# NATURAL FREQUENCIES OF VIBRATION
## OF SANDWICH PANELS

by

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# INTRODUCTION

Sandwich construction consists of two relatively thin sheets of a
material called facings, separated by and bonded to a relatively thick
internal member called the core. The facings are usually made of
material with high strength and stiffness whereas the core is usually
a material of less density and comparatively low strength and stiffness.
The resulting composite structure has an extremely high strength-weight
ratio as compared with that obtainable through the use of a single homo-
geneous material. For this reason, it is widely used in construction of
structural components in which weight is a major factor, such as guided
missiles and air frames. Recently, improved techniques of bonding and
fabrication are increasing its applications in many other industries such
as the domestic appliance and marine, etc.

Prior to fabrication, the core is extremely flexible with seemingly
little strength and the thin facings of the sandwich are incapable of
resisting reasonable design loads in their own plane because of their
elastic instability. However, once incorporated in a sandwich, the core
becomes the structural center of a rigid panel which has strength and
dimensional stability. Thus, the facings must be of a high strength
material that will withstand outside fiber stresses; the core must with-
stand maximum shear which occurs at the center. The bonding agent --
adhesive, brazing, alloy or weld -- must transmit stresses from core
to facing panels.

Although sandwich panels are rigid and strong, they are susceptible
to damages from concentrated loads (panels used as floors in aircraft

have been damaged by the heels of ladies' high-heeled shoes). It is difficult to fasten panels with conventional rivets or screws unless the internal core is sufficiently strengthened in the area to be fastened. Vibration hazards to structures are well known. The study of vibration in sandwich panels which go into making guided missiles, aircraft frames, etc. is important. Because of its complex layered system, many variables have to be taken into account, a fact which makes the analysis of the problem more difficult. Much work of both theoretical and experimental nature has been done in the field of statics of sandwich structures, but, to the knowledge of the author, there is nothing in the literature to suggest that work in the dynamics field has been undertaken.

In view of these facts, Raville and Kimel (1) presented a mathematical derivation for calculating the natural frequencies of vibration of sandwich panels, simply supported on two opposite ends and constructed with isotropic facings and orthotropic cores.

The purpose of this thesis is to present an experimental verification of Raville and Kimel's theory, and to lay the groundwork for the development of vibration techniques for future investigations in this field at Kansas State College.

A series of vibration tests was performed on four sandwich panels, 120 in. by 6 in. in size, and of two different thicknesses (0.25 in. and 1.0 in.) supported knife edges. Data of resonating frequencies for various modes were obtained, and presented in tabular and graphical forms.

The natural frequencies of the same four beams, for free-free and clamped end conditions, were also obtained and included in this thesis.

The latter investigations were necessary to develop the experimental technique.

This thesis describes the outline of the theory and the test procedure used, and presents the results of the test.

## THEORETICAL ANALYSIS

In their analysis, Raville and Kimel assumed that the core of the sandwich panel is capable of resisting only normal stresses perpendicular to the facings and shear stresses in planes perpendicular to the facings, which are treated by isotropic thin plate theory. It is also assumed that $E_c$, the transverse modulus of elasticity of core, is infinite.

The problem of the determination of natural frequencies of vibration associated with the cylindrical bending of a rectangular sandwich panel simply supported on two opposite edges and free on the other edges was analyzed. Modes of vibration corresponding to "face wrinkling" were not considered.

The dimensions of the sandwich panel strip and the co-ordinate system used in this analysis are shown in Fig. 1. Each facing is assumed to have the same physical properties.

### Core Equilibrium Equations

A differential element of the deformed core is shown in Fig. 2. Summation of forces in the x and z directions, respectively, yields
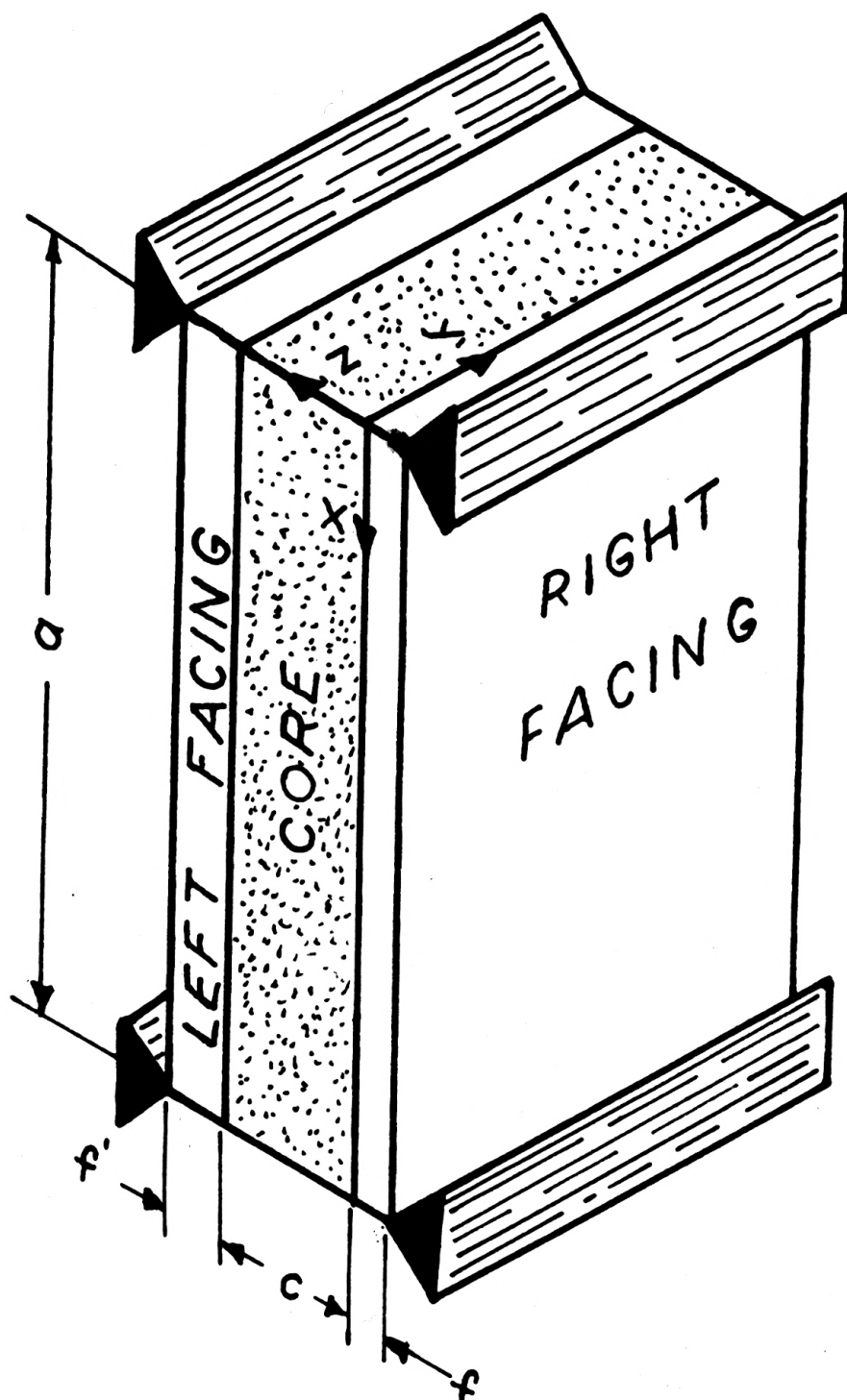
LEFT FACING

CORE

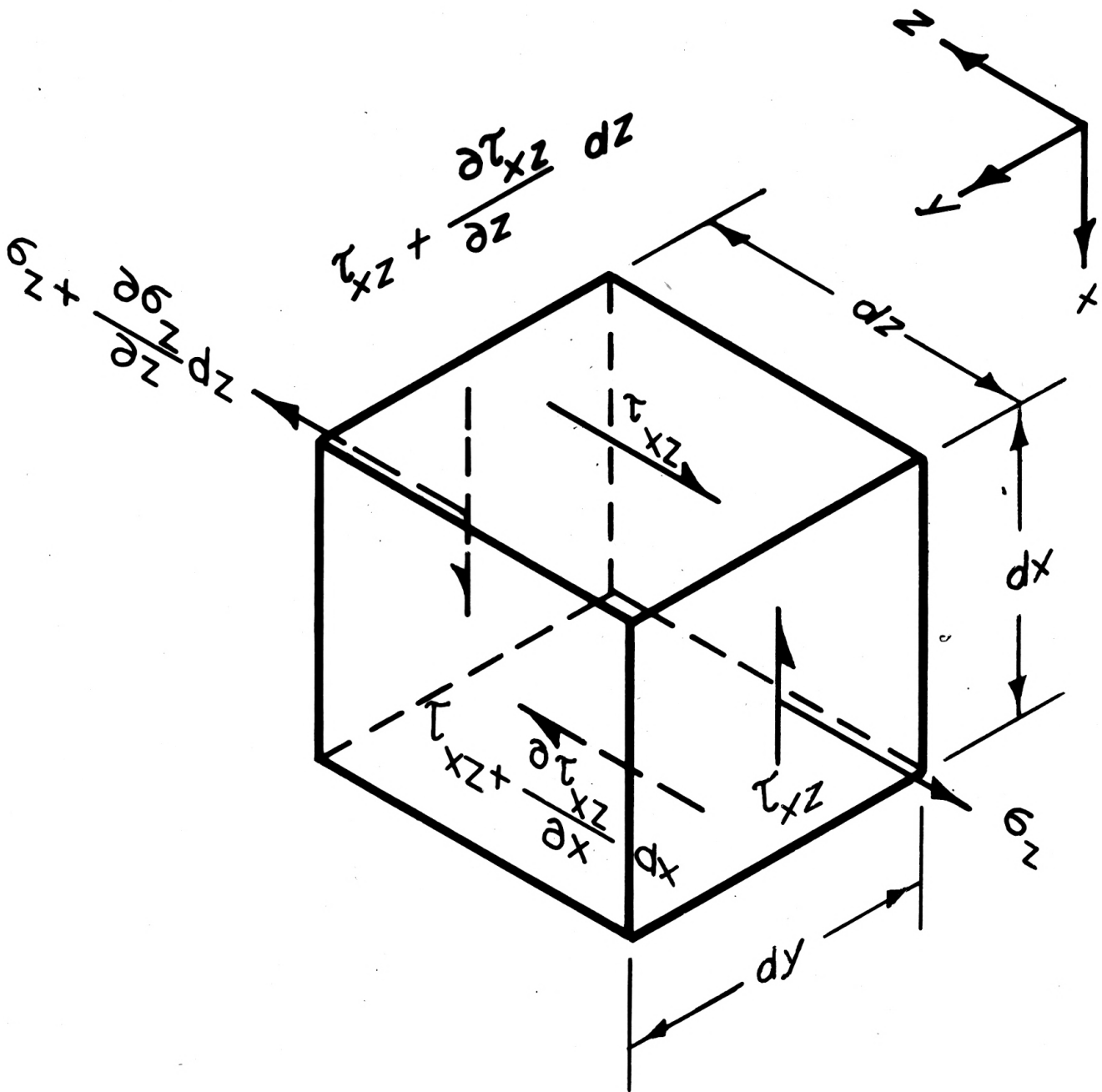RIGHT FACING

$a$

$f'$

$c$

$f$

Fig. 1. Sandwich Strip

Fig. 2. Differential Element of the Core

the two equilibrium equations

$$\frac{\partial \tau_{xz}}{\partial z} = 0 \tag{1}$$

and
$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \delta \frac{\partial^2 w_c}{\partial t^2} . \tag{2}$$

The boundary conditions are

$$w_c (0, a) \big|_t = \frac{\partial^2 w_c}{\partial x^2} (0, a) \big|_t = 0.$$

In order to satisfy the boundary conditions of the simply supported panel, as well as the core equilibrium equations, the core displacements are assumed to be of the forms

$$\tau_{xz} = \sum_{m=1}^{\infty} A_m \cos \frac{m \pi x}{a} \sin \omega_m t \tag{3}$$

and

$$w_c = \sum_{m=1}^{\infty} C_m \sin \frac{m \pi x}{a} \sin \omega_m t \tag{4}$$

where $A_m$ and $C_m$ are constants of integration.
Substituting (3) and (4) into (2) gives

$$\sigma_z = \sum_{m=1}^{\infty} \left[ \left( \frac{m \pi}{a} A_m - \delta \omega_m^2 C_m \right) z + B_m \right] \sin \frac{m \pi x}{a} \sin \omega_m t \tag{5}$$

Substitution of (3) and (4) into the equation

$$\tau_{xz} = G_{xz} \left( \frac{\partial u_c}{\partial z} + \frac{\partial w_c}{\partial x} \right) \tag{6}$$

gives

$$u_c = \sum_{m=1}^{\infty} \left[ \left( \frac{A_m}{G_{xz}} - \frac{m\pi}{a} C_m \right) z + F_m \right] \cos \frac{m\pi x}{a} \sin \omega_m t. \tag{7}$$

Equations (3), (4), (5) and (6) satisfy the core equilibrium equations (1) and (2), and the boundary conditions.

## Strain Energy in the Cores

From (4) and (7), the core strains are expressed as

$$\epsilon_{zc} = \frac{\partial w_c}{\partial z} = 0 \tag{8}$$

and

$$\gamma_{xzc} = \frac{\partial u_c}{\partial z} + \frac{\partial w_c}{\partial x} = \sum_{m=1}^{\infty} \frac{A_m}{G_{xz}} \cos \frac{m\pi x}{a} \sin \omega_m t. \tag{9}$$

The elastic strain energy of the core can be expressed as

$$V_c = \frac{1}{2} \int_0^a \int_0^c G_{xz} \gamma_{xzc}^2 \, dx \, dz. \tag{10}$$

Substituting (9) into (10) and integrating, gives

$$V_c = \frac{ac}{4G_{xz}} \sum_{m=1}^{\infty} A_m^2 \sin^2 \omega_m t. \tag{11}$$

### Strain Energy in the Facings Due to
### Membrane Strains in the Facings

The membrane strains in the right and left facings, $\epsilon_{xM}$ and $\epsilon'_{xM}$ respectively, can be expressed in terms of the core displacements as follows:

$$\epsilon_{xM} = (\frac{\partial u_c}{\partial x} + \frac{f}{2} \frac{\partial^2 w_c}{\partial x^2})_{z=0} \qquad (12)$$

and

$$\epsilon'_{xM} = (\frac{\partial u_c}{\partial x} - \frac{f'}{2} \frac{\partial^2 w_c}{\partial x^2})_{z=c}. \qquad (13)$$

The elastic strain energy, $V_{MF}$, associated with the membrane strain in the right facing can be expressed as:

$$V_{MF} = \frac{E}{2(1-\nu^2)} \int_0^a \int_{-f}^0 \epsilon^2_{xM} \, dx \, dz. \qquad (14)$$

Substituting (12) into (14) and integrating gives

$$V_{MF} = \frac{Eaf}{4(1-\nu^2)} \sum_{m=1}^\infty (\frac{m\pi}{a} F_m + \frac{f}{2} \frac{m^2\pi^2}{a^2} C_m)^2 \sin^2 \omega_m t. \qquad (15)$$

Similarly, for the left facing

$$V'_{MF} = \frac{Eaf'}{4(1-\nu^2)} \sum_{m=1}^\infty \left[ - \frac{A_m}{G_{xz}} \frac{m\pi c}{a} (c + \frac{f'}{2}) C_m - \frac{m\pi}{a} F_m \right]^2 \sin^2 \omega_m t. \qquad (16)$$

### Strain Energy of the Facings Associated with the Bending of the Facings About Their Own Middle Surfaces

The bending strains, $\epsilon_{xB}$ and $\epsilon'_{xB}$, in the right and left facings, respectively, can be expressed in terms of the core displacement as

$$\epsilon_{xB} = \left(z + \frac{f}{2}\right)\left(\frac{\partial^2 w_c}{\partial x^2}\right) \tag{17}$$

and

$$\epsilon'_{xB} = \left(z - c - \frac{f'}{2}\right)\left(\frac{\partial^2 w_c}{\partial x^2}\right). \tag{18}$$

From (17) and (4), the elastic strain energy $V_{BF}$, associated with strains due to bending of right facing about its middle surface, is

$$V_{BF} = \frac{E}{2(1 - \nu^2)} \int_0^a \int_{-f}^0 \epsilon^2_{xB}\, dx\, dz. \tag{19}$$

Therefore,

$$V_{BF} = \frac{E\, a\, f^3}{48(1 - \nu^2)} \sum_{m=1}^{\infty} \frac{m^4\, \pi^4}{a^4}\, C^2_m\, \sin^2 \omega_m\, t. \tag{20}$$

Similarly, the elastic strain energy, $V_{BF}$ of the left facing can be expressed as

$$V'_{BF} = \frac{E\, a\, (f')^3}{48(1 - \nu^2)} \sum_{m=1}^{\infty} \frac{m^4\, \pi^4}{a^4}\, C^2_m\, \sin^2 \omega_m\, t. \tag{21}$$

## Total Elastic Strain Energy

The total elastic strain energy, $V$, of the sandwich strip of unit width is

$$V = V_c + V_{MF} + V'_{MF} + V_{BF} + V'_{BF}. \qquad (22)$$

Substituting equations (11), (15), (16), (20) and (21) into equation (22) gives

$$V = \sum_{m=1}^{\infty} \left\{ \frac{a\,c}{4 G_{xz}} A_m^2 + \frac{E\,a\,f}{4(1 - \nu^2)} \left( \frac{m\,\pi}{a} F_m + \frac{f}{2} \frac{m^2 \pi^2}{a^2} C_m \right)^2 \right.$$

$$+ \frac{E\,a\,f'}{4(1 - \nu^2)} \left[ -\frac{A_m}{G_{xz}} \frac{m\,\pi c}{a} + \frac{m^2 \pi^2}{a^2} \left( c + \frac{f'}{2} \right) C_m - \frac{m\,\pi}{a} F_m \right]^2$$

$$\left. + \frac{E\,a \left[ f^3 + (f')^3 \right]}{48(1 - \nu^2)} \frac{m^4 \pi^4}{a^4} C_m^2 \right\} \sin^2 \omega_m\, t. \qquad (23)$$

## Kinetic Energy of the Vibrating Strip

Kinetic energy $T$, of the vibrating strip is given by

$$T = \frac{1}{2} \rho \int_0^a \left( \frac{\partial w}{\partial t} \right)^2 dx \qquad (24)$$

Since,        $w = w_c.$        (25)

Equations (4) and (25) satisfy the boundary conditions

$$w(0, a) \Big|_t = \frac{\partial^2 w}{\partial x^2} (0, a) \Big|_t = 0$$

Therefore,

$$T = \frac{\rho \, \omega_m^2 \, a}{4} \sum_{m=1}^{\infty} C_m^2 \cos^2 \omega_m\, t. \qquad (26)$$

## Frequency Criteria

The vibration system is assumed to be conservative so that

$$\frac{\partial}{\partial A_m} ( V_{max} - T_{max} ) = 0,$$  (27)

$$\frac{\partial}{\partial C_m} ( V_{max} - T_{max} ) = 0,$$  (28)

and

$$\frac{\partial}{\partial F_m} ( V_{max} - T_{max} ) = 0.$$  (29)

Substituting maximum values of $V$ and $T$ from equations (23) and (26), leads to three linear, homogeneous equations in which $A_m$, $C_m$ and $F_m$ are unknown.

Such a system of equations can yield for $A_m$, $C_m$ and $F_m$, solutions other than the trivial one in which $A_m = C_m = F_m = 0$, only if the determinant of these equations is equal to zero. This condition brings to the frequency equation from which the frequencies of the various modes of vibrations can be calculated. Thus, the resulting equation can be simplified to give

$$\omega^2_m = \frac{m^4 \pi^4 E}{a^4 (1 - \nu^2) \rho} ( \frac{I_T}{1 + m^2 s} + I_F ).$$  (30)

## OPERATION OF EQUIPMENT

A schematic diagram of the apparatus for a typical vibration test of a sandwich beam with free ends is shown on Plate I.

A sandwich beam is suspended by two strings $s$, at its nodal positions. The vibration exciter is placed under the beam at a position

# EXPLANATION OF PLATE I

A Schematic Diagram of Experimental Equipment

Reference numbers:

(1) Panel cabinet
(2) Vibration exciter
(3) Sandwich beam with sand Chladni figure
(4) Piezo electric pickup
(5) Amplifier
(6) Voltmeter
(7) Electronic frequency counter

Plate I
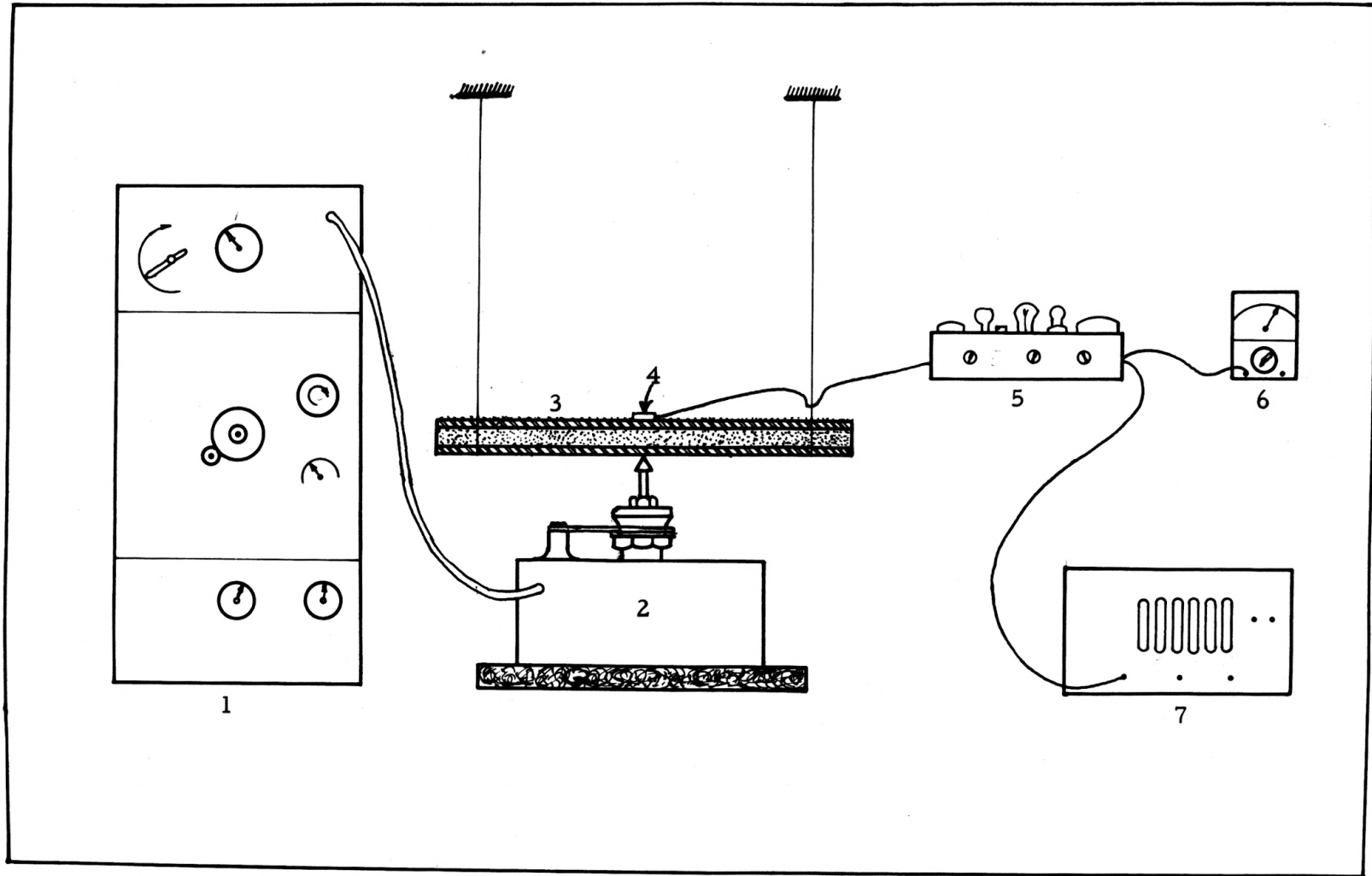
of maximum amplitude.

The Model C-31 vibration exciter by MB Manufacturing Company consists of a moving assembly including two flat spring flexures which support a threaded magnesium tube. Mounted on the tube are a driver coil and a signal generator coil, and a vibration pickup attachment table. The function of the vibration exciter is to generate a sinusoidal forcing function when the probe is properly matched to the specimen being vibrated. The driver coil which is supplied by a variable frequency alternating current, is suspended in a constant magnetic field. The amplitude and acceleration of the driver coil at a desired frequency are controlled by varying the applied current. The generated force depends upon the direct current in the field coil as well as the alternating current in the driver coil. The amplitude of the response depends not only on the generated force, but also on the dynamic properties of the system, including both the moving parts and the specimen. The motion of the force take-off (probe) is limited to plus or minus one-fourth-inch travel (giving a total excursion of one-half inch ) or an acceleration of 100 g, whichever applies.

A piezo electric crystal pickup was used to measure the vibrations of the beams. It was held on the beam manually at lower frequencies to make it follow the beam in spite of its inertia. It was fixed by means of an adhesive tape at higher frequencies where the amplitudes are small.

The amplified signal of the pickup is lead simultaneously to a voltmeter and an electronic frequency counter.

The piezo electric effect is utilized in this crystal pickup to convert mechanical force (due to vibration) into electrical voltage. In other words, the voltage generated by the crystal is proportional to the force applied to it, and is therefore also proportional to the acceleration. The voltmeter gives a needle deflection proportional to the voltage generated by the crystal pickup.

The high speed electronic counter has an accurate crystal controlled timing gate. The unit automatically counts and displays in integer form any number of events which have occurred during a precise one-second counting interval. The events may be optical, physical or electrical occurrences translatable into voltage changes. The unit can also display the accumulated number of events during any integral number of seconds desired. If the timing gate is kept open for ten seconds, and the accumulated number of events divided by ten, the resulting figure for one second gives an accuracy of one decimal place.

## EXPERIMENTAL PROCEDURE

The tests were carried out for the following end conditions:

1. Free-free,
2. Simply supported, and
3. Fixed-fixed.

In order to avoid complications due to end conditions, and to develop the technique of determining the natural frequencies of the beam, comparatively simple end conditions -- free ends -- were selected for the initial test.

Two symmetrical nodal positions and one antinodal position were calculated for each mode of vibration. These positions were carefully marked on the beam and used to support the beam and position the exciting probe, respectively. The beam was properly leveled to avoid any torsional effects while under transverse vibration test.

The vibration exciter with a rubber probe was placed under the specimen (Plate II) at the position of maximum amplitude.

After making proper electrical connections of the apparatus (see schematic diagram on Plate I ), the following preliminary procedure was adopted for the proper operation of the Vibration Exciter;

1. The amplifier was turned on and allowed 10 to 15 minutes warm-up time.

2. The field supply was turned on and the field current was adjusted with the powerstat control to 0.7 amps. as specified by the manufacturer.

3. The desired range of operating frequencies was selected with the help of the decade range switch at the right-hand side of the oscillator panel.

4. The Amplitude Control, located just above the range switch on the oscillator panel was set at a 80 to 90 percent of the total rotation.

The desired input to the vibrator was established by adjusting the AC output control (top panel) and the amplitude control (middle panel). The frequency of vibration was controlled by the frequency dial on the oscillator panel.

The frequencies were slowly varied and when the forced frequency coincided with the natural frequency of the sandwich beam, resonance

**EXPLANATION OF PLATE II**

Photograph of the vibration test setup
for free-free end conditions.

Plate II



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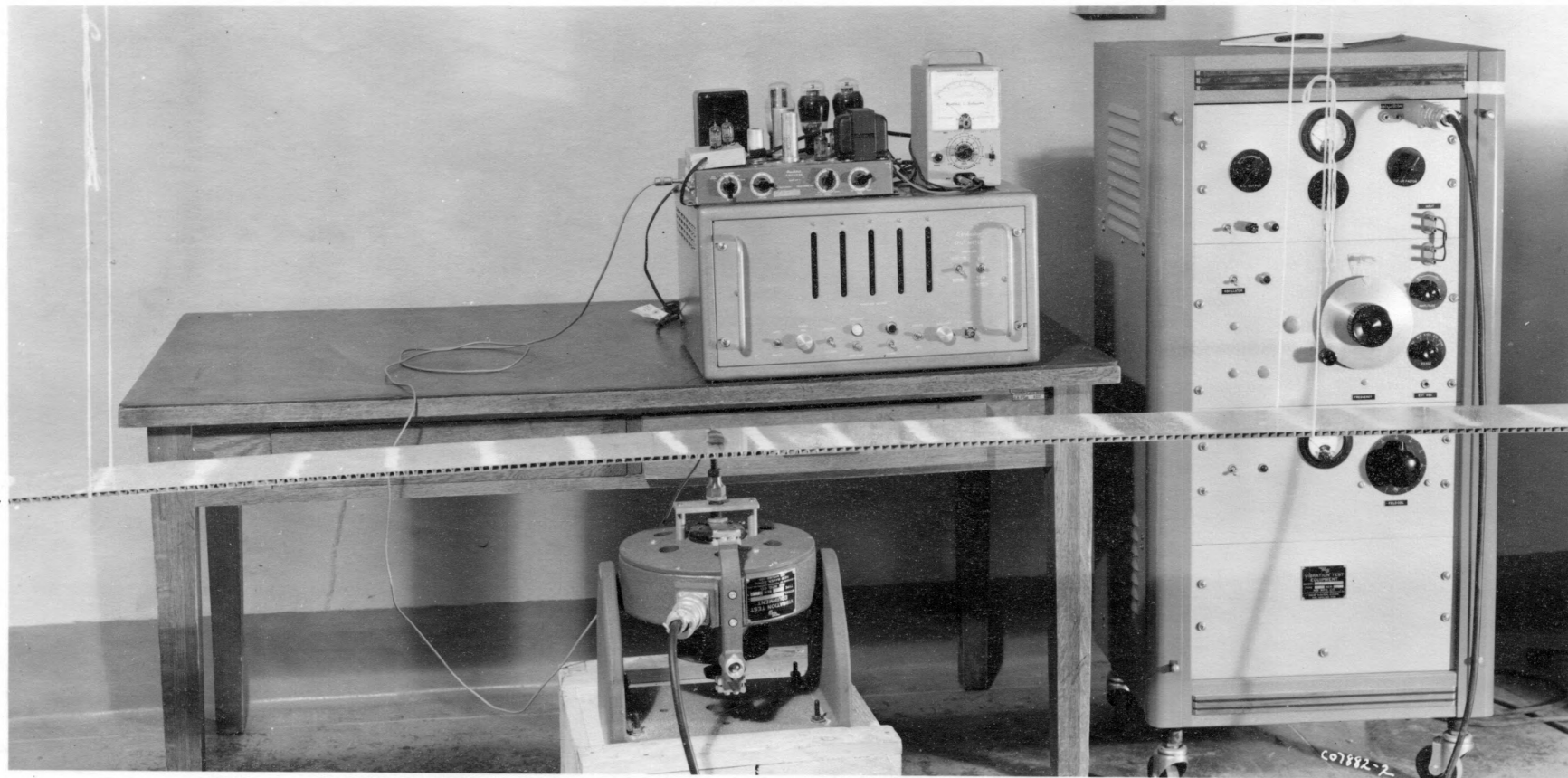
occurred. Resonance was detected by noticing a maximum voltmeter deflection, vigorous movement of sawdust or sand which was spread on the beam prior to the operation of the test, the formation of Chladni figures on the beam, and a peculiar change in the noise level (sound). The resonating frequency was read from the electronic counter panel. Sawdust collected at the nodal positions. The mode number in which the beam was resonating was determined by counting the nodal positions. The mode is one less than the number of nodes.

When the frequency was slightly altered, the Chladni pattern of sawdust was disturbed and the voltmeter deflection decreased. The next higher mode was excited by increasing the frequency until a new maximum voltmeter deflection was observed, and the Chladni figure corresponding to the higher frequency appeared.

The same technique for determining the natural frequencies of various modes was employed for the simply supported and clamped end conditions.

## COMMENTS ON THE PROCEDURE

The symmetrical nodal positions were calculated before the operation of the test because if the beam is suspended at any other positions, the reactions cause couples at the supports, and the beam no longer has free-free end conditions. The positions of maximum amplitude (antinodes) are calculated and marked on the beam in order to determine where to place the vibration exciter. Both nodal and antinodal positions were calculated for various end conditions and are included in the Appendix.

The sawdust, being comparatively light and soft, moved with the beam at the lower frequencies and did not respond to the vibrations of the beam at higher frequencies. In order to get better Chladni patterns, fine Ottawa sand of uniform size was substituted for sawdust. The sand worked very satisfactorily and some of the patterns formed by it can be seen on Plate III.

Difficulties were encountered in exciting modes beyond the 15th because of the fact that the rubber probe, like a soft (low stiffness) spring, absorbed most of the vibration energy of the generated force of the exciter. These difficulties were solved by substituting plastic and steel probes with which it was possible to excite modes up to the 25th and 35th, respectively.

The performance of the probes of different materials can be explained by the electromechanical analogy as shown in Plate IV. The probe corresponds to a capacitance $C$. The natural frequencies of the system are given by $f_n = \frac{\alpha}{c}$, where $\alpha$ is a constant. If the capacitance is too high, only current with low frequencies can pass through the circuit, or, in other words, the circuit can be tuned for comparatively low frequencies only. The rubber probe corresponds to a relatively high capacitance, therefore, it could excite modes up to 15th only. Similar explanation can be offered for the behaviour of the plastic and steel probes.

Modes beyond 35th could not be excited because, at these modes, the Chladni patterns became smaller than the length between the two free edges and the beams began to vibrate as plates giving complex

## EXPLANATION OF PLATE III

Photograph of the vibration test setup
for clamped end conditions.

Plate III
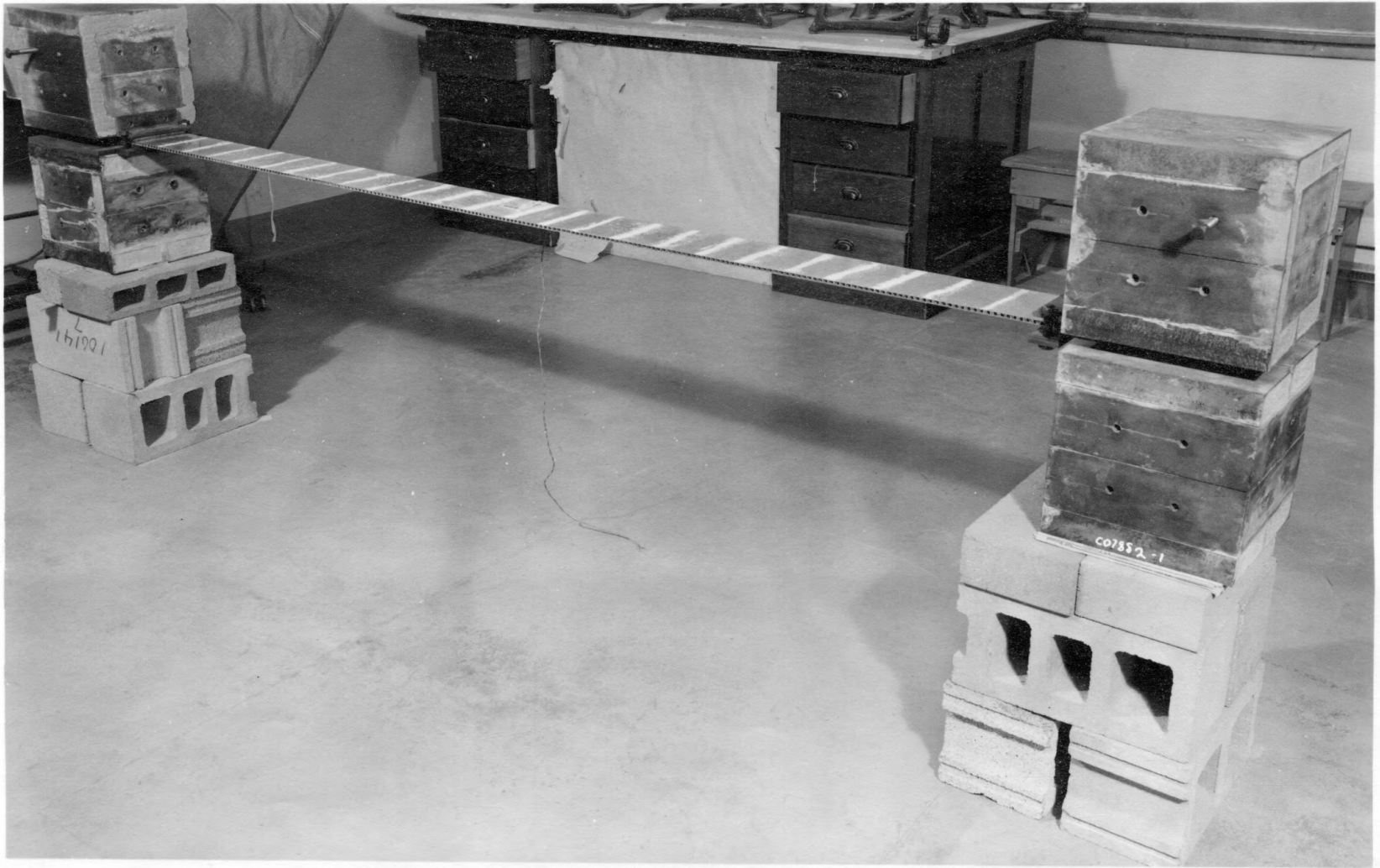
# EXPLANATION OF PLATE IV

**Electro-mechanical Analogy:**
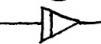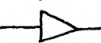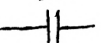
⊗— Multiplier      Inductance ⟿

▷— Integrater      Resistance ⟋⟍⟋

▷— Summer        Source of power ⊙

⊣⊢ Capacitance

Plate IV
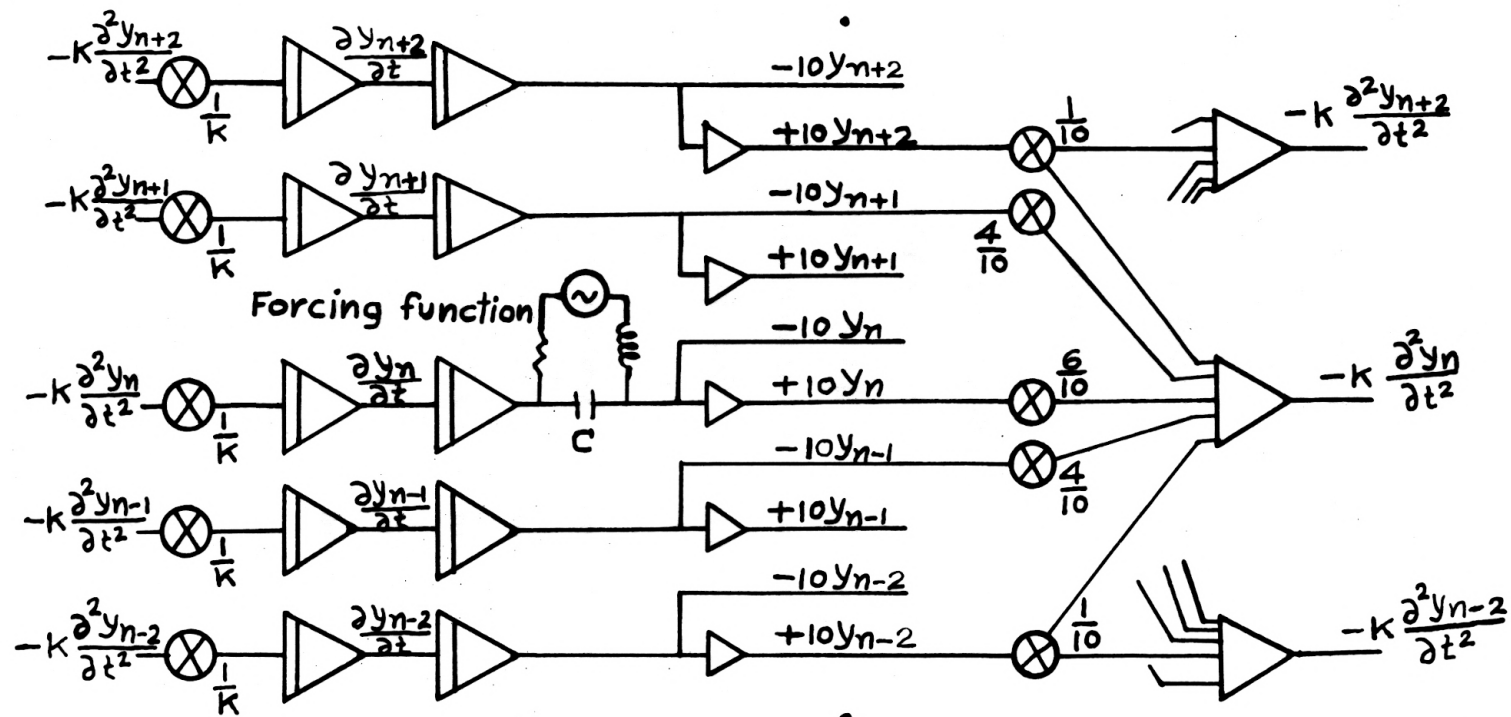
patterns not accounted for in the theory.

Another limiting factor was the force generating capacity of the vibration exciter; i.e., the acceleration at such frequencies was high, resulting in extremely small amplitudes to which the beam did not respond.

## PRESENTATION OF DATA AND SAMPLE CALCULATIONS

### Presentation of Data

Three different tests, corresponding to three different end conditions, were made on four samples of sandwich panels. Their physical dimensions and properties are given in Table 4.

The values of the natural frequencies which were determined experimentally for the different end conditions are recorded in Tables 1(a,b), 2(a,b), and 3(a,b).

Tables 1 (a,b) are for hinged or simply supported sandwich beams. A set of data for two beams (Nos. 2 and 5) of equal core thickness (0.25 in.) but of different core strength is presented in Table 1a. Beam No. 2 has a stronger core $(G_{xz} = 11,700)$ than that of No. 5 $(G_{xz} = 4375)$. Similarly, Table 1-b is for beams Nos. 10 and 7, of equal core thickness but of different strength. The facings are 0.016 in. thick in all the beams.

In all the cases, $\omega_e$ is the experimental value of the natural frequency. $\omega_m$ is obtained by solving the equation derived by Raville and Kimel (see sample calculations). $\omega_h$ is calculated by using the homogeneous beam theory (see sample calculations) with stiffness

Table 1-a. Frequencies of vibration of hinged beams Nos. 2 and 5.

| No. of mode: m | Beam #2, $G_{xz}$ = 11,700; Style 125-35; Type 20 (strong core) | | | | | Beam #5, $G_{xz}$ = 4375; Style 60-20; Type 40 (weak core) | | | | |
| | $\omega_e$ | $\omega_m$ | $\omega_h$ | $\omega_h/\omega_e$ | % deviation of expt.freq. | $\omega_e$ | $\omega_m$ | $\omega_h$ | $\omega_h/\omega_e$ | % deviation of expt.freq. |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 1 | — | 16. | 2.571 | | | | | 2.624 | | |
| 2 | - | | 10.283 | | | | | 10.496 | | |
| 3 | - | | 23.137 | | | | | 23.616 | | |
| 4 | 40 | | 41.132 | | | | | 41.986 | | |
| 5 | 61 | 65.613 | 64.269 | 1.0536 | -7.562 | | | 65.602 | | |
| 6 | 90 | | 92.547 | - | | 83 | | 94.467 | 1.1382 | |
| 7 | 114 | | 125.967 | 1.145 | | | | 128.581 | | |
| 8 | 178 | | 164.528 | - | 7.94 | 158 | 158.0 | 167.943 | 1.0629 | |
| 9 | 201 | | 208.231 | 1.170 | | 239 | | 212.552 | | |
| 10 | 254 | 250.487 | 251.075 | 1.012 | 1.383 | 273 | 235.5 | 26.410 | | 13.736 |
| 11 | 335 | | 311.061 | - | | 313 | | 317.516 | 1.0144 | |
| 12 | 353 | 351.555 | 370.188 | 1.049 | 8.687 | 357 | 320.773 | 377.871 | 1.0585 | 10.147 |
| 13 | 423 | 406.908 | 434.457 | 1.025 | 3.963 | 396 | | 443.473 | 1.1199 | |
| 14 | 479 | | 503.867 | 1.052 | | 445 | | 514.324 | 1.156 | |
| 15 | 532 | 525.835 | 578.419 | 1.087 | 1.158 | 498 | 458.759 | 590.423 | 1.186 | 7.879 |
| 16 | 585 | | 658.112 | 1.150 | | 551 | | 671.77 | | |
| 17 | 644 | | 742.946 | 1.1536 | | 597 | | 758.366 | | |
| 18 | 712 | | 832.923 | 1.1698 | | 646 | | 850.21 | 1.3161 | |
| 19 | 768 | 789.821 | 928.041 | 1.2084 | -1.389 | 701 | | 947.301 | 1.3514 | |
| 20 | 835 | | 1028.300 | 1.2315 | -3.071 | 754 | 702.10 | 1049.641 | 1.3921 | 6.883 |
| 21 | 897 | | 1133.701 | 1.2639 | | 810 | | 1157.229 | 1.4287 | |
| 22 | 966 | | 1244.243 | 1.2879 | | 868 | | 1270.06 | 1.4632 | |
| 23 | 1039 | | 1359.926 | 1.3089 | | 923 | | 1388.15 | 1.5039 | |
| 24 | 1111 | | 1480.751 | 1.333 | | 971 | | 1511.483 | 1.5566 | |
| 25 | 1172 | | 1606.72 | 1.371 | -4.81 | 1037 | 951.31 | 1640.064 | 1.5815 | 8.263 |
| 26 | 1251 | 1304.18 | 1737.827 | 1.389 | -4.25 | 1093 | | 1773.893 | 1.623 | -10.93 |
| 27 | 1320 | | 1874.077 | 1.4197 | | 1151 | | 1912.971 | 1.662 | |
| 28 | 1396 | | 2015.47 | 1.4437 | | 1198 | | 2057.296 | 1.7173 | |
| 29 | 1470 | | 2152.0 | 1.471 | | 1268 | | 2206.87 | 1.7404 | |
| 30 | 1536 | 1613.441 | 2313.675 | 1.51 | -5.041 | 1317 | 1200.0 | 2361.692 | 1.7932 | 8.883 |
| 31 | 1613 | | 2470.49 | 1.532 | | 1376 | | 2521.76 | 1.8327 | |
| 32 | 1691 | | 2632.448 | 1.556 | | 1431 | | 2687.081 | 1.878 | |
| 33 | 1759 | | 2799.55 | 1.5916 | | 1490 | | 2857.647 | 1.9179 | |
| 34 | 1833 | | 2927.786 | 1.6213 | | 1552 | | 3033.462 | 1.955 | |
| 35 | 1919 | 2005.839 | 3149.169 | 1.641 | | 1608 | | 3214.525 | 1.999 | |

Table 1-b. Frequencies of vibration of hinged beams Nos. 10 and 7.

| No. of mode: m | Beam #10, $G_{xz}$ = 9725; Style 125-35; Type 20 (strong core) | | | | | Beam #7, $G_{xz}$ = 3500; Style 60-20; Type 40 (weak core) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\omega_e$ | $\omega_m$ | $\omega_h$ | $\omega_h/\omega_e$ | % deviation of expt. freq. | $\omega_e$ | $\omega_m$ | $\omega_h$ | $\omega_h/\omega_e$ | % deviation of expt. freq. |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 1 | | | 8.015 | | | | | 8.6724 | | |
| 2 | | | 32.420 | | | | | 34.690 | | |
| 3 | | | 72.420 | | | | | 78.052 | | |
| 4 | 71 | | 129.676 | 1.823 | | 182 | | 138.759 | 1.823 | |
| 5 | | | 202.619 | | | | | 216.811 | | |
| 6 | 310 | 277.87 | 291.771 | | 10.362 | | | 312.208 | | |
| 7 | | | 397.133 | | | 359 | 355.16 | 424.950 | | 1.069 |
| 8 | 435 | | 518.705 | 1.1924 | -6.0 | 445 | 433.621 | 555.036 | 1.1924 | 2.557 |
| 9 | 595 | | 656.486 | 1.1033 | 5.529 | 578 | 513.124 | 702.467 | 1.1033 | 11.224 |
| 10 | 652 | | 810.476 | 1.243 | | | | 867.243 | 1.243 | |
| 11 | 758 | | 980.676 | 1.294 | -2.524 | 746 | | 1049.364 | 1.2938 | |
| 12 | 865 | | 1167.086 | 1.359 | | 831 | 754.76 | 1248.830 | 1.359 | 9.174 |
| 13 | 957 | | 1369.705 | 1.4312 | | 927 | | 1465.641 | 1.4312 | |
| 14 | 1062 | | 1588.534 | 1.4958 | | 1025 | | 1699.80 | 1.4958 | |
| 15 | 1097 | | 1823.572 | 1.6623 | -11.94 | 1143 | 994.54 | 1951.297 | 1.6623 | 12.988 |
| 16 | 1223 | | 2074.82 | | | 1211 | | 2220.142 | 1.833 | |
| 17 | 1354 | | 2342.276 | | | 1311 | | 2506.33 | 1.912 | |
| 18 | 1464 | | 2625.943 | 1.7936 | -3.3196 | 1443 | 1231.0 | 2809.867 | 1.947 | 14.69 |
| 19 | 1579 | | 2925.819 | 1.8529 | -7.156 | | | 3130.75 | | |
| 20 | 1677 | | 3241.905 | 1.9331 | -7.698 | 1639 | 1387.58 | 3468.973 | 2.1165 | 15.339 |
| 21 | 1794 | | 3574.201 | 1.9923 | | 1738 | 3824.542 | 3824.54 | 2.200 | |
| 22 | - | | 3922.705 | | | 1817 | 4197.457 | 4197.457 | 2.3101 | |
| 23 | 1933 | | 4287.42 | 2.2180 | | 1895 | 4587.716 | 4587.716 | 2.4209 | |

Table 2-a.  Frequencies of vibration of free-free beams Nos. 2 and 5.

| No. of mode: m | Beam #2, $G_{xz}$ = 11,700, C = 0.25 (strong core) | | | Beam #5, $G_{xz}$ = 4375, C = 0.25 (weak core) | | |
|---|---|---|---|---|---|---|
| | $\omega_e$ expt. | $\omega_h$ homo. | $\omega_h/\omega_e$ | $\omega_e$ expt. | $\omega_h$ homo. | $\omega_h/\omega_e$ |
| 1 | | | | | 5.947 | |
| 2 | | | | | 16.397 | |
| 3 | | | | | 32.137 | |
| 4 | | | | | 53.125 | |
| 5 | 74.0 | 77.75 | 1.0506 | | 79.359 | |
| 6 | 104.0 | 108.59 | 1.0441 | | 110.841 | |
| 7 | 142.0 | 144.57 | 1.0181 | | 147.569 | |
| 8 | 174.0 | 185.69 | 1.0672 | 189 | 189.544 | |
| 9 | 218.7 | 231.95 | 1.0606 | | 236.77 | |
| 10 | 260.0 | 283.35 | 1.0898 | | 289.235 | |
| 11 | 339.9 | 319.2 | 1.0648 | 333 | 346.951 | 1.0419 |
| 12 | 386.6 | 401.58 | 1.0388 | | 409.914 | |
| 13 | 423.5 | 468.40 | 1.1060 | 438 | 478.124 | 1.1493 |
| 14 | 489.6 | 540.37 | 1.1104 | | 551.581 | |
| 15 | 545.0 | 617.47 | 1.1330 | 524 | 630.284 | 1.2028 |
| 16 | 592.0 | 699.71 | 1.1820 | 564 | 714.234 | 1.266 |
| 17 | 661.8 | 787.10 | 1.1893 | 614 | 803.432 | 1.3085 |
| 18 | 732.4 | 879.62 | 1.2010 | 669 | 897.876 | 1.342 |
| 19 | 793.0 | 977.29 | 1.2324 | 722 | 997.567 | 1.3817 |
| 20 | 860.3 | 1080.09 | 1.2555 | 780 | 1102.506 | 1.4135 |
| 21 | 930.5 | 1188.03 | 1.277 | 834 | 1212.69 | 1.4541 |
| 22 | 1062.0 | 1301.12 | 1.2252 | - | 1328.12 | - |
| 23 | 1066.8 | 1419.34 | 1.3305 | 952 | 1448.80 | 1.5219 |
| 24 | 1138.5 | 1542.71 | 1.3550 | - | 1574.73 | - |
| 25 | 1211.7 | 1671.22 | 1.3792 | 1065 | 1705.90 | 1.602 |
| 26 | 1277.0 | 1804.86 | 1.4134 | 1121 | 1842.32 | 1.6435 |
| 27 | 1366.0 | 1943.65 | 1.4228 | 1174 | 1983.98 | 1.6899 |
| 28 | 1435.0 | 2087.57 | 1.4547 | - | 2130.89 | - |
| 29 | 1505.0 | 2236.63 | 1.4861 | 1292 | 2283.06 | 1.7671 |
| 30 | - | 2390.85 | - | - | 2440.47 | - |
| 31 | 1646.0 | 2550.19 | - | 1401 | 2603.12 | 1.858 |
| 32 | 1718.0 | 2714.68 | - | - | 2771.02 | - |
| 33 | 1791.0 | 2884.31 | - | 1504 | 2944.17 | - |
| 34 | 1864.0 | 3059.1 | - | - | 3122.56 | - |
| 35 | 1913.0 | 3238.98 | - | 1625 | 3306.21 | - |

Table 2-b.  Frequencies of vibration of free-free
beams Nos. 10 and 7.

| No. of mode: m | Beam #10, $G_{xz} = 9425$; $C = 1.0$ (strong core) | | | Beam #7, $G_{xz} = 3500$; $C = 1.00$ (weak core) | | |
|---|---|---|---|---|---|---|
| | $\omega_e$ expt. | $\omega_h$ homo. | $\omega_h/\omega_e$ | $\omega_e$ expt. | $\omega_h$ homo. | $\omega_h/\omega_e$ |
| 5 | 413 | 455.78 | 1. | 242 | 262.153 | 1.0833 |
| 6 | | | | 316 | 366.32 | 1.1592 |
| 7 | 413 | 455.78 | 1.1036 | 420 | 487.706 | 1.1368 |
| 8 | 513 | 585.424 | 1.1411 | 512 | 626.4315 | 1.2235 |
| 9 | 595 | 731.273 | 1.229 | 617 | 782.475 | 1.2681 |
| 10 | 693 | 893.326 | 1.2891 | 695 | 955.903 | 1.3754 |
| 11 | 808 | 1071.587 | 1.3262 | 789 | 1146.65 | 1.4533 |
| 12 | 890 | 1266.05 | 1.4225 | 880 | 1354.74 | 1.5395 |
| 13 | 1006 | 1476.73 | 1.4679 | 982 | 1580.166 | 1.6091 |
| 14 | 1107.0 | 1703.60 | 1.5389 | 1082 | 1822.94 | 1.6848 |
| 15 | 1239.6 | 1946.68 | 1.5704 | 1192 | 2083.05 | 1.7475 |
| 16 | 1306 | 2205.97 | 1.6891 | | | |
| 17 | - | 2481.46 | | | | |
| 18 | - | | | | | |

Table 3-a.  Frequencies of vibration of clamped beams Nos. 2 and 5.

| No. of mode m | Beam No. 2; $G_{xz}$ = 11,700; (strong core) | | | | | | | Beam No. 5; $G_{xz}$ = 4375; (weak core) | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | $\omega_e$ experimental | | | $\omega_h$ | $\omega_h/\omega_e$ | | | $\omega_e$ experimental | | | $\omega_h$ | $\omega_h/\omega_e$ | | |
| | Rubber | Plastic | Steel | Homo.beam | Rubber | Plastic | Steel | Rubber | Plastic | Steel | Homo.beam | Rubber | Plastic | Steel |
| 1 | | | | 9.10 | | | | | | | 9.29 | | | |
| 2 | | | | 25.10 | | | | | | | 25.62 | | | |
| 3 | | | | 49.19 | | | | | | | 50.21 | | | |
| 4 | | | | 81.32 | | | | 95 | | | 83.01 | | | |
| 5 | 116 | | | 121.48 | | | | 120 | | | 123.99 | 1.033 | | |
| 6 | | | | 169.67 | | | | | 168 | | 173.19 | | 1.031 | |
| 7 | 215 | 203 | | 225.89 | 1.051 | 1.113 | | | 204 | | 230.58 | 1.130 | | |
| 8 | | | | 290.14 | | | | | | | 296.16 | | | |
| 9 | 315 | | | 362.43 | 1.151 | | | 304 | 347 | | 369.95 | 1.217 | 1.066 | |
| 10 | 387 | | | 442.74 | 1.144 | | | 364 | | | 451.93 | 1.242 | | |
| 11 | 448 | 473 | | 531.09 | 1.186 | 1.123 | | 422 | 443 | | 542.11 | 1.285 | 1.224 | |
| 12 | 517 | 544 | | 627.47 | 1.214 | 1.153 | | 481 | 506 | | 640.49 | 1.332 | 1.266 | |
| 13 | 590 | 608 | | 731.88 | 1.241 | 1.204 | | 545 | 569 | | 747.07 | 1.371 | 1.313 | |
| 14 | 666 | 688 | | 844.32 | 1.268 | 1.227 | | 610 | 631 | | 861.84 | 1.413 | 1.366 | |
| 15 | 759 | 762 | | 964.79 | 1.295 | 1.266 | | 679 | 697 | | 984.82 | 1.450 | 1.413 | |
| 16 | | 837 | | 1093.30 | 1.306 | | | | 757 | | 1115.99 | | 1.474 | |
| 17 | 918 | 922 | | 1229.84 | 1.340 | 1.334 | | | 826 | | 1255.36 | | 1.52 | |
| 18 | | 1004 | | 1374.41 | | 1.369 | | | 903 | | 1402.93 | | 1.554 | |
| 19 | | 1090 | | 1527.01 | | 1.401 | | | 969 | | 1558.70 | | 1.610 | |
| 20 | | 1175 | | 1687.64 | | 1.436 | | | 1035 | | 1722.66 | | 1.664 | |
| 21 | | 1263 | | 1856.30 | | 1.470 | | | 1111 | | 1894.83 | | 1.706 | |
| 22 | | 1351 | | 2033.00 | | 1.505 | | | 1176 | | 2075.19 | | 1.765 | |
| 23 | | 1442 | | 2217.73 | | 1.538 | | | 1249 | | 2263.75 | | 1.812 | |
| 24 | | 1526 | | 2410.48 | | 1.580 | | | 1317 | | 2460.51 | | 1.868 | |
| 25 | | | 1616 | 2611.27 | | | 1.616 | | | 1378 | | | | 1.934 |
| 26 | | | 1710 | 2820.1 | | | 1.649 | | | 1455 | 2878.62 | | | 1.978 |
| 27 | | | 1793 | 3036.95 | | | 1.694 | | | 1520 | 3099.98 | | | 2.039 |
| 28 | | | 1882 | 3261.83 | | | 1.733 | | | 1597 | 3329.53 | | | 2.085 |
| 29 | | | 1966 | 3494.74 | | | 1.778 | | | 1657 | 3567.28 | | | 2.153 |
| 30 | | | | 3535.00 | | | | | | 1748 | 3813.23 | | | 2.182 |
| 31 | | | 2149 | 3984.68 | | | 1.854 | | | 1782 | 4067.37 | | | 2.282 |
| 32 | | | | | | | | | | 1875 | 4329.73 | | | 2.310 |
| 33 | | | | | | | | | | 1926 | 4600.26 | | | 2.389 |
| 34 | | | | | | | | | | | 4879.00 | | | |
| 35 | | | | | | | | | | 2125 | 5165.95 | | | 2.431 |

Table 3-b. Frequencies of vibration of clamped beams Nos. 10 and 7.

| No. of mode m | Beam No. 10; $G_{xz} = 9725$; (strong core) | | | | | | | Beam No. 7; $G_{xz} = 3500$; (weak core) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\omega_e$ experimental | | | $\omega_h$ | $\omega_h/\omega_e$ | | | $\omega_e$ experimental | | | $\omega_h$ | $\omega_h/\omega_e$ | | |
| | Rubber | Plastic | Steel | Homo.beam | Rubber | Plastic | Steel | Rubber | Plastic | Steel | Homo.beam | Rubber | Plastic | Steel |
| 3 | 131 | | | 155.09 | 1.184 | | | 141 | | | 165.95 | 1.177 | | |
| 4 | 218 | | | 256.38 | 1.176 | | | 248 | 233 | | 274.33 | 1.106 | 1.18 | |
| 5 | 321 | 375 | | 382.98 | 1.193 | 1.021 | | 325 | | | 409.81 | 1.261 | | |
| 6 | | 419 | | 534.91 | | 1.277 | | 422 | | | 572.37 | 1.356 | | |
| 7 | 526 | 555 | | 712.16 | 1.36 | 1.283 | | 554 | 547 | | 762.04 | 1.375 | 1.40 | |
| 8 | 642 | | | 914.73 | 1.425 | | | | 660 | | 978.79 | | 1.48 | |
| 9 | 765 | 784 | | 1142.62 | 1.494 | 1.457 | | | 760 | | 1222.64 | | 1.61 | |
| 10 | | 891 | | 1395.82 | | 1.566 | | | 865 | | 1493.59 | | 1.73 | |
| 11 | 1027 | 1047 | | 1674.36 | 1.63 | 1.599 | | | 976 | | 1791.63 | | 1.84 | |
| 12 | | 1146 | | 1978.21 | | 1.726 | | | 1095 | 1106 | 2116.76 | | 1.93 | |
| 13 | | 1290 | | 2307.39 | | 1.789 | | | 1205 | | 2468.99 | | 2.05 | |
| 14 | | 1452 | | 2661.88 | | 1.833 | | | | 1331 | 2848.30 | | 2.14 | |
| 15 | | 1570 | | 3041.70 | | 1.937 | | | | 1414 | 3254.70 | | | 2.302 |
| 16 | | | 1667 | 3446.84 | | | 2.067 | | | 1591 | 3668.20 | | | 2.318 |
| 17 | | | 1836 | 3877.29 | | | 2.112 | | | 1692 | 4148.86 | | | 2.450 |
| 18 | | | 1941 | 4333.1 | | | 2.232 | | | 1835 | 4636.57 | | | 2.530 |
| 19 | | | 2030 | 4814.18 | | | 2.371 | | | 1942 | 5151.37 | | | 2.653 |

Table 4. Dimensions and physical properties of test beams.

| Beam No. | Stiffness determined by static bending test EI lb-in.$^2$ | Face thickness f in. | Core thickness c in. | Supplied by mfg. $G_{xz}$ | Determined by vibration test $G_{xz}$ | Total wt. of panel lb. | Style and Type |
|---|---|---|---|---|---|---|---|
| 2 | 36,475 | .016 | 0.25 | 11,700 | 11,540 | 3.040 | 125-35; 20 |
| 5 | 35,906 | .016 | 0.25 | 4,375 | 6,630 | 2.875 | 60-20; 40 |
| 7 | 434,760 | .016 | 1.00 | 3,500 | 6,870 | 3.185 | 60-20; 40 |
| 10 | 511,990 | .016 | 1.00 | 9,725 | 9,220 | 4.300 | 125-35; 20 |

Modulus of Elasticity of aluminum facings,  $E = 10.3 \times 10^6$ psi.
Poisson's Ratio $= 0.33$
Length of the sandwich panel $= 120$ in.
Width of the sandwich panel $= 6$ in.

determined experimentally using static bending tests.

Tables 2(a, b) and 3(a, b) are for free and clamped end conditions, respectively. For these two conditions, there are no values for theoretical, natural frequencies available.

The ratios $(\frac{\omega_h}{\omega_e})^2$ and $(\frac{\omega_h}{\omega_m})^2$ are calculated and

$$\left[ (\frac{\omega_h}{\omega_e})^2 - 1 \right] \quad \text{and} \quad \left[ (\frac{\omega_h}{\omega_m})^2 - 1 \right]$$ are plotted against the number

of modes on log log graph paper. This representation was selected for the following reasons:

1. It exhibits the properties of the Raville-Kimel theory.

2. Graphical representation is a good way to detect scattering and trends in the observed data.

According to the theory, the natural frequencies of the simply supported sandwich beam are given by the following equation:

$$\omega^2_m = \frac{m^4 \pi^4 E}{a^4 (1 - \nu^2) \rho} \left( \frac{I_T}{1 + m^2 s} + I_F \right) \tag{a}$$

where,

$$s = \frac{\pi^2 c f f'}{a^2 (f + f')} \frac{E}{G_{xz}(1 - \nu^2)} . \tag{b}$$

Those of equivalent homogeneous beams are given by,

$$\omega^2_h = \frac{m^4 \pi^4 E_h I_h}{a^4 \rho} \tag{c}$$

where, $I_h$ = moment of inertia of the cross-section of equivalent homogeneous beam,

and $\qquad$ $E_h$ = modulus of elasticity of the equivalent homogeneous beam.

The parameter $s$ in (a) represents the effect of shear in the core of the sandwich. If $s$ is set to zero, equations (a) and (c) become identical. Therefore, taking the ratio of (c) to (a) and considering

$$E_h = \frac{E}{(1 - \nu^2)} \text{, gives}$$

$$\omega^2_m = \frac{\omega^2_h}{I_h} \left( \frac{I_T}{1 + m^2 s} + I_F \right). \tag{d}$$

since, $I_h \sim I_T$ and $I_F \ll I_h$ for usual panels,

$$\omega^2_m \cong \frac{\omega^2_h}{1 + m^2 s} . \tag{e}$$

Taking the logarithm of both sides and simplifying, gives

$$\log \left[ \left( \frac{\omega_h}{\omega_m} \right)^2 - 1 \right] = 2 \log m + \log s . \tag{f}$$

(f) is the equation of a straight line ( $y = mx + c$ ) on log log graph paper with a slope 2, and an intercept log $s$.

If the experimental values $\omega_e$ are to verify the theory, they should satisfy equation (f). Therefore, the theoretical and observed values of natural frequencies were substituted in equation (f) and the results plotted on log log graph paper (Fig. 3 (a,b) ).

As no theory exists for free-free and clamped beams, only the experimental data is presented in Figs. 4(a,b) and 5(a,b).

Columns 5 and 10 in Tables 1(a,b) for hinged beams show the percentage deviation of experimental values of natural frequencies from
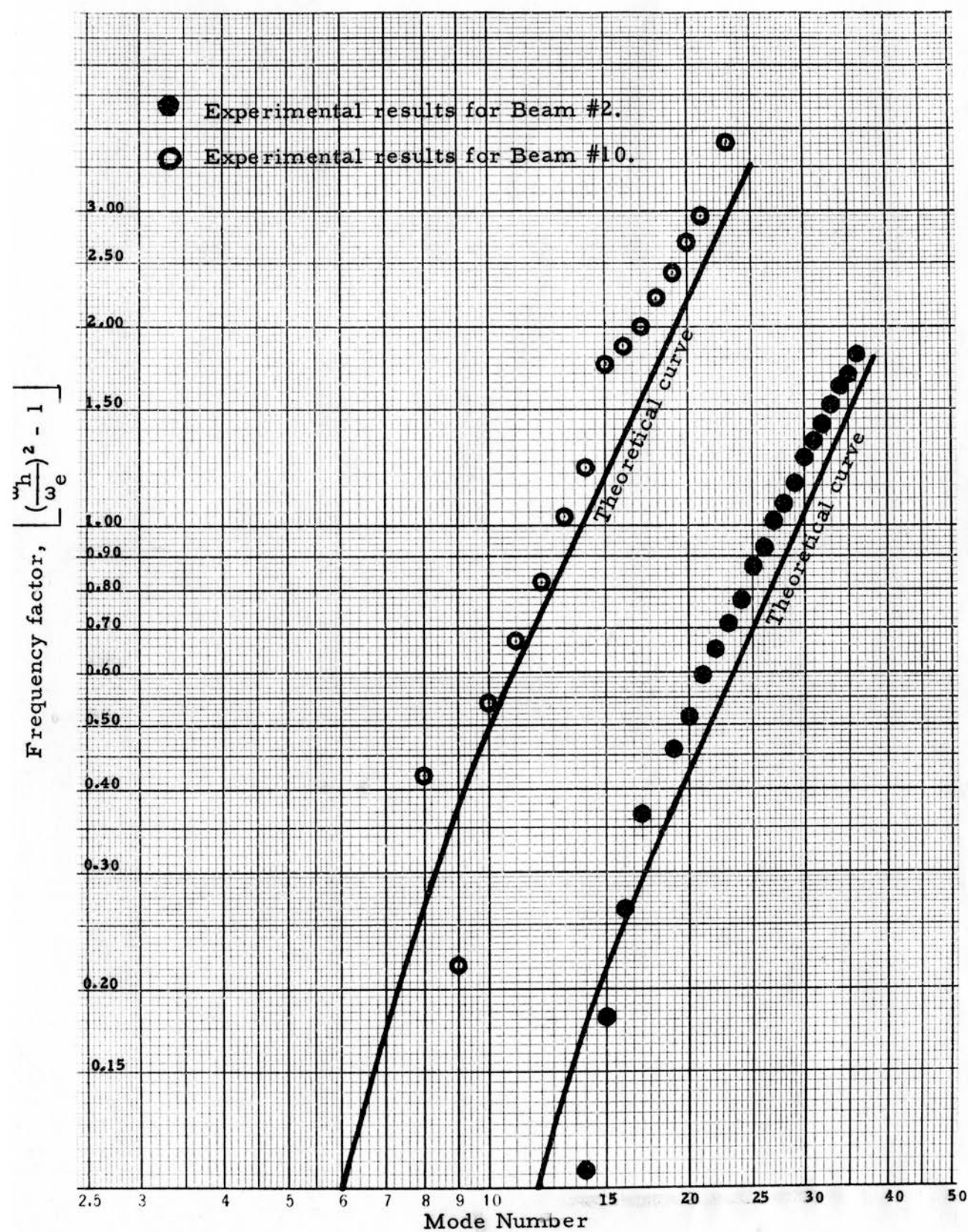
Fig. 3a.  Theoretical curves and experimental results
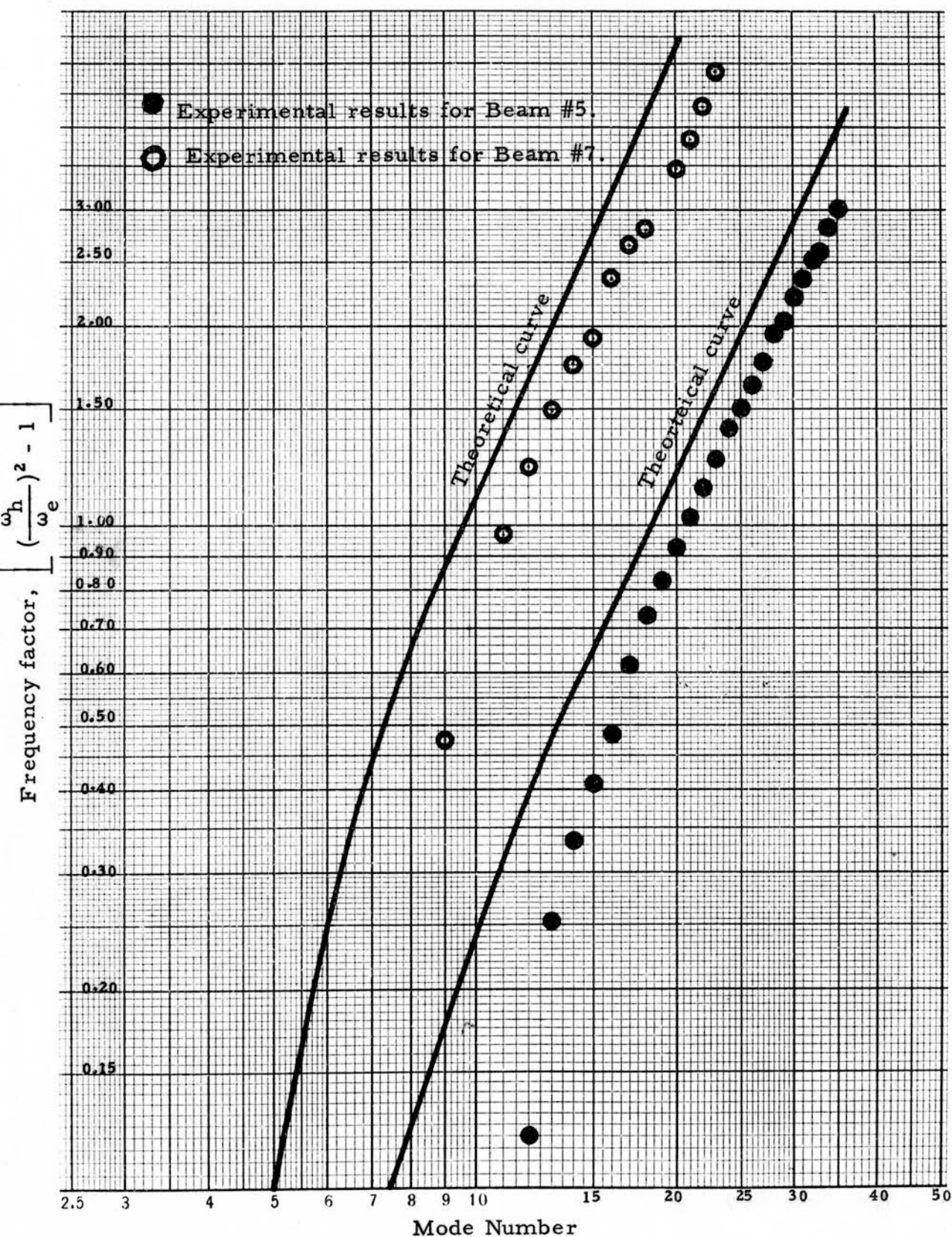of hinged beams Nos. 2 and 10.

Fig. 3b. Theoretical curves and experimental results
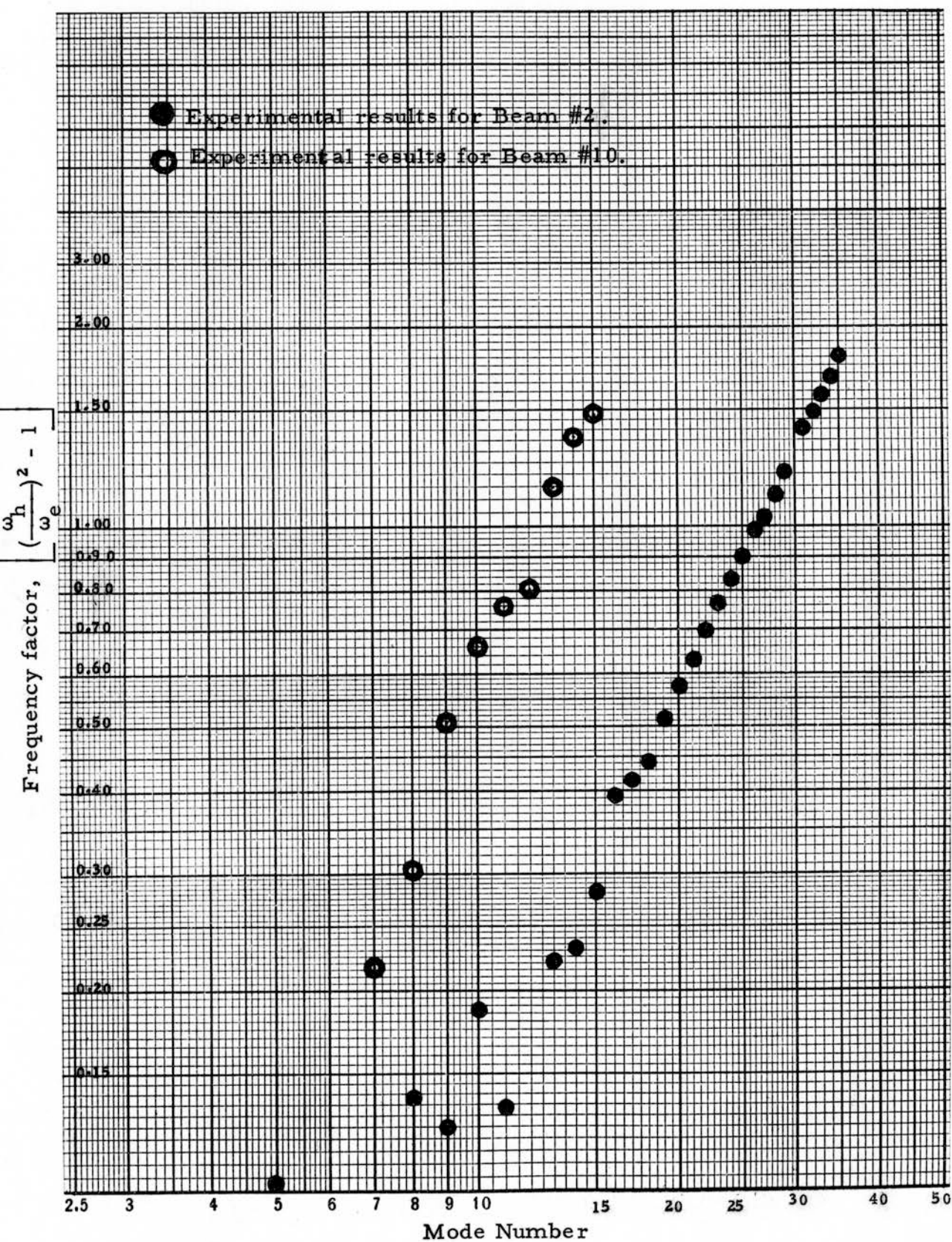of hinged beams Nos. 5 and 7.

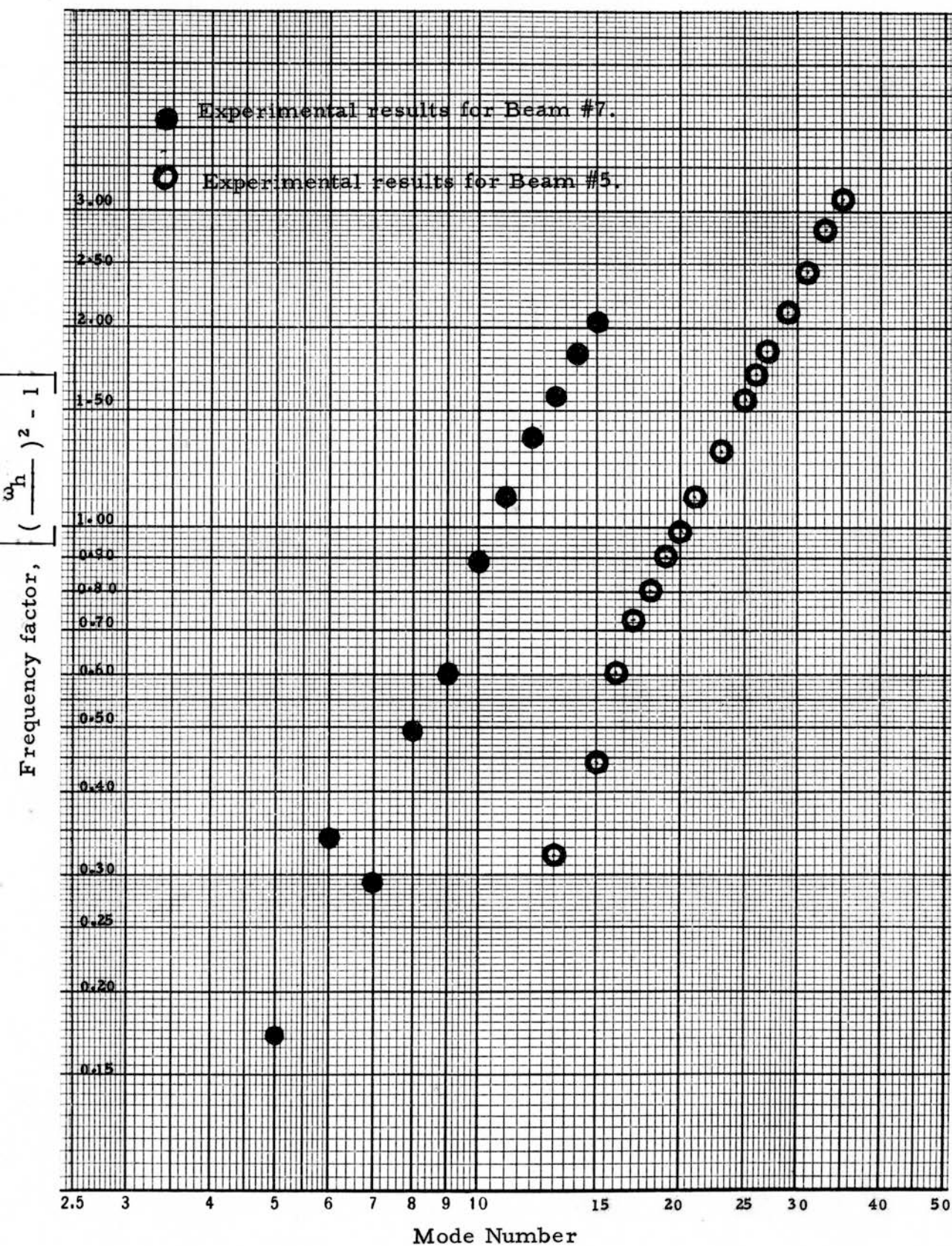Fig. 4a.  Experimental results of free-free beams Nos. 2 and 10.

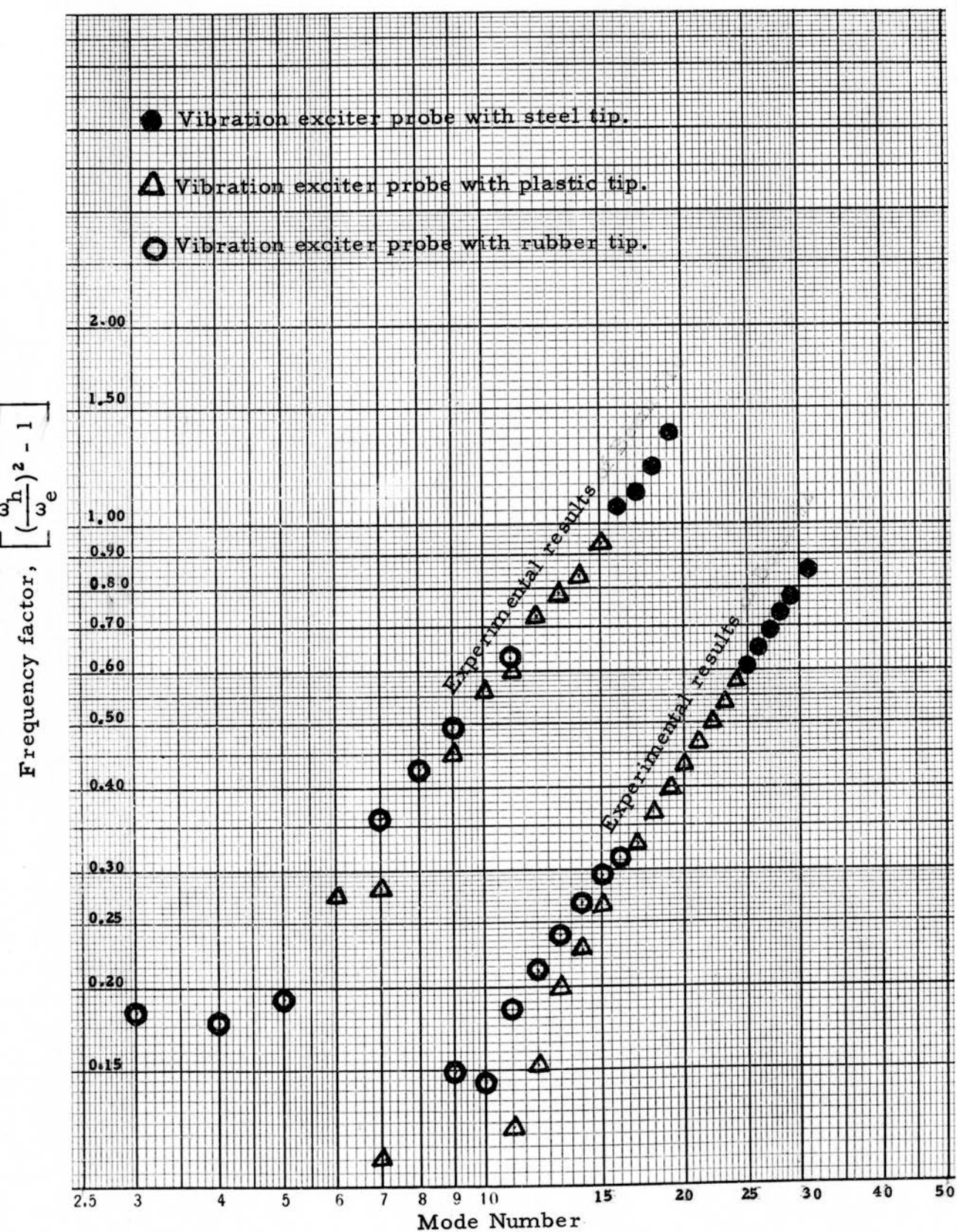Fig. 4b.  Experimental results of free-free beams Nos. 5 and 7.

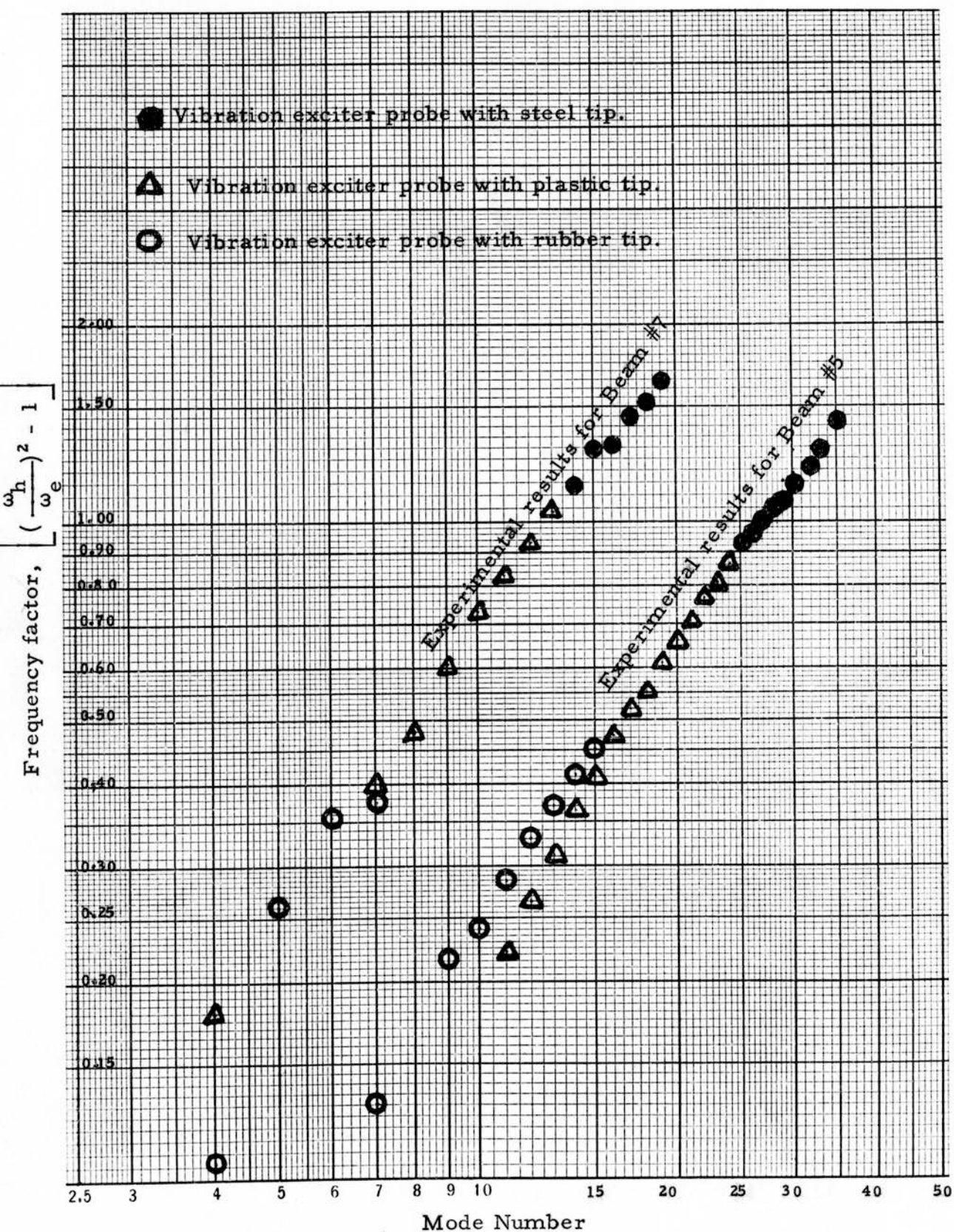Fig. 5a. Experimental results of clamped beams Nos. 2 and 10.

Fig. 5b. Experimental results of clamped beams Nos. 5 and 7.

the theoretical ones.

## Sample Calculations

I. To find the natural frequencies of the simply supported, homogeneous beam.

The natural frequency is obtained from the equation:

$$\omega_h = \frac{m^2 \pi^2}{2\pi} \sqrt{\frac{g\,E\,I_h}{\gamma\,\ell^4}} = c \sqrt{\frac{g\,E\,I_h}{\gamma\,\ell^4}} \qquad (g)$$

where

$$c = \frac{m^2 \pi^2}{2} = \frac{\pi}{2},\ \frac{4\pi}{2},\ \frac{9\pi}{2},\ \text{etc.}$$

$g = 386$ in. per sec.,

$E\,I_h = $ Flexural stiffness in lb-in.$^2$,

$\gamma = $ Beam weight in pounds per inch,

$\ell = $ Beam length in inches.

Considering beam No. 2 as a homogeneous beam, $\omega_h$ is determined by following substitutions in equation (g):

$E\,I_h = 36,475$ lb-in.$^2$ (determined experimentally),

$m = 15$, number of mode,

$\gamma = .02535$ lb per in.

$\ell = 120$ inches.

Therefore,

$$\omega_h = \frac{(15)^2 (\pi)^2}{2\pi} \sqrt{\frac{(386)(36,475)}{(.02535)(120)^4}}$$

and

$$\omega_h = \underline{\underline{578.419}}\ \text{c.p.s.}$$

II. To determine theoretical natural frequencies of simply supported sandwich beam.

From equation (a), for Beam No. 2,

$$\omega_m^2 = \frac{m^4 \pi^4 E}{\ell^4 (1 - \nu^2) \frac{\gamma}{6g}} \left( \frac{I_T}{1 + m^2 s} + I_F \right)$$

where $I_T = \frac{f f'}{f + f'} \left( c + \frac{f + f'}{2} \right)^2$ and $\rho = \frac{\gamma}{6g}$

For $f = f'$,

$$I_T = \frac{f}{2} (c + f)^2 = \frac{.016}{2} (0.25 + 0.016)^2 = \underline{\underline{5.6605 \times 10^{-4}}},$$

$$I_F = \frac{f^3 + f'^3}{12} = \frac{f^3}{6} = \left( \frac{0.016)^3}{6} = \underline{\underline{0.682666 \times 10^{-6}}}$$

$E = 10.3 \times 10^6$ p.s.i. ; $\nu = 0.33$

$\gamma = 0.02535$ lb per in. ; $\ell = 120$ in.,

and $s = \frac{\pi^2 c f f'}{\ell^2 (f + f')} \frac{E}{G_{xz} (1 - \nu^2)} = \frac{\pi^2 c f}{2 \ell^2} \frac{E}{G_{xz} (1 - \nu^2)}$ where

$G_{xz}$ is given by the manufacturer

$$= \frac{\pi^2 (0.25)(0.016)}{2 (120)^2} \frac{(10.3 \times 10^6)}{(11,700)(0.8911)} = \underline{\underline{1.35423 \times 10^{-3}}}.$$

On substitution of above values, and on further simplification

$$\omega_m^2 = \frac{(15)^4 \pi^4 (10.3 \times 10^6)(6)(386)}{(120)^4 (.8911)(0.02535)} \left[ \frac{5.6605 \times 10^{-4}}{1 + (15)^2 (1.35 \times 10^{-3})} + .6821 \times 10^{-6} \right].$$

Therefore,

$$\omega_m = \underline{\underline{525.835}} \text{ c.p.s.}$$

III. $\quad (\frac{\omega_h}{\omega_m})^2 = (\frac{578.419}{521.835})^2 = (1.1)^2$

$$= \underline{\underline{1.21}},$$

similarly,

$$(\frac{\omega_h}{\omega_e})^2 = (\frac{578.419}{532})^2 = (1.0873)^2$$

$$= \underline{\underline{1.1821}}$$

IV. Per cent deviation of experimental values of natural frequencies from the theoretical ones.

$$\text{Per cent deviation} = (\frac{\omega_e - \omega_m}{\omega_e})(100)$$

$$= (\frac{532 - 525.835}{532}) = \underline{\underline{1.158}} \text{ per cent}$$

V. To find the value of s and $G_{xz}$ for Beam No. 2.

Rewriting equation (f),

$$\log \left[ (\frac{\omega_h}{\omega_e})^2 - 1 \right] = 2 \log m + \log s. \tag{f}$$

For any mode number, say m = 20, it is seen from the curve for Beam No. 2 on Fig. 3a, that

$$\left[ (\frac{\omega_h}{\omega_e})^2 - 1 \right] = \underline{\underline{0.55}},$$

on substitution in (f)

$$\log (0.55) = 2 \log (20) + \log s,$$

$$\overline{1}.7404 = 2(1.3060) + \log s.$$

Therefore,

$$\log s = \overline{3}.1373 \text{ and } s = \underline{\underline{1.372 \times 10^{-3}}}$$

now, since $f = f'$

$$s = \frac{\pi^2 c f}{2 \ell^2} \frac{E}{G_{xz}(1 - \nu^2)} \; ,$$

therefore,

$$G_{xz} = \frac{\pi^2 c f}{2 \ell^2} \frac{E}{s (1 - \nu^2)}$$

$$= \frac{(\pi^2)(.25)(.016)}{2(14,400)} \frac{(10.3 \times 10^6)}{(1.372 \times 10^{-3})(.8911)}$$

$$= \underline{\underline{11,540}}$$

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## DISCUSSION OF TEST RESULTS

Graphical interpretation of the tests and calculated results are shown in Figs. 3 to 5 inclusive, for three different end conditions. Examination of the percentage deviation of the theoretical values from the observed values (in cols. 5 and 10 in table 1 (a,b), and Figs. 3(a,b), shows that the agreement between calculated and observed values is reasonable. A maximum deviation of 15 per cent was found at one calculated place; the majority of the results agreed within 10 per cent.

The deviation of the calculated results from observed ones can be explained if it is considered that the natural frequencies of given simply supported beams, of constant dimensions and physical properties depend on the value of the core parameter s, which in turn is a function of $G_{xz}$ (modulus of rigidity in x-z plane). In other words, the accuracy of s depends upon the accuracy with which the value of $G_{xz}$ is determined.

The determination of the value of $G_{xz}$ required a relatively complicated test set-up, and in view of the lack of time to perform such a test, the values of $G_{xz}$ which were reported by the manufacturer were used in calculating the natural frequencies at various modes. The theoretical curves in Figs. 3a and 3b were plotted from these values.

Therefore, no estimate of the accuracy of the values of s, and hence $\omega_m$, can be given. However, the theoretical and experimental curves are both asymptotic to a straight line with a slope 2 and an intercept log s. In view of this fact, straight lines are drawn through the plotted experimental results and the intercepts log s, and hence the values of $G_{xz}$ are

determined and listed in Table 4. These values are lower in case of strong core panels, and higher in weak core panels than the respective values reported by the manufacturer.

Smaller and larger values (because of different $G_{xz}$) of intercept log's, caused the shifting up and down of the theoretical curve of the weak and strong core beams, respectively. This implies that $G_{xz}$ can be determined by vibration tests without destroying the panels, without having a complicated experimental set-up, and perhaps with better accuracy.

Further examination of the percentage deviations in Tables 1(a, b) shows that the deviations between the calculated and observed values are mostly greater in case of beams with a weak core than those in beams with a strong core. These discrepancies are also due to higher differences in the values (between those reported by manufacturer and those determined from vibration tests) of $G_{xz}$ in case of weak core beams than those in strong core beams.

It is difficult to get beyond 35 modes in case of thin beams, partly because at higher modes the limiting factor is the force generating capacity of the vibration exciter, that is, the acceleration at such high frequencies is high, resulting in extremely small amplitudes to which the beam does not respond, and partly because of the fact that the Chladni pattern becomes smaller than the length between the other two free edges. Vibrations of this type are not accounted for by the theory.

Similarly, modes beyond 23 cannot be obtained for thick beams because their relatively higher stiffness requires higher frequencies for

resonance and the acceleration reached the maximum limit of the vibrator.

## CONCLUSIONS

Conclusions derived from the limited number of tests performed are as follows:

The theory for natural frequencies of vibration of sandwich panels, simply supported on two opposite ends is verified.

The experiments indicate that theoretical formulas, similar to those of the Raville-Kimel theory for simply supported ends, exist for the free-free and clamped end sandwich panels.

Assuming that the theory is valid, the testing procedure can be used as a non-destructive method for the determination $G_{xz}$. When used this way, it yields values for $G_{xz}$ which are consistent with those obtained by direct static methods.

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# NOMENCLATURE

| | |
|---|---|
| x, y, z | Rectangular co-ordinates (Fig. 1) |
| t | time co-ordinate |
| a | length of panel in direction of loading, (Fig. 1) |
| c | thickness of core, (Fig. 1) |
| $f, f'$ | thickness of right and left facings, respectively, (Fig. 1) |
| E | Modulus of Elasticity of facings |
| $\nu$ | Poisson's ratio of facings |
| $E_c$ | Modulus of Elasticity of core |
| $G_{xz}$ | Modulus of Rigidity of core in xz plane |
| $u_c \, w_c$ | displacements of core in x and z directions, respectively |
| w | displacement in z direction of any point in sandwich |
| $\epsilon_{zc}, \gamma_{xzc}$ | normal and shear strains, respectively in core |
| $\epsilon_{xM}, \epsilon'_{xM}$ | membrane strains in right and left facings, respectively |
| $\epsilon_{xB}, \epsilon'_{xB}$ | bending strains in right and left facings, respectively |
| $\sigma_z, \tau_{xz}$ | normal and shear stress, respectively, in core (Fig. 1) |
| $V_c$ | elastic energy per unit width of core |
| $V_{MF}, V'_{MF}$ | elastic energy per unit width associated with membrane strains in right and left facings, respectively |
| $V_{BF}, V'_{BF}$ | elastic energy per unit width associated with bending strains in right and left facings, respectively |
| V | total elastic energy per unit width of panel |
| T | kinetic energy per unit width of panel |

$\rho$   mass density of composite panel per unit length and width

$\delta$   mass density of core per unit volume

$\omega_m$   theoretical natural frequency of vibration of panel, radians per unit time

$\omega_e$   experimental (observed) natural frequency of vibration of panel radians per unit time

$\omega_h$   natural frequency of vibration of equivalent homogeneous beam, radians per unit time

$m$   integer

$A_m, B_m, C_m, F_m$   configuration parameters

$I_F$   $\dfrac{f^3 + (f')^3}{12}$

$I_T$   $\dfrac{f \, f'}{f + f'} \left(c + \dfrac{f + f'}{2}\right)^2$

$\gamma$   weight of the beam, pounds per inch

$s$   $\dfrac{\pi^2 \, c \, f \, f'}{a^2(f + f')} \quad \dfrac{E}{G_{xz}(1 - \nu^2)}$

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APPENDICES

## APPENDICES

To find the nodes and antinodes of a beam:

Let

$$X = c_1 \sin \beta x + c_2 \cos \beta x + c_3 \sin \beta x + c_4 \cosh \beta x \qquad (1)$$

where $\beta = \sqrt[4]{\dfrac{\gamma \omega^2}{EIg}}$ . ( (2) p. 325)

Equation (1) represents the deflection curve of a beam performing normal mode vibration.

The constants $c_1$, $c_2$, $c_3$ and $c_4$ can be determined from end conditions.

1. Beam with free-free ends:

On substitution of the end conditions, equation (1) reduces to the following:

$$X = \cosh \beta x - \sinh \beta x + \cos \beta x. \qquad (2)$$

(a) At nodal positions, $X = 0$; therefore,

$$\cosh \beta x - \sinh \beta x - \sin \beta x + \cos \beta x = 0. \qquad (3)$$

For even modes, the nodal position is always located at $x = 0.5$ for a beam of unit length, where $x$ is the distance of the node from either of the ends.

For 6th mode,

Since, $\beta = 20.42033$ and $x = 0.5$ ( (2) p. 336)

$\beta x = 10.21016$ radians.

Therefore, $x = \dfrac{10.21016}{29.845} = \underline{\underline{0.342}}$

Table 5 presents the nodal positions for various modes.

**(b)** To find antinodal positions or positions of maximum amplitude
for beams with free ends (for use in placing the vibration exciter).

Differentiate (2) with respect to x, gives

$$\frac{dX}{dx} = \beta (\sinh \beta x - \cosh \beta x - \sin \beta x - \cos \beta x). \qquad (4)$$

At antinode, $\frac{dX}{dx} = 0$, therefore, (4) becomes

$$\sinh \beta x - \cosh \beta x - \sin \beta x - \cos \beta x = 0. \qquad (5)$$

In the case of odd numbered modes, antinode always occurs
at x = 0.5, therefore, for 9th mode, since $\beta = 29.845$ and x = 0.5
then $\beta x = 14.923$.

So, for 16th mode, since $\beta = 51.836$ and $\beta x = 14.923$ from which

$$x = \frac{14.923}{51.836} = \underline{.2885}$$

2. **Beam with clamped ends:**

In this case, only antinodal positions are needed to place
the vibration exciter and they are the same as free-free end beam.

3. **Beam with simply supported ends:**

In this case, also, only antinodal positions are needed to
place the vibration exciter. On substituting the end conditions,
equation (1) reduces to $X = D \sin \beta x.$ $\qquad (6)$

Differentiating (6) gives

$$\frac{dX}{dx} = D \beta \cos \beta x. \qquad (7)$$

For antinodal positions, therefore,

$$\cos \beta x = 0. \qquad (8)$$

Since, the frequency equation in this case is

$$\sin \beta \ell = 0 \qquad (9)$$

The values of $\beta$ for unit length are

$$\pi, \ 2\pi, \ 3\pi, \ 4\pi, \ \text{etc.}$$

The antinode always occurs at $x = 0.5$ for odd numbered nodes.

For 4th mode, since $\beta = 4\pi$ therefore $x = \dfrac{3}{8}$ to satisfy equation (8).

Table 5. Nodal and antinodal positions on the beam of unit length.

| No. of modes | For free-free ends beam | | Clamped ends beam position for vibration exciter | Hinged ends beam position for vibration exciter |
|---|---|---|---|---|
| | nodal position for strings | antinodal position for vibration exciter | | |
| 1 | 0.244 | 0.5 | 0.5 | 0.5 |
| 2 | .132 | .308 | .308 | .25 |
| 3 | .356 | .5 | .5 | .5 |
| 4 | .277 | .389 | .389 | .375 |
| 5 | .227 | .5 | .5 | .5 |
| 6 | .192 | .423 | .423 | .5833 |
| 7 | .3 | .5 | .5 | .5 |
| 8 | .265 | .441 | .441 | .5625 |
| 9 | .342 | .5 | .5 | .5 |
| 10 | .309 | .454 | .454 | .45 |
| 11 | .283 | .5 | .5 | .5 |
| 12 | .26 | .321 | .381 | .5416 |
| 13 | .315 | .5 | .5 | .5 |
| 14 | .293 | .466 | .466 | .5357 |
| 15 | .274 | .5 | .5 | .5 |
| 16 | .318 | .41 | .41 | .5313 |
| 17 | .30 | .5 | .5 | .5 |
| 18 | .284 | .474 | .474 | .5278 |
| 19 | .269 | .5 | .5 | .5 |
| 20 | .256 | .428 | .428 | .525 |
| 21 | .245 | .5 | .5 | .5 |
| 22 | .278 | .433 | .433 | .5227 |
| 23 | .266 | .50 | .50 | .5 |
| 24 | .245 | .481 | .481 | .5218 |
| 25 | .284 | .5 | .50 | .5 |
| 26 | .274 | .444 | .444 | .5193 |
| 27 | .264 | .50 | .50 | .5 |
| 28 | .255 | .413 | .413 | .5179 |
| 29 | .280 | .50 | .50 | .5 |
| 30 | .271 | .484 | .484 | .5167 |
| 31 | .262 | .50 | .50 | .5 |
| 32 | .254 | .454 | .454 | .5156 |
| 33 | .276 | .500 | .500 | .5 |
| 34 | .268 | .428 | .428 | .5147 |
| 35 | .260 | .50 | .50 | .5 |

# NATURAL FREQUENCIES OF VIBRATION
## OF SANDWICH PANELS

by

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Much work of both theoretical and experimental nature has been done in the field of statics of sandwich structures, but, to the knowledge of the author, very little work has been done previously in the dynamics field.

In view of these facts, Raville and Kimel presented a theoretical analysis for calculating the natural frequencies of vibration of sandwich panels, simply supported at the ends and constructed with isotropic facings and orthotropic cores.

This thesis presents an experimental verification of the theory and reports the development of vibration techniques for future investigations in this field.

Vibration tests were conducted for three end conditions,

1.  Free-free,

2.  Simply supported, and

3.  Fixed-fixed.

In all cases the natural frequencies $\omega_m$, derived from the Raville-Kimel theory where applicable, $\omega_h$ derived from the homogeneous beam theory, and $\omega_c$, obtained experimentally, were determined. The Raville-Kimel theory predicts that certain functions of these quantities should be asymptotic to straight lines when plotted against the mode number on log-log paper. The slope of the straight line should be two, and the intercept determined by the value of $G_{xz}$, the shear modulus of the core.

The experimental data shows this character, and the value of $G_{xz}$ obtained from the intercept method is approximately that obtained from

static tests as reported by the manufacturer of the sandwich material tested.

It is concluded that the Raville-Kimel theory is verified, and that it can be used in combination with the vibration tests described here as a non-destructive method for determination of $G_{xz}$.

Although there is no theory at present for the free-free and fixed-fixed end conditions, analysis of the data shows that theoretical results similar to that for the simply supported beams should exist.