MODEL STUDY OF THE SHEAR STRENGTH

OF REINFORCED CONCRETE BEAMS

by

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INTRODUCTION

1. Statement of the Problem

Structural model analysis has been employed in recent years to study the strength and behavior of reinforced and prestressed concrete beams because models are much cheaper to fabricate and test than full-size members. Although the model test results have not always agreed with available theories or with the results of tests on full-size beams, there is some evidence that models can be used successfully in concrete research. The primary purpose of the research described in this thesis was to develop additional information on the applicability of model investigations in research on concrete structures.

2. Scope

The investigation was limited in scope to an experimental study of the shear strength of reinforced concrete model beams with rectangular cross section. For the models, "Ultracal 30" was used as a substitute for cement and threaded rods were used for tensile reinforcement. No other types of reinforcement were studied. The main test variables were the amount of tensile reinforcement and the length of the shear span.

In order to check the applicability of the model material and modeling technique, the test results will be compared with available shear strength theories for reinforced concrete beams without shear reinforcement which are based on tests on full-sized reinforced concrete beams.

LITERATURE SURVEY

The following review of literature is presented in two parts. First, some recent papers dealing with model concrete beam studies will be summarized to indicate the interest in this topic and to show how much success has been attained with concrete models. The results of these studies will also be used in designing the experimental program of this investigation. Next, the results of shear tests of full-size reinforced concrete beams will be described. Empirical shear strength formulas based on these tests will be compared with the results of the model tests of this investigation.

1. Model Tests of Concrete Beams

Burton developed a technique to study small scale prestressed concrete structures in the inelastic range.[1] It was necessary to find a substitute material for the model and it was concluded that a mix consisting of plaster and Ottowa sand exhibited the required compressive strength.

The results of a study on model reinforced concrete members in simple flexure and torsion has been presented by Fan.[2] In his investigation numerous control tests were conducted in connection with plaster mix design and the properties of the reinforcing steel. The experimental results of the bending tests compared quite well with the predicted behavior according to Hognestad's Theory.

Chao investigated the application of small scale model analysis to prestressed concrete.[3] The first part of his paper consists of a theoretical study of the similitude requirements for determining the ultimate flexural strength by models using Whitney's method, and a presentation of experimental results. The second part is concerned with the use of model beams to investigate the relationship between the ultimate flexural strength of prestressed beams and the degree of prestressing using both underreinforced and overreinforced beams. The results from this investigation were insufficient to draw definite conclusions.

Cardenas studied the behavior of rectangular reinforced plaster model beams subjected to combined bending and torsion.[4] The test specimens, which contained both longitudinal and web reinforcement, were analyzed by Lessig's Theory. The test results for plain and longitudinally reinforced specimens tested in torsion were also compared with the elastic and plastic theories. The experimental results agreed reasonably well with theoretical results calculated using Lessig's Theory, however, they did not agree with either the plastic or the elastic theories.

Mason used small scale models to determine if model reinforced concrete beams could be useful in predicting the behavior of prototype beams in flexure and in torsion.[5] He used "Ultracal 30" as a substitute for concrete. The flexure specimens contained only threaded rods for longitudinal reinforcement. The torsion specimens contained threaded rods for longitudinal reinforcement as well as smooth wire for transverse reinforcement. The results from the flexure tests agreed very well with Whitney's Theory but the torsion test results did not agree with the ultimate torsional strength predicted by Lessig's Theory.

Magura has published a paper which describes the fabrication, instrumentation, and testing of 16 ordinary reinforced and 14 prestressed portland cement mortar beams. Material investigations and the development of laboratory techniques for structural testing are presented.[6] Tests of small mortar beams showed that they behave in accordance with known performance of full-size members. The results of this work on simple beams indicate that with proper care it is feasible to use mortar models to study behavior of more complex full-size concrete structures.

2. Shear Tests on Full-Size Reinforced Concrete Beams and Empirical Shear Strength Formulas

Kani has reported the results of 132 shear tests on rectangular full-size beams to determine the influence of the three basic parameters (concrete compressive strength, f'_c ; percentage of reinforcement, p ; and the shear arm ratio a/d).[7] The results showed: (1) the influence of compressive strength, f'_c, on shear strength was insignficant and could be ignored in the analysis of diagonal failure load or allowable shear stress; (2) the influenec of the percentage of main reinforcement, p, on shear strength was considerable; (3) the minimum value of bending moment at failure for beams of identical cross section was obtained in the vicinity of a shear arm ratio, a/d, of 2.5, and this was not influenced by p or f'_c ; and (4) the "relative beam strength" $Mu/\overline{M}_{fl}$ (where Mu = maximum bending moment at failure, $\overline{M}_{fl}$ = comparative flexural moment) is a much more suitable indicator of the beam strength than the "ultimate shear strength".

In Kani's report, it is emphasized that an anchorage failure produces a crack which is similar to the diagonal crack associated with a shear failure. This particular type of failure must be excluded, therefore all reinforcing bars had anchor plates at the ends of the beam.

Krefeld and Thursten tested over 200 full-size reinforced concrete beams subjected to concentrated and distributed loads.[8] The specimens were tested with and without strirrups and with various values of concrete strengths, steel ratios, effective depths and span lengths. From the test data the following formula for estimating the critical shear intensity was determined

$$v_c = 1.8\sqrt{f'_c} + 2600\ pVd/M\ , \qquad (1)$$

where $v_c = V/bd$ = critical shear stress

$p = A_s/bd$ = ratio of longitudinal tension reinforcement

A_s = area of longitudinal tension reinforcement

b = width of rectangular beam

d = distance from extreme compression fiber to centroid of tension reinforcement

V = total shear at section

M = bending moment

Clark tested full-size beams of two cross sections (8" x 18", 6" x 15"), four span lengths (6', 8', 9'-7", 10') and concrete strengths ranging from 2,000 to 6,000 psi.[9] Five different positions of concentrated loads were studied. The results showed that the shear capacity of a beam increases with the strength of the concrete when other factors are the same. For the same concrete strength the resistance to failure in diagonal tension increased as the loads were shifted from the center of the span toward the supports. The strength in shear varied as the compressive strength multiplied by a factor representing the ratio of depth of beam to destance from the plane of load to the plane of support. The resistance to shear was found to vary as the square root of the ratio of web reinforcement and the first power of the ratio of tensile reinforcement. The values of the maximum shearing stress observed in the beams are in agreement with the formula

$$v_c = 7000\,p + (0.12\,f'_c)\,d/a + 2500\sqrt{r} \qquad (2)$$

where v_c = calculated shearing stress at maximum load

a = distance from plane of the nearest concentrated load point to plane of the support

$r = A_v/bs$ = ratio of web reinforcement

s = spacing of stirrups

Av = area of two legs of a stirrup

For the investigation described in this thesis, r = 0 for no web reinforcement, therefore Clark's formula reduces to

$v_c = 7000\ p + (0.12\ f'_c)\ d/a$ ---(3)

Mathey and Watstein used high strength steel deformed bars having six different yield strength ranging from 40,000 to 100,000 psi in their investigation of the behavior of reinforced concrete beams failing in shear.[10] In these tests the shear span-to-depth ratio and the ratio of reinforcement were varied. It was determined that a linear relationship appeared to exist between the terms V_c/bdp and $(f'_c/p)(d/a)$ for the shear strength of beams without web reinforcement and subjected to two equal concentrated loads symmetrically placed :

$v_c = V_c/bd = 3.1\sqrt{f'_c}\ d/a + 4000\ p = 3.1\sqrt{f'_c}\ V_c\ d/M_{max.} + 4000\ p$ ------------(4)

where V_c = external shear force corresponding to the diagonal tension cracking load, and

$M_{max.}$ = maximum bending moment in the shear span.

This equation is a modification of an expression developed by Clark,[9] who was the first to express the calculated shear strength in terms of a/d. Mathey and Watstein's results indicated that the shear strength decreased roughly linearly as the corresponding steel stress increased in beams having the same shear span to depth ratio.

The shear formula currently used by ACI[11] is very similar to that of Krefeld and Thursten (Eq. 1), the only difference between the two formulas being slightly different values of the constants.

The ACI formula

$v_c = 1.9\sqrt{f'_c} + 2500\ pVd/M$ --(5)

is not intended to be used when M is less than Vd.

The results of the model beam tests described in this thesis will be compared with the empirical formulas of Clark (Eq. 3), Mathey and Watstein (Eq. 4) and ACI (Eq. 5). In addition, the conclusions of Kani based on his experiments will also be examined in the light of the results of the model tests.

EXPERIMENTAL PROGRAM

The topic of this research is the shear strength of reinforced concrete beams. This relatively simple problem was selected because the models were relatively easy to construct and because there is a large body of data available from full-size beam tests which can be used to check the results of the model tests. A rectangular cross section of constant dimensions was used and the only reinforcement was longitudinal tensile bars. Because it has been successfully used in previous model concrete beam studies, "Ultracal 30" was utilized as a substitute for cement. A constant compressive strength was attained by controlling the proportions of the mix. Thus, the only variable in the cross sectional strength was the amount of steel reinforcement. With simple end supports and two equal concentrated loads placed equidistant from midspan, the effect of varying shear spans was investigated by varying the distance between the supports.

1. Properties of Materials

The model materials selected for this investigation were "Ultracal 30" as a substitute for cement and threaded rods for deformed reinforcing bars.

A. Mortar

Fan determined that the initial setting of Ultracal plaster occurs about 10 to 15 minutes after mixing with water and that the necessary compressive strength can be obtained in two hours.[2] The strength then remains nearly constant for the first 24 hours, after which it increases sharply as shown in Fig. 1. Therefore, in this investigation, the specimens were removed from the forms 2 hours after placing, and tested after curing in a moist room for 22 hours.

From the results of previous research on material properties of gypsum plasters at Cornell University,[1,3] "Ultracal 30" was found to be a possible model concrete if its high value of modulus of rupture could be decreased. Therefore, Ottawa sand and fine limestone aggregate were added in order to decrease the tensile strength.

Preliminary tests were conducted to determine the proper mix to keep f'_c approximately equal to 3,000 psi. In these tests the average compressive strength was obtained from 2" dia. x 4" long cylinders. Before the cylinders were cast, fine limestone aggregate (between #8 and #16 sieves) and Ottawa sand (20-30) were dried in an oven for 48 hours, then cooled to room temperature (68° F to 77° F).

The quantities of dried aggregates, Ultracal, and water calculated for each batch were placed in a bowl and mixed mechanically for two minutes. The mortar was then placed into the cylinder molds, tamping with 25 blows on each of three equal layers. The forms were removed after 2 hours and the cylinders cured for 22 hours in a moist room (humidity 99%). After the curing period, the cylinders were tested. Each batch consisted of three cylinders.

The system used to identify the test batches, and the various mixes and water-Ultracal ratios investigated in the preliminary cylinder tests are summarized in Table 1. The results of the first series of tests (batches I_1- I_4), in which the batches were mixed as described above, are presented in Table 2 and plotted in Fig. 2.

From Fig. 2, it can be seen that batches I_2 and I_3 had compressive strengths less than 3,000 psi for the entire range of water-Ultracal ratios tested, while batches I_1 and I_4 had f'_c greater than 3,000 psi for at least some values of the water-Ultracal ratio. Since the mix for batch I_4 required more Ultracal 30, it was decided that the mix used for batch I_1 , 40% Ultracal 30, 40% limestone (between #8 and #16 sieves) and 20% Ottawa sand (20-30), provided the most desirable results.

The reason for the downward slope of the curves in Fig. 2 for batches I_1 , I_2 and I_3 , where the water-Ultracal ratio was less than 25%, is that the water in the mix was absorbed by the dried aggregate; therefore, there was not sufficient water available for setting the Ultracal plaster. However, the curve for batch I_4 in the figure is upward, because the water absorbed by the small amount of aggregate present did not exceed that required for setting the plaster.

The maximum percent deviation from the average value of f_c' for each batch is listed in the last column of Table 2. For one batch the maximum deviation was almost 30% and in four other cases it was about 20%. It was felt that better agreement among the cylinder test results from each batch should have been obtained when the ingredients were measured very carefully and the cylinders cast using the same procedure each time. With this in mind, the mixing procedure was modified in an attempt to get more consistent results.

In the modified mixing procedure, the quantities of dried aggregates and water calculated for each batch were placed in a wetted container for 24 hours. The container was sealed with a cap in order to minimize evaporation during the immersing period. After wetting the mixer bowl, the Ultracal 30 and aggregates (with the absorbed water) were placed in the bowl and mixed for two minutes. Then the cylinders were cast using the same procedure which described previously.

Using the modified mixing procedure and the mix previously selected based on the first test series, a second series of cylinders was prepared and tested. The mix and water-Ultracal ratios for the four batches of this series are summarized in Table 1 while the test results are presented in Table 3 and plotted in Fig. 3.

The maximum deviation from the average value of f'_c for the second test series ranged from 3% to 15%, a considerable improvement over the values obtained from the first series. Based on these results, the modified mixing procedure was adopted for all of the later model beam tests. From the results of the second series of tests shown in Fig. 3, it was determined that a mix consisting of 40% Ultracal, 40% limestone aggregate, and 20% Ottawa sand (by volume) with a water-Ultracal ratio (by weight) of 32 : 100 is very workable and could be expected to have a compressive strength of about 3,000 psi. This mix was therefore adopted for all of the later beam tests.

B. Reinforcing Bars

The longitudinal reinforcement used in the models consisted of #8-32 or #6-32 threaded rods. The tensile area of a #8-32 bar is 0.0124 $in.^2$, while that of a #6-32 bar is 0.0078 $in.^2$ This type of steel rod is commercially available, as mentioned by Fan,[2] the threaded rods as obtained from the producer may not exhibit a definite yield point and yield plateau, and therefore it was necessary to anneal the rods. This was accomplished by placing them in an oven at 950° F for two hours.

Annealed threaded rod samples were tested in tension and a load-elongation curve was determined using an automatic recorder and a 2-in. extensometer. A cross head speed of 0.025 in./minute was used for these tests. The modulus of elasticity and the yield point were determined for each sample tested from the load-elongation curve. The stress-strain curves shown in Figs. 4 and 5 were constructed from the average values of three samples of #6 and #8 rods which were tested.

2. Fabrication Procedures for Model Beams

As previously mentioned, the modified mixing procedure developed for the second series of preliminary cylinder tests was used for all of the batches of the model beam test program. The mortar mix, based on the results of the same series of cylinder tests consisted of 40% Ultracal, 40% limestone aggregate, and 20% Ottawa sand (by volume), with a water-Ultracal ratio of 32:100 by weight. The beam specimens were fabricated in plexiglass forms and tamped on each of three layers. In order to avoid deflections of the threaded rods after setting them in the forms, a U-type wire was used to support them. Two beams and three quality control cylinders were cast at the same time from each batch. At the end of two hours the cylinders and the test beams were removed from the forms and cured in the moist room (humidity 99%) for 22 hours. After this curing period the specimens were ready for testing.

3. Design of Experiment

The experimental program consisted of 30 model beams and is summarized in Tables 4 and 5. The beams all had the same cross section, namely 1" x 2" (see Fig. 6). f'_c and f_y were maintained as constant as possible. f'_c was approximately 3,000 psi and f_y for the #6-32 threaded rods was 97,000 psi, and for the #8-32 rods was 80,500 psi. One main variable was the percentage of reinforcement, with the following values included in the program : p_1 = 0.92% (use two #6-32 threaded rods) ; p_2 = 1.46% (use two #8-32 threaded rods); and p_3 = 2.19% (use three #8-32 threaded rods). The values of 1.46% and 0.92% were chosen because they could be obtained with an even number of available reinforcement bars. The 2.19% value was chosen to ensure that enough space was available between threaded rods for the aggregate to pass through.

Two equal concentrated loads were applied equidistant from the center line of the span so that with varying span, the shear span "a" would vary (see Fig. 7). The distance between the two concentrated loads was 6", and the values of the shear span variable were : $A_1 = a/d = 1$, $A_2 = a/d = 2.5$, $A_3 = a/d = 3$, $A_4 = a/d = 5.5$, $A_5 = a/d = 6.5$. These values were chosen because, according to Kani's tests,[7] the tests should result in $Mu/\overline{M_{fl}} = 100\%$ at these values.

4. Test Procedure

The setup for the beam tests is shown in Fig. 7. Loads were applied with a Riehle Model FS-20 universal testing machine as shown in Fig. 8. The load was applied gradually at a cross head speed of 0.025 in./minute. The load required to initiate the first crack and the ultimate load were recorded. The development of cracks and the crack patterns were also observed and recorded.

In Kani's report,[7] it was emphasized that an anchorage failure produces a crack which is similar to the diagonal crack associated with shear failure. In order to exclude this particular type of failure, all reinforcing bars were anchored by nuts at the ends of the beams prior to testing.

TEST RESULTS

1. Cylinder Tests

Three quality control cylinder tests were conducted for each of the fifteen mortar batches prepared for the model beam investigation. The results of these tests are presented in Table 6. In the fourth column of the table, values of f_c' established for each batch as the average of the three cylinder test results are listed. These f_c' values range from 2,660 psi to 3,110 psi with an average of 2,935 psi, which is quite close to the desired 3,000 psi. Within each batch, the results were very consistent, as indicated by the maximum percent deviation from the batch f_c' , shown in the last column of the table.

2. Model Beam Tests

The model beam test results are summarized in Table 7. In the second and third columns of the table are listed the experimental values of P_{cr} , the load at which cracks were first observed, and P_u , the ultimate load. In many cases P_{cr} was the same as P_u , while in the other cases P_u was generally only slightly higher than P_{ur}. The average of the Pu-values for the two beams in each batch has been calculated for later comparison with the available shear strength formulas. The percentage deviations from the average Pu-values, shown in the last column in the table, range from 0.3% to 8.1%, which again indicates that the mixing and testing procedures resulted in very consistent results within each batch.

For the beam test setup in this investigation, the ultimate shear force Vu is equal to the ultimate load Pu (see Fig. 7). The ultimate shear stress v_u can be calculated from the relationship v_u = Vu/bd. In Fig. 9, a plot of the ultimate shear stress v_u versus the shear span-to-depth ratio a/d is shown for the model beam test results. As indicated by this figure, the ultimate shear stress v_u decreases with increasing a/d values for a constant steel reinforcement percentage p, and for a constant a/d ratio, v_u increases with increasing p-values. It should be noted that a point has not been plotted in Fig. 9 for batch A_5P_2 , since both beams in this batch failed in bending rather than shear.

Photographs of the specimens showing the crack patterns are shown in Figs. 10, 11 and 12. For the specimens in Fig. 10, the steel percentage was 0.92% and all of the specimens failed in shear. The specimens shown in Fig.11 had a p-value of 1.46% and all of them except specimen A_5P_2 failed in shear. The crack pattern for A_5P_2 developed in the pure moment region between the load points and is therefore classified as a bending failure. Finally, all of the beams in Fig. 12, with p = 2.19%, failed in shear. It should be noted that in each test the specimens were strained well beyond the ultimate load to exaggerate the crack patterns.

COMPARISON OF TEST RESULTS AND THEORY

There are three empirical shear strength formulas based on full scale beam tests (Eqs. 3, 4 and 5) which can be compared with the model beam test results. The predicted ultimate loads based on these formulas and calculated using the f'_c-values from the control cylinder tests for each batch (see Table 6) are shown in Table 8 along with the predicted ultimate load according to the Whitney's flexure formula.[5] As indecated by asterisks in the table, the Whitney's flexure formula predicts a lower ultimate load than the various shear strength formulas for several batches although a bending failure was observed in only one of these batches.

Table 9 presents a comparison of the test data with the formulas of Clark (Eq. 3), Mathey and Watstein (Eq. 4) and the ACI (Eq. 5). In each case the ratio of the experimentally determined ultimate load to the predicted ultimate load is also shown. From these correlation ratios it is seen that Clark's formula provides a much better prediction of the model test results than the formulas of Mathey and Watstein and the ACI. Specimens A_5P_2 have not been included in the table since they both failed in bending.

A graphical comparison of the model test results with the three shear strength formulas is shown in Fig. 13, which is a plot of the correlation ratios for each of the batches tested. The figure provides a further indication of the accuracy of Clark's formula in predicting the test results, with the plotted points falling quite close to a vertical line through a correlation ratio of 1.0. With regard to Mathey and Watsstein's formula, the points are scattered around a vertical line corresponding to a correlation ratio slightly less than 2.0. This indecates that Mathey and Watstein's formula predicts shear strengths which are on the order of one-half of the shear strengths of the test beams.

Perhaps one of the more interesting results of the entire investigation is the pattern of the correlation ratios corresponding to the ACI shear strength formula in Fig. 13. These points are clustered about three inclined lines corresponding to the three values of the steel reinforcement percentages included in the investigation. From this result it can be concluded that, at least for the range of variables included in this investigation, the form of the shear strength equation used by both Clark and Mathey and Watstein,

$$v_c = K_1 p + K_2 d/a \, f(f'_c),$$

where K_1 and K_2 are constants and $f(f'_c)$ indicates a function of f'_c , is a much more satisfactory form than that of the ACI shear strength equation,

$$v_c = K_3 f(f'_c) + K_4 p \, d/a \ .$$

As a further indication of the agreement between the test results and Clark's shear strength formula, a plot of shear strength v_c versus the shear span-to -depth ratio a/d is presented in Fig. 14. In the figure, three curves corresponding to the three p-values tested have been plotted using Clark's formula. The test results have also been plotted in the figure and the test points fall quite close to the appropriate curves in most cases.

Kani's test results showed that (1) the influence of compressive strength, f'_c , onshear strength was insignificant and could be ignored in the analysis of diagonal failure load or allowable shear stress, (2) the influence of the percentage of main reinforcement, p, was considerable, (3) the minimum value of bending moment at failure for beams of identical cross section was obtained in the vicinity of a shear arm ratio, a/d, of 2.5, and this was not influenced by p or f'_c , and (4) the "relative beam strength" $Mu/\overline{M}_{fl}$ is a much more suitable indicator of the beam strength than the "ultimate shear strength". Kani's conclusions (1), (3), and (4) can not be compared with the model tests described in this thesis because the f'_c values (2,500 psi, 3,800 psi, and 5,000 psi) and p values (0.5%, 0.8%, 1.88% and 2.8%) for Kani's tests are all different from the model tests (f'_c = constant = 3,000 psi, p = 0.92%, 1.46% and 2.19%). Hower, the model test results agree with Kani's second conclusion that p has a considerable influence on v_u.

CONCLUSIONS

The following conclusions can be drawn from the results of this investigation :

1. Very consistent results can be obtained from small scale model beam tests using "Ultracal 30" as a substitute for cement and threaded rods for tensile reinforcement, provided proper precautions are taken in preparing and testing the specimens.
2. The test results agreed very closely with Clark's formula for the shear strength of beams without web reinforcement.
3. The formula of Mathey and Watstein predicted shear strengths on the order of one-half of the experimentally observed values.
4. For the range of variables investigated, the test results indicate that the form of Clark's and Mathey and Watstein's shear strength formulas is superior to the form of the ACI shear strength formula.
5. The results of the shear tests agree with Kani's conclusion that the steel reinforcement percentage has a considerable influence on the ultimate shear strength.

RECOMMENDATIONS FOR FURTHER RESEARCH

After studying the problem of the shear strength of small scale model reinforced concrete beams having rectangular under-reinforced and over-reinforced sections without web reinforcement, it is felt that the following topics could be profitably investigated in furture research projects :

1. It appears desirable to extend these studies to include shear arm ratios a/d less than one.
2. Other values of f'_c , such as 2,500 psi, 3,800 psi, and 5,000 psi, could be tested, and the results compared with Kani's conclusion that the influence of compressive strength, f'_c , on shear strength is insignificant and can be neglected in the analysis of diagonal failure load or allowable shear stress.
3. It would be interesting to compare the results of tests on model rectangular beams with web reinforcement with Clark's shear strength formula.
4. Model T-beams and box-section beams could be tested to determine if model analysis gives reasonable results for these cross sections.

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NOTATION

a	= distance from plane of the nearest concentrated load point to plane of the support, in.
As	= area of longitudinal tension reinforcement, $in.^2$
Av	= area of two legs of a stirrup, $in.^2$
b	= width of rectangular beam, in.
d	= distance from extreme compression fiber to centroid of tension reinforcement, in.
E	= modulus of elasticity, psi
f'_c	= compressive strength of concrete, psi
f_y	= yield stress of longitudinal reinforcement, psi
K_1 , K_2 K_3 , K_4	= constants
L	= span of beam, in.
M	= bending moment, in-lbs
$\overline{M}_{fl}$	= comparative flexural moment, in-lbs
$M_{max.}$	= maximum bending moment in the shear span, in-lbs
Mu	= maximum bending moment at failure, in-lbs
P	= total load on beam, lbs
p	= As/bd = steel ratio of longitudinal tension reinforcement
Pa	= ultimate load calculated by ACI's formula, lbs
P_c	= ultimate load calculated by Clark's formula, lbs
P_{cr}	= load at which cracks were first observed, lbs
P_m	= ultimate load calculated by Mathey and Watstein's formula, lbs
P_t	= maximum test load, lbs
P_u	= ultimate load, lbs
r	= Av/bs = ratio of web reinforcement

v = shear stress, psi

V = total shear at section, lbs

v_c = shear stress corresponding to the diagonal tension cracking load, psi

v_u = shear stress corresponding to the ultimate load, psi

V_c = external shear force correspondig to the diagonal tension cracking load, lbs

Table 1. Designation of Preliminary Cylinder Tests

First Series (Aggregates Not Immersed)

I = percentage of Ultracal - limestone - sand (by volume)

I_1 = U : L : S = 40 : 40 : 20

I_2 = U : L : S = 40 : 30 : 30

I_3 = U : L : S = 50 : 30 : 20

I_4 = U : L : S = 60 : 25 : 15

A = weight of water = 1/3 of Ultracal by weight

B = weight of water = 1/4 of Ultracal by weight

C = weight of water = 1/5 of Ultracal by weight

Curing Period : 24 hours

Specimens : 2" dia. x 4" long cylinders

Second Series (Aggregates Immersed)

I_5 = U : L : S = 40 : 40 : 20

D = weight of water = 35% of Ultracal by weight

E = weight of water = 30% of Ultracal by weight

F = weight of water = 25% of Ultracal by weight

G = weight of water = 32% of Ultracal by weight

Curing Period : 24 hours

Specimens : 2" dia. x 4" long cylinders

Table 2. Results of First Series of Preliminary Cylinder Tests (Aggregates Not Immersed)

Cylinder No.	Load (lb.)	f'_c (psi)	Average f'_c (psi)	Deviation from Average f'_c (psi)	Max. % Deviation from Average f'_c
I_1-A-1 I_1-A-2 I_1-A-3	9320 9300 8800	2970 2960 2800	2910	60 50 110	3.8%
I_1-B-1 I_1-B-2 I_1-B-3	9700 11400 10400	3090 3630 3310	3343	253 287 33	8.6%
I_1-C-1 I_1-C-2 I_1-C-3	6700 7300 6500	2140 2320 2070	2177	37 143 107	6.6%
I_2-A-1 I_2-A-2 I_2-A-3	7500 6500 6700	2390 2070 2140	2200	190 130 60	8.6%
I_2-B-1 I_2-B-2 I_2-B-3	10000 7300 7800	3180 2320 2480	2660	520 340 180	19.5%
I_2-C-1 I_2-C-2 I_2-C-3	4500 3400 3200	1430 1080 1020	1180	250 100 160	21.2%
I_3-A-1 I_3-A-2 I_3-A-3	6900 6700 7000	2200 2130 2230	2187	13 57 43	2.6%
I_3-B-1 I_3-B-2 I_3-B-3	8900 6500 9600	2830 2070 3050	2650	180 580 400	21.8%
I_3-C-1 I_3-C-2 I_3-C-3	5000 7000 7000	1590 2230 2230	2010	420 220 220	20.9%
I_4-A-1 I_4-A-2 I_4-A-3	8200 8600 8300	2600 2740 2640	2660	60 80 20	3.0%
I_4-B-1 I_4-B-2 I_4-B-3	10400 8600 10000	3310 2740 3180	3076	234 336 104	11.0%
I_4-C-1 I_4-C-2 I_4-C-3	9900 16500 12000	3150 5250 3820	4070	920 1180 250	29.0%

Table 3. Results of Second Series of Preliminary Cylinder Tests (Aggregates Immersed)

Cylinder No.	Load (lb)	f'_c (psi)	Average f'_c (psi)	Deviation from Average f'_c (psi)	Max. % Deviation from Average f'_c
I_5-D-1	7000	2230		83	
I_5-D-2	7800	2480	2313	167	7.2%
I_5-D-3	7000	2230		83	
I_5-E-1	10200	3250		84	
I_5-E-2	10200	3250	3166	84	5.2%
I_5-E-3	9400	3000		166	
I_5-F-1	12500	3980		534	
I_5-F-2	10000	3180	3446	266	15.5%
I_5-F-3	10000	3180		266	
I_5-G-1	8900	2840		80	
I_5-G-2	9500	3020	2920	100	3.4%
I_5-G-3	9100	2900		20	

Table 4. Designation of Test Beams

p %	Span Length L (inches) (a/d)				
	9.4" (1.0)	14.5" (2.5)	16.2" (3.0)	24.7" (5.5)	28" (6.5)
0.92	A_1P_1-1	A_2P_1-1	A_3P_1-1	A_4P_1-1	A_5P_1-1
	A_1P_1-2	A_2P_1-2	A_3P_1-2	A_4P_1-2	A_5P_1-2
1.46	A_1P_2-1	A_2P_2-1	A_3P_2-1	A_4P_2-1	A_5P_2-1
	A_1P_2-2	A_2P_2-2	A_3P_2-2	A_4P_2-2	A_5P_2-2
2.19	A_1P_3-1	A_2P_3-1	A_3P_3-1	A_4P_3-1	A_5P_3-1
	A_1P_3-2	A_2P_3-2	A_3P_3-2	A_4P_3-2	A_5P_3-2

Table 5. Details of Test Beams

Beam No.	Average f'_c (psi)	Shear Arm Ratio a/d	As (one bar)($in.^2$)	Percentage of Reinforcement p (%)	f_y (Ksi)
A_1P_1	3110	1.0	#6 - 32 0.0078	0.92 (Two #6 - 32)	97.0
A_2P_1	2760	2.5	#6 - 32 0.0078	0.92 (Two #6 - 32)	97.0
A_3P_1	3033	3.0	#6 - 32 0.0078	0.92 (Two #6 - 32)	97.0
A_4P_1	2886	5.5	#6 - 32 0.0078	0.92 (Two #6 - 32)	97.0
A_5P_1	2780	6.5	#6 - 32 0.0078	0.92 (Two #6 - 32)	97.0
A_1P_2	3060	1.0	#8 - 32 0.0124	1.46 (Two #8 - 32)	80.5
A_2P_2	3030	2.5	#8 - 32 0.0124	1.46 (Two #8 - 32)	80.5
A_3P_2	2970	3.0	#8 - 32 0.0124	1.46 (Two #8 - 32)	80.5
A_4P_2	2750	5.5	#8 - 32 0.0124	1.46 (Two #8 - 32)	80.5
A_5P_2	3010	6.5	#8 - 32 0.0124	1.46 (Two #8 - 32)	80.5
A_1P_3	3080	1.0	#8 - 32 0.0124	2.19 (Three #8 - 32)	80.5
A_2P_3	2970	2.5	#8 - 32 0.0124	2.19 (Three #8 - 32)	80.5
A_3P_3	3050	3.0	#8 - 32 0.0124	2.19 (Three #8 - 32)	80.5
A_4P_3	2660	5.5	#8 - 32 0.0124	2.19 (Three #8 - 32)	80.5
A_5P_3	2880	6.5	#8 - 32 0.0124	2.19 (Three #8 - 32)	80.5

Table 6. Results of Control Cylinder Tests

Batch No.	Cylinder Failure Load (~~psi~~ lb.)	f'_c for Cylinder (psi)	f'_c for Batch (psi)	Deviation from Batch (psi)	Max. % Deviation from Batch f'_c
A_1P_1	10300 10200 8800	3280 3250 2800	3110	170 140 310	9.9%
A_2P_1	8600 8400 9000	2740 2680 2860	2760	20 80 100	3.6%
A_3P_1	9600 9200 9800	3050 2930 3120	3030	20 100 90	3.3%
A_4P_1	9800 8600 8800	3120 2740 2800	2890	230 150 90	8.0%
A_5P_1	9200 8600 8400	2930 2740 2680	2780	150 40 100	5.4%
A_1P_2	9600 9200 10000	3050 2930 3180	3060	10 130 120	4.3%
A_2P_2	10000 9200 9400	3180 2930 3000	3030	150 100 30	5.0%
A_3P_2	9600 8800 9600	3050 2800 3050	2970	80 170 80	5.7%
A_4P_2	9000 9600 8300	2870 2740 2650	2750	120 10 100	4.4%
A_5P_2	9400 9000 10000	3000 2870 3180	3010	10 140 170	5.7%
A_1P_3	9400 9600 10000	3000 3050 3180	3080	80 30 100	3.3%
A_2P_3	9000 9400 9600	2870 3000 3050	2970	100 30 80	3.4%
A_3P_3	9300 9500 10000	2960 3030 3180	3050	90 20 130	4.3%
A_4P_3	8600 8200 8200	2740 2610 2610	2660	80 50 50	3.0%
A_5P_3	9300 9000 8800	2960 2870 2800	2880	80 10 80	2.8%
Ave.			2935		

$A = 3.1416 \text{ in.}^2$

Table 7. Results of Model Beam Tests

Beam No.	P_{cr} (lb)	P_u (lb)	Average P_u (lb)	Deviation from Average P_u (lb)	% Deviation from Average P_u
A_1P_1-1 A_1P_1-2	751 793	797 838	818	21 20	2.5%
A_2P_1-1 A_2P_1-2	420 400	526 483	504	22 21	4.4%
A_3P_1-1 A_3P_1-2	350 334	362 343	352	10 9	2.8%
A_4P_1-1 A_4P_1-2	265 266	273 268	270	3 2	1.1%
A_5P_1-1 A_5P_1-2	208 227	209 227	218	9 9	4.1%
A_1P_2-1 A_1P_2-2	940 955	990 995	992	2 3	3.0%
A_2P_2-1 A_2P_2-2	466 480	490 541	515	25 26	5.1%
A_3P_2-1 A_3P_2-2	410 422	441 452	446	5 6	1.3%
A_4P_2-1 A_4P_2-2	324 311	324 311	318	6 7	2.2%
A_5P_2-1 A_5P_2-2	287 294	287 294	291 *	4 3	1.4%
A_1P_3-1 A_1P_3-2	1030 1005	1090 1025	1057	33 32	3.1%
A_2P_3-1 A_2P_3-2	550 560	580 606	593	13 13	2.2%
A_3P_3-1 A_3P_3-2	440 470	466 548	507	41 41	8.1%
A_4P_3-1 A_4P_3-2	390 388	390 388	389	1 1	0.3%
A_5P_3-1 A_5P_3-2	300 305	300 305	303	3 2	1.0%

* : Failure by Moment

Table 8. Predicted Ultimate Load Values

Batch No.	Moment	Shear		
	Whitney[5]	Clark (Eq. 3.)	M & W (Eq. 4.)	ACI (Eq. 5.)
	lb.	lb.	lb.	lb.
A_1P_1	1250	876	419.6	258.0
A_2P_1	491	393	203.6	217.4
A_3P_1	418	371	187.6	223.4
A_4P_1	228	255 *	134.2	212.4
A_5P_1	190	232 *	123.6	207.1 *
A_1P_2	1635	938	461.0	238.0
A_2P_2	650	496	253.0	237.2
A_3P_2	532	442	230.0	232.4
A_4P_2	273	324 *	176.2	213.3
A_1P_3	1785	1046	519.2	320.0
A_2P_3	695	592	310.6	251.8
A_3P_3	595	550	289.2	246.6
A_4P_3	290	422 *	233.4	216.0
A_5P_3	260	412 *	226.4	220.8

* : Predicted Failure Load Due to Bending Controls

Table 9. Comparison of Model Test Results with Shear Strength Formulas

Beam No.	Max. Test Load P_t	Clark P_c	$\frac{P_t}{P_c}$	M & W P_m	$\frac{P_t}{P_m}$	ACI P_a	$\frac{P_t}{P_a}$
A_1P_1	818 lb.	876 lb.	0.935	419.6 lb.	1.950	258 lb.	3.170
A_2P_1	504	393	1.280	203.6	2.480	217.4	2.320
A_3P_1	352	371	0.950	187.6	1.878	223.4	1.578
A_4P_1	270	255	1.058	134.2	2.010	212.4	1.270
A_5P_1	218	232	0.940	123.6	1.765	207.1	1.052
A_1P_2	992	938	1.058	461.0	2.150	283	3.510
A_2P_2	515	496	1.040	253.0	2.035	237.2	2.170
A_3P_2	446	442	1.010	230.0	1.940	232.4	1.920
A_4P_2	318	324	0.982	176.2	1.805	213.3	1.491
A_1P_3	1057	1046	1.010	519.2	2.038	320.0	3.300
A_2P_3	593	592	1.000	310.6	1.910	251.8	2.360
A_3P_3	507	550	0.922	289.2	1.755	246.6	2.050
A_4P_3	389	422	0.922	233.4	1.667	216.0	1.800
A_5P_3	303	412	0.735	226.4	1.335	220.8	1.370
Ave.			0.989		1.908		2.097

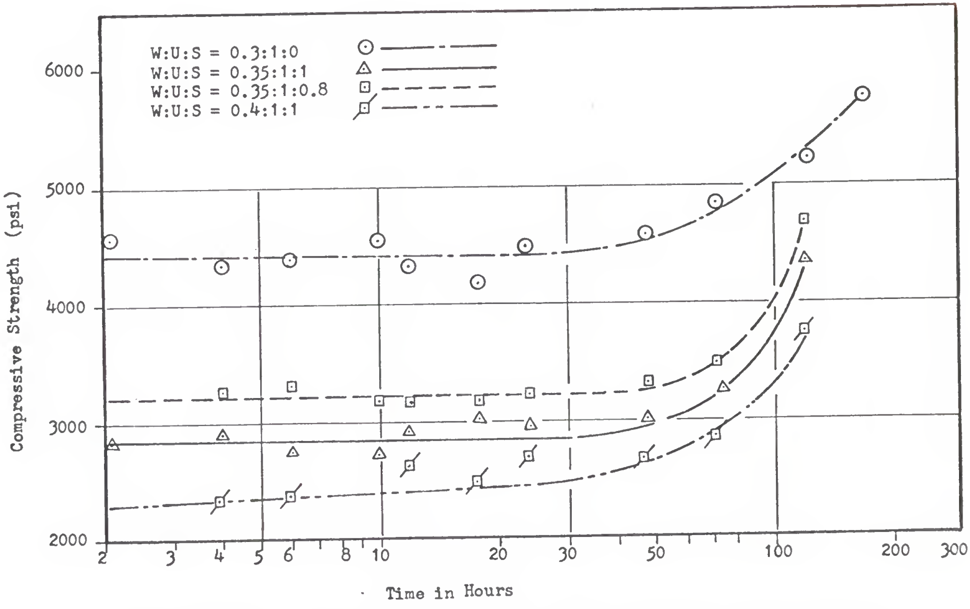


Fig. 1. Variation of Compressive Strength of Plaster Mix With Time (from Ref. 2)

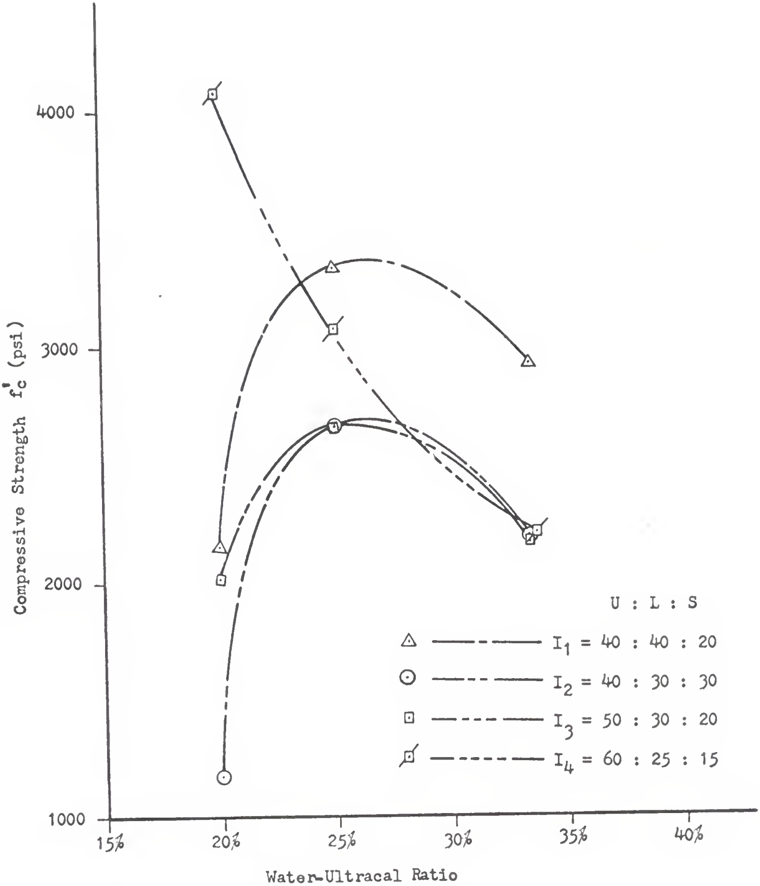


Fig. 2. Cylinder Strength Versus Water-Ultracal Ratio

(First Preliminary Test Series - Aggregates Not Immersed)

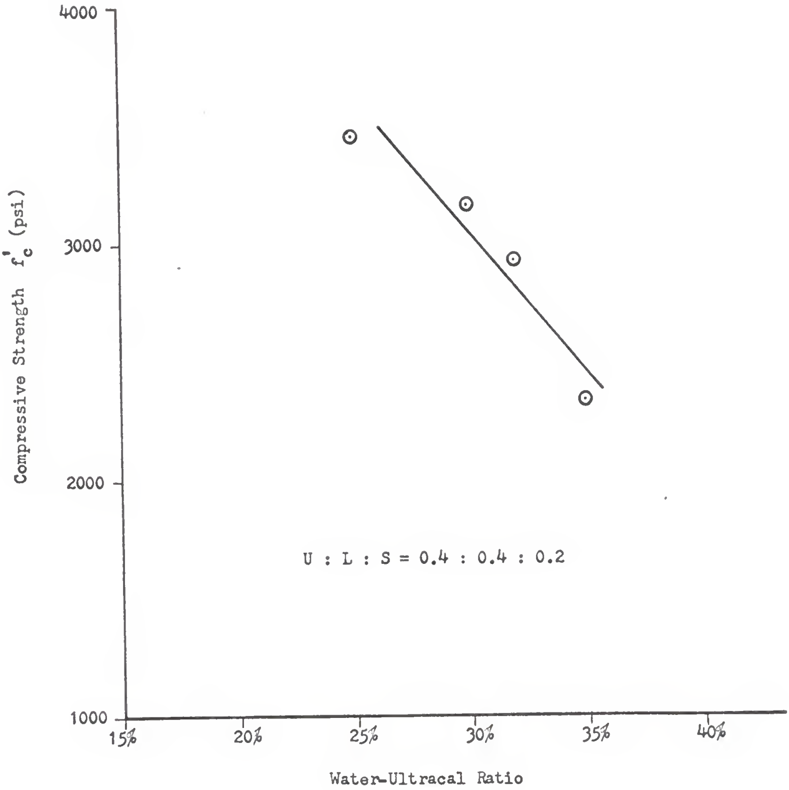


Fig. 3. Cylinder Strength Versus Water-Ultracal Ratio

(Second Preliminary Test Series - Aggregates Immersed)

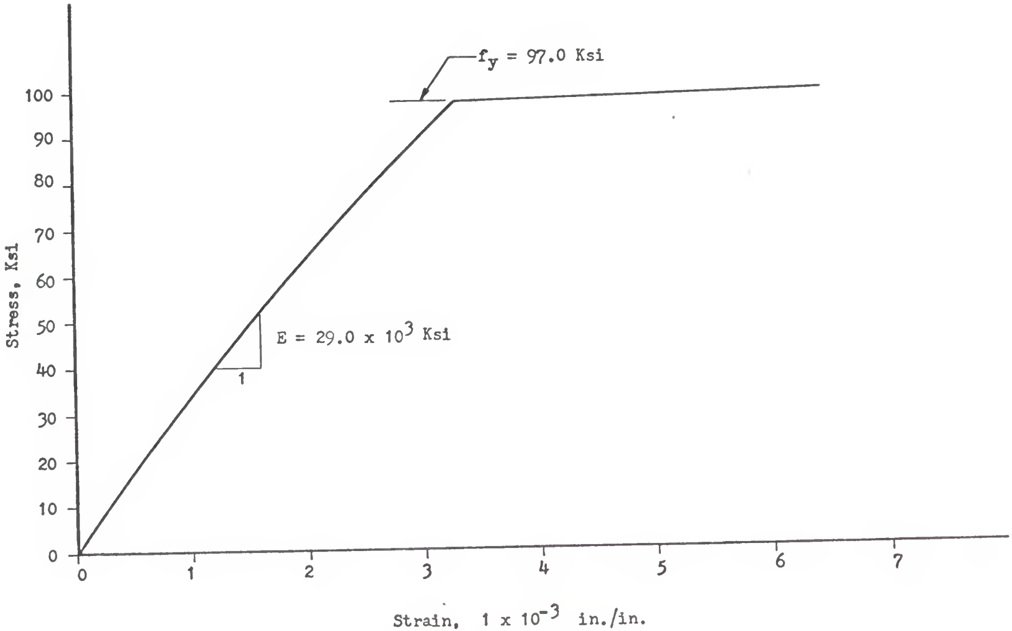


Fig. 4. Stress-Strain Curve for #6 Threaded Rod

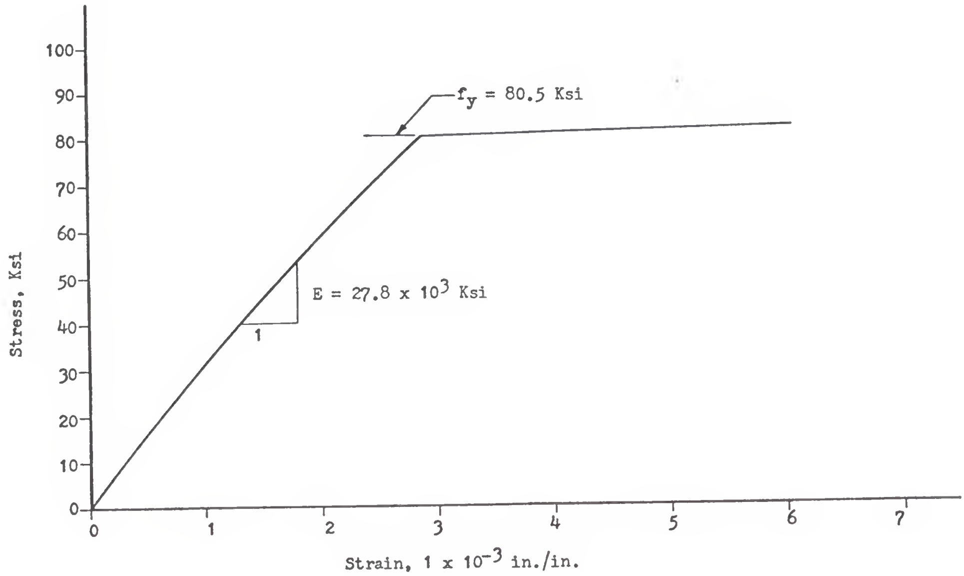
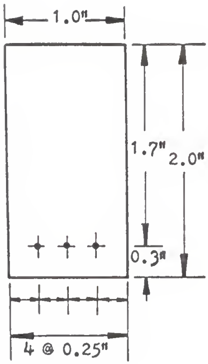
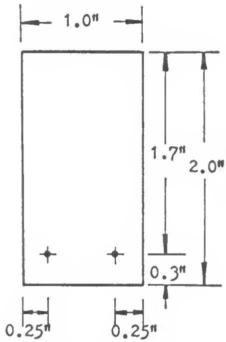


Fig. 5. Stress-Strain Curve for #8 Threaded Rod



(a). Beam Cross Section with Four Reinforcing Bars



(b). Beam Cross Section with Two Reinforcing Bars

Fig. 6. Cross Sections of Beam Specimens

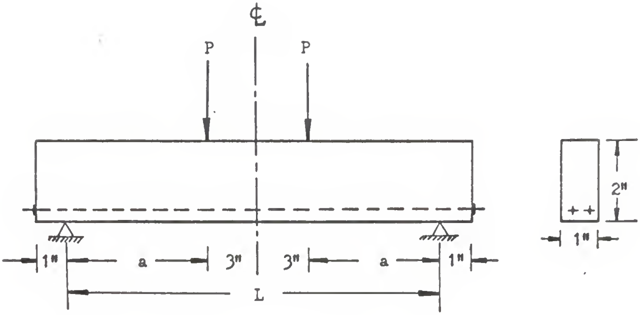


Fig. 7. Setup for Beam Tests

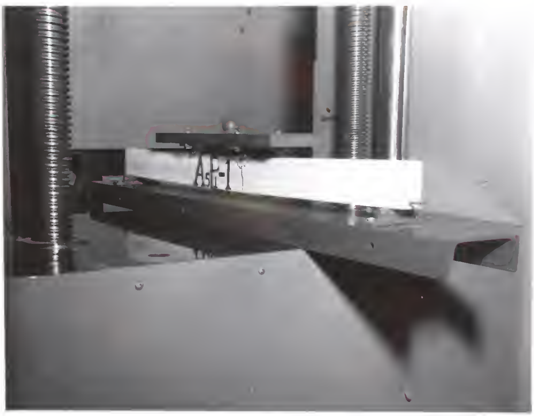


Fig. 8. Specimen In Testing Machine

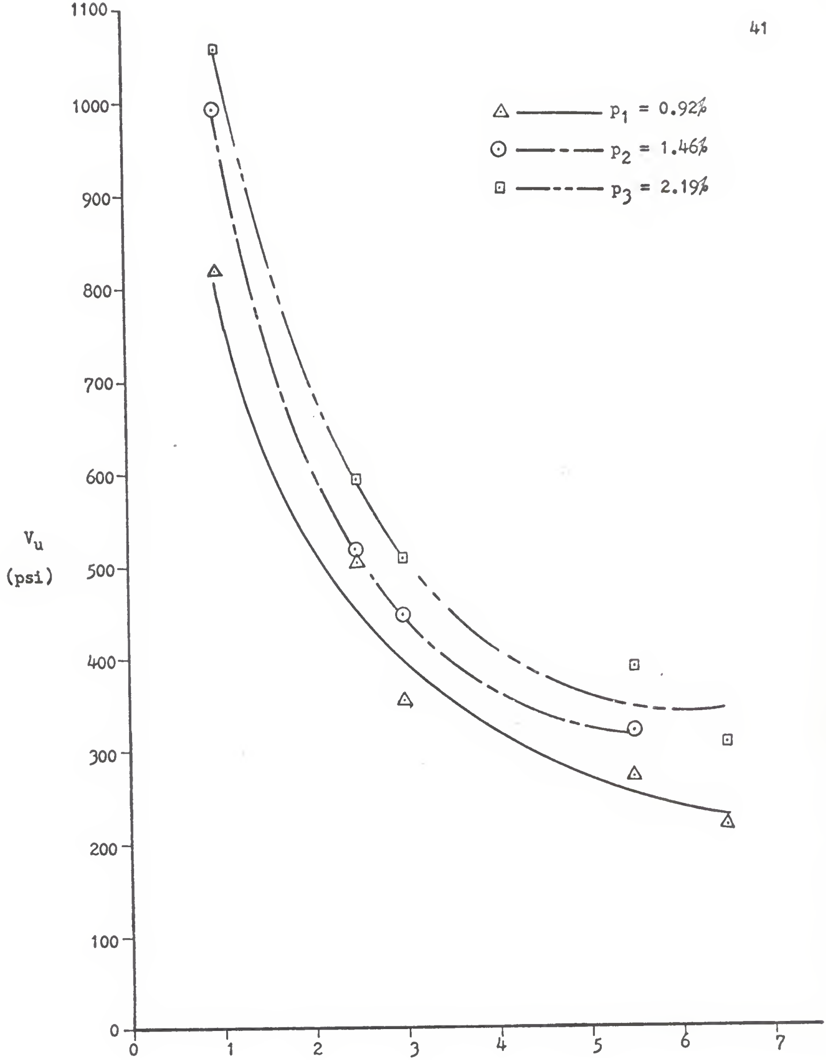


Fig. 9. Beam Test Results

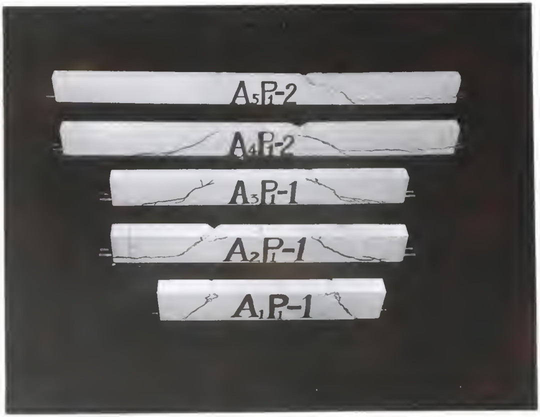


Fig. 10. Crack Formation of P_1 Series

Fig. 11. Crack Formation of P_2 Series

Fig. 12. Crack Formation of P_3 Series

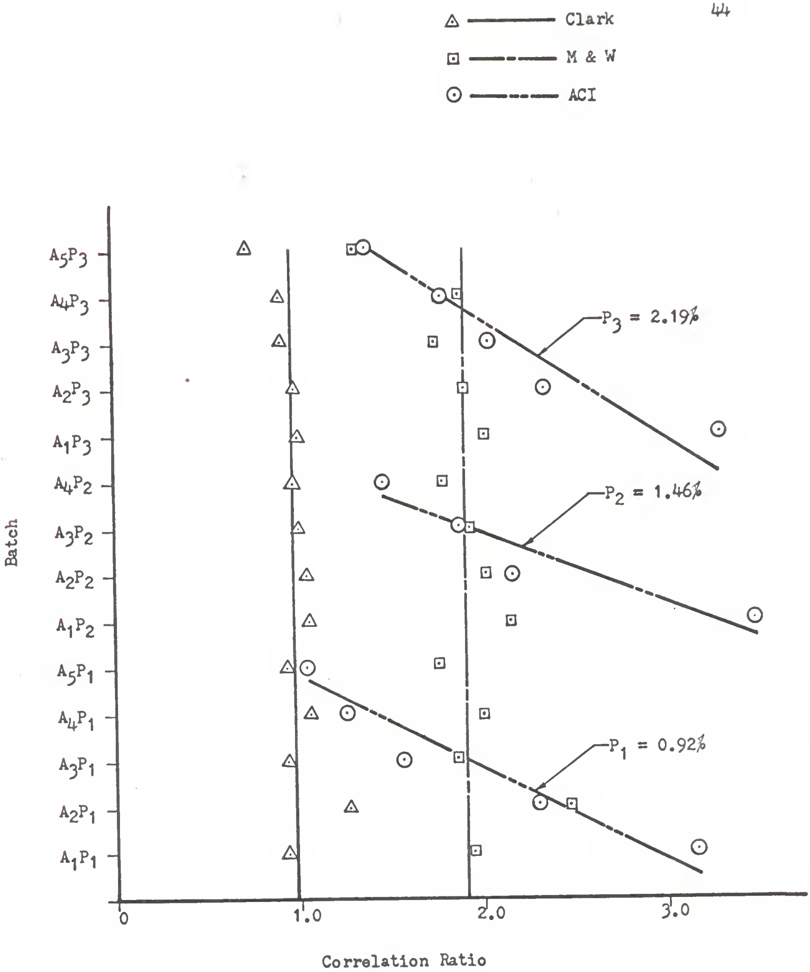


Fig. 13. Comparison of Model Test Results with Shear Strength Formulas

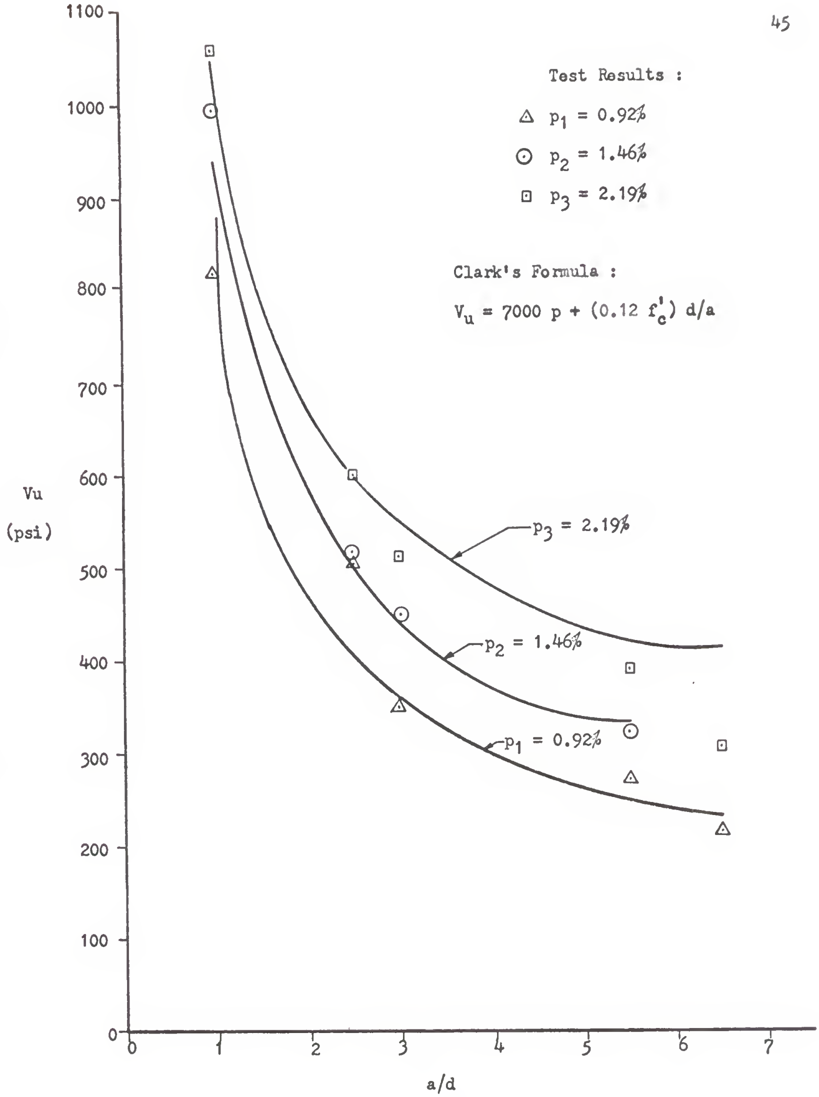


Fig. 14. Comparison of Test Results with Clark's Shear Strength Formula

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BIOGRAPHICAL SKETCH

The writer was born in Formosa, China, in 1940. He was graduated from the Tainan First High School in 1956 and attended National Taiwan University, receiving the Bachelor of Science Degree in 1963. During the following year, he served in the ROTC of Chinese Army.

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MODEL STUDY OF THE SHEAR STRENGTH

OF REINFORCED CONCRETE BEAMS

by

HSIEH, JUI-FU

B. S., National Taiwan University, 1963

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1969

ABSTRACT

The primary purpose of this investigation was to develop additional information on the applicability of model investigations in research on concrete structures. A total of 30 model beams were included in the investigation. For the models, "Ultracal 30" was used as a substitute for cement and No.6 & No.8 threaded rods were used for longitudinal tensile bars. The cross section of the model beams was rectangular with constant dimensions and no web reinforcement was used. The variables were the amount of tensile reinforcement and the length of shear span. A constant compressive strength was attained by controlling the proportions of the mix. All the beams were tested with simple end supports and two equal concentrated loads placed equidistant from midspan.

The test results are compared with three empirical shear strength formulas. From this comparison it is concluded that Clark's shear strength formula for beams without web reinforcement provided a good prediction of the test results, and that the form of the ACI shear strength formula is not as satisfactory as that of Clark's and Mathey and Watstein's formulas. It is also concluded that very consistent results can be obtained using small scale model beams provided that proper precautions are taken in preparing the specimens and conducting the tests.