INVESTIGATION OF AN AXISYMMETRICAL CHILLED VERTICAL JET PROJECTED INTO A STRATIFIED ENVIRONMENT

by

THOMAS F. BAILEY

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Approved by:

H. B. Ball
Major Professor

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NOMENCLATURE

A - area of the outlet nozzle, ft^2

a - velocity shape factor

b - radial distance from the centerline to the boundaries of the jet, ft

C - integration constant

c - temperature shape factor

C_2 - integration constant

C_p - constant pressure specific heat, $\frac{BTU}{lb\,^oF}$

C_t - hyperbolic decay constant, ^oF ft

C_{Tamb} - ambient temperature constant, ^oF

D_o - nozzle outlet diameter, ft

g - acceleration of gravity, ft/min^2

K_p - distance from apparent point source where velocity is constantly accelerating

M.F.E. - Mass Flow Entrained, $\frac{lbm}{min}$

P_{bar} - ambient barometric pressure, inches of mercury

p - atmospheric pressure, lb/sq ft

Q - axial flow rate, cubic ft/min

Q_o - initial axial flow rate, cubic ft/min

R - gas constant for air, ft - lbf per lbm per ^oR

r - radial distance from centerline, ft

T_a - ambient air temperature, ^oF

T_{amb} - ambient temperature, ^{o}F

T_c - centerline temperature, ^{o}F

T_{db} - ambient dry bulb temperature, ^{o}F

T_g - stratification factor, $^{o}F/ft$

T_{jet} - temperature of the jet, ^{o}F

T_o - initial jet temperature, ^{o}F

$T_{r=b}$ - temperature at the radial distance b, ^{o}F

T_{wb} - ambient wet bulb temperature, ^{o}F

V - time average velocity, ft/min

V_c - centerline time average velocity, ft/min

V_o - initial outlet velocity, ft/min

x - axial distance from the apparent point source, ft

x_n - axial distance from the nozzle outlet, ft

β - coefficient of volumetric expansion, $\frac{1}{^{o}F}$

ΔT - difference between the jet temperature and ambient temperature, ^{o}F

ΔT_c - difference between the centerline and ambient temperature, ^{o}F

ΔTo - initial temperature difference, ^{o}F

ϵ_m - eddy diffusivity, lb/min-ft

ρ_{air} - density of ambient at a specific axial location, ft^3/lbm

ρ_{jet} - density of the chilled jet at a specific axial location, ft^3/lbm

τ - time average shear, lbf/sq ft

INTRODUCTION

In the early 1950's, as the standard of living in America rose, so did what might be called the standard of comfort. People became more concerned with their personal comfort. As the concern for personal comfort grew, additional attention gave rise to research devoted to the development of new techniques, and implementation of existing methods, for ventilation and air conditioning.

During the 1950's the use of turbulent air jets for air conditioning and ventilation was investigated. H. B. Nottage (1) studied the velocity profiles, entrainment, shear, and boundary edges of isothermal jets, and their relationship with axial distance. Nottage's work is probably the most notable investigation concerning ventilation jets. His study is the basis for many following articles on air distribution jets (2), (4), (5), including the ASHRAE Fundamentals (6) chapter on air space distribution. While Nottage continued his work and began studying chilled horizontal jets, Linn Helander and his associates (3) at Kansas State College were compiling experimental data dealing with vertical heated jets. Helander, using the resultant data, was able to formulate empirical equations dealing with the given throw of a vertical heated jet at prescribed outlet velocities, diameters and temperatures. His data has long been accepted as the norm for heated jets and therefore is used as a basis for checking theory concerning heated jets. Following Helander's experimental investigation, A. Koestel (4) of Case Institute, attempted to predict the velocities and temperatures of vertical non-isothermal jets using a theoretical analysis. Comparing Koestel's theory with experimental data shows a general agreement between theory and experiment. The work of G. L. Tuve (5) should be mentioned. Tuve may be termed as the pioneer in the investigation of ventilation jets. His work, which dates back to the middle 1930's, concerns the fundamentals of jet theory, such as entrainment and velocity profiles. His work is still used as a basis for various air jet phenomena. Tuve was also the advisor and tutor of such engineers as Nottage and Koestel at Case Institute. These men, together with their associates have formulated the major basis of the existing knowledge concerning ventilation jets.

Today, instead of the basis of concern being for comfort, the major concentration of effort is on energy conservation. However, people are not generally willing to give up their comfort for energy conservation. Therefore, the need for new methods and improvement of existing techniques utilized in air conditioning has arisen. The increased activity in air conditioning studies will most probably include air jet studies for such purposes as spot cooling. The reasoning behind the use of spot cooling is the energy savings in conditioning small work spaces as compared with cooling large areas, such as an entire factory. The major purpose of this study is to predict centerline velocities of chilled vertical axisymetrical jets for possible use in spot cooling, utilizing theory and experimental background. In doing so, any deficiencies of theoretical or experimental data may be found to allow further, more detailed study at a later time.

REVIEW OF LITERATURE

The first comprehensive study of axisymmetrical jets by Corrsin (12) found that temperature diffuses much more rapidly than the velocity in a turbulent heated jet. In comparing his results to theory, Corrsin (12) found that none of the existing theories satisfactorily predicted the relationship between the spread of velocity and the spread of temperature, but that Prandtl's constant effective shear coefficient gave the best agreement for velocity profiles. In later experiments, Corrsin and Uberoi (13) found that the rate of spread of a turbulent jet increases with a decrease in jet density, while the turbulent Prandtl number, which approximately equals the laminar Prandtl number, remains constant in the fully developed region.

Following these preliminary investigations the idea of using jets for ventilation purposes became popular. Rydberg and Norback (14) theoretically analyzing an axisymmetrical jet, suggested several equations to calculate the centerline velocity and temperature distribution, utilizing simplified momentum equations.

Koestel, Herman and Tuve (15) made a comparative study of various types of outlets used in ventilation jets in which a number of semiempirical equations were proposed for use in isothermal jets. Experimental results indicate that the formulae afford a reasonable amount of accuracy for the jet from the axial distance $8\sqrt{A}$ to $50\sqrt{A}$. They also mention that no simple method seemed to be available to predict the main stream velocity in the near region of the jet.

Nottage, Slaby and Gojsza (2) in a series of papers written directly from Nottage's Ph.D. dissertation (1) investigate the fundamentals of isothermal ventilation jets. The authors discuss the various characteristics of isothermal jets such as boundary contours, axial velocities, outlet characteristics, cross jet profiles and jet flow rates. Their investigations serve as a basis for most of the established isothermal ventilation jet theory used today.

Helander, Yen and Crank (3) showed that the maximum throw of a heated axisymmetrical jet was a function of the buoyancy number, B_o.

$$B_o = \frac{V_o^2/gD_o}{T_o/T_a - 1}$$

They proposed an empirical equation to predict the maximum throw, utilizing the buoyancy number, which could be used for various diameter nozzles.

Koestel (4) analyzed the velocities and temperatures of a non-isothermal jet in a constant temperature environment by the use of integral momentum and energy equations. Assuming similarity of velocity and temperature profiles, Koestel (4) developed an equation used to describe the buoyancy forces in the jet. Simplifying this equation through the use of the assumptions of negligible differences in density and coefficient of expansion throughout the jet, Koestel (4) performed the integration necessary to derive the following equation describing the centerline velocity of the jet.

$$\left(\frac{V_c}{V_o}\right)^3 = \left(\frac{K_p}{x/D_o}\right)^3 \pm \frac{3}{8} C_2 \left(\frac{a}{c} + 1\right) \left(\frac{D_o \Delta T_o \beta g}{V_o^2}\right) \frac{1}{x/D_o}$$

Through a similar analysis of the convective energy within the jet, utilizing an energy balance, Koestel (4) derived an equation describing the centerline temperature difference in the jet. To complete his analysis, Koestel (4) rearranged the derived equations to predict the maximum throw of a heated jet. Comparing his results to Helander's (3), good agreement was found in predicting the maximum throw.

Kleinstein (16), modifying the techniques of Carrier and Lewis using the Von Mises Transformation, suitably linearized the momentum equations. From his analysis, Kleinstein (16) concluded that as a result of the linearization, the radial distribution of any fluid property at an axial station was found to be a function of the initial conditions and the value of the property on the axis.

Sforza, Steiger and Trentacoste (17), utilizing momentum equations describing three dimensional flow, investigated different types of jets, including round and planar jets. They concluded that each jet approached axisymmetrical decay as confirmed by velocity profile analysis.

Wang and du Plessis (18) developed a numerically explict method to analyze a jet, either in the near or far region. Using this method the velocity profiles were described by a cosine series which satisfies the vanishing velocity gradient at the centerline of the jet.

Mollendorf and Gebhart (19) showed that thermal buoyancy effects do not produce extreme changes, but that they may have an important influence on the stability of the jet.

Susarla, De and Dutta (20) investigated the temperature, velocity and entrainment characteristics of preheated and isothermal jets. Using experimental data, they formulated various empirical equations utilizing the initial conditions of the jet to calculate the entrainment, temperature and velocity of the jet.

Since previous investigation failed to deal with possible stratification of surrounding air and most assumed density to be constant, this work was undertaken.

ZONES OF A VENTILATION JET

A ventilation jet may be broken into four different zones. Each zone has distinguishing characteristics (6), as shown in Figure 1. Zone 1 is generally called the near zone. It is a short zone extending only to a distance of about four diameters from the outlet face. Zone 2 is termed a transition zone. It too, is a short zone extending only to about eight diameters from the outlet face. In this zone the effects of entrainment begin to be noticed. Zone 3 is called the long zone and may extend as much as one hundred diameters from the outlet. This zone is also known as the zone of fully established turbulent flow in which the mechanism of entrainment is completely operative. Zone 4 is the terminal zone where the jet dies quickly and reaches very low velocities which are generally accepted as still air. For the purposes of this study, the work was done in the third zone.

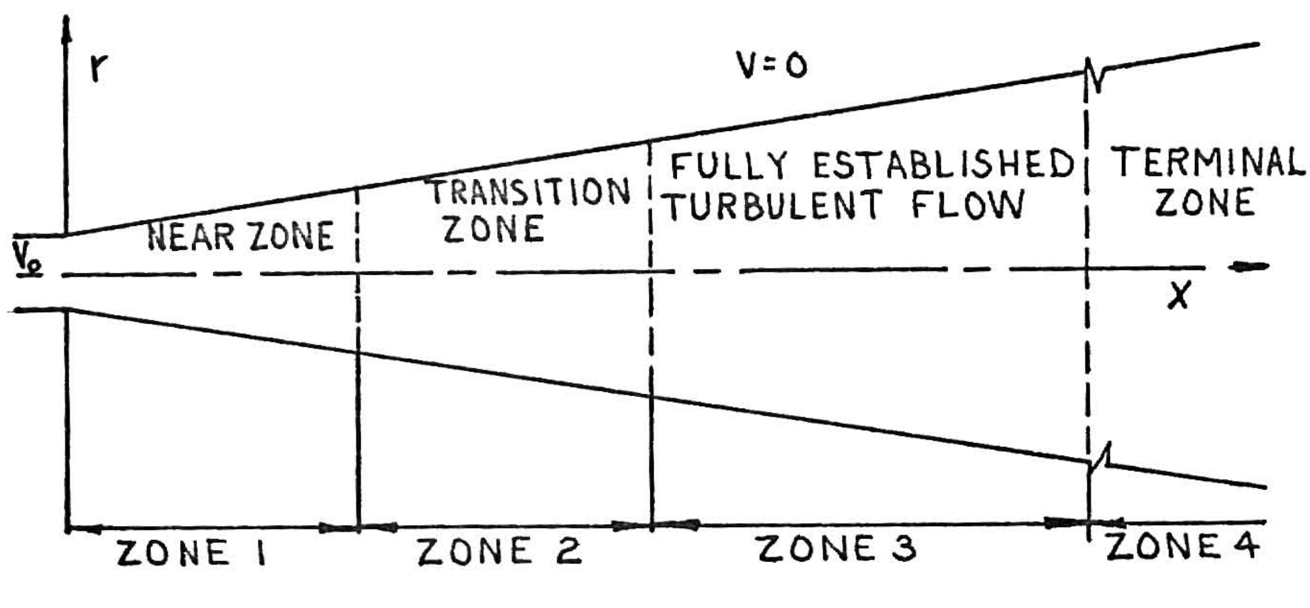


FIGURE 1

DIFFERENTIAL EQUATIONS DESCRIBING JET FLOW

To examine the flow of a chilled jet, the interactions at a typical cross section were studied, as is shown in Figure 2. To simplify and aid in the understanding of the problem, the approximate von Karman integral differential equation technique, similar to the method used by Holman (7) in his description of convection principles, was used.

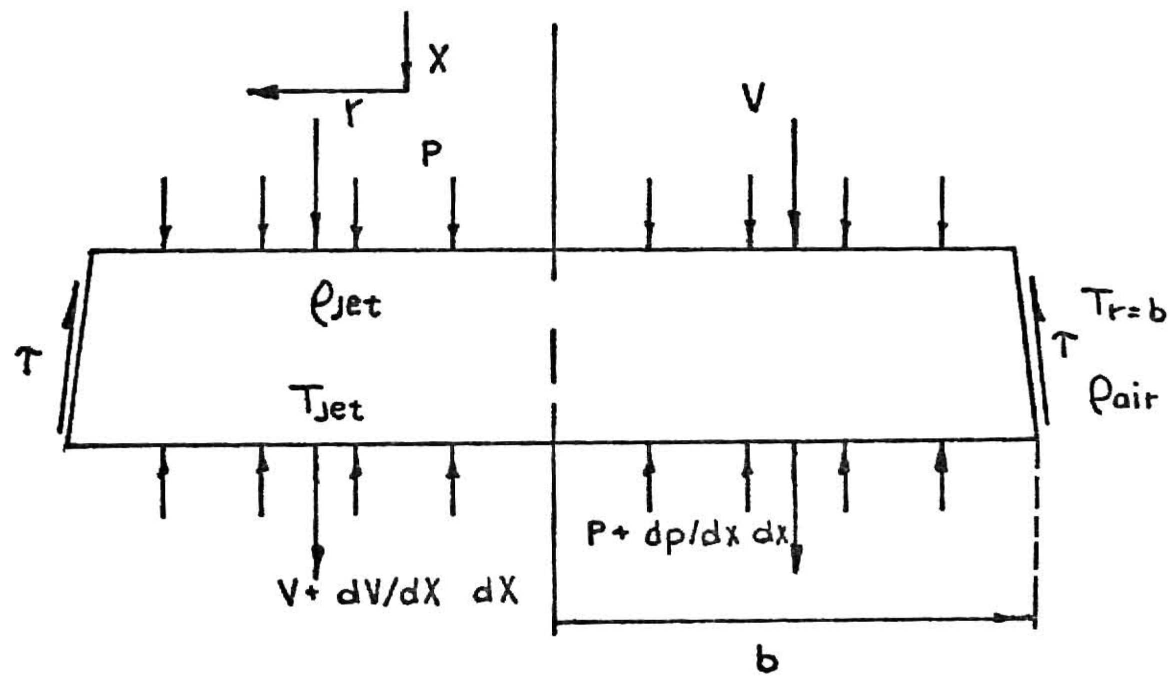


FIGURE 2

MOMENTUM EQUATION

The influx of momentum into the cross section of Figure 2, assuming the surroundings are at rest, is equal to:

$$\int_o^b \rho_{jet} V^2 2\pi r dr \qquad (1)$$

Utilizing a steady-state momentum balance, the momentum influx plus the net forces acting on the system equal the momentum outflux. The momentum outflux is equal to:

$$\int_o^b \rho_{jet} V^2 2\pi r dr + \frac{d}{dx}\left[\int_o^b \rho_{jet} V^2 2\pi r dr\right] dx \quad (2)$$

Substracting the momentum influx from the outflux, it is found that the net forces acting on the system are equal to:

$$\frac{d}{dx}\left[\int_o^b \rho_{jet} V^2 2\pi r dr\right] dx \quad (3)$$

Upon examination, Figure 2 reveals that there are three forces acting on the flow; shear force, pressure differential and buoyancy forces. Realizing the turbulent shear makes up the major portion of the shear force, the shear force τ is equal to (8):

$$\tau = \varepsilon_m \frac{dV}{dr} \quad (4)$$

It can be seen that the shear force on the cross section reduces to zero because the change in velocity at the width b is zero. The force due to pressure differential is equal to:

$$\int_o^b p 2\pi r dr - \int_o^b \left[p + \left(\frac{dp}{dx} dx\right)\right] 2\pi r dr \quad (5)$$

which equals:

$$\left[-\int_o^b \frac{dp}{dx} 2\pi r dr\right] dx \quad (6)$$

However, it is generally accepted in boundary layer work that $\frac{dp}{dx}$ is approximately equal to zero in air. Therefore, the force due to the pressure differential is negligible. The buoyancy forces result due to the differences in the densities of the jet and air surrounding it. The jet weighs more than the surrounding ambient so there is an accelerating effect on the jet. The buoyancy force is equal to:

$$g\int_o^b \left(\rho_{jet} - \rho_{air}\right) 2\pi r dr\ dx \tag{7}$$

which is assumed to be the only force acting on the jet. Combining equations 3 and 7:

$$\frac{d}{dx}\left[\int_o^b \rho_{jet} V^2 2\pi r dr\right] dx = g\int_o^b \left(\rho_{jet} - \rho_{air}\right) 2\pi r dr\ dx \tag{8}$$

dividing by dx and using that:

$$\beta = \frac{\left(\rho_{air} - \rho_{jet}\right)}{\rho_{jet}\ \Delta T} \tag{9}$$

where,

$$\Delta T = (T_{jet} - T_{amb}) \tag{10}$$

equation 8 can be simplified to:

$$\frac{d}{dx}\int_o^b \rho_{jet}\ V^2 2\pi r dr = -g\int_o^b \rho_{jet} \beta \Delta T 2\pi r dr \tag{11}$$

Thus, the change in momentum is due to the buoyancy forces caused by the temperature difference between the jet and the surroundings.

ENERGY EQUATION

As indicated in Figure 2, there are three major sources of energy exchanges in the flow; conduction, radiation and convection. The energy radiated and conducted is small in comparison to the convected energy and may be considered negligible. Therefore, the incoming energy is the convective influx of energy. The convective influx is the sum of the axial convective term and the entrained energy. The axial convective influx is equal to:

$$C_p\int_o^b \rho_{jet} VT_{jet} 2\pi r dr \tag{12}$$

The entrained energy is equal to:

$$C_p(\text{mass flow entrained})T_{r=b} \tag{13}$$

where $T_{r=b}$ is the temperature of the cross section at the radial distance b. However, the difference between $T_{r=b}$ and the ambient temperature is small. Since the difference is small, the entrained energy is approximately equal to:

$$C_p(\text{mass flow entrained})T_{amb} \tag{14}$$

For a steady-state flow, the incoming energy is equal to the energy leaving the system. The incoming energy is the sum of the entrained and axial energy which is:

$$C_p\int_o^b \rho_{jet} VT_{jet} 2\pi r dr + C_p(\text{mass flow entrained})T_{amb} \tag{15}$$

The energy flowing out is equal to:

$$C_p\int_o^b \rho_{jet} VT_{jet} 2\pi r dr + C_p \frac{d}{dx}\left[\int_o^b \rho_{jet} VT_{jet} 2\pi r dr\right] dx \tag{16}$$

Combining equations 15 and 16:

$$\begin{aligned} &C_p\int_o^b \rho_{jet} VT_{jet} 2\pi r dr + C_p(\text{M.F.E.})T_{amb} \\ &= C_p\int_o^b \rho_{jet} VT_{jet} 2\pi r dr + C_p \frac{d}{dx}\left[\int_o^b \rho_{jet} VT_{jet} 2\pi r dr\right] dx \end{aligned} \tag{17}$$

where M.F.E. is the mass flow entrained. Substracting like quantities equation 17 becomes:

$$C_p(\text{M.F.E.})T_{amb} = C_p\left[\frac{d}{dx}\int_o^b \rho_{jet} VT_{jet} 2\pi r dr\right] dx \tag{18}$$

CONTINUITY EQUATION

Equating the mass flows in Figure 2, using a steady-state mass blance, the incoming mass flow equals the outgoing mass flow. The mass flow out is equal to:

$$\int_o^b \rho_{jet} V 2\pi r dr + \frac{d}{dx}\left[\int_o^b \rho_{jet} V 2\pi r dr\right] dx \qquad (19)$$

The mass flow in, is the sum of the mass flow entrained and the axial mass flow. The mass flow in, is equal to:

$$\int_o^b \rho_{jet} V 2\pi r dr + \text{M.F.E.} \qquad (20)$$

Combining equations 19 and 20:

$$\int_o^b \rho_{jet} V 2\pi r dr + \text{M.F.E.} = \int_o^b \rho_{jet} V 2\pi r dr + \frac{d}{dx}\left[\int_o^b \rho_{jet} V 2\pi r dr\right] dx \qquad (21)$$

Subtracting like quanities and simplifying, equation 21 becomes:

$$\text{M.F.E.} = \frac{d}{dx}\left[\int_o^b \rho_{jet} V 2\pi r dr\right] dx \qquad (22)$$

CENTERLINE VELOCITY FROM SIMPLIFICATION OF MOMENTUM EQUATION

Equation 11 can be simplified using various assumptions to find the centerline velocity of chilled axisymmetrical jets. In order to integrate equation 11:

$$\frac{d}{dx}\int_o^b \rho_{jet} V^2 2\pi r dr = -g\int_o^b \rho_{jet} \Delta T \beta 2\pi r dr \qquad (11)$$

the relation of ρ_{jet}, V, ΔT, and β to r must be known. It is usually assumed that the radial temperature difference and velocity profiles may be approximated by error function shaped curves for axisymmetrical jets (5), (6), and (9). It is also usually assumed, that the curves are similar throughout the entire third zone. Therefore, assume that $\Delta T = \Delta T_c e^{-cr^2}$, where ΔT_c is the axial temperature difference between the centerline temperature and the surrounding ambient temperature and c is the temperature shape factor describing the error curve. Also assume the $V = V_c e^{-ar^2}$, where V_c is the centerline velocity and a is the velocity profile shape factor. Assuming that the jet fluid behaves as an ideal gas, the relationships for ρ_{jet} and β can be found. However, their variation with radial distance is negligible, so they were assumed to be functions of x alone.

Substituting the error function assumptions and moving the functions of x from within the integrals, equation 11 becomes:

$$\frac{d}{dx}\left[2\pi V_c^2 \rho_{jet}\int_o^b re^{-2ar^2}\,dr\right] = -g\Delta T_c \beta\rho_{jet}\int_o^b 2\pi re^{-cr^2}\,dr \tag{23}$$

dividing by 2π yields:

$$\frac{d}{dx}\left[V_c^2 \rho_{jet}\int_o^b re^{-2ar^2}\,dr\right] = -g\Delta T_c \beta\rho_{jet}\int_o^b re^{-cr^2}\,dr \tag{24}$$

Assuming that the velocity is zero at the jet edge b and that the temperature difference is sufficiently small as to be assumed zero, equation 24 can be integrated to yield:

$$\frac{d}{dx}\left[V_c^2 \rho_{jet}\frac{1}{4a}\right] = -g\rho_{jet}\Delta T_c \beta\frac{1}{2c} \tag{25}$$

Assuming that the apparent point source for the velocity and temperature difference boundaries are the same then (4):

$$\frac{c}{a} = .65 \tag{26}$$

for a turbulent Prandtl number of .7, which has been generally accepted in jet work (1). In order to find c, a must be known. From Koestel's (4) work:

$$a = \frac{C_2}{x^2} \tag{27}$$

where C_2 is an integration constant. Substituting equations 26 and 27 into equation 25 and simplifying:

$$\int d\left(\rho_{jet} V_c^{\,2} x^2 / (4C_2)\right) = \int -g\rho_{jet}\Delta T_c \beta x^2 / (1.3C_2) dx \tag{28}$$

Performing the left hand integration equation 28 becomes:

$$\frac{\rho_{jet} V_c^{\,2} x^2}{4C_2} + C = -\frac{g}{1.3C_2}\int \rho_{jet}\Delta T_c \beta x^2 dx \tag{29}$$

where C is the integration constant. To integrate the right hand side, an assumption of T_c as a function of x must be made. It has been found that the centerline temperature difference obeys a hyperbolic decay representation (10):

$$\Delta T_c = \frac{C_t}{x} \tag{30}$$

where C_t is an undetermined constant. Using ideal gas assumptions then:

$$\rho_{jet} = \frac{p}{RT_c} \tag{31}$$

and:

$$\beta = \frac{1}{T_c} \tag{32}$$

The definition of ΔT_c is:

$$\Delta T_c = T_c - T_{amb} \tag{33}$$

where T_c is the centerline temperature of the jet. Rearranging equation 33:

$$T_c = \Delta T_c + T_{amb} \tag{34}$$

Substituting equation 34 into equations 31 and 32:

$$\rho_{jet} = \frac{p}{R(T_{amb} + \Delta T_c)} \tag{35}$$

$$\beta = \frac{1}{(T_{amb} + \Delta T_c)} \tag{36}$$

Now utilizing equations 30, 35 and 36, equation 29 can be shown to equal:

$$\frac{\rho_{jet} V_c^2 x^2}{4} + C = \frac{-gp}{1.3R} \int \frac{C_t x^3 dx}{(T_{amb} x + C_t)^2} \tag{37}$$

To evaluate this integral, the stratification relationship between T_{amb} and x must be known. Assume that:

$$T_{amb} = C_{Tamb} - T_g x \tag{38}$$

where C_{Tamb} is the ambient temperature at the point source of the jet, and T_g is the change T_{amb} makes with the change in axial distance. Using equation 38, equation 37 can be shown to become:

$$\rho_{jet} V_c^2 \frac{x^2}{4} + C = -g \frac{C_t p}{1.3R} \int \frac{x^3 dx}{(-T_g x^2 + C_{Tamb} x + C_t)^2} \tag{39}$$

To solve this equation, an initial condition is necessary to find the integration constant C. Usually in isothermal jets, there is a cone of constant velocity which extends a distance F_p from the apparent point source of the jet (2). This distance is a characteristic of the outlet orfice. However, in a chilled jet, there would more likely be a cone of constant acceleration caused by constant buoyancy forces. Using this assumption as an initial condition, the constant C can be solved for and a solution found to equation 39. The closed form solution to equation 39 is:

$$
\begin{aligned}
V_c^{\,2} = {} & \frac{-gC_t 4(-T_g x^2 + C_{Tamb}x + C_t)}{x^3 1.3} \cdot \Bigg[\\
& \frac{1}{2T_g^{\,2}} \ln \left| (-T_g x^2 + C_{Tamb}x + C_t) \right| \\
& - \frac{C_{Tamb}}{2T_g^{\,2}} \frac{1}{(C_{Tamb}^{\,2} + 4C_t T_g)^{1/2}} \ln \left| \frac{\left\{-2T_g x + C_{Tamb} - (C_{Tamb}^{\,2} + 4T_g C_t)^{1/2}\right\}}{\left\{-2T_g x + C_{Tamb} + (C_{Tamb}^{\,2} + 4T_g C_t)^{1/2}\right\}} \right| \\
& + \frac{C_t}{T_g} \Bigg[\frac{-(C_{Tamb}x + 2C_t)}{(-4T_g C_t - C_{Tamb}^{\,2})(-T_g x^2 + C_{Tamb}x + C_t)} - \frac{C_{Tamb}}{(-4T_g C_t - C_{Tamb}^{\,2})} \cdot \\
& \left[\frac{1}{(C_{Tamb}^{\,2} + 4C_t T_g)^{1/2}} \cdot \ln \left| \frac{\left\{-2T_g x + C_{Tamb} - (C_{Tamb}^{\,2} + 4T_g C_t)^{1/2}\right\}}{\left\{-2T_g x + C_{Tamb} + (C_{Tamb}^{\,2} + 4T_g C_t)^{1/2}\right\}} \right| \right] \Bigg] \\
& + \frac{C_{Tamb}}{T_g} \Bigg[\frac{(C_{Tamb}^{\,2} + 4T_g C_t)x + C_{Tamb} C_t}{-T_g(-4T_g C_t - C_{Tamb}^{\,2})(-T_g x^2 + C_{Tamb}x + C_t)} \\
& + \frac{2C_t}{(-4T_g C_t - C_{Tamb}^{\,2})(C_{Tamb}^{\,2} + 4T_g C_t)^{1/2}} \ln \Bigg| \\
& \frac{\left\{-2T_g x + C_{Tamb} - (C_{Tamb}^{\,2} + 4T_g C_t)^{1/2}\right\}}{\left\{-2T_g x + C_{Tamb} + (C_{Tamb}^{\,2} + 4T_g C_t)^{1/2}\right\}} \Bigg| \Bigg] \Bigg] - \frac{4C}{\rho_{jet} x^2}
\end{aligned}
\qquad (40)
$$

where C is the integration constant solved for using the initial condition of constant acceleration to the distance K_p.

However, one problem remains: the determination of C_t. It is possible to use the energy and continuity equations to solve for the constant. This would be a lengthy and complex manipulation, with many indefinite assumptions to be made. Therefore, C_t was determined experimentally.

EXPERIMENTAL APPARATUS

The experimental goal of this research was to test the assumptions made in the theoretical analysis and compare theoretical with experimental results. To achieve this, required an apparatus to measure the temperature and velocity at point locations throughout the jet, while maintaining constant chilled jet properties.

A unit air conditioner connected to the jet plenum, as shown in Figure 3, by means of plastic PVC piping was used to supply the necessary chilled air to the 2 inch ASME nozzle. The 23,500 BTU/hr air conditioner was equipped with two plenums, as shown in Figure 3. One plenum supplied chilled air from the exhaust of the air conditioner, while the other returned air to the intake of the unit. A recirculatory piping system interconnecting the supply, return and jet plenums, as shown in Figure 3, was used to reduce the heat gain caused by long residence time in the supply piping. To further reduce heat gains, the entire system was insulated. The flow rate and temperature in the system was controlled through the use of a slide valve and cone reheater. In order to keep frost from forming on the evaporation coils, the return plenum was equipped with three cone preheaters. To further reduce the possibility of the formation of frost, the supply and return plenums were interconnected to maintain the highest possible flow across the evaporation coil. The cone heaters in both plenum were manually controlled, using variable AC transformers. By using the variable transformers and manual adjustment of the supply plenum slide valve, it was possible to maintain consistant chilled jet conditions.

The jet plenum was equipped with temperature and pressure probes as shown in Figure 3, to monitor the jet conditions. A Digitec temperature measuring unit, coupled with a thermister placed in the jet plenum provided the necessary

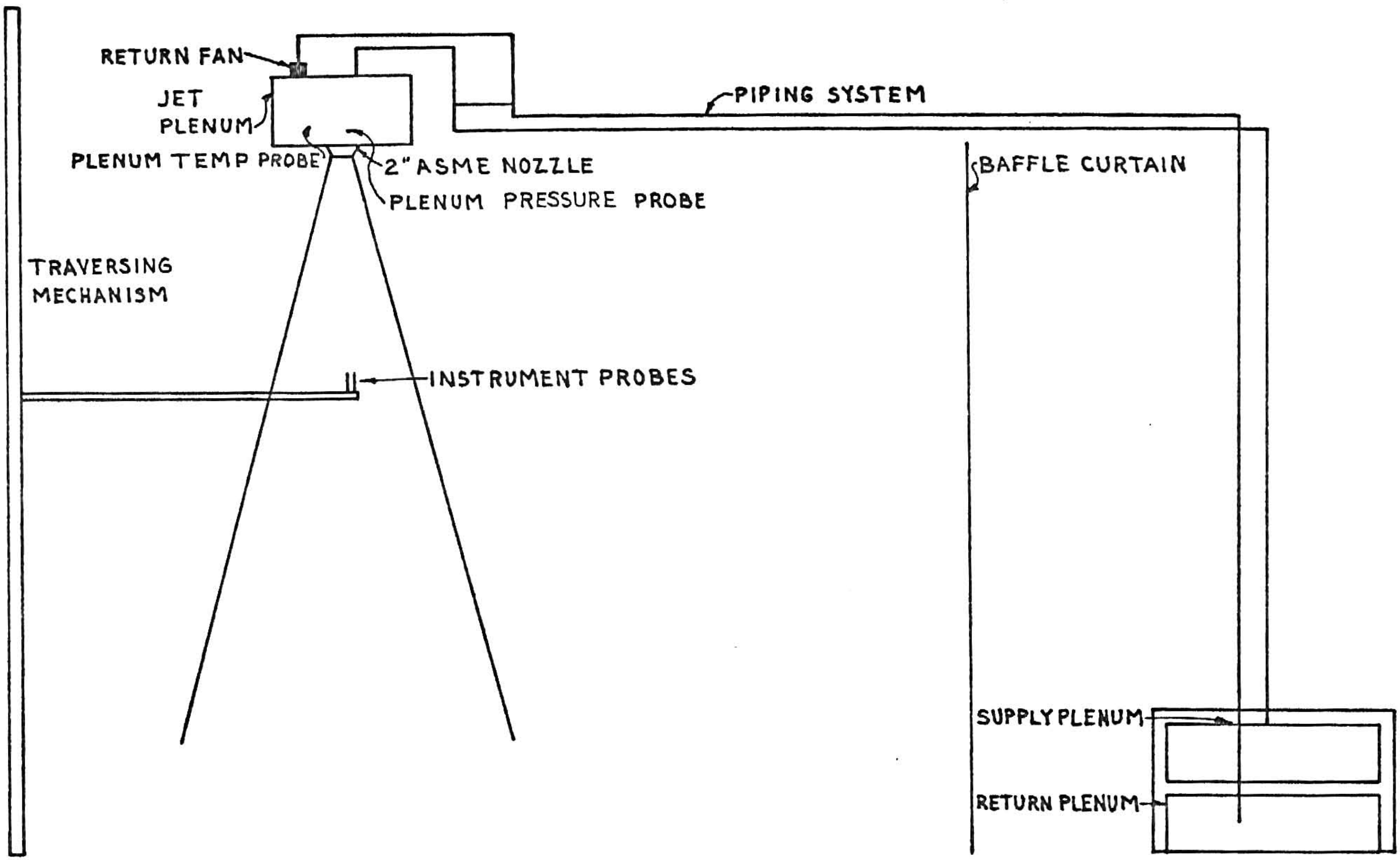
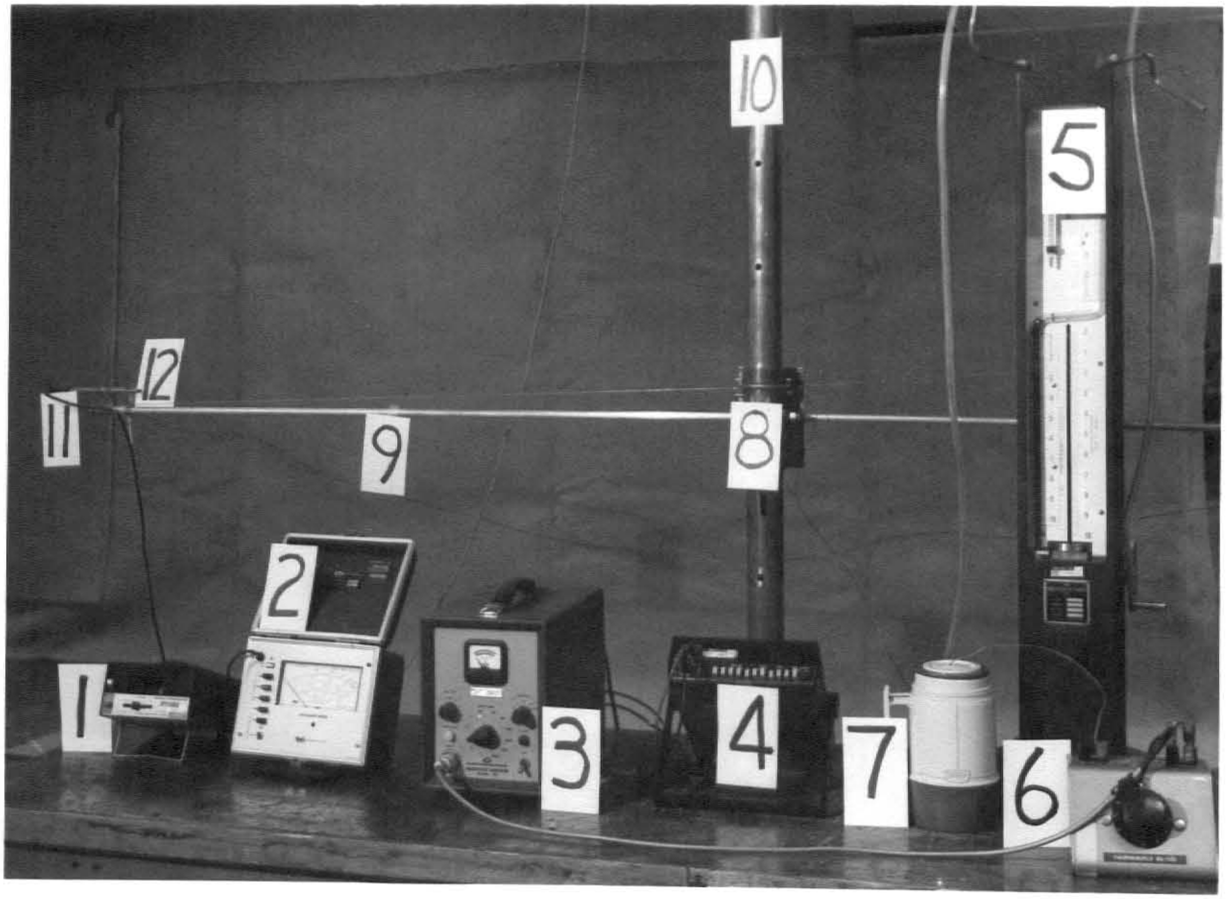

RETURN FAN
JET
PLENUM
PLENUM TEMP PROBE
2" ASME NOZZLE
PLENUM PRESSURE PROBE
PIPING SYSTEM
BAFFLE CURTAIN
TRAVERSING
MECHANISM
INSTRUMENT PROBES
SUPPLY PLENUM
RETURN PLENUM

FIGURE 3

PLATE I

1. Digitec Temperature Meter
2. Thermo Systems Inc. Velocity Meter
3. Nanovolt Amplifer
4. Digital Multimeter
5. Micromanometer
6. Thermocouple Switch
7. Ice Bath Reference Junction
8. Interlocking Collar
9. Horizontal Traverse Bar
10. Vertical Traverse Bar
11. Velocity Probe
12. Thermocouple Probe


10
5
12
11
9
8
2
1
3
4
7
6

information to maintain constant jet temperature. To keep the flow rate constant, the jet plenum was equipped with a static pressure tap coupled to a micromanometer. Observing the micromanometer and Digitec, it was possible through manual adjustment of the system to maintain constant jet conditions.

To achieve the primary goal of the experimentation, to check the theoretical assumptions and to compare theoretical with experimental results, a traversing mechanism was designed to measure temperature and velocity at each 1 inch vertical and 6 inch horizontal plan of the jet. The traversing device consisted of a horizontal pole, interconnected to a vertical pole using an interlocking collar as shown in Figure 3 . The jet temperature was measured, using a copper constant thermocouple directly connected to the horizontal bar of the transversing device. A similar thermocouple attached to the interlocking collar, was used to measure the ambient temperature. Through the use of a thermocouple switch, these non-shielded thermocouples used the same ice water bath reference junction and potientiometer system. The potientiometer system consisted of a high impedence nanovolt amplifier coupled with a digital multimeter reading the thermocouple outputs in millivolts. A Thermo Systems Incorporated hot wire anemometer, attached to the horizontal pole at the same location as the jet temperature probe, was used to measure the velocity. The velocity meter, calibrated in FPM, at 70° F and 14.7 psia, required corrections due to differences in temperature and pressure from the calibration conditions.

The temperature and velocity measurements within the jet were substantially influenced by random currents of the room air. To reduce these disturbances, a baffle curtain was placed between the unit air conditioner and the experimental apparatus to shield the jet from the condensor exhaust air. Also during test periods, all doors to the test location were locked.

DATA TAKING PROCEDURES

The micromanometer was balanced and zeroed. Then, the entire system was turned on and allowed to reach steady-state conditions which generally took 1 to 2 hours. During this time, the ice bath was readied and the thermocouple potientometer system zeroed. The plenum temperature was constantly checked to see if steady-state had been achieved. The test then began, provided steady-state conditions had been reached. The initial temperature and pressure difference of the jet plenum was measured and recorded. Then using a sling psychrometer, the wet and dry bulb temperatures were measured and recorded. Lastly, the velocity and temperature measurements of the jet were taken. Starting at the horizontal cross section closest to the jet outlet, a traverse was made until the edges of the jet were found. Having completed the first horizontal cross section, the traversing mechanism was moved down to the next horizontal cross section. This procedure was continued until 5 to 8 cross sections had been examined, depending upon the initial velocity. The entire system was then turned off and the traversing mechanism returned to its initial position. Finally for purposes of velocity correction, the barametric pressure was measured using a mercury barometer.

EXPERIMENTAL RESULTS AND DISCUSSION

The original data are contained in Appendix A. The errors associated with their measurement are discussed in Appendix D. All computer simulations and results were carried out on the KSU IBM 370 digital computer, utilizing WATFIV FORTRAN language.

To verify the assumption of error functions to describe the temperature and velocity profiles at each horizontal cross section of the jet, required that the apparent point source of the jet be found. The distances from the apparent point source to the edge of the outlet nozzle are listed in Appendix F. Using the experimental results for boundaries of the jet at each horizontal cross section, it was possible to graphically solve for the apparent point source of the jet. Having found the apparent point source, the dimensionless parameters $\frac{\Delta T}{\Delta T_c}$ and $\frac{V}{V_c}$ were plotted as functions of $\frac{r}{x}$, similar to the method used by Nottage (1) in his experimental work. Utilizing Nottage's (1) work on 500 FPM isothermal jets, a value of 65.3 for C_2 was used in the error curves of Figures 4, 5, 6, 7, and 8. Because of a lack of other data for vertically chilled jets, this value was used. The Tolmien curve used for comparative analysis with the error curve in Figures 4, 5, 6, 7, and 8, was obtained from Nottage's (1) analysis of a 500 FPM isothermal jet. The 500 FPM data was used because of the small variations with velocity. The Tolmien curve comes from Nottage's (1) study of Tolmien's solution for the velocity profiles of an isothermal jet using Prandtl's mixing length formula. Examining Figures 4, 5, 6, 7, and 8, representative of the different ranges of velocities studied, it can be seen that the error curve functions provide a satisfactory approximation of the velocity and temperature profiles.

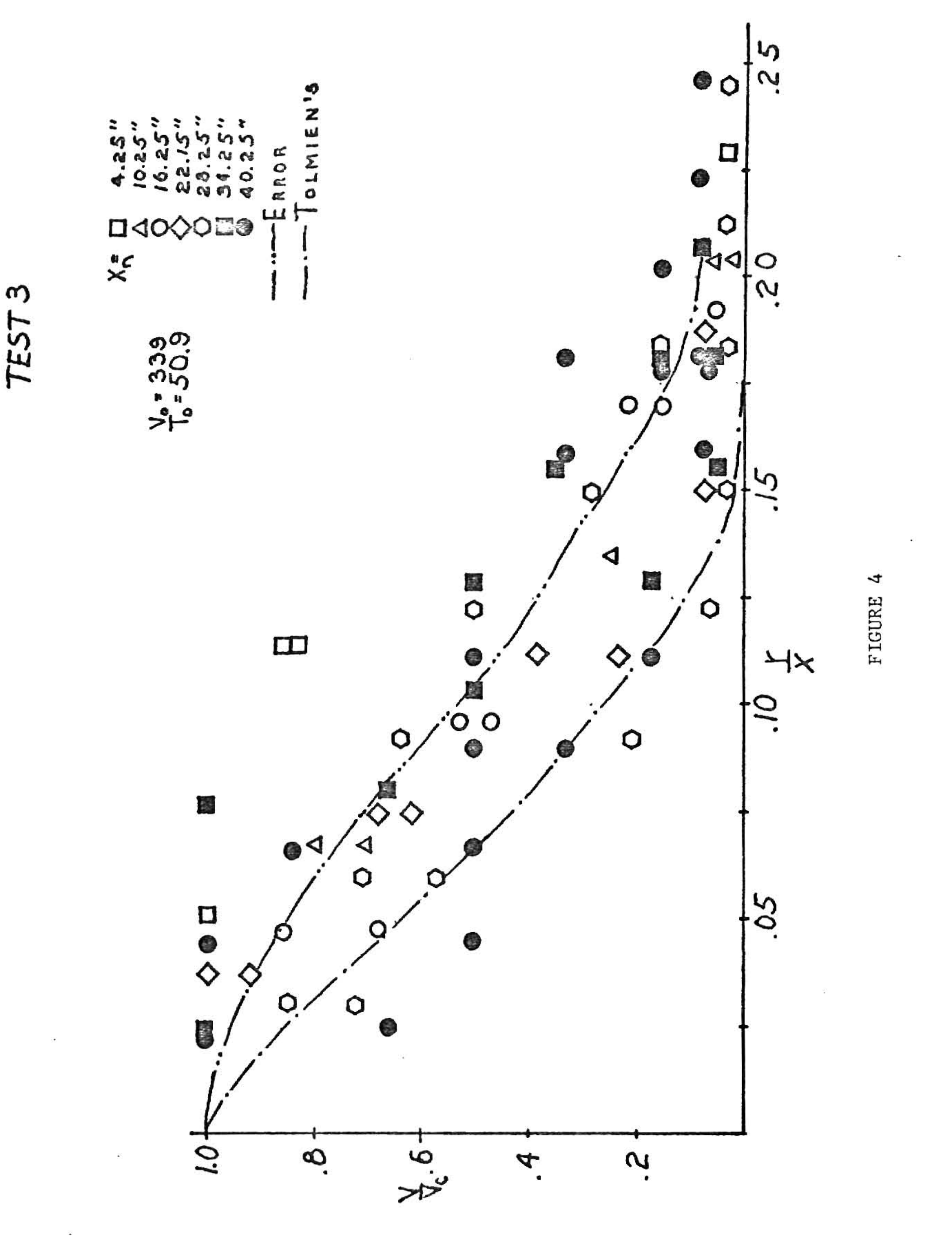

TEST 3
$V_o = 339$
$T_o = 50.9$
X_n = 4.25" 10.25" 16.25" 22.15" 28.25" 34.25" 40.25"
ERROR
TOLMIEN'S
V/V_c
1.0
.8
.6
.4
.2
.05
.10
.15
.20
.25
r/X

FIGURE 4

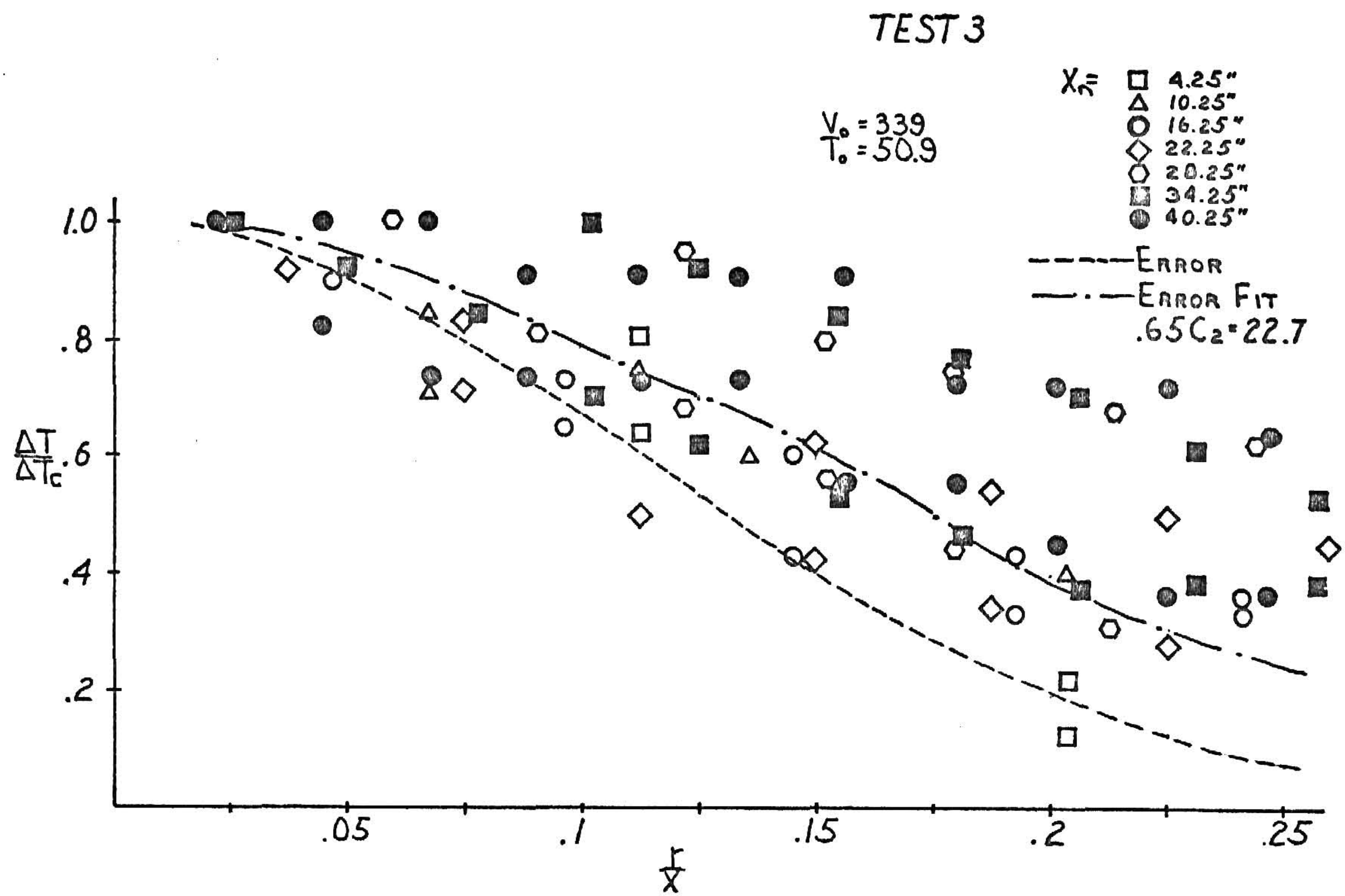

TEST 3
V₀ = 339
T₀ = 50.9
Xn = 4.25"
10.25"
16.25"
22.25"
28.25"
34.25"
40.25"
Error
Error Fit
.65C₂ = 22.7
ΔT/ΔTc
1.0
.8
.6
.4
.2
.05
.1
.15
.2
.25
r/X

FIGURE 5

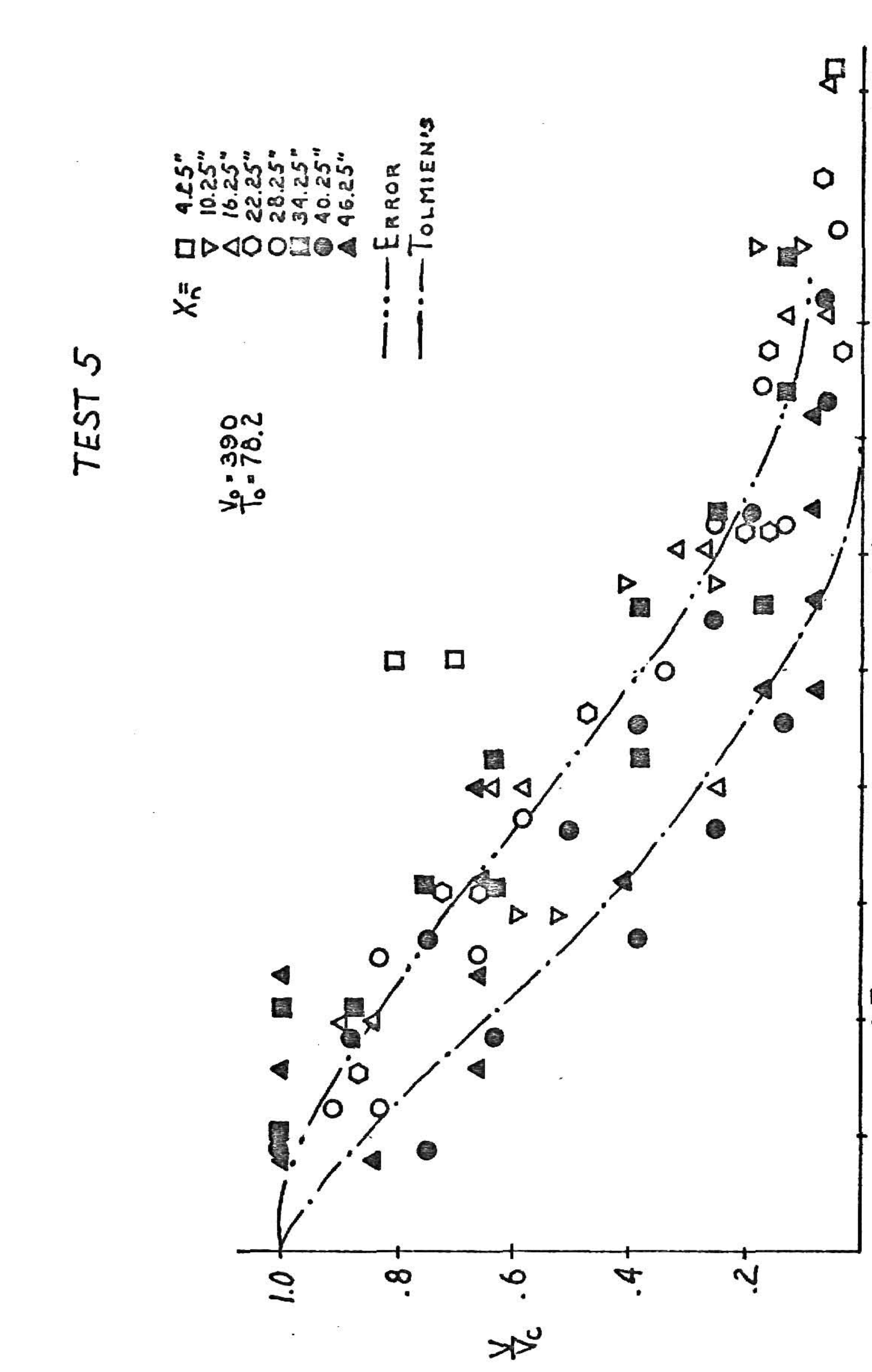

TEST 5
V₀ = 390
T₀ = 78.2
Xₙ=
4.25"
10.25"
16.25"
22.25"
28.25"
34.25"
40.25"
46.25"
ERROR
TOLMIEN'S
1.0
.8
.6
.4
.2
V/V_c
.05
.1
.15
.2
.25
r/x

FIGURE 6

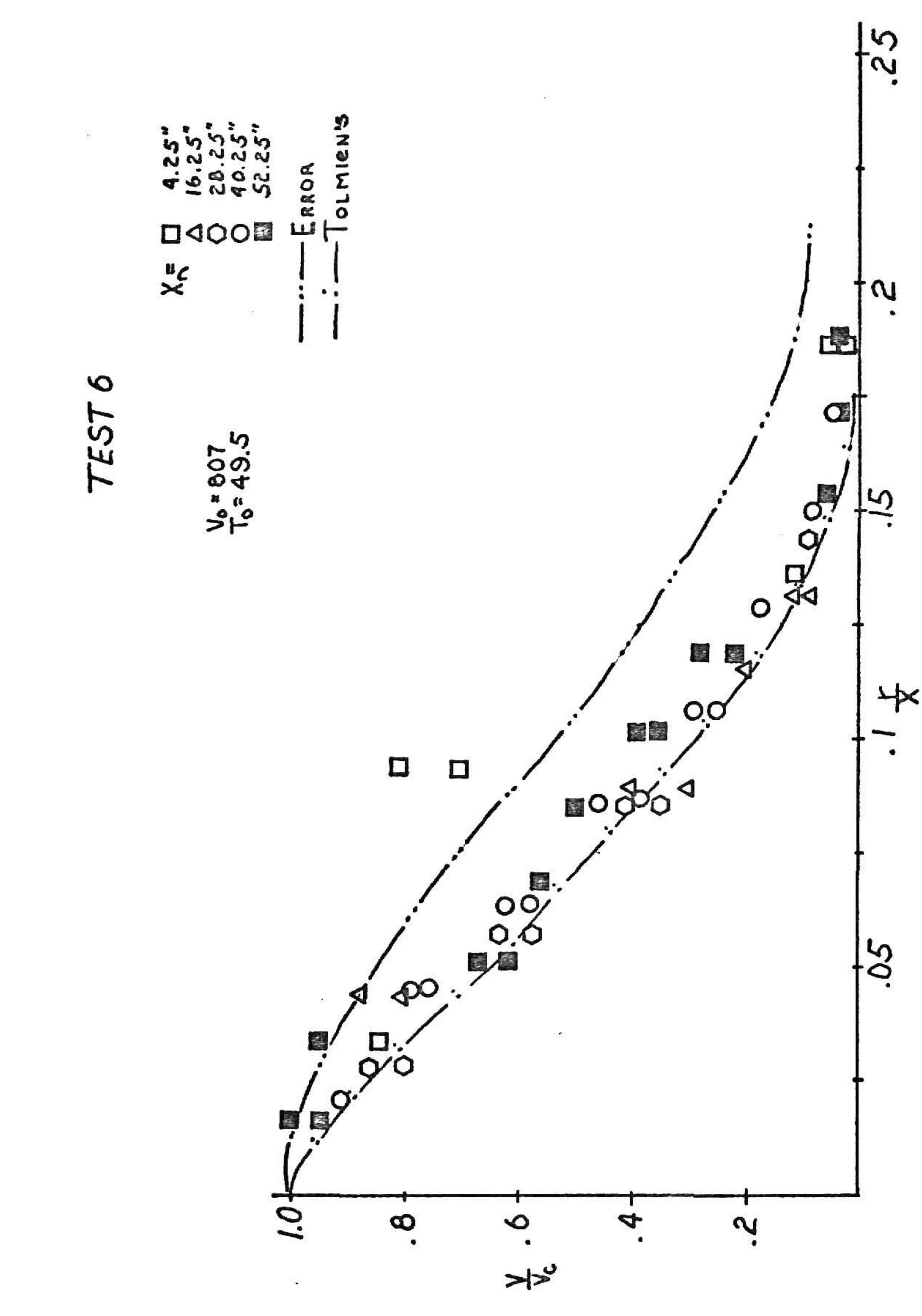

TEST 6
$V_o = 807$
$T_o = 49.5$
$X_n =$
4.25"
16.25"
28.25"
40.25"
52.25"
ERROR
TOLMIEN'S
1.0
.8
.6
.4
.2
$\frac{V}{V_c}$
.05
.1
.15
.2
.25
$\frac{r}{x}$

FIGURE 7

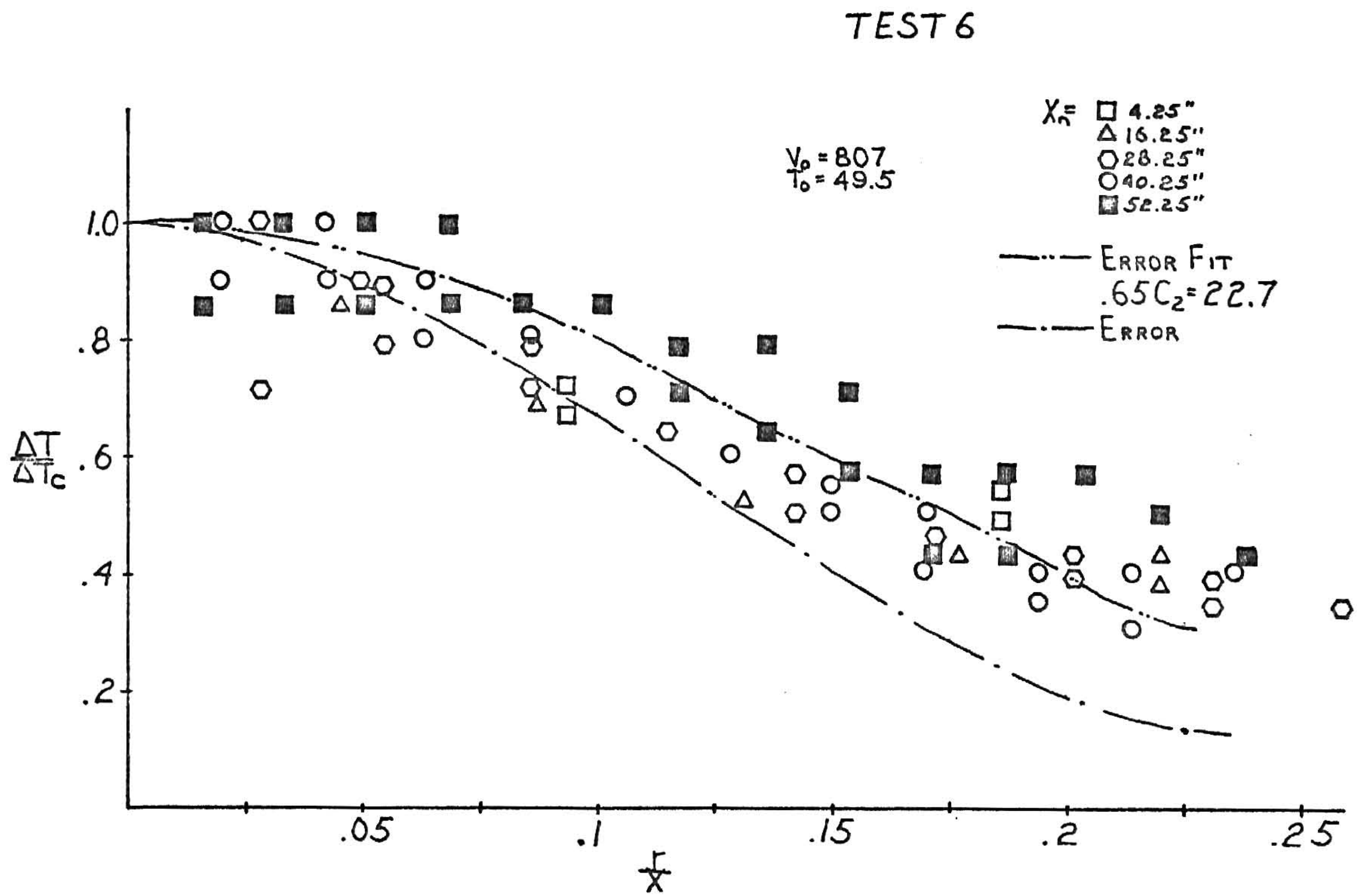

TEST 6
V_0 = 807
T_0 = 49.5
X_n = □ 4.25"
△ 16.25"
⬡ 28.25"
○ 40.25"
■ 52.25"
Error Fit
.65 C_2 = 22.7
Error
$\frac{\Delta T}{\Delta T_c}$
1.0
.8
.6
.4
.2
.05
.1
.15
.2
.25
$\frac{r}{x}$

FIGURE 8

Use of error function distributions required knowledge of the centerline temperature's variation with axial distance. It was assumed in the theoretical analysis that the centerline temperature difference decays *hyperbolically*. It was also assumed that the ambient air temperature could be approximated with a linear distribution. Using least squares curve fits, these assumptions were verified. The results are included in Appendix E. To complete the analysis of the centerline temperature of the jet, the experimental results for centerline temperature were plotted and compared with the theoretically experted results as calculated from the combination of equations 34 and 38 where

$$T_c = \frac{C_t}{x} + C_{Tamb} - T_g x \qquad (42)$$

In Figure 9, the values -30 and -50 for C_t were used to formulate the theoretical boundaries, as they represent the range of experimentally found C_t. In addition, -.7 for T_g and 6.0 inches for the distance from the apparent point source to the nozzle outlet were used in equation 42 because they represent the experimental data found. From Figure 9, it can be seen that the shape of the theoretical boundaries closely follows the experimental results, with a majority of the experimental points falling on or between the boundaries.

Having verified the assumptions used in the theoretical analysis, the theory predicting centerline velocity was compared with experimental results. Due to the excessive length of equation 40, a numerical integration of equation 25 with the appropriate substitutions for c, a, ΔT_c and β, was used to predict the centerline velocity. Performing the left hand differentiation, equation 25 can be shown to equal:

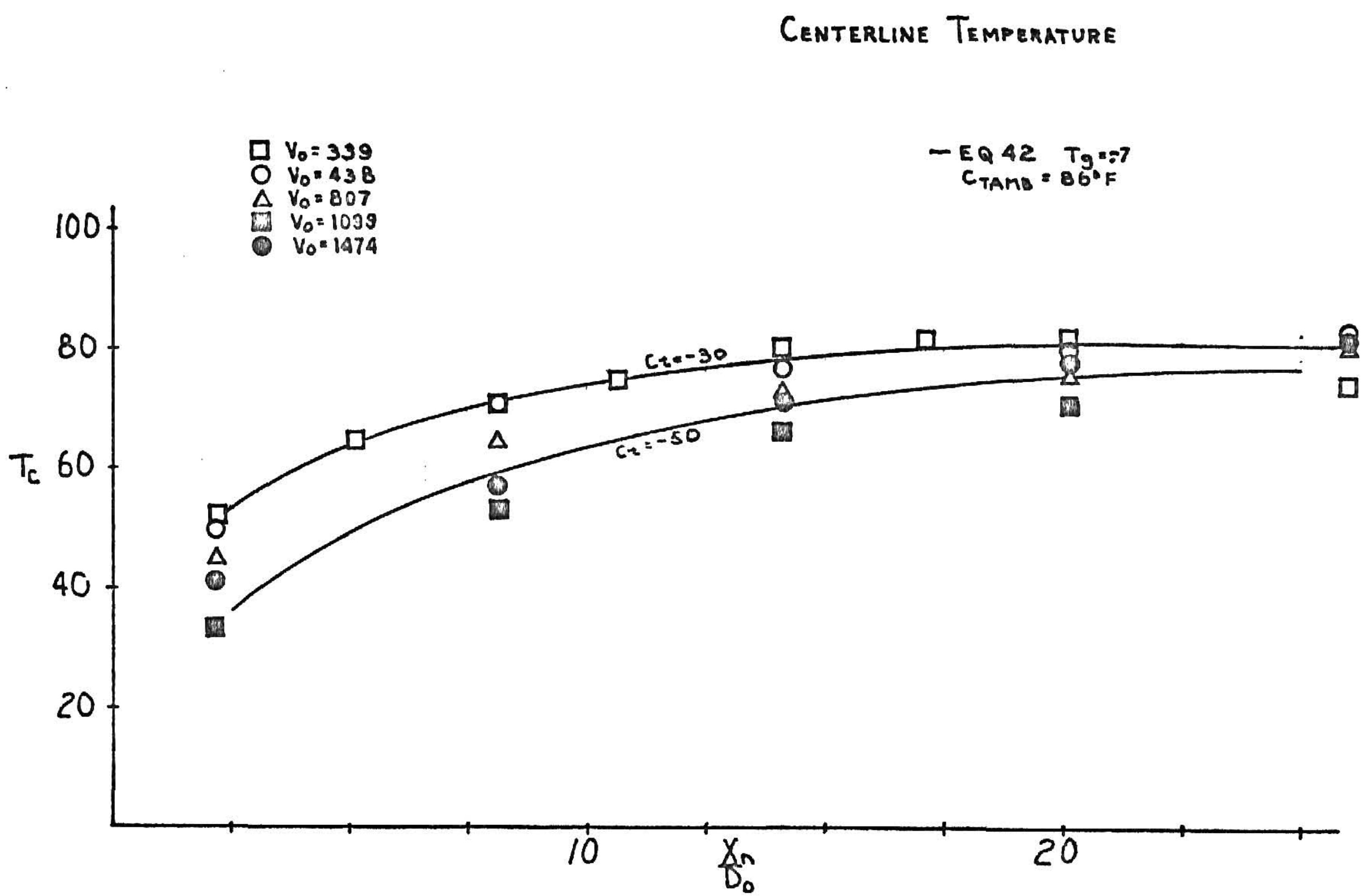

CENTERLINE TEMPERATURE
— EQ 42 Tg = 7
CTAMB = 86°F
Vo = 339
Vo = 438
Vo = 807
Vo = 1039
Vo = 1474
Ct = -30
Ct = -50
Tc
100
80
60
40
20
10
20
Xn/Do

FIGURE 9

$$\frac{dV_c}{dx} = \frac{2}{1.3}\,\frac{-gC_t}{V_c(-T_g x^2 + C_{Tamb} + C_t)} - \frac{3}{2}\,\frac{V_c}{x} + \frac{V_c(-2T_g x + C_{Tamb})}{2(-T_g x^2 + C_{Tamb} + C_t)} \quad (41)$$

Using a Runge Kutta numerical integration (11) with .01 foot steps and assuming constant acceleration to K_p as the initial point for the numeric method, it was possible to generate the theoretical centerline velocity. The numerical integration program is contained in Appendix B. The experimental results were formed into the dimensionless parameters $\frac{V_c}{V_o}$ and $\frac{x_n}{D_o}$ as shown in Table I and in Figure 10. To compare theoretical and experimental results, the boundaries for centerline velocity from the numerical integration of equation 25 were plotted in Figure 10. Also included in the figure, are the boundaries for isothermal data given by Nottage's (1) summary on previous isothermal experimentation. The experimental results closely follow the theoretical boundaries. For an initial velocity of 1099 FPM the experimental and theoretical results are approximately the same. It should also be noted, that all but three experimental points lay within the boundaries of isothermal data. The results indicate that the theoretical analysis provides a good approximation for the centerline velocity in the developed region of the jet.

Using the experimental results, it was possible to analyze the entrainment ratio, $\frac{Q}{Q_o}$. Breaking each horizontal cross section in a series of rectangles as shown in Figure 11, it was possible using average velocities to find flow rate at each rectangle, which represented a concentric flow area. Adding the flows from each of the rectangles, the flow rate at each horizontal cross section was found. The results of this graphical integration performed by the computer are contained in Appendix C. To analyze these results, the dimensionless parameter

TABLE I

V_o	338.780	437.710	390.370	807.000	1099.000	1390.160	1474.260
$\frac{x_n}{D_o}$				$\frac{V_c}{V_o}$			
2.125	.935	1.039	1.001	1.000	1.006	.988	1.006
5.125	.720		.755				
8.125	.579	.612	.512	.604	.593	.658	.693
11.125	.398		.404				
14.125	.430	.333	.325	.448	.428	.466	.466
17.125	.186		.215				
20.125	.186	.239	.215	.309	.320		.331
23.125			.161				
26.125		.169		.233	.246		.232

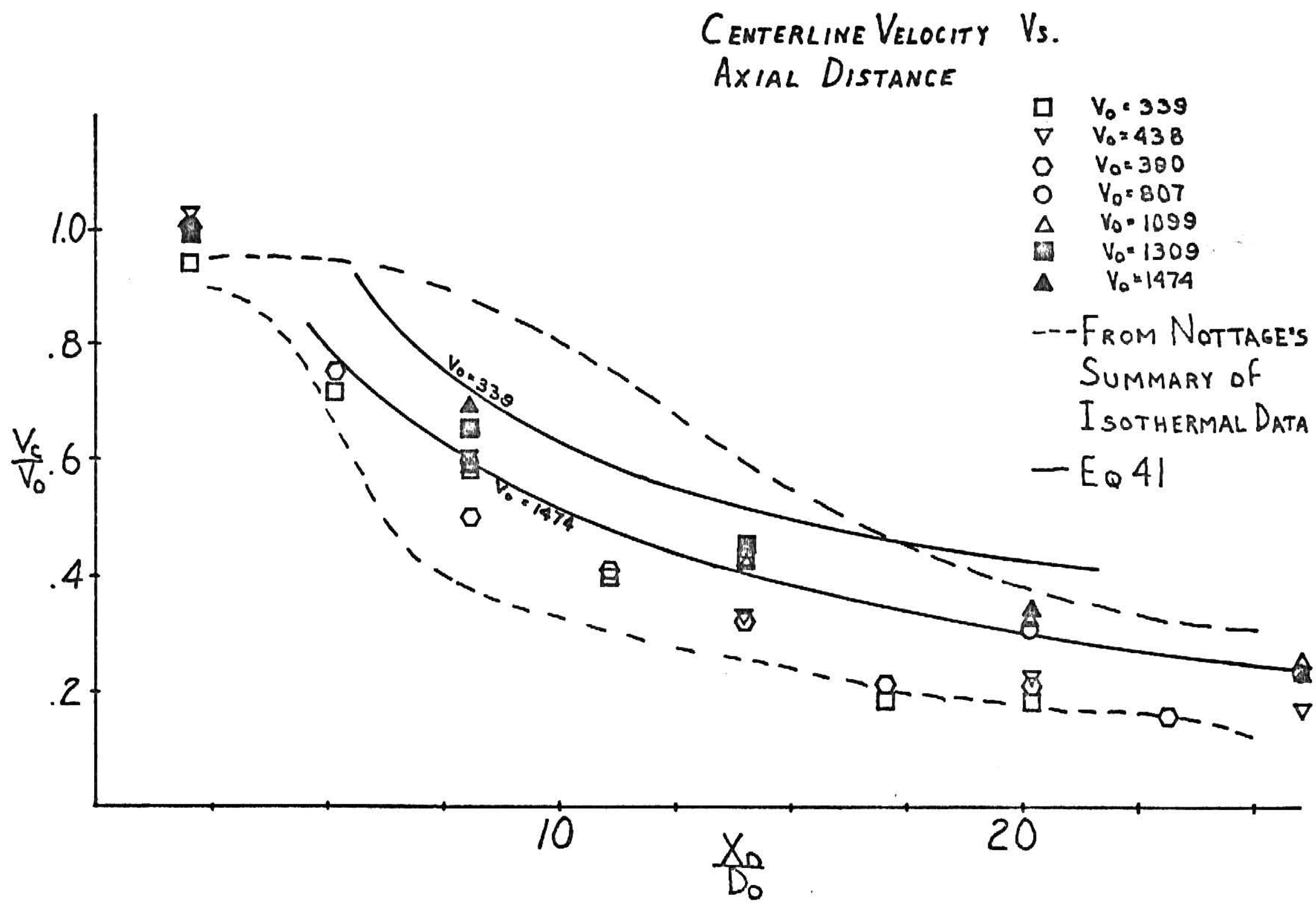

CENTERLINE VELOCITY VS.
AXIAL DISTANCE
Vo = 339
Vo = 438
Vo = 380
Vo = 807
Vo = 1099
Vo = 1309
Vo = 1474
FROM NOTTAGE'S SUMMARY OF ISOTHERMAL DATA
EQ 41
Vo = 339
Vo = 1474
Vc/Vo
1.0
.8
.6
.4
.2
10
20
Xo/Do

FIGURE 10

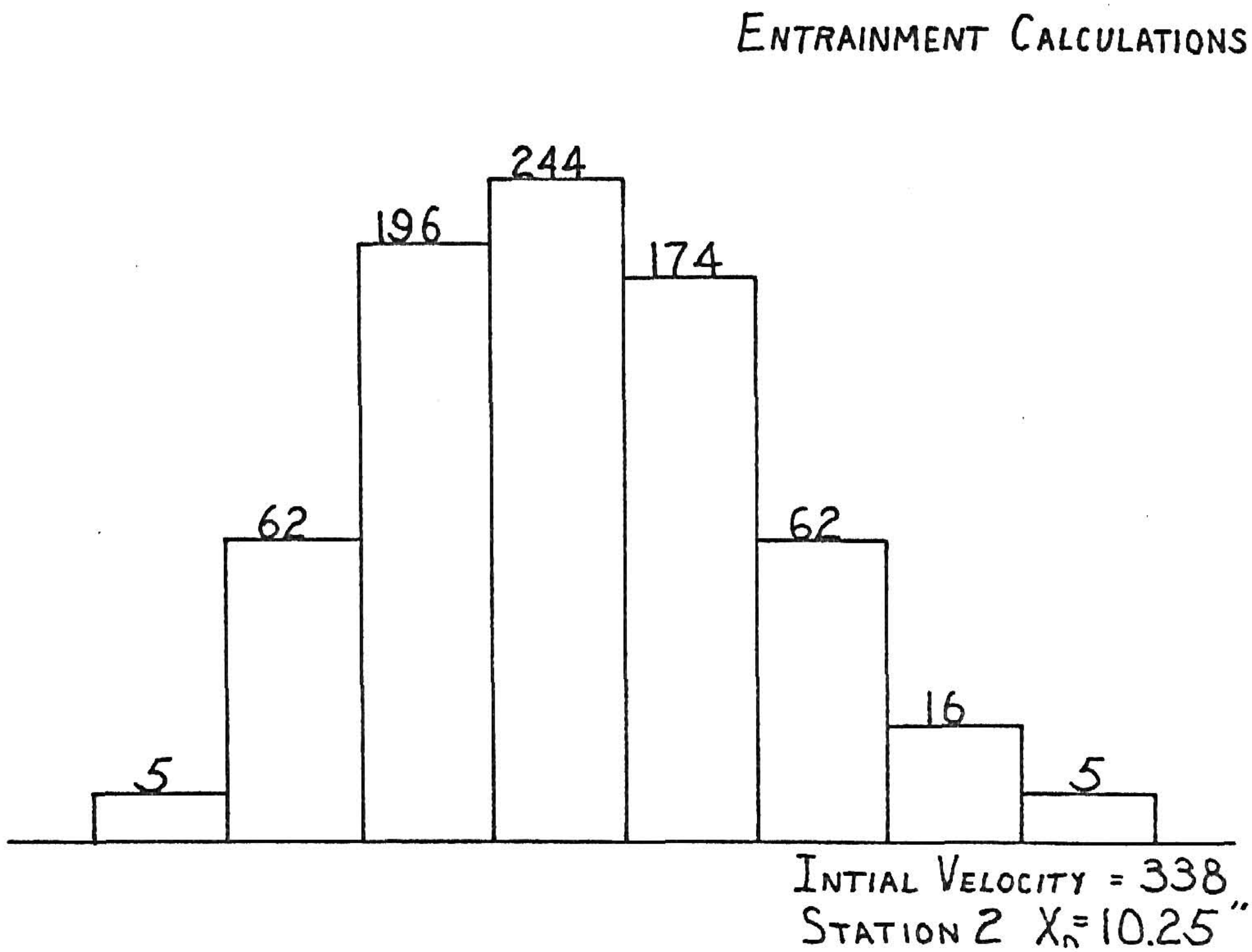

ENTRAINMENT CALCULATIONS
5
62
196
244
174
62
16
5
INTIAL VELOCITY = 338
STATION 2 Xn=10.25"

FIGURE 11

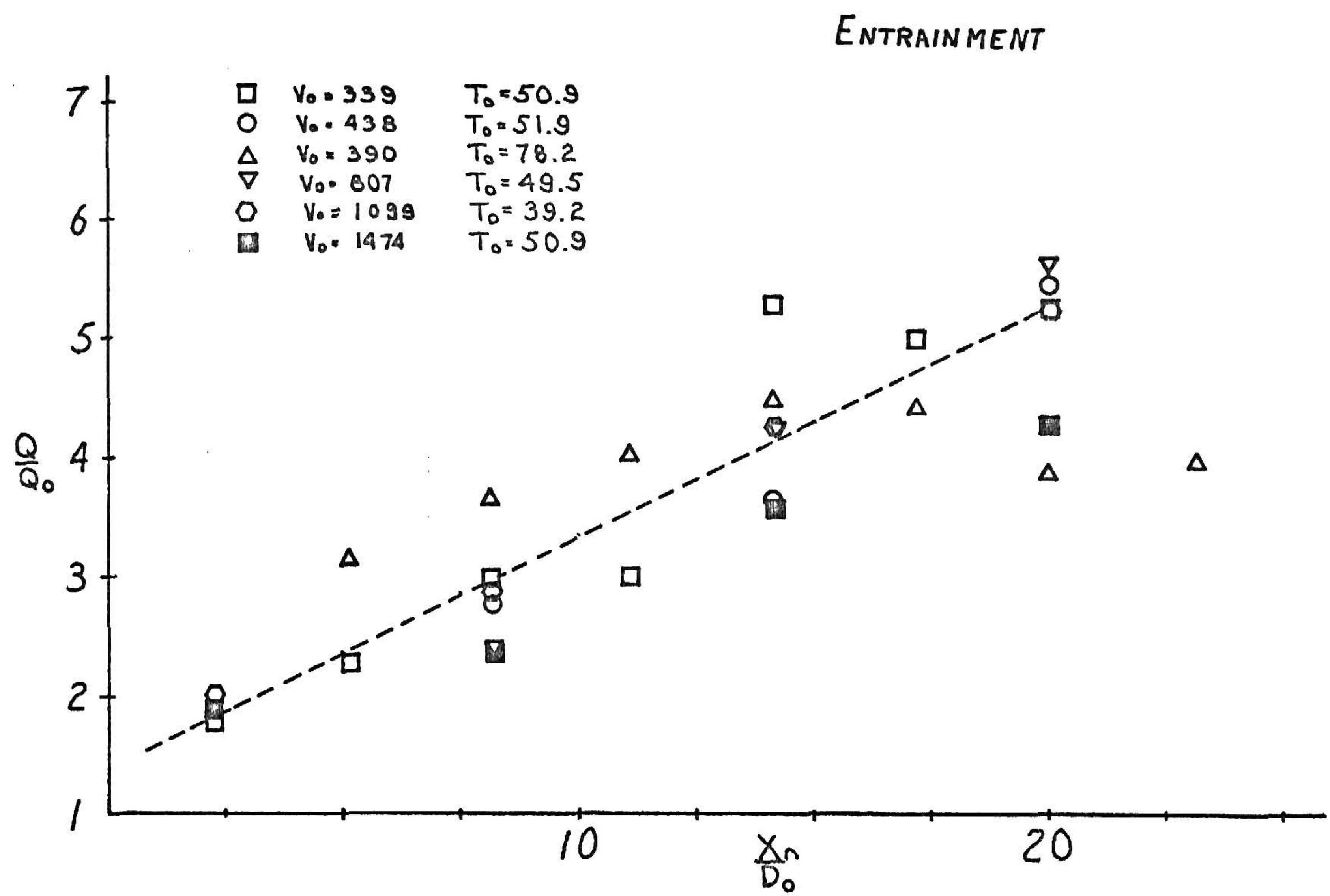

ENTRAINMENT
V₀ = 339 T₀ = 50.9
V₀ = 438 T₀ = 51.9
V₀ = 390 T₀ = 78.2
V₀ = 807 T₀ = 49.5
V₀ = 1099 T₀ = 39.2
V₀ = 1474 T₀ = 50.9
Q/Q₀
1
2
3
4
5
6
7
10
20
X/D₀

FIGURE 12

$\frac{Q}{Q_o}$ was plotted as a function of $\frac{x_n}{D_o}$. Figure 12 shows that distribution of $\frac{Q}{Q_o}$ is approximately linear with a slope of .196. The linear approximation agrees with Nottage's (1) results for a chilled horizontal jet, except the slope from his data was found to be .215. However, Susarla (20) found that the entrainment ratios for heated and isothermal jets could be approximated by exponential power curves. The difference in these results suggest the need for further investigations.

Finally, the effect of changing the stratification factor T_g upon the centerline velocity was investigated, using the numerical integration of equation 25. Varying T_g from -2 $^{o}F/ft$ to 2 $^{o}F/ft$ caused a difference of 2 to 4 FPM in the calculated centerline velocities. Since the difference in the centerline velocity was less than the error of the velocity meter, the effect of changing the stratification factor was assumed negligible within the range of T_g tested.

CONCLUSIONS

From this research it was found:

(1) The velocity and temperature profiles at each horizontal cross section could be approximated by error functions.

(2) The centerline temperature difference hyperbolically decayed.

(3) The centerline velocity decay is correctly predicted by the numerical integration of equation 25.

(4) The entrainment function is linear with a slope of .196.

(5) The effect of changes in the stratification factor with the range of -2 ^{o}F/ft to 2 ^{o}F/ft has a negligible effect upon the centerline velocity.

RECOMMENDATIONS

As a result of theoretical and experimental research done, the following recommendations are made:

(1) Further investigate the use of equation 25 to predict the centerline velocities in chilled jets for different nozzle sizes and different temperature ranges.

(2) Experimentally investigate the effect of stratification on a chilled jet.

(3) Obtain an empirical relationship between C_t and the factors that control its value such as V_o, T_g and the diameter of the outlet nozzle.

(4) Obtain an empirical relationship between the slope of the entrainment curve and the factors that control its value.

(5) Correlate the entrainment ratio to entrainment of pollutants.

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ACKNOWLEDGEMENTS

I would like to thank Dr. H. D. Ball for his patience during the preparation of this thesis. Also, I would like to thank my wife, Kathy, who without her data taking, grammar and spelling correcting abilities, and constant encouragement, this thesis would never have been completed.

APPENDIX A

Experimental Data

TEST 3

V_o = 338.78 FPM
T_o = 50.9° F
T_{db} = 83° F
T_{wb} = 59° F
P_{bar} = 29.13" H_g

Temperature - °F

	1	2	3	4	5	6	7	8	9
14						85.5	85.5		
13						85.0	85.0		
12						84.5	85.0		
11					85.0	84.5	84.5		
10					84.0	84.5	84.0		
9					84.0	84.0	84.0		
8				82.5	82.5	83.5	84.0		
7		81.0	82.0	81.5	82.0	83.0	83.0		
6		80.5	81.0	81.0	81.5	82.5	83.0		
5	78.0	80.0	81.0	80.5	81.0	82.0	83.0		
4	77.0	79.0	80.0	79.5	80.0	81.5	83.0		
3	75.5	77.5	77.5	78.0	79.0	81.0	82.5		
2	71.5	73.5	75.5	77.0	78.5	81.0	82.5		
1	58.5	68.0	73.0	76.0	77.5	81.0	82.5		
CL	52.0	65.0	71.5	75.0	79.5	81.5	82.5		
1	64.0	71.0	75.5	78.5	79.0	81.5	83.5		
2	75.0	73.5	76.5	78.5	79.5	82.0	84.0		
3	77.0	78.0	80.0	81.0	81.0	82.5	84.0		
4	79.0	80.5	81.0	82.0	82.0	83.5	84.0		
5	79.5	81.0	81.5	83.0	83.0	84.0	84.0		
6		81.0	82.0	83.5	84.0	84.5	85.0		
7					84.0	85.0	85.0		
8					85.0	85.5	85.5		
9					85.0	85.5	86.0		
10						85.5	86.0		
11							86.0		
12									
13									
14									
T_{amb}	86.0	86.0	86.5	87.0	87.5	88.0	88.0		

TEST 3

Velocity - FPM

	1	2	3	4	5	6	7	8	9
14						0	0		
13						0	5		
12						0	5		
11					0	5	5		
10					0	5	5		
9					0	5	10		
8				0	5	10	10		
7		0	0	0	5	10	21		
6		0	0	0	21	21	21		
5	0	0	0	10	42	32	32		
4	0	5	10	10	73	32	32		
3	0	16	42	52	94	63	53		
2	10	62	93	83	104	63	63		
1	271	174	134	135	125	63	63		
CL	317	244	196	135	146	63	63		
1	264	196	166	125	105	63	42		
2	10	62	104	94	84	63	32		
3	0	5	31	31	31	42	32		
4	0	0	0	0	10	32	21		
5	0	0	0	0	5	10	10		
6	0	0	0	0	5	5	5		
7					0	5	5		
8					0	0	5		
9					0	0	0		
10						0	0		
11							0		
12									
13									
14									

TEST 4

V_o = 437.71 FPM
T_o = 51.7° F
T_{db} = 83° F
T_{wb} = 60° F
P_{bar} = 29.13" H_g

Temperature - °F

	1	2	3	4	5	6	7	8	9
14									85.0
13							85.0		85.0
12							84.5		84.5
11							84.5		84.0
10					84.0		84.0		84.0
9					84.0		84.0		84.0
8					83.0		84.0		84.0
7			83.0		83.0		83.0		84.0
6			83.0		82.0		82.0		83.0
5	78.0		82.0		81.0		82.0		83.0
4	76.0		80.5		80.0		81.5		83.0
3	74.5		79.0		79.0		81.0		83.0
2	70.5		74.0		78.0		80.5		83.0
1	60.5		72.0		77.0		81.0		83.0
CL	50.5		71.0		77.0		80.0		83.0
1	63.0		74.0		80.0		83.0		84.0
2	70.0		76.0		81.0		83.0		84.5
3	77.0		79.5		82.0		84.0		84.0
4	79.0		81.0		83.0		84.0		84.5
5	80.0		82.0		85.0		84.5		85.0
6	80.0		83.0		85.0		85.0		85.0
7			83.0		85.0		85.0		85.0
8							85.0		85.0
9							86.0		85.5
10									85.5
11									
12									
13									
14									
T_{amb}	86.0		87.0		87.5		88.0		87.5

TEST 4

Velocity - FPM

	1	2	3	4	5	6	7	8	9
14									0
13							0		5
12							0		5
11							0		21
10					0		5		21
9					0		5		21
8					0		11		42
7			0		5		16		42
6			0		11		32		42
5	0		0		31		63		63
4	0		16		52		84		63
3	0		31		84		94		63
2	10		124		104		105		74
1	363		206		146		105		74
CL	455		268		146		105		74
1	355		207		115		105		63
2	10		114		94		84		42
3	5		21		42		42		42
4	0		11		16		32		21
5	0		0		0		16		21
6	0		0		0		5		11
7			0		0		0		11
8							0		0
9							0		0
10									0
11									
12									
13									
14									

TEST 5

V_o = 390.37 FPM
T_o = 78.2^o F - isothermal test
T_{db} = 78^o F
T_{wb} = 57^o F
P_{bar} = 28.89" H_g

Temperature - ^oF

	1	2	3	4	5	6	7	8	9
14									
13									
12							80.0	80.0	
11						80.0	80.0	80.0	
10					80.5	80.0	80.0	80.0	
9				80.0	80.0	80.0	80.0	80.0	
8			80.0	80.0	80.0	80.0	80.0	80.0	
7		80.0	80.0	80.0	80.0	80.0	80.0	80.0	
6		80.0	80.0	80.0	80.0	80.0	80.0	80.0	
5	78.0	79.5	80.0	79.5	79.5	80.0	80.0	80.0	
4	78.0	79.0	80.0	79.5	79.5	79.5	80.0	80.0	
3	77.5	79.0	79.5	79.0	79.5	79.5	80.0	80.0	
2	77.5	78.5	79.5	79.0	79.5	79.5	80.0	80.0	
1	77.0	78.5	79.5	79.0	79.5	79.5	80.0	80.0	
CL	77.0	78.0	79.0	79.0	79.5	79.5	80.0	80.0	
1	78.0	78.5	79.0	79.5	79.5	80.0	80.0	80.0	
2	78.0	79.0	79.0	79.5	79.5	79.5	80.0	80.0	
3	78.0	79.0	79.0	79.0	79.5	79.5	80.0	80.0	
4	78.0	79.0	79.5	79.5	79.5	79.5	80.0	80.0	
5	78.5	79.5	80.0	80.0	79.5	80.0	80.0	80.0	
6	78.5	80.0	80.0	80.0	80.0	80.0	80.0	80.0	
7		80.0	80.0	80.0	80.0	80.0	80.0	80.0	
8				80.0	80.0	80.0	80.0	80.0	
9								80.0	
10									
11									
12									
13									
14									

TEST 5

Velocity - FPM

	1	2	3	4	5	6	7	8	9
14									
13									
12									
11									
10									
9							5	5	
8						11	5	5	
7					5	11	16	11	
6				11	21	21	21	11	
5			11	11	32	32	32	42	
4		16	26	32	42	53	42	42	
3		53	63	74	74	63	63	63	
2	21	116	127	105	95	84	74	63	
1	315	232	168	137	116	84	84	63	
CL	391	295	200	158	127	84	84	63	
1	274	211	179	137	105	84	63	53	
2	21	74	116	116	84	74	53	42	
3		32	53	74	63	53	32	42	
4		5	11	26	42	32	21	26	
5				5	16	16	11	16	
6								5	
7								5	
8								5	
9									
10									
11									
12									
13									
14									

TEST 6

V_o = 807 FPM
T_o = 49.5° F
T_{db} = 82.5° F
T_{wb} = 59.5° F
P_{bar} = 29.13" H_g

Temperature - °F

	1	2	3	4	5	6	7	8	9
14									84.0
13									83.5
12									83.0
11							83.0		83.0
10					82.5		83.0		83.0
9					82.0		83.0		82.0
8					82.0		82.0		81.5
7			79.0		81.5		81.5		81.5
6			78.5		80.5		81.0		81.0
5	73.0		78.0		80.0		80.0		81.0
4	71.5		77.0		78.0		79.0		80.0
3	69.5		75.0		76.0		78.0		80.0
2	65.0		71.5		74.5		77.0		80.0
1	56.0		68.0		73.0		77.0		80.0
CL	45.0		65.0		73.0		77.0		80.0
1	58.0		68.0		77.0		78.0		81.0
2	63.0		71.5		76.0		78.0		81.0
3	74.0		75.0		77.0		79.0		81.0
4	75.0		77.0		78.0		79.0		81.0
5	76.0		77.0		79.0		80.0		81.0
6	77.0		78.5		81.0		81.0		81.0
7			79.0		81.0		82.0		82.0
8					81.5		83.0		82.5
9					82.0		83.5		83.0
10							84.0		84.0
11									84.0
12									
13									
14									
T_{amb}	84.0		86.0		87.0		87.0		87.0

TEST 6

Velocity - FPM

	1	2	3	4	5	6	7	8	9
14									
13									
12									
11									5
10									5
9									11
8							11		21
7					5		21		52
6					16		42		73
5					31		73		94
4			5		73		115		105
3			41		145		156		126
2	10		144		228		198		157
1	650		389		289		229		178
CL	807		488		362		250		188
1	562		430		313		229		189
2	30		197		208		188		178
3	10		52		125		146		115
4			5		73		94		105
5					31		63		94
6					5		42		63
7							11		42
8									21
9									
10									
11									
12									
13									
14									

TEST 7

V_o = 1099 FPM
T_o = 39.2^{o} F
T_{db} = 78^{o} F
T_{wb} = 56^{o} F
P_{bar} = 28.95" H_g

Temperature - ^{o}F

	1	2	3	4	5	6	7	8	9
14									80.0
13									80.0
12							79.0		79.5
11							79.0		79.0
10					78.0		78.0		79.0
9					78.0		78.0		78.5
8					77.0		78.0		78.0
7			76.0		77.0		77.0		77.0
6			75.0		76.0		76.0		77.0
5	73.5		74.0		75.0		75.0		76.0
4	73.0		73.0		74.0		74.0		75.0
3	72.0		72.0		71.0		73.0		74.0
2	71.0		66.0		68.0		72.0		74.0
1	49.0		58.0		66.0		71.0		74.0
CL	33.0		54.0		65.0		71.0		74.0
1	44.0		59.0		69.0		72.5		76.0
2	63.0		62.0		69.0		72.5		75.5
3	69.0		69.0		70.0		73.0		76.0
4	70.0		72.0		72.0		75.0		76.0
5	71.0		73.5		75.0		76.0		77.0
6	72.0		75.0		76.0		76.0		77.0
7			75.0		76.5		77.0		78.0
8					77.0		78.0		78.5
9					78.0		78.0		79.0
10							79.0		79.5
11									80.0
12									80.0
13									
14									
T_{amb}	81.0		82.0		83.0		84.0		83.5

TEST 7

Velocity - FPM

	1	2	3	4	5	6	7	8	9
14									
13									
12									
11									5
10									16
9							5		32
8							16		52
7					5		31		52
6					10		52		73
5					52		104		115
4			10		94		146		188
3			83		207		208		208
2	31		287		309		260		229
1	1093		556		452		311		250
CL	1106		652		471		352		271
1	688		608		454		353		251
2	41		326		310		270		188
3	5		103		186		187		157
4			5		83		115		115
5					31		73		84
6					5		31		42
7							10		21
8									11
9									11
10									
11									
12									
13									
14									

TEST 8

V_o = 1390.16 FPM
T_o = 46.9° F
T_{db} = 83.5° F
T_{wb} = 61.5° F
P_{bar} = 28.85" H_g

Temperature - °F

	1	2	3	4	5	6	7	8	9
14									
13									
12									
11									
10					83.0				
9					83.0				
8					82.5				
7			80.0		82.0				
6			80.0		82.0				
5	80.0		79.5		81.0				
4	80.0		79.0		80.0				
3	80.0		77.5		78.0				
2	79.5		75.0		74.0				
1	54.0		65.0		71.5				
CL	41.5		59.0		69.5				
1	52.0		62.0						
2	75.0		68.0						
3	76.0		73.5						
4	76.5		79.0						
5	79.0		79.5						
6	80.0		80.0						
7			82.0						
8									
9									
10									
11									
12									
13									
14									
T_{amb}									

TEST 8

Velocity - FPM

	1	2	3	4	5	6	7	8	9
14									
13									
12									
11									
10									
9									
8									
7					5				
6					11				
5					32				
4			5		116				
3			74		179				
2	42		419		355				
1	1409		720		562				
CL	1374		915		648				
1	701		715						
2	26		289						
3	5		73						
4			5						
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									

TEST 9

V_o = 1474.26 FPM
T_o = 50.9° F
T_{db} = 85° F
T_{wb} = 64.5° F
P_{bar} = 28.63" H_g

Temperature - °F

	1	2	3	4	5	6	7	8	9
14									
13									87.0
12									86.5
11							86.0		86.0
10							85.5		86.0
9					84.0		85.0		85.5
8					83.5		85.0		85.0
7					83.0		84.0		84.5
6			81.5		82.0		83.5		84.0
5	81.0		81.0		81.0		82.0		83.0
4	81.0		81.0		80.0		81.5		83.0
3	80.5		80.5		78.0		80.0		82.0
2	80.0		73.0		76.0		79.0		82.0
1	54.0		63.0		73.0		79.0		81.0
CL	41.0		57.5		71.5		78.0		81.0
1	50.0		65.0		75.5		79.0		82.0
2	74.0		70.0		76.0		79.0		82.0
3	75.5		75.0		78.0		80.0		83.0
4	76.0		78.0		80.0		81.0		83.0
5	77.0		79.0		82.0		82.0		84.0
6	78.0		80.0		83.0		83.0		85.0
7			80.0		83.0		84.0		85.0
8					84.0		84.5		86.0
9					84.0		85.0		86.0
10							85.0		86.0
11							86.0		87.0
12									87.0
13									
14									
T_{amb}	87.5		88.0		89.0		89.5		90.0

TEST 9

Velocity - FPM

	1	2	3	4	5	6	7	8	9
14									
13									
12									
11									
10									11
9									32
8							5		32
7							21		54
6					11		54		86
5					32		75		118
4					96		128		150
3			5		180		213		192
2	32		379		381		362		278
1	1420		774		578		425		320
CL	1483		1021		687		488		342
1	755		855		634		425		300
2	32		335		423		319		235
3	5		74		234		234		182
4			11		85		139		139
5					21		86		118
6					5		32		65
7							11		54
8							5		32
9									11
10									
11									
12									
13									
14									

APPENDIX B

Computer Numerical Integration

ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE

```
$JOB
      COMMON CT,GRAV,CTAMB,TG
      REAL KP
  501 FORMAT(7F10.4)
C *****************************************************************
C INITIAL VELOCITY EQUAL TO 1099
C INITIAL TEMPERATURE EQUAL TO 29.2 DEGREES
C *****************************************************************
      READ(5,501)CT,GRAV,KP,TG,CTAMB,VC,DELTO
      X=1/(CTAMB-DELTO)
      H=.01
      VC2=((VC**2)+(2*GRAV*X*DELTO*KP))**.5
      V=VC2
      X=[illegible]
   10 AK1=DV(X,V)*H
      AK2=DV((X+(H/2)),(V+(AK1/2)))*H
      AK3=DV((X+(H/2)),(V+(AK2/2)))*H
      AK4=DV((X+H),(V+AK3))*H
      V=V+((AK1+(2*AK2)+(2*AK3)+AK4)/6)
      VC=[illegible]
      X=X+H
  502 FORMAT(' ','THE CENTERLINE VELOCITY AT',F10.4,' IS',F10.4,' FPM')
      WRITE(6,502)X,VC
      IF(VC-[illegible])20,20,10
   20 CONTINUE
      STOP
      END

      FUNCTION DV(X,V)
      COMMON CT,GRAV,CTAMB,TG
      TC=(-TG*X*X)+(CTAMB*X)+CT
      A1=(-2)/1.5*CT*GRAV/(TC*V)
      A2=(-3)/X*V/2
      A3=V*((-2*TG*X)+CTAMB)/(2*TC)
      DV=A1+A2+A3
      RETURN
      END

$ENTRY
THE CENTERLINE VELOCITY AT    1.0920 IS 1098.7540 FPM
THE CENTERLINE VELOCITY AT    1.1030 IS 1089.2009 FPM
THE CENTERLINE VELOCITY AT    1.1130 IS 1079.3133 FPM
THE CENTERLINE VELOCITY AT    1.1230 IS 1070.5500 FPM
THE CENTERLINE VELOCITY AT    1.1330 IS 1061.5240 FPM
THE CENTERLINE VELOCITY AT    1.1430 IS 1052.6120 FPM
THE CENTERLINE VELOCITY AT    1.1530 IS 1043.9530 FPM
THE CENTERLINE VELOCITY AT    1.1630 IS 1035.2390 FPM
THE CENTERLINE VELOCITY AT    1.1730 IS 1026.7690 FPM
THE CENTERLINE VELOCITY AT    1.1830 IS 1018.4331 FPM
THE CENTERLINE VELOCITY AT    1.1930 IS 1010.2449 FPM
THE CENTERLINE VELOCITY AT    1.2030 IS 1002.1830 FPM
THE CENTERLINE VELOCITY AT    1.2130 IS  994.2524 FPM
THE CENTERLINE VELOCITY AT    1.2230 IS  986.4478 FPM
THE CENTERLINE VELOCITY AT    1.2330 IS  978.7464 FPM
THE CENTERLINE VELOCITY AT    1.2430 IS  971.2061 FPM
THE CENTERLINE VELOCITY AT    1.2530 IS  963.7637 FPM
THE CENTERLINE VELOCITY AT    1.2630 IS  956.4373 FPM
THE CENTERLINE VELOCITY AT    1.2730 IS  949.2234 FPM
THE CENTERLINE VELOCITY AT    1.2830 IS  942.1196 FPM
THE CENTERLINE VELOCITY AT    1.2930 IS  935.1235 FPM
```

APPENDIX C

Computer Entrainment Calculation

```
$JOB
      DIMENSION VEL(16,31),ETRAIN(16)
C ************************************************************************
C INTIAL VELOCITY EQUAL TO 339 FPM
C INTIAL TEMPERATURE EQUAL TO 52 DEGREES
C ************************************************************************
      DO 5 I=1,16
      ETRAIN(I)=0.0
      DO 6 J=1,30
      VEL(I,J)=0.0
    6 CONTINUE
    5 CONTINUE
  501 FORMAT(10F8.3)
  502 FORMAT(8F10.4)
   10 READ(5,501)((VEL(I,J),J=1,30),I=1,7)
      READ(5,502)(VEL(I,31),I=1,16)
      DO 15 I=1,16
      SUME=0.0
      DO 20 J=1,15
      VAVG=(VEL(I,J)+VEL(I,31-J))/2
      RAD=((16-J+.5)**2)-((15-J+.5)**2)
      PI=3.14159
      SUME=SUME+(PI*RAD*VAVG)
   20 CONTINUE
      VPLUS=VEL(I,31)*.5*.5*3.14159
      ETRAIN(I)=SUME+VPLUS
   15 CONTINUE
   25 DO 30 I=1,16
      ETRAIN(I)=ETRAIN(I)/144
   30 CONTINUE
      DO 35 I=1,16
  503 FORMAT(' ','THE FLOW RATE Q AT STATION',I3,   ' IS',F10.4,' CFM')
   35 WRITE(6,503)I,ETRAIN(I)
      STOP
      END

$ENTRY
THE FLOW RATE Q AT STATION  1 IS   13.0954 CFM
THE FLOW RATE Q AT STATION  2 IS   16.6242 CFM
THE FLOW RATE Q AT STATION  3 IS   21.8602 CFM
THE FLOW RATE Q AT STATION  4 IS   21.5275 CFM
THE FLOW RATE Q AT STATION  5 IS   39.6000 CFM
THE FLOW RATE Q AT STATION  6 IS   36.3409 CFM
THE FLOW RATE Q AT STATION  7 IS   38.5226 CFM
THE FLOW RATE Q AT STATION  8 IS    0.0000 CFM
THE FLOW RATE Q AT STATION  9 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 10 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 11 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 12 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 13 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 14 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 15 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 16 IS    0.0000 CFM

CORE USAGE      OBJECT CODE=    1304 BYTES,ARRAY AREA=     2048 BYTES,TOTAL AREA AVAILABLE=  174120  BYTES

DIAGNOSTICS       NUMBER OF ERRORS=       0, NUMBER OF WARNINGS=       0, NUMBER OF EXTENSIONS=       0

COMPILE TIME=     0.10 SEC,EXECUTION TIME=     0.19 SEC,  WATFIV - JUL 1973 V1L4      15.57.37    WEDNESDAY     3 MAR 76
```

```
$JOB
      DIMENSION VEL(16,31),ETRAIN(16)
C ***********************************************************************
C INITIAL VELOCITY EQUAL TO 438 FPM
C INITIAL TEMPERATURE EQUAL TO 51.7 DEGREES
C ***********************************************************************
      DO 5 I=1,16
      ETRAIN(I)=0.0
      DO 4 J=1,30
      VEL(I,J)=0.0
    4 CONTINUE
    5 CONTINUE
  501 FORMAT(10F8.3)
  502 FORMAT(9F10.4)
   10 READ(5,501)((VEL(I,J),J=1,30),I=1,9)
      READ(5,502)(VEL(I,31),I=1,16)
      DO 15 I=1,16
      SUMF=0.0
      DO 20 J=1,15
      VAVG=(VEL(I,J)+VEL(I,31-J))/2
      RAD=((15-J+.5)**2)-((15-J+.5)**2)
      PI=3.14159
      SUMF=SUMF+(PI*RAD*VAVG)
   20 CONTINUE
      VPLUS=VEL(I,31)*.5*.5*3.14159
      ETRAIN(I)=SUMF+VPLUS
   15 CONTINUE
   25 DO 30 I=1,16
      ETRAIN(I)=ETRAIN(I)/144
   30 CONTINUE
      DO 35 I=1,16
  503 FORMAT(' ','THE FLOW RATE Q AT STATION',I3,    ' IS',F10.4,' CFM')
   35 WRITE(6,503)I,ETRAIN(I)
      STOP
      END

$ENTRY
THE FLOW RATE Q AT STATION  1 IS   19.3459 CFM
THE FLOW RATE Q AT STATION  2 IS    0.0000 CFM
THE FLOW RATE Q AT STATION  3 IS   26.6162 CFM
THE FLOW RATE Q AT STATION  4 IS    0.0000 CFM
THE FLOW RATE Q AT STATION  5 IS   34.9956 CFM
THE FLOW RATE Q AT STATION  6 IS    0.0000 CFM
THE FLOW RATE Q AT STATION  7 IS   52.3215 CFM
THE FLOW RATE Q AT STATION  8 IS    0.0000 CFM
THE FLOW RATE Q AT STATION  9 IS   70.9147 CFM
THE FLOW RATE Q AT STATION 10 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 11 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 12 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 13 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 14 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 15 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 16 IS    0.0000 CFM

CORE USAGE      OBJECT CODE=   1304 BYTES,ARRAY AREA=    2048 BYTES,TOTAL AREA AVAILABLE=  174136  BYTES

DIAGNOSTICS        NUMBER OF ERRORS=       0, NUMBER OF WARNINGS=       0, NUMBER OF EXTENSIONS=       0

COMPILE TIME=    0.13 SEC,EXECUTION TIME=    0.24 SEC,  WATFIV - JUL 1973 V1L4      10.29.07    WEDNESDAY    10 MAR 76
```

```
$JOB
      DIMENSION VEL(16,31),ETRAIN(16)
C *****************************************************************************
C INTIAL VELOCITY EQUAL TO 390 FPM
C THE JET IS ISOTHERMAL
C *****************************************************************************
      DO 5 I=1,16
      ETRAIN(I)=0.0
      DO 6 J=1,30
      VEL(I,J)=0.0
    6 CONTINUE
    5 CONTINUE
  501 FORMAT(10F8.3)
  502 FORMAT(8F10.4)
   10 READ(5,501)((VEL(I,J),J=1,30),I=1,8)
      READ(5,502)(VEL(I,31),I=1,16)
      DO 15 I=1,16
      SUMF=0.0
      DO 20 J=1,15
      VAVG=(VEL(I,J)+VEL(I,31-J))/2
      RAD=((16-J+.5)**2)-((15-J+.5)**2)
      PI=3.14159
      SUMF=SUMF+(PI*RAD*VAVG)
   20 CONTINUE
      VPLUS=VEL(I,31)*.5*.5*3.14159
      ETRAIN(I)=SUMF+VPLUS
   15 CONTINUE
   25 DO 30 I=1,16
      ETRAIN(I)=ETRAIN(I)/144
   30 CONTINUE
      DO 35 I=1,16
  503 FORMAT(' ','THE FLOW RATE Q AT STATION',I3,    ' IS',F10.4,' CFM')
   35 WRITE(6,503)I,ETRAIN(I)
      STOP
      END

$ENTRY
THE FLOW RATE Q AT STATION  1 IS    16.9151 CFM
THE FLOW RATE Q AT STATION  2 IS    26.9599 CFM
THE FLOW RATE Q AT STATION  3 IS    31.2850 CFM
THE FLOW RATE Q AT STATION  4 IS    34.4156 CFM
THE FLOW RATE Q AT STATION  5 IS    38.3699 CFM
THE FLOW RATE Q AT STATION  6 IS    37.6113 CFM
THE FLOW RATE Q AT STATION  7 IS    32.6594 CFM
THE FLOW RATE Q AT STATION  8 IS    33.8533 CFM
THE FLOW RATE Q AT STATION  9 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 10 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 11 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 12 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 13 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 14 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 15 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 16 IS     0.0000 CFM

CORE USAGE        OBJECT CODE=    1304 BYTES,ARRAY AREA=    2048 BYTES,TOTAL AREA AVAILABLE=  174136  BYTES

DIAGNOSTICS        NUMBER OF ERRORS=       0, NUMBER OF WARNINGS=       0, NUMBER OF EXTENSIONS=      0

COMPILE TIME=     0.09 SEC,EXECUTION TIME=     0.16 SEC,  WATFIV - JUL 1973 V1L4      15.58.05     WEDNESDAY      3 MAR 76
```

```
$JOB
      DIMENSION VEL(16,31),ETRAIN(16)
C ***************************************************************************
C INTIAL TEMPERATURE EQUAL TO 49.5 DEGREES
C INTIAL VELOCITY EQUAL TO 907 FPM
C ***************************************************************************
      DO 5 I=1,16
      ETRAIN(I)=0.0
      DO 6 J=1,30
      VEL(I,J)=0.0
    6 CONTINUE
    5 CONTINUE
  501 FORMAT(10F8.3)
  502 FORMAT(F10.4)
   10 READ(5,501)((VEL(I,J),J=1,30),I=1,9)
      READ(5,502)(VEL(I,31),I=1,16)
      DO 15 I=1,16
      SUME=0.0
      DO 20 J=1,15
      VAVG=(VEL(I,J)+VEL(I,31-J))/2
      RAD=(((16-J+.5)**2)-((15-J+.5)**2)
      PI=3.14159
      SUME=SUME+(PI*RAD*VAVG)
   20 CONTINUE
      VPLUS=VEL(I,31)*.5*.5*3.14159
      ETRAIN(I)=SUME+VPLUS
   15 CONTINUE
   25 DO 30 I=1,16
      ETRAIN(I)=ETRAIN(I)/144
   30 CONTINUE
      DO 35 I=1,16
  503 FORMAT(' ','THE FLOW RATE Q AT STATION',I3,    ' IS',F10.4,' CFM')
   35 WRITE(6,503)I,ETRAIN(I)
      STOP
      END

$ENTRY
THE FLOW RATE Q AT STATION  1 IS    33.2430 CFM
THE FLOW RATE Q AT STATION  2 IS     0.0000 CFM
THE FLOW RATE Q AT STATION  3 IS    41.9315 CFM
THE FLOW RATE Q AT STATION  4 IS     0.0000 CFM
THE FLOW RATE Q AT STATION  5 IS    74.8199 CFM
THE FLOW RATE Q AT STATION  6 IS     0.0000 CFM
THE FLOW RATE Q AT STATION  7 IS    59.9400 CFM
THE FLOW RATE Q AT STATION  8 IS     0.0000 CFM
THE FLOW RATE Q AT STATION  9 IS   120.0130 CFM
THE FLOW RATE Q AT STATION 10 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 11 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 12 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 13 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 14 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 15 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 16 IS     0.0000 CFM

CORE USAGE        OBJECT CODE=    1304 BYTES,ARRAY AREA=    2048 BYTES,TOTAL AREA AVAILABLE=  174136  BYTES

DIAGNOSTICS         NUMBER OF ERRORS=       0, NUMBER OF WARNINGS=       0, NUMBER OF EXTENSIONS=       0

COMPILE TIME=     0.15 SEC,EXECUTION TIME=     0.29 SEC,  WATFIV - JUL 1973 V1L4      21.30.51     TUESDAY        2 MAR 76
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```
$JOB
      DIMENSION VEL(16,31),ETRAIN(16)
C ****************************************************************************
C INTIAL VELOCITY EQUAL TO 1099 FPM
C INTIAL TEMPERATURE EQUAL TO 39.2 DEGREES
C ****************************************************************************
      DO 5 I=1,16
      ETRAIN(I)=0.0
      DO 6 J=1,30
      VEL(I,J)=0.0
    6 CONTINUE
    5 CONTINUE
  501 FORMAT(10F8.3)
  502 FORMAT(3F10.4)
   10 READ(5,501)((VEL(I,J),J=1,30),I=1,9)
      READ(5,502)(VEL(I,31),I=1,16)
      DO 15 I=1,16
      SUME=0.0
      DO 20 J=1,15
      VAVG=(VEL(I,J)+VEL(I,31-J))/2
      RAD=((16-J+.5)**2)-((15-J+.5)**2)
      PI=3.14159
      SUME=SUME+(PI*RAD*VAVG)
   20 CONTINUE
      VPLUS=VEL(I,31)*.5*.5*3.14159
      ETRAIN(I)=SUME+VPLUS
   15 CONTINUE
   25 DO 30 I=1,16
      ETRAIN(I)=ETRAIN(I)/144
   30 CONTINUE
      DO 35 I=1,16
  503 FORMAT(' ','THE FLOW RATE Q AT STATION',I3,   ' IS',F10.4,' CFM')
   35 WRITE(6,503)I,ETRAIN(I)
      STOP
      END

$ENTRY
THE FLOW RATE Q AT STATION  1 IS   48.3565 CFM
THE FLOW RATE Q AT STATION  2 IS    0.0000 CFM
THE FLOW RATE Q AT STATION  3 IS   65.1804 CFM
THE FLOW RATE Q AT STATION  4 IS    0.0000 CFM
THE FLOW RATE Q AT STATION  5 IS  102.2925 CFM
THE FLOW RATE Q AT STATION  6 IS    0.0000 CFM
THE FLOW RATE Q AT STATION  7 IS  126.8416 CFM
THE FLOW RATE Q AT STATION  8 IS    0.0000 CFM
THE FLOW RATE Q AT STATION  9 IS  152.8633 CFM
THE FLOW RATE Q AT STATION 10 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 11 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 12 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 13 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 14 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 15 IS    0.0000 CFM
THE FLOW RATE Q AT STATION 16 IS    0.0000 CFM

CORE USAGE      OBJECT CODE=    1304 BYTES,ARRAY AREA=    2048 BYTES,TOTAL AREA AVAILABLE=  174136  BYTES

DIAGNOSTICS       NUMBER OF ERRORS=      0, NUMBER OF WARNINGS=      0, NUMBER OF EXTENSIONS=      0

COMPILE TIME=    0.11 SEC,EXECUTION TIME=    0.13 SEC,  WATFIV - JUL 1973 V1L4     10.29.22    WEDNESDAY    10 MAR 76
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```
$JOB
      DIMENSION VEL(15,31),ETRAIN(16)
C ***************************************************************************
C INITIAL VELOCITY EQUAL TO 1424 FPM
C INITIAL TEMPERATURE EQUAL TO 50.9 DEGREES
C ***************************************************************************
      DO 5 I=1,16
      ETRAIN(I)=0.0
      DO 6 J=1,30
      VEL(I,J)=0.0
    6 CONTINUE
    5 CONTINUE
  501 FORMAT(10F8.3)
  502 FORMAT(8F10.4)
   10 READ(5,501)((VEL(I,J),J=1,30),I=1,9)
      READ(5,502)(VEL(I,31),I=1,16)
      DO 15 I=1,16
      SUMF=0.0
      DO 20 J=1,15
      VAVG=(VEL(I,J)+VEL(I,31-J))/2
      RAD=(((15-J+.5)**2)-((15-J+.5)**2))
      PI=3.14159
      SUMF=SUMF+(PI*RAD*VAVG)
   20 CONTINUE
      VPLUS=VEL(I,31)*.5*.5*3.14159
      ETRAIN(I)=SUMF+VPLUS
   15 CONTINUE
   25 DO 30 I=1,16
      ETRAIN(I)=ETRAIN(I)/144
   30 CONTINUE
      DO 35 I=1,16
  503 FORMAT(' ','THE FLOW RATE Q AT STATION',I3,    ' IS',F10.4,' CFM')
   35 WRITE(6,503)I,ETRAIN(I)
      STOP
      END

$ENTRY
THE FLOW RATE Q AT STATION  1 IS    58.5294 CFM
THE FLOW RATE Q AT STATION  2 IS     0.0000 CFM
THE FLOW RATE Q AT STATION  3 IS    78.3924 CFM
THE FLOW RATE Q AT STATION  4 IS     0.0000 CFM
THE FLOW RATE Q AT STATION  5 IS   116.0370 CFM
THE FLOW RATE Q AT STATION  6 IS     0.0000 CFM
THE FLOW RATE Q AT STATION  7 IS   123.3553 CFM
THE FLOW RATE Q AT STATION  8 IS     0.0000 CFM
THE FLOW RATE Q AT STATION  9 IS   171.4392 CFM
THE FLOW RATE Q AT STATION 10 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 11 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 12 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 13 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 14 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 15 IS     0.0000 CFM
THE FLOW RATE Q AT STATION 16 IS     0.0000 CFM

CORE USAGE        OBJECT CODE=    1304 BYTES,ARRAY AREA=    2048 BYTES,TOTAL AREA AVAILABLE=  174104  BYTES

DIAGNOSTICS         NUMBER OF ERRORS=       0, NUMBER OF WARNINGS=       0, NUMBER OF EXTENSIONS=       0

COMPILE TIME=     0.17 SEC,EXECUTION TIME=     0.29 SEC,  WATFIV - JUL 1973 V1L4      21.31.24     TUESDAY       2 MAR 76
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APPENDIX D

Uncertainity Analysis

EXPERIMENTAL ERROR

Velocity Measurements

at V = 600 FPM $\lambda_{vm} \cong 2\%$

at V = 500 to 1000 $\lambda_{vm} \cong 4\%$

at very low velocities V < 25 $\lambda_{vm} \cong 150\%$

Temperature Measurements

digital multimeter error – $\lambda_M = .1\% + 1$ digit $\simeq .1\%$

nanovolt amplified error $\cong 0$

thermocouple error = 1 1/2 $^oF \simeq 2.5\%$ at T = 60 ^oF

total temperature error – $\lambda_{TT} = 2.5\%$

Positioning Error

horizontal location: resolution error for an 8 ft 1/16" division steel tape,

$$\lambda_H = \sqrt{\left(\frac{1/16}{2}\right)^2 + \left(\frac{1/16}{2}\right)^2} = .044194''$$

vertical location: resolution error for a 1/16" division steel tape, $\lambda_V = .044194''$

vertical positioning error, $\lambda_{VP} = 1/16'' = .0625''$

horizontal positioning error, $\lambda_{HP} \cong 0$

total error, $\lambda_{TP} = .0883'' \simeq 8.83\%$ for 1" increments

APPENDIX E

Hyperbolic Decay Constants

and

Stratification Factors

V_o

	339	438	807	1099	1474
C_t	-30.4567	-37.50704	-35.804928	-34.1206	-51.3495
T_g	-.78571	-.4000	-.71682936	-.7000	-.673396

APPENDIX F

Distance from

Apparent Point Source

to Outlet

	V_o					
	339	438	390	807	1099	1474
$x - x_n$	4.51"	5.54"	3.59"	6.48"	5.95"	6.68"

INVESTIGATION OF AN AXISYMMETRICAL CHILLED VERTICAL JET PROJECTED INTO A STRATIFIED ENVIRONMENT

by

THOMAS F. BAILEY

B.S., Kansas State University, 1975

AN ABSTRACT OF A MASTER THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1976

ABSTRACT

A numerically integrated equation, derived from the integral momentum equation, describing the centerline velocity of a axisymmetrical vertical chilled jet projected in a stratified environment, is experimentally verified. The momentum equation was simplified utilizing, similarity of temperature and velocity profiles, assumption of ideal gas relations and assuming the centerline temperature difference decays hyperbolically. The similarity and hyperbolical decay assumptions were experimentally verified. Several parameters were futher investigated using both the experimental and theoretical results. It was found the entrainment ratio is approximately linear and the effect of increasing the stratification of the environment upon the centerline velocity is negligible.

VITA

Thomas F. Bailey

Candidate for the Degree of

Master of Science

Thesis: INVESTIGATION OF AN AXISYMMETRICAL CHILLED VERTICAL JET PROJECTED INTO A STRATIFIED ENVIRONMENT

Biographical:

Personal Data: Born in Casper, Wyoming, April 1, 1953, the son of Thomas F. and Barbara W. Bailey.

Education: Attended grade school in Kansas City, Missouri; graduated from Topeka West High School in 1971; received the Bachelor of Science degree from Kansas State University, with a major in Mechanical Engineering in May, 1975.

Professional Experience: Designer, draftsman, surveyor, Black & Veatch Consultant Engineers, 1971-1975; graduate research assistant, Kansas State University, 1975-1976.