INVENTORY CONTROL FOR FINISHED FEEDS UNDER CONSTRAINT

by

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GLOSSARY

BINS = bin size needed for a feed type, tons.

C = per ton cost of feed, \$.

C_c = carrying cost per day as a percent of feed cost.

C_r = ordering cost per order, \$.

K = Tchebysheff's constant.

$\bar{K}$ = fixed, out of stock cost, \$/order.

L = Lagrangian.

LT = lead time between placing an order and receiving the quantity ordered, days.

n = number of types of feed in the A group.

P = probability.

R = average demand through the lead time, tons.

ROP = reorder point, tons.

S = standard deviation of demand through lead time, tons.

SS = total storage available for finished feed, in bulk, of group A feeds, without inventory for demand in lead time, tons.

S1 = standard deviation of the demand (in the same given time period as Z), tons.

T.C = total cost of an inventory policy in a given period of time, \$.

TS = total storage available for storing finished feed in bulk, tons.

TSA = total storage available for finished feed, in bulk, of group A feeds, tons.

TSM = total storage for mix-up, tons.

TSO = total storage available for finished feed, in bulk, for all feeds except group A, tons.

W = safety stock, tons.

X = lot size order, tons.

Z = average demand per type in a given time period, tons.

λ = Lagrange multiplier.

CONTENTS

INTRODUCTION

In recent years, there has been considerable development in the feed industry of techniques to achieve better efficiency in feed mills. Sophisticated operations research techniques together with computer applications give management better tools for decision making.

Two major decisions facing a production manager in the feed mill are: when to produce additional quantities of a specific feed type and how much of this material to produce.

While most of the development has been made in the field of nutrition and formulation of feeds (4,16), little has been published on using these techniques in Production and Inventory-Control.

The studies that have been published generally concern the designing and planning of new facilities (6), and the Inventory-Control of raw materials (17). Yet, the problems of the operation of an existing feed mill have not been emphasized, although these problems arise daily and must be resolved by management frequently.

This study is an attempt to give a solution to one problem which faces the production manager daily.

General feed mills produce a large variety of feed formulas, in various forms such as mash, pellets, crumbles or expanded products. These may be produced in several package forms or in bulk. A classification, resulting from a simple analysis, can determine the relative importance of each of the many products, as related to Production Scheduling and Inventory-Control.

Every feed mill has its total storage capacity for finished feed in bulk divided into bins. When designing the total storage capacity, the number of bins and their sizes, many factors must be taken into consideration. In an existing feed mill this storage becomes, in most cases, a constraint. The problem of assigning bins to types of feeds according to the constraints which exist in the system should not be solved according to a rule of thumb, such as: filling the first empty bin available.

It is desirable to have a mathematical form for describing costs and other relations found in the storage of finished feeds. Although it should be simple and easy to handle, it must be flexible enough to adequately approximate a wide variety of situations. It is further desirable to obtain decision rules from which actual decisions can be calculated by substituting numerical data into a formula. The substitution of the data from a particular situation would yield optimal decision for the particular problem. A simple program which is general enough for multi-formula feed mills would help in finding the optimal solution in a given storage situation. It should consider the problems of calculating for each product the physical quantities of production batches, safety stock levels and reorder points, together with the total cost.

After finding the optimal quantities, the task of assigning bins to feeds becomes a combinatorial problem. In a real life situation, however, according to the constraints of each bin or feed, the number of combinations reduces to a problem that can be handled and solved relatively easily.

The aim of this study was to reduce operational costs in a feed mill. Reducing the number of times a given type of feed will be produced to a minimum, will affect and smooth the production as a whole. In addition to other considerations, holding an optimal safety stock most probably will reduce the total cost to its optimal value.

The data used in this study is taken from "AMBAR" central feed mill in Israel. The author hopes that this work will contribute to improved decision making in the manufacturing of feeds.

REVIEW OF LITERATURE

Most of the Production Inventory Control Models suggested by several authors in the feed industry deal with four aspects:

1. Designing storage for raw materials and ingredients for a new feed mill.
2. Analyzing, according to Queueing Theory, an inventory policy of ordering raw materials in an existing feed mill.
3. Analyzing the demand distribution for feeds by customers, and developing an inventory control policy for finished feeds in designing a new feed mill or bin sizing.
4. Using a rule of thumb in order to design storage capacity for finished feed.

Stafford and Snyder (17), developed an Inventory Control Simulator which simulates three major operations:

1. Forecasting the magnitude of the ingredients required for a period of production.
2. Scheduling incoming ingredient shipments for this period's production according to forecasting requirements and safety stock policy.
3. Recording the cost effects of the resultant flow of ingredients into and out of inventory.

Pfost (14) developed, according to Queueing Theory, formulas for sizing raw material bins for a new feed mill. He also suggested an ordering policy of raw materials in an existing mill.

Carrillo (2) worked out methods of analyzing the demand distribution functions of several feeds. He approximated the order size distribution by polynomial expressions and the number of orders by Poisson distribution. According to these functions he calculated the optimal inventory policy quantities, without any constraints or limitation.

Some designers like Cory (3) use a rule of thumb in designing space for finished feed by providing space for at least a three day stock.

Much literature is published in the general area of Production and Inventory Control without constraints. Less is published in the case of shortage in capital investment, limited storage capacity, or other constraints.

Starr and Miller (19) show a way to solve dynamic inventory control problems under certainty in constrained situations such as working capital or number of orders to be produced. These problems have been solved using Lagrange Multipliers.

Hadley and Whitin (8), describe situations of constraints in dollar investments, floor space, number of back orders, number of orders and in volume. Almost all of those problems were worked out in deterministic models or with well behaved functions in Stochastic models. The approach toward solving these problems is dynamic programming or with Lagrange Multipliers. An explanation of the economical meaning of the Lagrange multiplier is also given.

Buchan and Königsberg (1) describe Inventory Control Models in various kinds of industries and businesses. By using analytical or simulation methods they improve total costs and inventory policies. Most of their models take into account that there are no limitations and that there is enough storage capacity for the optimal inventory.

Holt, Modigliani, Muth, and Simon (10), solved theoretically and numerically, some inventory problems with restrictions using Lagrange Multipliers. They provided mathematical models for decision rules in a large variety of situations in industries and warehouses. Many situations under certainty as well as stochastic conditions with well behaved function were described.

Eilon (5) used a linear programming model to optimize profit subject to production constraints of the individual products.

PROCEDURE

The problem of assigning bins to finished feeds in a feed mill may be a daily task of the production manager, if no technique of analyzing the demand is used. A careful examination of the situation will lead to a general approach of solving the problem.

In part I, a procedure for testing the importance of each individual feed type will be used. As a result of this procedure, a guide line to an inventory policy for groups of types of feeds will be given.

In part II, several Inventory Control policies will be described, some definitions offered, and the particular model will be formulated.

The constrained feed mill is described in part III, while solving for the optimal total cost and the corresponding physical quantities. The bin assignment procedure end result is described in part IV.

I. Classification of Feed Types.

Most Production Inventory Systems analyses show that a small percentage of produced and stored items account for a dominant percentage of the total production. Therefore the same degree of control is probably not justified for all items, and a common ABC or AB classification is often used. The procedure can be used for the number of types produced, the tonnage per type, the percent of total dollar demand, and the number of orders per type, etc.

The procedure is as follows: List all the products in order from the largest to the smallest value under consideration (descending order). Calculate the percentage each value is of the total value and calculate the cumulative percentage, Table 1. Plot the cumulative percentage against the products, Fig. 1.

Thus, the class A items might justify the most careful control system. Classes B and C ir B by itself are often treated in another way. In most general feed mills a classification according to percentage of the total number of orders and according to percentage of the total tons demanded, will show a small group of feeds in group A. It may show 20-25% of the total number of feeds accounting for 80-85% of the total demand. The author feels, therefore, that this group of feeds, named High-Volume feeds, group A, should be treated in a special Inventory Control policy, regardless of the actual demand for tomorrow, and according to the analysis of the demand over a period of time.

The other 75-80% of the types, which account for about 15-20% of the total demand, should be treated as special orders, each time a demand for them occurs.

II. Inventory Control Policies.

Most authors use a different way of dividing the set of Inventory Control problems into subsets. Let us use the method Starr and Miller (19) suggest and divide the Inventory Control problems into five sections, namely:

(1) Knowledge of demand: certainty, risk, and uncertainty.

(2) Method of obtaining commodity: outside supplier or self supplied.

(3) The decision process: One shot decision-static, or repetitive-dynamic.

(4) Fixed demand distribution over time or varying demand distribution.

(5) Constant time lag in replenishment of inventory or probability distribution.

Building according to the above mentioned criteria the Inventory Control Model of a feed mill depends on two major facts:

(1) The nature of the feed mill.

(2) The information available.

Table 1. Classification of Items According to Percentage of Total Demand.

Item	Demand (units)	Percentage of total	Cumulative percentage
C	98	49	49
L	66	33	82
D	12	6	88
A	10	5	93
B	4	2	95
G	3	1.5	96.5
F	2	1	97.5
E	2	1	98.5
H	2	1	99.5
I	1	0.5	100
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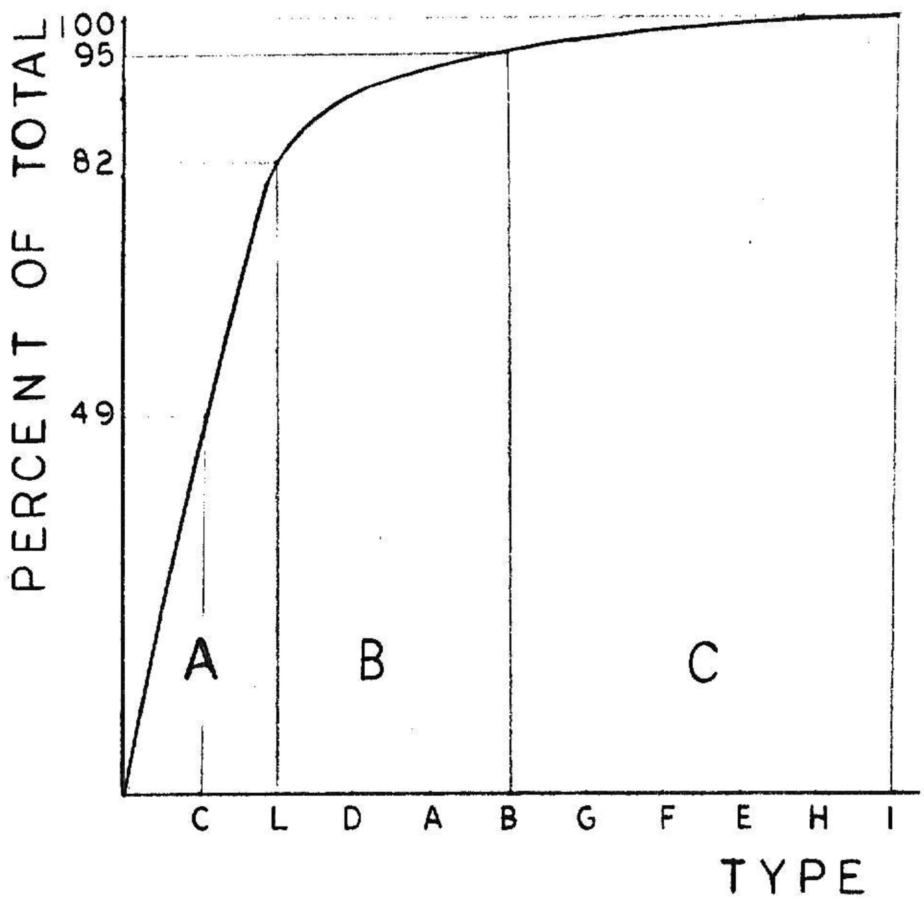

FIG I CLASSIFICATION OF ITEMS

A general feed mill, which produces many types of feed in varying quantities ordered in many time periods belong, of course, to the dynamic class of Inventory Control policy. Taking into account only the finished feeds, in most cases the feed mill is also self supplied.

The lead time policy may differ from one feed mill to another. One can get orders throughout the time before a certain "closing time", and from that time until the feed is supplied can be counted as the lead time.

On the other hand, one can get the orders all the time and count the lead time from the time of receiving the order from the customer up to the time of supplying the feed.

A main distinction must be made between "outside lead time" as described above, and lead time which is associated with production of feed for inventory. The second is the "inside lead time", which describes the relation of time to a type of feed to be produced for inventory. This lead time will be discussed later. Both "inside" and "outside" lead time may be constant or vary according to a function, depending in part or in whole, by types or by customer.

The accuracy of demand information is very important. A high accuracy will yield the lowest total cost of the policy. If the demand is known with certainty, no safety stock has to be held and the only costs are those which are associated with ordering replenishment stock and carrying inventory. If the demand varies according to a probability distribution, fixed or varying, and the function is completely known, a model under risk will be built. Partial information of the demand distribution will lead to a model under uncertainty. Knowing the exact demand distribution will yield a smaller safety stock and therefore reduce total cost when compared with partial information.

Many authors have built the inventory model under risk. The difficulties lie in the nature of the probability function of the demand. If the function is a well behaved continuous known function, the numerical solution can be found much easier than if it is a general function with nonregular shape.

The various techniques of fitting a given distribution function to a known one results in increasing the safety stock. The most accurate model will be the model which solves numerically from the actual demand distribution functions. If only partial knowledge exists for the demand, such as mean and variance, a model under uncertainty can be built. Of course, the safety stock level will be higher than in the case of risk. As will be discussed later, in the constrained feed mill description, a model under uncertainty will be formulated.

Three Inventory Control ordering and review systems are commonly used:

(1) Q system, or reorder point system, in which an additional fixed quantity of material, called lot size order, is produced whenever inventory is reduced to a particular value. The frequency of ordering is determined by the fluctuation in demand. This system is completely determined by knowing the order size and the minimal stock level which represents the signal to place the order (reorder point).

Fig. 2 illustrates the quantities referring to a storage bin. Precisely speaking, the size of the bin must be large enough to contain the three quantities in the extreme case when no demand is required in the lead time between placing an order and the time the feed is actually received in the bin.

Fig. 3 illustrates the fluctuation in the bin over a period of four days when the lead time is half a day. On the first day, rate of demand is constant. When the supply reaches the reorder point, an order for a fixed quantity is placed. During the lead time the demand continues and takes material from part R of the supply. In the beginning of the second day the demand, with a different rate, continues, depleting the R quantity and using from the safety stock quantity (W). Receipt of the ordered quantity of the second day will not fill the bin. Demand on the second day does not cause the supply to reach the reorder point. On the third day the reorder point is reached and the demand through the lead time is zero, and receipt of the lot order size fills the bin. During the third day the supply reached the reorder point again but during the lead time, all R and W, which are stock for demand during lead time and the safety stock, were used. This is a case of "out of stock". The time and quantity out of stock are shown on the fourth day, before the ordered quantity is received.

(2) P system, or reorder time method. (The method used to decide when to order is to look at the calendar.) The size of the order varies with fluctuations in demand. In this system the time between reorders is constant and equal and the size of an order is variable.

(2) Combined method (15), combination of method one and two. Two conditions must be satisfied before a new order can be placed. First, it must be time to reorder, and second, the inventory must have reached or dropped below the reorder point.

It seems but has not been proven that the Q system is the most applicable system for the Inventory Control policy of finished feeds of group A.

After this brief review on the theory, the model used in this study can be defined in Inventory Control terms as a dynamic inventory problem, under uncertainty, self supplied and with a constant lead time for each product in a Q system. The constraint will be discussed in the next part.

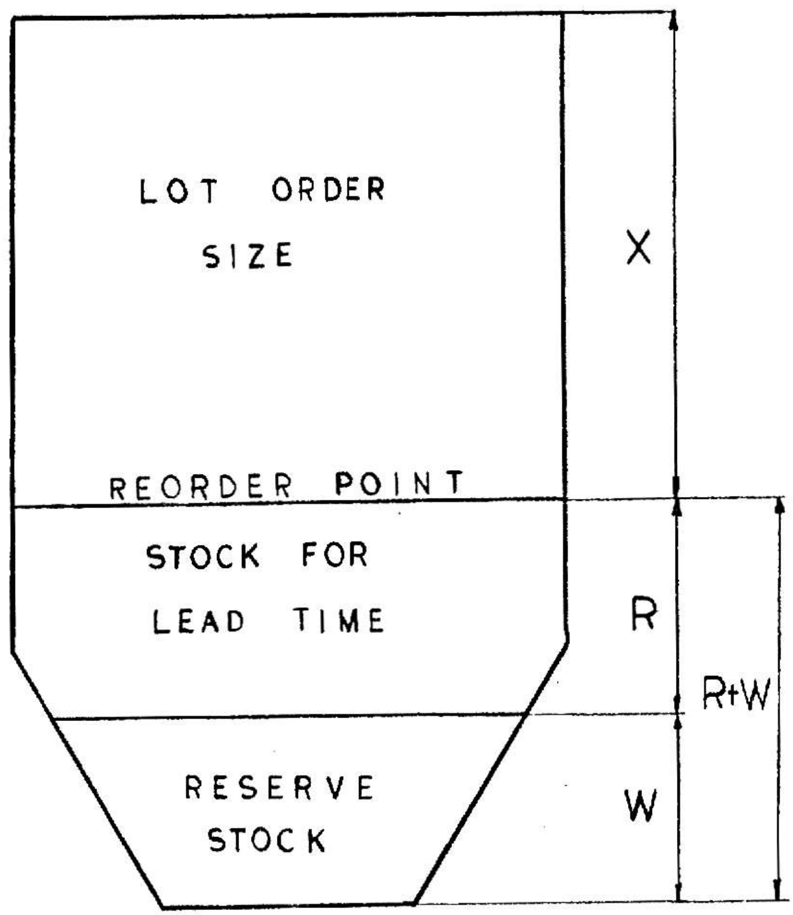

FIG. 2 A BIN SIZE FOR
Q SYSTEM

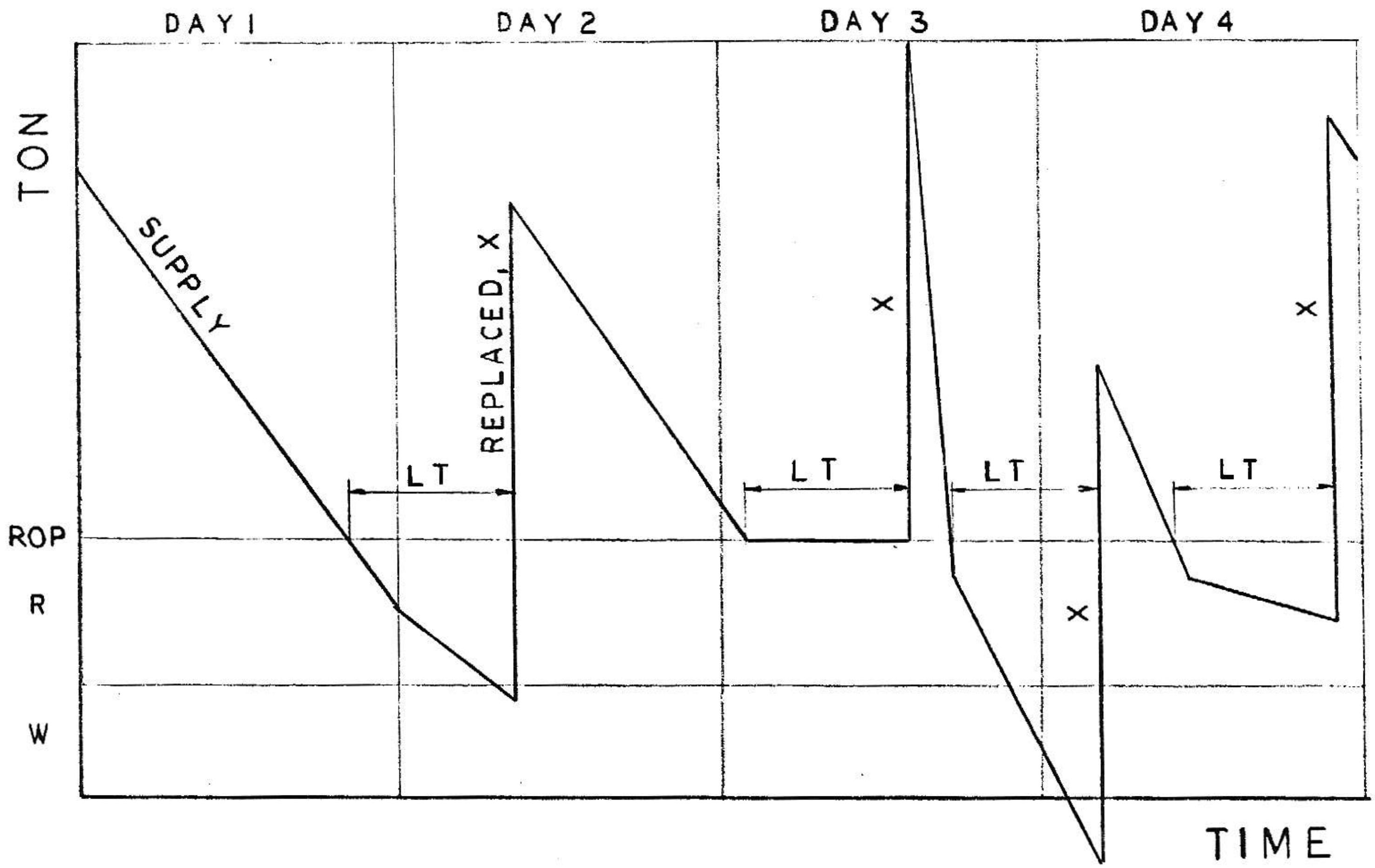

FIG. 3 Q SYSTEM WITH CONSTANT LEAD TIME

Before proceeding further, terms which will be used later need to be defined.

Definitions

Z = average demand per type in a given time period, tons.

S1 = standard deviation of the demand (in the same given time period as Z), tons.

LT = lead time between placing an order and receiving the quantity ordered, days.

S = standard deviation of demand through lead time, tons.

n = number of types of feed in the A group.

TS = total storage available for storing finished feed in bulk, tons.

TSA = total storage available for finished feed, in bulk, of group A feeds, tons.

TSO = total storage available for finished feed, in bulk, for all feeds except group A, tons.

SS = total storage available for finished feed, in bulk, of group A feeds, without inventory for demand in lead time, tons.

X = lot size order, tons.

R = average demand through the lead time, tons.

W = safety stock, tons.

ROP = reorder point, tons.

K = Tchebysheff's constant.

T.C = total cost of an inventory policy in a given period of time, $.

C_r = ordering cost per order, $.

C = per ton cost of a feed, $.

C_c = carrying cost per day as a percent of feed cost.

$\overline{K}$ = fixed, out of stock cost, \$/order.

L = Lagrangian.

λ = Lagrangian Multiplier.

BINS = bin size needed for a feed type, tons.

TSM = total storage for mix-up, tons.

A clear description of the order costs and out of stock costs were given by Carrillo (2), and are quoted in Appendix 1. There are two additional set up costs that are not discussed by Carrillo. The first one is simply the cost of changing a formula in the mixer and its system. In a semiautomated mill it means changing the card and waiting until the whole conveying system is cleaned (mostly by itself). The second is concerned with medicated or special feed, which requires cleaning of the system and additional manual work in the premix area and in the mixing system. Sometime caution has to be taken, which takes operation time, to produce two very different formulas each following the other.

LT, the lead time, is considered here as the "inside lead time". It is simply described as the time between indication of the signal that a type reaches the reorder point, and the time that the full lot size order was placed in the bin.

Each total cost equation in the Inventory Control Theory is essentially based on two components, which balance one another. Those two components are the carrying costs and the set up and out of stock costs. Carrying costs, in general, include the cost of carrying an average inventory, including safety stock, throughout the period of time in consideration and cost of facilities. Their opposing costs are built, in general, from two parts; the order cost and the out of stock cost. It is important, when dealing with the equations, to consistently use the same time unit in all the terms. The time unit that will be used here will be one day.

In the Q system, the number of times a feed is produced per day, i.e., the number of orders, is

$$\frac{Z}{X}.$$

The ordering costs, are therefore

$$\frac{Z\ Cr}{X}.$$

The average inventory which is being carried is $\frac{X}{2}$, and the carrying costs for the inventory are:

$$\frac{X}{2}\ CC_c\ .$$

The next terms are concerned with carrying the safety stock and the probability of being out of stock.

Completely knowing the demand distribution, (the case under risk), the costs of being out of stock are described by

$$\frac{Z\ \overline{K}}{X} \int_{R+W}^{\infty} f(y)\ dy$$

where f(y) is the probability distribution function of the demand during the lead time and the integrals' limits were from the reorder point to infinity (19). In this thesis, the demand distribution function is not known, but the mean and standard deviation of the demand are known.

A tool which helps in considering this situation is the Tchebyscheff inequality Theorem (7). If y is a discrete random variable whose distribution has the mean Z and the standard deviation S, then for any positive constant K the probability that y assumes a value less than Z - KS or greater than Z + KS is less or equal to $\frac{1}{K^2}$

$$P\ (|y - Z| \geq KS) \leq \frac{1}{K^2}\ .$$

This inequality indicates that the probability that demand will differ from the mean by more than KS is always less than or equal to $\frac{1}{K^2}$.

A more detailed discussion on Tchebyscheff's Theorem can be found in (7) and (19). When interested only in the probability of deviations from the mean in one direction, a better upper limit of the probability is Cramer's inequality:

$$P\ ((y - z) \geq KS) \leq \frac{1}{K^2+1}\ .$$

If the demand distribution is known to be symmetrical, the right-hand term will change to $\frac{1}{2K^2}$.

In this study the term $\frac{1}{2K^2}$ is used, namely:

$$P\ ((y - Z) \geq KS) \leq \frac{1}{2K^2}\ .$$

Returning to the total cost equation, it must be recognized that there should be enough stock for the demand through the lead time period, and a safety stock for fluctuation of the demand.

Assuming the demand during the lead time behaves like the demand in the whole period of time, the stock for demand in lead time will be the convuluted average demand

$$R = (LT)(Z)$$

The costs of carrying R are included in the cost of carrying the average inventory. The reserve stock will be KS,
where S is the convuluted standard deviation through the lead time, or

$$S = S1\sqrt{LT}\ .$$

The safety stock carrying cost is then

$$KSCC_c\ .$$

The probability of being out of stock is

$$P\ (y \geq R + KS).$$

using the Tchebysheff's Theorem, it can be approximated

$$P\ (y \geq R + KS) \leq \frac{1}{2K^2}\ .$$

If regardless of the number of units short, a fixed out of stock cost, $\overline{K}$, is assigned to the probability of being out of stock times $\frac{Z}{X}$, the number of times it may occur, the cost for being out of stock are,

$$\frac{Z\overline{K}}{X\ 2K^2}\ .$$

Collecting all the terms described above, the total cost equation for one day for each type of feed will be:

$$T.C_i = \frac{Z_i\ C_{ri}}{X_i} + \frac{X_i}{2} C_i C_{ci} + K_i S_i C_i C_{ci} + \frac{Z_i\ \overline{K}_i}{X_i} \frac{1}{2K_i^{\ 2}}$$

$$i = 1 \ldots n$$

where i stands for each type of feed.

The total cost of the whole operation will then be, for the Q system,

$$T.C = \sum_{i=1}^{n} T.C_i = \sum_{i=1}^{n} \left\{ \frac{Z_i}{X_i} C_{ri} + \frac{X_i}{2} C_i C_{ci} + K_i S_i C_i C_{ci} + \frac{Z_i \overline{K}_i}{X_i 2K_i^{\ 2}} \right\} .$$

The physical quantities will be:

$$BINS_i = X_i + R_i + W_i = X_i + (LT_i)\ (Z_i) + K_i \sqrt{LT_i}\ \ (Sl_i)$$

$$ROP_i = R_i + W_i = (LT_i)\ (Z_i) + K_i \sqrt{LT_i}\ \ (Sl_i) .$$

The optimal lot size order X_i can be found by the methods of maxima-minima of differentiating the total cost equation according to X_i and equating it to zero.

The problem of optimizing the total cost equation and for solving the physical quantities under constraints will be discussed in the next part.

III. The Constrained Feed Mill.

Constraints and Lagrange Multipliers (8,9).

Consider the problem of minimizing the continuous and differentiable function

$$TC = f(X_i \ldots X_n)$$

subject to the constraint

$$g\ (X_i \ldots X_n) = SS$$

where $g\ (X_i \ldots X_n)$ is also continuous and differentiable.

The optimal set of X_i values cannot be found by simply differentiating and equating to zero $\frac{\partial TC}{\partial X_i} = 0$ for $i = 1 \ldots n$ because of the dependence of the variables. A procedure for solving this problem would be to use the constraint to solve for one of the variables, say X_n, to yield $X_n = h\ (X_1 \ldots X_{n-1})$. This expression for X_n is then substituted into f to yield a function f^* of n-1 variables, which can then be optimized by

$$\frac{\partial f^*}{\partial X_i} = 0 \qquad i = 1 \ldots n-1$$

$$f^*\ (X_1 \ldots X_{n-1}) = f\ (X_1 \ldots X_{n-1},\ h(X_1 \ldots X_{n-1}))$$

$$g(X_1 \ldots X_n) = g(X_1 \ldots X_{n-1},\ h(X_i \ldots X_{n-1})) = SS.$$

In order to optimize

$$\frac{\partial f^*}{\partial X_i} = \frac{\partial f}{\partial X_i} + \frac{\partial f}{\partial X_n}\frac{\partial h}{\partial X_i} \qquad i = 1 \ldots n-1$$

and

$$\frac{\partial g}{\partial X_i} + \frac{\partial g}{\partial X_n}\frac{\partial h}{\partial X_i} = 0 \qquad i = 1 \ldots n-1$$

from which

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$$\frac{\partial h}{\partial X_i} = - \frac{\dfrac{\partial g}{\partial X_i}}{\dfrac{\partial g}{\partial X_n}} \qquad , \qquad \frac{\partial g}{\partial X_n} \neq 0$$

so

$$\frac{\partial f^*}{\partial X_i} = \frac{\partial f}{\partial X_i} - \frac{\partial f}{\partial X_n} \frac{\dfrac{\partial g}{\partial X_i}}{\dfrac{\partial g}{\partial X_n}} =$$

$$= \frac{\partial f}{\partial X_i} - \frac{\partial g}{\partial X_i} \frac{\dfrac{\partial f}{\partial X_n}}{\dfrac{\partial g}{\partial X_n}} \qquad i = 1 \ldots n-1,$$

let

$$\lambda = - \frac{\dfrac{\partial f}{\partial X_n}}{\dfrac{\partial g}{\partial X_n}} .$$

The optimal X_i values must then satisfy n + 1 equations

$$\frac{\partial f}{\partial X_i} + \lambda \frac{\partial g}{\partial X_i} = 0 \qquad i = 1\ldots n$$

$$g(X_i \ldots X_n) = SS$$

and can be found by solving them.

The procedure becomes then:

$$L = f(X_i \ldots X_n) + \lambda\,(g(X_i \ldots X_n) - SS)$$

$$\frac{\partial L}{\partial X_i} = 0 \qquad i = 1\ldots n$$

$$\frac{\partial L}{\partial \lambda} = 0$$

which provides n + 1 equations with n + 1 unknowns.

The economic interpretation of the Lagrange multiplier is, that λ is the amount by which the minimum TC can be reduced by adding an additional unit of resource SS. A proof is given in (8).

A brief description of the Newton Raphson's Method (12,13) with one variable will be given here. The method with many variables will be stated. Solving the root of an algebraic equation may sometimes become very difficult because of the complicated expressions. This method improves an approximation to a root of an equation in the form $f(X) = 0$. Figure 4 shows a function $f(X)$. Assume that $X = X_n$ is a first approximation of a root. A tangent line to the curve at $X = X_n$ intersects the X axis at the value X_{n+1} which is an improved approximation to the root (by ΔX).

It can be seen that

$$f'(X_n) = \frac{f(X_n)}{X_n - X_{n+1}} = \frac{f(X_n)}{\Delta X}$$

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$$

the same procedure repeated with the new approximation to get a better approximation to the root.

This procedure continues until successive values of the approximated root differ by less than a prescribed small amount, EPSI, which controls the allowance error in the root.

In the case of more than one variable and many functions, the expression becomes:

$$0 = f_1(X_1 + \Delta X_1,\ X_2 + \Delta X_2, \ldots X_n + \Delta X_n) =$$

$$= f_1(X_1 \ldots X_n) + \frac{\partial f_1}{\partial X_1}(X_1 \ldots X_n)\,\Delta X_1 + \ldots +$$

$$\ldots + \frac{\partial f_1}{\partial X_n}(X_1 \ldots X_n)\,\Delta X_n + \text{Higher orders}$$

$$\begin{matrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{matrix}$$

$$0 = f_n(X_1 + \Delta X_1,\ X_2 + \Delta X_2, \ldots X_n + \Delta X_n) =$$

$$= f_n(X_1 \ldots X_n) + \frac{\partial f_n}{\partial X_1}(X_1 \ldots X_n)\,\Delta X_1 + \ldots$$

$$+ \ldots\ldots \frac{\partial f_n}{\partial X_n}(X_1 \ldots X_n)\,\Delta X_n + \text{Higher orders.}$$

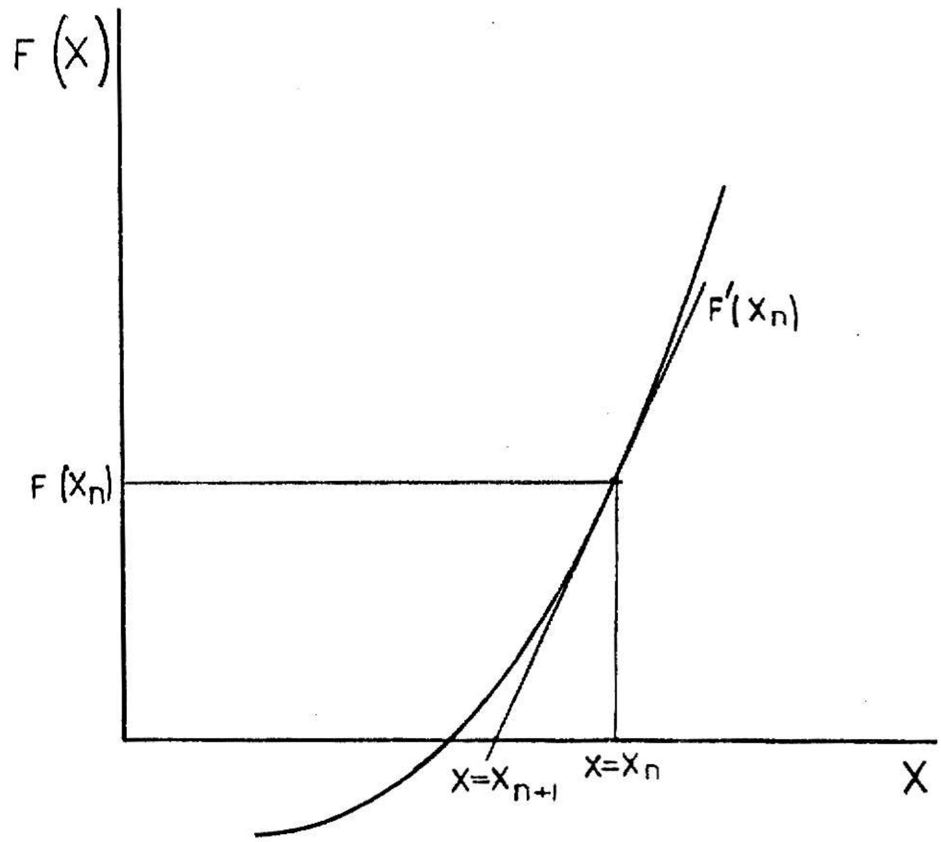


FIG.4 NEWTON-RAPHSON'S METHOD

Constraints in an Existing Feed Mill.

There are many constraints in an existing feed mill, associated with equipment, money, storage, raw materials or work force. As this work is concerned with the finished feed bins, we will analyze the constraints related to them. There are two main constraints:

1. Total storage capacity of all bins.

2. Number of bins.

Each individual bin has at least three constraints:

1. Is it a pellet or mash bin or both?

This may depend on the discharging equipment, flow angles, etc.

2. Connection of the bin to the system.

(As an example, a pellet bin may be connected only to one of many pellet mills.)

3. The size of the bin.

A general constraint is that one bin can be filled only with one type of feed at the same time, regardless of the size of the bin. (This constraint might cause a situation in which two tons of a particular feed are in a bin of ten ton capacity.) This constraint simply says, that until the material in the bin has been dispatched, the bin can not be used for another material.

In addition to the classification of types of feeds to "A" group and to "B" group, data analysis for each type has to be done. This analysis will provide the average and standard deviation for each feed type.

From now on the "A" group feed types will be called "Assigned feeds" and all the others "nonassigned feeds". The bins for assigned feeds will be called "Assigned bins". The bins for nonassigned feeds will be called "nonassigned bins". An assumption can be made here, that every nonassigned bin can be used twice a day. This assumption is, of course, particular. Any other assumption is relevant, and will not lead from the generalization of the problem. With this assumption and the knowledge of how many nonassigned feeds are demanded on the average per day and in what sizes, a group of bins have to be reserved for them. Another need for bins is for mix-up. A mix-up bin is used for material produced not according to the formulation, because of an error made by the operator. It is to be held for reprocessing. The number of bins for this purpose varies and is dependent on the system.

So

$$TS = TSA + TSO + TSM$$

$$TSA = TS - TSO - TSM \quad .$$

TSA now becomes the constraint to the optimization of the total cost for all assigned feeds.

Therefore, the formulation of the next part will deal only with the assigned feeds and assigned bins. By mentioning assignment of bins to feeds, it is assumed that some types will need a group of assigned bins while others may need only one bin.

Before formulating the problem, it is necessary to have some assumptions:

1. The average and standard deviation of the daily demand in tons per feed type are known.

2. The lead time is constant for each feed, (but need not be the same for all or part of the formulas), and is dependent on the average demand for that feed.

3. The total storage capacity available for all finished feeds in bulk is known, as well as the total storage available for assigned feeds.

4. The prices and costs for each feed are known, as well as the operation costs of ordering and set up.

5. None of the feeds under consideration can be omitted. The optimal solution has to include all feeds with nonzero values.

The size of the bin for a feed will be

$$BINS = X + W + R = X + KS + (LT)(Z)$$

and

$$TSA = \sum_{i=1}^{n} BINS = \sum_{i=1}^{n} (X_i + K_iS_i + (LT_i)(Z_i)) .$$

Assuming that the demand during the lead time is constant, it can be subtracted,

$$TSA = SS + R$$

$$SS = TSA - R = \sum_{i=1}^{n} (X_i + K_iS_i) .$$

The formulation of the problem will then become: minimize total cost where

$$T.C = \sum_{i=1}^{n} TC_i =$$

$$= \sum_{i=1}^{n} \frac{Z_i C_{ri}}{X_i} + \sum_{i=1}^{n} \frac{X_i}{2} C_i C_{ci} + \sum_{i=1}^{n} K_i S_i C_i C_{ci} + \sum_{i=1}^{n} \frac{Z_i \overline{K}_i}{X_i} \frac{1}{2K_i^2}$$

subject to

$$\sum_{i=1}^{n} (X_i + K_i S_i) = SS \; ;$$

It is important to recognize that the constraint is one total constraint for all assigned feeds, while the aim is to find the optimal quantities for each assigned feed which will yield an optimal total cost of the whole system. Of course, this formulation will not yield the optimal solution of each feed for itself. It will yield the optimal solution for the multiproduct system.

Using the Lagrangian Multiplier for optimizing the total cost equation subject to the constraint:

$$L = \sum_{i=1}^{n} \frac{Z_i C_{ri}}{X_i} + \sum_{i=1}^{n} \frac{X_i}{2} C_i C_{ci} + \sum_{i=1}^{n} K_i S_i C_i C_{ci} + \sum_{i=1}^{n} \frac{Z_i \overline{K}_i}{X_i} \frac{1}{2K_i^2}$$

$$+ \lambda \left\{ \sum_{i=1}^{n} (X_i + K_i S_i) - SS \right\} .$$

The unknowns are X , K , and λ .

In order to optimize:

$$\frac{\partial L}{\partial X_i} = 0 = -\frac{Z_i C_{ri}}{X_i^2} + \frac{1}{2} C_i C_{ci} - \frac{Z_i \overline{K}_i}{X_i^2} \frac{1}{2K_i^2} + \lambda$$

$$\frac{\partial L}{\partial K_i} = 0 = S_i C_i C_{ci} - \frac{Z_i \overline{K}_i}{X_i} \frac{1}{K_i^3} + \lambda S_i$$

$$\frac{\partial L}{\partial \lambda} = 0 = \sum_{i=1}^{n} (X_i + K_i S_i) - SS .$$

If n is the number of feeds involved in the total cost equation, then there exist 2n + 1 equations with 2n + 1 unknowns. The solution of those equations by algebraic methods will bring six ordered equations, in which two unknowns are involved in each. For a small number of equations, this may be solved by a trial and error method.

In this case, a numerical method was chosen, The Newton-Raphson's Method, for more than one variable.

In order to solve these 2n + 1 equations according to this method, the partial derivatives of the Lagrangian become the new functions.

$$f_i = \frac{\partial L}{\partial X_i} = 0 = -\frac{Z_i\ C_{ri}}{X_i^2} + \frac{1}{2} C_i C_{ci} - \frac{Z_i\ \overline{K}_i}{X_i^2}\ \frac{1}{2K_i^2} + \lambda$$

$$i = 1 \ldots n$$

$$g_i = \frac{\partial L}{\partial K_i} = 0 = S_i C_i C_{ri} - \frac{Z_i\ \overline{K}_i}{X_i}\ \frac{1}{K_i^3} + \lambda\ S_i$$

$$i = 1 \ldots n$$

$$h = \frac{\partial L}{\partial \lambda} = 0 = \sum_{i=1}^{n} (X_i + K_i S_i) - SS$$

There are 2n + 1 equations with 2n + 1 unknowns.

The partial derivative of each of these functions must be taken with respect to each (2n + 1) variable.

As will be shown, three partial derivatives for each function supply all needed information because each function is related to one type of feed only. Only λ and the X and K involved in the function yield a value other than zero.

The general expression will then be:

$$\frac{\partial f_i}{\partial X_i} = \frac{2Z_i\ C_{ri}}{X_i^3} + \frac{Z_i\ \overline{K}_i}{X_i^3}\ \frac{1}{K_i^2}$$

$$\frac{\partial f_i}{\partial K_i} = \frac{2Z_i\ \overline{K}_i}{X_i^2\ K_i^3}$$

$$\frac{\partial f_i}{\partial \lambda} = 1$$

$$\frac{\partial g_i}{\partial X_i} = \frac{Z_i \ \overline{K}_i}{X_i^2 \ K_i^3}$$

$$\frac{\partial g_i}{\partial K_i} = \frac{3Z_i \ \overline{K}_i}{X_i \ K_i^4}$$

$$\frac{\partial g_i}{\partial \lambda} = S_i$$

$$\frac{\partial h}{\partial X_i} = 1$$

$$\frac{\partial h}{\partial K_i} = S_i$$

$$\frac{\partial h}{\partial \lambda} = 0$$

$$i = 1 \ldots n \ .$$

The general expression of the set of equations to be solved will be given by:

$$-\left\{- \frac{Z_1 \ C_{r1}}{X_1^2} + \frac{1}{2} \ C_1 C_{c1} - \frac{Z_1 \ \overline{K}_1}{X_1^2} \ \frac{1}{2K_1^2} + \lambda \right\} =$$

$$= \left\{ \frac{2Z_1\, C_{r1}}{X_1^{\;3}} + \frac{Z_1\, K_1}{X_1^{\;3}}\, \frac{1}{K_1^{\;2}} \right\} \Delta X_1 + \left\{ \frac{Z_1\, \bar{K}_1}{X_1^{\;2}\, K_1^{\;3}} \right\} \Delta K_1 + 0 \Delta X_2 +$$

$$+ 0 \Delta K_2 + \ldots\ldots\ldots\, 0 \Delta X_n + 0 \Delta K_n + 1 \Delta \lambda\,.$$

$$- \left\{ S_1 C_1 C_{c1} - \frac{Z_1\, \bar{K}_1}{X_1}\, \frac{1}{K_1^{\;3}} + \lambda S_1 \right\} =$$

$$= \left\{ \frac{Z_1\, \bar{K}_1}{X_1^{\;2}\, K_1^{\;3}} \right\} \Delta X_1 + \left\{ \frac{3Z_1\, \bar{K}_1}{X_1\, K_1^{\;4}} \right\} \Delta K_1 + 0 \Delta X_2 + 0 \Delta K_2 + \ldots$$

$$\ldots + 0 \Delta X_n + 0 \Delta K_n + S_1 \Delta \lambda\,.$$

$$\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}$$

$$- \left\{ - \frac{Z_n\, C_{rn}}{X_n^{\;2}} + \frac{1}{2}\, C_n C_{cn} - \frac{Z_n\, \bar{K}_n}{X_n^{\;2}}\, \frac{1}{2K^{\;2}} + \lambda \right\} =$$

$$= 0 \Delta X_1 + 0 \Delta K_1 + \ldots \quad + 0 \Delta X_{n-1} + 0 \Delta K_{n-1} +$$

$$+ \left\{ \frac{2Z_n\, C_{rn}}{X_n^{\;3}} + \frac{Z_n\, \bar{K}_n}{X_n^{\;3}}\, \frac{1}{K_n^{\;2}} \right\} \Delta X_n + \left\{ \frac{Z_n\, \bar{K}_n}{X_n^{\;2}\, K_n^{\;3}} \right\} \Delta K_n + 1 \Delta \lambda\,.$$

$$-\left\{S_n C_n C_{cn} - \frac{Z_n \overline{K}_n}{X_n} \frac{1}{K_n^3} + \lambda S_n\right\} =$$

$$= 0\triangle X_1 + 0\triangle K_1 + \ldots + 0\triangle X_{n-1} + 0\triangle K_{n-1} +$$

$$+ \left\{\frac{Z_n \overline{K}_n}{X_n^2 \; K_n^3}\right\} \triangle X_n + \left\{\frac{3Z_n \overline{K}_n}{X_n \; K_n^4}\right\} \triangle K_n + S_n \triangle \lambda .$$

$$-\left\{X_1 + K_1 S_1 + X_2 + K_2 S_2 + \ldots X_n + K_n S_n - SS\right\} =$$

$$= 1\triangle X_1 + S_1 \triangle K_1 + 1\triangle X_2 + S_2 \triangle K_2 + \ldots$$

$$\ldots + 1\triangle X_n + S_n \triangle K_n + 0\triangle \lambda .$$

The set of the equations gives a symmetric matrix, which is shown in Fig. 5.

$$
\begin{bmatrix}
\left\{\frac{2Z_1\, C_{r1}}{X_1^{\,2}} + \frac{Z_1\, \overline{K}_1}{X_1^{\,3}\, K_1^{\,2}}\right\} & \left\{\frac{Z_1\, \overline{K}_1}{X_1^{\,2}\, K_1^{\,3}}\right\} & 0 & 0 & \cdot & 0 & 0 & 1 \\
\left\{\frac{Z_1\, \overline{K}_1}{X_1^{\,2}\, K_1^{\,3}}\right\} & \left\{\frac{3Z_1\, \overline{K}_1}{X_1\, K_1^{\,4}}\right\} & 0 & 0 & \cdot & 0 & 0 & S_1 \\
0 & 0 & \left\{\frac{2Z_2\, C_{r2}}{X_2^{\,2}} + \frac{Z_2\, \overline{K}_2}{X_2^{\,3}\, K_2^{\,2}}\right\} & \left\{\frac{Z_2\, \overline{K}_2}{X_2^{\,2}\, K_2^{\,3}}\right\} & \cdot & 0 & 0 & 1 \\
0 & 0 & \left\{\frac{Z_2\, \overline{K}_2}{X_2^{\,2}\, K_2^{\,3}}\right\} & \left\{\frac{3Z_2\, \overline{K}_2}{X_2\, K_2^{\,4}}\right\} & \cdot & 0 & 0 & S_2 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & 0 & 0 & \cdot & \left\{\frac{2Z_n\, C_{rn}}{X_n^{\,2}} + \frac{Z_n\, \overline{K}_n}{X_n^{\,3}\, K_n^{\,2}}\right\} & \left\{\frac{Z_n\, \overline{K}_n}{X_n^{\,2}\, K_n^{\,3}}\right\} & 1 \\
0 & 0 & 0 & 0 & \cdot & \left\{\frac{Z_n\, \overline{K}_n}{X_n^{\,2}\, K_n^{\,3}}\right\} & \left\{\frac{3Z_n\, \overline{K}_n}{X_n\, K_n^{\,4}}\right\} & S_n \\
1 & S_1 & 1 & S_2 & \cdot & 1 & S_n & 0
\end{bmatrix}
\begin{bmatrix}
\Delta X_1 \\ \Delta K_1 \\ \Delta X_2 \\ \Delta K_2 \\ \cdot \\ \cdot \\ \Delta X_n \\ \Delta K_n \\ \Delta \lambda
\end{bmatrix}
=
\begin{bmatrix}
f_1 \\ g_1 \\ f_2 \\ g_2 \\ \cdot \\ \cdot \\ f_n \\ g_n \\ h
\end{bmatrix}
$$

Fig. 5. Symmetric matrix of coefficients for n types of feed.

The procedure to be used for solving this set of equations will be the Gauss Jordan Reduction (12).

For all of the unknowns beginning trial values must be taken. The solutions will be the deltas of the X_i's, K_i's and the λ . Those solutions will be added to the previous corresponding unknowns. The end of the solution will occur when the deltas no longer change appreciably, or when both sides of the equations become approximately equal and zero. The final X's, K's and the λ are the values which optimize the Lagrangian, or more important, the total cost equation subject to the given constraint.

Break down of the costs for each feed together with its total cost can also be calculated. By knowing the X and K for each feed type, the optimal bin size and its relations can be calculated:

For each type of feed:

optimal BINS = X + W + R

R = (LT)(Z)

W = KS

X = X

ROP = R + W.

The value of λ indicates that one additional ton of storage will reduce the total cost by the value of λ in dollars per day.

In order to show how the technique works an example of two feeds will be given.

Appendix 2 contains a computer program which provides the inventory policy for an existing constrained feed mill.

Example of a Constrained Feed Mill.

For illustration of how the technique works, assume a feed mill has TSA = 100 ton for two types, P and M, with the following characteristics:

		Type p	Type m
Z	ton/day	40	20
S1	ton/day	15	10
C_r	\$/order	15	10
C	\$/ton	50	100
C_c	%/year	.10	.10
$\overline{K}$	\$/order	30	20
LT	days	0.5	0.5
Trial X	ton/order	40	20
Trial K		2	2

also EPSI = 0.0001

Trial λ = 1.000

The demand during lead time is

$$R = Z(LT)$$

$$R_p = (40)(0.5) = 20 \text{ ton/LT}$$

$$R_m = (20)(0.5) = 10 \text{ ton/LT}$$

The convuluted standard deviation in the lead time is

$$S = (S1)\sqrt{LT}$$

$$S_p = (15)\sqrt{0.5} = 10.6 \text{ ton/LT}$$
$$S_m = (10)\sqrt{0.5} = 7.1 \text{ ton/LT} .$$

Total storage for lead time demand is

$$\sum_{i=m}^{p} R_i = 10 + 20 = 30 \text{ ton.}$$

Total storage available without lead time inventory:

$$SS = TSA - \sum_{i=m}^{p} R_i = 100 - 30 = 70 \text{ ton.}$$

The total cost equation:

$$TC = \frac{Z_p\ C_{rp}}{X_p} + \frac{X_p}{2} C_p C_{cp} + K_p S_p C_p C_{cp} + \frac{Z_p\ \bar{K}_p}{X_p\ 2K_p^{\ 2}} +$$

$$+ \frac{Z_m\ C_{rm}}{X_m} + \frac{X_m}{2} C_m C_{cm} + K_m S_m C_m C_{cm} + \frac{Z_m\ \bar{K}_m}{X_m\ 2K_m^{\ 2}}$$

has to be minimized subject to the constraint

$$X_p + K_p S_p + X_m + K_m S_m = SS .$$

Substituting the values,

$$\text{minimize } TC = \frac{(40)(15)}{X_p} + \frac{X_p}{2} \frac{(50)(.10)}{360} + 10.6\ K_p \frac{(50)(.10)}{360} + \frac{(40)(30)}{(X_p)(2)(K_p^{\ 2})} +$$

$$+ \frac{(20)(10)}{X_m} + \frac{X_m}{2} \frac{(100)(.10)}{360} + 7.1\ K_m \frac{(100)(.10)}{360} + \frac{(20)(20)}{(X_m)(2)(K_m^{\ 2})}$$

subject to

$$X_p + 10.6\ K_m + X_m + 7.1\ K_m = 70\ .$$

The Lagrangian will be,

$$L = \frac{600}{X_p} + 0.0069\ X_p + 0.1473\ K_p + \frac{600}{X_p\ K_p^{\ 2}} +$$

$$+ \frac{200}{X_m} + 0.0139\ X_m + 0.1984\ K_m + \frac{200}{X_m\ K_m^{\ 2}} +$$

$$+ \lambda\ (X_p + 10.6\ K_p + X_m + 7.1\ K_m - 70)\ .$$

The partial derivatives are

$$f_p = \frac{\partial L}{\partial X_p} = 0 = -\frac{600}{X_p^{\ 2}} + 0.0069 - \frac{600}{X_p^{\ 2}\ K_p^{\ 2}} + \lambda$$

$$g_p = \frac{\partial L}{\partial K_p} = 0 = 0.1473 - \frac{1200}{X_p\ K_p^{\ 3}} + 10.6\ \lambda$$

$$f_m = \frac{\partial L}{\partial X_m} = 0 = -\frac{200}{X_m^{\ 2}} + 0.0139 - \frac{200}{X_m^{\ 2}\ K_m^{\ 2}} + \lambda$$

$$g_m = \frac{\partial L}{\partial K_m} = 0 = 0.1984 - \frac{400}{X_m\ K_m^{\ 3}} + 7.1\ \lambda$$

$$h = \frac{\partial L}{\partial \lambda} = 0 = X_p + 10.6\ K_p + X_m + 7.1\ K_m - 70\ .$$

Using Newton-Raphson's Method, the partial derivatives of the functions are

$$\frac{\partial f_p}{\partial X_p} = \frac{1200}{X_p^{\ 3}} + \frac{1200}{X_p^{\ 3}\ K_p^{\ 2}} = \frac{1200}{X_p^{\ 3}}\ (1 + \frac{1}{K_p^{\ 2}})$$

$$\frac{\partial f_p}{\partial X_m} = 0$$

$$\frac{\partial f_p}{\partial K_p} = \frac{1200}{X_p^2 \ K_p^3}$$

$$\frac{\partial f_p}{\partial K_m} = 0$$

$$\frac{\partial f_p}{\partial \lambda} = 1$$

$$\frac{\partial f_m}{\partial X_p} = 0$$

$$\frac{\partial f_m}{\partial X_m} = \frac{400}{X_m^3} + \frac{400}{X_m^3 \ K_m^2} = \frac{400}{X_m^3} \left(1 + \frac{1}{K_m^2}\right)$$

$$\frac{\partial f_m}{\partial K_p} = 0$$

$$\frac{\partial f_m}{\partial K_m} = \frac{400}{X_m^2 \ K_m^3}$$

$$\frac{\partial f_m}{\partial \lambda} = 1$$

$$\frac{\partial g_p}{\partial X_p} = \frac{1200}{X_p^2 \ K_p^3}$$

$$\frac{\partial g_p}{\partial X_m} = 0$$

$$\frac{\partial g_p}{\partial K_p} = \frac{3600}{X_p\ K_p^4}$$

$$\frac{\partial g_p}{\partial K_m} = 0$$

$$\frac{\partial g_p}{\partial \lambda} = 10.6$$

$$\frac{\partial g_m}{\partial X_p} = 0$$

$$\frac{\partial g_m}{\partial X_m} = \frac{400}{X_m^2\ K_m^3}$$

$$\frac{\partial g_m}{\partial K_p} = 0$$

$$\frac{\partial g_m}{\partial K_m} = \frac{1200}{X_m\ K_m^4}$$

$$\frac{\partial g_m}{\partial \lambda} = 7.1$$

$$\frac{\partial h}{\partial X_p} = 1$$

$$\frac{\partial h}{\partial X_m} = 1$$

$$\frac{\partial h}{\partial K_p} = 10.6$$

$$\frac{\partial h}{\partial K_m} = 7.1$$

$$\frac{\partial h}{\partial \lambda} = 0 .$$

Substituting into the $2n + 1 = 5$ equations with the trial values of X_p, X_m, K_p, K_m and λ :

from f_p

$$- \left(- \frac{600}{1600} + 0.0069 - \frac{600}{(1600)(4)} + 1\lambda \right) =$$

$$= \frac{1200}{64000}\left(1 + \frac{1}{4}\right)\Delta X_p + \frac{1200}{(1600)(8)}\Delta K_p + 0\Delta X_m + 0\Delta K_m + 1\Delta\lambda$$

from g_p

$$- \left(0.1473 - \frac{1200}{(40)(8)} + 10.6\lambda \right) =$$

$$= \frac{1200}{(1600)(8)}\Delta X_p + \frac{3600}{(40)(16)}\Delta K_p + 0\Delta X_m + 0\Delta K_m + 10.6\Delta\lambda$$

from f_m

$$- \left(- \frac{200}{400} + 0.0139 - \frac{200}{(400)(4)} + 1\lambda \right) =$$

$$= 0\Delta X_p + 0\Delta K_p + \frac{400}{8000}\left(1 + \frac{1}{4}\right)\Delta X_m + \frac{400}{(400)(8)}\Delta K_m + 1\,\Delta\,\lambda$$

from g_m

$$- \left(0.19838 - \frac{400}{(20)(8)} + 7.1\lambda \right) =$$

$$= 0\Delta X_p + 0\Delta K_p + \frac{400}{(400)(8)}\Delta X_m + \frac{1200}{(20)(16)}\Delta K_m + 7.1\,\Delta\,\lambda$$

from h

$$- \left(40 + (10.6)(2) + 20 + (7.1)(2) - 70 \right) =$$

$$= 1\Delta X_p + 10.6\Delta K_p + 1\Delta X_m + 7.1\Delta K_m + 0\,\Delta\lambda\,.$$

Arranging the equations:

$$\begin{aligned}
-0.5381 &= 0.0235\,\Delta X_p + 0.0938\,\Delta K_p + 0\,\Delta X_m \quad 0\,\Delta K_m + 1\,\Delta\lambda \\
-6.9973 &= 0.0938\,\Delta X_p + 5.6250\,\Delta K_p + 0\,\Delta X_m + 0\,\Delta K_m + 10.6\,\Delta\lambda \\
-0.3889 &= 0\,\Delta X_p + 0\,\Delta K_p + 0.0625\,\Delta X_m + 0.125\,\Delta K_m + 1\,\Delta\lambda \\
-4.7983 &= 0\,\Delta X_p + 0\,\Delta K_p + 0.125\,\Delta X_m + 3.75\,\Delta K_m + 7.1\,\Delta\lambda \\
-25.4 &= 1\,\Delta X_p + 10.6\,\Delta K_p + 1\,\Delta X_m + 7.1\,\Delta K_m + 0\,\Delta\lambda.
\end{aligned}$$

Checking the constraint yields:

$$40 + (2)\,(10.6) + 20 + (2)\,(7.1) = 95.4 \neq 70$$

indicating that the assumed values were not correct.

The Lagrangian is

$$L = \frac{600}{40} + (0.0069)(40) + (0.1473)(2) + \frac{600}{(40)(4)} +$$

$$+ \frac{200}{20} + (0.0139)(20) + (0.1984)(2) + \frac{200}{(20)(4)} +$$

$$+ 1 \quad (40 + (2)(10.6) + 20 + (2)(7.1) - 70) =$$

$$= 15 + 0.2760 + 0.2946 + 3.7500 + 10. + 0.2780 + 0.3968$$

$$+ 2.5 + 40 + 21.2 + 20 + 14.2 - 70 = 57.8954.$$

Solving the equations, the results are:

$$\Delta X_p = -11.2883$$

$$\Delta K_p = -0.6581$$

$$\Delta X_m = -1.1645$$

$$\Delta K_m = -0.8410$$

$$\Delta \lambda = -0.2110$$

By adding the deltas to the corresponding terms, the new variables for the next iteration will be:

$X_p = 40 - 11.2883 = 28.7117$ ton

$K_p = 2 - 0.6581 = 1.3419$

$X_m = 20 - 1.1645 = 18.8355$ ton

$K_m = 2 - 0.8410 = 1.1590$

$\lambda = 1 - 0.2110 = 0.7890.$

The new Lagrangian will then be

$$L = \frac{600}{28.7117} + (0.0069)(28.7117) + (0.1473)(1.3419) + \frac{600}{(28.7117)(1.3419^2)} +$$

$$+ \frac{200}{18.8355} + (0.0139)(18.8355) + (0.1984)(1.1590) + \frac{200}{(18.8355)(1.1590^2)} +$$

$$+ 0.7890\ (28.7117 + (1.3419)(10.6) + 18.8355 + (1.1590)(7.1) - 70) =$$

$$= 51.9132$$

which shows an improvement to the previous one.

Proceeding in this way until all the absolute value of the deltas are less than or equal to EPSI, will take five iterations. The end result will be:

$X_p = 27.42$ ton

$K_p = 1.534$

$X_m = 16.07$ ton

$K_m = 1.449$

$\lambda = 1.1300$

with optimal total cost of \$50.49/day.

For type p:

Lot size order = 27.42 ton.

Safety stock = $K_p S_p = 1.534 \times 10.6 = 16.27$ ton.

Stock for demand during lead time = 20 ton.

Reorder point = 20 + 16.27 = 36.27 ton.

Optimal bin size = 27.42 + 16.27 + 20 = 63.69 ton.

Cycle per day = $\frac{Z}{X} = \frac{40}{27.42} = 1.46$.

Order cost = \$21.88/day.

Carrying cost = \$0.19/day.

Safety stock cost = \$0.23/day.

Out of stock cost = \$9.30/day.

Total cost = \$31.60/day.

Probability of being out of stock = $\frac{1}{2K^2}$ = 0.2126.

For type m:

Lot size order = 16.07 ton.

Safety stock = $K_m\ S_m$ = 10.24 ton.

Stock for demand during lead time = 10 ton.

Reorder point = 10 + 10.24 = 20.24 ton.

Optimal bin size = 16.07 + 10 + 10.24 = 36.31 ton.

Cycle per day = $\frac{20}{16.07}$ = 1.449.

Order cost = \$12.45/day.

Carrying cost = \$0.22/day.

Safety stock cost = \$0.29/day.

Out of stock cost = \$5.93/day.

Total cost = \$18.89/day.

Probability of being out of stock = .2382.

Check the constraint:

63.69 + 36.31 = 100 ton.

IV. The Bin Assignment

Knowing from the previous procedure the optimal bin size needed for each assigned feed, the problem remains now of fitting those values as closely as possible into a given situation of bins. The total number of bins, their size and their restrictions is well known.

In most cases, the optimal bin size has a noninteger value. More than this, in most cases a combination of the sizes of a number of actual bins does not equal the integer part of the optimal bin size. It therefore becomes a task of "rounding" the optimal bin size to a combination near the actual bin sizes. This fact moves the actual solution from the theoretical solution calculated before, and it is desirable to have the actual total costs as close as possible to the theoretical total cost.

All finished feed bins must be taken into consideration although in the previous procedure only total storage capacity for assigned feeds was used. The average number of nonassigned feeds demanded per day and its standard deviation is known. Assuming that the nonassigned bins can be used twice a day, the number of these bins is supposed to be half of the average number of nonassigned feeds per day plus half its standard deviation. This number of bins for nonassigned feeds has to be determined, although they have not yet been marked for this purpose. The bins for mix-up have to be determined and marked because they are almost dependent on the system.

The total number of bins available for assigned bins and nonassigned bins, therefore equals the total number of finished feed bins, excluding the mix-up bins.

Two criteria were chosen for marking the assigned bins:

1. According to total cost per type per demand.
2. According to the probability of being out of stock.

The procedure will be as follows:

1. List all assigned feeds with the associated total cost, demand and probability of being out of stock.
2. Calculate for each of these feeds

$$t_i = \frac{TC_i}{Z_i} \qquad i=1...n.$$

3. Sequence these feeds in such an order that the largest t_i will be the first and the smallest t_i the last to be assigned.
4. List these feeds according to the probability of being out of stock when the largest probability will be the first in the list.

The assignment will be done by fitting the optimal bin size value to a combination of bins, as closely correlated as possible to the restriction of the bins and one priority list. The same procedure will be done with the second priority list.

The total cost has to be calculated for each assignment. The value closest to the theoretical one will then be the best selection. In either case, the predetermined number of bins for the nonassigned feeds has to remain unchanged. The remaining bins after marking the assigned bins then become the nonassigned bins.

The actual values of the lot size order, the safety stock, the reorder point and the costs have to be recalculated for the actual situation. It is assumed that the change of the K's will be so small that the theoretical K's can be used in calculating the actual total cost.

Although according to theory there are many combinations for bin assignment, according to the restriction described before, the available bins reduce the feasible solutions to only a few.

A small computer program for calculating the actual values after the bin assignment is supplied in Appendix 3.

RESULTS AND DISCUSSION

The AMBAR Feed Mill in Israel has an annual production of about 100,000 tons (metric). The Feed Mill produces 81 types of feed, about 76% in pellets and 24% in mash. About 80% of the sales are in bulk, the remainder in bags.

In order to evaluate this study, data from one month was collected. The data is based on the sales in May, 1969, where 8345 tons were dispatched. This quantity is built of 2580 Semiorders, where a semiorder is a demand per type per feed. (A customer's order may contain several semiorders.) The company has a very limited warehouse for bagged feeds, which has not been taken into consideration. The size of the bins is based on the average density of the feeds. The distribution of the bins are shown in Table 2. The total storage capacity for finished feed of 705 tons is distributed in 46 bins with the restriction as shown in Table 3.

Table 2. Distribution of the Finished Feed Bins.

Purpose	Size (ton)	No. of Bins	Percentage of Total Size
Pellets	423	27	60.00
Mash	214	14	30.35
Pellets or Mash	28	2	3.97
Mix-up	40	3	5.68
Total	705 tons	46 bins	

It is recognized that the bin distribution does not fit the demand distribution according to pellets or mash.

Table 3. Finished Feed Bins Characteristics.

Bin No.	Size Tons	For Pellets	For Mash	Bin No.	Size Tons	For Pellets	For Mash
1	14		+	24	16		+
2	14		+	25	10	Mix up	
3	14		+	26	9	+	
4	14		+	27	18	+	
5	14	+	+	28	18	+	
6	14	+	+	29	18	+	
7	14	+		30	18	+	
8	14		+	31	20	+	
9	14	+		32	20	+	
10	14	Mix up		33	20	+	
11	14	+		34	20	+	
12	14	+		35	9	+	
13	14	+		36	9	+	
14	14	+		37	16		+
15	14	+		38	16		+
16	14	+		39	16		+
17	14	+		40	16		+
18	14	+		41	16		+
19	14	+		42	24	+	
20	14	+		43	14	+	
21	16	Mix up		44	14	+	
22	16		+	45	24	+	
23	16		+	46	16		+

Data Processing

1. Classification of feeds.

An ABC analysis was performed and is shown in Tables 4 and 5 and in Figures 5 and 6. Table 4 and Fig. 5 show that 7059 tons, about 85% of the entire production, consists of 16 feed types. Another 10% contains 15 feed types, while the remaining 5% of the total production volume consists of 50 types. Table 5 and Fig. 6 illustrate the classification according to the semiorders, where 19 types of feed make up about 85% of the total number of semiorders, 12 types make up 10% of the orders and 50 types make up 5% of the total number of semiorders.

Table 6 compares the type assigned to group A in both classifications. Although the feed types sometimes are not sequenced in the same order, most of the feeds contained in group A according to one classification are also in the second group of A. It has then been decided that this study will emphasize the 16 types of group A to be assigned feeds. Group B and C were combined to nonassigned feeds.

Table 4. Classification of Feed Types by Total Tons Produced in May, 1969.

Type	Tons	Percentage of Total	Cumulative Percentage
1	960.230	11.51	11.51
2	930.880	11.15	22.66
3	583.830	7.00	29.66
4	578.740	6.93	36.59
5	577.980	6.92	43.51
6	503.340	6.03	49.54
7	384.140	4.60	54.14
8	370.900	4.45	58.59
9	333.020	4.00	62.59
10	326.200	3.91	66.50
11	323.980	3.88	70.38
12	317.120	3.80	74.18
13	282.590	3.39	77.57
14	229.730	2.75	80.32
15	184.890	2.22	82.54
16	171.310	2.05	84.59
17	120.540	1.44	86.03
18	106.410	1.28	87.31
19	90.360	1.08	88.52
20	80.060	0.96	89.48
21	61.670	0.74	90.22
22	61.560	0.74	90.96

Table 4. Continuation.

Type	Tons	Percentage of Total	Cumulative Percentage
23	57.690	0.69	91.65
24	54.060	0.65	92.30
25	50.190	0.60	92.90
26	46.400	0.56	93.46
27	44.950	0.54	94.00
28	42.400	0.51	94.51
29	33.850	0.41	94.92
30	32.290	0.39	95.31
31	30.640	0.37	95.68
32	18.210	0.22	95.90
33	17.650	0.21	96.11
34	17.220	0.21	96.32
35	17.200	0.20	96.52
36	15.460	0.18	96.70
37	15.400	0.18	96.88
38	15.400	0.18	97.06
39	15.380	0.18	97.24
40	12.580	0.15	97.39
41	12.740	0.15	97.54
42	12.440	0.15	97.69
43	11.800	0.13	97.82
44	11.040	0.13	97.95
45	10.620	0.12	98.07
46	10.160	0.12	98.19

Table 4. Continuation.

Type	Tons	Percentage of Total	Cumulative Percentage
47	9.240	0.11	98.30
48	8.950	0.11	98.41
49	8.100	0.10	98.51
50	7.900	0.10	98.61
51	7.620	0.09	98.70
52	7.350	0.09	98.79
53	6.460	0.08	98.87
54	6.340	0.08	98.95
55	6.000	0.08	99.03
56	5.900	0.07	99.10
57	5.800	0.07	99.17
58	5.780	0.07	99.24
59	5.300	0.07	99.31
60	5.000	0.07	99.38
61	4.600	0.06	99.44
62	4.400	0.06	99.49
63	4.300	0.05	99.58
64	4.100	0.05	99.63
65	3.600	0.04	99.67
66	3.480	0.04	99.71
67	3.450	0.04	99.75
68	2.460	0.03	99.79
69	2.340	0.03	99.82
70	1.900	0.03	99.85

Table 4. Continuation.

Type	Tons	Percentage of Total	Cumulative Percentage
71	1.800	0.02	99.88
72	1.300	0.02	99.90
73	1.280	0.01	99.92
74	1.250	0.01	99.93
75	1.200	0.01	99.94
76	1.200	0.01	99.95
77	1.000	0.01	99.96
78	1.000	0.01	99.97
79	.600	0.01	99.98
80	.600	0.01	99.99
81	.600	0.01	100.00

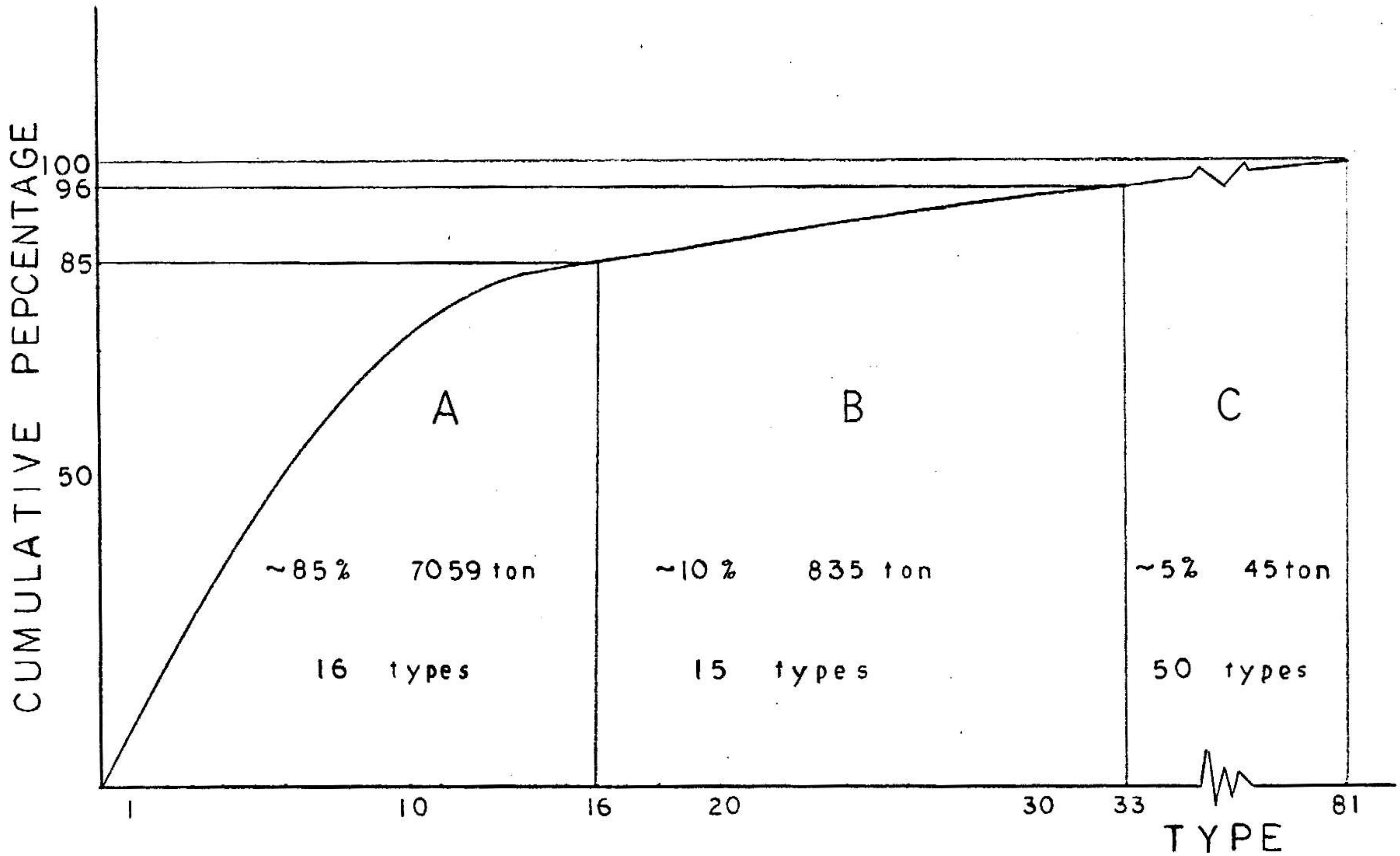


FIG. 5 FEED CLASSIFICATION ACCORDING TO PPODUCTION VOLUME. TOTAL = 8345 ton

Table 5. Classification of Feed Types by Number of Semi-Orders in May, 1969.

Type	No. of Orders	Percentage of Total	Cumulative Percentage
1	272	10.54	10.54
2	212	8.22	18.76
12	192	7.44	26.20
10	161	6.24	32.44
3	148	5.74	38.18
5	137	5.31	43.49
6	136	5.27	48.76
4	134	5.19	53.95
7	128	5.10	59.05
14	92	3.57	62.62
16	88	3.41	66.03
13	85	3.29	69.32
8	80	3.10	72.42
9	71	2.75	75.17
15	67	2.60	77.77
17	61	2.36	80.13
11	55	2.13	82.26
18	34	1.32	83.58
31	34	1.32	84.90
24	33	1.28	86.18
25	32	1.24	87.42
20	32	1.24	88.66
21	25	0.97	89.63

Table 5. Continuation.

Type	No. of Orders	Percentage of Total	Cumulative Percentage
30	23	0.89	90.52
19	23	0.89	91.41
23	19	0.74	92.15
27	18	0.70	92.85
22	18	0.70	93.55
34	14	0.54	94.09
32	10	0.39	94.48
33	9	0.35	94.83
29	9	0.35	95.18
51	7	0.27	95.45
55	7	0.27	95.72
28	7	0.27	95.99
35	6	0.23	96.22
67	5	0.19	96.41
41	5	0.19	96.60
37	4	0.15	96.75
44	4	0.15	96.90
56	4	0.15	97.05
45	4	0.15	97.20
47	4	0.15	97.35
43	3	0.11	97.46
52	3	0.11	97.57
48	3	0.11	97.68
.	.		.

Table 5. Continuation.

Type	No. of Orders	Percentage of Total	Cumulative Percentage
.	.		.
.	.		.
.	.		.
81	1		100.00

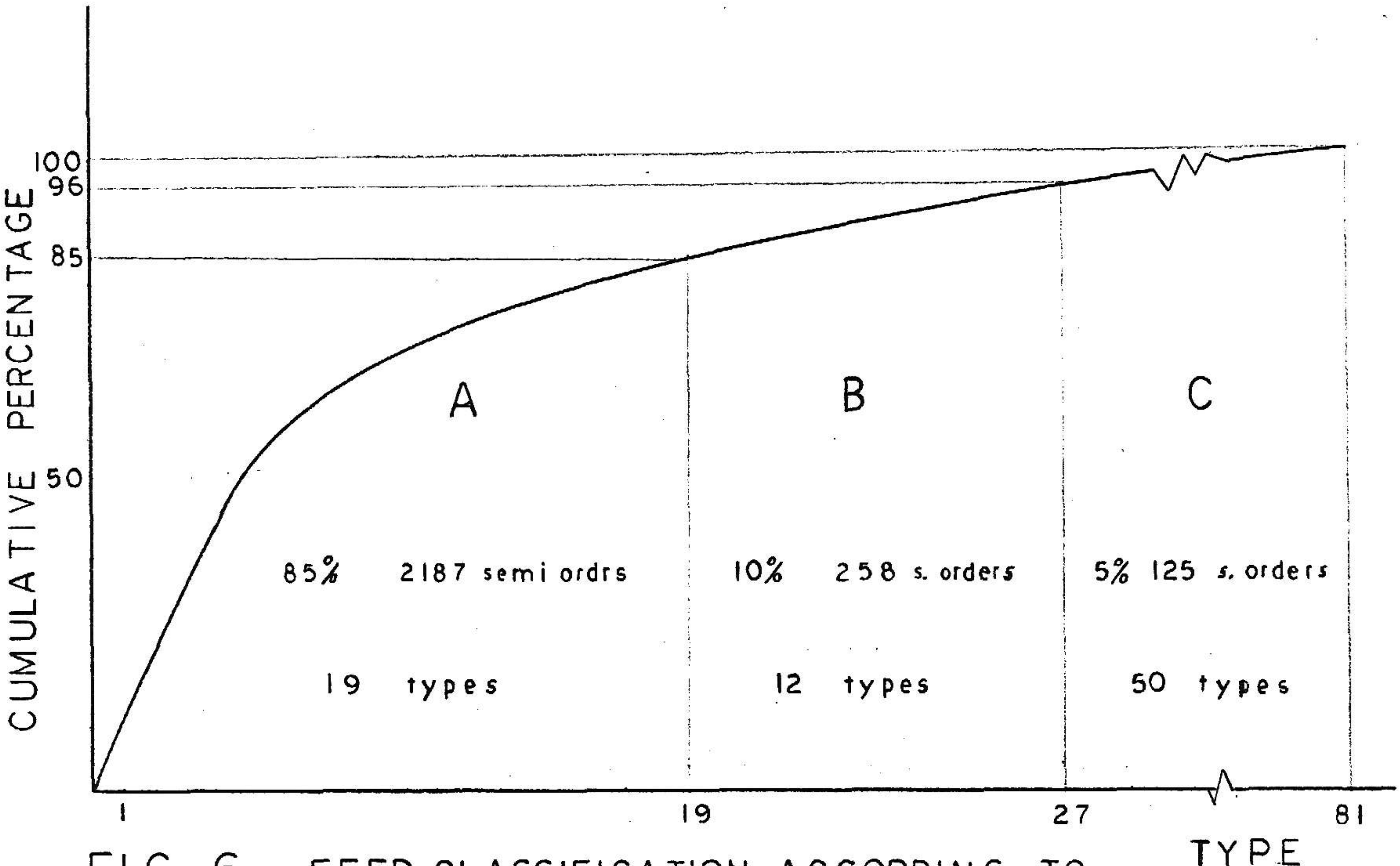


FIG. 6 FEED CLASSIFICATION ACCORDING TO NUMBER OF SEMI ORDERS. TOTAL = 2580 s. orders

Table 6. Comparison Between Group A as Classified by Size and by Number of Semiorders.

Place	According to Tons	According to Semiorders
1	1	1
2	2	2
3	3	12
4	4	10
5	5	3
6	6	5
7	7	6
8	8	4
9	9	7
10	10	14
11	11	16
12	12	13
13	13	8
14	14	9
15	15	15
16	16 85% of Total	17
17	17	11
18	18	18
19	19	31

2. Demand

For each of the sixteen assigned feeds the average demand per day and its standard deviation in tons was found as summarized in Table 7.

Table 7. Average and Standard Deviation of the Demand Per Type Per Day.

Type	Z Tons/Day	S1 Tons/Day
1	40.010	17.753
2	38.784	15.124
3	24.326	10.729
4	24.114	8.773
5	24.082	10.250
6	20.972	12.570
7	16.006	14.570
8	15.454	8.122
9	13.876	5.690
10	13.592	6.425
11	13.500	14.122
12	13.213	10.567
13	11.774	7.603
14	9.572	3.943
15	7.704	6.314
16	7.138	4.302

In order to know how many types from either assigned types or nonassigned types were demanded per day, the data was analyzed and is shown in Table 8.

Table 8. Number of Assigned and Nonassigned Types of Feeds Demanded Per Day.

Day	Number of Assigned Feeds	Number of Nonassigned Feeds
1	13	6
2	15	16
3	16	9
4	15	14
5	13	15
6	16	13
7	15	19
8	14	7
9	15	7
10	14	16
11	16	11
12	16	12
13	16	19
14	16	9
15	16	11
16	15	16
17	16	12
18	16	13
19	15	14
20	15	11
21	15	13
22	14	17
23	16	16
24	16	21
Total	364	317
Average	15.16 Types/Day	13.21 Types/Day
S.D	0.94 Types/Day	3.87 Types/Day

The results indicate that almost every day there was demand for almost all assigned feeds. The demand for the sixty-five nonassigned feeds varies much more. The average number of different feeds per day is about 28.

The Constrained Feed Mill.

The total storage capacity for finished feeds without the mix-up bins is 705 - 40 = 665 tons divided into 46 - 3 = 43 available bins, from Table 3.

Table 8 indicates that the average demand for nonassigned feeds 13.21 with a standard deviation of 3.87. Therefore, assuming each nonassigned bin will be used twice a day, $\frac{13.21 + 3.87}{2} \cong 9$ bins will be needed for this purpose. Nine bins were taken for nonassigned feeds but not yet marked. The smallest total capacity of nine bins is 3 x 9 + 6 x 14 = 111 ton. In order to have also some bigger bins as nonassigned bins, storage of 135 ton was taken into consideration for the nonassigned bin. The theoretical storage available for assigned feed, therefore will be 665 - 135 = 530 ton, distributed in 34 bins.

It is emphasized here, that the above assumptions are specific to a given system, while the approach is general.

Computer Program Minimizing Total Cost and Optimizing Bin Size for Assigned Feed.

A complete computer program is supplied in the Appendix. The user only has to change the DIMENSION cards. All arrays are dimensioned according to the number of types in consideration, except array A. Array A is the matrix to be solved for the deltas. It should be dimensioned as follows: DIMENSION A(2 * NTYPE + 1, 2 * NTYPE + 2). In order to write out the deltas further WRITE (3,102) (A(I N),I=1,..) statements may be added. The program gives an echo-check for the input.

In the first part, the input is read in. The input includes for each type the following values: Z, C_r, CC_c, $\bar{K}$, Sl, a trial X and a trial Tchebyseff's constant. Lead time for each type is also supplied. The input includes also a trial λ and the total storage available for these feeds. The value of CC_c in the input is $\frac{C_i\ C_{ci}}{360}$. The program computes in the second part the standard deviation and the demand during lead time.

In the third part the functions and their partial derivatives are calculated and the coefficients are arranged in matrix form, which will be used in the Gauss-Jordan reduction in the fourth part, and lastly the iteration process. In order to control the length of the run, two control statements are supplied. The first will stop the program after 20 iterations, the second will stop it if the deltas reach EPSI. The output contains the information of costs and physical quantities for each feed and for the whole system.

Table 9 shows a printout of the input and output. The values for Z and Sl are analyzed according to the data. Prices of the feeds were assumed to be \$100 per ton, and the yearly percentage for carrying one ton 0.10. C_r was assumed to be \$10 for mash feed and \$15 for a pellet feed. $\bar{K}$ is assumed to be two times C_r. The starting values for the X's were chosen to be near Z, the K's equal 2 and $\lambda = 1$.

Table 10 shows the results of the same program except an additional unit of storage, one ton, was added to the restriction. This has been done in order to show the meaning of λ. While in the first system $TSIZE_1 = 530$ ton, $T.C_1 = \$399.85$ and $\lambda_1 = 1.69$, in the second on $TSIZE_2 = 531$ ton, $T.C_2 = 398.17$ which shows,

$$TC_1 - TC_2 = 399.85 - 398.17 = 1.68 \approx \lambda_1 .$$

The computer time for one set of data, with 16 types was less than 3 minutes.

Table 9.

AN INVENTORY POLICY FCR AN EXISTING FEED MILL CONSRAINED
BY STORAGE FOR FINISHED FEED IN BULK.
BY REUBEN KALINHOFF

INPUT DATA
NUMBER OF TYPES=16

TYPE	LEAD TIME IN DAYS
1	0.5
2	0.5
3	0.5
4	0.5
5	0.5
6	0.5
7	0.5
8	0.5
9	0.5
10	0.5
11	0.5
12	0.5
13	0.5
14	0.5
15	0.5
16	0.5

TYPE	DEMAND PER DAY TONS	ORDER COST $	CARRYING COST $/TON/DAY	SET UP COST $/ORDER	ST.DEV.OF DEMAND TONS	TRIAL X	TRIAL TK
1	40.00	15.00	0.0279	30.00	17.75	31.00	2.000
2	38.79	15.00	0.0279	30.00	15.10	30.00	2.000
3	24.33	10.00	0.0279	20.00	10.73	19.00	2.000
4	24.11	15.00	0.0279	30.00	8.77	23.00	2.000
5	24.08	15.00	0.0279	30.00	10.25	23.00	2.000
6	20.97	15.00	0.0279	30.00	12.57	22.00	2.000
7	16.00	15.00	0.0279	30.00	14.57	12.00	2.000
8	15.45	10.00	0.0279	20.00	8.12	12.00	2.000
9	13.88	15.00	0.0279	30.00	5.67	12.00	2.000
10	13.59	10.00	0.0279	20.00	6.42	12.00	2.000
11	13.50	15.00	0.0279	30.00	14.12	12.00	2.000
12	13.21	15.00	0.0279	30.00	10.57	12.00	2.000
13	11.77	10.00	0.0279	20.00	7.60	10.00	2.000
14	9.57	15.00	0.0279	30.00	3.94	10.00	2.000
15	7.70	15.00	0.0279	30.00	6.31	10.00	2.000
16	7.14	10.00	0.0279	20.00	4.30	10.00	2.000

TRIAL LAMDA= 1.0000
TOTAL STORAGE AVAILABLE=530.0TONS
OUTPUT DATA
TOTAL STORAGE AVAILABLE WITHOUT LEAD TIME INVENTORY=383.0
THE DELTAS OF X, TK AND LAMDA

-0.00002	0.00001	-0.00002	0.00000	-0.00002
-0.00000	-0.00002	0.00000	-0.00002	-0.00000
-0.00001	-0.00000	-0.00002	-0.00000	-0.00002
-0.00000	-0.00002	-0.00000	-0.00001	0.00000
-0.00006	0.00003	-0.00001	-0.00000	-0.00001
-0.00000	-0.00002	-0.00000	-0.00001	-0.00000
-0.00001	-0.00000	0.00001		

TYPE	SET UP AND ORDER COST	INVENTORY COST	SAF.STOCK COST	O.O.H COST	TOTAL COST
1	25.62	0.33	0.47	14.41	40.82
2	25.71	0.32	0.42	12.96	39.41
3	16.43	0.21	0.29	8.89	25.81
4	21.00	0.24	0.27	8.43	29.94
5	20.63	0.24	0.30	9.30	30.47
6	18.60	0.24	0.33	10.29	29.47
7	15.59	0.21	0.35	10.71	26.86
8	13.18	0.16	0.22	6.86	20.42
9	16.19	0.18	0.19	5.78	22.34
10	12.62	0.15	0.19	5.78	18.74
11	14.20	0.20	0.33	10.17	24.90
12	14.65	0.19	0.27	8.47	23.58
13	11.40	0.14	0.20	6.25	17.99
14	13.67	0.15	0.14	4.29	18.24
15	11.55	0.14	0.18	5.55	17.41
16	9.23	0.11	0.13	3.99	13.45

Table 9. Continuation

TOTAL COSTS FOR ALL TYPES,PER DAY
TOTAL CCST= 399.85$
ORDER AND SET UP COST= 260.26
CARRYING COST= 3.20
SAFETY STOCK COST= 4.28
OUT OF HAND COST= 132.11

TYPE	LOT SIZE ORDER TONS	SAFETY STOCK	STOCK FOR LEAD TIME	REORDER POINT	CYCLE /DAY	CHEBYSHEV TK	PROBABILITY LE. O.O.H
1	23.42	16.73	20.00	36.73	1.71	1.333	0.2814
2	22.63	15.04	19.39	34.44	1.71	1.409	0.2520
3	14.81	10.32	12.17	22.48	1.64	1.360	0.2705
4	17.22	9.79	12.06	21.84	1.40	1.578	0.2007
5	17.51	10.79	12.04	22.83	1.38	1.489	0.2254
6	16.91	11.95	10.49	22.43	1.24	1.344	0.2767
7	15.39	12.43	8.00	20.43	1.04	1.207	0.3434
8	11.72	7.96	7.72	15.68	1.32	1.386	0.2602
9	12.86	6.71	6.94	13.65	1.08	1.674	0.1785
10	10.77	6.71	6.80	13.50	1.26	1.478	0.2289
11	14.26	11.80	6.75	18.55	0.95	1.182	0.3579
12	13.53	9.83	6.60	16.44	0.98	1.315	0.2890
13	10.33	7.26	5.89	13.14	1.14	1.350	0.2742
14	10.50	4.97	4.78	9.76	0.91	1.786	0.1568
15	10.00	6.44	3.85	10.29	0.77	1.443	0.2401
16	7.73	4.63	3.57	8.20	0.92	1.522	0.2159

TYPE	OPTIMAL BIN SIZE
1	60.155
2	57.065
3	37.292
4	39.064
5	40.346
6	39.343
7	35.823
8	27.409
9	26.508
10	24.271
11	32.810
12	29.962
13	23.468
14	20.264
15	20.290
16	15.932

VALUE OF LAMDA= 1.6949990
NUMBER OF ITERATIONS= 7

Table 10.

INPUT DATA
NUMBER OF TYPES=16

TYPE	LEAD TIME IN DAYS
1	0.5
2	0.5
3	0.5
4	0.5
5	0.5
6	0.5
7	0.5
8	0.5
9	0.5
10	0.5
11	0.5
12	0.5
13	0.5
14	0.5
15	0.5
16	0.5

TYPE	DEMAND PER DAY TONS	ORDER COST $	CARRYING COST $/TON/DAY	SET UP COST $/ORDER	ST.DEV.OF DEMAND TONS	TRIAL X	TRIAL TK
1	40.00	15.00	0.0279	30.00	17.75	31.00	2.000
2	38.79	15.00	0.0279	30.00	15.10	30.00	2.000
3	24.33	10.00	0.0279	20.00	10.73	19.00	2.000
4	24.11	15.00	0.0279	30.00	8.77	23.00	2.000
5	24.08	15.00	0.0279	30.00	10.25	23.00	2.000
6	20.97	15.00	0.0279	30.00	12.57	22.00	2.000
7	16.00	15.00	0.0279	30.00	14.57	12.00	2.000
8	15.45	10.00	0.0279	20.00	8.12	12.00	2.000
9	13.88	15.00	0.0279	30.00	5.67	12.00	2.000
10	13.59	10.00	0.0279	20.00	6.42	12.00	2.000
11	13.50	15.00	0.0279	30.00	14.12	12.00	2.000
12	13.21	15.00	0.0279	30.00	10.57	12.00	2.000
13	11.77	10.00	0.0279	20.00	7.60	10.00	2.000
14	9.57	15.00	0.0279	30.00	3.94	10.00	2.000
15	7.70	15.00	0.0279	30.00	6.31	10.00	2.000
16	7.14	10.00	0.0279	20.00	4.30	10.00	2.000

TRIAL LAMDA= 1.0000
TOTAL STORAGE AVAILABLE=531.0TONS
OUTPUT DATA
TOTAL STORAGE AVAILABLE WITHOUT LEAD TIME INVENTORY=384.0
THE DELTAS OF X, TK AND LAMDA

-0.00005	0.00000	-0.00004	-0.00000	-0.00003
-0.00000	-0.00003	-0.00000	-0.00003	-0.00000
-0.00002	-0.00000	-0.00002	-0.00000	-0.00002
-0.00000	-0.00002	-0.00000	-0.00002	-0.00000
-0.00006	0.00002	-0.00002	-0.00000	-0.00002
-0.00000	-0.00002	-0.00000	-0.00002	-0.00000
-0.00002	-0.00000	0.00001		

TYPE	SET UP AND ORDER COST	INVENTORY COST	SAF.STOCK COST	O.O.H COST	TOTAL COST
1	25.53	0.33	0.47	14.32	40.65
2	25.63	0.32	0.42	12.88	39.24
3	16.37	0.21	0.29	8.83	25.70
4	20.93	0.24	0.27	8.38	29.82
5	20.56	0.25	0.30	9.24	30.34
6	18.54	0.24	0.33	10.23	29.34
7	15.54	0.22	0.35	10.64	26.75
8	13.13	0.16	0.22	6.81	20.33
9	16.14	0.18	0.19	5.74	22.25
10	12.58	0.15	0.19	5.74	18.66
11	14.16	0.20	0.33	10.10	24.79
12	14.60	0.19	0.27	8.42	23.48
13	11.36	0.14	0.20	6.21	17.92
14	13.62	0.15	0.14	4.26	18.16
15	11.51	0.14	0.18	5.51	17.34
16	9.20	0.11	0.13	3.96	13.40

Table 10. Continuation

TOTAL COSTS FOR ALL TYPES,PER DAY
TOTAL COST= 398.17$
ORDER AND SET UP COST= 259.38
CARRYING COST= 3.21
SAFETY STOCK COST= 4.28
OUT OF HAND COST= 131.29

TYPE	LOT SIZE ORDER TONS	SAFETY STOCK	STOCK FOR LEAD TIME	REORDER POINT	CYCLE /DAY	CHEBYSHEV TK	PROBABILITY LE. O.O.H
1	23.50	16.76	20.00	36.76	1.70	1.335	0.2805
2	22.71	15.06	19.39	34.46	1.71	1.411	0.2512
3	14.86	10.33	12.17	22.50	1.64	1.362	0.2697
4	17.28	9.80	12.06	21.86	1.40	1.580	0.2002
5	17.57	10.81	12.04	22.85	1.37	1.491	0.2248
6	16.97	11.97	10.49	22.45	1.24	1.346	0.2759
7	15.44	12.45	8.00	20.45	1.04	1.208	0.3424
8	11.76	7.97	7.72	15.70	1.31	1.388	0.2594
9	12.90	6.72	6.94	13.66	1.08	1.676	0.1780
10	10.80	6.72	6.80	13.51	1.26	1.480	0.2283
11	14.31	11.82	6.75	18.57	0.94	1.184	0.3569
12	13.57	9.84	6.60	16.45	0.97	1.317	0.2882
13	10.36	7.27	5.89	13.15	1.14	1.352	0.2734
14	10.54	4.98	4.78	9.77	0.91	1.788	0.1564
15	10.04	6.45	3.85	10.30	0.77	1.445	0.2395
16	7.76	4.63	3.57	8.20	0.92	1.524	0.2153

TYPE	OPTIMAL BIN SIZE
1	60.258
2	57.163
3	37.357
4	39.137
5	40.421
6	39.417
7	35.892
8	27.460
9	26.562
10	24.318
11	32.874
12	30.022
13	23.514
14	20.308
15	20.334
16	15.965

VALUE OF LAMDA= 1.6818150
NUMBER OF ITERATIONS= 7

Bin Assignment.

Using the results from Table 9, the priority lists can be built in Table 11.

Table 11. Priority Lists for Assignment.

Type	Pellets or Mash	Z Ton/Day	T.C $/Day	$t = \frac{T.C}{Z}$ $/Ton	List 1 Priority	Probability	List 2 Priority
1	P	40.00	40.82	1.0205	15	0.2814	4
2	P	38.79	39.41	1.0160	16	0.2520	9
3	M	24.33	25.81	1.0608	14	0.2705	7
4	P	24.11	29.94	1.2211	13	0.2007	14
5	P	24.08	30.47	1.2654	12	0.2254	12
6	P	20.97	29.47	1.4053	9	0.2767	5
7	P	16.00	26.86	1.6788	6	0.3434	2
8	M	15.45	20.42	1.3217	11	0.2602	8
9	P	13.88	22.34	1.6095	7	0.1785	15
10	M	13.59	18.74	1.3790	10	0.2289	11
11	P	13.50	24.90	1.8444	4	0.3579	1
12	P	13.21	23.58	1.7850	5	0.2890	3
13	M	11.77	17.99	1.5285	8	0.2742	6
14	P	9.57	18.24	1.9060	2	0.1568	16
15	P	7.70	17.41	2.2610	1	0.2401	10
16	M	7.14	13.45	1.8838	3	0.2159	13

Table 12 illustrates two bin assignments, according to priority list 1 and 2. In both cases nine bins remain for the nonassigned types.

Table 12. Bin Assignment According to the Priority Lists.

Type	Pellets or Mash	Theoretical Bin Size	Bin assignment Priority list 1	Bin assignment Priority list 2
1	P	60.155	24 + 14 + 14 + 9 = 61	20 + 20 + 20 = 60
2	P	57.065	14 + 14 + 14 + 14 = 56	24 + 18 + 14 = 56
3	M	37.292	16 + 14 + 14 = 44	14 + 14 + 16 = 44
4	P	39.064	20 + 18 = 38	14 + 14 + 14 = 42
5	P	40.346	14 + 14 + 14 = 42	14 + 14 + 14 = 42
6	P	39.343	24 + 14 = 38	24 + 14 = 38
7	P	35.823	18 + 18 = 36	18 + 18 = 36
8	M	27.409	14 + 14 = 28	14 + 16 = 30
9	P	26.508	14 + 14 = 28	14 + 14 = 28
10	M	24.271	16 + 14 = 30	16 + 14 = 30
11	P	32.810	18 + 14 = 32	18 + 14 = 32
12	P	29.962	20 + 9 = 29	20 + 9 = 29
13	M	23.468	16 = 16	14 + 14 = 28
14	P	20.264	20 = 20	20 = 20
15	P	20.290	20 = 20	14 = 14
16	M	15.932	16 = 16	16 = 16

It can be seen from Table 13 that the total cost for the two assignments differ slightly. The assignment according to the probability of being out of stock is closer to the theoretical optimal solution and therefore will be the selection.

Table 13.

MODIFICATION OF TOTAL COST AND LOT SIZE ORDER AFTER BIN ASSIGNMENT

BY REUBEN KALINKOFF

TYPE	TOTAL COST	LOT SIZE ORDER	BIN CAPACITY	REORDER POINT	SAF.STOCK	CYC/ DAY
1	39.37	24.316	61.00	36.68	16.68	1.64
2	41.30	21.561	56.00	34.44	15.04	1.80
3	18.01	21.516	44.00	22.48	10.32	1.13
4	31.87	16.159	38.00	21.84	9.79	1.49
5	27.91	19.168	42.00	22.83	10.79	1.26
6	31.94	15.569	38.00	22.43	11.95	1.35
7	26.57	15.565	36.00	20.44	12.44	1.03
8	19.47	12.317	28.00	15.68	7.96	1.25
9	20.08	14.348	28.00	13.65	6.71	0.97
10	12.43	16.495	30.00	13.50	6.71	0.82
11	26.35	13.449	32.00	18.55	11.80	1.00
12	25.34	12.567	29.00	16.43	9.83	1.05
13	63.98	2.860	16.00	13.14	7.25	4.12
14	18.70	10.239	20.00	9.76	4.98	0.93
15	17.92	9.712	20.00	10.29	6.44	0.79
16	13.34	7.802	16.00	8.20	4.63	0.92

TOTAL COST FOR GROUP A FEEDS IN THIS ASSIGNMENT= 434.55

TYPE	TOTAL COST	LOT SIZE ORDER	BIN CAPACITY	REORDER POINT	SAF.STOCK	CYC/ DAY
1	41.09	23.269	60.00	36.73	16.73	1.72
2	41.30	21.561	56.00	34.44	15.04	1.80
3	18.01	21.516	44.00	22.48	10.32	1.13
4	25.70	20.159	42.00	21.84	9.79	1.20
5	27.91	19.168	42.00	22.83	10.79	1.26
6	31.94	15.569	38.00	22.43	11.95	1.35
7	26.57	15.565	36.00	20.44	12.44	1.03
8	16.83	14.317	30.00	15.68	7.96	1.08
9	20.08	14.348	28.00	13.65	6.71	0.97
10	12.43	16.495	30.00	13.50	6.71	0.82
11	26.35	13.449	32.00	18.55	11.80	1.00
12	25.34	12.567	29.00	16.43	9.83	1.05
13	12.68	14.860	28.00	13.14	7.25	0.79
14	44.68	4.239	14.00	9.76	4.98	2.26
15	46.30	3.712	14.00	10.29	6.44	2.07
16	13.34	7.802	16.00	8.20	4.63	0.92

TOTAL COST FOR GROUP A FEEDS IN THIS ASSIGNMENT= 430.52

The nonassigned bins therefore will be:

Number of Bins	Form	Size (tons)	Total
1	P	9	9
5	M	16	80
1	PM	14	14
2	P	17	28
9 bins			131 tons

The actual total costs as well as the actual physical quantities are illustrated in Table 12.

Figures 7, 8, 9, 10, 11 and 12 illustrate the relation of some variables as a function of the demand. Any one of those pairs of graphs shows the variable as a function of the demand before and after the actual assigning.

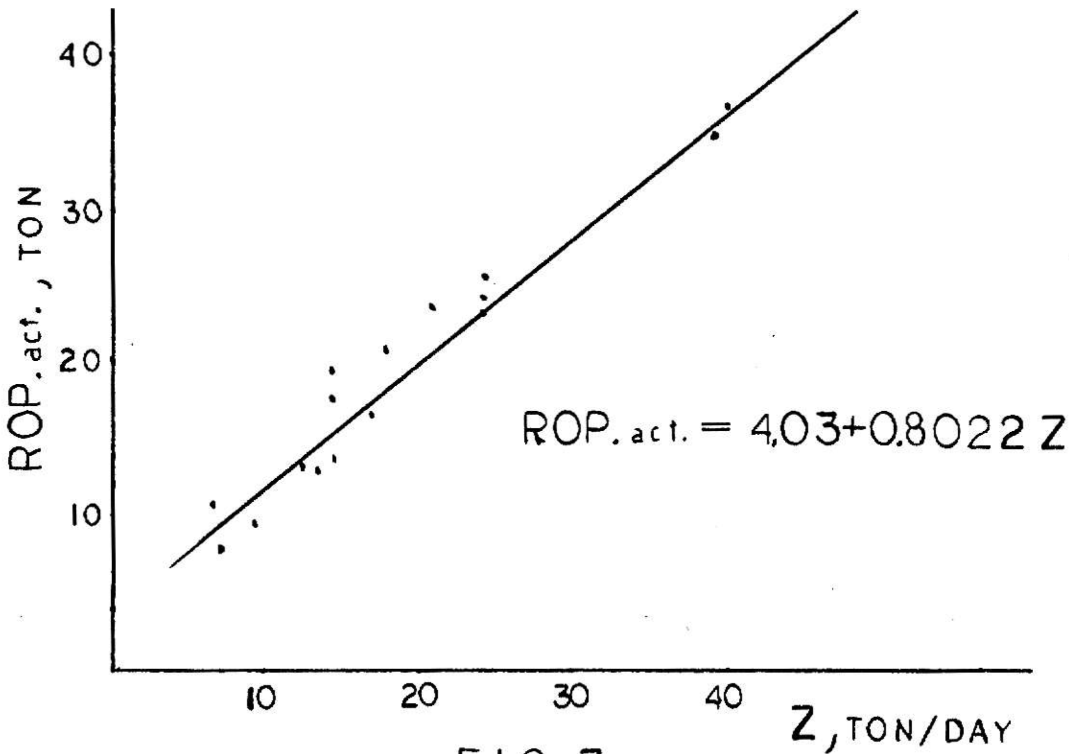
THIS BOOK CONTAINS NUMEROUS PAGES THAT WERE BOUND WITHOUT PAGE NUMBERS.

THIS IS AS RECEIVED FROM CUSTOMER.

EXPLANATION OF PLATE I

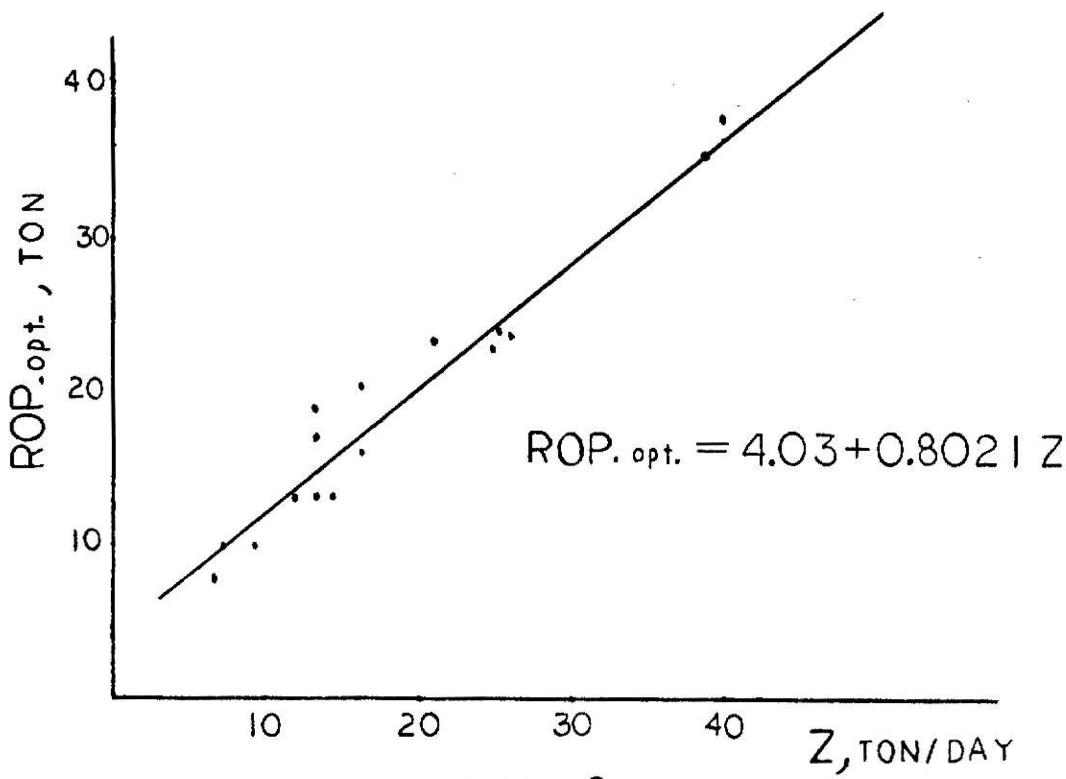
Fig. 7. Actual reorder point as a function of the demand per day.

Fig. 8. Optimal reorder point as a function of the demand per day.



40
30
20
10
ROP.act. , TON
ROP. act. = 4.03+0.8022 Z
10
20
30
40
Z, TON/DAY

FIG. 7



40
30
20
10
ROP.opt. , TON
ROP. opt. = 4.03+0.8021 Z
10
20
30
40
Z, TON/DAY

FIG. 8

EXPLANATION OF PLATE II

Fig. 9. Actual bin size as a function of the demand per day.

Fig. 10. Optimal bin size as a function of the demand per day.

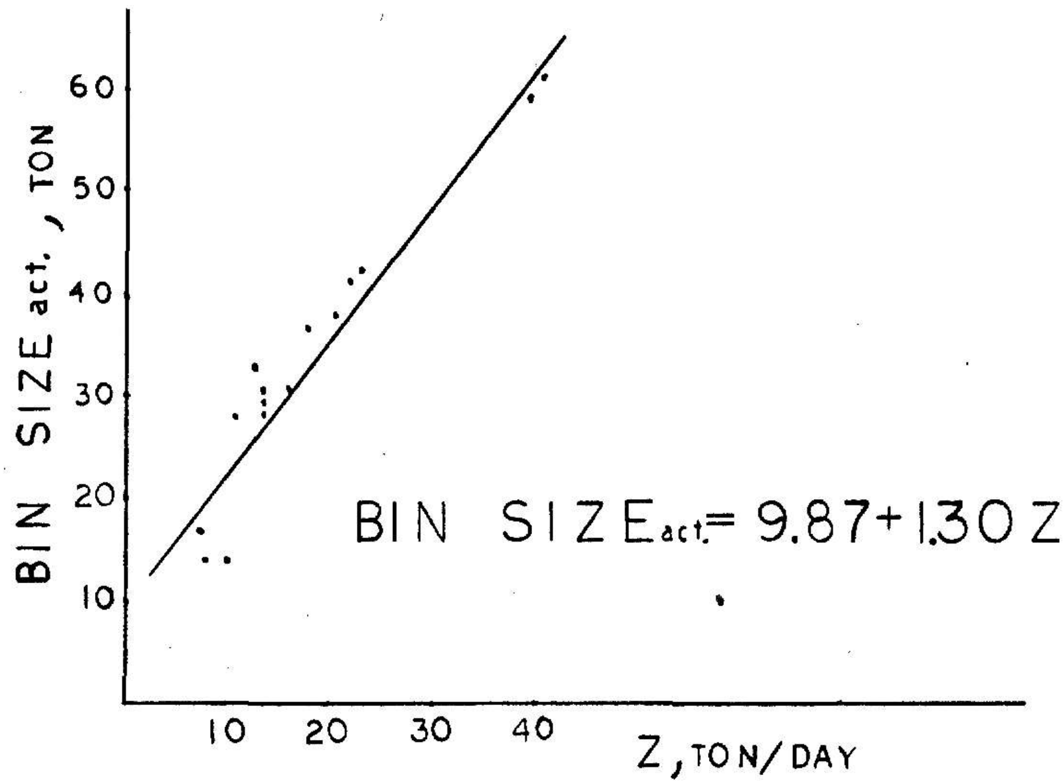

60
50
40
30
20
10
BIN SIZE act. , TON
10
20
30
40
Z, TON/DAY
BIN SIZEact.= 9.87+1.30Z

FIG. 9

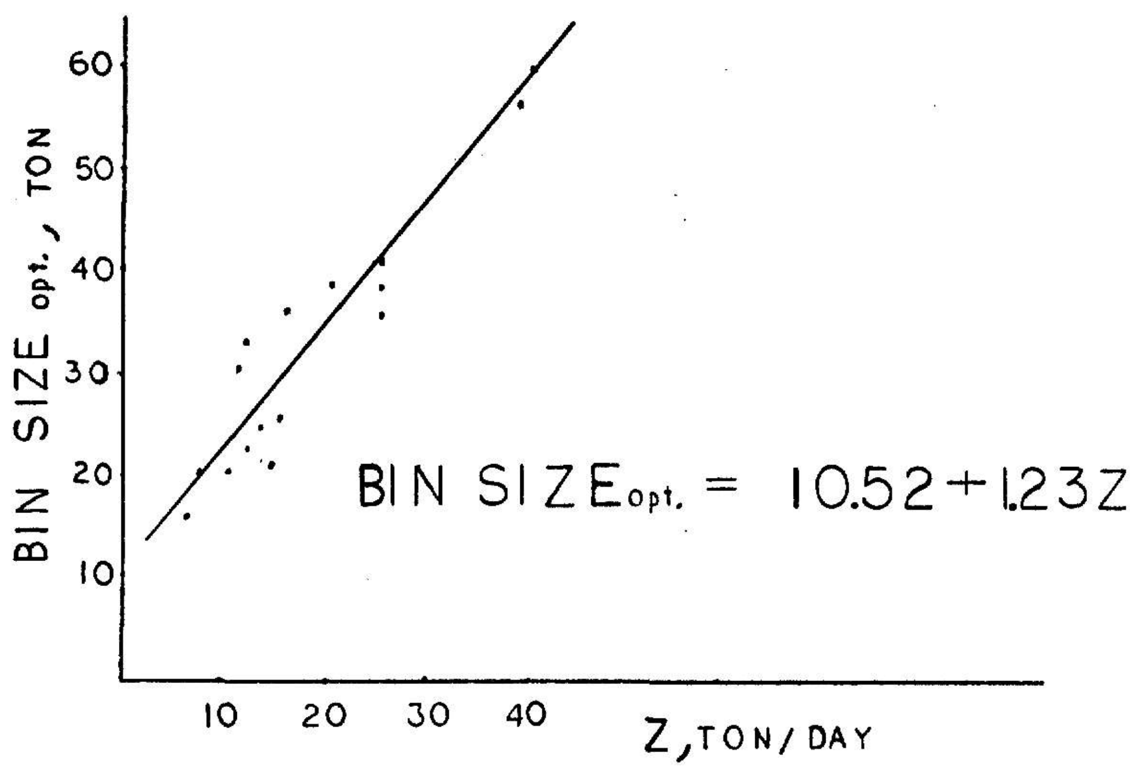

60
50
40
30
20
10
BIN SIZE opt. , TON
10
20
30
40
Z, TON/DAY
BIN SIZEopt. = 10.52+1.23Z

FIG. 10

EXPLANATION OF PLATE III

Fig. 11. Actual lot size order as a function of the demand per day.

Fig. 12. Optimal lot size order as a function of the demand per day.

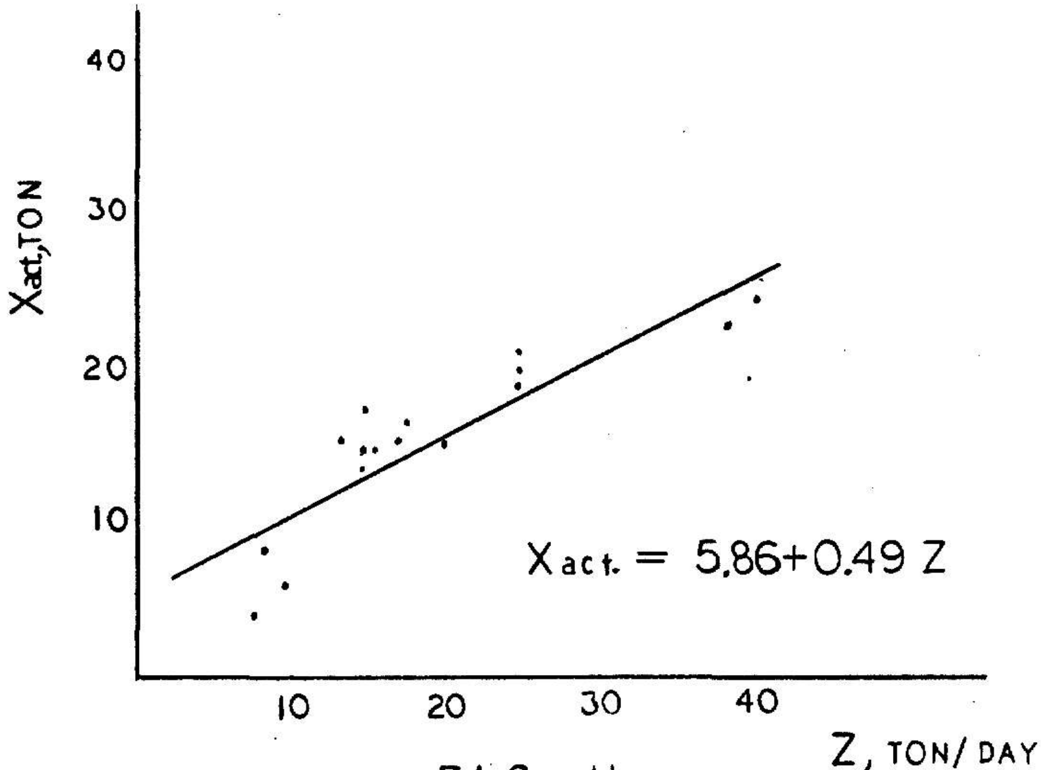

40
30
20
10
Xact,TON
Xact. = 5.86+0.49 Z
10
20
30
40
Z, TON/DAY

FIG. 11

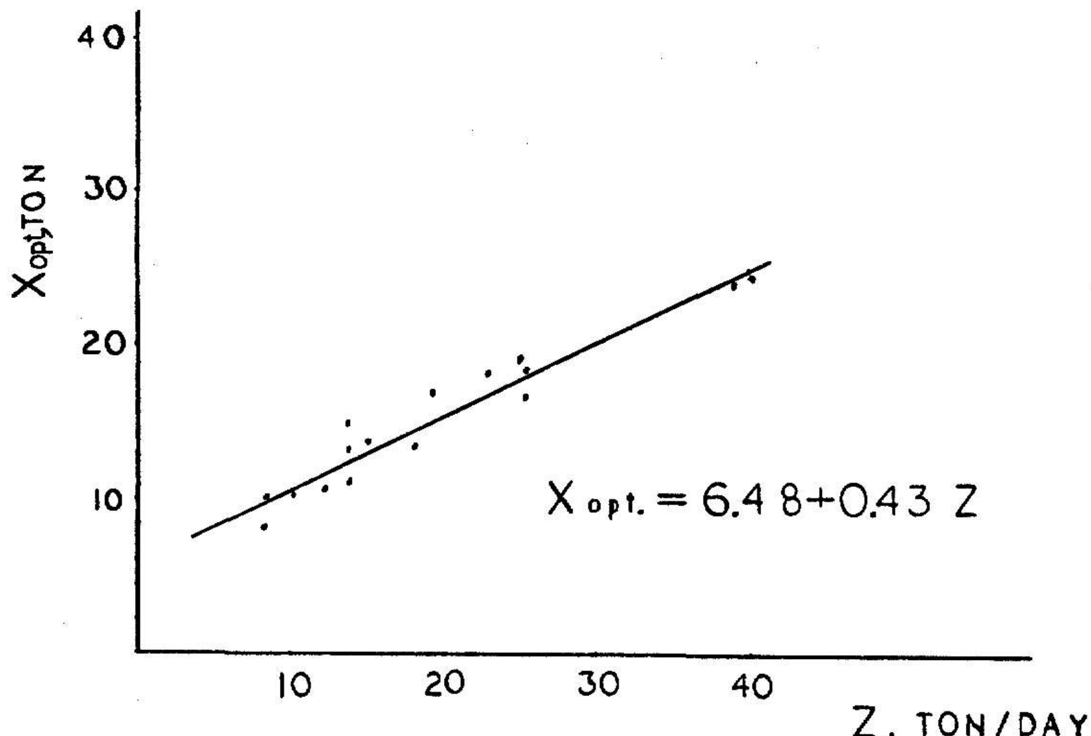

40
30
20
10
Xopt,TON
Xopt. = 6.4 8+0.43 Z
10
20
30
40
Z, TON/DAY

FIG. 12

SUMMARY

Optimizing the use of storage for finished feeds in bulk was the aim of this study. Classification of the types of feed according to an ABC analysis proves that about 20 percent of the number of types handled in the feed mill dominate about 85 percent of the production and inventory storage. These high volume feed types were treated separately from other feeds, in order to optimize their operational needs. The total storage available for these feeds becomes a constraint for the inventory of these feeds.

In order to minimize total cost an Inventory Control model under uncertainty was built with a constrained storage capacity. This model was solved by use of a Lagrange Multiplier and the nonlinear equations by Newton-Raphson's numerical method. The result is the optimal total cost and the physical quantities subject to the constraint, together with an optimal bin size for each type of feed under consideration. The actual bin assignment was done according to restrictions depending on the system and according to two priority criteria. The best actual inventory policy for finished feeds in bulk was calculated.

The approach of building the model is general and can be used for a general feed mill. Data of a specific feed mill was evaluated and the problem was solved according to its specific restrictions. A computer program for the optimal solution is supplied. Length of the computation was less than three minutes for 16 types. The computation of actual assignment takes about ten seconds per set of 16 types.

As a result of this study, the production manager is given a decision rule for the main question he faces: <u>when</u> to produce a type of feed (from this group of feeds) and <u>how much</u> to produce.

SUGGESTIONS FOR FURTHER RESEARCH

1. More detailed study of the exact classification of types as A, B or C. A simulation study would show the effect on cost of placing the cut-off point for the A-types at 80% or 90% rather than the 85% which has been suggested in the literature and which was used here.
2. Study of an Inventory-Control model for finished feeds under risk by use of the actual demand distribution functions.
3. Development of a mathematical tool for the actual bin assignment itself, after the optimal bin sizes were found by the methods presented in this study.
4. Study of the various effects of the lead time by inserting the bin assignment into a feed mill model (such as by simulation techniques) in order to study the production behaviour and its relationship to the assignment.

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APPENDIX 1.

From "Problems of Inventory Control in Feed Mills" by R. J. Carrillo, pp. 52-56.

Set-up Cost (C_r) (2)

In general set-up cost is simply the cost of the lost time during set-up. Within the feed mill it would be the time lost in making the necessary changes to produce a new formula when it has been producing a different one.

Whenever a change of formula occurs or even with the same formula but changing the shape of the final product (mash, pellets, crumbles) or changing from bulk to bags, some time is spent in setting the equipment so the new formula can be produced at full capacity. This operation is called "set-up", and the cost is the set-up cost.

Deriving the set-up cost involves determining the period of time spent between the time the scheduled production of the current formula is finished and the time the mill is producing the new formula at its regular capacity. Once this period of time is determined and knowing the cost per hour of the mill (labor, utilities, depreciation rates, etc.), the set-up cost is derived by multiplying the set-up time times the cost per hour.

Set-up cost for similarly processed formulas would be the same. In other words, independent of the formula itself if two formulas are produced in the same way (pellets in bulk or mash in bags, for example) they will have, on the average, the same set-up cost. By the same token, variation on these conditions will vary the set-up cost.

There are several operations that have to be made in changing formulas in a feed mill. The set-up time will be computed on the "bottle-neck" operation; that which will take more time in setting the mill up, since almost all of them can be made simultaneously.

In general time losses may occur at the following points.

a. Check if major ingredients are available in the ingredient bins. If not, order transfer from storage bins or warehouse of the quantity needed. Set grinders and grind if it is necessary. (Usually in normal operation this should have been done before the formula change.)

b. Check availability of minor ingredients, premix, etc. same observation as in (a) .

c. Check if the flow from ingredient bins to final destination of the mix is clear. Set the flow, spouts, conveyors, etc. For bagged feed: Arrange for availability of bags, labels, etc. For bulk feed: Set spouts and flow to the desired bin. For pelleted or crumblized feed: (1) change pellet mill die if it is necessary, (2) empty cooler, (3) set feed rate, steam rate, temperature to get optimal capacity in pellet mill and optimal pellet quality, and (4) fill cooler.

Only careful time studies can determine the extent of the time losses in an actual mill.

Cost of Being Out-of-Stock (K)

The out-of-stock cost is the cost of not carrying inventory. There are several situations to consider in regard to this cost. If an order for some amount of a particular feed arrives at the mill when this is out-of-stock:

a. If the customer is willing to wait the time required to fill the order, and the mill is not running 24 hours per day, the order could be filled by overtime production. The regular production schedule could be interrupted in order to make room for the particular order, and of course, overtime is needed to recuperate the normal production. Or the order could be run directly in overtime without interrupting the regular schedule. In any case the out-of-stock cost (K) would be: (Overtime production cost + set-up cost for interrupted normal schedule). The overtime production cost can be easily calculated, and usually amounts to the regular production cost plus the additional percentage of labor cost due to overtime.

Set-up cost was explained above.

b. If the customer is willing to wait some time to get his order filled but the mill is running 24 hours per day, the order could be lost (this case is discussed later), or the production schedule should be interrupted to make room for that particular order. In this latter case, recuperation of the lost production could be done over the week-end with its consequent costs (overtime cost plus set-up cost), or lost production could be carried on indefinitely with risk of going out-of-stock in other formulas and losing additional orders.

c. Assuming that the mill is running 24 hours, seven days a week, or if the customer is not willing to wait for his order to be filled in a given time. This depends on the emergency of the order itself from the customer's point of view. (Animals could starve, for instance.) In this case, the sale will be lost because the customer will go to some other feed supplier to get his order. The cost of out-of-stock would be then, the profit lost due to not making a sale.

But usually this is not all because a customer who has found one or more out-of-stocks in one given formula, will become less likely to return and therefore, the customer will be lost. It could happen that a particular customer usually buys not only the formula we have been talking about, but several others.

Thus, the cost of out-of-stock would be the profit lost due to not making all possible future sales in the different formulas the lost customer used to buy. It is very difficult to measure this cost and perhaps it could only be established by estimation. The way to do this is by establishing a policy of permissible percentage level of out-of-stocks, one out-of-stock in a hundred times or one in a thousand times for example.

APPENDIX 2.

Computer program for optimal bin assignment.

```
$JOB           RK,TIME=5
C      OPTIMAL  BIN  ASSIGNMENT.
C              BY  R. KALINHOFF.
C****** NTYPE= NUMBER OF TYPES.
C****** TSIZE= TOTAL STORAGE AVAILABLE.
C****** SS= TOTAL STORAGE AVAILABLE WITHOUT STORAGE FOR LEAD TIME DEMAND.
C****** RR= TOTAL STORAGE FOR LEAD TIME DEMAND.
C****** SLAMDA= LAGRANGIAN MULTIPLIER.
C****** NO= NUMBER OF ITERATIONS.
C****** M= NUMBER OF DATA SETS.
C************ FOR ALL TYPES PER DAY.**************
C****** TCC= TOTAL COST.
C****** ORCC= TOTAL COST OF REGULAR ORDER AND SET UP.
C****** CACC= TOTAL CARRYING COST.
C****** STCC= TOTAL SAFETY STOCK HCLDING COST.
C****** OHCC= TOTAL OUT OF STOCK COST.
C**************     FOR TYPE (K).  ***************
C****** Z(K)= AVERAGE DAILY DEMAND.
C****** TLT(K)= LEAD TIME ,DAYS.
C****** CR(K)= ORDER AND SET UP COST.
C****** CC(K)= CARRYING COST PER DAY OF A TON.
C****** FK(K)= FIXED COST FOR SPECIAL ORDER AND SET UP.
C****** TK(K)= CHEBYSHEV CONSTANT.
C****** SD(K)= STANDARD DEVIATION CF DAILY DEMAND.
C****** S(K)= CONVULUTED STANDARD CEVIATICN.
C****** X1(K)= CAPACITY OF BINS.
C****** X(K)= LOT SIZE ORDER.
C****** ORC(K)= ORDER AND REGULAR SET UP COST.
C****** CAC(K)= CARRYING COST.
C****** STC(K)= SAFETY STOCK COST.
C****** SSTOCK(K)= SAFETY STOCK,TONS.
C****** OHC(K)= OUT OF STOCK COST.
C****** TC(K)= TOTAL COST.
C****** ROP(K)=REORDER POINT,TONS.
C****** R(K)= DEMAND THROUGH LEAD TIME.
C****** TT(K)= CYCLES PER DAY.
C****** P(K)= PROBABILITY TO BE OUT OF STOCK.
C****** BIN(K)= BIN CAPACITY NEEDEC.
C
C******  DATA ARRANGING  ******
C
C     ONE   CARD  = NUMBER OF SETS.
C      FOR EACH SET OF DATA-
C     ONE   CARD  = NTYPE.
C     NTYPE CARDS = LEAD TIME FOR EACH FEEC.
C     ONE   CARD  =TSIZE.
C     ONE   CARD  = SLAMDA.
C     NTYPE CARDS = Z,CR,CC,FK,SD,X,TK, FOR EACH FEED.
      DIMENSION Z(16),CR(16),CC(16),FK(16),S(16),X(16),TK(16),A(33,34)
      DIMENSION ORC(16),CAC(16),STC(16),OHC(16),TC(16)
      DIMENSION SSTOCK(16),ROP(16),R(16),TT(16),P(16),SD(16),BIN(16)
      DIMENSION TLT(16)
   20 FORMAT(F5.2,F5.2,F6.4,3F5.2,F5.3)
   25 FORMAT(1H ,2X,I2,5X,F5.2,8X,F5.2,8X,F6.4,8X,F5.2,9X,F5.2,4X,F5.2,2
     1X,F5.3)
   30 FORMAT(6F14.4)
   40 FORMAT(F7.4)
   50 FORMAT(F5.1)
   60 FORMAT(2F14.5)
   70 FORMAT(1H ,'VALUE OF LAMDA=',F14.7)
```

```
   80 FORMAT(1H ,2X,I2,4X,F5.2,6X,F4.2,7X,F4.2,6X,F6.2,7X,F5.2,2X,F5.2)
   91 FORMAT(1H ,'TYPE',2X,'SET UP AND',2X,'INVENTORY',2X,'SAF.STOCK',2X
     1,'O.O.H',6X,'TOTAL')
   92 FORMAT(1H ,6X,'ORDER COST',4X,'COST',7X,'COST',6X,'COST',7X,'COST'
     1)
  102 FORMAT(5F14.5)
  111 FORMAT(1H ,'INPUT DATA')
  112 FORMAT(1H ,'NUMBER OF TYPES=',I2)
  113 FORMAT(I2)
  114 FORMAT(1H ,'TYPE',2X,'DEMAND PER',2X,'ORDER COST',2X,'CARRYING COS
     1T',2X,'SET UP COST',2X,'ST.DEV.OF',4X,'TRIAL',2X,'TRIAL')
  115 FORMAT(1H ,6X,'DAY TONS',8X,'$',8X,'$/TON/DAY',6X,'$/ORDER',4X,'DE
     1MAND TCNS',4X,'X',5X,'TK')
  117 FORMAT(1H ,'TRIAL LAMDA=',F7.4)
  118 FORMAT(1H ,'TOTAL STORAGE AVAILABLE=',F5.1,'TONS')
  119 FORMAT(1H ,'OUTPUT DATA')
  120 FORMAT(1H ,'TOTAL STORAGE AVAILABLE WITHOUT LEAD TIME INVENTORY=',
     1F5.1)
  121 FORMAT(1H ,'THE DELTAS OF X, TK AND LAMDA')
  122 FORMAT(1H ,'TOTAL COSTS FOR ALL TYPES,PER DAY')
  123 FORMAT(1H ,'TOTAL COST=',F10.2,'$')
  124 FORMAT(1H ,'ORDER AND SET UP COST=',F10.2)
  125 FORMAT(1H ,'CARRYING COST=',F10.2)
  126 FORMAT(1H ,'SAFETY STOCK COST=',F10.2)
  128 FORMAT(1H ,'OUT OF HAND COST=',F10.2)
  129 FORMAT(1H ,'TYPE',2X,'LOT SIZE',2X,'SAFETY',2X,'STOCK FOR',2X,'REO
     1RDER',2X,'CYCLE',2X,'CHEBYSHEV',2X,'PROBABILITY')
  130 FORMAT(1H ,5X,'ORDER TONS',2X,'STOCK',2X,'LEAD TIME',3X,'POINT',4X
     1,'/DAY',6X,'TK',7X,'LE. O.O.H')
  131 FORMAT(1H ,2X,I2,3X,F5.2,5X,F5.2,4X,F5.2,5X,F5.2,4X,F4.2,4X,F5.3,
     16X,F7.4)
  132 FORMAT(1H ,'NUMBER OF ITERATIONS=',I2)
  133 FORMAT(1H ,2X,I2,8X,F7.3)
  134 FORMAT(1H ,'TYPE',2X,'OPTIMAL BIN SIZE')
  135 FORMAT(1H ,'TYPE',2X,'LEAD TIME IN CAYS')
  136 FORMAT(1H ,1X,I2,10X,F5.1)
  222 FORMAT(1H ,'AN INVENTORY POLICY FOR AN EXISTING FEED MILL CONSRAIN
     1ED')
  223 FORMAT(1H ,10X,'BY STORAGE FCR FINISHED FEEC IN BULK.')
  224 FORMAT(1H ,20X,'BY REUBEN KALINHOFF')
      WRITE(3,222)
      WRITE(3,223)
      WRITE(3,224)
      READ(1,113) M
      MO=0
 2C00 WRITE(3,111)
      MO=MO+1
      NO=0
      EPSI=0.0001
      READ(1,113) NTYPE
      WRITE(3,112) NTYPE
      WRITE(3,135)
      DO 77 K=1,NTYPE
      READ(1,50) TLT(K)
   77 WRITE(3,136) K,TLT(K)
      WRITE(3,114)
      WRITE(3,115)
      N=2*NTYPE
      N1=N+1
      N2=N-1
```

```
      N3=N1+1
      RR=0.
      READ(1,50) TSIZE
      READ(1,40) SLAMDA
      DO 7 K=1,NTYPE
      READ(1,20) Z(K),CR(K),CC(K),FK(K),SD(K),X(K),TK(K)
      WRITE(3,25) K,Z(K),CR(K),CC(K),FK(K),SD(K),X(K),TK(K)
C******CONVULUTED STANDARD DEVIATION.
      S(K)=SD(K)*SQRT(TLT(K))
C******LEAD TIME DEMAND.
      R(K)=Z(K)*TLT(K)
      RR=RR+R(K)
    7 CONTINUE
      SS=TSIZE-RR
      WRITE(3,117) SLAMDA
      WRITE(3,118) TSIZE
C****** ARRANGING OF THE COEFFICIENTS MATRIX.******
      DO1I=1,N1
      DO2J=1,N3
      A(I,J)=0.
    2 CONTINUE
    1 CONTINUE
C******ELEMENTS OF DIAGONAL,ODD COLUMNS,ODD ROWS.
 1000 K=1
      NO=NO+1
      DO3I=1,N2,2
      J=I
      A(I,J)=2*Z(K)*CR(K)/X(K)**3+Z(K)*FK(K)/(X(K)**3*TK(K)**2)
      K=K+1
    3 CONTINUE
C******ELEMENTS OF DIAGONAL,ODD ROWS,EVEN COLUMNS.
      K=1
      DO4I=1,N2,2
      J=I+1
      A(I,J)=Z(K)*FK(K)/(X(K)**2*TK(K)**3)
      K=K+1
    4 CONTINUE
C******ELEMENTS OF DIAGONAL,EVEN ROWS,ODD COLUMNS.
      K=1
      DO5I=2,N1,2
      J=I-1
      A(I,J)=Z(K)*FK(K)/(X(K)**2*TK(K)**3)
      K=K+1
    5 CONTINUE
C******ELEMENTS OF DIAGONAL,EVEN ROWS,EVEN COLUMNS.
      K=1
      DO 6I=2,N1,2
      J=I
      A(I,J)=3*Z(K)*FK(K)/(X(K)*TK(K)**4)
      K=K+1
    6 CONTINUE
C******ELEMENTS IN COLUMN N1,ODD ROWS.
      J=N1
      DO8I=1,N,2
      A(I,J)=1.000
    8 CONTINUE
C******ELEMENTS IN COLUMN N1,EVEN ROWS.
      K=1
      DO 9 I=2,N,2
      A(I,J)=S(K)
```

```
      K=K+1
    9 CONTINUE
C******ELEMENTS IN LAST COLUMN,(THE FUNCTION),IN ODD ROWS.
      J=N1+1
      K=1
      DO10I=1,N,2
      A(I,J)=-(-Z(K)*CR(K)/X(K)**2+CC(K)/2
     1-Z(K)*FK(K)/(X(K)**2*2*TK(K)**2)+SLAMDA)
      K=K+1
   10 CONTINUE
C******ELEMENTS IN LAST COLUMN,(THE FUNCTION),IN EVEN ROWS.
      K=1
      DO11I=2,N,2
      J=N1+1
      A(I,J)=-(S(K)*CC(K)-Z(K)*FK(K)/(X(K)*TK(K)**3)+SLAMDA*S(K))
      K=K+1
   11 CONTINUE
C******LAST ELEMENT.
      T=0.
      I=N1
      J=N1+1
      DO12K=1,NTYPE
      Q=X(K)+TK(K)*S(K)
      T=T+Q
   12 CONTINUE
      A(I,J)=-(T-SS)
C******ELEMENTS IN LAST ROW,ODD COLUMNS.
      I=N1
      DO13J=1,N,2
      A(I,J)=1.
   13 CONTINUE
      I=N1
      J=I
      A(I,J)=0.
C******ELEMENTS IN LAST ROW,EVEN COLUMNS.
      K=1
      DO14J=2,N,2
      A(I,J)=S(K)
      K=K+1
   14 CONTINUE
C******PERFORM A GAUSS JORDAN REDUCTION.
      DO200J=1,N1
      DIV=A(J,J)
      S1=1.0/DIV
      DO201L=J,N3
  201 A(J,L)=A(J,L)*S1
      DO202I=1,N1
      IF(I-J) 203,202,203
  203 AIJ=-A(I,J)
      DO 204L=J,N3
  204 A(I,L)=A(I,L)+AIJ*A(J,L)
  202 CONTINUE
  200 CONTINUE
      J=N3
      IF(NO.EQ.20) GO TO 900
      DO18I=1,N1
      IF(ABS(A(I,N3)).LT.EPSI) GO TO 18
      GO TO 19
   18 CONTINUE
      GO TO 900
```

```
  900 WRITE(3,119)
      WRITE(3,120) SS
C******WRITE OUT THE DELTAX'S,K'S,AND DELTA LAMDA.
      WRITE(3,121)
      WRITE(3,102) (A(I,N3),I=1,5)
      WRITE(3,102) (A(I,N3),I=6,10)
      WRITE(3,102) (A(I,N3),I=11,15)
      WRITE(3,102) (A(I,N3),I=16,20)
      WRITE(3,102) (A(I,N3),I=21,25)
      WRITE(3,102) (A(I,N3),I=26,30)
      WRITE(3,102) (A(I,N3),I=31,N1)
      TCC=0.
      ORCC=0.
      CACC=0.
      STCC=0.
      OHCC=0.
      DO21K=1,NTYPE
      ORC(K)=0.
      CAC(K)=0.
      TC(K)=0.
      STC(K)=0.
      OHC(K)=0.
      TT(K)=0.
   21 CONTINUE
      WRITE(3,91)
      WRITE(3,92)
      DO17 K=1,NTYPE
      ORC(K)=Z(K)*CR(K)/X(K)
      ORCC=ORCC+ORC(K)
      CAC(K)=CC(K)*X(K)/2.
      CACC=CACC+CAC(K)
      STC(K)=TK(K)*S(K)*CC(K)
      STCC=STCC+STC(K)
      OHC(K)=Z(K)*FK(K)/(X(K)*2*TK(K)**2)
      OHCC=OHCC+OHC(K)
      TC(K)=ORC(K)+CAC(K)+STC(K)+OHC(K)
      TCC=TCC+TC(K)
      WRITE(3,80) K,ORC(K),CAC(K),STC(K),OHC(K),TC(K)
   17 CONTINUE
      WRITE(3,122)
      WRITE(3,123) TCC
      WRITE(3,124) ORCC
      WRITE(3,125) CACC
      WRITE(3,126) STCC
      WRITE(3,128) OHCC
      WRITE(3,129)
      WRITE(3,130)
      DO33K=1,NTYPE
      TT(K)=Z(K)/X(K)
      SSTOCK(K)=S(K)*TK(K)
      ROP(K)=SSTOCK(K)+R(K)
      P(K)=1./(2.*TK(K)**2.)
      WRITE(3,131) K,X(K),SSTOCK(K),R(K),ROP(K),TT(K),TK(K),P(K)
   33 CONTINUE
      WRITE(3,134)
      DO35K=1,NTYPE
      BIN(K)=X(K)+R(K)+SSTOCK(K)
      WRITE(3,133) K,BIN(K)
   35 CONTINUE
      WRITE(3,70) SLAMDA
```

```
      WRITE(3,132) NO
      GO TO 999
C******ADD THE DELTAS TO THE CORRESPONDING VARIABLES.
   19 K=1
      DO15I=1,N,2
      X(K)=X(K)+A(I,J)
      K=K+1
   15 CONTINUE
      J=N3
      K=1
      DO16I=2,N,2
      TK(K)=TK(K)+A(I,J)
      K=K+1
   16 CONTINUE
      SLAMDA=SLAMDA+A(N1,N3)
C******GO TO NEXT ITERATION.
      GO TO 1000
C*****MAKE SECOND RUN WITH NEW DATA.
  999 IF(MO.EQ.M) GO TO 888
      GO TO 2000
  888 STOP
      END
```

APPENDIX 3.

Computer program for actual bin assignment.

```
$JOB              RK,TIME=1
C         ACTLAL  BIN  ASSIGNMENT.
C
C              BY  R. KALINHOFF.
C******NTYPE=NO. OF TYPES.
C******TLT(K)=LEAD TIME.
C******TC(K)=TOTAL COST FOR TYPE K.
C******Z(K)VAVERAGE DAILY DEMAND FCR TYPE K.
C******CR(K)=ORDER AND SET UP COST FOR TYPE K.
C******CC(K)=CARRYING COST PER DAY OF A TCN OF TYPE K.
C******FK(K)=FIXED COST FOR SPECIAL SET UP AND ORDER OF TYPEK.
C******TK(K)=CHEBYSEV CCNSTANT FOR TYPE K.
C******SD(K)=STANDARD DEVIATION OF DAILY DEMAND FOR TYPE K.
C******S(K)=CINVULUTED ST. DEV. FOR TYPE K.
C******X1(K)=CAPACITY OF BINS ASSIGNED TC TYPT K.
C******X(K)=LOT SIZE ORDER FOR TYPE K.
C******R(K)=DEMAND THROUGH LEAD TIME FOR TYPE K.
C******ROP(K)=REORDER POINT FOR TYPE K.
C******SSTOCK(K)=RESERVE STOCK FOR TYPE K.
C******TT(K)=CYCLES PER DAY.
C******TCC=TCTAL COST PER DAY.
C******N=NUMBER OF DATA SETS.
C******M=COUNTER FOR N.
C         ARRANGING OF INPUT DATA.
C         ONE CARD = NUMBER OF DATA SETS.
C         FOR EACH DATA SET---
C         ONE CARD = NTYPE.
C         NTYPE CARDS = TLT.
C         NTYPE CARDS ON EACH  Z,CR,CC,FK,SC,X1,TK.
      DIMENSION TC(16),Z(16),CR(16),CC(16),FK(16),TK(16),SD(16),S(16)
      DIMENSION X(16),X1(16),ROP(16),R(16),SSTOCK(16)
      DIMENSION TT(16),TLT(16)
    1 FORMAT(I2)
    2 FORMAT(///,2X,'MODIFICATION CF TOTAL COST AND LOT SIZE ORDER AFTER
     1 BIN ASSIGNMENT')
    3 FORMAT(I2)
    4 FORMAT(F5.2,F5.2,F6.4,3F5.2,F5.3)
    5 FORMAT(F5.1)
    6 FORMAT(1H ,2X,I2,5X,F6.2,8X,F6.3,10X,F5.2,10X,F5.2,8X,F5.2,5X,F5.2
     1)
    7 FORMAT(///,2X,'TOTAL COST FOR GROUP A FEEDS IN THIS ASSIGNMENT=',F
     110.2)
    8 FORMAT(//,2X,'TYPE',2X,'TOTAL COST',2X,'LOT SIZE ORDER',2X,'BIN CA
     1PACITY',2X,'REORDER POINT',2X,'SAF.STOCK',2X,'CYC/ DAY')
    9 FORMAT(///,20X,'BY REUBEN KALINHCFF')
      WRITE(3,2)
      WRITE(3,9)
      READ(1,1) N
      M=0
  999 M=M+1
      READ(1,3) NTYPE
      DO 10 K=1,NTYPE
      READ(1,5) TLT(K)
   10 CONTINUE
      WRITE(3,8)
      DO15K=1,NTYPE
      READ(1,4) Z(K),CR(K),CC(K),FK(K),SD(K),X1(K),TK(K)
      S(K)=SC(K)*SQRT(TLT(K))
      R(K)=Z(K)*TLT(K)
      SSTOCK(K)=TK(K)*S(K)
```

```
      X(K)=X1(K)-R(K)-SSTOCK(K)
      IF(X(K).GT.0.) GO TO 20
      X(K)=X1(K)
      SSTOCK(K)=0.
      R(K)=X(K)
   20 ROP(K)=R(K)+SSTOCK(K)
      TT(K)=Z(K)/X(K)
   15 CONTINUE
      TCC=0.
      DO13K=1,NTYPE
      TC(K)=0.
   13 CONTINUE
      DO14K=1,NTYPE
      TC(K)=Z(K)*CR(K)/X(K)+X(K)*CC(K)/2.+TK(K)*S(K)*CC(K)+FK(K)*Z(K)/(
     1X(K)*2.*TK(K)**2)
   19 TCC=TCC+TC(K)
      WRITE(3,6) K,TC(K),X(K),X1(K),ROP(K),SSTOCK(K),TT(K)
   14 CONTINUE
      WRITE(3,7) TCC
      IF(M.EQ.N) GO TO 1000
      GO TO 999
 1000 STOP
      END
```

INVENTORY CONTROL FOR FINISHED FEEDS UNDER CONSTRAINT

by

REUBEN KALINHOFF

B. Sc., Technion, Israel Institute of Technology, 1965

AN ABSTRACT OF A MASTER'S THESIS

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Increased sales rate of feeds, failure in initial design and other effects cause in some cases the finished feed bins in a feed mill to become a constraint. A careful analysis of the demand will show that a small number of feed types dominate the production and the inventory, while a large number of feeds occupy a relative small production requirement and storage capacity. It is not justifiable to treat all types of feeds in the same manner. The high volume feeds should be treated separately according to an Inventory Control policy which will optimize the total cost of production and storing.

This study suggests a method, in which the high volume feeds will be treated in a policy which will take care of the average demand and its fluctuations, but not on the actual demand for "tomorrow". The low volume feeds will be produced whenever an order for them arrives. An Inventory-Control model under uncertainty in the Q system was built. The optimal total cost equation for this model has to be solved subject to the total storage capacity available for all of the high volume feeds. Solving this model will yield an optimal bin size, lot size order and safety stock as well as reorder point for the high volume feeds. According to the limitation of feeds and bins, the optimal results have to be fitted into a given bin system in order to assign the feeds to bins.

The analytical solution to the model has been done using a Lagrange Multiplier, while the numerical solution has been done by Newton-Raphson's method. Two computer programs are supplied for the calculations. The optimal solution was computed in less than three minutes for 16 types, and the actual assignment in less than ten seconds. As a result of these computations, the production manager is given a decision rule in order to know when and how much to produce of each of the high volume feeds.