ELASTIC BUCKLING OF RIGID FRAMES

by 4589

Bong Soo Park

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Approved by:

J Eugene Thorson

Major Professor

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SYNOPSIS

The objective of this report is to get more accurate values in the stability analysis of rigid frames by a matrix formulation of the "exact" method using an electronic digital computer.

As the basic equations, slope deflection equations for an axially loaded member are derived expressing the end moments and shears as a function of its end slopes and lateral displacement considering the influence of axial force in the member. Since the stiffness of the axially loaded member depends on the axial force, the stiffness coefficient is expressed as a function of the axial force.

The well-known displacement method for the first-order frame analysis is modified to include the influence of axial forces. In the relationship for equilibrium between external and internal forces, a second-order spring matrix is added to account for the effect of axial forces on the end moments due to lateral displacement. The deformation matrix does not need to be modified and is still the transpose of the statics matrix as has been proved [1]*. In the stiffness matrix, the stiffness coefficients for every axially loaded member are replaced with the modified values due to the axial forces. The buckling criterion is set up by equating the determinant of the load-displacement matrix to zero.

After discussing the computational procedure, a computer

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* Figures in brackets denote references in bibliography

program is described. The program is written in FORTRAN IV language for the IBM 1130 computer to get the buckling loads in the fundamental mode and the effective length ratio for every axially loaded member in the frame. A simple beam and two rigid frames are used as numerical examples, showing the input and output.

To compare the design by this "exact" method with the conventional design using the AISC Specification [2], a portal frame with an infinitely stiff horizontal member is analyzed for four different special conditions and a table for K values is made in each condition. Two typical numerical examples are designed by the two methods using the K factors offered by the existing AISC Specification [2] and the values in the tables provided by the "exact" method.

INTRODUCTION

1. Introduction

The American Institute of Steel Construction (AISC) Specification [2] accounts for the rather complex buckling problem of a column which is part of a rigid frame configuration by recommending certain K values which modify the actual unbraced length to an effective length. These values are derived from idealized theoretical considerations which assume all columns in the frame to be of identical stiffness and subjected to identical loading. In practice, however, frames are quite often encountered in the conditions in which either columns with unequal stiffness are subjected to equal loads or columns with identical stiffness are subjected to unequal loads.

In these circumstances the design will generally be based on the most unfavorable combination. In the first case the column with the least stiffness will be designed for the given load without utilizing the reserve strength of the stiffer column. In the second case it is usual to design the heavier loaded columns without regard to the inherent reserve strength furnished by the column with the smaller axial load.

An example of the first case is the practice of alternating the orientation of columns in steel buildings in earthquake regions to provide equal lateral stiffnesses in both directions. The second case is best examplified by a frame which supports a crane. Each column of such a frame has to be designed for the most unfavorable crane position which cannot occur simultaneously for both columns. Consequently one column has an unused reserve strength which cannot be utilized in the buckling analysis of the other column when using the K factors offered by the existing AISC Specification [2].

Many structural engineering problem which are theoretically solvable have not been feasible to solve until the advent of the electronic digital computer. Today, as the access to a computer is increasing rapidly, one may be able to use the "exact" method presented here to get more accurate values in the stability analysis of rigid frames.

2. Objective of Report

The objective of this report is to determine the elastic buckling loads of a rigid frame and the effective lengths for every axially loaded member in the frame by a matrix formulation of the "exact" method using a digital computer.

DERIVATION OF SLOPE-DEFLECTION EQUATIONS [3]

1. Moments due to End Rotation

Consider an axially loaded beam with a couple applied at one end as in Fig. 1.

The bending moment in the beam is

$$M = \frac{M_b}{L} x + Py \tag{1}$$

By using the expression for the curvature of the axis of the beam, the differential equation is

$$EI \frac{d^2y}{dx^2} = - M = - \left(\frac{M_b}{L} x + Py \right) \tag{2}$$

For simplification the following notation is introduced:

$$k^2 = \frac{P}{EI} \tag{3}$$

and then Eq. (2) becomes

$$\frac{d^2y}{dx^2} + k^2 y = - \frac{M_b}{EIL} x$$

The general solution of this equation is

$$y = A\cos kx + B\sin kx - \frac{M_b}{PL} x \tag{4}$$

The constants of integration A and B are determined from

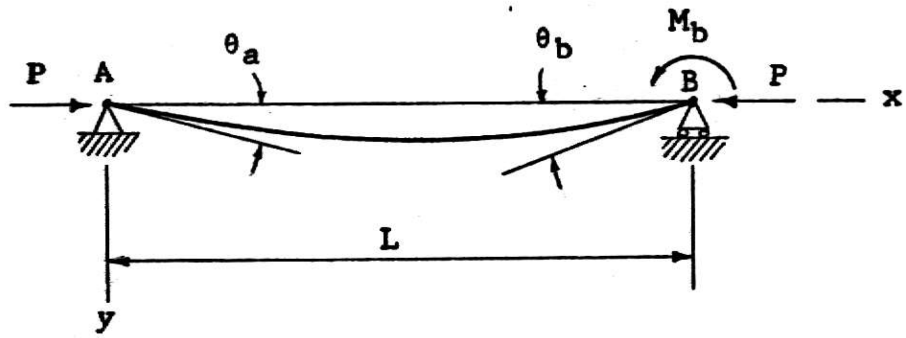

Fig. 1 Axially loaded beam with a couple at one end

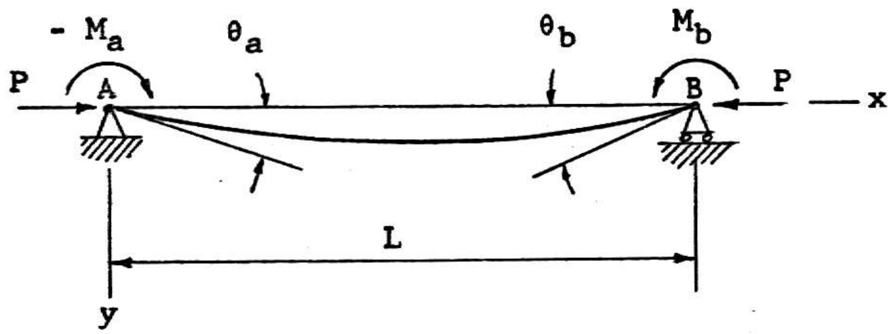

Fig. 2 Axially loaded beam with couples at both ends

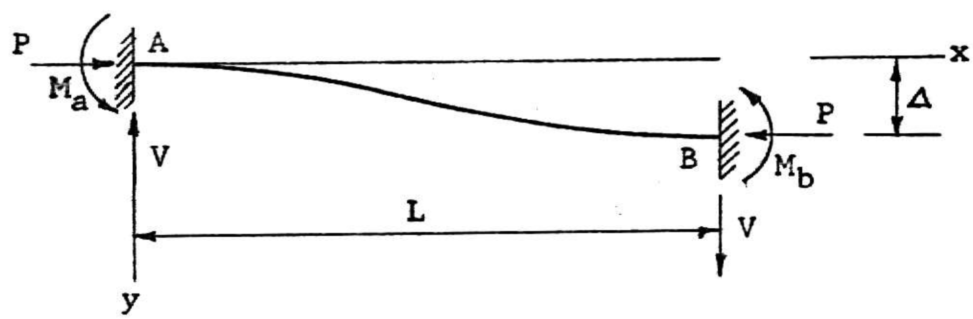

Fig. 3 Axially loaded beam with lateral displacement

the conditions at the ends of the beam. Since the deflections at the ends of the bar are zero, it is concluded that

$$A = 0 \qquad B = \frac{M_b}{P\sin kL}$$

Substituting into Eq. (4), the equation for the deflection curve is obtained as

$$y = \frac{M_b}{P}\left(\frac{\sin kx}{\sin kL} - \frac{x}{L}\right) \tag{5}$$

In the following discussion it will be necessary to have the formulas giving the angles of rotation θ_a and θ_b of the ends of the bar.

Taking the derivative of Eq. (5),

$$\begin{aligned}\theta_a = \left(\frac{dy}{dx}\right)_{x=0} &= \frac{M_b}{P}\left(\frac{k}{\sin kL} - \frac{1}{L}\right)\\ &= \frac{M_bL}{EI}\frac{1}{kL}\left(\frac{1}{\sin kL} - \frac{1}{kL}\right)\\ &= \frac{M_bL}{6EI}\frac{6}{kL}\left(\frac{1}{\sin kL} - \frac{1}{kL}\right)\end{aligned} \tag{6}$$

$$\begin{aligned}\theta_b = \left(\frac{dy}{dx}\right)_{x=L} &= \frac{M_b}{P}\left(\frac{k\cos kL}{\sin kL} - \frac{1}{L}\right)\\ &= -\frac{M_bL}{EI}\frac{1}{kL}\left(\frac{1}{kL} - \frac{1}{\tan kL}\right)\\ &= -\frac{M_bL}{3EI}\frac{3}{kL}\left(\frac{1}{kL} - \frac{1}{\tan kL}\right)\end{aligned} \tag{7}$$

It is to be noted that the known expressions $M_bL/6EI$ and $M_bL/3EI$, *for the angles produced by the couple* M_b acting alone, are multiplied by trigonometric factors representing the influence of the axial force P on the angles of rotation of the ends of the bar. In subsequent equations the following notation will be used in order to simplify the expression:

$$\phi_n = \frac{1}{kL}\left(\frac{1}{kL} - \frac{1}{\tan kL}\right) \tag{8}$$

$$\phi_f = \frac{1}{kL}\left(\frac{1}{\sin kL} - \frac{1}{kL}\right) \tag{9}$$

For the sign convention, moments will be regarded as positive if they turn the joint counter-clockwise, reactions will be positive if they form a couple turning counter-clockwise, but the rotation and deflection will be positive if they turn the joint or member clockwise.

If two couples M_a and M_b are applied at the ends A and B of the bar (Fig. 2), the deflection curve can be obtained by superposition.

By substituting $- M_a$ for M_b and (L-x) for x in the same equation, the deflections produced by the couple M_a are found.

Adding these results together, the deflection curve for the case represented in Fig. 2 is obtained as

$$y = \frac{M_b}{P}\left(\frac{\sin kx}{\sin kL} - \frac{x}{L}\right) - \frac{M_a}{P}\left(\frac{\sin k(L-x)}{\sin kL} - \frac{L-x}{L}\right) \tag{10}$$

The angles θ_a and θ_b giving the rotation of the ends of the beam in Fig. 2 are obtained by using Eqs. (6) and (7) and notations (8) and (9). Then, by superposition, the angles θ_a and θ_b produced by the two couples M_a and M_b are obtained as

$$\theta_a = -\frac{M_a L}{EI}\phi_n + \frac{M_b L}{EI}\phi_f \tag{11}$$

$$\theta_b = +\frac{M_a L}{EI}\phi_f - \frac{M_b L}{EI}\phi_n \tag{12}$$

Equations (11) and (12) can be expressed in matrix form as follows:

$$\begin{Bmatrix}\theta_a \\ \theta_b\end{Bmatrix} = \frac{L}{EI}\begin{bmatrix}-\phi_n & \phi_f \\ \phi_f & -\phi_n\end{bmatrix}\begin{Bmatrix}M_a \\ M_b\end{Bmatrix} \tag{13}$$

Using the inverse matrix,

$$\begin{Bmatrix}M_a \\ M_b\end{Bmatrix} = \frac{1}{D}\frac{EI}{L}\begin{bmatrix}-\phi_n & -\phi_f \\ -\phi_f & -\phi_n\end{bmatrix}\begin{Bmatrix}\theta_a \\ \theta_b\end{Bmatrix}$$

where,

$$D = \begin{vmatrix}-\phi_n & \phi_f \\ \phi_f & -\phi_n\end{vmatrix} = \phi^2{}_n - \phi^2{}_f \tag{14}$$

Using the notation

$$K = \frac{I}{L} \tag{15}$$

M_a and M_b are expressed in terms of θ_a and θ_b as

$$M_a = -\frac{EK}{D}(\phi_n\theta_a + \phi_f\theta_b) = -EK(S_n\theta_a + S_f\theta_b) \tag{16}$$

$$M_b = -\frac{EK}{D}(\phi_f\theta_a + \phi_n\theta_b) = -EK(S_f\theta_a + S_n\theta_b) \tag{17}$$

where,

$$S_n = \frac{\phi_n}{\phi_n^2 - \phi_f^2} \tag{18}$$

$$S_f = \frac{\phi_f}{\phi_n^2 - \phi_f^2} \tag{19}$$

Substituting Eqs. (8) and (9) into Eqs. (18) and (19) and using a notation $U = kL$ for simplification, the stiffness coefficient equations are expressed in terms of U as

$$S_n = \frac{U\sin U - U^2\cos U}{2 - U\sin U - 2\cos U} \tag{20}$$

$$S_f = \frac{U^2 - U\sin U}{2 - U\sin U - 2\cos U} \tag{21}$$

2. Moments due to Lateral Displacement

Consider an axially loaded beam undergoing a translation without rotation of the ends A and B.

The differential equation of the elastic curve is

$$EI\frac{d^2y}{dx^2} = -\left(Py - M_a + \frac{M_a + M_b - P\cdot\Delta}{L}x\right) \tag{22}$$

With the notation (3) and as $M_b = M_a$

$$\frac{d^2y}{dx^2} + k^2y = \frac{M_a}{EI} - \frac{2M_a}{EIL}x + \frac{P\cdot\Delta}{EIL}x$$

The general solution of this equation is

$$y = C\cos kx + D\sin kx + \frac{M_a}{P} - \frac{2M_a}{PL}x + \frac{\Delta}{L}x \tag{23}$$

Again the constants C and D are determined from the end conditions. Since the deflection at A is zero and Δ at B, it is concluded that

$$C = -\frac{M_a}{P} \qquad D = \frac{M_a}{P}\,\frac{\cos kL + 1}{\sin kL}$$

Substituting into Eq. (23) and rearranging

$$y = \frac{M_a}{P}\left[\left(\frac{\sin kx}{\sin kL} - \frac{x}{L}\right) - \left(\frac{\sin k(L-x)}{\sin kL} - \frac{L-x}{L}\right)\right] + \frac{\Delta}{L}x \tag{24}$$

Taking the derivative of Eq. (24) and from the end conditions

$$\left(\frac{dy}{dx}\right)_{x=0} = 0,$$

$$\left(\frac{dy}{dx}\right)_{x=L} = \frac{M_a}{P}\left(\frac{k\cos kL}{\sin kL} - \frac{1}{L}\right) + \left(\frac{k}{\sin kL} - \frac{1}{L}\right) + \frac{\Delta}{L} = 0$$

Rearranging and using the notations (8) and (9),

$$\frac{M_a}{P}k^2L(-\phi_n + \phi_f) + \frac{\Delta}{L} = 0 \tag{25}$$

Using the notations (3),(15),(18) and (19) and rearranging, M_a and M_b are expressed in terms of displacement Δ as

$$M_a = M_b = \frac{EK}{\phi_n - \phi_f}\cdot\frac{\Delta}{L} = \frac{EK}{L}(S_n + S_f)\cdot\Delta \tag{26}$$

3. Shear due to Lateral Displacement

The shear is obtained from the equilibrium equation in Fig. 3 as

$$VL - (M_a + M_b) + P\cdot\Delta = 0$$

from which follows that

$$V = \frac{M_a + M_b - P\cdot\Delta}{L} \tag{27}$$

Substituting into Eq. (27) the values from Eq. (26)

$$V = \frac{2EK}{L^2}(S_n + S_f)\cdot\Delta - \frac{P}{L}\cdot\Delta \tag{28}$$

APPLICATION OF THE DISPLACEMENT METHOD [7]

1. Definition of Problem

When axial deformation is neglected in applying the displacement method of rigid frame analysis, the degree of freedom is equal to the sum of the number of unknown joint rotations and the number of unknown sidesway displacements. While consideration of the axial deformation may simplify the formulation with an increase in the degree of freedom, the present work is limited to the former assumption so that the method herein is identical with the accepted "exact" method.

For example, consider the rigid frame of Fig. 4(a) with a degree of freedom of 3. When the three members are subjected to axial loads $\alpha_1 P$, $\alpha_2 P$ and $\alpha_3 P$ only, in the primary condition, as shown in Fig. 4(b), the frame is not deformed at all under the assumption of zero axial deformation. In the absence of any external loads, Q_1, Q_2 or Q_3, and using the assumption of an infinite elastic limit for the material, when the axial loads are increased to $\alpha_1 P_{cr}$, $\alpha_2 P_{cr}$ and $\alpha_3 P_{cr}$, as shown in Fig. 4(c), the frame may take a buckled shape, for which the displacements, X_1, X_2, and X_3, as well as the end moments, F_1 to F_6, may assume any set of proportional values. The ratio of P_{cr} to P is defined as the buckling load factor.

Hereafter, external moments acting clockwise at the joint and internal end moments acting clockwise on the member will be considered to be positive. Correspondingly, clockwise internal

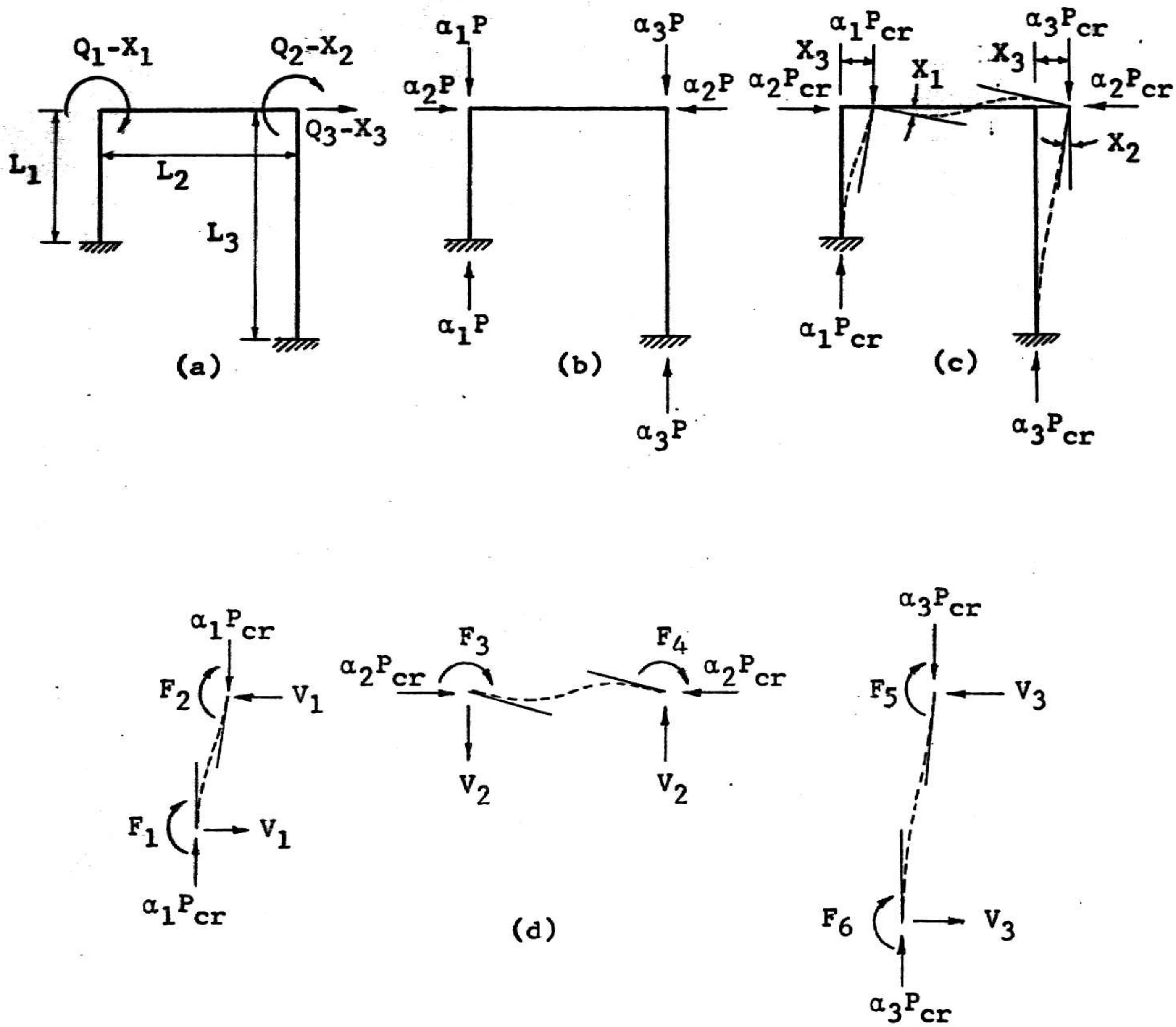

Fig. 4 Elastic buckling of a rigid frame:

(a) The degree of freedom
(b) Primary condition
(c) Buckled condition
(d) Members in buckled condition

end rotations are positive.

2. Equilibrium Equations

For equilibrium between external and internal forces in the frame of Fig. 4,

$$Q_1 = F_2 + F_3 \tag{29-a}$$

$$Q_2 = F_4 + F_5 \tag{29-b}$$

$$Q_3 = - V_1 - V_3 \tag{29-c}$$

From Fig. 4(d)

$$\begin{aligned} V_1 &= + \frac{F_1 + F_2}{L_1} + \frac{\alpha_1 P_{cr}}{L_1} X_3 \\ V_3 &= + \frac{F_5 + F_6}{L_3} + \frac{\alpha_3 P_{cr}}{L_3} X_3 \end{aligned} \tag{30}$$

Substituting Eq. (30) into Eq. (29-c), the external forces can be expressed in terms of the end moments and the sidesway displacement as

$$\begin{Bmatrix} Q_r \\ Q_s \end{Bmatrix} = [A]\{F\} + \begin{bmatrix} 0 & 0 \\ 0 & S2 \end{bmatrix} \begin{Bmatrix} X_r \\ X_s \end{Bmatrix} \tag{31}$$

in which the subscripts r and s refer to rotation and sidesway, respectively; [A] is the statics matrix as ordinarily used in first-order analysis; and [S2] may be defined as the second-order spring matrix because it expresses the forces per unit displacements, and assesses the second-order effects of deformation on equilibrium. In the present example

$$[A] = \begin{bmatrix} 0 & +1 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & +1 & 0 \\ -\dfrac{1}{L_1} & -\dfrac{1}{L_1} & 0 & 0 & -\dfrac{1}{L_3} & -\dfrac{1}{L_3} \end{bmatrix} \tag{32}$$

$$[S2] = -P_{cr}\left[\frac{\alpha_1}{L_1} + \frac{\alpha_3}{L_3}\right] \tag{33}$$

3. Compatibility Conditions

An angle of rotation at the end of a member is defined as the clockwise angle measured from the member axis, whether it is parallel to the original member axis or rotated to the tangent to the elastic curve at that end.

Then, for the frame of Fig. 4,

$$\begin{aligned} \theta_1 &= -\frac{X_3}{L_1} \\ \theta_2 &= +X_1 - \frac{X_3}{L_1} \\ \theta_3 &= +X_1 \\ \theta_4 &= +X_2 \\ \theta_5 &= +X_2 - \frac{X_3}{L_3} \\ \theta_6 &= -\frac{X_3}{L_3} \end{aligned} \tag{34}$$

In matrix notation

$$\{\theta\} = [B]\begin{Bmatrix} X_r \\ X_s \end{Bmatrix} \tag{35}$$

The matrix [B] has been defined as the deformation matrix and proved [1] to be the transpose of the statics matrix [A], expressed as

$$[B] = [A^T] \tag{36}$$

4. <u>Force-Deformation Relationship</u>

Let the primary and buckled shapes of the k^{th} member in a rigid frame be the straight line AB and the elastic curve A'B', respectively, as shown in Fig. 5. The F-θ relationship has been derived previously and was expressed by stiffness coefficients S_{ii} and S_{ij}.

From Eqs. (16) and (17),

$$\begin{aligned} F_i &= S_{ii}EK\theta_i + S_{ij}EK\theta_j \\ F_j &= S_{ij}EK\theta_i + S_{ii}EK\theta_j \end{aligned} \tag{37}$$

For a rigid frame with many members, all the values of F may be expressed in terms of all the values of θ by the matrix equation

$$\{F\} = [S]\{\theta\} \tag{38}$$

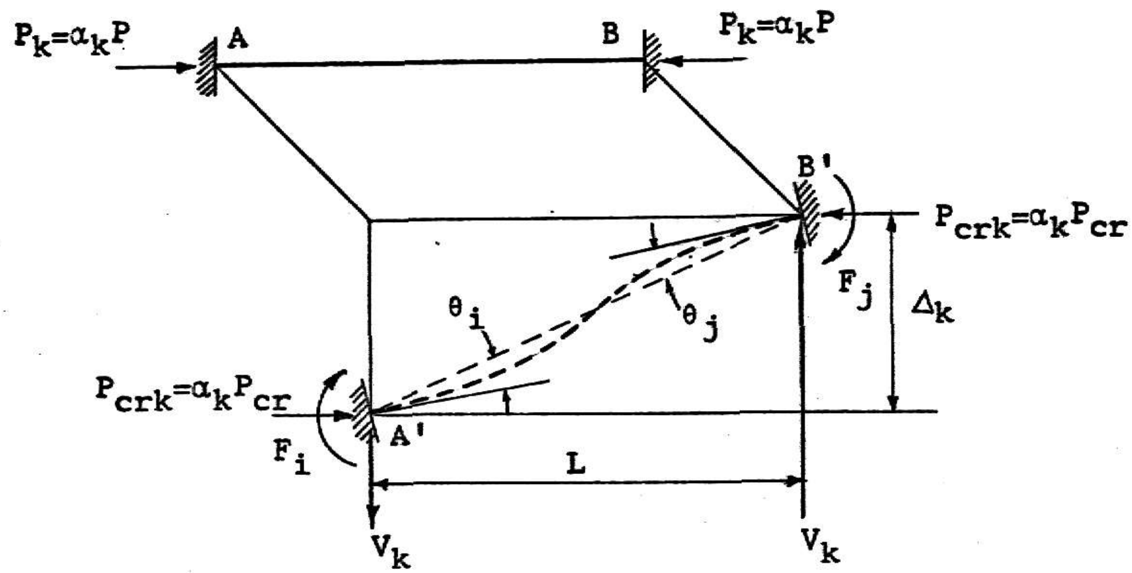

Fig. 5 Typical member in a buckled rigid frame

in which

$$[S]_{2m\times 2m} = \begin{bmatrix} S_{ii1}EK_1 & S_{ij1}EK_1 & 0 & 0 & 0 & 0 \\ S_{ij1}EK_1 & S_{ii1}EK_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{ii2}EK_2 & S_{ij2}EK_2 & 0 & 0 \\ 0 & 0 & S_{ij2}EK_2 & S_{ii2}EK_2 & 0 & 0 \\ & & & & & \\ 0 & 0 & 0 & 0 & S_{iim}EK_m & S_{ijm}EK_m \\ 0 & 0 & 0 & 0 & S_{ijm}EK_m & S_{iim}EK_m \end{bmatrix}$$

5. <u>Buckling Criterion</u>

When the external loads that produce a set of axial forces in the members of a rigid frame remain relatively small, any nodal displacements arbitrarily applied on the frame by some disturbing nodal forces will vanish when the source of disturbance is removed. If the loads increase gradually until they pass a certain limit (the critical loads), the frame will collapse suddenly under any disturbing force. At the critical load condition, any arbitrarily imposed nodal displacements will remain unchanged even after the disturbing forces are removed.

Combining Eqs. (31), (35) and (38)

$$\begin{Bmatrix} Q_r \\ Q_s \end{Bmatrix} = [ASA^T]\begin{Bmatrix} X_r \\ X_s \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & S2 \end{bmatrix}\begin{Bmatrix} X_r \\ X_s \end{Bmatrix}$$

$$= \left[[ASA^T] + \begin{bmatrix} 0 & 0 \\ 0 & S2 \end{bmatrix} \right]\begin{Bmatrix} X_r \\ X_s \end{Bmatrix} \tag{39}$$

When the loads reach the critical value, the displacements can assume any value without any change in the external forces. In other words, a small change in the displacements ΔX can be produced without any change of the external forces, that is,

$$\begin{Bmatrix} \Delta Q_r \\ \Delta Q_s \end{Bmatrix} = \left[[ASA^T] + \begin{bmatrix} 0 & 0 \\ 0 & S2 \end{bmatrix} \right] \begin{Bmatrix} \Delta X_r \\ \Delta X_s \end{Bmatrix} = 0 \qquad (40)$$

The requirement for a nontrivial solution of Eq. (40) is

$$\text{DET} \left[[ASA^T] + \begin{bmatrix} 0 & 0 \\ 0 & S2 \end{bmatrix} \right] = \text{DET}[\overline{S}] = 0 \qquad (41)$$

Equation (41) is the buckling criterion.

6. Second-Order Spring Matrix

The second-order spring matrix may be determined directly, as for the simple rigid frame of Fig. 4. For more complex frames, a general method will be derived here.

First $[C]$ is defined as a matrix expressing the values of Q_s in terms of the end shears on the member such that

$$\{Q_s\} = [C]\{V\} \qquad (42)$$

Next $[H]$ is defined as a matrix expressing the values of Δ of the members as defined in Fig. 5 in terms of the values of X_s such that

$$\{\Delta\} = [H]\{X_s\} \tag{43-a}$$

By the Principle of Virtual Work [1]

$$[H] = [C^T] \tag{43-b}$$

Equation (43-a) may be written as

$$\{\Delta\} = [C^T]\{X_s\} \tag{43-c}$$

The relationship between V_k (due to axial loads applied at distance Δ_k only) and Δ_k of the k^{th} member may be observed from Fig. 5 to be

$$V_k = -\frac{\alpha_k P_{cr}}{L_k}\Delta_k \tag{44}$$

Then all the values of V may be expressed in terms of all the values of Δ by the matrix equation

$$\{V\} = P_{cr}[G]\{\Delta\} \tag{45-a}$$

in which

$$[G] = \begin{bmatrix} -\dfrac{\alpha_1}{L_1} & 0 & 0 \\ 0 & -\dfrac{\alpha_2}{L_2} & 0 \\ 0 & 0 & -\dfrac{\alpha_k}{L_k} \end{bmatrix} \tag{45-b}$$

Combining Eqs. (42), (43-c) and (45-a),

$$\{Q_s\} = P_{cr}[C][G][C^T]\{X_s\} \tag{46}$$

and here the second-order spring matrix is expressed as

$$[S2] = P_{cr}[C][G][C^T] \tag{47}$$

PROGRAMMING

1. Computational Procedure [4]

Let the buckling load on the k^{th} member be $\alpha_k P_{cr}$, its length $(RL)_k L_o$ and its moment of inertia $(RI)_k I_o$. Also define

$$U_{cr} = L_o \sqrt{\frac{P_{cr}}{EI_o}} \tag{48}$$

Then

$$(U_{cr})_k = L_k \sqrt{\frac{\alpha_k P_{cr}}{EI_k}} = (RL)_k \sqrt{\frac{\alpha_k}{(RI)_k}}\, U_{cr} \tag{49}$$

Define

$$\beta_k = (RL)_k \sqrt{\frac{\alpha_k}{(RI)_k}} \tag{50}$$

Then

$$(U_{cr})_k = \beta_k U_{cr} \tag{51}$$

and before buckling

$$U_k = \beta_k U_o \tag{52}$$

Now the problem is to find the value of U_o which is U_{cr} and makes the determinant $[\bar{S}] = 0$.

As a first step to find the U_{cr}, the values of the determinant $[\bar{S}]$ are computed using the Dwyer's pivotal method [5] for as many values of U_o as necessary in a range where the first and several subsequent buckling modes should occur, and at a reasonably small increment apart.

The [A] matrix depends only on the configuration of the undeformed rigid frame and thus it remains constant.

Recalling that

$$[\bar{S}] = [ASA^T] + \begin{bmatrix} 0 & 0 \\ 0 & S2 \end{bmatrix}, \qquad (53)$$

it may be seen that only the elements in the [S] matrix and [S2] matrix defined by Eq. (33) change with the trial values of U_o.

To locate the roots of the determinant equation, it is desirable to obtain a graph of the values of the determinant versus the values of U_o. The fundamental mode occurs where the value of the determinant first changes from positive to negative. A change in the sign of the determinant is detected by determining where the sign of the product of its values at two successive trial values of U_o becomes negative. A Lagrange interpolation routine [6] is then used to find the values of U_{cr} using ten determinant values in this region.

Having found the values of U_{cr}, the critical load P_{cr} can be found from Eq. (48). Defining the effective length ratio K_j of the j^{th} member as

$$P_{crj} = \alpha_j P_{cr} = \frac{\pi^2 EI_j}{(K_j L_j)^2} \qquad (54)$$

it may be shown that

$$K_j = \frac{\pi}{\beta_j U_{cr}} \tag{55}$$

2. Computer Program

The program was written in FORTRAN IV language for the IBM 1130 computer and can be used for any plane rigid frames consisting of prismatic members subjected to axial loads only in the primary condition. A maximum of 20 members and 20 degrees of freedom can be handled with the IBM 1130 computer. These figures may be increased, however, simply by increasing the dimensions of the computer storage arrays or by using additional tapes.

The flow chart is shown in Appendix E and the source program is listed in Appendix F.

Although the present program gives results only for the fundamental mode of buckling which is usually the only required mode, it can be modified very easily for higher modes.

In using the program, it is necessary to choose an initial value of U which is usually 1.5 or smaller and the increment ΔU, usually 0.05. For the non-axially-loaded member within the frame, constant values of the ordinary flexural stiffness are set in the program.

In the output the machine first prints out all the input data for the users to check and then gives the effective length factor K and buckling loads P_{cr} for each axially loaded member.

3. Numerical Examples

For general numerical examples, a simple beam and two rigid frames, as shown in Fig. 6 to 8 with the assumed loading, properties and dimensions, were chosen and analyzed using the program. The computer outputs for the three examples are presented in Appendix I. The K factors obtained in the output can be used in conjunction with the other rules of the AISC Specification[2] to check if the assumed member size satisfies all AISC requirements [2]. With the given loading on the frame and the P_{cr} value from the output, a factor of safety against buckling can be evaluated.

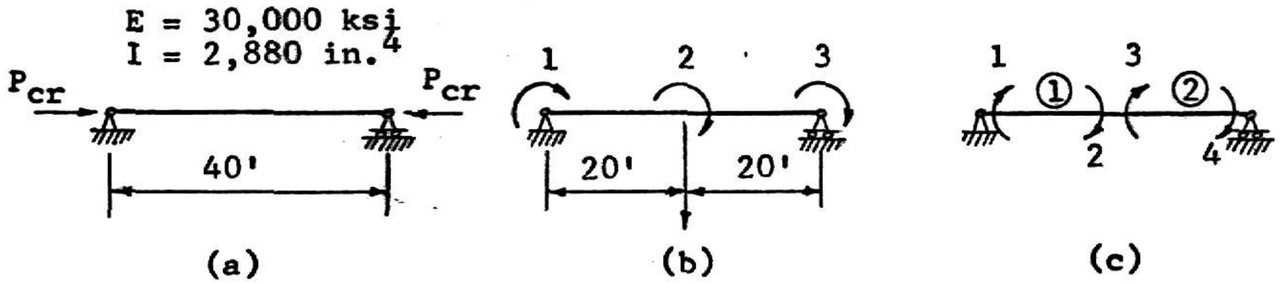

Fig. 6 Example 1: (a) Beam; (b) Q-X diagram; (c) F-θ diagram

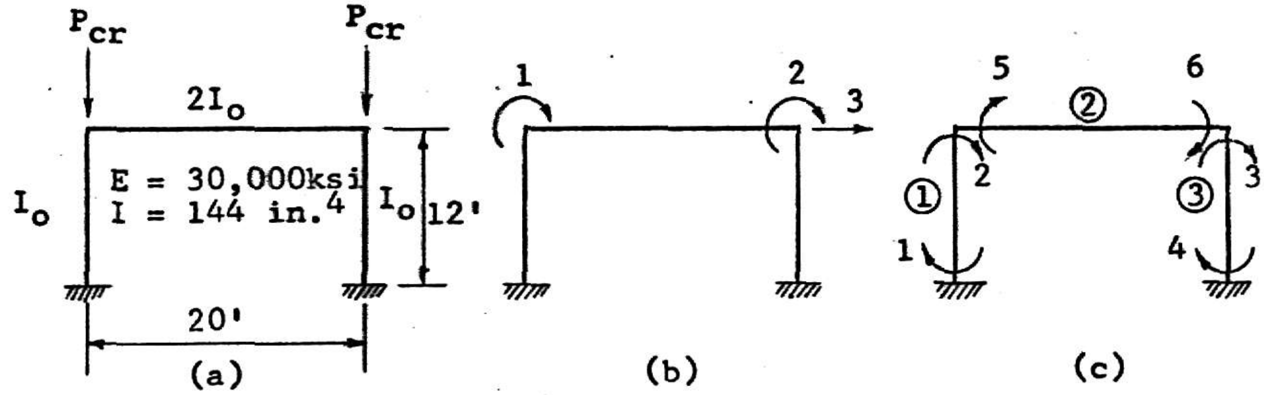

Fig. 7 Example 2: (a) Rigid Frame; (b) Q-X diagram; (c) F-θ diagram

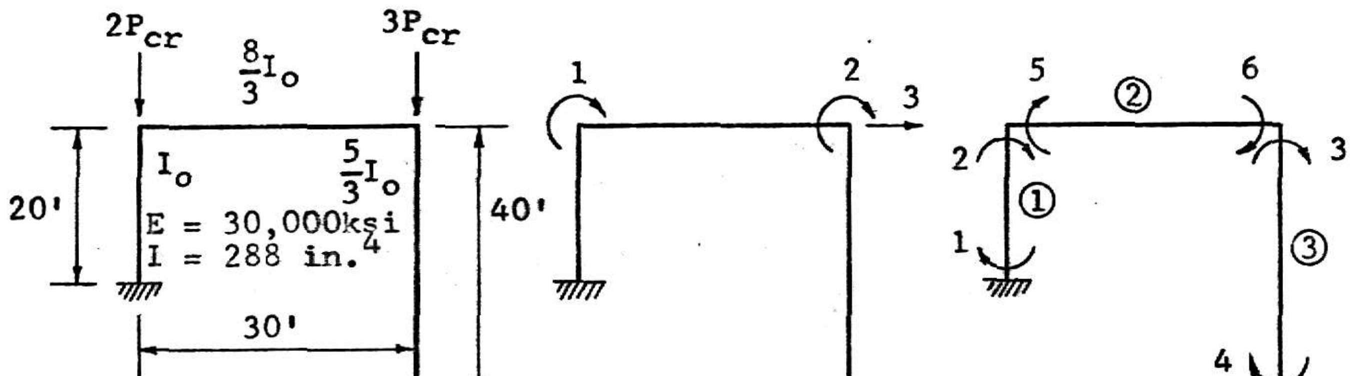

Fig. 8 Example 3: (a) Rigid Frame; (b) Q-X diagram; (c) F-θ diagram

APPLICATION OF NUMERICAL RESULTS

1. Portal Frame with Various Conditions

As a case of practical significance, a portal frame in which the moment of inertia of the horizontal member can be assumed to be infinitely stiff as compared to that of the columns was chosen. This case is frequently encountered in one-story factory or warehouse buildings with roof trusses which are so much stiffer than the supporting columns that for practical purpose they can be regarded as infinitely stiff.

Four special conditions were analyzed with the computer and a table of K values varying the ratio of column stiffnesses or the ratio of column loads by an increment was made in each condition as shown in Appendices A to D.

The four special conditions are:

1). A portal frame with equal column loads, different column stiffnesses and hinged bases (Fig. 9).

2). A portal frame with equal column loads, different column stiffnesses and fixed bases (Fig. 10).

3). A portal frame with different column loads, equal column stiffnesses and hinged bases (Fig. 11).

4). A portal frame with different column loads, equal column stiffnesses and fixed bases (Fig. 12).

The tabulated K values imply an ideally hinged or fixed column base. When this condition is not met in practice, for instance in the generally encountered pinned base plate with two anchor bolts, it may be permissible to use only 80% of the

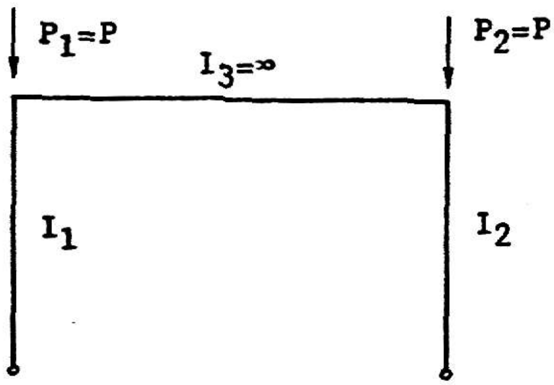

Fig. 9 Portal frame with equal column loads, different column stiffnesses and hinged bases

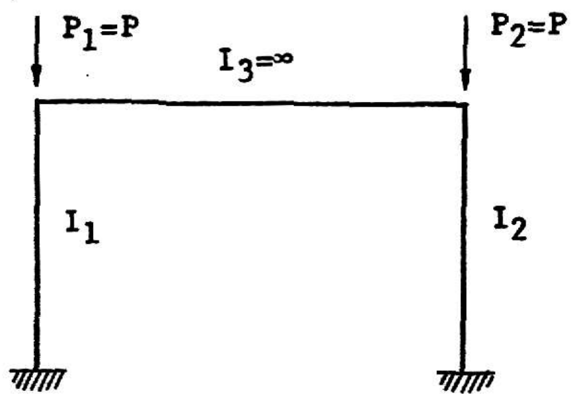

Fig. 10 Portal frame with equal column loads, different column stiffnesses and fixed bases

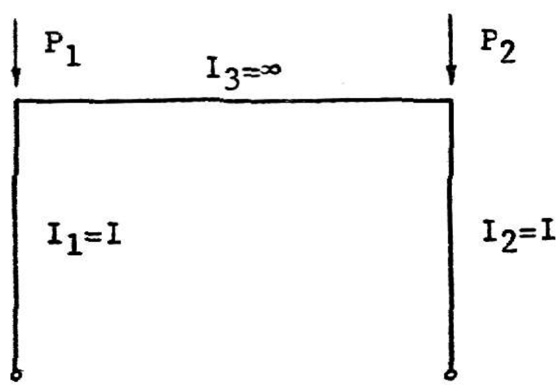

Fig. 11 Portal frame with different column loads, equal column stiffnesses and hinged bases

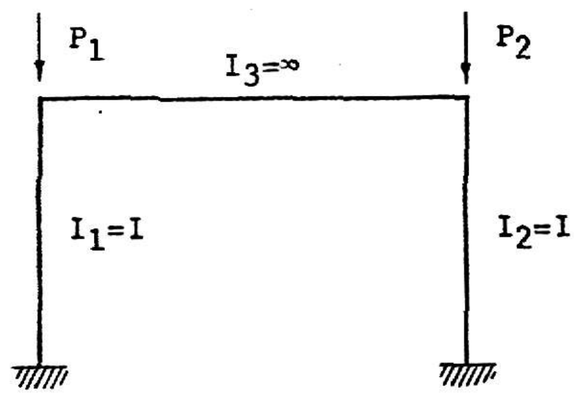

Fig. 12 Portal frame with different column loads, equal column stiffnesses and fixed bases

theoretical K values listed in Appendices A and C, or in the generally encountered fixed base plate it is recommended to multiply these theoretical K values listed in Appendices B and D with a factor of 1.2 as suggested in the Commentary to the AISC Specification [7] for similar cases.

2. Design Application and Discussion

a. Example 4

Design the columns for a factory or warehouse building 420 ft. by 420 ft. in plan with columns spaced 60 ft. on centers in each direction. The roof framing consists of trusses and carrying trusses 7 ft. 6 in. deep having a moment of inertia of 36,000 in^4. The unsupported column height to the underside of the trusses is 24 ft. A typical column load is 300 kips per column. No vertical wall bracing is permitted and stability must be achieved by frame action. The column base are assumed to be hinged.

Turning all the columns in the same direction, as is the general practice, will result, according to Fig. C.1.8.2. of the Commentary to the AISC Specification [7], in a K value of 1.65. This is based on G = 10 for a hinged bottom and G = 0 for a fully fixed top. The equivalent column height is therefore 24 x 1.65 = 39.6 ft., which requires for A36 steel a 14W119 column section [8].

Even if the adjacent columns are alternately turned 90°, the section would be the same if the design is based on the column of least stiffness and the K factors offered by the

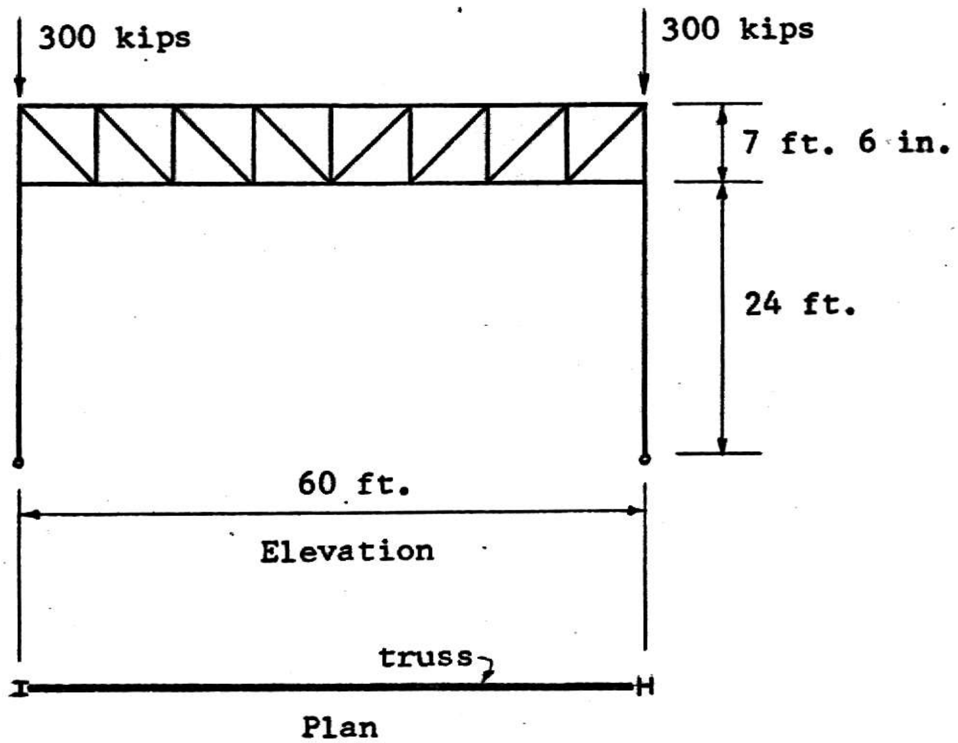

Fig. 13 Frame loading, elevation and plan for Example 4

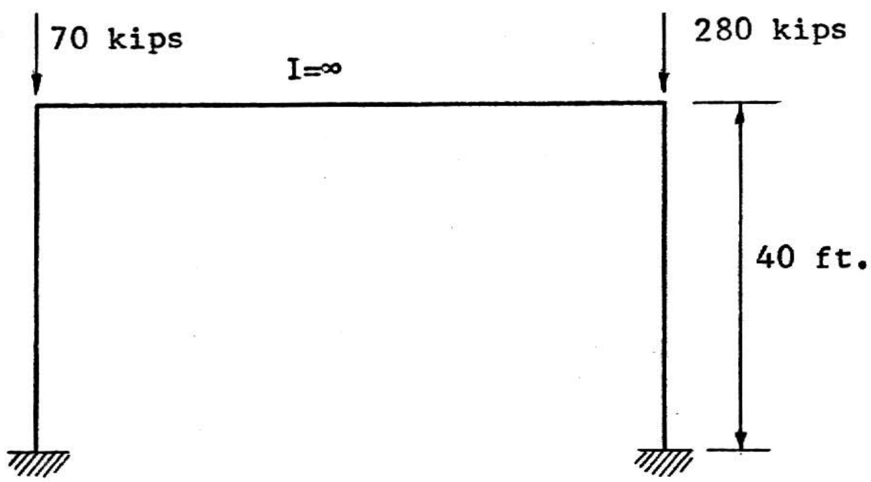

Fig. 14 Frame loading for Example 5

AISC Specification [2].

Consider an idealized frame, in which one column is turned 90° as shown in Fig. 13. Since the truss can be assumed to be infinitely stiff compared to the column, the K values in Appendix A may be used. Try a 14WF95(A36) for the column. The column stiffness ratio is I_y/I_x = 383.7/1063.5 = .360 and K = 1.4588 from Appendix A. The equivalent column height is 24 x 1.4588 = 35.0 ft. and the trial section 14WF95(A36) is good for the column load [8]. Note that in this comparision the theoretical K value is used without any reduction. As previously mentioned, such a reduction would be permissible and, in fact, was used for the conventional design in which the AISC Specification [2] permit a K value of 1.65 instead of the theoretical K = 2.0. Reducing the K value would permit the use of an even smaller column section.

b. Example 5

The columns of the frame as shown in Fig. 14 are subjected to roof and crane loads. If the crane hoist is near one column the maximum load in this column will be 280 kips, and the load in the other column at the same time will be only 70 kips and vice versa. Assume that the column bases are fixed and that the truss is infinitely stiff. For this comparision and to simplify the investigation, the influence of the moments in the column due to the eccentric application of the crane loading are neglected. The columns are assumed to be braced frequently perpendicular to the plane of the frame but in the plane of the frame buckling is to be resisted by the frame stiffness.

In the conventional analysis the columns would be designed under the assumption that the maximum load of 280 kips exists simultaneously in both columns. Under these circumstances the equivalent unbraced column height would be 40x1.2 = 48 ft. and a 14W74(A36) section would be required. In this case $A = 21.76$ in.^{2}, $r_x = 6.05$, $KL_x/r_x = (48 \times 12)/6.05 = 95.2$, $F_a = 13.57$ ksi, and the allowable load would be $21.76 \times 13.57 = 295.3$ kips.

Because the maximum load cannot exist simultaneously in both columns it is possible to utilize the reserve strength of the column with the smaller stress to help in the buckling resistance of the heavier stressed column. In this way it can be shown with the help of Appendix D that a 14W61(A36) section is sufficient. In Appendix D read for a column load ratio of 70/280 = 0.250 a theoretical K value of 0.7928 for a fixed column base. Increasing this figure by 20 % to account for nonideal conditions the equivalent column height is $40 \times 0.7928 \times 1.2 = 38.0$ ft. and for a 14W61(A36) with $r_x = 5.98$ and $A = 17.94$ in.^{2}, $KL_x/r_x = (38.0 \times 12)/5.98 = 76.3$, $F_a = 15.76$ ksi and the allowable load $P = 15.76 \times 17.94 = 282.7$ kips.

CONCLUSIONS

As observed in the numerical examples, the "exact" method of frame buckling analysis presented in this report resulted in the use of lighter sections because it utilized the unused reserve strength in the frame. This advantage is not possible by using the K values offered by the existing AISC Specification.

With the given loading on the frame and P_{cr} value from the computer output, the method also make it possible for investigators to obtain a factor of safety against buckling.

Although the use of the "exact" method is feasible only by using the electronic computer, it is not a major problem in today's design work in which most of the frame analysis is carried out on the electronic computer. Furthermore, once the tables for K values are made for some frequently encountered cases such as the case of portal frames presented in this report, the tables can be used without running the computer in every design of similar cases.

This method can also be extended to the stability analysis of rigid frames with non-prismatic members by dividing each member into a number of segments, each of which is considered to be uniform by itself [9].

Further investigation of the over-all stability of a rigid frame should include second-order effects, the plastic behavior, and the presence of primary bending moments.

NOTATION

[A]	Statics matrix
[B]	Deformation matrix
[C]	Sidesway force to shear matrix
E	Modulus of elasticity
F, {F}	Internal moment and its column matrix
[G]	Diagonal matrix of (α/L) values
[H]	Member end deflection to sidesway displacement matrix
I, I_o	Moment of inertia and its reference value
K	Stiffness ($K=I/L$), effective length ratio
L, L_o	Member length and its reference value
M	Bending moment, couple
P, P_{cr}	Axial load and its buckling load
Q, $\{Q\}$, $\{Q_r\}$, $\{Q_s\}$	External load, its column matrix, submatrix in rotation and in sidesway
RI	Ratio of moment of inertia to reference value (I/I_o)
RL	Ratio of member length to reference value (L/L_o)
S	Stiffness coefficient
[S2]	Second-order spring matrix
[S]	Member flexural stiffness matrix
U, U_{cr}	Axial load factor for beam-columns ($U=kL$, $U_{cr}=k_{cr}L$)

V, $\{V\}$	Shear force and its column matrix
X, $\{X\}$, $\{X_r\}$, $\{X_s\}$	External displacement, its column matrix, submatrix in rotation and in sidesway
k	Axial load factor for beam-columns ($k^2=P/EI$)
x, y	Rectangular coordinates
θ	Angle
ϕ	Amplification factor for beam-columns
Δ	Lateral displacement
α	Ratio of axial load to reference value
β	$RL\cdot\sqrt{\alpha/RI}$

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APPENDICES

A. Table for K values for a portal frame with equal column loads, different column stiffnesses and hinged bases.

B. Table for K values for a portal frame with equal column loads, different column stiffnesses and fixed bases.

C. Table for K values for a portal frame with different column loads, equal column stiffnesses and hinged bases.

D. Table for K values for a portal frame with different column loads, equal column stiffnesses and fixed bases.

E. Flow chart.

F. Computer program listing.

G. Computer program for values in Appendices A and B.

H. Computer program for values in Appendices C and D.

I. Computer outputs for Examples 1, 2 and 3.

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Appendix A.

PORTAL FRAME WITH
HORIZONTAL MEMBER INFINITELY STIFF
STIFFNESS OF BOTH COLUMNS DIFFERENT
BOTH COLUMN LOADS EQUAL
COLUMN BASE HINGED

COLUMN STIFFNESS RATIO LEFT/RIGHT	BETA COLUMN LEFT	BETA COLUMN RIGHT	UCR COLUMN LEFT	UCR COLUMN RIGHT	K COLUMN LEFT	K COLUMN RIGHT
0.100	3.1622	1.0000	3.6478	1.1535	0.8612	2.7234
0.110	3.0151	1.0000	3.4400	1.1409	0.9132	2.7534
0.120	2.8867	1.0000	3.3216	1.1506	0.9457	2.7302
0.130	2.7735	1.0000	3.2157	1.1594	0.9769	2.7095
0.140	2.6726	1.0000	3.1199	1.1673	1.0069	2.6911
0.150	2.5819	1.0000	3.0332	1.1747	1.0357	2.6742
0.160	2.5000	1.0000	2.9543	1.1817	1.0633	2.6584
0.170	2.4253	1.0000	2.8822	1.1883	1.0899	2.6436
0.180	2.3570	1.0000	2.8160	1.1947	1.1156	2.6294
0.190	2.2941	1.0000	2.7551	1.2009	1.1402	2.6159
0.200	2.2360	1.0000	2.6987	1.2069	1.1640	2.6029
0.210	2.1821	1.0000	2.6465	1.2128	1.1870	2.5903
0.220	2.1320	1.0000	2.5979	1.2185	1.2092	2.5781
0.230	2.0851	1.0000	2.5526	1.2242	1.2307	2.5662
0.240	2.0412	1.0000	2.5102	1.2297	1.2515	2.5546
0.250	2.0000	1.0000	2.4704	1.2352	1.2716	2.5432
0.260	1.9611	1.0000	2.4331	1.2406	1.2911	2.5321
0.270	1.9245	1.0000	2.3979	1.2460	1.3101	2.5213
0.280	1.8898	1.0000	2.3647	1.2513	1.3285	2.5106
0.290	1.8569	1.0000	2.3333	1.2565	1.3463	2.5001
0.300	1.8257	1.0000	2.3036	1.2617	1.3637	2.4898
0.310	1.7960	1.0000	2.2754	1.2668	1.3806	2.4797
0.320	1.7677	1.0000	2.2486	1.2720	1.3971	2.4697
0.330	1.7407	1.0000	2.2231	1.2770	1.4131	2.4599
0.340	1.7149	1.0000	2.1988	1.2821	1.4287	2.4503
0.350	1.6903	1.0000	2.1756	1.2871	1.4439	2.4407
0.360	1.6666	1.0000	2.1534	1.2920	1.4588	2.4314
0.370	1.6439	1.0000	2.1323	1.2970	1.4733	2.4221
0.380	1.6222	1.0000	2.1120	1.3019	1.4874	2.4130
0.390	1.6012	1.0000	2.0925	1.3068	1.5012	2.4039
0.400	1.5811	1.0000	2.0739	1.3116	1.5147	2.3950
0.410	1.5617	1.0000	2.0560	1.3165	1.5279	2.3863
0.420	1.5430	1.0000	2.0388	1.3213	1.5408	2.3776

0.430	1.5249	1.0000	2.0222	1.3260	1.5535	2.3690
0.440	1.5075	1.0000	2.0063	1.3308	1.5658	2.3606
0.450	1.4907	1.0000	1.9909	1.3355	1.5779	2.3522
0.460	1.4744	1.0000	1.9761	1.3402	1.5897	2.3439
0.470	1.4586	1.0000	1.9618	1.3449	1.6013	2.3357
0.480	1.4433	1.0000	1.9480	1.3496	1.6126	2.3277
0.490	1.4285	1.0000	1.9347	1.3542	1.6238	2.3197
0.500	1.4142	1.0000	1.9218	1.3589	1.6347	2.3118
0.510	1.4002	1.0000	1.9093	1.3635	1.6453	2.3040
0.520	1.3867	1.0000	1.8972	1.3681	1.6558	2.2962
0.530	1.3736	1.0000	1.8855	1.3726	1.6661	2.2886
0.540	1.3608	1.0000	1.8741	1.3772	1.6762	2.2810
0.550	1.3483	1.0000	1.8631	1.3817	1.6861	2.2735
0.560	1.3363	1.0000	1.8525	1.3862	1.6958	2.2661
0.570	1.3245	1.0000	1.8421	1.3907	1.7053	2.2588
0.580	1.3130	1.0000	1.8320	1.3952	1.7147	2.2515
0.590	1.3018	1.0000	1.8223	1.3997	1.7239	2.2444
0.600	1.2909	1.0000	1.8127	1.4041	1.7330	2.2373
0.610	1.2803	1.0000	1.8035	1.4086	1.7418	2.2302
0.620	1.2700	1.0000	1.7945	1.4130	1.7506	2.2232
0.630	1.2598	1.0000	1.7858	1.4174	1.7592	2.2163
0.640	1.2500	1.0000	1.7772	1.4218	1.7676	2.2095
0.650	1.2403	1.0000	1.7689	1.4261	1.7759	2.2027
0.660	1.2309	1.0000	1.7608	1.4305	1.7841	2.1960
0.670	1.2216	1.0000	1.7529	1.4348	1.7921	2.1894
0.680	1.2126	1.0000	1.7453	1.4392	1.8000	2.1828
0.690	1.2038	1.0000	1.7377	1.4435	1.8078	2.1763
0.700	1.1952	1.0000	1.7304	1.4478	1.8154	2.1698
0.710	1.1867	1.0000	1.7233	1.4521	1.8229	2.1634
0.720	1.1785	1.0000	1.7163	1.4563	1.8303	2.1571
0.730	1.1704	1.0000	1.7095	1.4606	1.8376	2.1508
0.740	1.1624	1.0000	1.7028	1.4648	1.8448	2.1446
0.750	1.1547	1.0000	1.6963	1.4690	1.8519	2.1384
0.760	1.1470	1.0000	1.6900	1.4733	1.8589	2.1323
0.770	1.1396	1.0000	1.6837	1.4775	1.8657	2.1262
0.780	1.1322	1.0000	1.6776	1.4816	1.8725	2.1202
0.790	1.1250	1.0000	1.6717	1.4858	1.8792	2.1143
0.800	1.1180	1.0000	1.6659	1.4900	1.8858	2.1084
0.810	1.1111	1.0000	1.6602	1.4941	1.8922	2.1025
0.820	1.1043	1.0000	1.6546	1.4983	1.8986	2.0967
0.830	1.0976	1.0000	1.6491	1.5024	1.9049	2.0909
0.840	1.0910	1.0000	1.6437	1.5065	1.9111	2.0852
0.850	1.0846	1.0000	1.6385	1.5106	1.9173	2.0796
0.860	1.0783	1.0000	1.6333	1.5147	1.9233	2.0740
0.870	1.0721	1.0000	1.6283	1.5188	1.9293	2.0684

0.880	1.0660	1.0000	1.6234	1.5228	1.9351	2.0629
0.890	1.0599	1.0000	1.6185	1.5269	1.9409	2.0574
0.900	1.0540	1.0000	1.6138	1.5309	1.9467	2.0520
0.910	1.0482	1.0000	1.6091	1.5350	1.9523	2.0466
0.920	1.0425	1.0000	1.6045	1.5390	1.9579	2.0412
0.930	1.0369	1.0000	1.6000	1.5430	1.9634	2.0359
0.940	1.0314	1.0000	1.5956	1.5470	1.9688	2.0307
0.950	1.0259	1.0000	1.5913	1.5510	1.9742	2.0254
0.960	1.0206	1.0000	1.5870	1.5550	1.9794	2.0203
0.970	1.0153	1.0000	1.5828	1.5589	1.9847	2.0151
0.980	1.0101	1.0000	1.5787	1.5629	1.9898	2.0100
0.990	1.0050	1.0000	1.5747	1.5668	1.9949	2.0050
1.000	1.0000	1.0000	1.5707	1.5707	2.0000	2.0000

Appendix B

PORTAL FRAME WITH
HORIZONTAL MEMBER INFINITELY STIFF
STIFFNESS OF BOTH COLUMNS DIFFERENT
BOTH COLUMN LOADS EQUAL
COLUMN BASE FIXED

COLUMN STIFFNESS RATIO	BETA		UCR		K	
	COLUMN		COLUMN		COLUMN	
LEFT/RIGHT	LEFT	RIGHT	LEFT	RIGHT	LEFT	RIGHT
0.100	3.1622	1.0000	7.1393	2.2576	0.4400	1.3915
0.110	3.0151	1.0000	6.8783	2.2812	0.4567	1.3771
0.120	2.8867	1.0000	6.6431	2.3012	0.4729	1.3651
0.130	2.7735	1.0000	6.4312	2.3188	0.4884	1.3548
0.140	2.6726	1.0000	6.2398	2.3347	0.5034	1.3455
0.150	2.5819	1.0000	6.0664	2.3495	0.5178	1.3371
0.160	2.5000	1.0000	5.9086	2.3634	0.5316	1.3292
0.170	2.4253	1.0000	5.7644	2.3767	0.5449	1.3218
0.180	2.3570	1.0000	5.6321	2.3895	0.5578	1.3147
0.190	2.2941	1.0000	5.5102	2.4018	0.5701	1.3079
0.200	2.2360	1.0000	5.3975	2.4138	0.5820	1.3014
0.210	2.1821	1.0000	5.2931	2.4256	0.5935	1.2951
0.220	2.1320	1.0000	5.1959	2.4371	0.6046	1.2890
0.230	2.0851	1.0000	5.1052	2.4484	0.6153	1.2831
0.240	2.0412	1.0000	5.0205	2.4595	0.6257	1.2773
0.250	2.0000	1.0000	4.9409	2.4704	0.6358	1.2716
0.260	1.9611	1.0000	4.8662	2.4813	0.6455	1.2660
0.270	1.9245	1.0000	4.7959	2.4920	0.6550	1.2606
0.280	1.8898	1.0000	4.7295	2.5026	0.6642	1.2553
0.290	1.8569	1.0000	4.6667	2.5131	0.6731	1.2500
0.300	1.8257	1.0000	4.6072	2.5234	0.6818	1.2449
0.310	1.7960	1.0000	4.5508	2.5337	0.6903	1.2398
0.320	1.7677	1.0000	4.4972	2.5440	0.6985	1.2348
0.330	1.7407	1.0000	4.4462	2.5541	0.7065	1.2299
0.340	1.7149	1.0000	4.3976	2.5642	0.7143	1.2251
0.350	1.6903	1.0000	4.3512	2.5742	0.7219	1.2203
0.360	1.6666	1.0000	4.3069	2.5841	0.7294	1.2157
0.370	1.6439	1.0000	4.2646	2.5940	0.7366	1.2110
0.380	1.6222	1.0000	4.2240	2.6038	0.7437	1.2065
0.390	1.6012	1.0000	4.1851	2.6136	0.7506	1.2019
0.400	1.5811	1.0000	4.1478	2.6233	0.7573	1.1975
0.410	1.5617	1.0000	4.1120	2.6330	0.7639	1.1931
0.420	1.5430	1.0000	4.0776	2.6426	0.7704	1.1888

0.430	1.5249	1.0000	4.0445	2.6521	0.7767	1.1845
0.440	1.5075	1.0000	4.0126	2.6616	0.7829	1.1803
0.450	1.4907	1.0000	3.9819	2.6711	0.7889	1.1761
0.460	1.4744	1.0000	3.9522	2.6805	0.7948	1.1719
0.470	1.4586	1.0000	3.9236	2.6899	0.8006	1.1678
0.480	1.4433	1.0000	3.8960	2.6992	0.8063	1.1638
0.490	1.4285	1.0000	3.8694	2.7085	0.8119	1.1598
0.500	1.4142	1.0000	3.8436	2.7178	0.8173	1.1559
0.510	1.4002	1.0000	3.8186	2.7270	0.8226	1.1520
0.520	1.3867	1.0000	3.7944	2.7362	0.8279	1.1481
0.530	1.3736	1.0000	3.7710	2.7453	0.8330	1.1443
0.540	1.3608	1.0000	3.7483	2.7544	0.8381	1.1405
0.550	1.3483	1.0000	3.7263	2.7635	0.8430	1.1367
0.560	1.3363	1.0000	3.7050	2.7725	0.8479	1.1330
0.570	1.3245	1.0000	3.6842	2.7815	0.8526	1.1294
0.580	1.3130	1.0000	3.6641	2.7905	0.8573	1.1257
0.590	1.3018	1.0000	3.6446	2.7994	0.8619	1.1222
0.600	1.2909	1.0000	3.6255	2.8083	0.8665	1.1186
0.610	1.2803	1.0000	3.6071	2.8172	0.8709	1.1151
0.620	1.2700	1.0000	3.5891	2.8260	0.8753	1.1116
0.630	1.2598	1.0000	3.5716	2.8348	0.8796	1.1081
0.640	1.2500	1.0000	3.5545	2.8436	0.8838	1.1047
0.650	1.2403	1.0000	3.5379	2.8523	0.8879	1.1013
0.660	1.2309	1.0000	3.5217	2.8610	0.8920	1.0980
0.670	1.2216	1.0000	3.5059	2.8697	0.8960	1.0947
0.680	1.2126	1.0000	3.4906	2.8784	0.9000	1.0914
0.690	1.2038	1.0000	3.4755	2.8870	0.9039	1.0881
0.700	1.1952	1.0000	3.4609	2.8956	0.9077	1.0849
0.710	1.1867	1.0000	3.4466	2.9042	0.9114	1.0817
0.720	1.1785	1.0000	3.4326	2.9127	0.9151	1.0785
0.730	1.1704	1.0000	3.4190	2.9212	0.9188	1.0754
0.740	1.1624	1.0000	3.4057	2.9297	0.9224	1.0723
0.750	1.1547	1.0000	3.3927	2.9381	0.9259	1.0692
0.760	1.1470	1.0000	3.3800	2.9466	0.9294	1.0661
0.770	1.1396	1.0000	3.3675	2.9550	0.9328	1.0631
0.780	1.1322	1.0000	3.3553	2.9633	0.9362	1.0601
0.790	1.1250	1.0000	3.3434	2.9717	0.9396	1.0571
0.800	1.1180	1.0000	3.3318	2.9800	0.9429	1.0542
0.810	1.1111	1.0000	3.3204	2.9883	0.9461	1.0512
0.820	1.1043	1.0000	3.3092	2.9966	0.9493	1.0483
0.830	1.0976	1.0000	3.2983	3.0048	0.9524	1.0454
0.840	1.0910	1.0000	3.2875	3.0131	0.9555	1.0426
0.850	1.0846	1.0000	3.2770	3.0213	0.9586	1.0398
0.860	1.0783	1.0000	3.2667	3.0294	0.9616	1.0370
0.870	1.0721	1.0000	3.2567	3.0376	0.9646	1.0342

0.880	1.0660	1.0000	3.2468	3.0457	0.9675	1.0314
0.890	1.0599	1.0000	3.2371	3.0538	0.9704	1.0287
0.900	1.0540	1.0000	3.2276	3.0619	0.9733	1.0260
0.910	1.0482	1.0000	3.2182	3.0700	0.9761	1.0233
0.920	1.0425	1.0000	3.2091	3.0780	0.9789	1.02C6
0.930	1.0369	1.0000	3.2001	3.0860	0.9817	1.0179
0.940	1.0314	1.0000	3.1913	3.0940	0.9844	1.0153
0.950	1.0259	1.0000	3.1826	3.1020	0.9871	1.0127
0.960	1.0206	1.0000	3.1741	3.1100	0.9897	1.01C1
0.970	1.0153	1.0000	3.1657	3.1179	0.9923	1.C075
0.980	1.0101	1.0000	3.1575	3.1258	0.9949	1.0050
0.990	1.0050	1.0000	3.1495	3.1337	0.9974	1.0025
1.000	1.0000	1.0000	3.1415	3.1415	1.0000	1.00C0

Appendix C.

PORTAL FRAME WITH
HORIZONTAL MEMBER INFINITELY STIFF
LOAD OF BOTH COLUMNS DIFFERENT
COLUMN STIFFNESS EQUAL
COLUMN BASE HINGED

COLUMN LOAD RATIO	BETA		UCR		K	
	COLUMN		COLUMN		COLUMN	
LEFT/RIGHT	LEFT	RIGHT	LEFT	RIGHT	LEFT	RIGHT
0.100	0.3162	1.0000	0.6661	2.1066	4.7158	1.4912
0.110	0.3316	1.0000	0.6956	2.0975	4.5158	1.4977
0.120	0.3464	1.0000	0.7235	2.0886	4.3421	1.5041
0.130	0.3605	1.0000	0.7498	2.0797	4.1895	1.5105
0.140	0.3741	1.0000	0.7749	2.0710	4.0541	1.5169
0.150	0.3872	1.0000	0.7987	2.0623	3.9331	1.5232
0.160	0.3999	1.0000	0.8215	2.0538	3.8240	1.5296
0.170	0.4123	1.0000	0.8433	2.0453	3.7252	1.5359
0.180	0.4242	1.0000	0.8642	2.0370	3.6351	1.5422
0.190	0.4358	1.0000	0.8843	2.0287	3.5525	1.5485
0.200	0.4472	1.0000	0.9036	2.0205	3.4765	1.5547
0.210	0.4582	1.0000	0.9222	2.0125	3.4064	1.5610
0.220	0.4690	1.0000	0.9402	2.0045	3.3413	1.5672
0.230	0.4795	1.0000	0.9575	1.9966	3.2808	1.5734
0.240	0.4898	1.0000	0.9743	1.9888	3.2243	1.5796
0.250	0.4999	1.0000	0.9905	1.9811	3.1715	1.5857
0.260	0.5099	1.0000	1.0062	1.9734	3.1219	1.5918
0.270	0.5196	1.0000	1.0215	1.9659	3.0753	1.5980
0.280	0.5291	1.0000	1.0363	1.9584	3.0314	1.6041
0.290	0.5385	1.0000	1.0506	1.9510	2.9900	1.6101
0.300	0.5477	1.0000	1.0646	1.9437	2.9508	1.6162
0.310	0.5567	1.0000	1.0782	1.9365	2.9137	1.6222
0.320	0.5656	1.0000	1.0914	1.9293	2.8784	1.6283
0.330	0.5744	1.0000	1.1042	1.9222	2.8449	1.6343
0.340	0.5830	1.0000	1.1167	1.9152	2.8130	1.6403
0.350	0.5916	1.0000	1.1289	1.9083	2.7826	1.6462
0.360	0.5999	1.0000	1.1408	1.9014	2.7536	1.6522
0.370	0.6082	1.0000	1.1524	1.8946	2.7259	1.6581
0.380	0.6164	1.0000	1.1637	1.8879	2.6994	1.6640
0.390	0.6244	1.0000	1.1748	1.8812	2.6740	1.6699

0.400	0.6324	1.0000	1.1856	1.8746	2.6497	1.6758
0.410	0.6403	1.0000	1.1961	1.8681	2.6263	1.6816
0.420	0.6480	1.0000	1.2064	1.8616	2.6038	1.6875
0.430	0.6557	1.0000	1.2165	1.8552	2.5823	1.6933
0.440	0.6633	1.0000	1.2264	1.8489	2.5615	1.6991
0.450	0.6708	1.0000	1.2360	1.8426	2.5415	1.7049
0.460	0.6782	1.0000	1.2455	1.8364	2.5222	1.7107
0.470	0.6855	1.0000	1.2547	1.8302	2.5037	1.7164
0.480	0.6928	1.0000	1.2638	1.8241	2.4857	1.7221
0.490	0.6999	1.0000	1.2726	1.8181	2.4684	1.7279
0.500	0.7071	1.0000	1.2813	1.8121	2.4517	1.7336
0.510	0.7141	1.0000	1.2899	1.8062	2.4355	1.7393
0.520	0.7211	1.0000	1.2982	1.8003	2.4198	1.7449
0.530	0.7280	1.0000	1.3064	1.7945	2.4046	1.7506
0.540	0.7348	1.0000	1.3144	1.7887	2.3899	1.7562
0.550	0.7416	1.0000	1.3223	1.7830	2.3757	1.7618
0.560	0.7483	1.0000	1.3301	1.7774	2.3619	1.7674
0.570	0.7549	1.0000	1.3376	1.7718	2.3485	1.7730
0.580	0.7615	1.0000	1.3451	1.7662	2.3354	1.7786
0.590	0.7681	1.0000	1.3524	1.7607	2.3228	1.7842
0.600	0.7745	1.0000	1.3596	1.7553	2.3105	1.7897
0.610	0.7810	1.0000	1.3667	1.7499	2.2986	1.7952
0.620	0.7874	1.0000	1.3736	1.7445	2.2870	1.8008
0.630	0.7937	1.0000	1.3804	1.7392	2.2757	1.8063
0.640	0.7999	1.0000	1.3871	1.7339	2.2647	1.8117
0.650	0.8062	1.0000	1.3937	1.7287	2.2540	1.8172
0.660	0.8124	1.0000	1.4002	1.7235	2.2436	1.8227
0.670	0.8185	1.0000	1.4066	1.7184	2.2334	1.8281
0.680	0.8246	1.0000	1.4128	1.7133	2.2235	1.8335
0.690	0.8306	1.0000	1.4190	1.7083	2.2138	1.8389
0.700	0.8366	1.0000	1.4251	1.7033	2.2044	1.8443
0.710	0.8426	1.0000	1.4310	1.6983	2.1952	1.8497
0.720	0.8485	1.0000	1.4369	1.6934	2.1862	1.8551
0.730	0.8544	1.0000	1.4427	1.6885	2.1775	1.8604
0.740	0.8602	1.0000	1.4484	1.6837	2.1689	1.8658
0.750	0.8660	1.0000	1.4540	1.6789	2.1606	1.8711
0.760	0.8717	1.0000	1.4595	1.6742	2.1524	1.8764
0.770	0.8774	1.0000	1.4649	1.6695	2.1444	1.8817
0.780	0.8831	1.0000	1.4703	1.6648	2.1366	1.8870
0.790	0.8888	1.0000	1.4756	1.6601	2.1290	1.8923
0.800	0.8944	1.0000	1.4808	1.6555	2.1215	1.8975
0.810	0.8999	1.0000	1.4859	1.6510	2.1142	1.9028

0.820	0.9055	1.0000	1.4909	1.6465	2.1070	1.9080
0.830	0.9110	1.0000	1.4959	1.6420	2.1000	1.9132
0.840	0.9165	1.0000	1.5008	1.6375	2.0932	1.9184
0.850	0.9219	1.0000	1.5056	1.6331	2.0864	1.9236
0.860	0.9273	1.0000	1.5104	1.6287	2.0798	1.9288
0.870	0.9327	1.0000	1.5151	1.6244	2.0734	1.9339
0.880	0.9380	1.0000	1.5197	1.6200	2.0671	1.9391
0.890	0.9433	1.0000	1.5243	1.6158	2.0609	1.9442
0.900	0.9486	1.0000	1.5288	1.6115	2.0548	1.9494
0.910	0.9539	1.0000	1.5333	1.6073	2.0488	1.9545
0.920	0.9591	1.0000	1.5377	1.6031	2.0430	1.9596
0.930	0.9643	1.0000	1.5420	1.5990	2.0373	1.9647
0.940	0.9695	1.0000	1.5463	1.5948	2.0316	1.9697
0.950	0.9746	1.0000	1.5505	1.5907	2.0261	1.9748
0.960	0.9797	1.0000	1.5546	1.5867	2.0207	1.9799
0.970	0.9848	1.0000	1.5587	1.5827	2.0154	1.9849
0.980	0.9899	1.0000	1.5628	1.5787	2.0101	1.9899
0.990	0.9949	1.0000	1.5668	1.5747	2.0050	1.9949
1.000	0.9999	1.0000	1.5707	1.5707	2.0000	2.0000

Appendix D

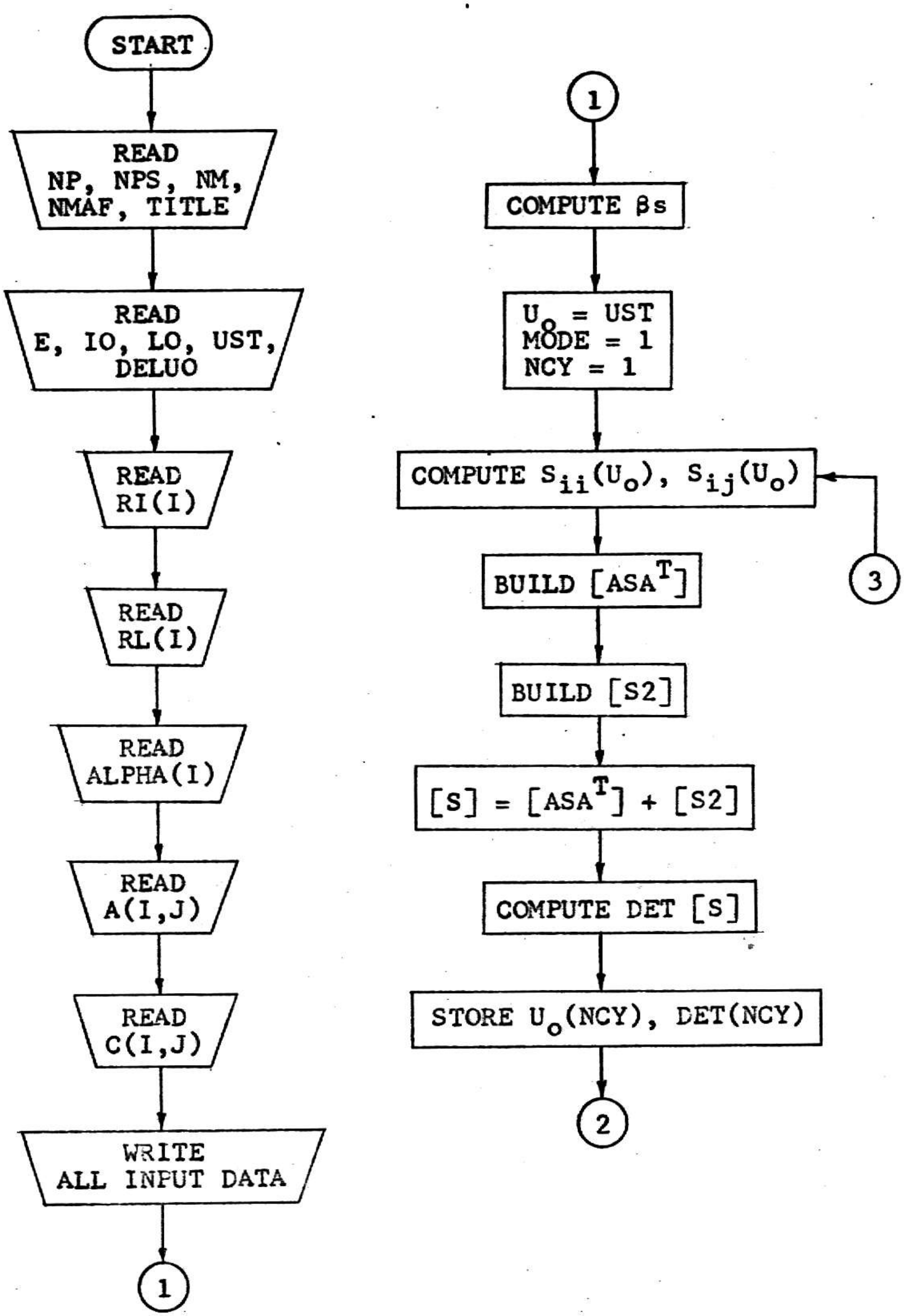
PORTAL FRAME WITH
HORIZONTAL MEMBER INFINITELY STIFF
LOAD OF BOTH COLUMNS DIFFERENT
COLUMN STIFFNESS EQUAL
COLUMN BASE FIXED

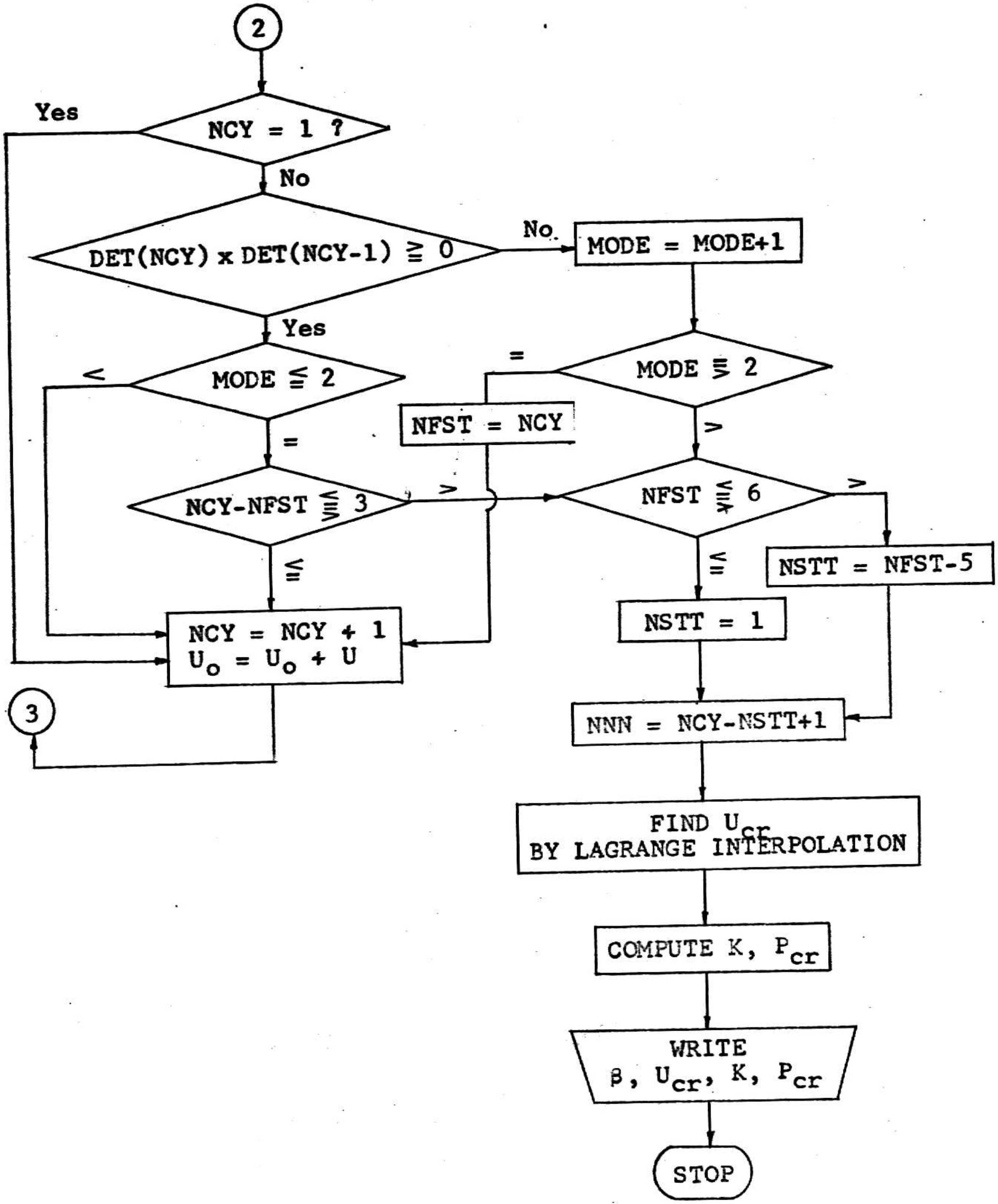
COLUMN LOAD RATIO	BETA		UCR		K	
	COLUMN		COLUMN		COLUMN	
EFT/RIGHT	LEFT	RIGHT	LEFT	RIGHT	LEFT	RIGHT
0.100	0.3162	1.0000	1.3323	4.2132	2.3579	0.7456
0.110	0.3316	1.0000	1.3913	4.1951	2.2579	0.7488
0.120	0.3464	1.0000	1.4470	4.1772	2.1710	0.7520
0.130	0.3605	1.0000	1.4997	4.1595	2.0947	0.7552
0.140	0.3741	1.0000	1.5498	4.1420	2.0270	0.7584
0.150	0.3872	1.0000	1.5975	4.1247	1.9665	0.7616
0.160	0.3999	1.0000	1.6430	4.1076	1.9120	0.7648
0.170	0.4123	1.0000	1.6866	4.0907	1.8626	0.7679
0.180	0.4242	1.0000	1.7284	4.0740	1.8175	0.7711
0.190	0.4358	1.0000	1.7686	4.0575	1.7762	0.7742
0.200	0.4472	1.0000	1.8072	4.0411	1.7382	0.7773
0.210	0.4582	1.0000	1.8445	4.0250	1.7032	0.7805
0.220	0.4690	1.0000	1.8804	4.0090	1.6706	0.7836
0.230	0.4795	1.0000	1.9151	3.9933	1.6404	0.7867
0.240	0.4898	1.0000	1.9486	3.9776	1.6121	0.7898
0.250	0.4999	1.0000	1.9811	3.9622	1.5857	0.7928
0.260	0.5099	1.0000	2.0125	3.9469	1.5609	0.7959
0.270	0.5196	1.0000	2.0430	3.9318	1.5376	0.7990
0.280	0.5291	1.0000	2.0726	3.9169	1.5157	0.8020
0.290	0.5385	1.0000	2.1013	3.9021	1.4950	0.8050
0.300	0.5477	1.0000	2.1292	3.8875	1.4754	0.8081
0.310	0.5567	1.0000	2.1564	3.8730	1.4568	0.8111
0.320	0.5656	1.0000	2.1828	3.8587	1.4392	0.8141
0.330	0.5744	1.0000	2.2085	3.8445	1.4224	0.8171
0.340	0.5830	1.0000	2.2335	3.8305	1.4065	0.8201
0.350	0.5916	1.0000	2.2579	3.8166	1.3913	0.8231
0.360	0.5999	1.0000	2.2817	3.8028	1.3768	0.8261
0.370	0.6082	1.0000	2.3049	3.7892	1.3629	0.8290
0.380	0.6164	1.0000	2.3275	3.7758	1.3497	0.8320
0.390	0.6244	1.0000	2.3496	3.7625	1.3370	0.8349

0.400	0.6324	1.0000	2.3712	3.7493	1.3248	0.8379
0.410	0.6403	1.0000	2.3923	3.7362	1.3131	0.8408
0.420	0.6480	1.0000	2.4129	3.7233	1.3019	0.8437
0.430	0.6557	1.0000	2.4331	3.7105	1.2911	0.8466
0.440	0.6633	1.0000	2.4528	3.6978	1.2807	0.8495
0.450	0.6708	1.0000	2.4721	3.6852	1.2707	0.8524
0.460	0.6782	1.0000	2.4910	3.6728	1.2611	0.8553
0.470	0.6855	1.0000	2.5095	3.6605	1.2518	0.8582
0.480	0.6928	1.0000	2.5276	3.6483	1.2428	0.8610
0.490	0.6999	1.0000	2.5453	3.6362	1.2342	0.8639
0.500	0.7071	1.0000	2.5627	3.6243	1.2258	0.8668
0.510	0.7141	1.0000	2.5798	3.6124	1.2177	0.8696
0.520	0.7211	1.0000	2.5965	3.6007	1.2099	0.8724
0.530	0.7280	1.0000	2.6129	3.5890	1.2023	0.8753
0.540	0.7348	1.0000	2.6289	3.5775	1.1949	0.8781
0.550	0.7416	1.0000	2.6447	3.5661	1.1878	0.8809
0.560	0.7483	1.0000	2.6602	3.5548	1.1809	0.8837
0.570	0.7549	1.0000	2.6753	3.5436	1.1742	0.8865
0.580	0.7615	1.0000	2.6902	3.5325	1.1677	0.8893
0.590	0.7681	1.0000	2.7049	3.5215	1.1614	0.8921
0.600	0.7745	1.0000	2.7193	3.5106	1.1552	0.8948
0.610	0.7810	1.0000	2.7334	3.4998	1.1493	0.8976
0.620	0.7874	1.0000	2.7473	3.4890	1.1435	0.9004
0.630	0.7937	1.0000	2.7609	3.4784	1.1378	0.9031
0.640	0.7999	1.0000	2.7743	3.4679	1.1323	0.9058
0.650	0.8062	1.0000	2.7875	3.4575	1.1270	0.9086
0.660	0.8124	1.0000	2.8004	3.4471	1.1218	0.9113
0.670	0.8185	1.0000	2.8132	3.4369	1.1167	0.9140
0.680	0.8246	1.0000	2.8257	3.4267	1.1117	0.9167
0.690	0.8306	1.0000	2.8380	3.4166	1.1069	0.9194
0.700	0.8366	1.0000	2.8502	3.4066	1.1022	0.9221
0.710	0.8426	1.0000	2.8621	3.3967	1.0976	0.9248
0.720	0.8485	1.0000	2.8739	3.3869	1.0931	0.9275
0.730	0.8544	1.0000	2.8854	3.3771	1.0887	0.9302
0.740	0.8602	1.0000	2.8968	3.3675	1.0844	0.9329
0.750	0.8660	1.0000	2.9080	3.3579	1.0803	0.9355
0.760	0.8717	1.0000	2.9191	3.3484	1.0762	0.9382
0.770	0.8774	1.0000	2.9299	3.3390	1.0722	0.9408
0.780	0.8831	1.0000	2.9406	3.3296	1.0683	0.9435
0.790	0.8888	1.0000	2.9512	3.3203	1.0645	0.9461
0.800	0.8944	1.0000	2.9616	3.3111	1.0607	0.9487
0.810	0.8999	1.0000	2.9718	3.3020	1.0571	0.9514

0.820	0.9055	1.0000	2.9819	3.2930	1.0535	0.9540
0.830	0.9110	1.0000	2.9919	3.2840	1.0500	0.9566
0.840	0.9165	1.0000	3.0017	3.2751	1.0466	0.9592
0.850	0.9219	1.0000	3.0113	3.2662	1.0432	0.9618
0.860	0.9273	1.0000	3.0209	3.2575	1.0399	0.9644
0.870	0.9327	1.0000	3.0303	3.2488	1.0367	0.9669
0.880	0.9380	1.0000	3.0395	3.2401	1.0335	0.9695
0.890	0.9433	1.0000	3.0487	3.2316	1.0304	0.9721
0.900	0.9486	1.0000	3.0577	3.2231	1.0274	0.9747
0.910	0.9539	1.0000	3.0666	3.2146	1.0244	0.9772
0.920	0.9591	1.0000	3.0754	3.2063	1.0215	0.9798
0.930	0.9643	1.0000	3.0840	3.1980	1.0186	0.9823
0.940	0.9695	1.0000	3.0926	3.1897	1.0158	0.9848
0.950	0.9746	1.0000	3.1010	3.1815	1.0130	0.9874
0.960	0.9797	1.0000	3.1093	3.1734	1.0103	0.9899
0.970	0.9848	1.0000	3.1175	3.1654	1.0077	0.9924
0.980	0.9899	1.0000	3.1256	3.1574	1.0050	0.9949
0.990	0.9949	1.0000	3.1336	3.1494	1.0025	0.9974
1.000	0.9999	1.0000	3.1415	3.1415	1.0000	1.0000

Appendix E. Flow Chart




2
NCY = 1 ?
Yes
No
DET(NCY) x DET(NCY-1) ≧ 0
No
MODE = MODE+1
Yes
MODE ≦ 2
<
=
MODE ≧ 2
=
NFST = NCY
>
NCY-NFST ≦ 3
>
NFST ≦ 6
>
≦
≦
NSTT = NFST-5
NCY = NCY + 1
U_o = U_o + U
NSTT = 1
3
NNN = NCY-NSTT+1
FIND U_cr
BY LAGRANGE INTERPOLATION
COMPUTE K, P_cr
WRITE
β, U_cr, K, P_cr
STOP

Appendix F.

```
PAGE   1    12/23/69

// JOB

LOG DRIVE    CART SPEC    CART AVAIL   PHY DRIVE
  0000         1213          1213        0002
                             5400        0000
                             5401        0001

V2 M06   ACTUAL 16K  CONFIG 16K

// FOR
*IOCS(CARD,1132 PRINTER)
*ONE WORD INTEGERS
*EXTENDED PRECISION
*LIST SOURCE PROGRAM
C     THIS PROGRAM COMPUTES THE ELASTIC BUCKLING LOADS OF AN AXIALLY-LOA
C     DED RIGID FRAME AND THE EFFECTIVE LENGTH FOR EVERY AXIALLY-LOADED
C     MEMBER IN THE FRAME BY A MATRIX FORMULATION OF THE 'EXACT' METHOD
C     PROGRAMMED BY BONG SOO PARK  12-05-69
      REAL IO,LO
      DIMENSION RL(20),RI(20),A(30,40),ALPHA(20),C(20,20),BETA(20),S(40)
     C,ASAT(20,20),S2(20,20),TEMP(20,20),UU(150),DETT(150),X(12),Y(12),B
      (12),TITLE(10)
  777 READ (2,5) NP,NPS,NM,NMAF,TITLE
    5 FORMAT (4(I2,8X),10A4)
      NMT2=2*NM
      READ (2,6) E,IO,LO,UST,DELUU
    6 FORMAT (5F10.4)
      READ (2,7) (RI(I),I=1,NM)
    7 FORMAT (8F10.4)
      READ (2,7) (RL(I),I=1,NM)
      READ (2,7) (ALPHA(I),I=1,NMAF)
      READ (2,7) ((A(I,J),J=1,NMT2),I=1,NP)
      READ (2,7) ((C(I,J),J=1,NM),I=1,NPS)
      WRITE (3,90) TITLE
   90 FORMAT (1H1,9X,'STRUCTURE',2X,10A4//)
      WRITE (3,91) NP
   91 FORMAT (14X,'DEGREE OF FREEDOM',17X,I2)
      WRITE (3,92) NPS
   92 FORMAT (14X,'DEGREE OF FREEDOM IN SIDESWAY',5X,I2)
      WRITE (3,93) NM
```

```
   93 FORMAT (14X,'NUMBER OF MEMBERS',17X,I2)
      WRITE (3,94)
   94 FORMAT (14X,'NUMBER OF MEMBERS HAVING')
      WRITE (3,95) NMAF
   95 FORMAT (14X,'PRIMARY AXIAL LOADS',15X,I2//)
      WRITE (3,96) E
   96 FORMAT (14X,'MODULUS OF ELASTICITY',19X,F7.1,1X,'KSI')
      WRITE (3,97) IO
   97 FORMAT (14X,'REFERENCE VALUE OF MOMENT OF INERTIA',4X,F7.1,1X,'IN*
     *4')
      WRITE (3,98) LO
   98 FORMAT (14X,'REFERENCE VALUE OF MEMBER LENGTH',8X,F7.1,1X,'IN')
      WRITE (3,99) UST
   99 FORMAT (14X,'STARTING VALUE OF U',24X,F4.1)
      WRITE (3,61) DELUO
   61 FORMAT (14X,'INCREMENT OF U',29X,F5.2//)
      WRITE (3,62)
   62 FORMAT (14X,'RATIO OF MOMENT OF INERTIA TO REFERENCE'/)
      WRITE (3,63) (RI(I),I=1,NM)
   63 FORMAT (14X,7F10.4)
      WRITE (3,64)
   64 FORMAT (/14X,'RATIO OF MEMBER LENGTH TO REFERENCE'/)
      WRITE (3,63) (RL(I),I=1,NM)
      WRITE (3,65)
   65 FORMAT (/14X,'RATIO OF AXIAL LOAD TO REFERENCE'/)
      WRITE (3,63) (ALPHA(I),I=1,NMAF)
      WRITE (3,66)
   66 FORMAT (/14X,'THE STATICS MATRIX A'/)
      WRITE (3,63) ((A(I,J),J=1,NMT2),I=1,NP)
      WRITE (3,67)
   67 FORMAT (/14X,'THE SIDESWAY FORCE TO SHEAR MATRIX C'/)
      WRITE (3,63) ((C(I,J),J=1,NM),I=1,NPS)
C     COMPUTE BETAS'
      DO 100 I=1,NMAF
      BETA(I)=RL(I)*SQRT(ALPHA(I)/RI(I))
  100 CONTINUE
C     SET STARTING VALUE OF U
      MODE=1
      UO=UST
      NCY=1
```

```
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        NFST=1
C       COMPUTE STIFFNESS FOR EACH MEMBER
   300  DO 101 I=1,NM
        L=2*I-1
        M=2*I
        RK=RI(I)*IO/(RL(I)*LO)
        IF (I-NMAF) 200,200,201
   200  U=BETA(I)*UO
        DENOM=2.-U*SIN(U)-2.*COS(U)
        S(L)=RK*(U*SIN(U)-U**2*COS(U))/DENOM
        S(M)=RK*(U**2-U*SIN(U))/DENOM
        GO TO 101
   201  S(L)=4.*RK
        S(M)=2.*RK
   101  CONTINUE
C       BUILD ASAT
        DO 102 I=1,NP
        DO 102 J=1,NP
        ASAT(I,J)=0.
        DO 102 K=1,NM
        L=2*K-1
        M=2*K
        ASAT(I,J)=ASAT(I,J)+A(I,L)*(S(L)*A(J,L)+S(M)*A(J,M))
        ASAT(I,J)=ASAT(I,J)+A(I,M)*(S(M)*A(J,L)+S(L)*A(J,M))
   102  CONTINUE
C       BUILD S2
        PPCR=(UO/LO)**2*IO
        DO 103 I=1,NPS
        DO 103 J=1,NPS
        S2(I,J)=0.
        DO 104 K=1,NM
        S2(I,J)=S2(I,J)-C(I,K)*ALPHA(K)/(RL(K)*LO)*C(J,K)
   104  CONTINUE
        S2(I,J)=PPCR*S2(I,J)
   103  CONTINUE
C       COMBINE ASAT AND S2 TO BUILD SBAR
        NPR=NP-NPS
        DO 105 I=1,NPS
        M=I+NPR
        DO 105 J=1,NPS
```

```
      N=J+NPR
      ASAT(M,N)=ASAT(M,N)+S2(I,J)
  105 CONTINUE
C     COMPUTE DETERMINANT OF SBAR
      IF (NP-1) 400,400,401
  400 DET=ASAT(1,1)
      GO TO 402
  401 DET=1.
      NP1=NP-1
      DO 106 K=1,NP1
      NN=NP-K
      DO 107 I=1,NN
      DO 107 J=1,NN
      TEMP(I,J)=ASAT(I+1,J+1)-ASAT(1,J+1)*ASAT(I+1,1)/ASAT(1,1)
  107 CONTINUE
      DET=DET*ASAT(1,1)
      DO 108 I=1,NN
      DO 108 J=1,NN
      ASAT(I,J)=TEMP(I,J)
  108 CONTINUE
  106 CONTINUE
      DET=DET*ASAT(1,1)
C     COMPUTE AND STORE TEN DETERMINENT VALUES OF SBAR IN THE REGION OF
C     FUNDAMENTAL MODE OF BUCKLING
  402 UU(NCY)=UO
      DETT(NCY)=DET
      IF (NCY-1) 202,202,203
  203 IF (DETT(NCY)*DETT(NCY-1)) 204,210,210
  210 IF (MODE-2) 202,206,205
  204 MODE=MODE+1
      IF (MODE-2) 202,211,205
  206 IF (NCY-NFST-3) 202,202,205
  211 NFST=NCY
  202 NCY=NCY+1
      UO=UO+DELUO
      GO TO 300
  205 IF (NFST-6) 207,207,208
  207 NSTT=1
      GO TO 301
  208 NSTT=NFST-5
```

```
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  301 NNN=NCY-NSTT+1
C       FIND UCR IN FUNDAMENTAL MODE BY USING LAGRANGE INTERPOLATION ROUTI
C       NE
        DO 109 I=1,NNN
        LL=NSTT+I-1
        X(I)=DETT(LL)
        Y(I)=UU(LL)
  109 CONTINUE
        UFST=0.0
        DO 110 K=1,NNN
        B(K)=1.0
        DO 111 J=1,NNN
        IF (J-K) 209,111,209
  209 B(K)=B(K)* (-X(J)/(X(K)-X(J)))
  111 CONTINUE
        UFST=UFST+B(K)*Y(K)
  110 CONTINUE
C       COMPUTE PCR AND EFFECTIVE LENGTH RATIO
        PPCR=E*IO*(UFST/LO)**2
        WRITE (3,50) TITLE
   50 FORMAT (1H1,9X,'RESULTS FOR STRUCTURE',2X,10A4//)
        WRITE (3,51)
   51 FORMAT (14X,'MEMBER',7X,'BETA',10X,'UCR',13X,'K',9X,'PCR(KIPS)'/)
        DO 112 I=1,NMAF
        U=BETA(I)*UFST
        ELRK=3.141592/U
        PCR=ALPHA(I)*PPCR
        WRITE (3,52) I,BETA(I),U,ELRK,PCR
   52 FORMAT (16X,I2,3(5X,F9.4),6X,F9.2)
  112 CONTINUE
        GO TO 777
        END

FEATURES SUPPORTED
 ONE WORD INTEGERS
 EXTENDED PRECISION
 IOCS

CORE REQUIREMENTS FOR
 COMMON       0  VARIABLES   9874  PROGRAM   1912
```

```
PAGE   6    12/23/69
 END OF COMPILATION
// XEQ
```

Appendix G.

```
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// JOB

LOG DRIVE   CART SPEC   CART AVAIL  PHY DRIVE
  0000        1213         1213        0000
                           1111        0001
                           0001        0002

V2 M06   ACTUAL 16K  CONFIG 16K

// FOR
*IOCS(CARD,1132 PRINTER)
*ONE WORD INTEGERS
*EXTENDED PRECISION
*LIST SOURCE PROGRAM
      REAL IO,LO
      DIMENSION RL(20),RI(20),A(30,40),ALPHA(20),C(20,20),BETA(20),S(40)
     C,ASAT(20,20),S2(20,20),TEMP(20,20),UU(150),DETT(150),X(12),Y(12),B
     C(12)
  777 READ (2,5) NP,NM,NPS,NMAF
    5 FORMAT (4(I2,8X))
      NMT2=2*NM
      READ (2,6) E,IO,LO,UST,DELUO
    6 FORMAT (5F10.4)
      READ (2,7) (RL(I),I=1,NM)
    7 FORMAT (8F10.4)
      READ (2,7) (RI(I),I=1,NM)
      READ (2,7) (ALPHA(I),I=1,NMAF)
      READ (2,7) ((A(I,J),J=1,NMT2),I=1,NP)
      READ (2,7) ((C(I,J),J=1,NM),I=1,NPS)
      WRITE(3,71)
   71 FORMAT (1H1,40X,'PORTAL FRAME WITH')
      WRITE(3,72)
   72 FORMAT (33X,'HORIZONTAL MEMBER INFINITELY STIFF')
      WRITE(3,73)
   73 FORMAT (33X,'STIFFNESS OF BOTH COLUMNS DIFFERENT')
      WRITE (3,74)
   74 FORMAT (39X,'BOTH COLUMN LOADS EQUAL')
      IF (NP-1) 500,500,501
  501 WRITE(3,75)
   75 FORMAT (41X,'COLUMN BASE HINGED'//)
      GO TO 502
  500 WRITE(3,79)
   79 FORMAT (41X,'COLUMN BASE FIXED'//)
```

```
502 WRITE (3,76)
 76 FORMAT (12X,'COLUMN',14X,'BETA',20X,'UCR',22X,'K')
    WRITE (3,77)
 77 FORMAT (10X,'STIFFNESS')
    WRITE (3,78)
 78 FORMAT (12X,'RATIO',14X,'COLUMN',18X,'COLUMN',18X,'COLUMN')
    WRITE (3,80)
80  FORMAT (10X,'LEFT/RIGHT',7X,'LEFT',6X,'RIGHT',9X,'LEFT',6X,'RIGHT'
   C,9X,'LEFT',6X,'RIGHT'/)
302 DO 100 I=1,NMAF
    BETA(I)=RL(I)*SQRT(ALPHA(I)/RI(I))
100 CONTINUE
    MODE=1
    UO=UST
    NCY=1
    NFST=1
300 DO 101 I=1,NM
    L=2*I-1
    M=2*I
    RK=RI(I)*IO/(RL(I)*LO)
    IF (I-NMAF) 200,200,201
200 U=BETA(I)*UO
    DENOM=2.-U*SIN(U)-2.*COS(U)
    S(L)=RK*(U*SIN(U)-U**2*COS(U))/DENOM
    S(M)=RK*(U**2-U*SIN(U))/DENOM
    GO TO 101
201 S(L)=4.*RK
    S(M)=2.*RK
101 CONTINUE
    DO 102 I=1,NP
    DO 102 J=1,NP
    ASAT(I,J)=0.
    DO 102 K=1,NM
    L=2*K-1
    M=2*K
    ASAT(I,J)=ASAT(I,J)+A(I,L)*(S(L)*A(J,L)+S(M)*A(J,M))
    ASAT(I,J)=ASAT(I,J)+A(I,M)*(S(M)*A(J,L)+S(L)*A(J,M))
102 CONTINUE
    PPCR=(UO/LO)**2*IO
    DO 103 I=1,NPS
    DO 103 J=1,NPS
    S2(I,J)=0.
    DO 104 K=1,NM
```

```
      S2(I,J)=S2(I,J)-C(I,K)*ALPHA(K)/(RL(K)*LO)*C(J,K)
  104 CONTINUE
      S2(I,J)=PPCR*S2(I,J)
  103 CONTINUE
      NPR=NP-NPS
      DO 105 I=1,NPS
      M=I+NPR
      DO 105 J=1,NPS
      N=J+NPR
      ASAT(M,N)=ASAT(M,N)+S2(I,J)
  105 CONTINUE
      IF (NP-1) 400,400,401
  400 DET=ASAT(1,1)
      GO TO 402
  401 DET=1.
      NP1=NP-1
      DO 106 K=1,NP1
      NN=NP-K
      DO 107 I=1,NN
      DO 107 J=1,NN
      TEMP(I,J)=ASAT(I+1,J+1)-ASAT(1,J+1)*ASAT(I+1,1)/ASAT(1,1)
  107 CONTINUE
      DET=DET*ASAT(1,1)
      DO 108 I=1,NN
      DO 108 J=1,NN
      ASAT(I,J)=TEMP(I,J)
  108 CONTINUE
  106 CONTINUE
      DET=DET*ASAT(1,1)
  402 UU(NCY)=UO
      DETT(NCY)=DET
      IF (NCY-1) 202,202,203
  203 IF (DETT(NCY)*DETT(NCY-1)) 204,210,210
  210 IF (MODE-2) 202,206,205
  204 MODE=MODE+1
      IF (MODE-2) 202,211,205
  206 IF (NCY-NFST-3) 202,202,205
  211 NFST=NCY
  202 NCY=NCY+1
      UO=UO+DELUO
      GO TO 300
  205 IF (NFST-6) 207,207,208
  207 NSTI=1
```

```
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      GO TO 301
  208 NSTT=NFST-5
  301 NNN=NCY-NSTT+1
      DO 109 I=1,NNN
      LL=NSTT+I-1
      X(I)=DETT(LL)
      Y(I)=UU(LL)
  109 CONTINUE
      UFST=0.0
      DO 110 K=1,NNN
      B(K)=1.0
      DO 111 J=1,NNN
      IF (J-K) 209,111,209
  209 B(K)=B(K)* (-X(J)/(X(K)-X(J)))
  111 CONTINUE
      UFST=UFST+B(K)*Y(K)
  110 CONTINUE
      RATIO=RI(1)+.00001
      BETAL=BETA(1)
      BETAR=BETA(2)
      UL=BETA(1)*UFST
      UR=BETA(2)*UFST
      ELRKL=3.141592/UL
      ELRKR=3.141592/UR
      WRITE (3,70) RATIO,BETAL,BETAR,UL,UR,ELRKL,ELRKR
   70 FORMAT (10X,F7.3,8X,F7.4,3X,F7.4,2(7X,F7.4,3X,F7.4))
      IF (RATIO-1.0) 212,213,213
  212 RI(1)=RI(1)+.01
      GO TO 302
  213 GO TO 777
      END

FEATURES SUPPORTED
 ONE WORD INTEGERS
 EXTENDED PRECISION
 IOCS

CORE REQUIREMENTS FOR
 COMMON      0  VARIABLES   9858  PROGRAM   1566

 END OF COMPILATION

// XEQ
```

Appendix H.

```
// FOR
*IOCS(CARD,1132 PRINTER)
*ONE WORD INTEGERS
*EXTENDED PRECISION
*LIST SOURCE PROGRAM
      REAL IO,LO
      DIMENSION RL(20),RI(20),A(30,40),ALPHA(20),C(20,20),BETA(20),S(40)
     C,ASAT(20,20),S2(20,20),TEMP(20,20),UU(150),DETT(150),X(12),Y(12),B
     C(12)
  777 READ (2,5) NP,NM,NPS,NMAF
    5 FORMAT (4(I2,8X))
      NMT2=2*NM
      READ (2,6) E,IO,LO,UST,DELUD
    6 FORMAT (5F10.4)
      READ (2,7) (RL(I),I=1,NM)
    7 FORMAT (8F10.4)
      READ (2,7) (RI(I),I=1,NM)
      READ (2,7) (ALPHA(I),I=1,NMAF)
      READ (2,7) ((A(I,J),J=1,NMT2),I=1,NP)
      READ (2,7) ((C(I,J),J=1,NM),I=1,NPS)
      WRITE(3,71)
   71 FORMAT (1H1,40X,'PORTAL FRAME WITH')
      WRITE(3,72)
   72 FORMAT (33X,'HORIZONTAL MEMBER INFINITELY STIFF')
      WRITE(3,73)
   73 FORMAT (35X,'LOAD OF BOTH COLUMNS DIFFERENT')
      WRITE (3,74)
   74 FORMAT (39X,'COLUMN STIFFNESS EQUAL')
      IF (NP-1) 500,500,501
  501 WRITE(3,75)
   75 FORMAT (41X,'COLUMN BASE HINGED'//)
      GO TO 502
  500 WRITE(3,79)
   79 FORMAT (41X,'COLUMN BASE FIXED'//)
  502 WRITE (3,76)
   76 FORMAT (12X,'COLUMN',14X,'BETA',20X,'UCR',22X,'K')
      WRITE (3,77)
   77 FORMAT (13X,'LOAD')
      WRITE (3,78)
   78 FORMAT (12X,'RATIO',14X,'COLUMN',18X,'COLUMN',18X,'COLUMN')
      WRITE (3,80)
```

```
 80   FORMAT (10X,'LEFT/RIGHT',7X,'LEFT',6X,'RIGHT',9X,'LEFT',6X,'RIGHT'
     C,9X,'LEFT',6X,'RIGHT'/)
 302  DO 100 I=1,NMAF
      BETA(I)=RL(I)*SQRT(ALPHA(I)/RI(I))
 100  CONTINUE
      MODE=1
      UO=UST
      NCY=1
      NFST=1
 300  DO 101 I=1,NM
      L=2*I-1
      M=2*I
      RK=RI(I)*IO/(RL(I)*LO)
      IF (I-NMAF) 200,200,201
 200  U=BETA(I)*UO
      DENOM=2.-U*SIN(U)-2.*COS(U)
      S(L)=RK*(U*SIN(U)-U**2*COS(U))/DENOM
      S(M)=RK*(U**2-U*SIN(U))/DENOM
      GO TO 101
 201  S(L)=4.*RK
      S(M)=2.*RK
 101  CONTINUE
      DO 102 I=1,NP
      DO 102 J=1,NP
      ASAT(I,J)=0.
      DO 102 K=1,NM
      L=2*K-1
      M=2*K
      ASAT(I,J)=ASAT(I,J)+A(I,L)*(S(L)*A(J,L)+S(M)*A(J,M))
      ASAT(I,J)=ASAT(I,J)+A(I,M)*(S(M)*A(J,L)+S(L)*A(J,M))
 102  CONTINUE
      PPCR=(UO/LO)**2*IO
      DO 103 I=1,NPS
      DO 103 J=1,NPS
      S2(I,J)=0.
      DO 104 K=1,NM
      S2(I,J)=S2(I,J)-C(I,K)*ALPHA(K)/(RL(K)*LO)*C(J,K)
 104  CONTINUE
      S2(I,J)=PPCR*S2(I,J)
 103  CONTINUE
```

```
      NPR=NP-NPS
      DO 105 I=1,NPS
      M=I+NPR
      DO 105 J=1,NPS
      N=J+NPR
      ASAT(M,N)=ASAT(M,N)+S2(I,J)
  105 CONTINUE
      IF (NP-1) 400,400,401
  400 DET=ASAT(1,1)
      GO TO 402
  401 DET=1.
      NP1=NP-1
      DO 106 K=1,NP1
      NN=NP-K
      DO 107 I=1,NN
      DO 107 J=1,NN
      TEMP(I,J)=ASAT(I+1,J+1)-ASAT(1,J+1)*ASAT(I+1,1)/ASAT(1,1)
  107 CONTINUE
      DET=DET*ASAT(1,1)
      DO 108 I=1,NN
      DO 108 J=1,NN
      ASAT(I,J)=TEMP(I,J)
  108 CONTINUE
  106 CONTINUE
      DET=DET*ASAT(1,1)
  402 UU(NCY)=UD
      DETT(NCY)=DET
      IF (NCY-1) 202,202,203
  203 IF (DETT(NCY)*DETT(NCY-1)) 204,210,210
  210 IF (MODE-2) 202,206,205
  204 MODE=MODE+1
      IF (MODE-2) 202,211,205
  206 IF (NCY-NFST-3) 202,202,205
 211  NFST=NCY
  202 NCY=NCY+1
      UD=UD+DELUD
      GO TO 300
  205 IF (NFST-6) 207,207,208
  207 NSTT=1
      GO TO 301
```

```
  208 NSTT=NFST-5
  301 NNN=NCY-NSTT+1
      DO 109 I=1,NNN
      LL=NSTT+I-1
      X(I)=DETT(LL)
      Y(I)=UU(LL)
  109 CONTINUE
      UFST=0.0
      DO 110 K=1,NNN
      B(K)=1.0
      DO 111 J=1,NNN
      IF (J-K) 209,111,209
  209 B(K)=B(K)* (-X(J)/(X(K)-X(J)))
  111 CONTINUE
      UFST=UFST+B(K)*Y(K)
  110 CONTINUE
      RATIO=ALPHA(1)+.00001
      BETAL=BETA(1)
      BETAR=BETA(2)
      UL=BETA(1)*UFST
      UR=BETA(2)*UFST
      ELRKL=3.141592/UL
      ELRKR=3.141592/UR
      WRITE (3,70) RATIO,BETAL,BETAR,UL,UR,ELRKL,ELRKR
   70 FORMAT (10X,F7.3,8X,F7.4,3X,F7.4,2(7X,F7.4,3X,F7.4))
      IF (RATIO-1.0) 212,213,213
  212 ALPHA(1)=ALPHA(1)+.01
      GO TO 302
  213 GO TO 777
      END

FEATURES SUPPORTED
 ONE WORD INTEGERS
 EXTENDED PRECISION
 IOCS

CORE REQUIREMENTS FOR
 COMMON       0  VARIABLES     9858  PROGRAM    1558
```

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 END OF COMPILATION
// XEQ
```

Appendix I.

```
STRUCTURE   EXAMPLE NO.1

    DEGREE OF FREEDOM                          4
    DEGREE OF FREEDOM IN SIDESWAY              1
    NUMBER OF MEMBERS                          2
    NUMBER OF MEMBERS HAVING
    PRIMARY AXIAL LOADS                        2

    MODULUS OF ELASTICITY                          30000.0 KSI
    REFERENCE VALUE OF MOMENT OF INERTIA            2880.0 IN**4
    REFERENCE VALUE OF MEMBER LENGTH                 240.0 IN
    STARTING VALUE OF U                                1.0
    INCREMENT OF U                                     0.05

    RATIO OF MOMENT OF INERTIA TO REFERENCE

        1.0000    1.0000

    RATIO OF MEMBER LENGTH TO REFERENCE

        1.0000    1.0000

    RATIO OF AXIAL LOAD TO REFERENCE

        1.0000    1.0000

    THE STATICS MATRIX A

        1.0000    0.0000    0.0000    0.0000    0.0000    1.0000    1.0000
        0.0000    0.0000    0.0000    0.0000    1.0000   -0.0041   -0.0041
        0.0041    0.0041

    THE SIDESWAY FORCE TO SHEAR MATRIX C

       -1.0000    1.0000
```

RESULTS FOR STRUCTURE EXAMPLE NO.1

MEMBER	BETA	UCR	K	PCR(KIPS)
1	1.0000	1.5707	1.9999	3701.10
2	1.0000	1.5707	1.9999	3701.10

```
STRUCTURE  EXAMPLE NO.2

    DEGREE OF FREEDOM                       3
    DEGREE OF FREEDOM IN SIDESWAY           1
    NUMBER OF MEMBERS                       3
    NUMBER OF MEMBERS HAVING
    PRIMARY AXIAL LOADS                     2

    MODULUS OF ELASTICITY                       30000.0 KSI
    REFERENCE VALUE OF MOMENT OF INERTIA          144.0 IN**4
    REFERENCE VALUE OF MEMBER LENGTH              144.0 IN
    STARTING VALUE OF U                             2.5
    INCREMENT OF U                                  0.05

    RATIO OF MOMENT OF INERTIA TO REFERENCE

       1.0000     1.0000     2.0000

    RATIO OF MEMBER LENGTH TO REFERENCE

       1.0000     1.0000     1.6666

    RATIO OF AXIAL LOAD TO REFERENCE

       1.0000     1.0000

    THE STATICS MATRIX A

       0.0000     1.0000     0.0000     0.0000     1.0000     0.0000     0.0000
       0.0000     1.0000     0.0000     0.0000     1.0000    -0.0069    -0.0069
      -0.0069    -0.0069     0.0000     0.0000

    THE SIDESWAY FORCE TO SHEAR MATRIX C

      -1.0000    -1.0000     0.0000
```

RESULTS FOR STRUCTURE EXAMPLE NO.2

MEMBER	BETA	UCR	K	PCR(KIPS)
1	1.0000	2.7738	1.1325	1602.97
2	1.0000	2.7738	1.1325	1602.97

```
STRUCTURE   EXAMPLE NO.3

    DEGREE OF FREEDOM                       3
    DEGREE OF FREEDOM IN SIDESWAY           1
    NUMBER OF MEMBERS                       3
    NUMBER OF MEMBERS HAVING
    PRIMARY AXIAL LOADS                     2

    MODULUS OF ELASTICITY                      30000.0 KSI
    REFERENCE VALUE OF MOMENT OF INERTIA         288.0 IN**4
    REFERENCE VALUE OF MEMBER LENGTH             240.0 IN
    STARTING VALUE OF U                            1.5
    INCREMENT OF U                                 0.05

    RATIO OF MOMENT OF INERTIA TO REFERENCE

       1.0000     1.6666     2.6666

    RATIO OF MEMBER LENGTH TO REFERENCE

       1.0000     2.0000     1.5000

    RATIO OF AXIAL LOAD TO REFERENCE

       2.0000     3.0000

    THE STATICS MATRIX A

       0.0000     1.0000     0.0000     0.0000     1.0000     0.0000     0.0000
       0.0000     1.0000     0.0000     0.0000     1.0000    -0.0041    -0.0041
      -0.0020    -0.0020     0.0000     0.0000

    THE SIDESWAY FORCE TO SHEAR MATRIX C

      -1.0000    -1.0000     0.0000
```

RESULTS FOR STRUCTURE EXAMPLE NO.3

MEMBER	BETA	UCR	K	PCR(KIPS)
1	1.4142	2.3331	1.3465	816.50
2	2.6832	4.4267	0.7096	1224.76

ELASTIC BUCKLING OF RIGID FRAMES

by

Bong Soo Park

B.S., Seoul National University, 1962

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF ARCHITECTURE

Option-Architectural Structures

College of Architecture and Design

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1970

An elastic stability analysis of rigid frames subjected to axial loads only in the primary condition is performed by a matrix formulation of the "exact" method using an electronic digital computer.

As the basic equations, the slope-deflection equations are derived including the influence of axial forces, and the stiffness coefficient for an axially-loaded member is expressed as a function of axial force.

The well-known displacement method for first-order frame analysis is then modified to include the influence of axial force; a second-order spring matrix is added to the statics matrix to account for the effect of axial forces on the end moments due to the lateral displacement, and in the stiffness matrix the stiffness coefficients for every axially loaded member are replaced with the modified values. The buckling criterion is set up by equating the determinant of the load-displacement matrix to zero.

A computer program was written in FORTRAN IV language for the IBM 1130 computer to get the effective length ratio K and the fundamental buckling load P_{cr} for each axially loaded member in the frame.

The advantage of using the "exact" method is demonstrated by the numerical examples in which the more accurate approach resulted in the use of lighter sections since it utilized the unused reserve strength in the frame by performing an over-all frame stability analysis.