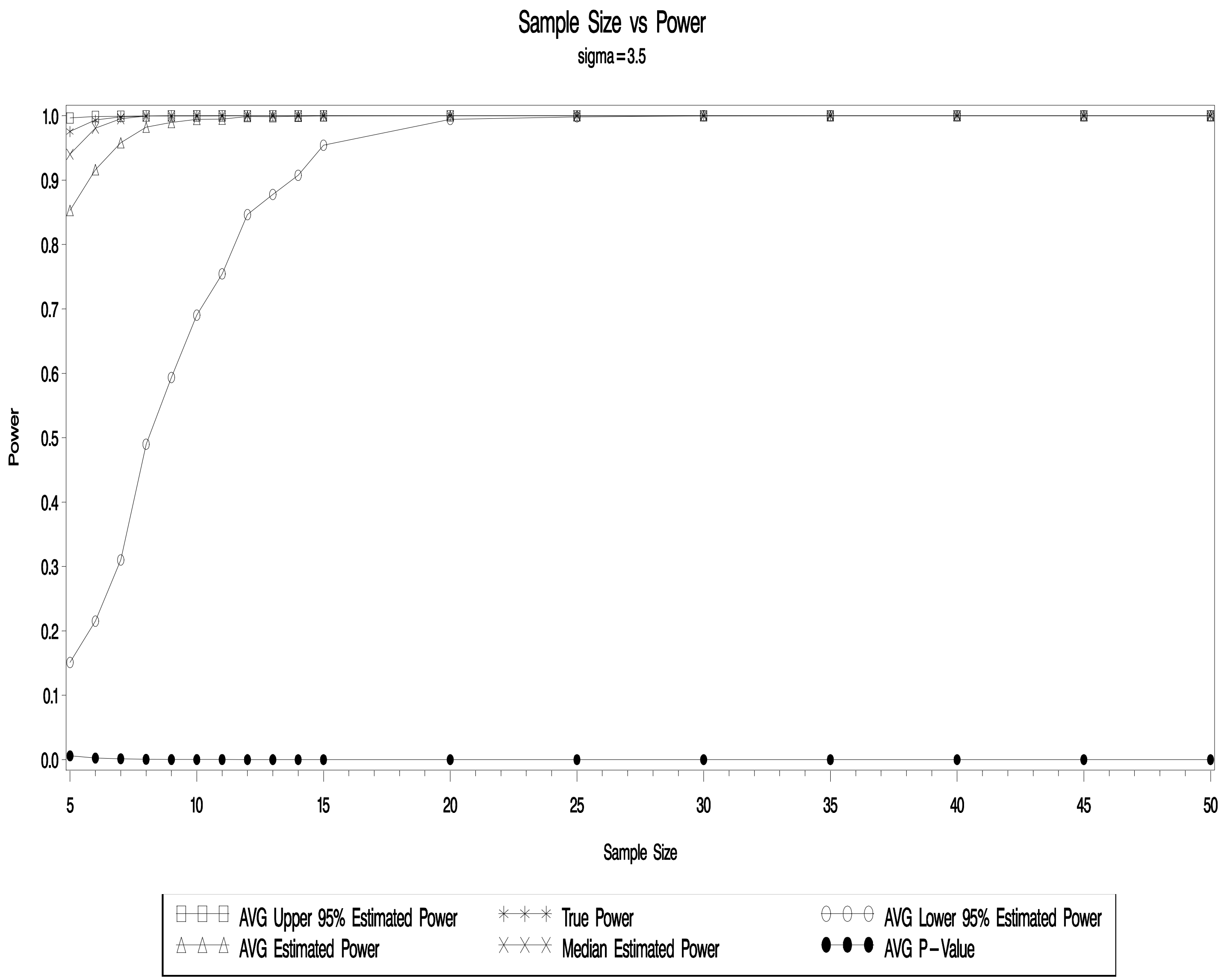
Appendix C: Graphs of Treatment Arrangement 30, 40



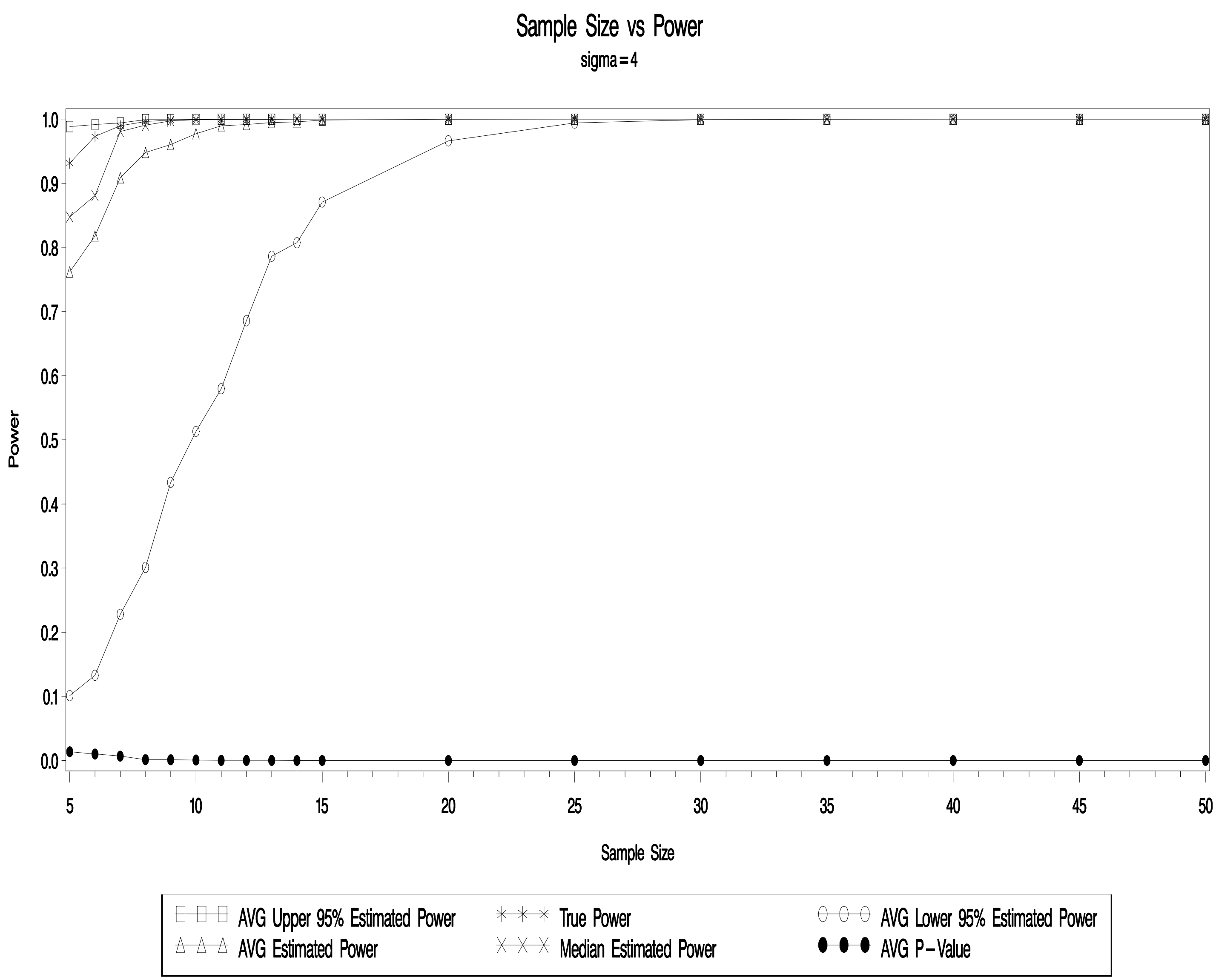
Appendix C: Graphs of Treatment Arrangement 30, 40



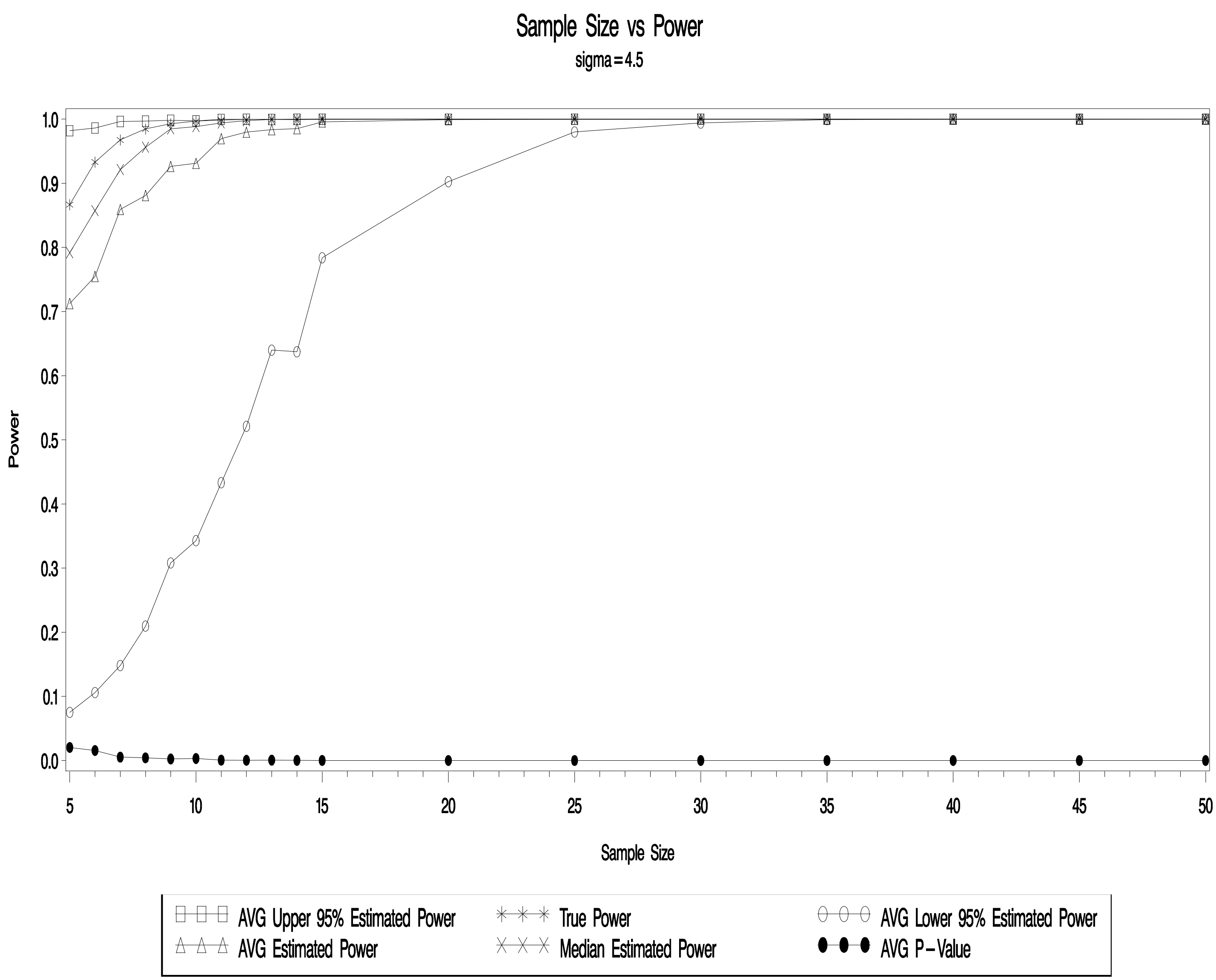
Appendix C: Graphs of Treatment Arrangement 30, 40



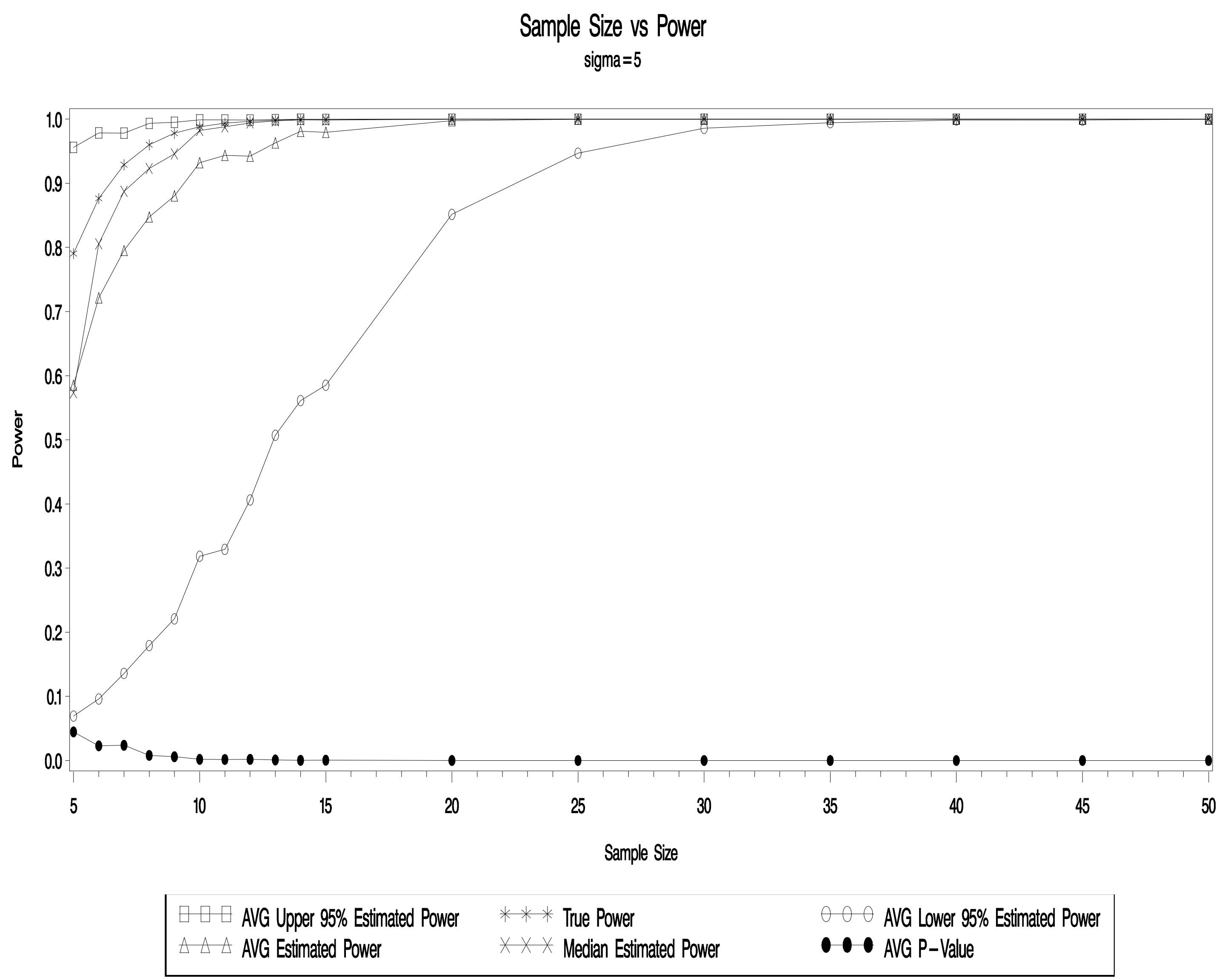
Appendix C: Graphs of Treatment Arrangement 30, 40



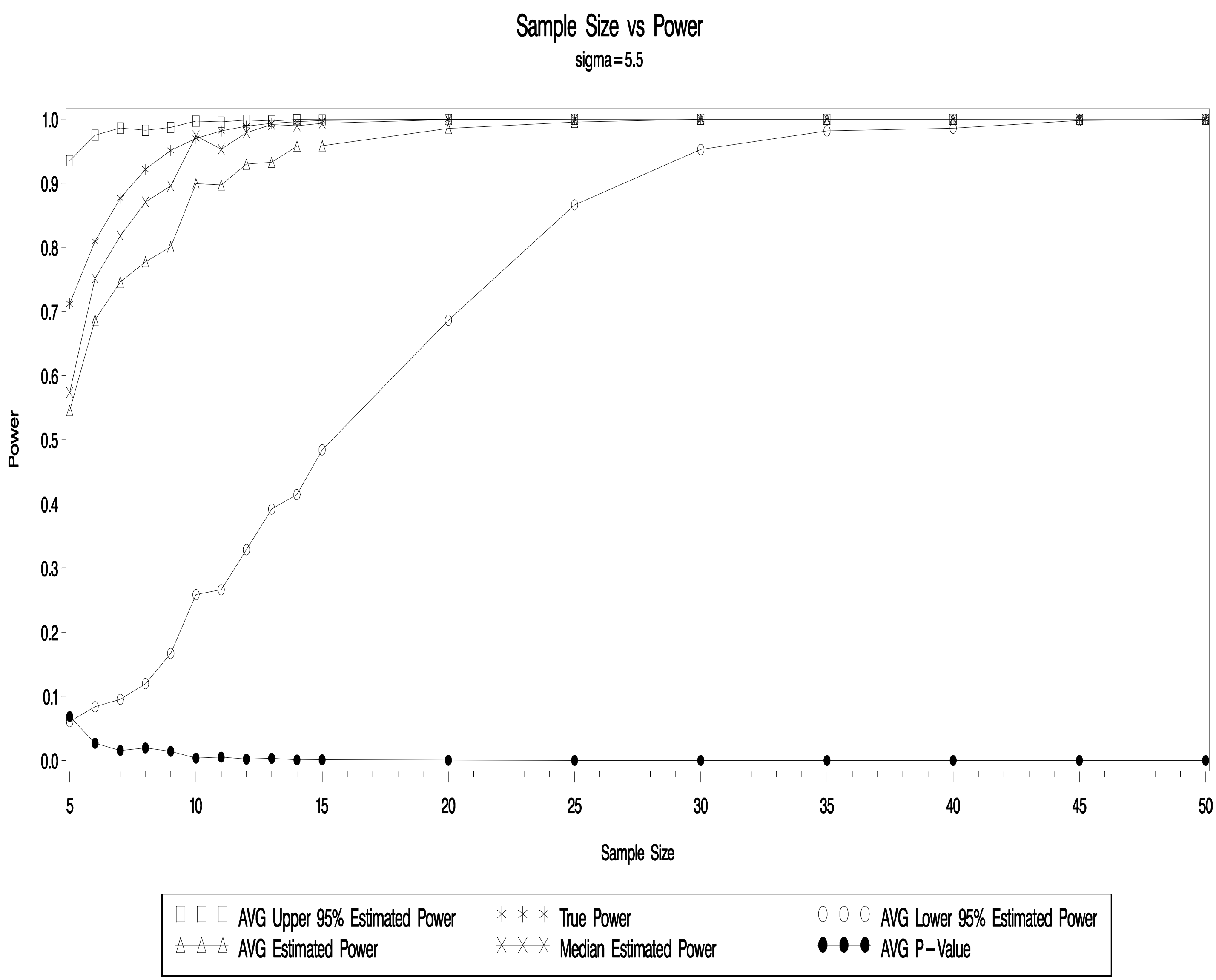
Appendix C: Graphs of Treatment Arrangement 30, 40



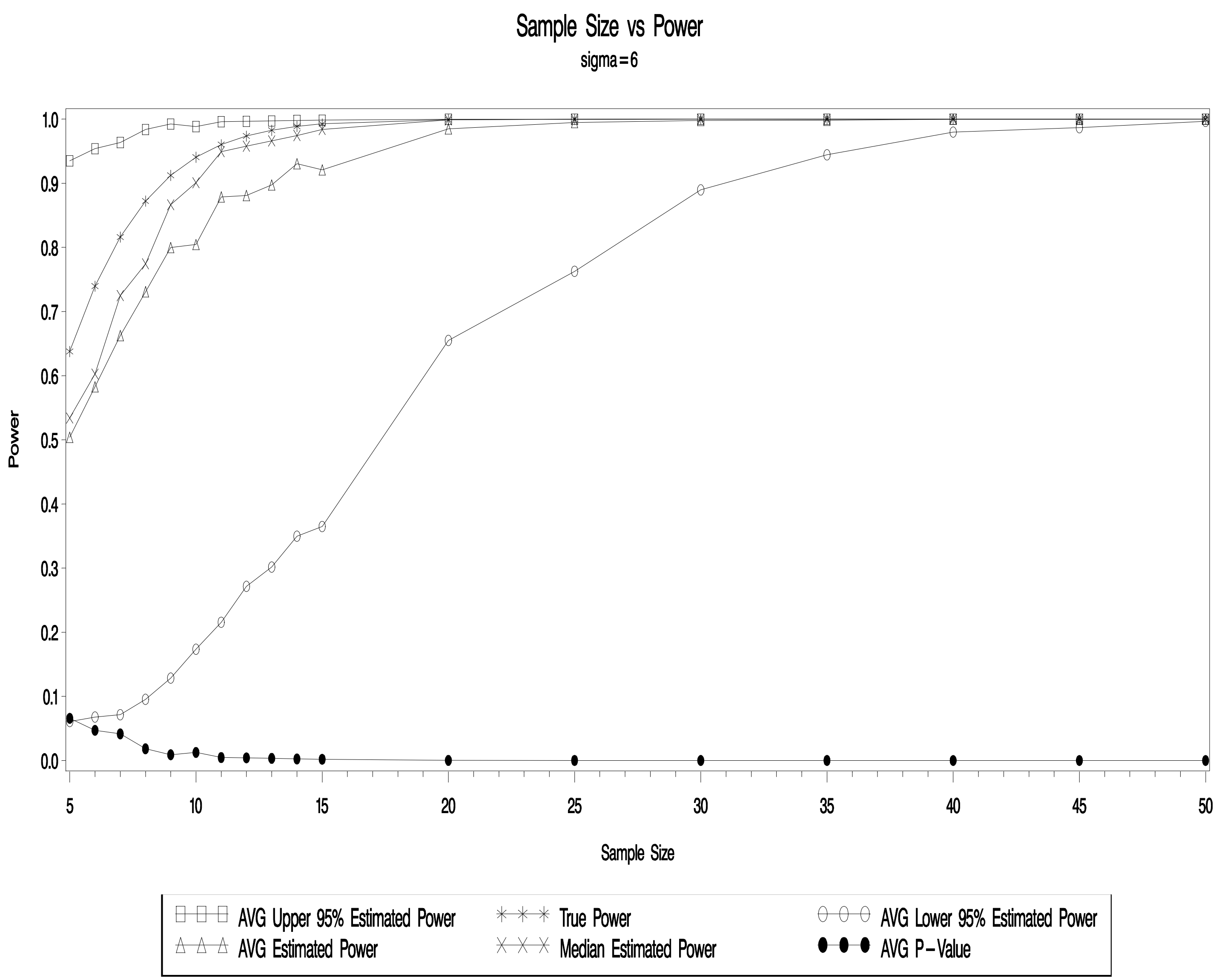
Appendix C: Graphs of Treatment Arrangement 30, 40



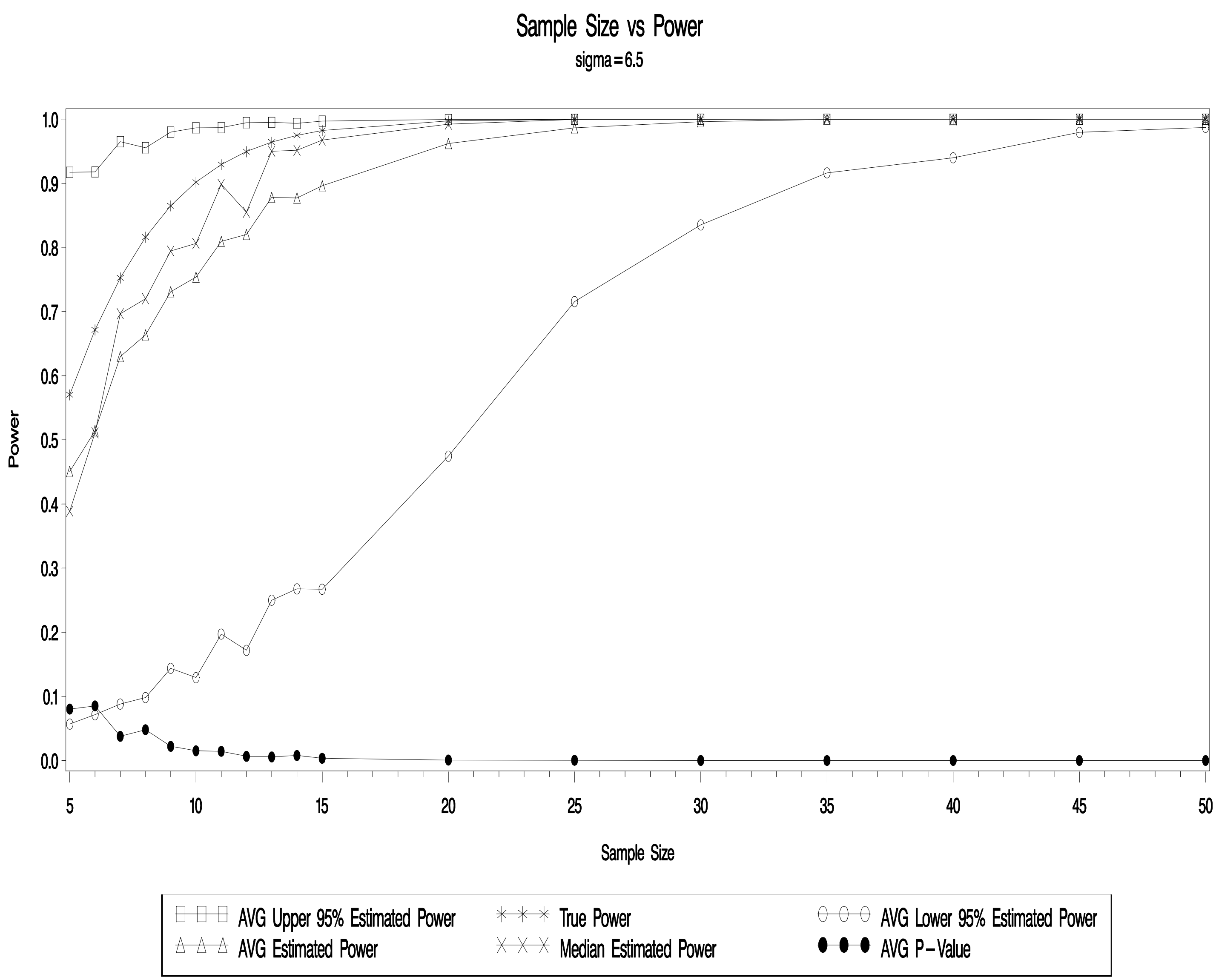
Appendix C: Graphs of Treatment Arrangement 30, 40



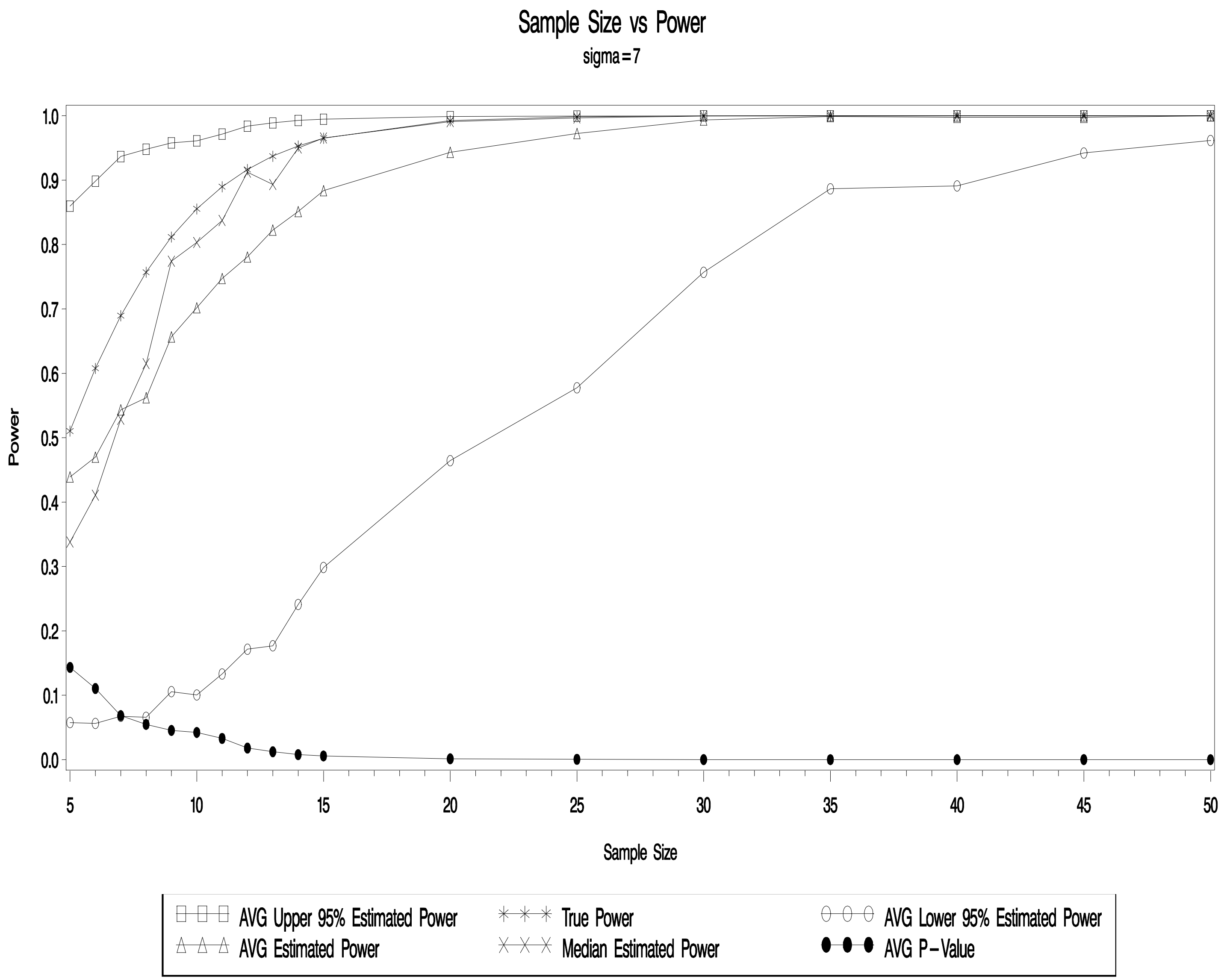
Appendix C: Graphs of Treatment Arrangement 30, 40



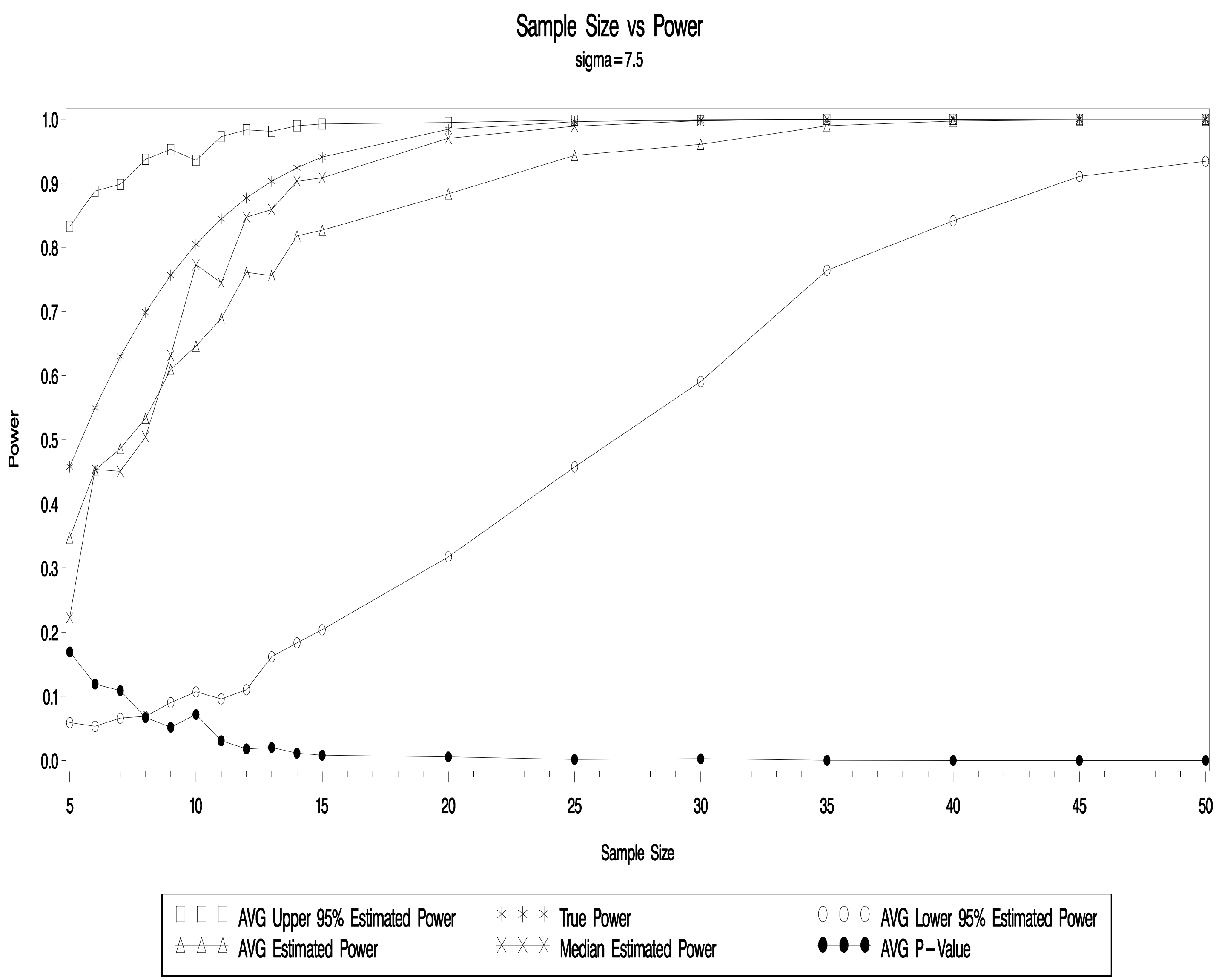
Appendix C: Graphs of Treatment Arrangement 30, 40



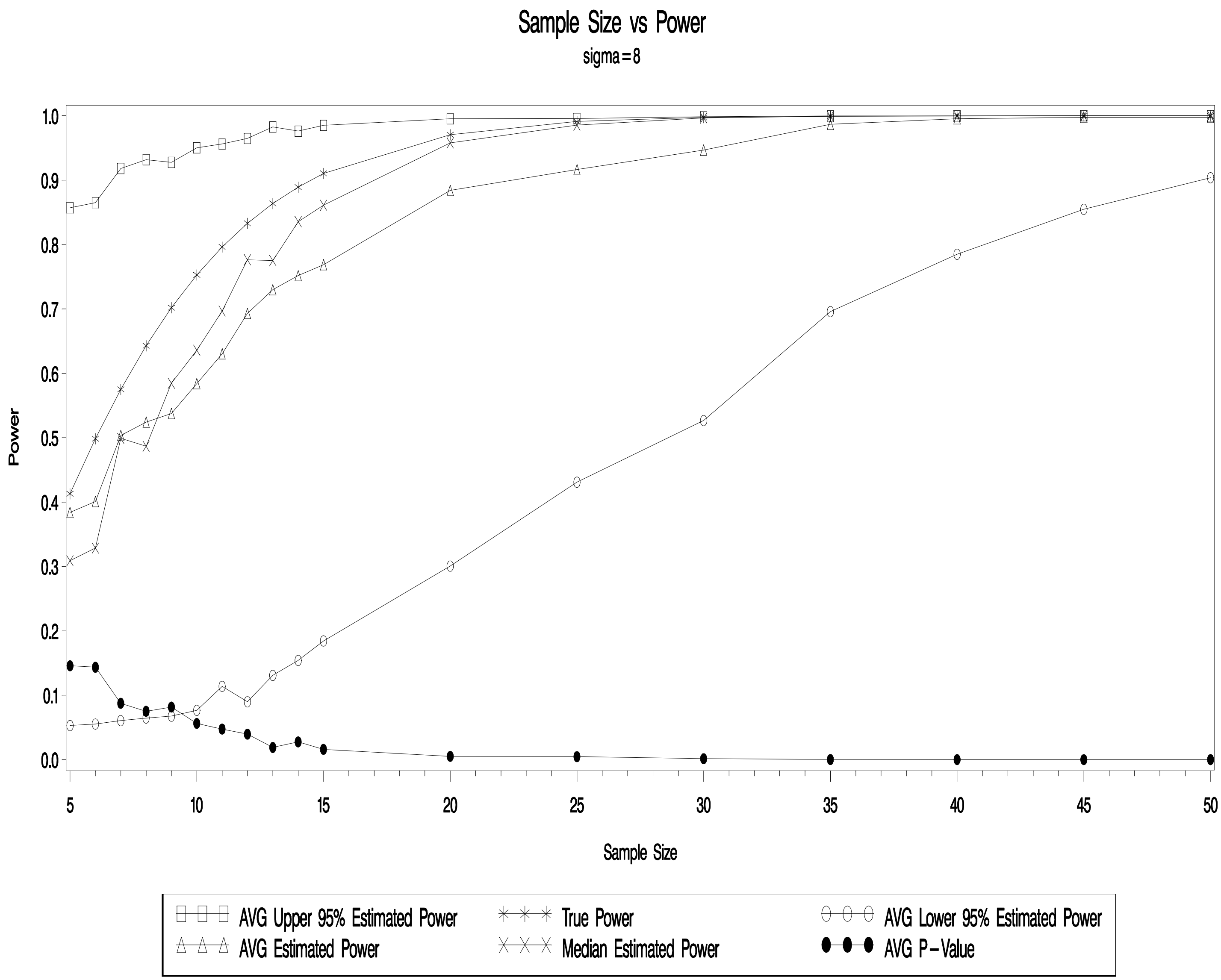
Appendix C: Graphs of Treatment Arrangement 30, 40



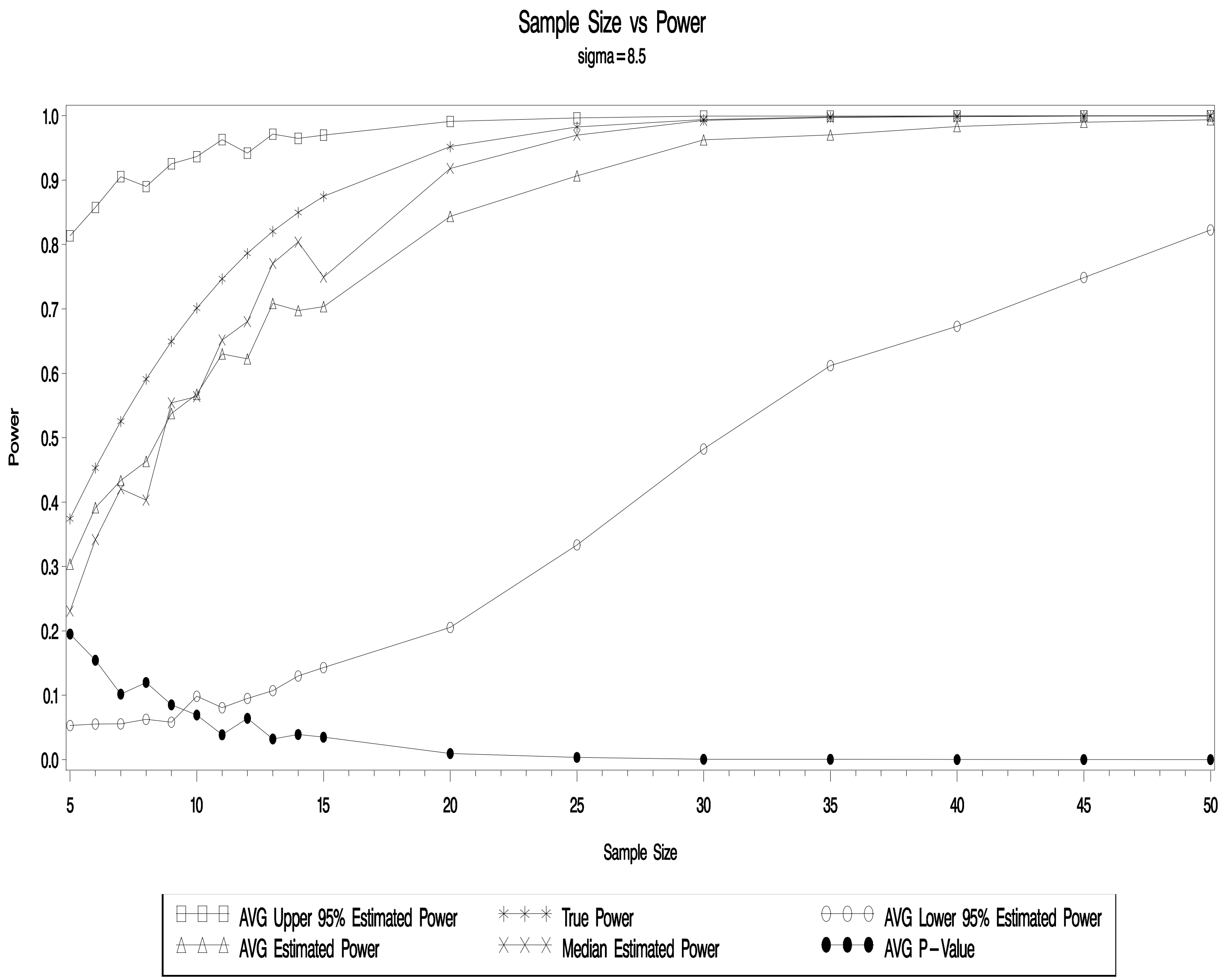
Appendix C: Graphs of Treatment Arrangement 30, 40



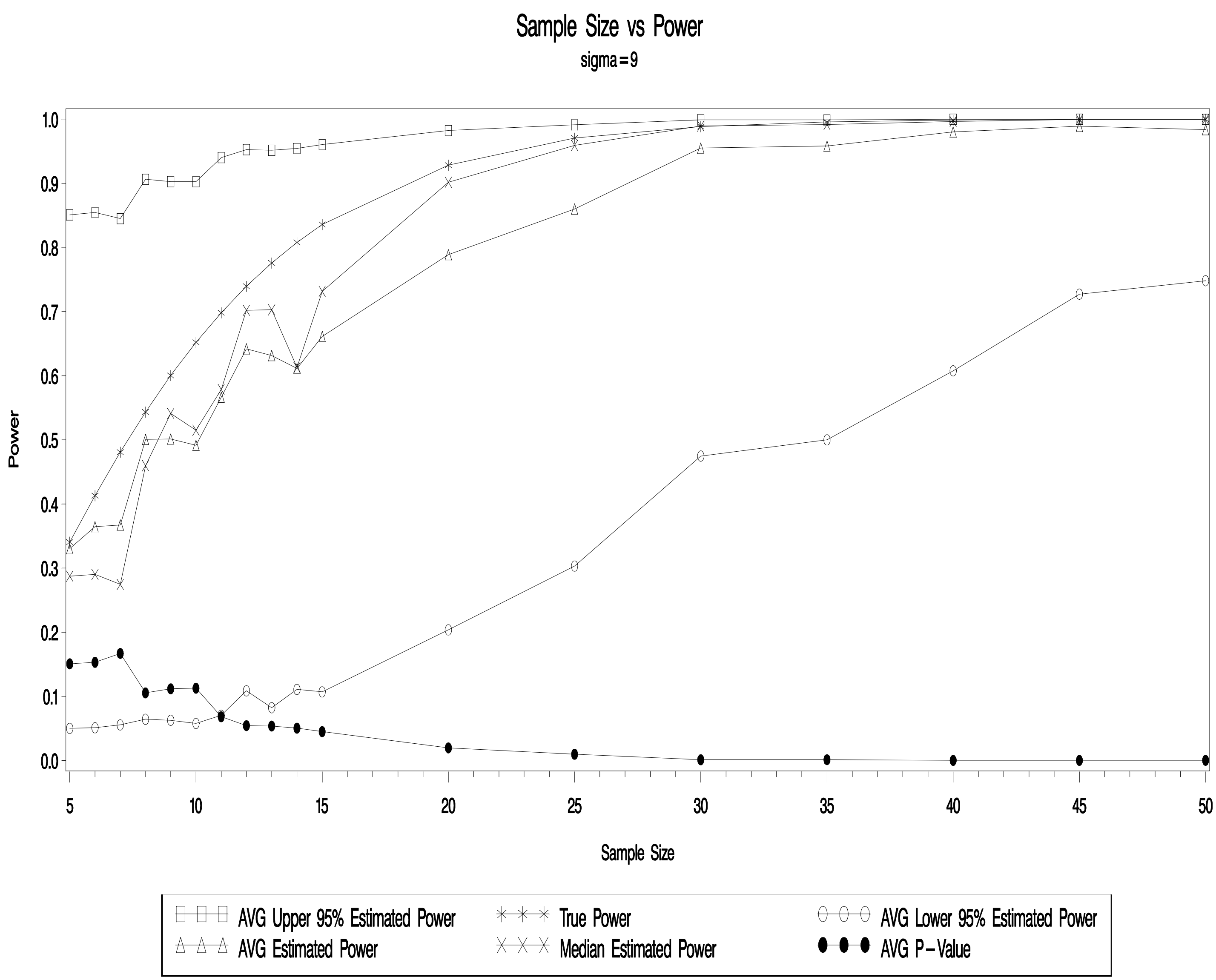
Appendix C: Graphs of Treatment Arrangement 30, 40



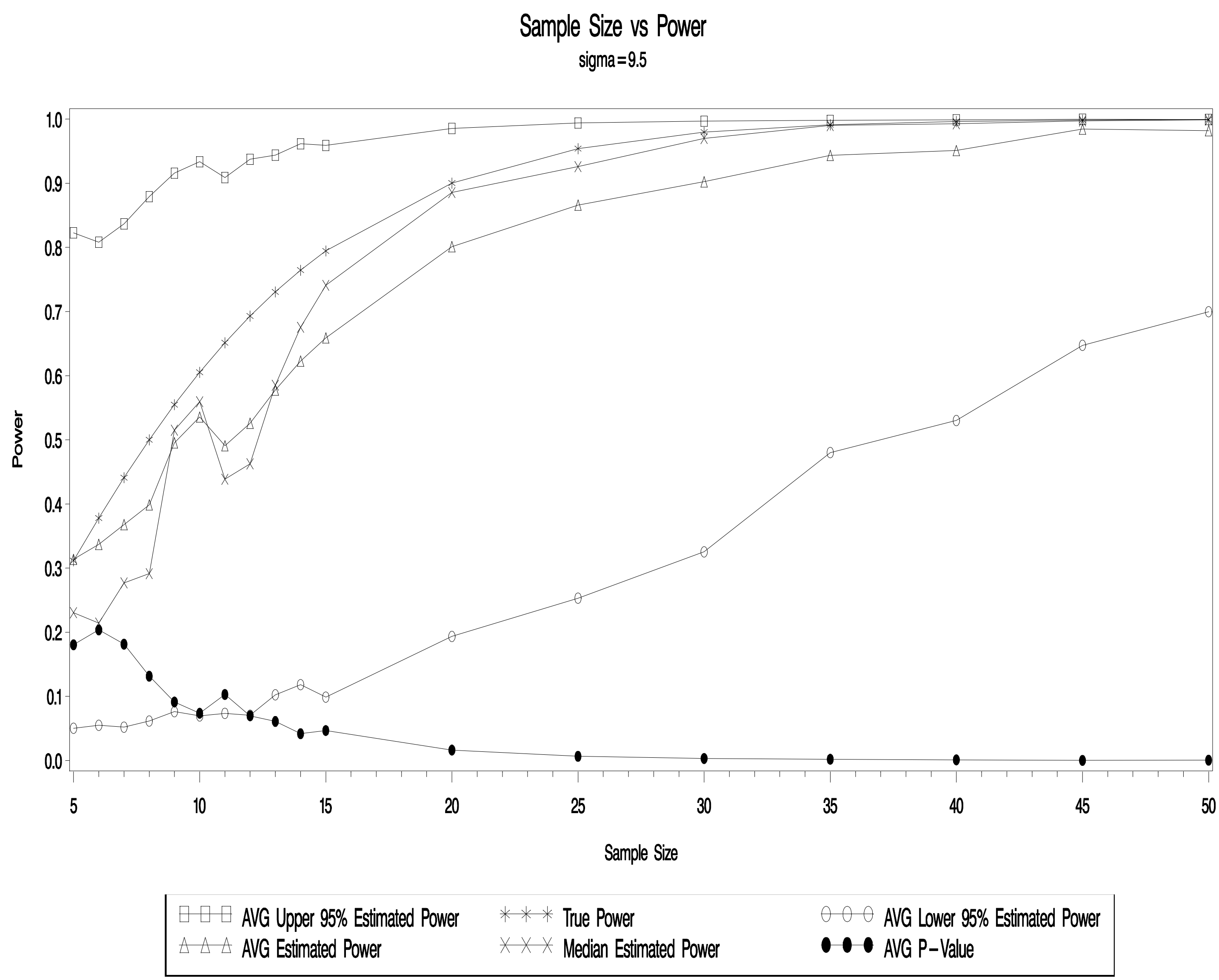
Appendix C: Graphs of Treatment Arrangement 30, 40



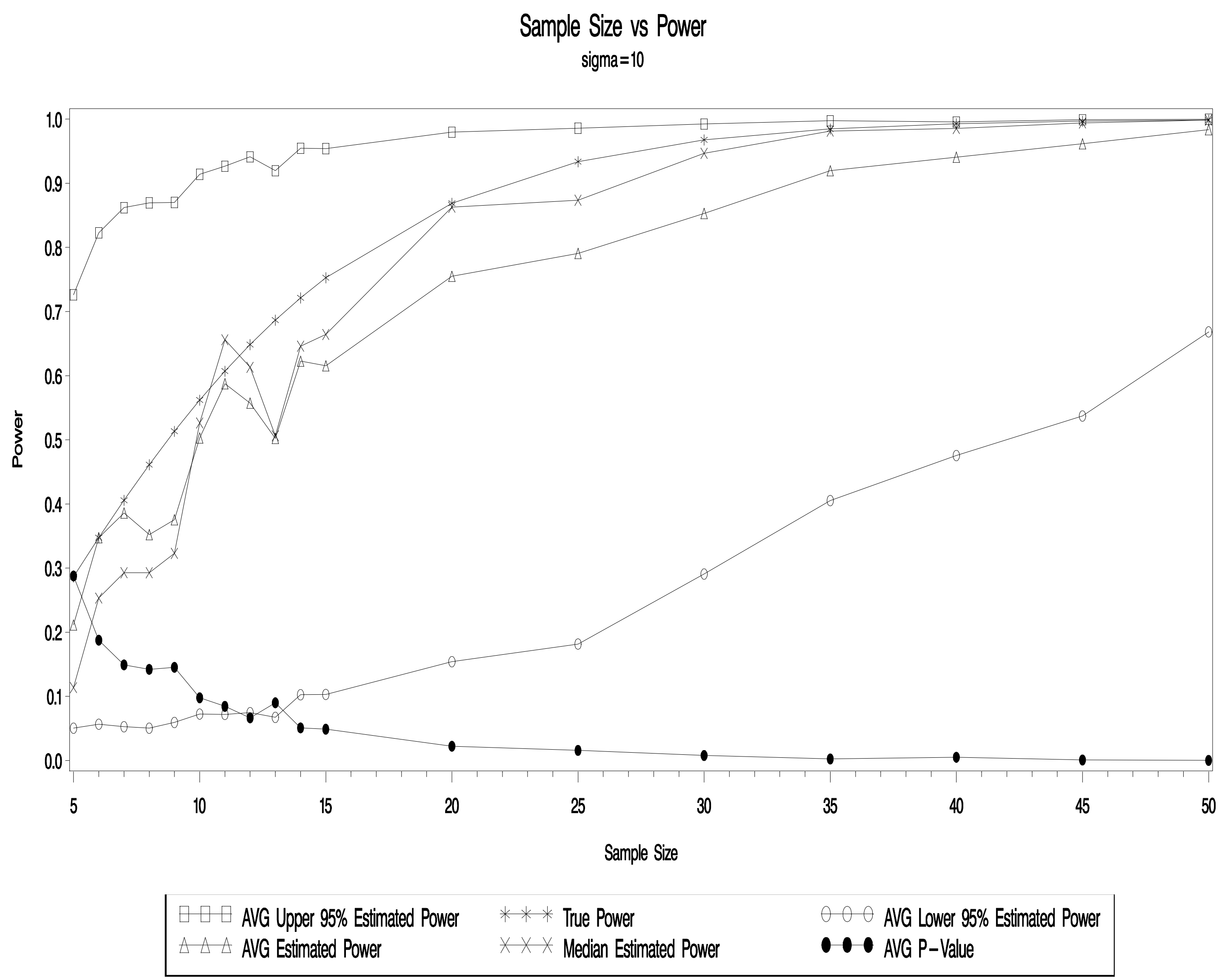
Appendix C: Graphs of Treatment Arrangement 30, 40



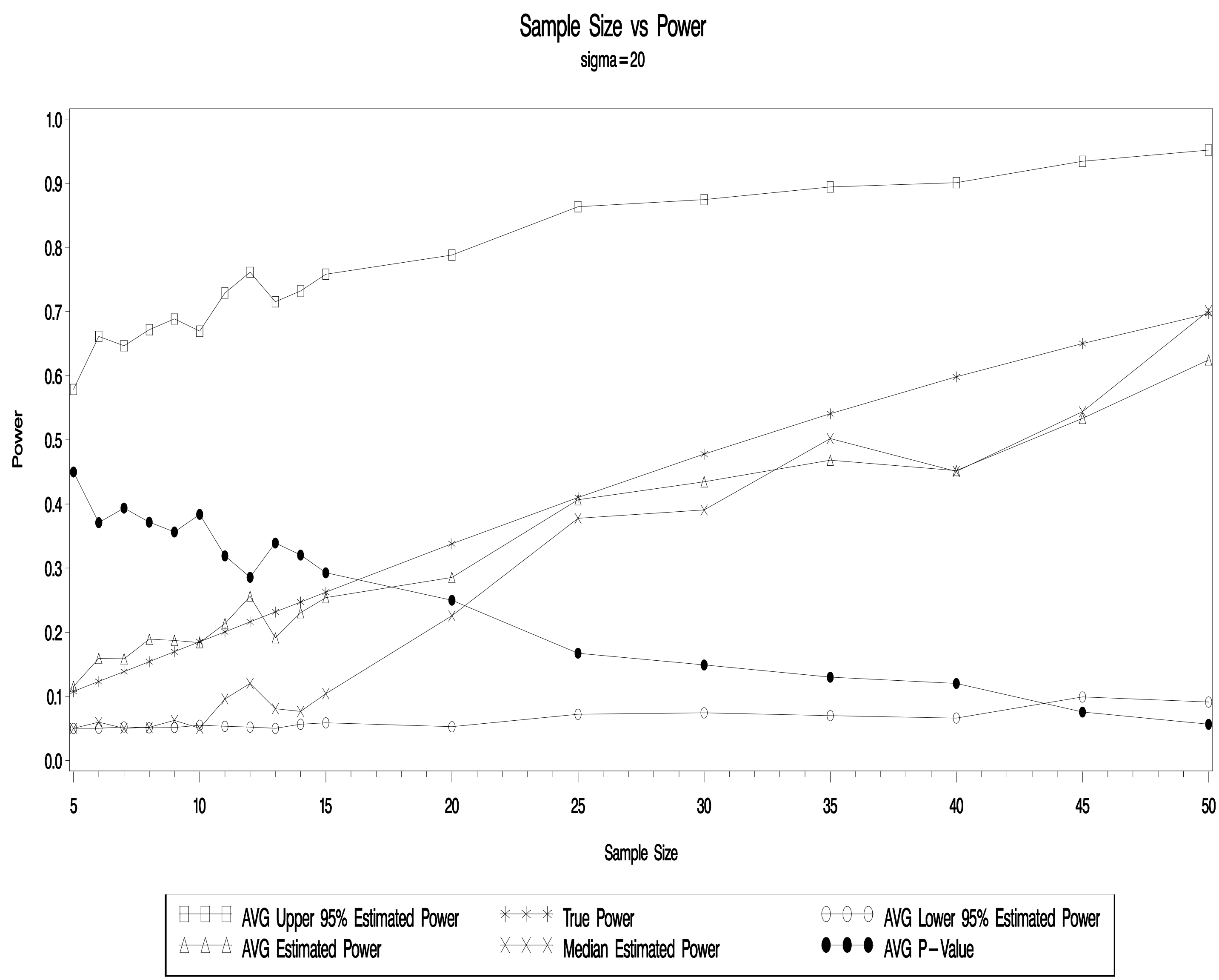
Appendix C: Graphs of Treatment Arrangement 30, 40



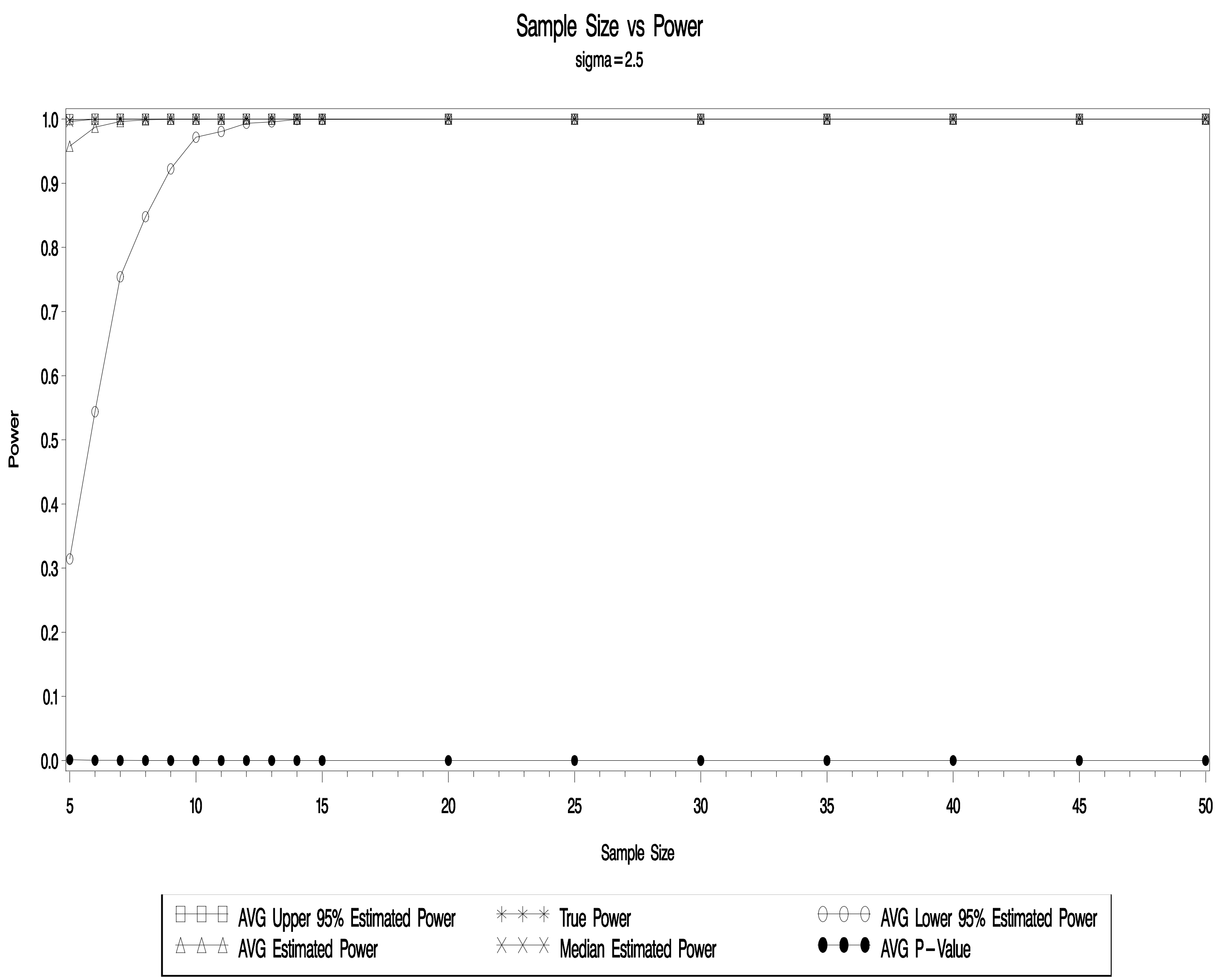
Appendix C: Graphs of Treatment Arrangement 30, 40



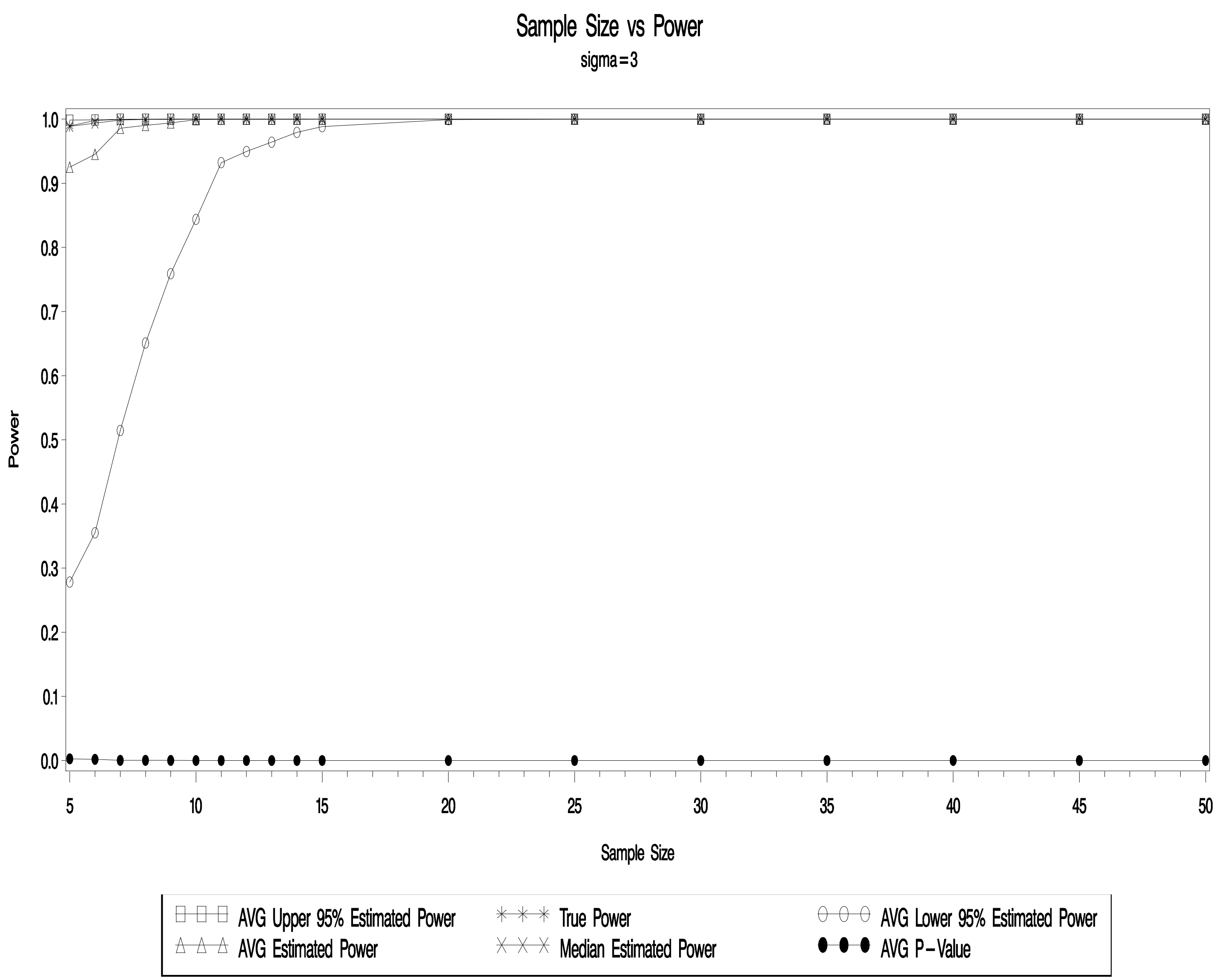
Appendix C: Graphs of Treatment Arrangement 30, 40



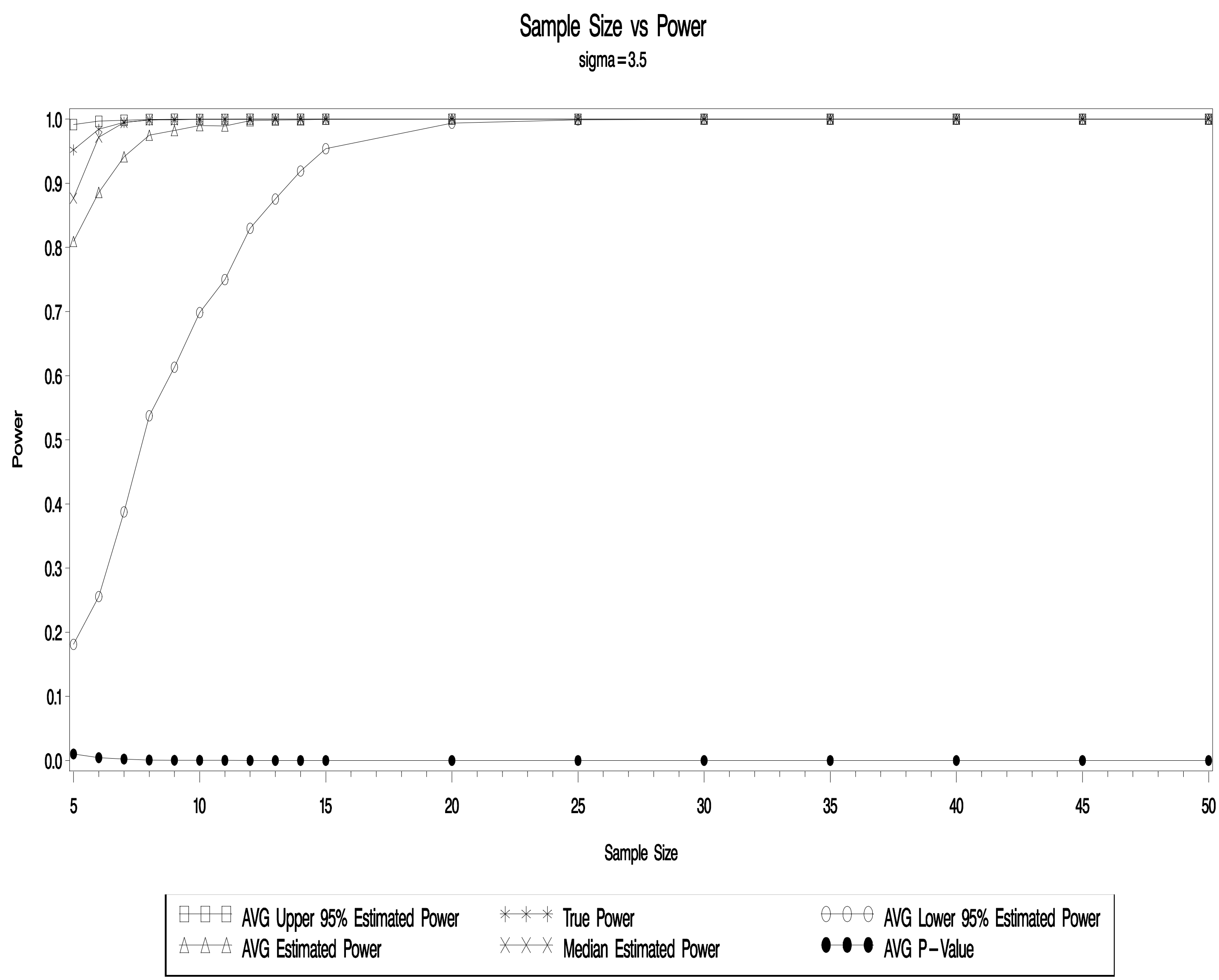
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



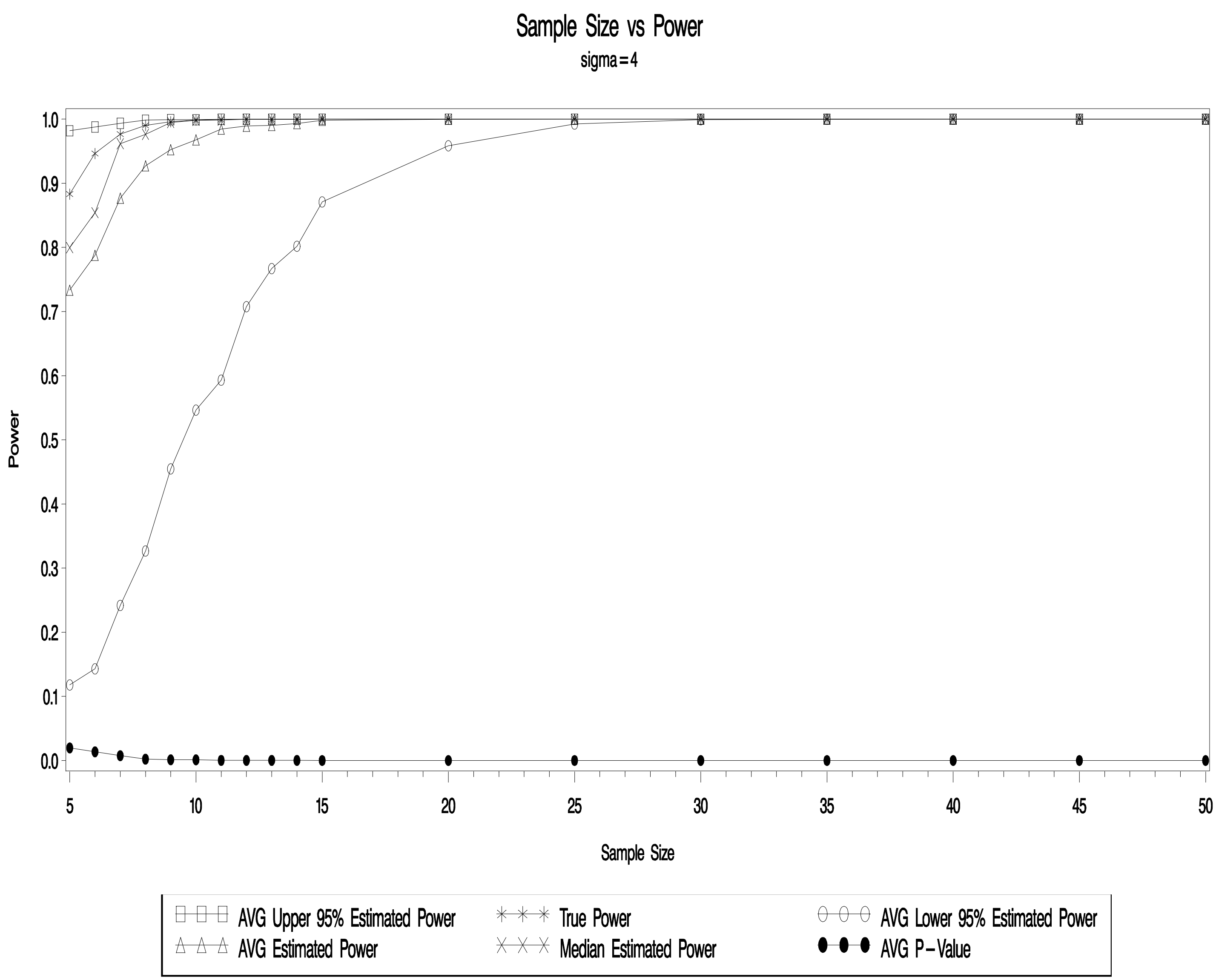
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



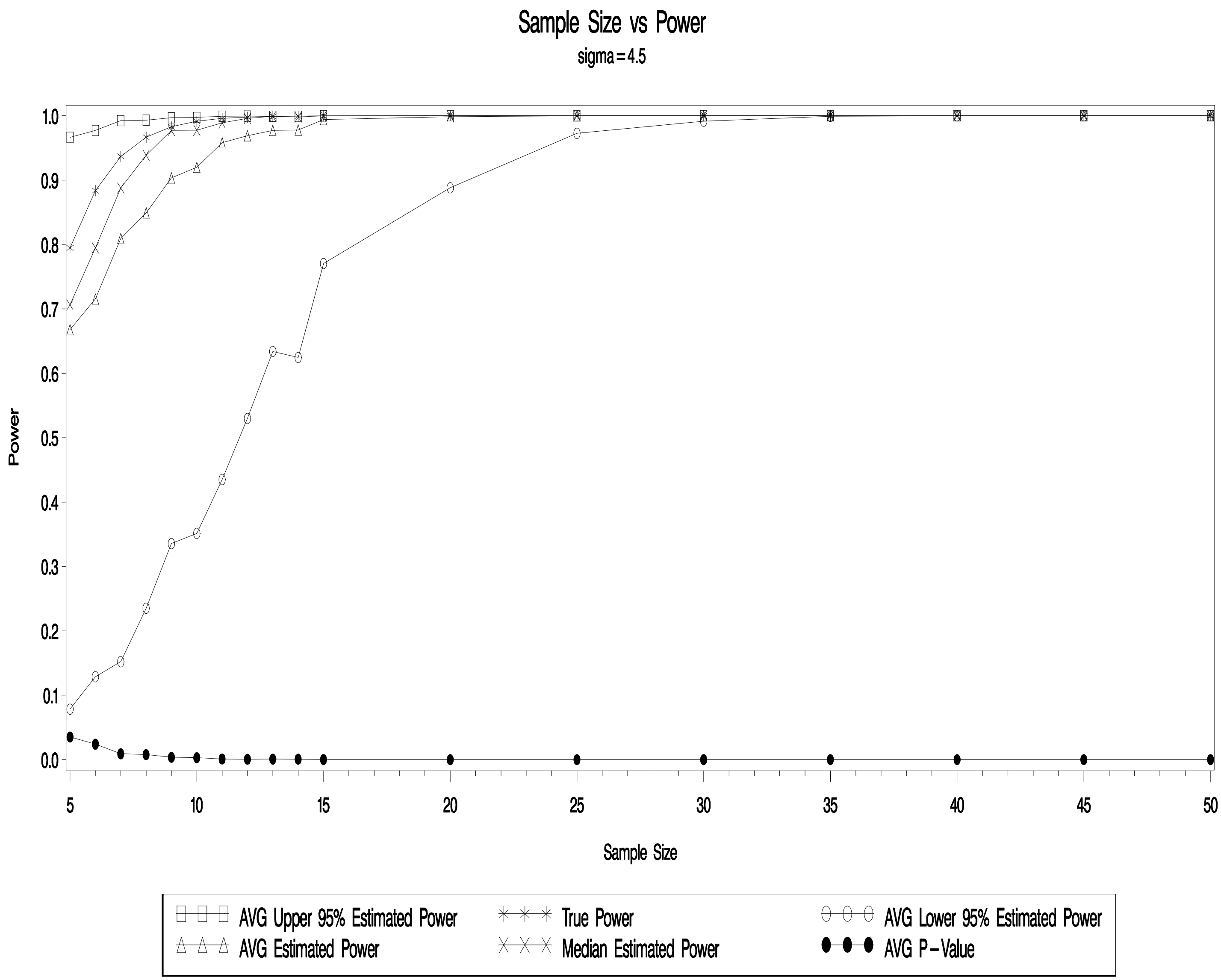
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



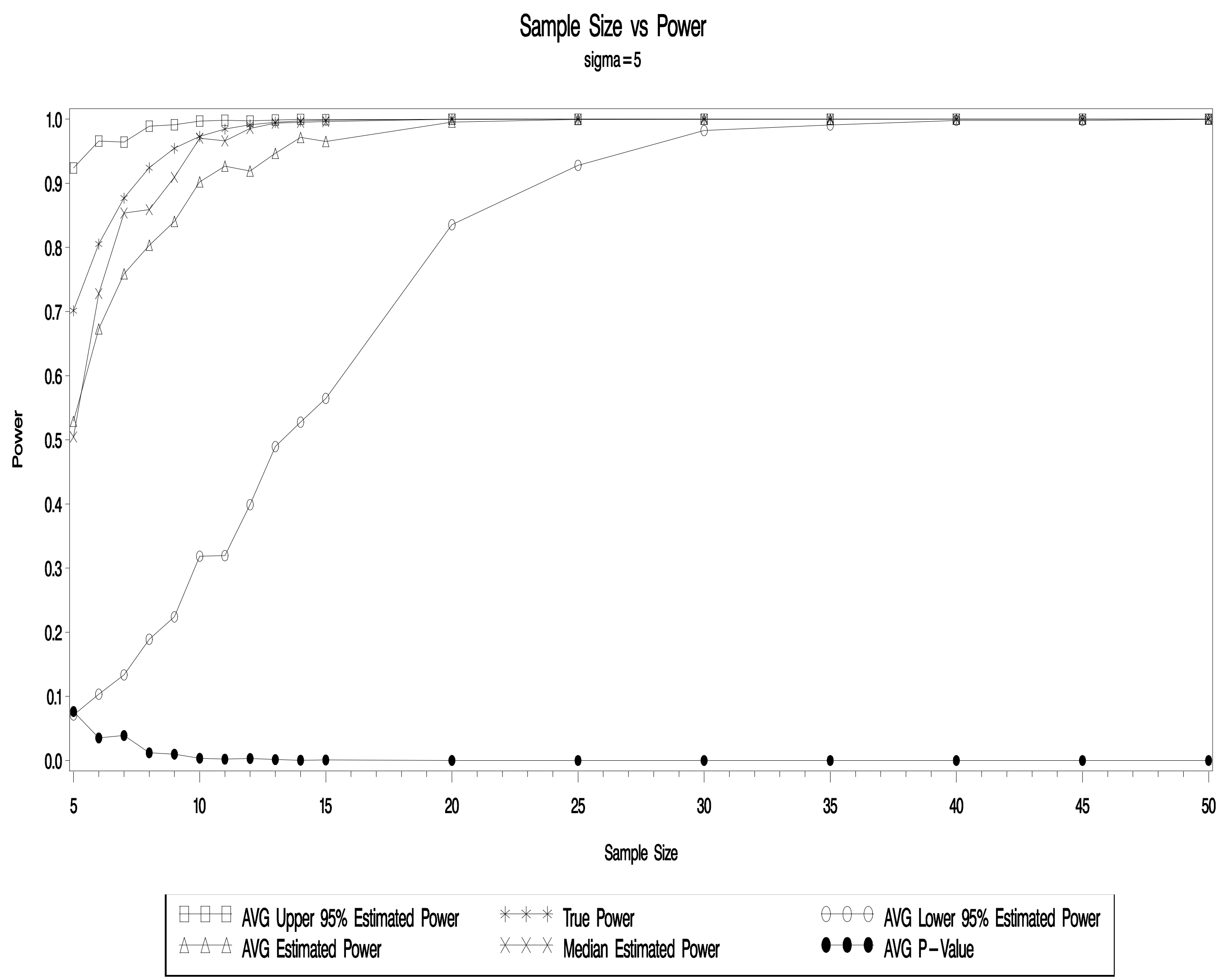
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



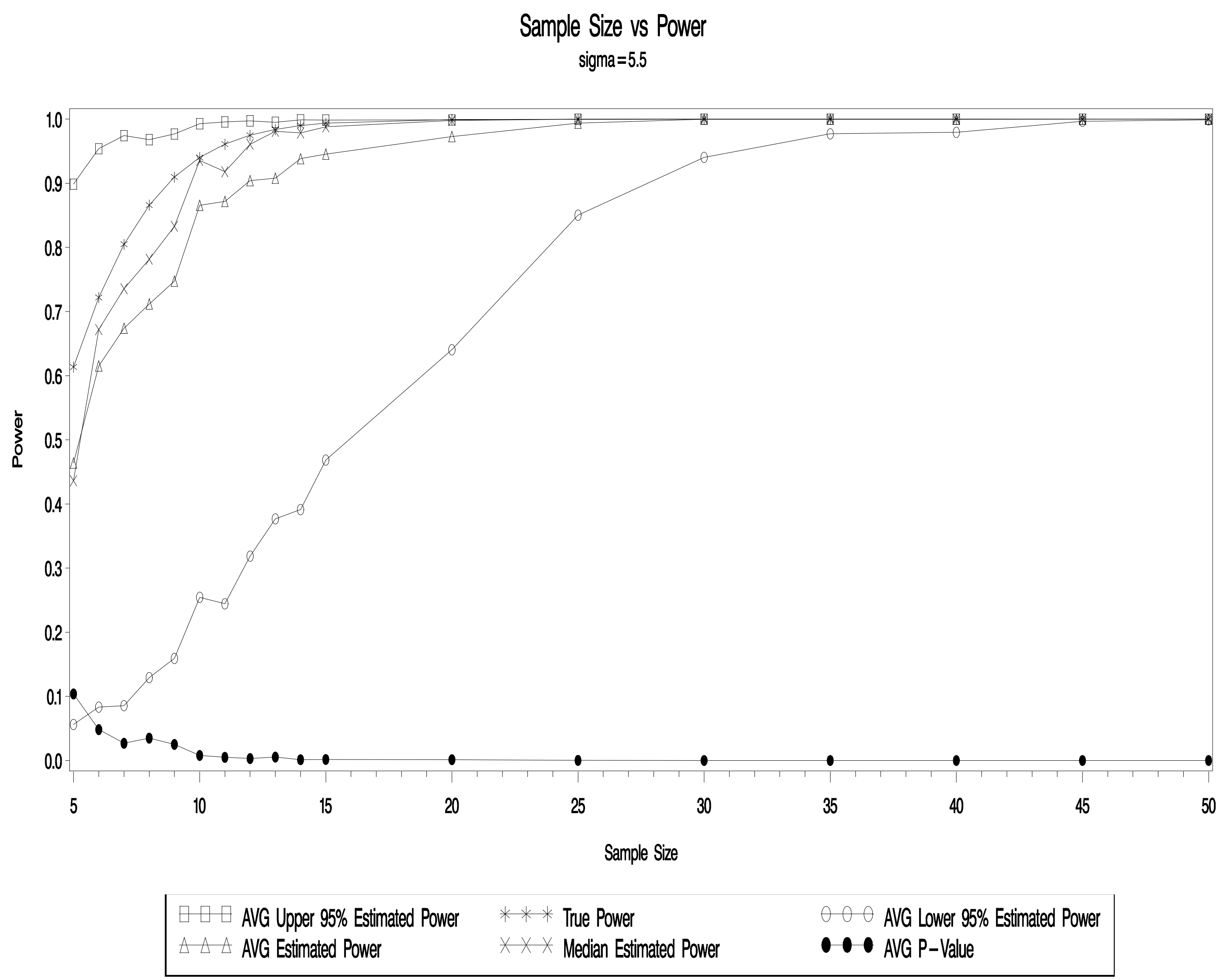
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



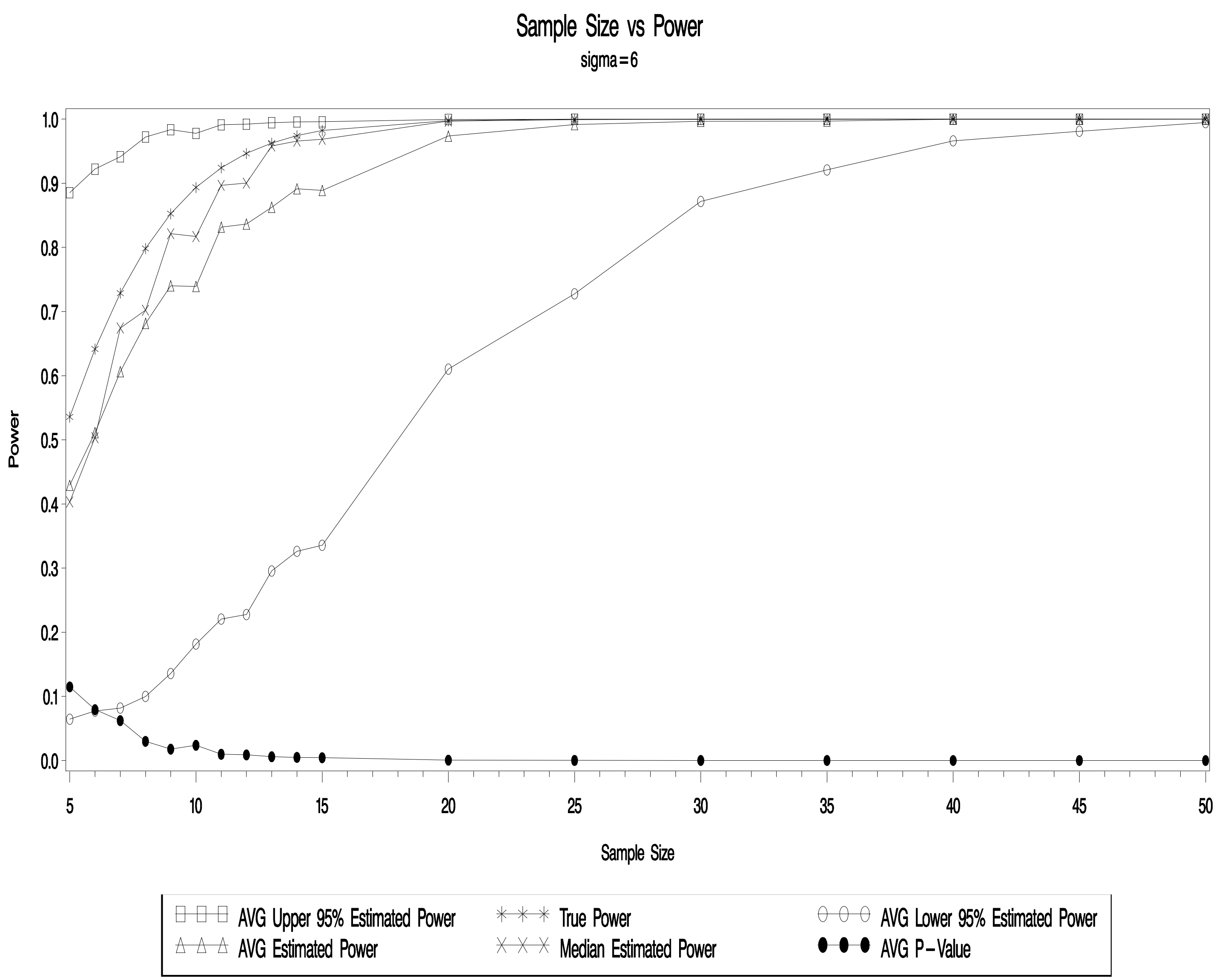
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



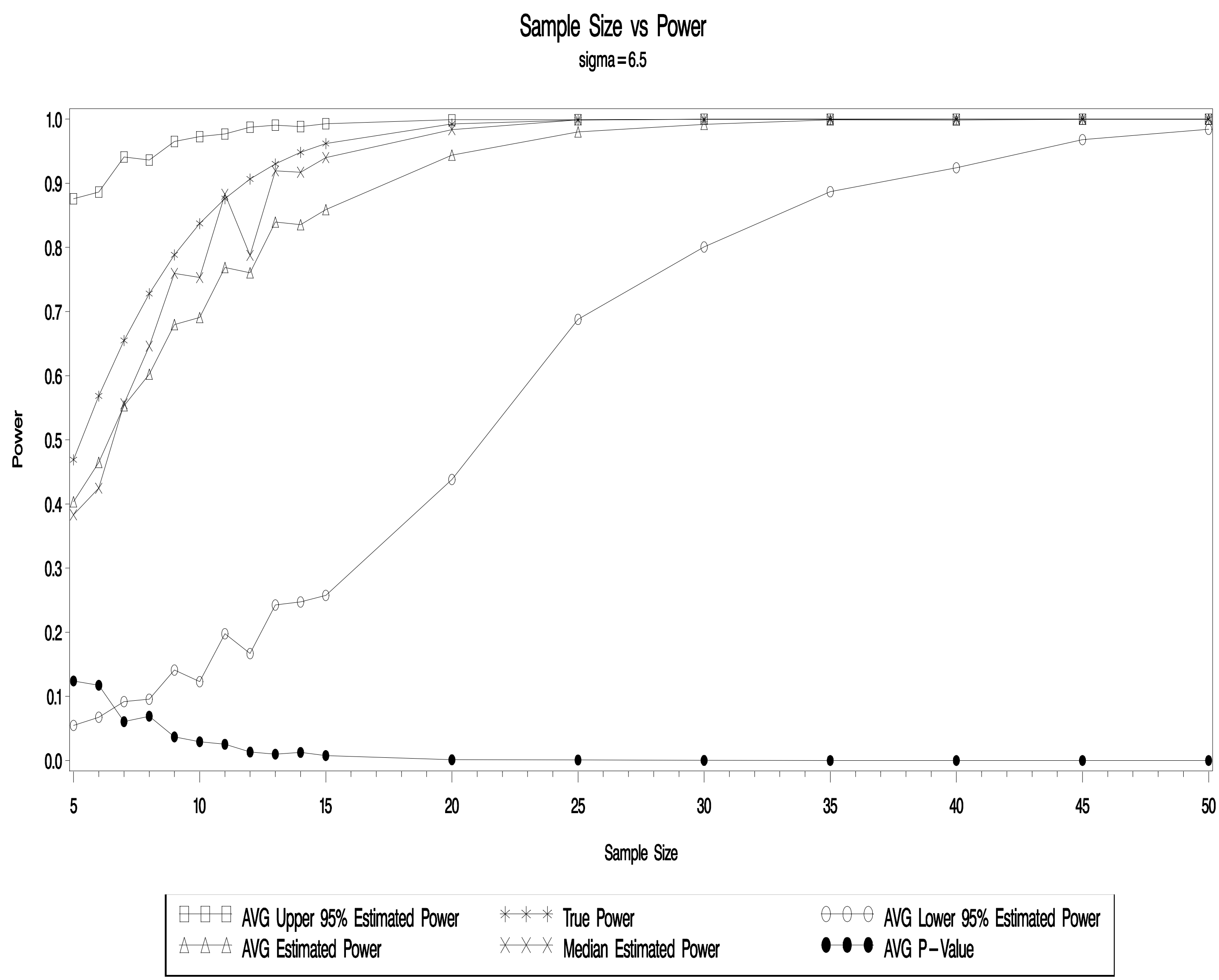
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



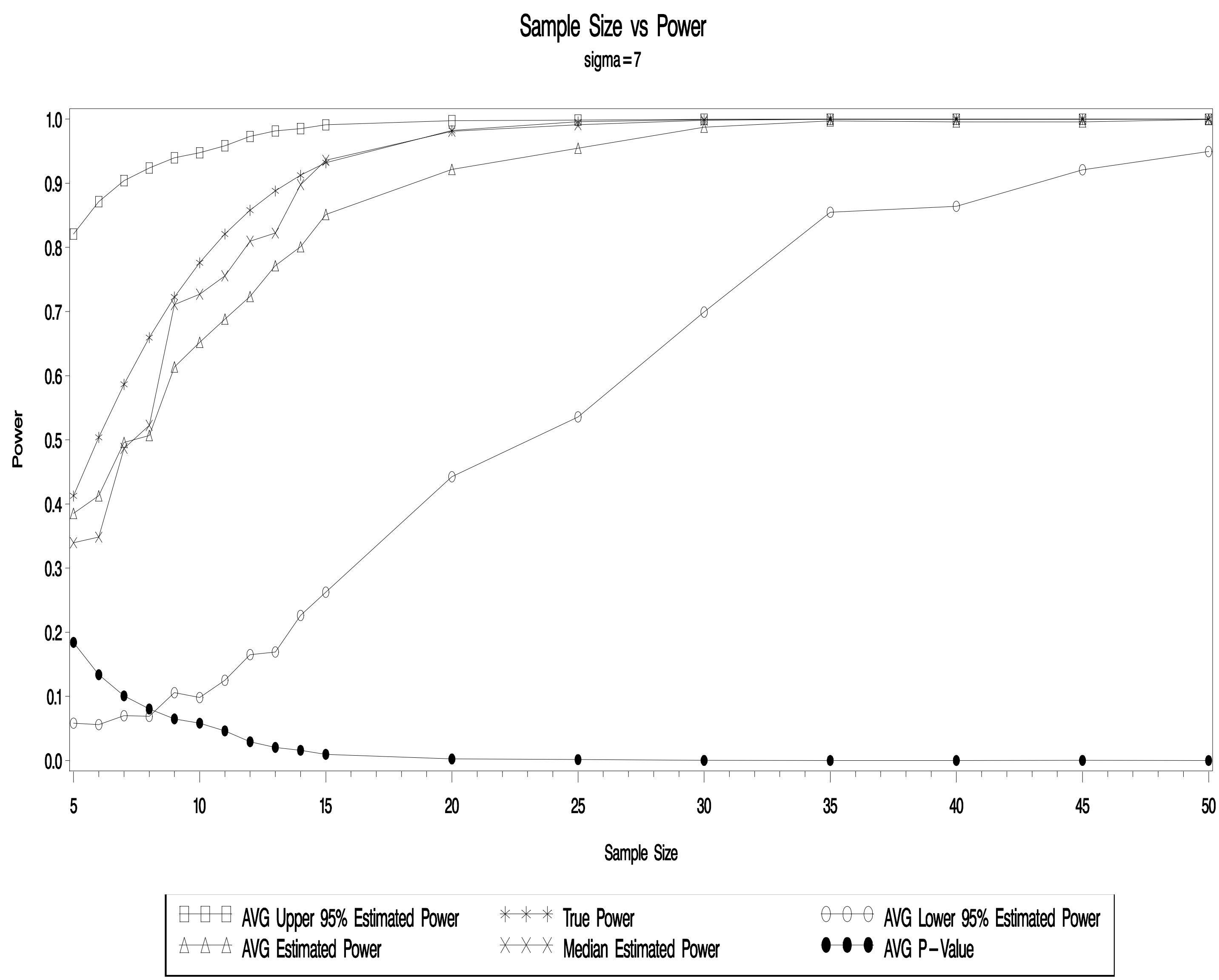
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



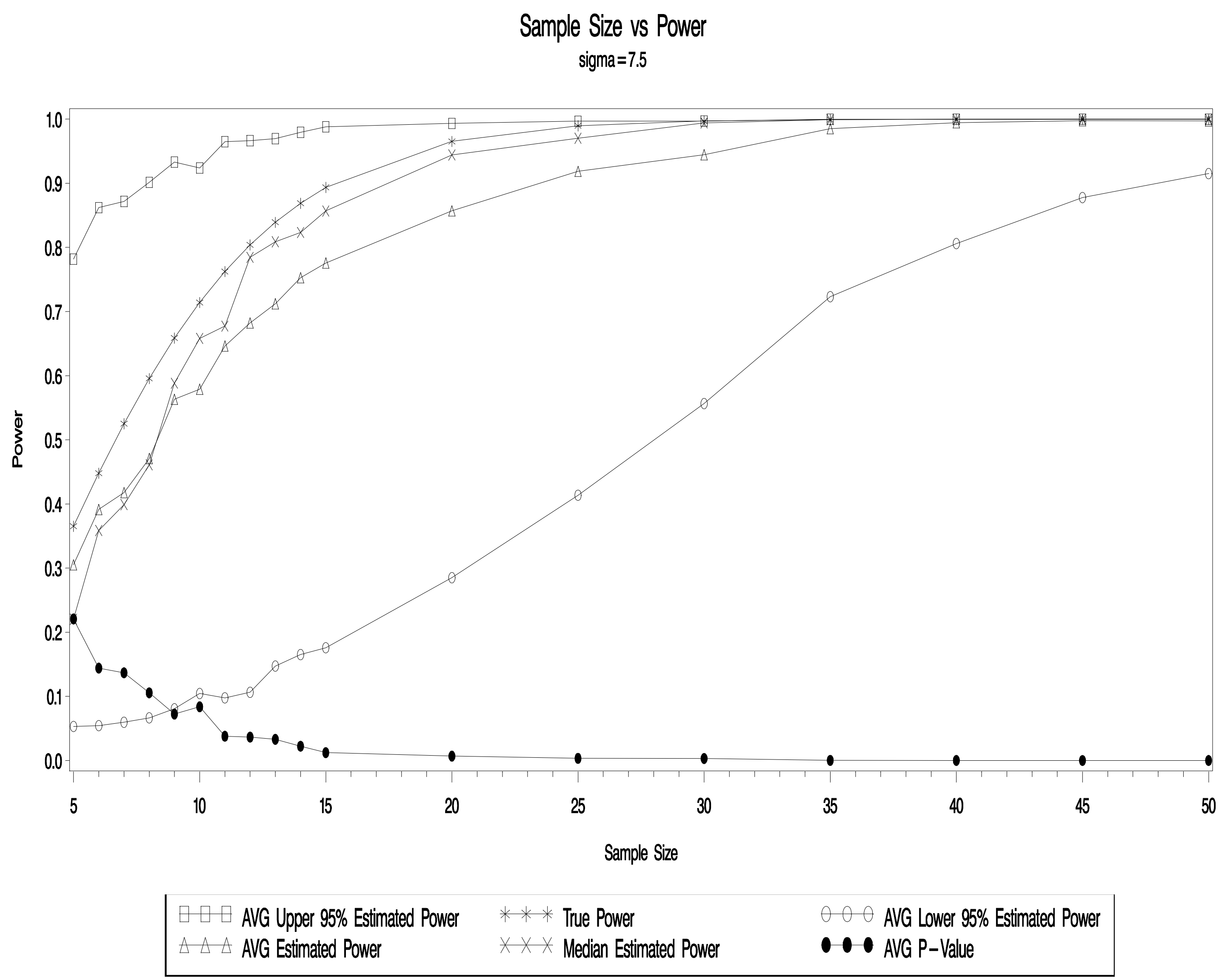
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



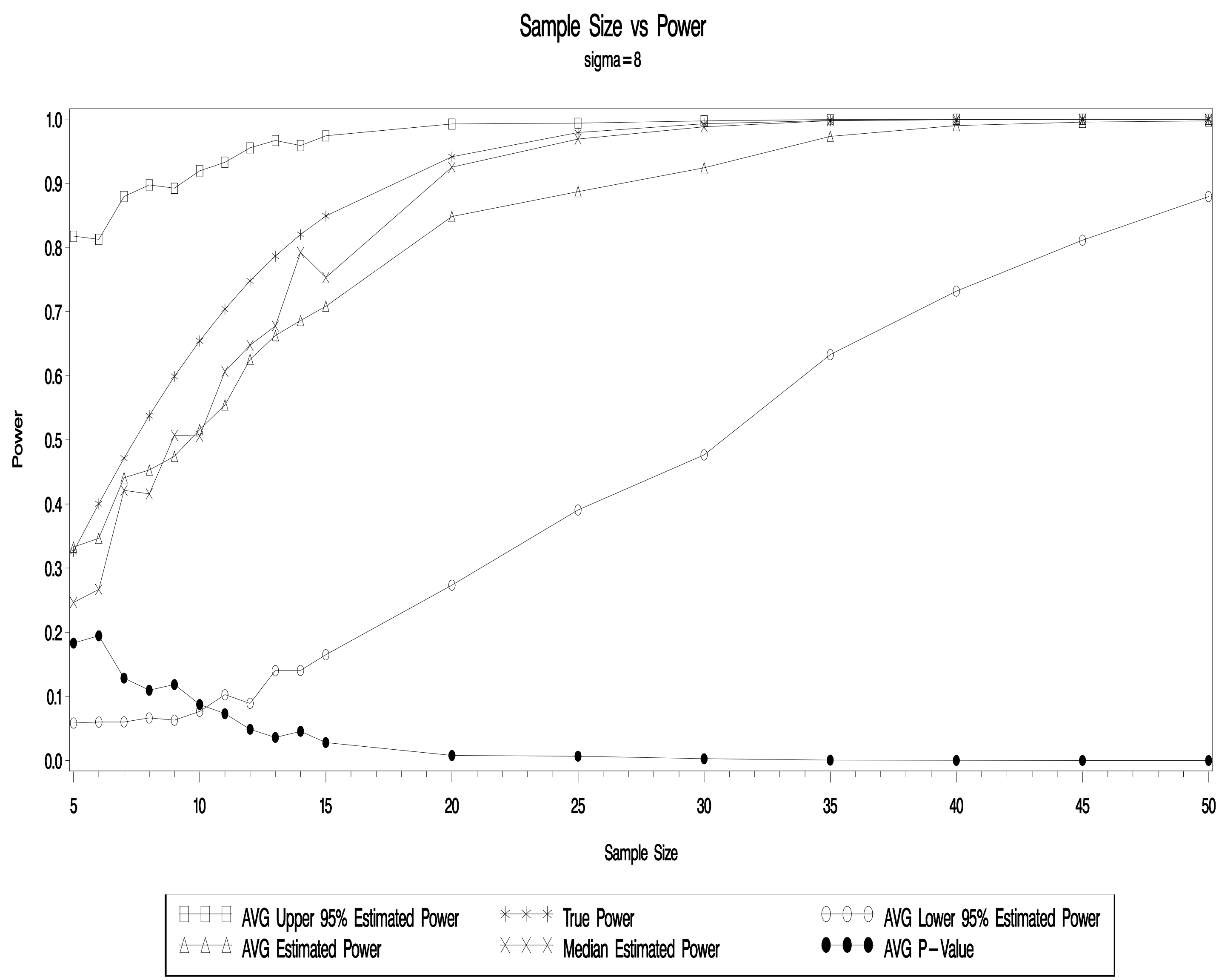
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



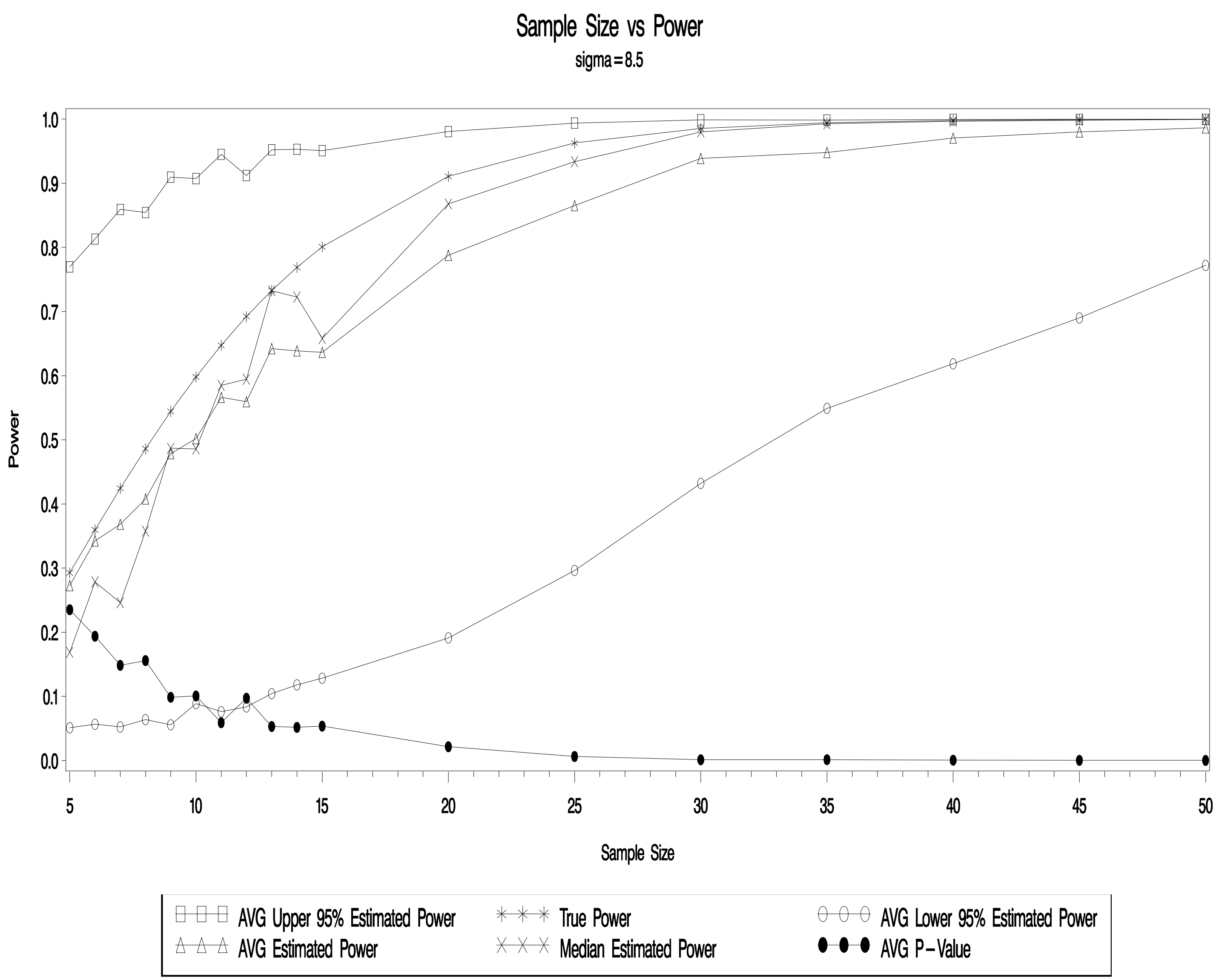
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



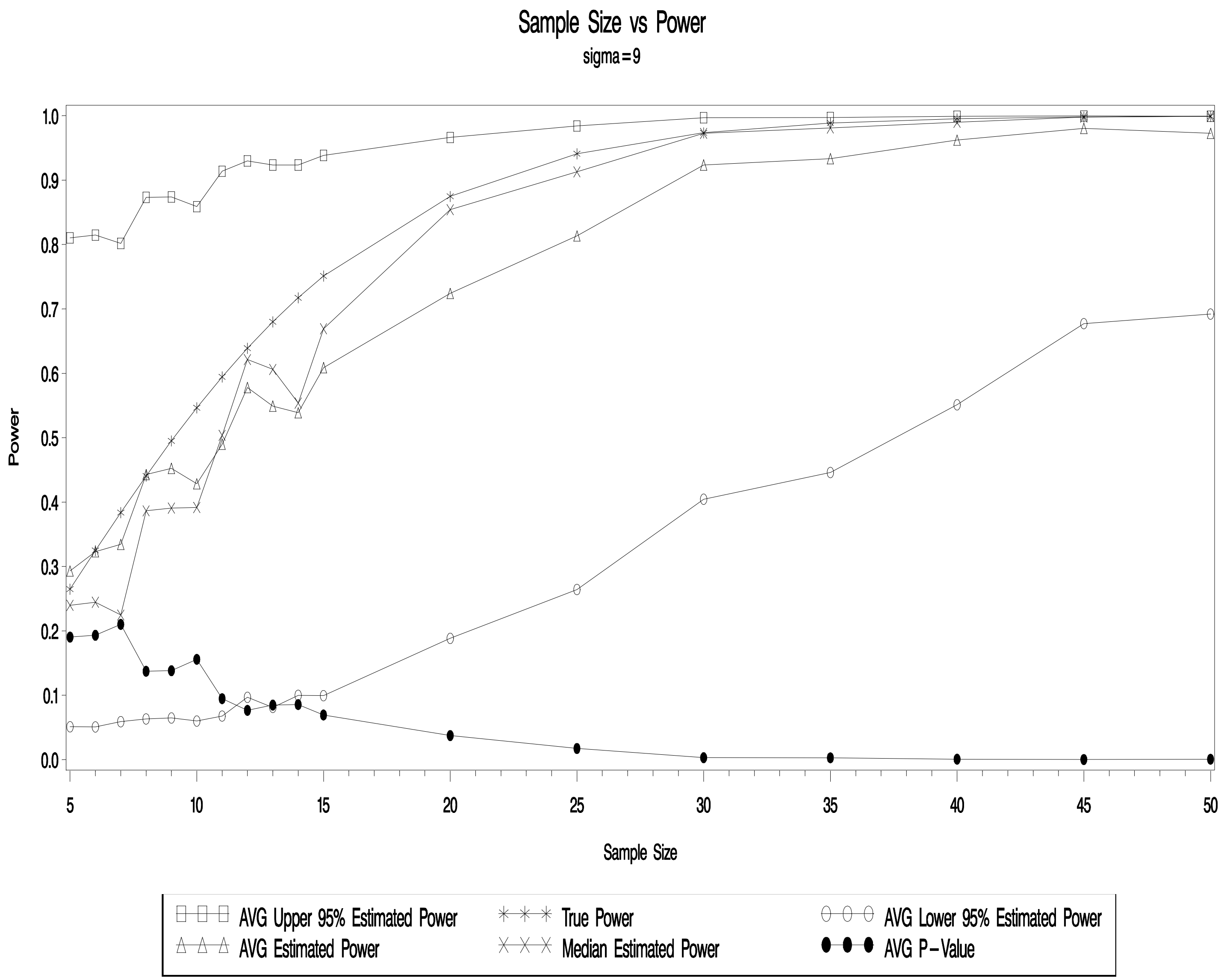
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



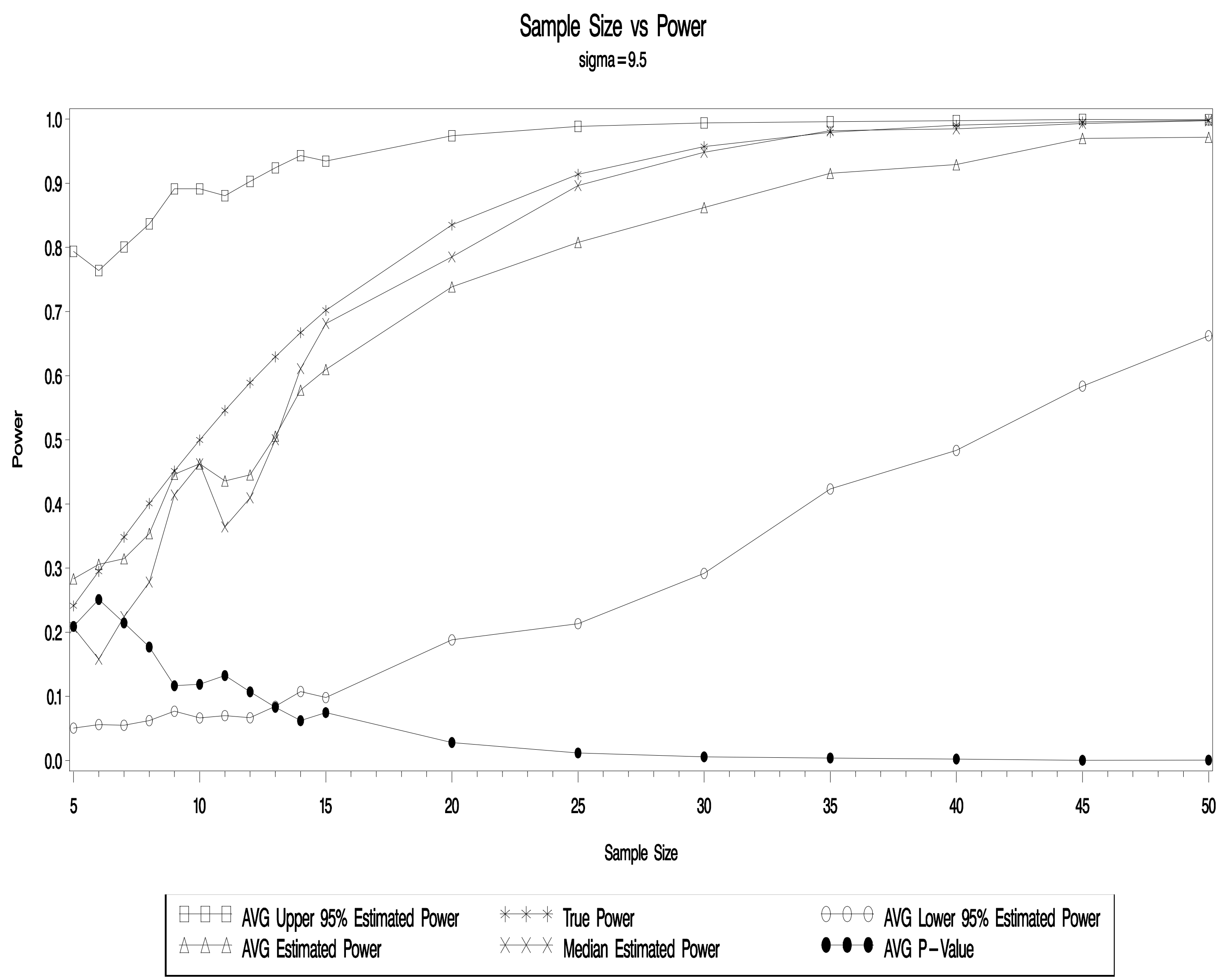
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



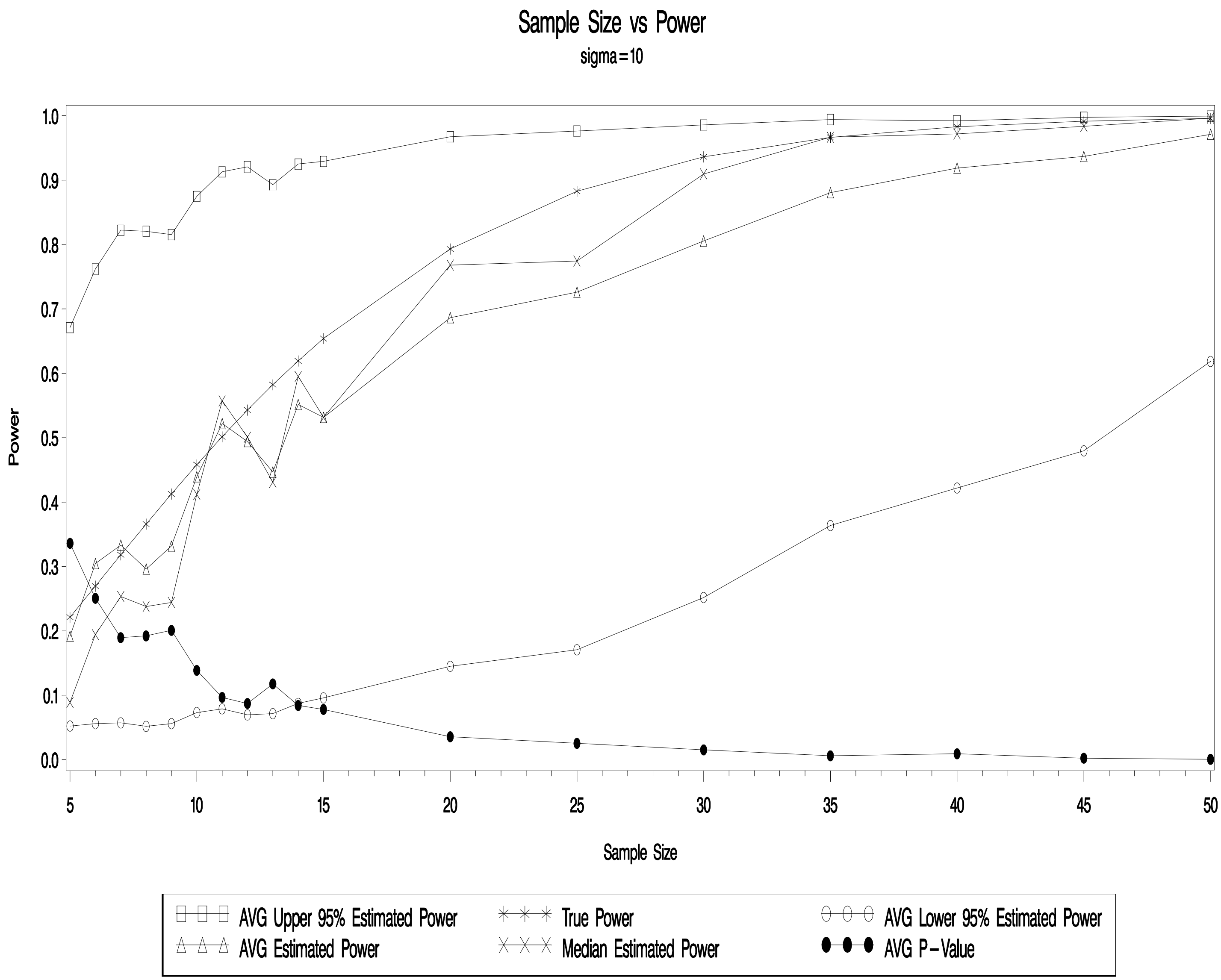
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



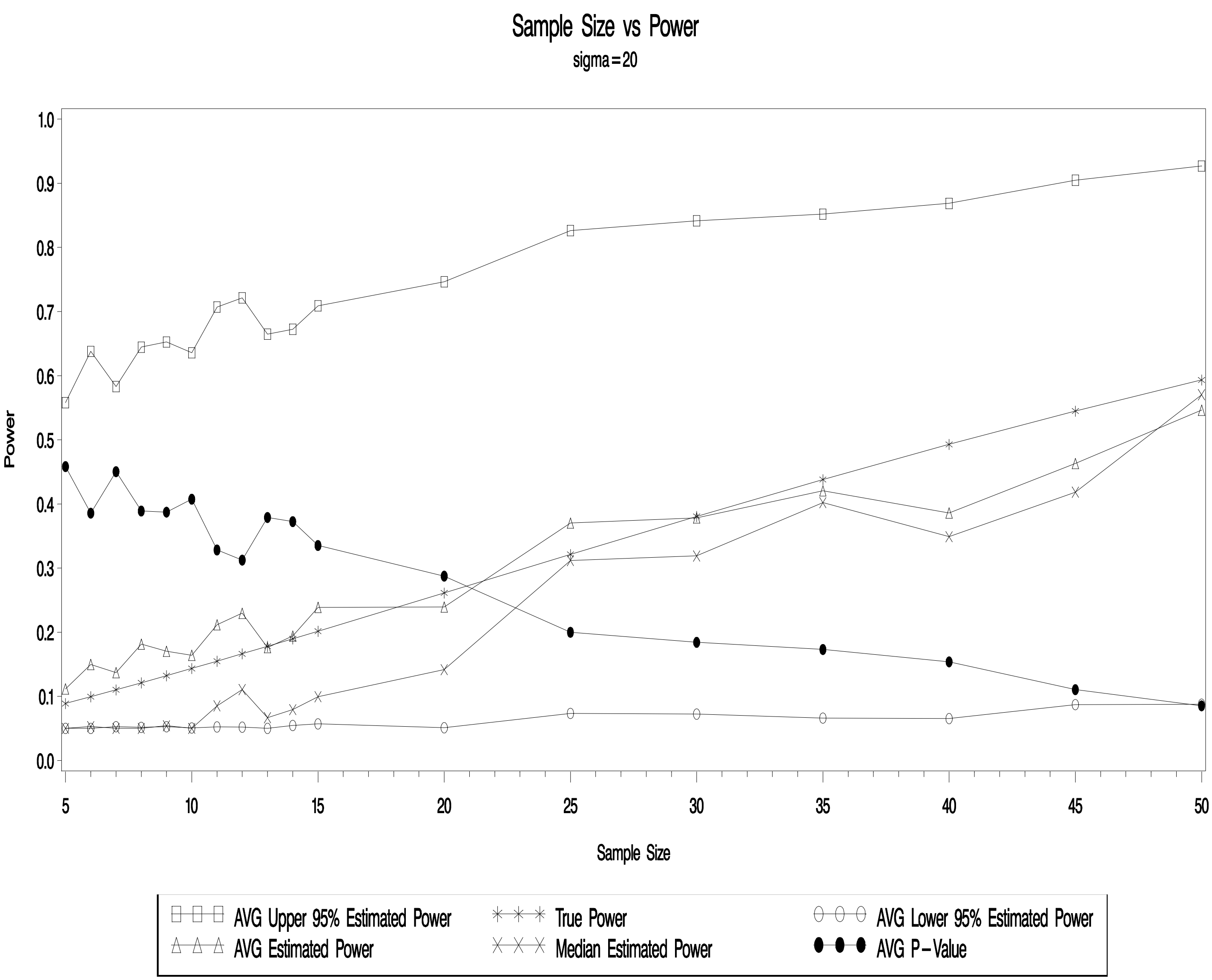
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



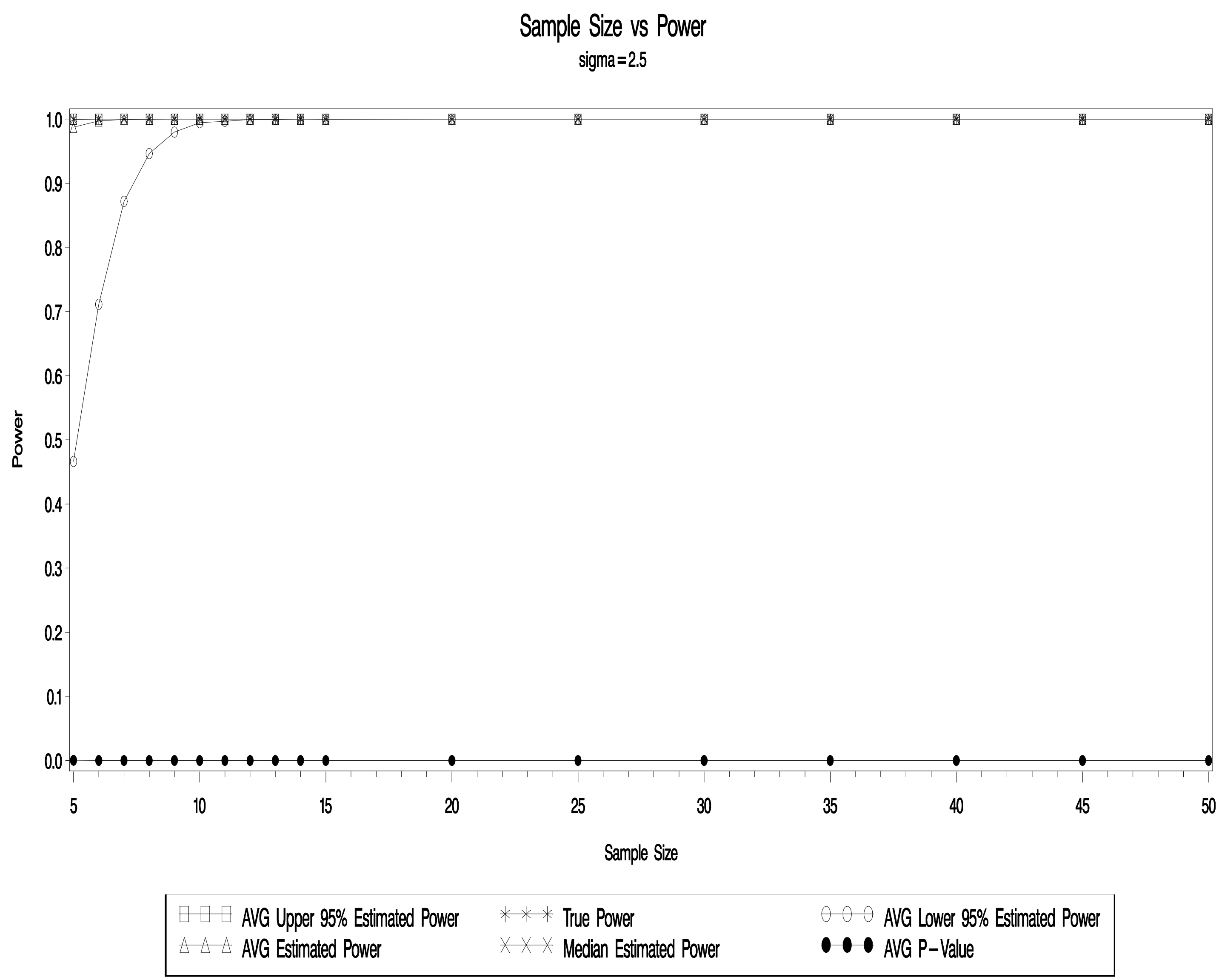
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



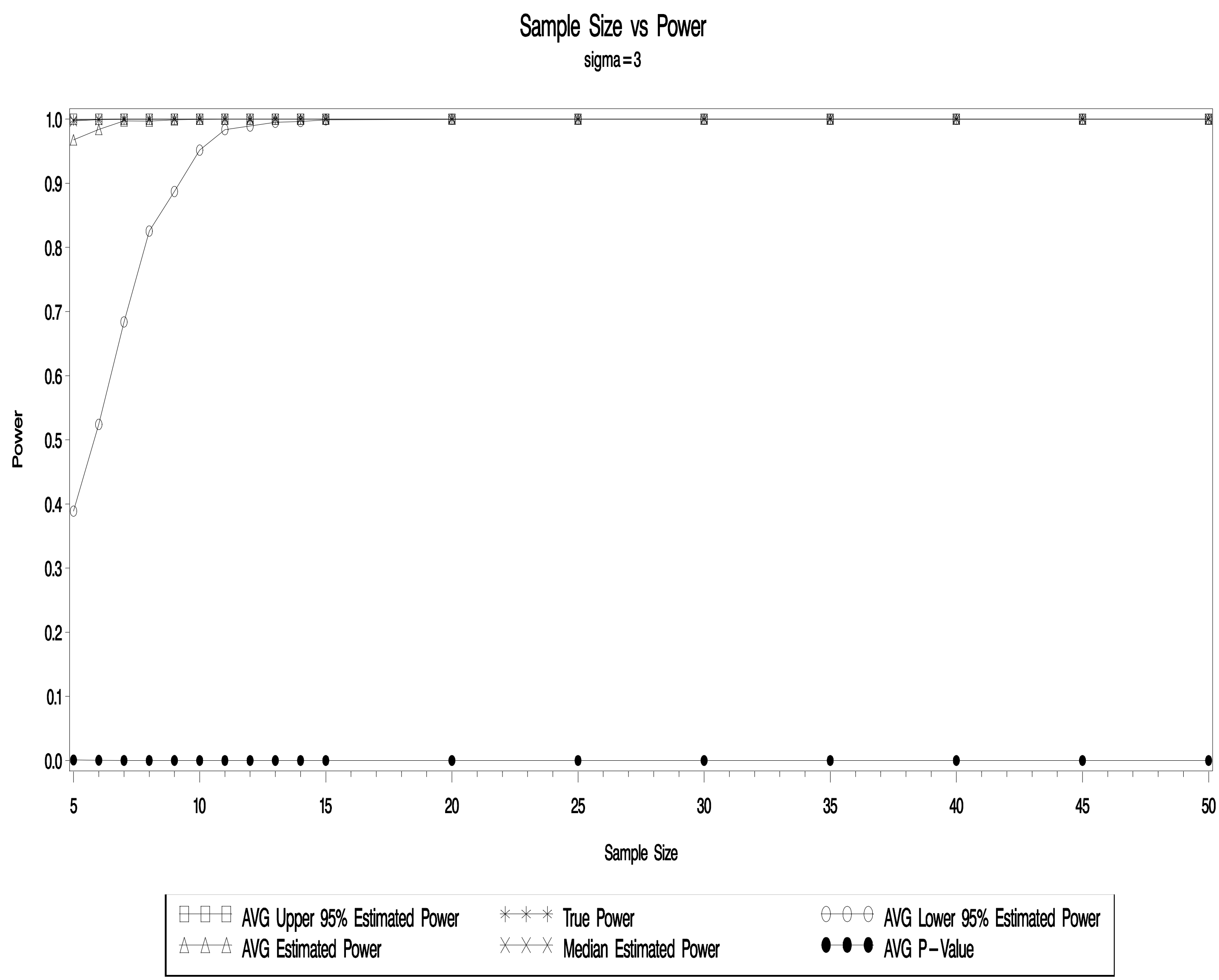
Appendix C: Graphs of Treatment Arrangement 30, 35, 40



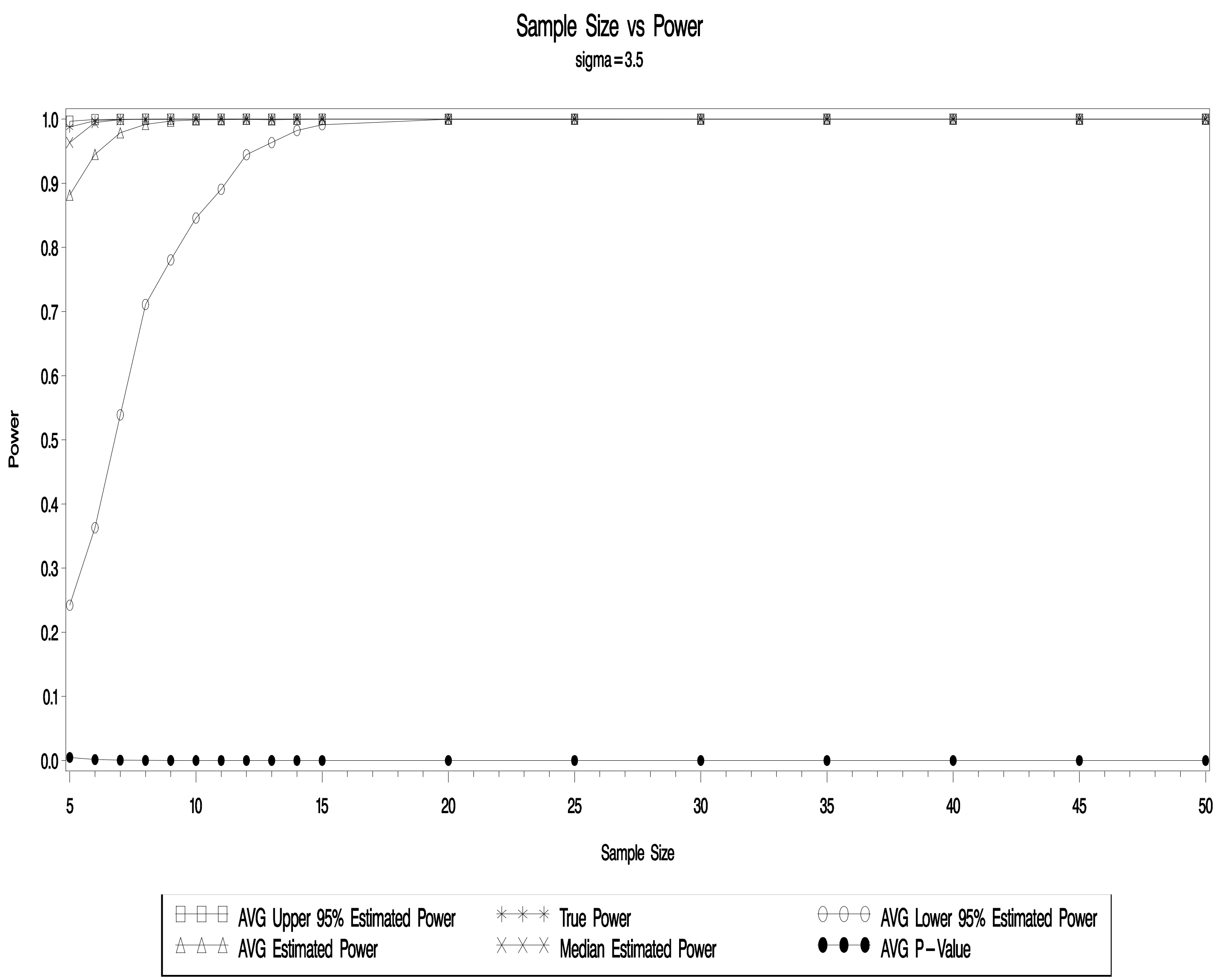
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



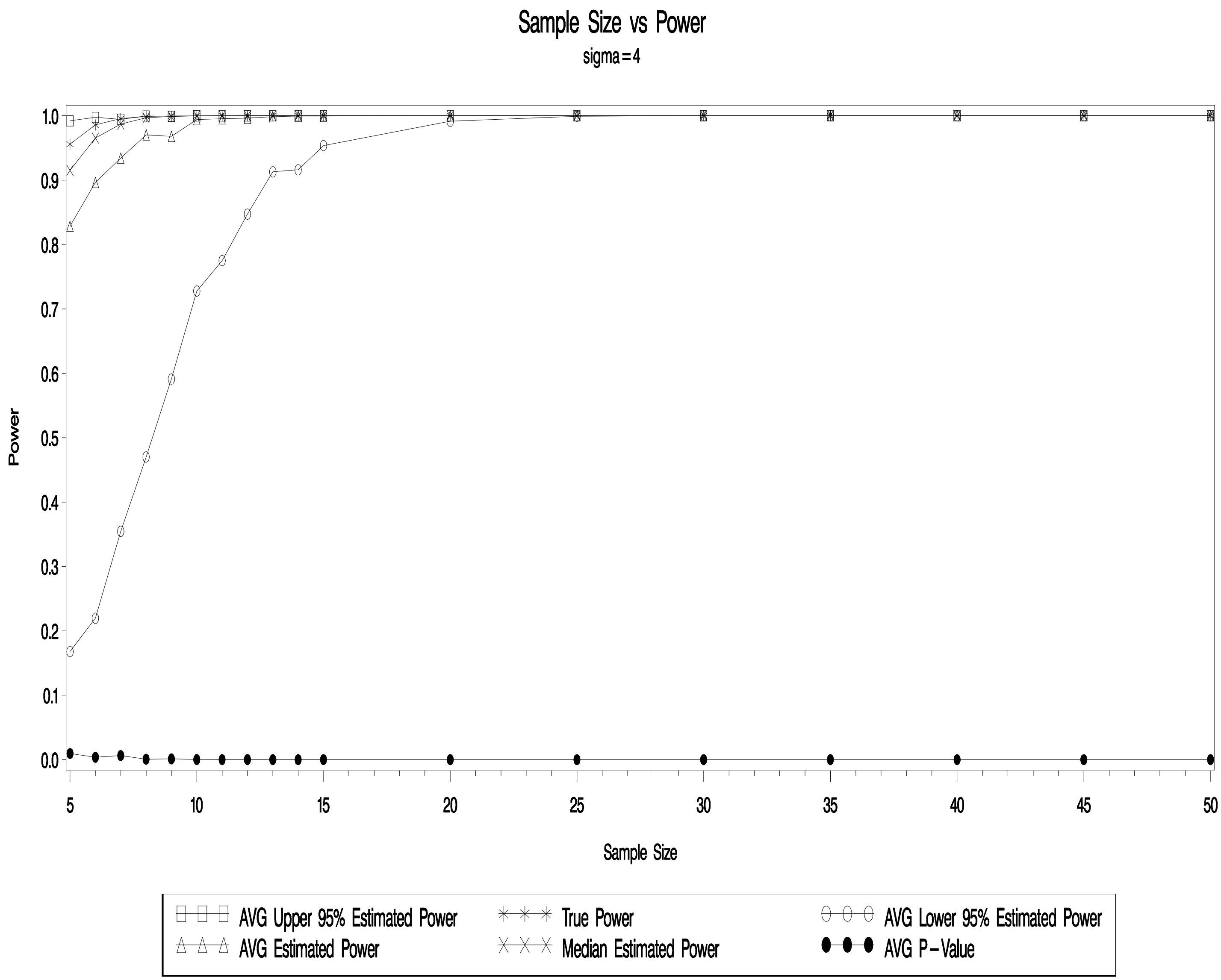
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



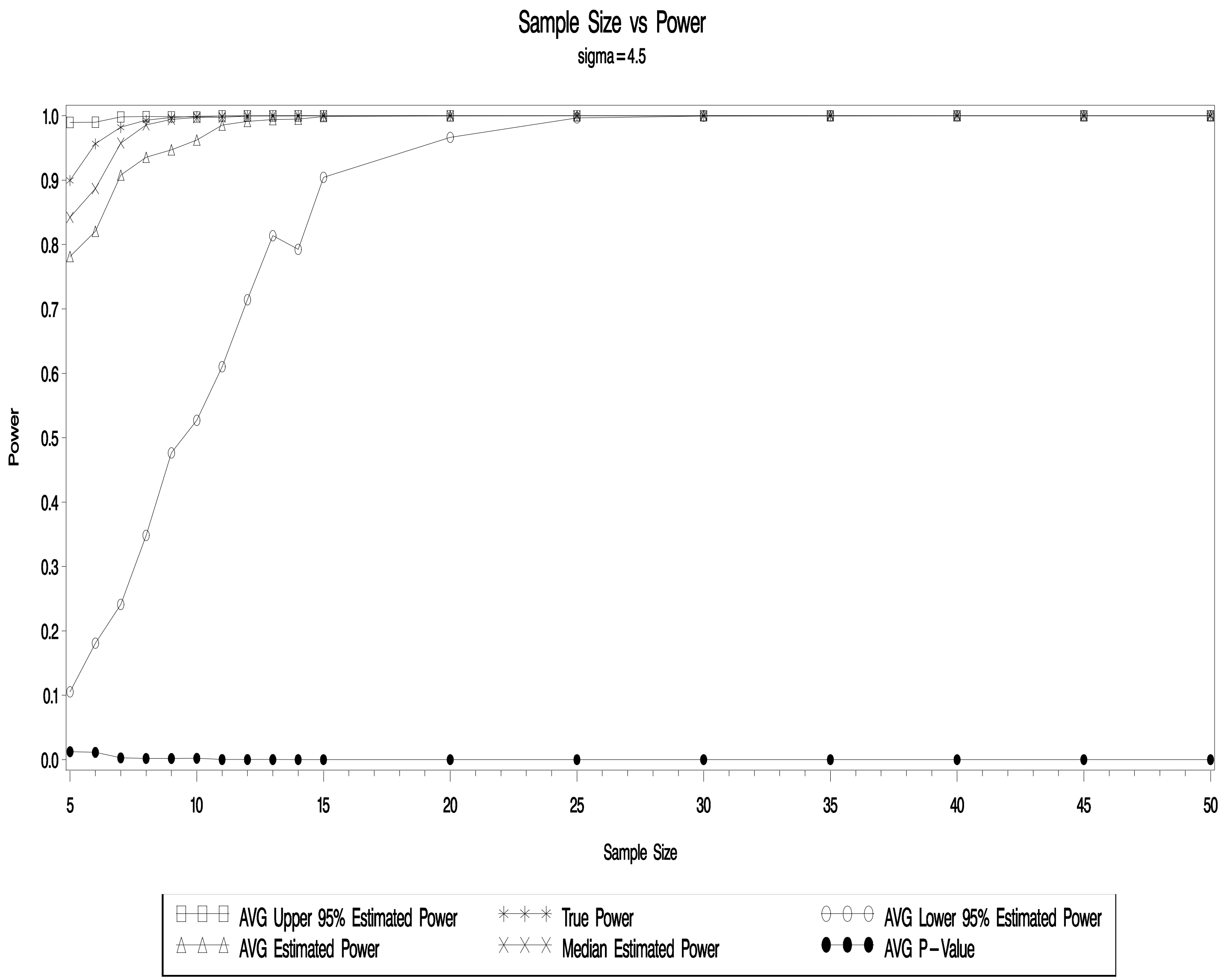
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



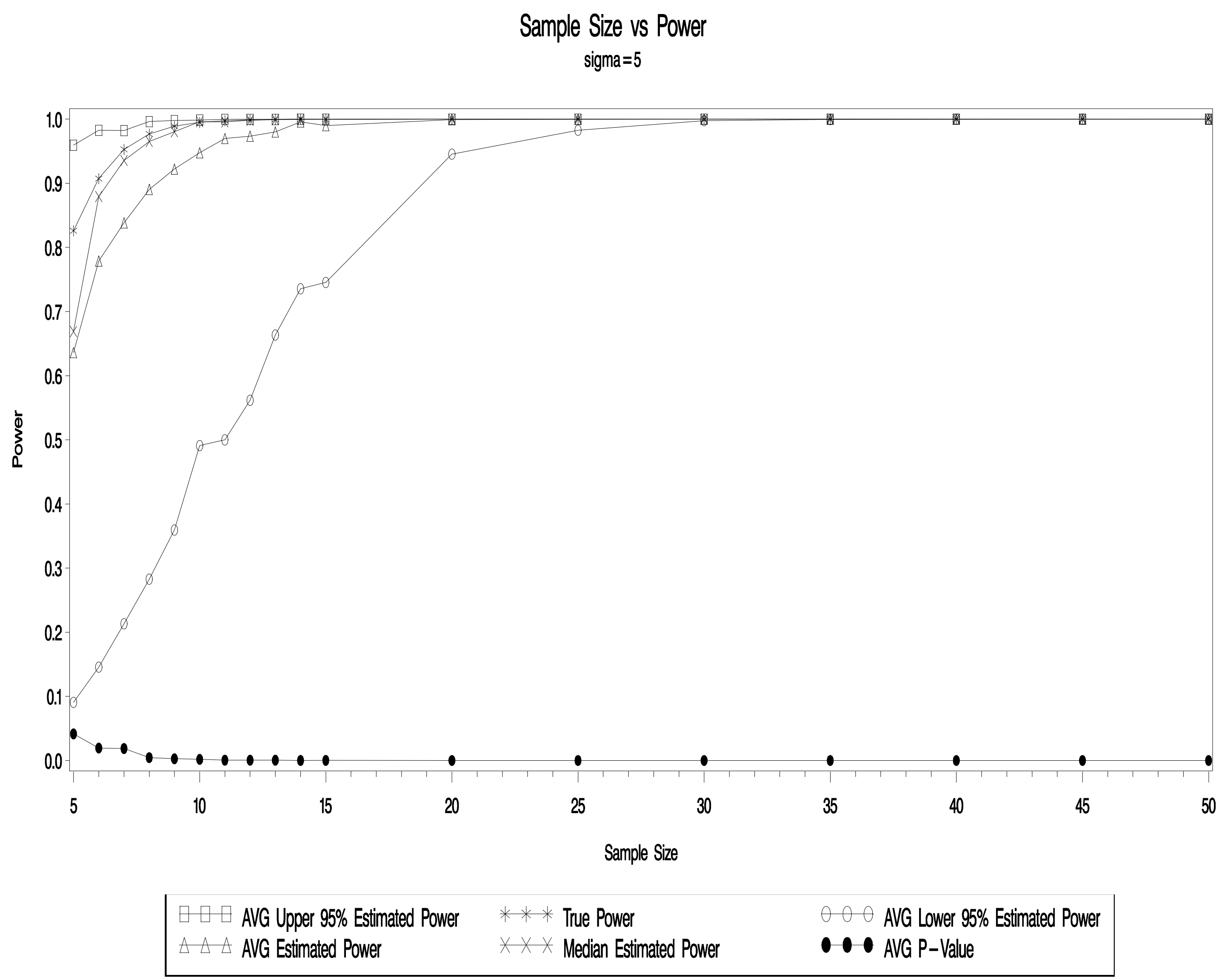
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



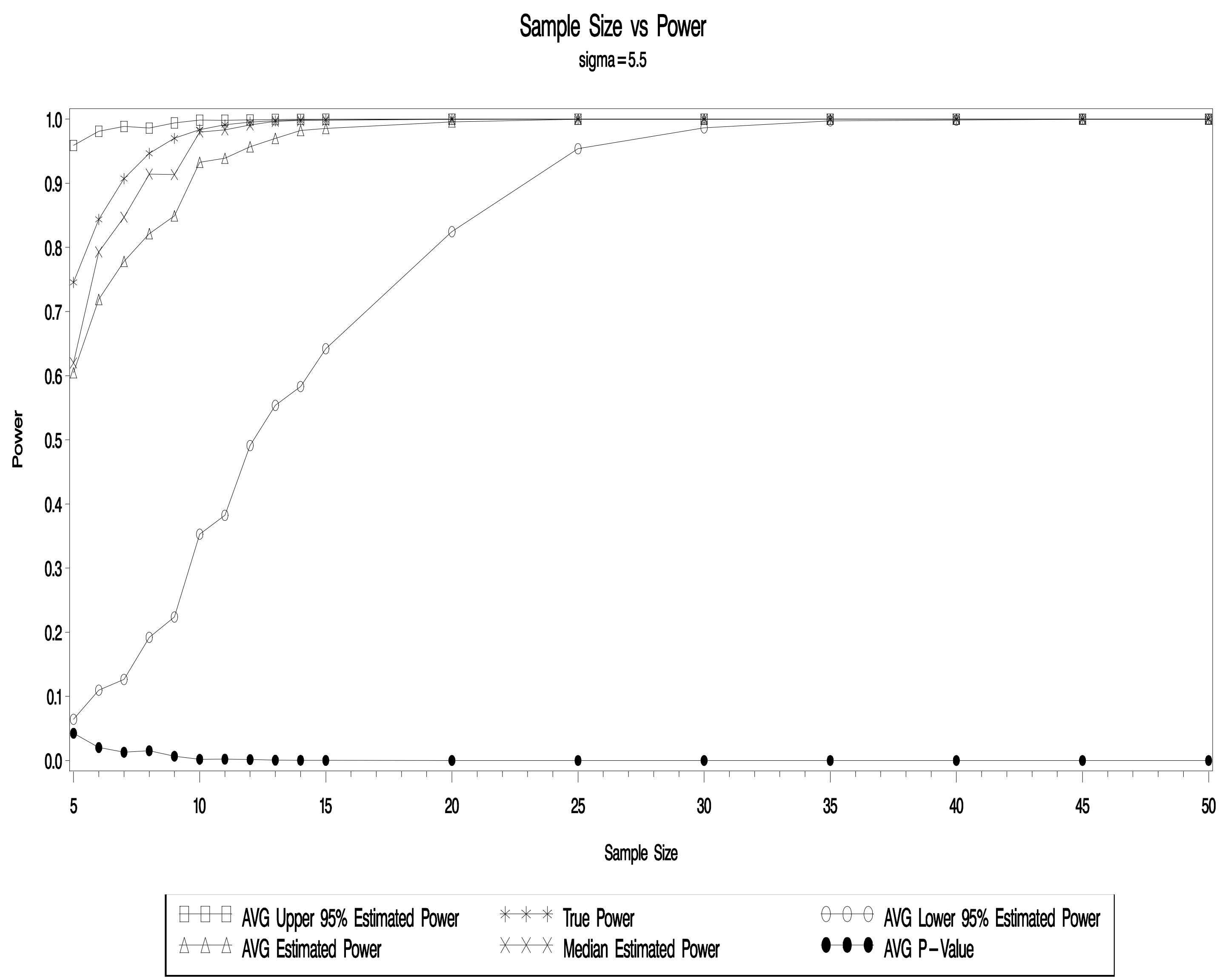
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



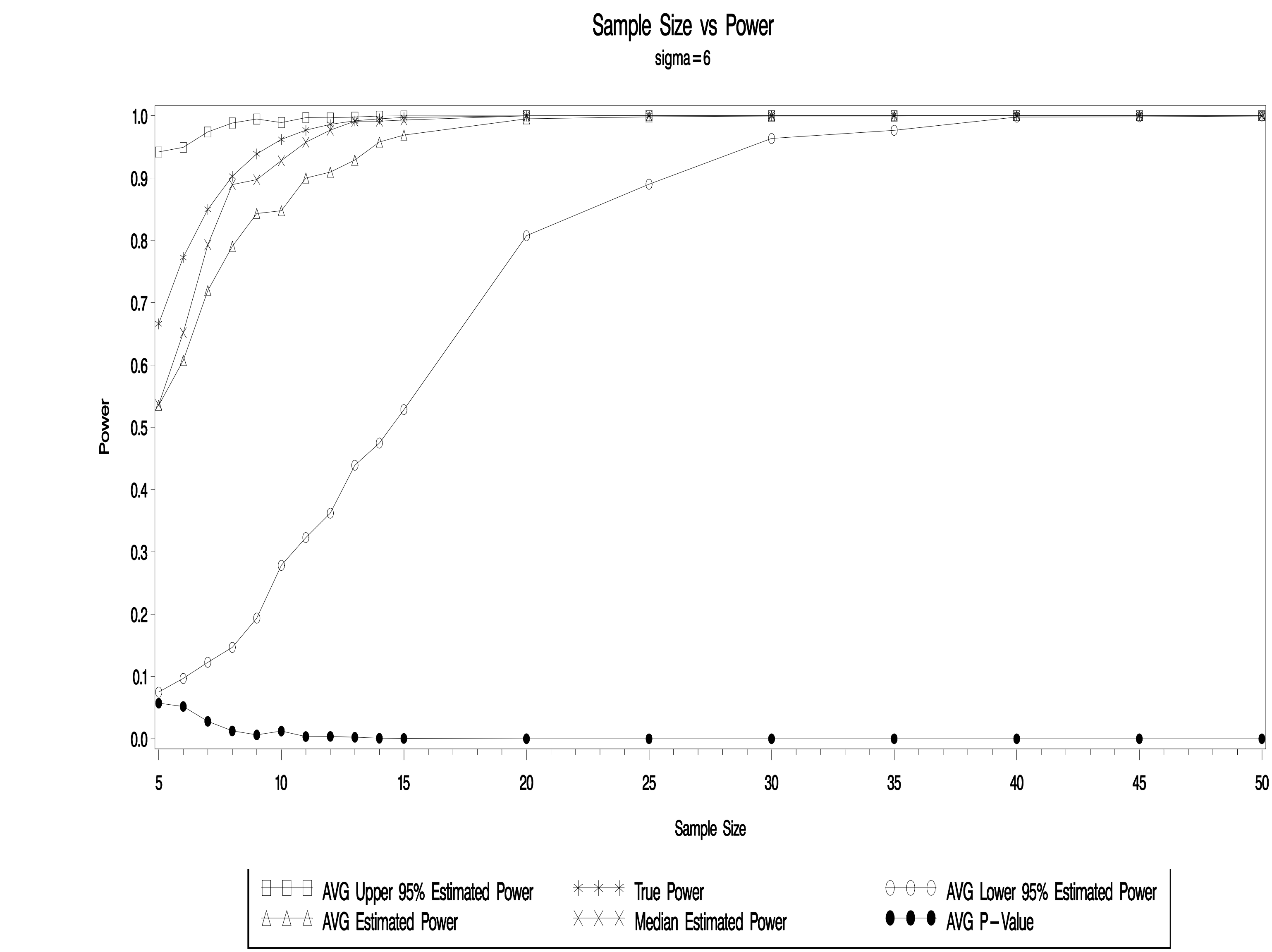
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



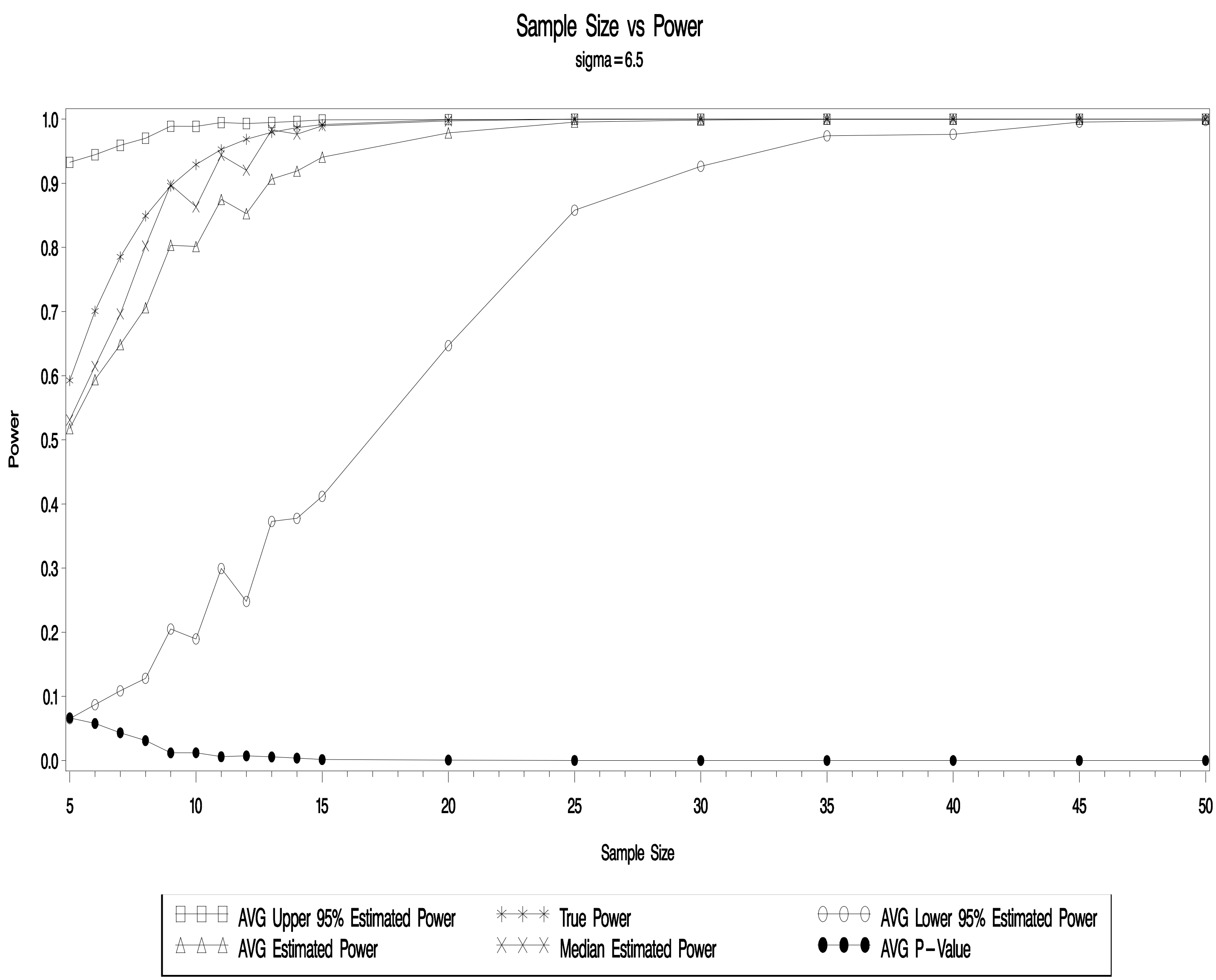
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



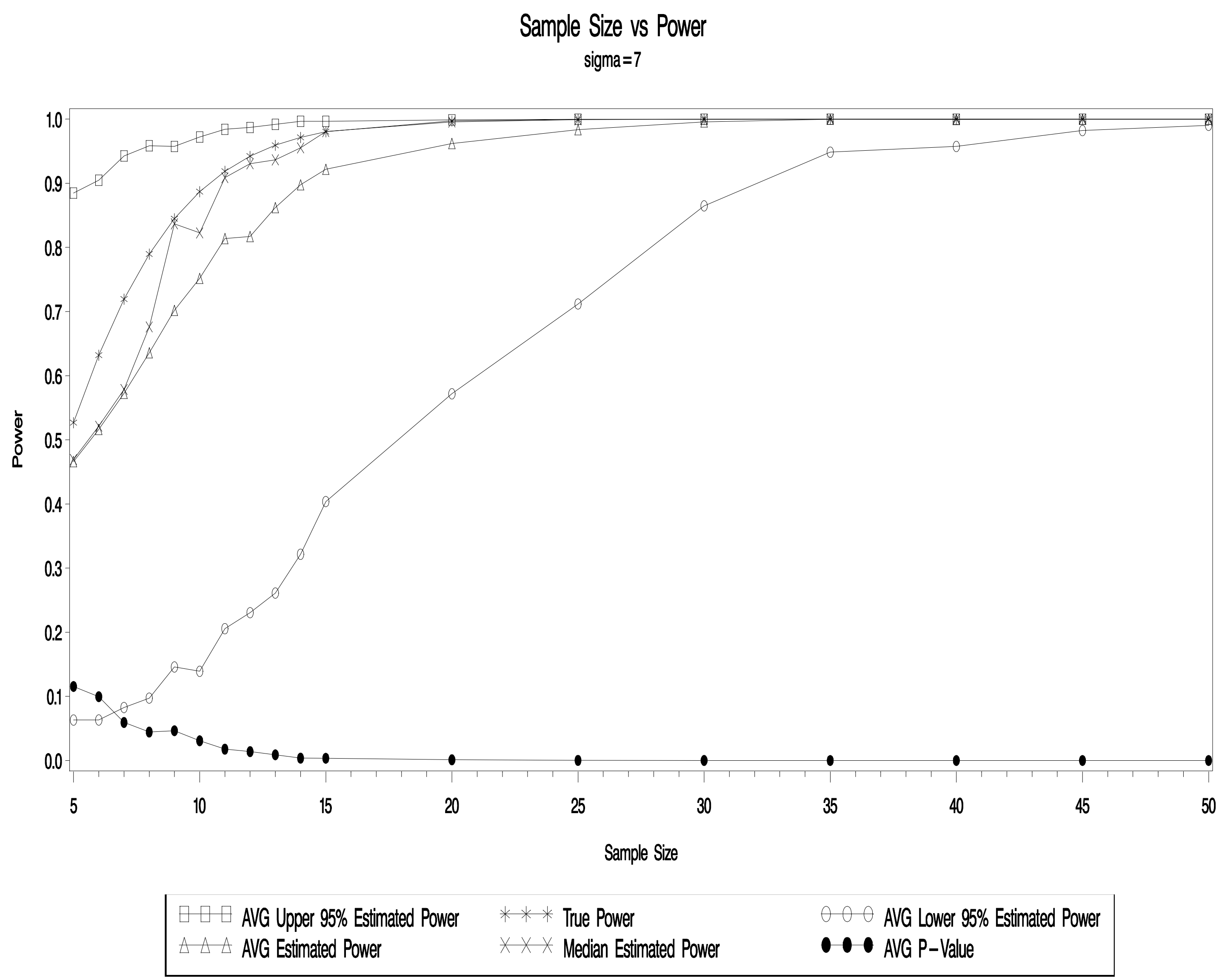
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



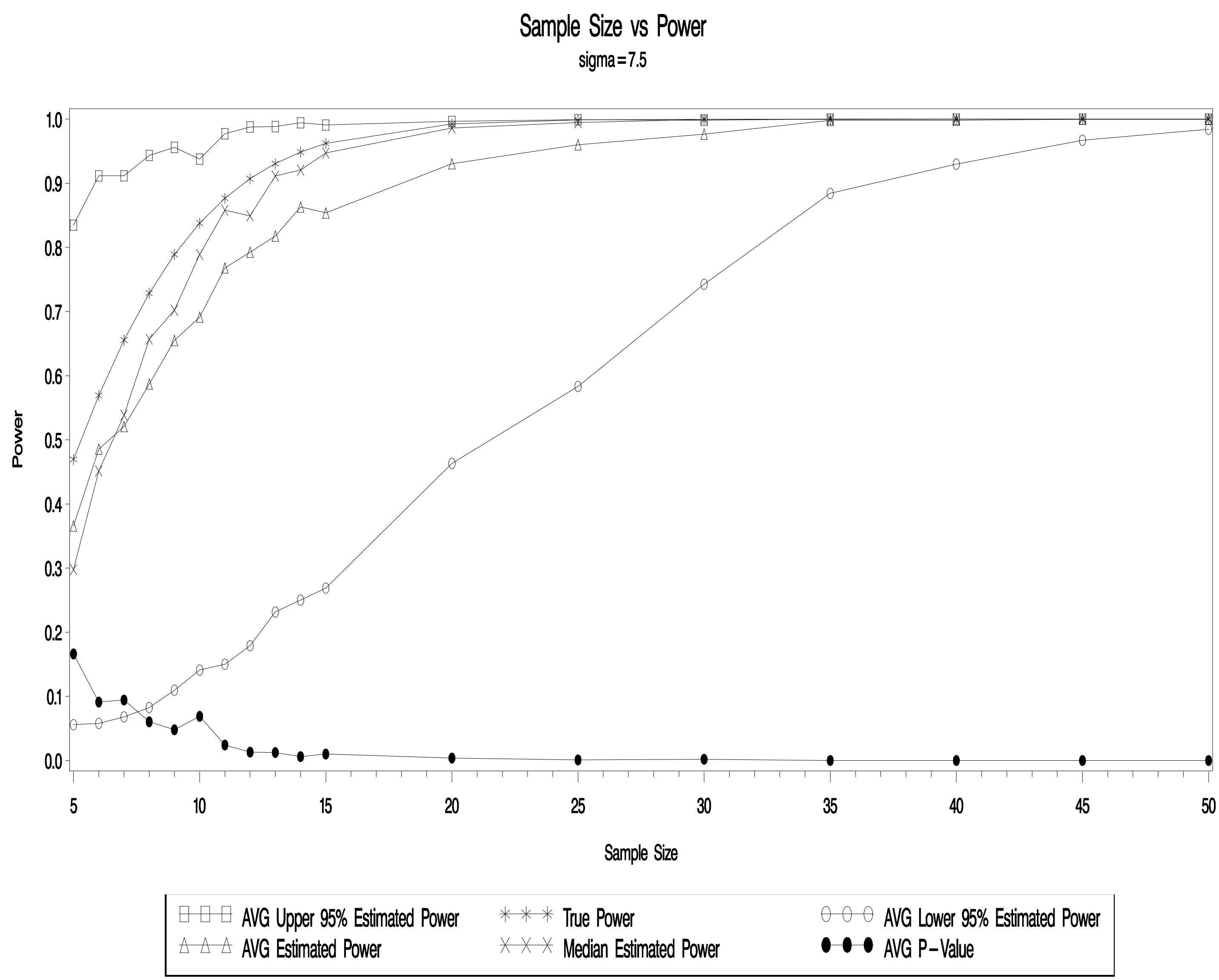
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



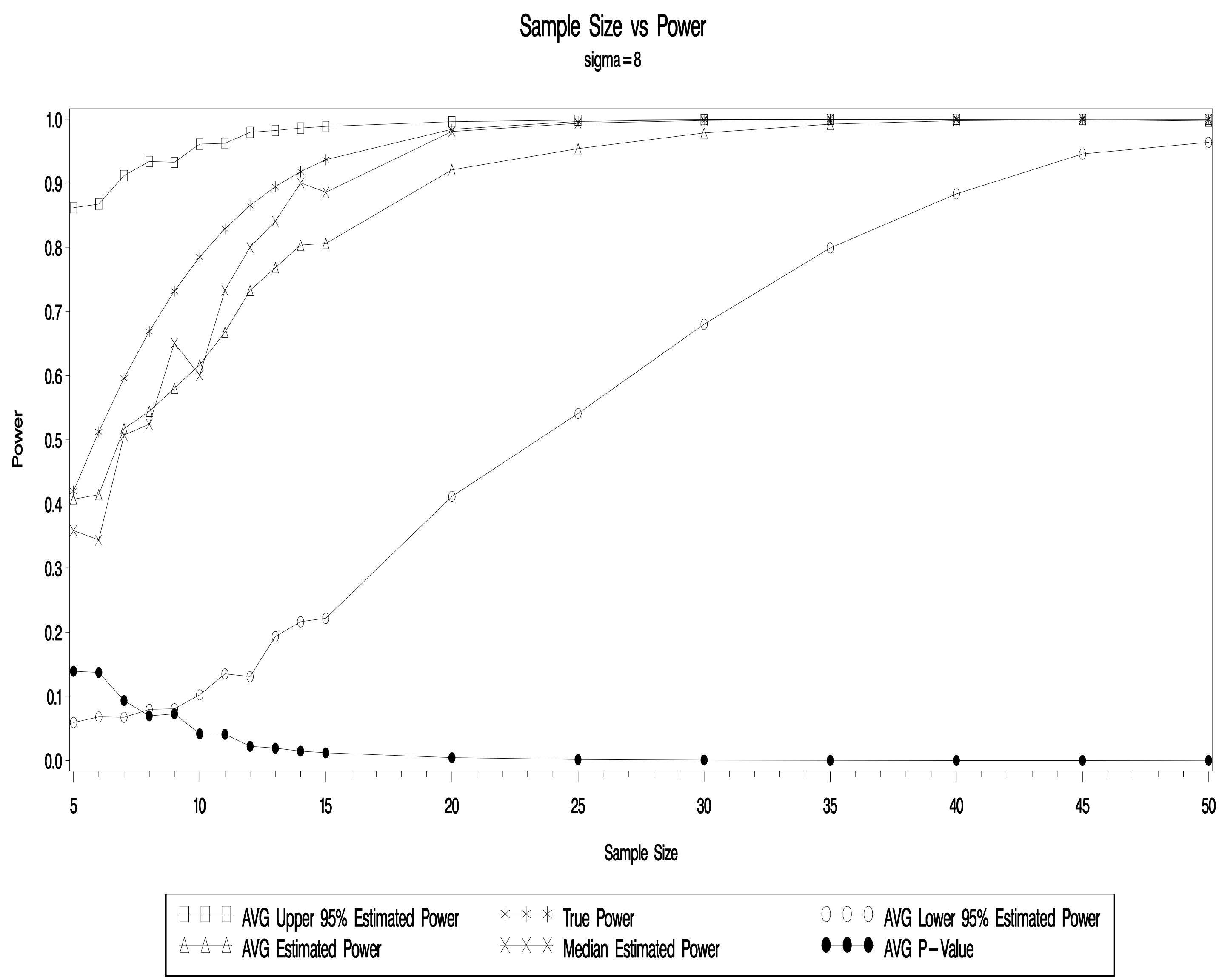
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



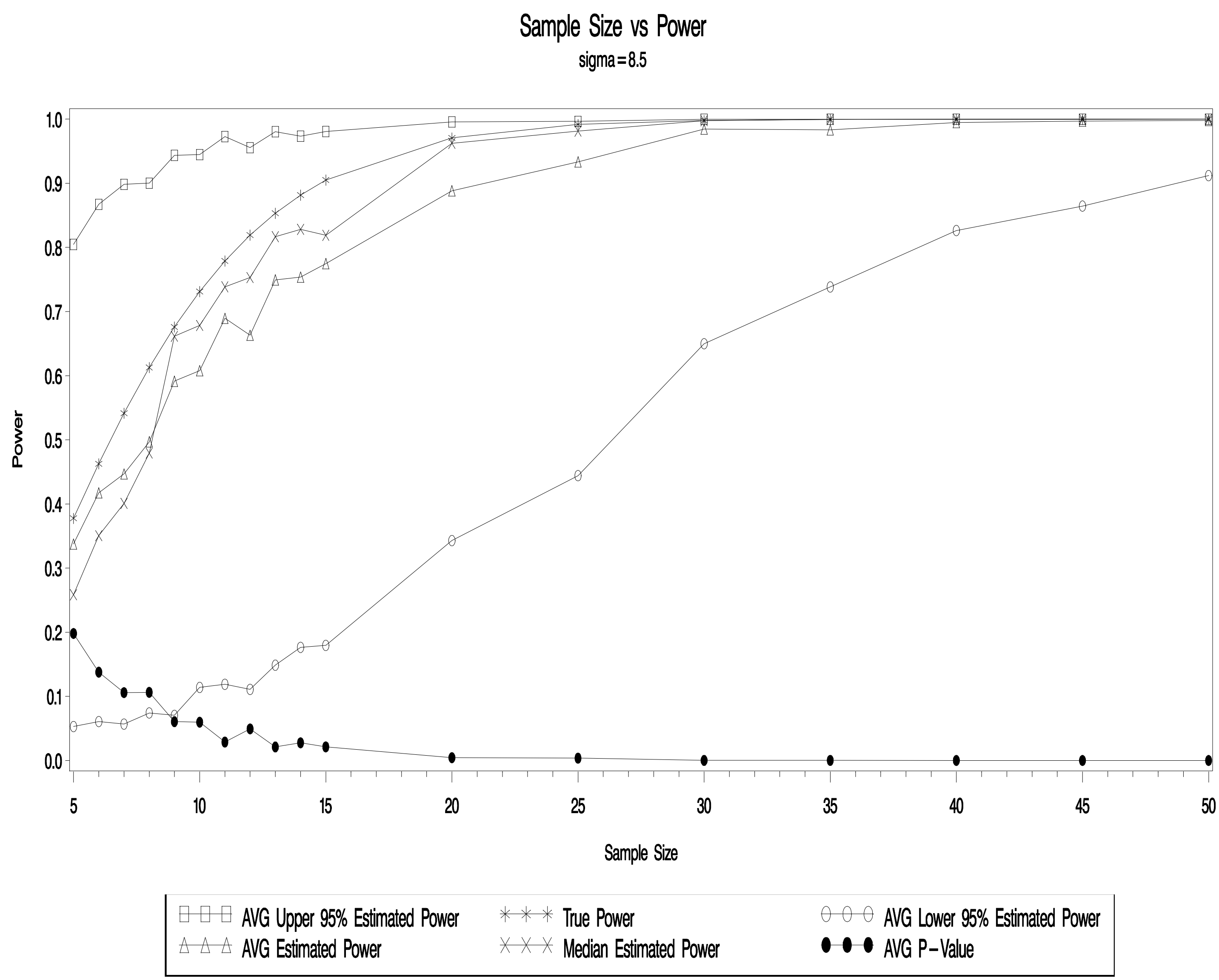
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



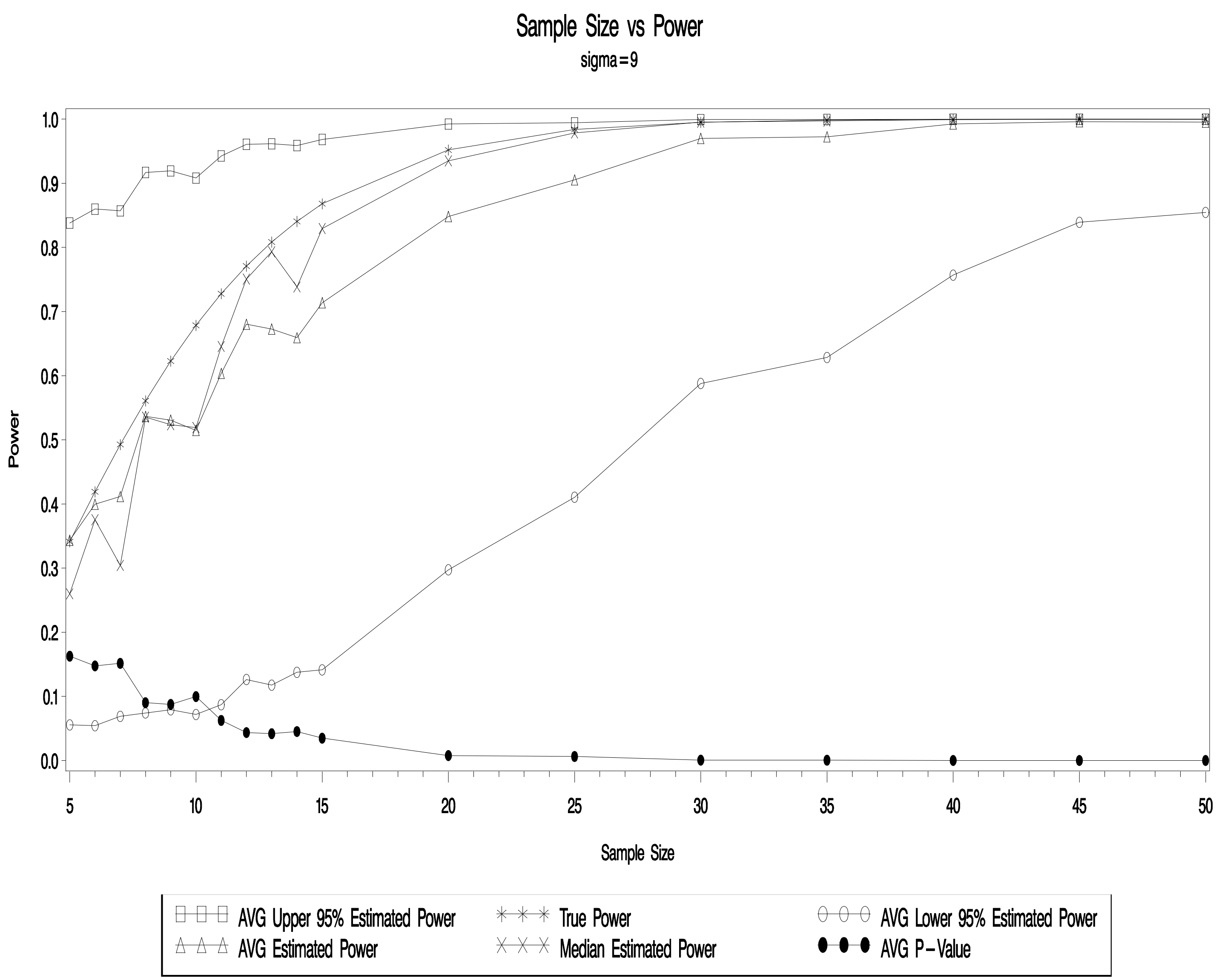
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



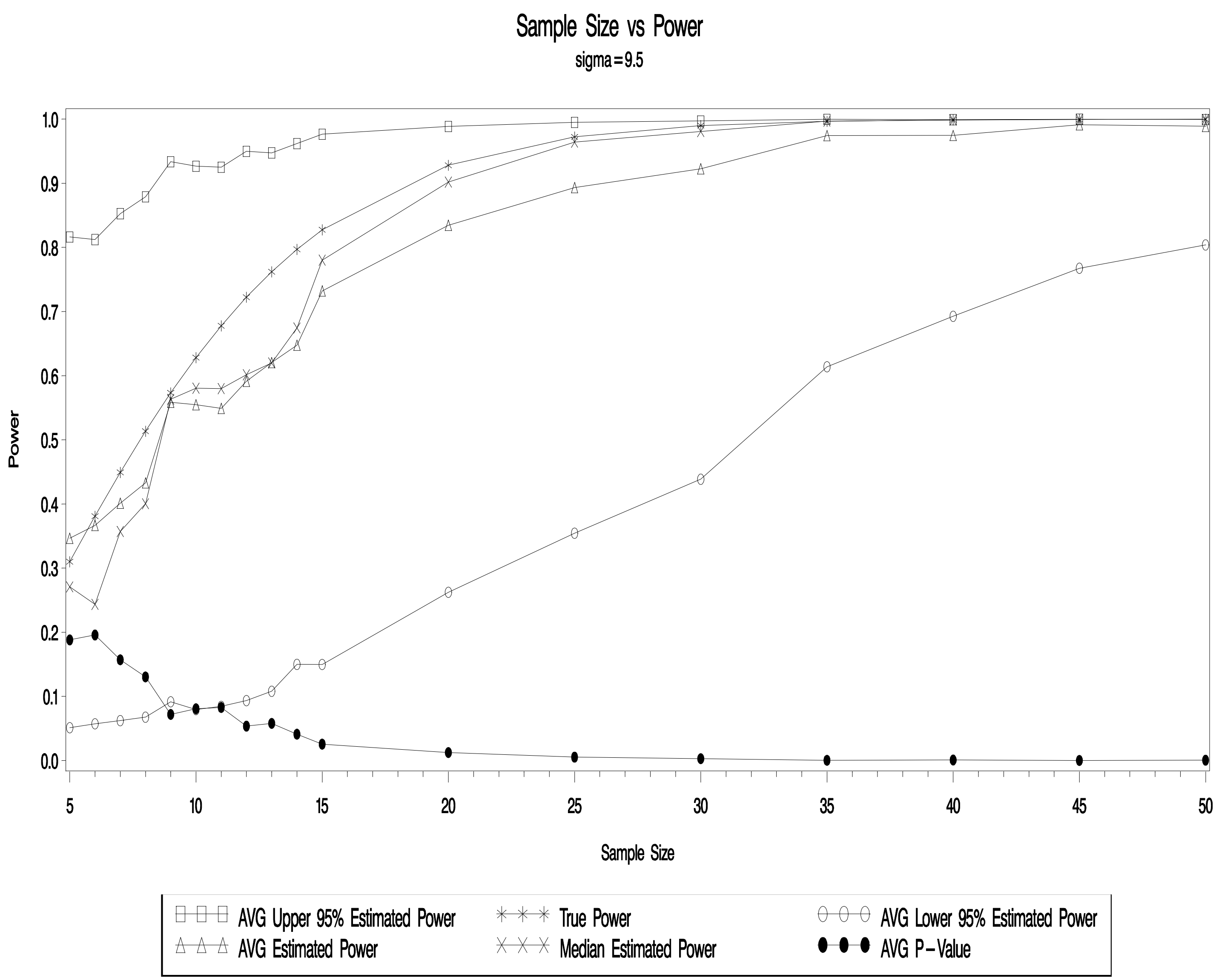
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



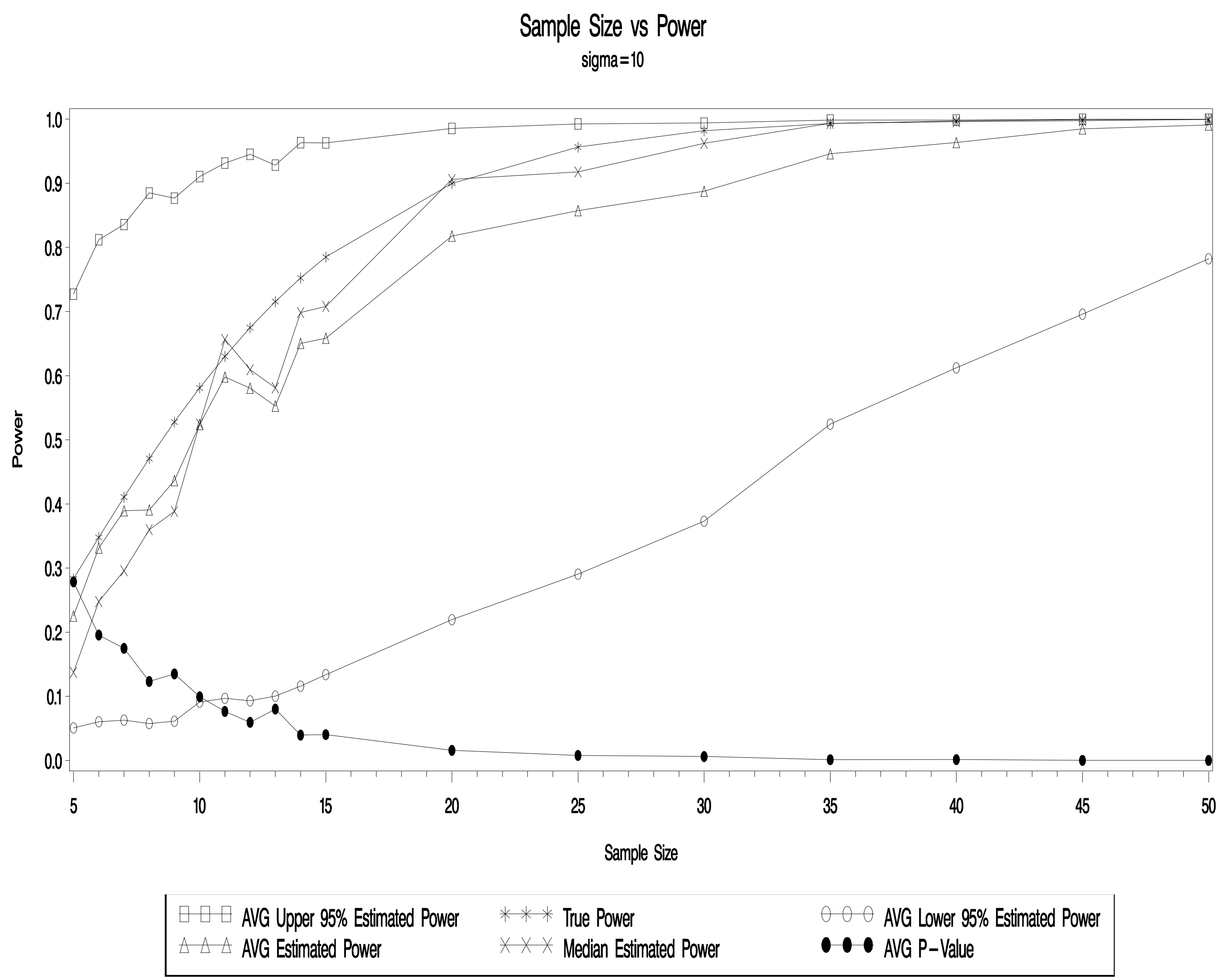
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



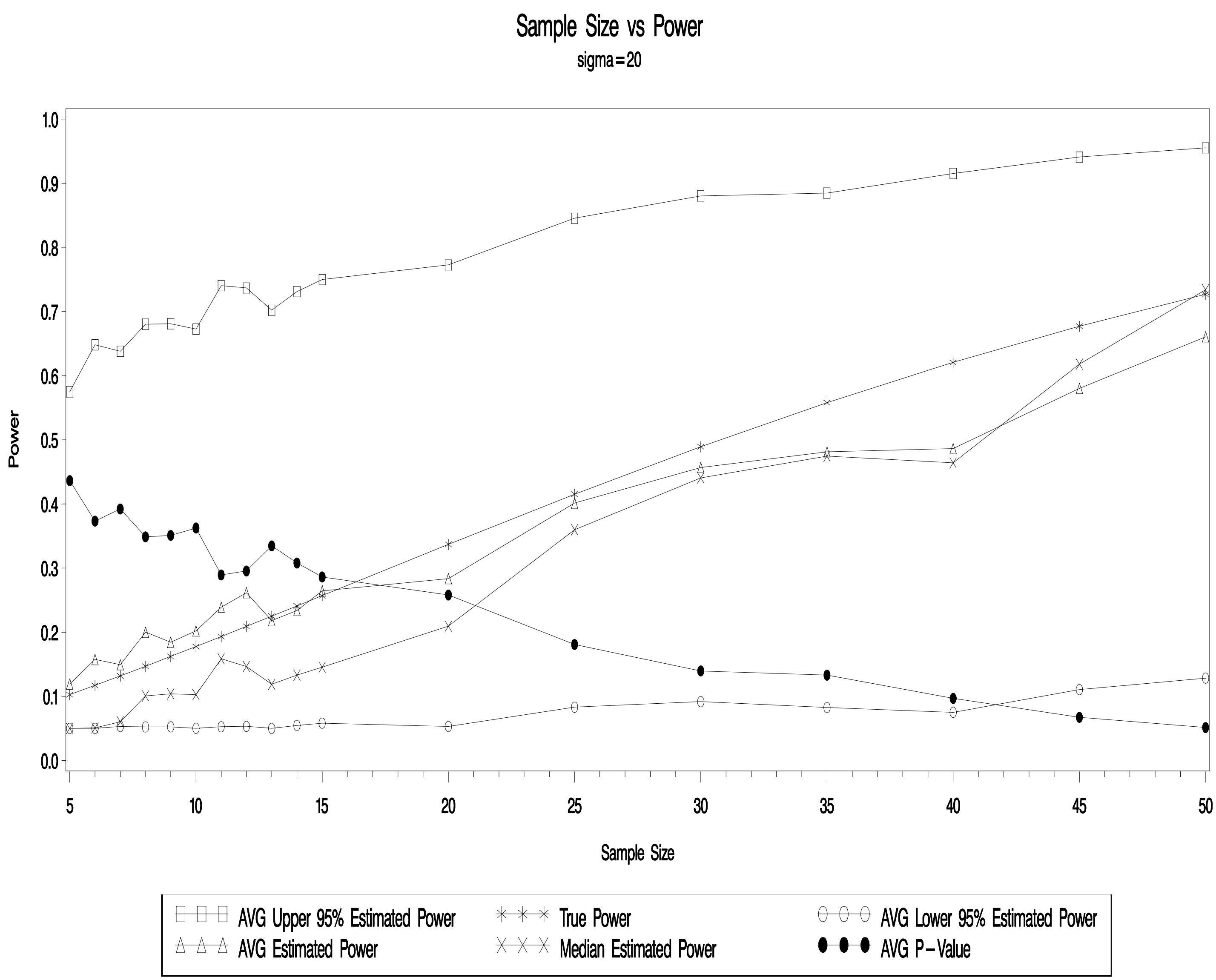
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



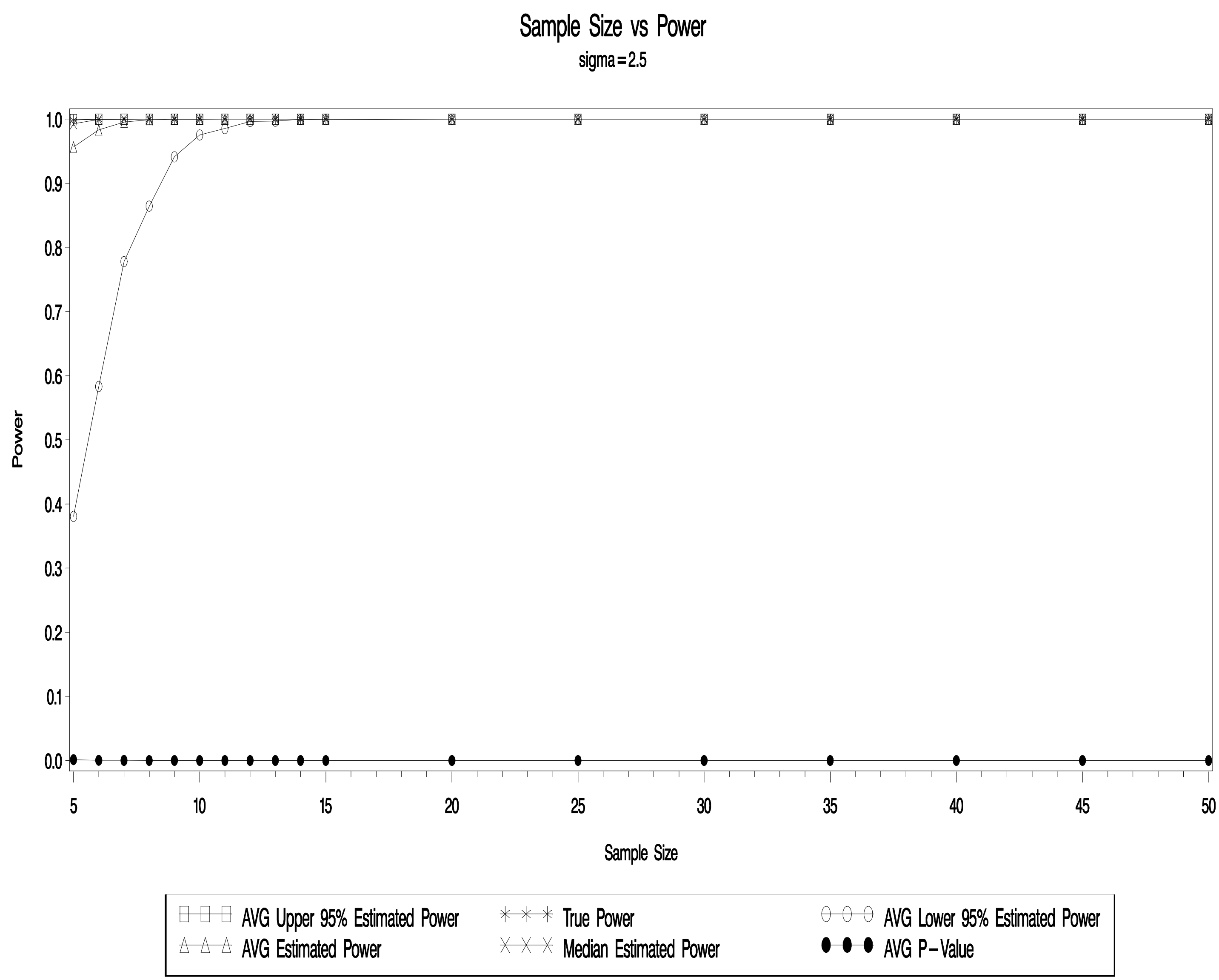
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



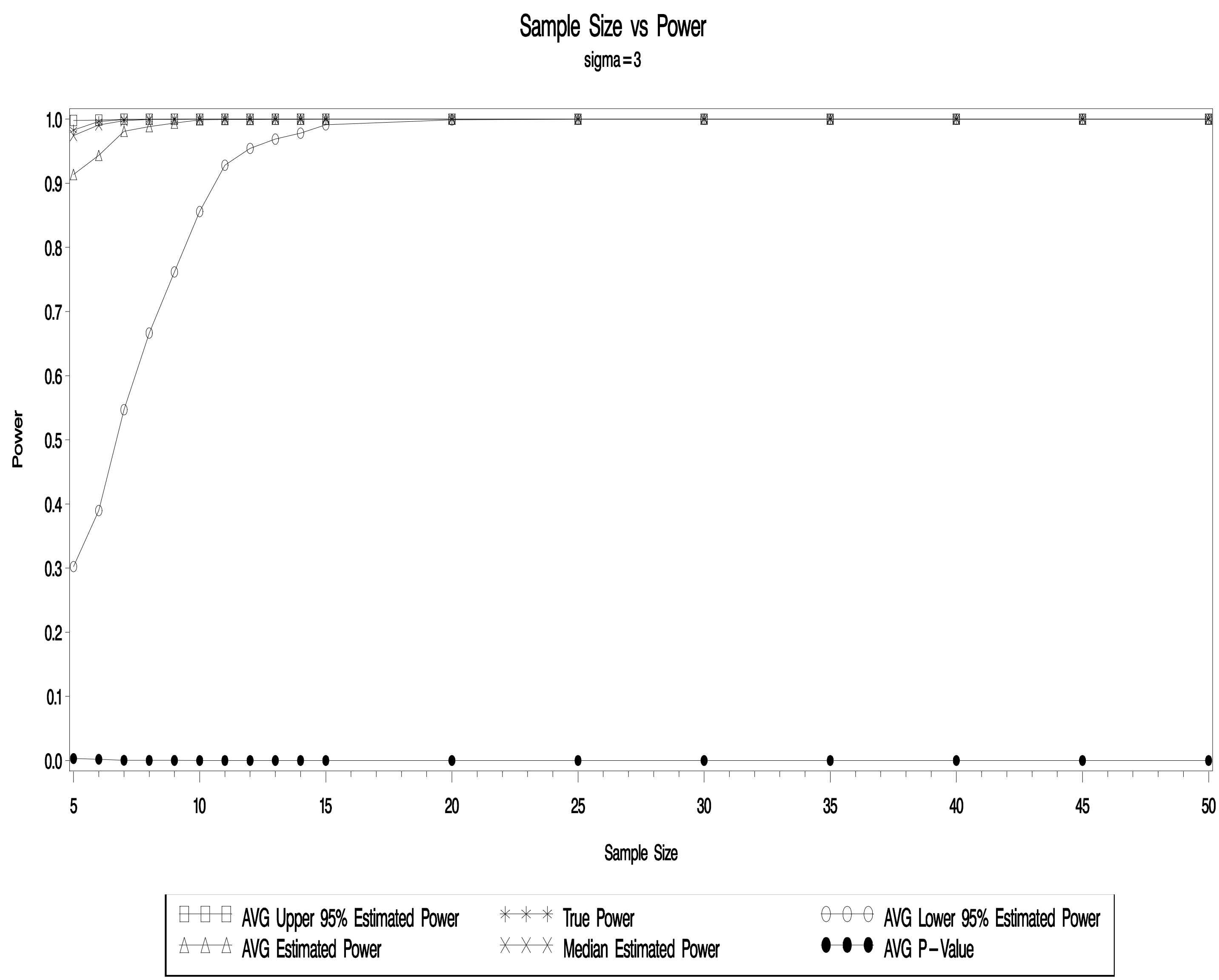
Appendix C: Graphs of Treatment Arrangement 30, 40, 40



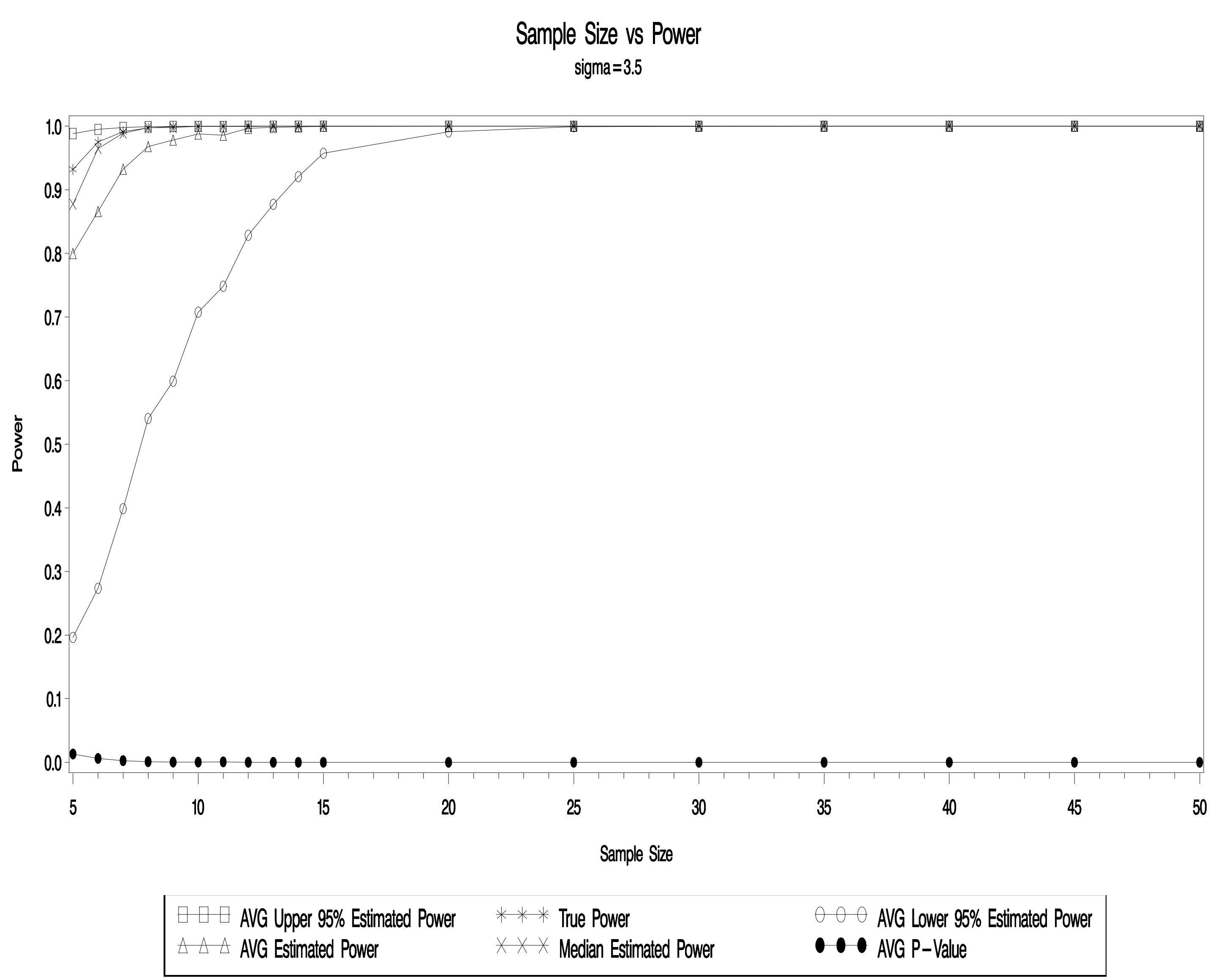
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



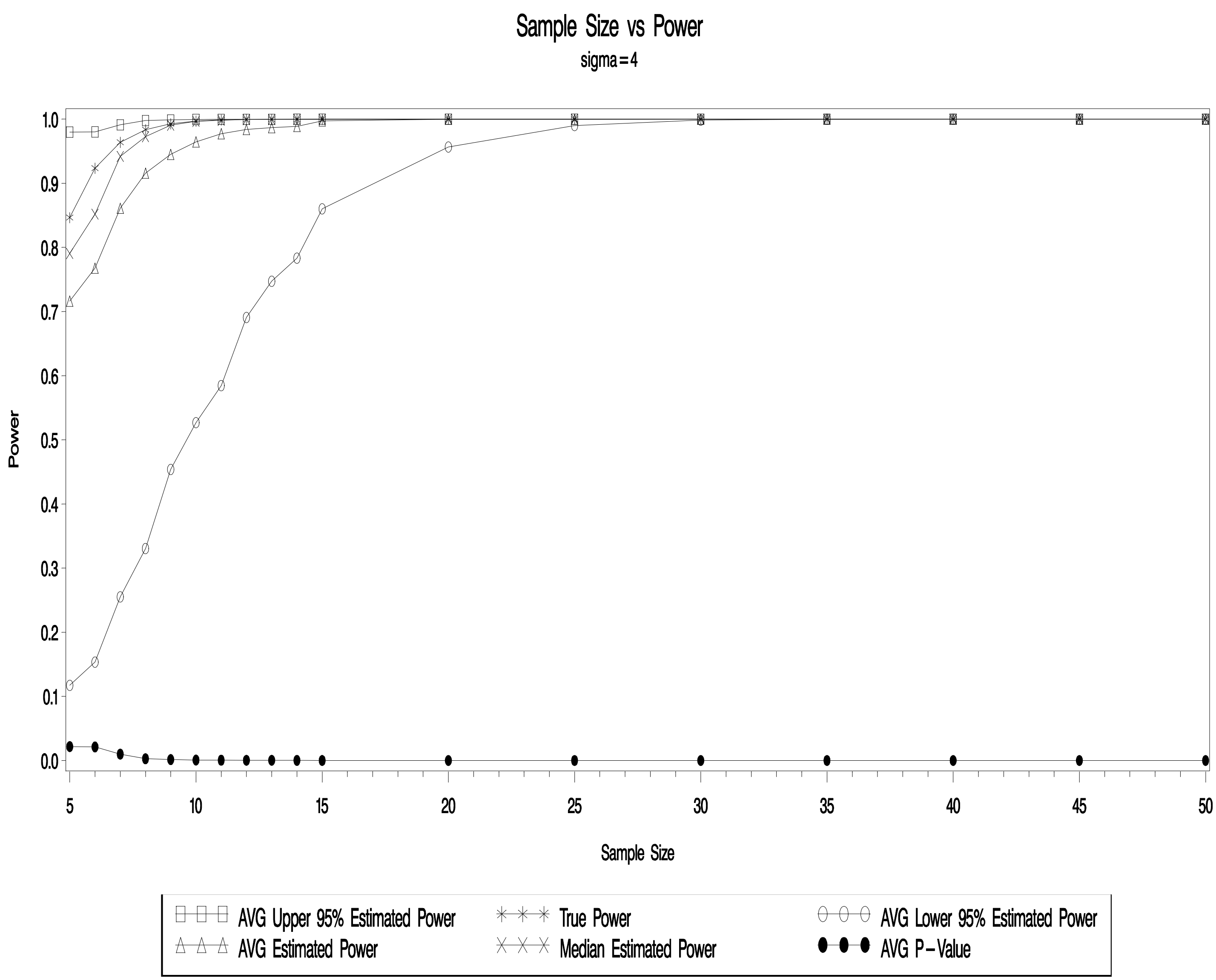
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



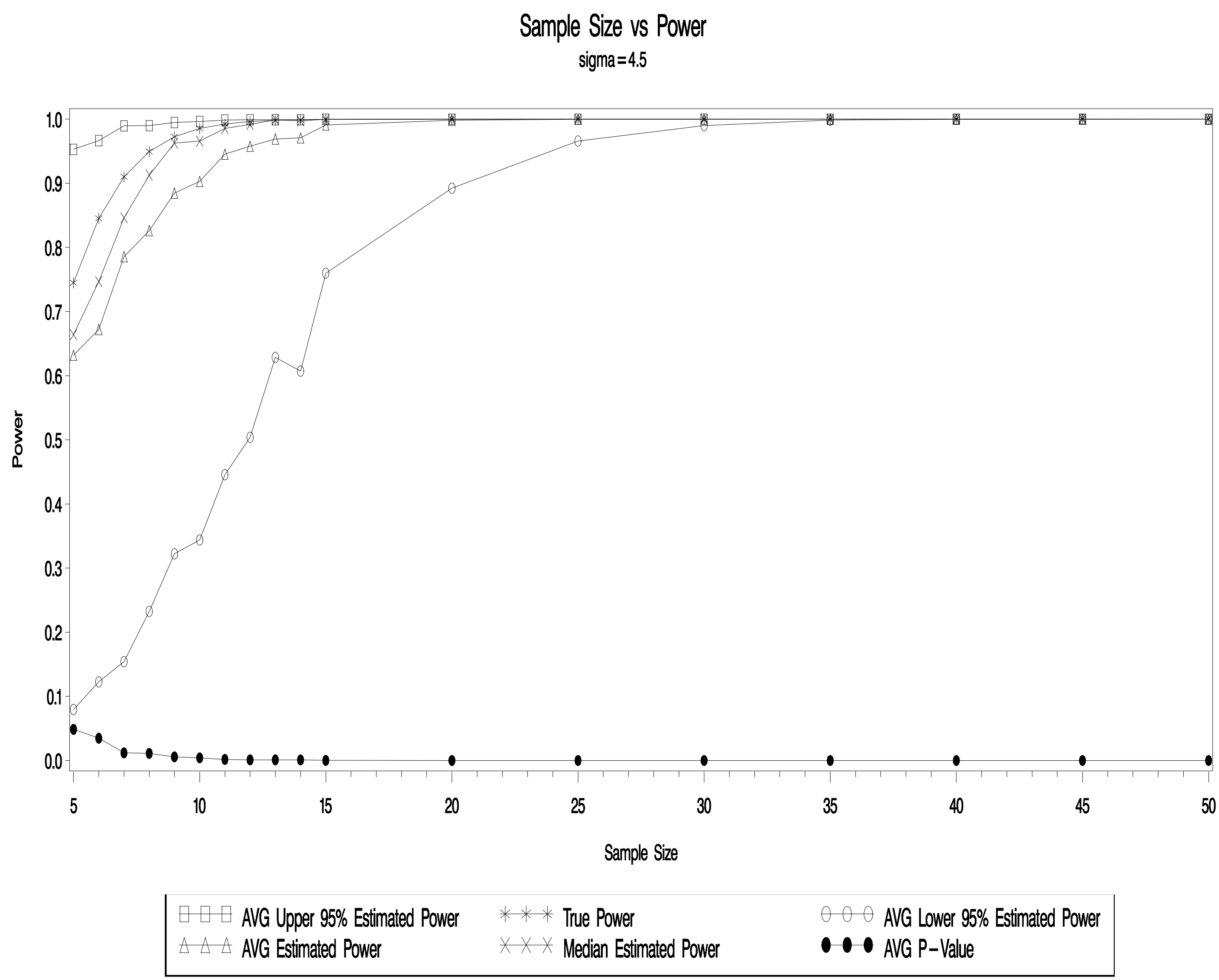
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



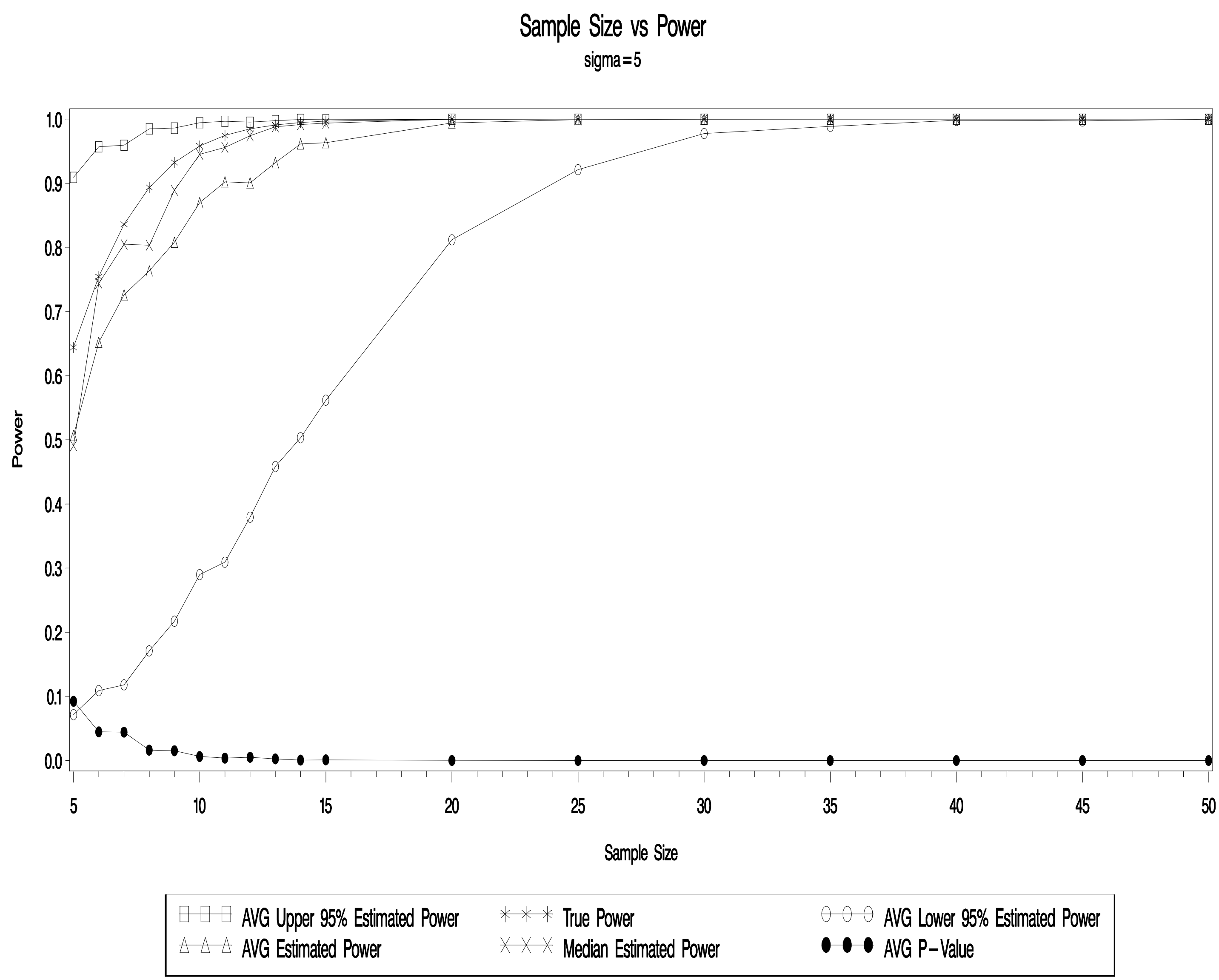
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



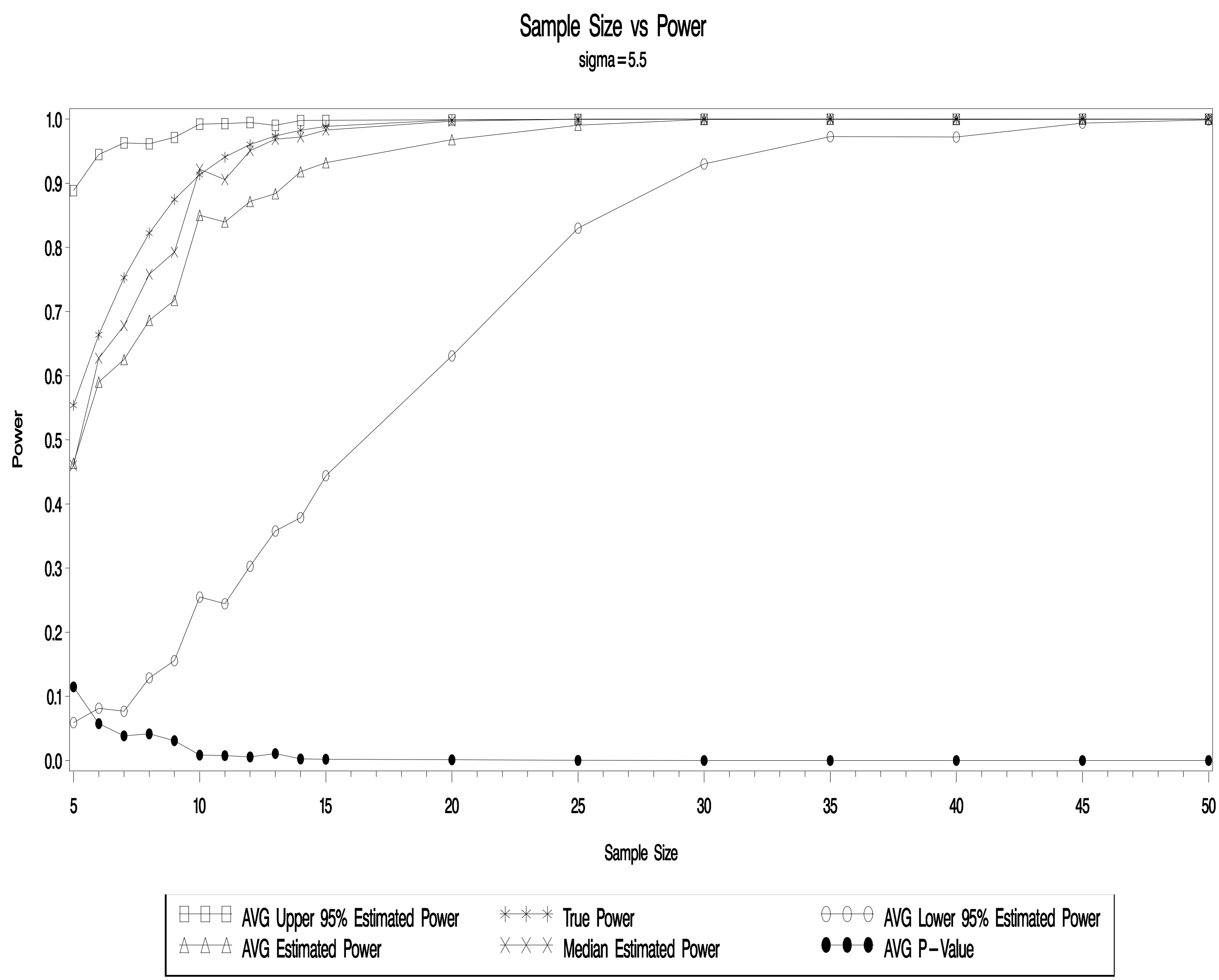
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



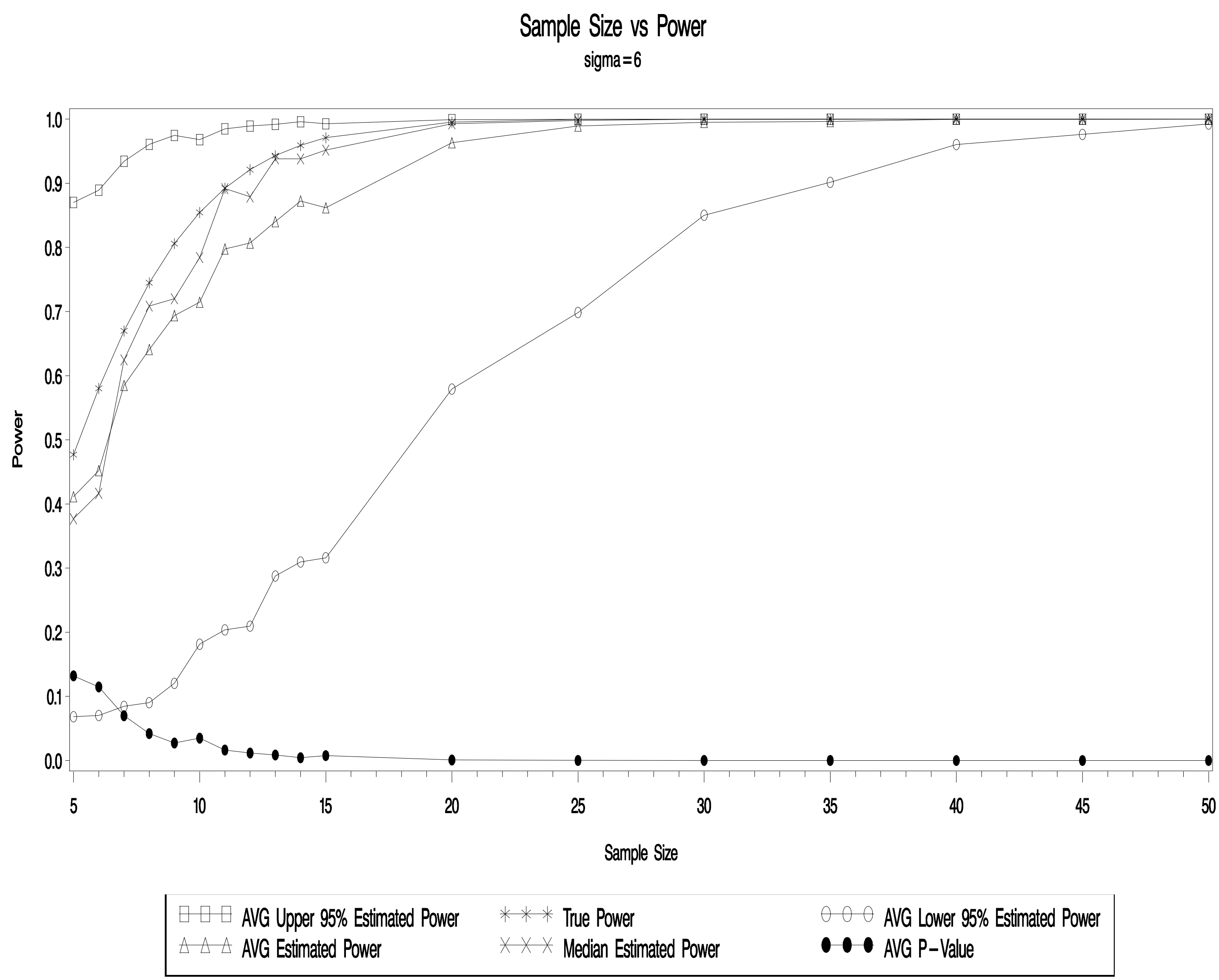
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



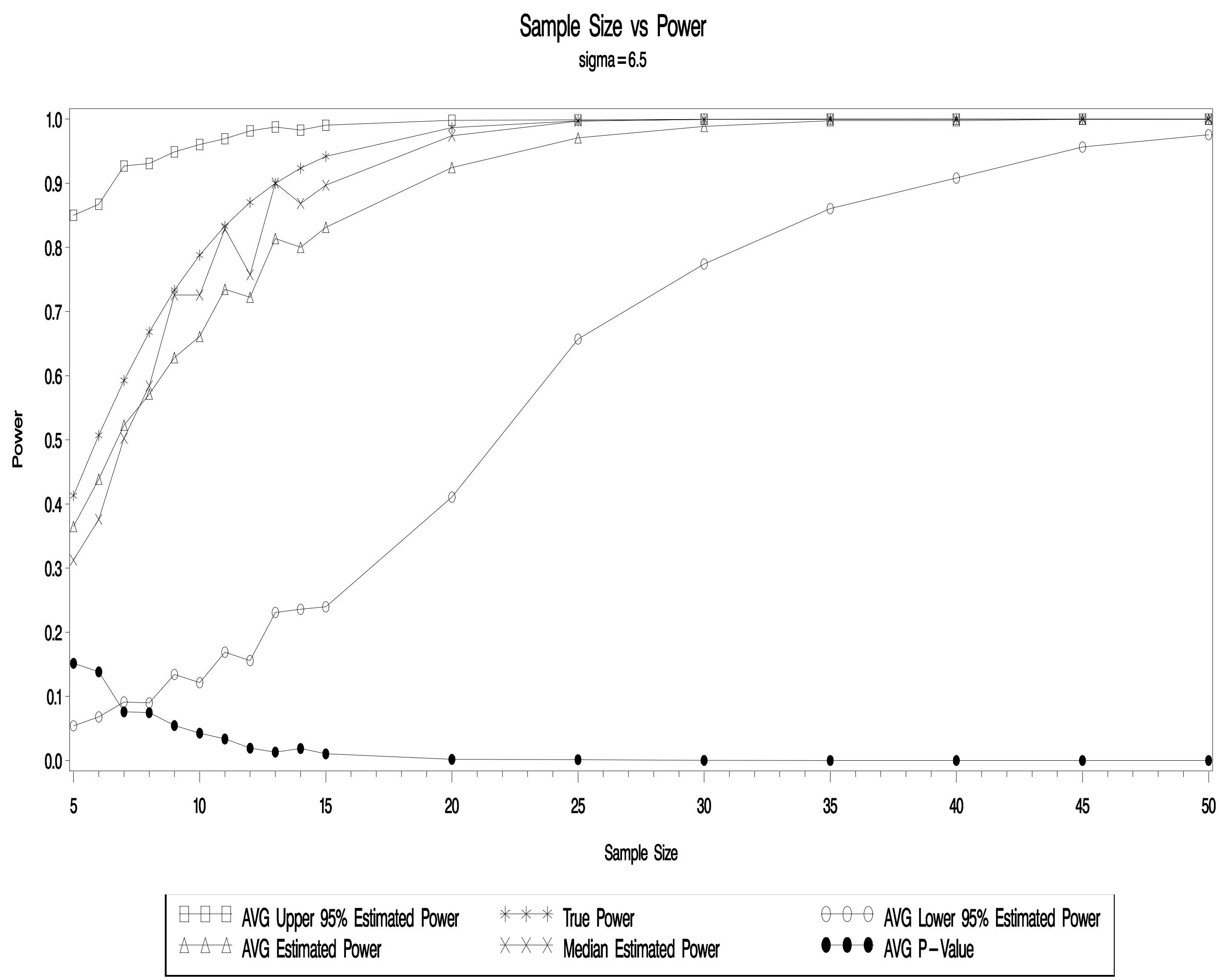
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



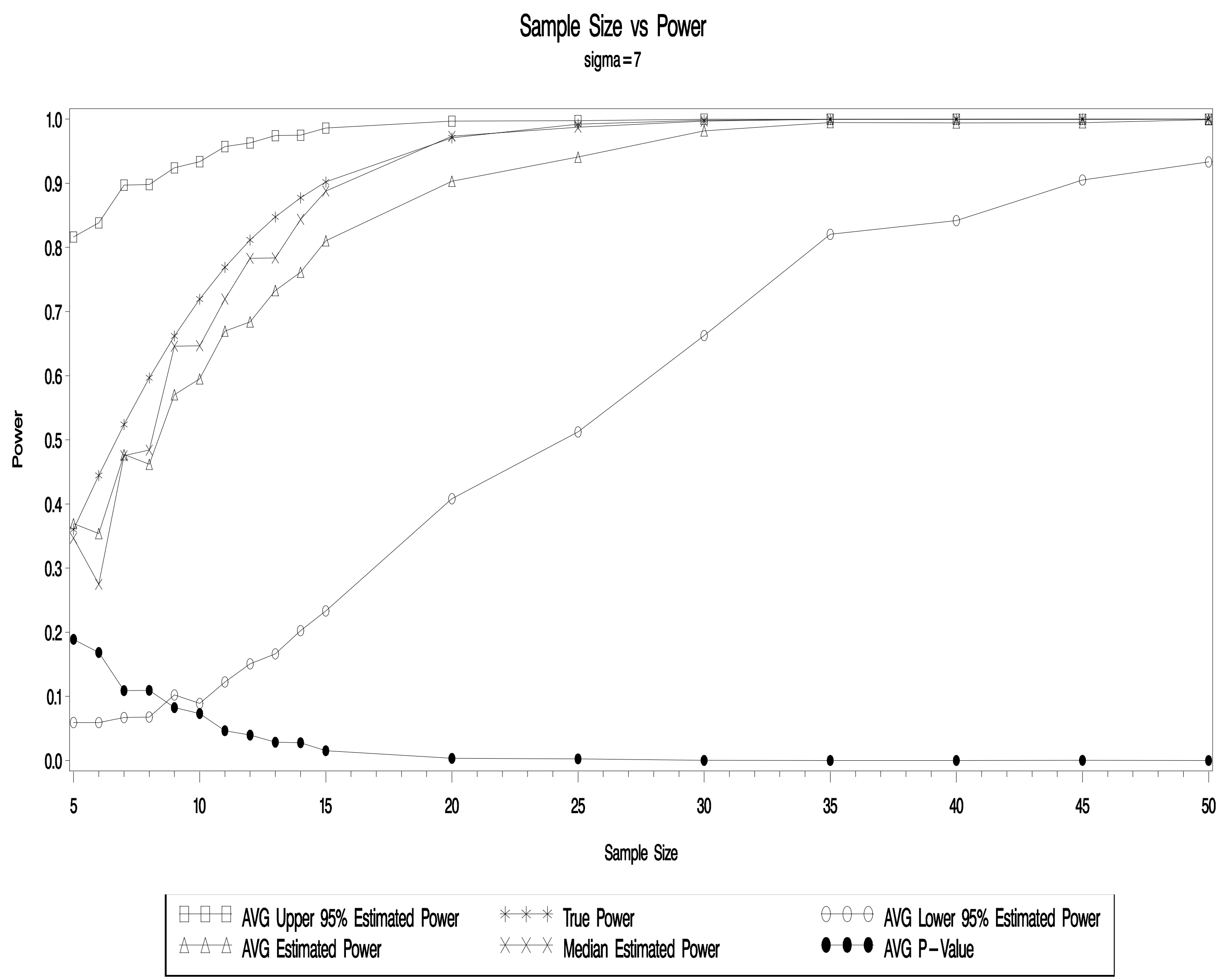
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



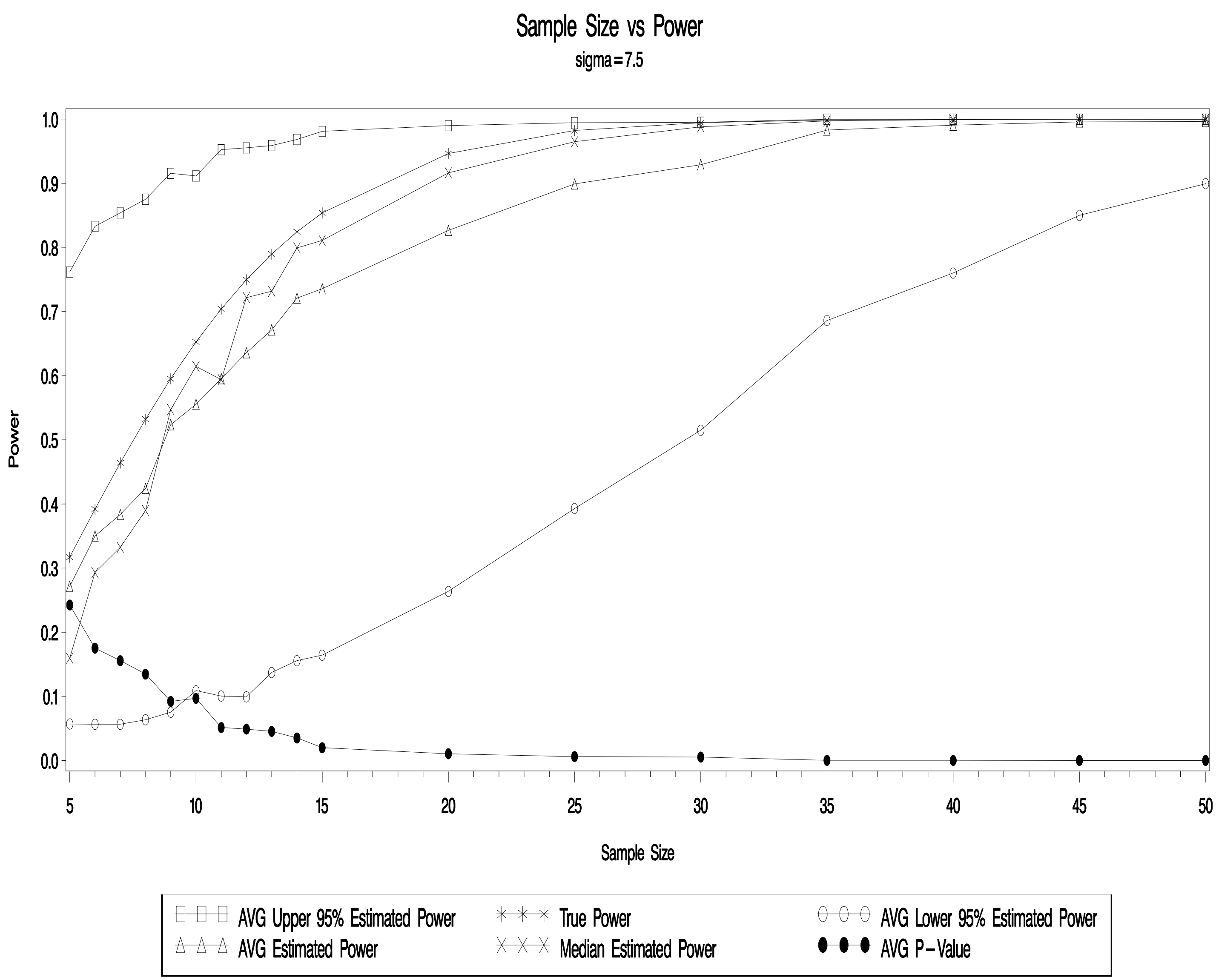
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



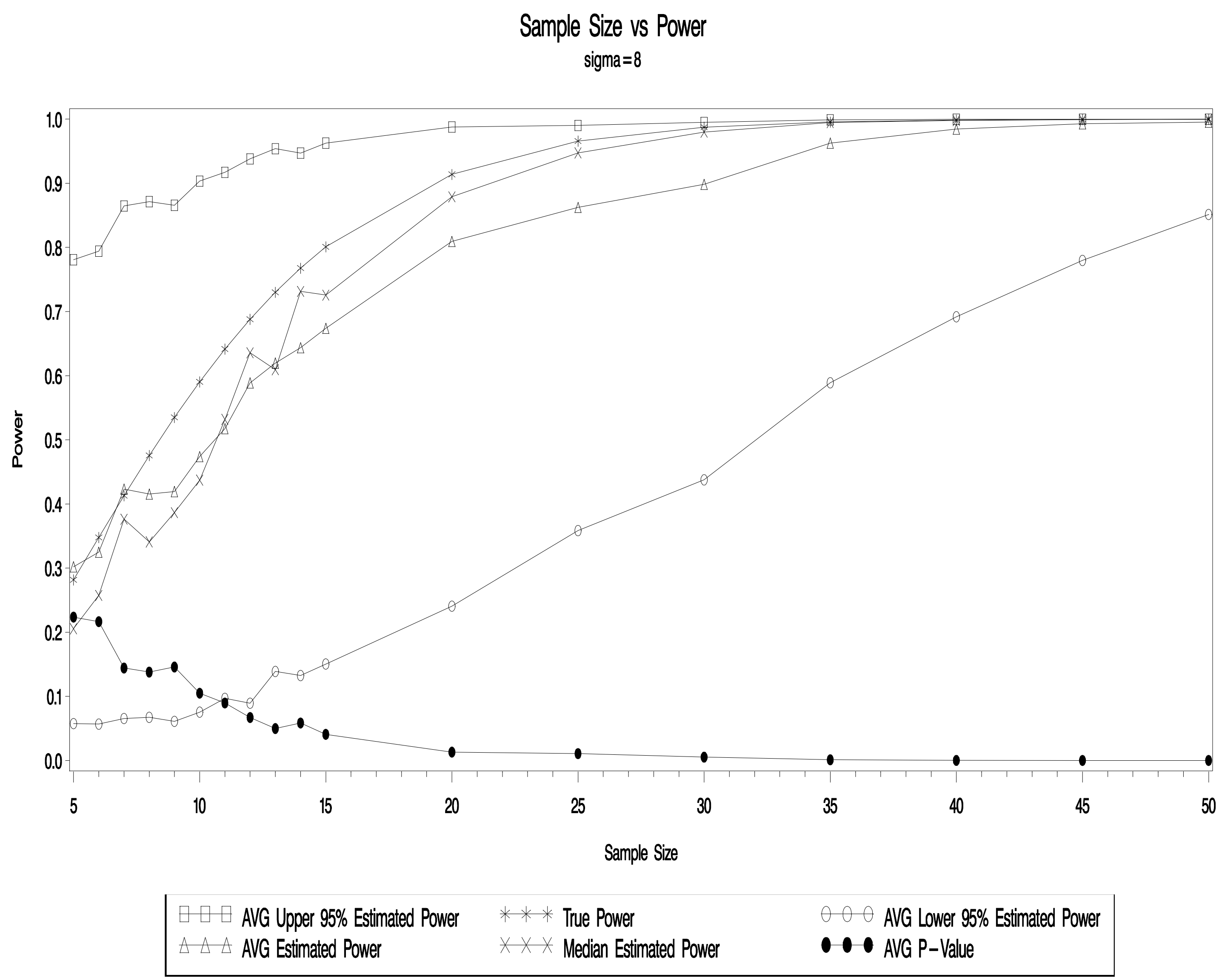
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



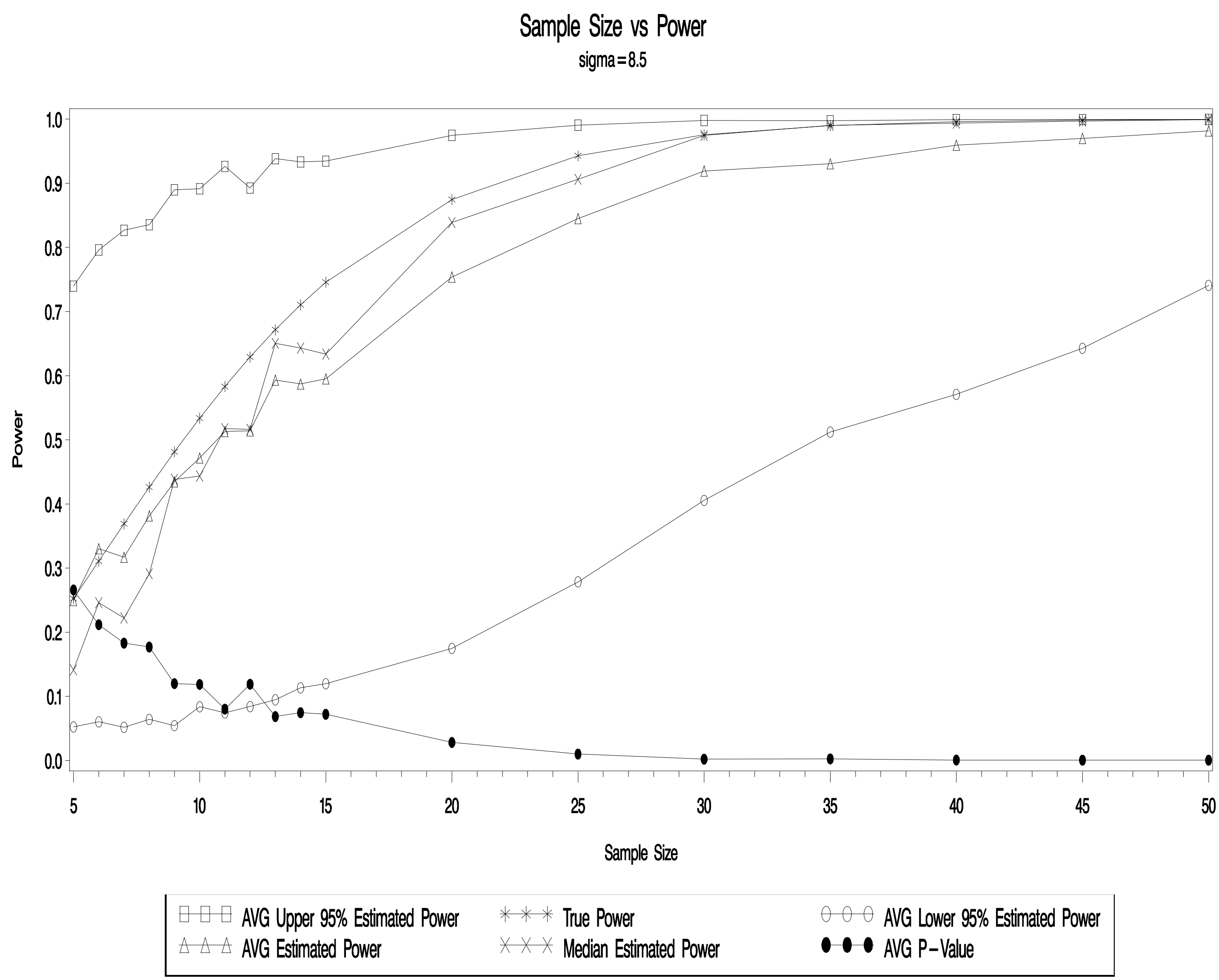
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



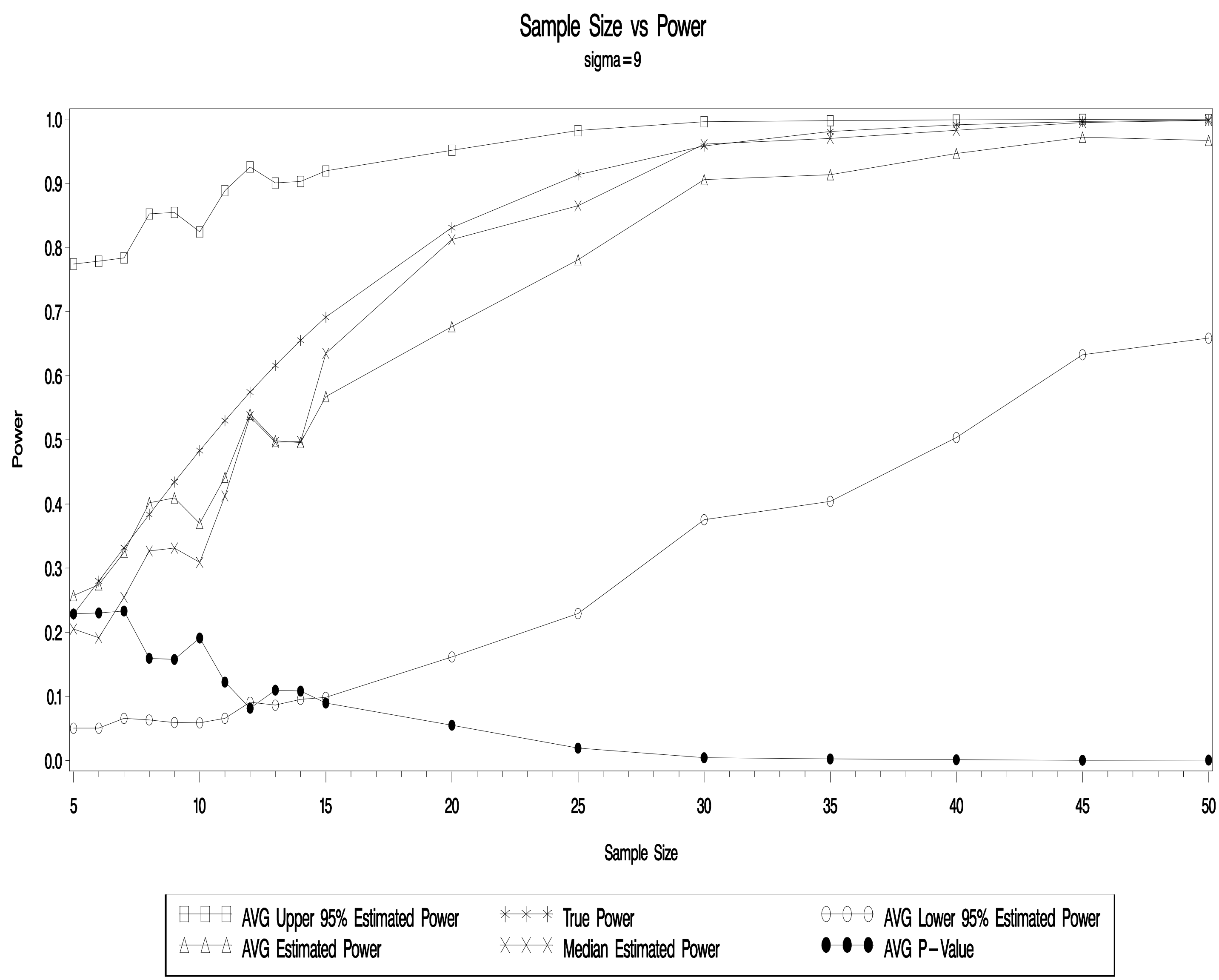
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



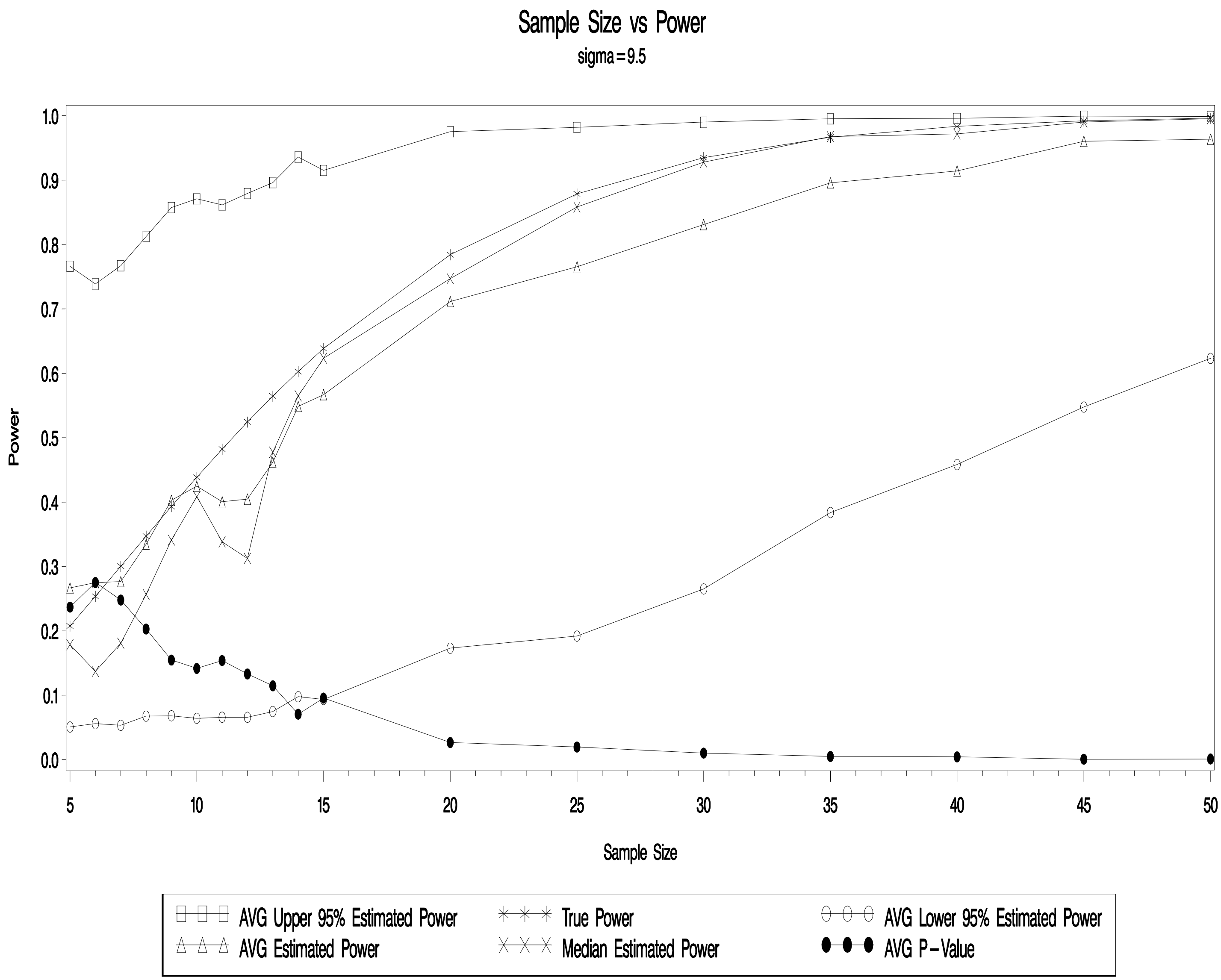
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



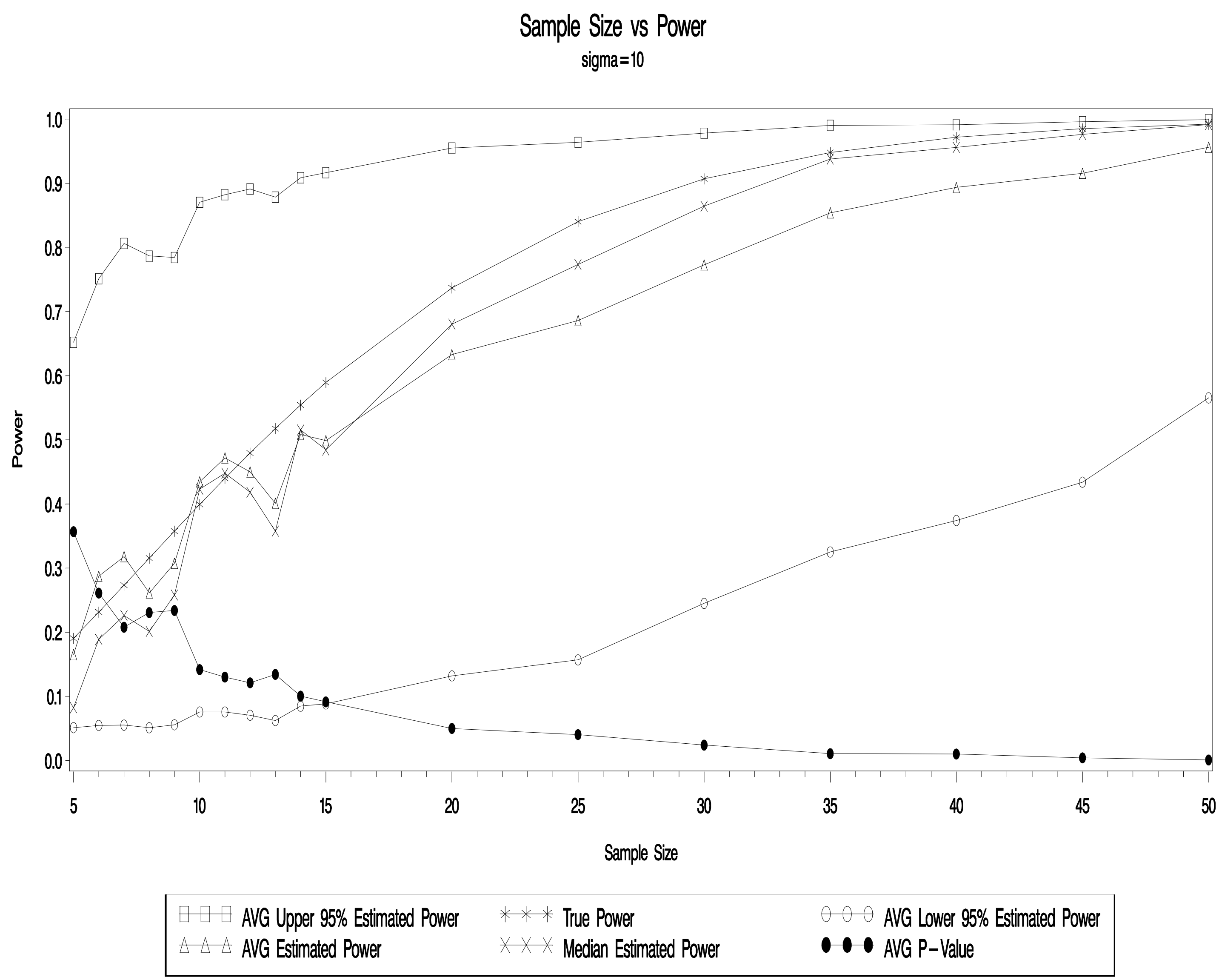
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



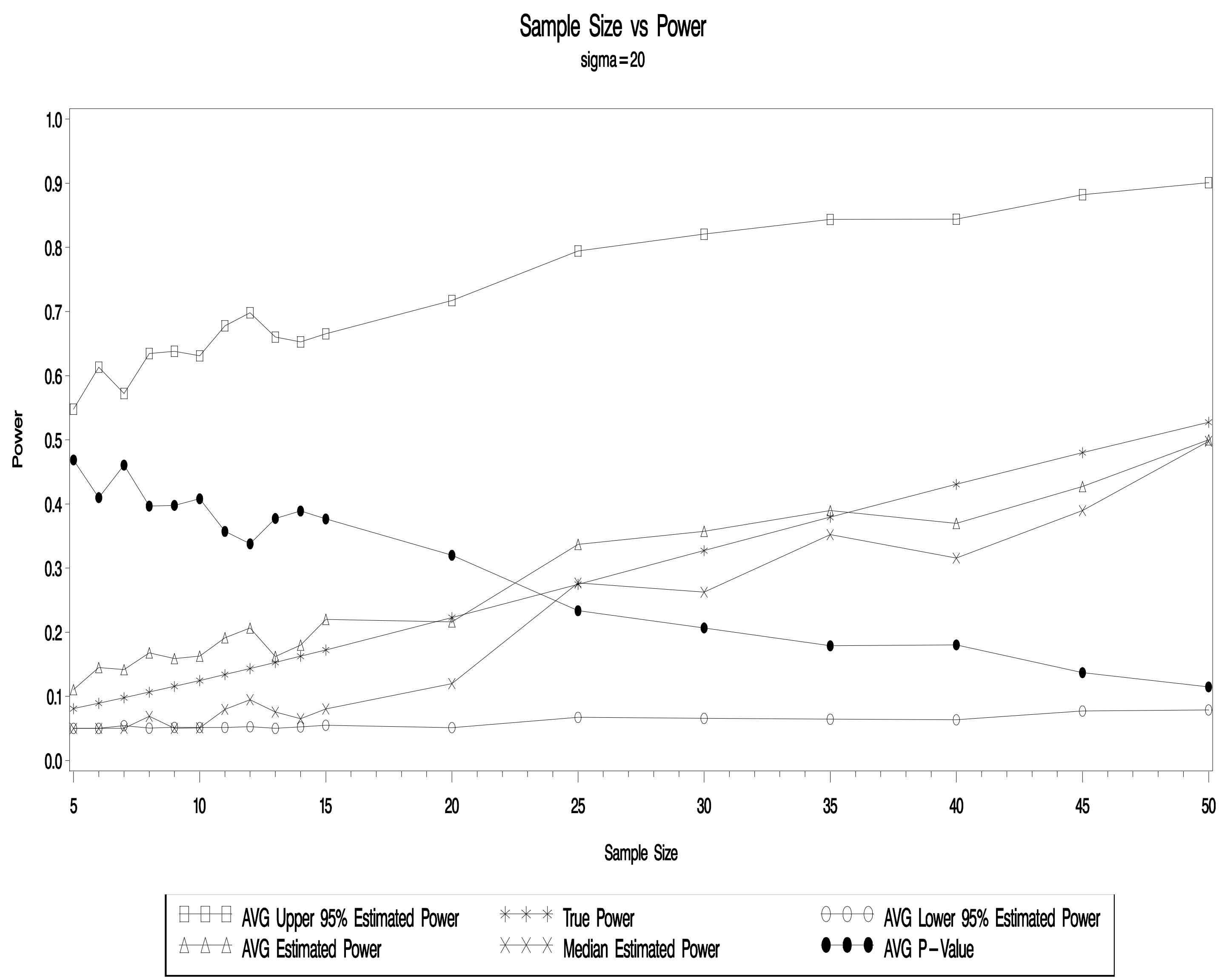
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



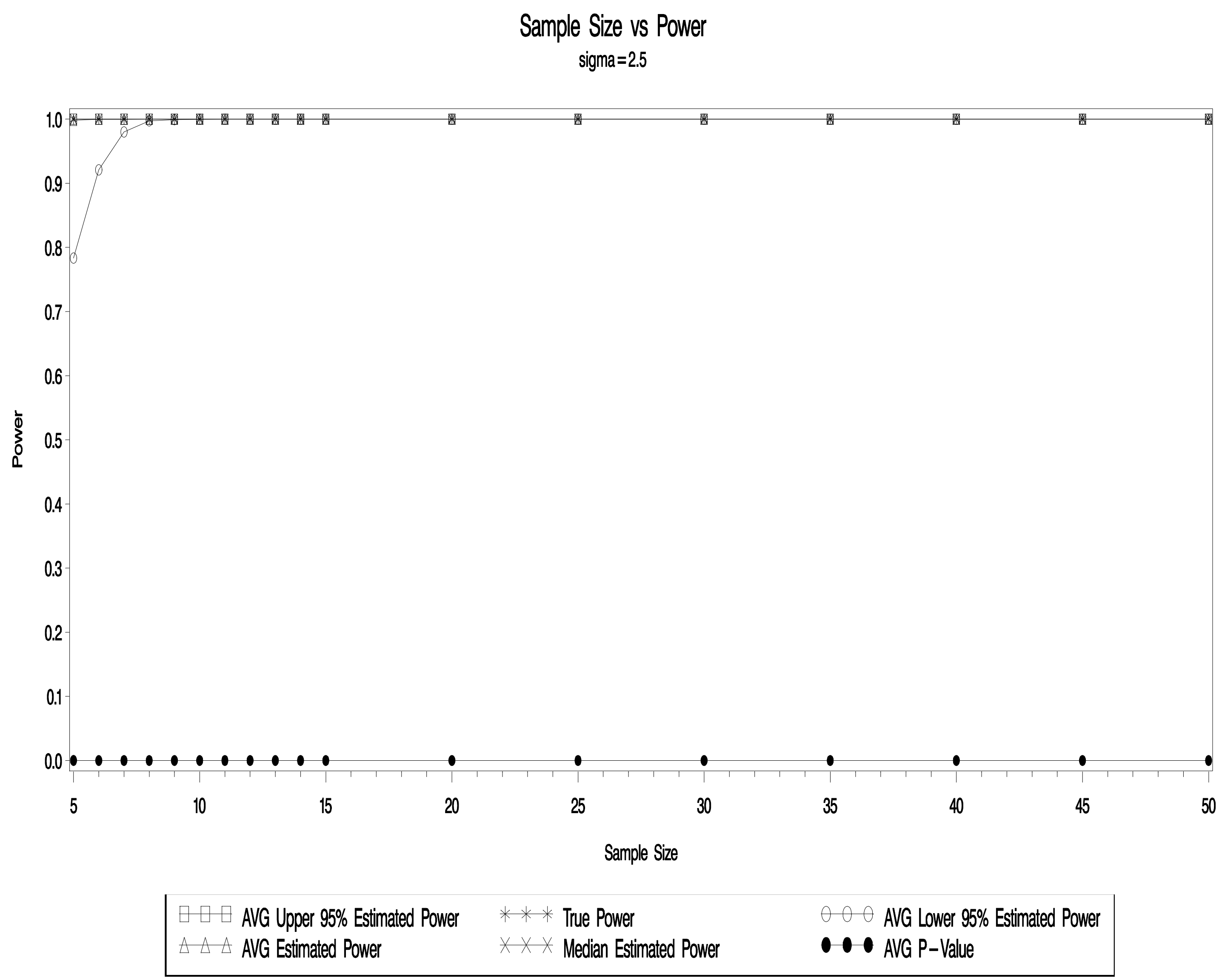
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



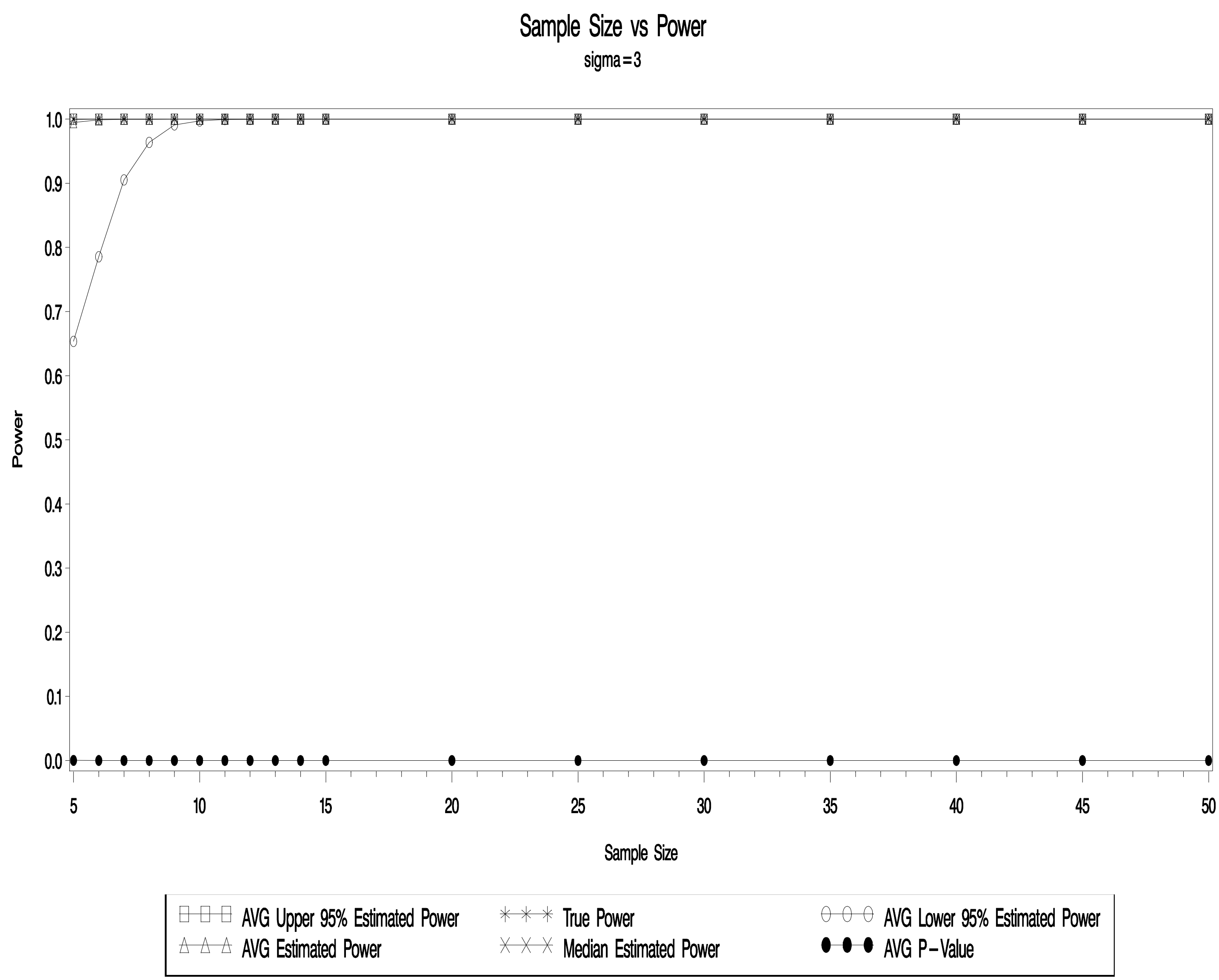
Appendix C: Graphs of Treatment Arrangement 30, 35, 35, 40



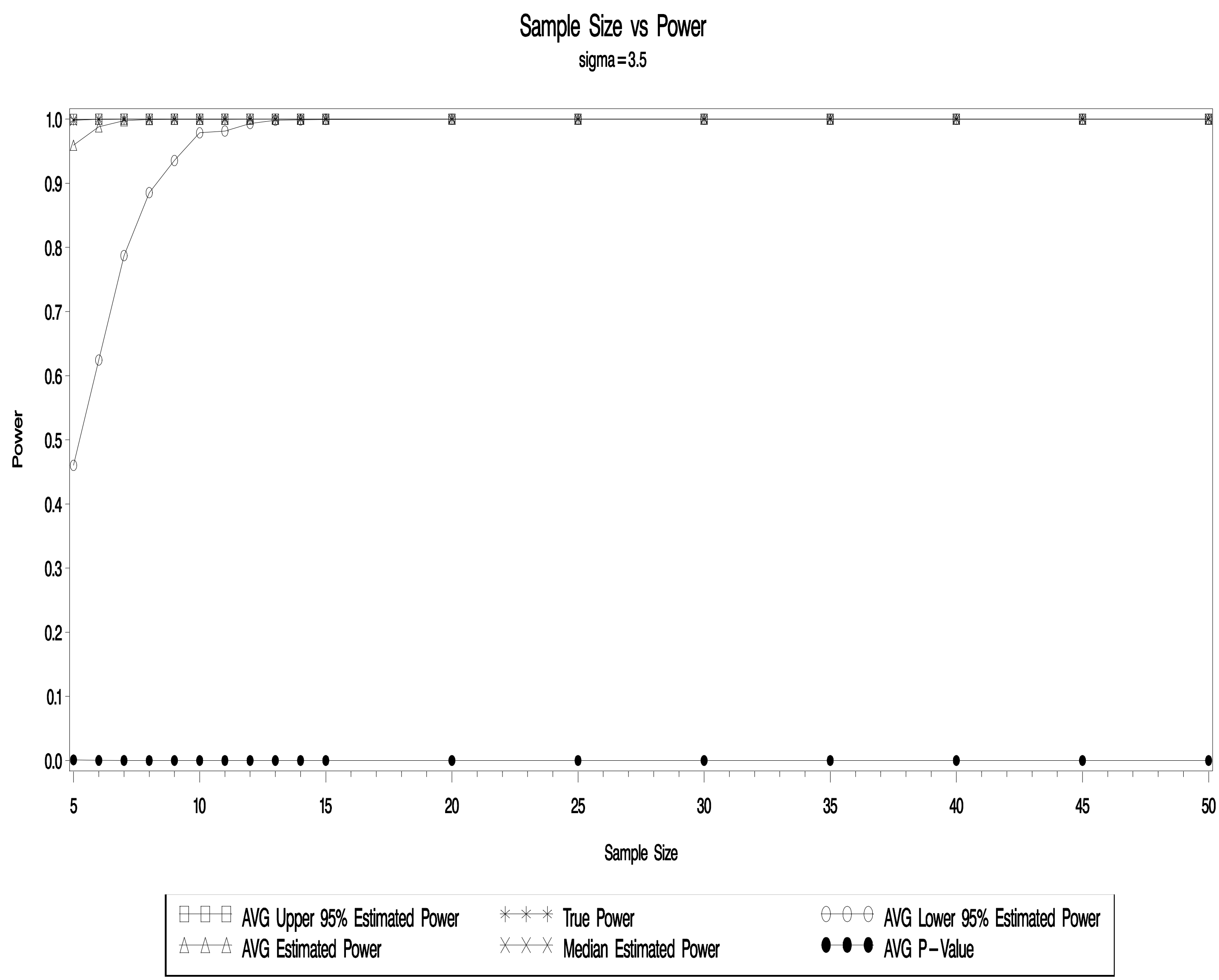
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



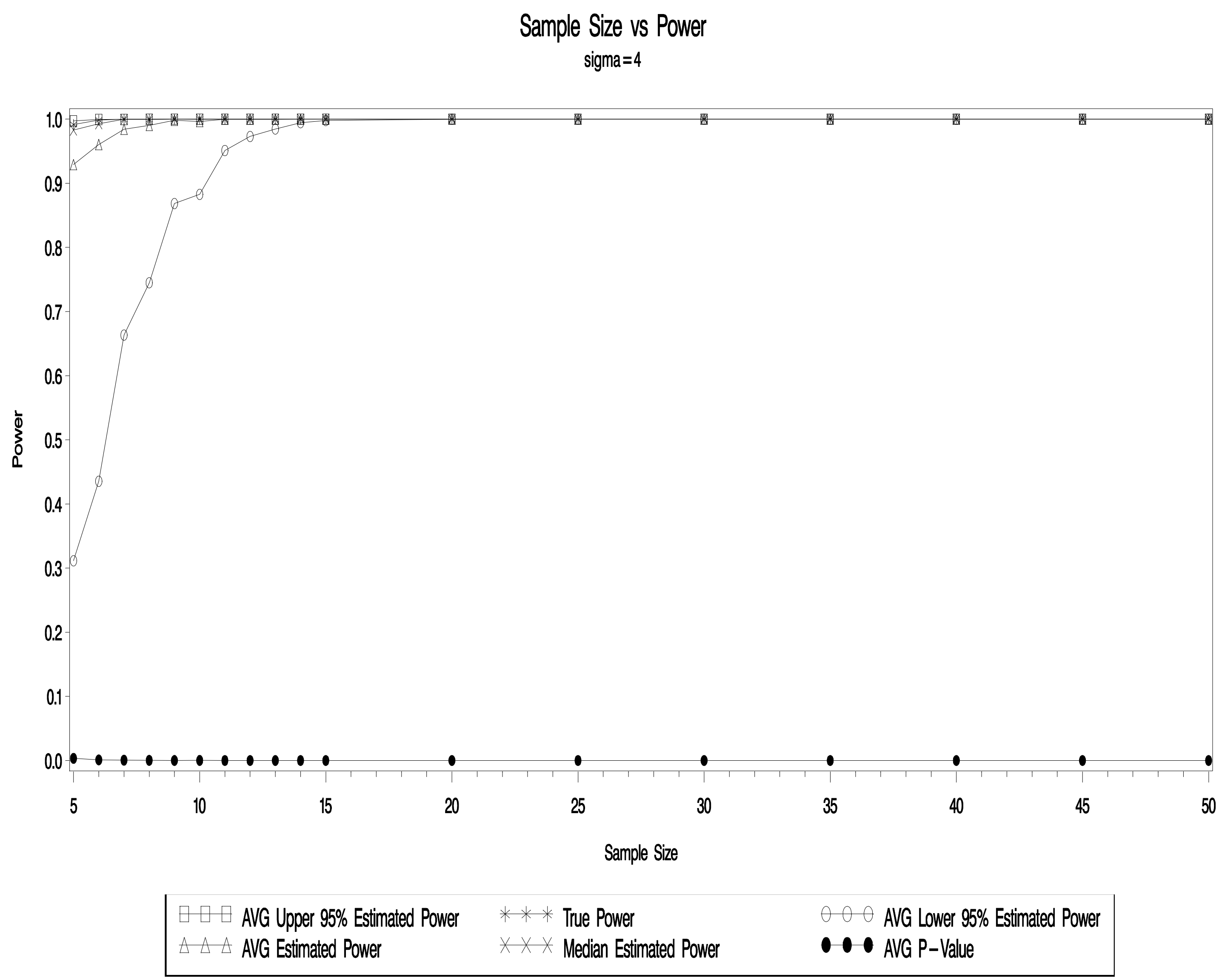
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



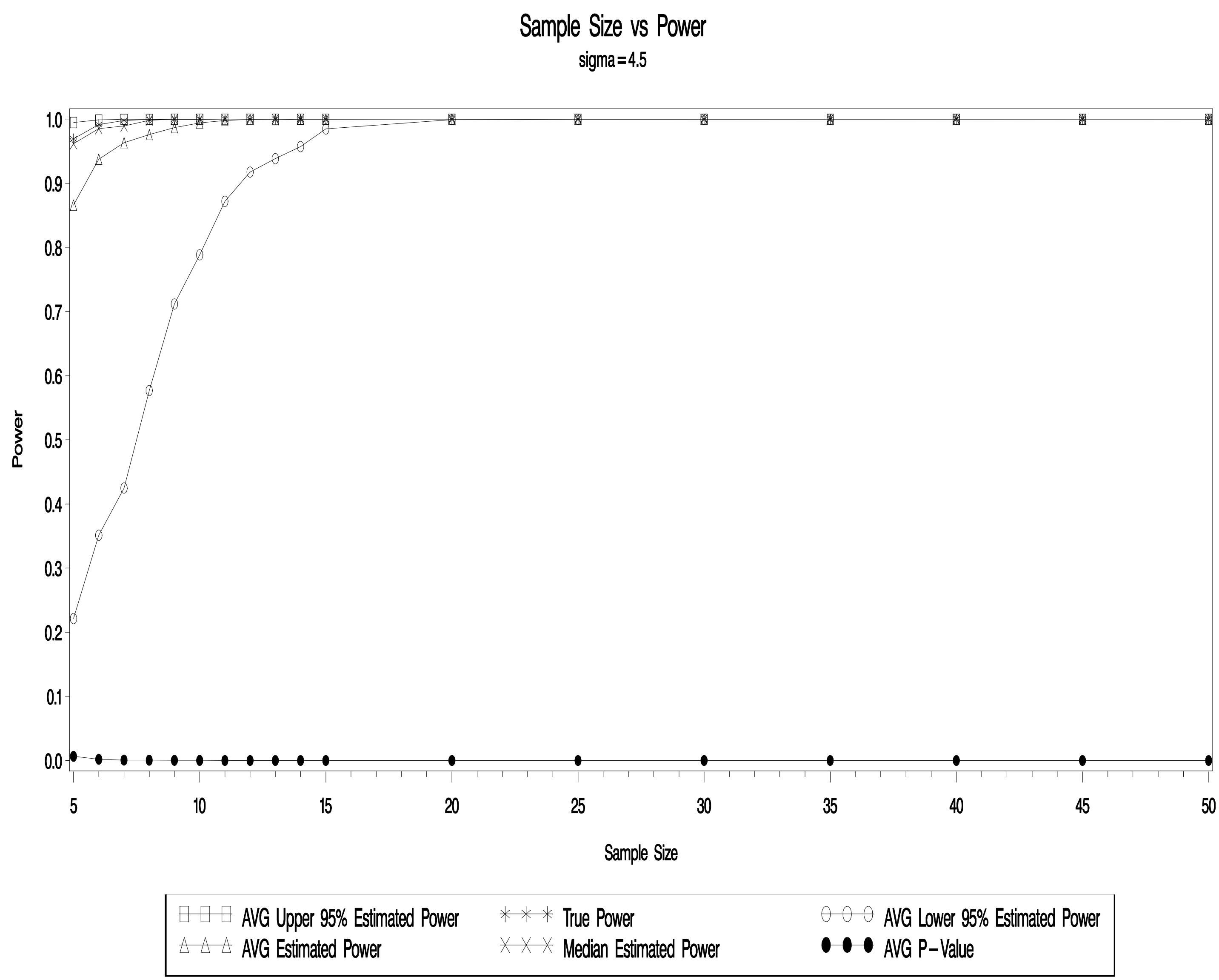
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



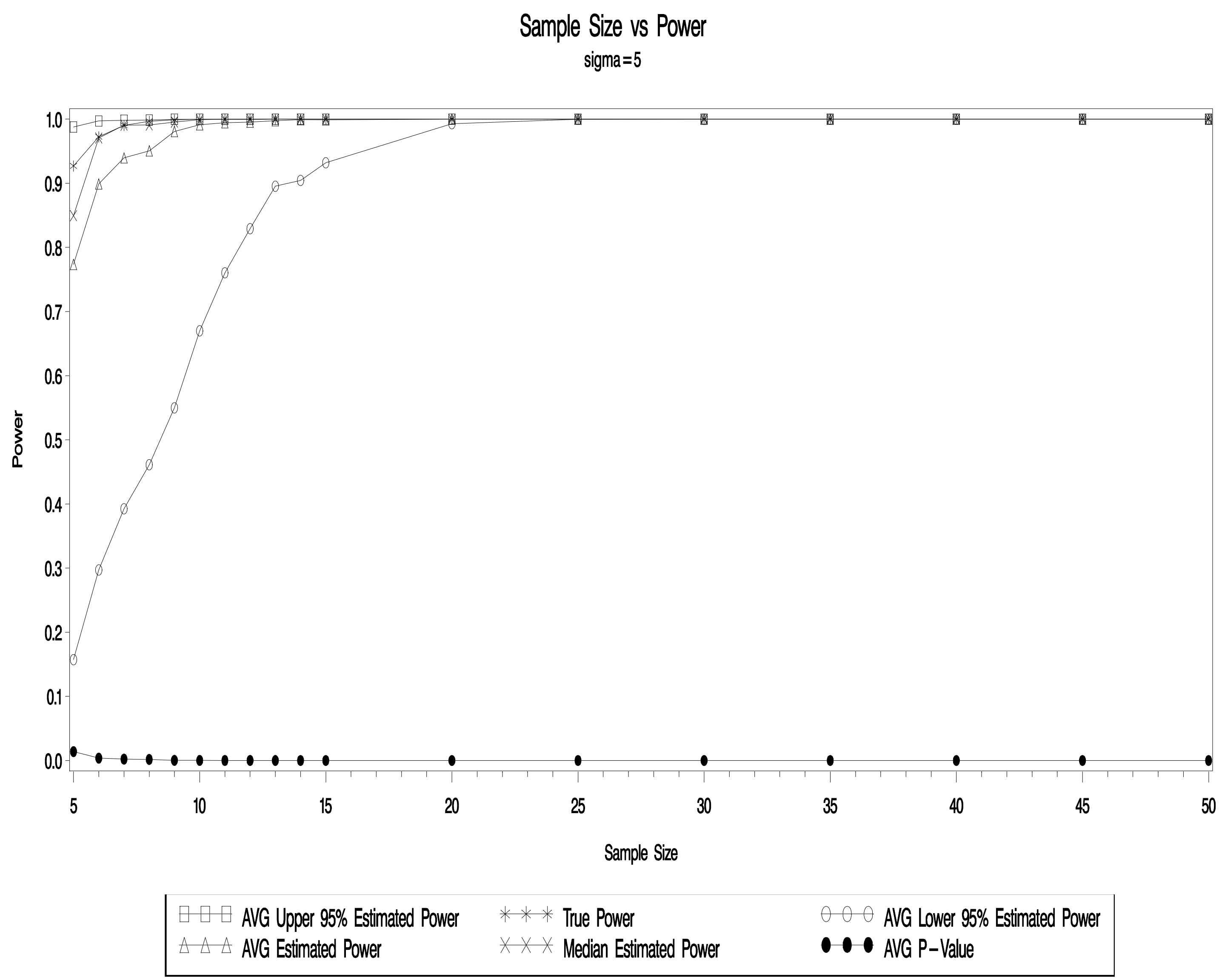
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



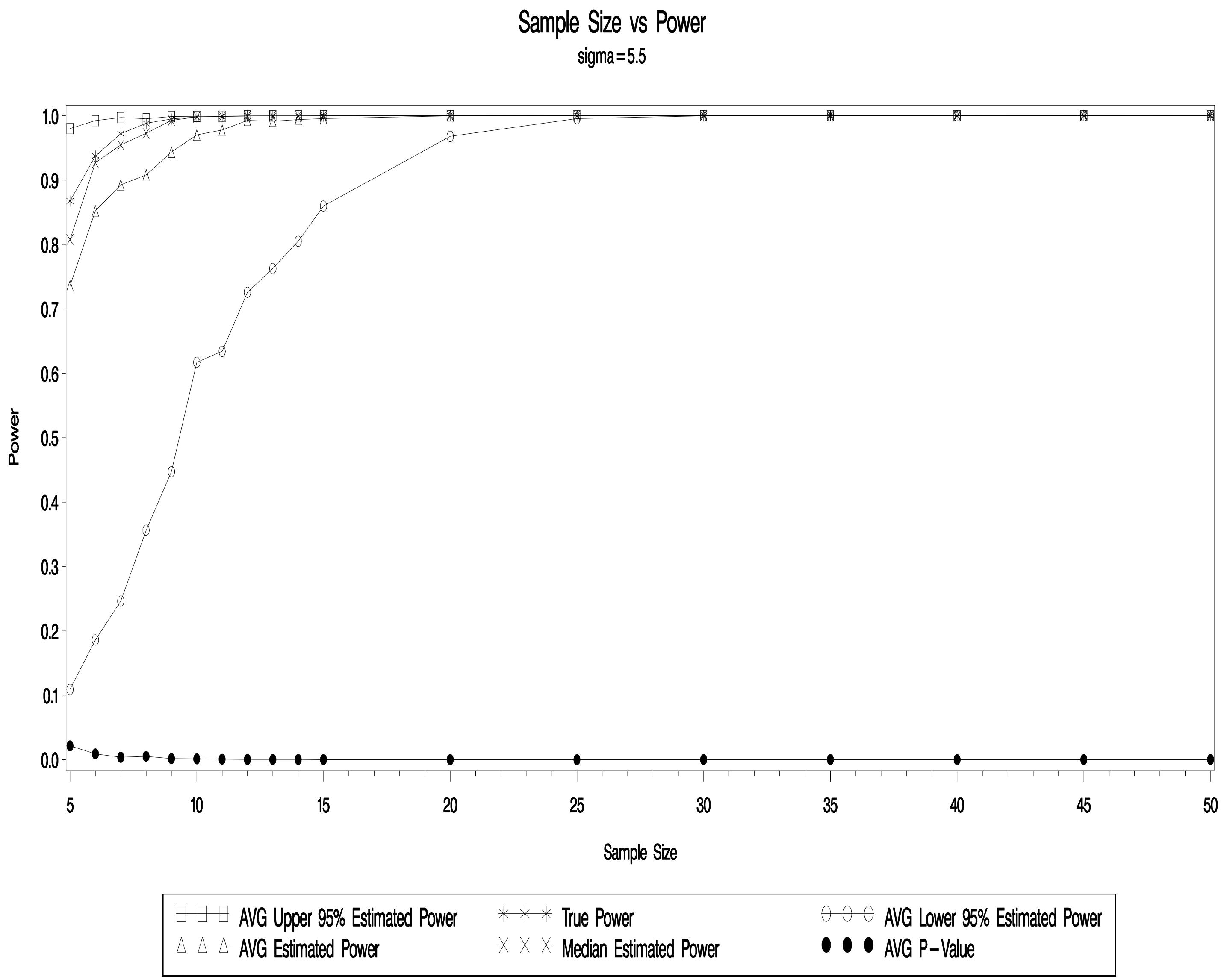
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



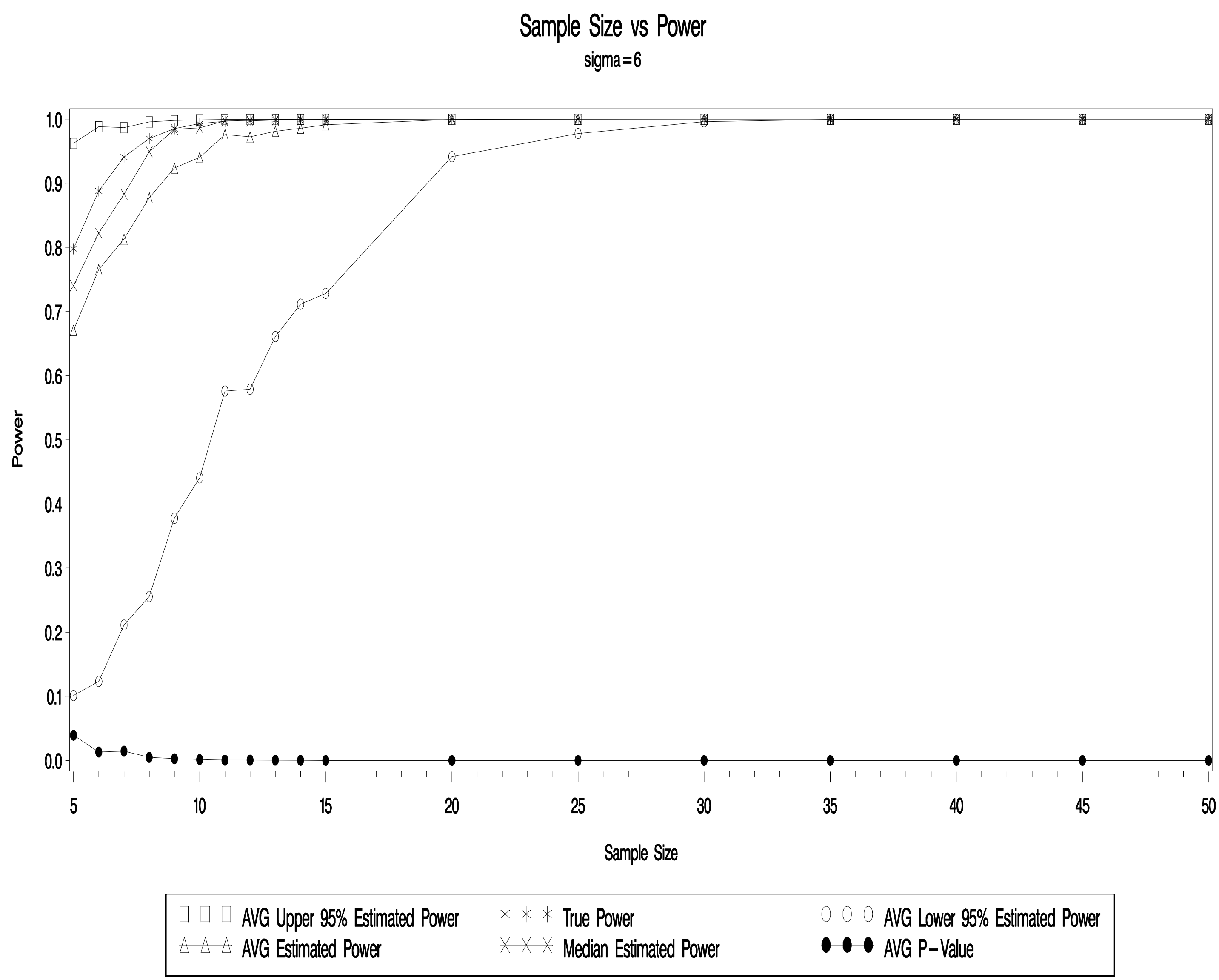
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



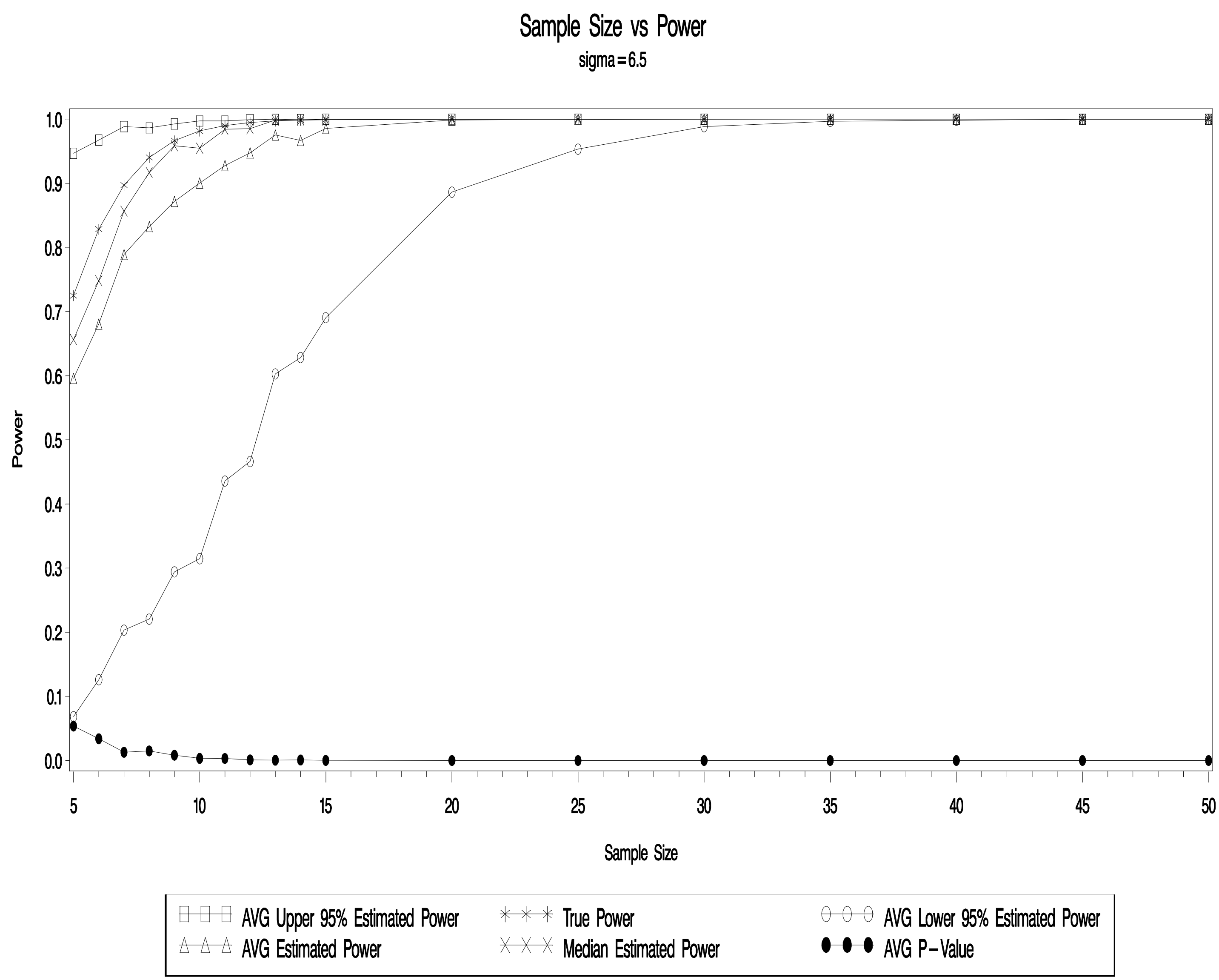
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



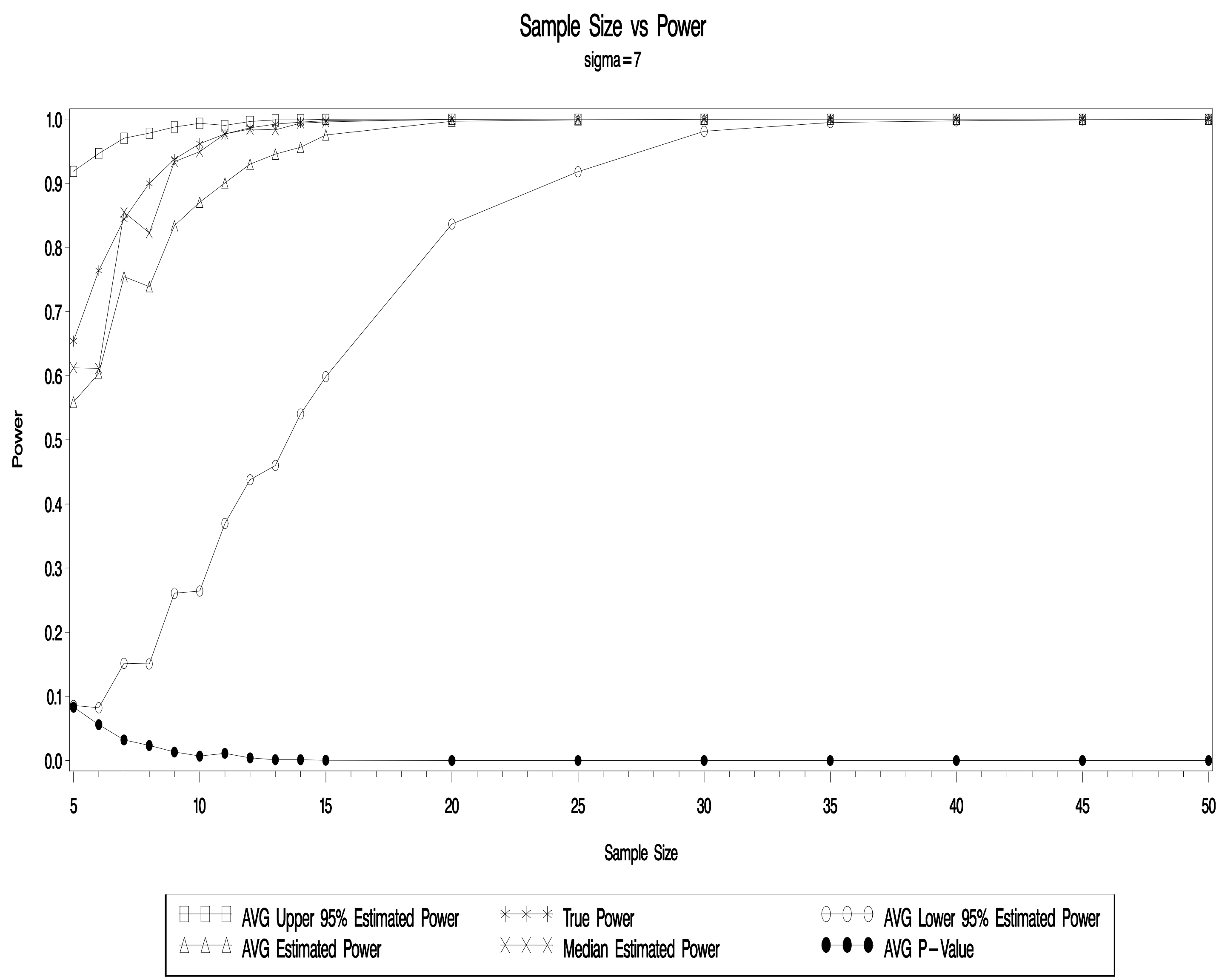
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



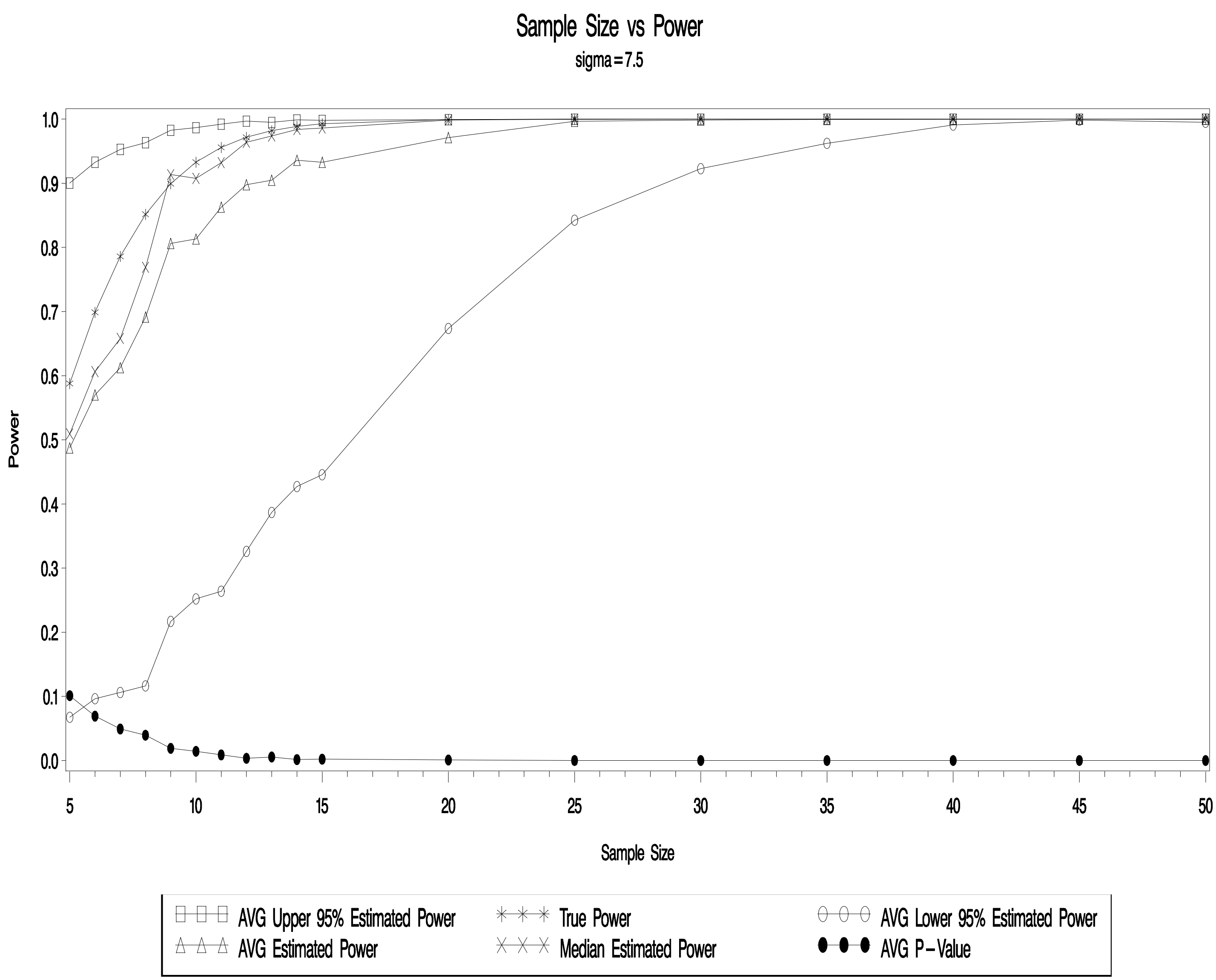
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



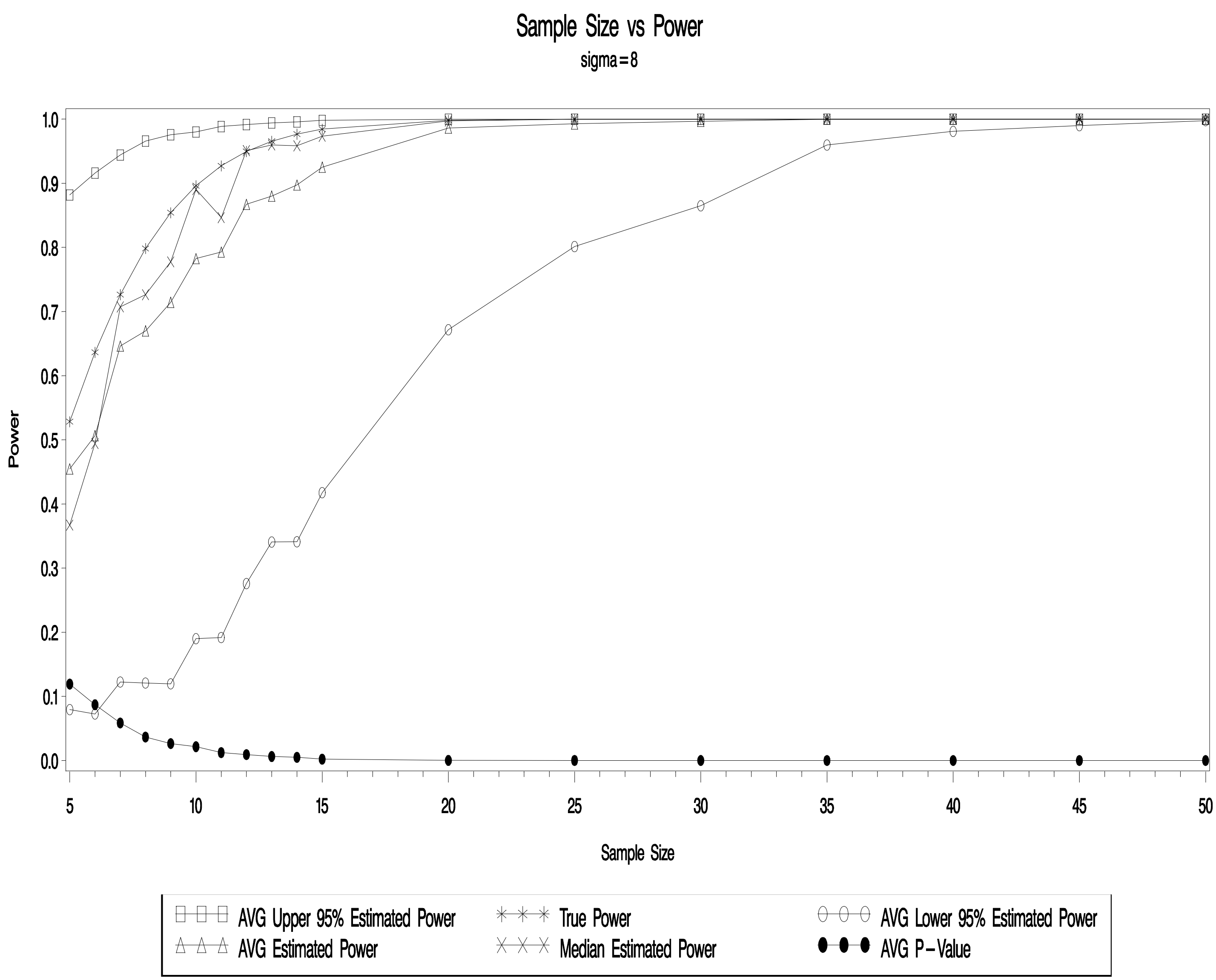
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



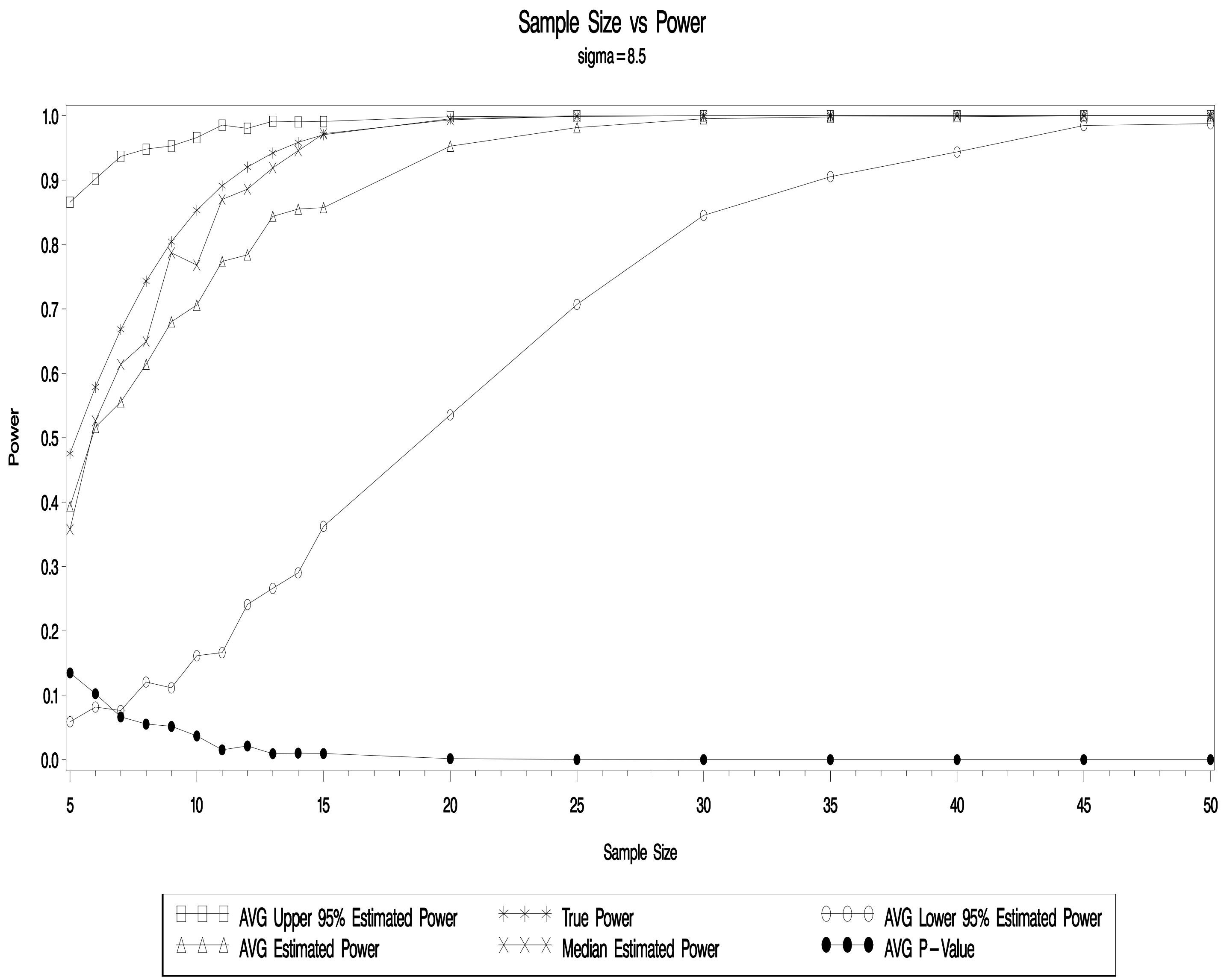
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



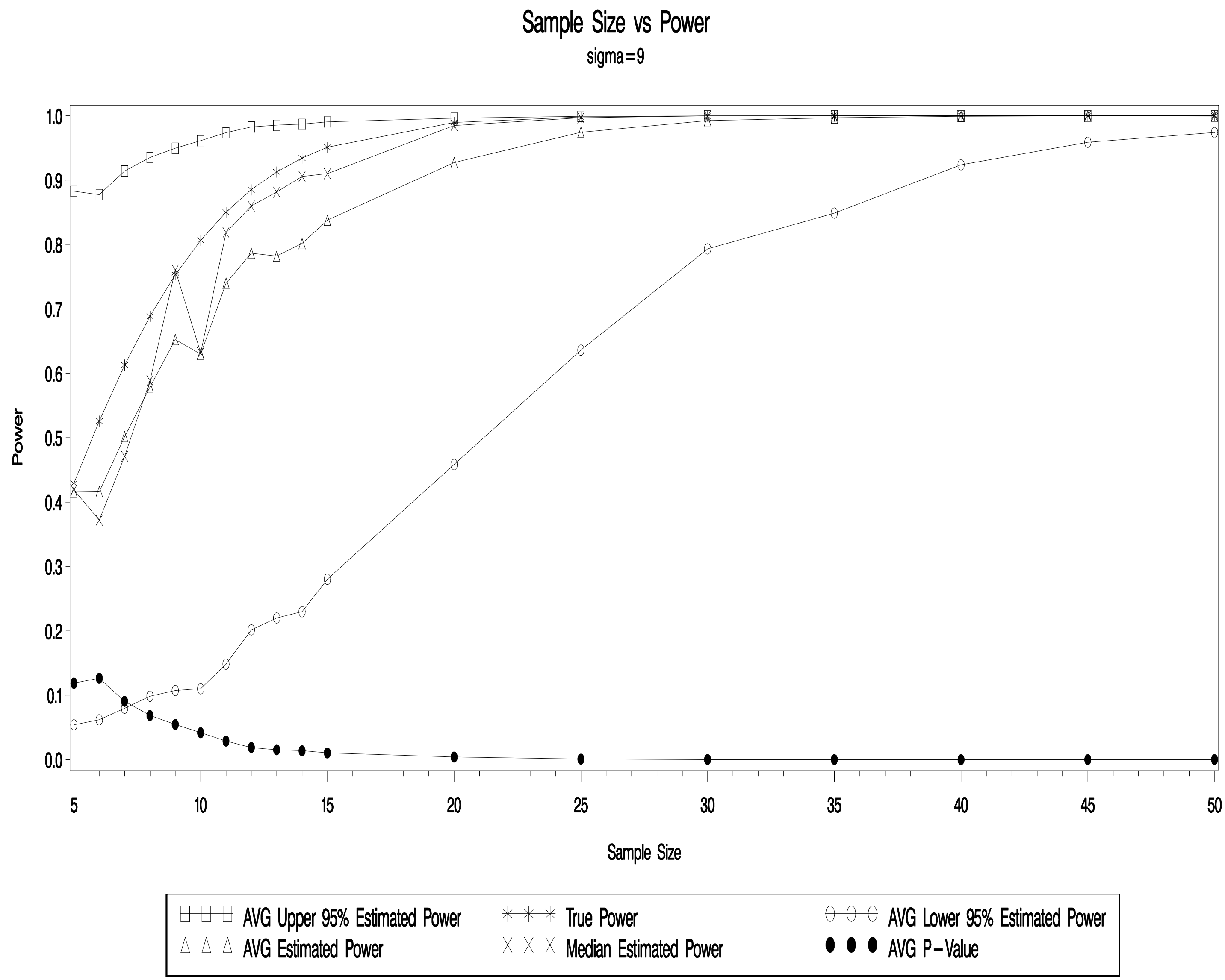
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



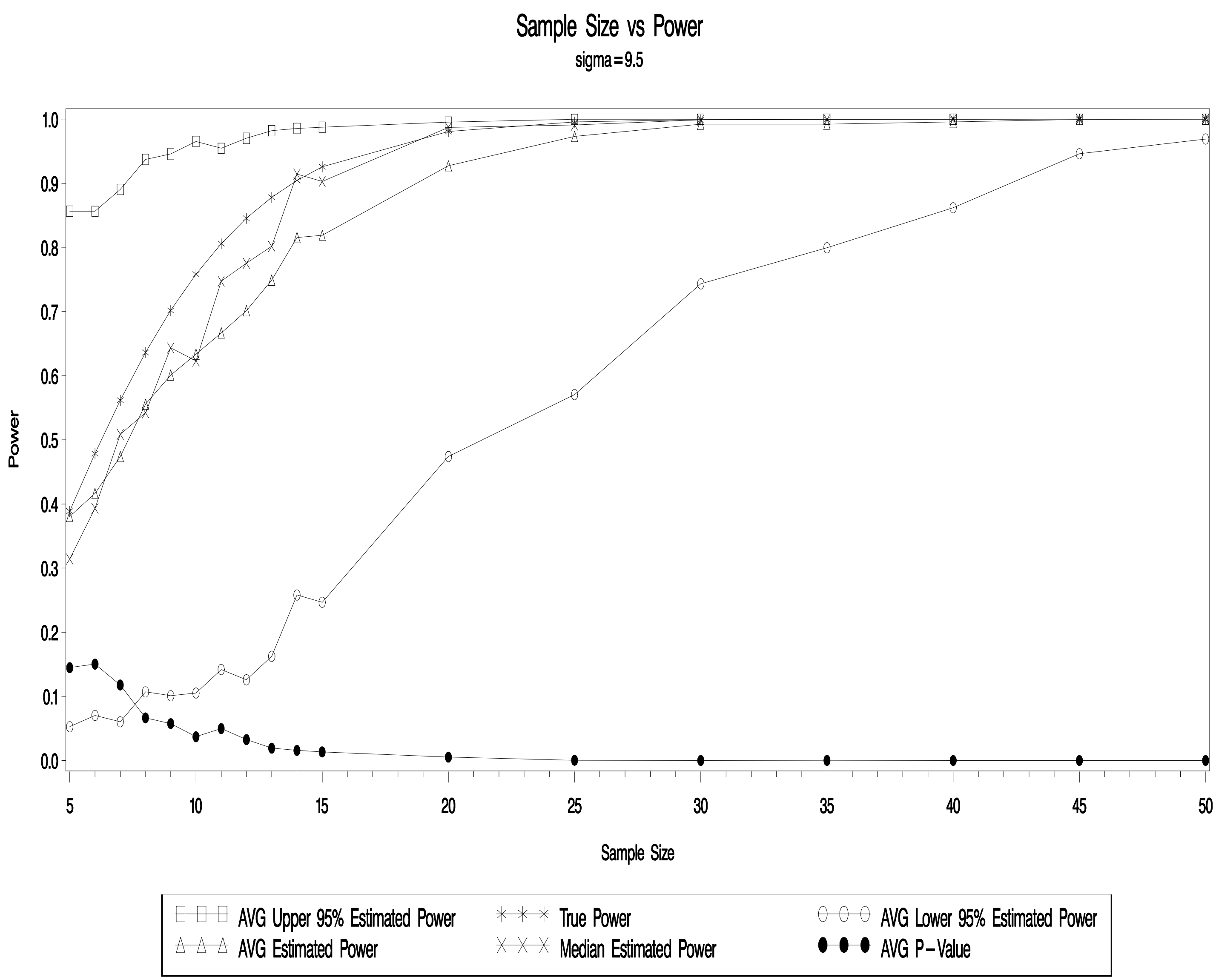
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



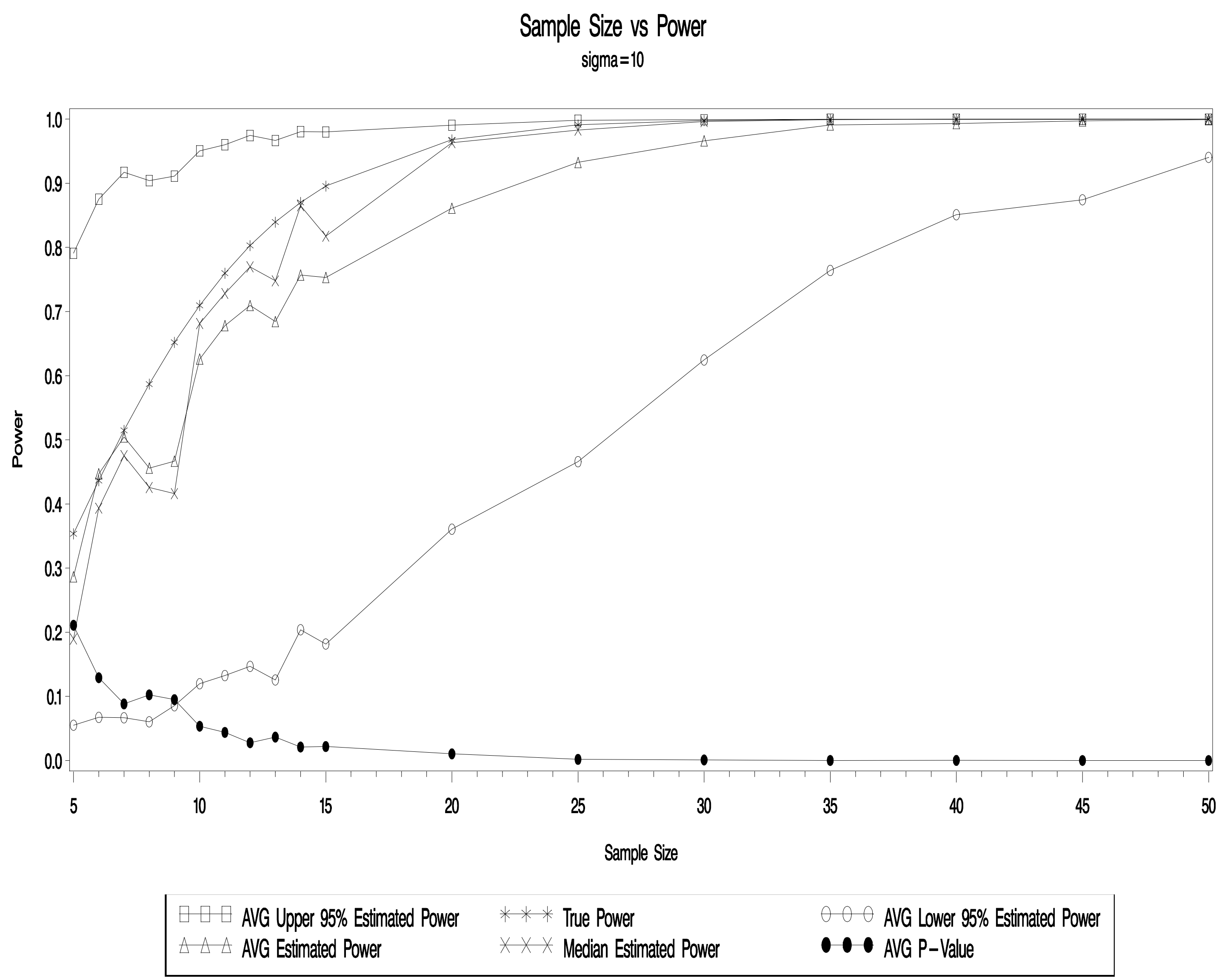
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



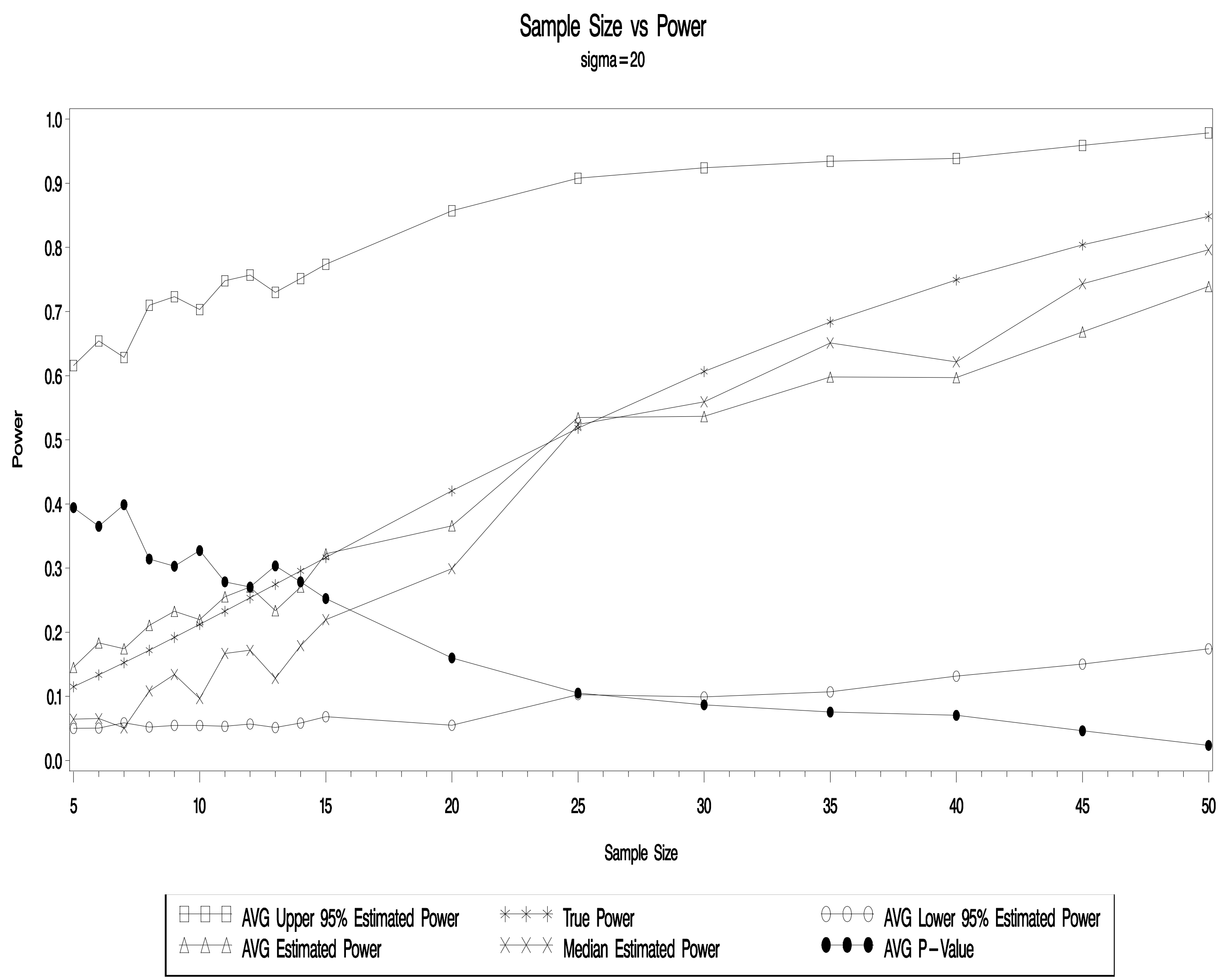
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



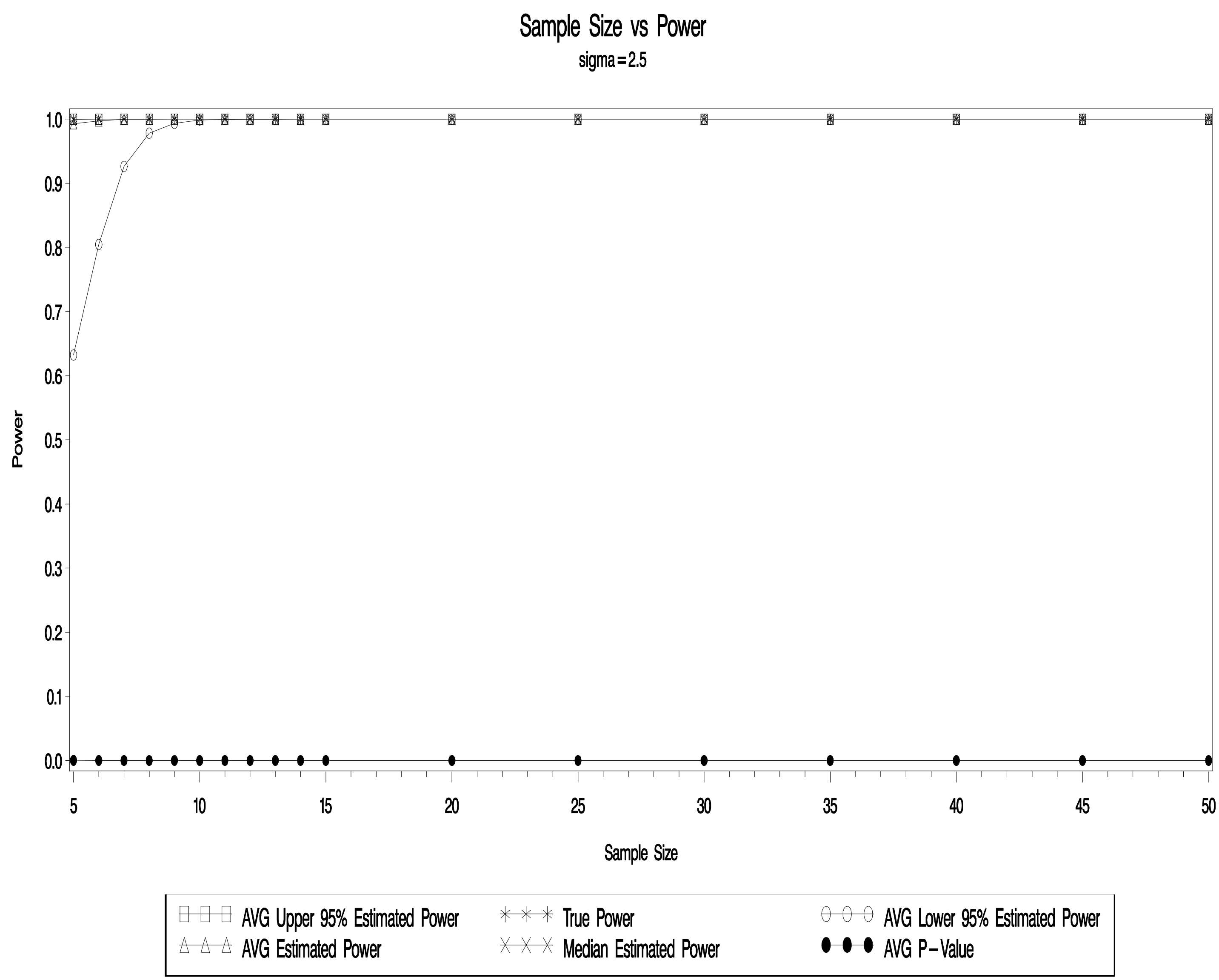
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



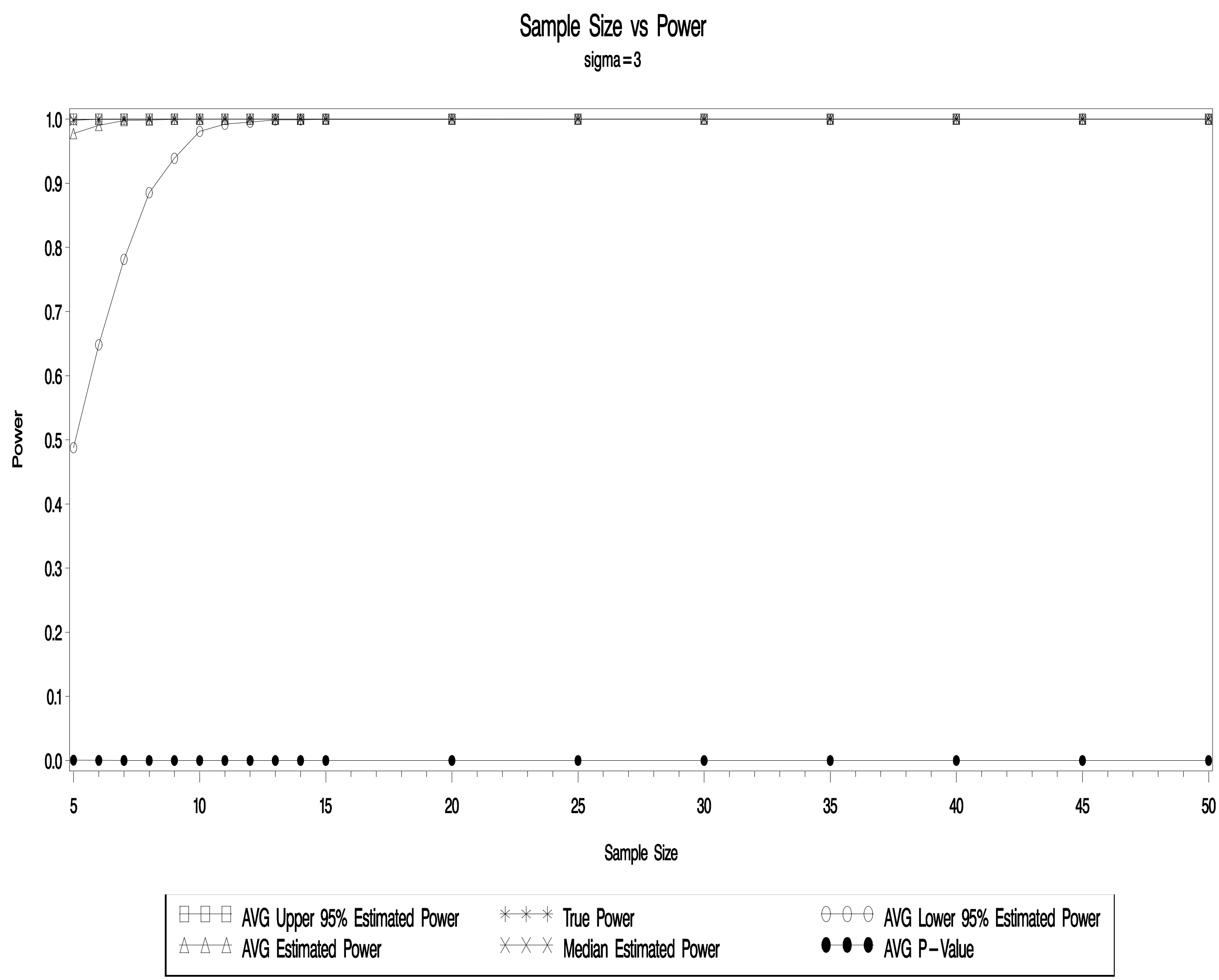
Appendix C: Graphs of Treatment Arrangement 30, 30, 40, 40



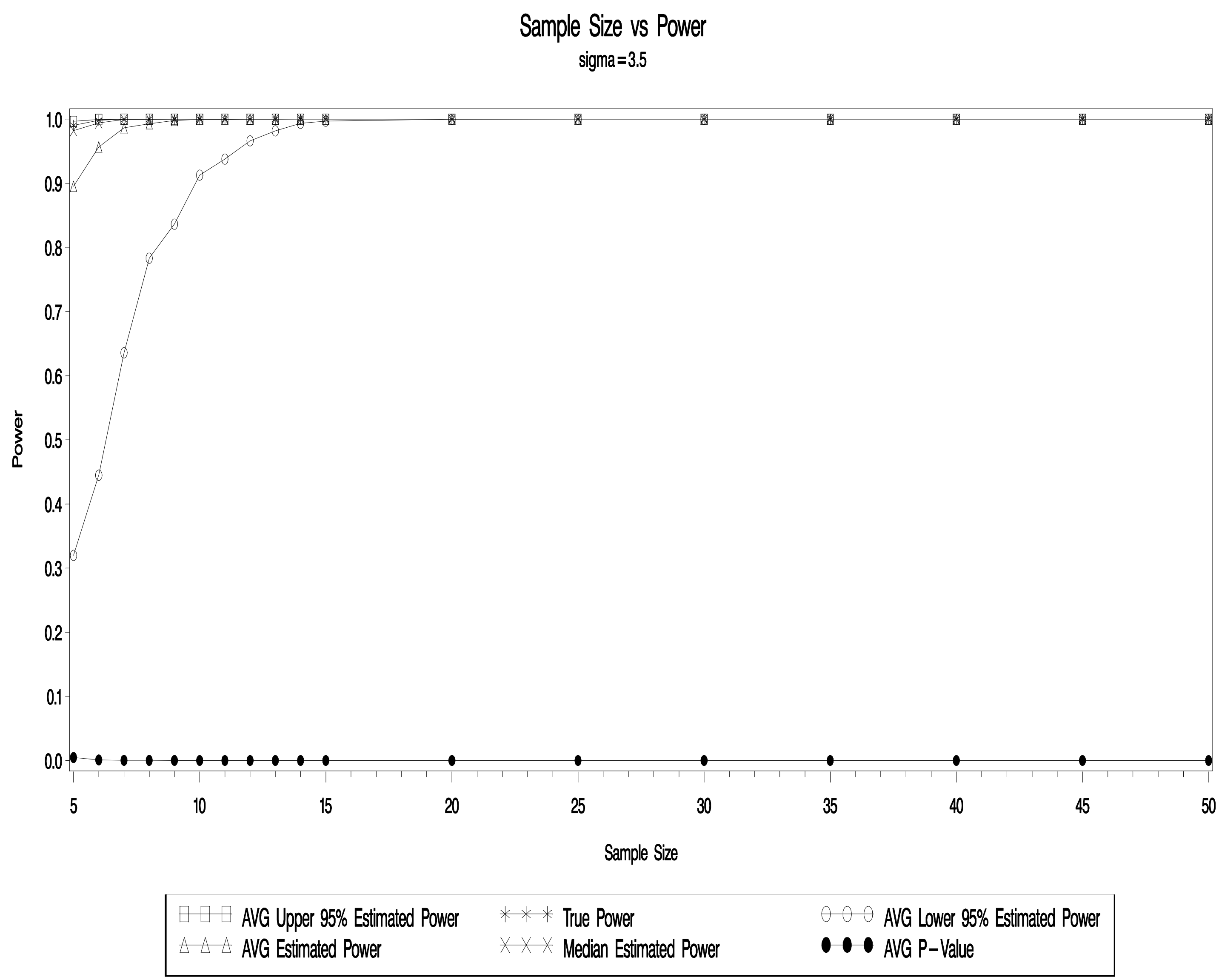
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



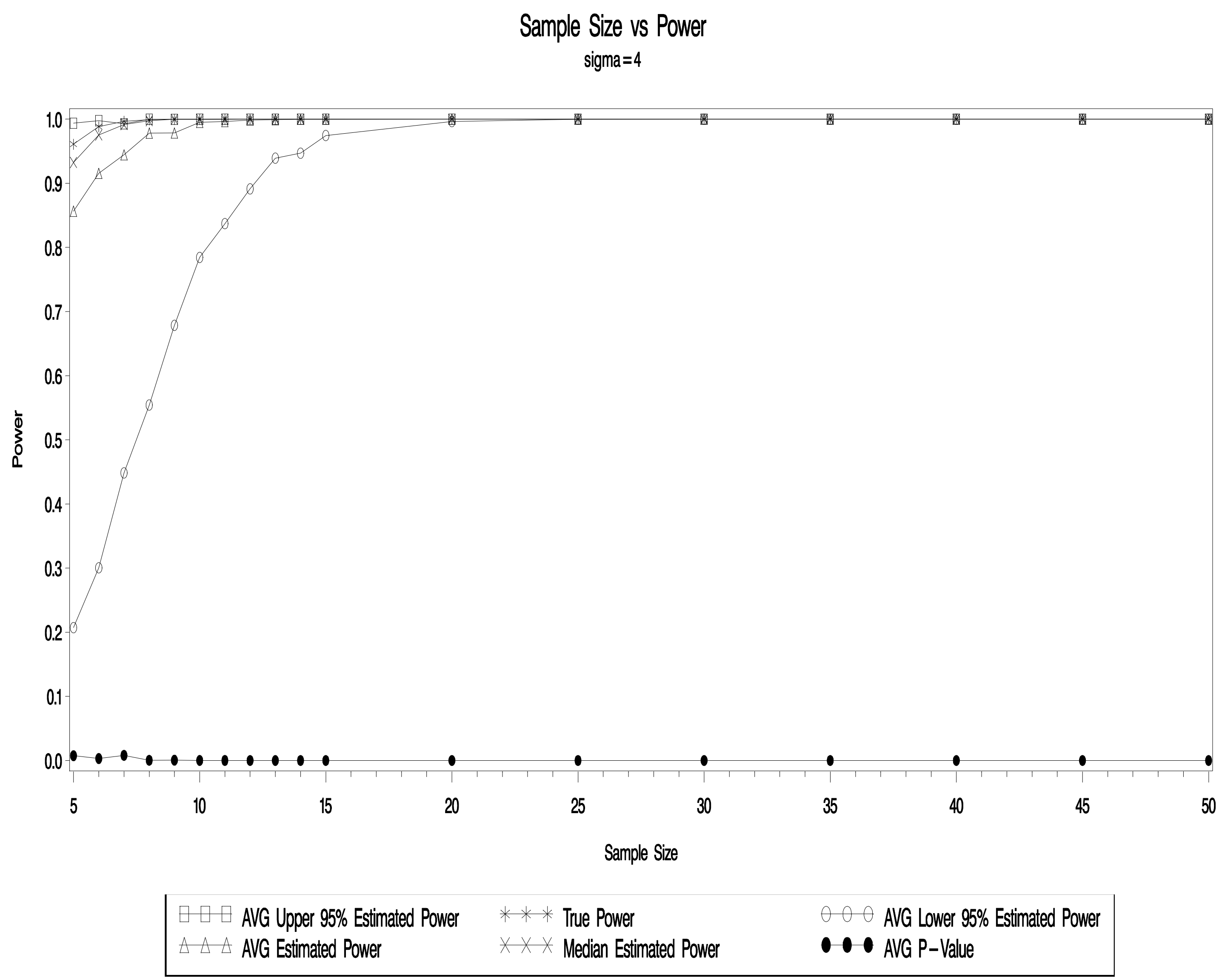
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



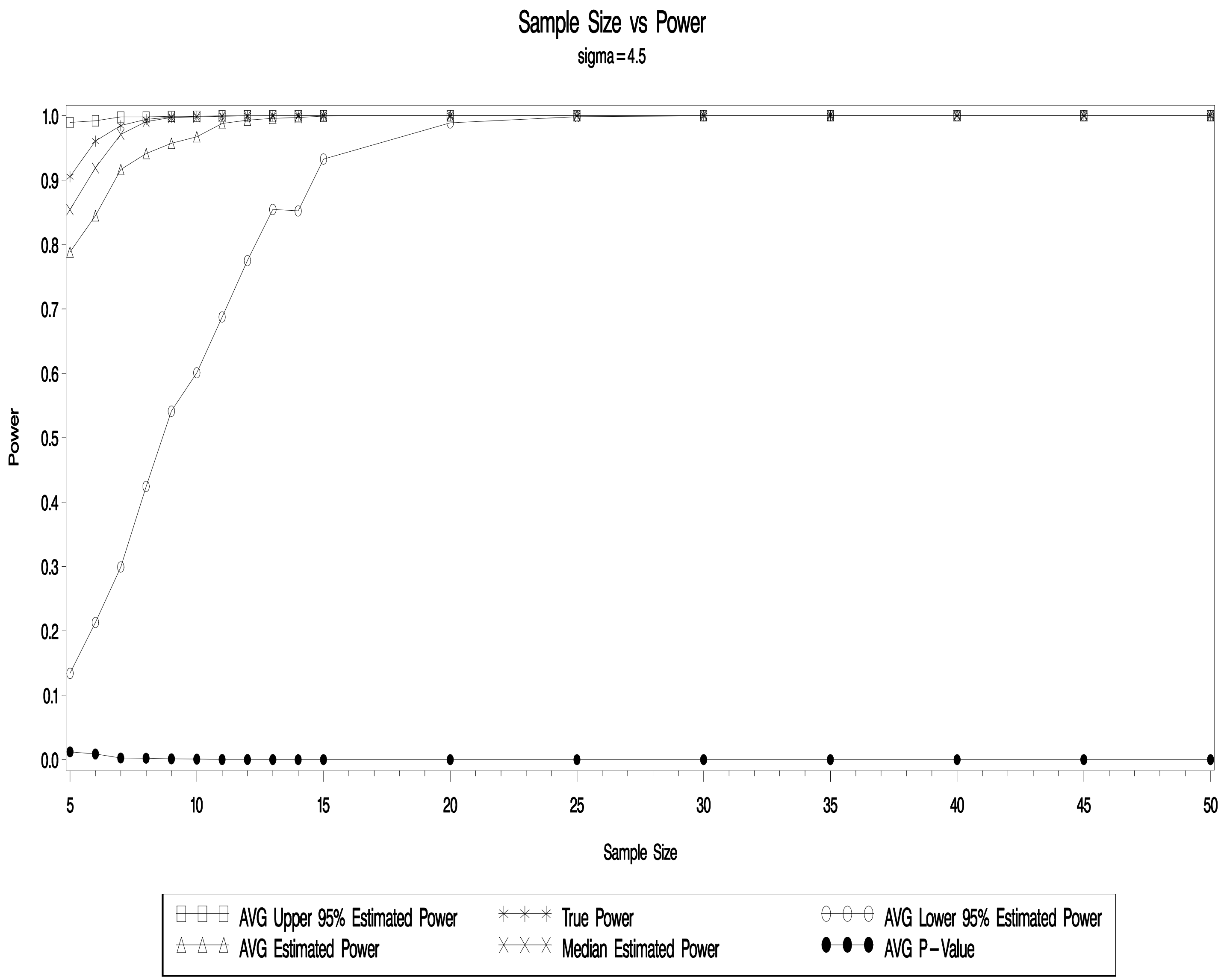
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



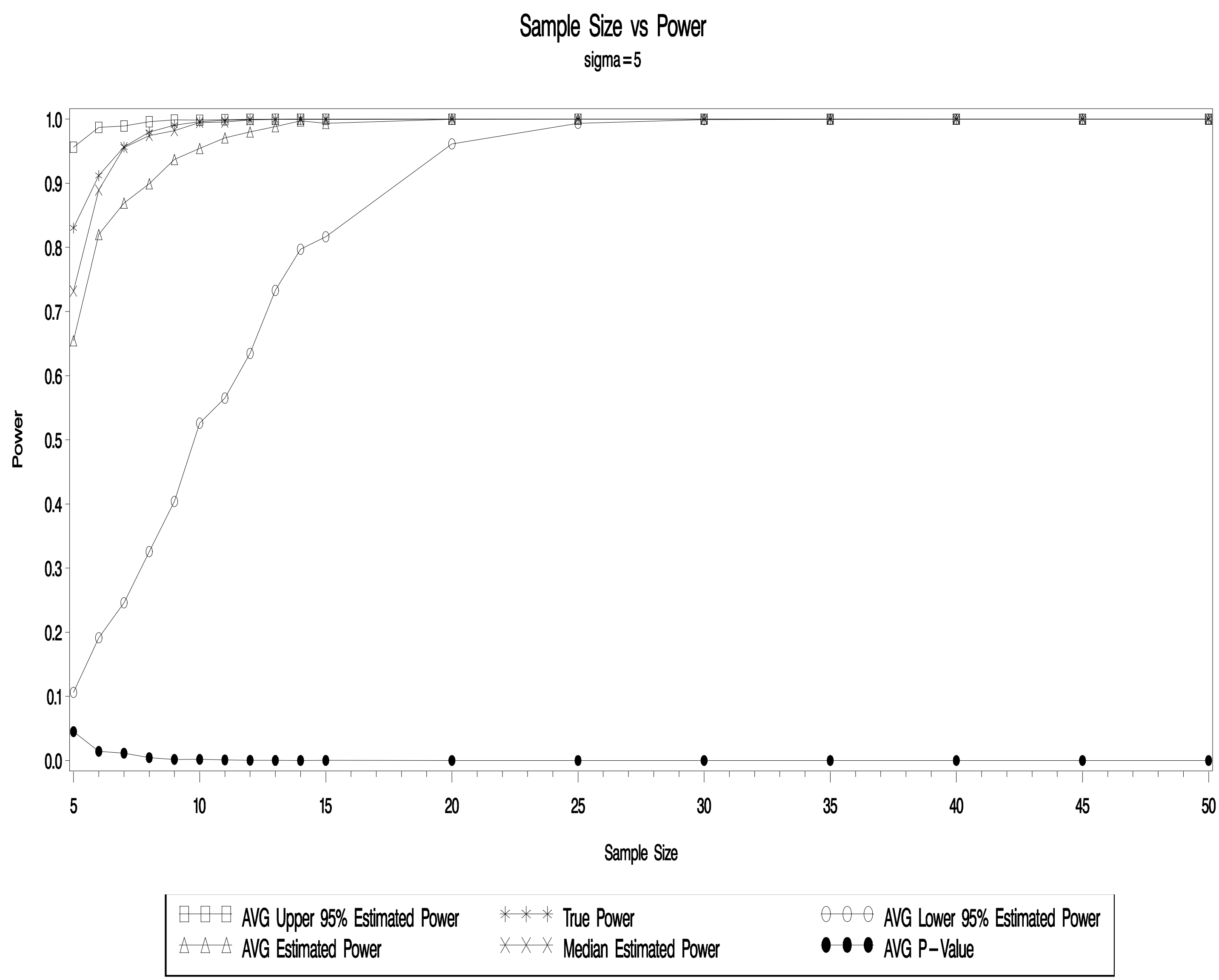
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



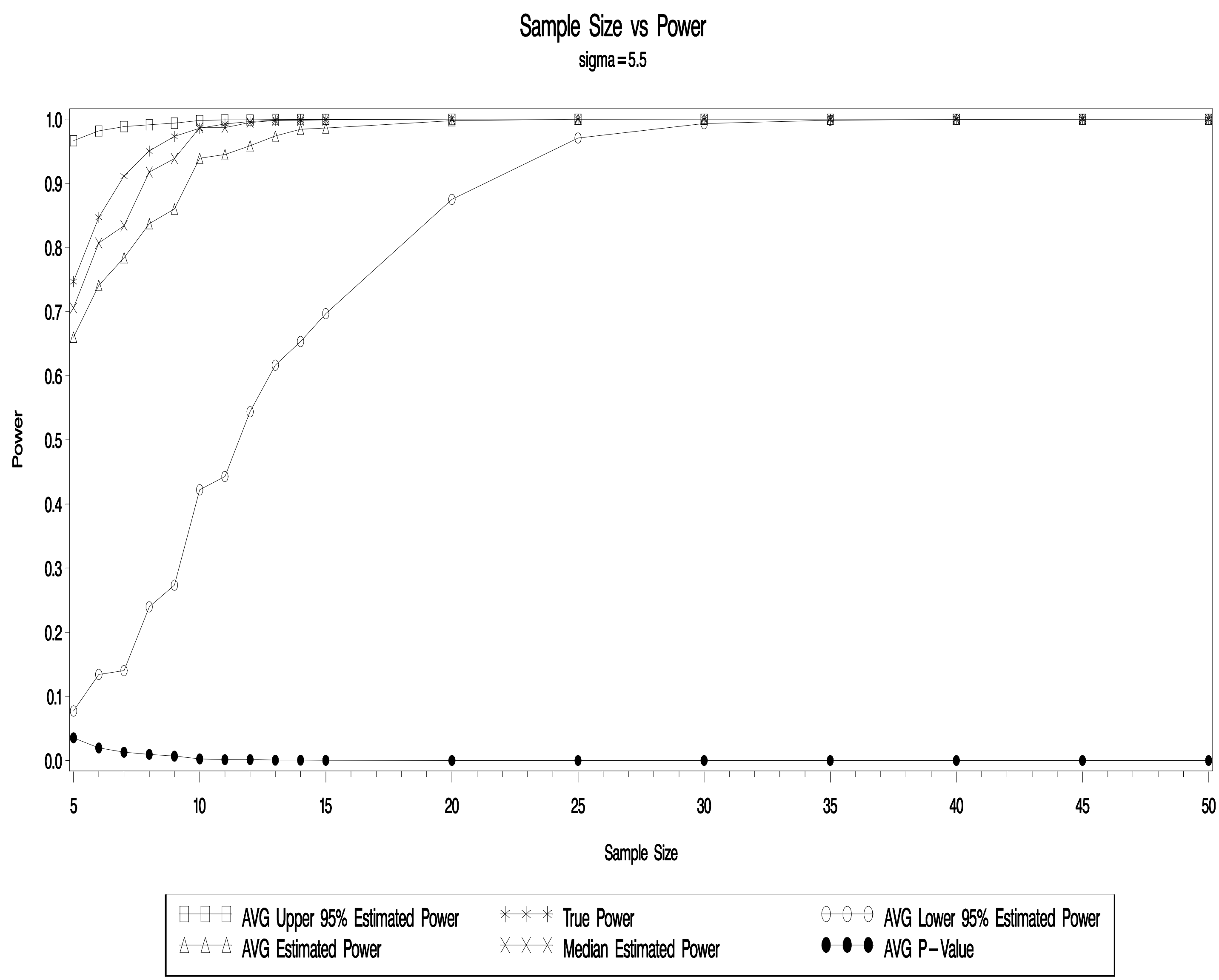
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



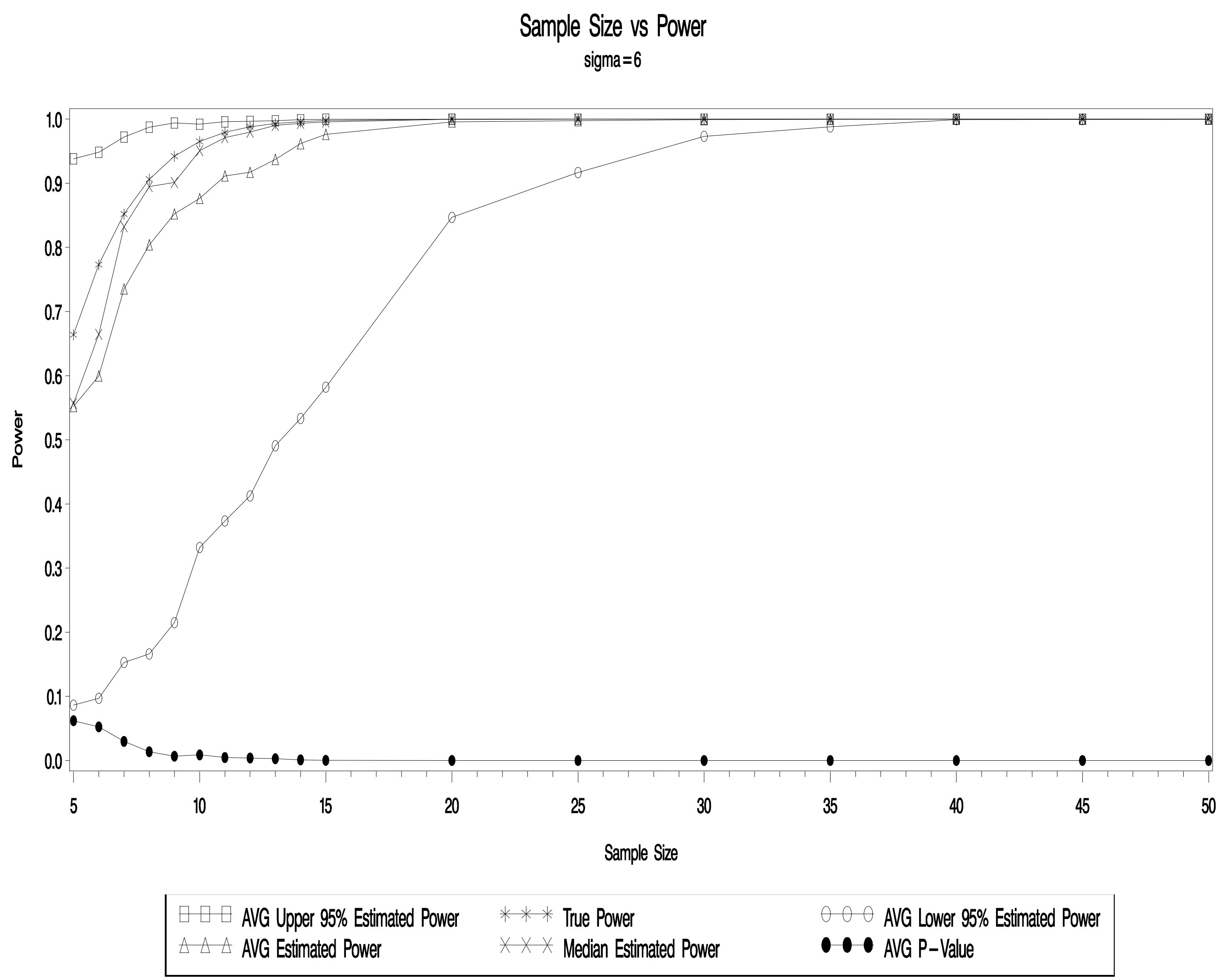
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



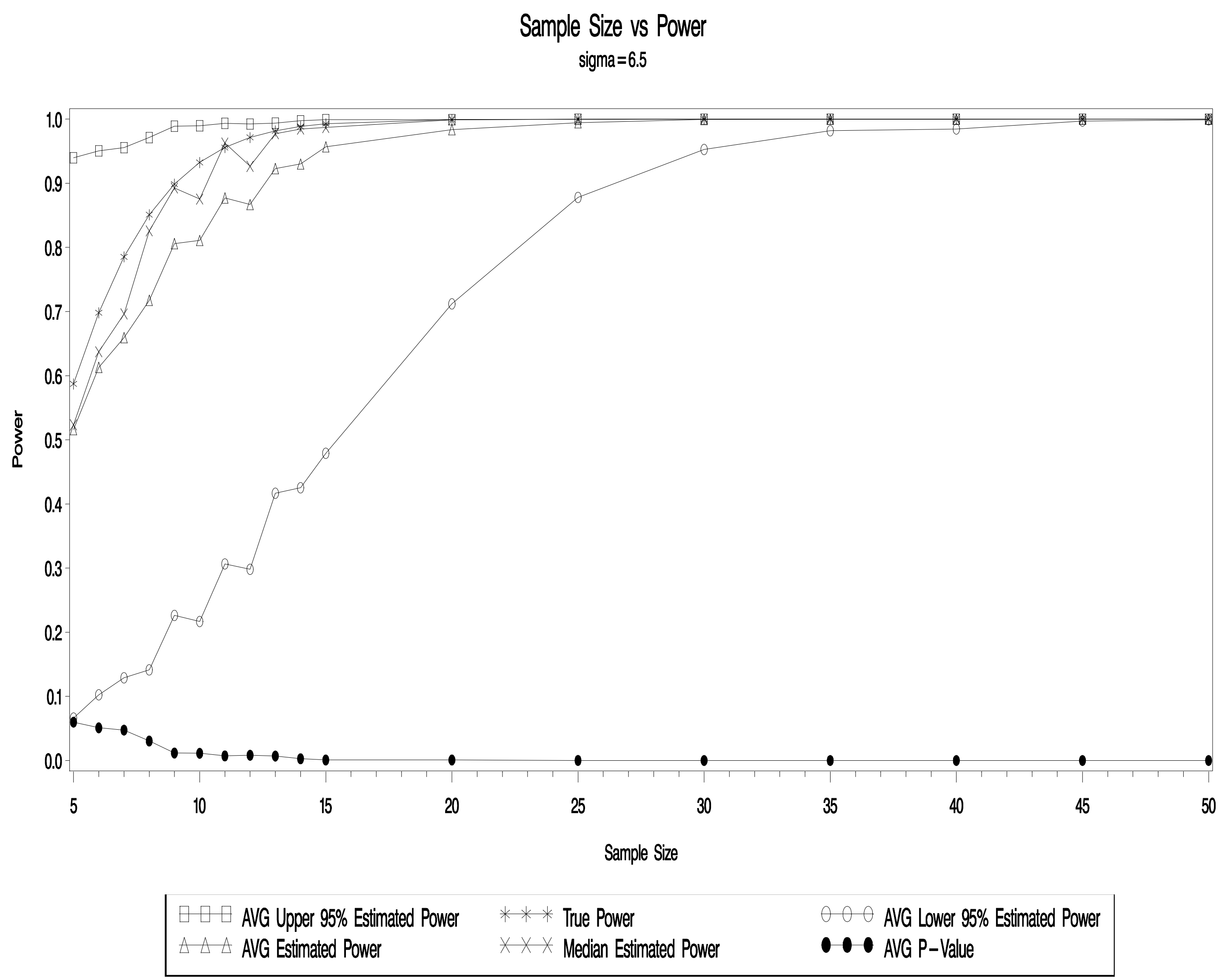
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



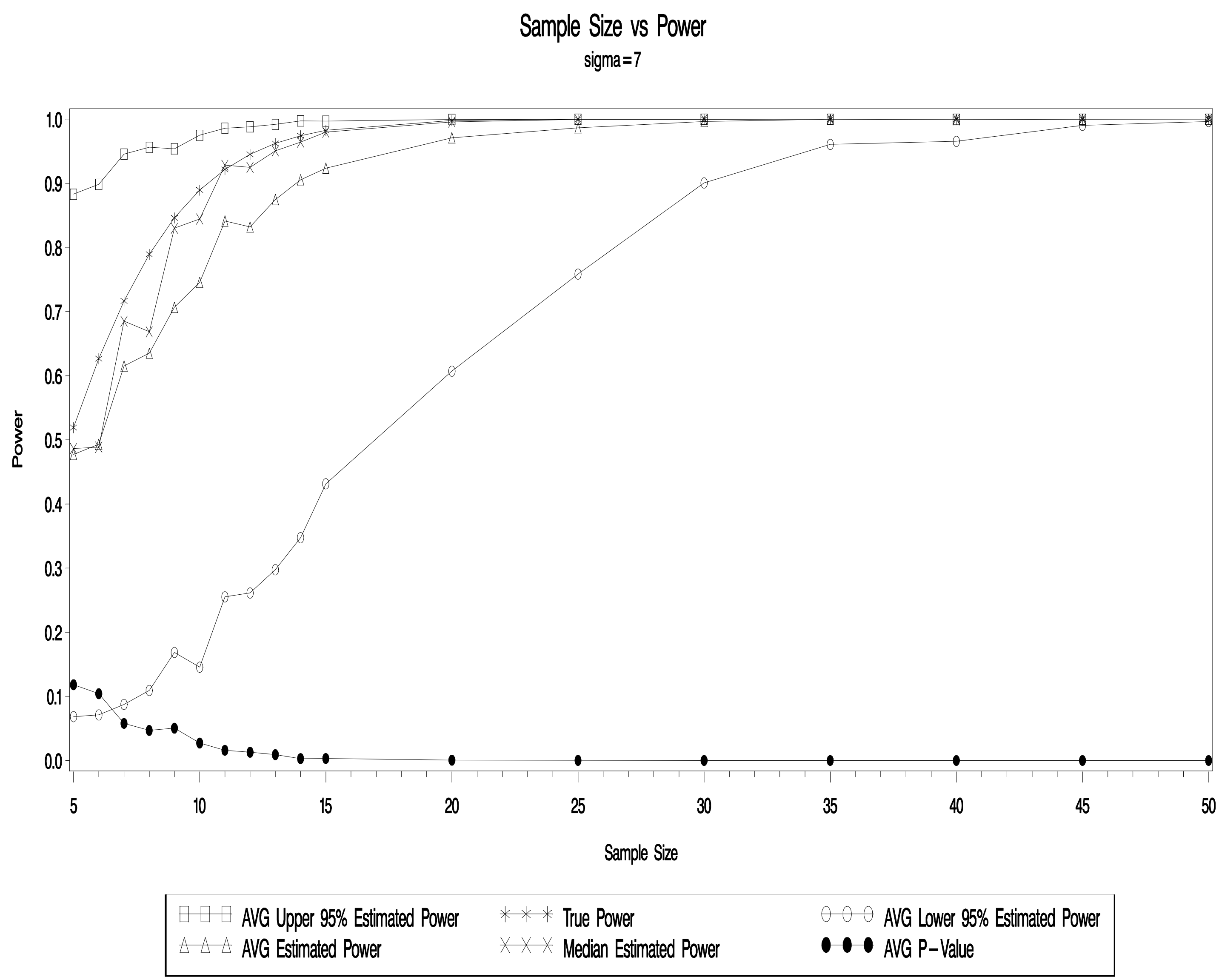
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



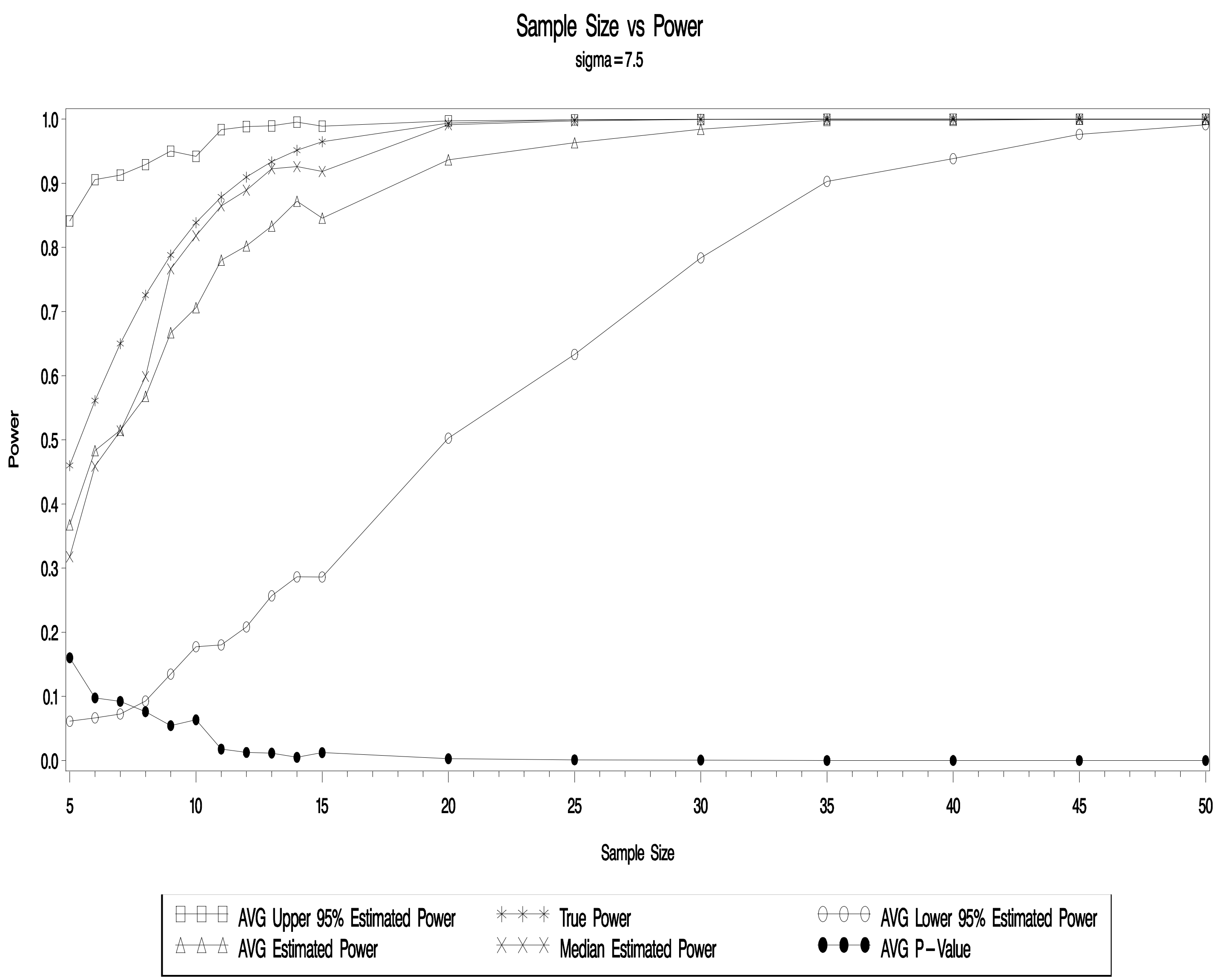
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



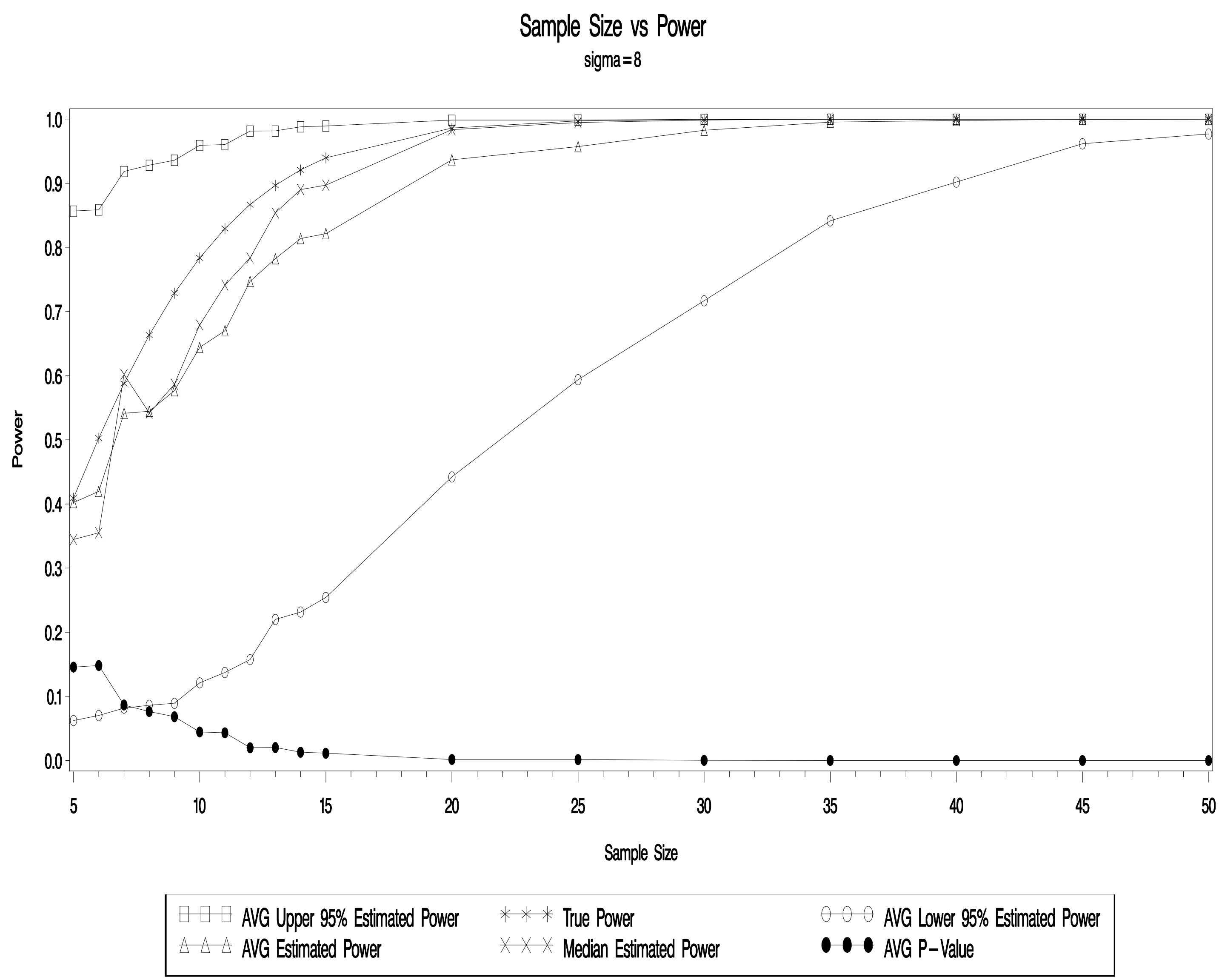
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



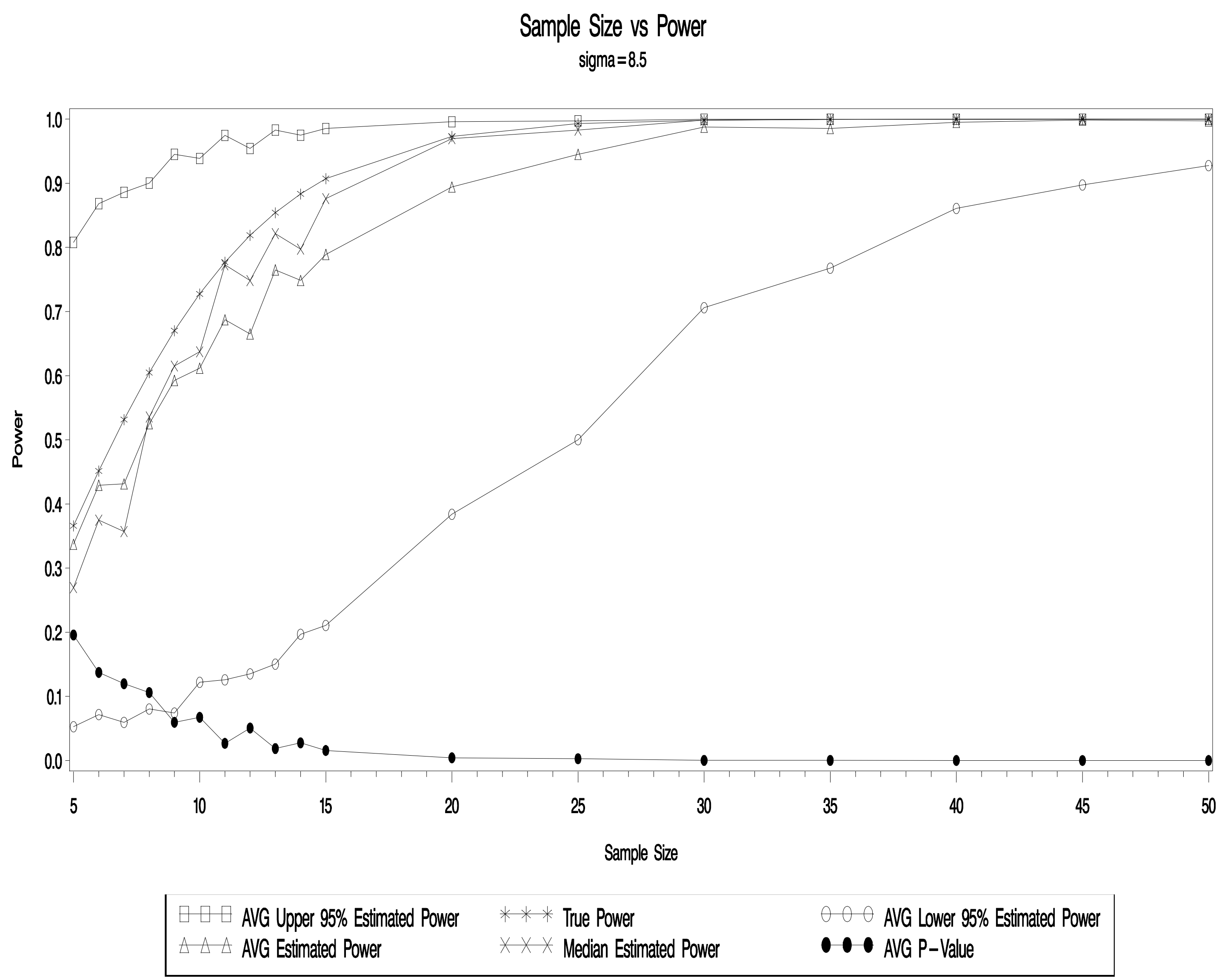
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



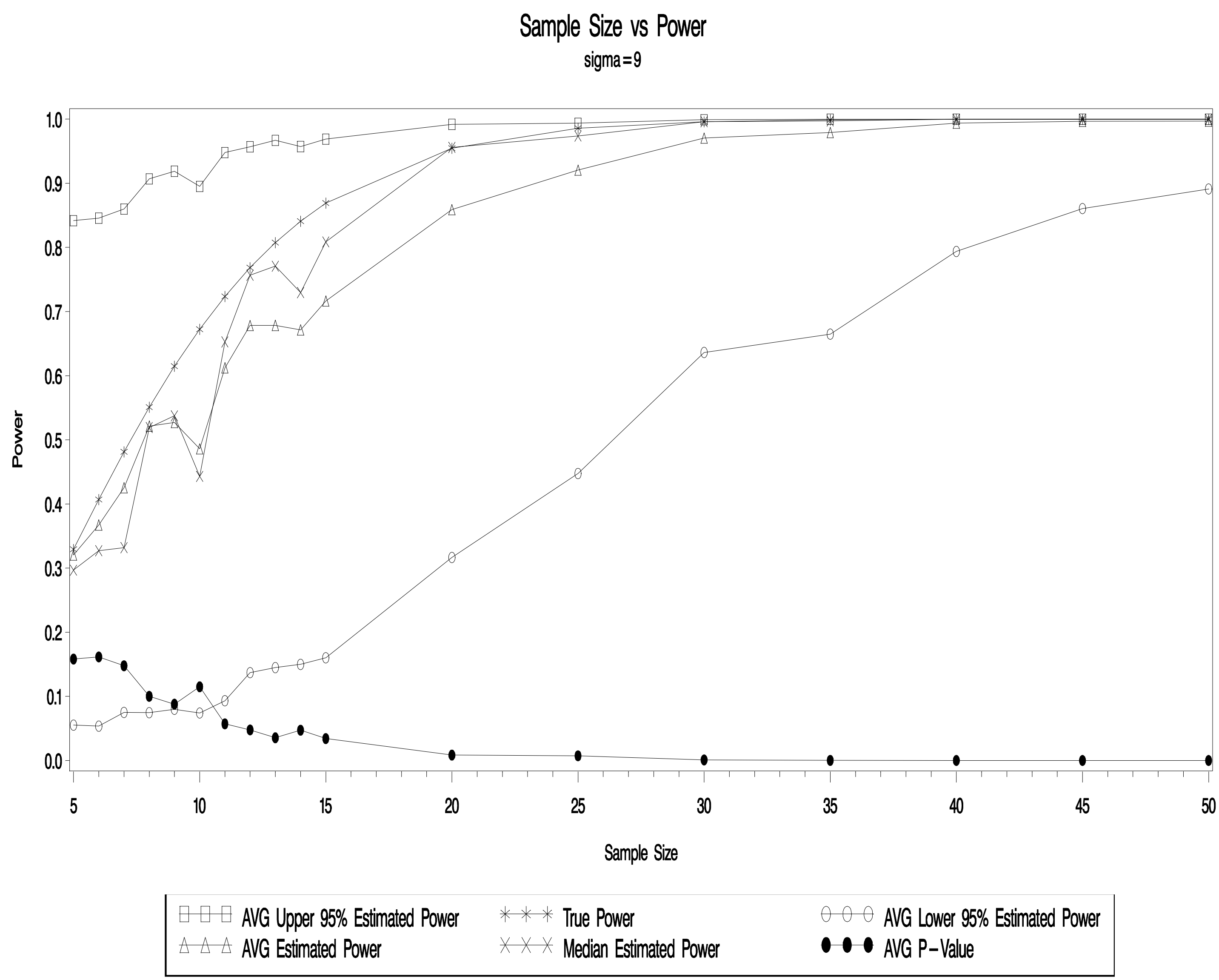
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



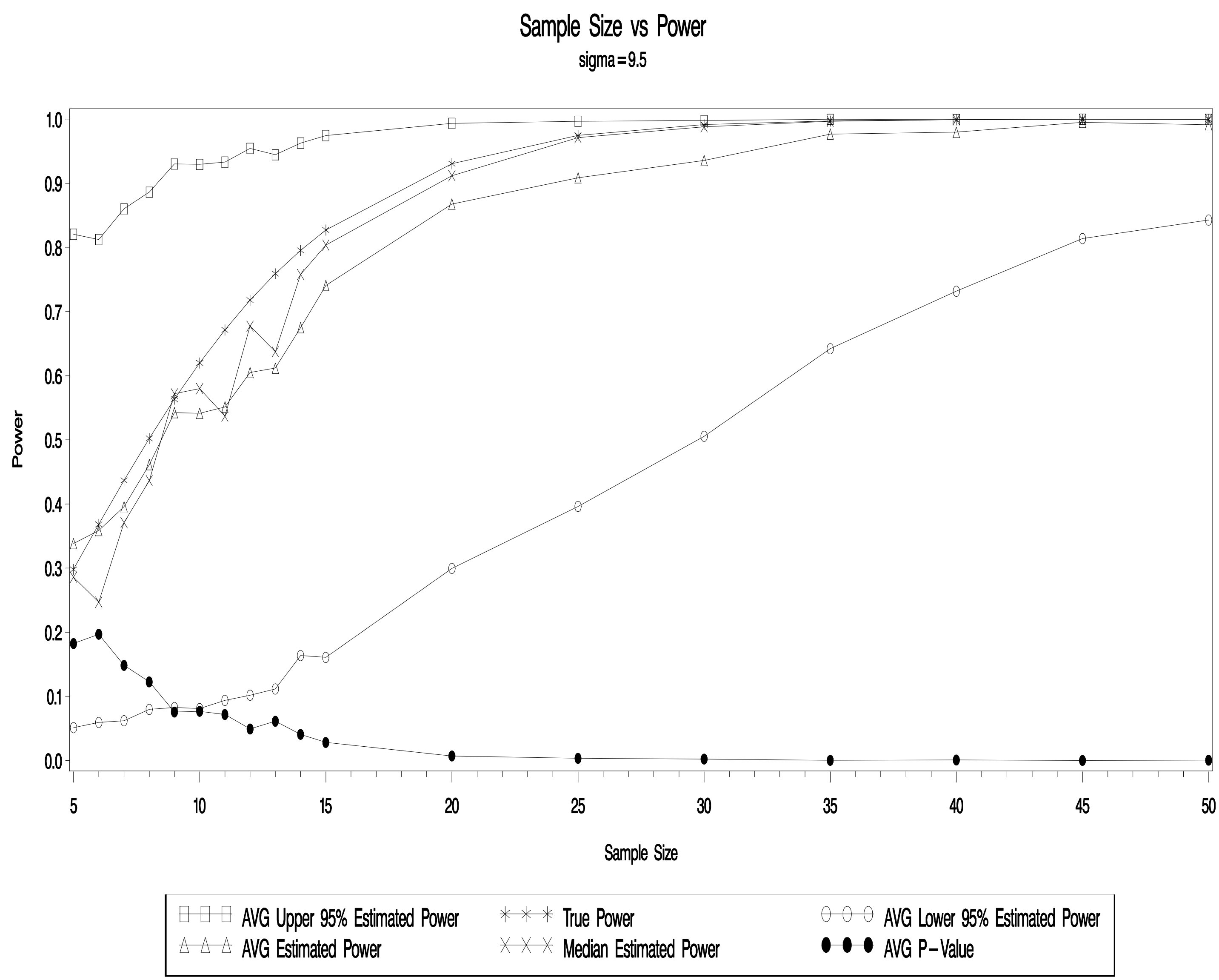
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



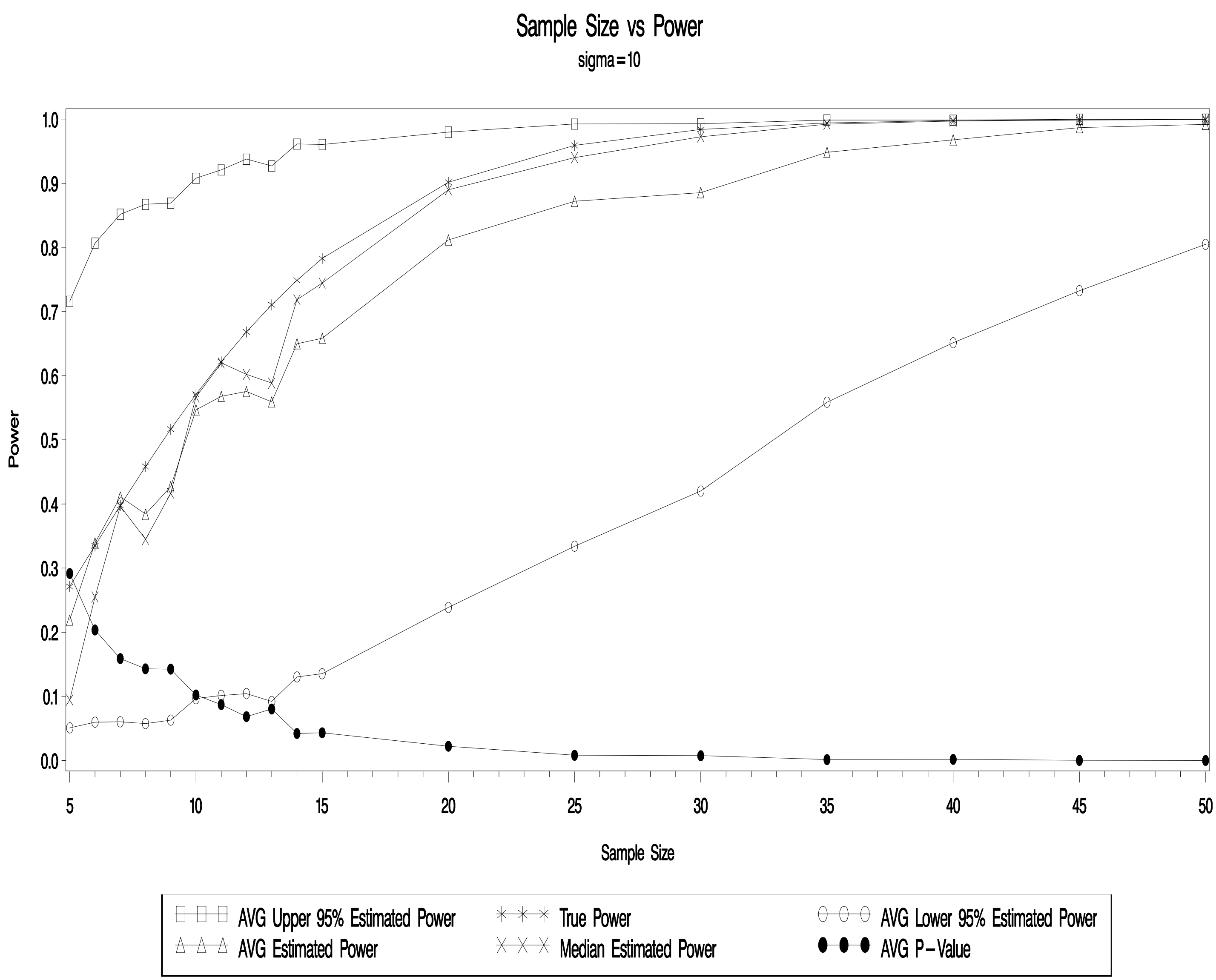
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



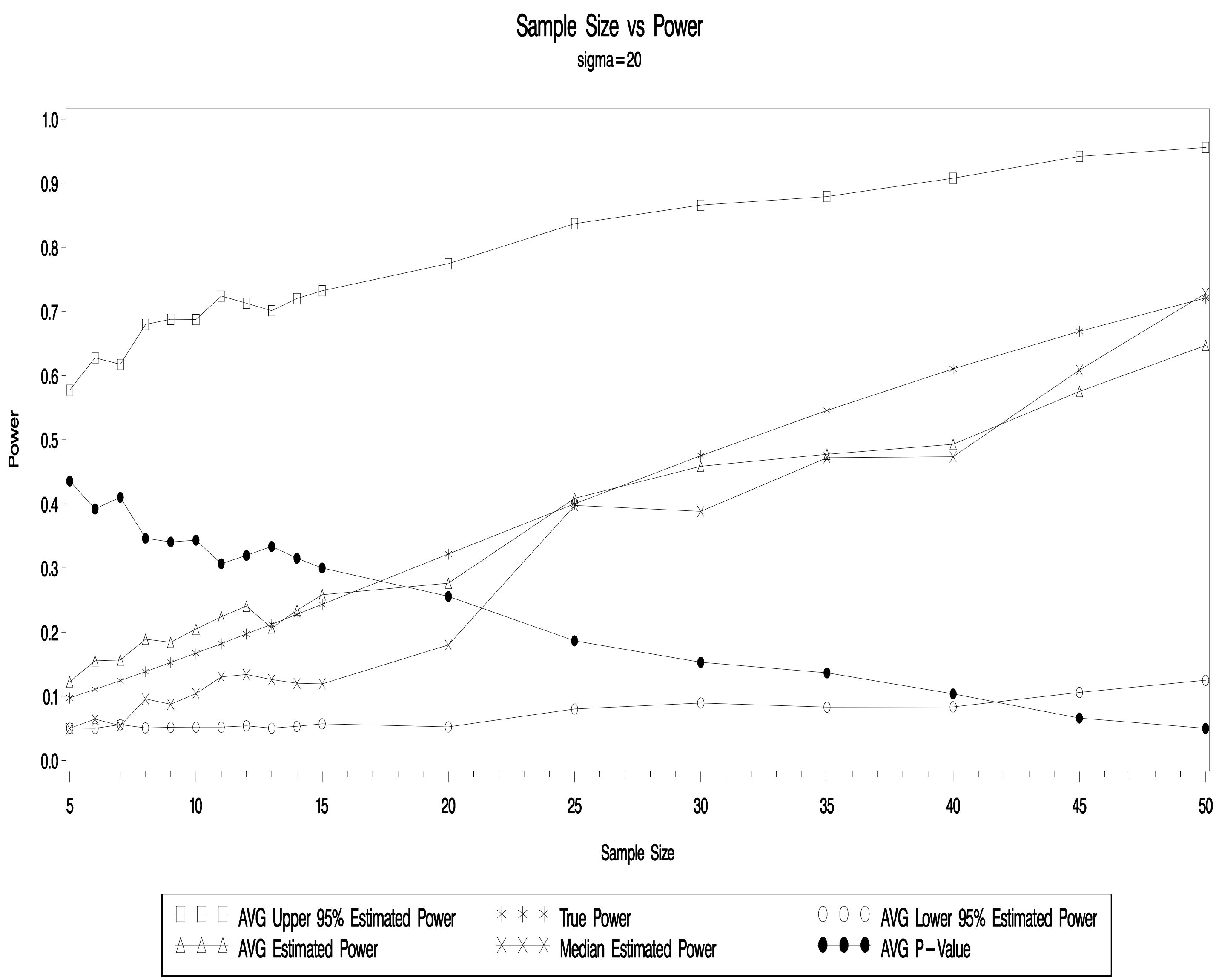
Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



Appendix C: Graphs of Treatment Arrangement 30, 40, 40, 40



Appendix D: SAS Code for Estimating Power for an Observed *F*-Statistic

```
/* Application **************************************************************/
/*                                                                          */
/* Input: significance level, observed F-statistic, and                     */
/* numerator and denominator degrees of freedom.                            */
/*                                                                          */
/* Output: estimate for power (1-alpha)% confidence interval for power, and */
/* p-value of the test                                                      */
/*                                                                          */
/****************************************************************************/

%macro application1(F=,df1=,df2=,alpha=);
proc iml;
    reset nolog;

        /* Function ***********************************************************/
        /*                                                                    */
        /* Return the confidence interval width for a noncentral              */
        /* f-dsitribution                                                     */
        /*                                                                    */
        /**********************************************************************/

        start w(alphaLow,ndf,ddf,ncent,sl);

            width = quantile('F',1-sl+alphaLow,ndf,ddf,ncent)-quantile('F',alphaLow,ndf,ddf,ncent);
            return(width);

        finish w;
```

```
/* Function: optimal lower critical region ***************************/
/*                                                                    */
/* Finds the minimum confidence interval width using an              */
/* implementation of the Golden Section Search method by             */
/* Kincaid and Cheney 2009, and Jan Verschelde                        */
/* http://www.math.uic.edu/~jan/mcs471/Lec9/gss.pdf                  */
/*                                                                    */
/* Note(s):                                                           */
/* The Golden Section Search method finds a value which minimizes     */
/* a function.  The function must be one dimensional and unimodal.    */
/*                                                                    */
/* Imagine a 'typical' pdf of an F distribution.  Say we want to      */
/* setup an interval such that the area covered by the sum of the two */
/* tails is alpha.  For each alpha we can find the                    */
/* associated quantile using the SAS function 'quantile'.             */
/* Since alpha is specified, knowing the lower area easily leads to   */
/* the upper area.  Here we consider only the lower                   */
/* area.  As such we can relate interval width as a function of       */
/* lower alpha size.  As we increase or decrease the size of the      */
/* lower area we increase or decrease the width of the interval.      */
/* Assuming a minimum interval width exists, we optimize over the     */
/* values [0.00, 0.05].                                               */
/*                                                                    */
/**********************************************************************/

start alphaLU(ndf1,ddf2,ncp,slAlpha,tol);

    alphaA = 0;                                         /* Initial lower bracket on optimal region */
    alphaB = slAlpha;                                   /* Initial upper bracket on optimal region */

                                                        /* Note 1 **********************************/
                                                        /*                                        */
                                                        /* For each iteration an interval         */
                                                        /* [alphaA, alphaB] is retained           */
                                                        /*                                        */
                                                        /******************************************/

    phi = (-1+sqrt(5))/2;                               /* Golden ratio constant reduction factor */
    a1 = alphaA+phi*(alphaB-alphaA);                    /* Set test bracket a1 */
    wa1 = w(a1,ndf1,ddf2,ncp,slAlpha);                  /* Calculate Interval width for a1 */
    a2 = alphaA+phi*phi*(alphaB-alphaA);                /* Set test bracket a2 */
    wa2 = w(a2,ndf1,ddf2,ncp,slAlpha);                  /* Calculate interval width for a2 */
```

```
                                                            /* Note 2 ************************************/
                                                            /*                                           */
                                                            /* The intial setup invokes two function     */
                                                            /* calls.  The following loop uses one       */
                                                            /* function call per iteration.              */
                                                            /*                                           */
                                                            /*********************************************/

        do while (abs(alphaB-alphaA)>tol);                  /* Note 3 ************************************/
                                                            /*                                           */
                                                            /* We continue until the absolute difference */
                                                            /* between an upper and a lower bracket is   */
                                                            /* negligable. Tolerance is set to1e-4.      */
                                                            /*                                           */
                                                            /*********************************************/

                                                            /* Note 4 ************************************/
                                                            /*                                           */
                                                            /* Four brackets are maintained at any       */
                                                            /* given time.                               */
                                                            /*                                           */
                                                            /*********************************************/

            if wa1>wa2 then do;                             /* If wa1 > wa2 then minimum width is between */
                                                            /* alphaA and a1                               */

                                                            /* Note 5 ************************************/
                                                            /*                                           */
                                                            /* Within the interval [alphaA, a1] we have  */
                                                            /* the width wa2.  a2 is situated by the     */
                                                            /* expression alphaA+phi*(a1-alphaA).        */
                                                            /*                                           */
                                                            /*********************************************/

                alphaB = a1;                                /* Updated alphaB */
                a1 = a2;                                    /* Save a2 */
                wa1 = wa2;                                  /* Save wa2 */
                a2 = alphaA+phi*phi*(alphaB-alphaA);        /* Set new test bracket a2 */
                wa2 = w(a2,ndf1,ddf2,ncp,slAlpha);          /* Calculate interval width for new a2 */
            end;
            else do;                                        /* If wa1 < wa2 then minimum width is between */
                                                            /* a2 and alphaB                               */
```

```
                                                        /* Note 6 ************************************/
                                                        /*                                           */
                                                        /* Within the interval [a2, alphaB] we have  */
                                                        /* the width wa1.  a1 is situated by the     */
                                                        /* expression alphaA+phi*phi*(alphaB-a2).    */
                                                        /*                                           */
                                                        /*********************************************/

            alphaA = a2;                                /* Updated alphaA */
            a2 = a1;                                    /* Save a1 */
            wa2 = wa1;                                  /* Save wa1 */
            a1 = alphaA+phi*(alphaB-alphaA);            /* Set new test bracket a1 */
            wa1 = w(a1,ndf1,ddf2,ncp,slAlpha);          /* Calculate interval width for new a1 */
        end;
    end;

    out = (alphaA+alphaB)/2;
    return(out);

finish alphaLU;


/* Calculations */

eps = 1e-4;                                             /* Stopping criteria for GSS method */

fCrit = quantile('F',1-&alpha,&df1,&df2);                   /* F critical value */

pValue = 1-cdf('F',&F,&df1,&df2);                       /* P-Value */

estLambda = &F*&df1;                                /* Estimated lambda */
adjEstLambda = (estLambda*(&df2-2)/&df2)-&df1;              /* Adjusted estimated lambda */

if adjEstLambda < 0 then adjEstLambda = 0;              /* Note 7 ***********************/
                                                        /*                              */
                                                        /* For some random seeds the    */
                                                        /* noncentrality parameter is   */
                                                        /* negative.                    */
                                                        /*                              */
                                                        /* A negative noncentrality     */
                                                        /* parameter occurs when        */
                                                        /* the expression               */
                                                        /* estLambda*(df2-2)/df2 is     */
                                                        /* less than df1.               */
                                                        /*                              */
                                                        /********************************/
```

```

        alphaL = alphaLU(&df1,&df2,adjEstLambda,&alpha,eps);    /* Lower significance */
        alphaU = &alpha-alphaL;                                 /* Upper significance */

        adjAlphaL = 1-alphaL;                                   /* Note 8 ********************************/
        probCenF = cdf('F',&F,&df1,&df2);                       /*                                      */
                                                                /* fnonct reports an error for a supplied */
                                                                /* probability greater than the           */
                                                                /* probability from the cdf of a central  */
                                                                /* F distribution.                        */
                                                                /*                                        */
                                                                /* As the probability supplied gets large */
                                                                /* fnonct tends toward zero.  probCenF    */
                                                                /* with fnonct is small around zero.      */
                                                                /* Using adjAlphaL would then be smaller  */
                                                                /* about zero.  Set to zero.              */
                                                                /*                                        */
                                                                /******************************************/


        if adjAlphaL > probCenF then lambdaL = 0;               /* See note 8 */
            else lambdaL = fnonct(&F,&df1,&df2,adjAlphaL);
        if alphaU > probCenF then lambdaU = 0;                  /* See note 8 */
            else lambdaU = fnonct(&F,&df1,&df2,alphaU);


        estPower = 1-cdf('F',fCrit,&df1,&df2,adjEstLambda);     /* Estimated power */
        powerL = 1-cdf('F',fCrit,&df1,&df2,lambdaL);                /* Lower limit for power */
        powerU = 1-cdf('F',fCrit,&df1,&df2,lambdaU);                /* Upper limit for power */

        print estPower;
        print powerL, powerU, pValue;

quit;
%mend application1;

%application1(F=1.5,df1=4,df2=10,alpha=0.05);

```

Appendix D: Section 5 Output for Example 1

```
                     The SAS System           15:16 Thursday, November 10, 2005  80

                        ESTPOWER

                       0.0975334


                         POWERL

                            0.05
                         POWERU

                       0.8017428
                         PVALUE

                       0.2278353
```

Appendix D: Section 5 Output for Example 2

```
                                The SAS System           15:16 Thursday, November 10, 2005  81

                                   ESTPOWER

                                  0.0777412


                                    POWERL

                                       0.05
                                    POWERU

                                  0.6713596
                                    PVALUE

                                  0.2741814
```