ANALYSIS ON AN ONLINE PLACEMENT EXAM FOR COLLEGE ALGEBRA

by

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A REPORT

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Abstract

An online placement exam was administered to 2800 entering freshmen, 700 of whom enrolled in College Algebra during the succeeding Fall semester. Problems on the placement exam were clustered using several different techniques including both expert analysis and item response theory. Student scores on these groupings of problems were then compared to their scores on the first two hour exams in the course (representing the first half of the material in the course) and also on ACT data. Based on this comparison, certain problems were selected as more or less informative for purposes of placement. A model was created using previously available ACT data along with the new placement data to predict initial student success in the course. This model explains 50% more of the variance in student scores than the previously available ACT data alone. Suggestions for improvements to the test and the placement methodology are made based on our analysis

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Acknowledgements

I would like to thank my committee, Professors Bennett, Muenzenberger, and Volok for their help in carrying out this project, and I would also like to thank the students at Kansas State University.

Finally, I could not have completed this thesis without the love, help, and support of my husband, Xuehua Hu.

CHAPTER 1 - Background

College Algebra is a required course for almost all students at Kansas State University. Prior to recent course revisions, the C or better rate has historically ranged between 55% and 66% depending on the semester. This is above the national average for College Algebra, but is low compared to most other freshmen courses at KSU. During the process of revising the course, questions were raised about whether students were being properly placed into College Algebra (as well as other mathematics courses).

Historically, placement of students into mathematics classes was based on ACT scores and high-school transcripts. The ACT's website reports:

> The ACT, or American College Testing, is a standardized collegiate examination. In use since 1959, it is commonly used as an indicator of academic aptitude and readiness to enter college. Although the ACT is not as well-known as the SAT, it is almost as widespread; as of 2008, nearly all four-year colleges and universities in the United States accept the ACT, although every school factors the results into admission decisions differently.
>
> The test itself consists of four subject multiple-choice examinations; the exams cover English, mathematics, science and reading. A fifth exam, an essay writing test, was added in 2005, though not all schools require the essay portion of the test. Textbook reviews and national surveys of teachers and other educators are also used in determining exam content.
>
> The ACT is given only at set time periods during the year, generally four to six times per year, depending on the state where the testing takes place. The amount of time allotted for the standard exam is roughly 3.5 hours, which generally includes two 15-minute breaks. Students that take the ACT Plus Writing exam, which includes the essay writing portion, are allotted just over four hours for testing. Students who take the ACT can send their score reports to up to four different colleges or universities.

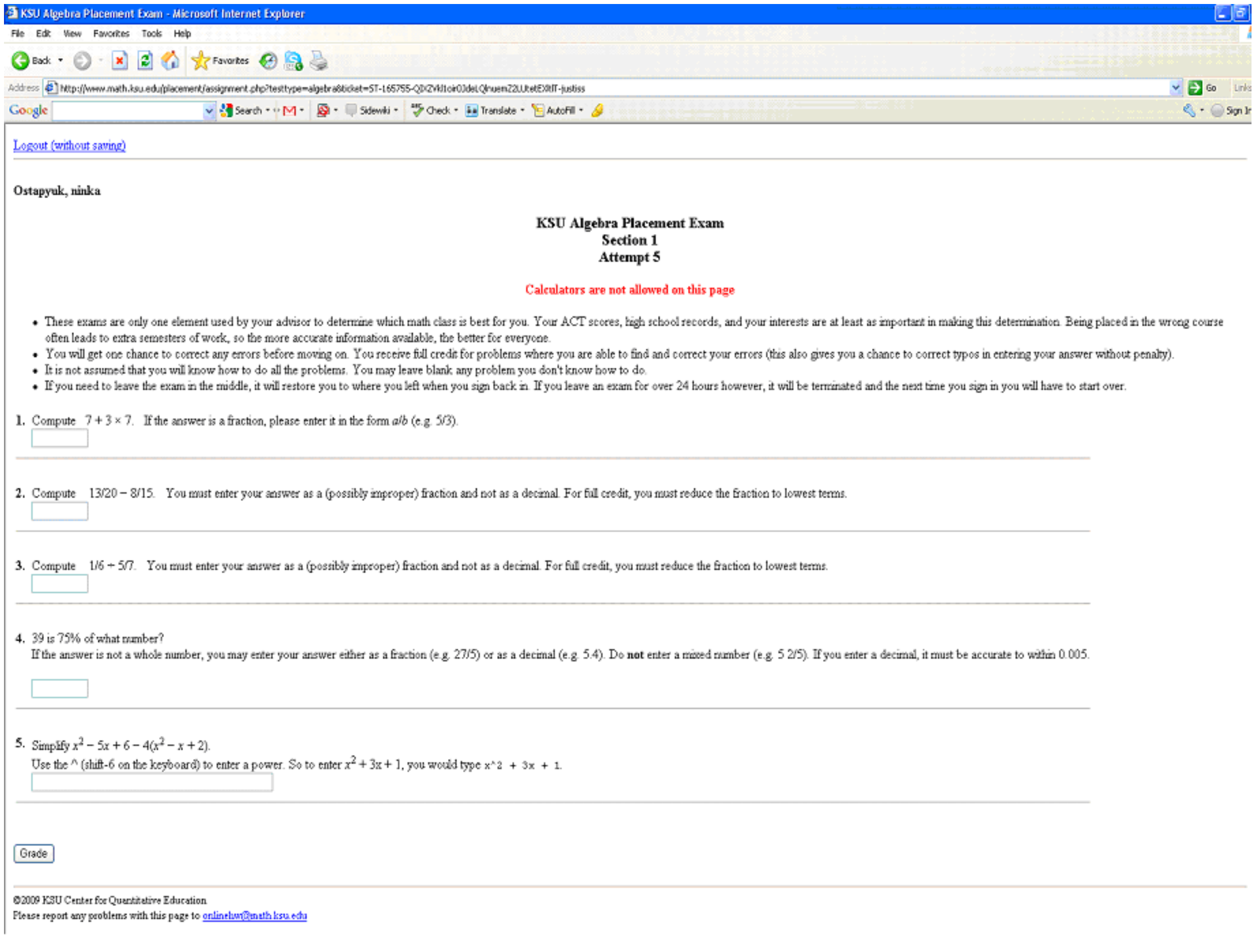
For most students, the ACT exam was taken either at the end of the Junior year or beginning of the Senior year of high school, and so reflected their mathematical understanding a year or more before the beginning of college. This raised issues about how accurate it could be. During this time students could improve their math skills (or make them worse), increase or decrease their volume of appropriate knowledge and generally change their attitude toward mathematics. Secondly, the ACT is a standardized test and doesn't exactly match our specific courses. High-school transcripts list names of courses taken, but a variety of different topics might be covered at different schools under the same name. In any event, anecdotal information from instructors suggested placement was a problem.

To improve our ability to properly place students in college algebra the mathematics department decided to improve placement by creating our own placement exam. In discussions with New Student Services about how to offer the exam to all students, the department was informed that there was insufficient free time available during registration to require students to take an exam on campus. Therefore, it was decided to offer the exam online so students could take it at home prior to their arrival for registration in June. The exam was built on the framework of the department's current online homework system. As such, most problems were not multiple-choice but required students to type in numbers or formulas. Students were given one chance to fix errors in problems they missed, allowing them to correct simple computational or typographical errors. The system randomly generates different but similar problems for each student, so students could try multiple times if they felt their initial score was not reflective of their ability. Since the exam covered a wide range of material from basic algebra through calculus, it was split into two sections: Algebra and Calculus (with Trigonometry included in the Calculus exam). Students were on their honor not to cheat on the exam. Since they were taking it at home, we have no way to proctor. However, they are also warned that, "Being place in the wrong course often leads to extra semesters of work, so the more accurate information available, the better for everyone." In other words, if they cheat they only hurt themselves.

Sample Algebra Placement Exam

Since each student gets a different, randomly generated exam, we can only show one example to indicate the type of problems asked. The specific values in the questions vary for each student.

Figure 1-1Sample first section of algebra placement exam



Students are given one possibility to correct their typos and mistakes. Incorrect answers are highlighted with red message as illustrated in Figure 1-2.

Figure 1-2 Indication of incorrect answers

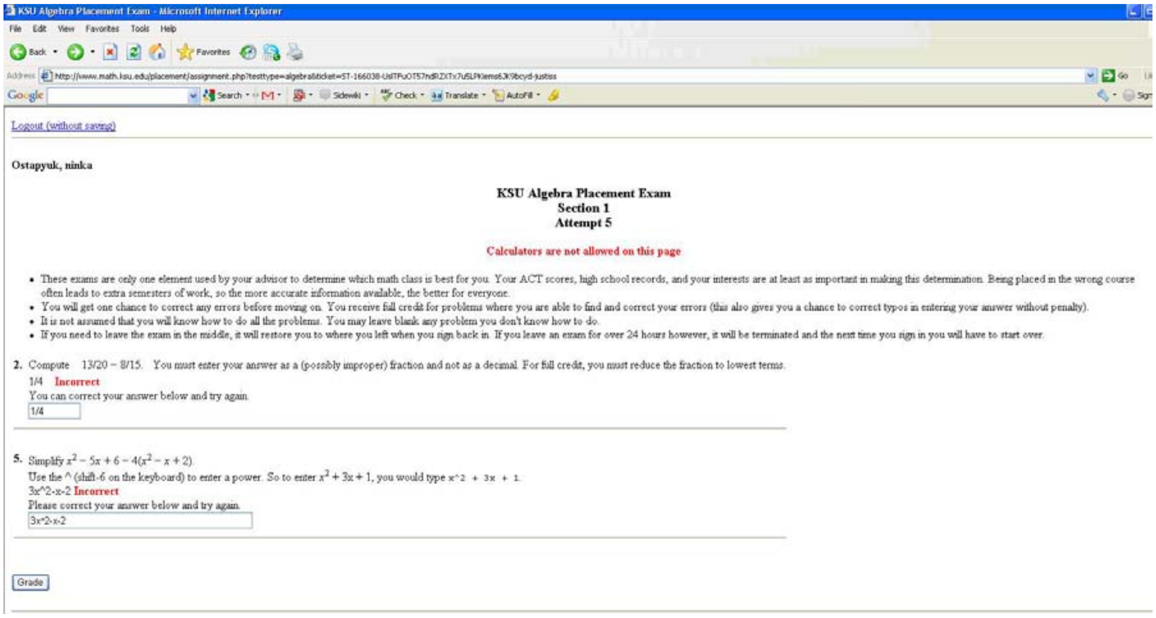


Figure 1-3 Sample second section of algebra exam

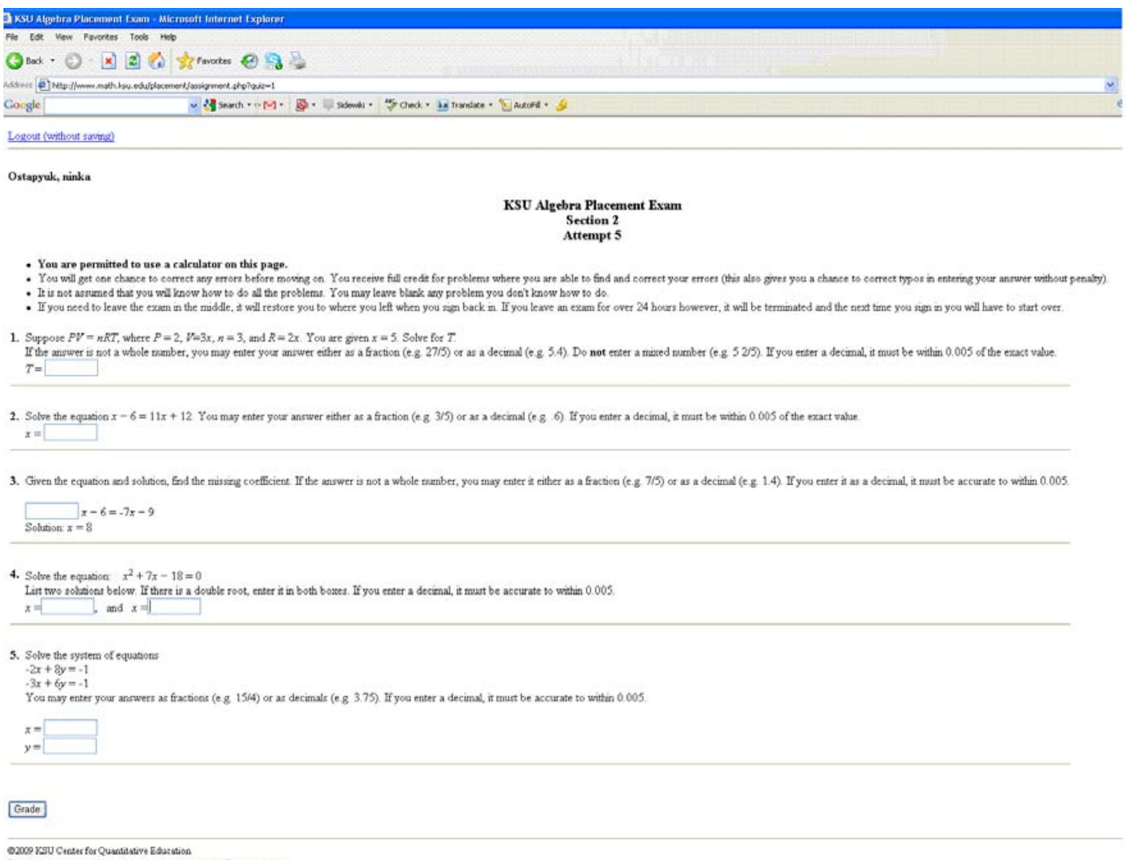
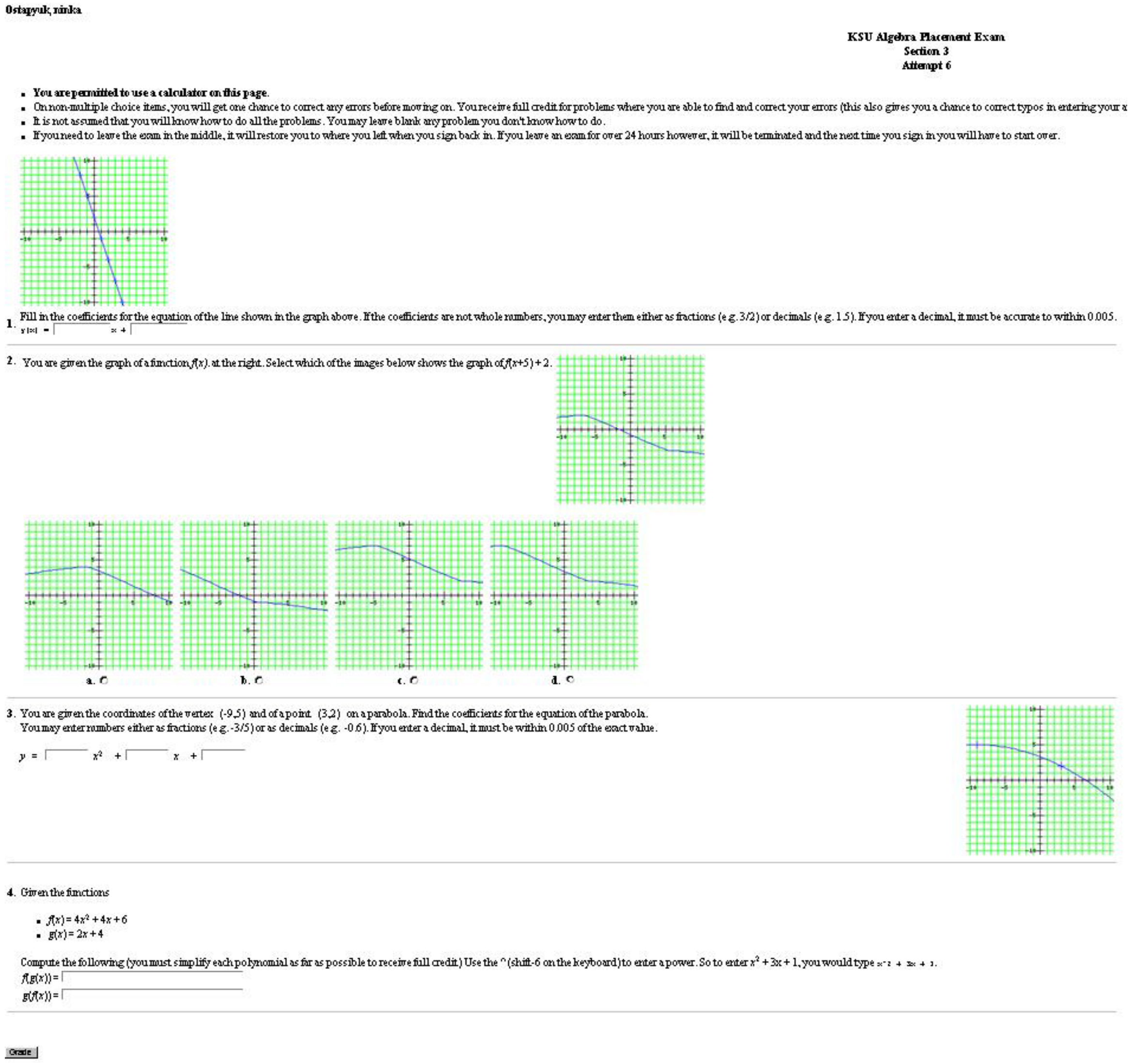


Figure 1-4 Sample third section of algebra exam



Ostapyuk, ninka

KSU Algebra Placement Exam
Section 3
Attempt 6

- **You are permitted to use a calculator on this page.**
- On non-multiple choice items, you will get one chance to correct any errors before moving on. You receive full credit for problems where you are able to find and correct your errors (this also gives you a chance to correct typos in entering your a
- It is not assumed that you will know how to do all the problems. You may leave blank any problem you don't know how to do.
- If you need to leave the exam in the middle, it will restore you to where you left when you sign back in. If you leave an exam for over 24 hours however, it will be terminated and the next time you sign in you will have to start over.

1. Fill in the coefficients for the equation of the line shown in the graph above. If the coefficients are not whole numbers, you may enter them either as fractions (e.g. 3/2) or decimals (e.g. 1.5). If you enter a decimal, it must be accurate to within 0.005.
$y(x) =$ [ ] $x +$ [ ]

2. You are given the graph of a function $f(x)$. at the right. Select which of the images below shows the graph of $f(x+5)+2$.

a. ○ b. ○ c. ○ d. ○

3. You are given the coordinates of the vertex (-9,5) and of a point (3,2) on a parabola. Find the coefficients for the equation of the parabola.
You may enter numbers either as fractions (e.g. -3/5) or as decimals (e.g. -0.6). If you enter a decimal, it must be within 0.005 of the exact value.

$y =$ [ ] x^2 + [ ] x + [ ]

4. Given the functions

- $f(x) = 4x^2 + 4x + 6$
- $g(x) = 2x + 4$

Compute the following (you must simplify each polynomial as far as possible to receive full credit) Use the ^ (shift-6 on the keyboard) to enter a power. So to enter $x^2 + 3x + 1$, you would type x^2 + 3x + 1.
$f(g(x)) =$ [ ]
$g(f(x)) =$ [ ]

Grade

Figure 1-5 Sample fourth page of algebra exam

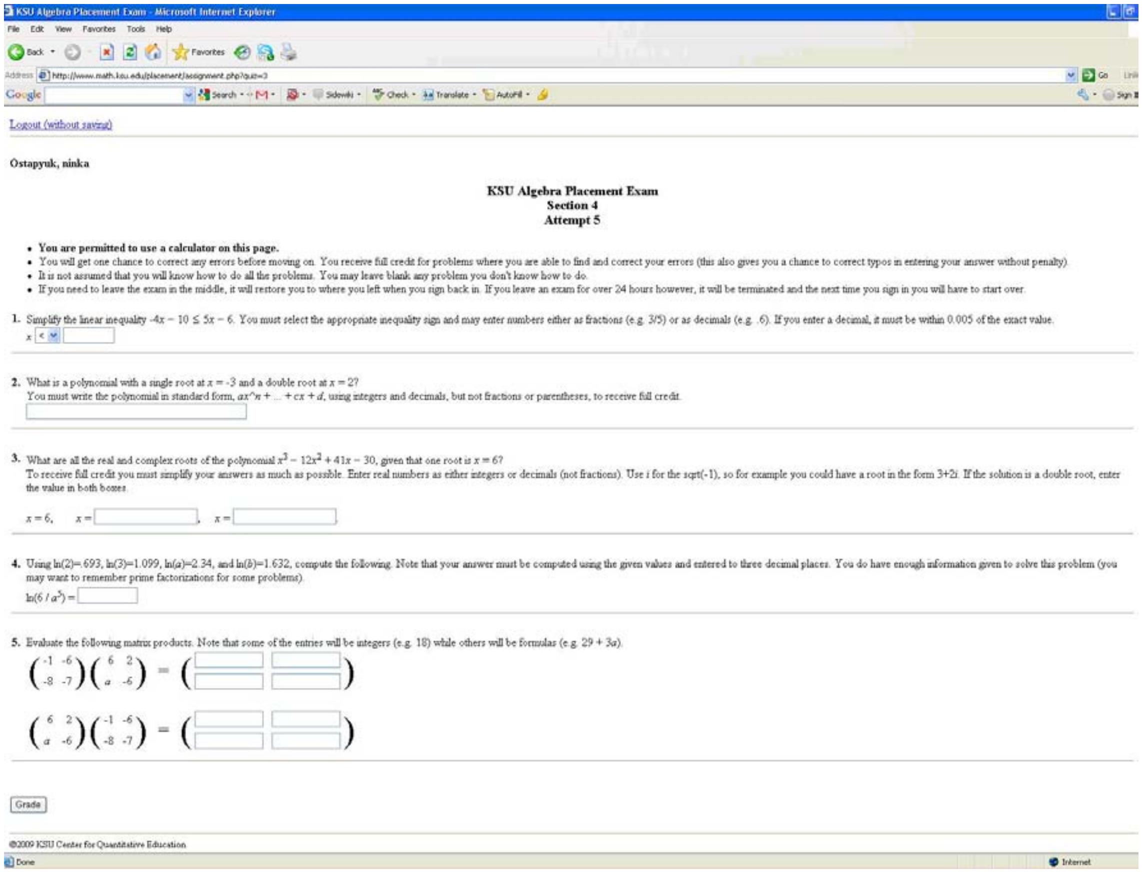
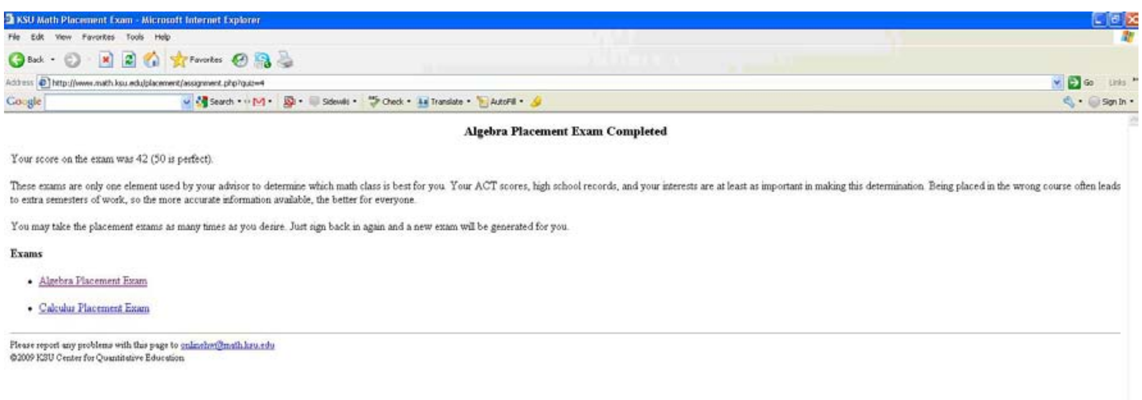


Figure 1-6 Sample page of earned grade



Scores are reported out of 50 points possible, with the minimum possible score being 10. This was chosen for several reasons. The exam as written had 40 points possible. However, in transferring scores to the iSIS system where advisors could find and use them, it was discovered that iSIS does not allow 0 as a possible value for a placement exam. This is in line with the usual practice of such exams, for example the minimum possible score on the ACT is 15. By adding 10 points to all scores, we avoided this issue. And by making both the Algebra and the Calculus exams set to 50 points possible, we ended with an even 100 points possible, which was satisfying to the advisors and students.

Research Questions

Having given this exam, we are now faced with the issue of properly interpreting the scores, with a goal of eventually improving placement of students. To make this process manageable, we will focus just on placement into College Algebra in this thesis. This thesis will consider the following two research questions:

- Will this online exam provide more information about student abilities and help us to improve our placement? Should the exam be continued or edited?
- How should we advise students, based on our results?

The first question is whether giving the exam actually provides any benefit. It is not enough that we show performance on the placement exam is correlated to performance in class, we need to consider whether this gives us additional information beyond that available previously. If not, the exam should not be continued. A sub-question of this is whether all parts of the placement exam are useful. It is possible that certain topics and questions provide useful information while others do not. In this case the exam may need to be edited to emphasize the useful portions.

The second question is, assuming the exam does provide more information, how should that information be used. The simplest version of this would be to suggest what minimum score should be required for students to be placed into College Algebra. However, depending on the amount of information we gain (or, in other words, the strength of the correlation between performance on the placement exam and performance in the class), it may be more appropriate to provide an estimate of success and allow the student and advisor to make individual decisions using this estimate.

CHAPTER 2 - Analysis of Placement Exam

In the spring of 2009, all entering students were asked to take an online placement exam covering algebra prior to enrollment in June. Students who wished to take Math 220 or higher were also asked to take a separate online calculus placement exam. On these exams, problems were randomly generated and each student gets different exam, with the same types of problems, but different numbers.

The Algebra placement exam consisted of 19 problems, divided in to 4 sections which appeared as different pages in the online exam. The third section had 4 problems and every other has 5. The first page of the algebra placement exam consisted of mostly computational problems, which were to be solved without using a calculator. The second page had different equations to be solved. The third page was mostly dedicated to graphs, and last page had an assortment of problems about inequalities, polynomials, logarithms and matrices. Pages 1 and 2 dealt mostly with topics from Intermediate Algebra while pages 3 and 4 addressed topics from College Algebra.

The Calculus placement exam consisted of 18 problems, divided in to 4 sections (pages). The first and second pages had 4 problems each and last two had 5 problems each. The first and second pages were dedicated to trigonometry. The third page consisted of limits problems, and last page was about calculating derivatives and integrals of functions.

During both exams students had the possibility to edit and resubmit once any problem, which was solved incorrectly at first attempt. Calculators were allowed on all pages after the first page of the Algebra exam. After the exam, students had the option to go back and generate a new exam and try again. In total, 2792 students took the Algebra exam and 528 took the Calculus exam. Because the exams were available to anyone with KSU eID and WID, some people took the exam who were not students, for example advisors, curious about the exam contents.

To analyze results on the exam prior to students starting work in the fall, we used several different techniques to decide which sorts of problems addressed similar issues. In addition to a content analysis, two different data-mining techniques were used in an attempt to locate unexpected correlations between problems.

1) Pages were grouped by similar topic according to expert analysis.

2) Principal component analysis (PCA, also referred to as SVD for singular value decomposition, the matrix decomposition used to carry out PCA) was used to decide which problems should be grouped together

3) Item response curves were developed for each problem. The coefficients of these curves were plotted and clusters were detected by inspection as a third approach to grouping the problems. This approach is sometimes labeled IRT for Item Response Theory.

The content was carried out during the design of the exam and the breakdown of topics for each page are discussed above. We discuss the results of the data-mining approaches below:

PCA was applied both to page scores and problem scores. The results of the PCA on the individual problem scores carried out in the R statistical programming language are given below (the formatting has been changed slightly to fit on the page):

```
Call:
princomp(x = algprob)

Standard deviations:
   Comp.1    Comp.2    Comp.3    Comp.4    Comp.5    Comp.6    Comp.7    Comp.8
2.2206654 1.0330883 0.9995743 0.8955682 0.7150584 0.7009453 0.6951191 0.6498476
   Comp.9   Comp.10   Comp.11   Comp.12   Comp.13   Comp.14   Comp.15   Comp.16
0.6286761 0.6040348 0.5905315 0.5757066 0.5568504 0.5402935 0.5225764 0.5171451
  Comp.17   Comp.18   Comp.19
0.4747651 0.4677953 0.4410588

 19  variables and  2792 observations.

Loadings:
    Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9 Comp.10
p11                                                         -0.156
p12               -0.171                             0.137 -0.271
p13 -0.133        -0.193                             0.177 -0.357  0.173
p14         0.108 -0.152               -0.208        0.304 -0.482 -0.183
p15 -0.198  0.154 -0.280         0.783 -0.357 -0.159 -0.214  0.149
```

```
p21 -0.107        -0.188        -0.130
p22 -0.108        -0.193
p23 -0.169  0.189 -0.257        -0.568 -0.442 -0.220 -0.487  0.115
p24 -0.206  0.128 -0.264                0.125         0.355         0.180
p25 -0.251  0.172 -0.261                0.714        -0.478 -0.192
p31 -0.219        -0.192        -0.162                0.359  0.227 -0.146
p32 -0.152  0.350  0.447  0.803
p33 -0.123  0.192  0.183 -0.160               -0.160               -0.274
p34 -0.732 -0.640  0.208
p41 -0.188  0.106 -0.135                0.175         0.221  0.603 -0.259
p42 -0.144  0.231  0.251 -0.247               -0.163                0.116
p43 -0.178  0.294  0.281 -0.300               -0.354  0.150  0.103  0.556
p44 -0.140  0.202  0.208 -0.216               -0.247        -0.180 -0.629
p45 -0.186  0.280  0.170 -0.325        -0.251  0.814
    Comp.11 Comp.12 Comp.13 Comp.14 Comp.15 Comp.16 Comp.17 Comp.18 Comp.19
p11         -0.143                                  0.144   0.824   0.275   0.369
p12         -0.310  -0.246          -0.113  -0.102   0.260          -0.774
p13         -0.307  -0.489          -0.155  -0.228  -0.308           0.477
p14 -0.487           0.518                          -0.170
p15          0.111
p21  0.117          -0.181  -0.162   0.457   0.715  -0.247   0.234
p22  0.161                                   0.195   0.134  -0.905   0.142
p23                                         -0.168
p24  0.636   0.113   0.387   0.212          -0.148  -0.125   0.171
p25 -0.169   0.119
p31 -0.196   0.649  -0.397  -0.132                   0.163
p32
p33  0.227                  -0.619   0.420  -0.405
p34
p41 -0.297  -0.550   0.116
p42                  0.125  -0.417  -0.660   0.360
p43 -0.260                   0.279   0.292
p44  0.191          -0.205   0.512
p45

               Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9
SS loadings     1.000  1.000  1.000  1.000  1.000  1.000  1.000  1.000  1.000
Proportion Var  0.053  0.053  0.053  0.053  0.053  0.053  0.053  0.053  0.053
Cumulative Var  0.053  0.105  0.158  0.211  0.263  0.316  0.368  0.421  0.474
               Comp.10 Comp.11 Comp.12 Comp.13 Comp.14 Comp.15 Comp.16 Comp.17
SS loadings      1.000   1.000   1.000   1.000   1.000   1.000   1.000   1.000
Proportion Var   0.053   0.053   0.053   0.053   0.053   0.053   0.053   0.053
Cumulative Var   0.526   0.579   0.632   0.684   0.737   0.789   0.842   0.895
               Comp.18 Comp.19
SS loadings      1.000   1.000
Proportion Var   0.053   0.053
       Cumulative Var   0.947   1.000
```

The remaining 9 components all have small variance and so we omit their loadings. Analyzing the results, we find the primary component is, as expected, a slightly weighted overall average on the exam. However, the first page contributes relatively little to this component, suggesting that page is too easy and provides little information. After factoring out this first component of overall score, the second component contrasts results on page 3 problem 4 to an average of scores over many other problems. This suggests there is something special about the problem, which asks the students to compute the composition of a linear and a quadratic function. The problem was worth twice as much as other problems since it had two parts. Since the two parts were scored independently, there was also more opportunity for “partial credit” on this problem. We will return to this problem below when we discuss the item response curves.

The third component contrasts scores on the first two pages against scores on the last two pages. From the content analysis, this means it contrasts skills in Intermediate Algebra against those in College Algebra. However, there are two loadings out of place, in that problem 1 on pages 3 and 4 both match up with the Intermediate Algebra problems rather than the College Algebra problems. The problems cover graphing a line and linear inequalities. Graphing a line is covered in both Intermediate and College Algebra at KSU, while linear inequalities are only briefly mentioned in Intermediate Algebra at KSU, but apparently get more coverage in high school algebra. Because these two problems appear to be more geared toward Intermediate Algebra skills than College Algebra skills, it suggests the exam may be weighted more toward Intermediate Algebra than we intended. Added to the comments about the ease of the first page and it appears we might get better separation of students by making the exam somewhat harder.

	Comp.11	Comp.12	Comp.13	Comp.14	Comp.15	Comp.16	Comp.17	Comp.18	Comp.19
p11		-0.143				0.144	0.824	0.275	0.369
p12		-0.310	-0.246		-0.113	-0.102	0.260		-0.774
p13		-0.307	-0.489		-0.155	-0.228	-0.308		0.477
p14	-0.487		0.518				-0.170		
p15		0.111							
p21	0.117		-0.181	-0.162	0.457	0.715	-0.247	0.234	
p22	0.161					0.195	0.134	-0.905	0.142
p23						-0.168			
p24	0.636	0.113	0.387	0.212		-0.148	-0.125	0.171	
p25	-0.169	0.119							
p31	-0.196	0.649	-0.397	-0.132			0.163		
p32									
p33	0.227			-0.619	0.420	-0.405			
p34									
p41	-0.297	-0.550	0.116						
p42			0.125	-0.417	-0.660	0.360			
p43	-0.260			0.279	0.292				
p44	0.191		-0.205	0.512					
p45									

```
Calculus
Call:
princomp(x = calc)

Standard deviations:
  Comp.1   Comp.2   Comp.3   Comp.4
5.853423 2.572125 1.761938 1.596139

 4  variables and  520 observations.

Loadings:
```

	Comp.1	Comp.2	Comp.3	Comp.4
c1	-0.343	0.144	-0.207	0.905
c2	-0.533	0.784		-0.308
c3	-0.638	-0.489	-0.522	-0.284

```
c4 -0.437 -0.354  0.823

               Comp.1 Comp.2 Comp.3 Comp.4
SS loadings      1.00   1.00   1.00   1.00
Proportion Var   0.25   0.25   0.25   0.25
Cumulative Var   0.25   0.50   0.75   1.00
Call:
princomp(x = calcprob)

Standard deviations:
   Comp.1    Comp.2    Comp.3    Comp.4    Comp.5    Comp.6    Comp.7    Comp.8 
2.9562898 1.4559836 1.1239335 0.9748880 0.8961765 0.7938829 0.7407666 0.7090392 
   Comp.9   Comp.10   Comp.11   Comp.12   Comp.13   Comp.14   Comp.15   Comp.16 
0.7020121 0.6479064 0.6344151 0.6147035 0.6067864 0.5890691 0.5830478 0.5404385 
  Comp.17   Comp.18 
0.5095181 0.3469772 

 18  variables and  528 observations.

Loadings:
    Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9 Comp.10
p51 -0.187        -0.233 -0.301                             -0.172  0.310 
p52                                                                       
p53 -0.281        -0.458 -0.676  0.129 -0.124                             
p54 -0.162         0.163                                            0.136 
p61 -0.313 -0.558         0.148 -0.301 -0.140  0.629 -0.120               
p62 -0.337 -0.555         0.282               -0.648        -0.124        
p63 -0.316 -0.103  0.321         0.654  0.405  0.203  0.319  0.152        
p64 -0.120         0.231         0.189 -0.405 -0.126 -0.172 -0.268 -0.509 
p71 -0.477  0.464 -0.391  0.483        -0.189         0.170  0.118        
p72 -0.227  0.150               -0.212  0.591        -0.566  0.145 -0.203 
p73 -0.165         0.203         0.229               -0.404 -0.102  0.271 
p74 -0.226  0.164  0.166        -0.227  0.160         0.235 -0.655  0.265 
p75 -0.153                             -0.318        -0.165  0.214  0.389 
p81 -0.217  0.195 -0.176        -0.177  0.167               -0.156 -0.209 
p82 -0.187  0.101  0.216 -0.197 -0.372                0.429  0.194 -0.251 
p83 -0.134         0.255 -0.106 -0.170 -0.108 -0.309         0.457        
p84 -0.168  0.114  0.311 -0.185 -0.245                       0.106  0.309 
p85 -0.136         0.276 -0.125        -0.252        -0.189 -0.203 -0.257 
    Comp.11 Comp.12 Comp.13 Comp.14 Comp.15 Comp.16 Comp.17 Comp.18
p51  0.513   0.228  -0.241   0.321  -0.290   0.319                 
p52                                                           0.985
p53 -0.317                  -0.206   0.165  -0.107                 
p54 -0.214                   0.103          -0.142  -0.907         
p61                                  0.147                         
p62         -0.131   0.111                                         
p63                                                                
p64  0.203   0.306  -0.253                  -0.370   0.104         
p71                 -0.190  -0.122           0.149                 
p72          0.204  -0.115  -0.224  -0.142                         
p73 -0.274  -0.527  -0.272   0.345                   0.258         
p74 -0.298   0.310                   0.193           0.173         
p75          0.308   0.458  -0.105  -0.465  -0.241   0.154         
p81  0.284  -0.296   0.562   0.411          -0.304                 
p82 -0.223  -0.129  -0.211   0.217  -0.532                         
p83          0.274   0.114   0.368   0.503   0.231   0.136         
p84  0.479  -0.309  -0.172  -0.445   0.161  -0.271                 
p85         -0.199   0.336  -0.281           0.654                 

               Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9
SS loadings     1.000  1.000  1.000  1.000  1.000  1.000  1.000  1.000  1.000
Proportion Var  0.056  0.056  0.056  0.056  0.056  0.056  0.056  0.056  0.056
Cumulative Var  0.056  0.111  0.167  0.222  0.278  0.333  0.389  0.444  0.500
               Comp.10 Comp.11 Comp.12 Comp.13 Comp.14 Comp.15 Comp.16 Comp.17
SS loadings      1.000   1.000   1.000   1.000   1.000   1.000   1.000   1.000
Proportion Var   0.056   0.056   0.056   0.056   0.056   0.056   0.056   0.056
Cumulative Var   0.556   0.611   0.667   0.722   0.778   0.833   0.889   0.944
               Comp.18
SS loadings      1.000
Proportion Var   0.056
Cumulative Var   1.000
```

```
Call:
princomp(x = calc)

Standard deviations:
  Comp.1   Comp.2   Comp.3   Comp.4 
5.853423 2.572125 1.761938 1.596139 

 4  variables and  520 observations.
```

An item response curve shows how likely students were to get a particularl problem right against their overall score on the exam. This curve should be an S-shape where students who do poorly on the exam do poorly on that problem while students who do well on the exam do well on the problem. Failure to fit an S-shape suggests the problem is measuring something different from the overall exam. We divided students into groups which represented 10% grade bounds. We then plotted the mean score on each problem from the students in each grade band, and also fit a logistic curve for item response. The graphs below show our results, with the actual data labeled “mean##” where ## denotes the page and problem and the fitted logistic curve labeled “b”. Note that the colors are not always consistent between graphs, so you should check the legend to be sure which graph is which. Also note that while most problems had a maximum score of 2, problem 4 on page 3 was a two-part problem with each part worth 2 points for a maximum score of 4.

Algebra Exam

Figure 2-1 Problem 1 page 1

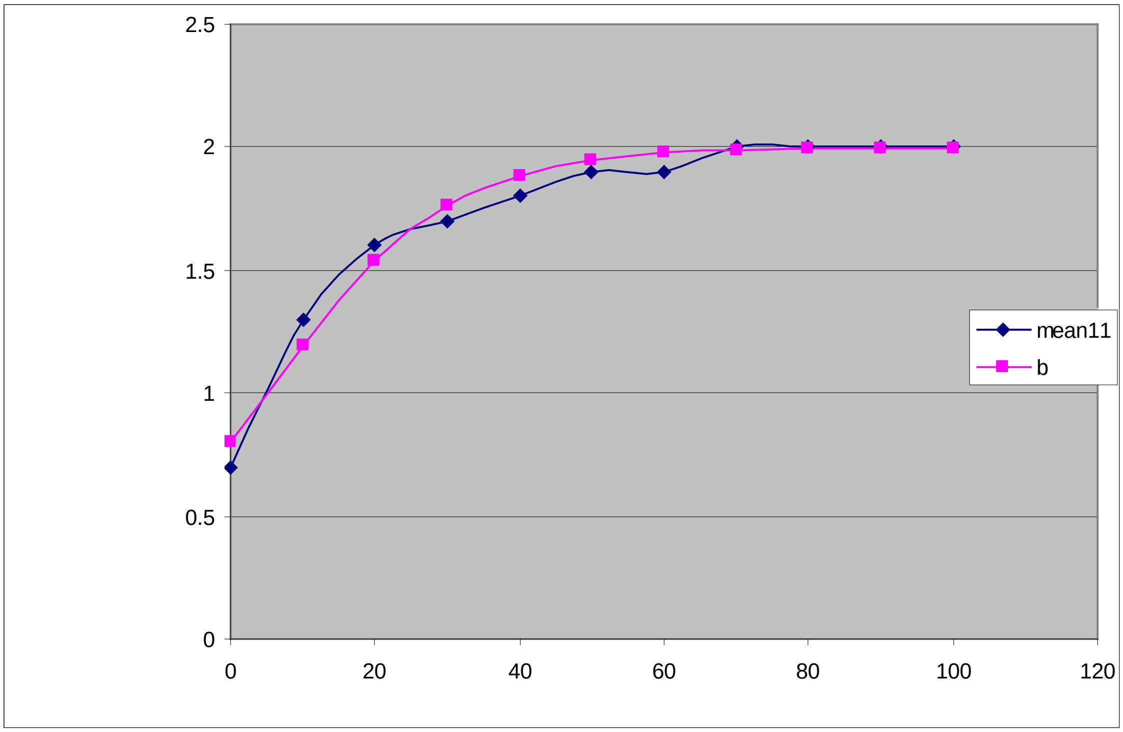


Figure 2-2 Problem 2 page 1

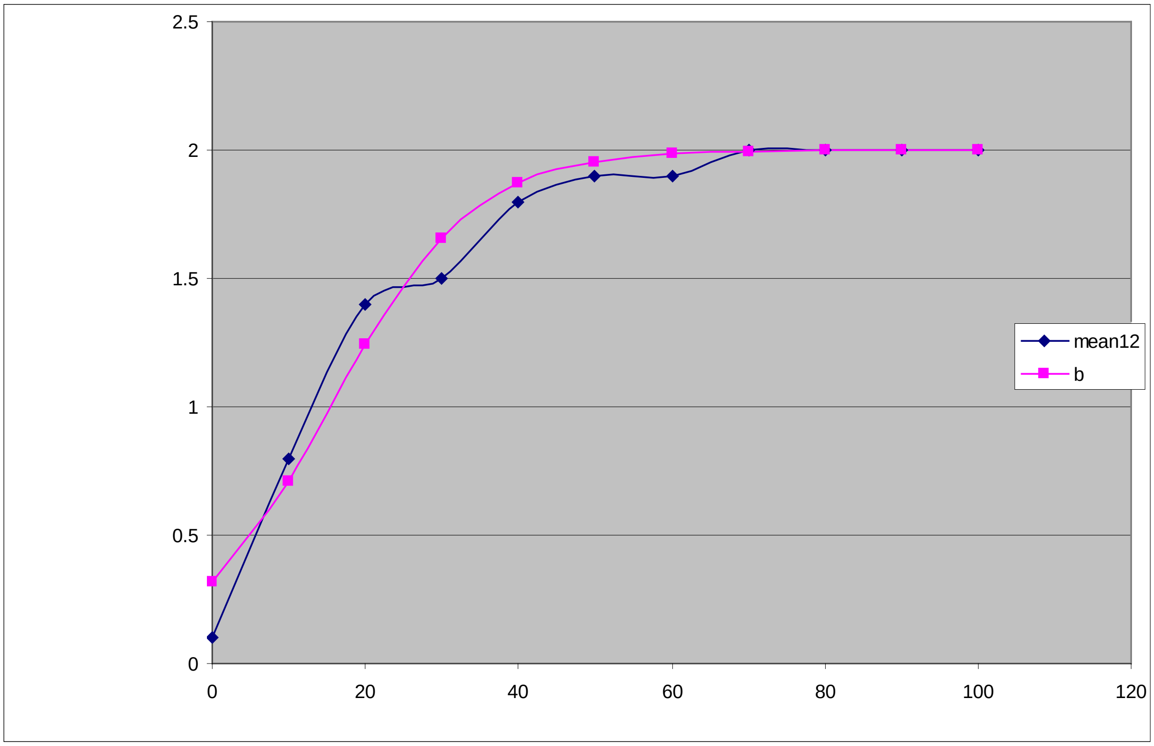


Figure 2-3 Problem 3 page 1

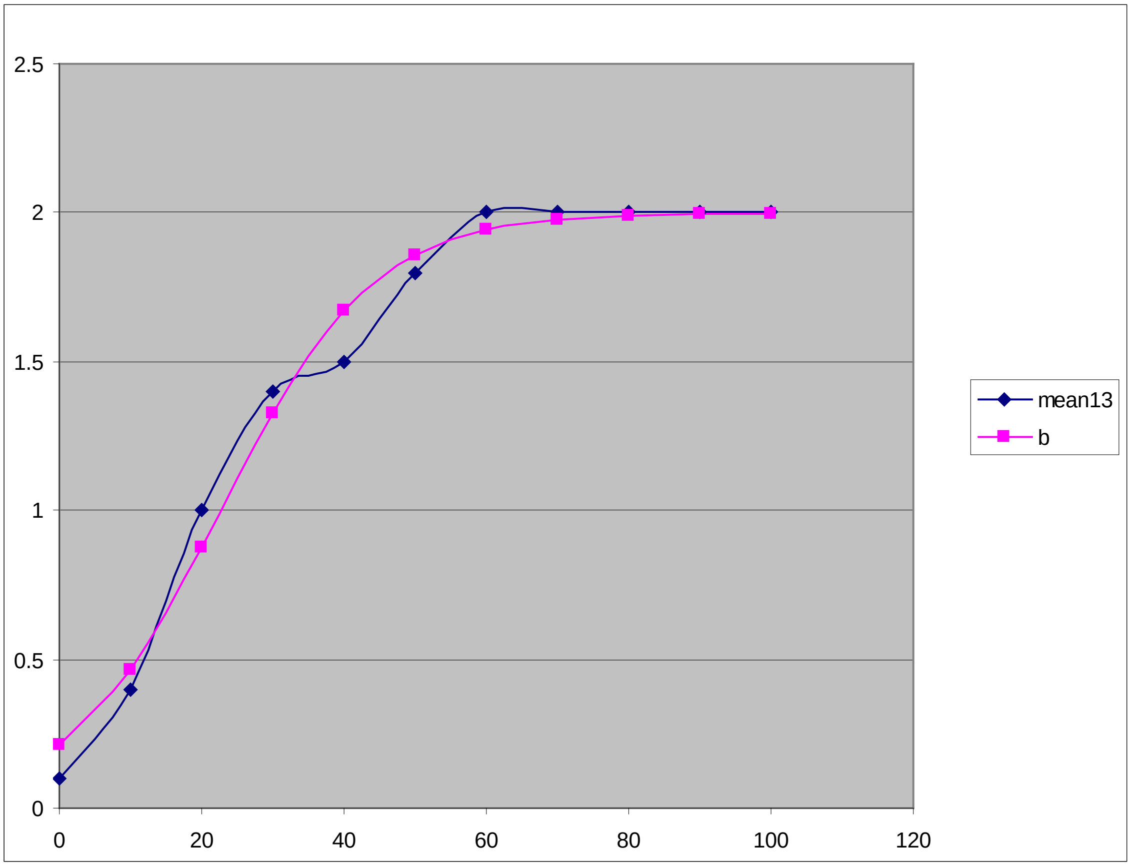


Figure 2-4 Problem 4 page1

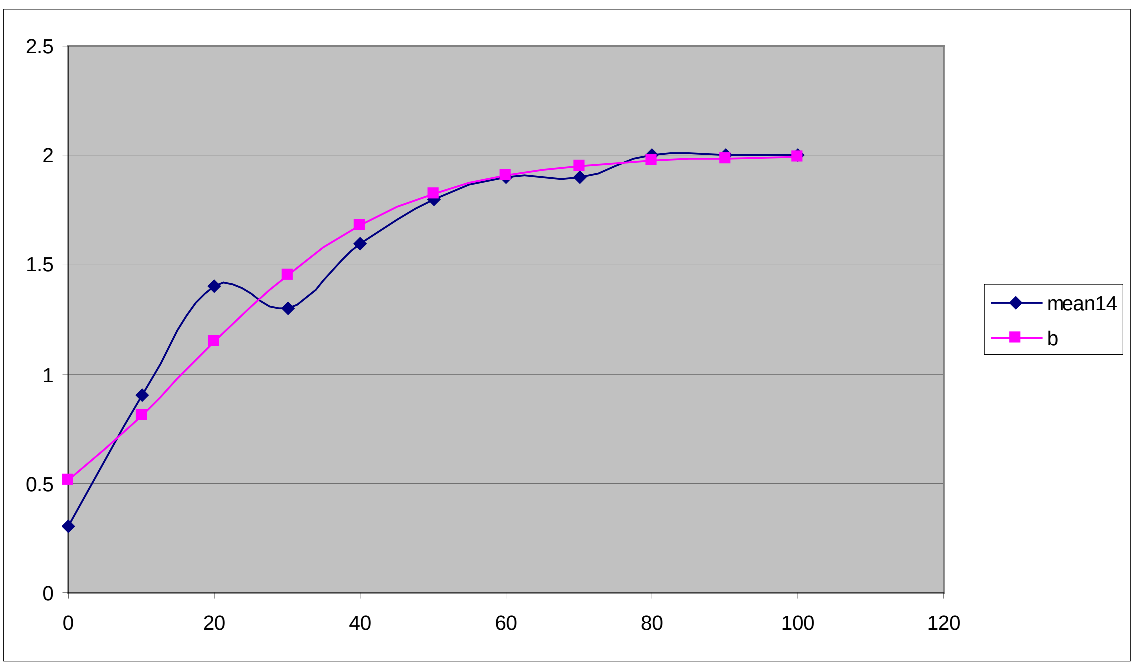


Figure 2-5 Problem 5 page 1

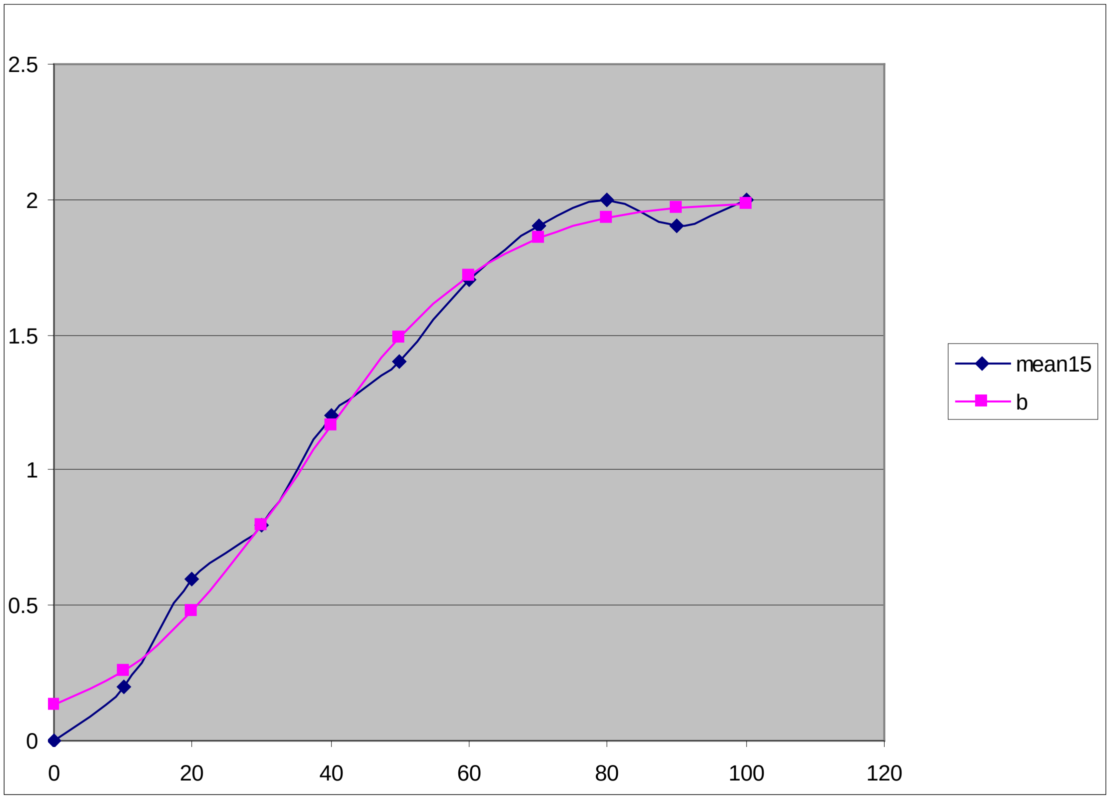


Figure 2-6 Problem 1 page 2

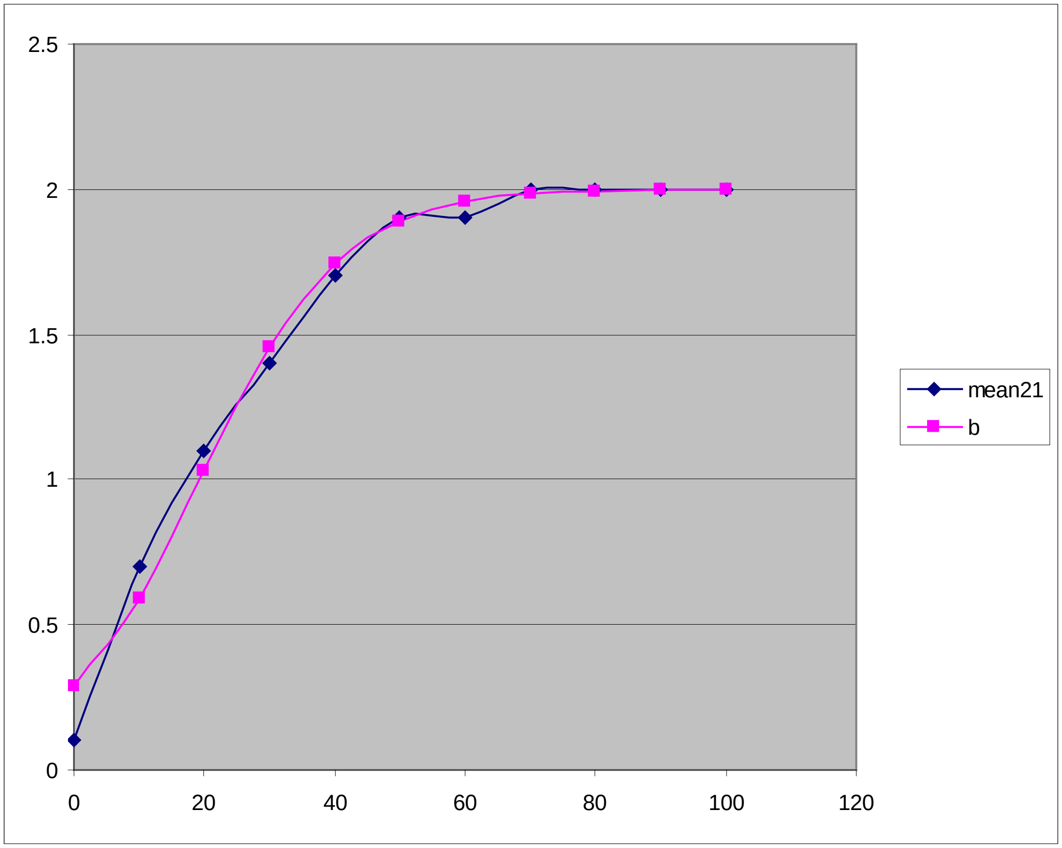
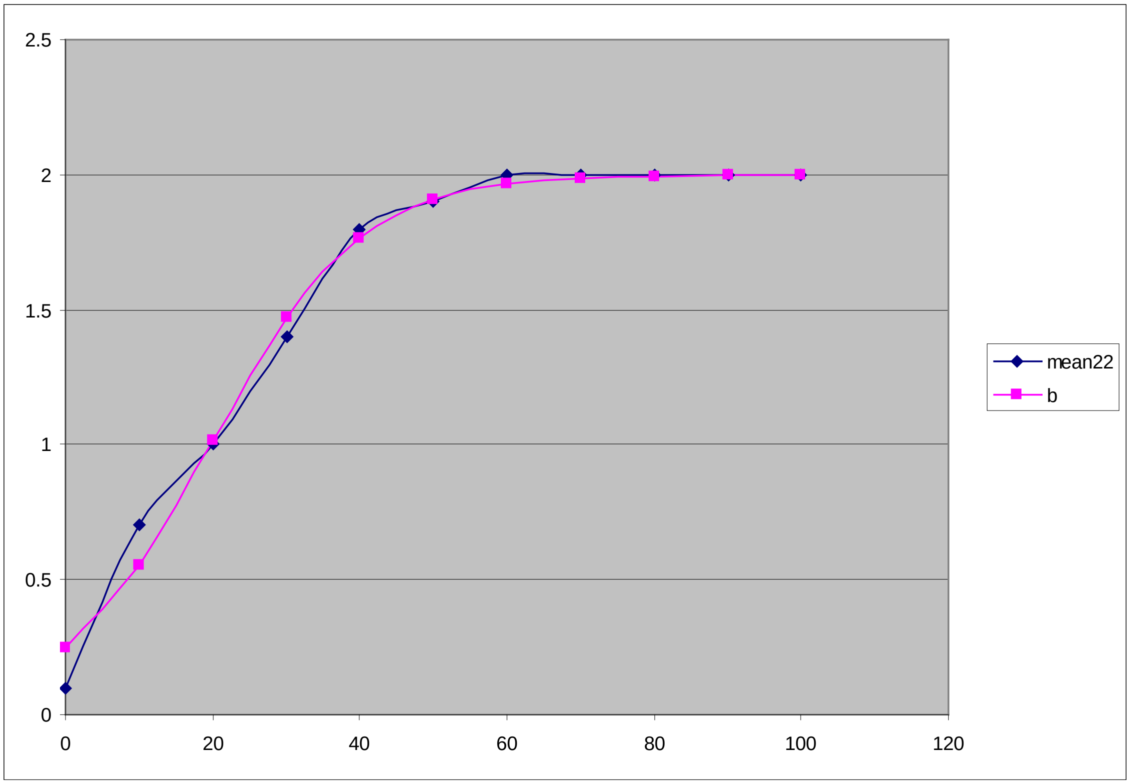


Figure 2-7 Problem 2 page 2



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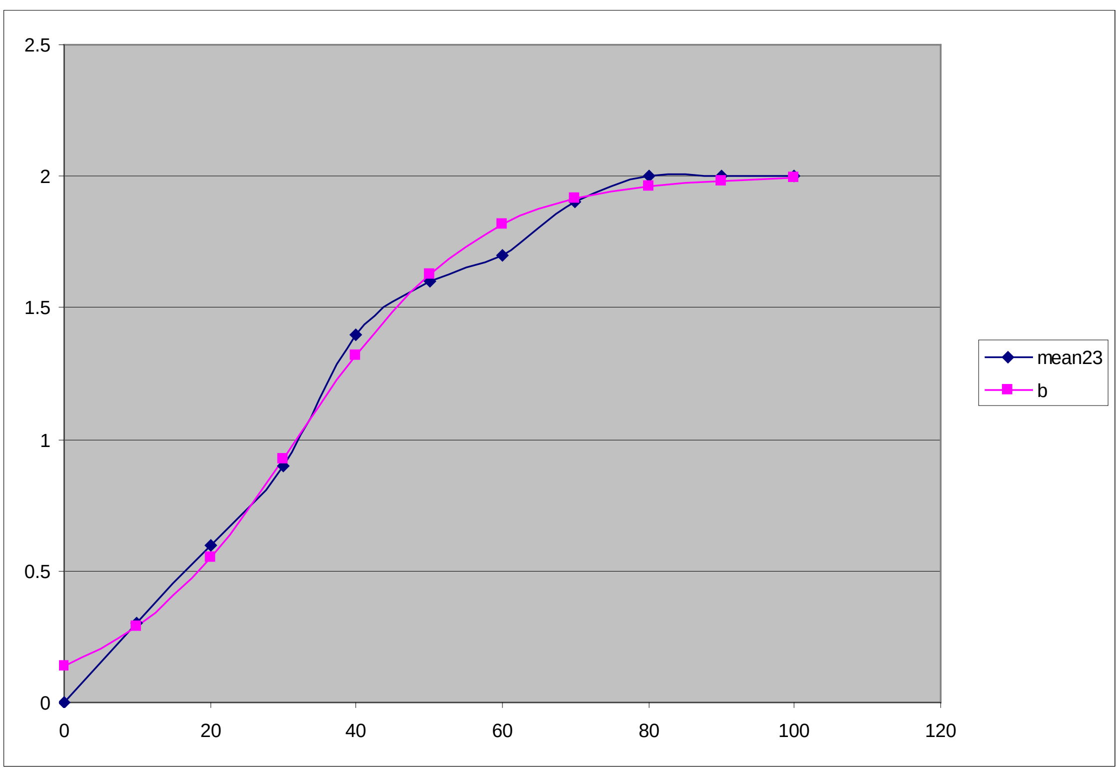


Figure 2-9 Problem 4 page 2

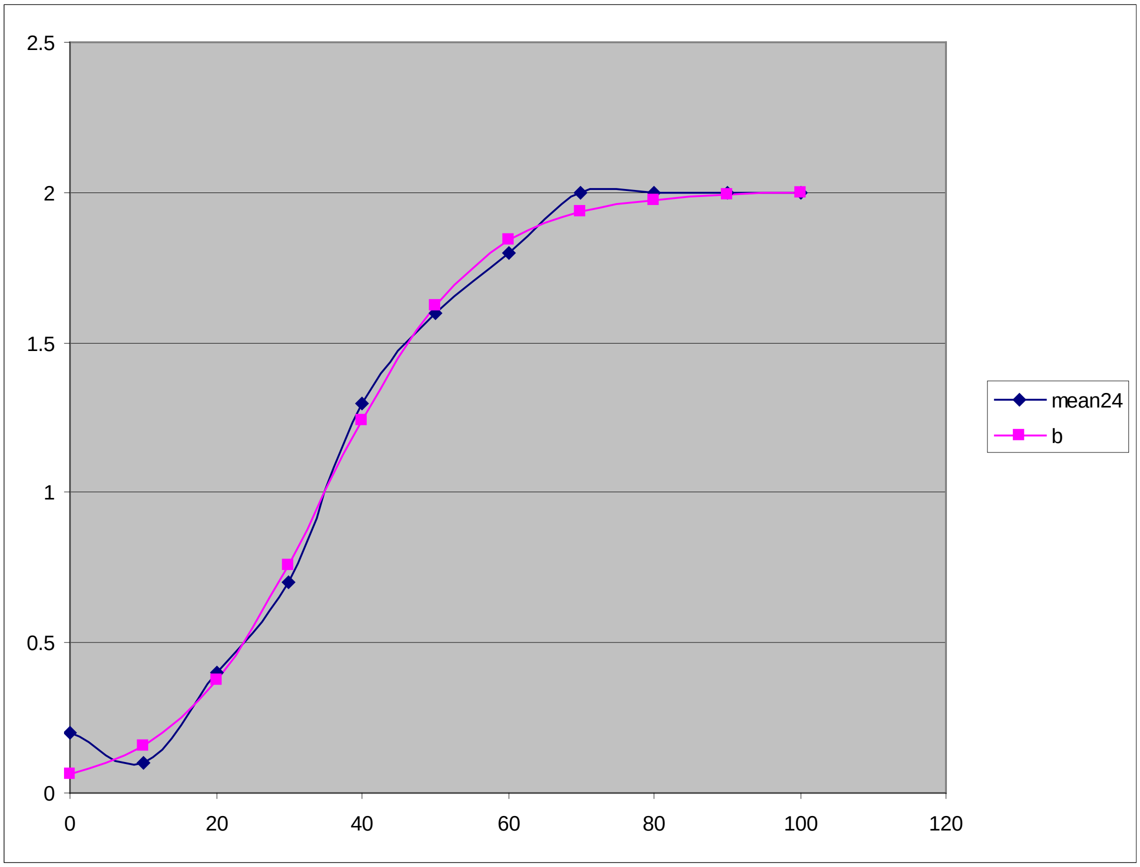


Figure 2-10 Problem 5 page 2

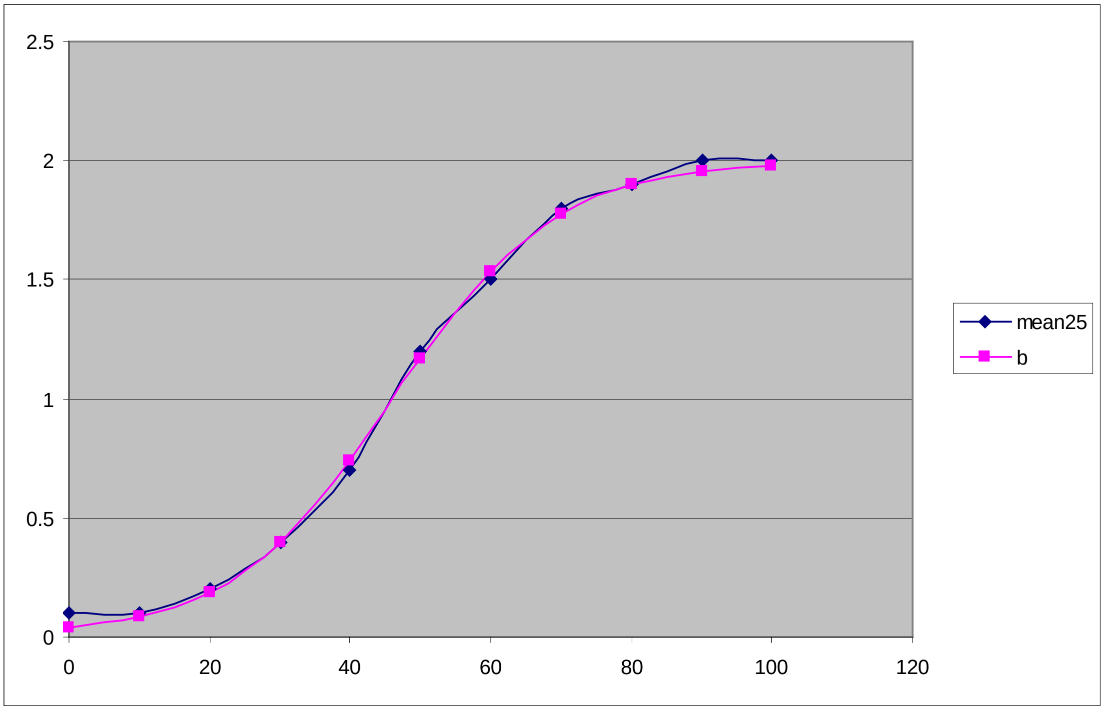


Figure 2-11 Problem 1 page 3

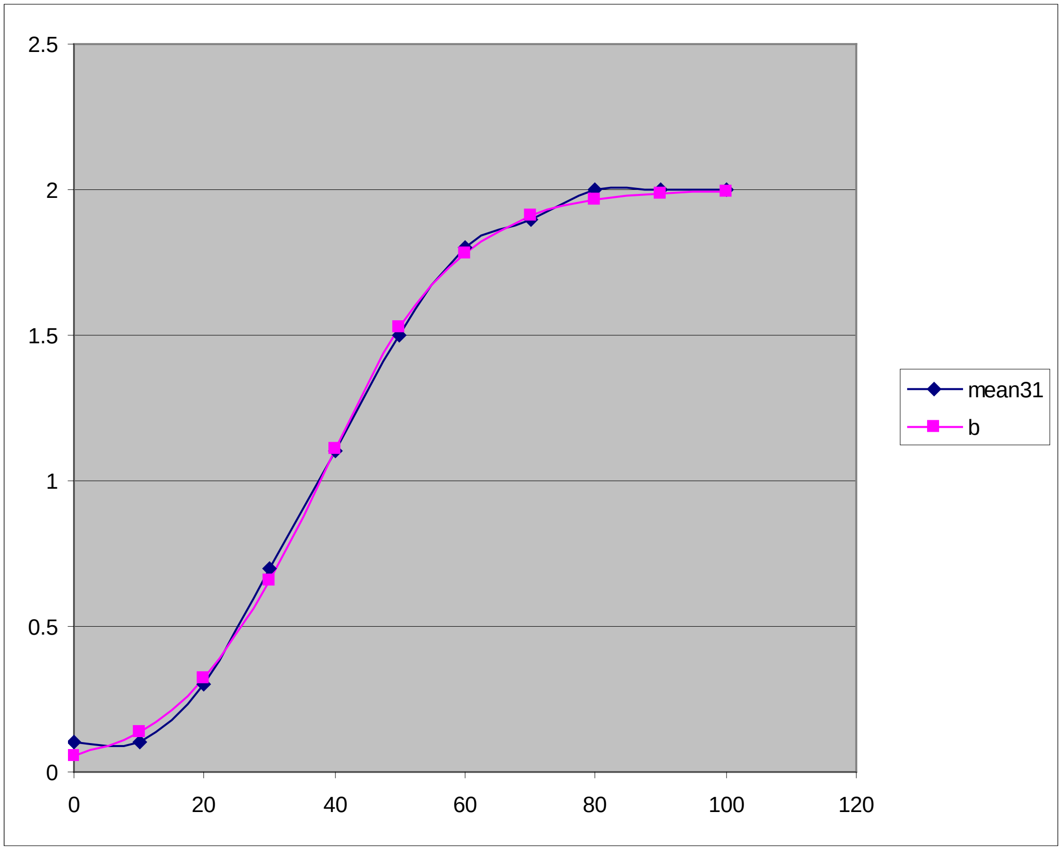


Figure 2-12 Problem 2 page 3

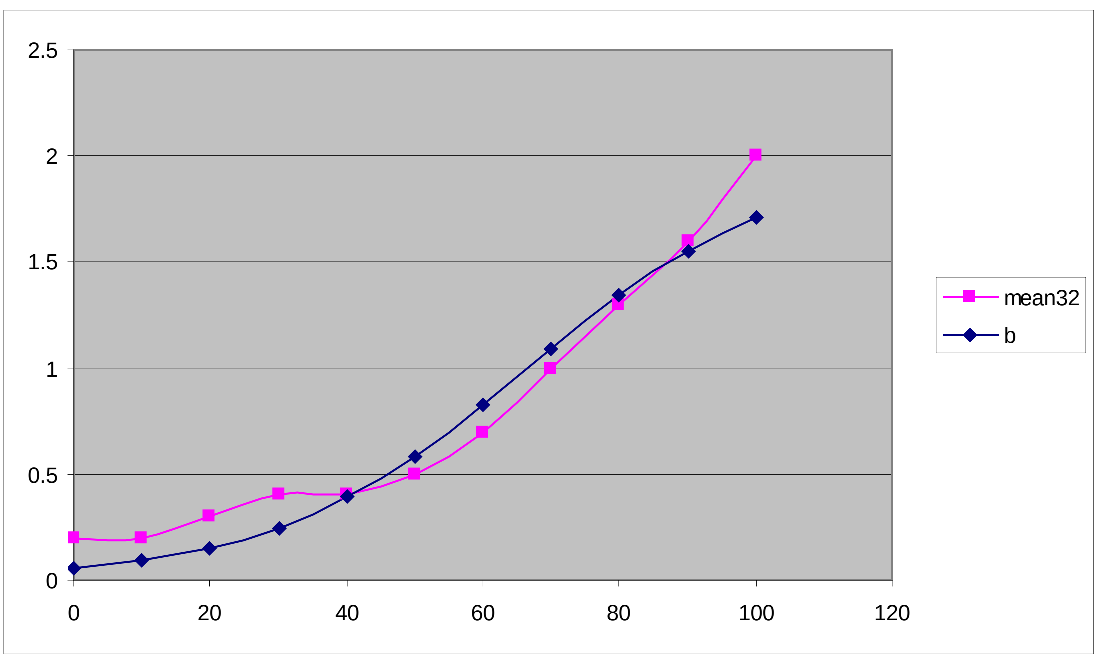


Figure 2-13 Problem 3 page 3

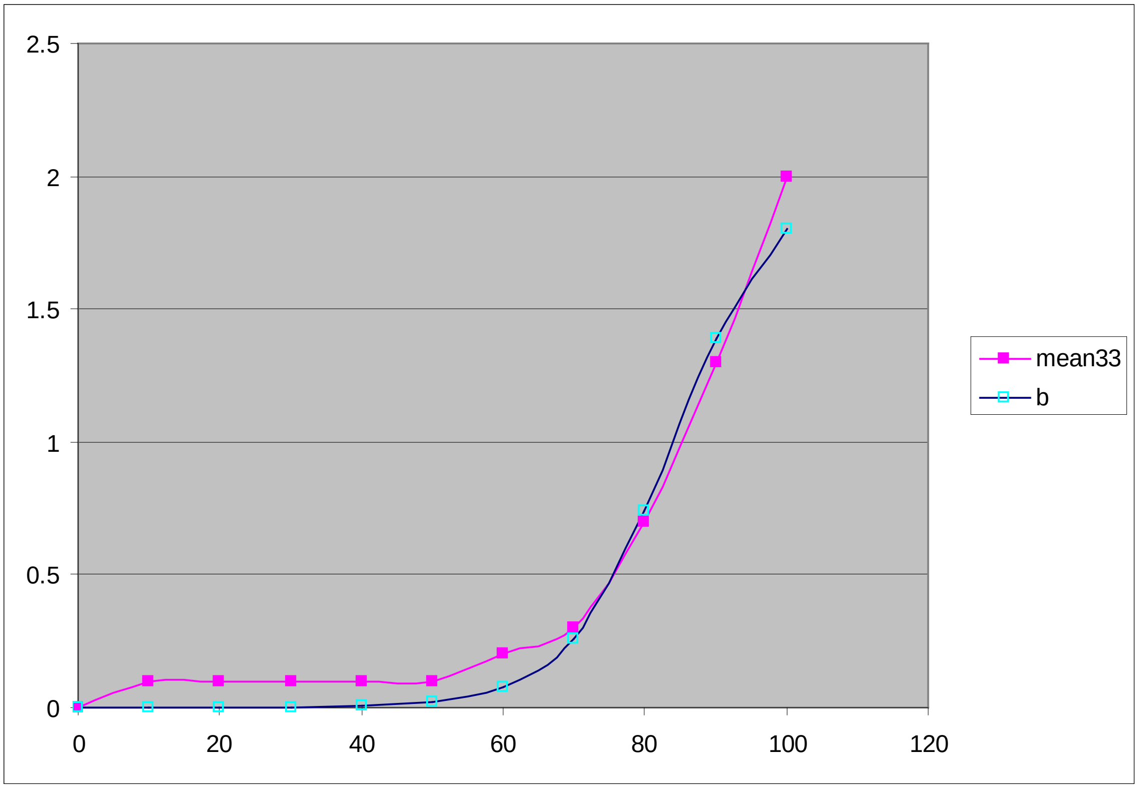


Figure 2-14 Problem 4 page 3

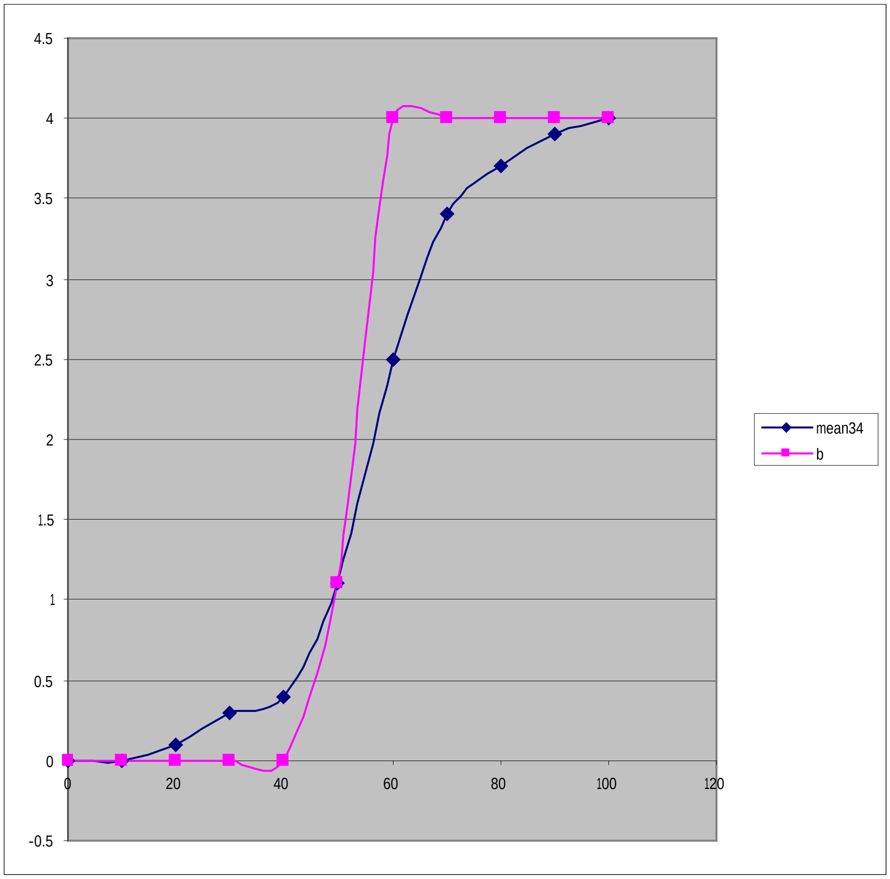


Figure 2-15 Problem 1 page 4

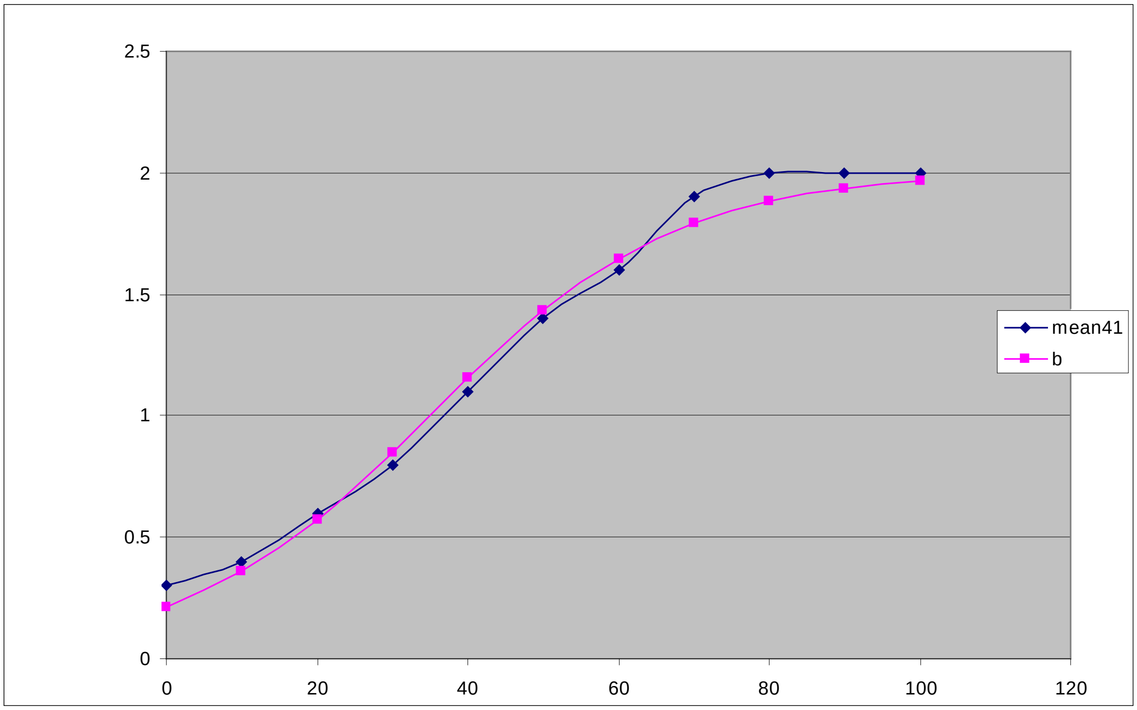


Figure 2-16 Problem 2 page 4

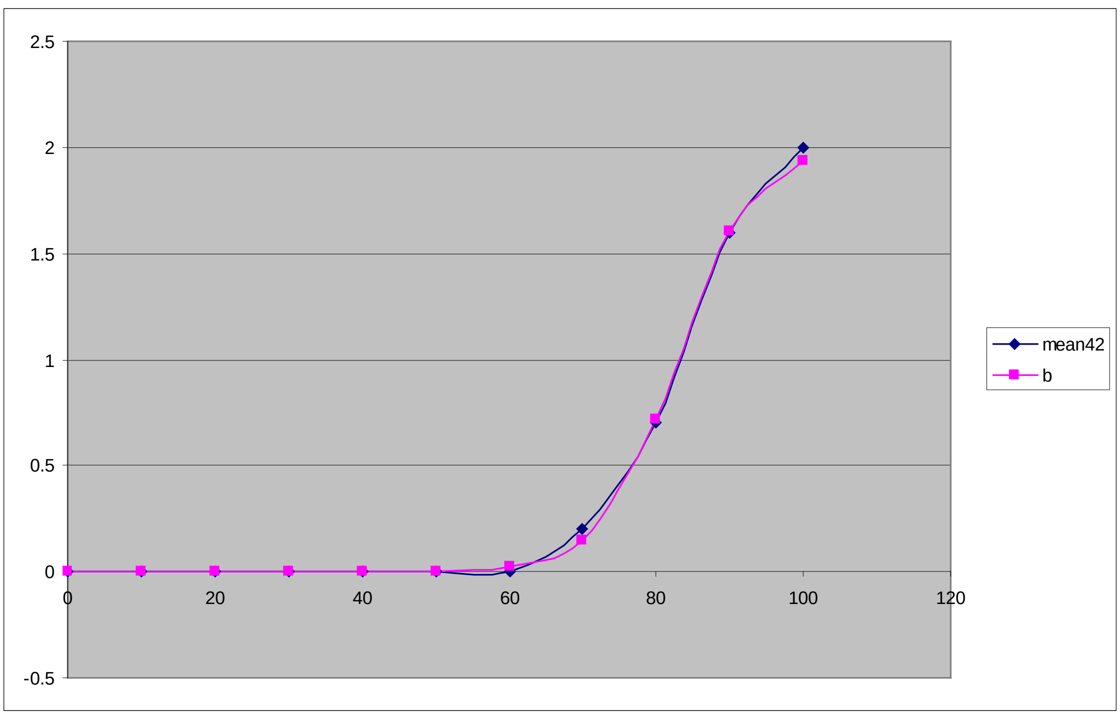


Figure 2-17 Problem 3 page 4

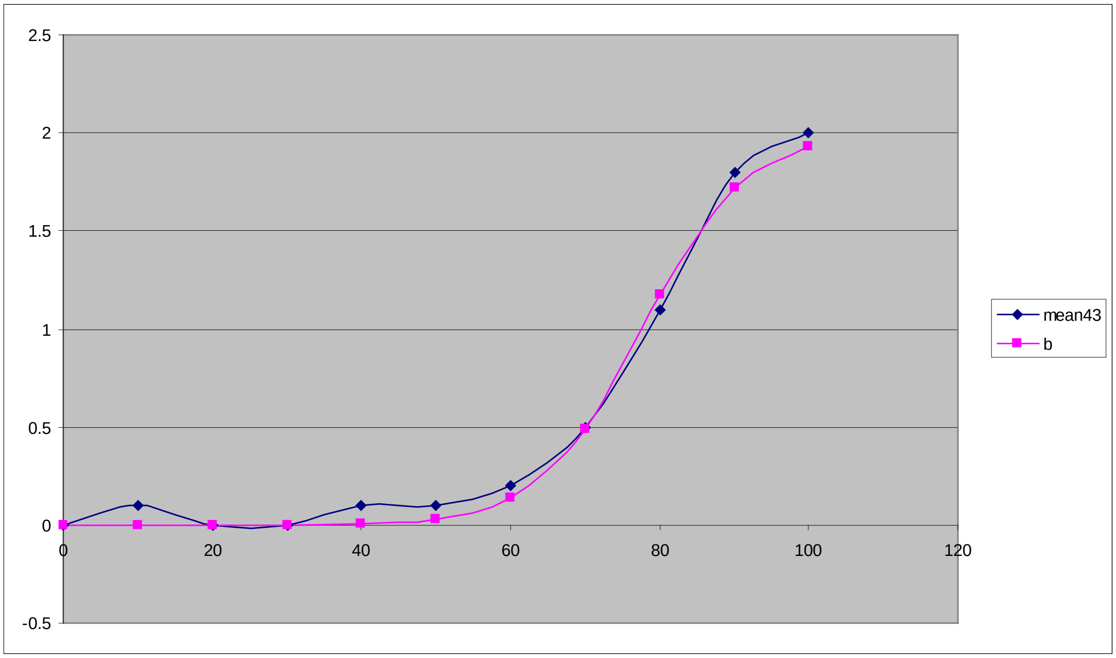


Figure 2-18 Problem 4 page 4

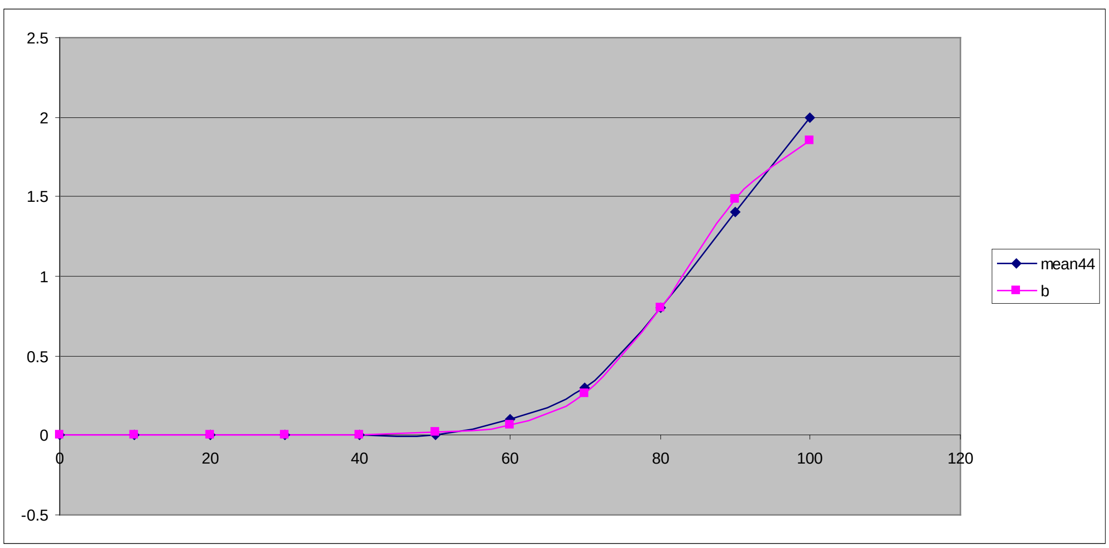
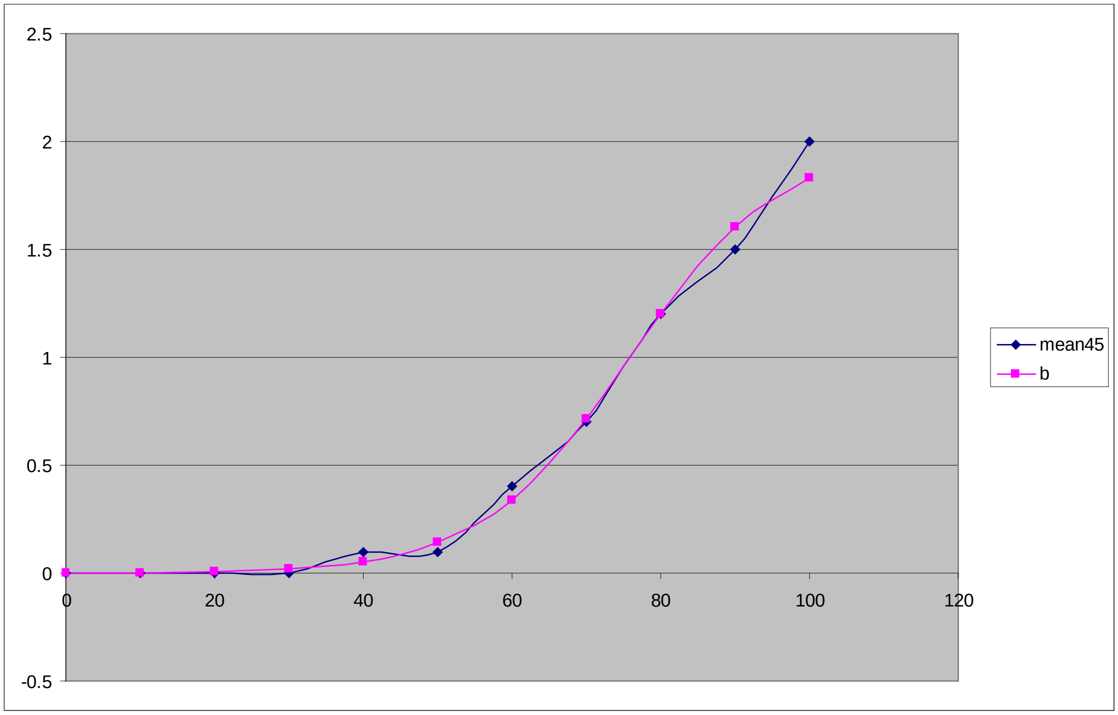


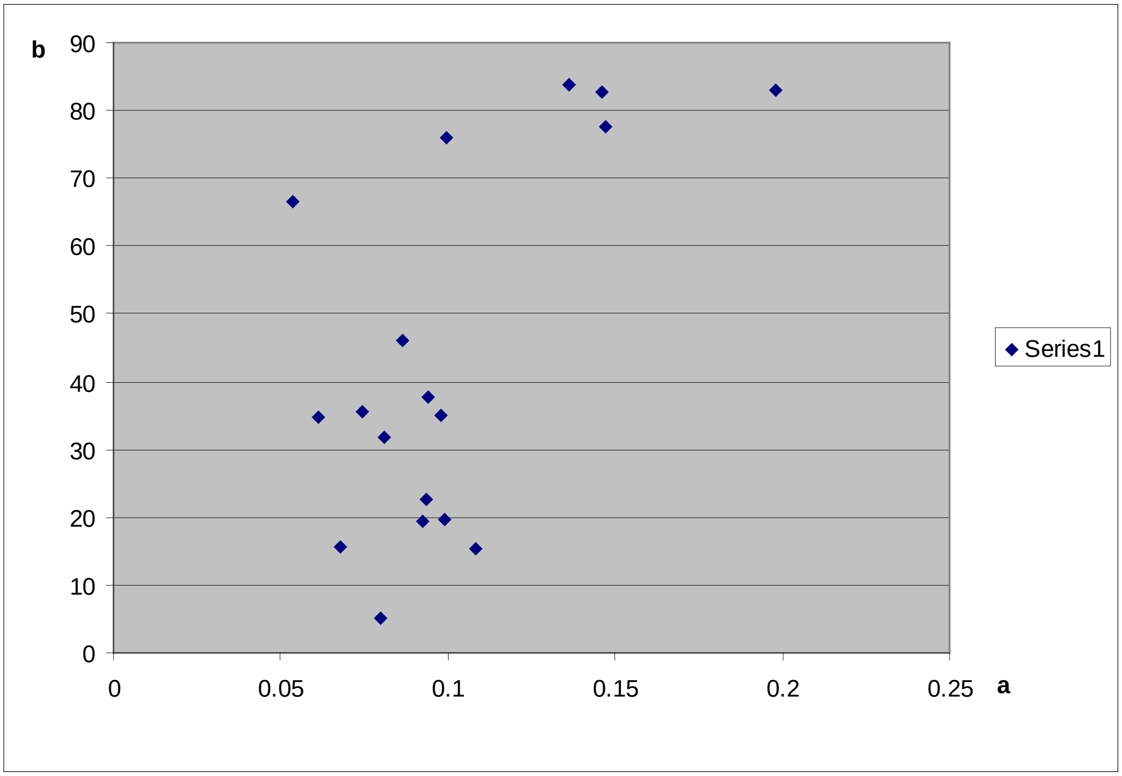
Figure 2-19 Problem 5 page 4



The item response curves all fit quite well with the logistic models. The PCA analysis identified problem 4 on page 3 as special. Looking at the item response curve, it appears that problem had an exceptionally sharp bend in the "S" where good students almost all got it right and mediocre students almost all got it wrong.

We next plotted the coefficients of the fitted logistic curves for each problem, obtaining the following graph

Figure 2-20 Coefficients of fitted logistic curves



By inspection, it appears the problems are divided into 3 groups:

- Basic algebra: problems 1-4 from section 1, problems 1 and 2 from section 2;
- Intermediate algebra: problem 5 from section 1, problems 3-5 from section 2, problem 1 from section 3, problem 1 from section 4.
- College algebra: everything else.

Problem 1 on page 1 (general calculation addressing order of operations) does not give any useful information, because almost all students solved it correctly. Comparing to results from the PCA, it is possible that we should reduce the Basic Algebra problems and increase the number of College Algebra problems. However, the real test will be which problems prove the best predictors of success in later courses.

Figure 2-21 Problem 1 page 1

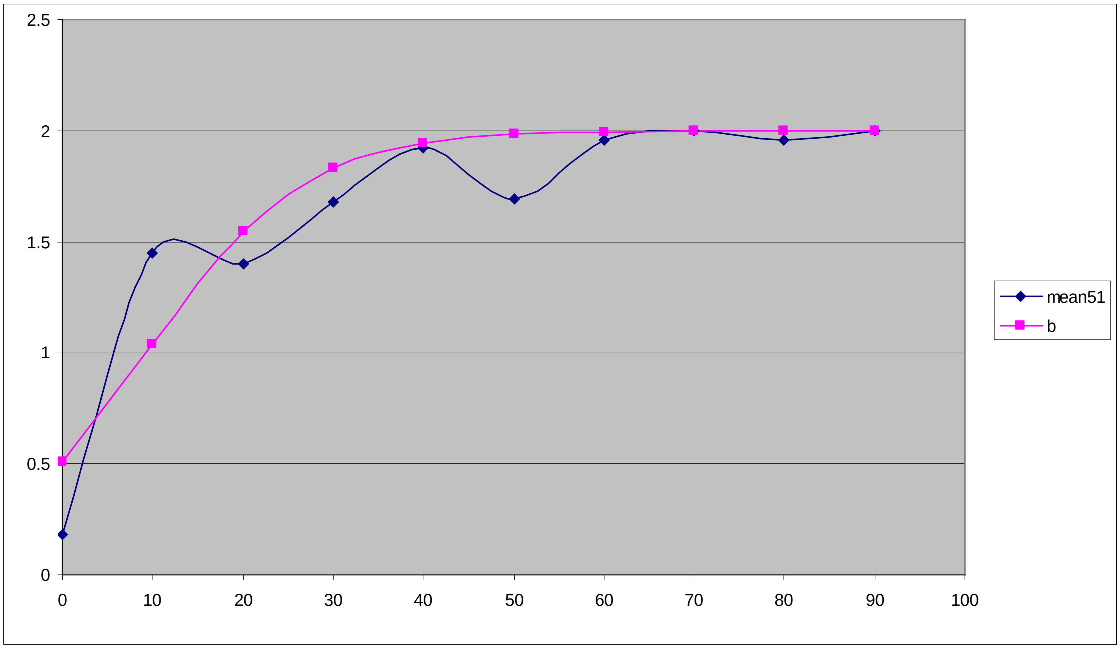


Figure 2-22 Problem 2 page 1

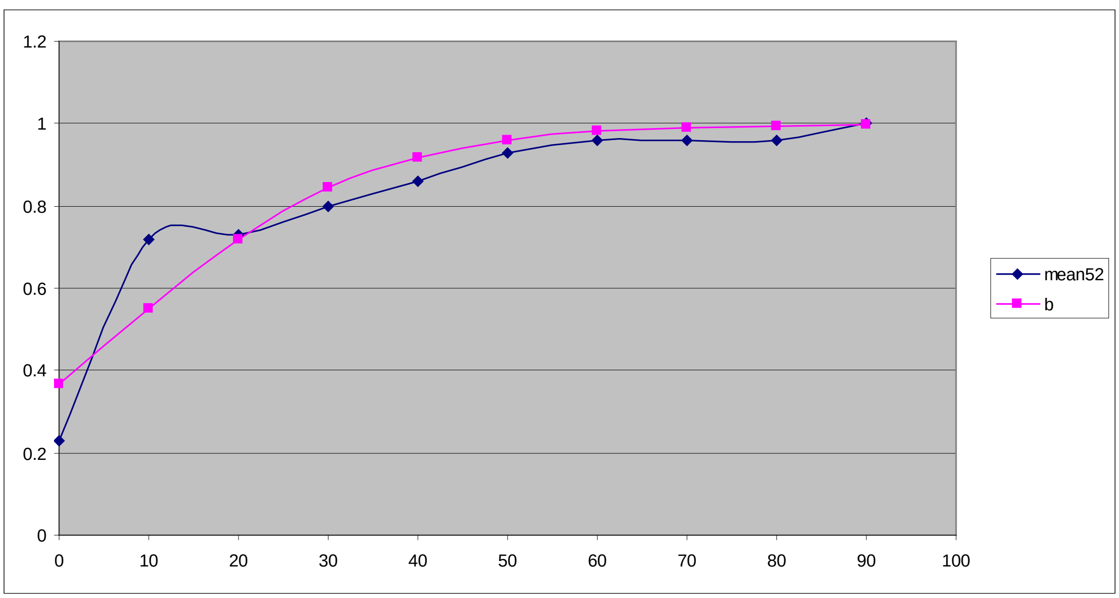


Figure 2-23 Problem 3 page 1

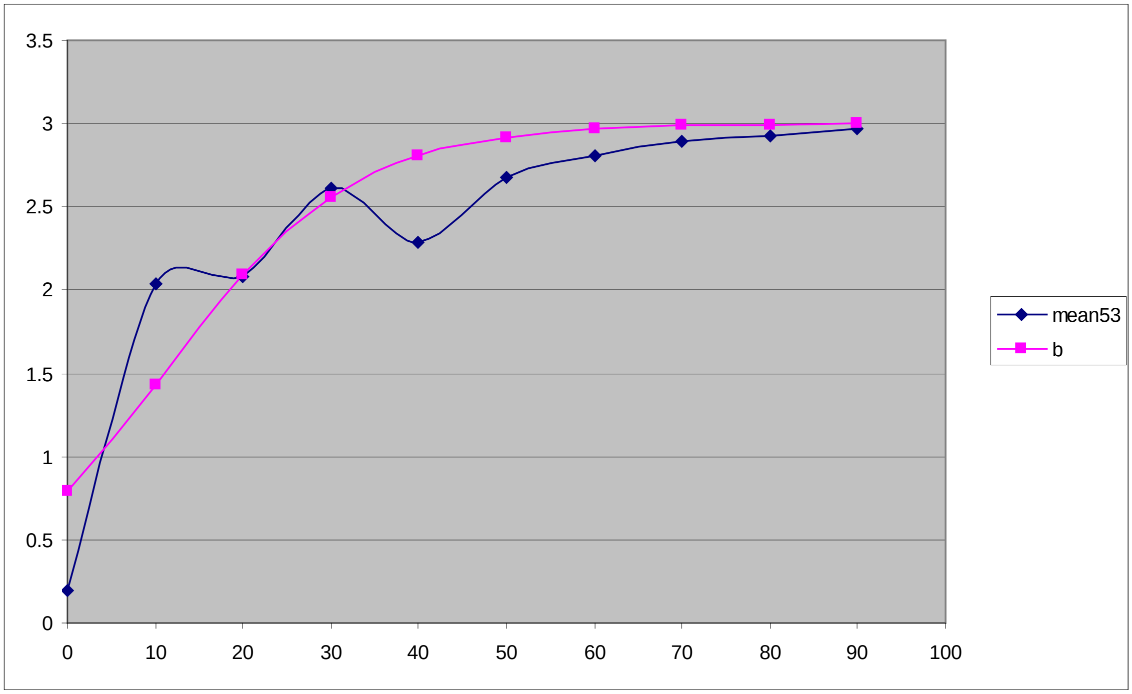


Figure 2-24 Problem 4 page 1

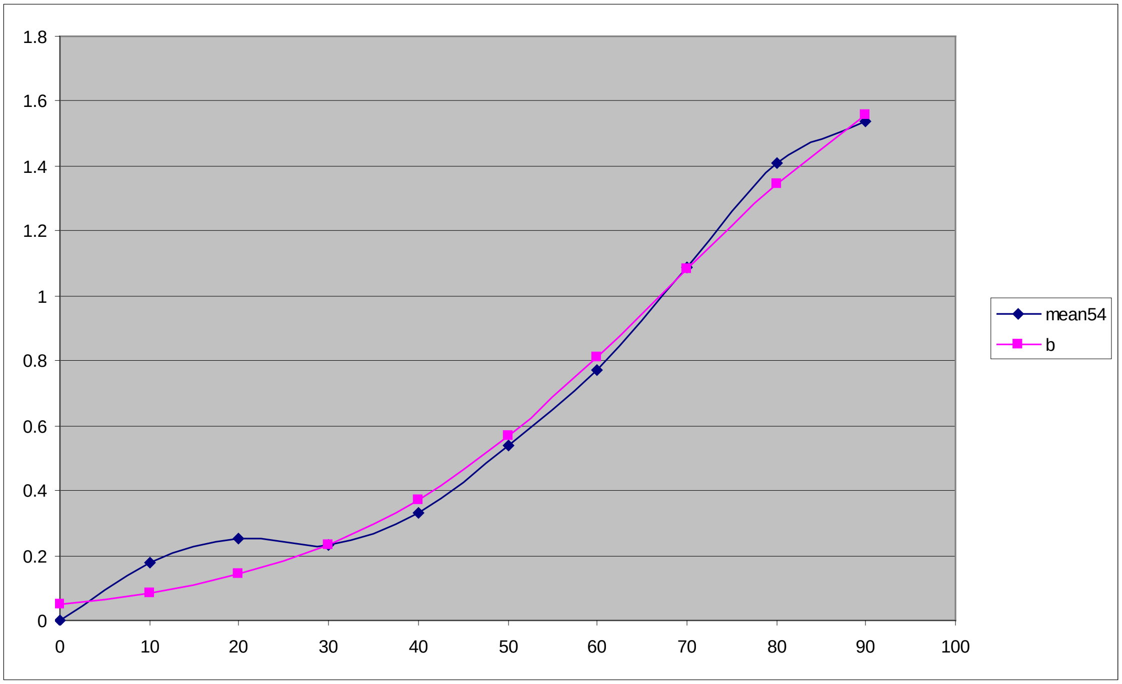


Figure 2-25 Problem 1 page 2

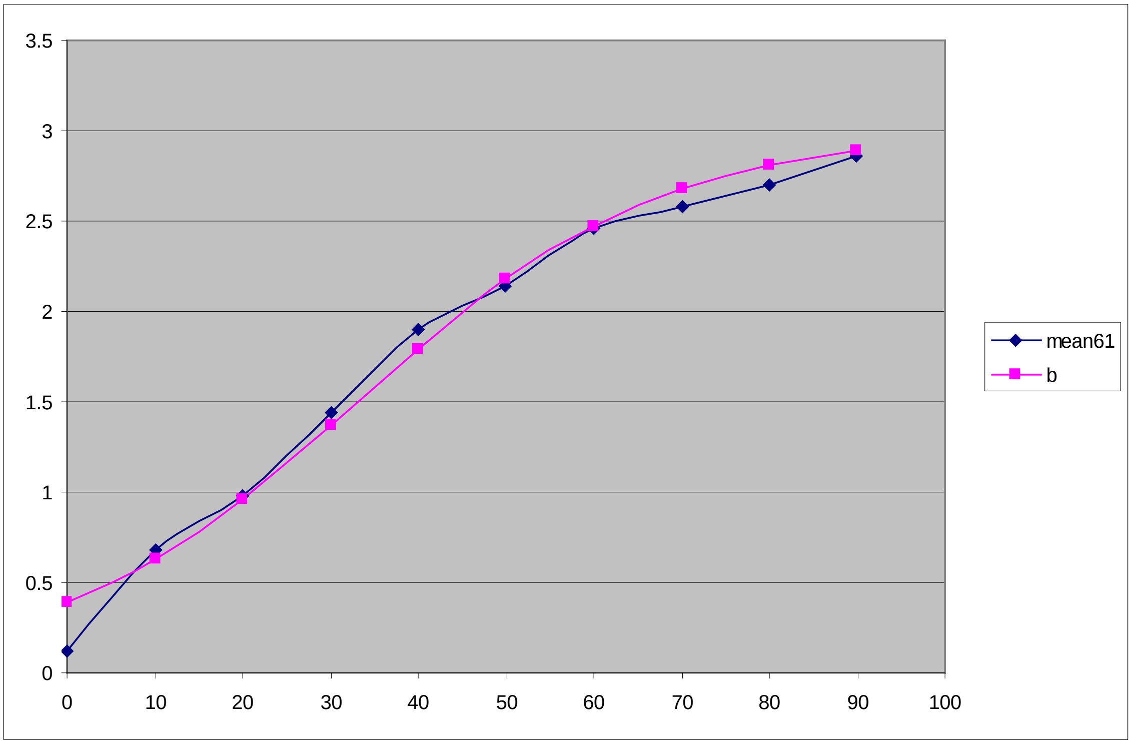


Figure 2-26 Problem 2 page 2

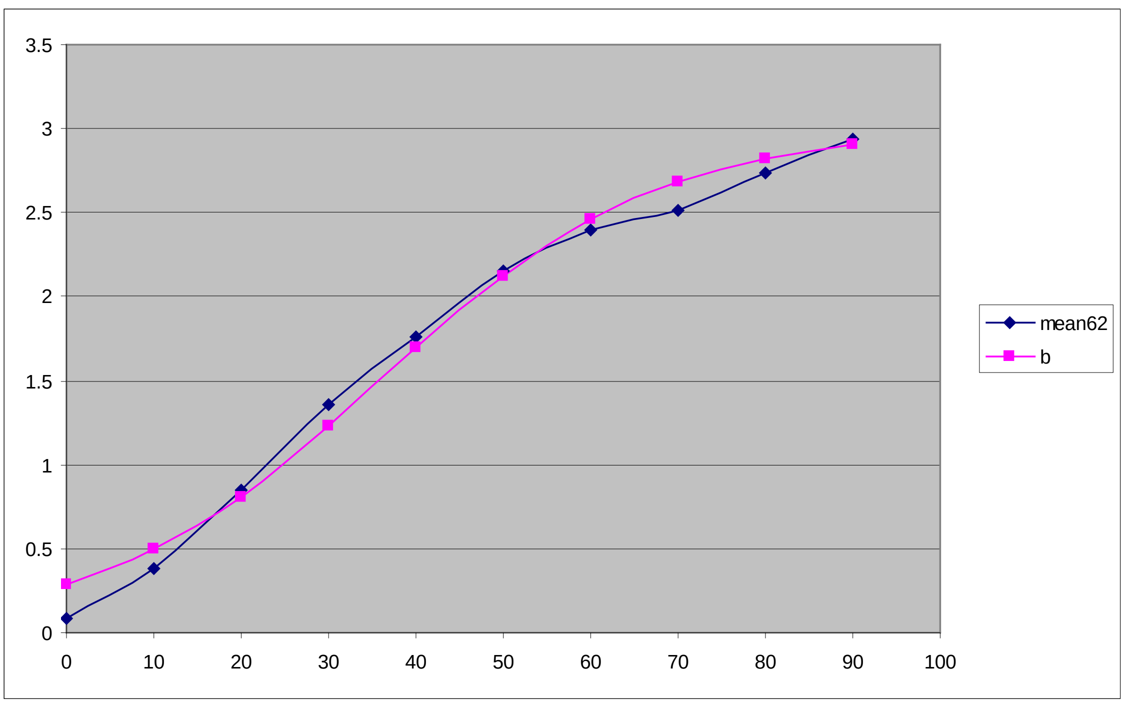


Figure 2-27 Problem 3 page 2

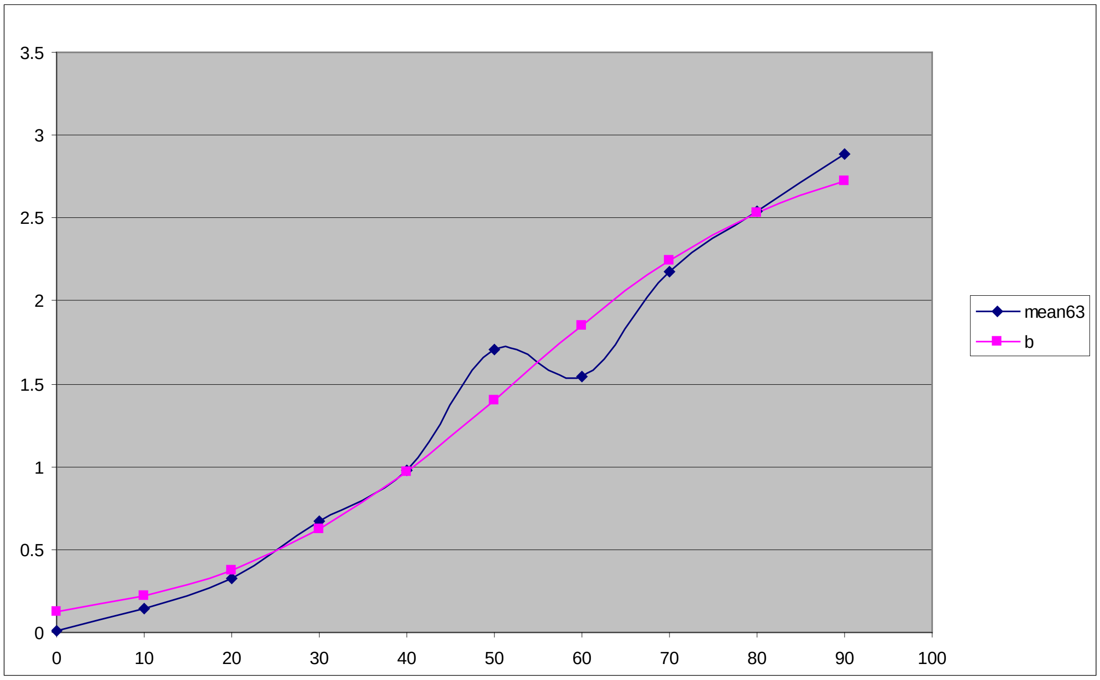


Figure 2-28 Problem 4 page 2

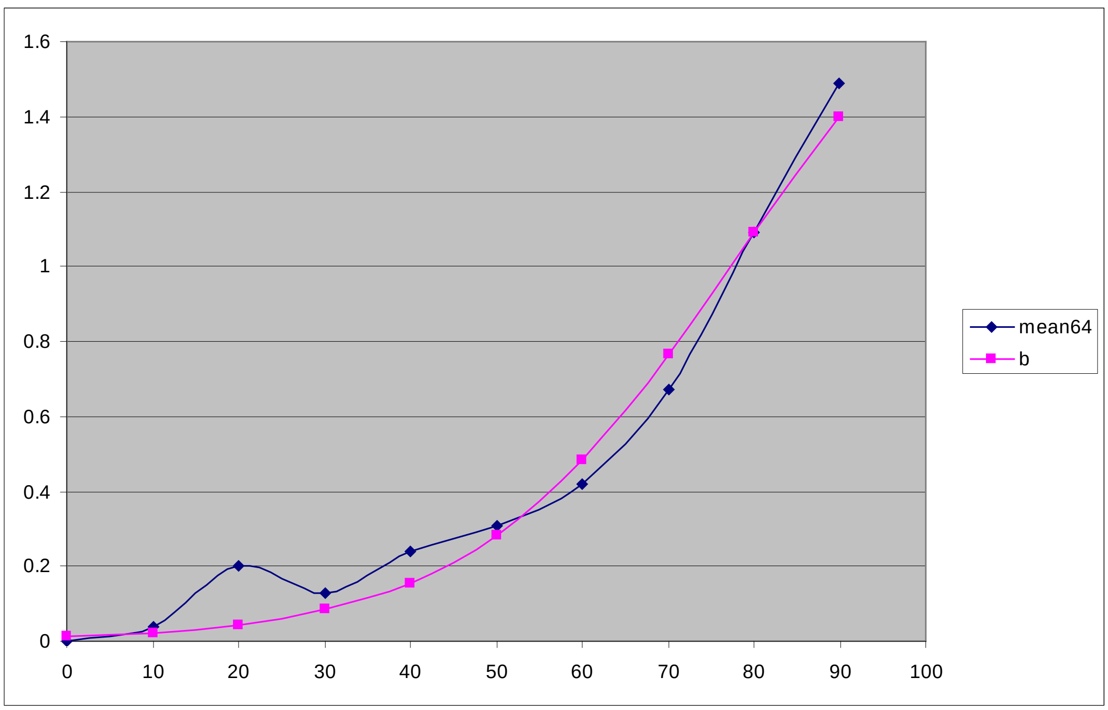


Figure 2-29 Problem 1 page 3

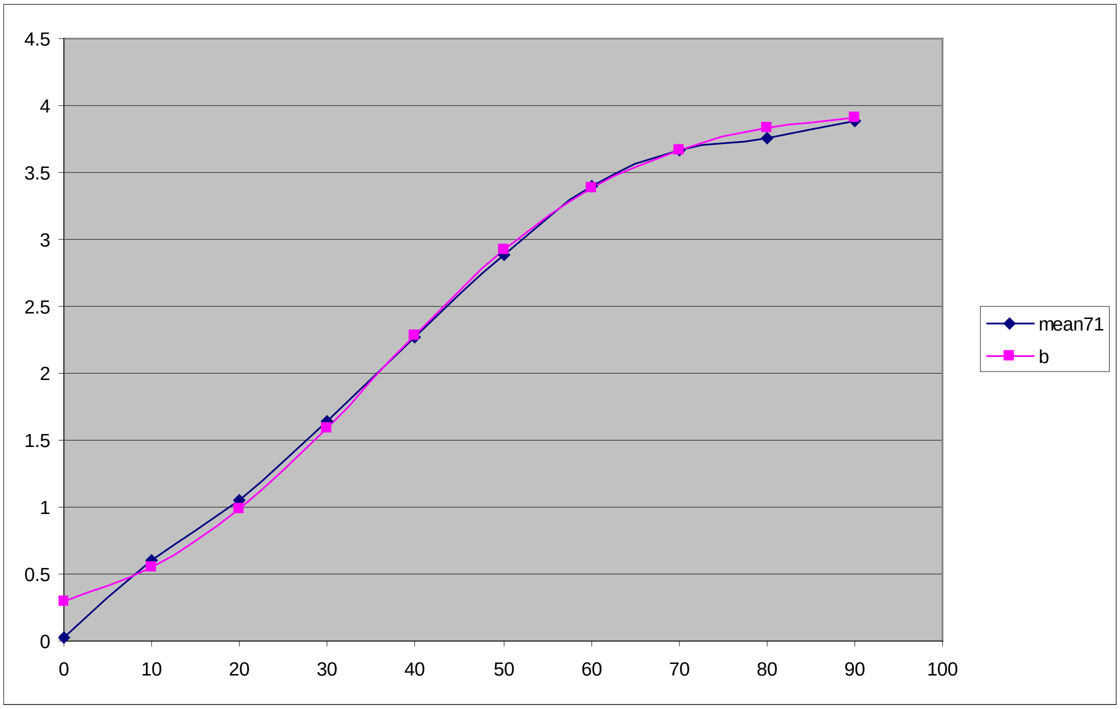


Figure 2-30 Problem 2 page 3

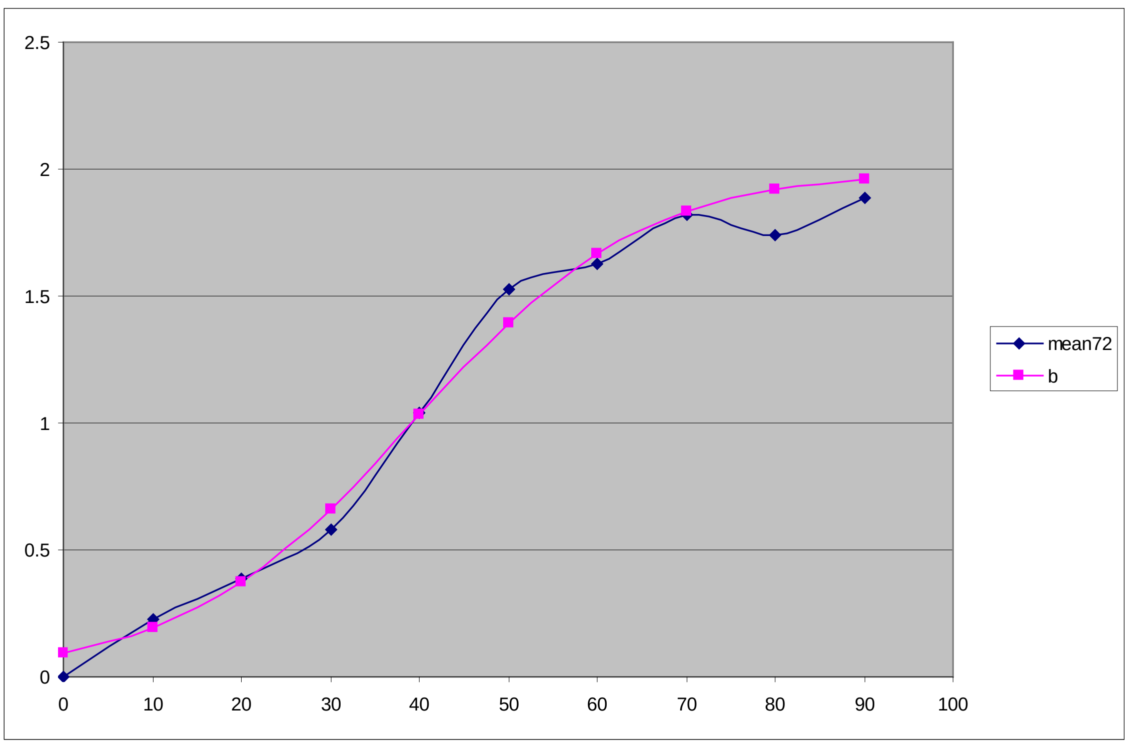


Figure 2-31 Problem 3 page 3

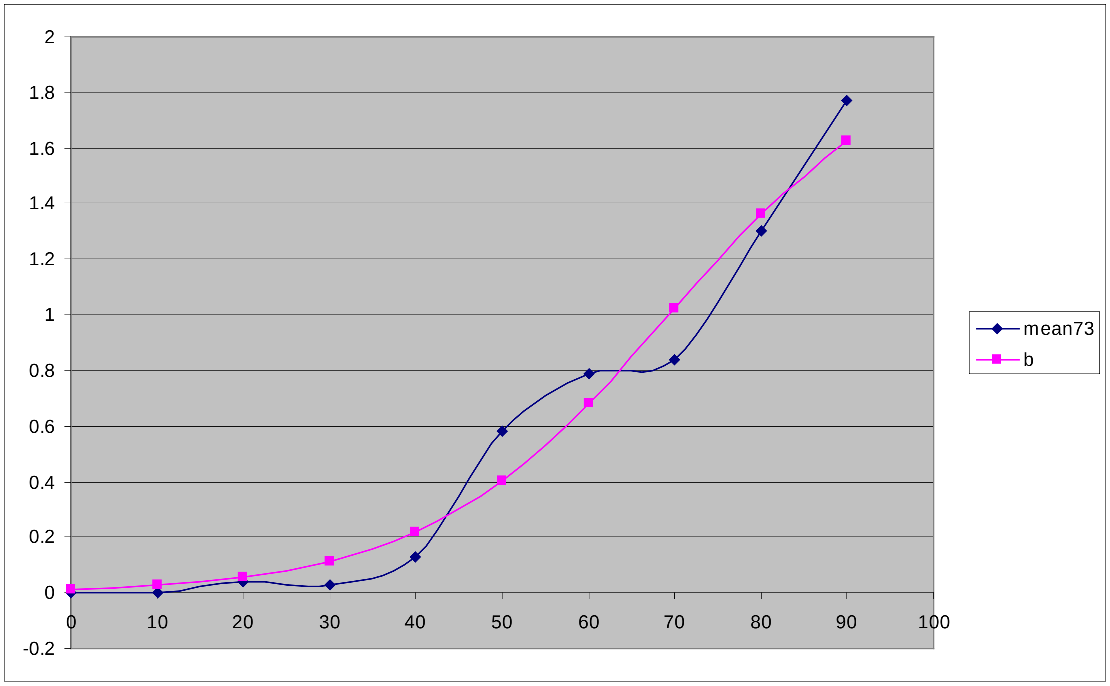


Figure 2-32 Problem 4 page 3

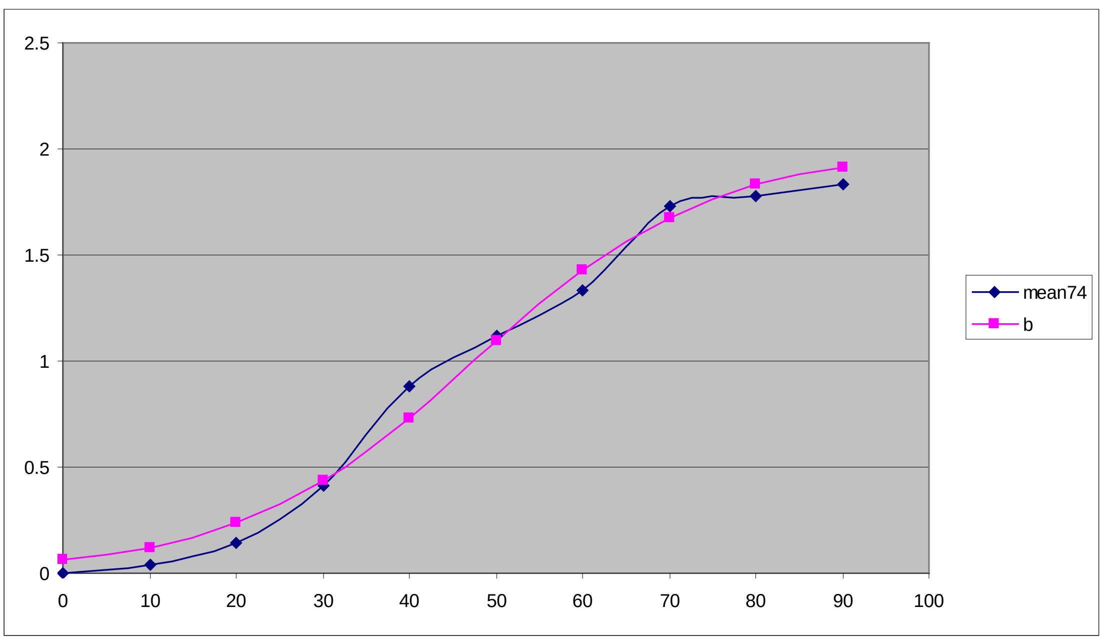


Figure 2-33 Problem 5 page 3

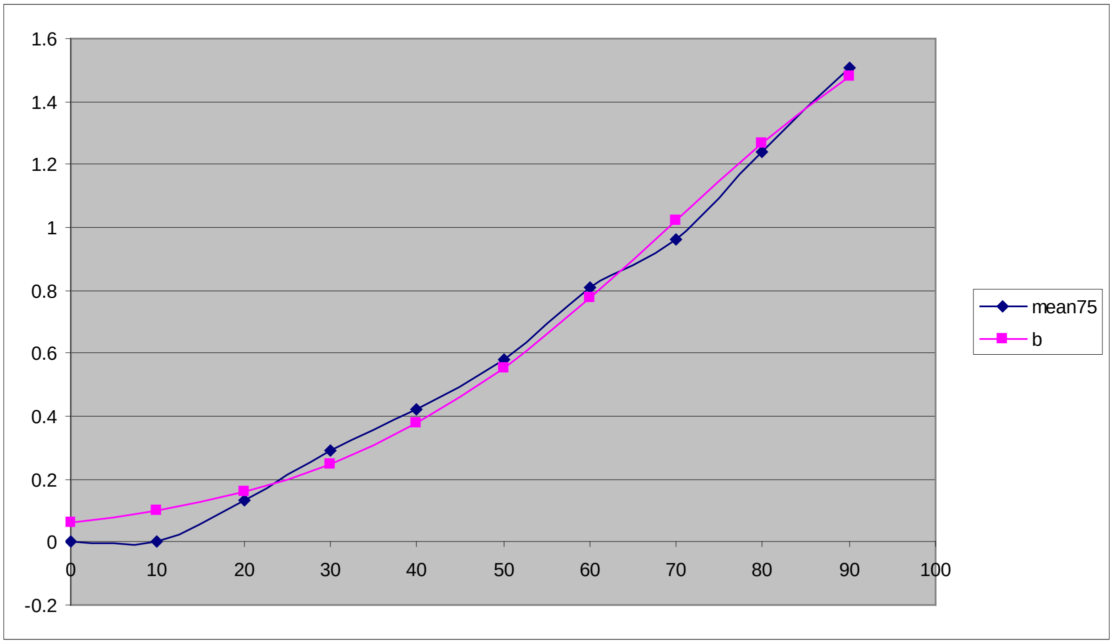


Figure 2-34 Problem 1 page 4

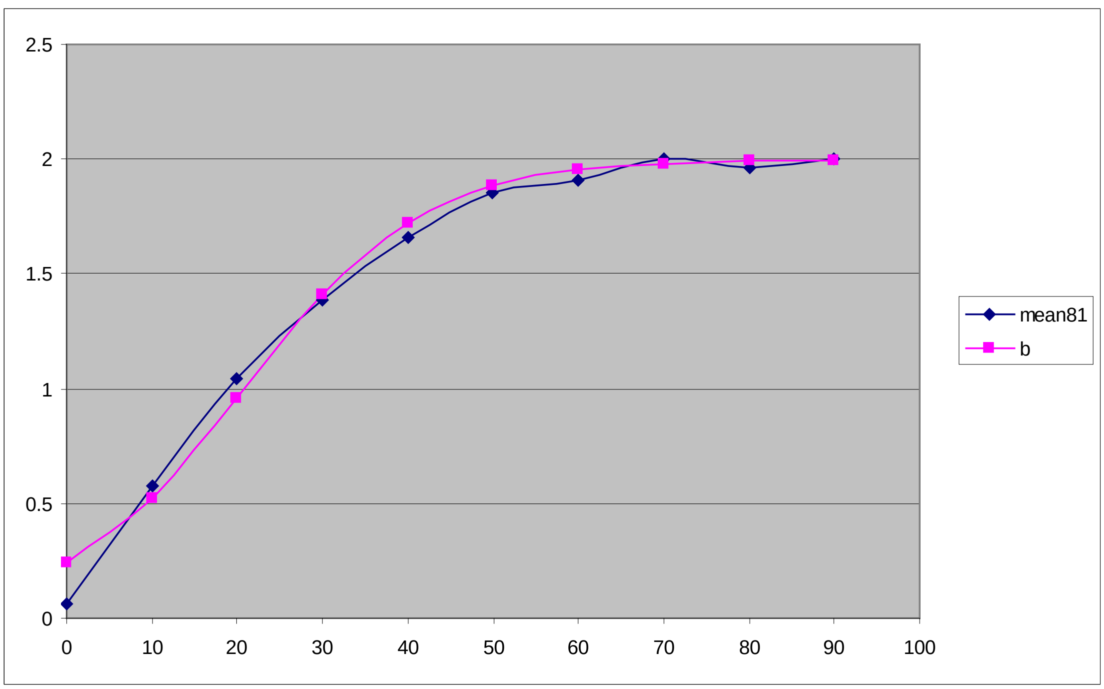


Figure 2-35 Problem 2 page 4

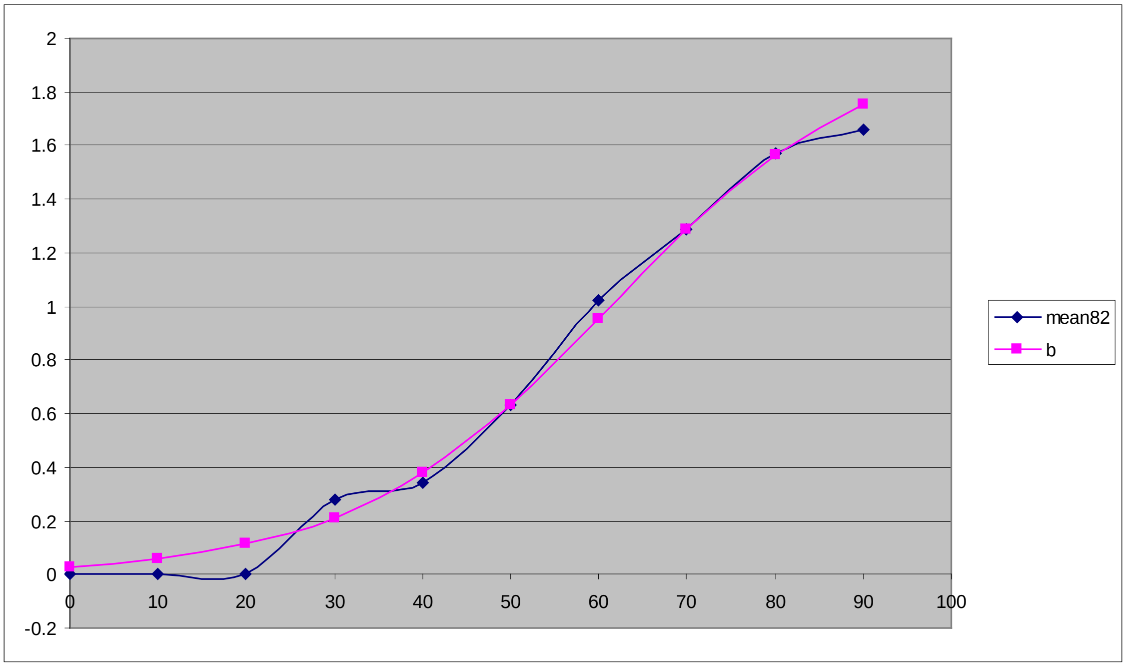


Figure 2-36 Problem 3 page 4

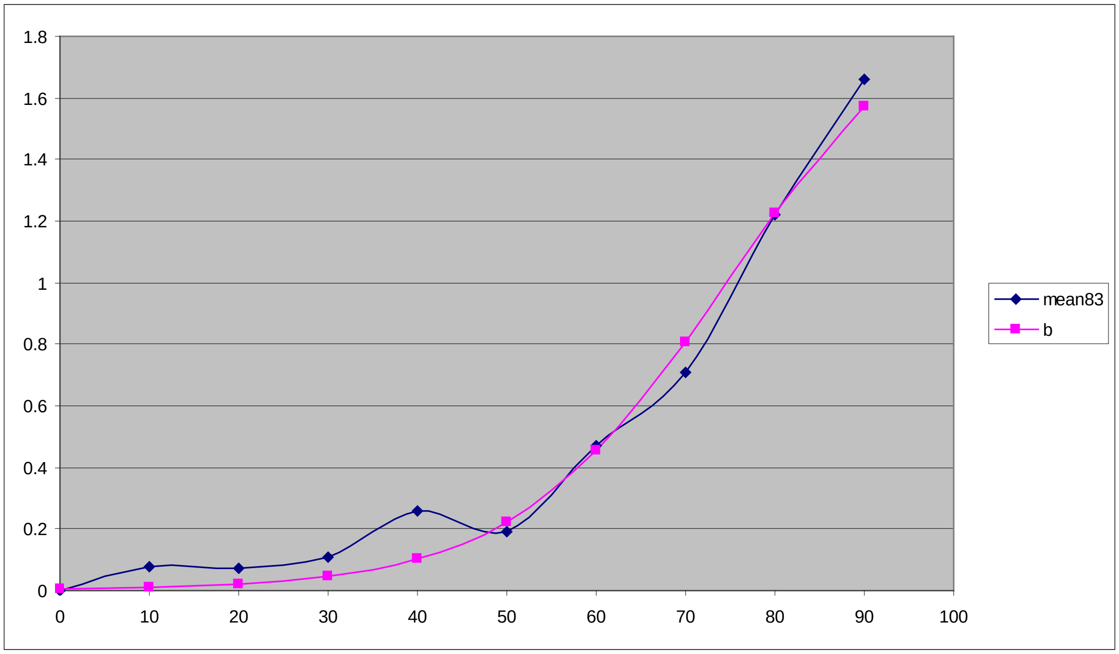


Figure 2-37 Problem 4 page 4

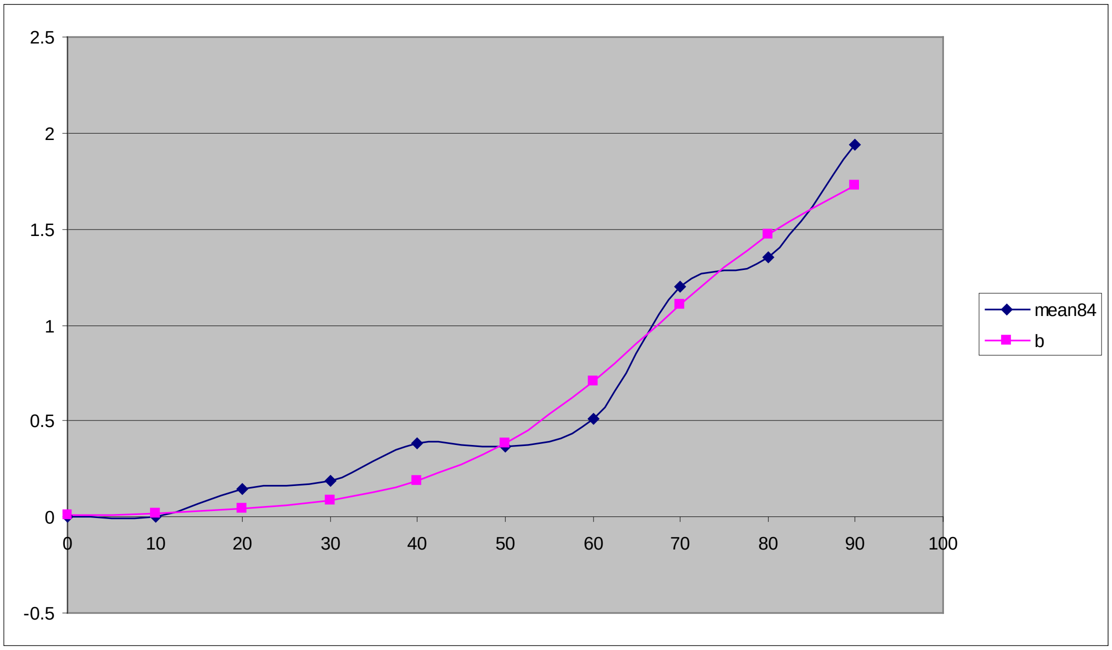
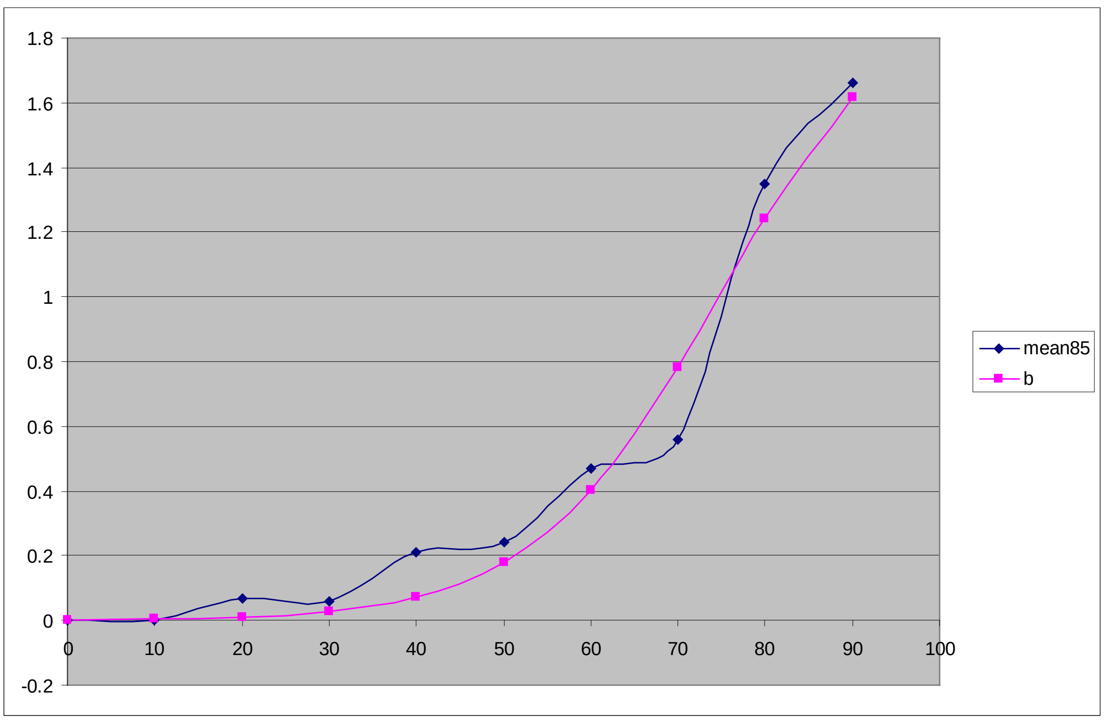
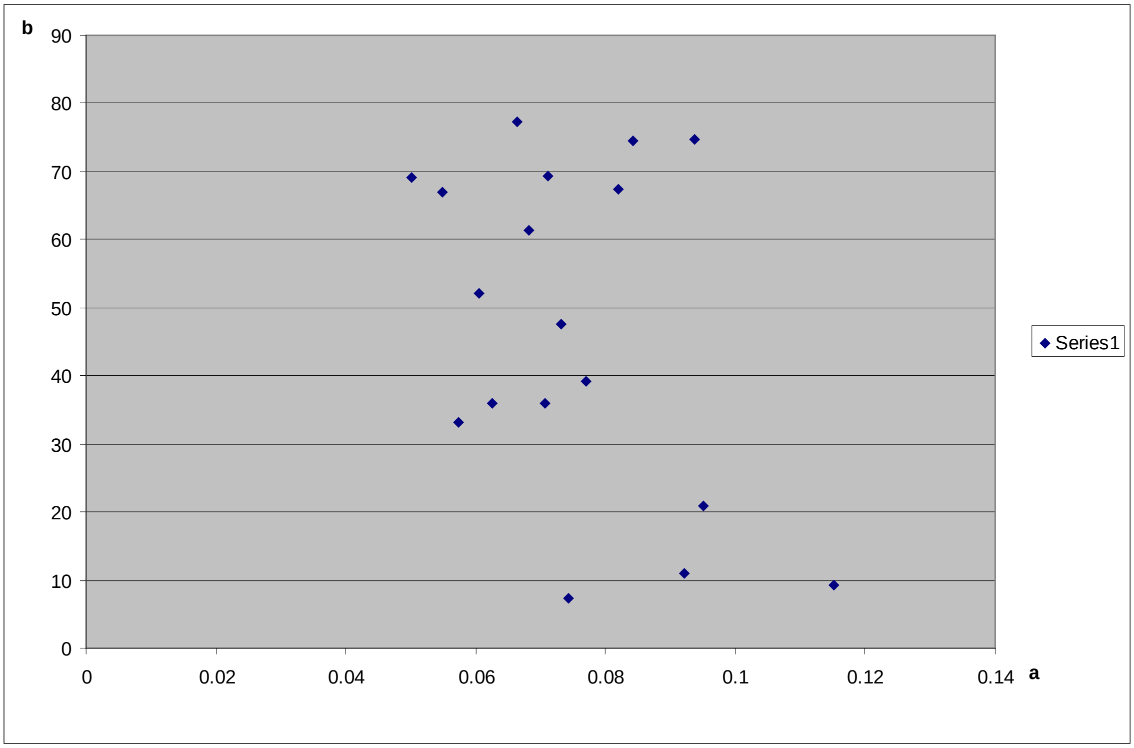


Figure 2-38 Problem 5 page 4



Next we plotted coefficients of the logistic curves for each problem.

Figure 2-39 Coefficients of logistic curves



Problem 4 on page 3 of Calculus exam (evaluating definite integral) and problem 2 on page 1 (finding derivative of function) provide little information about students' abilities since it was solved correctly by almost all students.

By inspection, it appears the problems are divided into 3 groups:

- Problems 1-3 from section 1, problems 1 from section 4;
- Problems 1 and 2 from section 2, problems 1,2,4 from section 3
- Everything else

CHAPTER 3 - Comparison with initial success in College Algebra

In order to evaluate the placement exam, we compared placement scores to scores on the first two midterm exams. We chose to focus on the first two exams both because we could get this data quicker and because the placement exam measures readiness at the start of class, so it should be a better predictor for the initial exams. By averaging over two exams, we improve the confidence that the exam scores properly represent student success.

During the fall semester, students enrolled in the Traditional College Algebra course took midterm exams on Sept. 15 and Oct. 13. For every midterm exam they could earn maximum 100 points and curves for grades were the following:

Exam 1:

A: 85-100

B: 73-84

C: 53-72

D: 40-52

Exam 2

A: 80-100

B: 67-79

C: 43-66

D: 32-42

Studio College Algebra students took midterm exams on the same dates and were able to earn 80 points with next grading curves:

Exam 1:

A:70-80

B:60-69

C:50-59

D:35-49

Exam 2:

A:65-80

B:55-64

C:40-54

D:25-39

Student performance was analyzed against the different placement variables as defined in chapter 2. We also included the ACT data that has traditionally been used for placement. The best results, which gave us most information, were given by IRT variables paired with the ACT data.

- actm is the math score on ACT
- actc is composite score on ACT
- basic is score of Basic Algebra component of placement exam
- interm is score of Intermediate Algebra component of placement exam
- college is score of College Algebra component of placement exam

A linear regression model for total score on the two exams in traditional college algebra as a function of the data above was computed using the R statistical language. The output ANOVA table is given below.

```
Call:
lm(formula = xtotal ~ actm + actc + basic + interm + college)

Residuals:
    Min      1Q  Median      3Q     Max 
-81.204 -16.760   2.028  21.025  73.600 
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -8.2074    13.8120  -0.594 0.552818
actm          3.0896     0.7023   4.399 1.52e-05 ***
actc          1.1463     0.7608   1.507 0.132938
basic         0.9214     1.0344   0.891 0.373820
interm        2.4368     0.6744   3.613 0.000355 ***
college       2.9060     0.6117   4.751 3.15e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 28.15 on 298 degrees of freedom
Multiple R-Squared: 0.4259,
Adjusted R-squared: 0.4162,
F-statistic: 44.21 on 5 and 298 DF,
      p-value: < 2.2e-16.
```

ANOVA table for exam 1 plus exam 2 as function of ACT data only

```
Call:
lm(formula = xtotal ~ actm + actc)

Residuals:
    Min      1Q  Median      3Q     Max
-83.798 -18.945   2.585  22.039  72.913

Coefficients:
            Estimate    Std.Error  t value  Pr(>|t|)
(Intercept)   0.1899    12.8227     0.015    0.988
actm          4.4125     0.7580     5.822    1.49e-08 ***
actc          1.2355     0.8412     1.469    0.143
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 31.34 on 301 degrees of freedom
Multiple R-Squared: 0.2811,
Adjusted R-squared: 0.2764
F-statistic: 58.86 on 2 and 301 DF,
p-value: < 2.2e-16
```

ANOVA table for exam 1 plus exam 2 as function of placement data alone (IRT variables)

```
Call:
lm(formula = xtotal ~ basic + interm + college)

Residuals:
    Min   1Q        Median      3Q      Max
-94.581   -20.288   4.359       21.442  78.280

Coefficients:
            Estimate    Std. Error t value  Pr(>|t|)
(Intercept)  67.0785    10.5991    6.329    8.99e-10 ***
basic         1.9789     1.1340    1.745    0.082 .
interm        3.2699     0.7360    4.443    1.25e-05 ***
```

```
college       3.5138     0.6696    5.247    2.92e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 31.18 on 300 degrees of freedom
Multiple R-Squared: 0.2909,
Adjusted R-squared: 0.2838
F-statistic: 41.02 on 3 and 300 DF,
p-value: < 2.2e-16
```

Based on these results, we dropped the actc and basic variables as contributing too little information and ran the analysis again with just actm, intermediate and college algebra.

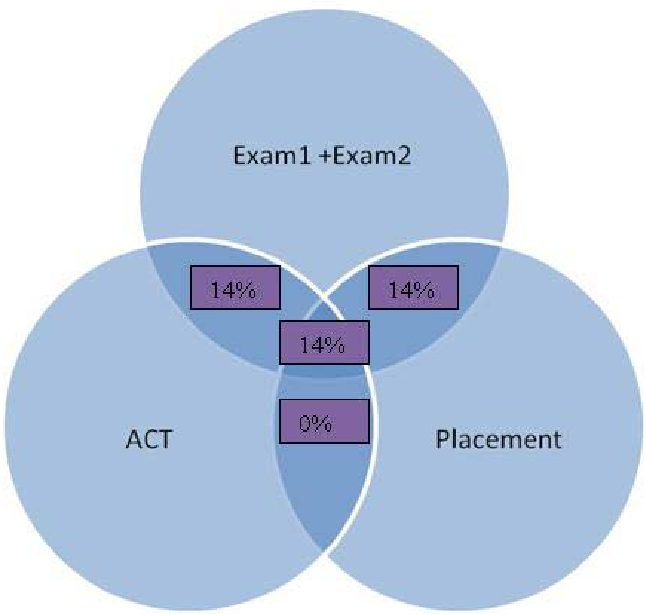
ANOVA table for exam 1 plus exam 2 as function of actm, interm and college

```
      Call:
lm(formula = xtotal ~ actm + interm + college)

Residuals:
 Min     1Q       Median  3Q      Max
-81.579 -18.097   1.786   20.314  73.702

Coefficients:
            Estimate    Std.Error   t value Pr(>|t|)
(Intercept)   6.3827    10.3069     0.619   0.536
actm          3.9263     0.4672     8.405   1.75e-15 ***
interm        2.7908     0.6023     4.634   5.36e-06 ***
college       2.8756     0.6094     4.718   3.65e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 28.2 on 300 degrees of freedom
Multiple R-Squared: 0.4202,
Adjusted R-squared: 0.4144
F-statistic: 72.47 on 3 and 300 DF,
p-value: < 2.2e-16
```

Here we have a model where all variables are highly significant and which explains 42% of the variation in student test scores. As both ACT alone and Placement data alone only explained about 28% of the variation in student test scores, adding the placement exams provides about half again as much about future students’ performance, than is given by ACT alone.

As can be seen, there is overlap between the information provided by the ACT data and the Placement exam. An analysis of this overlap by computing models with placement data and ACT data separately leads to the Venn Diagram below showing how much information each test provides. In particular, there is no overlap in information between the ACT data and the Placement data outside the information about College Algebra performance.

Figure 3-1 Venn Diagram for Traditional College Algebra



The results for studio were not as strong. The ANOVA table for the analysis of exam scores as a function of the ACT and Placement data is given below.

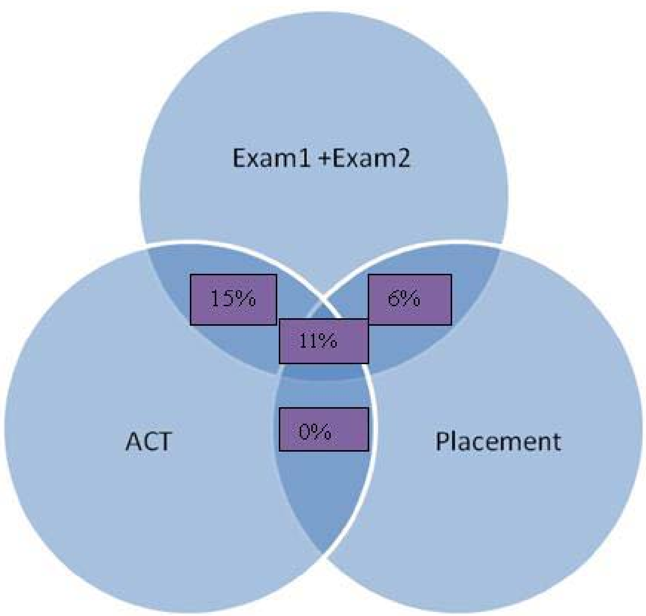
```
Call:
lm(formula = xtotal ~ actm + actc + basic + interm + college)

Residuals:
    Min      1Q  Median      3Q     Max
-62.181  -3.869   1.375   7.392  15.878

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.07847    6.33123   1.750 0.081669 .
actm         1.12869    0.32181   3.507 0.000558 ***
```

```
actc          0.69102    0.30940   2.233 0.026621 *
basic         0.06919    0.41516   0.167 0.867810
interm        1.03704    0.28372   3.655 0.000328 ***
college       0.11862    0.30362   0.391 0.696435
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10 on 202 degrees of freedom
Multiple R-Squared: 0.3397,     Adjusted R-squared: 0.3234
F-statistic: 20.79 on 5 and 202 DF,  p-value: < 2.2e-16
```

So for Studio College Algebra, the ACT and Placement data only explained about 33% of the variation, as compared to 42% for the Traditional sections. Analysis of the overlap provides a Venn diagram as above.

According to this data, adding the placement exams only added about a third more information about students' performance in Studio College algebra than the ACT data alone. Furthermore, the college variable is no longer significant, while the ACT composite score becomes mildly significant.

Figure 3-2 Venn Diagram for Studio College Algebra



After running the analysis above for exam scores in Traditional College Algebra as function of actm, intermediate and college algebra only, we found out, that estimate for ACTM is 3.9263 with a standard error of .4672. The estimate for intermediate is 2.7908 with a standard error of .6023. The estimate for college algebra is 2.8756 with a standard error of .6094. For ease in advising, we prefer a simple formula, and the coefficients 4, 3, and 3 are each well within a single standard deviation of the measure values. So we defined the "Placement Score" for students as 4*actm + 3*interm + 3*college. Also for simplicity in advising, we want to use the same formula for both Studio and Traditional sections and since most students take traditional and the results were stronger for traditional, we have used this one Placement Score for both versions of the course.

We now compute the Z-scores for student placement scores (Z scores are the values normalized to have mean 0 and variance 1). We plot the Z-scores on the x-axis and the probability of a student getting 100+ points on the sum of the first two exams on the y-axis. Actual data is shown in blue. We fit a logistic model for this data and plotted the model values in purple.

Figure 3-3 Z-scores of Traditional College Algebra

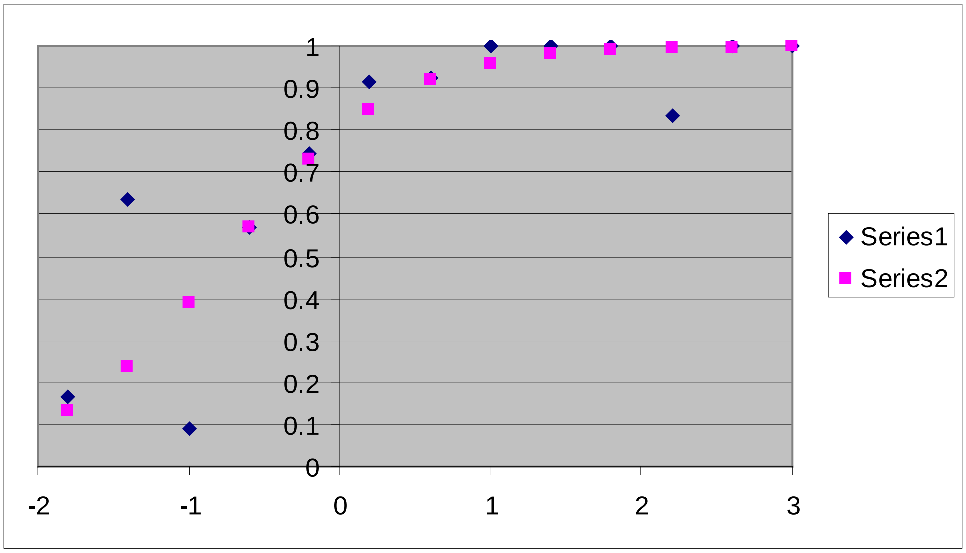


Table 3-1 Values

Z Score	Number	Over 50%	Zscore	Percent	Model	Norm SE
-1.8	6	1	-1.8	0.166667	0.131863	0.002967
-1.4	11	7	-1.4	0.636364	0.2379	0.526592
-1	22	2	-1	0.090909	0.39082	0.421886
-0.6	44	25	-0.6	0.568182	0.568686	1.69E-06
-0.2	43	32	-0.2	0.744186	0.730439	0.001239
0.2	46	42	0.2	0.913043	0.84777	0.028897
0.6	39	36	0.6	0.923077	0.919648	7.34E-05
1	34	34	1	1	0.95922	0.009697
1.4	27	27	1.4	1	0.979733	0.002134
1.8	18	18	1.8	1	0.990035	0.000421
2.2	6	5	2.2	0.833333	0.995126	0.06412
2.6	4	4	2.6	1	0.997623	1.13E-05
3	1	1	3	1	0.998842	1.34E-06

As it appears on graph, two points at -1 and -1.4 are located off the curve. We believe this happened because they involve data from relatively few students. The model is y=1/(1+e^(-a(x-b))), where a= 1.80090134126757 and b= -0.75353008323706

This model predicts that a student whose placement z-score is b has a 50-50 chance of scoring at least 50% total on the first two exams.

We repeat this analysis for students in the Studio College Algebra sections. Since the exams were out of 80 points instead of 100, we defined success on the exams as a total score of 90+ points (which was the minimum C).

Figure 3-4 Z-scores of Studio College Algebra

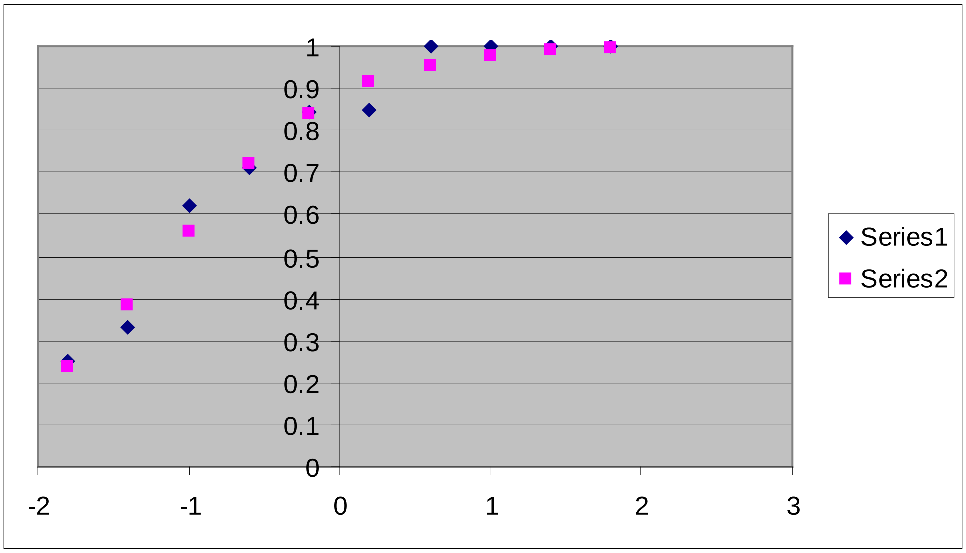


Table 3-2 Values

Z Score	Number	C or better	Z Score	Percent	Model	Norm SE
-1.8	4	1	-1.8	0.25	0.237707	0.000302
-1.4	15	5	-1.4	0.333333	0.386111	0.010788
-1	21	13	-1	0.619048	0.559199	0.016414
-0.6	31	22	-0.6	0.709677	0.719001	0.000484
-0.2	32	27	-0.2	0.84375	0.837687	0.000208
0.2	33	28	0.2	0.848485	0.912354	0.023433
0.6	19	19	0.6	1	0.954537	0.009009
1	24	24	1	1	0.976931	0.002607
1.4	7	7	1.4	1	0.988428	0.000354
1.8	3	3	1.8	1	0.994229	5.77E-05

Oddly, while we are using the Z-scores with coefficients derived from the traditional sections, the fit is better without any outliers for the studio sections.

The model is y=1/(1+e^(-a(x-b))), where a= 1.75400463617888 and b= -1.13563927254269

CHAPTER 4 - Conclusions

Our first question was whether adding a placement exam could improve our ability to properly place students in college algebra. Based on the results in the last chapter, we conclude that the answer to this question is yes. We are able to explain about half again as much variation in initial exam performance using the placement exam in addition to ACT data alone. However, we are only able to explain about 42% of the overall variation. Because of this, placement of students into classes shouldn't be based solely on the exam data. Cutoffs should serve advisory purposes only.

We can conclude that adding placement exam improved placement students in Studio College algebra. However, we had even less information, then for Traditional College Algebra We explained about one third again as much variation in initial exam performance using the placement exam in addition to ACT data alone. We were able to explain only about 33% of the overall variation.

Our second question was that if we could provide more information, what cutoffs should be recommended. According to received results, students have to have at least a 50-50 shot of initial success (by success we mean receiving grade C or higher on two midterm exams).Since we don't recommend mandatory cutoffs, it will be wise to consider the probability of success instead of a cut score. It appears that weaker students have greater probability of success in Studio College Algebra. Actually, it is true for all students, but the difference is significantly larger for weaker students.

Recommendations for future

While the exam provides useful information, it appears the exam could be improved by editing the problem selection.

Since problems which cover basic algebra skills, appeared to be easy and were correctly solved by majority of students, they don't provide any significant information about students abilities. Furthermore, what information they do provide is already available in the ACT data. So the Algebra placement exam should be rewritten with following modifications:

a) basic algebra problems need to be removed in order to save time and effort of students

b) more intermediate and college algebra problems should be added, which hopefully will help us to improve prediction of students success in algebra, therefore our recommendations on placement in different classes will be more helpful.

Students with weaker preparation may be advised into Studio College algebra section.

We hope that with these additions the predictions become more accurate. If so, it may be appropriate to reconsider at that point making a minimum placement score mandatory for students to enroll in College Algebra

References Or Bibliography

ACT Website: URL: http://www.actexampracticetests.com/icetests.com/ downloaded on 12/14/09.