

UNCOLLIDED FLUX FROM FINITE
RIGHT-CIRCULAR CYLINDER
VIEWED ENDWISE

by

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INTRODUCTION

The cylinder is a shape quite frequently encountered in engineering work, and the nuclear field is no exception. Many reactors, many fuel elements, and many other sources of radiation have shapes approximating cylinders.

It is not uncommon to need to know the flux from the end of such a cylinder, whether the flux is composed of neutrons or gamma rays or both. This information is required for such reasons as the design of adequate shielding and for calculation of dose rates. To the author's knowledge, there is no method of calculation available at this time which can be performed without a computer in reasonable time periods and give answers with predictable accuracy.

An approximate solution has been developed by Rockwell (6) for determining upper and lower limits on the flux from the end of a cylinder. This technique, while bracketing the flux, in many cases provides such a large bracket that the actual flux is still very much in doubt.

An IBM 704 Computer code has been developed by Gillis et al. (3), which calculates the uncollided flux at any point outside a cylinder.

It is the purpose of this paper to develop empirical techniques enabling one to determine the centerline uncollided flux from the end of a finite cylinder without the aid of a computer.

NOMENCLATURE

Φ	Scalar flux ($\text{cm}^{-2} \text{ sec}^{-1}$)
S_A	Source strength of plane source ($\text{cm}^{-2} \text{ sec}^{-1}$)
S_V	Source strength of volume source ($\text{cm}^{-3} \text{ sec}^{-1}$)
μ_i	Macroscopic cross section of i th shield material (cm^{-1})
μ_s	Macroscopic cross section of source material (cm^{-1})
t_i	Thickness of i th shield material (cm)
t_o	Thickness of source disk (cm)
a	Distance from end of cylinder to observation point (cm)
$\bar{\mu}$	$\sum_i^n \mu_i t_i / a \quad (\text{cm}^{-1})$
b_1	$\sum_i^n \mu_i t_i$
z	Effective self-attenuation distance (cm)
b_2	$b_1 + \mu_s z$
h	Cylinder height (cm)
b_3	$b_1 + \mu_s h$
h'	Cone height having same volume as cylinder (cm)
b_3'	$b_1 + \mu_s h'$
R_o	Cylinder radius (cm)
θ_1	$\tan^{-1} R_o / a$
θ_2	$\tan^{-1} R_o / (a + h)$
NMAX+1	Number of disks in cylinder
δ	Depression of equivalent circular plane source (cm)
θ_N	$\tan^{-1} R_o / (a + Nt_o + \delta)$
θ_z	$\tan^{-1} R_o / (a + z)$
$E_n(b)$	$b^{n-1} \int_b^{\infty} \frac{e^{-t}}{t^n} dt \quad n > 0, \quad E_o(b) = \frac{e^{-b}}{b}$

THEORETICAL DEVELOPMENT

Problem

The equations for calculation of uncollided flux from some geometrically simple sources are developed in Rockwell and those used in this work are listed here. From a circular plane source (Fig. 1) of radius R_o emitting S_A particles per square centimeter - second,

$$\Phi(a) = \frac{S_A}{2} \left\{ E_1(b_1) - E_1(b_1 \sec \theta) \right\} . \quad (1)$$

From a truncated cone (Fig. 2) of height h emitting S_V particles per cubic centimeter - second the flux at P , the imaginary apex of the cone is,

$$\Phi(a) = \frac{S_V}{2\mu_s} \left\{ E_2(b_1) - \frac{E_2(b_1 \sec \theta)}{\sec \theta} - E_2(b_3) + \frac{E_2(b_3 \sec \theta)}{\sec \theta} \right\} \quad (2)$$

The flux from the end of a cylinder (Fig. 3) is not so easily obtained and must be calculated by approximation techniques. There are several approximate solutions available and these are listed and discussed. In all following discussion the orientation of the cylinder will be assumed to be such that the centerline is vertical and the detection point P is above the cylinder. All the solutions require that the point P be on the centerline of the cylinder. All solutions also require that the source S_V be of constant strength throughout the cylinder.

There are six parameters which must be considered in this problem area. Those associated with the cylinder are μ_s , R_o , and h . The parameters associated with the shield are b_1 , a , and $\bar{\mu}$. The shield is assumed to be composed of slabs perpendicular to the cylinder centerline.

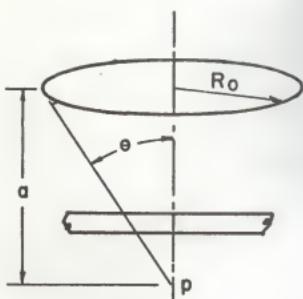


Fig.1. Circular plane source.

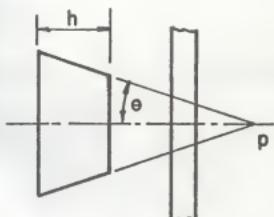


Fig.2. Truncated right-circular cone source.

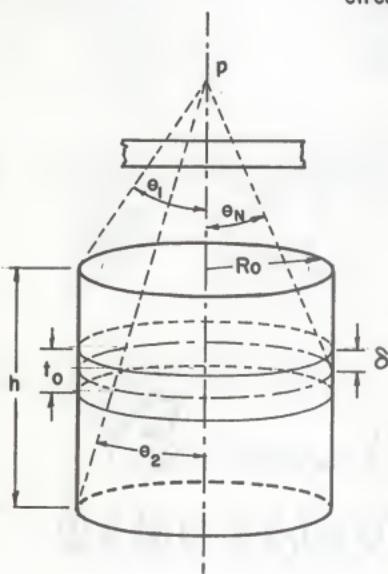


Fig.3. Right-circular cylindrical source viewed endwise

Solutions

Stacked Disk Solution. In order to determine empirical relationships for the flux, it was necessary to determine the flux accurately and this solution served that purpose. A cylinder is assumed to be made up of a series of thin stacked disks. The flux at \underline{P} from each disk will be calculated by assuming each disk to be an equivalent circular plane source with $\frac{S_v t_o}{A} = S_v t_o$. The shielding of all disks above the one under consideration must be considered so each disk will "see" a different amount of shielding.

The self-shielding of each disk is considered by locating the plane source some distance b beneath the upper surface of the disk as shown in Fig. 3. Since the location of the plane source within the disk is actually a weighting function applied to the source distributed within the disk, b is expected to be less than $t_o/2$ because the upper portion of the disk is more important due to the fact that it "sees" less shielding.

A second reason exists for choosing $b < t_o/2$; that being the desire to always make conservative calculations where possible exposure to nuclear radiation may be involved. As the plane source is moved closer to the upper surface of the disk, it "sees" less of the shielding in the disk than is actually the case so the resultant flux calculation will be too high. As the disks are made thinner, the neglected shielding in the source will decrease and the solution becomes more accurate.

The mathematical expression for the flux is

$$\Phi(a) = \frac{S_v t_o}{2} \sum_{N=0}^{N_{MAX}} \left\{ E_1(b_1 + N\mu_s t_o + \mu_s b) - E_1[(b_1 + N\mu_s t_o + \delta\mu_s) \sec \theta_N] \right\}, \quad (3)$$

where NMAX, t₀, and δ will be chosen to assure the calculated flux is within 1% higher than the true flux. Equation 3 is identical to Eq. A-1 in Appendix A which explains the computer code used to solve the equation.

Large Cone Approximation. This solution assumes the cylinder to be a truncated cone the same height as the cylinder with P at the apex. The angle subtended by the cone is θ₁ on Fig. 3, and the cone is obviously larger than the cylinder it represents. The flux from this approximation is calculated by

$$\Phi(a) = \frac{S_V}{2\mu_s} \left\{ E_2(b_1) - \frac{E_2(b_1 \sec \theta_1)}{\sec \theta_1} - E_2(b_3) + \frac{E_2(b_3 \sec \theta_1)}{\sec \theta_1} \right\} . \quad (4)$$

This solution provides an upper limit on the flux from the cylinder. Equation 4 is identical to Eq. A-3 in Appendix A which explains the computer code used to solve the equation.

Small Cone Approximation. This solution also assumes the cylinder to be a truncated cone which is the same height as the cylinder with P at the apex. The angle subtended by the cone is θ₂ on Fig. 3, and the cone is obviously smaller than the cylinder it represents. The flux from this approximation is calculated by

$$\Phi(a) = \frac{S_V}{2\mu_s} \left\{ E_2(b_1) - \frac{E_2(b_1 \ sec \theta_2)}{\sec \theta_2} - E_2(b_3) + \frac{E_2(b_3 \ sec \ theta_2)}{\sec \theta_2} \right\} . \quad (5)$$

This solution provides a lower limit on the flux from the cylinder and when coupled with the large cone approximation provides a bracket on the expected flux. Equation 5 is identical to Eq. A-5 in Appendix A which explains the computer code used to solve the equation.

Equivalent Volume Cone Approximation. This solution also assumes the cylinder to be a truncated cone with P at the apex. The angle subtended

by the cone is θ_1 on Fig. 3, and the height of the truncated cone is such that the volume is equal to that of the cylinder it represents. The flux from this approximation is calculated by

$$\Phi(a) = \frac{S_V}{2\mu_s} \left\{ E_2(b_1) - \frac{E_2(b_1 \sec \theta_1)}{\sec \theta_1} - E_2(b_3) + \frac{E_2(b_3 \sec \theta_1)}{\sec \theta_1} \right\}. \quad (6)$$

Equation 6 is identical to Eq. A-4 in Appendix A which explains the computer code used to solve the equation.

Equivalent Circular Plane Source Approximation. This solution assumes that the cylinder can be represented by a single circular plane source the same radius as the cylinder. The equivalent source is assumed to be located somewhere within the space occupied by the cylinder it represents so that the shielding it "sees" is that of the cylinder within the confines of the cylinder. To allow for self-shielding within the source, the strength of the equivalent source must be considered in two parts. For cylinders with small mean free path height ($\mu_s h$) the equivalent source strength will be assumed to be $S_A = S_V h$. For cylinders with a mean free path height greater than some value K , which is chosen empirically, the source strength will be assumed to be $S_A = \frac{KS_V}{\mu_s}$. The flux from this approximation is calculated by

$$\Phi(a) = \frac{S_V h}{2} \left\{ E_1(b_2) - E_1(b_2 \sec \theta_z) \right\}, \text{ if } \mu_s h \leq K. \quad (7)$$

or

$$\Phi(a) = \frac{KS_V}{2\mu_s} \left\{ E_1(b_2) - E_1(b_2 \sec \theta_z) \right\}, \text{ if } \mu_s h > K. \quad (8)$$

Equations 7 and 8 are identical to Eqs. A-6 and A-7 respectively in Appendix A which explains the computer code used to solve the equations.

NUMERICAL ANALYSIS

Method of Solution

This problem has six parameters which may vary; R_o , μ_s , h , $\bar{\mu}$, b_1 , and a . In order to consider all possible combinations of parameters adequately, a large number of problems had to be solved. To do this an IBM 650 Computer code was written which solved the problems by each of the five previously listed solutions. This code is described in Appendix A.

By proper choice of disk thickness the stacked disk solution could be used to solve the problem with any desired degree of accuracy. This solution was used to provide an answer accurate to within 1%. The choice of disk thickness to assure this accuracy will be discussed later.

The three conical approximate solutions were compared with the stacked disk solutions and the relative error of each solution was calculated. The results were examined in an effort to determine some simple method of empirically presenting the data to enable one to calculate the correct flux within some set limits of accuracy without the use of a computer.

The plane source approximation was used as a trial and error calculation to determine the self-absorption distance which forced the solution to approximate the stacked disk solution within specified limits. The results were examined in an effort to determine a simple empirical relationship between the self-absorption distance and the flux.

Preliminary Investigations

Accuracy Analysis of Stacked Disk Solution. Using the computer code, it was first necessary to determine the disk thickness which gave sufficient

accuracy. As the disks became thinner the solution became more accurate, so very thin disks were preferred. On the other hand, computer running time increased as disk thickness decreased since more disks were then required for any given problem. A compromise was made such that the flux was always calculated to within 1% of the asymptotic value, the result being always higher than the true value. The required disk thickness could then be determined for different combinations of parameters.

In order to better understand the problems associated with the work undertaken, it was necessary to approach the problems of disk thickness and location of the equivalent plane source within each disk in an oblique fashion. It was first assumed that the plane source was on the upper surface of each disk, i.e. $\delta = 0$, and disk thicknesses were determined. Problems solved with this choice of δ were quite lengthy, due to the necessity of using extremely thin disks since no self-absorption in each disk was considered. This extreme location of δ was used initially since the results would satisfy the original requirement that the flux always approach the correct flux from some higher value.

With the information provided from the initial set of problems, it was possible to compare the results with those obtained when δ was varied. Three facts were immediately apparent when these comparisons were made. First, the optimum location for δ was quite near $t_0/2$. Second, the optimum location for δ varied as the parameters of the cylinder and shield varied. Third, the major portion of the advantage of the depression of the plane source was observed as δ varied from 0 to $0.4t_0$. The choice was made for $\delta = 0.45t_0$ and proved to be satisfactory. The advantage of this depression of the plane source was a decrease in the number of disks

required for any given cylinder, the decrease being at least a factor of ten which saved considerable computer time.

In the search to determine the proper disk thickness for each problem, three parameters were shown to have an appreciable effect on the thickness necessary to provide the 1% accuracy required. The results of this search are shown in Table 1 with the variation of a single parameter from some intermediate value shown as the cause with the change of disk thickness while still maintaining 1% accuracy of the calculated flux shown as the effect.

Table 1. Effect on disk thickness required to determine flux within 1% of the asymptotic value as parameters are varied from intermediate values.

Cause	:	Effect
	:	
Increase b_1		Thicker
Increase R_o		Thicker
Increase μ_s		Thicker

The disk thickness was determined in terms of mean free paths, $\mu_s t_o$. Use of mean free paths makes it easier to explain the dependence on μ_s while not affecting the other results.

By the choice of δ , the location of the circular plane source within each disk representing the disk, some small amount of the shielding within each disk has not been considered. This was done, as previously explained, to force the flux calculation to always approach the asymptotic value from some higher value. Certainly one expects this neglected shielding to become less important as the total amount of shielding is increased. This expectation is borne out by the observed effect that increasing b_1 has on disk thickness.

In considering the dependence on R_o , it is only logical to assume that to represent a disk by a plane the disk must "look like" a plane, that is, $R_o \gg t_o$. This being the case, one expects that disks with larger radii could be thicker and still "look like" a plane. This expectation is borne out by the observed effect that increasing R_o has on disk thickness.

The effect of μ_s on disk thickness is again one of geometry. As μ_s increases, the disk thickness in terms of mean free paths is expected to increase because this tends to hold the shape of the disk constant. If the thickness did not increase, the disk would approximate a plane even more closely than necessary. The expected results were observed and disk thickness did increase as μ_s increased.

Parameter Analysis. Having chosen δ it was possible to calculate the flux from cylinders whose parameters were chosen at random, as were the parameters of the shields. These problems were solved in an effort to determine which approximate method of solution or combination of methods showed the greatest promise as a tool to be eventually used in the simple empirical presentation of data which was desired.

The three conical solutions were shown to have little promise. As $\mu_s R_o$ decreased no single solution or average combination of the approximate conical solutions could provide accurate results. Errors were greater than a factor of 10 in some cases. The errors varied in such inconsistent fashion that no pattern which might make the application of correction factors to the approximate solution(s) feasible could be observed. In most cases the large cone approximation and the equivalent volume cone approximation were more nearly correct and the large cone approximation was the more reliable of these two because it always gave a conservative result.

The equivalent plane source approximation was chosen as the only approach which showed some promise of lending itself to the empirical fit of large amounts of data. It was observed that for large $\mu_s R_0$, the self-absorption distance in terms of mean free paths, $\mu_s z$, depended only on b_1 for a given cylinder height and it was decided to present the results as a family of curves for different $\mu_s h$ with $\mu_s z$ plotted versus b_1 .

It was possible at this point to choose a value for K , the mean free path cylinder height that fixed the constant source strength, for cylinders whose height exceeded K mean free paths. The artificial circular plane source having $S_A = S_V h$ kept increasing as h increased, although the flux from the cylinder did not. The self-absorption distance z necessarily increased as the plane source increased and these compensating effects were undesirable since the flux did not continue to increase. The value of K was fixed at 1.5 so that all cylinders more than 1.5 mean free paths tall were assumed to have an equivalent circular plane source strength such that $S_A = 1.5 S_V / \mu_s$. The plane source strength then fell into one of two categories: if $\mu_s h \leq 1.5$, $S_A = S_V h$; if $\mu_s h > 1.5$, $S_A = 1.5 S_V / \mu_s$. The figure of $\mu_s h = 1.5$ was chosen as the minimum value which could be used to fix a constant source strength without a risk of developing a negative z , which is not anticipated in the computer program. The minimum value was desired since the accuracy of the empirical relationships improved as the cylinder height used to fix a constant source strength decreased.

Data Analysis

Typical values for R_0 and μ_s were chosen and problems with varying K , a , b_1 , and h were solved and the resultant curves in Figs. 4a and 4b were

determined. The self-absorption distance $\mu_s z$, expressed in terms of mean free paths, refers to the equivalent circular plane source which is assumed to replace the cylinder. Presentation of the data in this manner requires that $\mu_s z$ be dependent only on the shield mean free paths, b_1 , for a given cylinder height. This requirement is satisfied for large $\mu_s R_o$.

For the cases of small $\mu_s R_o$, marked deviation of $\mu_s z$ from that suggested by Figs. 4a and 4b was observed. In general, $\mu_s z$ increased as $\mu_s R_o$ first dropped below some critical value dependent on b_1 , and eventually decreased rapidly as $\mu_s R_o$ continued to decrease. Figure 5 indicates the area where fluxes may be calculated to within 10% of the actual value without the use of correction factors. As $\mu_s R_o$ exceeds the critical value shown for any b_1 , correction factors are not required.

The maximum deviation of $\mu_s z$ from the values predicted by Figs. 4a and 4b is observed for large $\bar{\mu}$. For a given value of $\mu_s R_o$ the deviation was greater for smaller μ_s as $\bar{\mu}$ was held constant. The relative deviation was slightly dependent on $\mu_s h$ and quite dependent on b_1 . It was necessary to develop correction factors for $\mu_s z$ which depend on b_1 , $\bar{\mu}$, and μ_s . This has been done in Figs. 6 through 12. These correction factors were determined by assuming cylinders having $\mu_s h = 2$. and the results are most accurate for this case. For values of $\mu_s h$ not equal to two the error in the calculated flux for small $\mu_s R_o$ increases. Largest errors are noted when μ_s is small, R_o is small, and $\bar{\mu}$ is large. Errors are greatest for the smallest $\mu_s h$. Errors are positive, i.e. the empirically determined flux is greater than the flux determined by the stacked disk solution, for most cases, and the estimated maximum negative error is <25%. The estimated maximum

positive error is < 175% and these extreme errors apply only in isolated cases. Table 2 lists the estimated maximum errors anticipated for varying μ_s and varying $\mu_s h$ at values of $\mu_s h$ other than 2. The comparable errors resulting from the use of the large cone approximation are included in Table 2. Appendix B explains the computer program used to calculate the error in flux determination by the empirical method.

Table 2. Study of maximum percentage errors in flux determination compared to stacked disk flux determination, by the curves in this paper and by the large cone approximation for varying source and shield parameters.

μ_s	μ_{sh}	b_1	Percentage errors in flux determination											
			$\bar{\mu} = 0.02$				$\bar{\mu} = 0.1$				$\bar{\mu} = 1.0$			
			Curves	Large	Curves	Large	Curves	Large	Curves	Large	Curves	Large	Curves	Large
0.02	0.25	0.1	36.7	228.9	90.5	854.5	145.7	1312.5	155.8	1213.1	1.0	1.5	23.6	20.0
			In this	cone	In this	cone	In this	cone	In this	cone	In this	cone	In this	cone
			paper *	approx.	paper *	approx.	paper *	approx.	paper *	approx.	paper *	approx.	paper *	approx.
0.02	0.25	0.1	36.7	228.9	90.5	854.5	145.7	1312.5	155.8	1213.1	1.0	1.5	23.6	20.0
			0	8.9	3.2	29.6	99.2	756.3	139.9	779.8	4.0	0	8.9	3.2
			10	-3.9	0.1	10.6	45.0	268.5	97.1	531.1	30	1.1	-3.9	0.7
						-5.0**	19.4	114.1	57.3	330.8				
						4.5	39.0	25.0	25.0	138.9				
0.02	1.00	0.1	10.3	690.7	10.9	2370.8	9.8	2756.6	10.2	2498.1	1.0	-2.6	79.1	1.3
			4.0	-2.1	21.1	362.5	7.0	2118.8	3.2	2021.1	10	2.6	8.2	-4.5
			30	0.3	8.2	97.8	-2.3	808.1	0.3	1539.2				
						40.0	-6.8	356.1	-5.8	995.7				
						9.3	-7.3	127.8	-6.5	432.8				
0.02	1.25	0.1	10.7	785.5	8.5	2645.7	5.0	2943.8	5.1	2664.4	1.0	0.4	92.1	1.2
			4.0	4.3	24.1	414.8	2.9	2367.6	-2.6	2217.9	10	0.3	8.2	-2.7
			30	0.3	8.2	113.7	2.7	917.9	2.8	1727.6				
						47.0	-8.3	407.6	-10.4	1128.5				
						14.0	-7.1	148.2	-9.1	494.5				
0.02	1.50	0.1	2.9	859.9	-1.8	2851.0	-6.5	3071.6	-6.3	2777.8	1.0	-1.8	103.0	-5.0
			4.0	0.6	26.9	456.2	-7.6	2554.1	-13.8	2358.6	10	1.4	10.6	-4.5
			30	1.4	10.6	126.8	-14.2	1003.7	-16.6	1870.1				
						53.0	-15.7	448.5	-20.2	1231.7				
						16.1	-12.1	164.7	-17.2	543.3				
0.02	1.75	0.1	4.3	918.0	-0.4	3004.5	-4.8	3160.2	-4.6	2856.5	1.0	0.4	111.9	-1.4
			4.0	1.4	29.8	488.9	-2.6	2693.9	-9.1	2459.9	10	0.2	11.4	-1.7
			30			137.5	-6.1	1070.7	-8.0	1977.9				
						58.1	-6.6	480.7	-9.7	1311.7				
						18.9	-4.1	178.1	-6.7	581.7				

Table 2. (cont.).

μ_s	μ_{sh}	b_1	Percentage errors in flux determination											
			$\mu = 0.02$				$\mu = 0.1$				$\mu = 1.0$			
			Curves	Large cone	Curves in this paper *	Large cone approx.	Curves	Large cone	Curves in this paper *	Large cone approx.	Curves	Large cone	Curves in this paper *	Large cone approx.
0.02	3.00	0.1	8.1	1064.9	4.2	3353.1	-0.6	3338.0	-0.6	304.3	2675.6	2230.0	1510.6	679.5
	1.0	4.8	136.4	9.6	572.7	11.2	3013.9	4.8	3013.9	4.8	2675.6	2230.0	1510.6	679.5
	4.0	4.2	38.1	13.1	166.3	18.8	1239.0	19.5	1239.0	19.5	2675.6	2230.0	1510.6	679.5
	10	0.3	15.4	11.1	72.6	23.2	563.3	24.6	563.3	24.6	2675.6	2230.0	1510.6	679.5
	30			6.6	24.7	22.0	213.7	29.6	213.7	29.6	2675.6	2230.0	1510.6	679.5
0.05	0.25	0.1	13.1	92.4	57.5	341.5	107.4	509.3	113.3	463.9	292.3	203.6	130.0	55.4
	1.0	-0.1	9.3	7.8	46.9	58.9	299.8	96.0	299.8	96.0	292.3	203.6	130.0	55.4
	4.0	0.02	5.2	0.4	11.9	20.5	108.1	63.9	108.1	63.9	292.3	203.6	130.0	55.4
	10	0.3	-4.7	0.2	3.5	6.1	45.9	26.9	45.9	26.9	292.3	203.6	130.0	55.4
	30			0.3	-7.0	0.3	15.6	9.9	15.6	9.9	292.3	203.6	130.0	55.4
0.05	1.00	0.1	3.3	289.9	14.4	973.9	18.4	1099.1	17.3	988.6	806.2	625.1	411.1	180.4
	1.0	-1.1	32.4	3.2	156.1	7.2	869.1	9.1	869.1	9.1	988.6	806.2	625.1	411.1
	4.0	0.3	8.7	-2.4	40.2	-1.5	337.9	14.2	337.9	14.2	988.6	806.2	625.1	411.1
	10	2.7	3.4	-1.1	16.1	-6.2	149.2	-5.4	149.2	-5.4	988.6	806.2	625.1	411.1
	30			-0.2	1.4	-6.8	52.8	-3.2	52.8	-3.2	988.6	806.2	625.1	411.1
0.05	1.25	0.1	5.4	332.2	13.1	1090.4	14.1	1176.5	13.0	1057.2	888.8	708.4	470.6	208.8
	1.0	2.5	38.2	5.9	172.9	4.9	975.0	3.9	975.0	3.9	1057.2	888.8	708.4	470.6
	4.0	2.9	9.7	0.5	47.4	-2.3	387.5	10.3	387.5	10.3	1057.2	888.8	708.4	470.6
	10	0.8	2.6	1.6	19.0	-6.1	172.9	-8.0	172.9	-8.0	1057.2	888.8	708.4	470.6
	30			1.8	4.6	-5.4	62.0	-3.5	62.0	-3.5	1057.2	888.8	708.4	470.6
0.05	1.50	0.1	0.3	365.9	4.3	1177.8	2.2	1229.4	0.7	1104.2	948.0	767.3	515.1	230.9
	1.0	2.4	43.1	2.6	195.1	-4.5	1054.6	-7.6	1054.6	-7.6	1104.2	948.0	767.3	515.1
	4.0	3.3	10.7	-1.1	53.3	-9.5	425.2	-1.0	425.2	-1.0	1104.2	948.0	767.3	515.1
	10	0.4	4.2	1.3	21.7	-11.6	191.5	-16.9	191.5	-16.9	1104.2	948.0	767.3	515.1
	30			0.9	5.3	5.3	69.6	-9.3	69.6	-9.3	1104.2	948.0	767.3	515.1

Table 2. (cont.).

		Percentage errors in flux determination									
μ_s	μ_{gA}	b_1	$\bar{\mu} \quad 0.02$	$\bar{\mu} \quad 0.1$	$\bar{\mu} \quad 1.0$	$\bar{\mu} \quad 4.0$	$\bar{\mu} \quad 1.0$	$\bar{\mu} \quad 1.0$	$\bar{\mu} \quad 1.0$	$\bar{\mu} \quad 4.0$	
			Curves in this paper *	Large cone approx.							
0.05	1.75	0.1	0.8	392.5	5.5	1263.2	3.7	1266.2	2.0	1136.8	
		1.0	1.5	47.2	5.2	210.5	-0.1	1114.5	-2.2	990.7	
	4.0	2.6	12.0	1.0	58.3	-2.4	455.9	8.9	814.8		
	10	-1.5	4.1	2.3	24.0	-3.7	206.8	-7.1	551.3		
	30				7.0	-3.1	76.0	-0.2	248.9		
0.05	3.00	0.1	3.7	460.7	10.2	1392.5	5.5	1340.1	0.4	1202.4	
	1.0	1.8	59.4	13.7	251.2	11.5	1251.1	6.2	1081.8		
	4.0	1.4	15.7	9.4	72.7	20.5	532.8	28.1	923.4		
	10	-3.2	6.2	6.5	30.9	19.2	246.8	18.0	640.2		
	30			0.8	9.5	14.3	93.8	31.6	295.7		
0.1	0.25	0.1	5.5	46.3	28.7	169.3	60.9	241.7	69.2	214.6	
	1.0	-0.3	4.5	2.1	23.5	36.5	147.3	57.1	131.1		
	4.0	0.5	4.0	0.3	5.9	9.1	54.1	31.0	95.1		
	10	0.1	-4.9	0.2	1.0	1.5	22.7	11.0	62.7		
	30			-0.6	-7.9	-0.02	7.6	1.8	27.3		
0.1	1.00	0.1	3.8	150.0	4.7	498.1	3.4	541.2	14.8	481.1	
	1.0	0.7	16.4	-0.6	78.4	11.7	443.7	12.2	395.9		
	4.0	0.8	4.6	-0.6	20.4	-0.9	174.9	4.9	314.4		
	10	0.5	1.2	0.02	7.8	-6.0	76.6	-7.5	209.4		
	30			-1.2	-1.4	-3.3	26.7	-8.5	92.3		
0.1	1.25	0.1	7.4	173.4	4.8	560.3	5.9	581.4	11.5	516.8	
	1.1	4.6	19.4	2.7	91.4	12.8	500.4	6.6	439.7		
	4.0	4.0	4.9	2.8	24.1	-1.6	202.2	2.7	359.0		
	10	3.2	0.5	3.0	9.3	-4.8	89.4	-9.2	241.3		
	30			0.6	11.8	-1.0	31.5	-8.2	107.7		

Table 2. (cont.).

		Percentage errors in flux determination																			
		$\bar{\mu}=0.02$					$\bar{\mu}=0.1$					$\bar{\mu}=1.0$					$\bar{\mu}=4.0$				
μ_s	μ_h	b_1	Curves in this cone paper *	Large cone approx.	Curves in this paper *	Large cone approx.															
0.1	1.50	0.1	4.5	192.2	-2.2	607.1	-4.2	609.1	-4.2	541.3	-4.7	469.3	-4.7	541.3	-4.7	469.3	-4.7	391.4	-6.3	391.4	
		1.0	5.4	21.9	0.6	102.0	2.9	541.9	2.9	223.3	-7.2	223.3	-7.2	100.1	-16.1	100.1	-16.1	267.0	-12.5	267.0	
		4.0	4.5	5.2	2.6	27.3	-7.2													120.4	
		10	3.2	1.8	3.1	10.8	-8.2														
		30			3.0	1.7	-2.1														
0.1	1.75	0.1	4.5	207.3	-1.6	642.4	-3.0	628.3	-3.0	574.7	7.3	574.7	0.1	558.4	0.1	558.4	0.1	492.7	1.8	492.7	
		1.0	3.0	24.2	2.1	110.9	-0.7	240.7	-0.7	240.7	20.1	240.7	20.1	108.7	-7.4	108.7	-7.4	417.3	1.8	417.3	
		4.0	1.9	5.9	2.8	30.1	-0.7													286.6	
		10	0.9	1.7	2.6	11.9	-2.4													130.5	
		30			1.7	2.8	0.7														
0.1	3.00	0.1	5.9	247.0	2.4	723.4	1.2	667.2	1.2	592.9	6.2	592.9	6.2	524.9	0.1	524.9	0.1	477.5	17.1	477.5	
		1.0	-0.3	31.1	7.5	135.2	20.9	650.8	20.9	650.8	17.1	650.8	17.1	337.5	-0.7	337.5	-0.7	337.5	28.4	337.5	
		4.0	-2.0	7.9	5.6	38.6	18.6	285.2	18.6	285.2	28.4	285.2	28.4	132.5	-0.5	132.5	-0.5	158.2	20.4	158.2	
		10	-1.3	2.8	3.0	15.7	15.4														
		30			0.2	4.1	12.2														
1.0	0.25	0.1	0.1	4.3	-0.6	15.8	1.0	14.9	1.0	14.9	1.1	14.9	1.1	7.0	-0.8	7.0	-0.8	4.3	4.3	4.3	
		1.0	-0.1	0.07	-0.06	2.1	0.3	13.0	0.3	13.0	-0.8	13.0	-0.8	5.1	-0.7	5.1	-0.7	6.5	-0.4	6.5	
		4.0	0.1	2.8	0	0.4	-0.2	5.1	-0.2	5.1	-0.7	5.1	-0.7	2.0	-0.4	2.0	-0.4	5.6	-0.5	5.6	
		10	-0.3	-5.3	-0.2	-1.2	-0.2	0.5	-0.2	0.5	-0.5	0.5	-0.5	-0.4	-0.4	-0.4	-0.4	2.5	-0.5	2.5	
		30			-0.1	-9.0	-0.4														
1.0	1.00	0.1	5.6	15.7	-0.6	50.9	-0.6	34.1	-0.6	34.1	-0.8	34.1	-0.8	23.1	-3.3	23.1	-3.3	21.4	-3.9	21.4	
		1.0	1.9	1.4	1.9	7.9	-2.1	43.4	-2.1	43.4	-0.8	43.4	-0.8	18.3	-1.2	18.3	-1.2	25.2	-3.9	25.2	
		4.0	1.0	0.8	1.0	1.8	-0.5	1.8	-0.5	1.8	-0.5	1.8	-0.5	7.7	-1.2	7.7	-1.2	20.3	-3.4	20.3	
		10	0.3	0.3	0.3	0.3	-0.2	0.3	-0.2	0.3	-0.2	0.3	-0.2	2.5	-0.7	2.5	-0.7	9.3	-2.7	9.3	
		30			0.9	-0.9	-4.2	-0.9	-4.2	-0.9	-4.2	-0.9	-4.2								

Table 2. (concl)

		Percentage errors in flux determination								
		$\bar{\mu} = 0.02$			$\bar{\mu} = 0.1$			$\bar{\mu} = 1.0$		
μ_s	μ_{sh}	b_1	Curves in this paper *	Large cone approx.						
1.0	1.25	0.1	11.0	18.6	3.5	58.6	3.3	37.8	3.1	26.5
	1.0	5.7	5.7	1.7	6.0	9.4	1.3	50.3	-0.5	25.7
	4.0	4.1	0.3	4.3	2.2	2.2	2.7	21.6	-1.5	30.1
	10	2.5	-0.2	2.4	0.4	1.5	9.2	-1.1	24.1	
	30			1.6	-2.0	0.9	3.0	-0.3	11.1	
1.0	1.50	0.1	13.4	21.1	3.0	64.8	1.6	40.7	1.2	29.1
	1.0	7.4	7.4	2.0	7.4	10.8	-0.1	55.9	-2.9	29.2
	4.0	4.4	0	5.3	2.6	2.7	24.5	-3.0	34.1	
	10	2.7	1.3	2.9	0.4	1.4	10.3	-2.4	27.0	
	30			2.1	-2.2	0.8	3.2	-0.9	12.3	
1.0	1.75	0.1	9.0	23.3	1.4	69.7	1.1	42.8	1.1	31.0
	1.0	2.8	2.2	3.9	12.0	0.5	60.4	-1.6	31.9	
	4.0	2.1	0.2	2.4	2.9	2.8	27.0	-0.7	37.5	
	10	-0.4	0.4	0.9	0.8	1.2	11.7	-0.1	30.1	
	30			0	-1.1	-0.2	3.9	-0.3	14.0	
1.0	3.00	0.1	1.7	29.9	0.3	82.1	2.3	47.8	2.1	35.6
	1.0	-6.2	3.0	-2.2	15.7	4.2	71.9	3.7	38.8	
	4.0	-4.4	0.4	-3.1	3.7	4.9	34.1	7.1	46.3	
	10	-3.4	0.6	-3.0	1.2	1.7	15.3	38.4	18.0	
	30			-2.5	-1.2	-1.3	5.0	2.8		

* The interpolation required to determine the self-absorption distance may give rise to errors in the flux determination which exceed these estimated maximum errors. In no case is the error expected to increase by more than 5%.

** For large $\bar{\mu}$, i.e. large b_1 and small $\bar{\mu}$, the limitations of the computer were such that the large cone approximate calculation of \bar{F} flux was less than the stacked disk solution.

DESIGN CONSIDERATIONS

Use of Design Curves

The following curves are to be used to determine the location or self-absorption distance of a circular plane source assumed to represent a cylinder for purposes of calculating the centerline uncollided flux at some point P , at distance a beyond an end of the cylinder. The self-absorption distance is measured from the end of the cylinder closest to P . Necessary input data are $\bar{\mu}$, a , μ_s , R_o , h , and b_1 . The following step-by-step procedure enables one to determine the self-absorption distance and the resultant flux. The following limits on the values of the parameters must be observed:

$$\begin{aligned} R_o &\geq 1 \text{ cm} \\ 0.1 \leq b_1 &\leq 40 \\ 0.02 \text{ cm}^{-1} &\leq \mu_s \\ 0.02 \text{ cm}^{-1} &\leq \bar{\mu} \end{aligned} .$$

To determine the self-absorption distance proceed as follows:

Step 1. Calculate $\mu_s h$.

Determine $\mu_s z$ from Fig. 4a or 4b.

Step 2. Calculate $\frac{\mu_s R_o}{s_o}$.

From Fig. 5 determine necessity of correction factor application. If correction factor is necessary go to step 3; if not, go to step 6.

Step 3. Using Figs. 6b, 7b, 8b, 9b, 10b, 11b, or 12b select the Figures at the b_1 values immediately larger and smaller than the given b_1 . From these two Figures determine a relation to the appropriate F_i , G_i , H_i , J_i ,

K_i, L_i, or M_i curves. For example, at b₁ = 4, on Fig. 8b at μ = 0.3 and μ_s = 0.1 a location between H₂ and H₃ is observed approximately one-third of the distance from H₂ towards H₃.

Step 4. With the relations determined in Step 3, use Figs 6a, 7a, 8a, 9a, 10a, 11a, or 12a as appropriate, and by interpolation determine correction factors at the two values for b₁ used in Step 3.

Step 5. Interpolating between b₁ values, determine correction factor, CF, to be used at the given b₁, and multiply CF by $\mu_s z$ which is then the $\mu_s z$ to be used in succeeding steps.

Step 6. Calculate $\sec \theta_z$ and z from $\mu_s z$,
where $\sec \theta_z = \sqrt{1 + \frac{R_o^2}{(a+z)^2}}$.

Step 7. Calculate $b_2 = b_1 + \mu_s z$.
Calculate $\{E_1(b_2) - E_1(b_2 \sec \theta_z)\}$,
where $E_1(x) = -0.5772 - \ln x + x - \frac{x^2}{4} + \frac{x^3}{18} \dots$

if $x < 1$, Refs. (1,2,6).

$E_1(x) = \frac{e^{-x}}{x} \left\{ \frac{0.251 + 2.335x + x^2}{1.082 + 3.331x + x^2} \right\}$,
if $x \geq 1$, Ref.(3).

Step 8. If $\mu_s h \leq 1.5$, $S_A = S_V h$
If $\mu_s h > 1.5$, $S_A = S_V \frac{(1.5)}{\mu_s}$

Step 9. $\Phi(a) = \frac{S_A}{2} \{E_1(b_2) - E_1(b_2 \sec \theta_z)\}$

If the flux is desired at some point P not on the centerline of the cylinder a conservative estimate can be obtained by assuming the point P is on the centerline of the cylinder at the same vertical distance above

the source, and solving the problem in the manner outlined above.

Sample Problem

The following problem will serve to illustrate the use of the design curves. The parameter values have been assumed to be as follows:

$$\mu_s = 0.07, \bar{\mu} = 0.3, b_1 = 6, R_o = 10 \text{ cm}, h = 40 \text{ cm}, a = 20 \text{ cm}.$$

Step 1. $\mu_s h = 2.8$

From Fig. 4b, $\mu_{sZ} = 0.497$

Step 2. $\mu_s R_o = 0.7$ From Fig. 5, correction factor is necessary.

Step 3. From Fig. 8b, observe a location between

H_2 and H_3 close to H_3 .

From Fig 9b, observe a location between

J_1 and J_2 close to J_2 .

Step 4. From fig. 8a, by interpolation, the correction factor is estimated as 0.97. From Fig. 9a, by interpolation, the correction factor is estimated as 1.01.

Step 5. Correction factor is 0.983 for $b_1 = 6$.

Corrected μ_{sZ} is 0.489.

Step 6. $z = 6.96 \text{ cm}$.

$\sec \theta_z = 1.0665$.

Step 7. $b_2 = 6.489$.

$b_2 \sec \theta_z = 6.920$.

$E_1(6.489) = 2.078 \times 10^{-4}$.

$E_1(6.921) = 1.273 \times 10^{-4}$.

Step 8. $S_A = 21.428 \text{ cm}^{-2} \text{ sec}^{-1}$

Step 9. $\Phi(a) = 8.625 \times 10^{-4} \text{ cm}^{-2} \text{ sec}^{-1}$

The stacked disk solution to this problem is $7.8 \times 10^{-4} \text{ cm}^{-2} \text{ sec}^{-1}$.

The error in the empirical solution is 10.6%. The error in the large cone approximate solution is 64.7%.

DATA PRESENTATION

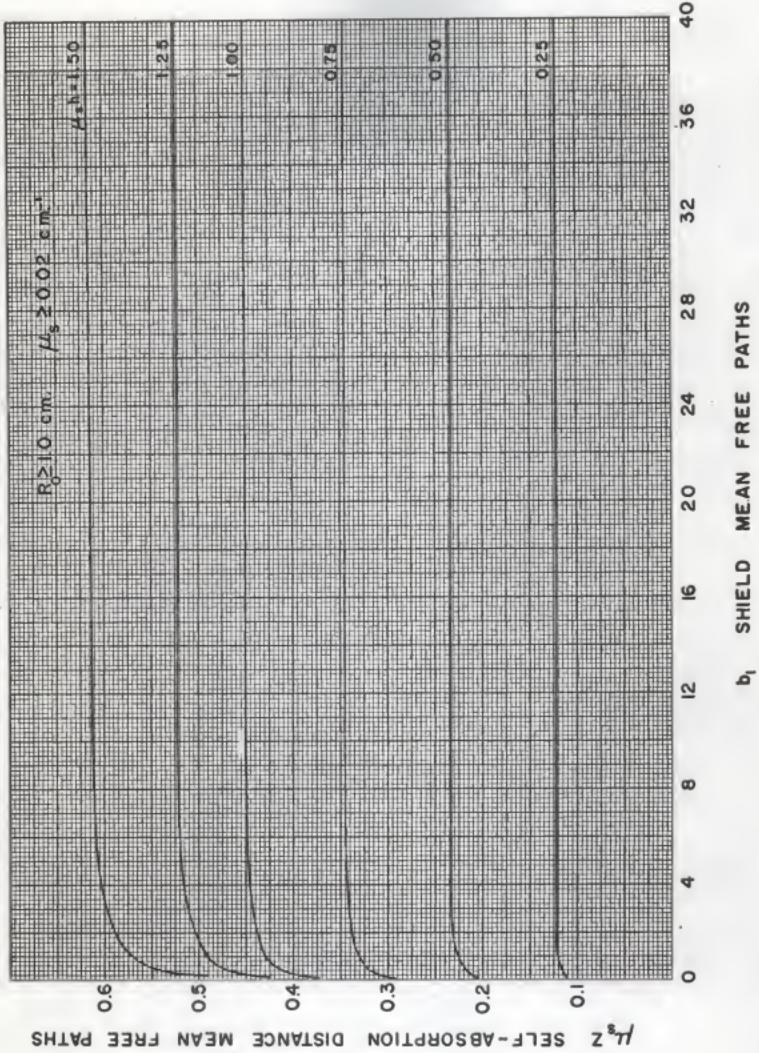


Fig. 4 a. Equivalent circular plane source self-absorption distance for $\mu_s h \leq 1.5$

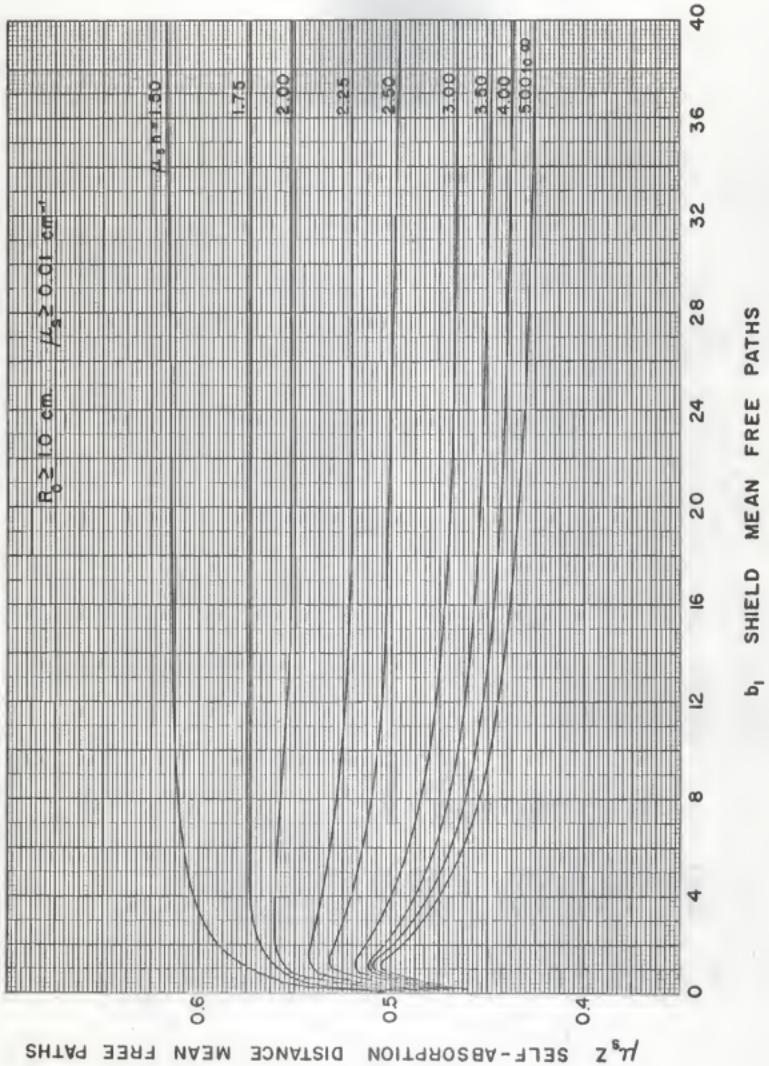


Fig. 4b. Equivalent circular plane source self-absorption distance for $\mu_s h > 1.5$

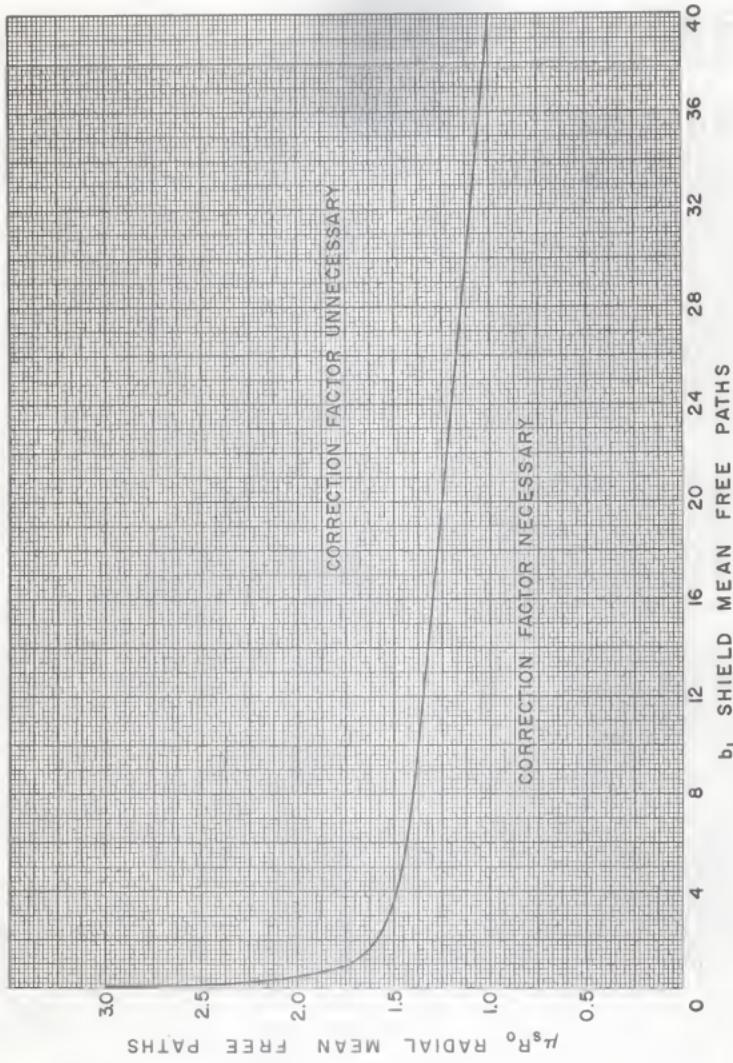


Fig. 5. Correction factor applicability.

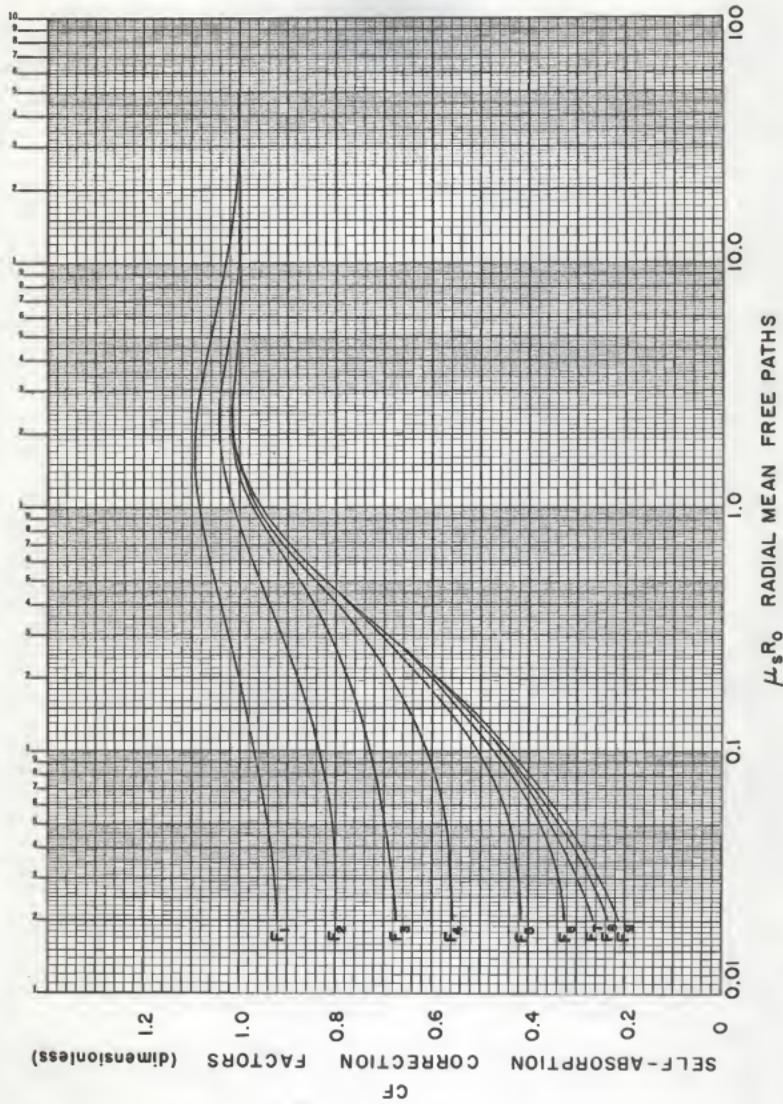


Fig. 6a. Correction factors for equivalent circular plane source self-absorption distance for $b_i = 0.1$.

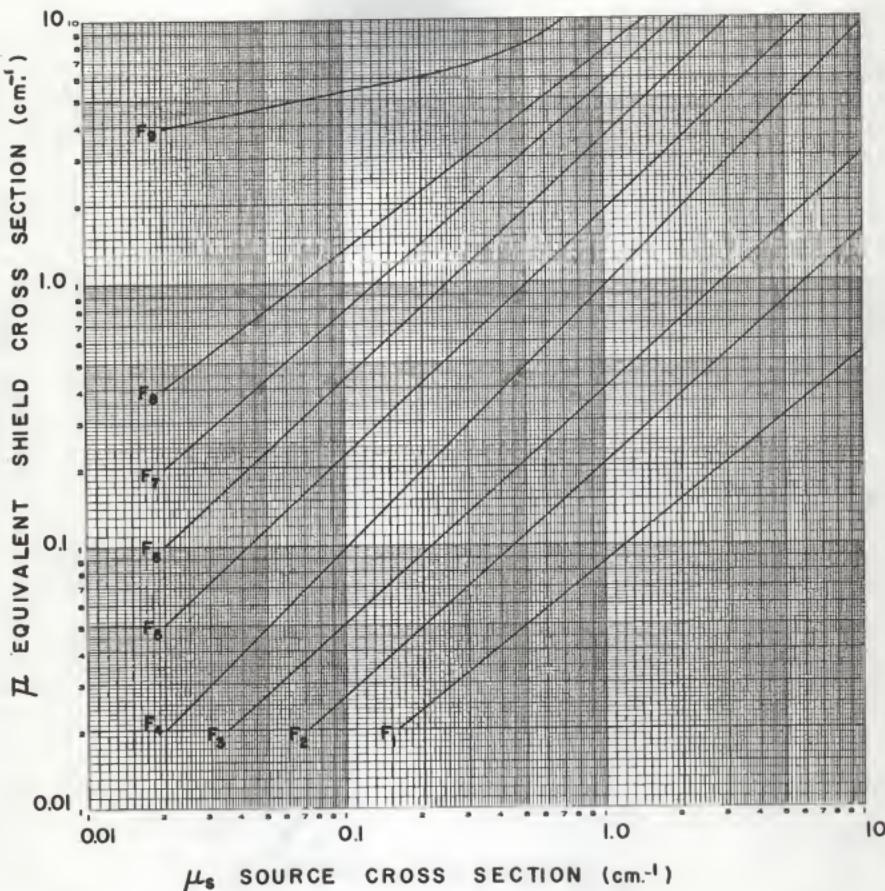


Fig. 6b. Source and shield cross sections correlation for $b_1 = 0.1$.

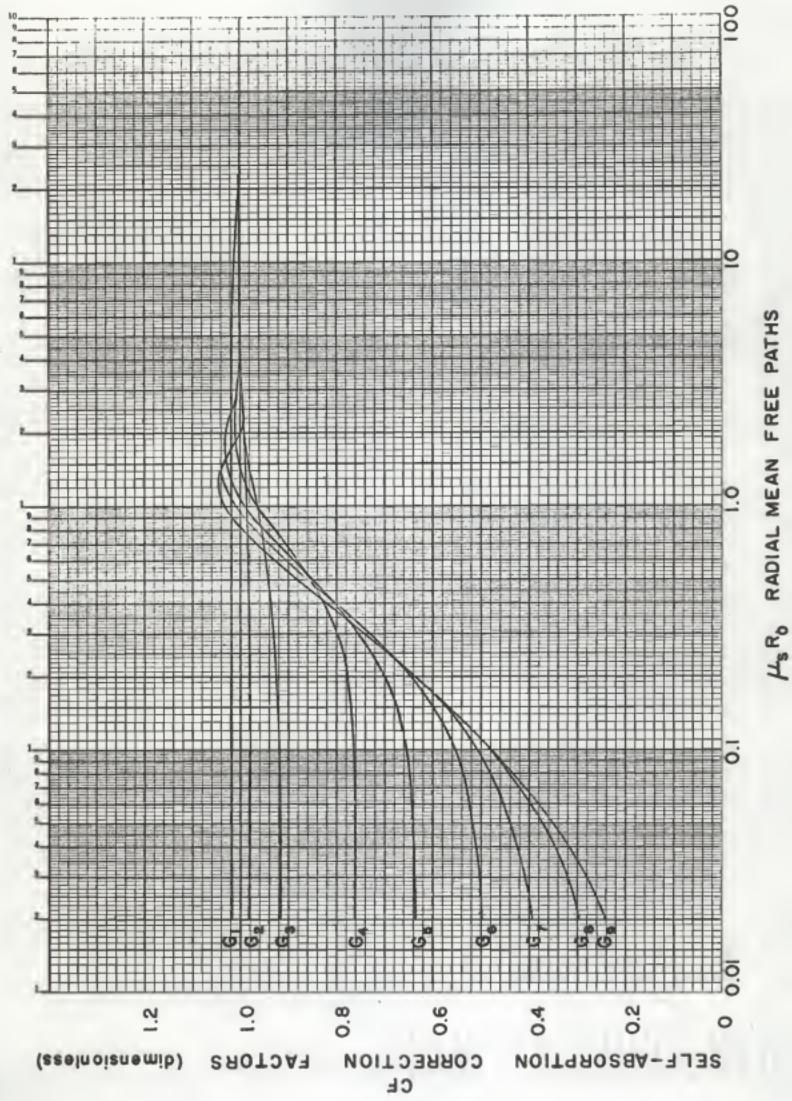


Fig. 7a. Correction factors for equivalent circular plane source self-absorption distance for $b = 1.0$.

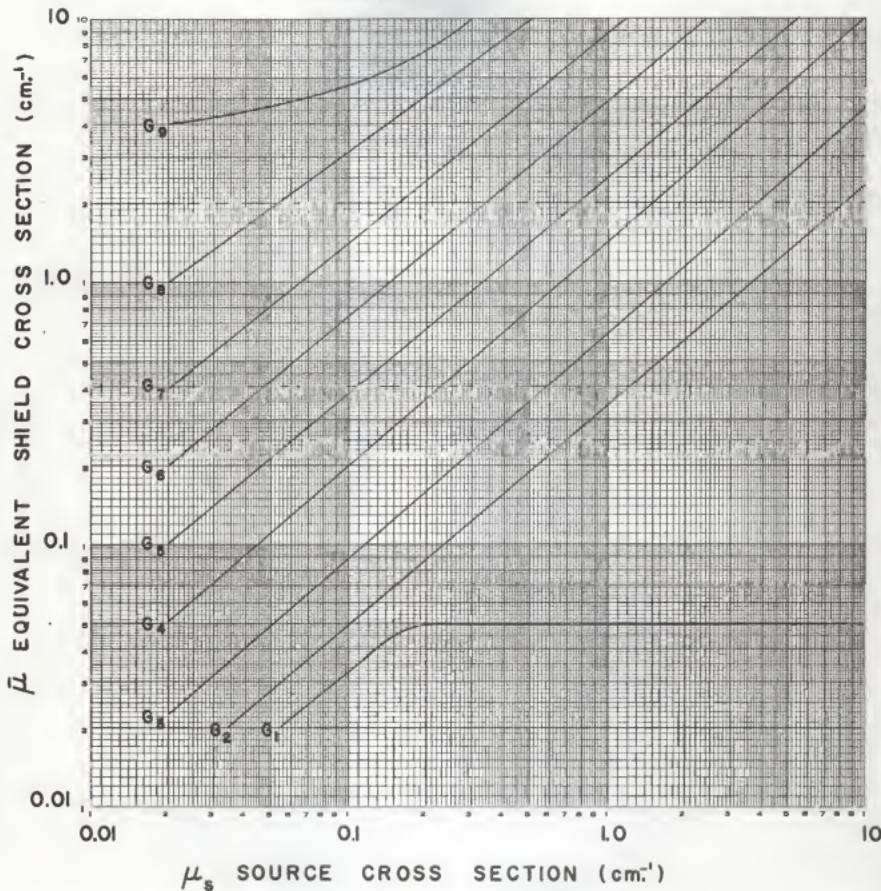


Fig. 7b. Source and shield cross sections correlation for $b_1 = 1.0$.

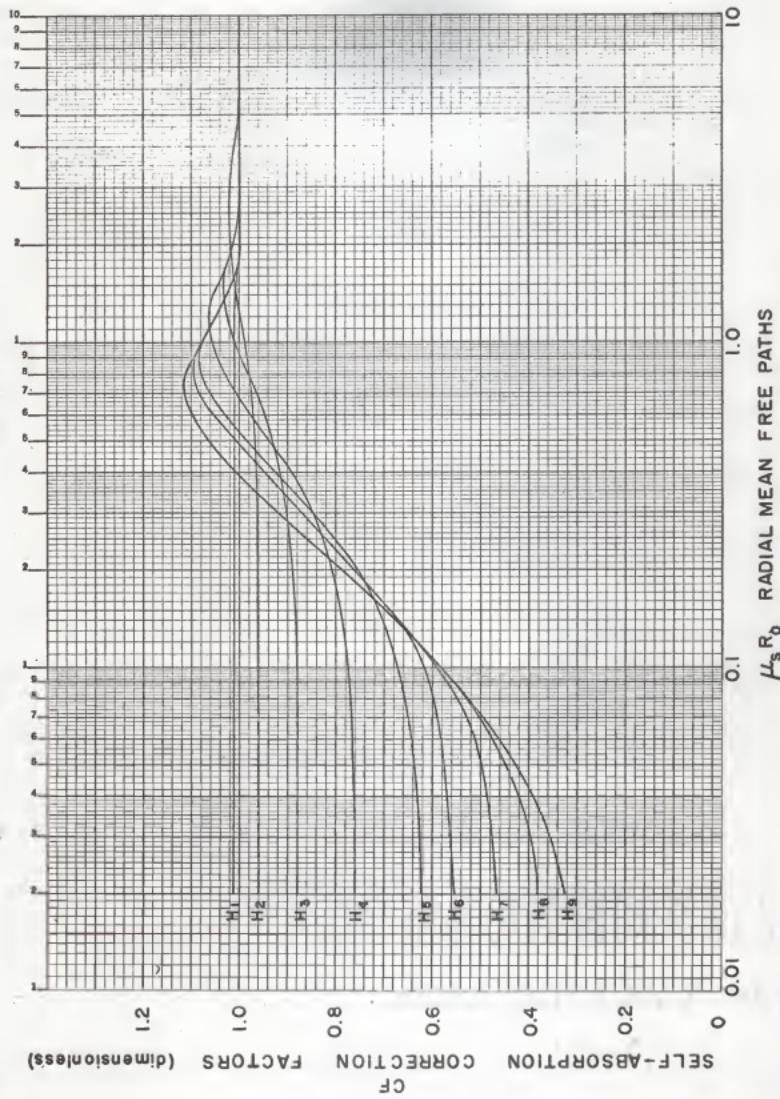


Fig. 8a. Correction factors for equivalent circular plane source self-absorption distance for $b_r = 4.0$.

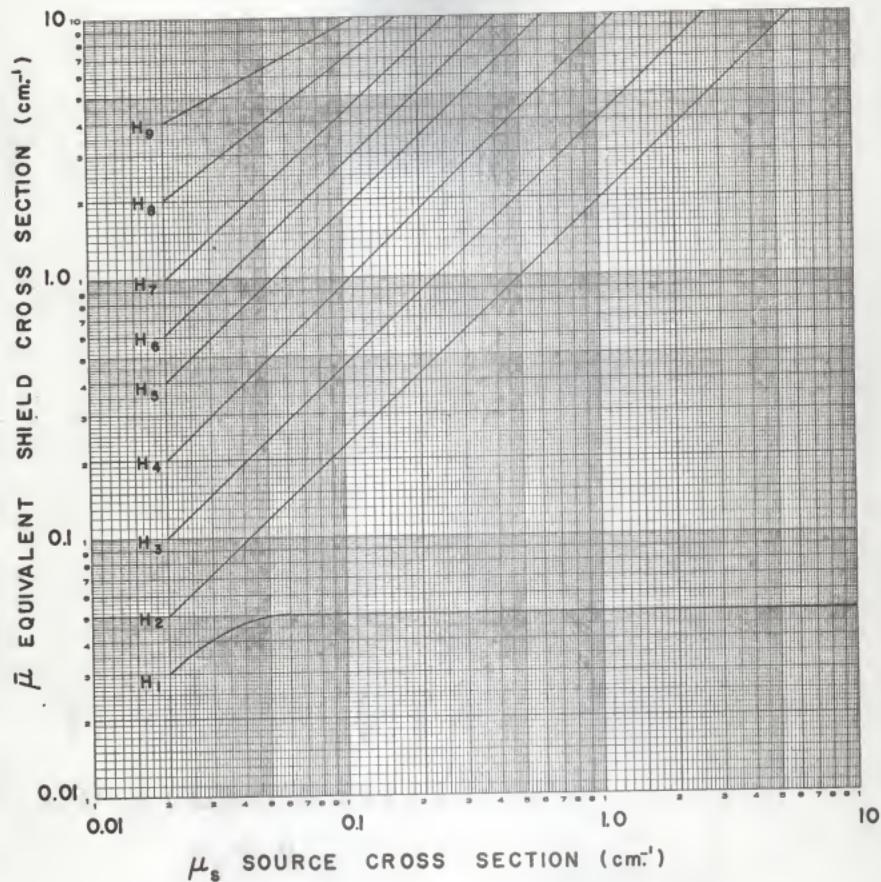


Fig. 8b. Source and shield cross sections correlation for $b_1 = 4.0$.

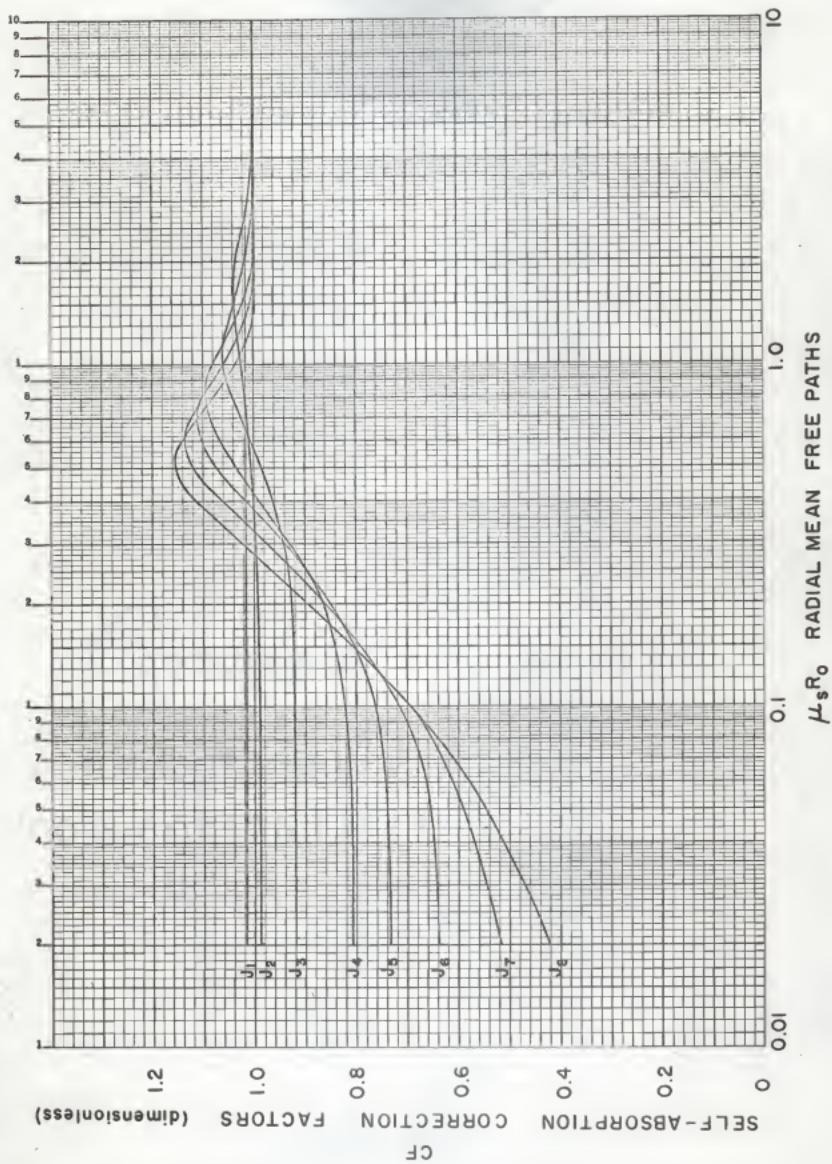


Fig. 9a. Correction factors for equivalent circular plane source self-absorption distance for $h = 10$.

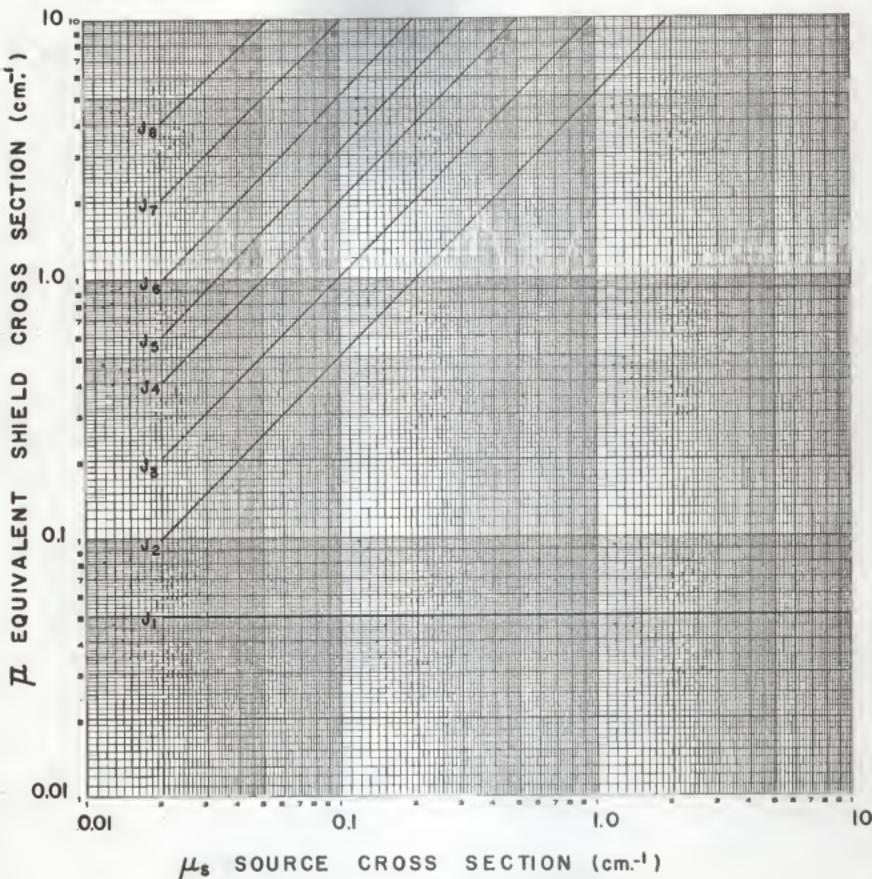


Fig. 9b. Source and shield cross sections correlation for b₁=10.

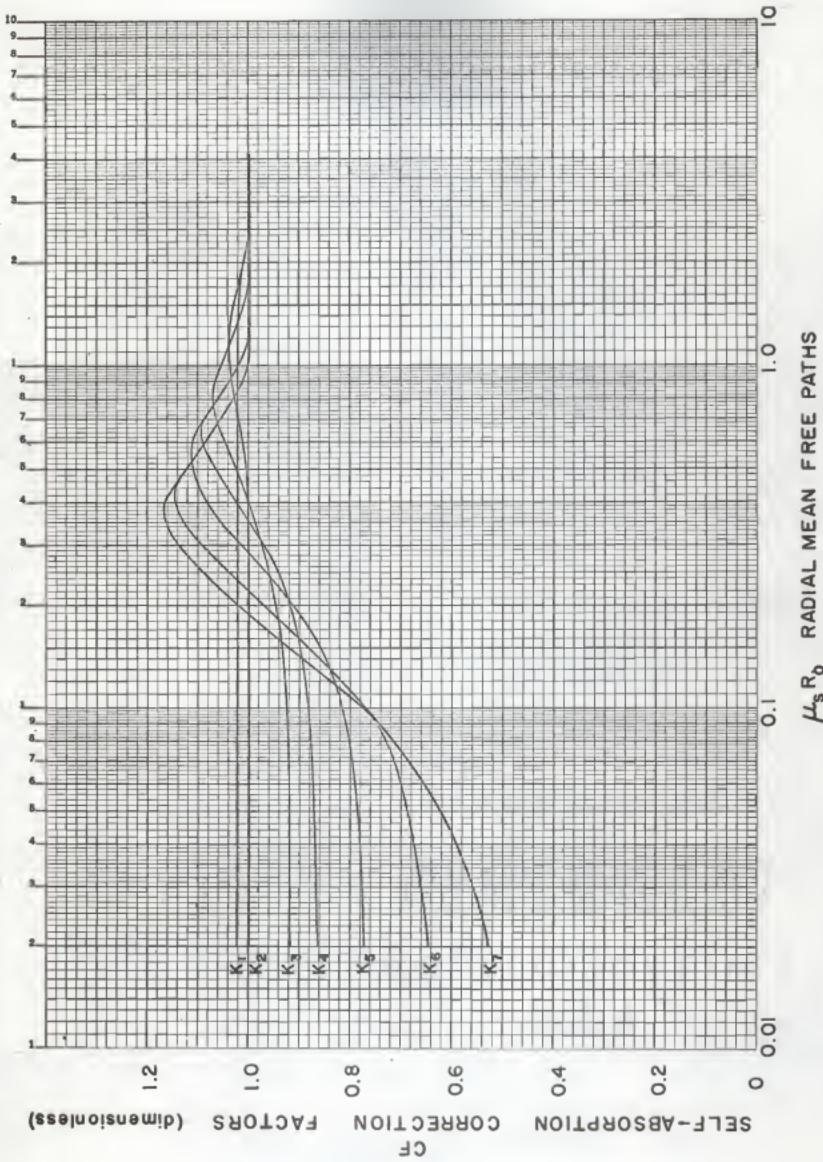


Fig. 10a. Correction factors for equivalent circular plane source self-absorption distance for $b_1 = 20$.

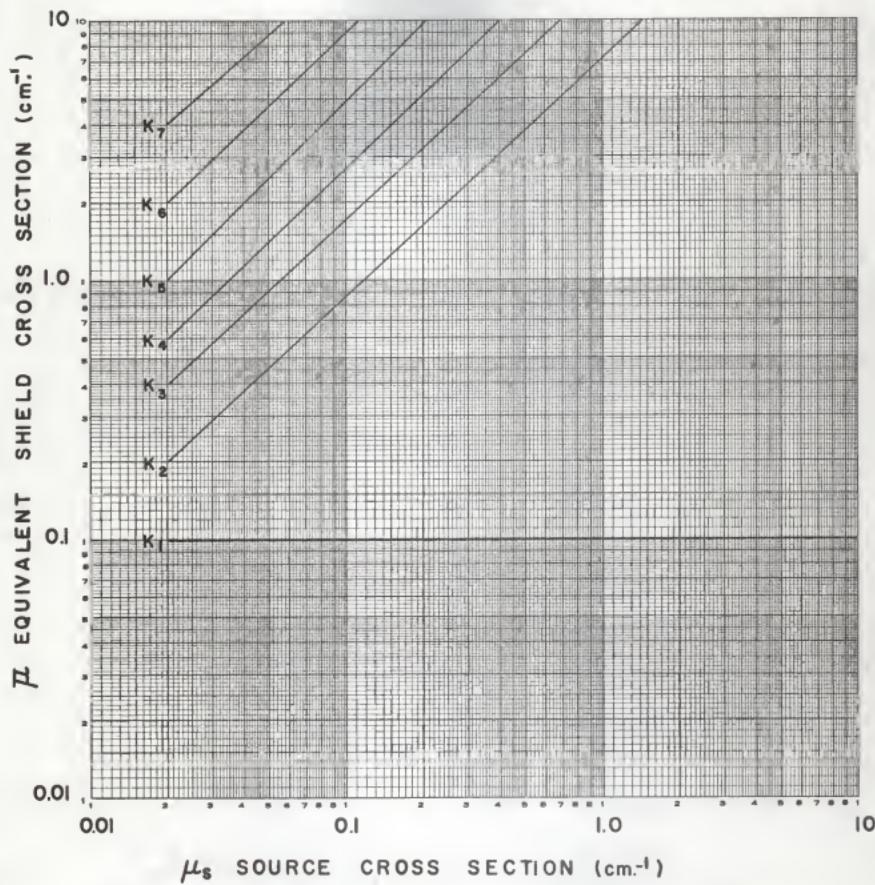


Fig. 10b. Source and shield cross sections correlation for $b_t = 20$.

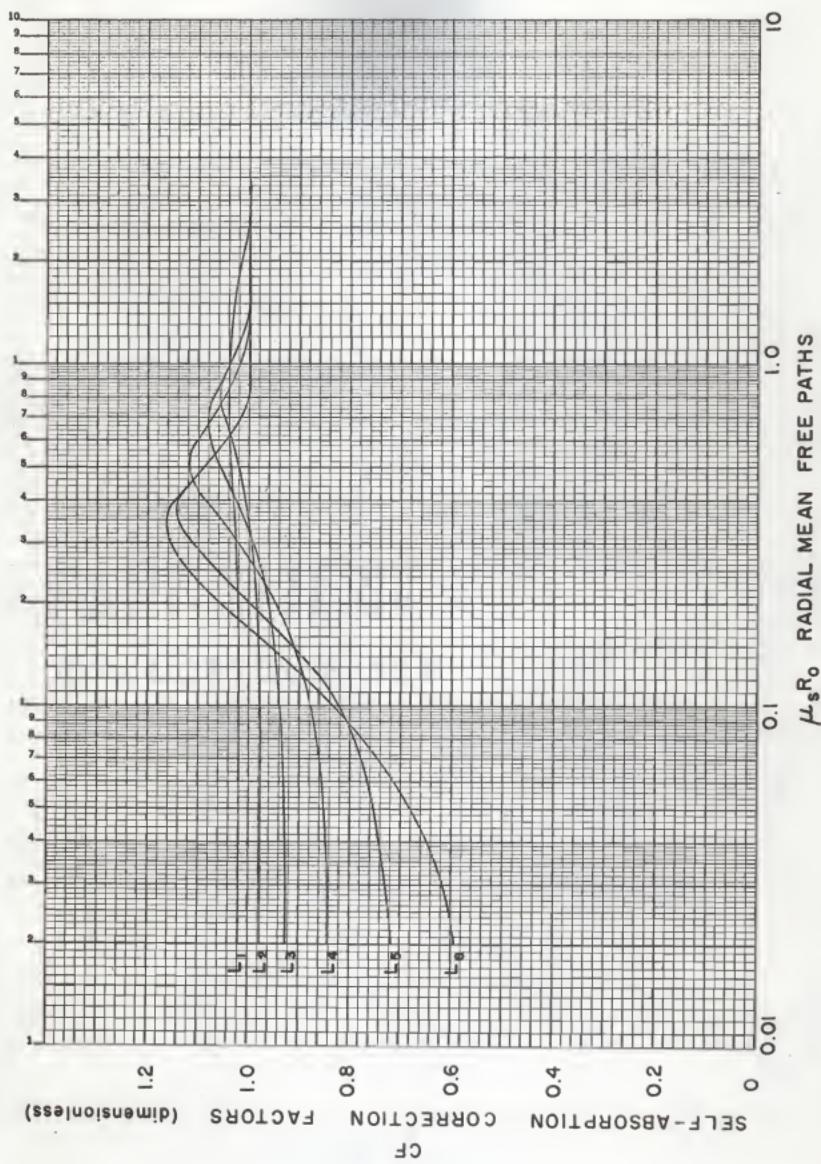


Fig. IIa. Correction factors for equivalent circular plane source self-absorption distance for $b_i = 30$.

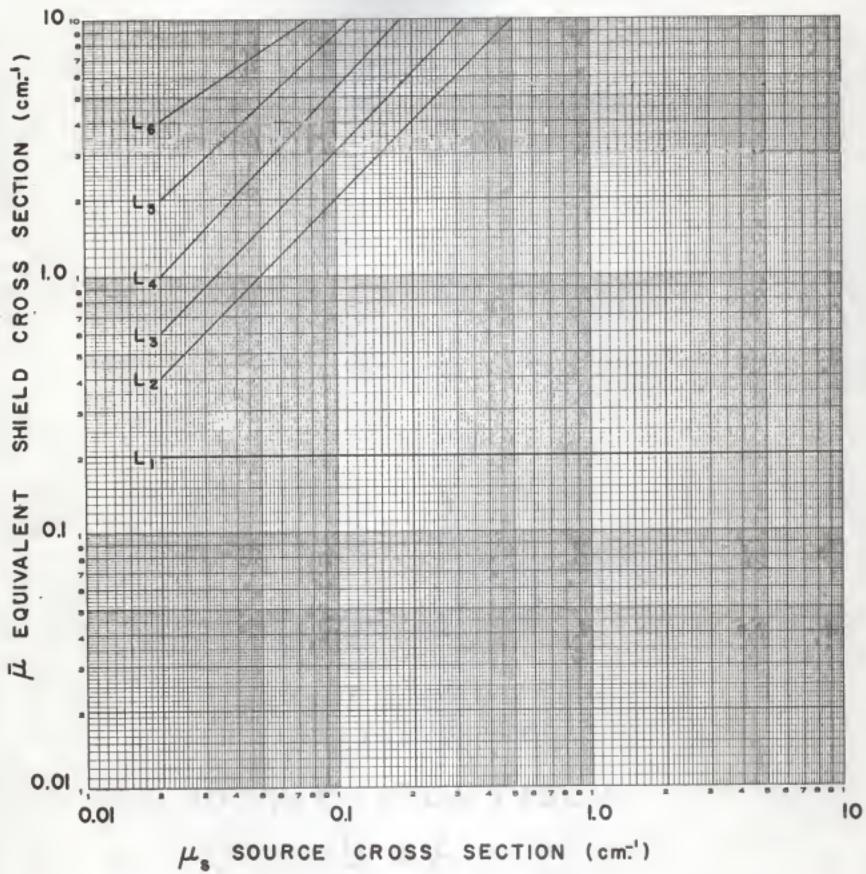


Fig. IIb. Source and shield cross sections correlation for b₁ = 30.

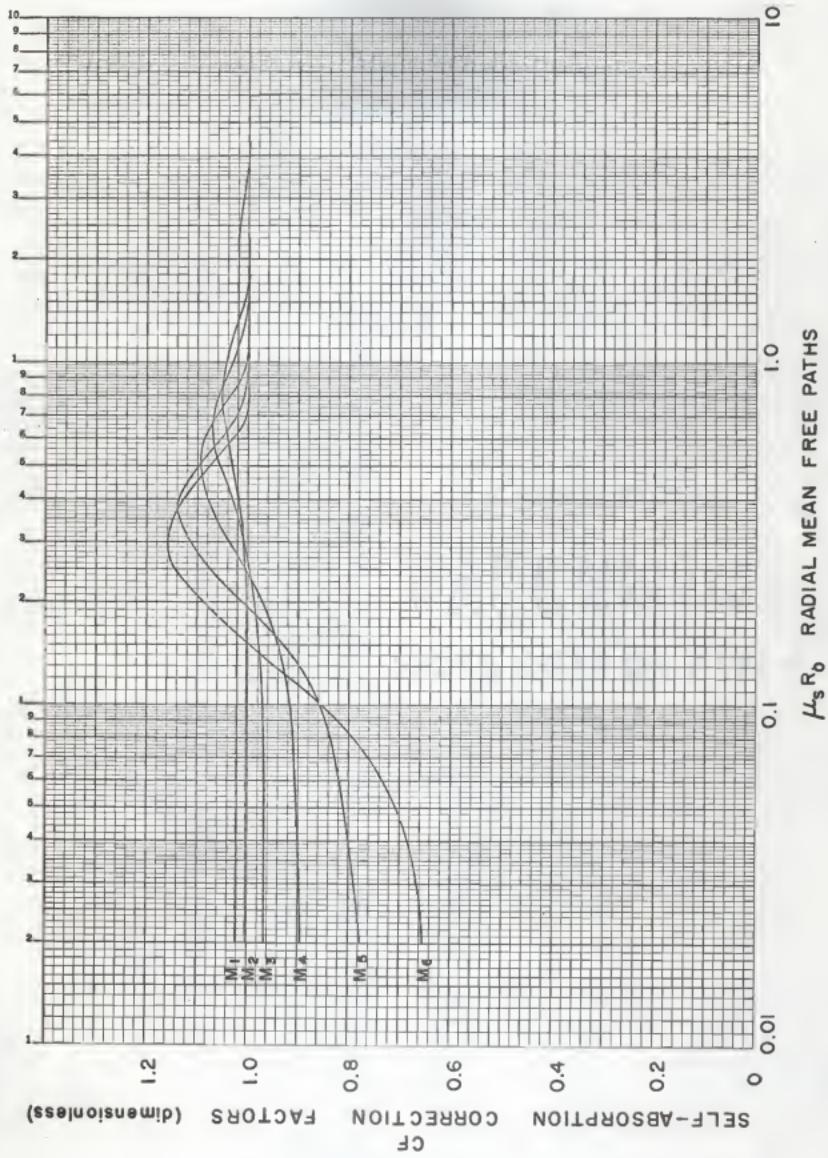


Fig. 12a. Correction factors for equivalent circular plane source self-absorption distance for $b_1 = 40$.

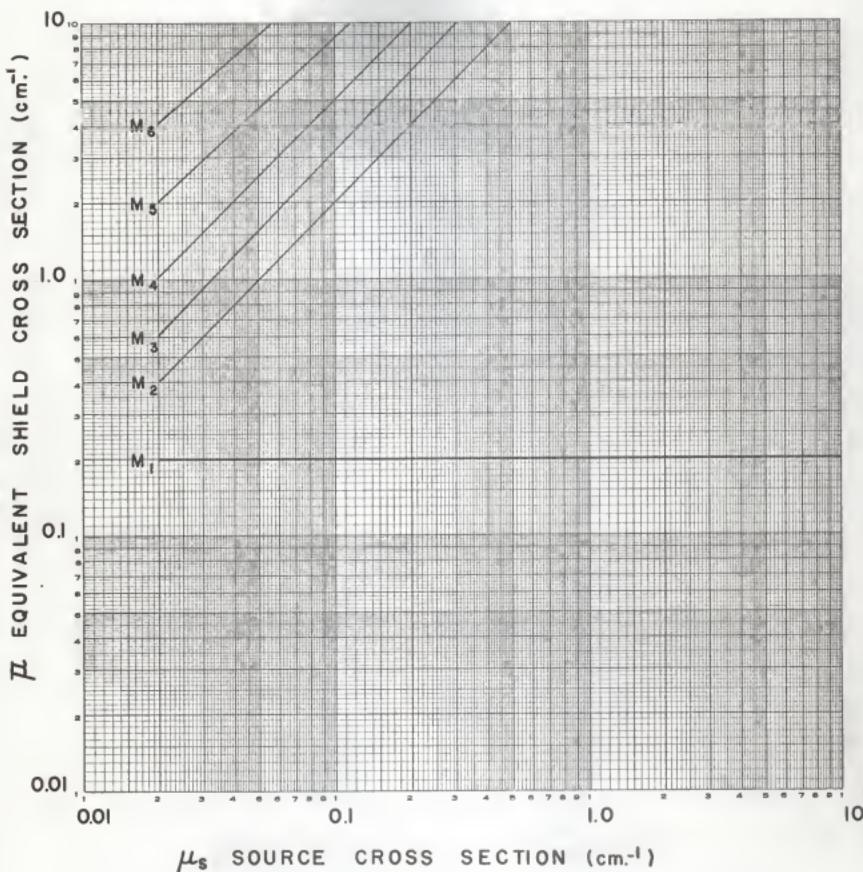


Fig. I2b. Source and shield cross sections correlation for $b_i = 40$.

CONCLUSIONS

The largest errors in determination of fluxes by use of the curves presented in this paper will occur in the region where correction factors are necessary, due to the interpolation required to determine correction factors. Even so, this is the region where the greatest benefit is derived from this work, for it is here that currently used conical approximations are most in error.

For any given cylinder height, h , the cylinder ceases to look like a disk and becomes a slender rod as R_o decreases. Similarly, for a fixed mean free path height, $\mu_s h$, as μ_s decreases the cylinder becomes longer and for a fixed radius the cylinder again appears rod-like. Certainly the artificial nature of the plane source approximation would be expected to have adverse effects in this region, if at all, and the correction factors necessary for small $\mu_s R_o$ were no surprise although their shape could not be anticipated.

An empirical solution similar to the one presented herein is presented in Rockwell (6) which enables one to determine the flux from the side of a cylinder by approximating the cylinder by a line source located within the cylinder. This empirical solution provides a method amenable to hand calculation to reproduce the results shown by Taylor and Obenshain (7). This original work is applicable only for cylinders having $h \gg R_o$ and $h > 1/\mu_s$. These limitations are not mentioned by Rockwell and therefore his empirical solution may be frequently applied in instances which exceed the restrictions placed on the original data.

It is recommended that work be done in the area of accurate flux solutions from the side of a cylinder for cases excluded by Taylor and Obenshain (7), and that an effort be made to combine the results. Further effort should be

made to present these results in some empirical form which lends itself to hand calculation. In particular, it is recommended that an effort be made to present the results in a manner analogous to that used in this paper.

ACKNOWLEDGMENT

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APPENDICES

APPENDIX A

Description and Explanation of the IBM 650 Computer Program
 Used to Calculate the Centerline Uncollided Flux
 from the End of a Finite Right-circular Cylinder

This program was written to calculate the centerline uncollided flux from the end of a finite cylinder, both accurately and by existing approximation methods, and to calculate the self-absorption distance of an equivalent source. This program was written in Soap II computer language using floating point operations. The logic diagram and the object program are given in this appendix.

The accurate solution is designated FLUX and the solution programmed was

$$\Phi(a) = \frac{S_V t_o}{2} \sum_{N=0}^{NMAX} \left\{ E_1(b_1 + (N + 0.45) \mu_s t_o) - E_1[(b_1 + (N + 0.45) \mu_s t_o) \sec \theta_N] \right\} \quad (A-1)$$

$$\text{where } \sec \theta_N = \sqrt{1 + \frac{R_o^2}{(a + (N + .45) t_o)^2}} \quad (A-2)$$

Equation A-1 is derived as Eq. 3, the stacked disk solution on page 5, with $\underline{\delta}$ set at 0.45.

The solution to the large cone approximation was designated FLUXB and the solution programmed was

$$\Phi(a) = \frac{S_V}{2\mu_s} \left\{ E_2(b_1) - \frac{E_2(b_1 \sec \theta_1)}{\sec \theta_1} - E_2(b_3) + \frac{E_2(b_3 \sec \theta_1)}{\sec \theta_1} \right\}. \quad (A-3)$$

Equation A-3 appears as Eq. 4 on page 6.

The solution to the equivalent volume cone approximation was designated FLUXM and the solution programmed was

$$\Phi(a) = \frac{S_V}{2\mu_s} \left\{ E_2(b_1) - \frac{E_2(b_1 \sec \theta_1)}{\sec \theta_1} - E_2(b_3') + \frac{E_2(b_3' \sec \theta_1)}{\sec \theta_1} \right\} \quad (A-4)$$

Equation A-4 appears on page 7 as Eq. 6.

The solution to the small cone approximation was designated FLUXS and the solution programmed was

$$\Phi(a) = \frac{S_V}{2\mu_s} \left\{ E_2(b_1) - \frac{E_2(b_1 \sec \theta_2)}{\sec \theta_2} - E_2(b_3) + \frac{E_2(b_3 \sec \theta_2)}{\sec \theta_2} \right\}. \quad (A-5)$$

Equation A-5 appears on page 6 as Eq. 5.

The solution to the equivalent circular plane source approximation was designated FLUXZ and the solutions programmed were

$$\Phi(a) = \frac{S_V h}{2} \left\{ E_1(b_2) - E_1(b_2 \sec \theta_z) \right\}, \text{ if } \mu_s h \leq 1.5 \quad (A-6)$$

and

$$\Phi(a) = \frac{S_V(1.5)}{2\mu_s} \left\{ E_1(b_2) - E_1(b_2 \sec \theta_z) \right\} \text{ if } \mu_s h > 1.5. \quad (A-7)$$

This was a trial and error solution where z was altered as necessary to cause FLUXZ to approximate FLUX within a specified accuracy. Equations A-6 and A-7 appear on page 7 as Eqs. 7 and 8 with K set at 1.5.

The program subroutine for $E_1(x)$ used different solutions for values of $x \leq 1$ and for $x \geq 1$.

For $x \leq 1$

$$E_1(x) = -0.57721566 - \ln x + x - \frac{x^2}{4} + \frac{x^3}{18} - \dots \quad (A-8)$$

For $x \geq 1$

$$E_1(x) = \frac{e^{-x}}{x} \left\{ \frac{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + x^4}{b_0 + b_1 x + b_2 x^2 + b_3 x^3 + x^4} \right\} \quad (A-9)$$

where

$$a_0 = 0.26777373$$

$$b_0 = 3.9584969$$

$$a_1 = 8.6347609$$

$$b_1 = 21.099653$$

$$a_2 = 18.059017$$

$$b_2 = 25.632956$$

$$a_3 = 8.5733287$$

$$b_3 = 9.5733223$$

Equation A-8 is an infinite series and the choice of number of terms used was set by requiring that the ratio of the absolute value of the last term to the complete series be less than some specified value. Equation A-9 is an approximate solution developed by Hastings (5), accurate to within the limitations of the computer.

Input and output data associated with this program are listed in Tables A-1 and A-2 respectively.

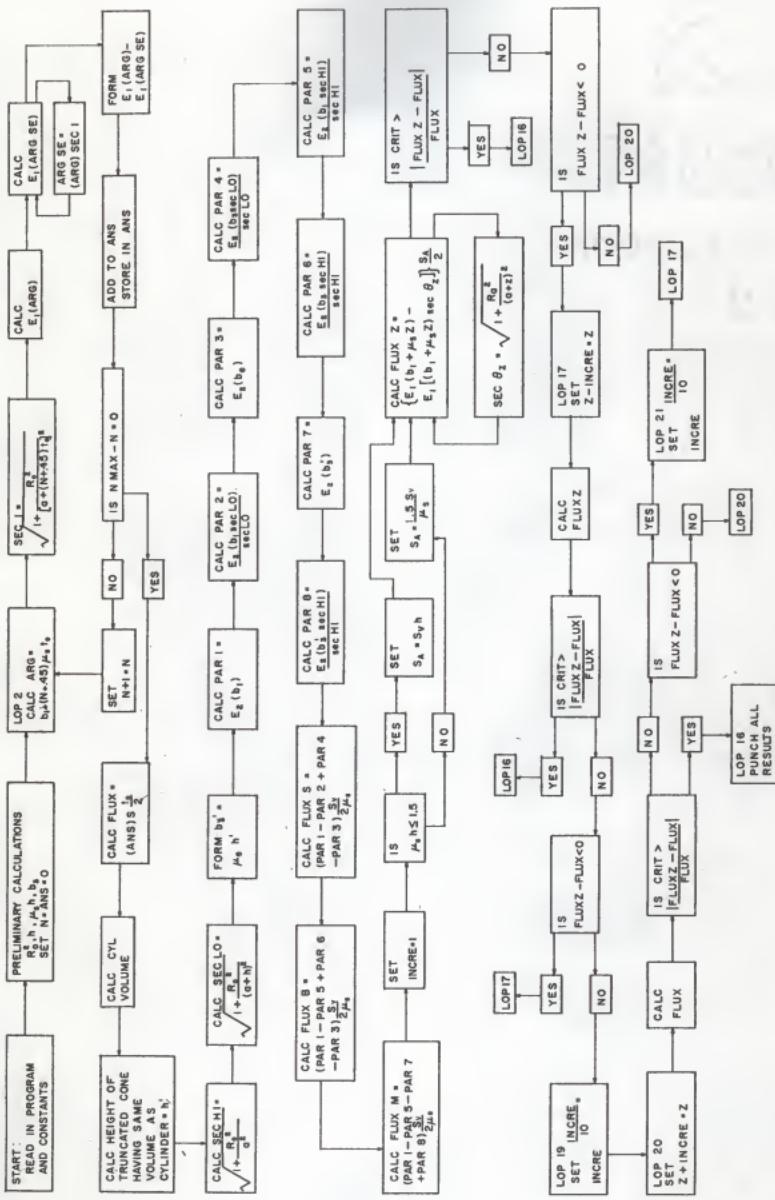
Table A-1. Input data required for use of the IBM 650 Computer program which calculates the centerline uncollided flux from the end of a finite right-circular cylinder.

Symbol	Explanation	Drum Storage Location
NMAX	One less than the number of disks in the cylinder	0100
TZRO	Disk thickness (cm)	0101
MUS	Cylinder cross section (cm^{-1})	0102
BONE	Shield mean free paths	0103
A	Shield thickness (cm)	0104
R	Cylinder radius (cm)	0105
S	Source strength ($\text{cm}^{-3} \text{ sec}^{-1}$)	0106
CHK	Check to calculate or pass by a given cylinder	0107
ZZRO	Initial self-absorption values (cm)	0108
CRIT	Required precision of equivalent source flux calculation	0500
CRIT1	Cylinder height (mean free paths) for constant equivalent source strength	0550
PT45	Ratio of δ to disk thickness	0350

Table A-2. Identification table for print outs for centerline uncollided flux calculation from the end of a finite cylinder.

Card	Word	Symbol and/or Explanation
1	1	FLUX
	2	FLUXB
	3	FLUXM
	4	FLUXS
	5	FLUXZ
	6	z Self-absorption distance (cm)
	7	$(FLUX - FLUXZ) / FLUX$
	8	$\mu_s z$ Self-absorption distance mean free paths
	1	FLUXB - FLUX
	2	$(FLUXB - FLUX) / FLUX$
2	3	FLUXM - FLUX
	4	$(FLUXM - FLUX) / FLUX$
	5	FLUX - FLUXS
	6	$(FLUX - FLUXS) / FLUX$
	1	HITE Cylinder height (cm)
	2	NMAX
	3	TZRO
	4	MUS
3	5	BONE
	6	A
	7	R
	8	S
	1	HITE + A
	2	$(HITE) (MUS)$ Cylinder height mean free paths
	3	$(TZRO) (MUS)$ Disk thickness mean free paths
	4	$(R) (MUS)$ Radial mean free paths
	5	MU Shield cross section (cm^{-1})
4	6	$A + R$
	7	A/R
	8	$A/HITE$

Approximate running time for the program is 5 minutes for NMAX equal to 100. The program is faster when $b_1 > 1$ due to the difference in the series calculations to be made.



AAA17 FOY AAA13 AAA1 EXIT INSTR 506 0953 34 0256 0853
 STO AAA18 507 0929 34 1056 1003
 HAU 5 508 1070 34 0256 0853
 FAO AAA2 509 1003 34 0342 0419
 FAO AAA19 510 0764 34 0342 0419
 LDU AAA27 511 0777 60 1130 0737
 STO AAA20 512 0733 24 0486 0339
 HAU 513 0916 34 0486 0339
 FAO AAA2 514 0445 60 0342 0397
 FMP AAA2 515 0397 39 0342 0442
 AAA23 AAA28 NUMERATOR 516 0392 34 0392 0492
 FOY AAA21 NTH TERM 516 0492 34 0392 0449
 STU AAA20 517 0492 34 0392 0449
 FAO AAA19 TOTAL 518 0551 31 0224 1627
 STU AAA24 519 1627 60 0696 0601
 FAO AAA23 520 0551 31 0224 1627
 FSG AAA25 CRITERIA 521 0274 33 1677 1053
 DMI AAA26 AAA26 522 1053 46 1556 0707
 DMI AAA26 523 1053 46 1556 0707
 HAU AAA20 524 1053 46 1556 0707
 FAO AAA20 525 1053 46 1556 0707
 STU AAA20 NEW N 526 1053 34 0486 0397
 FMP AAA21 NEW DENOM 527 1053 34 0486 0397
 FAO AAA23 528 1053 34 0486 0397
 HAU AAA23 529 1053 34 0486 0397
 FAO AAA23 530 1053 34 0486 0397
 FMP AAA23 531 0495 60 0546 0551
 STU AAA23 532 0495 60 0546 0551
 FAO AAA23 533 0651 39 0342 0592
 DMI AAA23 534 1053 46 1556 0707
 STU AAA23 535 1053 46 1556 0707
 FAO AAA23 ONE 536 0246 10 0000 0051
 FAO AAA23 ETYPE 537 0112 21 1750 1103
 FAO AAA23 ETYPE 537 0112 21 1750 1103
 AAA9, 538 0112 21 1750 1103
 STT 16526 539 0112 21 1750 1103
 FAO AAA23 FAINT TWO 540 0116 00 0000 0000
 AAA12 16526 FAINT TWO 541 0116 00 0000 0000
 FAO AAA23 ZERO 542 1130 30 0000 0051
 AAA16 00000 00000 CRITERIA 543 1130 30 0000 0051
 FAO AAA23 544 0051 34 0083 0939
 STT 16523 AAA22 SORT1 545 0083 24 0536 0939
 DMI SORT3 546 0112 21 1750 1103
 STU SORT3 547 0112 21 1750 1103
 FAO SORT3 SORT5 548 0112 21 1750 1103
 FOY SORT4 549 0112 21 1750 1103
 STT 16523 SORT5 550 0112 21 1750 1103
 FAO SORT5 551 0112 21 1750 1103
 STU SORT6 552 0112 21 1750 1103
 FAO SORT6 553 0112 21 1750 1103
 DMI SORT6 554 0112 21 1750 1103
 FAO SORT6 555 0112 21 1750 1103
 FSG SIZEB 556 0112 21 1750 1103
 DMI SORT7 557 0112 21 1750 1103
 FAO SORT7 558 0112 21 1750 1103
 STU SORT7 559 0112 21 1750 1103
 FAO SORT7 560 1159 60 1750 1109
 DMI SORT3 561 1159 60 1750 1109
 FAO SORT3 562 1159 60 1750 1109
 STU SORT2 563 0545 01 0806 0806
 FSG SORT2 564 0545 01 0806 0806
 RGT 00600 565 0545 01 0806 0806
 STT 16524 566 0545 01 0806 0806
 FAO SORT1 567 0545 01 0806 0806
 STU SORT1 568 0545 01 0806 0806
 FAO SORT1 569 0545 01 0806 0806
 RSG E22 570 0545 01 0806 0806
 LDU 571 0449 69 0582 1500
 FAO E23 E81 EXIT INSTR 572 0449 34 0989 0922
 STO E21 573 0036 34 0989 0922
 FAO E22 574 0692 69 0595 0298
 STT 16525 575 0692 69 0595 0298
 FAO E22 576 0692 69 0595 0298
 RSG E22 577 1551 24 1979 1551
 STT 16526 578 1551 24 1979 1551
 RSG E22 579 0833 24 1961 1284
 LDU 580 0833 24 1961 1284
 FAO E23 E81 581 0585 24 1983 0282
 STO E23 E81 582 0586 24 1984 0989
 FAO E23 E81 583 0586 24 1984 0989
 DMI E23 584 0586 24 1984 0989
 STO E23 585 0586 24 1984 0989
 FAO E23 586 0586 24 1984 0989
 RSG E23 587 0586 24 1984 0989
 STT 16527 588 0586 24 1984 0989
 FAO E23 589 0586 24 1984 0989
 RSG E23 590 0586 24 1984 0989
 DMI E23 591 0586 24 1984 0989
 STO E23 592 0586 24 1984 0989
 FAO E23 593 0586 24 1984 0989
 RSG E23 594 0586 24 1984 0989
 STT 16528 595 0586 24 1984 0989
 FAO E23 596 0586 24 1984 0989
 RSG E23 597 0586 24 1984 0989
 DMI E23 598 0586 24 1984 0989
 STO E23 599 0623 69 1556 1259
 FAO E23 600 0623 69 1556 1259
 RSG E23 601 0715 61 1356 1661
 STU EEE5 602 1261 39 0412 0462
 DMI EEE5 603 1261 39 0412 0462
 STU EEE5 LNX51 604 1261 39 0412 0462
 DMI EEE5 605 1261 39 0412 0462
 FAO EEE5 606 1261 39 0412 0462
 STU EEE5 607 1261 39 0412 0462
 DMI EEE5 608 1261 39 0412 0462
 FAO EEE5 609 1261 39 0412 0462
 STU EEE5 610 0601 34 0586 0936
 DMI EEE5 611 0601 34 0586 0936
 FAO EEE5 612 0601 34 0586 0936
 STU EEE5 613 0601 34 0586 0936
 DMI EEE5 614 0601 34 0586 0936
 FAO EEE5 615 0425 34 030 0356
 STU EEE5 616 0425 34 030 0356
 DMI EEE5 617 0425 34 030 0356
 FAO EEE5 618 0413 32 0050 1927
 STU EEE5 619 0413 32 0050 1927
 DMI EEE5 620 1211 39 0003 0665
 FAO EEE5 621 0665 39 0220 0623
 STU EEE5 622 0665 39 0220 0623
 DMI EEE5 623 0665 39 0220 0623
 FAO EEE5 624 0991 10 0000 0044
 STU EEE5 625 0991 10 0000 0044
 DMI EEE5 626 0991 10 0000 0044
 FAO EEE5 627 0991 10 0000 0044
 STU EEE5 628 1211 39 0003 0665
 DMI EEE5 629 1361 32 0014 1041
 FAO EEE5 630 1361 32 0014 1041
 FAO EEE5 631 1406 32 0209 1406
 FAO EEE5 632 0635 39 1356 1456
 FAO EEE5 633 1486 39 1356 1456
 FAO EEE5 634 0685 39 1356 1506
 FAO EEE5 635 1506 38 1409 0735

STU EEE3	DENOMINATOR	636	0735	31	0390	0693
RTE EEE2		637	0732	32	0384	0691
FAD CCC3		638	1411	32	0244	1091
FMP EEE2		639	1093	39	1356	1556
FCD CEE2		640	0756	35	0250	0695
FMP EEE2		641	0785	39	1356	1606
FAO CCC1		642	1606	33	1350	0835
FCD CEE2		643	1352	32	0250	0695
FAD CCC0		644	1656	32	1559	0885
FOT EEE3	RATIO	645	0885	34	0390	0440
FOT EEE2		646	0740	32	0244	0695
BTU EEE4	ALLBUTETOX	647	1756	21	0280	0443
RSU EEE2		648	0483	35	0250	0695
LDR EEE1		649	1659	32	1354	1500
FMP EEE4	ETOX	650	0314	39	0260	0348
RSU EEE3	EONEOFEEE2	651	1159	21	0280	0440
CCC0		652	1509	36	1747	0931
CCCC1		653	1459	38	0590	1753
CCCC2		654	1454	35	0590	1751
10 0590		655	1409	39	5849	6931
10 0590		656	1359	21	0996	5352
0001		657	0289	32	0250	0695
0002		658	0214	95	7332	8351
0003		659	0214	95	7332	8351

APPENDIX B

Description of IBM 650 Computer Program
 Used to Calculate the Error in Centerline Uncollided Flux
 Determination from the End of a Finite Right-circular Cylinder
 by the Use of an Equivalent Circular Plane Source

This program was written to calculate the error of a flux determination by the empirical method developed in this thesis. The program was written in Soap II computer language using floating point operations. The logic diagram and the object program are given in this appendix.

The solution to the equivalent circular plane source approximation was designated ABC9 and the solutions programmed were

$$\Psi(a) = \frac{S_V}{2} \left\{ E_1(b_2) - E_1(b_2 \sec \theta_z) \right\}, \quad \text{if } \mu_{sh} \leq 1.5; \quad (\text{B-1})$$

$$\Psi(a) = \frac{S_V(1.5)}{2\mu_s} \left\{ E_1(b_2) - E_1(b_2 \sec \theta_z) \right\}, \quad \text{if } \mu_{sh} > 1.5. \quad (\text{B-2})$$

Equations (B-1) and (B-2) appear as Eqs. A-6 and A-7 in Appendix A. The program subroutine for $E_1(x)$ is identical to that described in Appendix A.

Input data and output data associated with this program are listed in Tables B-1 and B-2 respectively.

Table B-1. Input data required for use of the IBM 650 Computer program which calculates the error in the empirical solution of the centerline uncollided flux from the end of a finite right-circular cylinder.

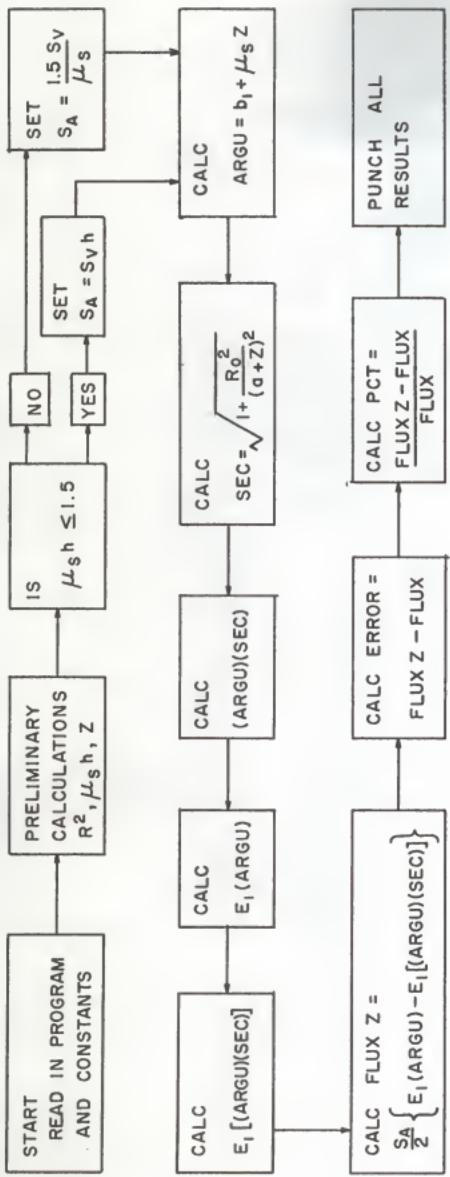
Symbol	Explanation	Drum Storage Location
HITE	Cylinder height (cm)	0000
BONE	Shield mean free paths	0001
FLUX	Actual flux ($\text{cm}^{-2} \text{ sec}^{-1}$)	0003
S	Source strength ($\text{cm}^{-3} \text{ sec}^{-1}$)	0004

Table B-1. (concl)

Symbol	Explanation	Drum Storage Location
R	Cylinder radius (cm)	0005
A	Shield thickness (cm)	0006
MUS	Source cross section (cm^{-1})	0008
MUSZ	Empirically determined self-absorption distance (mean free paths)	0009

Table B-2. Identification table for print outs for error calculation.

Card	Word	Symbol and/or Explanation
1	1	FLUX accurate flux solution
	2	ABC9 approximate flux solution
	3	ERROR ABC9 - FLUX
	4	PCT $(\text{ABC9} - \text{FLUX})/\text{FLUX}$
	5	z self-absorption distance
	6	A
	7	R
	8	BONE
2	1	HITE
	2	MUS
	3	μ_{sh} Cylinder height (mean free paths)
	4	μ Shield cross section
	5	MUSZ
	6	MUSZ + BONE
	7	A + R
	8	A/R



LOGIC DIAGRAM APPENDIX B

**OBJECT PROGRAM FOR CALCULATION OF ERROR IN EQUIV-
ALENT CIRCULAR PLANE SOURCE FLUX DETERMINATION**

```

HLR 1951 1960          1 0000 00 0000 0000 -0000000000000000
HLH 19577 19840          2 0000 00 0000 0000 -0000000000000000
HLH 19770 19745          3 0000 00 0000 0000 -0000000000000000
HYN HITE 0000            4 0000 00 0000 0000 -0000000000000000
HYN DOME 0001            5 0000 00 0000 0000 -0000000000000000
HYN ZHOZ 0002            6 0000 00 0000 0000 -0000000000000000
HYN S 0004              7 0000 00 0000 0000 -0000000000000000
HYN R 0005              8 0000 00 0000 0000 -0000000000000000
HYN A 0006              9 0000 00 0000 0000 -0000000000000000
HYN MUB 0008              10 0000 00 0000 0000 -0000000000000000
HYN WHT 0009              11 0000 00 0000 0000 -0000000000000000
HYN START 10000            12 0000 00 0000 0000 -0000000000000000
FP100 10 0000 0053      13 0100 10 0000 0000 -0000000000000000
ONE 0000 0001            14 0000 00 0000 0000 -0000000000000000
TWD 0000 0051            15 0000 00 0000 0000 -0000000000000000
CRITI 15 0000 0051      16 0000 00 0000 0000 -0000000000000000
START 0000000000000000
FBR 1700                  17 0000 00 0000 0000 -0000000000000000
NZE LOP1                  18 0000 00 0000 0000 -0000000000000000
LNU LOP1                  19 0027 45 0030 0031
PCH 1977                  20 0000 00 0000 0000 -0000000000000000
PCH 1977                  21 0033 71 1977 0077
PCH 1977                  22 0077 71 1977 0147
PCH 1977                  23 0177 46 0090 0081
PCH 1977                  24 0100 39 0090 0099
PCH 1977                  25 0059 39 0003 0013
PCH 1977                  26 0013 39 0000 0011
PCH 1977                  27 0021 60 0009 0063
PCH 1977                  28 0063 34 0008 0058
PCH 1977                  29 0100 34 0012 0052
PCH 1977                  30 0015 60 0000 0105
PCH 1977                  31 0150 34 0008 0048
PCH 1977                  32 0100 33 0200 0177
PCH 1977                  33 0177 46 0090 0081
PCH 1977                  34 0059 39 0090 0099
PCH 1977                  35 0109 39 0000 0250
PCH 1977                  36 0250 34 0150 0300
PCH 1977                  37 0059 39 0000 0250
PCH 1977                  38 0001 30 0200 0155
PCH 1977                  39 0155 34 0008 0158
PCH 1977                  40 0180 34 0012 0164
PCH 1977                  41 0104 34 0150 0350
PCH 1977                  42 0350 21 0054 0007
PCH 1977                  43 0077 21 1977 0147
PCH 1977                  44 0017 69 0020 0023
PCH 1977                  45 0020 69 0009 0051
PCH 1977                  46 0056 69 0000 0051
PCH 1977                  47 0130 69 0083 0086
PCH 1977                  48 0001 69 0000 0001
PCH 1977                  49 0131 29 0034 0037
PCH 1977                  50 0037 24 1979 0038
PCH 1977                  51 0022 24 1979 0038
PCH 1977                  52 0038 24 1980 0133
PCH 1977                  53 0133 69 0012 0065
PCH 1977                  54 0152 69 0009 0046
PCH 1977                  55 0094 69 0006 0159
PCH 1977                  56 0059 29 0002 0055
PCH 1977                  57 0250 29 0000 0208
PCH 1977                  58 0208 24 1983 0136
PCH 1977                  59 0164 24 1984 0144
PCH 1977                  60 0154 24 1984 0087
PCH 1977                  61 0087 24 1700 0033
PCH 1977                  62 0037 24 1984 0075
PCH 1977                  63 0287 20 0000 0205
PCH 1977                  64 0205 21 1977 0180
PCH 1977                  65 0180 21 1977 0180
PCH 1977                  66 0258 24 1978 0181
PCH 1977                  67 0381 21 1979 0229
PCH 1977                  68 0020 20 0001 0255
PCH 1977                  69 0255 34 0006 0106
PCH 1977                  70 0166 29 0001 0013
PCH 1977                  71 0183 29 0009 0062
PCH 1977                  72 0062 24 1981 0134
PCH 1977                  73 0142 29 0000 0040
PCH 1977                  74 0040 24 1982 0135
PCH 1977                  75 0135 60 0006 0011
PCH 1977                  76 0131 60 0006 0011
PCH 1977                  77 0231 21 1983 0186
PCH 1977                  78 0186 60 0005 0209
PCH 1977                  79 0210 21 1984 0180
PCH 1977                  80 0308 21 1984 0187
PCH 1977                  81 0147 72 1977 0277
PCH 1977                  82 0177 61 1977 0264
PCH 1977                  83 0230 71 1977 0000
PCH 1977                  84 0230 24 1983 0136
PCH 1977                  85 0029 29 0009 0158
PCH 1977                  86 0358 32 0001 0327
PCH 1977                  87 0079 24 1984 0090
PCH 1977                  88 0090 69 0043 0046
PCH 1977                  89 0043 21 0048 0051
PCH 1977                  90 0141 32 0012 0051
PCH 1977                  91 0061 32 0012 0039
PCH 1977                  92 0039 39 0003 0093
PCH 1977                  93 0137 34 0009 0111
PCH 1977                  94 0010 60 0018 0073
PCH 1977                  95 0073 34 0098 0148
PCH 1977                  96 0180 29 0000 0077
PCH 1977                  97 0377 69 0280 0233
PCH 1977                  98 0210 24 1984 0090
PCH 1977                  99 0237 31 0042 0045
PCH 1977                  100 0045 33 0198 0025
PCH 1977                  101 0025 33 0198 0025
PCH 1977                  102 0028 60 0042 0047
PCH 1977                  103 0047 69 0000 0046
PCH 1977                  104 0013 21 1977 0277
PCH 1977                  105 0079 69 0050 0103
PCH 1977                  106 0033 26 0024 0057
PCH 1977                  107 0033 26 0024 0057
PCH 1977                  108 0153 33 0204 0231
PCH 1977                  109 0154 33 0204 0231
PCH 1977                  110 0054 21 0083 0236
PCH 1977                  111 0236 33 0003 0139
PCH 1977                  112 0141 33 0204 0231
PCH 1977                  113 0287 34 0003 0203
PCH 1977                  114 0203 21 0035 0226
CALC 0000 0001            115 0100 10 0000 0000
ONI 0000 0051            ONE

```


UNCOLLIDED FLUX FROM FINITE
RIGHT-CIRCULAR CYLINDER
VIEWED ENDWISE

by

LARRY A. RASH

B. S., Kansas State University, 1957

AN ABSTRACT OF A THESIS

submitted in partial fulfillment of the

requirement for the degree

MASTER OF SCIENCE

Department of Nuclear Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1962

An IBM 650 Computer code was written to solve for the flux from the end of a finite circular cylinder. This code was then used to solve for the flux from cylinders of varying size and absorption coefficient as well as for varying shield thickness and absorption coefficients.

From the information obtained, a method was developed to determine fluxes from cylinders by considering them to be replaced by a circular plane source. This circular plane source is located within the confines of the cylinder and has the same radius as the cylinder. This method makes it possible to determine the uncollided flux from the end of a cylinder more accurately than has previously been possible with only a desk calculator.

Curves and supporting information as necessary are presented to enable users to determine the source strength of the circular plane source and its location within the cylinder.