APPLICATION OF EXPONENTIAL SMOOTHING TO FORECASTING SALES OF FEED

by

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INTRODUCTION

Modern industry must know projections of its different functions into the future to accomplish the requirements of an efficient planning. In order to meet this, industry can guess, assume or predict what is going to happen from now on. But these are only subjective ways to estimate future values.

Forecasting, on the other hand, is a technique of estimating future values of any system by the use of objectives computations.

Forecasts are derived by use of an adequate model and from data out of the past. If past data is unavailable, predictions, not forecasts, are made using subjective techniques to estimate future values.

A forecasting model provides a reasonably accurate forecast as long as the actual process does not change. However, should the process change with time, the forecasting system must detect the changes when they occur, and it must either modify the parameters of the model automatically or alert the investigator that something has changed within the forecasting system. Whenever it is known that the process is changing or will change in the future the forecaster should modify his model accordingly.

Forecasting systems normally produce a difference between the forecasted value and the one actually observed in the future. The forecasting techniques try to minimize these differences, which are called forecast errors. Using error-theory it is possible to adjust the forecasting system while it is being tested or is in operation.

Forecasting must distinguish between the errors produced by the purely random fluctuations of the forecasted system itself and the errors produced by a pattern which is irrelevant to past data. As an example, Brown (3), in 1963, cited that on January 25, 1962, the United States launched a rocket that was supposed to land cameras on the moon. The path was very carefully calculated (forecasted); all known factors were taken into account. Even though astronomical orbits can be calculated with extreme accuracy, the rocket missed the moon by some 20,000 miles or about 10% of the distance between the moon and the earth. As it turned out, very small controllable variations in factors governing the initial path of the space craft were amplified to produce a substantial error in the forecasting.

Almost a decade after that mishap we can see how scientists and engineers have learned from experience. Rockets today are sent back and forth from the moon on a very precise schedule, and nobody is surprised.

As general rules, applicable to any forecast system, Plossl and Wight (12) fix the following principles about forecast error:

1) Forecasts are more accurate for larger groups of items.

2) Forecasts are more accurate for shorter periods of time.

3) Every forecast should include an estimate of errors.

Forecasts are made using several methods, each of which is adequate for any kind of activity and over any range of future time (forecasting lead time).

The late Charles S. Ross*, one of the founders of the Econometric Society, classified business forecast into one of five categories:

1) Naive. These are unsophisticated, scientifically uninstructed projections. They include. . .random methods, guesses, straight-line or mathematical trend projections, autocorrelations, and harmonic analysis.

2) Leading Indexes. These are indexes or time series which usually (or always) change before a change in the index or aggregate to be forecasted. For examples, shipments of goods precede earnings; and industrial contract awards always precede industrial construction.

3) Comparative Pressures. These methods usually involve ratios or differences. For examples, the ratio of inventory to sales, production to capacity, new orders to production, or shipments to new orders. They may involve the difference between demand and supply.

4) Opinions Polls. These are the weighted or unweighted averages of naive forecasts, or of forecasts made by people who have information and techniques of forecasting not available to the polltake.

5) "Econometric" is a word coined to mean the union of economic theory and mathematics, statistics, and accounting. Econometrics methods of forecasting, to be successful, must be concerned with the truly dynamic theories of economics.

According to the time period involved, Plossl and Wight (12) classified forecasts and illustrated their applications, principally for inventory control, as follows:

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*From R. C. Brown (3, pp. 5).

(1) Long-range forecasts: Used in plant expansion and new machine and equipment acquisition in order to plan capital investment five years or more in advance.

(2) Intermediate-range forecasts: Used in the procurement of long lead time materials or planning of operating rates with adjustment for seasonal or cyclical products one or two years in advance.

(3) Short-range forecasts: Used to determine the proper order-timing for purchasing of manufacturing components, and to plan the proper manufacturing capacity by taking into consideration the desirability of leveling workloads three to six months in advance.

(4) Immediate-future forecasts: Used for assembly schedules and finished goods inventory distribution on a weekly or daily basis.

Before selecting a method of forecasting, one should consult with a systems analyst. In general, it is necessary to take into consideration factors such as, the capacity of the computer facilities available, and the desired frequency of the forecasts, the number of items to be forecasted, and the expected accuracy of the forecasts.

The costs of forecasting must be carefully balanced with the economic advantages which might be gained from the forecast information. Computer costs rise for forecasting with the accuracy and number of objects forecasted and with the frequency with which these forecasts are made.

In this paper techniques to forecast discrete time series will be developed through the method known as Exponential Smoothing.

Exponential Smoothing is a procedure for calculating a weighted moving average, in which the heaviest weight is assigned to the most recent data. In addition, the computations involved in exponential smoothing are particularly well suited to automatic data-processing equipment. Exponential Smoothing can yield estimates of future values that are as accurate as those derived from any other method of averaging data.

At any time the smoothed function is formed as a linear combination of all past observations; its mathematical representation is a linear relation which takes into account the most recent observation as it repeatedly forecasts future values.

This method meets the requirements for computer efficiency and for a way of auto-control during the computations.

Exponential Smoothing can be classified as a naive method in the Ross classification whereas it is in the immediate-future forecasting classification of Plossl and Wight.

This paper also describes several modifications of the original Exponential Smoothing Method developed by Robert C. Brown (3).

Two time series were examined in the application of forecasting techniques. The first is the same one used by Brown (3), who applied a quadratic model to forecast the Closing Prices on the New York Exchange of the I. B. M. Corporation Common Stock, between July 1, 1960 and March 24, 1961. Through this data the methods were compared.

The second time series examined (an attempt to apply the exponential smoothing method of forecasting to the feed industry) was sales-data of one product manufactured by the firm Protinal C. A. of Valencia, Venezuela.

Computer programs developed to do the forecasting are shown in this paper. The programs were written in FORTRAN IV. The I. B. M. 360/50 system of the Computing Center facilities at Kansas State University was used to run these programs.

CHAPTER I

The main purpose of this thesis is to develop methods to forecast future sales in the feed industry by using Exponential Smoothing.

Forecasting processes begin with the analysis of the time series or data, taken from the process to be forecasted, in order to test models that can represent the time series and determine which models are adequate. Those pre-selected models are checked using the known time series or data to determine which one best represents the behavior of the time series.

When the best model is found, it is applied to the forecasting system to obtain future values of the time series. Finally, every new observation taken is used to control the forecasting system in order to take, when necessary, adequate corrective actions before the next event occurs.

The following is a review of some concepts about time series and models which will be used in the development of this thesis.

Time Series

A time series is defined as a set of observations taken sequentially in time.

A time series is said to be continuous, if it is generated from a continuous process and its set of observations is recorded continuously. If the set of observations is taken at some time intervals (fixed or not), the time series from both continuous or discrete processes is said to be discrete.

Continuous time series must arise from a continuous process whereas discrete time series may arise in two ways (3),

1) By sampling a continuous process, and

2) By accumulating a variable over a period of time.

If any value of a time series can be exactly determined by some mathematical function, the time series is said to be deterministic. If future values can be described only in terms of a probability distribution the time series is said to be non-deterministic or simply a statistical time series.

It is possible to forecast future values of deterministic time series with high accuracy whereas in statistical time series it is impossible to exactly forecast future values.

Statistical time series are obtained from stochastic processes which are statistical phenomena that evolve in time according to probabilistic laws.

A very special class of stochastic processes, called strictly stationary processes, is based on the assumption that the process is in a particular state of "statistical equilibrium" which means that the joint distribution of any set of observations must be unaffected by shifting all the time of observations forward or backward by any integer interval.

In this thesis two discrete-statistical time series obtained from stochastic processes non-strictly stationary were used for application purposes.

Representation of a Time Series

For the purpose of analysis, a time series can be considered to be made up of two elements:

(1) The process which generates the time series, and

(2) Some superimposed random noise (fluctuations).

Thus the time series may be represented in the following manner

$$Y(t) = \xi(t) + \epsilon(t)$$

where: $Y(t)$ is the observed value at time (t)

$\xi(t)$ is the process at time "t"

$\epsilon(t)$ is the random noise in the t^{th} observation.

The distribution of the random noise samples has the following properties:

(1) The expected value is zero,

$$E\ \epsilon(t) = 0$$

(2) The noise sample has no serial correlation,

$$E\ \epsilon(i)\,\epsilon(j) = 0 \qquad \text{for } i \neq j$$

(3) The variance (unknown) of the noise distribution is,

$$E\ \epsilon(i)\,\epsilon(j) = \sigma^2 \qquad \text{for } i = j$$

Representation of the process is made through mathematical expressions called "models", whereas no attempt has been made to represent the noise because it is random.

Models

Miller and Starr (10) define models as "a representation of reality intended to explain the behavior of some aspects of it."

Since a model is an explicit representation it is generally less complex than the reality itself. But it must be sufficiently complete to approximate those aspects of reality which are being investigated.

Forecasting models are built on the basis of past data which is generated from a sequence of observations. Then the probability distribution from which future observations will be drawn is forecasted. In order to meet this goal, systematic changes in the sequence of past observations to date must be represented by the model which can be projected into the future.

Models to be used in Exponential Smoothing must be a good representation of the time series only in a segment of time around the date when the forecast is made. Models need not represent the time series beyond the future forecast lead time, nor the irrelevant past data.

The time ahead used to forecast the time series (forecast lead time) may determine the characteristics of the model and how much past data is needed. If the forecast lead time is long, the model should represent the data well for a long historical period. A simple model may be a reasonably good representation over short enough time intervals. An elaborate model should be an equally good representation over much longer intervals. In general, if the model chosen is a good representation of the process the distribution of forecast errors will have a small variance; if not, it will have a large one.

General models describe the processes in terms of two components,

(1) The trend, which the mean of the series is following, and

(2) Cyclical or seasonal components, which are superimposed upon this trend.

To represent those components, Brown (3) described the following functions:

(1) Algebraic Models. These are used to obtain the trend of the mean. Polynomial models, such as constant, linear or quadratic models are in this group of models.

(2) Transcendental Models. These are used when simpler models are inadequate to represent growing time series or/and seasonal or cyclical fluctuations. Exponential and trigonometric models are in this type of models.

(3) Composite Models combine both algebraic and transcendental models.

(4) Regression Models are formed from linear combinations of several functions.

For short lead time forecasting as applied in this thesis, the following algebraic models were used:

Constant Models:

$$X(t) = a + \epsilon(t)$$

Linear Models:

$$X(t) = a + b.t + \epsilon(t)$$

Quadratic Models:

$$X(t) = a + b.t + 1/2\ c.t^2 + \epsilon(t)$$

Exponential Smoothing

Exponential Smoothing is a procedure to obtain a weighted moving average of a time series in which the heaviest weight is assigned to the most recent observation.

The smoothed function of a time series at time "t" is defined by the relation:

$$S_t(X) = \alpha X_t + (1-\alpha)\ S_{t-1}(X) \qquad \text{I-1}$$

where: $S_t(X)$ = Value of the smoothed function at time "t".

α = A positive constant less than one (called Smoothing Constant).

X_t = Value of observation at time "t".

$S_{t-1}(X)$ = Previous value of the smoothed function.

Writing equation I-2 in the equivalent form:

$$S_t(X) = S_{t-1}(X) + \alpha(X_t - S_{t-1}(X)) \qquad \text{I-2}$$

one can see that the value of the smoothed function at time "t" is equal to the value of the previous smoothed function plus a fraction α of the difference between the value of the observation at time "t" and the previous smoothed value.

It can be shown that the smoothed function at time "t" is a linear combination of all past observations. By a repeated process of substitution in equation I-1 of the previously smoothed value for the early one, one obtains:

$$S_t(X) = \alpha X_t + (1-\alpha)S_{t-1}(X)$$

$$= \alpha X_t + (1-\alpha)\left[\alpha X_{t-1} + (1-\alpha)S_{t-2}(X)\right]$$

$$= \alpha X_t + \alpha(1-\alpha)X_{t-1} + (1-\alpha)^2\left[\alpha X_{t-2} + (1-\alpha)S_{t-3}(X)\right]$$

$$\vdots$$

$$S_t(X) = \alpha \sum_{k=0}^{t-1}(1-\alpha)^k X_{t-k} + (1-\alpha)^t X_o \qquad \text{I-3}$$

For convenience, a factor β (called the discount factor) is defined by the relation

$$\beta = 1 - \alpha$$

Using this factor, equation I-3 can be written as a linear combination of all past observations

$$S_t(X) = \alpha \sum_{k=0}^{t-1}\beta^k X_{t-k} + \beta^t X_o \qquad \text{I-4}$$

From equation I-4 one can see how the heaviest weight is assigned to the most recent data. The term " $\beta^t X_o$ " is the weighted value at time "t" of the initial value, its weight in the smoothed function is a fraction β^t of its original value. From the other term it is easy to appreciate that the weight given by previous observations decreases geometrically with age.

Furthermore, the weight of a past datum in the smoothed function at any time, depends on the value assigned to the smoothing constant. Smaller values of α gives a datum which retains more time weighting in the smoothed function.

Table I shows the weight of the data according to the value of the smoothing constant and the age of the datum. As example, observe that for $\alpha = 0.5$, a datum taken at time "t-6" is discounted at time "t" to weight less than 1% in the smoothed function, at time "t-9" less than 1/1,000th, and at time "t-13" less than 1/10,000th. On the other hand for $\alpha = 0.1$, a datum 21 units of time old is still weighted in the smoothed function at more than 1%.

This property of the smoothed function provides great flexibility for responses to changes in the time series. In practice when there is an interest in using a long historical data, a small value of α is used; while if the interest is to respond as fast as possible to the effects of recent data a large value of α is used.

Brown (3) demonstrated that the expected value of the smoothed function in a constant model is the expected value (average) of the data

$$\xi\left[S(X)\right] = \alpha \sum_{k=0}^{\infty} \beta^k \; \xi\left[X_{t-k}\right]$$

$$= \xi(X)\alpha \sum_{k=0}^{\infty} \beta^k = \xi(X) \quad \frac{\alpha}{1-\beta}\xi(X)$$

$$= S_t(X) = \xi(X) \qquad \text{I-5}$$

TABLE I WEIGHT OF DATA ACCORDING TO AGE OF DAT A AND EXPONENTIAL SMOOTHING CONSTANT

ALPHA AGE OF DATA	0.1	0.2	0.3	0.4	0.5
0	0.10000	0.20000	0.30000	0.40000	0.50000
1	0.09000	0.16000	0.21000	0.24000	0.25000
2	0.08100	0.12800	0.14700	0.14400	0.12500
3	0.07290	0.10240	0.10290	0.08640	0.06250
4	0.06561	0.08192	0.07203	0.05184	0.03125
5	0.05905	0.06554	0.05042	0.03110	0.01562
6	0.05314	0.05243	0.03529	0.01866	0.00781
7	0.04783	0.04194	0.02471	0.01120	0.00391
8	0.04305	0.03355	0.01729	0.00672	0.00195
9	0.03874	0.02684	0.01211	0.00403	0.00098
10	0.03487	0.02147	0.00847	0.00242	0.00049
11	0.03138	0.01718	0.00593	0.00145	0.00024
12	0.02824	0.01374	0.00415	0.00087	0.00012
13	0.02542	0.01100	0.00291	0.00052	0.00006
14	0.02288	0.00880	0.00203	0.00031	0.00003
15	0.02059	0.00704	0.00142	0.00019	0.00002
16	0.01853	0.00563	0.00100	0.00011	0.00001
17	0.01668	0.00450	0.00070	0.00007	0.00000
18	0.01501	0.00360	0.00049	0.00004	0.00000
19	0.01351	0.00288	0.00034	0.00002	0.00000
20	0.01216	0.00231	0.00024	0.00001	0.00000
21	0.01094	0.00184	0.00017	0.00001	0.00000
22	0.00985	0.00148	0.00012	0.00001	0.00000
23	0.00886	0.00118	0.00008	0.00000	0.00000
24	0.00798	0.00094	0.00006	0.00000	0.00000
25	0.00718	0.00076	0.00004	0.00000	0.00000
26	0.00646	0.00060	0.00003	0.00000	0.00000
27	0.00581	0.00048	0.00002	0.00000	0.00000
28	0.00523	0.00039	0.00001	0.00000	0.00000
29	0.00471	0.00031	0.00001	0.00000	0.00000
30	0.00424	0.00025	0.00001	0.00000	0.00000
31	0.00382	0.00020	0.00000	0.00000	0.00000
32	0.00343	0.00016	0.00000	0.00000	0.00000
33	0.00309	0.00013	0.00000	0.00000	0.00000
34	0.00278	0.00010	0.00000	0.00000	0.00000
35	0.00250	0.00008	0.00000	0.00000	0.00000
36	0.00225	0.00006	0.00000	0.00000	0.00000
37	0.00203	0.00005	0.00000	0.00000	0.00000
38	0.00182	0.00004	0.00000	0.00000	0.00000
39	0.00164	0.00003	0.00000	0.00000	0.00000
40	0.00148	0.00003	0.00000	0.00000	0.00000

Among other applications the smoothed function is used to forecast future values of a time series for a particular case in which the time series follows a constant model represented by the following relation:

$$\hat{X}(t) = \hat{a} + \epsilon(t) \qquad \text{1-6}$$

Since the expected value of the noise $\epsilon(t)$ is zero, the expected values of equation (1-6) become:

$$\xi(X) = \xi(a) = \hat{a}_t \qquad \text{1-7}$$

where: $\hat{a}_t$ (read "a hat sub t") is the estimated value of the coefficient of the model based on data through time "t".

Now the value of the time series "tau" units of time ahead is forecasted by assuming that its future value (X_{t+tau}) must be equal to its expected value at time "t", which is equal to the estimated value of the coefficient also at time "t". That is

$$\hat{X}_{t+\tau} = \hat{a}_t = \epsilon(X) \qquad \text{1-8}$$

By identification of equations 5 and 8 it is seen that the forecasted value of the time series is equal to the smoothed function at time "t"

$$\hat{X}_{t+\tau} = \hat{a}_t = S_t(X) \qquad \text{1-9}$$

The process of obtaining successive values of the smoothed function and then forecasting by using the relationship given in equation 1-9 is called single exponential smoothing.

Multiple Exponential Smoothing

Next equation defines the smoothed function of order "k". This relation will be applied to obtain forecasting values using higher order polynomial models:

$$S_t^{(k)}(X) = \alpha S_t^{(k-1)}(X) + \beta S_{t-1}^{(k)}(X) \qquad \text{1-10}$$

where: $S_t^{(k)}(X)$: Value at time "t" of the smoothed function of order "k"

$S_t^{(k-1)}(X)$: Value at time "t" of the smoothed function of order "k-1"

$S_{t-1}^{(k)}(X)$: Previous value of the smoothed function of order "k"

Brown (3, page 133) demonstrates that it is possible to estimate the (n+1) coefficients in any nth order polynomials models by linear combination of the first (n+1) orders of the smoothed functions. By application of this property he achieves the follows formulation to apply in linear and quadratic models. We have already shown the formulation for a constant model.

Double Exponential Smoothing

Given the forecasting linear model

$$\hat{X}_{t+\tau} = \hat{A}_t + \tau\hat{B}_t \qquad \text{I-11}$$

the coefficients of this model are obtained from the following set of equations:

$$\hat{A}_t = 2.S_t(X) - S_t^{(2)}(X) \qquad \text{(SET 1)}$$

$$\hat{B}_t = \frac{\alpha}{\beta}(S_t(X) - S_t^{(2)}(X)$$

where: $\hat{X}_{t+\tau}$: Forecasted value made " τ " units of time in advance.

$\hat{A}_t$, $\hat{B}_t$: Estimate values of the coefficients based on data through time "t".

τ : forecast lead time.

α, β, $S_t(X)$ as usual

$S_t^{(2)}(X) = \alpha S_t(X) + \beta S_{t-1}^{(2)}(X)$ according to the definition (I-10).

Triple Exponential Smoothing

Brown achieves the following set of equation to calculate the estimate of the coefficients of the quadratic model:

$$\hat{X}_{t+\tau} = \hat{A}_t + \tau\hat{B}_t + 1/2\ \tau^2 . \hat{C}_t \qquad \text{(SET 2)}$$

$$\hat{A}_t = 3S_t(X) - 3S_t^{(2)}(X) + S_t^{(3)}(X)$$

$$\hat{B}_t = \frac{\alpha}{2\beta^2}\ (6-5\alpha)S_t(X) - 2(5-4\alpha)S_t^{(2)} + (4-3\alpha)S_t^{(3)}(X)$$

$$\hat{C}_t = \frac{\alpha^2}{\beta^2}\ S_t(X) - 2S_t^{(2)}(X) + S_t^{(3)}(X)$$

where: $S_t^{(3)} = \alpha S_t^{(2)}(X) + \beta S_{t-1}^{(3)}(X)$

and the other terms as defined previously.

Forecast Errors

Forecasting errors were defined as the difference between the forecasted value and the one actually observed in the future; this is

$$\epsilon_t = \hat{Y}_{t-\tau} - Y_t \qquad \text{I-12}$$

where: ϵ_t = forecasting error at time "t",

$\hat{Y}_{t-\tau}$ = the forecasted value for time "t", which has been determined at time "t- τ "

Y_t = the observed value at time "t"

τ = forecasting lead time.

It is common to assume that the forecasting errors (simply called errors) are normally distributed with mean equal to zero, unknown standard deviation and no serial correlation.

To include an estimate of the errors in the control of a forecasting system, it is necessary to fix their standards of measure.

When exponential smoothing is applied to forecast statistical-discrete time series, the mean of errors may be different from zero but should be close to zero if the model is a good representation of the process.

Standard deviation of errors is obtained by the relation:

$$\sigma \doteq \sqrt{\frac{\sum_{i=1}^{n} (\hat{Y}i - Yi)^2}{n - 1}} \qquad \text{I-13}$$

which is equivalent to

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (\epsilon_i - \bar{\epsilon}_i)^2}{n - 1}} \qquad \text{I-14}$$

since: $\epsilon = 0$ by assumption and $\epsilon_i = \hat{Y}i - Yi$

To compare methods or models used in exponential smoothing we selected among them, as the best, the one that shows the smallest standard deviation. This is based on the fact that the standard deviation of errors is a value proportional to the sum of the squared errors (equation I-14), and the fact that exponential smoothing itself is based in the *minimization* of this sum squared.

Although it is out of the scope of this thesis, it is important to note here that the smallest standard deviation obtained among tested models, or methods, does not guarantee that an appropriate forecasting system was achieved. It is possible, for example, that the process is not at all forecastable with any of the models used so the best of them is still not a good one.

Other ways to measure errors are mean absolute deviation and tracking signals, which we define as follows.

Mean Absolute Deviation

The mean absolute deviation (MAD) is defined by the relation:

$$MAD_t = \alpha \text{ error } + \beta MAD_{t-1}$$

where: MAD_t = Mean absolute deviation at time "t"

MAD_{t-1} = Previous mean absolute deviation.

This value is a measure of the scatter of the errors around their mean, and is by definition a positive value. MAD is a linear relation widely used in exponential smoothing because it is simpler to calculate than the standard deviation.

Tracking Signal

Tracking signal is defined by the relation:

$$\text{Tracking signal}_t = \frac{\sum_{i=1}^{t} e_i}{MAD}$$

where: Tracking signal_t = Tracking Signal computed at time "t"

$$\sum_{i=1}^{t} e_i = \text{accumulative error through time "t"}$$

MAD_t = Mean absolute deviation computed at time "t"

Tracking signals are used as a way to detect changes in the processes. If the forecasting system is doing exact forecasts the sum of the errors will be zero and also the tracking signal. If the processes change, the forecasts are no longer exact and then the sum of the errors becomes large and the tracking signal attains large values.

By specifying acceptable limits of the tracking signal values, it can be detected significant changes of the processes.

To obtain these limits of the models applied in this thesis, with 95% confidence, Brown gives the following relation:

$$2\,\sigma_y = \sqrt{\frac{\Pi\,(1 + \beta)}{1 - \beta^{2N}}}\ (MAD)$$

where: σ_y = standard deviation of the sum of errors

N = degree of the model

The definition of tracking signal by Brown was modified by Trigg, who found that there are two disadvantages to Brown's definition. To quote Trigg (14, p. 271) the disadvantages are:

"1) Once the tracking signal has gone out of limits it will not necessarily return within limits even though the forecasting system itself comes back in control. Consequently, intervention is necessary to set the sum of forecast errors back to zero if future false alarms are to be avoided. Such interventions can be tedious and may tend to be neglected when several hundred items are being forecasted.

"2) Ironically if the system starts to give exceptionally accurate forecasts the tracking signal may go out of limits. For example, if perfect forecasts begin to occur, the MAD will tend to zero while the sum of errors will remain unaltered. This clearly leads the tracking signal to infinity."

CHAPTER II

This chapter is devoted to developing the method to test forecasting systems using the method of Brown and two other methods which modify it. One of the modified methods was developed by D'Amico (6) and the second was obtained in the research done for this thesis.

All three methods are based on the theory developed in Chapter I. The modified methods use the same theory but modifications were done to fix automatically, at the time that each observation is taken, the best value of the smoothing constant. In Brown's method any modification to the smoothing constant requires the intervention of the forecaster.

To do the forecast in exponential smoothing, the smoothed statistics $S_p^{(k)}(X)$ are revised with each new observation by a simple recursion relation

$$S_t^{(k)}(X) = \alpha S_t^{(k-1)}(X) + \beta S_{t-1}^{(k)}(X)$$

then, the coefficients of the forecast model

$$X_{t+\tau} = X_t^{(0)} + \tau X_t^{(1)} + 1/2\ \tau^2 X_t^{(2)} + \dots + \frac{1}{n!}\ \tau^n X_t^{(n)}$$

are calculated as a linear combination of the smoothed statistics. Formulations for the models used in this thesis are given in equation I-9 and set I,1 and I,2 of Chapter I.

In order to perform the calculations by means of computer facilities it is necessary to write a program in any of the known computer languages. For this thesis we used FORTRAN IV which, in our opinion, is the best for this kind of calculations. The programs were organized according to each of the following methods.

Brown's Method

The program to test the forecasting system was organized according to the following steps:

1) Stating the model. We restricted ourselves to algebraic models no higher than the second degree but in general, also with this restriction, it is not easy from the inspection of the data to know *a priori* which model follows the time series; thus it is necessary to try first with different models and then select among them the one which best represents the time series. In this thesis we used a quadratic model to forecast three days in advance the closing price of the IBM common stock. Brown used this model to forecast this time series, and we also used the same model in order to compare the behavior of the modified methods with Brown's.

Testing the forecasting of our feed sales data we first tried the three models and then, from the results we selected the linear model as the best to forecast future sales.

Since formulation differs with the model used, computer programs must be written accordingly.

2) Stating general program statements. The program must provide the usual declaration statements as well as those to fix or read specific values such as the number of data to be analyzed, number of values necessary to calculate initial conditions, etc.

3) Determining initial values of the smoothed functions and the estimate of the coefficients. Since we worked on the basis that past data were known, we used part of the data to obtain an initial average, then we handled the equations of the smoothed functions and coefficients to obtain this average as the forecasting of the first observation to take after the initial period.

When past data are not known, several criteria are used to fix initial conditions. Sometimes they are fixed setting values of similar products or processes known; sometimes they are fixed by simple estimation of management, and so on. If the prediction of initial conditions has much confidence a low value of alpha may be used; if not, use a high one in order to discount these conditions as fast as possible.

For computer computations, values such as sum of errors, sum square of errors, counters, etc., must be initialized. Others, like mean absolute deviation and common factors in calculations will be analyzed later in this chapter.

4) Fix the value of the smoothing constant. Brown uses a pre-fixed value of the smoothing constant, which can be changed only by the intervention of the forecaster when, as we will see later, certain conditions are found.

To select which value of alpha guarantees us the best results, we must collect statistics of the errors by testing the data with various values of alpha, selecting the one which shows the smallest standard deviation of the errors. Brown recommended the use of values of alpha between 0.1 and 0.4; values greater than 0.4 discount a datum from the smoothed function so fast that if they are used, practically the next forecast is equal to the value of the last observation taken; and if values of alpha smaller than 0.1 are used, the forecasting becomes practically the average of the past data.

Table II shows the value of the standard deviation of the errors for different values of the smoothed constant obtained from testing the forecasts of the IBM common stock prices, one and three days in advance.

Table II. Standard deviation of the forecasting errors for different values of the smoothing constant.

Alpha	Standard Deviation	
	$\tau=1$	$\tau=3$
0.1	9.57	13.26
0.2	13.01	13.68
0.3	13.60	14.57
0.4	13.69	17.03

From this table it is evident that a value of alpha equal to 0.1 is the best to forecast these prices.

5) Pre-calculated values to be used repetitively in calculations. The values function of the smoothing constant that will not change during the testing phase, must be pre-calculated in order to save computer time.

Pre-calculations are performed in the sets I-1 and I-2 for the linear and the quadratic models:

Set I-1 is written now in the form:

$$\hat{A}_t = 2S_t^{(1)}(X) - S_t^{(2)}(X)$$

$$\hat{B}_t = FACT\ (S_t^{(1)}(X) - S_t^{(2)}(X))$$

where: $FACT = \alpha/\beta$

Set I-2 is written as follows:

$$\hat{A}_t = 3S_t^{(1)}(X) - 3S_t^{(2)}(X) + S_t^{(3)}(X)$$

$$\hat{B}_t = FACT1(FACT2*S_t^{(1)}(X) - FACT3*S_t^{(2)}(X) + FACT4*S_t^{(3)}(X))$$

$$\hat{C}_t = FACT5\ (S_t^{(1)}(X) - 2S_t^{(2)}(X) + S_t^{(3)}(X))$$

where: $FACT1 = \alpha/2\beta^2$

$FACT2 = (6-5\alpha)$

$FACT3 = 2(5-8\alpha)$

$FACT4 = (4 - 3\alpha)$

$FACT5 = \alpha^2 / \beta^2$

6) Fix initial value of Mean Absolute Deviation (MAD). Using the same part of data that we used to determine the initial conditions in part 3), we can determine the standard deviation of that part of data, then we can establish the initial value of MAD by using the fact (3, pp. 289 & 292) that mean absolute deviation is a proportional value of the standard deviation, that is:

$$MAD(0) = 2\sigma_x \sqrt{\frac{1}{\Pi(2-\alpha)}} \qquad II.1$$

where: MAD(0) = initial value of mean absolute deviation.

σ_x = standard deviation of initial data around their mean, which is calculated by the relation:

$$\sigma_x = \sqrt{\frac{\Sigma(Xi)^2 - \frac{(\Sigma Xi)^2}{n}}{n - 1}}$$

where: Xi = values of initial data (i=1,2,.....,n)

n = number of data used to determine initial conditions.

7) Test forecasting system. Testing phase consists in using the known data to forecast successively the next value "tau" units of time ahead; that means to determine whether or not the forecasting is effective, we do not have to wait for the process forecasted to materialize before learning whether this technique is or is not valid. We can pretend that we are making our forecast one or two units of time earlier and then test the forecasting method against what really happened.

Since data is known, when each forecasted value is determined, we compare the actual value with its forecasted value to calculate error, MAD and tracking signal; also statistics of errors are collected in this part.

Then we use the observed value to smooth statistics and to determine the estimate of the coefficients to be used in the forecasting of the next event.

The tracking signal used to control the behavior of the forecasting system is calculated every time a forecast is done in that phase. As we saw in Chapter I, if the system is out of control, that is, tracking signal is greater than a prefixed value, twice in succession we must modify the value of the smoothing constant and, to avoid future false alarms, reset to zero the value of the accumulated error. This operation required the intervention of the forecaster. Modifications that we will propose are based on the fact that such forecaster intervention must be avoided.

In our computer program this control works in the following way: if the system is out of control at some time and regains control in the next time interval, no actions are taken, but an asterisk (*) is printed-out to show this fact. If the program is out of control twice in succession the sum of errors up to time "t" are reset to zero. Note that in the computer program the accumulative error is collected under two statements, the first (SUMERR) used in the control and the second one (ACCERR) used to calculate the standard deviation of errors.

The forecasting testing phase was written in our computer programs inside the DO-loop number 100, which is closed with the statement that prints-out the results of this phase. Out-put shows: actual time (T), observed value at time "t+tau", forecast made at time "t" of the value at time "t+tau", error, accumulative error, mean absolute deviation, tracking signal, value of the smoothing constant to be used at next time ("t+1"), and a control column in which an asterisk means that the system was out of control.

8) Finally, the computer program provided instructions to calculate mean and standard deviation of the errors and to print out these results.

Figure 1 and Fig. 2 show the plotted IBM price and their forecasts made one and three days in advance.

D'Amico's Method

D'Amico (6) modified the method of Brown. The methods provide for modifying alpha, without any intervention from the forecaster, by the considering relation between the mean absolute deviation at time "t" and the previous one calculated at time "t-1". He formulated the change of alpha as follows:

$$\alpha = (\gamma - \delta)\,\Delta MAD_t + \delta \qquad \text{(II-2-a)}$$

$$\alpha = (\gamma - \delta)\,\frac{0.1}{\Delta MAD_t} + \delta \qquad \text{(II-2-b)}$$

$$\Delta MAD_t = \frac{MAD_t - MAD_{t-1}}{MAD_{t-1}} \qquad \text{(II-3)}$$

where: γ = Upper limit prefixed for alpha

δ = Lower limit prefixed for alpha

ΔMAD_t = Factor for changing alpha

MAD_t and MAD_{t-1} = Mean absolute deviation calculated at times "t" and "t-1".

Next we give our comments about this method.

1) Since it is necessary to select between the direct (II.2.a) and the indirect relations (II.2.b) to obtain the new value of alpha, the forecaster intervention is not entirely avoided. D'Amico explained

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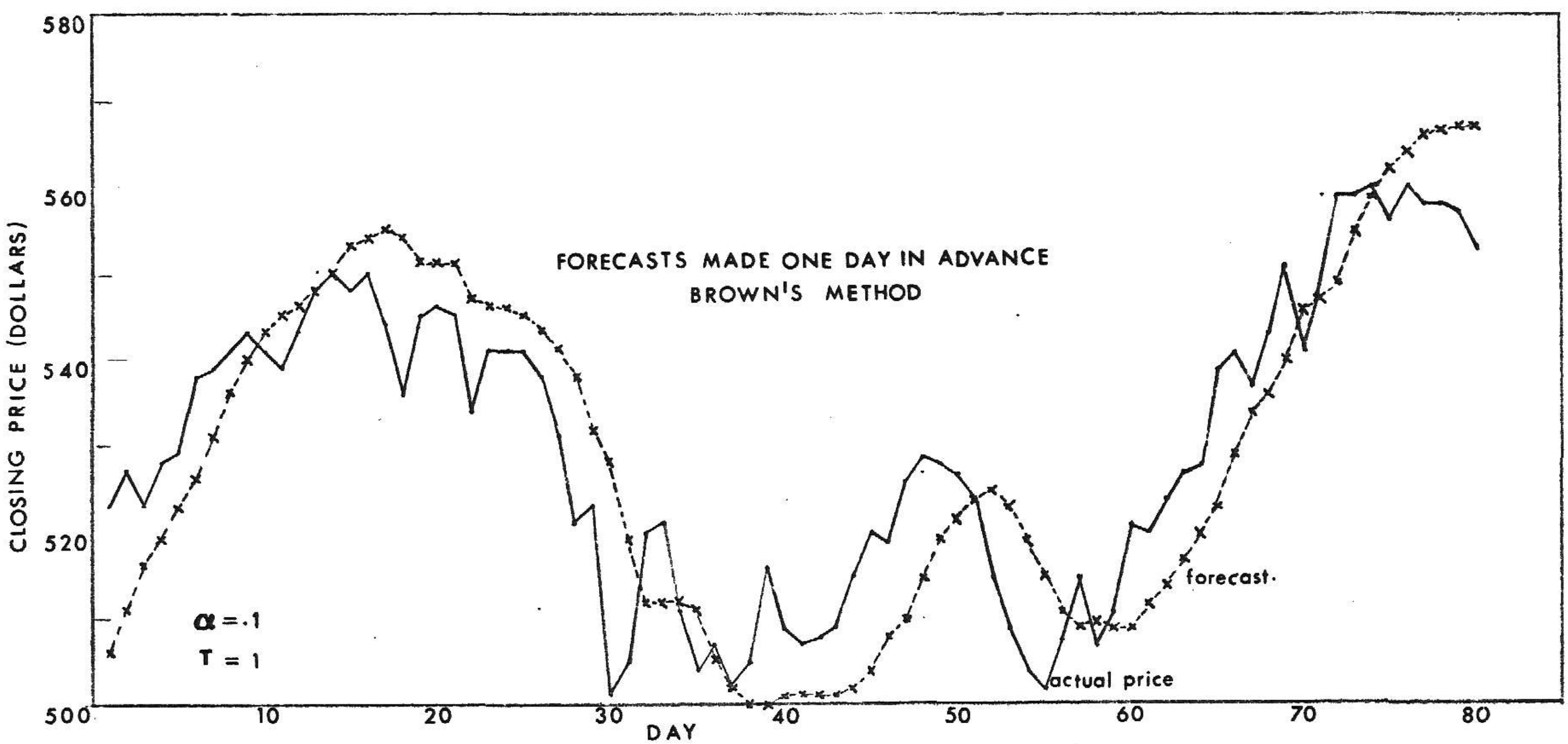

FIG. 1 IBM common stock trading on the New York Exchange,

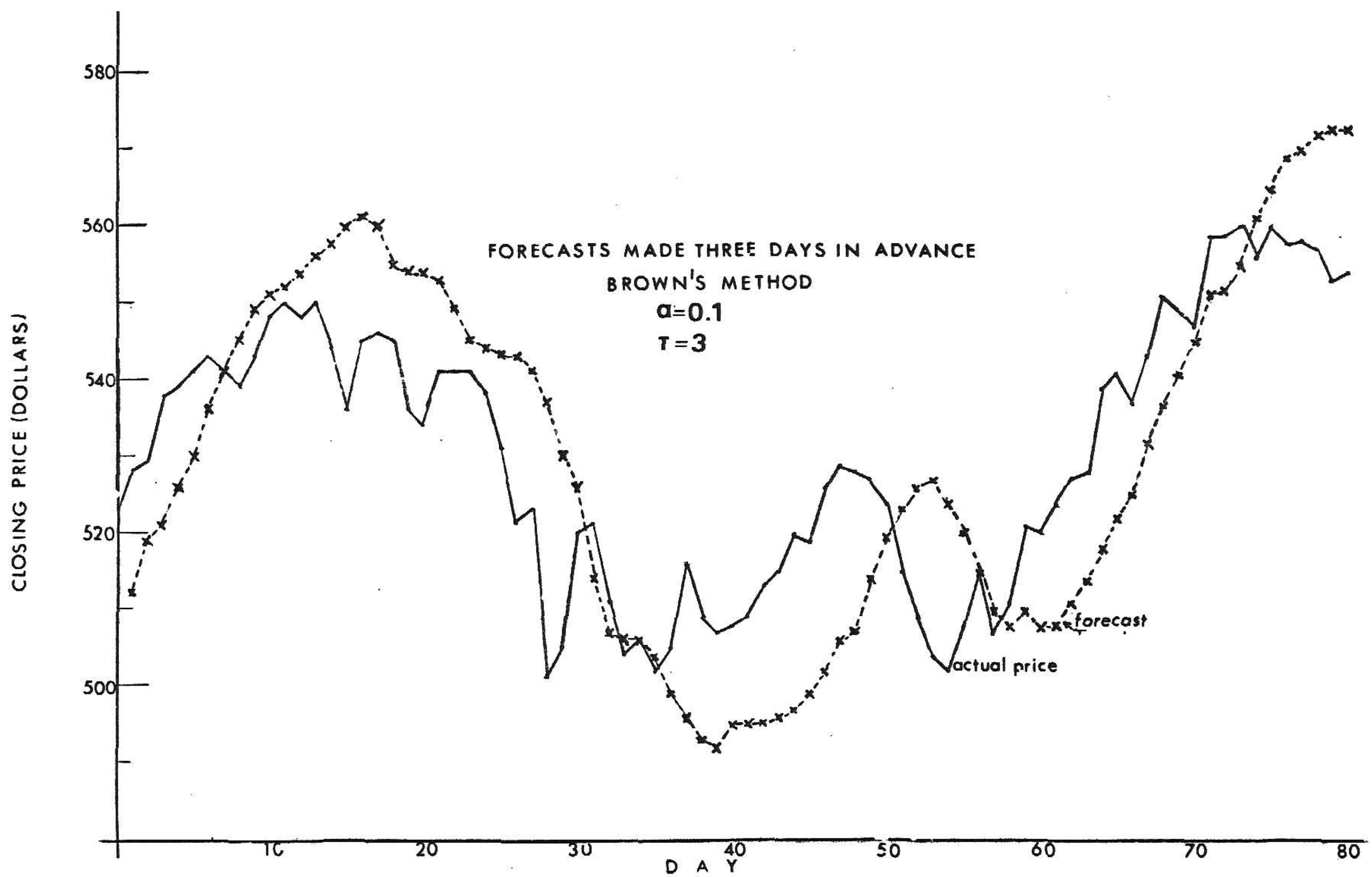

FIG. 2 IBM common stock trading on the New York Exchange.

that if demand level increases or decreases considerably, a high alpha is needed; and if demand pattern becomes random, a low value of alpha is desirable, but he did not establish what "considerably" and "random" mean in terms of rational values.

2) Formulation of this method is mathematically inconsistent, at least as the equations were printed, because of:

First, according to this formulation, negative values of the smoothing constant are obtained when the previous value of MAD is greater than the present one. By definition the smoothing constant is a positive value less than one.

Second, if the value of the actual MAD becomes much greater than the previous one, when equation II.2.a is used, alpha may become greater than its prefixed upper limit and eventually greater than one. Conversely, in equation II.2.b for smaller values of ΔMAD alpha can be also greater than one. In those cases it is logical to use the respectively inverse forms to avoid those inconsistencies but in our opinion that is contradictory because when the calculation tends to set a high value of alpha, using inverse form, a small value is obtained.

In order to avoid the inconsistencies we reformulated the equations as follows:

$$\alpha = (\gamma - \delta)\,\Delta MAD_t + \delta$$

$$\Delta MAD_t = \frac{\left| MAD_t - MAD_{t-1} \right|}{DIV}$$

Where: $\left| MAD_t - MAD_{t-1} \right|$ = Absolute value of the difference between present and previous values of MAD,

DIV = The greatest value of present and previous MAD.

With this formulation we avoid any intervention of the forecaster since only one equation is necessary to determine the new value of alpha. Furthermore the value of alpha is always proportional to the variation in the mean absolute deviations which is the goal in D'Amico's method.

Appendix A shows the computer program used for this method, which is the same as Brown's but the added statements make the changes in the value of the smoothing constant. This appendix also shows the out-put obtained in the testing phase of the method.

Floating Alpha Method

This method to forecast time series is based on Brown's method. It permits alpha to float between prefixed limits in order to achieve values that reflect variations on the process as the time series develops.

When forecasting is working properly and the time series follows a constant pattern, the errors are small, whereas if changes do occur during the process and the parameter alpha does not reflect the changes, large errors are obtained. Both Brown's and D'Amico's methods respond to those changes but because of their smoothed basis that response is too slow, or to quote Trigg & Leach (15) "the forecasting system will take an unacceptably long time to home into the new level."

In this thesis we tried to obtain a faster response along with comparable accuracy of the forecasting than the ones obtained by Brown. In order to achieve these goals we tested this method where alpha is changed directly proportional to the variations of the errors registered at the actual time and the one registered previously. This method also avoids forecaster intervention.

The smoothing constant floats between prefixed limits. When large values of errors are found alpha becomes large, reflecting the fact that a change had occurred in the process.

Formulation to change alpha according to this method is given as follows:

$$\alpha_{t+1} = (\gamma - \delta)\,\Delta\text{ERROR} + \delta \qquad (|e|_t > \text{INDEX})$$

$$\alpha_{t+1} = \alpha_t \qquad (|e|_t \leqslant \text{INDEX})$$

$$\Delta\text{ERROR} = \frac{|e|_t - \text{INDEX}}{|e|_t}$$

Where: α_{t+1} = Smoothing constant to use to forecast next event.

γ = Upper limit prefixed for alpha.

δ = Lower limit prefixed for alpha.

ΔERROR = Positive value less than one used as a factor to change alpha.

$|e|_t$ = Absolute value of error at time "t".

INDEX = Prefixed value used as low limit of error to change alpha (explained below).

Testing this method, we found that the best results for forecasting time series were obtained when the smoothing constant is changed if the absolute value of the error becomes greater than a prefixed value (called INDEX), and otherwise alpha is set equal to its previous value. For each of the time series used in this thesis, index values were tested at different levels selecting for each case the one that showed the smallest standard deviation of forecasting errors.

Figure 3 shows the value of the standard deviation of errors at different levels of index for the testing of the forecasts of both the IBM common stock price data and the sales data of feed.

In Fig. 4 one can see how fast this method responds to variations of data if it is compared with the other two methods studied, which is one of our goals.

On the other hand, our goal of obtaining comparable accuracy also was achieved, perhaps improved, for short forecasting lead times. Table III shows the standard deviation of errors obtained for each of the methods developed in this thesis in the testing of the forecasts of the IBM common stock prices made one and three days in advance.

Table III. Standard deviation of errors in the forecasts of the IBM common stock prices.

	STANDARD DEVIATION		
Method	$\tau = 1$	$\tau = 3$	α-Value
Brown's	9.57	13.98	= 0.1
D'Amico	9.16	13.67	$\geq$ 0.1
Floating	8.14	13.54	$\geq$ 0.05

Appendix A shows the computer program used for this method along with the out-puts obtained. Figure 5 and Fig. 6 show plots of the forecasts made one and three days in advance of the IBM price data.

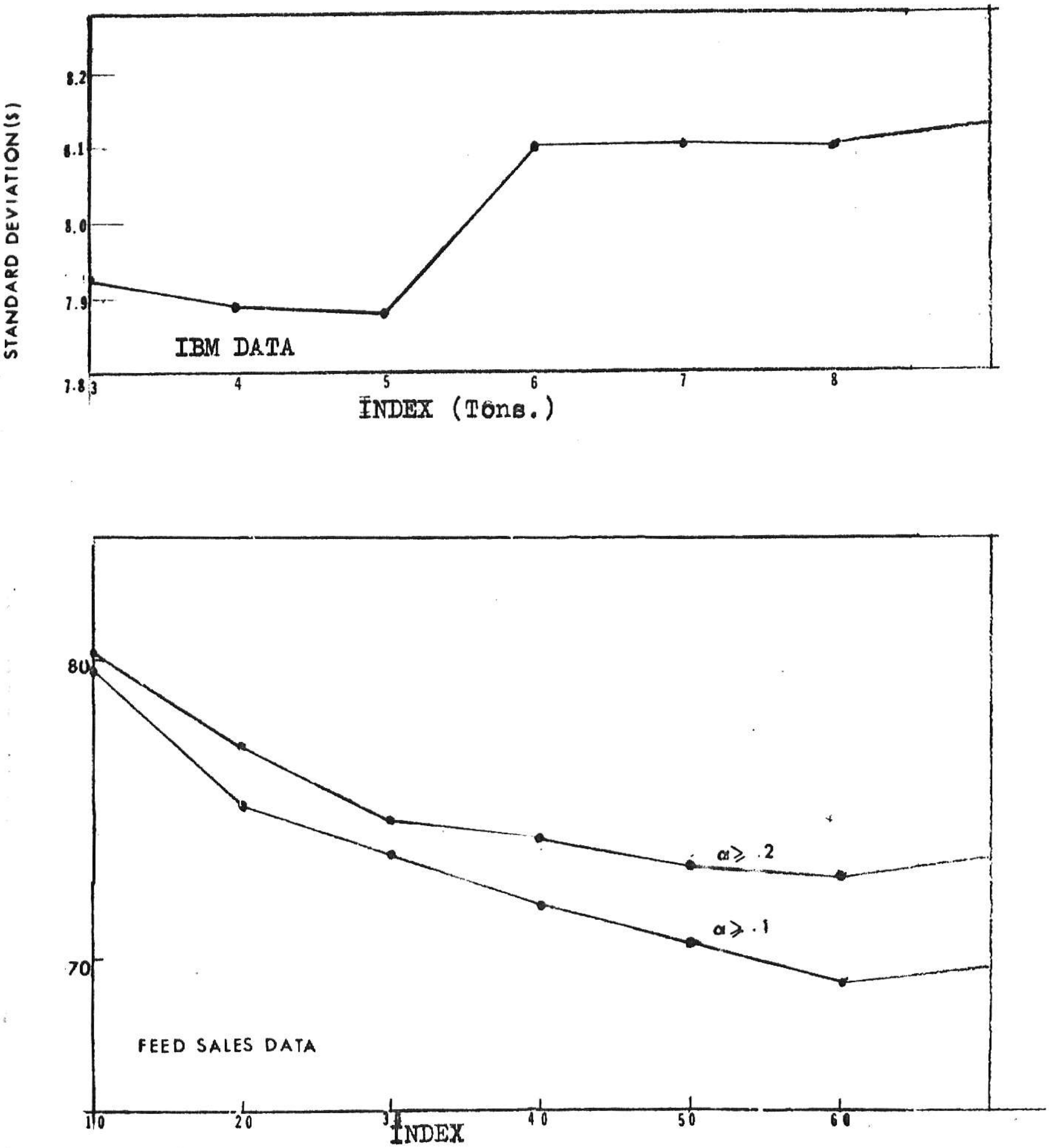

FIG. 3 Value of σ at differents values of low limits of errors
FLOATING α METHOD

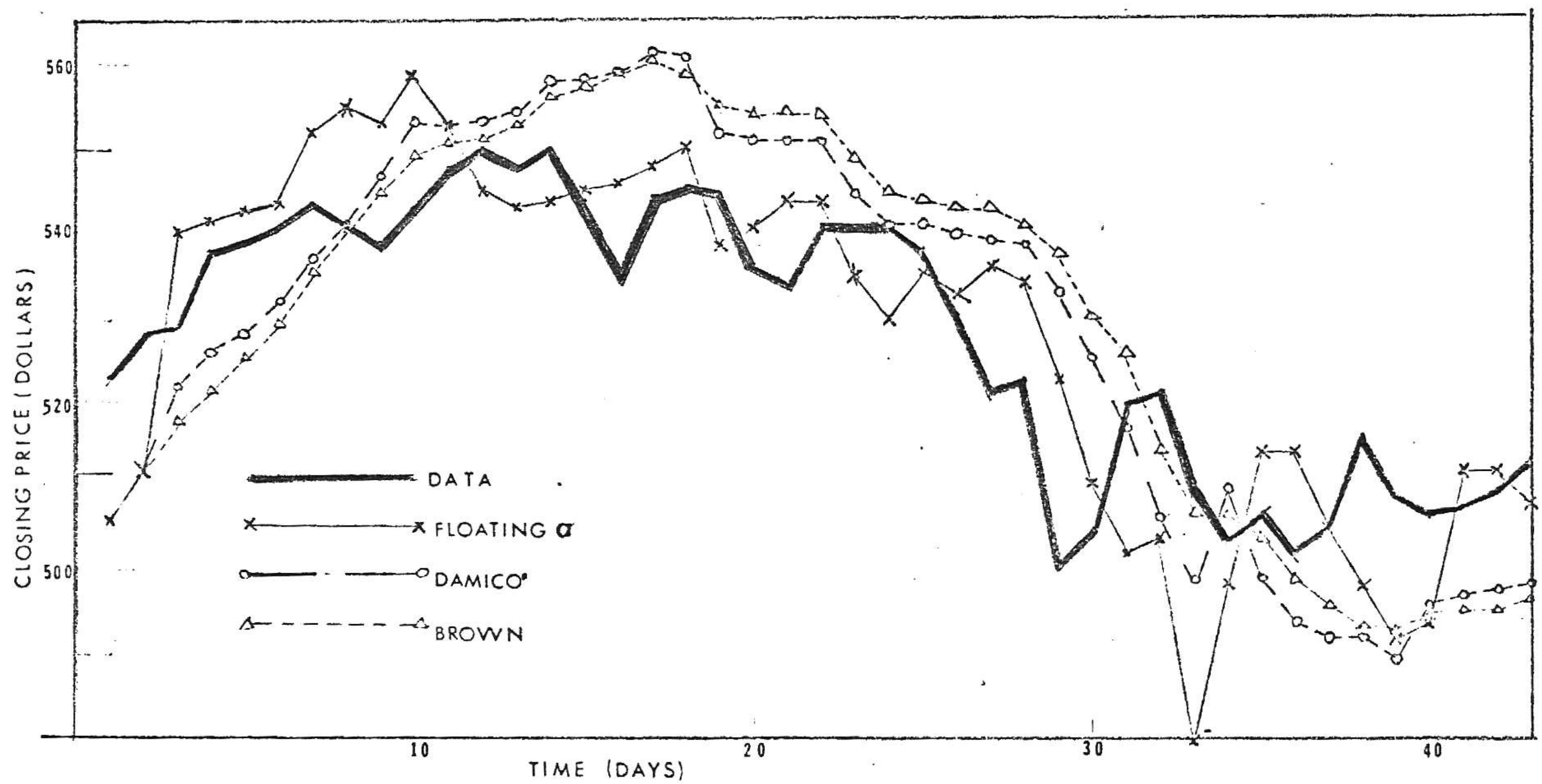

FIG. 4 IBM common stock trading on the New York Exchange.

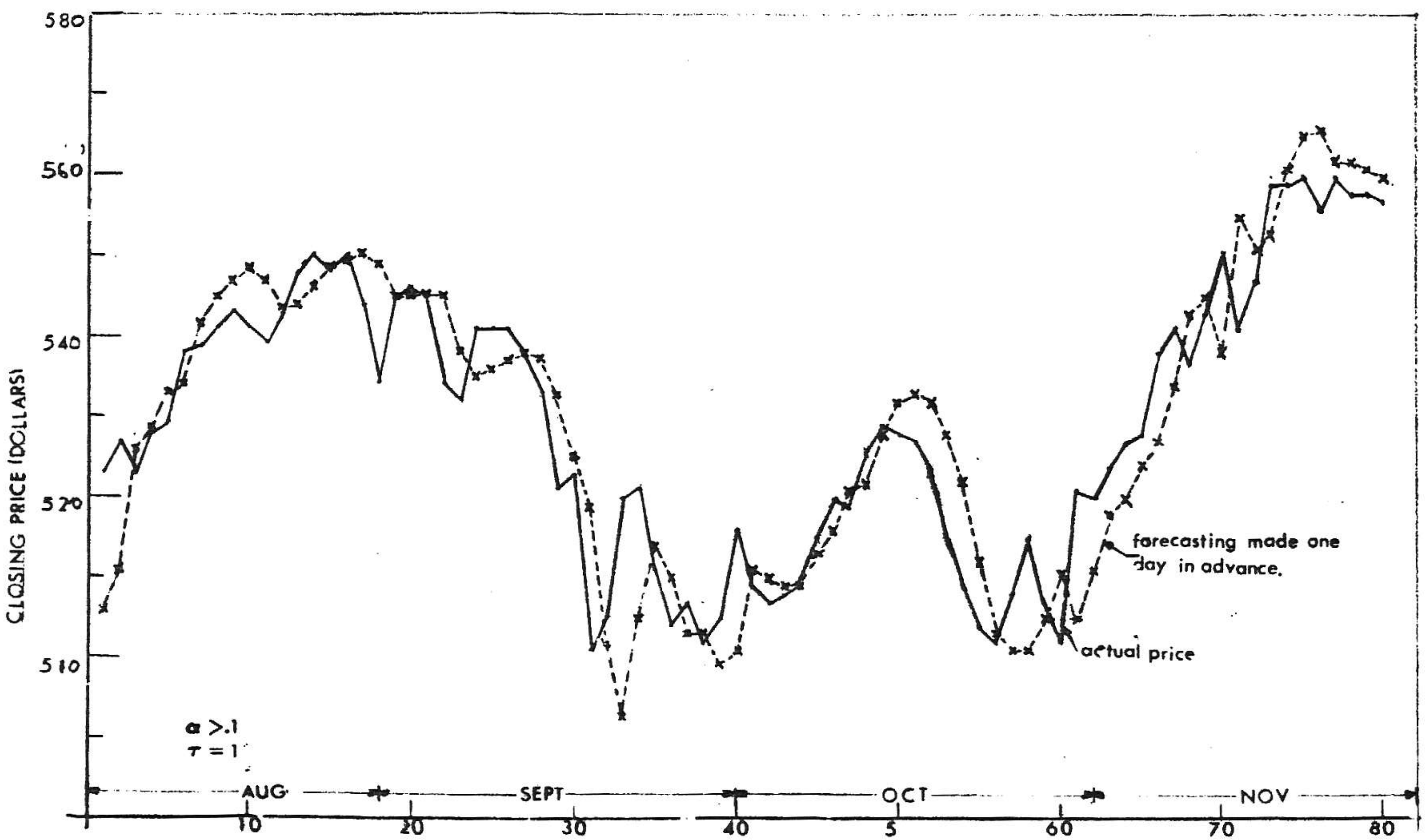

FIG. 5 IBM common stock trading on the New Yark Exchange
Floating alpha method.

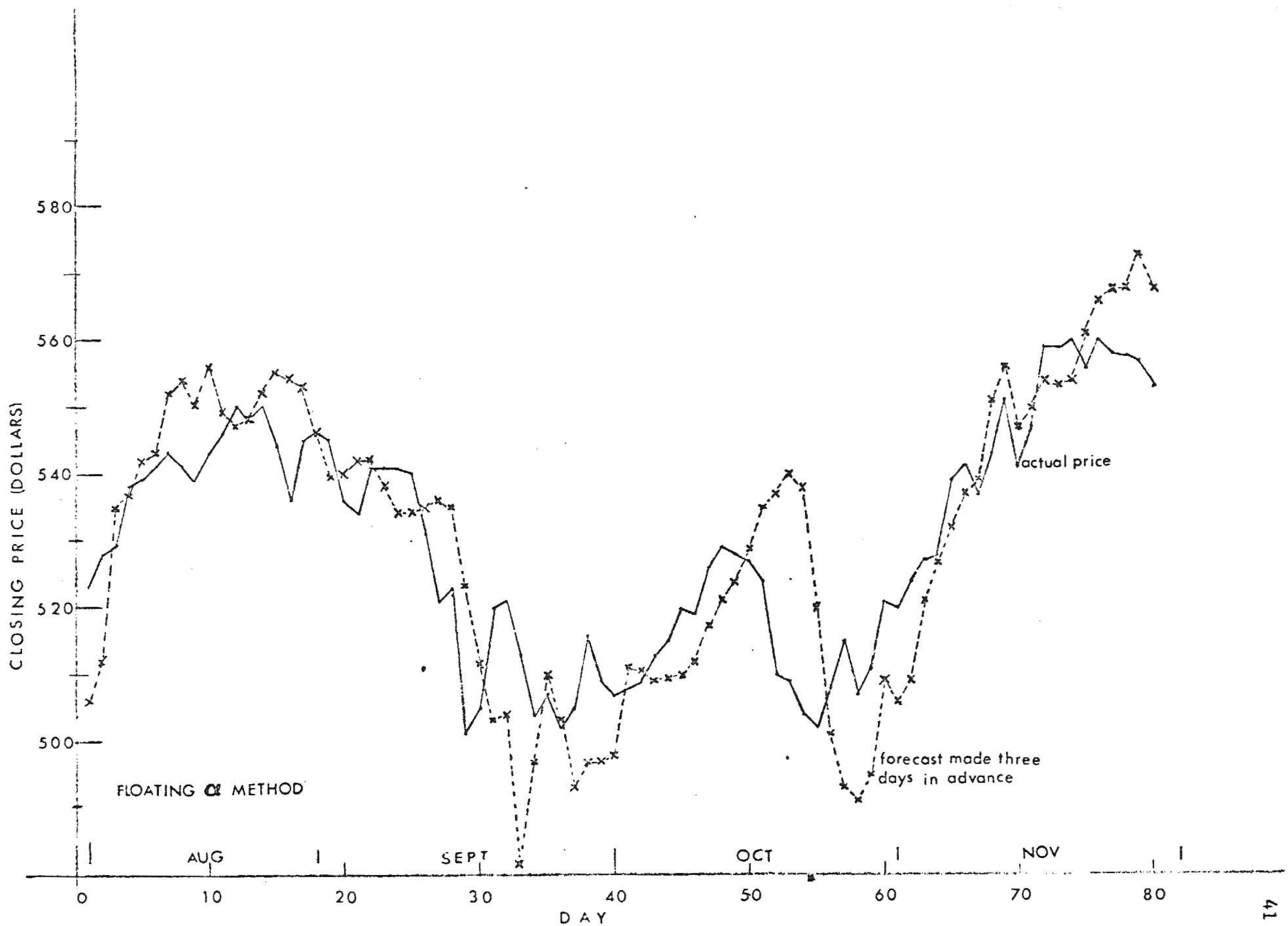

FIG 6 IBM common stock on the New York Exchange

CHAPTER III

Four steps must be followed to apply a forecasting system. They are as follows:

1) Preparing the data.
2) Testing the forecasting system.
3) Making the forecasts.
4) Utilizing the forecasts.

We will review separately each of these steps.

Preparing the Data

This step is the hardest problem. Making a specific decision in any system depends on the analysis, interpretation, and evaluation of the information that is available to the decision-maker. Every organization has a "memory system" in which data can be stored. The most obvious example of these memories are the organization files but the time required to obtain information stored in files can in some applications be prohibitively high. Computer memories have particular advantages but even the largest computers lack the flexibility and sizes that characterize cerebral storage of information.

It is quite apparent that the decision-maker can be deluged with information if he does not know how to select data that are pertinent to his problem. For this reason information must be carefully categorized. Decision problems exist as to what information should be collected and in what form; where and how long it should be stored, when and by whom it should be called for; how it should be evaluated; when it should be updated, supplemented, and so on.

To forecast time series using exponential smoothing, all that is required is a sequence of homogeneous values such as sales, production orders, etc., taken at regular time intervals. This time series must be selected accordingly with the process to be forecasted; for example, the structure of sales data of one item may differ from the production data of the same product although the same quantities are basically involved. That is because sales are in general stochastic processes produced for a factor beyond the company's control (consumer demand); conversely production is a planned process under the company's control.

Time intervals in which data were taken must be consistent with the forecasting lead time; for example, to do a weekly forecast, data must be taken in week periods or in daily periods which can be integrated into weeks; monthly or yearly data probably will not be helpful in forecasting the same process.

Before the selected data is applied it must be inspected to determine those aspects that show particular or abnormal events occurring in the process; for example, sales of a produce are affected during the time of a strike or during promotion campaigns. Events like these do not reflect a normal behavior of the process because of their abnormalities; it is up to the forecaster to either use or change these values, but if they are used the forecaster must take them into consideration when he interprets the resulting forecast.

Before time series is applied it is very convenient to determine its statistical properties, such properties as the range, the mean, and the standard deviation. In general if the time series varies widely, or shows wild random variations in short intervals or has a large standard deviation, its forecasting errors will be larger than the one that is more stable.

To show the application of the Exponential Smoothing techniques to forecast time series we selected from the computerized files of a feed company a daily sales data of one product recorded for a period of eighty-six (86) weeks. Then the data were summarized in a weekly and a monthly basis in order to forecast sales one (or "n") weeks or one month in advance.

Since only one product is forecasted we can expect that the errors will be relatively greater than for larger groups of products. Also we can expect more accurate forecasts for shorter periods of time than for larger.

Main characteristics of this data are as follows:

Mean = 367.5 tons/week

Standard deviation = 88.61 tons/week

Minimum sales in a week = 170.6 tons

Maximum sales in a week = 567.0

Data did not show serial correlation

General aspects of data show large variations between weeks.

With the help of a calendar one can observe that the greatest variations in the time series occurred in the "holiday weeks" and the weeks before and after holidays. The lowest values of data occurred during the weeks of Christmas and Easter which is explained by the fact that in those weeks the company worked 2 or 3 days/week as compared to the normal 5 days/week. In the weeks around the holidays the largest values of sales are observed. This is due to the fact that the customer either bought the feeds he needed to feed his animals in the previous week or bought the feeds in the following week in order to complete his minimal stock in inventory.

A forecaster can not expect accurate forecasts for these weeks and furthermore since the values observed during these times will enter into the smoothed statistics that abnormality will affect the accuracy of the whole forecasting system. It is up to the forecaster to determine how to use the values of data during the holidays. He may prefer not to use them and to fix by subjective estimation a more realistic level of sales rather than the ones the forecasting showed. He may prefer to bias the forecast or correct the data, or he may prefer to use the data just as it is, taking into consideration that forecast during the holiday weeks does not have a great confidence.

For the purpose of this thesis we used the data on its original form in order to show the facts explained above. For examples, in Figs. 7, 8, and 9 one can see the abnormal levels of sales during the 63rd week which is the week right before Christmas; in the 64th week, the Christmas, and so on. Also during the 42nd week, the Independence Day week, an "out of trend" level of sales was registered.

Testing the Forecasting System

The testing phase of forecasting systems includes the selection among the known techniques those which are the most convenient to apply. It is possible, for example, that for some applications a high

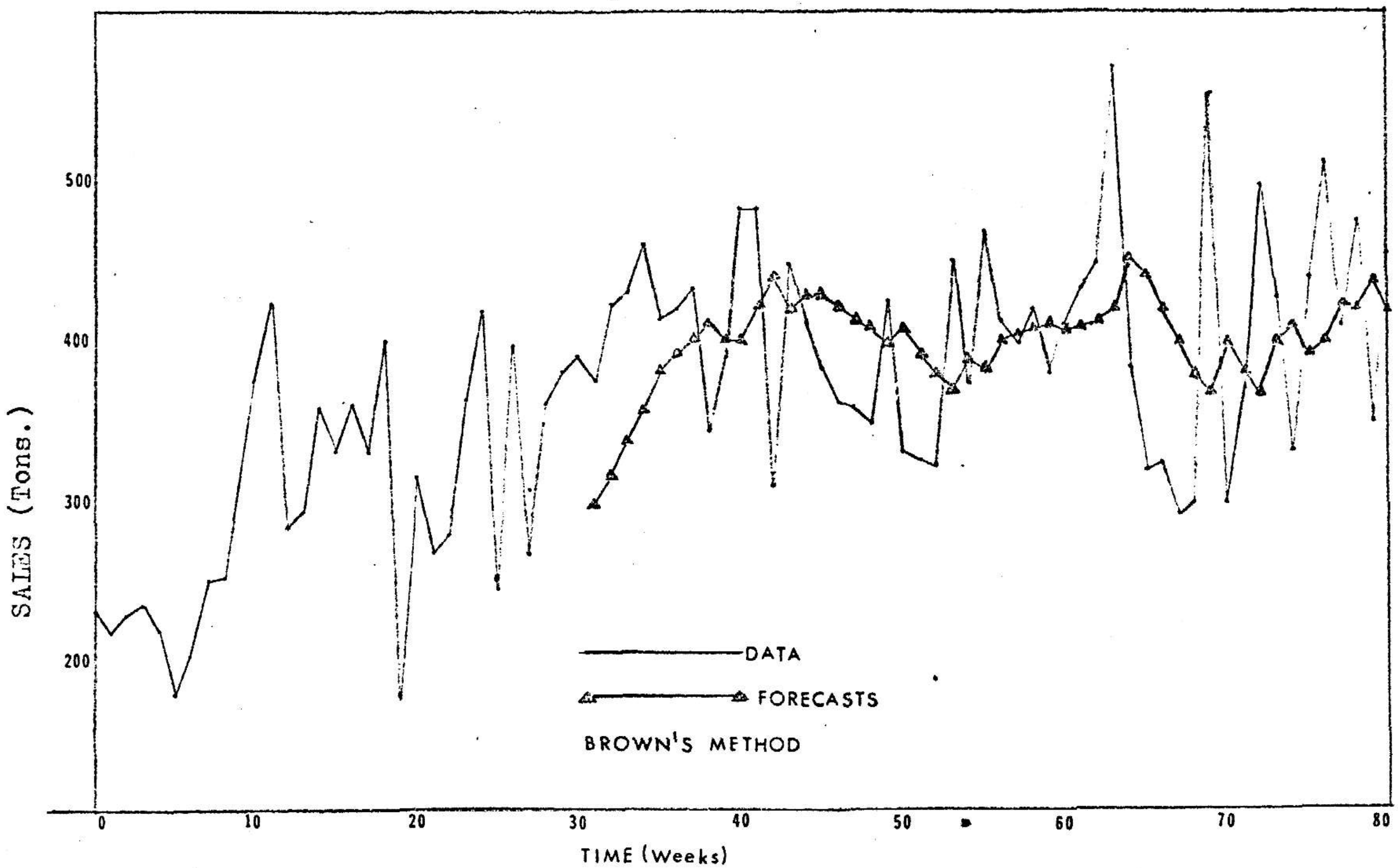

FIG. 7 Forecasts made one week in advance.
FEED DATA

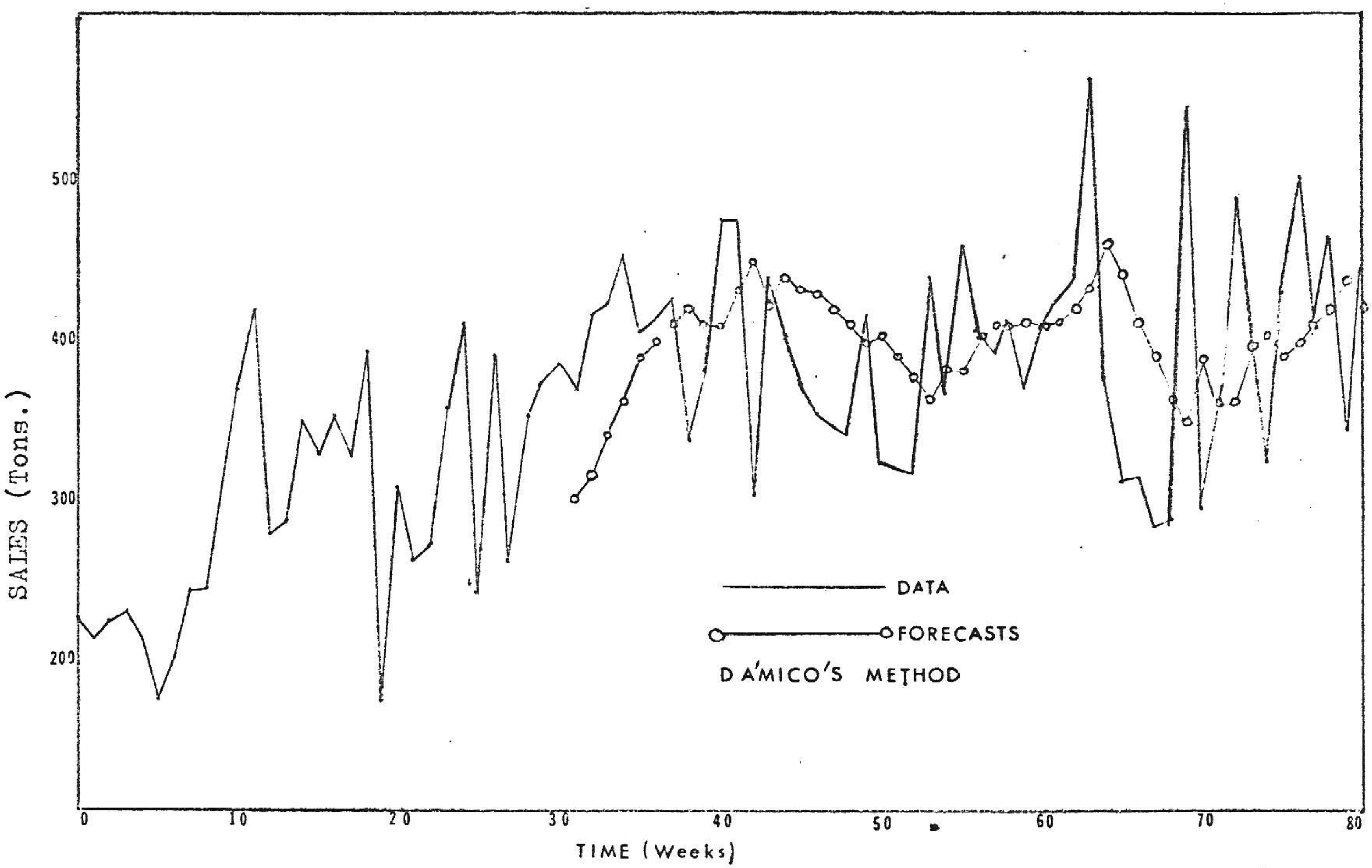

FIG. 8 FORECASTS MADE ONE WEEK IN ADVANCE.
FEED DATA.

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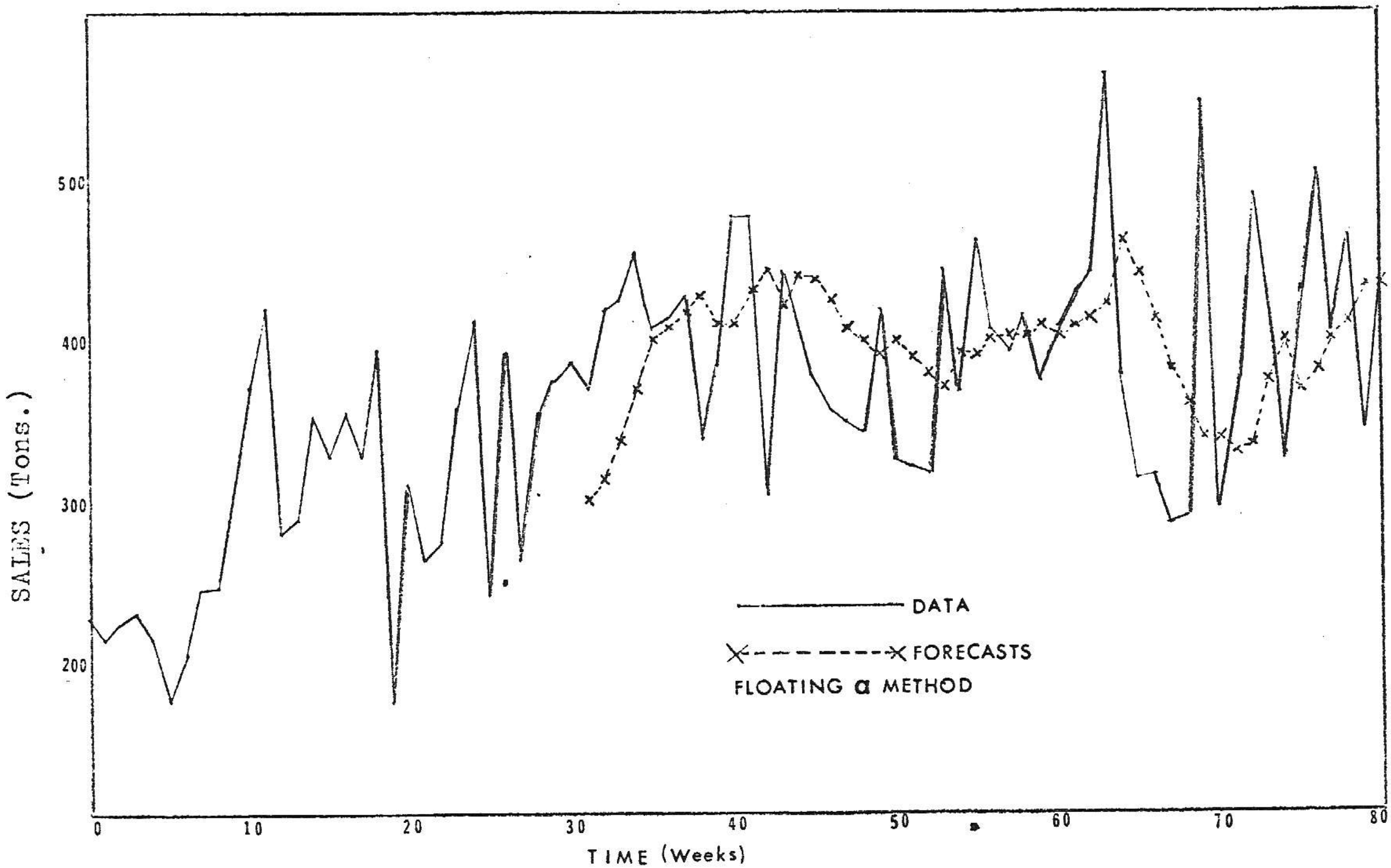

FIG. 9 Forecasts made one week in advance.
FEED DATA

accuracy is required which must be obtained by the use of regression analysis techniques by using several independent variables to forecast the dependent one. For some applications this method might be prohibitive because of its relative high cost. It is also possible that for other applications the cyclical or seasonal variation of data requires the use of spectral analysis techniques, which can be expensive.

We limited ourselves to use exponential smoothing for this thesis. We know that this technique does not have the accuracy of regression analysis to forecast the trend of the processes nor the ability to forecast seasonal variations that spectral analysis has, but other characteristics of this method can make it very useful for a variety of applications in the feed industry.

Exponential Smoothing operates by using smoothed statistics in which all past data are integrated with recent data weighted more in the statistics. On the other hand, only a few future observations can be forecasted with relative accuracy.

In general, exponential smoothing is applied when many items must frequently be forecasted. This method spends short computer time to do necessary calculations since only a few linear relations must be calculated. Also, since this method does not require carrying all the details of the data in memory, the requirement of memory is also small. These advantages mean in practice lower operation costs than the costs of running the other methods. On the other hand, this method can be applied to hand calculations without major problems by using (or not) desk calculators.

Once the advantages and limitations of this method are analyzed and it is selected to be used, the next step is the selection of the best procedure (Brown's, Floating Alpha Method) and the best model to do the forecasts.

A good forecasting system will always be in a state of flux as the forecaster learns more about his art. He may develop new techniques and wish to test them against actual company data. If the forecaster wants to determine whether or not a new forecasting technique is effective he does not have to wait for sales to materialize before learning whether this technique is valid or not. As we did in Chapter II, he can pretend he is making his forecasts a year or two earlier and then test the forecasting method against what really happened.

From the testing phase the forecaster obtains the estimate of future errors by assuming that the behavior of the process will continue in the same trend as before. If the errors obtained in the testing phase appear to be reasonable the method or system can be applied; if not, the forecaster must test another method or try with another model. It is important to take into consideration that sometimes some items are not forecastable at all since errors obtained using any model or method will be always high.

We tested in this thesis the forecasting system of our feed sales data by using the three methods developed and by using the three models that we restricted ourselves to use.

Appendix B includes the computer programs written for this testing phase as well as the resulting outputs obtained; they are as follows:

1) Constant Model or single exponential smoothing used to test the forecasts of the time series by using floating alpha method.

2) Linear Model or double exponential smoothing used to test the forecasting system by using all the three methods.

To test the forecasts using a quadratic model it was used the same program as for the IBM prices forecasted in Chapter I (see Appendix A).

The following initial conditions were assumed in all alternatives studied:

1) Initial period was extended up to thirty-six (36) weeks in order to pre-smooth the wild varieties observed in the same series.

2) Smoothed statistics was set equal to the average of sales in the initial period.

3) The forecasting of the first tested week (the 37th) was set equal to the average of sales in the initial period.

Mean absolute deviation was initialized by using equation II-1.

Then we must compute all the possible alternatives by using the three methods and the three models studied. Then it is convenient to tabulate the results and to obtain from these tables the best strategy to apply.

Table IV shows the standard deviations of errors obtained by testing the forecasts made one week in advance at different levels of the smoothing constant and by using the Brown's and floating alpha methods.

Table IV. Standard deviation of the forecasting errors (tons/week) for different levels of the smoothing constant and by using the three models.

Model	Brown's Method			Floating Alpha Method*		
	Alpha =					
	0.1	0.2	0.3	0.05	0.10	0.15
Constant	72.73	74.39	-	72.26	74.90	76.96
Linear	72.36	74.53	76.57	69.07	72.04	72.52
Quadratic	75.57	81.78	-	117.169	122.783	123.803

*Maximum value of alpha is 0.4.

The first conclusion one can obtain from the analysis of the results is that the time series under study does not follow a quadratic model since the standard deviation of the forecasting errors is significantly high.

The best result obtained in this phase occurred by using the floating alpha method and a linear model. This model was the one which best represents the time series. The forecasts resulting from using this model were plotted for each method in Figs. 7, 8, and 9. In the next table (Table V) one can see the influence of the lead time on the resulting forecast errors.

Analysis of results shows:

1. These tests confirm that forecasting is more accurate for shorter lead time.

2. The floating alpha method is the best to forecast this time series when short lead time (up to two) is required.

3. Brown's method is the most convenient one to forecast longer lead times.

Table V. Standard deviation of forecasting errors for different lead times and for the three methods using a linear model.

Lead time	Floating Alpha Method	Brown's Method	D'Amico's Method
1 week	69.074	73.356	73.540
2 weeks	73.899	76.132	78.026
3 weeks	81.935	76.881	78.361
4 weeks	91.340	79.076	82.011
5 weeks	91.031	79.049	81.695

When the testing phase is completed, the forecaster must decide which method he will apply to do the forecast. For this time series our suggestion is to use floating alpha method for short lead time forecasting. While for larger lead time we may prefer D'Amico's method although it was not shown to be better than Brown's, but the greater error obtained can be compensated by an automatized control of the behavior of the forecasting system.

Figure 10 shows the effects of the lead time on the standard deviation of forecasting errors.

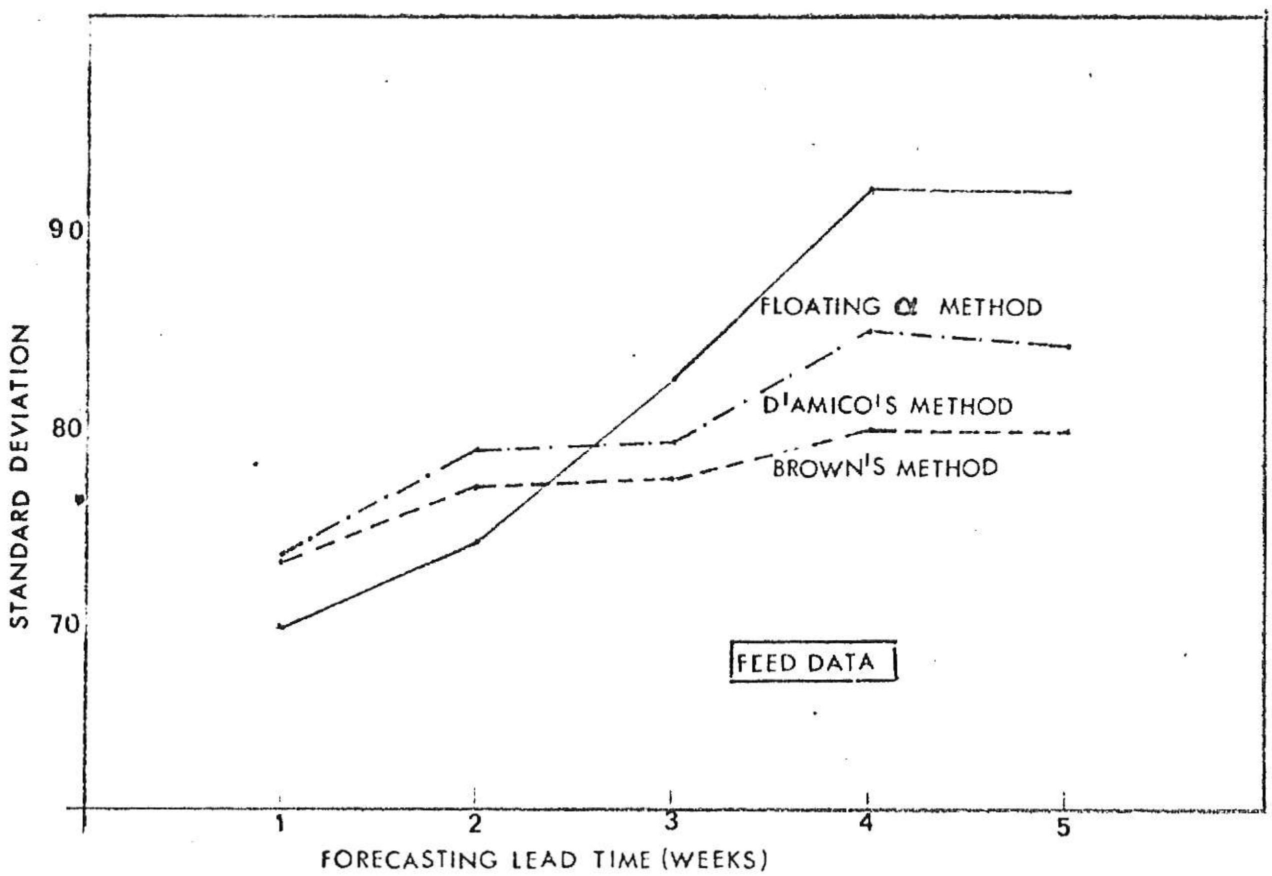

FIG. 10 Affect of forecasting lead time on standard deviation of forecasting errors

The resulting statistics should be plotted in order to analyze their overall behavior as affected by time. For example, Fig. 11 shows the mean absolute deviation for D'Amico's and the floating alpha methods against time. One can observe that in the interval between the 50th and the 60th day tested, the mean absolute deviation was greater when using the Floating Alpha Method than when using D'Amico's. If the forecaster had stopped the test at this interval and used the results obtained up to this point, he probably would have made the wrong selection since, overall, the Floating Alpha Method is better.

Making the Forecasts

The next step of forecasting is making the forecast. The forecaster has already obtained and analyzed the data, he has tested the different models and methods and he has made his decision about which model and method he will use. Then he needs only to apply them to obtain the information of the value of the process in the future. During this phase he first has to rewrite the program he will use which is basically the same one used in the testing phase but some modifications must be made for a continuous application. For example, in this phase he does not need to carry in memory all the past data but he needs to read in every new observation as it happens; probably he does not need to carry statistics of the errors to calculate the standard deviation since he will control the system by using the mean absolute deviation.

The forecaster must send the information collected to sections of the company that will use it for their specific purposes, such as the

Inventory Control, Production Control, and Sales. People that use the forecasting information must understand that the forecasting value is only the best possible prediction that can be determined and that a known margin of error must be allowed in such a prediction.

Utilizing the Forecasts

When the forecasting system is in operation, the forecaster must control the behavior of the system to see that it is accomplishing the goals for which it originally was designed.

When too many items are being forecasted it is really hard to control every one of them. D'Amico's and Floating Alpha Methods were designed to be autocontrolled in such a way that if the system is out of control at any time the program automatically will set it back into control by changing the smoothing constant parameter.

For effects of control a grouping of items in relation to their costs can improve the whole system. For example, in some products or items a relatively large error produces only relatively small losses but in others it is possible that small errors can produce large losses; so for the first group a measure of the effectiveness of the forecasting for all the items in the group may be enough; for the second group the control might be made item by item. We think that the control resulting from forecasts and the measure of the effect of the errors are not a function of the forecaster but of the people that used this information.

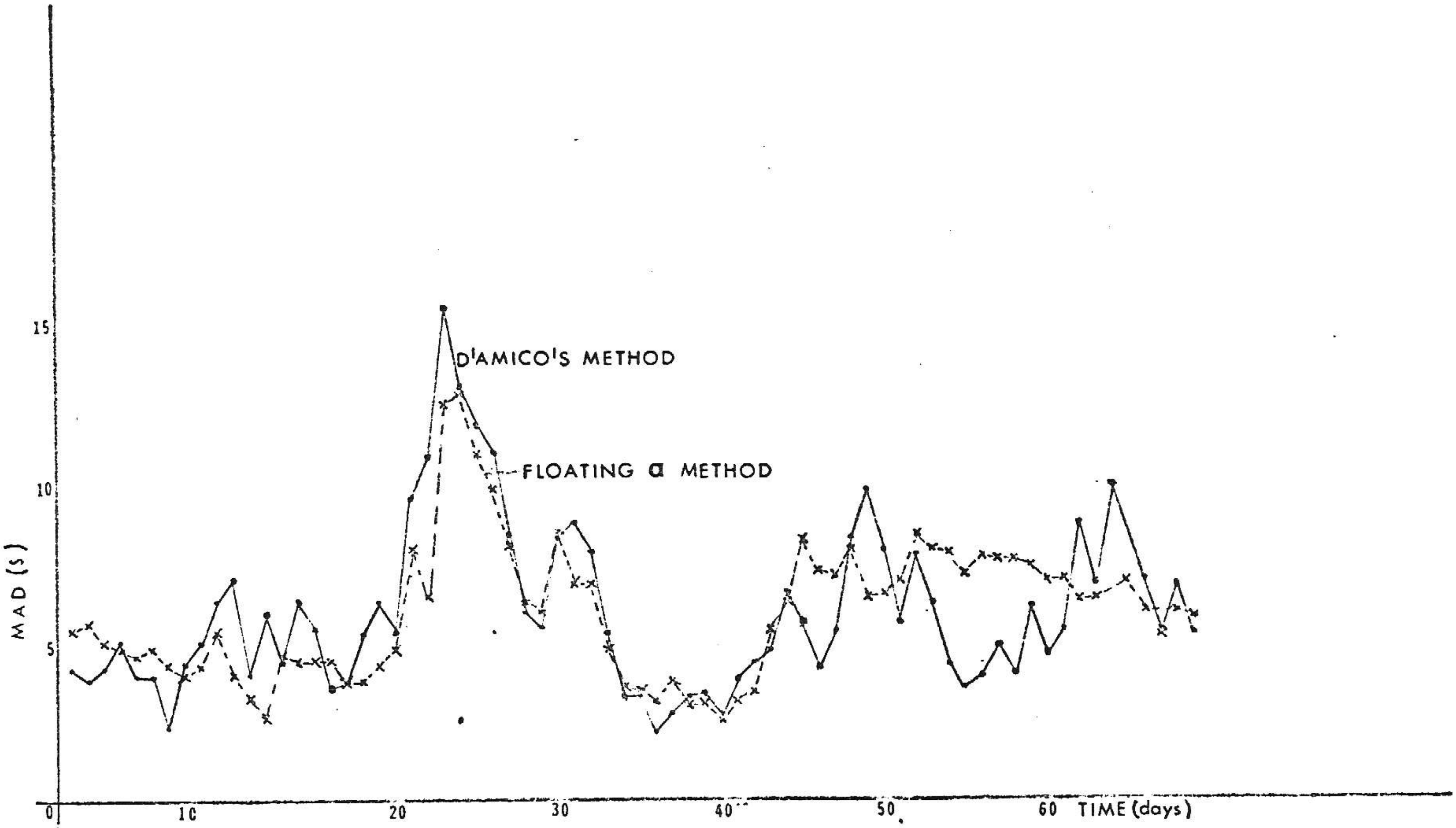

FIG. 11 The MAD in IBM stock price forecasts.

SUMMARY

Feed manufacturing firms, like others, must estimate the future values of their different functions to achieve their planning goals. Exponential smoothing is a device to forecast the future levels of these functions. Three methods using this technique were analyzed and tested in this thesis.

The first of them was developed by Robert G. Brown. It has the disadvantage that it can not be fully computerized since eventually it requires the forecaster's intervention. Such intervention can be cumbersome if many items are being forecasted.

The second method was developed by Peter D'Amico, who applied a relation between the mean absolute deviation at time "t" and the previous one calculated at time "t-1" to change automatically the smoothing constant when changes in the process are detected.

The third method called Floating Alpha Method was proposed in this thesis. In this method the forecasting errors are used to detect changes in the process. The forecasting system is revised with each new observation; if a change is detected in the time series the smoothing constant is changed accordingly to the magnitude of the observed change, otherwise if no change is detected the smoothing constant retains its previous value. This method as well as D'Amico's can be fully computerized.

By using actual feed sales data, we tested all possible alternatives by combining the three methods and the three models studied; furthermore, we tested them using different values of the parameters and by using different lead times.

From the results obtained we concluded that for shorter lead times the Floating Alpha Method was an improvement over that of Brown since a more accurate and faster response to changes in the time series was obtained. The main advantage of this method, in our opinion, is that it can be fully computerized. Although it was shown by the two time series analyzed in this thesis that the floating alpha method was the best, we cannot expect that this will always occur for all the time series but for comparable accuracy one must prefer it because of its autocontrol approach.

In practice it can be prohibitive to test, as we did here, all the possible alternatives for every time series. We suggest the use of the standardized limits of errors currently applied by the company as a measure to accept or reject the model or method tested. Then after it is in operation it can be improved.

ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to his major professor, Dr. Harry B. Pfost, for his advice and guidance during the preparation of this work. Acknowledgments are also extended to Drs. Louis E. Grosh and Paul E. Seib for their valuable suggestions and for being in the advisory committee.

Acknowledgment must also be extended to Dr. William J. Hoover, Head of the Department of Grain Science and Industry for providing the means and conditions for the accomplishment of this work.

Special thanks to Protinal C. A. and its Directors for giving me the time required, and the economic support which made this work possible.

Many thanks are given to Fred R. Anstaett and Dr. Oscar A. Romer for their valuable contribution and help which they rendered, and to Mrs. Helyn Marshall and Miss Joann Wells for having typed and graphed the mathematical models.

Last but not least, I want to thank my wife, Ada, and children, for their moral support and lost weekends.

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A P P E N D I C E S

APPENDIX A

```
$JOB              V.ALVARADO.
C
C              ***********************************************
C              *                                             *
C              * IBM COMMON STOCK TRADING ON THE NEW YORK    *
C              * EXCHANGE. DATA: DAILY CLOSING PRICE.        *
C              * FROM R. G. BROWN,TABLE C.5                  *
C              *                                             *
C              ***********************************************
C
C                THIS PROGRAM PERFORMS THE FORECASTING OF THE
C                DATA TAU   DAYS IN ADVANCE ACCORDING WITH
C                BROWN'S METHOD,
C                ALPHA IS EQUAL TO 0.1
C
      DIMENSION X(200),XHAT(200)
      REAL MAD
      DATA BLANK/' '/,ASTER/'*'/
C              * READ IN DATA.
      READ(5,1)(X(I),I=1,180)
    1 FORMAT(20F3.0)
C              * INITIALIZE TIME-OF FORECASTING.
      TAU=3.
      ITAU=3
C              * USE THE TWENTY INITIAL VALUES TO DETERMINE
C              * INITIAL CONDITIONS.
      NN=20
      SUM=0.
      ACC=0.
      DO 2 I=1,NN
      SUM=SUM+X(I)
    2 ACC=ACC+X(I)*X(I)
      STD=SQRT((ACC-SUM*SUM/NN)/(NN-1.))
      XMAD=STD*2.*SQRT(1./(3.1416*1.9))
      ALPHA=0.1
      BETA=1.-ALPHA
C                PRINT OUT TITLES.
      WRITE(6,3) ITAU
    3 FORMAT('1'//10X'IBM COMMON STOCK TRADING'/10X'ON THE'
     1' NEW YORK EXCHANGE'//10X'FORECAST MADE'I3' DAYS IN ADVANCE',/10X
     2'BROWNS METHOD',    /3X,'T'2X'X(T+TAU)'6X'XHAT'
     35X'ERROR'5X'ACC.ERR'9X'MAD'7X'TRACK'7X'ALPHA'2X'CONT.'/)
      ST1=SUM/20.
      ST2=SUM/20.
      ST3=SUM/20.
      XHAT0=3.*ST1-3.*ST2+ST3
      XHAT1=.15927*ST1-.26591*ST2+.10664*ST3
      XHAT2=(ST1-2.*ST2+ST3)/361.
C              * PERFORM INITIAL CALCULATIONS.
      FACT1=(ALPHA)/(2.*BETA*BETA)
      FACT2=6.-5.*ALPHA
      FACT3=10.-8.*ALPHA
      FACT4=4.-3.*ALPHA
      FACT5=(ALPHA*ALPHA)/(BETA*BETA)
      ACCERR=0.
      SUMERR=0.
      SACERR=0.
      MAD=XMAD
      CHECK=BLANK
      ICONT=0
```

```
            XN=0.
      C             * PERFORM THE FORECASTING.
            DO 100 I=21,125
            XN=XN+1
      C             * FORECAST X TAU  DAYS AHEAD.
            XHAT(I)=XHATO+TAU*XHAT1+0.5*TAU*TAU*XHAT2
      C
      C             * SMOOTH STATISTICS.
            ST1=ALPHA*X(I)+BETA*ST1
            ST2=ALPHA*ST1+BETA*ST2
            ST3=ALPHA*ST2+BETA*ST3
      C             * DETERMINE COEFFICIENTS.
            XHATO=3.*ST1-3.*ST2+ST3
            XHAT1=FACT1*(FACT2*ST1-FACT3*ST2+FACT4*ST3)
            XHAT2=FACT5*(ST1-2.*ST2+ST3)
      C             * DETERMINE ERROR AND SAVE ITS STATISTICS.
            ERROR=XHAT(I)-X(I+ITAU-1)
            ACCERR=ACCERR+ERROR
            SUMERR=SUMERR+ERROR
            SACERR=SACERR+ERROR*ERROR
      C             * DETERMINE MEAN ABSOLUTE DEVIATION.
            MAD=ALPHA*ABS(ERROR)+BETA*MAD
      C             * DETERMINE TRACKING SIGNAL.
            TRAC =SUMERR/MAD
            TRACK=ABS(TRAC)
      C             * CHECK IF THE PROGRAM IS  UNDER CONTROL.
      C              IF SYSTEM IS OUT OF CONTROL TWICE IN
      C              SUCCESSION : RESET SUMERR.
            IF(ICONT.EQ.1.AND.TRACK.GE.4.)GO TO 300
            IF(TRACK.GT.4.)GO TO 310
            CHECK=BLANK
            ICONT=0
            GO TO 100
      C     GO TO 320 (FLOATING AND DAMICOS'S METHODS)
        300 SUMERR=0.
            CHECK=ASTER
            ICONT=0
            GO TO 100
      C     GO TO 320 (FLOATING AND DAMICOS'S METHODS)
        310 CHECK=ASTER
            ICONT=1
      C
      C   **********************************************
      C         IF FLOATING ALPHA METHOD IS DESIRED
      C         REMOVE C IN COLUMN 1 OF NEXT LINES.
      C
      C 320 DIV=ABS(ERROR)
      C     IF(DIV.LE.INDEK)GO TO 100
      C     DERROR=(DIV-INDEX)/DIV
      C     ALPHA=(GAMMA-DELTA)*DERROR+DELTA
      C
      C   **********************************************
      C   *     D'AMICO'S METHOD.                      *
      C   *     IF D'AMICO'S METHOD IS DESIRED         *
      C         REMOVE C IN COLUMN 1 OF NEXT LINES.
      C 320 DIV=SMAD
      C     IF(SMAD.LT.MAD)DIV=MAD
      C     DMAD=ABS((SMAD-MAD)/DIV)
      C     ALPHA=(GAMMA-DELTA)*DMAD+DELTA
      C     BETA=1.-ALPHA
```

```
C       SMAD=MAD
C     ***********************************************
C       FACT1=(ALPHA)/(2.*BETA*BETA)
C       FACT2=6.-5.*ALPHA
C       FACT3=10.-8.*ALPHA
C       FACT4=4.-3.*ALPHA
C       FACT5=(ALPHA*ALPHA)/(BETA*BETA)
C     ***********************************
C              * PRINT OUT PARTIAL  RESULTS.
  100 WRITE(6,4)I,X(I+ITAU-1),XHAT(I),ERROR,SUMERR,MAD,TRACK,
     1ALPHA,CHECK
    4 FORMAT(I4,3F10.1,4F12.3,4X,A1)
C              * CALCULATE MEAN AND STANDARD DEVIATION OF ERRORS.
      EMEAN=ACCERR/XN
      STDEV=SQRT((SACERR-ACCERR*ACCERR/XN)/(XN-1.))
      WRITE(6,5)EMEAN,STDEV,ALPHA
    5 FORMAT(/20X'MEAN OF ERRORS ='F7.3/20X'STANDARD DEVIATION='F7.2/
     120X'ALPHA IS EQUAL TO='F5.3/)
      STOP
      END

$ENTRY
```

IBM COMMON STOCK TRADING
ON THE NEW YORK EXCHANGE

FORECAST MADE 3 DAYS IN ADVANCE .
BROWNS METHOD

T	X(T+TAU)	XHAT	ERROR	ACC.ERR	MAD	TRACK	ALPHA	CONT.
21	523.0	506.1	-16.9	-16.900	7.859	2.151	0.100	
22	528.0	512.2	-15.8	-32.700	8.653	3.779	0.100	
23	529.0	518.5	-10.5	-43.243	8.842	4.891	0.100	*
24	538.0	521.9	-16.1	0.000	9.568	6.203	0.100	*
25	539.0	526.3	-12.7	-12.695	9.881	1.285	0.100	
26	541.0	530.0	-11.0	-23.675	9.991	2.370	0.100	
27	543.0	536.0	-7.0	-30.638	9.688	3.162	0.100	
28	541.0	541.0	-0.0	-30.679	8.723	3.517	0.100	
29	539.0	545.3	6.3	-24.344	8.485	2.869	0.100	
30	543.0	549.2	6.2	-18.097	8.261	2.191	0.100	
31	548.0	551.3	3.3	-14.775	7.767	1.902	0.100	
32	550.0	551.9	1.9	-12.845	7.183	1.788	0.100	
33	548.0	553.5	5.5	-7.303	7.019	1.041	0.100	
34	550.0	556.3	6.3	-0.991	6.948	0.143	0.100	
35	544.0	558.9	14.9	13.924	7.745	1.798	0.100	
36	536.0	559.9	23.9	37.857	9.364	4.043	0.100	*
37	545.0	561.1	16.1	0.000	10.041	5.377	0.100	*
38	546.0	559.6	13.6	13.605	10.398	1.308	0.100	
39	545.0	555.2	10.2	23.803	10.378	2.294	0.100	
40	536.0	554.7	18.7	42.491	11.209	3.791	0.100	
41	534.0	554.4	20.4	62.907	12.129	5.186	0.100	*
42	541.0	553.6	12.6	0.000	12.179	6.202	0.100	*
43	541.0	549.5	8.5	8.546	11.816	0.723	0.100	
44	541.0	545.4	4.4	12.970	11.076	1.171	0.100	
45	538.0	544.6	6.6	19.529	10.625	1.838	0.100	
46	531.0	543.8	12.8	32.312	10.840	2.981	0.100	
47	521.0	543.1	22.1	54.398	11.965	4.546	0.100	*
48	523.0	541.4	18.4	0.000	12.607	5.773	0.100	*
49	501.0	537.5	36.5	36.454	14.992	2.432	0.100	
50	505.0	530.7	25.7	62.159	16.063	3.870	0.100	
51	520.0	526.1	6.1	68.261	15.067	4.531	0.100	*
52	521.0	514.6	-6.4	0.000	14.201	4.355	0.100	*
53	511.0	507.0	-4.0	-3.953	13.177	0.300	0.100	
54	504.0	506.7	2.7	-1.245	12.130	0.103	0.100	
55	507.0	507.1	0.1	-1.193	10.922	0.109	0.100	
56	502.0	504.0	2.0	0.766	10.026	0.076	0.100	
57	505.0	499.2	-5.8	-5.010	9.601	0.522	0.100	
58	516.0	496.8	-19.2	-24.182	10.558	2.290	0.100	
59	509.0	493.4	-15.6	-39.767	11.060	3.595	0.100	
60	507.0	492.1	-14.9	-54.666	11.444	4.777	0.100	*
61	508.0	495.3	-12.7	0.000	11.566	5.821	0.100	*
62	509.0	495.7	-13.3	-13.315	11.740	1.134	0.100	
63	513.0	495.5	-17.5	-30.814	12.316	2.502	0.100	
64	515.0	496.0	-19.0	-49.842	12.987	3.838	0.100	
65	520.0	497.0	-23.0	-72.891	13.994	5.209	0.100	*
66	519.0	499.4	-19.6	0.000	14.555	6.355	0.100	*
67	526.0	502.3	-23.7	-23.739	15.473	1.534	0.100	
68	529.0	506.5	-22.5	-46.229	16.175	2.858	0.100	
69	528.0	509.6	-18.4	-64.582	16.393	3.940	0.100	
70	527.0	514.8	-12.2	-76.827	15.978	4.808	0.100	*
71	524.0	519.9	-4.1	0.000	14.786	5.470	0.100	*

72	515.0	523.7	8.7	8.694	14.177	0.613	0.100	
73	509.0	526.3	17.3	25.968	14.486	1.793	0.100	
74	504.0	527.2	23.2	49.139	15.355	3.200	0.100	
75	502.0	524.6	22.6	71.698	16.075	4.460	0.100	*
76	508.0	520.3	12.3	0.000	15.693	5.349	0.100	*
77	515.0	515.0	-0.0	-0.006	14.124	0.000	0.100	
78	507.0	510.1	3.1	3.103	13.023	0.238	0.100	
79	511.0	508.4	-2.6	0.552	11.976	0.046	0.100	
80	521.0	509.7	-11.3	-10.715	11.905	0.900	0.100	
81	520.0	507.9	-12.1	-22.780	11.921	1.911	0.100	
82	524.0	508.0	-16.0	-38.754	12.326	3.144	0.100	
83	527.0	511.8	-15.2	-53.973	12.615	4.278	0.100	*
84	528.0	514.5	-13.5	0.000	12.709	5.313	0.100	*
85	539.0	518.0	-21.0	-20.965	13.534	1.549	0.100	
86	541.0	522.0	-19.0	-40.010	14.085	2.841	0.100	
87	537.0	525.4	-11.6	-51.626	13.838	3.731	0.100	
88	543.0	532.0	-11.0	-62.621	13.554	4.620	0.100	*
89	551.0	537.9	-13.1	0.000	13.512	5.606	0.100	*
90	541.0	540.9	-0.1	-0.093	12.170	0.008	0.100	
91	547.0	545.3	-1.7	-1.803	11.124	0.162	0.100	
92	559.0	551.4	-7.6	-9.370	10.768	0.870	0.100	
93	559.0	552.4	-6.6	-15.932	10.348	1.540	0.100	
94	560.0	555.1	-4.9	-20.808	9.801	2.123	0.100	
95	556.0	561.3	5.3	-15.505	9.351	1.658	0.100	
96	560.0	565.9	5.9	-9.610	9.005	1.067	0.100	
97	558.0	569.6	11.6	1.943	9.260	0.210	0.100	
98	558.0	570.6	12.6	14.589	9.599	1.520	0.100	
99	557.0	572.6	15.6	30.175	10.197	2.959	0.100	
100	553.0	573.0	20.0	50.209	11.181	4.491	0.100	*
101	554.0	573.0	19.0	0.000	11.966	5.786	0.100	*
102	555.0	572.3	17.3	17.320	12.502	1.385	0.100	
103	562.0	570.0	8.0	25.305	12.050	2.100	0.100	
104	569.0	568.2	-0.8	24.501	10.925	2.243	0.100	
105	585.0	566.9	-18.1	6.372	11.646	0.547	0.100	
106	590.0	568.1	-21.9	-15.522	12.671	1.225	0.100	
107	596.0	571.4	-24.6	-40.140	13.865	2.895	0.100	
108	586.0	579.5	-6.5	-46.620	13.127	3.552	0.100	
109	583.0	587.5	4.5	-42.115	12.265	3.434	0.100	
110	584.0	595.7	11.7	-30.444	12.205	2.494	0.100	
111	597.0	598.2	1.2	-29.290	11.100	2.639	0.100	
112	591.0	598.6	7.6	-21.663	10.753	2.015	0.100	
113	591.0	599.0	8.0	-13.705	10.473	1.309	0.100	
114	589.0	603.5	14.5	0.822	10.879	0.076	0.100	
115	581.0	604.6	23.6	24.411	12.150	2.009	0.100	
116	588.0	605.0	17.0	41.447	12.638	3.279	0.100	
117	592.0	604.3	12.3	53.730	12.603	4.263	0.100	*
118	597.0	600.4	3.4	0.000	11.686	4.892	0.100	*
119	594.0	599.6	5.6	5.585	11.076	0.504	0.100	
120	593.0	600.1	7.1	12.654	10.675	1.185	0.100	
121	583.0	602.0	19.0	31.639	11.506	2.750	0.100	
122	592.0	602.2	10.2	41.794	11.371	3.675	0.100	
123	591.0	601.7	10.7	52.463	11.301	4.642	0.100	*
124	597.0	597.4	0.4	0.000	10.214	5.178	0.100	*
125	597.0	597.1	0.1	0.106	9.203	0.012	0.100	
126	599.0	596.3	-2.7	-2.582	8.551	0.302	0.100	

MEAN OF ERRORS = 0.769
STANDARD DEVIATION= 13.95
ALPHA IS EQUAL TO=0.100

```
$JOB            V.ALVARADO.
C
C              *********************************************
C              *                                           *
C              * IBM COMMON STOCK TRADING ON THE NEW YORK  *
C              * EXCHANGE. DATA: DAILY CLOSING PRICE.      *
C              * FROM R. G. BROWN,TABLE C.5                *
C              *                                           *
C              *********************************************
C
C                THIS PROGRAM PERFORMS THE FORECASTING OF THE
C                DATA TAU   DAYS IN ADVANCE ACCORDING WITH
C                FLOATING ALPHA METHOD.
      DIMENSION X(200)
      REAL MAD,INDEX
      DATA BLANK/' '/,ASTER/'*'/
C              * READ IN DATA.
      READ(5,1)(X(I),I=1,180)
    1 FORMAT(20F3.0)
C              * FIX LOW LIMIT OF ERROR TO CHANGE ALPHA.
      INDEX=5.
C              * FIX LIMITS OF ALPHA.
      DELTA=0.05
      GAMMA=0.4
C              * USE THE TWENTY INITIAL VALUES TO
C              * DETERMINE INITIAL CONDITIONS.
      NN=20
      TAU=1.
      ITAU=1.
      DO 321 K=1,2
      SUM=0.
      ACC=0.
      DO 3 I=1,NN
      SUM=SUM+X(I)
    3 ACC=ACC+X(I)*X(I)
      STD=SQRT((ACC-SUM*SUM/NN)/(NN-1.))
      XMAD=STD*2.*SQRT(1./(3.1416*1.9))
C              * INITIALIZE TIME OF FORECASTING.
C              * FIX INITIAL VALUES.
      ALPHA=0.1
      BETA=1.-ALPHA
      ST1=SUM/NN
      ST2=ST1
      ST3=ST1
      XHAT0=3.*ST1-3.*ST2+ST3
      XHAT1=.15927*ST1-.26591*ST2+.10664*ST3
      XHAT2=(ST1-2*ST2+ST3)/361.
      FACT1=(ALPHA)/(2.*BETA*BETA)
      FACT2=6.-5.*ALPHA
      FACT3=10.-8.*ALPHA
      FACT4=4.-3.*ALPHA
      FACT5=(ALPHA*ALPHA)/(BETA*BETA)
      ACCERR=0.
      SACERR=0.
      MAD=XMAD
      XN=0.
      CHECK=BLANK
      ICONT=0
      SUMERR=0.
C              * PRINT OUT TITLES.
```

```
      WRITE(6,2) ITAU
    2 FORMAT('1'//10X'IBM COMMON STOCK TRADING'/10X'ON THE'
     1' NEW YORK EXCHANGE'//10X'FORECAST MADE'I3' DAYS IN ADVANCE',/)
     2'FLOATING ALPHA METHOD'/3X'I'2X'X(I+TAU)'6X'XHAT'
     35X'ERROR'5X'ACC.ERR'9X'MAD'7X'TRACK'7X'ALPHA'2X'CONT.'/)
C             * PERFORM THE FORECASTING.
      N1=126
      NN1=NN+1
      DO 100 I=NN1,N1
      XN=XN+1.
C             * FORECAST  X  TAU DAYS AHEAD.
      XHAT=XHAT0+TAU*XHAT1+.5*TAU*TAU*XHAT2
C             * SMOOTH STATISTICS.
      ST1=ALPHA*X(I)+BETA*ST1
      ST2=ALPHA*ST1+BETA*ST2
      ST3=ALPHA*ST2+BETA*ST3
C             * DETERMINE COEFFICIENTS.
      XHAT0=3.*ST1-3.*ST2+ST3
      XHAT1=FACT1*(FACT2*ST1-FACT3*ST2+FACT4*ST3)
      XHAT2=FACT5*(ST1-2.*ST2+ST3)
C             * DETERMINE ERROR AND SAVE ITS STATISTICS.
      ERROR=XHAT-X(I+ITAU-1)
      ACCERR=ACCERR+ERROR
      SUMERR=SUMERR+ERROR
      SACERR=SACERR+ERROR*ERROR
C             * DETERMINE  MEAN ABSOLUTE DEVIATION.
      MAD=ALPHA*ABS(ERROR )+BETA*MAD
C             * DETERMINE  TRACKING SIGNAL.
      TRAC =SUMERR/MAD
      TRACK=ABS(TRAC)
C             * CHECK IF PROGRAM IS UNDER CONTROL.
C          IF SYSTEM IS OUT OF CONTROL TWICE IN
C          SUCCESSION : RESET ACCUMULATE ERROR.
      IF(ICONT.EQ.1.AND.TRACK.GE.4.)GO TO 300
      IF(TRACK.GT.4.)GO TO 310
      CHECK=BLANK
      ICONT=0
      GO TO 320
  300 SUMERR=0.
      CHECK=ASTER
      ICONT=0
      GO TO 320
  310 CHECK=ASTER
      ICONT=1
C             * CHECK ERRORS FOR LESS THAN ITS LOW LIMIT.
C             * IF YES, KEEP VALUE OF ALPHA.
C             * IF NOT, CHANGE ALPHA.
  320 DIV=ABS(ERROR)
      IF(DIV.LE.INDEX)GO TO 100
C             * CHANGE ALPHA.
      DERROR=(DIV-INDEX)/DIV
      ALPHA=(GAMMA-DELTA)*DERROR+DELTA
      BETA=1.-ALPHA
      FACT1=(ALPHA)/(2.*BETA*BETA)
      FACT2=6.-5.*ALPHA
      FACT3=10.-8.*ALPHA
      FACT4=4.-3.*ALPHA
      FACT5=(ALPHA*ALPHA)/(BETA*BETA)
C             * PRINT OUT PARTIAL RESULTS.
  100 WRITE(6,101)I,X(I+ITAU-1),XHAT,ERROR,SUMERR,MAD,TRAC,
```

```
     1ALPHA,CHECK
 101 FORMAT(I4,3F10.1,4F12.3,4X,A1)
C             * CALCULATE MEAN AND STANDARD DEVIATION OF ERRORS.
      EMEAN=ACCERR/XN
      STDEV=SQRT((SACERR-ACCERR*ACCERR/XN)/(XN-1.))
      WRITE(6,102)EMEAN,STDEV,DELTA
 102 FORMAT(//20X'MEAN OF ERRORS='F7.3/ 20X'STANDARD DEVIATION='F7.
     120X'ALPHA IS RESTRICT TO BE .GE.'F5.2/)
      ITAU=3
 321 TAU=3.
      STOP
      END

$ENTRY
```

IBM COMMON STOCK TRADING
ON THE NEW YORK EXCHANGE

FORECAST MADE 1 DAYS IN ADVANCE
FLOATING ALPHA METHOD

T	X(T+TAU)	XHAT	ERROR	ACC.ERR	MAD	TRACK	ALPHA	CONT
21	523.0	506.1	-16.9	-16.900	7.859	-2.151	0.296	
22	527.0	511.2	-15.8	-32.731	10.222	-3.202	0.289	
23	523.0	527.7	4.7	-28.032	8.623	-3.251	0.289	
24	528.0	529.6	1.6	-26.400	6.600	-4.000	0.289	*
25	529.0	533.7	4.7	-21.697	6.051	-3.586	0.289	
26	538.0	535.2	-2.8	-24.536	5.121	-4.791	0.289	*
27	539.0	542.4	3.4	0.000	4.627	-4.565	0.289	*
28	541.0	545.3	4.3	4.263	4.521	0.943	0.289	
29	543.0	546.9	3.9	8.161	4.341	1.880	0.289	
30	541.0	548.1	7.1	15.239	5.133	2.969	0.153	
31	539.0	545.7	6.7	21.951	5.374	4.084	0.139	*
32	543.0	543.0	-0.0	0.000	4.632	4.729	0.139	*
33	548.0	543.5	-4.5	-4.548	4.621	-0.984	0.139	
34	550.0	545.8	-4.2	-8.703	4.556	-1.910	0.139	
35	548.0	548.3	0.3	-8.392	3.965	-2.117	0.139	
36	550.0	549.1	-0.9	-9.254	3.533	-2.620	0.139	
37	544.0	550.4	6.4	-2.819	3.937	-0.716	0.128	
38	536.0	548.7	12.7	9.508	5.062	1.957	0.263	
39	545.0	544.4	-0.6	9.266	3.902	2.375	0.263	
40	546.0	544.3	-1.7	7.586	3.319	2.286	0.263	
41	545.0	545.1	0.1	7.657	2.466	3.105	0.263	
42	536.0	544.6	8.6	16.214	4.065	3.989	0.195	
43	534.0	537.1	3.1	19.355	3.884	4.983	0.195	*
44	541.0	534.0	-7.0	12.335	4.497	2.743	0.151	
45	541.0	535.8	-5.2	7.160	4.600	1.557	0.062	
46	541.0	537.2	-3.8	3.406	4.547	0.749	0.062	
47	538.0	538.1	0.1	3.485	4.271	0.816	0.062	
48	531.0	537.8	6.8	10.268	4.426	2.320	0.142	
49	521.0	536.2	15.2	25.502	5.961	4.278	0.285	*
50	523.0	528.4	5.4	0.000	5.799	5.328	0.075	*
51	501.0	518.0	17.0	16.951	6.641	2.553	0.297	
52	505.0	516.7	11.7	28.660	8.145	3.519	0.251	
53	520.0	496.0	-24.0	4.670	12.114	0.386	0.327	
54	521.0	505.7	-15.3	-10.606	13.149	-0.807	0.285	
55	511.0	512.2	1.2	-9.426	9.732	-0.968	0.285	
56	504.0	508.4	4.4	-4.997	8.219	-0.608	0.285	
57	507.0	501.3	-5.7	-10.696	7.499	-1.426	0.093	
58	502.0	501.7	-0.3	-11.042	6.834	-1.616	0.093	
59	505.0	502.7	-2.3	-13.357	6.414	-2.082	0.093	
60	516.0	502.4	-13.6	-26.914	7.078	-3.802	0.271	
61	509.0	505.4	-3.6	-30.525	6.139	-4.972	0.271	*
62	507.0	506.4	-0.6	0.000	4.629	-6.716	0.271	*
63	508.0	506.4	-1.6	-1.618	3.814	-0.424	0.271	
64	509.0	507.5	-1.5	-3.136	3.191	-0.983	0.271	
65	513.0	509.0	-4.0	-7.093	3.399	-2.087	0.271	
66	515.0	513.1	-1.9	-8.556	2.983	-3.003	0.271	
67	520.0	516.6	-3.4	-12.308	3.083	-3.992	0.271	
68	519.0	522.1	3.1	-9.203	3.089	-2.979	0.271	
69	526.0	523.4	-2.6	-11.787	2.952	-3.992	0.271	
70	529.0	529.1	0.1	-11.651	2.179	-5.366	0.271	*
71	528.0	533.5	5.5	-6.156	3.088	-1.994	0.084	

72	527.0	534.0	7.0	0.794	3.411	0.233	0.148	
73	524.0	530.9	6.9	7.703	3.930	1.960	0.147	
74	515.0	529.9	14.9	22.604	5.539	4.081	0.283	*
75	509.0	524.5	15.5	0.000	8.357	4.561	0.287	*
76	504.0	511.9	7.9	7.896	8.224	0.960	0.178	
77	502.0	501.6	-0.4	7.508	6.827	1.100	0.178	
78	508.0	500.2	-7.8	-0.318	7.005	-0.045	0.176	
79	515.0	501.1	-13.9	-14.187	8.216	-1.727	0.274	
80	507.0	505.8	-1.2	-15.341	6.282	-2.442	0.274	
81	511.0	503.8	-7.2	-22.589	6.546	-3.451	0.159	
82	521.0	507.2	-13.8	-36.413	7.700	-4.729	0.273	*
83	520.0	513.5	-6.5	0.000	7.371	-5.821	0.131	*
84	524.0	520.1	-3.9	-3.950	6.924	-0.570	0.131	
85	527.0	521.6	-5.4	-9.344	6.724	-1.390	0.076	
86	528.0	525.0	-3.0	-12.369	6.446	-1.922	0.076	
87	539.0	525.9	-13.1	-25.466	6.947	-3.666	0.266	
88	541.0	529.8	-11.2	-36.672	8.081	-4.538	0.244	*
89	537.0	545.6	8.6	-28.064	8.209	-3.419	0.197	
90	543.0	546.1	3.1	-24.987	7.200	-3.470	0.197	
91	551.0	548.2	-2.8	-27.755	6.336	-4.387	0.197	*
92	541.0	554.4	13.4	-14.351	7.734	-1.856	0.270	
93	547.0	551.7	4.7	--9.677	6.909	-1.401	0.270	
94	559.0	554.4	-4.6	-14.323	6.298	-2.274	0.270	
95	559.0	563.1	4.1	-10.222	5.705	-1.792	0.270	
96	560.0	566.0	6.0	-4.207	5.789	-0.727	0.109	
97	556.0	566.8	10.8	6.602	6.336	1.042	0.238	
98	560.0	562.7	2.7	9.310	5.473	1.701	0.238	
99	558.0	564.4	6.4	15.717	5.695	2.760	0.127	
100	558.0	562.2	4.2	19.888	5.502	3.615	0.127	
101	557.0	560.8	3.8	23.673	5.284	4.480	0.127	*
102	553.0	559.8	6.8	0.000	5.473	5.563	0.142	*
103	554.0	557.4	3.4	3.413	5.181	0.659	0.142	
104	555.0	555.8	0.8	4.169	4.554	0.915	0.142	
105	562.0	555.0	-7.0	-2.864	4.905	-0.584	0.151	
106	569.0	557.4	-11.6	-14.501	5.923	-2.448	0.250	
107	585.0	562.3	-22.7	-37.157	10.100	-3.679	0.323	
108	590.0	580.1	-9.9	-47.042	10.030	-4.690	0.223	*
109	596.0	596.9	0.9	0.000	7.987	-5.781	0.223	*
110	586.0	600.7	14.7	14.685	9.481	1.549	0.281	
111	583.0	597.3	14.3	28.940	10.822	2.674	0.277	
112	584.0	591.8	7.8	36.707	9.975	3.680	0.175	
113	597.0	588.0	-9.0	27.713	9.803	2.827	0.205	
114	591.0	593.1	2.1	29.817	8.222	3.627	0.205	
115	591.0	593.6	2.6	32.399	7.063	4.587	0.205	*
116	589.0	593.3	4.3	0.000	6.495	5.650	0.205	*
117	581.0	591.6	10.6	10.570	7.332	1.442	0.234	
118	588.0	585.3	-2.7	7.914	6.236	1.269	0.234	
119	592.0	585.7	-6.3	1.574	6.260	0.251	0.124	
120	597.0	588.8	-8.2	-6.617	6.500	-1.018	0.186	
121	594.0	591.8	-2.2	-8.771	5.690	-1.542	0.186	
122	593.0	593.3	0.3	-8.446	4.690	-1.801	0.186	
123	583.0	593.5	10.5	2.081	5.778	0.360	0.234	
124	592.0	587.9	-4.1	-1.979	5.376	-0.368	0.234	
125	591.0	589.5	-1.5	-3.515	4.479	-0.785	0.234	
126	597.0	585.8	-7.2	-10.708	5.113	-2.094	0.157	

MEAN OF ERRORS= 0.057
STANDARD DEVIATION= 8.143
ALPHA IS RESTRICT TO BE .GE. 0.05

IBM COMMON STOCK TRADING
ON THE NEW YORK EXCHANGE

FORECAST MADE 3 DAYS IN ADVANCE
FLOATING ALPHA METHOD

T	X(T+TAU)	XHAT	ERROR	ACC.ERR	MAD	TRACK	ALPHA	CON
21	523.0	506.1	-16.9	-16.900	7.859	-2.151	0.296	
22	528.0	512.2	-15.8	-32.700	10.213	-3.202	0.289	
23	529.0	540.9	11.9	-20.808	10.698	-1.945	0.253	
24	538.0	541.1	3.1	-17.755	8.765	-2.026	0.253	
25	539.0	542.1	3.1	-14.620	7.342	-1.991	0.253	
26	541.0	543.2	2.2	-12.423	6.041	-2.056	0.253	
27	543.0	552.0	9.0	-3.413	6.792	-0.503	0.206	
28	541.0	555.0	14.0	10.613	8.280	1.282	0.275	
29	539.0	553.0	14.0	24.614	9.855	2.498	0.275	
30	543.0	558.8	15.8	40.418	11.491	3.517	0.289	
31	548.0	553.3	5.3	45.671	9.687	4.715	0.067	*
32	550.0	545.6	-4.4	0.000	9.331	4.426	0.067	*
33	548.0	543.2	-4.8	-4.797	9.028	-0.531	0.067	
34	550.0	544.7	-5.3	-10.125	8.781	-1.153	0.072	
35	544.0	546.4	2.4	-7.762	8.322	-0.933	0.072	
36	536.0	547.5	11.5	3.724	8.548	0.436	0.248	
37	545.0	548.8	3.8	7.534	7.375	1.022	0.248	
38	546.0	550.1	4.1	11.648	6.567	1.774	0.248	
39	545.0	538.1	-6.9	4.720	6.657	0.709	0.147	
40	536.0	541.9	5.9	10.652	6.550	1.626	0.105	
41	534.0	544.3	10.3	20.924	6.940	3.015	0.230	
42	541.0	544.6	3.6	24.527	6.174	3.973	0.230	
43	541.0	535.6	-5.4	19.108	6.001	3.184	0.077	
44	541.0	528.9	-12.1	7.042	6.468	1.089	0.255	
45	538.0	535.3	-2.7	4.335	5.509	0.787	0.255	
46	531.0	533.7	2.7	7.021	4.789	1.466	0.255	
47	521.0	536.4	15.4	22.396	7.488	2.991	0.286	
48	523.0	534.5	11.5	33.889	8.634	3.925	0.248	
49	501.0	523.6	22.6	56.488	12.094	4.671	0.323	*
50	505.0	511.5	6.5	0.000	10.289	6.122	0.131	*
51	520.0	501.8	-18.2	-18.208	11.325	-1.608	0.304	
52	521.0	504.9	-16.1	-34.279	12.767	-2.685	0.291	
53	511.0	473.8	-37.2	-71.458	19.874	-3.596	0.353	
54	504.0	498.3	-5.7	-77.202	14.887	-5.186	0.095	*
55	507.0	514.5	7.5	0.000	14.182	-4.915	0.167	*
56	502.0	514.0	12.0	12.039	13.826	0.871	0.255	
57	505.0	505.4	0.4	12.424	10.403	1.194	0.255	
58	516.0	498.5	-17.5	-5.119	12.221	-0.419	0.300	
59	509.0	492.6	-16.4	-21.518	13.475	-1.597	0.293	
60	507.0	494.0	-13.0	-34.528	13.339	-2.589	0.265	
61	508.0	514.0	6.0	-28.504	11.397	-2.501	0.109	
62	509.0	512.3	3.3	-25.237	10.507	-2.402	0.109	
63	513.0	509.4	-3.6	-28.866	9.754	-2.959	0.109	
64	515.0	509.0	-6.0	-34.823	9.338	-3.729	0.106	
65	520.0	509.2	-10.8	-45.612	9.492	-4.805	0.238	*
66	519.0	510.9	-8.1	0.000	9.164	-5.863	0.184	*
67	526.0	518.4	-7.6	-7.568	8.870	-0.853	0.169	
68	529.0	522.5	-6.5	-14.039	8.465	-1.658	0.130	
69	528.0	524.2	-3.8	-17.834	7.860	-2.269	0.130	
70	527.0	527.0	0.0	-17.817	6.844	-2.603	0.130	
71	524.0	531.5	7.5	-10.281	6.933	-1.483	0.168	

72	515.0	534.1	19.1	8.794	8.970	0.980	0.308	
73	509.0	537.7	28.7	37.506	15.056	2.491	0.339	
74	504.0	542.2	38.2	75.720	22.908	3.305	0.354	
75	502.0	520.5	18.5	94.203	21.340	4.414	0.305	*
76	508.0	499.1	-8.9	0.000	17.529	4.869	0.202	*
77	515.0	490.4	-24.6	-24.594	18.960	-1.297	0.329	
78	507.0	493.6	-13.4	-38.027	17.142	-2.218	0.270	
79	511.0	493.7	-17.3	-55.289	17.175	-3.219	0.299	
80	521.0	510.0	-11.0	-66.289	15.331	-4.324	0.241	*
81	520.0	506.0	-14.0	0.000	15.016	-5.349	0.275	*
82	524.0	509.8	-14.2	-14.218	14.796	-0.961	0.277	
83	527.0	525.1	-1.9	-16.084	11.216	-1.434	0.277	
84	528.0	529.8	1.8	-14.277	8.611	-1.658	0.277	
85	539.0	535.5	-3.5	-17.740	7.185	-2.469	0.277	
86	541.0	540.1	-0.9	-18.668	5.453	-3.424	0.277	
87	537.0	541.2	4.2	-14.512	5.094	-2.849	0.277	
88	543.0	553.8	10.8	-3.686	6.681	-0.552	0.238	
89	551.0	559.1	8.1	4.453	7.029	0.634	0.185	
90	541.0	551.4	10.4	14.823	7.647	1.939	0.231	
91	547.0	550.7	3.7	18.545	6.739	2.752	0.231	
92	559.0	561.8	2.8	21.323	5.823	3.662	0.231	
93	559.0	554.1	-4.9	16.443	5.605	2.934	0.231	
94	560.0	554.8	-5.2	11.195	5.522	2.027	0.067	
95	556.0	566.2	10.2	21.363	5.831	3.664	0.228	
96	560.0	559.1	-0.9	20.424	4.716	4.330	0.228	*
97	558.0	572.1	14.1	0.000	6.850	5.037	0.276	*
98	558.0	568.4	10.4	10.383	7.824	1.327	0.231	
99	557.0	570.6	13.6	23.970	9.158	2.617	0.271	
100	553.0	565.2	12.2	36.182	9.986	3.623	0.257	
101	554.0	562.0	8.0	44.221	9.487	4.661	0.182	*
102	555.0	558.0	3.0	0.000	8.309	5.687	0.182	*
103	562.0	554.1	-7.9	-7.856	8.234	-0.959	0.178	
104	569.0	552.0	-17.0	-24.856	9.790	-2.539	0.297	
105	585.0	551.7	-33.3	-58.201	16.782	-3.468	0.348	
106	590.0	557.9	-32.1	-90.335	22.117	-4.084	0.346	*
107	596.0	574.8	-21.2	0.000	21.816	-5.115	0.318	*
108	586.0	606.6	20.6	20.577	21.422	0.961	0.315	
109	583.0	615.8	32.8	53.358	25.000	2.134	0.347	
110	584.0	623.4	39.4	92.763	29.993	3.093	0.356	
111	597.0	604.0	7.0	99.725	21.803	4.574	0.149	*
112	591.0	584.4	-6.6	0.000	19.545	4.764	0.135	*
113	591.0	585.1	-5.9	-5.950	17.706	-0.336	0.106	
114	589.0	590.8	1.8	-4.111	16.026	-0.257	0.106	
115	581.0	591.3	10.3	6.170	15.418	0.400	0.230	
116	588.0	591.8	3.8	9.931	12.739	0.780	0.230	
117	592.0	591.0	-1.0	8.914	10.046	0.887	0.230	
118	597.0	581.3	-15.7	-6.832	11.355	-0.602	0.289	
119	594.0	583.2	-10.8	-17.604	11.187	-1.574	0.238	
120	593.0	588.9	-4.1	-21.719	9.507	-2.285	0.238	
121	583.0	596.9	13.9	-7.849	10.544	-0.744	0.274	
122	592.0	597.1	5.1	-2.762	9.049	-0.305	0.056	
123	591.0	595.5	4.5	1.652	8.792	0.192	0.056	
124	597.0	592.3	-4.7	-3.021	8.564	-0.353	0.056	
125	597.0	592.2	-4.8	-7.801	8.352	-0.934	0.056	
126	599.0	592.0	-7.0	-14.833	8.278	-1.792	0.151	

MEAN OF ERRORS= 0.324
STANDARD DEVIATION= 13.539
ALPHA IS RESTRICT TO BE .GE. 0.05

```
$JOB              V.ALVARADO.
C
C               **********************************************
C               *                                            *
C               * IBM COMMON STOCK TRADING ON THE NEW YORK   *
C               * EXCHANGE. DATA: DAILY CLOSING PRICE.       *
C               * FROM R. G. BROWN,TABLE C.5                 *
C               *                                            *
C               **********************************************
C
C                 THIS PROGRAM PERFORMS THE FORECASTING OF THE
C                 DATA TAU   DAYS IN ADVANCE ACCORDING WITH
C                 D'AMICO'S METHOD.
      DIMENSION X(200),XHAT(200)
      REAL MAD
      DATA BLANK/' '/,ASTER/'*'/
C               * READ IN DATA.
      GAMMA=.4
      DELTA=.1
      N=180
      NN=20
      N1=126
      NN1=NN+1
      READ(5,1)(X(I),I=1,N)
    1 FORMAT(20F3.0)
C               * USE THE TWENTY INITIAL VALUES TO DETERMINE
C               * INITIAL CONDITIONS.
      SUM=0.
      ACC=0.
      DO 2 I=1,NN
      SUM=SUM+X(I)
    2 ACC=ACC+X(I)*X(I)
      STD=SQRT((ACC-SUM*SUM/NN)/(NN-1.))
      XMAD=STD*2.*SQRT(1./(3.1416*1.9))
C               * INITIALIZE TIME OF FORECASTING.
      ITAU=1.
      TAU=1.
      DO 321 K=1,2
C               * PERFORM INITIAL CALCULATIONS.
      ST1=SUM/NN
      ST2=ST1
      ST3=ST1
      XHATO=3.*ST1-3.*ST2+ST3
      XHAT1=.15927*ST1-.26591*ST2+.10664*ST3
      XHAT2=(ST1-2.*ST2+ST3)/361.
      ALPHA=0.1
      BETA=1.-ALPHA
      FACT1=(ALPHA)/(2.*BETA*BETA)
      FACT2=6.-5.*ALPHA
      FACT3=10.-8.*ALPHA
      FACT4=4.-3.*ALPHA
      FACT5=(ALPHA*ALPHA)/(BETA*BETA)
      ACCERR=0.
      SACERR=0.
      MAD=XMAD
      XN=0.
      CHECK=BLANK
      ICONT=0
      SUMERR=0.
C               * PRINT OUT TITLES.
```

```
      WRITE(6,3)ITAU
    3 FORMAT('1'//10X'IBM COMMON STOCK TRADING'/10X'ON THE'
     1' NEW YORK EXCHANGE'//10X'FORECAST MADE'I3' DAYS IN ADVANCE',/10X
     2'METHOD OF D AMICO'/3X,'T'2X'X(T+TAU)'6X'XHAT'
     35X'ERROR'5X'ACC.ERR'9X'MAD'7X'TRACK'7X'ALPHA'2X'CONT.'/)
C              * PERFORM THE FORECASTING.
      DO 100 I=NN1,N1
      XN=XN+1.
C              * FORECAST X TAU  DAYS AHEAD.
      XHAT(I)=XHAT0+TAU*XHAT1+0.5*TAU*TAU*XHAT2
C
C              * SMOOTH STATISTICS.
      ST1=ALPHA*X(I)+BETA*ST1
      ST2=ALPHA*ST1+BETA*ST2
      ST3=ALPHA*ST2+BETA*ST3
C              * DETERMINE COEFFICIENTS.
      XHAT0=3.*ST1-3.*ST2+ST3
      XHAT1=FACT1*(FACT2*ST1-FACT3*ST2+FACT4*ST3)
      XHAT2=FACT5*(ST1-2.*ST2+ST3)
C              * DETERMINE ERROR AND SAVE ITS STATISTICS.
      ERROR=XHAT(I)-X(I+ITAU-1)
      ACCERR=ACCERR+ERROR
      SACERR=SACERR+ERROR*ERROR
      SUMERR=SUMERR+ERROR
C                SAVE OLD MAD
      SMAD=MAD
C              * DETERMINE MEAN ABSOLUTE DEVIATION.
      MAD=ALPHA*ABS(ERROR)+BETA*MAD
C              * DETERMINE TRACKING SIGNAL.
      TRAC =SUMERR/MAD
      TRACK=ABS(TRAC)
C                IF SYSTEM IS OUT OF CONTROL TWICE IN
C                SUCCESSION : RESET SUM OF ERRORS.
      IF(ICONT.EQ.1.AND.TRACK.GE.4.)GO TO 300
      IF(TRACK.GT.4.)GO TO 310
      CHECK=BLANK
      ICONT=0
      GO TO 320
  300 SUMERR=0.
      CHECK=ASTER
      ICONT=0
      GO TO 320
  310 CHECK=ASTER
      ICONT=1
C     CHANGE ALPHA
  320 DIV=SMAD
      IF(SMAD.LT.MAD)DIV=MAD
      DMAD=ABS((SMAD-MAD)/DIV)
      ALPHA=(GAMMA-DELTA)*DMAD+DELTA
      BETA=1.-ALPHA
      FACT1=(ALPHA)/(2.*BETA*BETA)
      FACT2=6.-5.*ALPHA
      FACT3=10.-8.*ALPHA
      FACT4=4.-3.*ALPHA
      FACT5=(ALPHA*ALPHA)/(BETA*BETA)
C              * PRINT OUT PARTIAL  RESULTS.
  100 WRITE(6,101)I,X(I+ITAU-1),XHAT(I),ERROR,SUMERR,MAD,TRAC,
     1ALPHA,CHECK
  101 FORMAT(I4,3F10.1,4F12.3,4X,A1)
C              * CALCULATE MEAN AND STANDARD DEVIATION OF ERRORS.
```

```
          EMEAN=ACCERR/XN
          STDEV=SQRT((SACERR-ACCERR*ACCERR/XN)/(XN-1.))
          WRITE(6,5)EMEAN,STDEV,DELTA
        5 FORMAT(/20X'MEAN OF ERRORS ='F7.3/20X'STANDARD DEVIATION='F7.2/
         120X'ALPHA IS .GE.  =   'F5.3/)
          ITAU=3
      221 TAU=3.
          WRITE(6,333)
      333 FORMAT('1')
C
          STOP
          END

$ENTRY
```

IBM COMMON STOCK TRADING
ON THE NEW YORK EXCHANGE

FORECAST MADE 1 DAYS IN ADVANCE
METHOD OF D AMICO

T	X(T+TAU)	XHAT	ERROR	ACC.ERR	MAD	TRACK	ALPHA	CONT.
21	523.0	506.1	-16.9	-16.900	7.859	-2.151	0.138	
22	527.0	511.2	-15.8	-32.731	8.962	-3.652	0.137	
23	523.0	518.6	-4.4	-37.103	8.333	-4.452	0.121	*
24	528.0	522.1	-5.9	0.000	8.042	-5.350	0.110	*
25	529.0	525.8	-3.2	-3.236	7.511	-0.431	0.120	
26	538.0	528.5	-9.5	-12.704	7.745	-1.640	0.109	
27	539.0	534.3	-4.7	-17.423	7.415	-2.350	0.113	
28	541.0	538.1	-2.9	-20.367	6.911	-2.947	0.120	
29	543.0	541.9	-1.1	-21.470	6.211	-3.457	0.130	
30	541.0	545.7	4.7	-16.809	6.009	-2.797	0.110	
31	539.0	547.7	8.7	-8.066	6.309	-1.278	0.114	
32	543.0	547.4	4.4	-3.710	6.086	-0.610	0.111	
33	548.0	548.8	0.8	-2.927	5.500	-0.532	0.129	
34	550.0	551.1	1.1	-1.831	4.932	-0.371	0.131	
35	548.0	554.2	6.2	4.374	5.099	0.858	0.110	
36	550.0	555.1	5.1	9.471	5.099	1.858	0.100	
37	544.0	555.5	11.5	21.003	5.742	3.658	0.134	
38	536.0	554.0	18.0	39.041	7.385	5.287	0.167	*
39	545.0	549.8	4.8	0.000	6.954	6.305	0.117	*
40	546.0	549.7	3.7	3.745	6.577	0.569	0.116	
41	545.0	549.2	4.2	7.925	6.299	1.258	0.113	
42	536.0	548.7	12.7	20.609	7.018	2.936	0.131	
43	534.0	545.1	11.1	31.750	7.557	4.201	0.121	*
44	541.0	541.1	0.1	0.000	6.652	4.788	0.136	*
45	541.0	540.8	-0.2	-0.203	5.776	-0.035	0.140	
46	541.0	540.4	-0.6	-0.770	5.049	-0.153	0.138	
47	538.0	540.2	2.2	1.386	4.650	0.298	0.124	
48	531.0	538.7	7.7	9.113	5.031	1.811	0.123	
49	521.0	535.3	14.3	23.384	6.165	3.793	0.155	
50	523.0	529.0	6.0	29.353	6.134	4.785	0.101	*
51	501.0	523.5	22.5	0.000	7.791	6.650	0.164	*
52	505.0	515.7	10.7	10.655	8.260	1.290	0.117	
53	520.0	504.7	-15.3	-4.597	9.078	-0.506	0.127	
54	521.0	507.6	-13.4	-18.028	9.631	-1.872	0.117	
55	511.0	508.8	-2.2	-20.206	8.758	-2.307	0.127	
56	504.0	506.9	2.9	-17.303	8.013	-2.159	0.126	
57	507.0	502.4	-4.6	-21.889	7.583	-2.887	0.116	
58	502.0	500.8	-1.2	-23.123	6.846	-3.378	0.129	
59	505.0	498.4	-6.6	-29.771	6.820	-4.365	0.101	*
60	516.0	497.2	-18.8	0.000	8.035	-6.049	0.145	*
61	509.0	501.0	-8.0	-8.033	8.035	-1.000	0.100	
62	507.0	500.8	-6.2	-14.199	7.848	-1.809	0.107	
63	508.0	501.6	-6.4	-20.550	7.688	-2.673	0.106	
64	509.0	501.9	-7.1	-27.604	7.621	-3.622	0.103	
65	513.0	502.7	-10.3	-37.863	7.891	-4.798	0.110	*
66	515.0	504.7	-10.3	0.000	8.154	-5.903	0.110	*
67	520.0	507.1	-12.9	-12.892	8.673	-1.486	0.118	
68	519.0	510.8	-8.2	-21.135	8.622	-2.451	0.102	
69	526.0	513.5	-12.5	-33.598	9.013	-3.728	0.113	
70	529.0	517.5	-11.5	-45.086	9.293	-4.852	0.109	*
71	528.0	522.2	-5.8	0.000	8.917	-5.712	0.112	*

72	527.0	525.2	-1.8	-1.821	8.121	-0.224	0.127	
73	524.0	527.3	3.3	1.470	7.508	0.196	0.123	
74	515.0	528.1	13.1	14.941	8.190	1.775	0.125	
75	509.0	525.0	16.0	30.503	9.162	3.329	0.132	
76	504.0	520.3	16.3	46.777	10.099	4.632	0.128	*
77	502.0	514.5	12.5	0.000	10.407	5.696	0.109	*
78	508.0	509.6	1.6	1.556	9.443	0.165	0.128	
79	515.0	508.5	-6.5	-4.940	9.067	-0.545	0.112	
80	507.0	510.1	3.1	-1.883	8.394	-0.224	0.122	
81	511.0	508.6	-2.4	-4.271	7.660	-0.558	0.126	
82	521.0	508.8	-12.2	-16.479	8.234	-2.001	0.121	
83	520.0	512.8	-7.2	-23.657	8.106	-2.918	0.105	
84	524.0	515.4	-8.6	-32.238	8.156	-3.953	0.102	
85	527.0	518.3	-8.7	-40.891	8.206	-4.983	0.102	*
86	528.0	521.5	-6.5	0.000	8.030	-5.898	0.106	*
87	539.0	524.4	-14.6	-14.645	8.734	-1.677	0.124	
88	541.0	530.2	-10.8	-25.428	8.988	-2.829	0.108	
89	537.0	536.5	-0.5	-25.949	8.070	-3.216	0.131	
90	543.0	538.6	-4.4	-30.329	7.588	-3.997	0.118	
91	551.0	543.6	-7.4	-37.722	7.565	-4.987	0.101	*
92	541.0	548.9	7.9	-29.861	7.595	-3.932	0.101	
93	547.0	548.9	1.9	-28.008	7.014	-3.993	0.123	
94	559.0	550.9	-8.1	-36.101	7.146	-5.052	0.106	*
95	559.0	557.6	-1.4	0.000	6.537	-5.733	0.126	*
96	560.0	560.8	0.8	0.802	5.817	0.138	0.133	
97	556.0	565.0	9.0	9.827	6.244	1.574	0.121	
98	560.0	566.0	6.0	15.872	6.220	2.552	0.101	
99	558.0	567.2	9.2	25.098	6.524	3.847	0.114	
100	558.0	566.8	8.8	33.946	6.789	5.000	0.112	*
101	557.0	567.1	10.1	0.000	7.162	6.154	0.116	*
102	553.0	566.4	13.4	13.406	7.884	1.700	0.127	
103	554.0	564.3	10.3	23.718	8.194	2.895	0.111	
104	555.0	562.8	7.8	31.473	8.145	3.864	0.102	
105	562.0	561.5	-0.5	30.982	7.366	4.206	0.129	*
106	569.0	562.7	-6.3	24.681	7.229	3.414	0.106	
107	585.0	566.8	-18.2	6.512	8.384	0.777	0.141	
108	590.0	573.8	-16.2	-9.724	9.494	-1.024	0.135	
109	596.0	584.0	-12.0	-21.715	9.831	-2.209	0.110	
110	586.0	592.4	6.4	-15.297	9.455	-1.618	0.111	
111	583.0	593.2	10.2	-5.135	9.533	-0.539	0.102	
112	584.0	593.2	9.2	4.048	9.498	0.426	0.101	
113	597.0	593.0	-4.0	0.013	8.945	0.001	0.117	
114	591.0	596.7	5.7	5.703	8.563	0.666	0.113	
115	591.0	598.2	7.2	12.904	8.409	1.535	0.105	
116	589.0	598.5	9.5	22.441	8.528	2.632	0.104	
117	581.0	597.8	16.8	39.267	9.392	4.181	0.128	*
118	588.0	594.7	6.7	0.000	9.046	5.079	0.111	*
119	592.0	594.4	2.4	2.379	8.305	0.286	0.125	
120	597.0	594.8	-2.2	0.224	7.539	0.030	0.128	
121	594.0	597.3	3.3	3.495	6.994	0.500	0.122	
122	593.0	597.7	4.7	8.163	6.711	1.216	0.112	
123	583.0	597.3	14.3	22.446	7.560	2.969	0.134	
124	592.0	593.5	1.5	23.902	6.744	3.544	0.132	
125	591.0	593.5	2.5	26.406	6.183	4.271	0.125	*
126	597.0	592.9	-4.1	22.328	5.920	3.772	0.113	

MEAN OF ERRORS = 0.223
STANDARD DEVIATION= 9.16
ALPHA IS .GE. = 0.100

IBM COMMON STOCK TRADING
ON THE NEW YORK EXCHANGE

FORECAST MADE 3 DAYS IN ADVANCE
METHOD OF D AMICO

T	X(T+TAU)	XHAT	ERROR	ACC.ERR	MAD	TRACK	ALPHA	CONT
21	523.0	506.1	-16.9	-16.900	7.859	-2.151	0.138	
22	528.0	512.2	-15.8	-32.700	8.957	-3.651	0.137	
23	529.0	522.1	-6.9	-39.623	8.679	-4.565	0.109	*
24	538.0	526.1	-11.9	0.000	9.029	-5.704	0.112	*
25	539.0	528.9	-10.1	-10.144	9.153	-1.108	0.104	
26	541.0	532.5	-8.5	-18.662	9.087	-2.054	0.102	
27	543.0	537.7	-5.3	-23.971	8.701	-2.755	0.113	
28	541.0	542.1	1.1	-22.903	7.840	-2.921	0.130	
29	539.0	547.4	8.4	-14.479	7.916	-1.829	0.103	
30	543.0	553.4	10.4	-4.042	8.175	-0.494	0.110	
31	548.0	552.2	4.2	0.163	7.741	0.021	0.116	
32	550.0	552.9	2.9	3.036	7.176	0.423	0.122	
33	548.0	554.7	6.7	9.766	7.122	1.371	0.102	
34	550.0	558.0	8.0	17.816	7.217	2.469	0.104	
35	544.0	558.5	14.5	32.331	7.975	4.054	0.129	*
36	536.0	559.3	23.3	0.000	9.951	5.595	0.160	*
37	545.0	562.4	17.4	17.433	11.145	1.564	0.132	
38	546.0	561.1	15.1	32.524	11.667	2.788	0.113	
39	545.0	552.6	7.6	40.085	11.201	3.579	0.112	
40	536.0	551.4	15.4	55.443	11.666	4.752	0.112	*
41	534.0	551.2	17.2	0.000	12.282	5.912	0.115	*
42	541.0	550.4	9.4	9.448	11.956	0.790	0.108	
43	541.0	545.9	4.9	14.363	11.196	1.283	0.119	
44	541.0	541.7	0.7	15.055	9.945	1.514	0.134	
45	538.0	541.0	3.0	18.058	9.018	2.002	0.128	
46	531.0	540.2	9.2	27.304	9.048	3.018	0.101	
47	521.0	539.8	18.8	46.151	10.037	4.598	0.130	*
48	523.0	538.8	15.8	0.000	10.787	5.745	0.121	*
49	501.0	533.4	32.4	32.424	13.402	2.419	0.159	
50	505.0	525.9	20.9	53.346	14.594	3.655	0.125	
51	520.0	517.3	-2.7	50.607	13.118	3.858	0.130	
52	521.0	506.3	-14.7	35.952	13.318	2.699	0.105	
53	511.0	497.7	-13.3	22.603	13.322	1.697	0.100	
54	504.0	502.0	-2.0	20.635	12.185	1.693	0.126	
55	507.0	504.1	-2.9	17.751	11.017	1.611	0.129	
56	502.0	499.3	-2.7	15.061	9.945	1.514	0.129	
57	505.0	494.3	-10.7	4.325	10.047	0.430	0.103	
58	516.0	492.8	-23.2	-18.920	11.407	-1.659	0.136	
59	509.0	492.8	-16.2	-35.079	12.052	-2.911	0.116	
60	507.0	489.2	-17.8	-52.866	12.718	-4.157	0.116	*
61	508.0	496.0	-12.0	0.000	12.634	-5.134	0.102	*
62	509.0	497.1	-11.9	-11.910	12.561	-0.948	0.102	
63	513.0	498.1	-14.9	-26.823	12.800	-2.096	0.106	
64	515.0	498.8	-16.2	-43.033	13.160	-3.270	0.108	
65	520.0	499.8	-20.2	-63.255	13.924	-4.543	0.116	*
66	519.0	502.4	-16.6	0.000	14.234	-5.609	0.107	*
67	526.0	505.6	-20.4	-20.417	14.892	-1.371	0.113	
68	529.0	510.2	-18.8	-39.229	15.336	-2.558	0.109	
69	528.0	513.5	-14.5	-53.653	15.241	-3.523	0.102	
70	527.0	518.8	-8.2	-61.891	14.524	-4.261	0.114	*
71	524.0	523.6	-0.4	0.000	12.917	-4.826	0.133	*

72	515.0	527.9	12.9	12.923	12.918	1.000	0.100	
73	509.0	531.7	22.7	35.645	13.898	2.565	0.121	
74	504.0	530.3	26.3	61.948	15.401	4.022	0.129	*
75	502.0	527.4	25.4	0.000	16.691	5.232	0.123	*
76	508.0	521.3	13.3	13.311	16.275	0.818	0.107	
77	515.0	514.4	-0.6	12.703	14.591	0.871	0.131	
78	507.0	509.3	2.3	15.046	12.986	1.159	0.133	
79	511.0	507.0	-4.0	11.033	11.792	0.936	0.128	
80	521.0	509.2	-11.8	-0.735	11.789	-0.062	0.100	
81	520.0	507.1	-12.9	-13.587	11.896	-1.142	0.103	
82	524.0	507.9	-16.1	-29.643	12.323	-2.406	0.110	
83	527.0	511.9	-15.1	-44.712	12.626	-3.541	0.107	
84	528.0	515.0	-13.0	-57.698	12.665	-4.556	0.101	*
85	539.0	518.9	-20.1	0.000	13.416	-5.800	0.117	*
86	541.0	522.7	-18.3	-18.328	13.990	-1.310	0.112	
87	537.0	527.1	-9.9	-28.192	13.527	-2.084	0.110	
88	543.0	534.3	-8.7	-36.900	12.997	-2.839	0.112	
89	551.0	540.3	-10.7	-47.612	12.742	-3.737	0.106	
90	541.0	543.4	2.4	-45.254	11.642	-3.887	0.126	
91	547.0	547.1	0.1	-45.135	10.191	-4.429	0.137	*
92	559.0	556.2	-2.8	0.000	9.180	-5.225	0.130	*
93	559.0	557.4	-1.6	-1.573	8.193	-0.192	0.132	
94	560.0	559.1	-0.9	-2.463	7.227	-0.341	0.135	
95	556.0	566.6	10.6	8.173	7.688	1.063	0.118	
96	560.0	571.7	11.7	19.895	8.164	2.437	0.117	
97	558.0	572.6	14.6	34.525	8.924	3.869	0.126	
98	558.0	572.5	14.5	49.022	9.624	5.094	0.122	*
99	557.0	574.5	17.5	0.000	10.584	6.286	0.127	*
100	553.0	573.5	20.5	20.520	11.848	1.732	0.132	
101	554.0	572.9	18.9	39.439	12.781	3.086	0.122	
102	555.0	571.3	16.3	55.785	13.216	4.221	0.110	*
103	562.0	567.1	5.1	0.000	12.329	4.942	0.120	*
104	569.0	564.4	-4.6	-4.616	11.403	-0.405	0.123	
105	585.0	563.1	-21.9	-26.561	12.695	-2.092	0.131	
106	590.0	564.7	-25.3	-51.822	14.335	-3.615	0.134	
107	596.0	569.4	-26.6	-78.378	15.977	-4.906	0.131	*
108	586.0	580.8	-5.2	0.000	14.565	-5.737	0.127	*
109	583.0	590.5	7.5	7.473	13.668	0.547	0.118	
110	584.0	599.4	15.4	22.855	13.871	1.648	0.104	
111	597.0	600.0	3.0	25.817	12.732	2.028	0.125	
112	591.0	598.0	7.0	32.837	12.020	2.732	0.117	
113	591.0	599.1	8.1	40.973	11.567	3.542	0.111	
114	589.0	603.1	14.1	55.117	11.854	4.650	0.107	*
115	581.0	603.2	22.2	0.000	12.958	5.963	0.126	*
116	588.0	602.8	14.8	14.822	13.192	1.123	0.105	
117	592.0	602.6	10.6	25.447	12.922	1.969	0.106	
118	597.0	597.0	0.0	25.479	11.554	2.205	0.132	
119	594.0	596.1	2.1	27.604	10.311	2.677	0.132	
120	593.0	597.7	4.7	32.264	9.564	3.374	0.122	
121	583.0	600.2	17.2	49.456	10.493	4.713	0.127	*
122	592.0	599.8	7.8	0.000	10.151	5.640	0.110	*
123	591.0	599.1	8.1	8.139	9.930	0.820	0.107	
124	597.0	594.0	-3.0	5.115	9.195	0.556	0.122	
125	597.0	593.8	-3.2	1.902	8.464	0.225	0.124	
126	599.0	593.1	-5.9	-4.013	8.148	-0.492	0.111	

MEAN OF ERRORS = 0.639
STANDARD DEVIATION= 13.67
ALPHA IS .GE. = 0.100

APPENDIX B

```
$JOB              V.ALVARADO.
C
C              * * * * * * * * * * * * * * * * * * *
C               DOUBLE EXPONENTIAL SMOOTHING.
C              FEED SALES DATA
C               MODEL LINEAR : X(T) = A + B*T
C               BROWNS METHOD.
C               FEED DATA.
C              * * * * * * * * * * * * * * * * * * *
C
      DIMENSION X(100)
      REAL MAD
      DATA ASTER/'*'/,BLANK/' '/
C               READ IN DATA.
      N=86
      NN=36
      READ(5,1)(X(I),I=1,N)
    1 FORMAT(20X,F6.3)
C               INITIALIZE
      ACC=0.
      SUM=0.
      DO 3 I=1,NN
      SUM=SUM+X(I)
    3 ACC=ACC+X(I)*X(I)
      STD=SQRT((ACC-SUM*SUM/NN)/(NN-1.))
      XMAD=STD*2.*SQRT(1./(3.1416*1.9))
      ALPHA=0.1
      BETA=1.-ALPHA
      TAU=1
      ITAU=1
      ST1=SUM/NN
      ST2=ST1
      AHAT=ST1
      BHAT=0.
      MAD=XMAD
C               PRINT OUT HEADING.
      WRITE(6,32)ALPHA,ITAU
   32 FORMAT('1'//20X'ALPHA ='F5.2/20X'FORECASTED MADE'I3
     1'WEEKS IN ADVANCE'///3X'T'8X'AHAT'8X'BHAT'6X
     2'X(T+I)'8X'XHAT'7X'ERROR'9X'MAD'5X'SUM.ERR'7X'TRACK')
      ACCERR=0.
      SACCER=0.
      XN=0.
      CHECK=BLANK
      ICONT=0
      SUMERR=0.
C               MAKE FORECASTING.
      NN1=NN+1
      N1=N-ITAU
      FACT=ALPHA/BETA
      DO 100 I=NN1,N1
      XN=XN+1.
C               FORECAST TAU PERIODS AHEAD.
      XHAT=AHAT+BHAT*TAU
C               SMOOTH STATISTICS.
      ST1=ALPHA*X(I)+BETA*ST1
      ST2=ALPHA*ST1+BETA*ST2
C               DETERMINE COEFFICIENTS.
      AHAT=2.*ST1-ST2
      BHAT=FACT*(ST1-ST2)
```

```
C                 DETERMINE ERROR.
          ERROR=X(I+ITAU-1)-XHAT
          ACCERR=ACCERR+ERROR
          SACCER=SACCER+ERROR*ERROR
          SUMERR=SUMERR+ERROR
C                 DETERMINE MAD
          MAD=ALPHA*ABS(ERROR)+BETA*MAD
C                 DETERMINE TRACKING SIGNAL.
          TRAC =SUMERR/MAD
          TRACK=ABS(TRAC)
C                 IF SYSTEM IS OUT OF CONTROL TWICE IN
C                 SUCCESSION : RESET SUM ERR.
          IF(ICONT.EQ.1.AND.TRACK.GE.4.)GO TO 300
          IF(TRACK.GT.4)GO TO 310
          CHECK=BLANK
          ICONT=0.
          GO TO 100
      300 SUMERR=0.
          CHECK=ASTER
          ICONT=0
          GO TO 100
      310 CHECK=ASTER
          ICONT=1
C               PRINT OUT RESULTS.
      100 WRITE(6,101)I,AHAT,PHAT,X(I+ITAU-1),XHAT,ERROR,MAD,SUMERR,TRAC,
         1ALPHA,CHECK
      101 FORMAT(I4,9F12.3,2A1)
C                 CALCULATE MEAN AND STANDARD DEVIATION.
          EMEAN=ACCERR/XN
          STDEV=SQRT((SACCER-ACCERR*ACCERR/XN)/(XN-1.))
          WRITE(6,33)EMEAN,STDEV
       33 FORMAT(//20X'MEAN OF ERRORS='F10.3/20X'STANDARD DEVIATION='F 7.3)
          STOP
          END

$ENTRY
```

ALPHA = 0.10
FORECASTED MADE 1WEEKS IN ADVANCE

T	AHAT	BHAT	X(T+1)	XHAT	ERROR	MAD	SUM.ERR	TRACK	
37	336.680	1.139	428.920	315.044	113.876	72.359	113.876	1.574	0.100
38	338.263	1.162	340.160	337.819	2.341	65.357	116.217	1.778	0.100
39	347.758	1.601	383.280	339.425	43.855	63.207	160.072	2.533	0.100
40	373.823	2.888	478.120	349.358	128.762	69.762	288.834	4.140	0.100*
41	395.979	3.902	478.120	376.711	101.409	72.927	0.000	5.351	0.100*
42	381.496	2.935	303.120	399.881	-96.761	75.310	-96.761	-1.285	0.100
43	395.248	3.504	441.360	384.431	56.929	73.472	-39.832	-0.542	0.100
44	400.083	3.574	405.760	398.751	7.009	66.826	-32.823	-0.491	0.100
45	398.980	3.328	379.040	403.657	-24.617	62.605	-57.440	-0.918	0.100
46	393.653	2.873	356.760	402.308	-45.548	60.899	-102.988	-1.691	0.100
47	388.271	2.438	353.080	396.526	-43.446	59.154	-146.434	-2.475	0.100
48	382.229	1.992	345.080	390.709	-44.629	57.701	-191.062	-3.311	0.100
49	391.141	2.356	420.640	384.221	36.419	55.573	-154.644	-2.783	0.100
50	380.520	1.673	325.200	393.497	-68.297	56.845	-222.940	-3.922	0.100
51	370.433	1.054	320.300	382.192	-61.892	57.350	-284.833	-4.967	0.100*
52	361.423	0.524	318.520	371.487	-52.967	56.912	0.000	-5.936	0.100*
53	377.925	1.365	446.040	361.947	84.093	59.630	84.093	1.410	0.100
54	377.426	1.267	368.480	379.290	-9.810	54.648	74.283	1.359	0.100
55	394.841	2.117	463.680	378.693	84.987	57.682	159.270	2.761	0.100
56	398.911	2.220	407.240	396.958	10.282	52.942	169.552	3.203	0.100
57	399.774	2.149	393.990	401.131	-7.141	48.362	162.412	3.358	0.100
58	404.050	2.260	413.120	401.922	11.198	44.645	173.610	3.889	0.100
59	400.672	1.964	376.640	406.310	-29.670	43.148	143.940	3.336	0.100
60	402.948	1.980	404.280	402.636	1.644	38.997	145.584	3.733	0.100
61	409.236	2.207	427.600	404.928	22.672	37.365	168.256	4.503	0.100*
62	417.370	2.519	442.640	411.443	31.197	36.748	0.000	5.428	0.100*
63	447.893	3.993	567.280	419.889	147.391	47.812	147.391	3.083	0.100
64	437.870	3.255	378.120	451.885	-73.765	50.408	73.626	1.461	0.100
65	416.835	1.977	313.280	441.125	-127.845	58.151	-54.220	-0.932	0.100
66	400.189	0.997	320.800	418.811	-98.011	62.137	-152.231	-2.450	0.100
67	379.847	-0.126	288.880	401.186	-112.306	67.154	-264.537	-3.939	0.100
68	363.768	-0.966	295.760	379.721	-83.961	68.835	-348.498	-5.063	0.100*
69	398.476	0.912	550.560	362.802	187.758	80.727	-160.740	-1.991	0.100
70	380.010	-0.108	297.400	399.387	-101.988	82.853	-262.727	-3.171	0.100
71	377.747	-0.222	368.560	379.902	-11.342	75.702	-274.069	-3.620	0.100
72	399.222	0.920	491.720	377.525	114.195	79.551	-159.874	-2.010	0.100
73	404.470	1.148	422.920	400.142	22.778	73.874	-137.095	-1.856	0.100
74	390.855	0.371	327.920	405.618	-77.698	74.256	-214.793	-2.893	0.100
75	399.270	0.794	433.560	391.226	42.334	71.064	-172.460	-2.427	0.100
76	420.257	1.857	506.343	400.064	106.279	74.586	-66.181	-0.887	0.100
77	418.931	1.690	405.360	422.114	-16.754	68.802	-82.935	-1.205	0.100
78	430.497	2.209	472.600	420.620	51.980	67.120	-30.955	-0.461	0.100
79	416.027	1.332	344.920	432.706	-87.786	69.187	-118.741	-1.716	0.100
80	423.909	1.676	451.840	417.358	34.482	65.716	-84.259	-1.282	0.100
81	418.817	1.320	389.960	425.586	-35.626	62.707	-119.885	-1.912	0.100
82	428.015	1.735	461.600	420.137	41.463	60.583	-78.421	-1.294	0.100
83	443.295	2.448	501.040	429.749	71.291	61.654	-7.131	-0.116	0.100
84	456.249	3.001	501.040	445.742	55.298	61.018	48.167	0.789	0.100
85	448.965	2.459	405.120	459.249	-54.129	60.329	-5.962	-0.099	0.100

MEAN OF ERRORS= 5.019
STANDARD DEVIATION= 74.525

```
$JOB             V.ALVARADO.
C
C             * * * * * * * * * * * * * * * * * * *
C               DOUBLE EXPONENTIAL SMOOTHING.
C             FEED SALES DATA
C               MODEL LINEAR  : X(T) = A + B*T
C               FLOATING ALPHA METHOD.
C             * * * * * * * * * * * * * * * * * * *
C
      DIMENSION X(100)
      REAL MAD,INDEX
      DATA ASTER/'*'/,BLANK/' '/
      N=86
      NN=36
C               READ IN DATA.
      READ(5,1)(X(I),I=1,N)
    1 FORMAT(20X,F6.3)
C               DETERMINE INITIAL CONDITIONS.
      SUM=0.
      ACC=0.
      INDEX=60.
      GAMMA=0.3
      DELTA=0.05
      DO 3 I=1,NN
      SUM=SUM+X(I)
    3 ACC=ACC+X(I)*X(I)
      STD=SQRT((ACC-SUM*SUM/NN)/(NN-1.))
      XMAD=STD*2.*SQRT(1./(3.1416*1.9))
      TAU=1.
      ITAU=1
C               PRINT OUT HEADING.
      WRITE(6,32)DELTA,ITAU
   32 FORMAT('1'//20X'ALPHA IS RESTRICTED TO BE .GE.'F5.2/20X'ITAU='I3//
     13X,'T',8X,'AHAT',8X,'BHAT',6X,'X(T+I)',8X,'XHAT',7X,'ERROR',9X,
     2'MAD',5X,'SUM.ERR',7X,'TRACK',7X,'ALPHA')
      NN1=NN+1
      N1=N-ITAU
C               INITIAL CONDITIONS.
      ST1=SUM/NN
      ST2=ST1
      AHAT=ST1
      BHAT=0.
      MAD=XMAD
      ACCERR=0.
      SACCER=0.
      SUMERR=0.
      ICONT=0
      CHECK=BLANK
      XN=0.
      ALPHA=.1
      BETA=1.-ALPHA
C               MAKE THE FORECASTING.
      DO 100 I=NN1,N1
      XN=XN+1.
      FACT=ALPHA/BETA
C               FORECAST TAU PERIODS AHEAD.
      XHAT=AHAT+BHAT*TAU
C               SMOOTH STATISTITICS.
      ST1=ALPHA*X(I)+BETA*ST1
      ST2=ALPHA* ST1+BETA*ST2
```

```
C                DETERMINE COEFFICIENTS.
      AHAT=2.*ST1-ST2
      BHAT=FACT*(ST1-ST2)
C                DETERMINE ERROR AND SAVE ITS STATISTICS.
      ERROR=X(I+ITAU-1) -XHAT
      ACCERR=ACCERR+ERROR
      SACCER=SACCER+ERROR*ERROR
      SUMERR=SUMERR+ERROR
C                DETERMINE MEAN ABSOLUTE DEVIATION.
      MAD=ALPHA*ABS(ERROR)+BETA*MAD
C                DETERMINE TRACKING SIGNAL AND MONITOR SYSTEM.
      TRAC =SUMERR/MAD
      TRACK=ABS(TRAC)
C              * CHECK IF PROGRAM IS UNDER CONTROL.
C            IF SYSTEM IS OUT OF CONTROL TWICE IN
C            SUCCESSION : RESET ACCUMULATE ERROR.
      IF(ICONT.EQ.1.AND.TRACK.GE.4.)GO TO 300
      IF(TRACK.GT.4.)GO TO 310
      CHECK=BLANK
      ICONT=0
      GO TO 320
  300 SUMERR=0.
      CHECK=ASTER
      ICONT=0
      GO TO 320
  310 CHECK=ASTER
      ICONT=1
C                CHECK FOR ERROR LESS THAN ITS LOWER LIMIT
C                IF YES, KEEP THE VALUE OF ALPHA.
C                IF NOT, CHANGE ALPHA.
  320 DIV=ABS(ERROR)
      IF(DIV.LE.INDEX)GO TO 100
C                CHANGE ALPHA.
      DERROR=(DIV-INDEX)/DIV
      ALPHA=(GAMMA-DELTA)*DERROR+DELTA
      BETA=1.-ALPHA
C                PRINT OUT RESULTS.
  100 WRITE(6,101)I,AHAT,BHAT,X(I+ITAU-1),XHAT,ERROR,MAD,SUMERR,TRAC,
     1ALPHA,CHECK
  101 FORMAT(I4,9F12.3,2A1)
C                DETERMINE MEAN & STANDARD DEVIATION OF ERRORS.
C                PRINT THEM OUT.
      EMEAN=ACCERR/XN
      STDEV=SQRT((SACCER-ACCERR*ACCERR /XN)/(XN-1.))
      WRITE(6,33)EMEAN,STDEV
   33 FORMAT(//,20X,'MEAN OF ERRORS=',F10.3,/,
     120X,'STANDARD DEVIATION='F7.3)
      WRITE(6,333)
  333 FORMAT('1')
      STOP
      END

$ENTRY
```

ALPHA IS RESTRICTED TO BE .GE. 0.05
ITAU= 1

T	AHAT	BHAT	X(T+I)	XHAT	ERROR	MAD	SUM.ERR	TRACK	ALPHA
37	336.680	1.139	428.920	315.044	113.876	72.359	113.876	1.574	0.168
38	339.187	2.113	340.160	337.819	2.341	60.576	116.217	1.919	0.168
39	354.240	3.302	383.280	341.300	41.980	57.447	158.197	2.754	0.168
40	394.709	6.717	478.120	357.542	120.578	68.070	278.775	4.095	0.175*
41	426.236	9.425	478.120	401.425	76.695	69.585	0.000	5.108	0.104*
42	406.005	3.761	303.120	435.661	-132.541	75.159	-132.541	-1.740	0.137
43	422.881	8.386	441.360	409.765	31.595	67.833	-100.946	-1.488	0.137
44	422.626	7.495	405.760	431.267	-25.507	59.925	-126.453	-2.110	0.137
45	412.817	5.712	379.040	430.121	-51.081	58.273	-177.534	-3.047	0.137
46	397.605	3.556	356.760	418.530	-61.770	58.926	-239.303	-4.061	0.057*
47	393.494	0.790	353.080	401.161	-48.081	58.306	0.000	-4.929	0.057*
48	398.931	0.632	346.080	394.284	-48.204	57.729	-48.204	-0.835	0.057
49	393.014	0.734	420.640	389.563	31.077	56.205	-17.127	-0.305	0.057
50	386.135	0.510	325.200	393.748	-68.548	56.911	-85.676	-1.505	0.031
51	376.508	0.304	320.300	385.645	-66.345	57.677	-152.021	-2.636	0.074
52	368.489	-0.043	318.520	376.812	-58.292	57.722	-210.313	-3.644	0.074
53	379.491	0.381	446.040	368.446	77.594	59.191	-132.719	-2.242	0.107
54	377.924	0.449	369.480	379.872	-10.392	53.985	-143.110	-2.651	0.107
55	395.604	1.420	463.680	378.373	85.307	57.326	-57.803	-1.008	0.124
56	399.607	1.839	407.240	397.024	10.216	51.477	-47.587	-0.924	0.124
57	399.709	1.724	393.990	401.446	-7.456	46.011	-55.043	-1.196	0.124
58	404.156	1.904	413.120	401.434	11.687	41.749	-43.356	-1.038	0.124
59	399.207	1.451	376.640	406.060	-29.420	40.218	-72.776	-1.810	0.124
60	401.501	1.507	404.280	400.658	3.622	35.674	-69.154	-1.938	0.124
61	408.736	1.886	427.600	403.008	24.592	34.298	-44.562	-1.299	0.124
62	418.079	2.379	442.640	410.622	32.018	34.015	-12.544	-0.369	0.124
63	454.655	4.643	567.280	420.459	146.821	48.022	134.278	2.796	0.193
64	432.565	4.766	378.120	459.298	-81.178	54.581	53.100	0.973	0.115
65	408.630	0.899	313.280	437.331	-124.051	62.585	-70.951	-1.134	0.179
66	381.004	-1.359	320.800	409.529	-88.729	67.267	-159.680	-2.374	0.131
67	357.748	-2.502	288.880	379.646	-90.766	70.344	-250.446	-3.560	0.135
68	340.233	-3.664	295.760	355.246	-59.486	68.881	-309.932	-4.500	0.135*
69	340.085	-3.410	350.560	336.569	13.991	61.485	0.000	-4.813	0.135*
70	326.804	-4.123	297.400	336.675	-39.275	58.493	-39.275	-0.671	0.135
71	334.211	-3.290	368.560	322.681	45.879	56.793	6.604	0.116	0.135
72	371.333	-0.371	491.720	330.921	160.799	70.807	167.402	2.364	0.207
73	390.066	1.610	422.920	370.962	51.958	66.910	219.360	3.278	0.207
74	368.042	-1.114	327.920	391.676	-63.756	66.258	155.604	2.348	0.055
75	375.990	-0.020	433.560	366.927	66.633	66.282	222.236	3.353	0.075
76	394.761	0.708	506.343	375.970	130.373	71.082	352.610	4.961	0.185*
77	399.636	2.278	405.360	395.469	9.891	59.765	0.000	6.065	0.185*
78	425.642	4.696	472.600	401.914	70.666	61.785	70.686	1.144	0.088
79	413.748	1.354	344.920	430.338	-85.418	63.860	-14.732	-0.231	0.124
80	424.168	2.557	451.840	415.102	36.738	60.486	22.006	0.364	0.124
81	418.147	1.989	389.960	426.725	-36.765	57.535	-14.760	-0.257	0.124
82	429.810	2.630	461.600	420.136	41.464	55.536	26.705	0.481	0.124
83	448.445	3.692	501.040	432.440	68.600	57.161	95.304	1.657	0.081
84	458.595	2.634	501.040	452.137	48.903	56.489	144.208	2.553	0.081
85	452.472	2.262	405.120	461.229	-56.109	56.458	88.099	1.560	0.031

MEAN OF ERRORS= 4.546
STANDARD DEVIATION= 71.039

```
$JOB            V.ALVARADO.
C
C              * * * * * * * * * * * * * * * * * * *
C               DOUBLE EXPONENTIAL SMOOTHING.
C              FEED SALES DATA
C               MODEL LINEAR    X(T) = A + B * T
C               D'AMICO'S METHOD.
C              * * * * * * * * * * * * * * * * * * *
C
      DIMENSION X(100)
      REAL MAD
      DATA ASTER/'*'/,BLANK/' '/
      N=86
      NN=36
C               READ IN DATA.
      READ(5,1)(X(I),I=1,N)
    1 FORMAT(20X,F6.3)
C             INITIALIZE LOW AND HIGH VALUES OF SMOOTHING CONSTANT.
      DELTA=0.1
      GAMMA=0.4
C             DETERMINE INITIAL CONDITIONS.
      SUM=0.
      ACC=0.
      DO 2 I=1,NN
      SUM=SUM+X(I)
    2 ACC=ACC+X(I)*X(I)
      STD=SQRT((ACC-SUM*SUM/NN)/(NN-1.))
      XMAD=STD*2.*SQRT(1./(3.1416*1.9))
C             FIX TIME AHEAD TO FORECAST.
      TAU=1.
      ITAU=1
C               PRINT OUT HEADING.
      WRITE(6,32)DELTA,ITAU
   32 FORMAT('1'//20X'ALPHA IS RESTRICTED TO BE .GE.'F5.2/20X'ITAU='I3//
     13X,'T',8X,'AHAT',8X,'BHAT',6X,'X(T+I)',8X,'XHAT',7X,'ERROR',9X,
     2'MAD',5X,'SUM.ERR',7X,'TRACK',7X,'ALPHA')
C             FIX INITIAL VALUES.
      ALPHA=0.1
      BETA=1.-ALPHA
      ACCERR=0.
      SACERR=0.
      XN=0.
      MAD=XMAD
      CHECK=BLANK
      ICONT=0
      SUMERR=0.
      ST1=SUM/NN
      ST2=ST1
      AHAT=ST1
      BHAT=0.
C             FORECAST X TAU PERIODS OF TIME AHEAD.
      NN1=NN+1
      N1=N-ITAU
      DO 100 I=NN1,N1
      XN=XN+1.
C            FORECAST NEXT PIECE OF INFORMATION.
      XHAT=AHAT+BHAT*TAU
C             SMOOTH STATISTICS.
      ST1=ALPHA*X(I)+BETA*ST1
      ST2=ALPHA*ST1+BETA*ST2
```

```
C               DETERMINE COEFFICIENTS.
      AHAT=2.*ST1-ST2
      BHAT=(ALPHA/BETA)*(ST1-ST2)
C               DETERMINE FORECAST ERROR.
      ERROR=X(I+ITAU-1)-XHAT
C               SAVE ERRORS STATISTICS.
      ACCERR=ACCERR+ERROR
      SACERR=SACERR+ERROR*ERROR
      SUMERR=SUMERR+ERROR
C               SAVE MEAN APSOLUTE DEVIATION.
      SMAD=MAD
C               DETERMINE NEW MEAN ABSOLUTE DEVIATION.
      MAD=ALPHA*ABS(ERROR)+BETA*MAD
C               DETERMINE TRACKING SIGNAL.
      TRAC =SUMERR/MAD
      TRACK=ABS(TRAC)
C               MONITORIN BEHAVIOR OF FORECASTING SYSTEM.
C               IF SYSTEM IS OUT OF CONTROL TWICE IN
C               SUCCESSION : RESET SUM OF ERRORS.
      IF(ICONT.EQ.1.AND.TRACK.GE.4.)GO TO 300
      IF(TRACK.GT.4.)GO TO 310
      CHECK=BLANK
      ICONT=0
      GO TO 320
  300 SUMERR=0.
      CHECK=ASTER
      ICONT=0
      GO TO 320
  310 CHECK=ASTER
      ICONT=1
  320 DIV=SMAD
C               CHANGE VALUE OF ALPHA BY D'AMICOS'METHOD.
      IF(SMAD.LT.MAD)DIV=MAD
      DMAD=ABS((SMAD-MAD)/DIV)
      ALPHA=(GAMMA-DELTA)*DMAD+DELTA
      BETA=1.-ALPHA
C               PRINT OUT RESULTS.
  100 WRITE(6,101)I,AHAT,BHAT,X(I+ITAU-1),XHAT,ERROR,MAD,SUMERR,TRAC,
     1ALPHA,CHECK
  101 FORMAT(I4,9F12.3,2A1)
C               DETERMINE AND PRINT OUT STATISTICS OF ERRORS.
      EMEAN=ACCERR/XN
      STDEV=SQRT((SACERR-ACCERR*ACCERR/XN)/(XN-1.))
      WRITE(6,102)EMEAN,STDEV,DELTA
  102 FORMAT(//20X'MEAN OF ERRORS='F10.3/20X'STANDARD DEVIATION='F15.3//
     120X'LOW VALUE OF ALPHA ='F6.2///)
      STOP
      END

$ENTRY
```

ALPHA IS RESTRICTED TO BE .GE. 0.10
ITAU= 1

T	AHAT	BHAT	X(T+1)	XHAT	ERROR	MAD	SUM.ERR	TRACK	ALPHA
37	336.680	1.139	428.920	315.044	113.876	72.359	113.876	1.574	0.119
38	338.535	1.416	340.160	337.819	2.341	64.018	116.217	1.815	0.135
39	350.987	2.409	383.280	339.950	43.330	61.233	159.547	2.606	0.113
40	379.659	3.574	478.120	353.396	124.724	68.411	284.271	4.155	0.131*
41	407.049	5.873	478.120	383.233	94.887	71.892	0.000	5.274	0.115*
42	388.542	3.589	303.120	412.922	-109.802	76.234	-109.802	-1.440	0.117
43	403.054	4.354	441.360	392.130	49.230	73.072	-60.573	-0.829	0.112
44	405.905	4.141	405.760	407.408	-1.648	65.041	-62.220	-0.957	0.133
45	403.755	4.431	379.040	411.045	-32.005	60.648	-94.226	-1.554	0.120
46	396.188	3.213	356.760	408.186	-51.426	59.539	-145.652	-2.446	0.105
47	389.791	2.261	353.080	399.401	-46.321	58.145	-191.973	-3.302	0.107
48	382.767	1.771	346.080	392.052	-45.972	56.842	-237.944	-4.186	0.107*
49	391.828	2.177	420.640	384.538	36.102	54.628	-201.843	-3.695	0.112
50	379.584	1.431	325.200	394.005	-68.805	56.212	-270.648	-4.815	0.103*
51	368.522	0.671	320.300	381.015	-60.715	56.700	0.000	-5.844	0.103*
52	359.298	0.098	318.520	369.193	-50.673	56.082	-50.673	-0.904	0.103
53	376.367	1.023	446.040	359.395	86.645	59.238	35.971	0.607	0.115
54	375.772	1.057	369.480	377.390	-7.910	53.285	28.062	0.527	0.130
55	398.077	2.674	463.680	376.829	86.851	57.653	114.913	1.993	0.123
56	402.113	2.600	407.240	400.751	6.489	51.374	121.402	2.363	0.133
57	402.239	2.650	393.990	404.713	-10.723	45.980	110.679	2.407	0.131
58	406.891	2.766	413.120	404.889	8.231	41.016	118.910	2.899	0.132
59	401.510	2.208	376.640	409.657	-33.017	39.967	85.893	2.150	0.108
60	403.465	1.760	404.280	403.718	0.562	35.713	86.455	2.421	0.132
61	411.079	2.595	427.600	405.225	22.375	33.954	108.831	3.205	0.115
62	419.543	2.601	442.640	413.673	28.967	33.382	137.797	4.128	0.105*
63	450.920	3.959	567.280	422.244	145.036	45.112	0.000	6.270	0.179*
64	432.243	4.765	378.120	454.879	-76.759	50.745	-76.759	-1.513	0.133
65	405.182	1.210	313.280	437.008	-123.728	60.474	-200.486	-3.315	0.148
66	383.009	-0.516	320.800	406.392	-85.592	64.198	-286.078	-4.456	0.117*
67	361.897	-1.686	288.880	382.493	-93.613	67.651	0.000	-5.612	0.115*
68	346.230	-2.510	295.760	360.210	-64.450	67.282	-64.450	-0.958	0.102
69	393.896	-0.045	550.560	343.720	206.840	81.466	142.389	1.748	0.152
70	359.513	-2.074	297.400	383.851	-86.451	82.225	55.939	0.680	0.103
71	360.212	-1.214	368.560	357.439	11.121	74.918	67.060	0.895	0.127
72	390.244	0.598	491.720	358.999	132.721	82.239	199.781	2.429	0.127
73	398.456	1.113	422.920	390.842	32.078	75.884	231.860	3.055	0.123
74	382.976	-0.009	327.920	399.569	-71.649	75.362	160.211	2.126	0.102
75	392.769	0.520	433.560	382.967	50.593	72.834	210.804	2.894	0.110
76	416.841	1.934	506.343	393.288	113.055	77.261	323.859	4.192	0.117*
77	415.926	1.890	405.360	418.776	-13.416	69.779	0.000	4.449	0.129*
78	431.210	3.019	472.600	417.816	54.784	67.844	54.784	0.808	0.108
79	415.497	1.433	344.920	434.229	-89.309	70.169	-34.525	-0.492	0.110
80	424.203	1.879	451.840	416.930	34.910	65.292	0.385	0.006	0.117
81	418.251	1.515	389.960	426.082	-36.122	62.775	-35.737	-0.559	0.115
82	428.895	2.067	461.600	419.766	41.834	60.348	6.097	0.101	0.112
83	445.662	2.855	501.040	430.962	70.078	61.434	76.176	1.240	0.105
84	458.352	3.259	501.040	448.517	52.523	60.495	128.699	2.127	0.105
85	450.793	2.611	405.120	462.111	-56.991	60.129	71.708	1.193	0.102

MEAN OF ERRORS= 6.798
STANDARD DEVIATION= 74.714

```
$JOB              V.ALVARADO.
           DIMENSION X(200)
           REAL MAD
C                    SINGLE EXPONENTIAL SMOOTHING.
C                     FEED DATA.
           N=86
           NN=36
           READ(5,2)(X(I),I=1,N)
         2 FORMAT(20X,F6.3)
           TAU=1.
           ITAU=1
           WRITE(6,99)ITAU
        99 FORMAT('1',///,10X,'FEED DATA',/,10X,'FORECASTING MADE',
          1I3,' DAYS IN ADVANCE',//,12X,'I   X(I+ITAU)',6X,'XHAT',
          25X,'ERROR',3X,'ACC.ERR',//)
           SUM=0.
           ACC=0.
           INDEX=60.
           GAMMA=0.3
           DELTA=0.05
           DO 1 I=1,NN
           SUM=SUM+X(I)
         1 ACC=ACC+X(I)*X(I)
           STD=SQRT((ACC-SUM*SUM/NN)/(NN-1.))
           XMAD=STD*2.*SQRT(1./(3.1416*2.9))
           NN1=NN+1
           N1=N-ITAU
           ACCERR=0.
           SACERR=0.
           XHAT=SUM/NN
           MAD=XMAD
           ALPHA=0.1
           BETA=0.9
           XN=0.
           DO 100 I=NN1,N1
           XN=XN+1.
           ERROR=X(I+ITAU)-XHAT
           DIV=ABS(ERROR)
           ACCERR=ACCERR+ERROR
           SACERR=SACERR+ERROR*ERROR
           XHAT=ALPHA*X(I)+BETA*XHAT
           IF(DIV.LE.INDEX)GO TO 100
           DERROR=(DIV-INDEX)/DIV
           ALPHA=(GAMMA-DELTA)*DERROR+DELTA
           BETA=1.-ALPHA
       100 WRITE(6,101)I,X(I+ITAU),XHAT,ERROR,ACCERR
       101 FORMAT(10X,I4,4F10.3)
           EMEAN=ACCERR/XN
           STDEV=SQRT((SACERR-ACCERR*ACCERR/XN)/(XN-1.))
           WRITE(6,36)EMEAN,STDEV
        36 FORMAT(//,20X,'MEAN OF ERRORS=',F10.3,/,
          120X,'STANDARD DEVIATION=',F7.2,/,20X,
          3'SINGLE EXPONENTIAL SMOOTHING')
           WRITE(6,333)
       333 FORMAT('1')
           STOP
           END

$ENTRY
```

FEED DATA
FORECASTING MADE 1 DAYS IN ADVANCE

I	X(I+ITAL)	XHAT	ERROR	ACC.ERR
37	340.160	326.581	24.949	24.949
38	383.580	327.939	56.999	81.948
39	478.120	333.503	150.181	232.129
40	478.120	362.444	144.617	376.746
41	303.120	385.148	-59.323	317.423
42	441.360	369.048	56.212	373.635
43	405.760	383.241	36.712	410.347
44	379.040	387.661	-4.201	406.146
45	356.760	385.968	-30.901	375.246
46	353.080	380.235	-32.888	342.358
47	346.080	374.905	-34.155	308.203
48	420.640	369.247	45.735	353.938
49	325.200	379.334	-44.047	309.890
50	320.300	368.708	-59.034	250.856
51	318.520	359.207	-50.188	200.668
52	446.040	351.221	86.833	287.501
53	369.480	363.287	18.259	305.760
54	463.680	364.074	100.393	406.154
55	407.240	379.073	43.166	449.319
56	393.990	383.315	14.917	464.236
57	413.120	384.922	29.805	494.041
58	376.640	389.168	-8.282	485.759
59	404.280	387.281	15.112	500.871
60	427.600	389.841	40.319	541.190
61	442.640	395.526	52.799	593.990
62	567.280	402.621	171.754	765.743
63	378.120	437.638	-24.501	741.242
64	313.280	424.980	-124.358	616.885
65	320.800	404.943	-104.180	512.704
66	288.880	391.815	-116.063	396.641
67	295.760	374.238	-96.055	300.585
68	350.560	362.949	-23.678	276.908
69	297.400	361.167	-65.550	211.358
70	368.560	356.629	7.393	218.751
71	491.720	357.478	135.091	353.842
72	422.920	382.844	65.442	419.284
73	327.920	385.681	-54.924	364.360
74	433.560	381.592	47.879	412.239
75	506.040	385.271	124.448	536.687
76	405.360	406.945	20.090	556.777
77	472.600	406.660	65.656	622.432
78	344.920	411.377	-61.740	560.692
79	451.840	407.585	40.463	601.156
80	389.960	410.110	-17.625	583.530
81	461.600	408.960	51.490	635.021
82	501.040	411.963	92.080	727.100
83	501.040	424.175	89.077	816.178
84	405.120	434.291	-19.055	797.123
85	580.640	430.451	146.349	943.472

MEAN OF ERRORS= 19.255
STANDARD DEVIATION= 72.85
SINGLE EXPONENTIAL SMOOTHING

```
$JOB              V.ALVARADO.
C
C                 DOUBLE SPONENTIAL SMOOTHING.
C                 DATA : FEED SALES (MONTHLY).
C                 MODEL LINEAR :   X(T) = A + B*T
C                 METHOD : CHANGE ALPHA.
      DIMENSION X(100),Y(100),SUM(100)
      REAL MAD,INDEX
      DATA ASTER/'*'/,BLANK/' '/
C                 READ IN DATA.
      READ(5,1)(Y(I),I=1,86)
    1 FORMAT(20X,F6.3)
C                 BUILT IN FOUR WEEKS DATA.
      DO 2 I=1,84,4
      SUM(I)=0.
      DO 3 J=1,4
    3 SUM(I)=SUM(I)+Y(I+J-1)
    2 CONTINUE
      DO 4 I=1,21
      K=4*I-3
    4 X(I)=SUM(K)
C                 INITIALIZE.
      TAU=1.
      ITAU=1
      DELTA=0.
      INDEX=200.
      DO 5 IMON=1,2
      GAMMA=0.5
      DELTA=DELTA+0.1
C                 INITIAL CONDITIONS.
      ST1=X(1)
      ST2=ST1
      AHAT=ST1
      BHAT=0.
      MAD=0.
      ACCERR=0.
      SACERR=0.
      XN=0.
      ALPHA=0.1
      BETA=1-ALPHA
C                 PRINT OUT HEADING
      WRITE(6,32)DELTA
   32 FORMAT('1'//20X'ALPHA IS RESTRICTED TO BE .GE.'F5.2///
     1 3X'T'8X'AHAT'8X'BHAT'6X'X(T+I)'8X'XHAT'7X'ERROR'9X
     2'MAD'5X'SUM.ERR'7X'TRACK'7X'ALPHA')
C                 MAKE THE FORECAST.
      DO 100 I=2,21
      XN=XN+1.
      CHECK=BLANK
      FACT=ALPHA/BETA
      XHAT=AHAT+BHAT*TAU
C                 SMOOTH STATISTICS.
      ST1=ALPHA* X(I)+BETA*ST1
      ST2=ALPHA*ST1+BETA*ST2
C                 DETERMINE COEFFICIENTS.
      AHAT=2.*ST1-ST2
      BHAT=FACT*(ST1-ST2)
C                 DETERMINE ERROR & ITS STATISTICS.
      ERROR=X(I+ITAU-1)-XHAT
      DIV=ABS(ERROR)
```

```
      ACCERR=ACCERR+ERROR
      SACERR=SACERR+ERROR*ERROR
C              DETERMINE MEAN ABSOLUTE DEVIATION.
      MAD=ALPHA*ABS(ERROR)+BETA*MAD
C              DETERMINE TRACKING SIGNAL.
      TRACK=ACCERR/MAD
      IF(ABS(ACCERR).GT.4.*MAD)CHECK=ASTER
C              CHANGE ALPHA.
      IF(DIV.LE.INDEX)GO TO 100
      DERROR=(DIV-INDEX)/DIV
      ALPHA=(GAMMA-DELTA)*DERROR+DELTA
      BETA=1.-ALPHA
C              PRINT OUT RESULTS.
  100 WRITE(6,101)I,AHAT,PHAT,X(I+ITAU-1),XHAT,ERROR,
     1MAD,ACCERR,TRACK,ALPHA,CHECK
  101 FORMAT(I4,9F12.3,2A1)
C              DETERMINE MEAN & STANDARD DEVIATION.
      EMEAN=ACCERR/XN
      STDEV=SQRT((SACERR-ACCERR*ACCERR/XN)/(XN-1.))
      WRITE(6,33)EMEAN,STDEV
   33 FORMAT(//20X'MEAN OF ERRORS='F10.3/20X
     1'STANDARD DEVIATION='F7.3)
    5 CONTINUE
      STOP
      END

$ENTRY
```

T	AHAT	BHAT	X(T+I)	XHAT	ERROR	MAD	SUM.ERR	TRACK	ALPHA
2	877.047	-0.648	824.560	889.360	-64.800	6.480	-64.800	-10.000	0.100*
3	957.491	3.620	1303.200	876.399	426.801	48.512	362.000	7.462	0.313*
4	1151.806	48.062	1312.640	961.111	351.529	143.223	713.530	4.982	0.272*
5	1295.081	55.931	1411.720	1199.867	211.852	161.919	925.382	5.715	0.122*
6	1314.020	20.707	1307.680	1351.012	-43.332	147.406	882.050	5.984	0.122*
7	1314.688	19.401	1247.520	1334.726	-87.206	140.039	794.844	5.676	0.122*
8	1383.084	22.594	1547.320	1334.088	213.232	148.996	1008.076	6.766	0.125*
9	1474.410	27.650	1697.640	1405.678	291.962	166.842	1300.037	7.792	0.225*
10	1570.892	61.685	1630.480	1502.061	128.419	158.158	1428.457	9.032	0.226*
11	1630.835	61.469	1628.360	1632.576	-4.216	123.369	1424.241	11.545	0.226*
12	1589.163	48.324	1434.960	1692.355	-257.395	153.658	1166.846	7.594	0.139*
13	1544.490	29.917	1384.660	1637.486	-252.825	172.420	914.020	5.301	0.184*
14	1611.039	32.641	1686.440	1574.406	112.034	161.334	1026.053	6.360	0.184*
15	1625.124	30.766	1588.030	1643.680	-55.650	141.933	970.403	6.837	0.134*
16	1709.159	36.149	1815.640	1655.889	159.751	145.204	1130.154	7.783	0.184*
17	1569.715	18.403	1218.720	1745.308	-526.588	215.217	603.566	2.804	0.348
18	1667.939	55.188	1708.240	1588.118	120.122	182.117	723.687	3.974	0.343
19	1704.506	51.265	1690.743	1723.127	-32.384	129.998	691.303	5.318	0.348*
20	1709.167	41.445	1674.720	1755.771	-81.051	112.960	610.253	5.402	0.348*
21	1827.103	57.562	1883.640	1750.612	133.029	119.946	743.281	6.197	0.348*

MEAN OF ERRORS= 37.164
STANDARD DEVIATION=226.213

T	AHAT	BHAT	X(T+I)	XHAT	ERROR	MAD	SUM.ERR	TRACK	ALPHA
2	877.047	-0.648	824.560	889.360	-64.800	6.480	-64.800	-10.000	0.100*
3	957.491	3.620	1302.200	876.399	426.801	48.512	362.000	7.462	0.359*
4	1174.408	61.798	1312.640	961.111	351.529	157.422	713.530	4.533	0.329*
5	1329.300	73.952	1411.720	1236.206	175.514	163.380	889.044	5.442	0.329*
6	1350.669	63.587	1307.680	1403.251	-95.572	141.050	793.472	5.625	0.329*
7	1322.521	45.505	1247.520	1414.256	-166.736	149.509	626.736	4.192	0.329*
8	1466.670	64.949	1547.320	1368.025	179.295	159.318	806.031	5.059	0.329*
9	1622.961	82.954	1697.640	1531.619	166.021	161.525	972.052	6.018	0.329*
10	1664.412	74.773	1630.480	1705.916	-75.436	133.174	896.616	6.733	0.329*
11	1678.211	62.754	1628.360	1739.185	-110.825	125.814	785.791	6.246	0.329*
12	1572.606	29.568	1434.960	1740.966	-306.005	185.154	479.786	2.591	0.304
13	1488.463	6.504	1384.660	1602.174	-217.514	194.989	262.271	1.345	0.224
14	1569.861	14.035	1585.440	1494.966	191.474	194.201	453.745	2.336	0.224
15	1585.541	14.243	1588.030	1583.896	4.134	151.597	457.879	3.020	0.224
16	1685.708	25.088	1815.640	1599.784	215.856	166.001	673.736	4.059	0.222*
17	1516.353	0.539	1218.720	1710.797	-492.077	238.402	181.659	0.762	0.378
18	1634.462	28.411	1708.240	1516.892	191.348	220.612	373.007	1.691	0.378
19	1679.962	32.395	1690.743	1662.873	27.870	147.743	400.877	2.713	0.378
20	1689.278	27.015	1674.720	1712.357	-37.637	106.115	363.240	3.423	0.378
21	1818.910	50.935	1883.640	1716.293	167.347	129.265	530.587	4.105	0.378*

MEAN OF ERRORS= 26.529
STANDARD DEVIATION=226.466

CORE USAGE OBJECT CODE= 2760 BYTES,ARRAY AREA= 1200 BYTES,TOTAL AREA AVAILABLE= 29584 BYTES

COMPILE TIME= 1.94 SEC,EXECUTION TIME= 1.87 SEC, WATFIV - VERSION 1 LEVEL 2 AUGUST 1970 DATE= 71/291

APPLICATION OF EXPONENTIAL SMOOTHING TO FORECASTING SALES OF FEED

by

VICTOR J. ALVARADO

Ingeniero Industrial, Universidad de Carabobo
Venezuela, 1963

AN ABSTRACT OF A MASTER'S THESIS

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requirements for the degree

MASTER OF SCIENCE

Department of Grain Science and Industry

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1972

A forecasting system is a device by which estimation of future values of a specific process can be determined with some statistical accuracy by using objective computations.

Exponential smoothing is one of the known techniques to forecast time series. It is used to forecast future values within short range intervals. The main advantage of this procedure consists of its relative low cost if computer facilities are used. This low cost makes it possible for exponential smoothing to be widely used to forecast systems in which many time series must be forecasted frequently.

Robert Goodel Brown developed a basic exponential smoothing method to forecast time series; after being published this became a standard reference for almost all the papers written about this particular subject. Brown's Method has the disadvantage in that it requires forecaster intervention to modify the system when changes in the process are detected in order to set a proper value of the parameter that rules the forecasting system. As a product of the research done for this thesis a modified method in which the smoothing constant (the parameter of the system) is changed, when necessary, automatically within the computer program is proposed.

In the modified method, called Floating Alpha Method, the forecasting system is analyzed with each new observation, then if a change in the pattern of the time series is detected the smoothing constant is changed accordingly with the magnitude of the detected change; if no change is observed, the smoothing constant retains its previous value.

All these operations are computer programmed and consequently any forecaster intervention is avoided.

While testing the modified method we obtained a faster response to changes in the time series along with comparable and perhaps greater accuracy in the forecasts than the ones obtained by using Brown's Method.

This method was applied to forecast sales of feedstuffs on a weekly and monthly basis.