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EFFECT OF CONTROL PARAMETERS ON
ENERGY CONSUMPTION OF A ROOM HEATING SYSTEM

by

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B.Tech., Indian Institute of Technology, Madras, India

1978

A MASTER'S THESIS

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

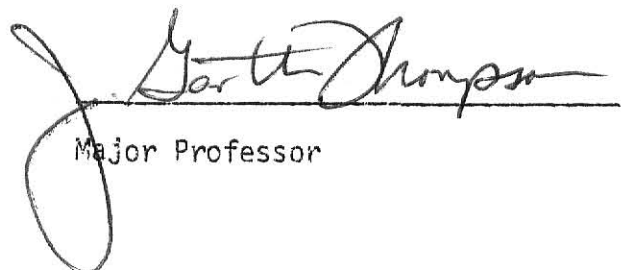
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KANSAS STATE UNIVERSITY

Manhattan, Kansas

1980

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I N T R O D U C T I O N

Rapidly rising energy costs increasingly and continuously demand more efficient ways of using present available energy. As the cost of producing energy continues to rise more forms of energy and methods of converting them into usable forms become economically acceptable. Nevertheless the importance of employing efficiently managed energy systems cannot be overstated.

A significant amount of energy is used in conditioning the air of buildings. The way a building air conditioning system is designed has a significant impact on energy use. It is doubtful that design procedures employed in an era of inexpensive energy, placed enough stress on the energy efficiency of the system. Today, much work is being done in developing new tools for better design. The primary concern remains the accurate prediction of heating and cooling loads which directly affect the sizing of heating, cooling, and air conditioning equipment to be installed.

Refined and sophisticated calculation procedures are both time consuming and expensive. Without the use of advanced computer methods, they are impossible. The development of calculation procedures must be justified on the basis that the more accurate calculations will result in overall savings in energy usage, in owning and operating costs and consequently in total life cycle costs. This will be achieved through better design of the building systems, and more efficiently controlled operation of the heating and cooling systems. There are many indications that such a justification is warranted.

The simulation of the heat transfer mechanism of a room or building

may be made on a digital computer. These simulations are based on solutions to simultaneous heat balance equations at all interior surfaces of a room or space. They contain different subroutines, each handling a specific part of the physical problem. For example, one subroutine may find heat loss due to conduction heat transfer through walls, while another may simulate room temperature at a given time step, etc.

Simulations may be made of steady rate or transient operation of the system. Simulations of the dynamic response of a system would be more complicated and informative, as it would represent the true functioning and nature of the system. Dynamic simulations are available which consider only the long term dynamics of the system, hour-by-hour simulations. These hourly simulations make use of weather bureau tapes for hourly weather data. Simulations with an hourly step size consider the dynamics of the building shell, but neglect air and water distribution system and the control system. To obtain hourly heat loss, the inside temperature is assumed to be fixed at a value determined by the control strategy. These programs compute the amount of energy that must be supplied by the HVAC system to maintain the specified temperature.

Hour-by-hour simulation programs are not well suited for studies of the dynamics of HVAC control systems. They are well suited for and employed to studies of the effect of the buildings' physical characteristics on energy use. In order to evaluate the effect of the short term dynamics of the control system and room, on energy consumption, simulation programs should be generated with shorter time step size and with the addition of models of the short term dynamics of the system.

This thesis is divided into four major parts.

Part I: Formulation and Improvements in Modified Thermal Response Factor Method. This part is an investigation of the effect of reducing step size on the terms

Part I: Formulation and Improvements in Modified Thermal Response Factor Method.

This part is an investigation of the effect of reducing step size on the terms of the conduction transfer functions. It also includes an improvement in the calculation procedure for heat fluxes at inside and outside surfaces of walls.

Part II: Generalization of Thermostat, Valve, and Coil Models.

Part III: Micro-Computer Implementation of the Thermal Simulation of the Room.

Part IV: A Study of the Effect of Control System Parameters on Energy Consumption.

Part 1FORMULATION AND IMPROVEMENTS IN MODIFIED THERMAL
RESPONSE FACTOR METHOD

It is the aim of this part of the thesis, to study the Modified Thermal Response factors applied in the calculation of transient heat conduction. Also, a method is proposed for improvement in the calculation procedure for obtaining surface temperatures and heat fluxes.

An algorithm for the Modified Thermal Response Factor Method is developed. It is desired that the algorithm (1) be able to handle different walls of varying number of layers, (2) has capability for variation in time step increments, which are short enough to be used with faster dynamic elements of the system, (3) be amenable to computer usage, and (4) have no restrictive input, such as constant or periodic boundary conditions.

A computer program for the above algorithm has been written which (1) is able to handle up to ten layered walls, (2) has variable time step, (3) is fast enough to be acceptable for practical application, (4) computes values as accurately as values reported by other investigators, and (5) has structured programming approach with good documentation for easy understanding.

Prediction of the rate of heat flow through walls and roofs, made of several layers of different materials and subject to arbitrary variations of ambient conditions, is a problem of practical importance in the design of buildings. One of the earliest studies of the subject was made by Nessi and Nisolle [1] . Their approach involved such a great amount of computations that it was of very little practical value as long as the

calculations had to be made by hand. Mackey and Wright [2] worked on the same problem with restricted boundary conditions where outside temperature varied periodically and conditions were constant inside. Their work has been the basis for air-conditioning design calculations made in America during the past 20 years. Mitalas and Stephenson [3] pioneered development of procedures suited for digital computers. Their approach was based on the Nessi-Nisolle procedure. It was called Thermal Response Factor Method. Mitalas [4] used the method to solve the problems with variable surface heat transfer coefficients and Kusuda [5] used the same technique to determine the flux through cylindrical and spherical walls. Mitalas and Stephenson [6] applied z-transforms to the problem of transient heat conduction. They observed that the resulting z-transfer function were similar to the response factors, but they were much more economical to use in terms of computer memory and running time. The new procedure was called Modified Thermal Response Factor Method. Mitalas and Arseneault [7] produced a computer program for the modified thermal response factors.

Other approaches to the problem of transient conduction could be analytical and finite difference methods. Analytical solutions to one-dimensional unsteady state systems with restricted boundary conditions are extensively investigated. Yet a solution for the problem of transient heat conduction through walls or roofs made up of different materials and subject to arbitrary variations of ambient conditions are not possible. Finite difference method is frequently used, but this approach is not preferred in the study of effects of dynamics of control system. The advantage in using the Modified Thermal Response Factors is use of experimental results for structures of complicated or nonhomogeneous composition.

The development of the algorithm for the Modified Thermal Response Factor Method for a multi-layered wall is accomplished through the following steps:

1. The Partial Differential equations for one dimensional heat flux and temperature in an infinite homogeneous are considered.
2. Time variable is eliminated by taking Laplace transforms of these equations to obtain Ordinary Differential equations.
3. The Ordinary Differential equations are solved to obtain a matrix form relating temperature and heat flux at one surface to that at the other surface.
4. The above matrix form for one layer is extended to obtain a matrix form for a multi-layer homogeneous semi-infinite slab. This relates temperature and heat flux at the interior surface of the slab to that at the exterior surface.
5. These equations are manipulated to obtain fluxes at the interior and exterior surfaces in terms of the inside and outside surface temperatures, and the corresponding transfer functions between these surface temperatures and fluxes.
6. A finite set of the smallest poles of these transfer functions is obtained. The number of poles considered depends on the solution time step size, and thermo-physical properties of the materials of the wall. A large number of poles must be considered if the time step size is small and/or the wall has high thermal storage capacity and high thermal resistivity.
7. The resulting finite approximations to the Laplace transfer function are transformed to the z -domain, using an appropriate hold function to account for the values of the temperatures during sampling intervals.
8. The z -transforms of the transfer functions are expressed in ratios of negative powers of " z 's", such that numerator and denominator polynomials are of the same order.
9. Inverse z -transforms are obtained using the signal delay form to pro-

duce a recursion formula to express the fluxes at the interior and exterior surfaces at the present time step in terms of weighted sums of surface temperatures at the present and previous time steps and fluxes at previous time steps. The weighting factors are the Modified Thermal Response Factors.

1.1.1

PROCEDURE FOR FORMULATION OF WALL MODEL

It is assumed that a wall is made up of several uniform layers of homogeneous materials and that the physical and thermal properties of the materials do not change with change in temperature.

An approach to modeling of walls includes the resistances of surface films at the interior and exterior surfaces of the wall. The results obtained are in terms of fluid temperatures and the radiation is ignored. Another approach does not include the film resistance in the Modified Response Factors and obtain the results in terms of surface temperatures, providing the opportunity to include the effect of radiation.

1.1.2

SINGLE LAYERED SLAB AS A WALL

An infinite slab of homogeneous material is considered, figure (1.1.1). The one dimension conduction equation for the slab is given by [8].

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha} \frac{\partial \theta}{\partial t} \quad \text{where} \quad \alpha = \frac{k}{\rho \cdot c} \quad \text{and} \quad q = -k \cdot \frac{\partial \theta}{\partial x}, (1.1.1)$$

where,

$\theta = \theta(x, t)$ - temperature at time t and distance x as shown in figure (1.1.1).

t - time

x - local position with respect to x coordinate

α - thermal diffusivity

k - thermal conductivity

ρ - density

c - specific heat of the material of the slab

q - heat flux

Figure (1.1.1) shows the application of heat flux, q , and placing of coordinate system.

Equations (1.1.1) are partial differential equations.

$$\text{Define, } T(x, t) = \theta(x, t) - \theta(x, 0) \quad (1.1.2)$$

Taking Laplace transform of equations (1.1.1) and (1.1.2) yields

$$\begin{aligned} \frac{d^2 T(x, s)}{dx^2} &= \frac{s}{\alpha} \cdot T(x, s) \\ \text{and } Q(x, s) &= -k \frac{dT(x, s)}{dx} \end{aligned} \quad (1.1.3)$$

The solutions to the above set of ordinary differential equations are,

$$\begin{aligned} T(x, s) &= - \frac{Q(0, s)}{k} \sqrt{\alpha/s} \sinh(\sqrt{s/\alpha} \cdot x) + T(0, s) \cdot \cosh(\sqrt{s/\alpha} \cdot x) \\ Q(x, s) &= Q(0, s) \cosh(\sqrt{s/\alpha} \cdot x) - T(0, s) \cdot k \cdot \sqrt{s/\alpha} \cdot \sinh(\sqrt{s/\alpha} \cdot x) \end{aligned} \quad (1.1.4)$$

where $T(0, s)$ and $Q(0, s)$ are conditions at the boundary $x = 0$. These expressions may be evaluated at the boundary $x = L$ to give,

$$T(L, s) = - \frac{Q(0, s)}{k} \sqrt{\alpha/s} \sinh(\sqrt{s/\alpha} \cdot L) + T(0, s) \cdot \cosh(\sqrt{s/\alpha} \cdot L)$$

$$Q(L, s) = Q(0, s) \cosh(\sqrt{s/\alpha} \cdot L) - T(0, s) \cdot k \cdot \sqrt{s/\alpha} \cdot \sinh(\sqrt{s/\alpha} \cdot L) \quad (1.1.5)$$

Transforming hyperbolic functions to normal trigonometric functions and arranging them into a matrix form, we obtain,

$$\begin{pmatrix} T(L,s) \\ Q(L,s) \end{pmatrix} = \begin{pmatrix} \cos(\sqrt{\kappa/\alpha} \cdot L) & -\frac{1}{k} \sqrt{\alpha/\kappa} \sin(\sqrt{\kappa/\alpha} \cdot L) \\ k\sqrt{\kappa/\alpha} \sin(\sqrt{\kappa/\alpha} \cdot L) & \cos(\sqrt{\kappa/\alpha} \cdot L) \end{pmatrix} \begin{pmatrix} T(0,s) \\ Q(0,s) \end{pmatrix} \quad (1.1.6)$$

The "s" is real and negative (Appendix A), hence $s = -\kappa$ is substituted.

The square 2 X 2 matrix on the right hand side, known as the transmission matrix for a slab, relates temperature and flux at one surface with that at the other.

1.1.3 MULTI-LAYERED SEMI-INFINITE SLAB

The single layered slab formulation is extended to an n-layered slab, figure (1.1.2). The derivations are carried out in s-domain.

The transmission matrix equation (1.1.6) is generalized for a pth layer in a multi-layered slab,

$$\begin{pmatrix} T_p(s) \\ Q_p(s) \end{pmatrix} = \begin{pmatrix} \cosh(\sqrt{s/\alpha_p} \cdot \Delta x_p) & -\frac{1}{k_p} \sqrt{\alpha_p/s} \cdot \sinh(\sqrt{s/\alpha_p} \cdot \Delta x_p) \\ -k_p \sqrt{s/\alpha_p} \cdot \sinh(\sqrt{s/\alpha_p} \cdot \Delta x_p) & \cosh(\sqrt{s/\alpha_p} \cdot \Delta x_p) \end{pmatrix} \begin{pmatrix} T_{p-1}(s) \\ Q_{p-1}(s) \end{pmatrix} \quad (1.1.7)$$

which is,

$$\begin{pmatrix} T_p(s) \\ Q_p(s) \end{pmatrix} = \begin{pmatrix} A_p(s) & B_p(s) \\ C_p(s) & D_p(s) \end{pmatrix} \begin{pmatrix} T_{p-1}(s) \\ Q_{p-1}(s) \end{pmatrix} \quad (1.1.8)$$

where, subscript p denotes that the pth layer is considered.

$$\begin{aligned} A_p(s) &= \cosh(\sqrt{s/\alpha_p} \cdot \Delta x_p) \\ B_p(s) &= -\frac{1}{k_p} \cdot \sqrt{\alpha_p/s} \sinh(\sqrt{s/\alpha_p} \cdot \Delta x_p) \end{aligned}$$

$$\begin{aligned} C_p(s) &= -k_p \cdot \sqrt{s/\alpha_p} \sinh(\sqrt{s/\alpha_p} \cdot \Delta x_p) \\ D_p(s) &= \cosh(\sqrt{s/\alpha_p} \cdot \Delta x_p) \end{aligned} \quad (1.1.9)$$

Since the temperature and heat flux out of the surface of one layer correspond to the temperature and heat flux into the next layer the transmission matrices of different layers may be combined in the order of occurrence. For an n -layered slab assuming perfect thermal contact between each different layer,

$$\begin{aligned} \begin{pmatrix} T_{ns}(s) \\ Q_{ns}(s) \end{pmatrix} &= \begin{pmatrix} A_n(s) & B_n(s) \\ C_n(s) & D_n(s) \end{pmatrix} \begin{pmatrix} A_{n-1}(s) & B_{n-1}(s) \\ C_{n-1}(s) & D_{n-1}(s) \end{pmatrix} \cdot \\ &\quad \begin{pmatrix} A_1(s) & B_1(s) \\ C_1(s) & D_1(s) \end{pmatrix} \begin{pmatrix} T_{1s}(s) \\ Q_{1s}(s) \end{pmatrix} \end{aligned} \quad (1.1.10)$$

Subscript ns and $1s$ denote consideration of surfaces exposed to air of n th and 1st layers.

1.1.4

MODELING OF SURFACE FILMS

The surface films on the open surfaces of a wall are modeled as,

- (1) having negligible thickness
- (2) having negligible capacitance
- (3) having a linear resistance R to the flow of heat.

In matrix notation this can be written as [refer to figure (1.1.2)]

$$\begin{pmatrix} T_a(s) \\ Q_a(s) \end{pmatrix} = \begin{pmatrix} 1 & -R \\ 0 & 1 \end{pmatrix} \begin{pmatrix} T_s(s) \\ Q_s(s) \end{pmatrix} \quad (1.1.11)$$

Where subscripts a and s denote the temperature and heat flux at ambient conditions and surface respectively.

A surface film is present of each side of the semi-infinite multi-

layered slab.

1.1.5

TRANSMISSION MATRIX FOR A WALL

The matrix expression (1.1.10) is now written, including the surface film resistances as

$$\begin{pmatrix} T_0(s) \\ Q_0(s) \end{pmatrix} = \begin{pmatrix} 1 & -R_n \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A_n(s) & B_n(s) \\ C_n(s) & D_n(s) \end{pmatrix} \cdots \begin{pmatrix} A_1(s) & B_1(s) \\ C_1(s) & D_1(s) \end{pmatrix} \cdot \begin{pmatrix} 1 & -R_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} T_i(s) \\ Q_i(s) \end{pmatrix} \quad (1.1.12)$$

The resultant matrix after the multiplication is

$$\begin{pmatrix} T_0(s) \\ Q_0(s) \end{pmatrix} = \begin{pmatrix} A(s) & B(s) \\ C(s) & D(s) \end{pmatrix} \begin{pmatrix} T_i(s) \\ Q_i(s) \end{pmatrix} \quad (1.1.13)$$

The square 2 X 2 matrix on the right hand side is transmission matrix for the wall.

Equation (1.1.13) can be written as

$$T_0(s) = A(s) \cdot T_i(s) + B(s) \cdot Q_i(s) \quad (1.1.14)$$

$$Q_0(s) = C(s) \cdot T_i(s) + D(s) \cdot Q_i(s) \quad (1.1.15)$$

In further derivation the argument "s" is dropped for convinience, except where necessary.

Since heat flux can not be measured directly, it is expressed in directly measurable temperature. Equations (1.1.14) and (1.1.15) are written as

$$Q_i = -\frac{A}{B} \cdot T_i + \frac{1}{B} T_0 \quad (1.1.16)$$

$$Q_0 = -\frac{(AD - BC)}{B} T_i + \frac{D}{B} T_0 \quad (1.1.17)$$

The term $(AD - BC)$ is the determinant of the matrix,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad (1.1.18)$$

The determinant is evaluated as follows

$$\begin{aligned} \Gamma &= \begin{vmatrix} A & B \\ C & D \end{vmatrix} = \Gamma_{Rn} \cdot \Gamma_n \cdot \Gamma_{n-1} \cdot \dots \cdot \Gamma_2 \cdot \Gamma_1 \cdot \Gamma_{R1} \\ &= \begin{vmatrix} 1 & -R_n \\ 0 & 1 \end{vmatrix} \begin{vmatrix} A_n & B_n \\ C_n & D_n \end{vmatrix} \begin{vmatrix} A_{n-1} & B_{n-1} \\ C_{n-1} & D_{n-1} \end{vmatrix} \cdot \dots \cdot \begin{vmatrix} A_1 & B_1 \\ C_1 & D_1 \end{vmatrix} \begin{vmatrix} 1 & -R_1 \\ 0 & 1 \end{vmatrix} \end{aligned} \quad (1.1.19)$$

where $\Gamma_{Rn} = \Gamma_{R1} = 1$

The determinant of the transmission matrix for a layer p is considered as follows,

$$\begin{aligned} \Gamma_p &= \begin{vmatrix} \cosh(\sqrt{s/\alpha_p} \cdot \Delta x_p) & -\frac{1}{k_p} \sqrt{\alpha_p/s} \cdot \sinh(\sqrt{s/\alpha_p} \cdot \Delta x_p) \\ -k_p \sqrt{s/\alpha_p} \sinh(\sqrt{s/\alpha_p} \cdot \Delta x_p) & \cosh(\sqrt{s/\alpha_p} \cdot \Delta x_p) \end{vmatrix} \\ &= \cosh^2(\sqrt{s/\alpha_p} \cdot \Delta x_p) + \sinh^2(\sqrt{s/\alpha_p} \cdot \Delta x_p) = 1 \end{aligned} \quad (1.1.20)$$

Hence, the determinant of matrix (1.1.20) is equal to 1.

Thus equations are simplified to

$$Q_i = -\frac{A}{B} T_i + \frac{1}{B} T_0 \quad (1.1.21)$$

$$Q_0 = -\frac{1}{B} T_i + \frac{D}{B} T_0 \quad (1.1.22)$$

These are re-arranged in the following forms, each suited to a particular kind of boundary condition ,

Case 1. Temperatures T_i and T_0 given

$$\begin{pmatrix} Q_i \\ Q_0 \end{pmatrix} = \frac{1}{B} \begin{pmatrix} -A & 1 \\ -1 & D \end{pmatrix} \begin{pmatrix} T_i \\ T_0 \end{pmatrix} \quad (1.1.23)$$

Case 2. flux Q_i and Q_0 given

$$\begin{pmatrix} T_i \\ T_0 \end{pmatrix} = \frac{1}{C} \begin{pmatrix} -D & 1 \\ -1 & A \end{pmatrix} \begin{pmatrix} Q_i \\ Q_0 \end{pmatrix} \quad (1.1.24)$$

Case 3. flux Q_i and temperature T_0 given

$$\begin{pmatrix} T_i \\ Q_0 \end{pmatrix} = \frac{1}{A} \begin{pmatrix} -B & 1 \\ 1 & C \end{pmatrix} \begin{pmatrix} Q_i \\ T_0 \end{pmatrix} \quad (1.1.25)$$

Case 4. flux Q_0 and temperature T_i given

$$\begin{pmatrix} T_0 \\ Q_i \end{pmatrix} = \frac{1}{D} \begin{pmatrix} B & 1 \\ 1 & -C \end{pmatrix} \begin{pmatrix} Q_0 \\ T_i \end{pmatrix} \quad (1.1.26)$$

Similarly it is also possible to express the temperature and flux at any plane inside the infinite slab in terms of a transfer function matrix and column matrix of boundary conditions.

It is important to note that all the s -transfer functions associated with one kind of boundary conditions have the same denominator. The denominator is B , when the surface temperatures are specified; C , when the surfaces fluxes are specified; A or D , when the boundary conditions are of the mixed type.

1.2.1

THE Z-TRANSFORM TECHNIQUE

The z -transform technique provides a means of finding the output of a system at time t , in terms of the input at time t , the input, and output at previous times $t - i \cdot \Delta$ and the co-efficients in the z -transfer function. This characteristic is applied to the problem of finding the heat flux through a wall in terms of the temperatures of the air on both the

sides of the wall. The z-transfer technique is demonstrated below. 1.14

A time function $x(t)$, shifted in time by an amount $n\Delta$ is $x(t - n\Delta)$. The Laplace transform of $x(t - n\Delta)$ is

$$L[x(t - n\Delta)] = \int_0^{\infty} x(t - n\Delta) e^{-st} dt \quad (1.2.1)$$

Substituting,

$$\lambda = t - n\Delta \quad \text{yields}$$

$$L[x(t - n\Delta)] = \int_{-n\Delta}^{\infty} x(\lambda) \cdot e^{-s(\lambda + n\Delta)} d\lambda \quad (1.2.2)$$

Since $x(\lambda) = 0$ for $\lambda < 0$. Equation (1.2.2) may be written as

$$L[x(t - n\Delta)] = e^{-s \cdot n\Delta} \int_0^{\infty} x(\lambda) \cdot e^{-s\lambda} d\lambda \quad (1.2.3)$$

$$= X(s) \cdot e^{-s \cdot n\Delta}$$

The z-transform is obtained from the Laplace transform by substituting $z = e^{s\Delta}$ thus equation (1.2.3) gives the Z-transform of $x(t - n\Delta)$ as $Z x(t - n\Delta) = X(z) z^{-n}$ (1.2.3a)

Taking the inverse Z-transform yields $x(t - n\Delta) = Z^{-1}[X(z) z^{-n}]$ (1.2.3a) which is known as the shifting theorem of Z-transforms. The Z-transfer function of a system which represents the Z-transform of the output over the Z-transform of the input is a ratio of polynomials in z^{-1} , thus

$$\frac{X(z)}{Y(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_m z^{-m}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-n}} \quad (1.2.4)$$

Where $X(z)$ and $Y(z)$ are the transforms of the output and input respectively. Cross multiplying yields

$$(a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_m z^{-m}) Y(z) =$$

$$(1 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-n}) X(z) \quad (1.2.5)$$

Taking the inverse z-transform and using the shifting theorem yields,

$$\begin{aligned}
 & x(t) + b_1 x(t - \Delta) + b_2 x(t - 2\Delta) + \dots + b_n x(t - n\Delta) \\
 = & a_0 y(t) + a_1 y(t - \Delta) + a_2 y(t - 2\Delta) + \dots + a_m y(t - m\Delta)
 \end{aligned} \tag{1.2.6}$$

Now the output of function $x(t)$, at time t can be written in terms of output and input at previous times $t - i\Delta$ as,

$$\begin{aligned}
 x(t) = & a_0 y(t) + a_1 y(t - \Delta) + \dots + a_m y(t - m\Delta) \\
 & - \left[b_1 x(t - \Delta) + b_2 x(t - 2\Delta) + \dots + b_n x(t - n\Delta) \right]
 \end{aligned} \tag{1.2.7}$$

The following sections develop the Z-transfer functions for the wall.

1.2.2 PROCEDURE TO OBTAIN Z-TRANSFER FUNCTIONS

The following steps are involved in deriving the Z-transfer function from a Laplace transfer function.

1. An input form is selected (step or ramp).
2. The Laplace transform of the output for the selected input form is the product of Laplace transform of the input function and the Laplace transfer function.
3. The z-transform of the output is found from tables of Laplace and z-transforms.
4. The required z-transfer function is obtained by dividing z-transform of the output by z-transform of the input.

It is observed that the z-transfer function is not obtained simply by taking the z-transform of the Laplace transfer function. If the z-transfer were to be obtained that way and then multiplied by the z-transform of the input, output obtained would be the response due to a sampled input rather than a continuous input, [9] . The input form selected [6] determines the nature of the continuous input between sample points. If a step input form is selected, the continuous input is constant between sample points. If the ramp input form is selected, the input is a ramp function between

sample points.

The z-transfer function will have poles at the zero of the z-transfer of the input form, as the output is divided by the z-transform of the input form. The z-transforms of time functions of order higher than ramps have zero on or outside the unit circle which become poles of the z-transfer function. Systems with poles on or outside the unit circle are unstable.

Hence, inputs of order higher than ramp yield unstable z-transfer functions, figures (1.2.1)-(1.2.2) show the step and ramp approximations to an input function. It can be seen that the step requires only previous values of the input function, while the ramp approximation requires the present value as well. Yet, in the following developments, ramp approximation to the input function is used since, as seen from the figures, the ramp approximation fits the input function more closely than the step approximation.

1.2.3

Z-TRANSFER FUNCTIONS FOR THE WALL

The s-transfer function whose z-transfer function are required, are $-A/B$, $1/B$, and D/B . The z-transfer function for $-1/B$ would be opposite in sign to that of $1/B$. Generalizing them as $\frac{G(s)}{B(s)}$, where $G(s) = -A(s)$, 1 , $D(s)$ and following the procedure mentioned in section (1.2.2)

$$O(s) = \frac{G(s)}{B(s)} I(s) = F(s) \cdot I(s)$$

$$I(s) = \frac{1}{s^2} \quad (\text{ramp approximation})$$

Gives,

$$O(s) = \frac{G(s)}{s^2 B(s)} \quad (1.2.8)$$

$O(s)$ has a pole of multiplicity two at $s = 0$ and other poles at the roots of B . Refer to Appendix A for discussion on roots of B .

Let,

$$\begin{aligned} O(s) &= \frac{a}{s^2} + \frac{b}{s} + \frac{c_1}{s+s_1} + \frac{c_2}{s+s_2} + \dots + \frac{c_n}{s+s_n} \\ &= \frac{a+bs}{s^2} + \sum_{j=1}^n \frac{c_j}{s+s_j} \end{aligned} \quad (1.2.9)$$

where, $s_j, j = 1, 2, \dots, n$ are the roots of B.

a, b , and c_j 's are constants given by

$$a = \left. \frac{G(s)}{B(s)} \right|_{s=0}, \quad b = \left. \left[\frac{B(s) \cdot \frac{dG(s)}{ds} - G(s) \cdot \frac{dB(s)}{ds}}{B^2(s)} \right] \right|_{s=0}$$

$$c_j = \left. \frac{G(s)}{s^2 B(s)} (s + s_j) \right|_{s = -s_j} \quad (1.2.10)$$

Equation(1.2.10) gives $c_j = \frac{0}{0}$

Applying L'Hospital's rule,

$$c_j = \left. \frac{G(s)}{s^2 \frac{dB(s)}{ds}} \right|_{s = -s_j}$$

The z-transform of the partial fraction expansion of $O(s)$ is obtained as follows,

$$O(z) = \frac{a \Delta z^{-1}}{(1-z^{-1})} + \frac{b}{(1-z^{-1})} + \sum_{j=1}^{\infty} \frac{c_j}{(1 - e^{-s_j \Delta} z^{-1})} \quad (1.2.11)$$

Δ is the sampling interval in hrs.

Then,

$$I(z) = \frac{\Delta z^{-1}}{(1 - z^{-1})^2} \quad (1.2.12)$$

$$\begin{aligned}
 F(z) &= \frac{O(z)}{I(z)} \\
 &= \frac{(1 - z^{-1})^2}{\Delta z^{-1}} \left[\frac{a \Delta z^{-1}}{(1 - z^{-1})^2} + \frac{b}{(1 - z^{-1})} + \right. \\
 &\quad \left. + \sum_{j=1}^{\infty} \frac{c_j}{(1 - e^{-s_j \Delta} z^{-1})} \right] \quad (1.2.13)
 \end{aligned}$$

$F(z)$ may be written as (1.2.14)

$$\begin{aligned}
 F(z) &= \frac{1}{\Delta \cdot z^{-1} \prod_{j=1}^{\infty} (1 - e^{-s_j \Delta} z^{-1})} \left[a \Delta z^{-1} \prod_{j=1}^{\infty} (1 - e^{-s_j \Delta} z^{-1}) \right. \\
 &\quad \left. + b(1 - z^{-1}) \prod_{j=1}^{\infty} (1 - e^{-s_j \Delta} z^{-1}) \right] + \sum_{j=1}^{\infty} \frac{c_j (1 - z^{-1})^2}{\Delta z^{-1} (1 - e^{-s_j \Delta} z^{-1})}
 \end{aligned}$$

By gathering co-efficients of the same powers of z^{-1} equation (1.2.14) may be written as a ratio of polynomials in z^{-1} . $B(s)$ has an infinite number of roots on the negative real axis (Appendix A). In practice, only a finite number of roots are calculated to make the polynomials finite.

1.2.4

EVALUATION OF Z-TRANSFORM COEFFICIENTS

Equation (1.2.14) may be written in the form,

$$F(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots}{1 + b_1 z^{-1} + b_2 z^{-2} + \dots} \quad (1.2.15)$$

The z-transfer co-efficients a_0, a_1, a_2, \dots and b_1, b_2, b_3, \dots can be evaluated by calculating a, b and c_j 's of $F(s)$ as described earlier, equation (1.2.10).

Equation (1.2.10) requires the values of $G(s)$, $B(s)$, $dG(s)/ds$, $dB(s)/ds$ at $s = 0$ and $dB(s)/ds$, and $G(s)$ at $s = -s_j$.

Recall that $G(s)/B(s)$ was a generalized transfer function for $-A(s)/B(s)$, $1/B(s)$, and $D(s)/B(s)$ where, $A(s)$, $B(s)$, $C(s)$, and $D(s)$ are the elements of the transmission matrix for the wall, equation (1.1.13). Since the transmission matrix is a product of transmission matrices in a particular order, the following procedure is adopted to find the required values.

$$\begin{bmatrix} A_p(s) & B_p(s) \\ C_p(s) & D_p(s) \end{bmatrix} = \begin{bmatrix} \cosh(\sqrt{s/\alpha_p} \cdot \Delta x_p) & -\frac{1}{k_p} \sqrt{\alpha_p/s} \sinh(\sqrt{s/\alpha_p} \cdot \Delta x_p) \\ -k_p \sqrt{s/\alpha_p} \sinh(\sqrt{s/\alpha_p} \cdot \Delta x_p) & \cosh(\sqrt{s/\alpha_p} \cdot \Delta x_p) \end{bmatrix} \quad (1.2.16)$$

Substituting $s = 0$ and applying L'Hospital's rule as necessary.

$$\begin{bmatrix} A_p(0) & B_p(0) \\ C_p(0) & D_p(0) \end{bmatrix} = \begin{bmatrix} 1 & -\frac{\Delta x_p}{k_p} \\ 0 & 1 \end{bmatrix} \quad (1.2.17)$$

Multiplying transmission matrices for each layer at $s = 0$ yields

$$\begin{bmatrix} A(0) & B(0) \\ C(0) & D(0) \end{bmatrix} = \begin{bmatrix} 1 & -R_1 - \frac{\Delta x_1}{k_1} - \frac{\Delta x_2}{k_2} - \dots - \frac{\Delta x_n}{k_n} - R_n \\ 0 & 1 \end{bmatrix} \quad (1.2.18)$$

Differentiation of equation (1.2.16) yields

$$\begin{bmatrix} \frac{dA_p}{ds} & \frac{dB_p}{ds} \\ \frac{dC_p}{ds} & \frac{dD_p}{ds} \end{bmatrix} = \begin{bmatrix} \frac{\Delta x_p}{2\sqrt{\alpha_p/s}} \sinh(\sqrt{s/\alpha_p} \cdot \Delta x_p) & -\frac{k_p}{2\sqrt{\alpha_p/s}} \sinh(\sqrt{s/\alpha_p} \Delta x_p) \\ -\frac{k_p \cdot \Delta x_p}{2\alpha_p} \cosh(\sqrt{s/\alpha_p} \Delta x_p) & -\frac{k_p}{2\sqrt{\alpha_p/s}} \sinh(\sqrt{s/\alpha_p} \Delta x_p) \end{bmatrix}$$

$$- \frac{\Delta x_p}{2k_p s} \cosh(\sqrt{s/\alpha_p} \Delta x_p) + \frac{1}{2k_p s} \sqrt{\alpha_p/s} \sinh(\sqrt{s/\alpha_p} \Delta x_p) \left. \begin{array}{l} \\ \frac{\Delta x_p}{2\sqrt{\alpha_p/s}} \sinh(\sqrt{s/\alpha_p} \Delta x_p) \end{array} \right\}$$

(1.2.19)

for $s = 0$

$$\left. \begin{array}{cc} \frac{dA_p}{ds} & \frac{dB_p}{ds} \\ \frac{dC_p}{ds} & \frac{dD_p}{ds} \end{array} \right|_{s=0} = \begin{pmatrix} \frac{\Delta x_p^3}{2\alpha_p} & -\frac{\Delta x_p^3}{6\alpha_p \cdot k_p} \\ -\frac{k_p \Delta x_p}{\alpha_p} & \frac{\Delta x_p^2}{2\alpha_p} \end{pmatrix} \quad (1.2.20)$$

Since,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & -R_n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \begin{bmatrix} A_{n-1} & B_{n-1} \\ C_{n-1} & D_{n-1} \end{bmatrix} \dots \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} 1 & -R_1 \\ 0 & 1 \end{bmatrix} \quad (1.2.21)$$

The derivative of equation (1.2.21) is,

$$\begin{aligned} \begin{bmatrix} \frac{dA}{ds} & \frac{dB}{ds} \\ \frac{dC}{ds} & \frac{dD}{ds} \end{bmatrix} &= \begin{bmatrix} 1 & -R_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{dA_n}{ds} & \frac{dB_n}{ds} \\ \frac{dC_n}{ds} & \frac{dD_n}{ds} \end{bmatrix} \begin{bmatrix} A_{n-1} & B_{n-1} \\ C_{n-1} & D_{n-1} \end{bmatrix} \dots \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} 1 & -R_1 \\ 0 & 1 \end{bmatrix} \\ &+ \begin{bmatrix} 1 & -R_n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \begin{bmatrix} \frac{dA_{n-1}}{ds} & \frac{dB_{n-1}}{ds} \\ \frac{dC_{n-1}}{ds} & \frac{dD_{n-1}}{ds} \end{bmatrix} \dots \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} 1 & -R_1 \\ 0 & 1 \end{bmatrix} \\ &+ \dots + \begin{bmatrix} 1 & -R_n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \dots \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} \frac{dA_1}{ds} & \frac{dB_1}{ds} \\ \frac{dC_1}{ds} & \frac{dD_1}{ds} \end{bmatrix} \begin{bmatrix} 1 & -R_1 \\ 0 & 1 \end{bmatrix} \end{aligned} \quad (1.2.22)$$

To obtain,

$$\left[\begin{array}{cc} \frac{dA}{ds} & \frac{dB}{ds} \\ \frac{dC}{ds} & \frac{dD}{ds} \end{array} \right]_{s=0} \quad \text{equations (1.2.20)}$$

and (1.2.17) are substituted into equation (1.2.22).

The multiplication of the above matrices and evaluation of residues is accomplished by a computer program, which is described in Appendix B. The evaluation of $G(s)$ and $B(s)/ds$ at $s=-s_j$ follows the same procedure.

1.2.5 MODIFIED THERMAL RESPONSE FACTOR METHOD

Consider the equation,

$$F(z) = \frac{O(z)}{I(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \dots}{1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + \dots} \quad (1.2.23)$$

Separating out the terms $O(z)$ yields,

$$O(z) = (a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots) \cdot I(z) - (b_1 z^{-1} + b_2 z^{-2} + \dots) \cdot O(z) \quad (1.2.24)$$

The above infinite series in powers of z^{-1} are truncated when their coefficients become sufficiently small.

Taking the inverse z-transform of the above equation

$$O(t) = a_0 I(t) + a_1 I(t - \Delta) + a_2 I(t - 2\Delta) + \dots + a_n I(t - n\Delta) - [b_1 O(t - \Delta) + b_2 O(t - 2\Delta) + \dots + b_n O(t - n\Delta)] \quad (1.2.25)$$

where Δ is the sampling interval, $I(t - n\Delta)$ and $O(t - n\Delta)$ are the values of input and output at time $(t - n\Delta)$.

This method of obtaining the output value at time t , as a function of

present as well as the previous values of output is used to calculate the heat flux at the inner or outer surfaces of a wall.

The equations in the Laplace transform

$$q_i(s) = -\frac{A(s)}{B(s)} T_i(s) + \frac{1}{B(s)} T_0(s) \quad (1.2.26)$$

$$\text{and } q_0(s) = -\frac{1}{B(s)} T_i(s) + \frac{D(s)}{B(s)} T_0(s) \quad (1.2.27)$$

may be written in time domain as

$$q_i(s) = \sum_{k=0}^N A_k T_i(t-k) - \sum_{k=0}^N B_k T_0(t-k) - \sum_{k=1}^N D_k q_i(t-k) \quad (1.2.28)$$

and

$$q_0(s) = -\sum_{k=0}^N B_k T_i(t-k) + \sum_{k=0}^N C_k T_0(t-k) - \sum_{k=1}^N D_k q_0(t-k) \quad (1.2.29)$$

This method of calculating $q_i(s)$ and $q_0(s)$ is called Modified Thermal Response Factor Method.

The advantage in using the above method over the Thermal Response Factor Method is in saving of computer memory. The input variables are sampled values of temperature.

The z-transfer coefficients for walls and roofs that are made of homogeneous materials can be determined by calculations. For a more complicated construction, these coefficients may be determined by test [10].

1.3.1

IMPROVEMENT IN COMPUTATION OF HEATING OR COOLING LOAD

For precise sizing of building air-conditioning systems and estimation of energy consumption, accurate prediction of building heating and cooling load is essential. Various computer programs like NBSLD, BLAST, CAL-ERDA, DOE-2 [11 to 14] have been developed which carry out extensive calculations

for the same. Two approaches, namely, heat balance equations and ASHRAE weighting factor method[15] are available. The NBSLD and BLAST incorporate meticulous solutions of various heat balance equations written for most types of heat transfer phenomena at each node of the thermal network. These equations are solved at each time step increment. Hence, this procedure is very accurate in predicting the cooling or heating loads and energy consumption. CAL-ERDA and DOE-2 use ASHRAE weighting factor method. Computational economy is achieved but at the expense of accuracy and flexibility. This method segregates loads into established categories for which weighting factors are established. It avoids non-linear phenomena such as variation in convection heat transfer coefficients. Flexibility is lost due to the weighting factor data, whose description for a particular building under analysis, is often not satisfactory.

Sowell and Walton[16] proposed improvements in the method of calculating cooling or heating load by use of heat balance equations. It is claimed that with these improvements the method retains the desired flexibility and accuracy (through solution of heat balance equations) and takes only about 10% - 15% more computer processing unit (C.P.U.) time than the quicker but less accurate ASHRAE weighting factor method.

The heatbalance method consists of selection and manipulation of heat balance equations in such a way as to form a matrix equation of the form,

$$V \times T = W$$

where V is an $(NS + 1) \times (NS + 1)$ coefficient matrix. NS is the total number of surfaces in the room or building envelope considered. T is an $(NS + 1)$ vector of temperatures and W is an $(NS + 1)$ vector of known quantities.

The solution for T involves inversion of the V matrix and multiplication by W . If the conditions remain such that the coefficients of the

equation remain unchanged than V need not be re-inverted. If a few coefficients vary, the new inverse of V can be obtained without going through all the extensive calculations previously carried out.

In the calculation of z-transfer coefficients for Modified Thermal Response Factors, a ramp approximation to the input function is used. The ramp approximation requires the present value of the input function as well as the previous value. This poses a problem in the simulation programs as cited by Thompson and Chen[17], the value of heat flux at the i th instant of time is not known until the room temperature at the i th instant of time is known and vice versa. In most simulation programs this problem is circumvented by slipping the temperature data back by one time step increment. The effect of such a procedure is shown in figure (1.3.1).

An algorithm is developed in the next section, which circumvents the problem mentioned above without slipping temperature data back by one time step increment. This algorithm uses Modified Thermal Response Factors to calculate heat fluxes through walls.

1.3.2

DERIVATION

Heat balance equations are written for various surfaces and for room air mass. The heat flux at the t th instant of time through the j th inner surface may be expressed in terms of Modified Thermal Response Factor.

$$q_{ji}(t) = \sum_{k=0}^N A_{jk} T_{ji}(t-k) - \sum_{k=0}^N B_{jk} T_{j0}(t-k) - \sum_{k=1}^N D_{jk} q_{ji}(t-k) \quad (1.3.1)$$

where N : number of z-transfer coefficients and NS : number of surfaces.

The heat flux onto the j th inner surface from the room may be

expressed

$$q_{ji}(t) = h_{ji} [T_n(t) - T_{ji}(t)] + \sum_{m=1}^{NS} g_{jm} [T_{mi}(t) - T_{ji}(t)] + R_{ji}(t) \quad (1.3.2)$$

where the first term is the heat flux by conduction through the film, the second term is long wave radiation from other surfaces and the last term is short wave radiation from the sun.

The heat flux at time t at the j th outer surface expressed in terms of Modified Thermal Response Factors,

$$q_{j0}(t) = - \sum_{k=0}^N B_{jk} T_{ji}(t-k) + \sum_{k=0}^N C_{jk} T_{j0}(t-k) - \sum_{k=1}^N D_{jk} q_{j0}(t-k) \quad (1.3.3)$$

The heat flux onto the j th outer surface from the outside atmosphere is written as

$$q_{j0}(t) = h_{j0} [T_a(t) - T_{j0}(t)] + R_{j0}(t) \quad (1.3.4)$$

where the first term is the heat flux by conduction through the film and the second term is short wave radiation from the sun.

The energy balance for the room air yields

$$m_a c_a \frac{dT_n(t)}{dt} = \sum_{j=0}^{NS} s_j h_{ji} [T_{ji}(t) - T_n(t)] + \dot{M}_1 c_a T_a(t) + \dot{M}_s c_a T_s(t) - \dot{M}_e c_a T_e(t) + QS(t) \quad (1.3.5)$$

where

- m_a : mass of room air
- \dot{M}_1 : infiltration air flow rate
- \dot{M}_s : supply air flow rate
- \dot{M}_e : exhaust air flow rate
- c_a : specific heat of air

- $T_r(t)$: room temperature
 $T_{ji}(t)$: inside surface temperature of j th surface
 $T_{j0}(t)$: outside surface temperature of j th surface
 h_{ji} : film coefficient on inside of j th surface
 h_{j0} : film coefficient on outside of j th surface
 s_j : area of j th surface
 $T_a(t)$: outside ambient temperature
 $T_s(t)$: supply air temperature
 $T_e(t)$: exhaust air temperature
 and $QS(t)$: heat convected to room air by occupants, lights, equipment, etc.

Dividing equation(1.3.5) by $m_a c_a$ and rearranging terms yields,

$$\frac{d T_r(t)}{dt} = -v T_r(t) + \sum_{j=1}^{NS} \frac{s_j h_{ji}}{m_a c_a} T_{ji}(t) + F(t) \quad (1.3.6)$$

where,

$$F(t) = \frac{\dot{M}_1}{m_a} T_a(t) + \frac{\dot{M}_s}{m_a} T_s(t) - \frac{\dot{M}_e}{m_a} T_e(t) + \frac{QS(t)}{m_a} \quad (1.3.7)$$

and

$$v = \sum_{j=1}^{NS} \frac{s_j h_{ji}}{m_a c_a} \quad (1.3.8)$$

The solution of differential equation (1.3.6) may be discretized as follows:

$$\begin{aligned}
 T_r(t) = & e^{-v\Delta} T_r(t-\Delta) + \frac{v\Delta - 1 + e^{-v\Delta}}{v^2 \Delta m_a c_a} \sum_{j=1}^{NS} s_j h_{ji} T_{ji}(t) + \frac{1 - e^{-v\Delta} - v\Delta e^{-v\Delta}}{v^2 \Delta m_a c_a} \sum_{j=1}^{NS} s_j h_{ji} T_{ji}(t-\Delta) + \\
 & \frac{v\Delta - 1 + e^{-v\Delta}}{v^2 \Delta} F(t) + \frac{1 - e^{-v\Delta} - v\Delta e^{-v\Delta}}{v^2 \Delta} F(t-\Delta)
 \end{aligned} \quad (1.3.9)$$

Rewriting equation (1.3.1) separating out current variables from previous values,

$$A_{j0}T_{ji}(t) - B_{j0}T_{j0}(t) + \sum_{k=1}^N Q_{ji}(t-k) = q_{ji}(t) \quad (1.3.10)$$

where,

$$\begin{aligned} \sum_{k=1}^N Q_{ji}(t-k) &= \sum_{k=1}^N A_{jk}T_{ji}(t-k) - \sum_{k=1}^N B_{jk}T_{j0}(t-k) \\ &\quad - \sum_{k=1}^N D_{jk}q_{ji}(t-k) \end{aligned}$$

or

$$Q_{ji}(t-k) = A_{jk}T_{ji}(t-k) - B_{jk}T_{j0}(t-k) - D_{jk}q_{ji}(t-k)$$

Equating (1.3.10) with (1.3.2) yields

$$\begin{aligned} [A_{j0} + h_{ji} + \sum_{m=1}^{NS} g_{jm}]T_{ji}(t) - \sum_{m=1}^{NS} g_{jm}T_{mi} - h_{ji}T_{ji}(t) \\ = B_{j0}T_{j0}(t) - \sum_{k=1}^N Q_{ji}(t-k) - R_{ji}(t) \end{aligned} \quad (1.3.11)$$

Similarly rewriting equation (1.3.3) separating current variables from previous values,

$$q_{j0}(t) = -B_{j0}T_{ji}(t) + C_{j0}T_{j0}(t) + \sum_{k=1}^N Q_{j0}(t-k) \quad (1.3.12)$$

where

$$\begin{aligned} \sum_{k=1}^N Q_{j0}(t-k) &= - \sum_{k=1}^N B_{jk}T_{ji}(t-k) + \sum_{k=1}^N C_{jk}T_{j0}(t-k) \\ &\quad - \sum_{k=1}^N D_{jk}q_{j0}(t-k) \end{aligned}$$

or

$$Q_{j0}(t-k) = -B_{jk}T_{ji}(t-k) + C_{jk}T_{j0}(t-k) - D_{jk}q_{j0}(t-k)$$

$T_{j0}(t)$ may be found in terms of $T_{ji}(t)$ and other terms by equating (1.3.12) with (1.3.4).

$$T_{j0}(t) = \frac{B_{j0}T_{ji}(t) - \sum_{k=1}^N Q_{j0}(t-k) + h_{j0}T_a(t) + R_{j0}(t)}{C_{j0} + h_{j0}} \quad (1.3.13)$$

Substituting equation (1.3.13) into equation (1.3.11) to obtain an equation free of $T_{j0}(t)$ but containing $T_{ji}(t)$, $T_a(t)$ and $T_n(t)$:

$$\begin{aligned} \frac{[(A_{j0} + h_{ji} + \sum_{m=1}^{NS} g_{jm})(C_{j0} + h_{j0}) - B_{j0}^2]T_{ji}(t)}{C_{j0} + h_{j0}} - \sum_{m=1}^{NS} g_{jm}T_{mi}(t) - h_{ji}T_n(t) &= -\sum_{k=1}^N Q_{ji}(t-k) - R_{ji}(t) \\ &+ \frac{B_{j0} - \sum_{k=1}^N Q_{j0}(t-k) + h_{j0}T_a(t) + R_{j0}(t)}{C_{j0} + h_{j0}} \end{aligned} \quad (1.3.14)$$

And rewriting equation (1.3.9)

$$\begin{aligned} -\left(\frac{v_\Delta - 1 + \bar{e}^{v_\Delta}}{v_\Delta^2 m_a c_a}\right) \sum_{j=1}^{NS} S_j h_{ji} T_{ji}(t) + T_n(t) &= \bar{e}^{v_\Delta} T_n(t-\Delta) + \left(\frac{1 - \bar{e}^{v_\Delta} - v_\Delta \bar{e}^{v_\Delta}}{v_\Delta^2 m_a c_a}\right) \sum_{j=1}^{NS} S_j h_{ji} T_{ji}(t-\Delta) \\ &+ \left(\frac{v_\Delta - 1 + \bar{e}^{v_\Delta}}{v_\Delta^2}\right) F(t) \\ &+ \left(\frac{1 - \bar{e}^{v_\Delta} - v_\Delta \bar{e}^{v_\Delta}}{v_\Delta^2}\right) F(t-\Delta) \end{aligned} \quad (1.3.15)$$

These equations can be put into a matrix form given by

$$V \times T = W$$

$$\begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1, NS+1} \\ v_{21} & v_{22} & & \\ \vdots & & & \\ v_{NS+1, 1} & \cdots & v_{NS+1, NS+1} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_{NS+1} \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_{NS+1} \end{bmatrix}$$

$$(1.3.16)$$

The elements are identified as follows,

$$v_{jj} = \frac{(A_{jo} + h_{ji} + \sum_{m=1}^{NS} g_{jm})(C_{jo} + h_{jo}) - B_{jo}^2}{C_{jo} + h_{jo}}$$

Observe that $g_{jj}=0$

$$v_{ij} = -g_{ij}$$

$$v_{j,NS+1} = -h_{ji}$$

$$v_{NS+1,} = - \left(\frac{v_{\Delta}-1+\bar{e}^{v_{\Delta}}}{v_{\Delta}^2 m_a c_a} \right) S_j h_{ji}$$

$$v_{NS+1,NS+1} = 1$$

w_j , for $j=1$ to NS is the right hand side of equation (1.3.14) and

w_{NS+1} is the right hand side of equation (1.3.15)

Observe that the values of the diagonal elements of matrix V are large relative to the values of the off diagonal elements.

It is suggested that Gauss-Seidal iteration [16] be used to solve the matrix equation. Temperatures $T_{ji}(t)$ and $T_h(t)$ are obtained on solving equation (1.3.16). These can be substituted into equation (1.3.13) to obtain $T_{jo}(t)$. Using these temperatures heat fluxes $q_{ji}(t)$ and $q_{jo}(t)$ at current time can be obtained from equations (1.3.10) and (1.3.12)

This procedure circumvents the problem cited in [17].

PART 2

2.1 GENERALIZATION AND MODIFICATION OF COIL, VALVE, THERMOSTAT AND RETURN AIR MODELS

A model has been developed to simulate the performance of the heating system for a room. The model was developed to match the characteristics of an experimental room at Kansas State University. Figure (2.1.1) is a schematic diagram of the room heating system. Models have been developed for the thermostat, valve, coil, ducts, room, wall, etc. Details of the work done previously is reported in reference [18]. The work to be reported here details improvements in the models of the temperature. These improvements were found necessary when the simulation runs were made for more general weather patterns and for longer periods of time. The verification experiment was run for 3 hours and 12 minutes. Consequently, the simulation was run for 64 three minute time steps. The modified simulation runs were made up to 1440 time steps (or three full days).

2.2 COIL MODEL

It is desired to have a coil model which relates the temperature of air leaving the coil, with the rate of air flow, the temperature of air entering the coil, the rate of water flow through the coil, and the temperature of water entering the coil achieved through the parameter, effectiveness (E4). The model used for the coil in reference [18] was limited to only one air flow rate, 95 cfm. The model also had a sharp discontinuity in the effectiveness versus flow curve, figure (2.2.1).

The model presented here eliminates the discontinuity and allows variations in the air and, water flow rates, and entering air and water

temperatures. The operating characteristics provided by the manufacturer of the coil were in the form of curves for Heat Transfer Index (HTI) versus Air Flow Quantity for various water flow rates (GPM), figure (2.2.2).

Heat Transfer Index is defined as,

$$HTI = \frac{\text{Total Heating Load (THL)}}{G2(1) - T3}, \text{ (Btu/hr.}^\circ\text{F)} \quad (2.2.1)$$

For this part of the thesis symbols used are the actual symbols used in the computer program, SIM 7:

$G2(1)$ = Temperature of water entering coil ($^\circ\text{F}$)

$T3$ = Dry bulb temperature of air entering coil ($^\circ\text{F}$)

The total heating load,

$$THL = M(0) * C1(0) * (J1 - T3) \quad (2.2.2)$$

where,

$M(0)$ = Mass flow rate of air through coil (lbs/hr)

$C1(0)$ = Specific heat of air (Btu/lb. $^\circ\text{F}$)

$J1$ = Temperature of air leaving coil ($^\circ\text{F}$)

Effectiveness of coil is defined as,

$$E4 = \frac{J1 - T3}{G2(1) - T3} \quad (2.2.3)$$

we have,

$$HTI = M(0) * C1(0) * E4 \quad (2.2.4)$$

Mass flow rate of air is given by,

$$M(0) = 60 * D9 * C \quad (2.2.5)$$

where,

$D9$ = Density of air (lbs/ft³)

C = Air Quantity (cfm)

The values in figure (2.2.3) were extracted from figure (2.2.2).

A cubic equation is fitted for HTI versus Air Quantity for each

water flow rate.

$$HTI = a + b*C + c*C^2 + d*C^3 \quad (2.2.6)$$

where the values of a's, b's, c's, and d's for the various values of water flow rate are given in figure (2.2.4).

Effectiveness now can be obtained for a given air flow rate and water flow rate from equations (2.2.4), (2.2.5), and (2.2.6).

$$\begin{aligned} HTI &= M(0)*C1(0)*E4 \\ &= 60*D9*C1(0)*C*E4 \\ &= 1.08*CE4 \\ &= b*C + c*C^2 + d*C^3 \quad (a = 0) \end{aligned} \quad (2.2.7)$$

Therefore effectiveness is given by

$$E4 = \frac{b}{1.08} + \frac{c*C}{1.08} + \frac{d*C^2}{1.08} \quad (2.2.8)$$

To make equation (2.2.8) general enough we realize that each of the $b/1.08$, $c/1.08$ and $d/1.08$ are functions of water flow rate $F0(1)$.

Fitting a forth order polynomial in water flow rate to each coefficient yields,

$$\begin{aligned} \frac{b}{1.08} &= b_1*F0(1) + b_2*F0(1)^2 + b_3*F0(1)^3 + b_4*F0(1)^4 \\ \frac{c}{1.08} &= c_1*F0(1) + c_2*F0(1)^2 + c_3*F0(1)^3 + c_4*F0(1)^4 \\ \frac{d}{1.08} &= d_1*F0(1) + d_2*F0(1)^2 + d_3*F0(1)^3 + d_4*F0(1)^4 \end{aligned} \quad (2.2.9)$$

The values of the b's, c's, and d's we give in figure (2.2.5).

The model for E4 is obtained by combining equations (2.2.8) and (2.2.9), thus:

$$\begin{aligned} E4 &= [b_1*F0(1) + b_2*F0(1)^2 + b_3*F0(1)^3 + b_4*F0(1)^4] \\ &+ [c_1*F0(1) + c_2*F0(1)^2 + c_3*F0(1)^3 + c_4*F0(1)^4]*C \\ &+ [d_1*F0(1) + d_2*F0(1)^2 + d_3*F0(1)^3 + d_4*F0(1)^4]*C^2 \end{aligned} \quad (2.2.10)$$

Rearranging yields:

$$\begin{aligned}
 E4 = & (b_1 + c_1 * C + d_1 * C^2) * F0(1) + (b_2 + c_2 * C + d_2 * C^2) * F0(1)^2 \\
 & + (b_3 + c_3 * C + d_3 * C^2) * F0(1)^3 + (b_4 + c_4 * C + d_4 * C^2) * F0(1)^4
 \end{aligned}
 \tag{2.2.11}$$

A plot of E4 versus F0(1) for various values of C computed from equation (2.2.11) is shown in figure (2.2.6). The undulatory nature of the plots between the data points is typical of polynomial curve fits, and is undesirable. To avoid this quality in the model, equation (2.2.11) will be used in the range $0 \leq F0(1) \leq 0.5$ and linear interpolation between the data points will be used for $0.5 \leq F0(1)$. Figure (2.2.7) illustrates the final model.

2.3.1

VALVE MODEL

Four test runs were made to investigate valve characteristics at different maximum flow rates. All the four test results were satisfactory and in agreement. Hence, only one set of test readings and corresponding plots are reported. Refer to figure (2.3.1) for readings of valve actuator pressure, the normalized valve travel, pressure drop across the valve, and % flow rate of water. The schematic diagram of the test set-up is shown in figure (2.3.2).

Three major discrepancies were observed on comparison with the model reported in reference [18] as detailed below.

2.3.2

VALVE TRAVEL VS. ACTUATOR PRESSURE

Figure (2.3.3) is a plot of valve travel vs. pressure. Hyterisis was not found to be as great as reported in reference [18]. Also, the valve travel did not enter or come out of the fully closed position as sharply

as was previously assumed. The new model consist of straight line segments for values of valve travel $Y9 \geq 0.1$ and quadratic curves for $0 \leq Y9 \leq 0.1$. The coefficients in the linear and quadratic models were obtained by least squares fits of the data from the four tests. Appendix C is a flow chart of the logic used to implement the model.

2.3.3

FLOW CHARACTERISTICS

Valves used to control the flow of water through heating or cooling coils are usually "equal percentage" valves. The characteristics of an equal percentage valve is that it provides an equal percentage relationship between valve opening and flow at a constant pressure drop across the valve. For the water circuit used in this project the pressure drop across the valve is not maintained constant. Figure (2.3.4) shows the relationship between water flow rate and pressure drop across the valve. Figure (2.3.5) shows the relationship between water flow rate and valve travel. The model developed from these data is:

$$\begin{aligned} F8 &= Y9 \\ F0(1) &= F8 \cdot F9 \end{aligned} \tag{2.3.1}$$

where,

$$\begin{aligned} Y9 &= \text{normalized valve travel} \\ F8 &= \text{normalized water flow rate} \\ F9 &= \text{maximum water flow rate} \end{aligned}$$

2.4

THERMOSTAT

A test was set up to investigate the behavior of the thermostat. Figure (2.4.1) shows the schematic of the experimental setup. Temperature was varied by controlling the power of the heater. The temperature of the thermostat and the pressure output from the thermostat were mea-

sured. Input pressure to the thermostat was maintained constant. The temperature was varied slowly to avoid the dynamic response characteristic of the thermostat. Readings were taken for several different temperature settings of the thermostat.

The purpose of these tests was to establish the value of gain (change in output pressure per unit change in temperature). The value of gain is influenced somewhat by the temperature setting, but this dependence is small. The value of gain which most accurately describes the thermostat is

$$P_6 = 2.1 \text{ psi/}^{\circ}\text{F}$$

2.5

RETURN AIR MODEL

In order to compute an energy balance on the room air it is necessary to establish the return air temperature. In a general sense the relationship between the return air temperature and temperatures at other points depends on the location of supply and return grills, sources of infiltration, the location of windows, etc. In a heating system, the air temperature will be higher near diffusers and lower near windows or in locations subject to air infiltration. In the energy balance equation, an "average" room temperature is computed. The return air temperature depends on the air circulation patterns and on the location of hot and cold air sources as well as the location of the return port. In the model previously used, the return air temperature was modelled as a weighted sum of supply temperature, average room temperature, and wall temperature.

Examination of the data taken in the verification experiment shows that the return air temperature was consistently lower than the temperature used in the weighted sum. This is likely due to air infiltration around the doors which are near the return air port. It was recommended in reference 18 that the model of return air be extended to include the effect of in-

filtration by adding the outside temperature to the weighted sum. A model of the form,

$$T1 = A2 * T1(1) + B2 * T3(N2) + C2 * T7 + D2 * T9(1,N2) \quad (2.5.1)$$

is used, where

N2 = number of z-coefficients in a time series

T1 = return air temperature

T1(1) = supply air temperature

T3(N2) = average room temperature at current time instant

T7 = inside temperature of the wall

T9(1,N2) = temperature outside the wall at current time instant

The method of least squares is used to obtain the values of A2, B2, C2, and D2. The values obtained were:

A2 = 0.004114866 C2 = 0.7848157

B2 = 0.1478111 D2 = 0.06874663

Figure (2.5.1) shows a comparison of the return air temperature computed by the two models and measured in the verification test. Note should be made of the fact that this dependent on the specific physical arrangement of the room under consideration and on the conditions of the test. A more general methodology for determining a return air model is badly needed. It was observed in the project that a suitable model of the return air temperature is a very important aspect of the system model.

PART 3MICROCOMPUTER IMPLEMENTATION OF THE
THERMAL SIMUALTION OF THE ROOM3.1

Development of microcomputers make practical use of complex control strategies based on the values of many system parameters. The extensive arithmetical and logical capabilities of microcomputers make them suitable for executing complex and extensive simulation programs. Microcomputers are being increasingly used in the field of air conditioning (HVAC) for efficient control of equipment.

Besides the advantages of cost effectiveness and high speed computations, the single, most important advantage of the microcomputer would be its ability to communicate directly with the programmer through a CRT and a keyboard. The simulation program written for the IBM 370 Computer in the FORTRAN language and described in [18] was modified and implemented in the BASIC language on a Z-80 microcomputer. The results obtained by this program were perfectly identical to those obtained by the simulation program described in [18].

Due to memory limitations on the microcomputer all the programs were designed accordingly, avoiding dimensioning of variables and storage of data, etc. to an extent necessary. Most of the experimental data were put on data files. The software support for creating data files, INDIAN, and reading them back SBREAD was also created.

Program SIM2 includes the modifications in the models mentioned earlier. Both SIM1 and SIM2 were written for model verification. Program SIM7 is a modified version of SIM2 to accommodate a sinusoidal weather pattern. The consistency of SIM7 with SIM2 was checked. In SIM7, the outside temperature may be varied between prescribed limits. This program can handle

an infinite number of time step increments. SIM8 is identical to SIM7 except that it makes printer plots of the outside and room temperatures at specified time intervals.

The BASIC language used on the microcomputer is of the interpretive type, and is therefore, not very fast. Execution time for the model verification runs take about 7 minutes. The microcomputer version of the program was used to conduct studies of the effect on energy consumption of system dynamics. These studies are described in the 4th part of the thesis.

PART 4EFFECT OF CONTROL SYSTEM PARAMETERS ON
THE ENERGY CONSUMPTION

The simulation program described earlier was modified to accommodate a sinusoidal weather pattern in the ambient temperatures outside the wall. The temperatures above the ceiling and below the floor were assumed to remain constant. The simulation programs was modified to run as many increments in time as desired.

4.1

SIMULATIONS RUNS

The simulation was initially run for as long as three daily weather cycles. Ambient temperature was varied from 52 °F to 70 °F in a sinusoidal pattern, while the temperatures below the floor and above the ceiling were maintained at 76 °F. It was observed that the response of the room temperature, due to initial conditions in the temperatures settled out very quickly. Hence, a one and a quarter day cycle (600 time steps of 3 minutes each) was used for study.

The temperature range 52 °F to 70 °F for daily weather cycle was used since this was within the control system range. During this range, for the base run, the valve was never fully closed or fully open. Wider range could be accommodated by raising the water temperature. The control parameters identified for variation were; mass flow rate of supply air keeping infiltration ate constant, temperature of hot water, thermostat time constant, thermostat gain, and the set temperature of the thermostat. Refer to figure (4.1.1) for the amount of variation in the parameters, the average room temperature and the energy consumed in one day's cycle.

Refer to figures (4.2.2) through (4.2.12) for the effect of variation in control parameters on the room temperature.

The average temperatures calculated from the plots are estimated to be 0.25 °F lower than the actual average temperatures, due to the type of plotting function used. Hence, the average temperatures noted in figure (4.1.1) are the adjusted values of temperatures, calculated from the plots.

In general, higher average room temperature accounted for higher heat energy used by the room. An exception was observed when the mass flow rate of the supply air was increased or decreased by 25%. The energy supplied for 24 hours was lower in both the cases than the base line run in one case, and lower in the other.

A substantial reduction in the energy supplied to the room was observed when the set temperature of the theormostat was decreased to 69.19 °F. An explanation to this behavior could be that, as the room temperature went below the constant temperatures of the space above the ceiling and the space below the floor, heat transfer from these spaces to the room in a 24 hour cycle are based on the raise in temperature of supply air as it passes through the heating coils.

A similar explanation could be given to account for above average supply of energy to the room when the set temperature was raised from 74.5 °F to 79.84 °F. Due to the higher room temperature, heat transfer took place not only through the walls, but also through the ceiling and floor to the spaces above and below. Heat was also lost from the supply air ducts to the spaces through which these ducts pass.

In other cases the average room temperature remains in close vicinity to the 76 °F constant temperatures of spaces above and below the room. Because of unknown energy required to keep the attic and cellar at constant

temperatures, the results obtained must be viewed with caution.

It is suggested that a complete simulation of the thermal behavior of the attic be implemented.

4.3

CONCLUSIONS

The following conclusions can be drawn from figure (4.2.13) which is a plot of percentage change in temperature difference between the room and outside air from the base line run versus the change in energy supplied per 24 hours with respect to the base line run.

1. The increased energy supplied caused an increase in the average room temperature except in one case.
2. There is a proportionate relationship between the percentage change in energy supplied with the percentage change in the difference in average temperature of room over the outside temperature.

SUMMARY AND SUGGESTIONS

5.1

SUMMARY

The algorithm for calculation of Modified Thermal Response Factors was developed in the first part of the thesis. A computer program was made to execute this algorithm. The algorithm has variable sampling time intervals. The inputs required to this program are, the number of layers in the wall, sampling time interval and thermal properties of the material of each layer of the multi-layer wall. The results are given in Appendix B.

It is observed from the results that a decrease in sampling time interval results in a larger number of z-transfer coefficients and larger number of roots of B to be calculated. A 95% reduction in the sampling time interval caused a 71% increase in the number of z-coefficients required to maintain the same order of truncation error. It may be observed that the required number of roots and of z-coefficients, for the same order of truncation error, is inversely related to the sampling time interval.

An algorithm was developed for the simulation of heat fluxes and temperatures in the thermal network. This algorithm is more accurate and general than the ASHRAE weighting factor method, though the latter would take less computing time. The algorithm also provides a more satisfactory description of the thermal network of the room. The improved method uses heat balance equation at each node of the thermal network.

Subroutines COIL and VALVE TRAVEL were modified and generalized to improve the simulation program [18]. Change was also made in the gain of the thermostat. The COIL subroutine was expanded to be able to simulate the effectiveness of the coil at different water and air flow rates. the VALVE TRAVEL SUBROUTINE was changed to predict normalized valve travel for a valve having the nonlinearity represented in figure (2.3.3). The THERMOSTAT subroutine was modified to accommodate the value of thermostat gain

observed in an experimental investigation. The simulation program was further modified to handle an indefinite number of time steps. The modified program was used to study the effect on energy supply to the room due to variations in control parameters. A sinusoidal weather pattern was introduced in the outside temperature. It was generally observed that changes in control parameters which resulted in higher average room temperatures resulted in higher energy use. A proportional relationship was observed between the percentage change in the difference of average temperatures of the room and the ambient and the percentage change in energy usage.

5.2

SUGGESTIONS

The following suggestions are made for improving the accuracy and calculation procedure of the simulation program and for exploring the effect of various control parameters on the energy supply to the room.

1. It is suggested that the algorithm developed in Section (1.3) be implemented in the modified simulation program for the following reasons:
 - a. The proposed algorithm would avoid stepping back in the temperature data by one time step.
 - b. The proposed algorithm would enable more accurate predictions of heat fluxes and temperatures of the thermal network including average room temperature.
 - c. The proposed algorithm would allow direct inclusion of radiation effects in the calculation of heat transfer at the outside and inside surfaces of the walls.
 - d. The proposed algorithm allows for variation in film convection coefficients without recomputing the values of the wall z-transfer coefficients.
2. It is recommended that the effect of each control parameter on the

energy supply to the room be studied independently and that mathematical relationships be developed for each. This may be achieved by making more runs of the modified simulation program, SIM7, with additional values of each control parameter.

3. It is recommended that a more detailed accounting of the energy flows to and from the room be made. This should include the heat transfer from the floor, ceiling and walls as well as the heat supplied to the room through the supply air, and the heat loss by air infiltration.
4. It is recommended that more realistic temperatures be used for the space above the ceiling and below the floor. This may be accomplished by providing a detailed model of these spaces.

It is hoped that the follow up of these recommendations would help produce a more realistic and reliable simulation for the study of effect of control parameters on the energy supply to the room.

APPENDIX A

A.1

DISCUSSION ON ROOTS OF FUNCTION B

A wall being a physical system is a highly stable system from the conduction heat transfer point of view. Hence, it may be expected that the roots of $B(s)$ should lie in the left half of the s -plane.

Through the following mathematical investigation for a single layer of semi-infinite slab, it will be shown that the roots of B are all negative real numbers.

For a single layer,

$$B_p = -\frac{1}{k_p} \sqrt{\alpha/s} \sinh(\sqrt{s/\alpha_p} L_p), \text{ for } n=0, s=0 \text{ and } \sqrt{\alpha_p/s} = \infty. \quad (\text{A.1.1})$$

where L_p is the thickness of the layer

For roots of B_p

$B_p = 0$, gives

$$\sinh(\sqrt{s/\alpha_p} L_p) = 0 \quad (\text{A.1.2})$$

Hence,

$$\sqrt{s/\alpha_p} L_p = i \pi n$$

where,

$$i = \sqrt{-1}, \quad n = 1, 2, 3, \dots$$

gives,

$$s = -\alpha_p \frac{\pi^2 n^2}{L_p^2} \quad (\text{A.1.3})$$

Since, $\alpha \geq 0$, $L_p \geq 0$, $\pi^2 > 0$, $n^2 \geq 0$, $s \leq 0$

The above derivation shows that roots of B_p , are all real negative numbers and cannot be complex numbers.

The roots of B are obtained from equation (A.1.3) as n is increased in value.

The first root of B, from equation (A.1.3) is,

$$s_1 = -\pi \alpha_p / L_p^2$$

It may be proven for a multi-layered wall, which is essentially a multi-layered infinite slab, that roots of B are real and negative.

A.2 DETERMINATION OF THE STEP SIZE ON x -AXIS FOR ITERATIVE METHOD OF CONVERGENCE ON ROOTS OF B

A method for finding the roots of B is developed based on stepping out the axis until a root is passed, then converging on it. The process is repeated until the necessary roots are found. It is more convenient to use trigonometric functions than to use hyperbolic trigonometric functions since some computer libraries do not have the hyperbolic functions. h

Since it is known that the roots of B are negative and real, $s = -\kappa$ is substituted in the formula derived in Part 1 of this thesis, and B_p becomes:

$$B_p = -\frac{1}{k_p} \sqrt{\alpha_p / \kappa} \sin(\sqrt{\kappa / \alpha_p} \Delta x_p) \quad (\text{A.2.1})$$

The step size for searching for roots should be determined by the layer which has roots most closely placed, since that layer will have the tendency to bring the roots of B (multi-layered wall) closer. Hence, equation (A.2.1) is investigated as follows. To find roots of B equating (A.2.1) with zero,

$$\sin(\sqrt{\kappa / \alpha_p} \Delta x_p) = 0$$

$$\sqrt{\kappa / \alpha_p} \Delta x_p = n\pi, \quad \kappa = \frac{n^2 \pi^2 \alpha_p}{(\Delta x_p)^2}, \quad n = 1, 2, 3, \dots$$

which means that the layer which has the smallest value to $\alpha_p/(\Delta x_p)^2$ is the critical layer. The computer program listed in Appendix B finds the smallest value of the $\alpha_p/(\Delta x_p)^2$ and uses it as the step size to step out on the x -axis.

APPENDIX BB.1 EXPLANATION TO THE COMPUTER PROGRAM FOR MODIFIED
THERMAL RESPONSE FACTORS

The procedure is described here for the computer program written, to find the z-transfer coefficients. The data supplied are, thickness - (ft), thermal conductivity (Btu/hr. ft. °F), density (lbs/ft³), specific heat (Btu/lb. °F) and thermal resistance (hr. ft.² °F/Btu). The thermal resistance for all the layers except surface films is given as zero, while all the other properties except thermal resistance are given zero for the surface films.

The residues a's, b's, and c's are obtained by knowing the roots of B, equation (1.2.10). Following steps are made in calculating z-transfer coefficients to different s-transfer functions.

1. A root of B is calculated. This requires use of subroutine COMP.
 - a. Steps of the r-axis are made, until the function B changes sign. Step size is determined as explained in Appendix A. Subroutine COMP computes the value of B, at different values of r, using matrix multiplication of transmission matrices.
 - b. When B changes sign, it is clear that a root has been crossed in that step. Hence, a reverse step is taken and then the step size is halved. Once again steps are taken on the r-axis until B changes sign. The above procedure is repeated until the root is approached so close that value of B reaches a near zero value. The value of r at that step is taken as the root of B.
 - c. To calculate more roots of B, step size (Appendix A) is used to take steps from the previously found root. Step 1.b is repeated.
2. Once a root is calculated, z-transfer coefficients are calculated in the following manner.

- a. Subroutine GZ makes use of subroutines DERIV and COEFF to calculate residues of the s-transfer functions $A(s)/B(s)$, $1/B(s)$ and $D(s)/B(s)$.
- b. Subroutine DERIV evaluates the derivatives of equation (1.1.18) using equation (1.2.22) at different roots of B.
- c. $A(s)$, $B(s)$, $C(s)$, and $D(s)$ are evaluated at roots of $B(s)$ by subroutine COMP.
- d. The above values are used by subroutine COEFF to calculate the residues of s-transfer functions.
- e. Subroutine GZ uses these residues to calculate z-transfer coefficients.

With each new root the number of z-transfer coefficients increases by one. A check is made to see if the last coefficient of the series falls below significance level. If so, the calculations of more roots is stopped and the Modified Thermal Response Factors are printed out. If the last coefficient has not reduced below the significance level, a new root is calculated. The procedure from steps 1 to 2.e are repeated. The Modified Thermal Response Factors are calculated using equation (1.2.14). Subroutine POLYM multiplies two polynomials to obtain the product polynomial.

An example of the input and output of this program follows.

B.2

COMPUTER PROGRAM FOR THE MODIFIED THERMAL RESPONSE FACTORS

In the following pages, the computer program for the calculation of Modified Thermal Response Factors is produced. The program was run for two sampling time intervals of one hour and 0.05 hours. The results of these two runs are also printed after the program.

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```

52 550 FORMAT (//,5X,'J',6X,'L',11X,'K',11X,'DEN',6X,'CP',10X,'RES')
53 575 FORMAT (10X,'F1',5X,'R10/(10.F1)',4X,'L0/F13',3X,'B10/(10.F1)',3X,
54 610 'F12,H,F10',//)
55 55 STOP
56 55 END

C ***** SUBROUTINE COMP COMPUTES THE VALUE OF B FOR
C GIVEN VALUES OF K, ALPHA, L AND RES

57 SUBROUTINE COMP (S,A,B,C,D,RL)
58 REAL L,K
59 DOUBLE PRECISION AHS ,AA(10),BB(10),CC(10),DD(10),A,B,C,D,S,R,
60 1SINE,AM,DM,CM,DM,ADUM,BDUM,CDUM,DDUM,SUM
61 COMMON ALPHA(10),L(10),K(10),ADLS(10),RES(10)
62 IF (S.NE.0.0) GO TO 100
63 A=1.0
64 C=0.0
65 D=1.0
66 SUM=0.0
67 DO 101 I=1,NL
68 IF (L(I).EQ.0.0) GO TO 5
69 SUM=SUM-L(I)/K(I)
70 GO TO 101
71 SUM=SUM-RES(I)
72 CONTINUE
73 B=SUM
74 GO TO 105
75 R=-S
76 DO 10 I=1,NL
77 IF (L(I).EQ.0.0) GO TO 9
78 SINE=DSINH(SORT(R/ALPHA(I))*L(I))
79 DD(I)=DCOS(DSORT(R/ALPHA(I))*L(I))
80 AA(I)=DD(I)
81 BB(I)=-DSORT(ALPHA(I)/R/K(I))*SINE
82 CC(I)=K(I)*DSORT(R/ALPHA(I))*SINE
83 GO TO 10
84 AA(I)=DD(I)=1.0
85 BB(I)=-RES(I)
86 CC(I)=0.0
87 CONTINUE
88 AM=AA(I)
89 BM=BB(I)
90 CM=CC(I)
91 DM=DD(I)
92 IF (NL.EQ.1) GO TO 25
93 DO 20 I=2,NL
94 ADUM=AA(I)*A+BB(I)*C+
95 DDUM=AA(I)*BM+BB(I)*DM
96 CDUM=CC(I)*AM+DD(I)*CM
97 DDUM=CC(I)*HA+DD(I)*DA
98 AM=ADUM
99 BM=BDUM
100 CM=CDUM
101 DM=DDUM
102 CONTINUE
103 A=AM
104 B=BM
105 C=CM
106 D=DM
107 RETURN

```



```

163 AC(1)=0.0
164 CONTINUE
165 DO 207 N=1,NL
166 IF U(N).EQ. 0.0) GO TO 200
167 AM=1.0
168 BM=0.0
169 CM=0.0
170 DM=1.0
171 DUMA=AA(1)
172 DUMB=AB(1)
173 DUMC=AC(N)
174 SOT=DSORT(R/AULS(1))
175 AA(N)=DSIN(SOT)/(2.0*DSORT(AULS(1)*R))
176 AB(N)=(L(N)*COS(SOT)-DSIN(SOT)/5.0*(12.0*K(1)*R)
177 AC(N)=-K(N)*(COS(SOT)+DSIN(SOT)/5.0*(12.0*AULS(1)*L(N))
178 DO 150 J=1,NL
179 ADUM =AA(1)*AM+AB(1)*CM
180 BDUM =AA(1)*BM+AB(1)*DM
181 CDUM =AC(1)*AM+AA(1)*CM
182 DDUM =AC(1)*BM+AA(1)*DM
183 AD=ADUM
184 BD=BDUM
185 CM=C.DUM
186 DM=DDUM
187 CONTINUE
188 AA(1)=DUMA
189 AB(1)=DUMB
190 AC(1)=DUMC
191 DA=DA*ADUM
192 DB=DB*BDUM
193 DC=DC*CDUM
194 DD=DD*DDUM
195 CONTINUE
196 RETURN
197 END
C ***** SUBROUTINE COEFFICIENTS, CALCULATES RESIDUES IN
C THE LAPLACE TRANSFORM OF G(Z). CA, CB, ARE THE COEFFICIENTS IN
C THE LAPLACE TRANSFORM OF G(Z). CC 'S ARE THE RESIDUES
C IN THE SUBSCRIPTS DENOTE THE CASE BEING CONSIDERED.
C THE CASES ARE 1 FOR A/B .2 FOR 1/B .3 FOR D/Z
198 SUBROUTINE COEFFS,CR,CA,CB,CC,RU1
199 COMMON ALPHA (10),L(10),K(10),AULS(10),RES(10)
200 DOUBLE PRECISION AULS ,S,CA(3),CB(3),CC(3),SUM,DS,DA,DB,DC,DZ,A
1,B,C,D,SB,SDR
INTEGER CR
REAL L,K
IF (CR.GT.1) GO TO 100
DS=0.0
SUM=0.0
DO 10 I=1,10
IF (L(I).EQ. 0.0) GO TO 5
SUM=SUM-L(I)/K(I)
GO TO 10
5 SUM=SUM-RES(I)
CONTINUE
CA(1)=L.0/SDA
CA(2)=CA(1)
CA(3)=CA(1)
CALL DECTV (DS,DA,DB,DC,DD,RU1)

```



```

265      CONTINUE
266      POL2(1)=-C(1)
267      MM=99
268      CALL PMYM (POL2,POL1,MM,MM)
269      POL2(1)=POL2(1)+C(1)/Y
270      POL2(2)=POL2(2)+C(1)/Z
271      CALL PMYM (POL2,C(1),MM,C(1))
272      DO 60 J=1,199
273      C(1,J)=POL2(J)
274      IF (DABS(POL2(1))-6.1E-15) C(1)=J
275      CONTINUE
276      CONTINUE
277      WRITE (6,601)
278      WRITE (6,602) (C(1),I=1,3)
279      WRITE (6,603) (C(1),I=1,3)
280      WRITE (6,604) (C(1),I=1,3)
281      WRITE (6,605)
282      DO 80 J=1,100
283      BRITF (6,606) J,(C(1,J),I=1,3)+J*(C(1,J))
284      CONTINUE
285      FORMAT (7X,10X,'A/ZH',30X,'1/0',30X,'0/B',7)
286      FORMAT (10X,'CA=',1X,15,'0,2(15X,15,0))
287      FORMAT (10X,'C0=',1X,15,'0,2(15X,15,0))
288      FORMAT (10X,'C1=',1X,15,'0,2(15X,15,0))
289      FORMAT (7X,10X,'COEFFS OF RATIONALS',70X,'COEFFS OF DENOMINATORS',
        1,7)
290      FORMAT(1X,'C(1',12,')=',2X,15,'0,2(15X,15,0),2X,'C(1',12,')=',2
        1X,15,'0)
291      RETURN
292      END
C ***** SUBROUTINE POLYM ADJUSTS TWO POLYNOMIALS
C TO OBTAIN THE RESULT IN A POLYNOMIAL FORM
SUBROUTINE POLYM (A,B,D,H)
DOUBLE PRECISION A(100),B(100),C(100)
K=H+4
IF (K-11,99) GO TO 60
K=99
KK=K+1
DO 10 I=1,100
C(1)=0.0
DO 25 I=1,KK
KKK=I
L=9+I
IF (L-11,1) GO TO 50
L=1
J=L-3
IF (J-11,1) GO TO 60
J=1
DO 30 MM=J,L
MM=1-MM
C(1)=C(1)+C(1)+C(1)
IF (I-11,5) GO TO 25
IF (DABS(C(1)-1.0-9999999)) GO TO 45
CONTINUE
KK=KKK
DO 30 I=1,100
A(1)=C(1)
Q=KK-1
RETURN

```

320 END

SENTRY

NUMBER OF LAYERS= 6 TIME INTERVAL 1.4 HOURS= 1.03609

J	I	K	DEM	CP	RTS
FT	RTU/10,FT,FT	10/113	RTU/10,FT,FT	RT2,RT,FT/10	
1	0.00000	0.00000	0.00000	0.00000	0.83300
2	0.33300	0.62000	100.41990	0.22000	0.00000
3	0.33300	0.77000	125.00000	0.22000	0.00000
4	0.00000	0.00000	0.00000	0.00000	0.33300
1	66	-0.174632800 00	0.275352190-09		1
2	67	-0.844634370 00	-0.879619520-19		2
3	67	-0.256933300 01	0.023992610-09		1
4	60	-0.486086210 01	-0.390558930-09		2
5	66	-0.886160810 01	0.282336300-09		1
6	53	-0.128524580 02	0.500649430-09		2
7	79	-0.191546820 02	0.419565910-10		1
8	77	-0.250120340 02	0.226137470-09		2
9	91	-0.333393230 02	-0.527695690-10		1

A/R	1/R	0/0
CA= -0.418170270 00	-0.418170270 00	-0.418170270 00
CB= -0.231229750 01	0.336567170 01	-0.247092240 01
CC= 0.218953610-03	-0.376129790-03	0.066135120-03

COEFFS. OF NUMERATORS

CN(1, 1)= 0.000000000 00	0.000000000 00	0.000000000 00	0.000000000 00	0.000000000 00	0.000000000 00
CN(1, 2)= -0.919704560 00	-0.125963590-03	-0.198458270 01	0.0421=	0.130003070 01	-0.135395430 01
CN(1, 3)= -0.1412470 01	-0.736625760-02	0.323921510 01	0.0421=	0.468703530 00	0.468703530 00
CN(1, 4)= -0.577689740 00	-0.202439770-01	-0.122253580 01	0.0421=	-0.412519360-01	-0.412519360-01
CN(1, 5)= -0.507925630-01	-0.692955320-02	0.144265160 03	0.0421=	0.218517130-03	0.218517130-03
CN(1, 6)= -0.732258010-03	-0.256209660-03	-0.237494730-02	0.0421=	-0.309050430-07	-0.309050430-07
CN(1, 7)= 0.123279770-05	-0.620157770-06	0.599727350-05	0.0421=	0.000000000 00	0.000000000 00
CN(1, 8)= -0.07629609-13	-0.533373500-10	-0.339134360-03	0.0421=	0.000000000 00	0.000000000 00

COEFFS. OF DENOMINATORS

CORE USAGE	OBJECT CODE=	14000	BYTES, TOTAL	159360	BYTES
DIAGNOSTICS	NUMBER OF ERRORS=	0	NUMBER OF WORDINGS=	0	1
COMPUTE TIME=	1.05	SELECTION TIME=	3.52	SEC	15.37.61
			MONDAY	20 FEB 83	MATHEV - JAN 1976 VIUS

ENTRY

NUMBER OF LAYERS= 4 TIME INTERVAL IN HOURS= 0.05000

J	I	K	DEK	CP	RES
FT	BTU/(H.F.T)	LB/FT ³	BTU/(H.F.T)	FT ² .H.F/UB	
1	0.00000	0.00000	0.00000	0.00000	0.83300
2	0.33300	0.42000	100.41900	0.22000	0.00000
3	0.33300	0.77000	125.00000	0.22000	0.00000
4	0.00000	0.00000	0.00000	0.00000	0.33300
1	44	-0.174632030 00	-0.174632030 00	0.295162190-09	1
2	47	-0.844634370 00	-0.844634370 00	-0.879619520-05	2
3	47	-0.256933300 01	-0.256933300 01	0.823992610-09	1
4	50	-0.46086219 01	-0.46086219 01	-0.308558930-09	2
5	66	-0.886160810 01	-0.886160810 01	0.202336380-09	1
6	63	-0.128524580 02	-0.128524580 02	0.580669430-07	2
7	79	-0.191546070 02	-0.191546070 02	0.419565910-10	1
8	77	-0.250120340 02	-0.250120340 02	0.224137470-07	2
9	91	-0.33393230 02	-0.33393230 02	-0.527495690-10	1
10	86	-0.414573270 02	-0.414573270 02	-0.659175590-09	2
11	101	-0.513717950 02	-0.513717950 02	-0.311551590-09	1
12	103	-0.621878690 02	-0.621878690 02	0.911033740-09	2
13	107	-0.73127750 02	-0.73127750 02	0.567522430-09	1
14	114	-0.870766210 02	-0.870766210 02	-0.362976050-09	2
15	113	-0.993252700 02	-0.993252700 02	-0.943051800-09	1
16	139	-0.115890300 03	-0.115890300 03	0.229273520-09	2
17	118	-0.129627310 03	-0.129627310 03	-0.146969660-09	1
18	148	-0.148453340 03	-0.148453340 03	-0.118143550-09	2
19	134	-0.164395810 03	-0.164395810 03	-0.260667970-09	1
20	154	-0.184664590 03	-0.184664590 03	0.661396230-07	2
21	156	-0.203651620 03	-0.203651620 03	0.232636210-09	1
22	167	-0.224619620 03	-0.224619620 03	-0.459952240-07	2

23	174	-0.294721460 03	-0.352029300-13	1
24	166	-0.268594600 03	-0.750796700-09	2
25	176	-0.299708160 03	0.417251360-09	1
26	166	-0.316752360 03	-0.311691050-09	2
27	219	-0.366006360 03	0.271760730-19	1
28	179	-0.369591110 03	-0.269230000-09	2
29	223	-0.400667620 03	0.203737110-09	1
30	191	-0.427070260 03	0.590606060-09	2
31	226	-0.458807610 03	0.112152300-09	1

A/B			1/B			0/B		
CA=	-0.410170270 00		-0.410170270 00			-0.410170270 00		
CB=	-0.231220750 01		0.346554170 01			-0.751032260 01		
CC=	0.119516790-05		-0.119750280-05			0.33345770-03		

COEFFS. OF NUMERATORS			COEFFS. OF DENOMINATORS			
CN1. 11=	-0.719562760-16		-0.177635600-13		CN1. 11=	0.100039630 01
CN1. 21=	-0.112532010 01		-0.270271360 01		CN1. 21=	-0.593061360 01
CN1. 31=	0.676027020 01		0.162605420 02		CN1. 31=	0.1540923620 02
CN1. 41=	-0.117609510 02		-0.277637160-03		CN1. 41=	-0.236043620 02
CN1. 51=	0.270597350 02		0.657013370 02		CN1. 51=	0.226633620 02
CN1. 61=	-0.266661510 02		-0.660371060 02		CN1. 61=	-0.167923430 02
CN1. 71=	0.173915660 02		0.429355030 02		CN1. 71=	0.658639250 01
CN1. 81=	-0.705115240 01		-0.195125440 02		CN1. 81=	-0.202517060 01
CN1. 91=	0.2246312300 01		0.611667760 01		CN1. 91=	0.425301160 01
CN1. 101=	-0.520955500 00		-0.131260160 01		CN1. 101=	-0.599192030-01
CN1. 111=	0.744265990-01		0.107672110 00		CN1. 111=	0.551337130-02
CN1. 121=	-0.6377915020-02		-0.179086500-01		CN1. 121=	-0.319945850-03
CN1. 131=	0.519043150-03		0.100631500-02		CN1. 131=	0.112279630-06
CN1. 141=	-0.150322140-04		-0.433365730-04		CN1. 141=	-3.227160120-06
CN1. 151=	0.316153220-06		0.367109460-06		CN1. 151=	0.254601430-08
CN1. 161=	-0.376174240-08		-0.106360860-07		CN1. 161=	0.000109630 00

CONF. USAGE	CONF. CODE=	14000 BYTES, ARRAY AREA=	7352 BYTES, TOTAL AREA AVAILABLE=	159840 BYTES
DIAGNOSTICS	NUMBER OF ERRORS=	0, NUMBER OF WARNINGS=	0, NUMBER OF CALCULATIONS=	1
COMPUT. TIME=	1.06 SEC, EXECUTION TIME=	21.27 SEC,	15.39-20	MON. DAY 23 FEB 69
				WALLY - JAD 1979 V11.5

APPENDIX CC.1 EXPLANATION OF LOGIC FOR SUBROUTINE VALVE TRAVEL

Figure (2.3.3) is a schematic diagram of valve travel characteristic. Figure (C.1.1) is the flow chart for subroutine VALVE TRAVEL to determine the normalized valve travel from the valve actuator pressure which is the output of the thermostat.

A flag F1 is set up which conveys the state of valve in the previous time step. F1=3 indicates constant valve travel, that is the horizontal line between two curves. Curve 1 is the valve opening characteristic, and Curve 2 is the valve closing characteristic. F1=2 indicates that on the previous time step the valve was on the closing curve while F1=1 indicates that valve was on the opening curve on the previous time step. Using the following equations, valve travel is determined from given valve actuator pressure.

Curve 1:

$$\begin{aligned}
 Y9 &= 1.0 && \text{for } P7 < 1.68 ; Y9 = 0 && \text{for } P7 < 7.00 \\
 \left. \begin{aligned} Y9 &= -0.277*P7 + 1.466 \\ P7 &= -3.610*Y9 + 5.292 \end{aligned} \right\} && \text{for } 1.68 < P7 \leq 4.93 \\
 \left. \begin{aligned} Y9 &= 0.111 - 0.0765 \sqrt{P7 - 4.911} \\ P7 &= 170.9*Y9^2 - 37.79*Y9 + 7 \end{aligned} \right\} && \text{for } 4.93 < P7 \leq 7.00
 \end{aligned}$$

Curve 2:

$$\begin{aligned}
 Y9 &= 1.0 && \text{for } P7 < 2.00 \\
 \left. \begin{aligned} Y9 &= -0.300*P7 + 1.600 \\ P7 &= -3.333*Y9 + 5.333 \end{aligned} \right\} && \text{for } 2.00 < P7 \leq 5.00 \\
 \left. \begin{aligned} Y9 &= 0.110 - 0.0755 \sqrt{P7 - 4.983} \\ P7 &= 166 - 7*Y9^2 - 36.67*Y9 + 7 \end{aligned} \right\} && \text{for } 5.00 < P7 \leq 7.00 \\
 Y9 &= 0 && \text{for } P7 > 7.00
 \end{aligned}$$

APPENDIX D

D.1 LISTING OF SIM7

In the following pages a listing of program SIM7 is produced. The listing includes subroutine SBREAD which is also an independent file by itself. SIM7 is the program for simulation of room heating system with variable number of time step increments.

Listing of nomenclature used for SIM7 , KEY, follows the listing of SIM7.

```

1 REM ** THIS IS INDEPENDENT SUBROUTINE "SBREAD".
2 REM ** READS DATA FROM A GIVEN FILE "G$".
3 REM ** DATA FILE SHOULD END WITH "*", LEAVING NO SPACE
4 REM ** BETWEEN THE LAST VALUE STORED AND THE *
5 REM ** STATEMENTS 1 TO 70 RESERVED FOR "SBREAD"
6 REM ** IN THE MAIN PROGRAM
7 REM ** "SBREAD" INITIALIZES ITS OWN VARIABLES. DIMENSION
8 REM ** P$( 256 ), B$( 1024 ), C$( 64 ), Z( 100 ) IN MAIN PROGRAM
9 REM ** DATA FILES ARE ASSUMED STORED ON DRIVE UNIT #1
10 REM ** "SBREAD" MAYBE CALLED FROM STMT. NO. 22
22 P$="P" : B$="1"
24 FOR X=1 TO 8
26 P$=P$+P$ : B$=B$+B$
28 NEXT X
30 Q7=CALL( 2560 )
32 DEF FNF( C$ )
34 U=ASC( C$( 6, 6 ) )-48
36 P$( U*64+1, U*64+64 )=C$
38 RETURN CALL( 2675, U )
40 FNEND
42 Q7=FNF( "FILE#1 OPEN, INPUT, "+G$+".DA" )
44 J=1
46 Q7=FNF( "FILE#1 READ" )
48 IF Q7=0 THEN 68
50 I=1
52 Z( J )=VAL( B$( I+1, 1024 ) )
54 I=I+1
56 IF B$( I, I )=" " THEN J=J+1 : GOTO 52
58 IF B$( I, I )="*" THEN 66
60 IF ASC( B$( I, I ) ) > 1 THEN 54
62 GOTO 46
64 PRINT " NO MORE ROOM " : GOTO 68
66 PRINT "FILE ", G$, " READING COMPLETED"
68 RETURN
70 DIM P$( 256 ), B$( 1024 ), C$( 64 ), Z( 100 )
71 DIM T9( 3, 11 ), A9( 2, 11 ), B9( 2, 11 ), C9( 2, 11 )
72 DIM C1( 1 ), M( 2 ), C2( 5 ), A9( 5 )
73 DIM T7( 2 ), G9( 10 ), G6( 10 ), P9( 5, 11 )
74 DIM T3( 11 ), P7( 1 ), E( 5 ), F9( 1 )
75 DIM T1( 1 ), G2( 1 ), B( 5 )
101 REM ** READ DATA 101-150
110 READ N3, T, N1, N2
112 READ M( 0 ), M( 2 ), U, H9
114 READ I9, C1( 0 ), C1( 1 ), M9
116 READ P2, Q9, W, E1
118 READ E2, E3, A9, P9
120 READ P8, T8, T7( 1 ), T7( 2 )
122 READ A7, B7, T9, U1
124 READ T5, T6, Y, H
126 READ A2, B2, C2, D2
128 READ F9, T4, T3( 0 ), F1
151 REM ** DATA AND READ DATA FILES 151-400
152 DATA 600, 0, 0.5, 2, 11
154 DATA 427.5, 776.5, 2126.6, 0.6
156 DATA 1, 0.24, 0.8, 100
158 DATA 1, 0, 1060, 0.5
160 DATA 0.6, 0.6, 0, 0.2
162 DATA 3.45, 74.5, 0.571, 0.26
164 DATA 0.725, 0.275, 155, 52

```

```

166 DATA 78.8, 77.9, 0.256, 0.265
168 DATA 4.114866E-3, 1.478111E-1, 7.848157E-1, 6.874663E-2
170 DATA 2, 78.8, 78.8, 1
190 FOR I=1 TO N2
191 T9(3, I)=76.00
192 T9(2, I)=76.00
193 T3( I)=78.8
195 NEXT I
196 G2( 1)=135.00
200 G$="A00"
209 GOSUB 22
210 FOR I=1 TO N2
211 A0( 1, I)=Z( I)
213 NEXT I
214 G$="B00"
215 GOSUB 22
216 FOR I=1 TO N2
217 B0( 1, I)=Z( I)
219 NEXT I
220 G$="C00"
221 GOSUB 22
222 FOR I=1 TO N2
223 C0( 1, I)=Z( I)
225 NEXT I
226 G$="A01"
227 GOSUB 22
228 FOR I=1 TO N2
229 A0( 2, I)=Z( I)
231 NEXT I
232 G$="B01"
233 GOSUB 22
234 FOR I=1 TO N2
235 B0( 2, I)=Z( I)
237 NEXT I
238 G$="C01"
239 GOSUB 22
240 FOR I=1 TO N2
241 C0( 2, I)=Z( I)
243 NEXT I
244 G$="C2"
245 GOSUB 22
246 FOR I=1 TO 5
247 C2( I)=Z( I)
249 NEXT I
250 G$="A9"
251 GOSUB 22
252 FOR I=1 TO 5
253 A9( I)=Z( I)
255 NEXT I
280 PRINT
281 PRINT
282 PRINT
285 PRINT "**** SIMULATION WITH MODIFIED OUTSIDE TEMP. AND RETURN AIR
286 PRINT
287 PRINT "T9( 1, N2)=61+9*SIN( 0.0131*N ), T9( 2, N2)=T9( 3, N2)=76.0"
288 PRINT

```



```

269 PRINT "      O=OUTSIDE TEMP.OF WALLS ; R=ROOM TEMPERATURE"
290 PRINT "***** SIM7 *****" : PRINT
401 REM ** PROGRAM BEGINS
402 PRINT
403 PRINT "N          T3(11)          T9(1,11)          Y9          E          T4
404 PRINT"-----
407 T1(1)=T3(0) : E=0
408 M(1)=M(2)-M(0)
409 C=M(0)/60/0.075
410 FOR N=1 TO N3
411 T9(1,N2)=61+9*SIN(0.0131*N)
412 IF N<>1 THEN 420
414 FOR I=1 TO N2
415 T9(1,I)=T9(1,N2) : T3(I)=T3(0)
416 NEXT I
420 FOR I=1 TO N2-1
421 T9(1,I)=T9(1,I+1)
422 T3(I)=T3(I+1)
423 P0(1,I)=P0(1,I+1)
424 P0(2,I)=P0(2,I+1)
425 T9(2,I)=T9(2,I+1)
426 NEXT I
427 IF N=120 THEN E=0
432 IF N=1 THEN 434
433 GOSUB 501
434 GOSUB 551
435 GOSUB 601
436 GOSUB 651
437 Q1=0
438 FOR I=4 TO 5
439 GOSUB 701
440 Q1=Q1+P0(I,N2)*A9(I)
441 NEXT I
442 Q1=Q1+C2(3)*A9(3)*(T9(3,N2)-T3(N2))
443 GOSUB 751
444 GOSUB 801
445 GOSUB 851
446 GOSUB 901 : E=E+M(0)*C1(0)*(J1-T3)/20
447 GOSUB 951
451 X1=INT((T9(1,N2)-40)*1.5) : X2=INT((T3(N2)-40)*1.5)
452 PRINT N; TAB(5); T3(11); TAB(17); T9(1,11); TAB(29); Y9; TAB(41); E; TAB(53);
454 P3=P7(1)
456 NEXT N
458 END
501 REM ** SUBROUTINE "ROOM TEMP.PREDICTION" *****
510 IF N=1 THEN 530
512 D9=14.696*144*P2/53.352/(460+T3(N2))
514 J2=(D9*U*C1(0)+M9*C1(1))/(M(2)*C1(0)*B2)
516 J3=1/J2
518 S1=(M(0)-A2*M(2))*T1(1)/(M(2)*B2)
520 S2=(M(1)-M(2)*I2)*T9(1,N2)/(M(2)*B2)
522 S3=(Q1+Q+Q9)/(M(2)*C1(0)*B2)
524 S4=-(C2*T7)/B2
526 S=S1+S2+S3+S4
528 T3(N2)=T3(N2)*EXP(-J3*T)+(1-EXP(-J3*T))*S
530 RETURN

```

```

551 REM ** SUBROUTINE "SFLUX" *****
560 Q=0
562 IF N=1 THEN 574
564 FOR I=1 TO N1
566 GOSUB 1001
568 Q=Q+P0(1,N2)*A9(I)
570 NEXT I
572 RETURN
574 FOR I=1 TO N1
576 P0(1,N2)=C2(I)*(T9(1,N2)-T3(N2))
578 Q=Q+A9(I)*P0(1,11)
580 NEXT I
582 IF N>1 THEN 594
584 FOR I=1 TO (N2-1)
586 FOR I2=1 TO N1
588 P0(I2,I)=P0(I2,N2)
590 NEXT I2
592 NEXT I
594 RETURN
601 REM ** SUBROUTINE "WALL TEMPERATURE" *****
610 T7=P0(1,N2)/H0+T3(N2)
612 RETURN
651 REM ** SUBROUTINE "THERMOSTAT" *****
660 P5=5.2446
662 P6=2.1
664 IF N=1 THEN 674
666 T5=T5*(2)+T3(N2)*(4)
668 T6=T6*(3)+T7*(5)
670 T4=T5*B7+T6*A7
672 GOTO 692
674 T4=T5*B7+T6*A7
676 P7(1)=11.65 : P3=P7(1)
678 E(0)=EXP(-P5*T)
680 E(1)=1-E(0)
682 E(2)=EXP(-T/T7(2))
684 E(3)=EXP(-T/T7(1))
686 E(4)=1-E(2)
688 E(5)=1-E(3)
690 GOTO 696
692 P7(1)=P7(1)*E(0)+E(1)*(P6*T4-74.5*P6+3.45)
694 IF P7(1)<=0 THEN P7(1)=0
696 RETURN
701 REM ** SUBROUTINE "FLUX" *****
710 P0(1,N2)=C2(1)*(T9(1,N2)-T3(N2))
712 RETURN
751 REM ***** VALUE TRAVEL *****
752 IF F1<>1 THEN 765
753 IF P7(1)<=P3 THEN 763
754 IF P7(1)<5 THEN P=-3.333*Y9+5.333 : GOTO 761
755 P=166.67*Y9^2-36.67*Y9+7
756 IF P7(1)<=P THEN I=3 : GOTO 797
760 Y9=0.11-0.0775*(P7(1)-4.983)^0.5 : GOTO 797
761 IF P7(1)<=P THEN I=3 : GOTO 800
762 Y9=-0.3*P7(1)+1.6 : I=2 : GOTO 797
763 IF P7(1)>4.93 THEN Y9=0.111-0.0765*(P7(1)-4.911)^0.5 : GOTO 797

```

```

764 Y9=-0.277*P7(1)+1.466 : GOTO 797
765 IF F1<>2 THEN 783
770 IF P7(1)>=P3 THEN 781
771 IF P7(1)>4.93 THEN P=170.9*Y9^2-57.79*Y9+7 : GOTO 775
772 P=-3.61*Y9+5.292
773 IF P7(1)>P THEN I=3 : GOTO 800
774 Y9=-0.277*P7(1)+1.466 : I=1 : GOTO 797
775 IF P7(1)>P THEN I=3 : GOTO 800
780 Y9=0.111-0.0765*(P7(1)-4.911)^0.5 : GOTO 797
781 IF P7(1)<5 THEN Y9=-0.3*P7(1)+1.6 : GOTO 797
782 Y9=0.11-0.0775*(P7(1)-4.983)^0.5 : GOTO 797
783 IF Y9>0.1 THEN P=-3.61*Y9+5.292 : P1=-3.333*Y9+5.333 : GOTO 785
784 P=170.9*Y9^2-37.79*Y9+7 : P1=166.67*Y9^2-36.67*Y9+7
785 IF P7(1)<=P THEN 793
790 IF P7(1)<=P1 THEN 800
791 IF P7(1)>5 THEN Y9=0.11-0.0775*(P7(1)-4.983)^0.5 : GOTO 797
792 Y9=-0.3*P7(1)+1.6 : I=2 : GOTO 797
793 IF P7(1)<4.93 THEN Y9=-0.277*P7(1)+1.466 : I=1 : GOTO 797
794 Y9=0.111-0.0775*(P7(1)-4.911)^0.5 : I=1 : GOTO 797
797 IF Y9<0 THEN Y9=0
798 IF Y9>1 THEN Y9=1
799 P3=P7(1)
800 RETURN
801 REM ***** WATER FLOW RATE *****
810 F8=Y9
812 F0(1)=F8*F9
814 RETURN
851 REM ***** INLET FAN DUCT *****
860 IF N=2 THEN T1(1)=T3(0)
862 T1=A2*T1(1)+B2*T3(N2)+C2*T7+D2*T9(1,N2)
864 T2=T1+P9*E1*E3*W/M(2)/C1(0)
866 T3=A9*T9(1,N2)+(1-A9)*T2
868 RETURN
901 REM ** SUBROUTINE "COIL" *****
910 C1=C/100
912 A3=C1*(0.2070*C1-1.4811)+3.8257
914 A4=C1*(0.3246*C1-2.2450)+5.1568
916 A5=C1*(0.1938*C1-1.3222)+2.9018
918 A6=C1*(0.0396*C1-0.2685)+0.5763
920 FOR I=1 TO 4
921 F=I/2
922 E4=F*(F*(F*(-A6*F+A5)-A4)+A3)
924 B(1)=E4
926 NEXT I
928 B(5)=B(4)
930 F=F0(1)
932 IF F<.5 THEN E4=F*(F*(F*(-A6*F+A5)-A4)+A3) : GOTO 944
934 IF F<1 THEN I=1 : GOTO 942
936 IF F<1.5 THEN I=2 : GOTO 942
938 IF F<2 THEN I=3 : GOTO 942
940 I=4
942 E4=B(I)+(2*F-I)*(B(I+1)-B(I))
944 J1=E4*(G2(1)-T3)+T3
946 RETURN

```

```

951 REM ** SUBROUTINE "EXIT FAN DUCT" ****
960 D8=M(0)*C1(0)+U1*0.5
962 T1(1)=(J1*(M(0)*C1(0)-U1*0.5)+(1-P9)*E1*E3*W+U1*T9(3,N2))/D8
964 RETURN
1001 REM ** SUBROUTINE "HEAT FLUX" ****
1010 P0(1,N2)=0
1012 I2=N2
1014 FOR I1=1 TO I2
1020 I8=N2-I1+1
1022 IF I8<=0 THEN I8=1
1024 P0(1,N2)=P0(1,N2)+C0(1,I1)*T9(1,I8)
1026 P0(1,N2)=P0(1,N2)-A0(1,I1)*T3(I8)
1028 NEXT I1
1030 I2=N2
1032 FOR I1=2 TO I2
1038 I8=N2-I1+1
1040 IF I8<=0 THEN I8=1
1042 P0(1,N2)=(P0(1,N2)-P0(1,I8)*B0(1,I1))/B0(1,1)
1044 NEXT I1
1046 RETURN

```

D.2

File : KEY, nomenclature for variables used in SIM7
and nomenclature for DATA FILES

```

1 ***** " KEY " TO VARIABLES OF SIM7 PROGRAM *****
5 ***** DIMENSIONED VARIABLES *****
10 A9(1)      : AREA OF WALL (FT.SQ.)
12 A9(2)      : AREA OF FLOOR
14 A9(3)      : AREA OF CEILING
16 A9(4)      : AREA OF DOORS
18 A9(5)      : AREA OF WINDOWS
20 A0(1,11)   : Z-COEFF. OF A/B FOR WALL
22 A0(2,11)   : Z-COEFF. OF A/B FOR FLOOR
24 B0(1,11)   : Z-COEFF. OF DENOMINATOR FOR WALL
26 B0(2,11)   : Z-COEFF. OF DENOMINATOR FOR FLOOR
28 C0(1,11)   : Z-COEFF. OF 1/B FOR WALL
30 C0(2,11)   : Z-COEFF. OF 1/B FOR FLOOR
32 C1(0)      : HEAT CAPACITY OF AIR
34 C1(1)      : HEAT CAPACITY OF FURNITURE
36 C2(1)      : OVERALL CONDUCTANCE OF WALL
38 C2(2)      : OVERALL CONDUCTANCE OF FLOOR
40 C2(3)      : OVERALL CONDUCTANCE OF CEILING
42 C2(4)      : OVERALL CONDUCTANCE OF DOORS
44 C2(5)      : OVERALL CONDUCTANCE OF WINDOWS
46 F0(1)      : FLOW RATE OF WATER (GPM)
48 G2(1)      : TEMP. OF WATER ENTERING THE COIL (F)
50 M(0)       : SUPPLY AIR MASS FLOW RATE (LBS/HR)
52 M(1)       : INFILTRATION MASS FLOW RATE (LBS/HR)
54 M(2)       : RETURN AIR MASS FLOW RATE (LBS/HR)
56 P7(1)      : VALVE ACTUATOR PRESSURE (PSI)
58 T1(1)      : SUPPLY AIR TEMP.(F)
60 T3(11)     : ROOM TEMP. AT CURRENT TIME (F)
62 T7(1)      : TIME CONSTANT EFFECTED BY WALL
64 T7(2)      : TIME CONSTANT EFFECTED BY ROOM AIR
66 T9(1,11)   : TEMP. OUTSIDE WALL AT CURRENT TIME (F)
68 T9(2,11)   : TEMP. OF SPACE BELOW FLOOR (F)
70 T9(3,11)   : TEMP. OF SPACE ABOVE CEILING (F)
80 ***** UNDIMENSIONED VARIABLES *****
82 A7         : WEIGHTING FACTOR TO COMPUTE EFFECTIVE TEMP.
84 B7         : -DO-
86 C          : SUPPLY AIR FLOW RATE (CFM)
88 D9         : DENSITY OF AIR
90 E1         : MECH. EFFICIENCY OF MOTOR DRIVING HOT AIR FAN
92 E3         : FAN STATIC ENERGY/ENERGY TO FAN SHAFT
94 E4         : EFFECTIVENESS OF COIL
96 F1         : FLAG IN VALVE TRAVEL
98 F8         : WATER FLOW RATE AS FRACTION OF MAX FLOW RATE
100 F9        : MAX. WATER FLOW RATE
102 H0        : CONV. HEAT TR.COEFF. BETWEEN WALL AND ROOM
104 J1        : TEMP. OF AIR LEAVING THE COIL
106 J2        : ROOM TIME CONSTANT IN ROOM TEMP. PREDICTION
108 M9        : MASS OF FURNITURE (LBS)
110 N         : NUMBER OF TIME STEPS CONSIDERED
112 N1        : NUMBER OF SURFACES WITH HEAT STORAGE
114 N2        : NUMBER OF Z-COEFF. FOR A TIME SERIES

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116 P2   : ROOM ATM. PRESSURE (ATM)
117 P5   : INVERSE OF THERMOSTAT TIME CONSTANT
118 P6   : GAIN OF THERMOSTAT
119 P8   : SET PRESSURE OF THERMOSTAT (PSI)
120 Q    : HEAT TR.BY CONDUCTION FROM SURFACES WITH HEAT STORAGE
122 Q1   : HEAT TR.BY CONDUCTION FROM SURFACES WITHOUT
124 Q9   : HEAT FROM LIGHTS
126 T    : TIME STEP (SAMPLING INTERVAL) (0.05 HRS)
128 T1   : RETURN AIR TEMP.(F)
130 T3   : TEMP.OF AIR ENTERING THE COIL (F)
132 T4   : EFFECTIVE TEMPERATURE AT THERMOSTAT
134 T7   : TEMP.OF WALL (F)
136 T8   : SET TEMP. OF THERMOSTAT (F)
140 ***** DATA FILES *****
142 TM1   : RETURN AIR TEMPERATURE
144 TM2   : OUTSIDE TEMPERATURE OF NORTH WALL
146 TM3   : OUTSIDE TEMPERATURE OF WEST WALL
148 TM4   : OUTSIDE TEMPERATURE OF SOUTH WALL
150 TM5   : OUTSIDE TEMPERATURE OF EAST WALL
152 TM6   : VENTILATION AIR TEMPERATURE
154 TM7   : OUTSIDE TEMPERATURE BELOW THE FLOOR
156 TM8   : OUTSIDE TEMPERATURE ABOVE THE CEILING
158 TM9   : TEMPERATURE OF NORTH DIFFUSER
160 TM10  : TEMPERATURE OF SOUTH DIFFUSER
162 TM11  : TEMPERATURE AT THE INLET OF THE FAN
164 TM12  : TEMPERATURE OF AIR ENTERING THE COIL
166 GM1   : TEMPERATURE OF AIR LEAVING THE COIL
168 GM2   : TEMPERATURE OF WATER ENTERING COIL
170 GM3   : TEMPERATURE OF WATER LEAVING COIL
172 GM4   : INSIDE TEMPERATURE OF NORTH WALL
174 GM5   : INSIDE TEMPERATURE OF WEST WALL
176 GM6   : INSIDE TEMPERATURE OF SOUTH WALL
178 GM7   : INSIDE TEMPERATURE OF EAST WALL
178 GM8   : TEMPERATURE OF FLOOR
180 GM9   : AVERAGE TEMPERATURE OF ROOM
182 GM10  : TEMPERATURE AT THERMOSTAT
184 GM11  : TEMPERATURE OF AIR AT OUTLET OF FAN
186 A00   : Z-COEFF OF A/B FOR WALL
188 A01   : Z-COEFF OF A/B FOR FLOOR
190 B00   : Z-COEFF OF DENOMINATOR FOR WALL
192 B01   : Z-COEFF OF DENOMINATOR FOR FLOOR
194 C00   : Z-COEFF OF 1/B FOR WALL
196 C01   : Z-COEFF OF 1/B FOR FLOOR
198 A9    : AREAS OF WALL, FLOOR, CEILING, DOORS, WINDOWS
200 C2    : OVERALL CONDUCTUNCE OF ABOVE FIVE ITEMS
202 EXP7  : EXPERIMENTAL VALVE ACTUATOR PRESSURE IN PSI
204 EXVTR : EXPERIMENTAL NORMALIZED VALVE TRAVEL
206 EXWFLO: EXPERIMENTAL WATER FLOW IN GPM
210 ***** END *****

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**THIS BOOK
CONTAINS
NUMEROUS PAGES
WITH DIAGRAMS
THAT ARE CROOKED
COMPARED TO THE
REST OF THE
INFORMATION ON
THE PAGE.**

**THIS IS AS
RECEIVED FROM
CUSTOMER.**

Figure (1.1.1). Single Layer slab with surface film.

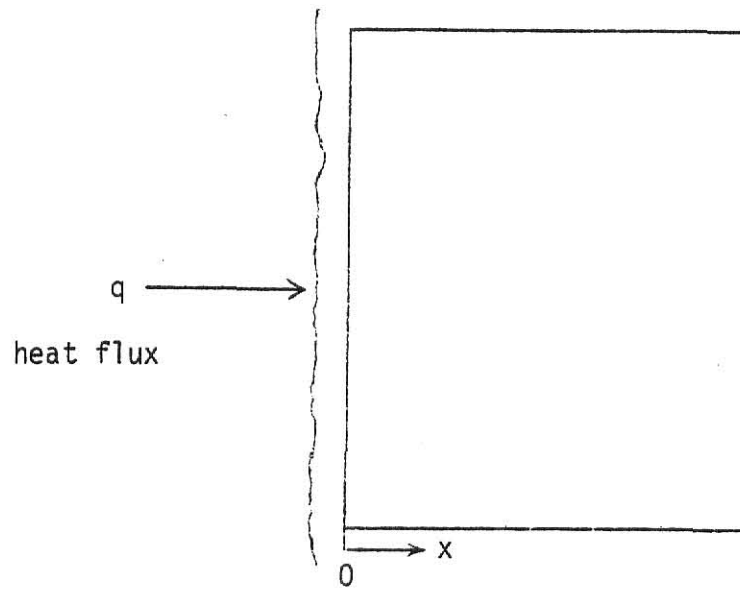


Figure (1.1.2). Multi-Layer Slab with surface films.

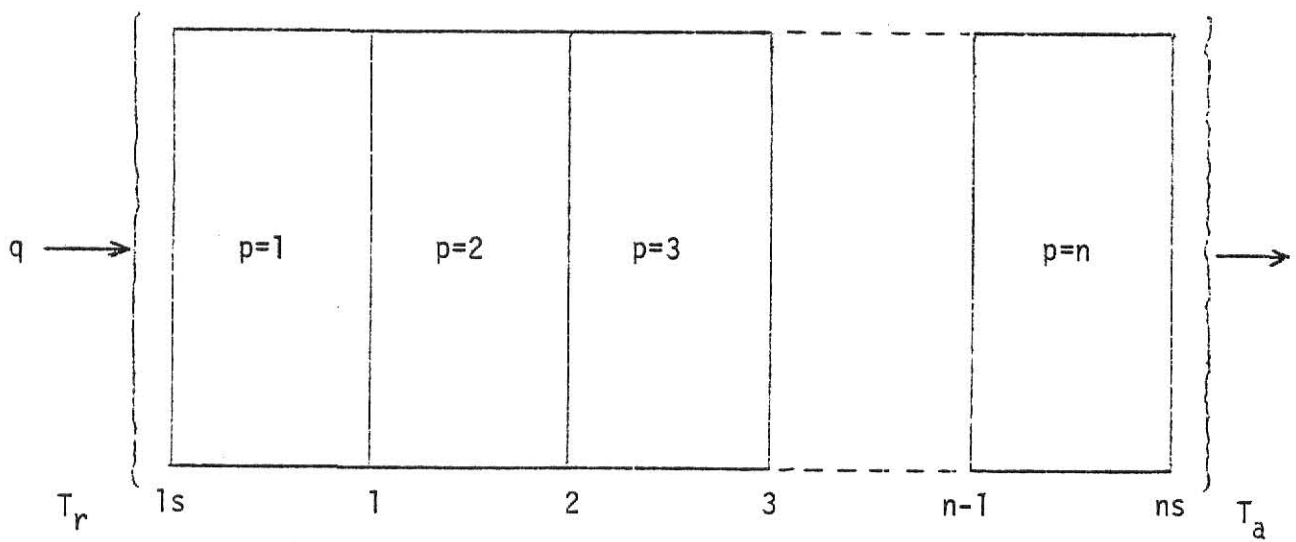


Figure 1.2.1 Rectangular pulse (step) form

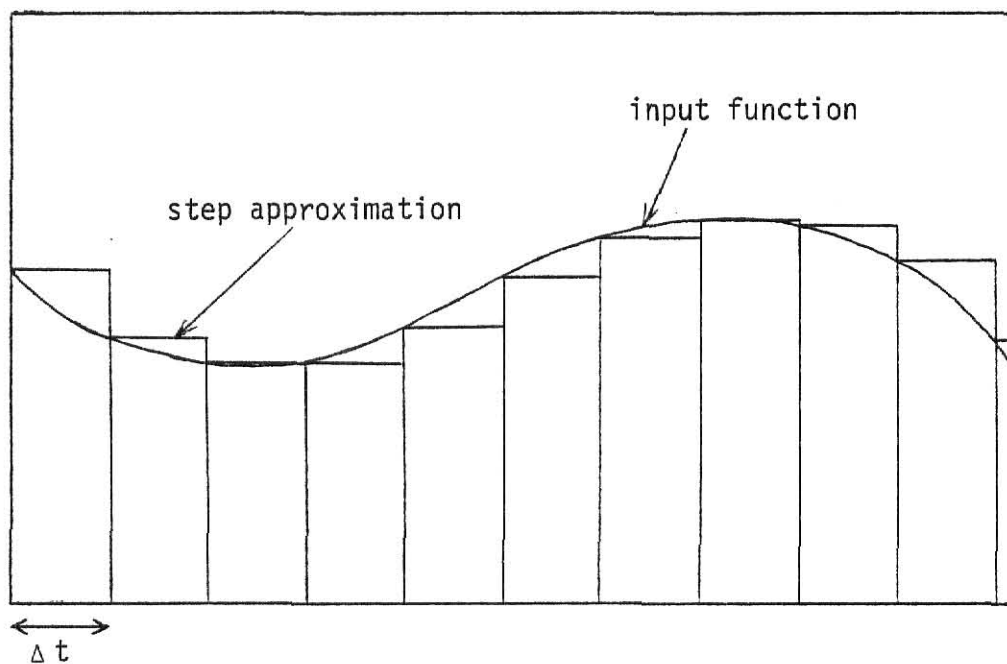
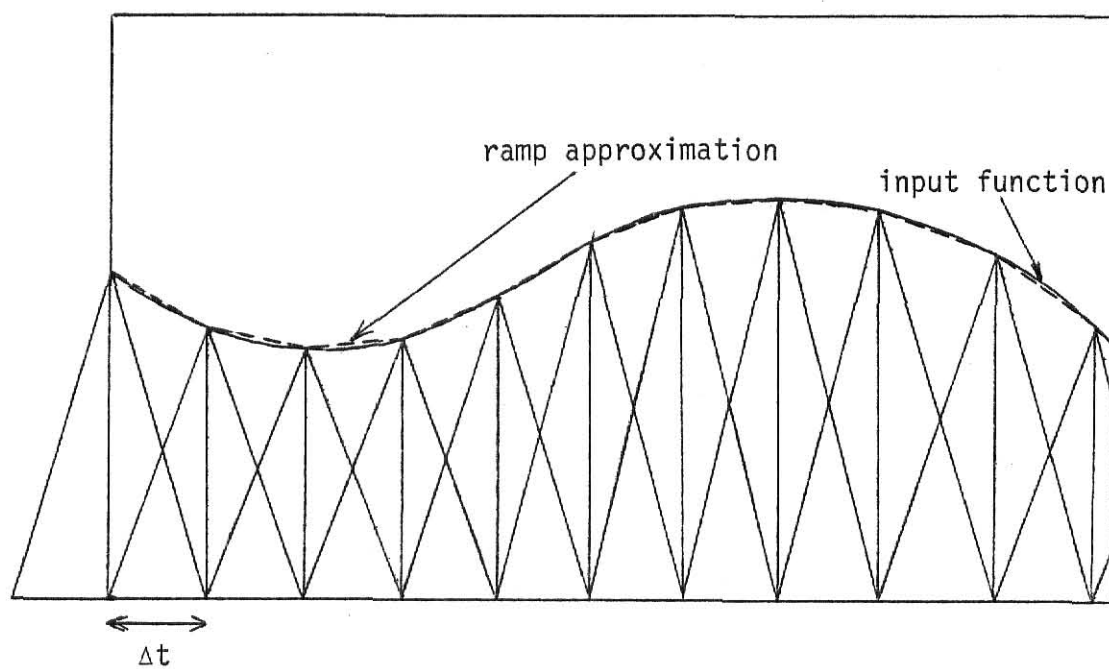


Figure 1.2.2 Triangular pulse (ramp) form



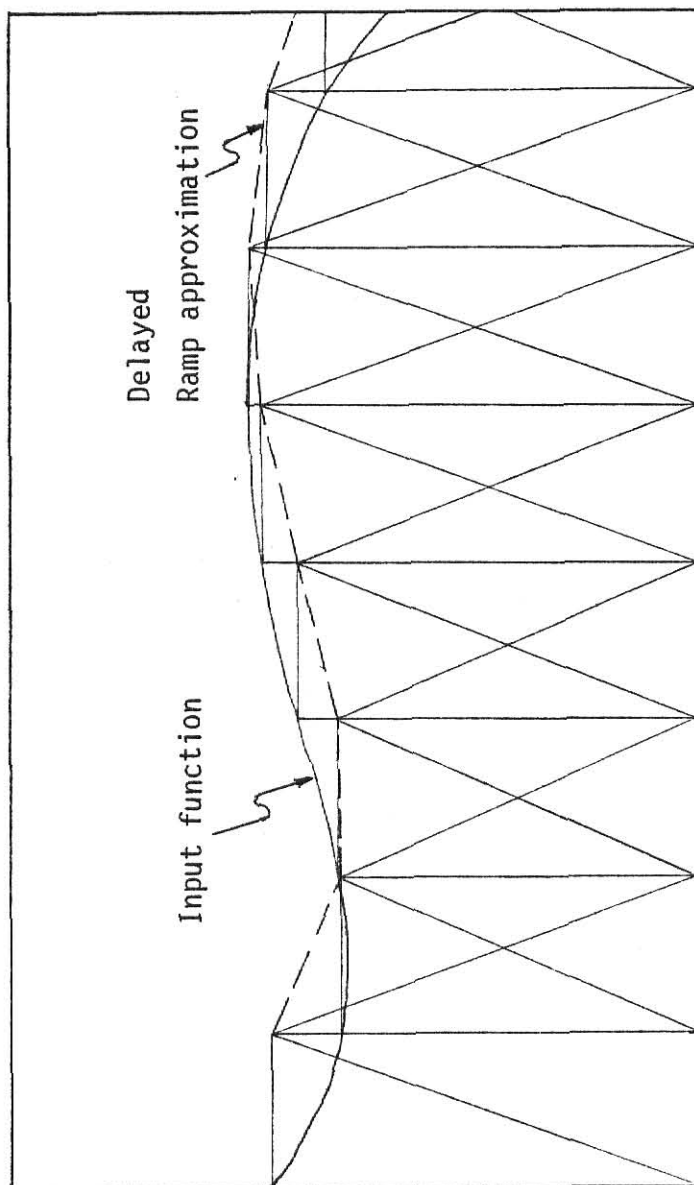


Figure I.3.1 Delayed ramp approximation

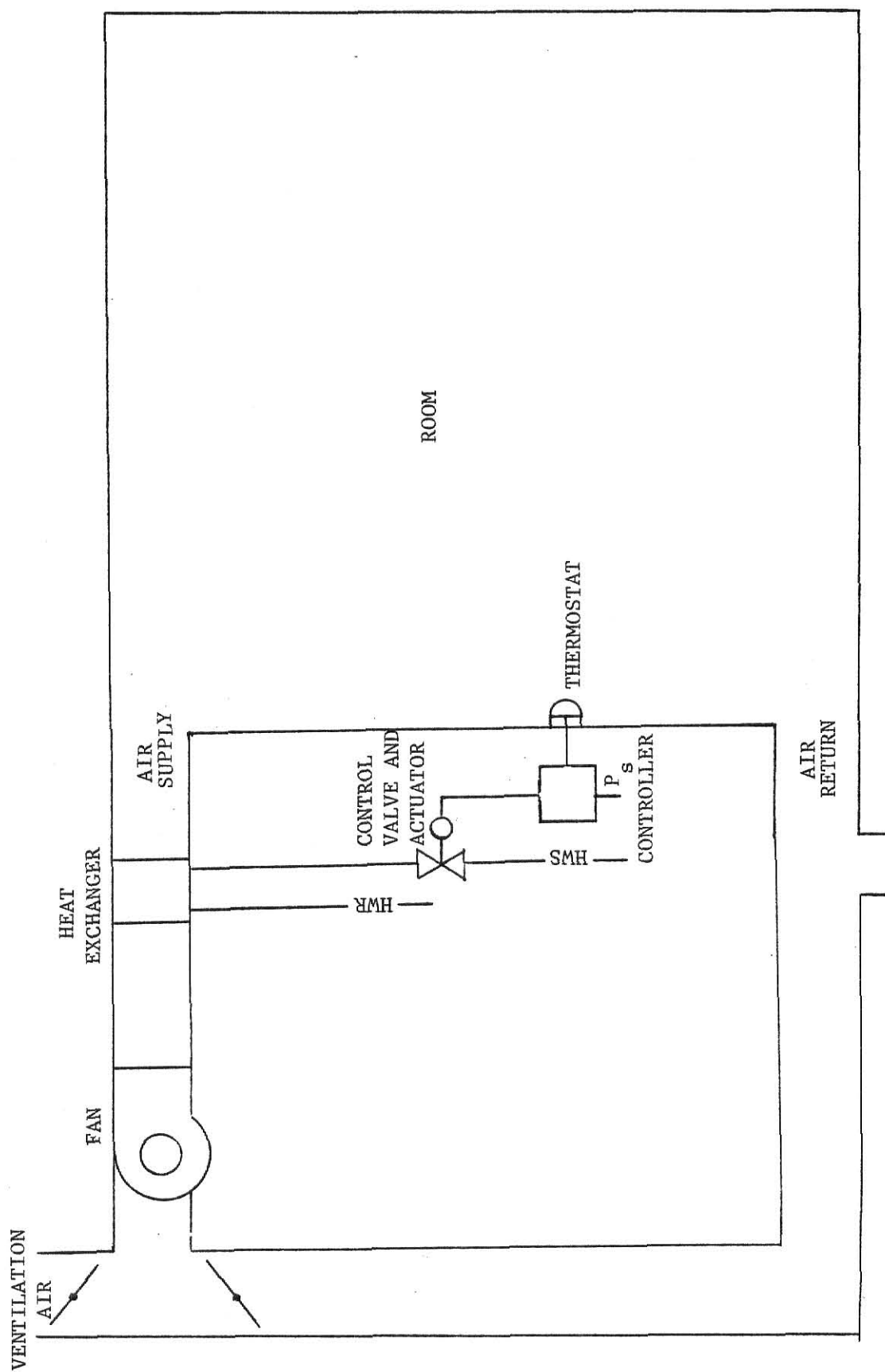


Figure (2.1.1) Schematic Representation of System Being Modeled.

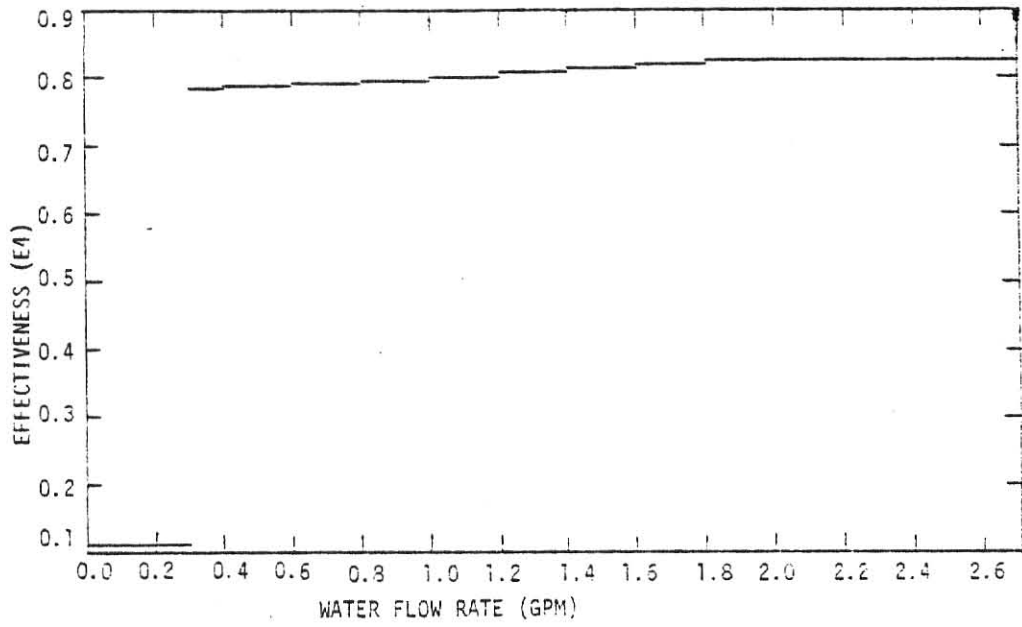


Figure (2.2.1) Coil Effectiveness - Reference(16)

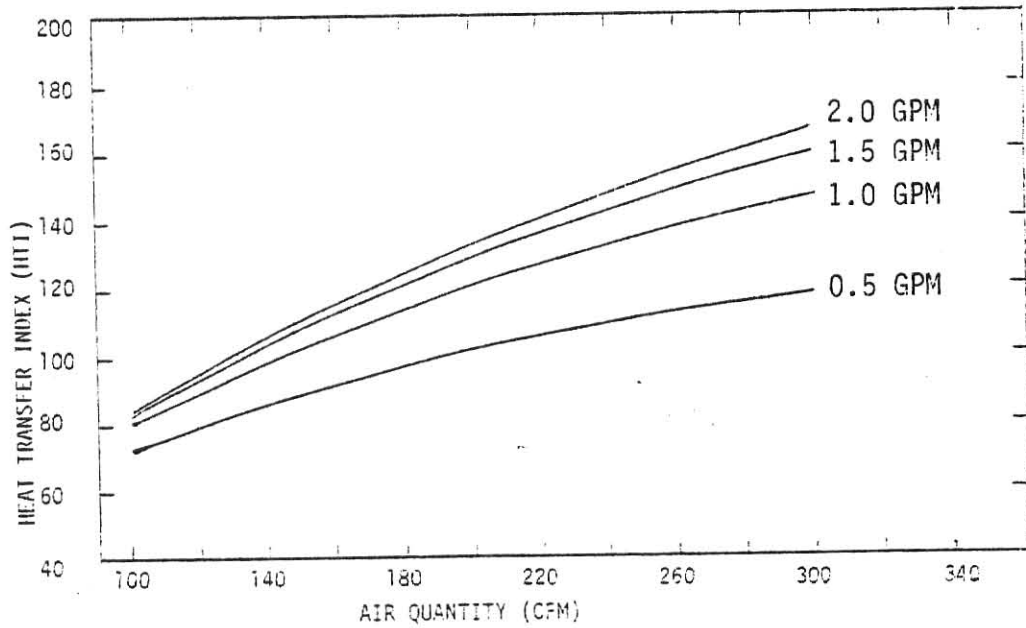


Figure (2.2.2) Coil Characteristics

Figure (2.2.3) HTI for various values of Air Quantity and Water flow rates.

(GPM) water flow	Air Quantity (cfm)			
	0	100	200	300
0.5	0	72.00	101.0	118.1
1.0	0	80.00	121.1	147.0
1.5	0	83.25	129.3	160.0
2.0	0	84.70	133.4	167.1

Figure (2.2.4) Coefficients for Equation (2.2.6)

(GPM) Water flow	a	b	c	d
0.5	0	1.0263	-35.4×10^{-4}	4.76×10^{-6}
1.0	0	1.0735	-31.3×10^{-4}	3.95×10^{-6}
1.5	0	1.0913	-29.5×10^{-4}	3.65×10^{-6}
2.0	0	1.0962	-28.5×10^{-4}	3.50×10^{-6}

Figure (2.2.5) Coefficients for equation (2.2.9)

i	b_i	c_i	d_i
1	3.8257	1.4811×10^{-2}	0.2070×10^{-4}
2	-5.1568	-2.2450×10^{-2}	-0.3246×10^{-4}
3	2.9014	1.3222×10^{-2}	0.1938×10^{-4}
4	-0.5763	-0.2685×10^{-2}	-0.0396×10^{-4}

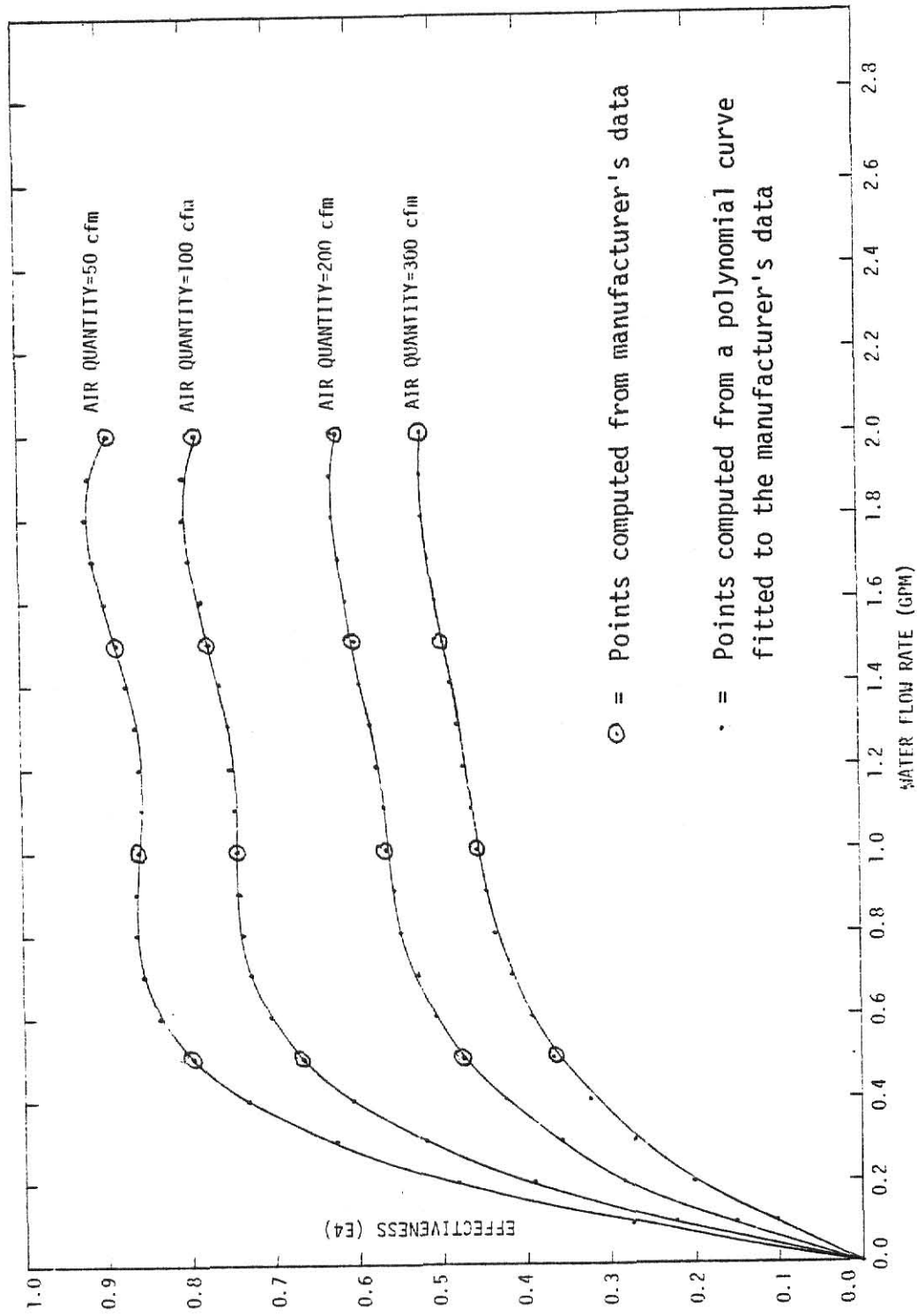


Figure (2.2.6) Effectiveness of Coil by equation (2.2.11)

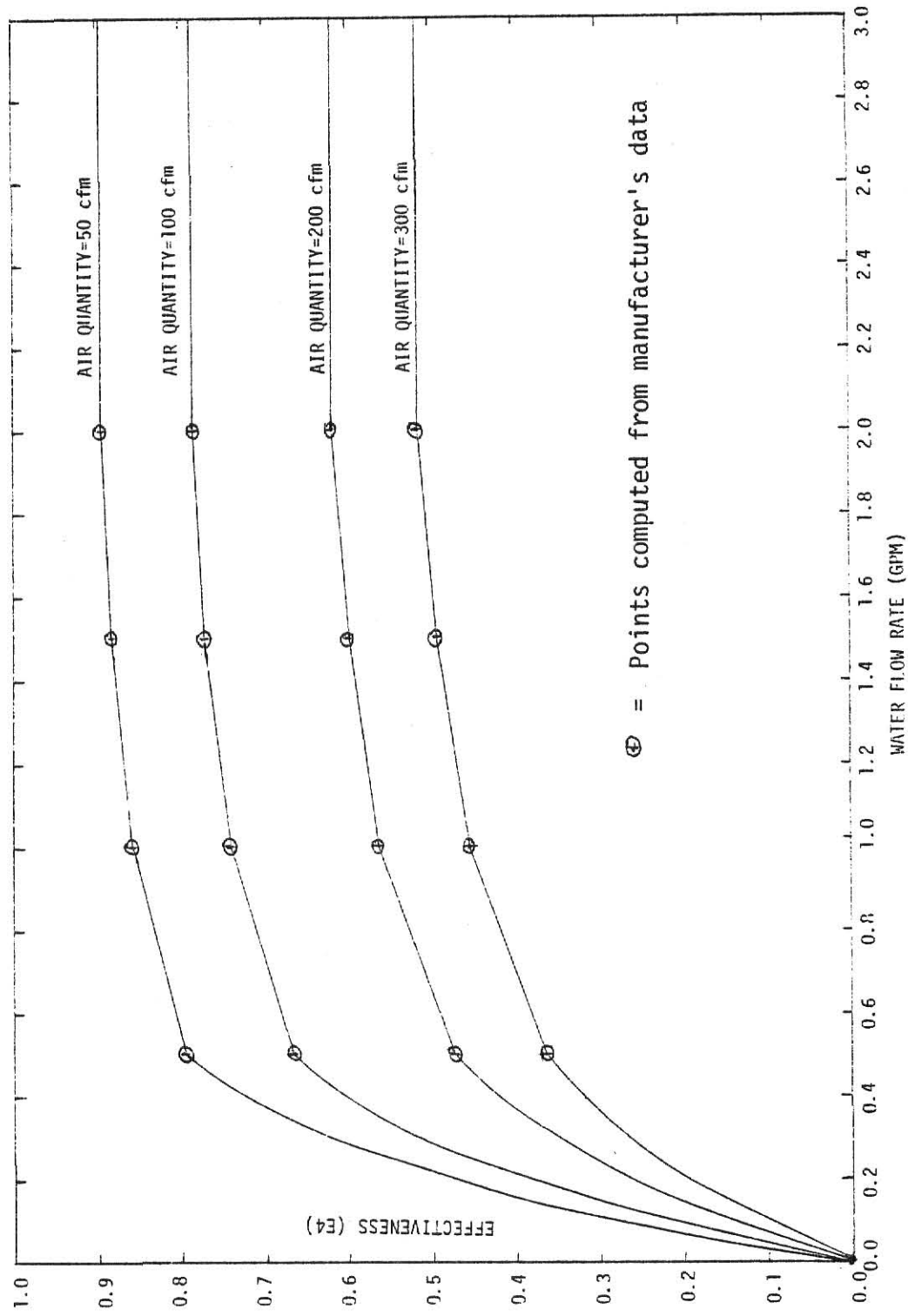


Figure (2.2.7) Coil Effectiveness as in Subroutine Coil

Figure (2.3.1) Experimental readings for investigative test of valve

	Y9	P8	P7	ΔP	P_s
1	0.0	0.0	7.0	15.88	18.13
2	2.36	0.0	6.0	15.88	18.13
3	7.87	12.20	5.0	15.50	17.88
4	21.26	29.27	4.5	14.50	17.63
5	34.65	39.02	4.0	13.38	17.50
6	49.61	48.78	3.5	12.00	17.50
7	62.20	62.20	3.0	10.13	17.25
8	76.38	80.49	2.5	6.68	17.00
9	90.55	93.90	2.0	3.73	16.93
10	100.0	97.56	1.5	2.85	16.83
11	100.0	100.0	1.0	2.50	16.80
12	100.0	100.0	0.5	2.35	16.75
13	100.0	100.0	0.0	2.30	16.75
14	100.0	100.0	0.5	2.30	16.80
15	100.0	100.0	1.0	2.23	16.83
16	100.0	100.0	1.5	2.23	16.83
17	100.0	100.0	2.0	2.28	16.83
18	87.40	90.24	2.5	4.38	16.93
19	70.87	71.95	3.0	8.10	17.15
20	53.54	53.66	3.5	11.18	17.48
21	41.73	44.88	4.0	12.45	17.53
22	22.83	30.49	4.5	14.20	17.85
23	11.02	25.37	5.0	14.70	18.00
24	5.51	0.0	6.0	15.90	18.35
25	0.0	0.0	7.0	15.95	18.45

NOTE: Bypass waterflow rate about 10.6 GPM, according to pump curves.

Max water flow rate 2.05 GPM.

Y9 : Normalized valve travel
P8 : Normalized water flow rate
P7 : Valve actuator pressure
 ΔP : Pressure drop across the valve
 P_s : Supply pressure head

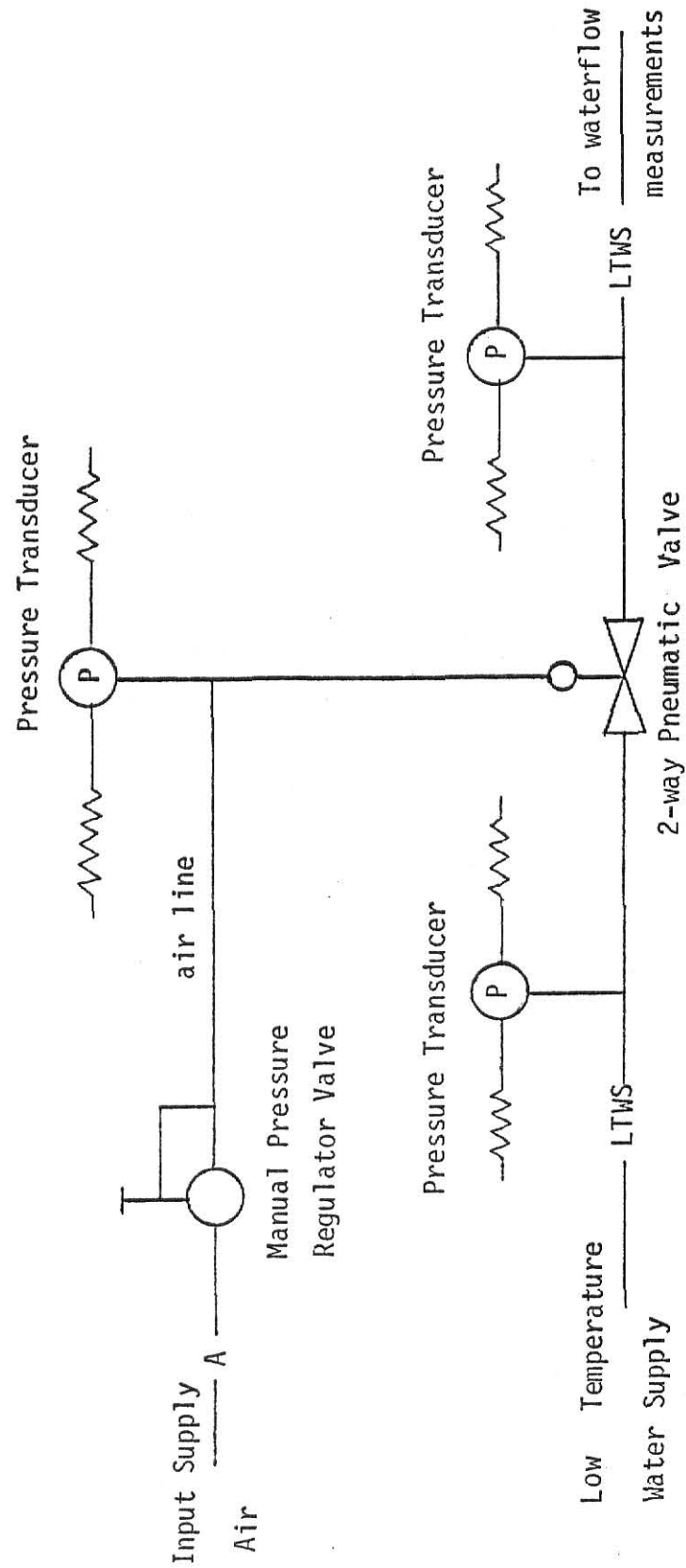


Figure (2.3.2) Schematic of Test Set Up for Valve Characteristic

Measurements were, Valve Travel, Pressure Drop Across The Valve, Water Flow Rate

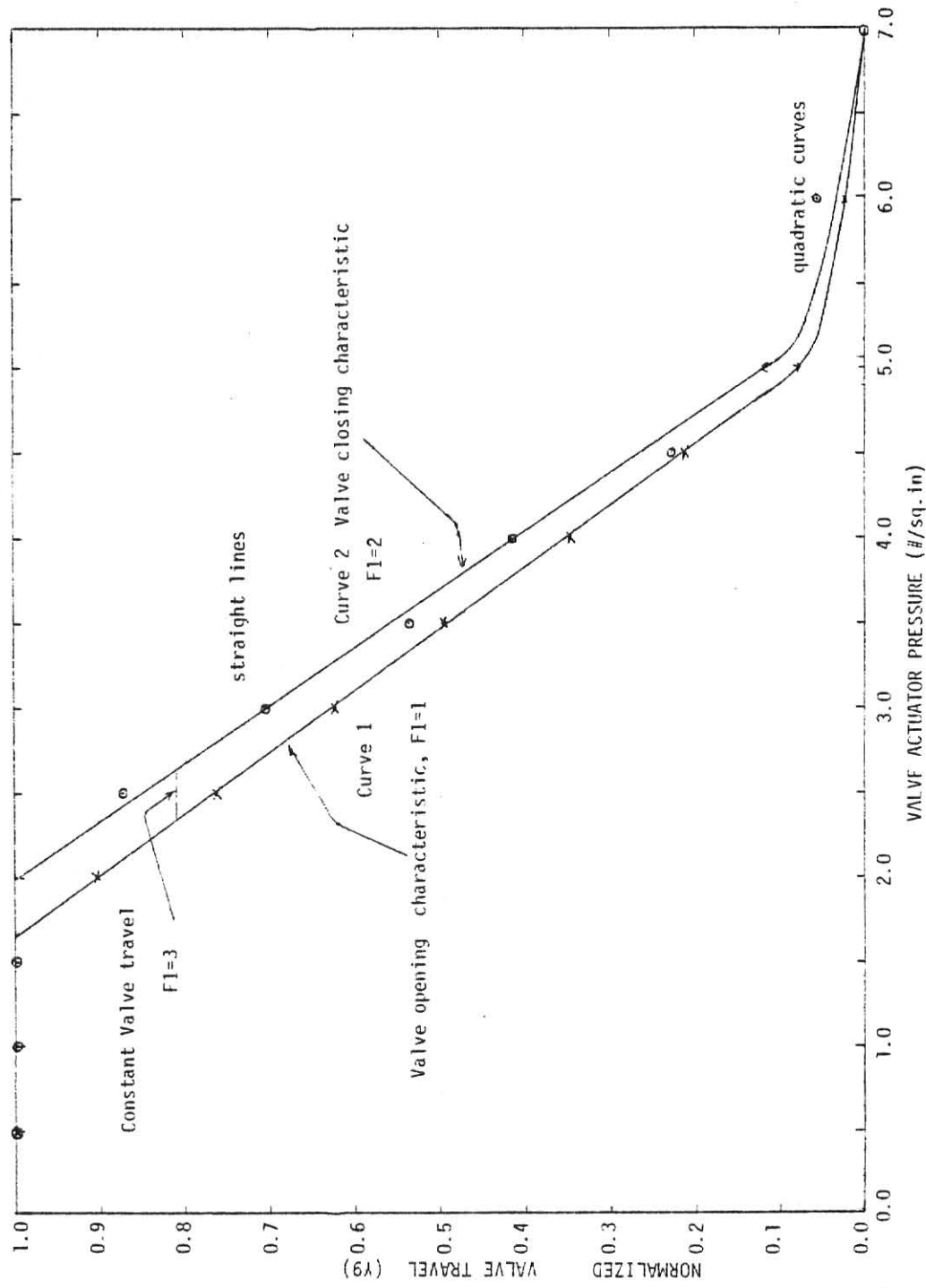


Figure (2.3.3) Valve Travel vs Actuator Pressure

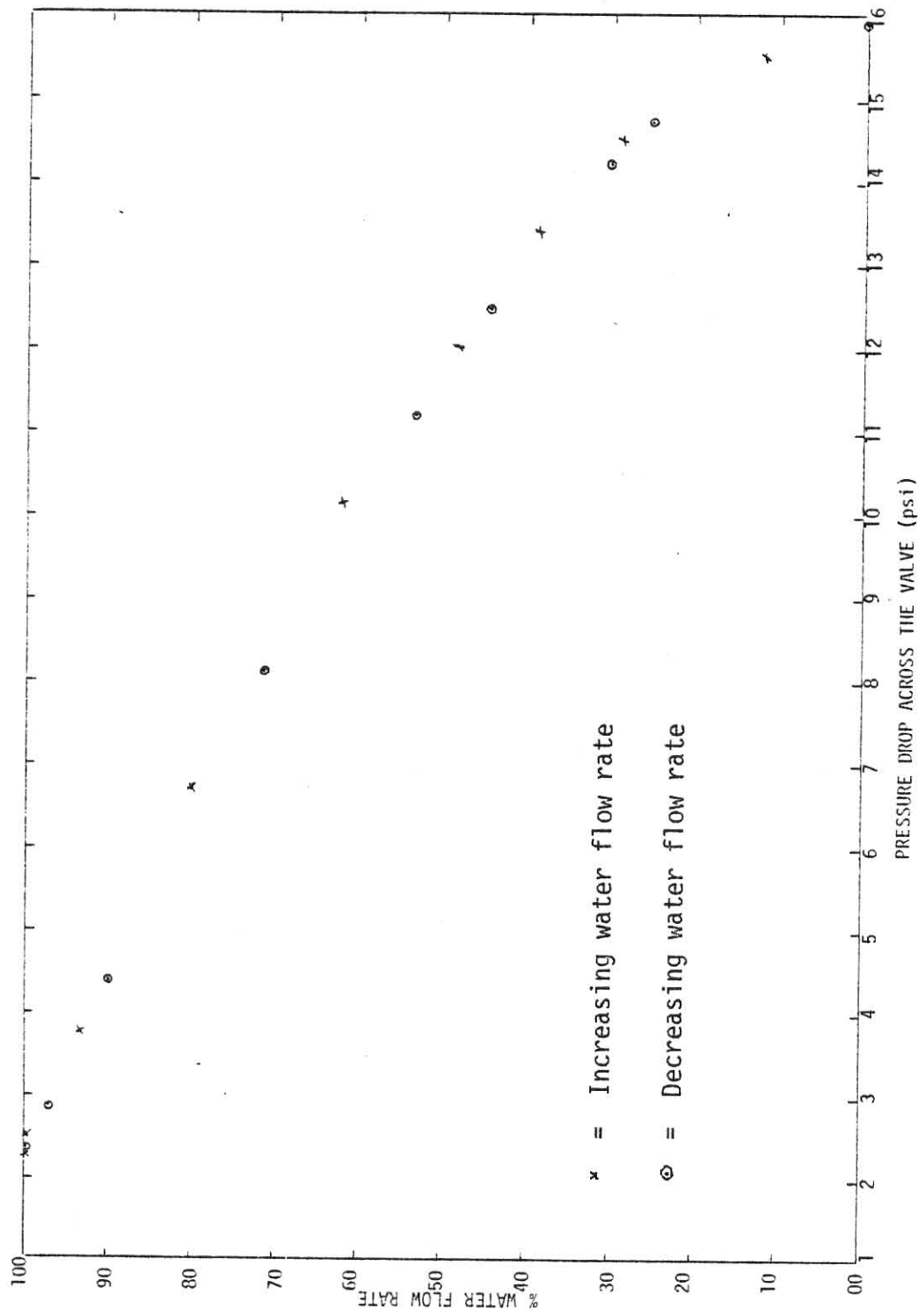


Figure (2.3.4) Water Flow Rate vs Pressure Drop Across Valve

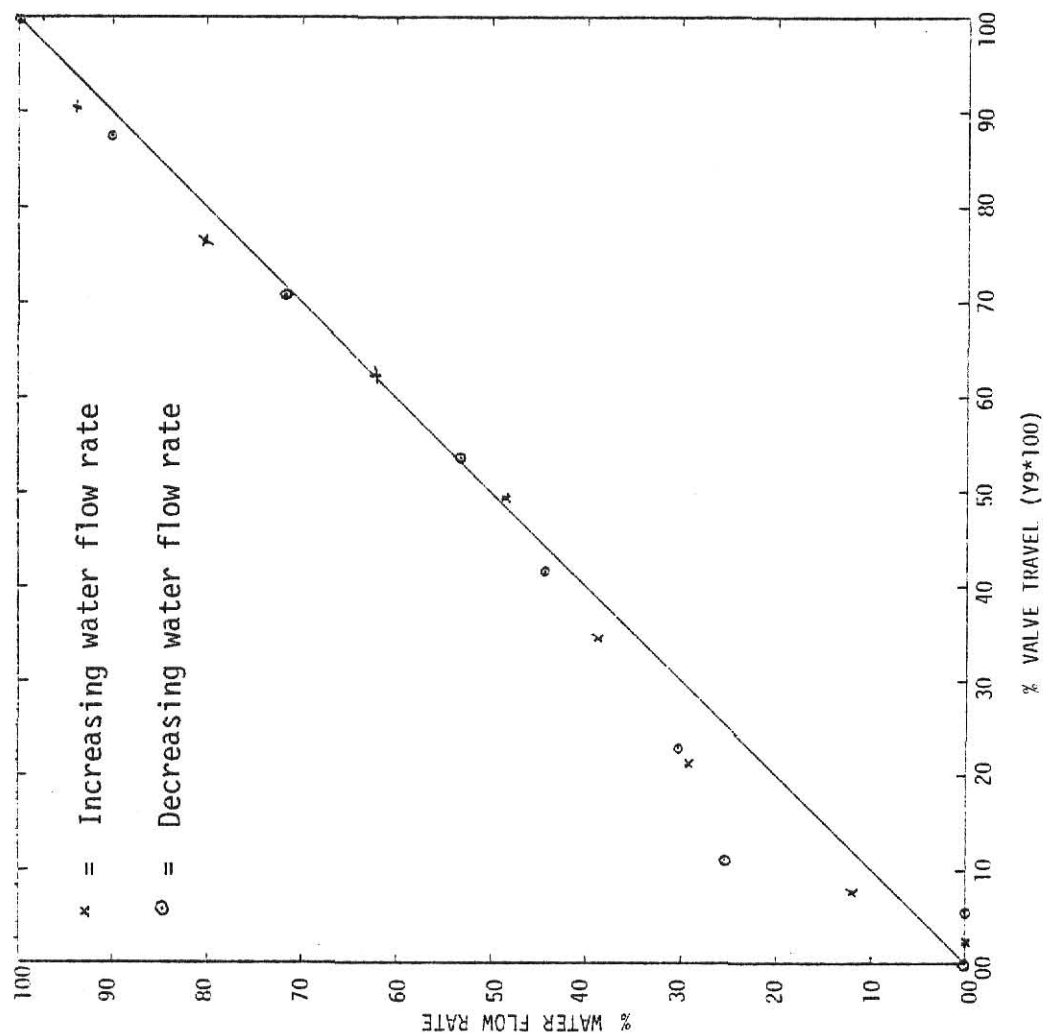


Figure (2.3.5) Water Flow Rate vs Valve Travel

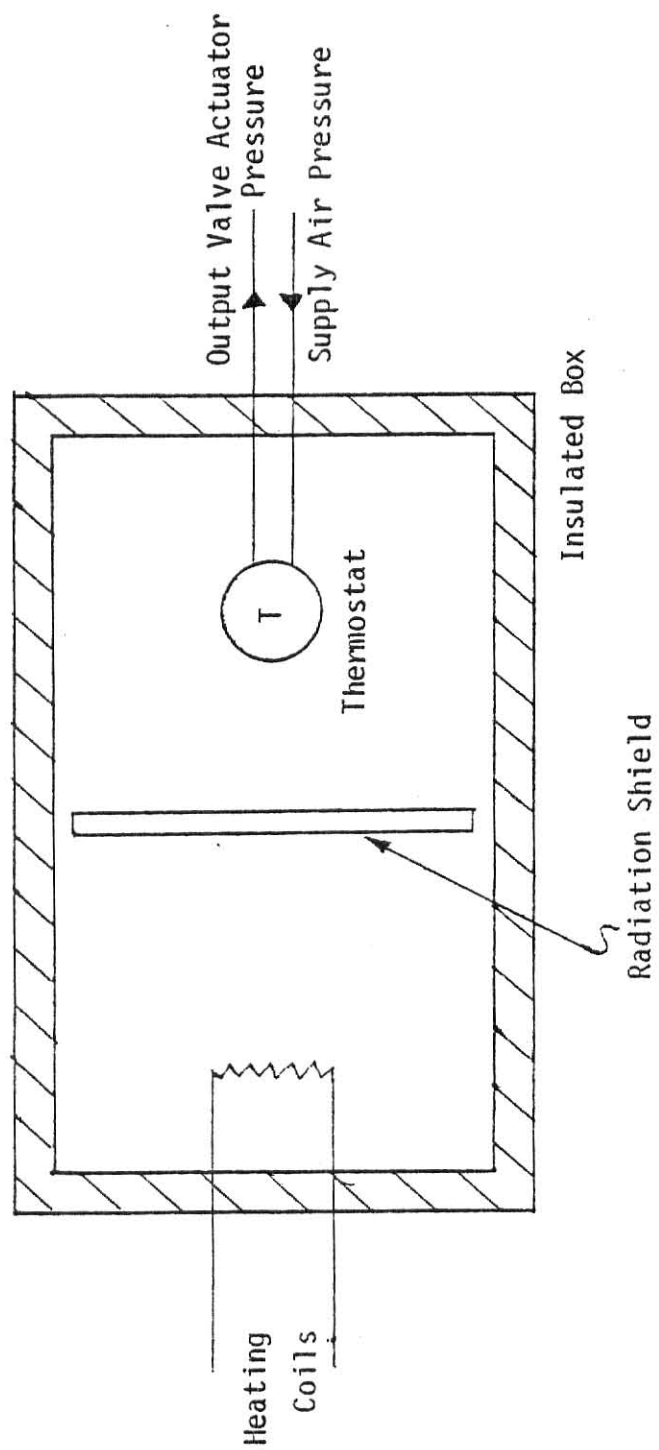


Figure (2.4.1) Schematic of Test Set Up for Thermostat Calibration

Figure (2.5.1) Return air temperature of SIM1 , SIM2 and experimental readings.

SIMULATED VALUES SIM2					SIM1		EXPERIMENTAL VALUES	
	ROOM TEMP	RETURN TEMP	ROOM TEMP	RETURN TEMP	ROOM TEMP	RETURN TEMP	ROOM TEMP	RETURN TEMP
1	78.8	77.480584	78.8	76.613627	78.8	77		
2	78.316239	77.236972	78.335617	76.734775	78.26	77		
3	77.806531	76.764003	77.817924	76.61887	77.72	76.28		
4	77.305544	76.388422	77.309798	76.393933	77.72	75.92		
5	76.819346	76.038002	76.816038	76.124349	77.54	75.74		
6	76.413453	75.709652	76.404802	75.81462	77.36	75.2		
7	76.03486	75.34179	76.016417	75.526707	77	75.2		
8	75.687363	75.027633	75.661694	75.239953	76.64	74.48		
9	75.311121	74.637116	75.270721	74.977552	76.46	74.3		
10	75.011052	74.427947	74.970258	74.670155	76.46	73.94		
11	74.733043	74.15322	74.687894	74.397162	75.56	73.78		
12	74.44479	73.825274	74.390556	74.142148	74.84	73.04		
13	74.130419	73.613448	74.074913	73.882906	75.02	73.04		
14	73.859871	73.295323	73.798022	73.599682	74.84	72.5		
15	73.580417	73.017233	73.513962	73.330891	74.66	72.32		
16	73.357749	72.781419	73.291287	73.041411	74.3	71.96		
17	73.149741	72.537118	73.082613	72.777716	74.12	71.96		
18	72.949654	72.265593	72.879563	72.525603	73.76	71.78		
19	72.733661	72.027665	72.65875	72.283763	73.58	71.6		
20	72.538182	71.798989	72.45672	72.029139	73.58	71.24		
21	72.35203	71.561509	72.259869	71.781353	73.22	71.06		
22	72.163006	71.346689	72.065653	71.537566	73.22	71.06		
23	72.016217	71.123248	71.946794	71.281457	72.68	70.52		
24	72.460753	70.87621	72.49425	70.957788	73.22	70.88		
25	73.570596	70.805391	73.733282	70.858405	74.48	70.88		
26	74.369854	71.133825	74.585457	71.265414	75.56	71.06		
27	74.97522	71.472713	75.230668	71.67038	76.82	71.24		
28	75.464722	71.802383	75.750182	72.029215	77.18	71.42		
29	75.846627	72.039247	76.135526	72.354124	77.54	71.6		
30	76.048941	72.289366	76.33284	72.636632	77.9	71.78		
31	76.114733	72.461476	76.387022	72.83259	78.08	71.78		
32	76.181022	72.540672	76.443347	72.926119	77.9	71.96		
33	76.247049	72.623127	76.504763	72.984239	78.26	71.96		
34	76.319787	72.664517	76.574701	73.024367	78.08	71.96		
35	76.377755	72.693792	76.630966	73.062444	78.26	71.96		
36	76.424275	72.721972	76.677003	73.089632	78.08	71.96		
37	76.456459	72.735483	76.708765	73.108738	78.26	71.96		
38	76.476529	72.730723	76.727999	73.118575	78.08	71.96		
39	76.46868	72.689244	76.716436	73.124001	77.72	71.78		
40	76.441222	72.689591	76.688477	73.112558	77.72	71.78		
41	76.446092	72.663163	76.694618	73.076372	78.26	71.96		
42	76.467037	72.644215	76.718679	73.048001	77.9	71.96		
43	76.45197	72.611841	76.705316	73.039964	77.9	71.6		
44	76.436987	72.586157	76.692721	73.014316	77.9	71.78		
45	76.419668	72.542505	76.676994	72.986483	77.54	71.6		
46	76.412148	72.538542	76.674585	72.952641	77.72	71.78		

47	76.41097	72.478048	76.675071	72.923639	77.54	71.6
48	76.381827	72.450186	76.64749	72.9039	77.54	71.42
49	76.322877	72.435263	76.601275	72.874291	77.54	71.6
50	76.309423	72.357618	76.59403	72.822706	77.72	71.42
51	76.289659	72.324845	76.579096	72.789033	77.54	71.42
52	76.280299	72.26926	76.572171	72.754258	77.54	71.42
53	76.254861	72.244688	76.548376	72.727064	77.54	71.24
54	76.201351	72.104246	76.485953	72.702458	77.18	71.24
55	76.14877	72.138163	76.434061	72.651297	77.36	71.06
56	76.09864	72.038129	76.378852	72.601507	77.18	71.06
57	76.06193	72.049092	76.343339	72.542336	77.36	71.06
58	76.092347	72.014468	76.375502	72.477114	77.54	71.24
59	76.096905	71.970483	76.378496	72.457137	77.36	71.24
60	76.077662	71.920562	76.354721	72.438316	77.18	70.88
61	76.03642	71.913435	76.310898	72.410999	77.18	70.88
62	76.024519	71.867746	76.295435	72.363403	77.18	71.06
63	76.014191	71.813838	76.279666	72.327298	77.18	71.06
64	75.989726	71.772356	76.249668	72.29786	77	70.88

Change in parameter	Supply air (lbs/hr)	Thermostat gain (psi/'F)	Inverse of Thermostat time constant (1/hr)	Supply water (°F)	Set Temp. (°F)	Energy (Btu/24hr)	% Change in Energy	Average room temp. (°F)	% Change in ave. ($T_r - T_a$)
Base line run	427.5	2.1	5.2446	135.0	74.5	80548.35		76.52	0
+25%	534.37					77527.23	-3.75	76.9	2.45
-25%	320.62					77175.91	-4.19	75.48	-6.70
+10%		2.31				79861.79	-0.85	76.48	-0.26
-10%		1.89				81300.11	0.93	76.57	0.32
+10%			5.76906			80506.22	-0.05	76.52	0.00
-10%			4.72014			80532.31	-0.02	76.53	0.06
+ 5%				164.75		88880.15	10.34	77.01	3.16
- 5%				105.25		54167.65	-32.75	74.95	-10.12
+ 1%					79.84	115191.26	43.01	78.57	13.21
- 1%					69.16	16986.03	-78.91	72.73	-24.42

Figure (4.1.1) Control Parameters and Their Effects

Figure (4.2.2)

Base line run

$$M(0) = 427.5$$

$$M(2) = 776.5$$

$$P6 = 2.1$$

$$P5 = 5.2446$$

$$T8 = 74.5$$

$$G2(1) = 135.0$$

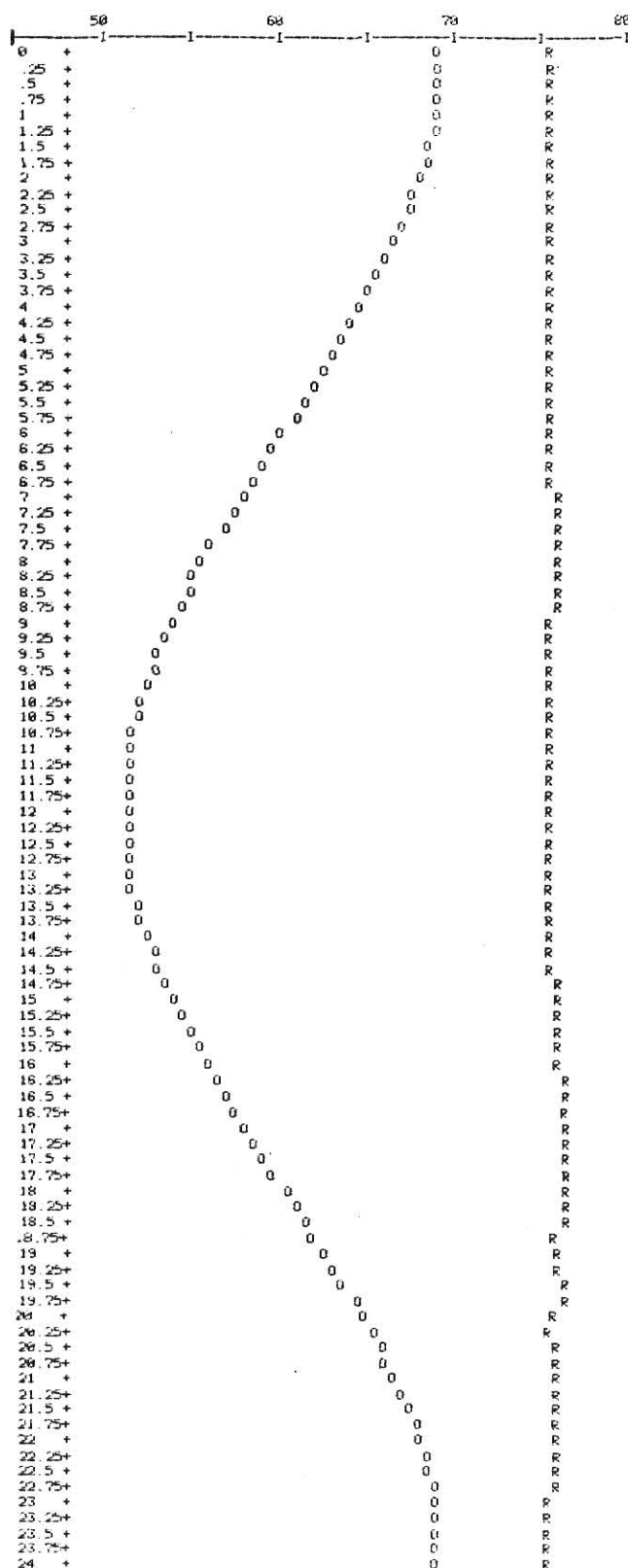


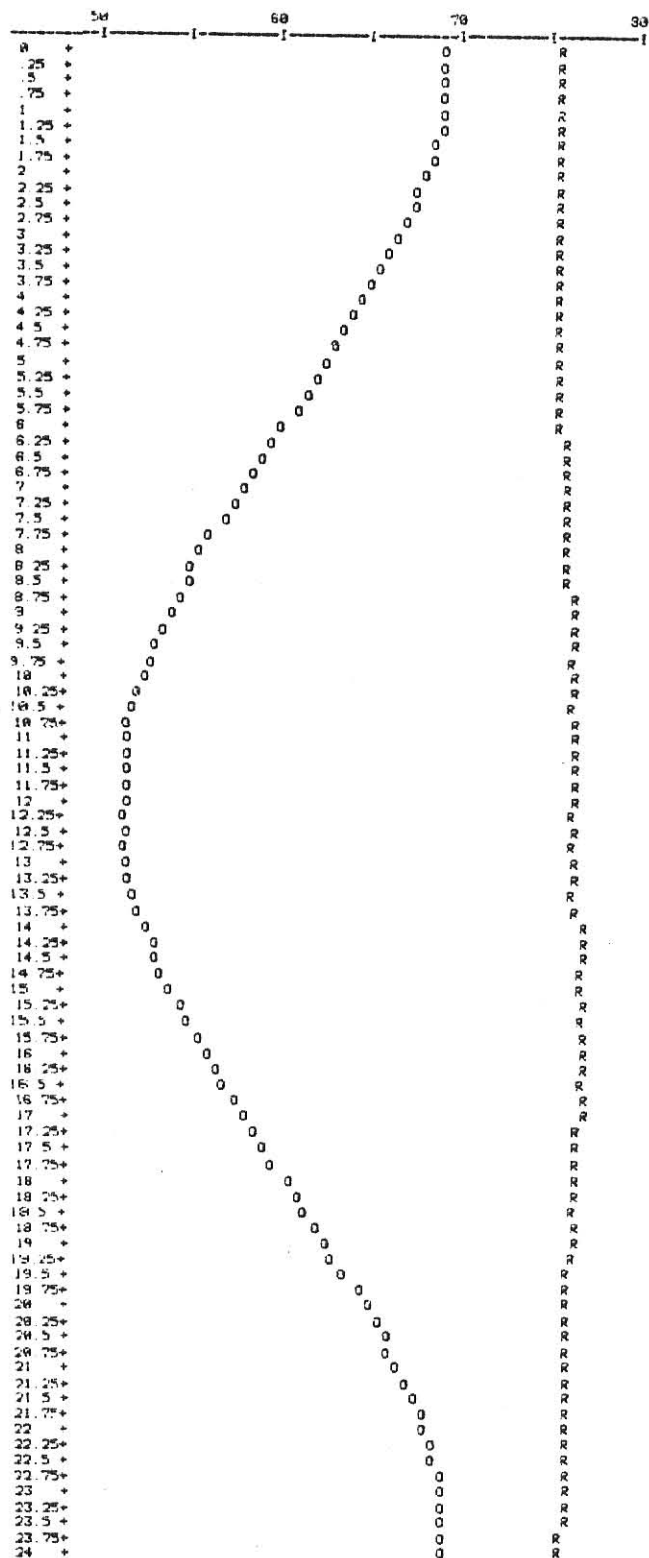
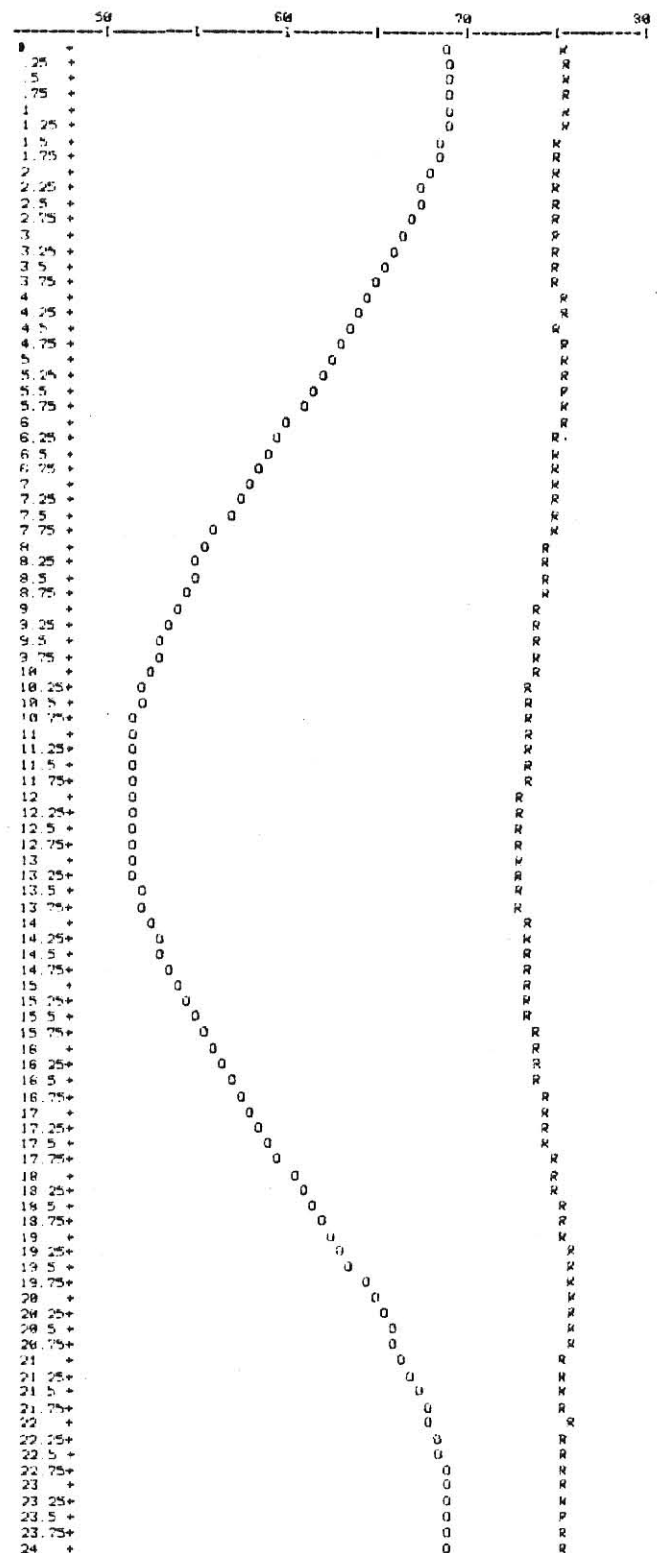
Figure (4.2.3) $M(0)=534.37$ Figure (4.2.4) $M(0)=320.62$ 

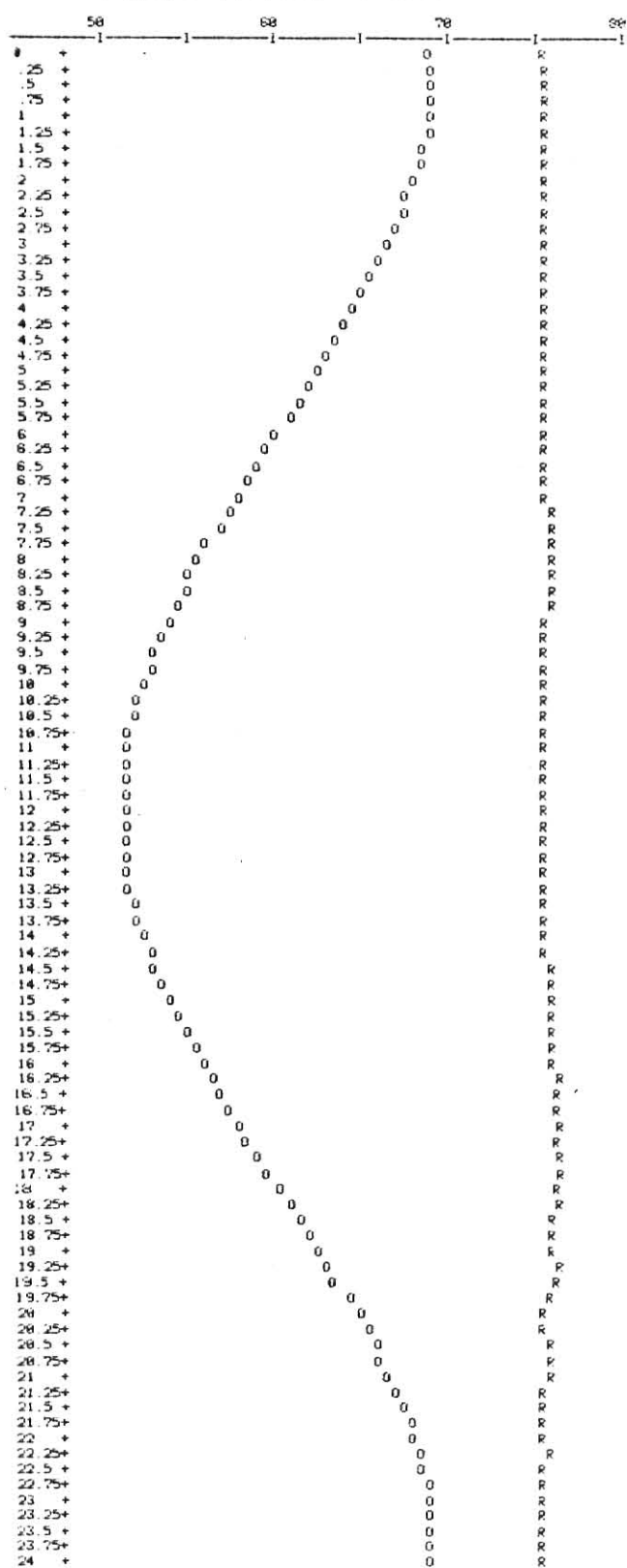
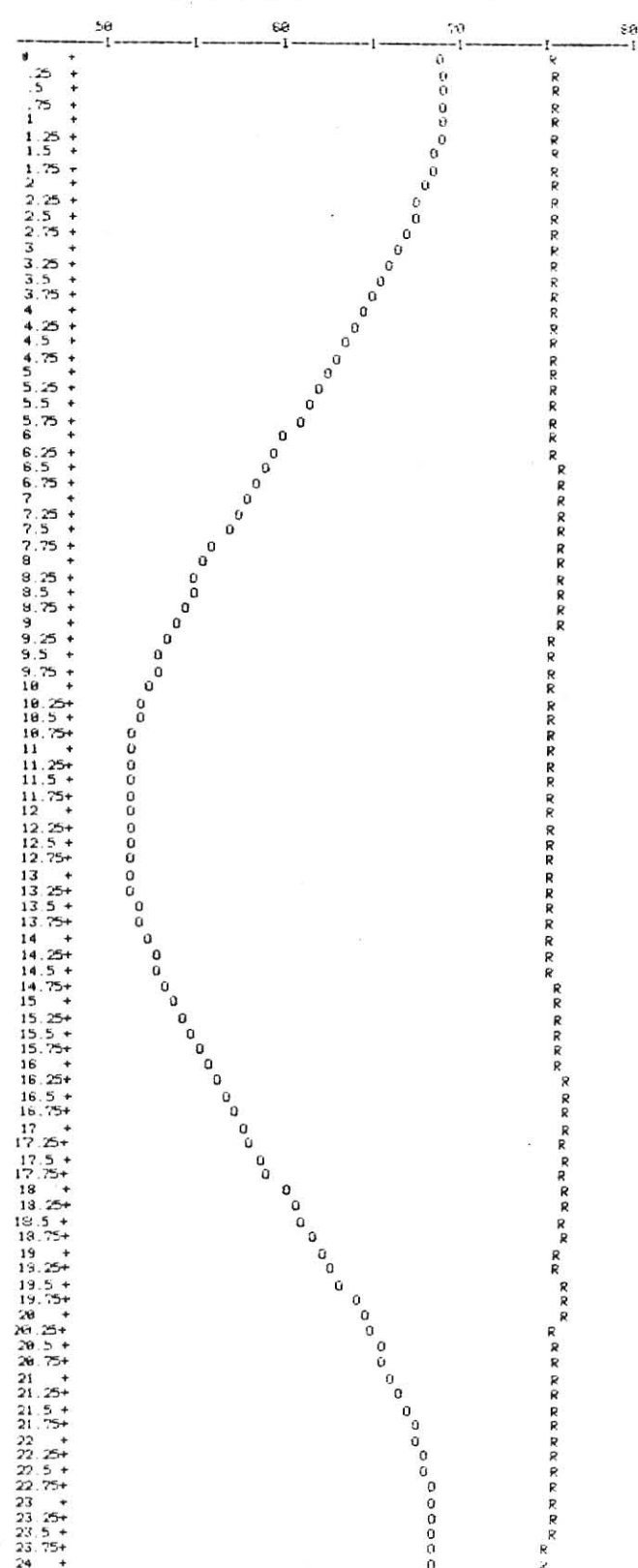
Figure (4.2.5) $P6=2.31$ Figure (4.2.6) $P6=1.89$ 

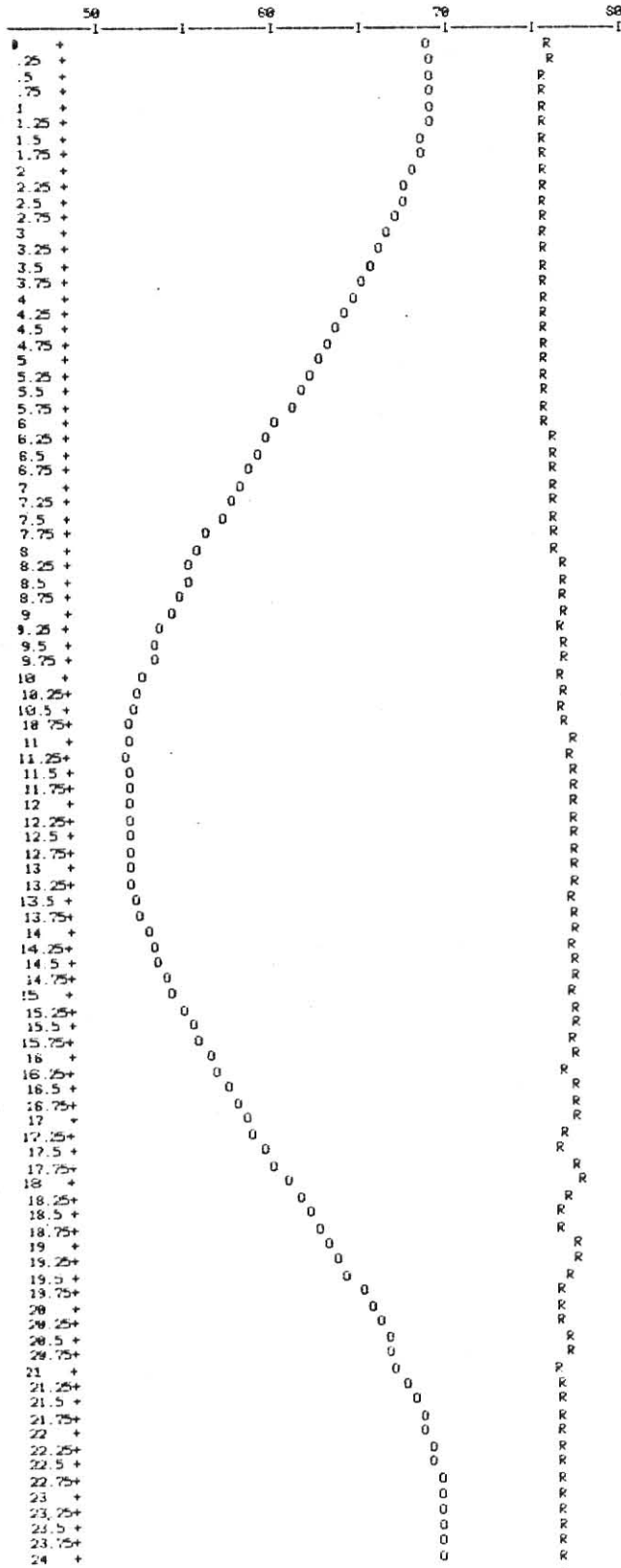
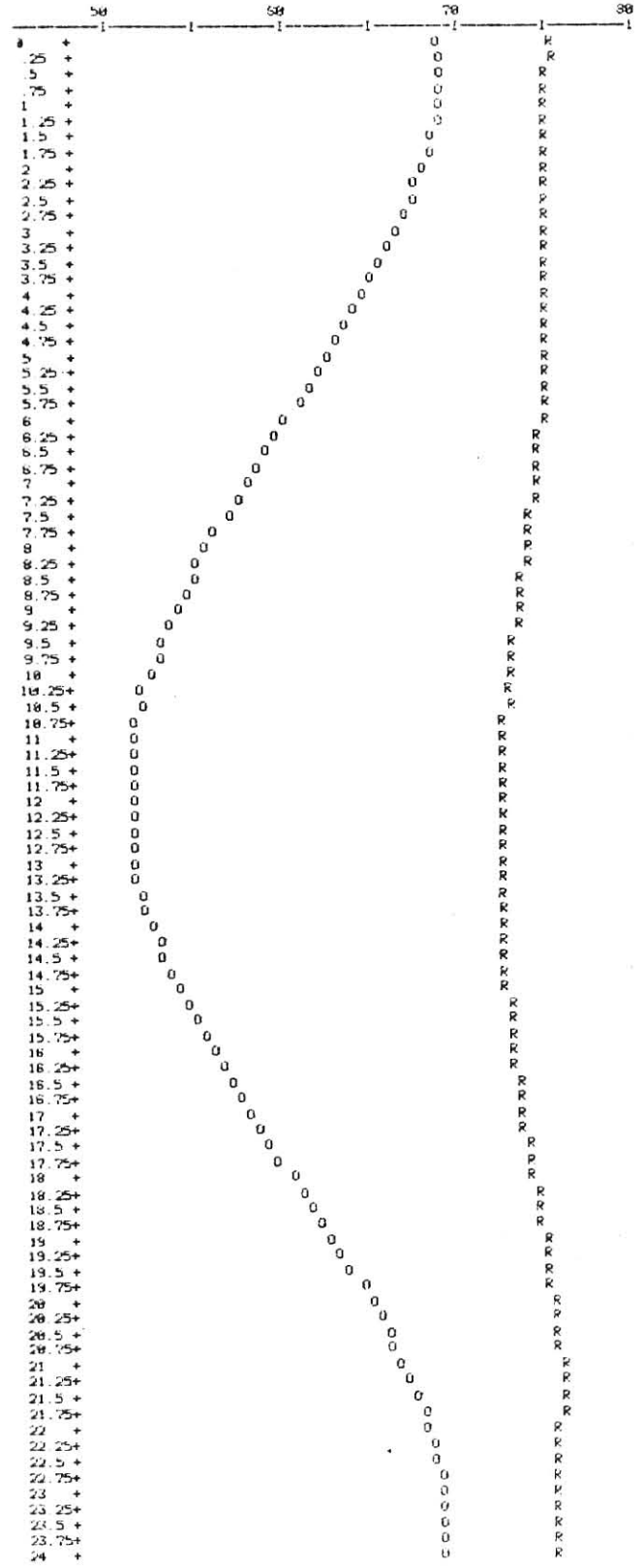
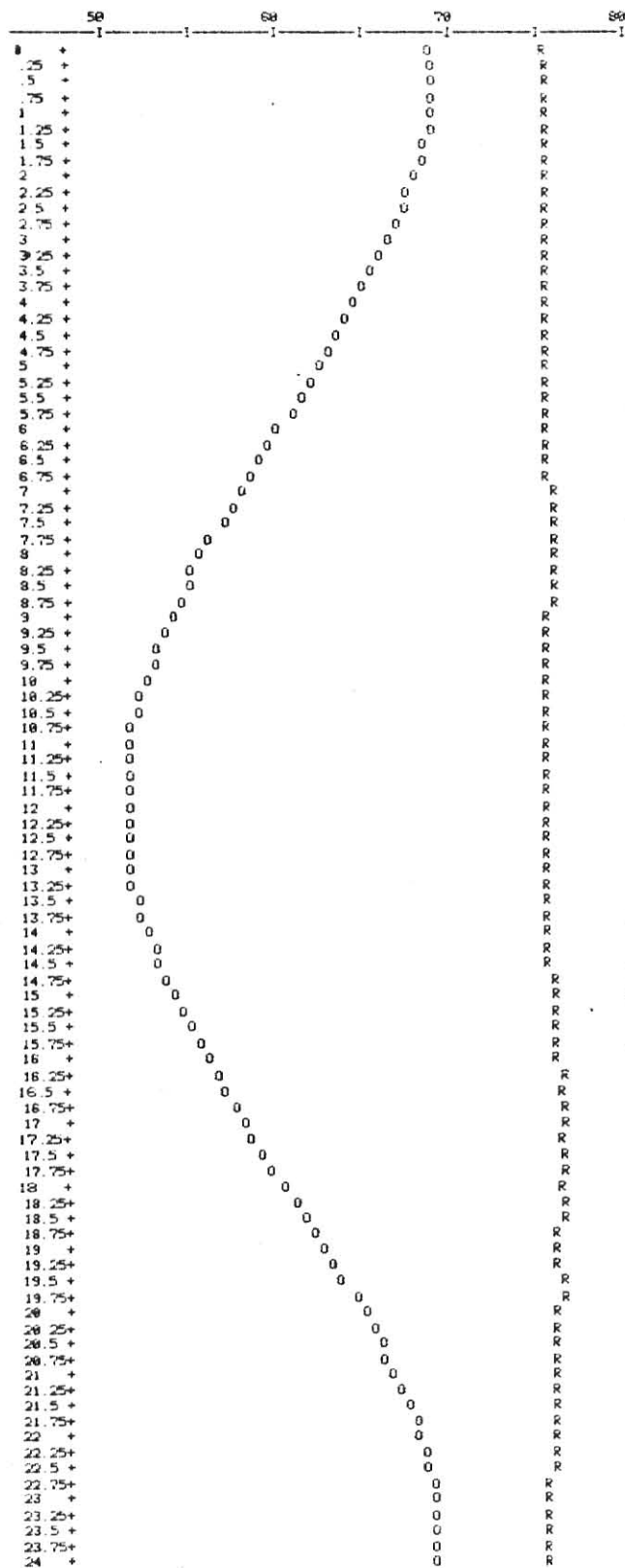
Figure (4.2.7) $G2(1)=164.75$ Figure (4.2.8) $G2(1)=105.25$ 

Figure (4.2.9) P5=5.76906



Figure(4.2.10) P5=4.72014

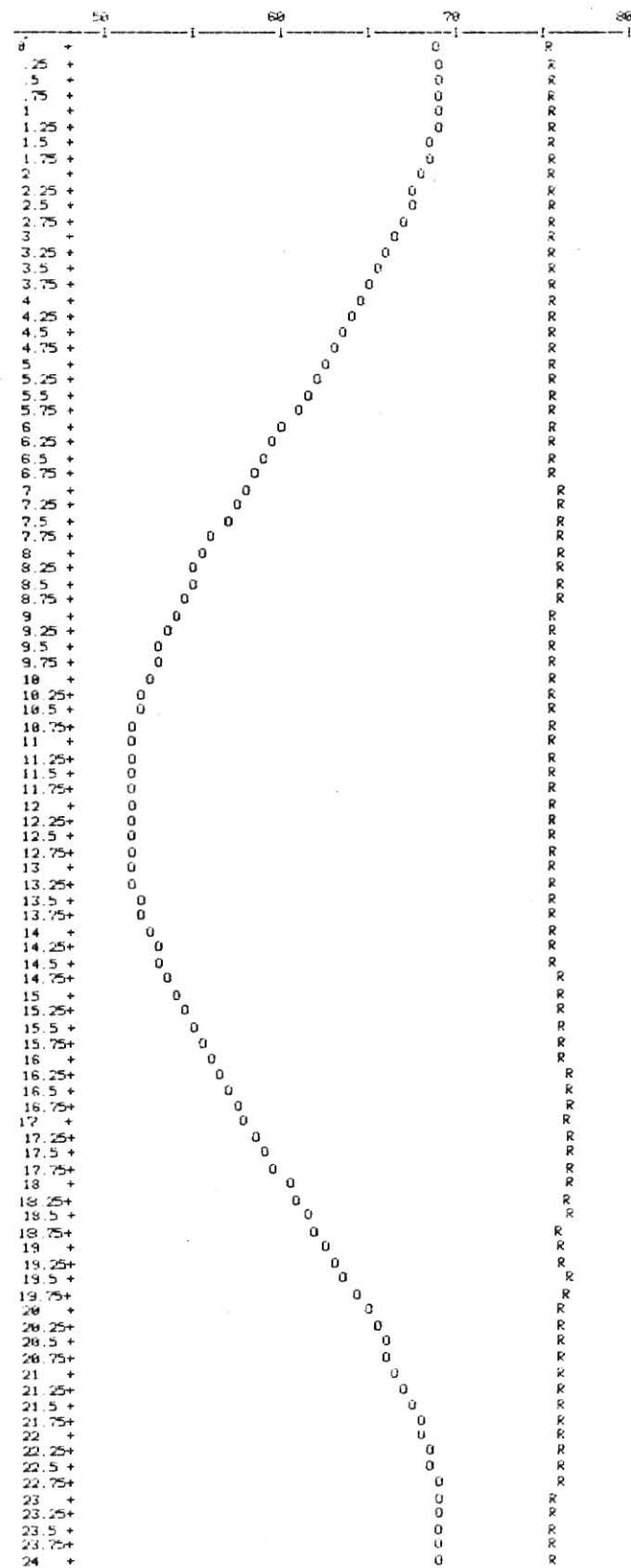


Figure (4.2.11) T8=79.84

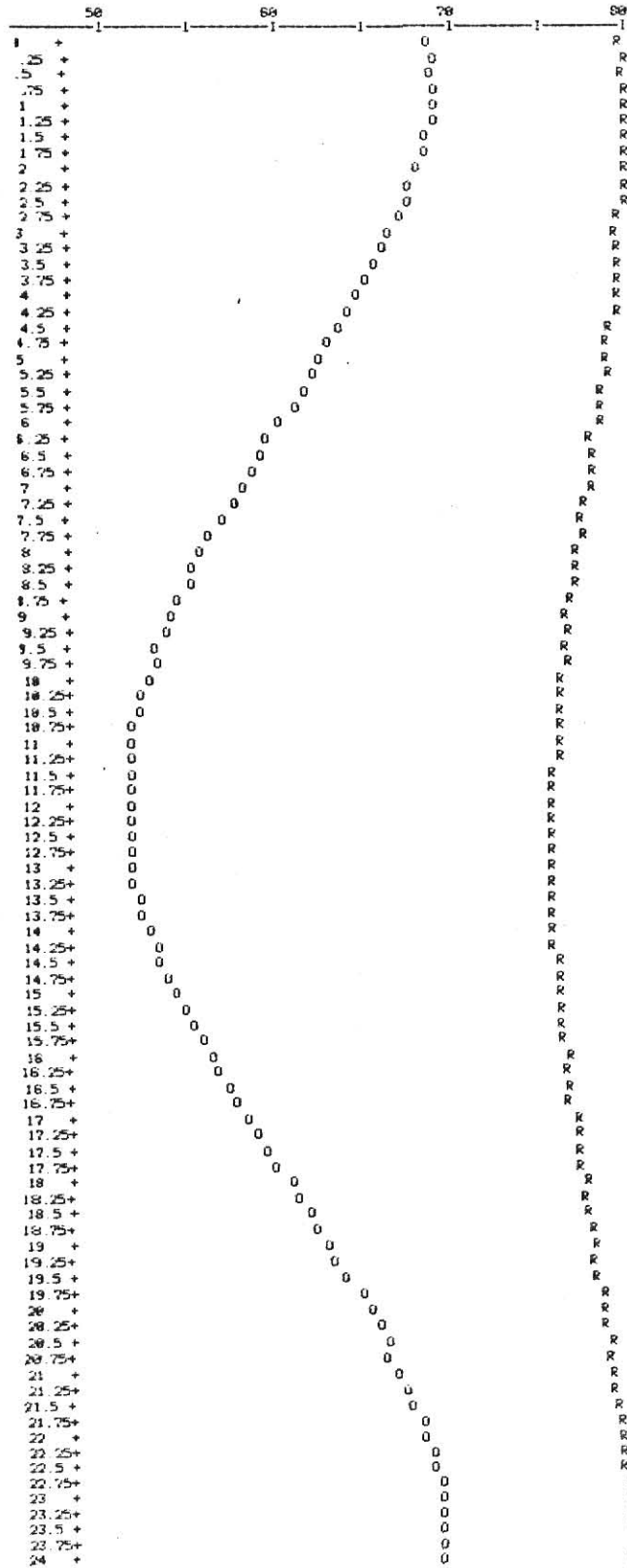
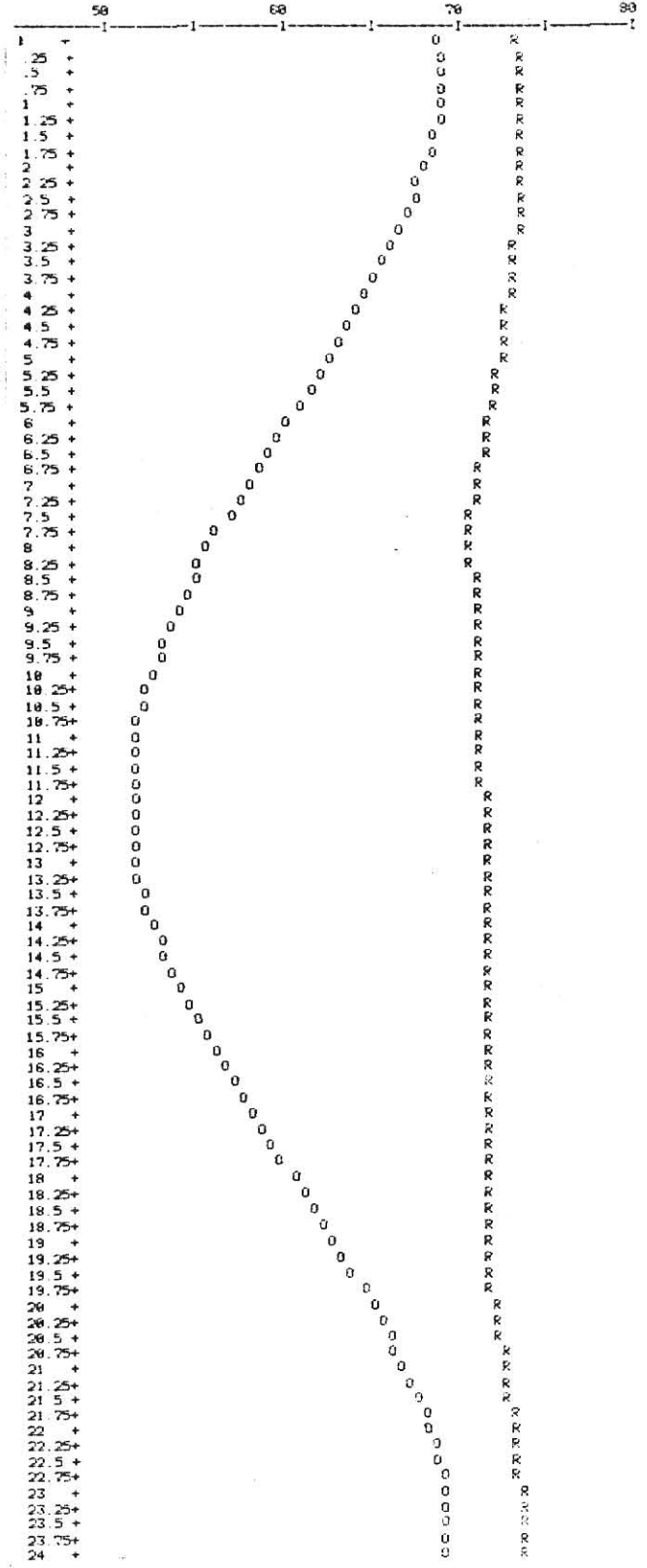


Figure (4.2.12) T8=69.16



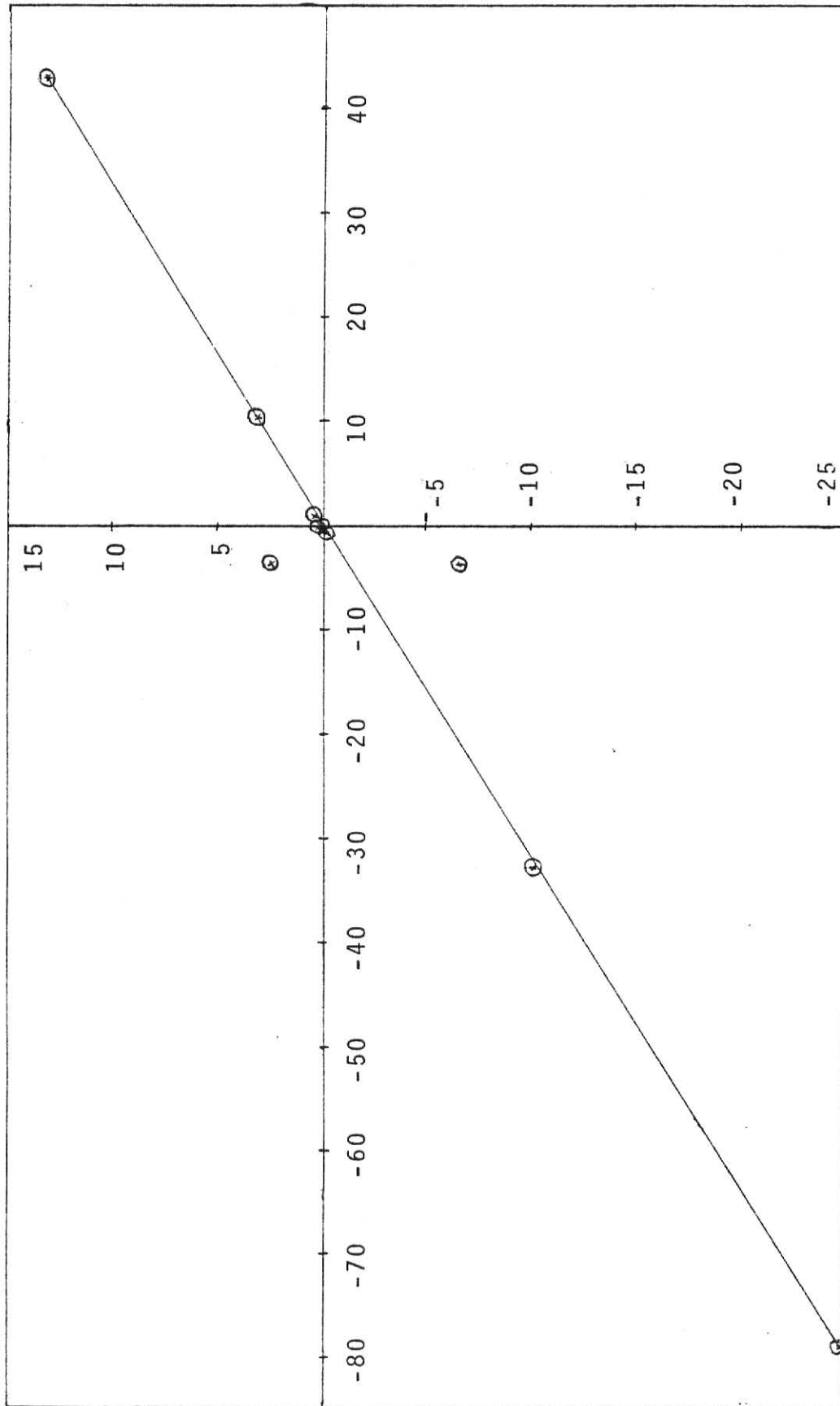
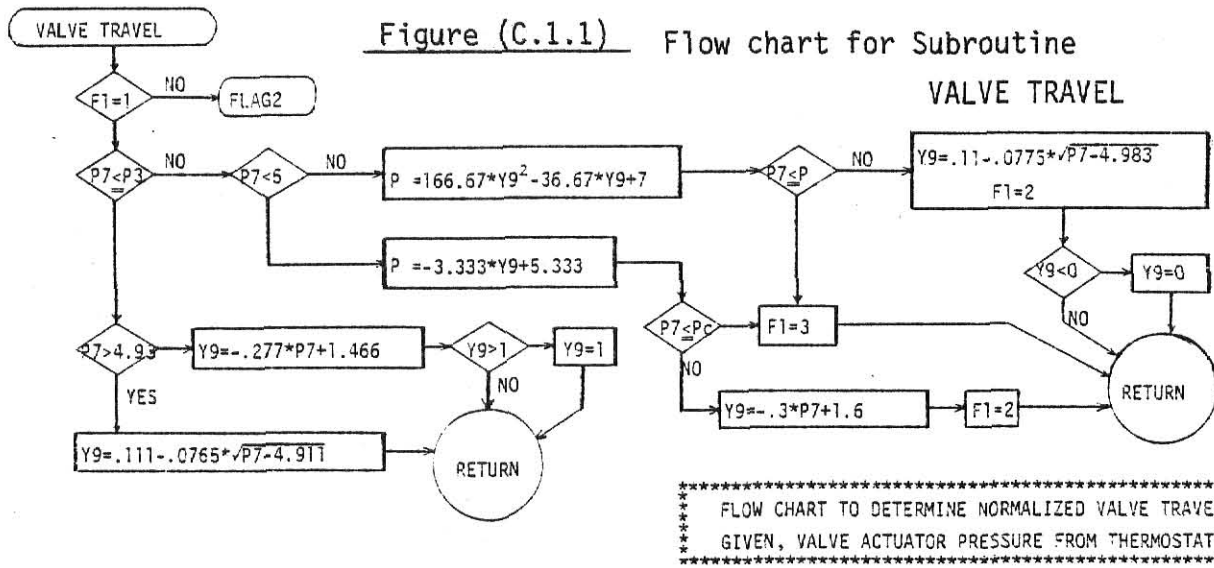


Figure (4.2.13) % Change in Difference of Average Temperatures of Room and Ambient vs. % Change in Energy Supplied to the Room



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ACKNOWLEDGEMENTS

The author wishes to extend his sincere appreciation to Dr. J. Garth Thompson for his continuous guidance and advise throughout the graduate program. Also, the author would like to thank Dr. N.Z. Azer and Dr. W.W. Koepsel for serving as committee members.

Furthermore, the author wishes to extend thanks to Dr. Paul L. Miller, Professor and Head, Department of Mechanical Engineering, Kansas State University, for extending financial support during the graduate studies.

EFFECT OF CONTROL PARAMETERS ON
ENERGY CONSUMPTION OF A ROOM HEATING SYSTEM

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1978

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1980

ABSTRACT

The study of the effect of control system parameters on the energy consumption requires a simulation with a time step short enough to manifest the dynamics of the control system. An algorithm to calculate Modified Thermal Response Factors for multi-layered walls was developed. The computer program for this algorithm has a variable sampling time interval. The algorithm was used for two different time steps to observe the change in the number of z-transfer coefficients. The computer program produced requires thermal properties of the materials of the wall, the time step size and the number of layers in the wall.

An algorithm was also developed for accurate prediction of inside and outside temperatures and heat fluxes in a room heating system simulation.

A study of the effect of control parameters on the energy supply to the room was made. Five parameters studied were mass flow rate of supply air keeping infiltration rate constant, temperature of hot water, set temperature of thermostat, thermostat gain, and time constant of thermostat. It was observed that higher average room temperature required larger energy supply. Results and graphs are produced for the room temperature for a daily cycle where the outside temperature was varied sinusoidally from 52 °F to 70 °F.