

A PATTERN RECOGNITION APPROACH TO GRAIN SAMPLE ANALYSIS

by

ANIL HARIKANT VYAS

B. E., M. S. University, Baroda, India, 1970

613-8301

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1973

Approved by:

Rasid Ahmed
Major Professor

LD
2668
R4
1973
V93
c.2
Docu
ment

TABLE OF CONTENTS

Chapter		Page
I	INTRODUCTION	1
	1.1 Introductory Remarks.	1
	1.2 General Remarks Pertaining to Pattern Recognition Problems.	1
	1.3 Signal Representation For Grain Classification.	3
II	THE TWO-DIMENSIONAL BIFORE TRANSFORM	6
	2.1 Definition.	6
	2.2 The 2-BT Power Spectrum	8
	2.3 A Numerical Example	9
	2.4 Motivation For Using The 2-BT Power Spectrum	11
III	EXPERIMENTAL RESULTS	15
	3.1 Data Collection	15
	3.2 Appendix 3-1.	19
	3.3 Appendix 3-2.	32
IV	GRAIN CLASSIFICATION RESULTS	36
	4.1 Elements of Pattern Classification.	36
	4.2 The Training Algorithm.	38
	4.3 Classification of Corn, Wheat, Barley, Oats and Milo	38
	4.4 Classification Results.	39
	4.5 Appendix 4-1	42
V	CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK.	50
	5.1 Conclusions	50

5.2 Recommendations For Future Work	51
REFERENCES	52
ACKNOWLEDGEMENTS.	53

CHAPTER I

INTRODUCTION

1.1. Introductory Remarks

This report concerns an initial feasibility study of grain sample analysis. All grain sample analysis includes a determination of the amount of material in the sample which is not the grain being analyzed. The methods which are available at present make use of appropriate sieves, followed by a manual hand picking of the remainder. Since this hand picking is both tedious and time consuming, it is performed separately. Clearly, it would be desirable to have a device that can replace the hand picking part of the analysis. Such a device should be capable of distinguishing between long grains such as wheat, barley, oats, rye, etc. with perhaps the greatest challenge being to distinguish between wheat and rye.

The approach entertained in this study is based on pattern recognition techniques. Such techniques may be able to do equally well on separating other types of grain such as corn and soybeans in addition to the long grains mentioned above. Thus, a device which is based on pattern recognition techniques could conceivably be placed ahead of the sieves, except perhaps the sand sieve. To this end, the main objective of this study is to consider some basic aspects pertaining to the feasibility of a pattern recognition system for grain identification. The types of grain used for the study are: (1) corn, (2) wheat, (3) barley, (4) oats and (5) milo.

1.2. General Remarks Pertaining to Pattern Recognition Problems

The basic ideas associated with pattern recognition problems are best introduced by referring to Figure 1-1.

**THIS BOOK
CONTAINS
NUMEROUS PAGES
WITH DIAGRAMS
THAT ARE CROOKED
COMPARED TO THE
REST OF THE
INFORMATION ON
THE PAGE.**

**THIS IS AS
RECEIVED FROM
CUSTOMER.**

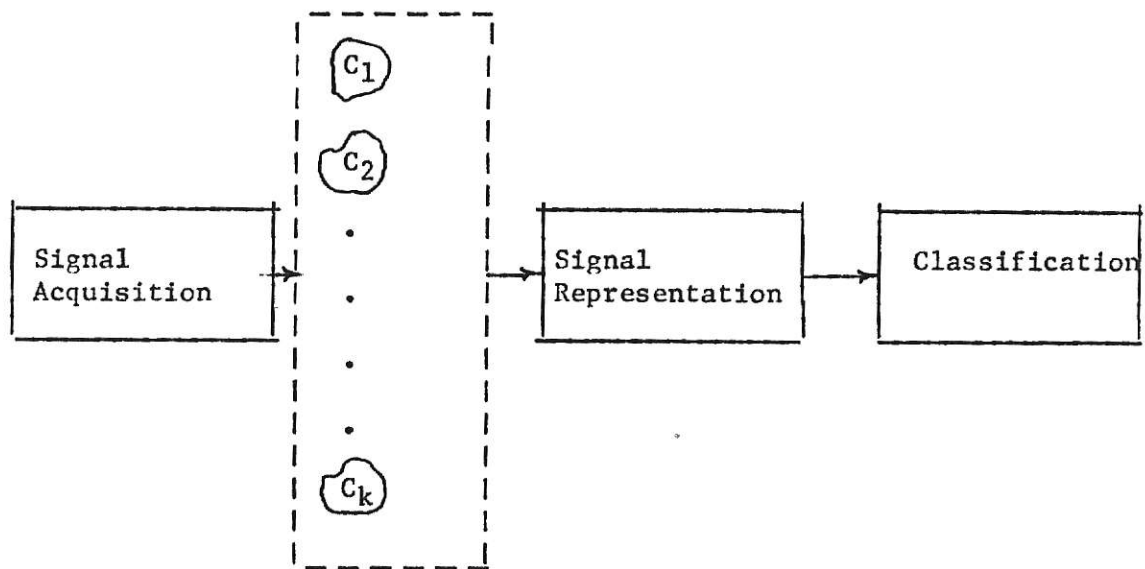


Figure 1-1. Block diagram representation of a pattern recognition problem.

As the name implies, the signal acquisition stage acquires a set of signals from each of the categories or classes which are to be classified. The signals acquired may be one-dimensional or multi-dimensional and hence the complexity of the signal acquisition stage varies from one pattern recognition problem to another.

The output of the signal acquisition stage is denoted by C_i , $i=1, 2, \dots, k$, where C_i represents the i th category or class, each of which consists of N_i signals. The j th signal belonging to class i is denoted by x_{ij} . Thus for each i , j varies from 1 to N_i .

Consider a typical signal x_{ij} which is fed into a signal representation stage is shown in Figure 1-1. The role of the signal representation stage is to seek some "measurements", "features" or "attributes" which will help discriminate a signal x_{ij} from another signal x_{mj} , where, $i \neq m$. The output of this stage corresponding to the input x_{ij} is denoted by P_{ij} which may be

in the form of a finite n -dimensional vector or a finite multi-dimensional array. Thus P_{ij} is generally referred to as a pattern corresponding to the signal x_{ij} .

The classification stage in Figure 1-1 is in essence a device which is "trained" to recognize a set of patterns $\{P_{ij}\}$. Consequently the set of patterns $\{P_{ij}\}$ whose classification is a known a-priori is referred to as the training set. Once the classifier has been trained using the training set, it is conceivable that it will make errors while classifying patterns not belonging to the training set. Therefore the game is to train the classifier in such a way that it makes as few errors as possible. There is a large number of training procedures available in the literature.¹ The procedure which is best suited for a specific application is generally dictated by the nature of the signal representation stage.

In conclusion it is remarked that although the signal representation stage in Figure 1-1 plays a crucial role with respect to the overall system complexity and performance, very little theory is available which enables one to select the "best" measurements or features to represent the set of signals $\{x_{ij}\}$. Feature selection techniques vary from one pattern recognition problem to another. Some aspects of signal representation for the problem at hand are briefly considered in what follows.

1.3. Signal Representation for Grain Classification

The signal representation technique used in this study is simple in that it concerns only the size and shape of a given kernel. Consider a black

1. For an excellent summary, see "A Survey of Pattern Recognition," by W.G.Wee, IEEE Proc. of the Seventh Symposium on Adaptive Processing, December 1968, pp. 2-E-13.

and white image of a grain kernel as shown in Figure 1-2.

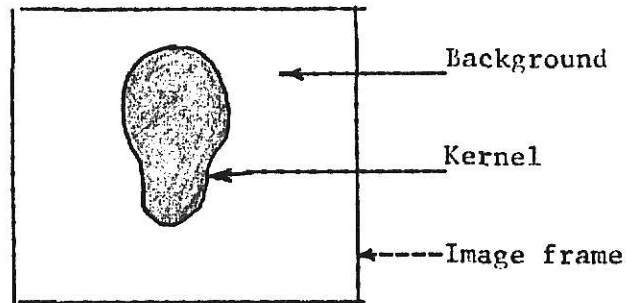
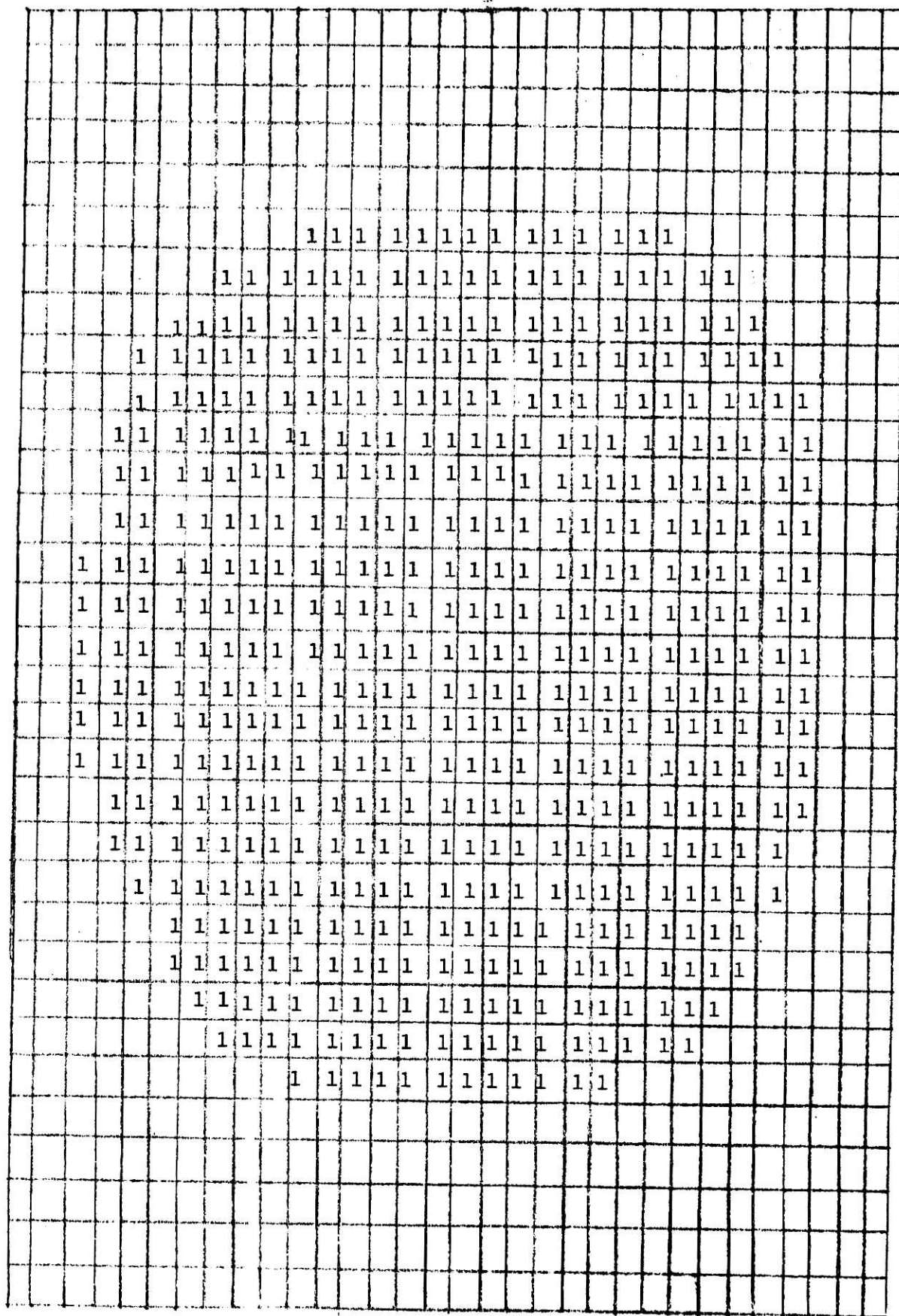


Figure 1-2. Black and White Image of a Kernel

The image frame in Figure 1-2 is now divided into an (32×32) array. Again, each element of this array which contains the kernel is represented by a "1" while that which does not is represented by a "0". An example of the coded image which results by this process is shown in Figure 1-3. Information pertaining to the size and shape of this image frame is extracted by means of a two-dimensional spectral analysis. The transform used for the spectral analysis is analogous to the familiar discrete Fourier transform and is called the Walsh Hadamard or BIFORE (Binary Fourier Representation) transform . A brief introduction to the two-dimensional BIFORE transform is provided in Chapter II.



1-3 Typical output from micro film reader for corn.

(Blank elements of the array consist of zeros).

CHAPTER II

THE TWO-DIMENSIONAL BIFORE TRANSFORM

2.1. Definition

The (32x32) array of zeros and ones of a typical coded image as shown in Figure 3 can be represented by a matrix as follows:

$$[\underline{f}(x_1, x_2)] = \begin{bmatrix} f(0,0) & f(0,1) & \dots\dots\dots & f(0, N_2-1) \\ f(1,0) & f(1,1) & \dots\dots\dots & f(1, N_2-1) \\ & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ f(N_1-1,0) & f(N_1-1,1) & \dots\dots\dots & f(N_1-1, N_2-1) \end{bmatrix} \quad (2-1)$$

where ¹ $N_1=N_2=32$ and each of the $f(i,j)$ is a zero or a one². Then the two dimensional BIFORE transform (2-BT) of the data matrix $\underline{f}(x_1, x_2)$ in (2-1) is defined as

$$[F(u_1, u_2)] = \frac{1}{N_1 N_2} \sum_{x_1=0}^{N_1-1} \sum_{x_2=0}^{N_2-1} f(x_1, x_2) (-j)^{\langle x, u \rangle} \quad (2-2)$$

where

$F(u_1, u_2)$ is a transform coefficient

$f(x_1, x_2)$ is an input data point

$u_i = 0, 1, \dots, (N_i-1); i=1, 2$

$$\langle x_1, u_1 \rangle = \sum_{m=0}^{n_1-1} u_1(m) x_1(m) \quad (2-3)$$

1. Note that N_1 need not be equal to N_2

2. In general $f(i,j)$ can be any finite real number

$$\langle x, u \rangle = \langle x_1, u_1 \rangle + \langle x_2, u_2 \rangle$$

and

$$n_i = \log_2 N_i, i=1, 2.$$

The terms $u_i(m)$ and $x_i(m)$ in (2-3) are the binary representations of u_i and x_i respectively. For example,

$$\left[u_i \right]_{\text{decimal}} = \left[u_i(k_i-1), u_i(k_i-2), \dots, u_i(1), u_i(0) \right]_{\text{binary}} \quad (2-4)$$

where $u_i(k) \in \{0, 1\}$.

Alternately, (2-2) can be written in the form of a matrix to obtain

$$\left[F(u_1, u_2) \right] = \frac{1}{N_1 N_2} \left[H(n_1) \right] \left[f(x_1, x_2) \right] \left[H(n_2) \right] \quad (2-5)$$

where

$\left[F(u_1, u_2) \right]$ is a $(N_1 \times N_2)$ transform matrix corresponding to the data matrix $\left[f(x_1, x_2) \right]$

$\left[H(n_1) \right]$ and $\left[H(n_2) \right]$ are $(N_1 \times N_1)$ and $(N_2 \times N_2)$ Hadamard matrices with $n_i = \log N_i, i=1, 2$.

The Hadamard matrices in (2-5) are defined by the recurrence relation

$$\left[H(k+1) \right] = \begin{bmatrix} H(k) & \vdots & H(k) \\ \hline H(k) & \vdots & -H(k) \end{bmatrix} \quad k=0, 1, \dots, n_i \quad (2-6)$$

$$\left[H(0) \right] = 1$$

From (2-6) it follows that the elements of a Hadamard matrix are either +1 or -1.

Using the fact that the 2-BT is an orthogonal transform, it can be shown that (2-7) the corresponding inverse transform (2-IBT) is defined as

$$\underline{f}(x_1, x_2) = \sum_{u_1=0}^{N_1-1} \sum_{u_2=0}^{N_2-1} F(u_1, u_2) \quad (-1)^{\langle x, u \rangle} \quad (2-7)$$

or alternately as

$$[\underline{f}(x_1, x_2)] = [H(n_1)] [\underline{F}(u_1, u_2)] [H(n_2)] \quad (2-8)$$

2.2. The 2-BT Power Spectrum

A BIFORE power spectrum which is analogous to a two-dimensional Fourier power spectrum can be defined as

$$P(z_1, z_2) = \sum_{u_1=\lfloor 2^{z_1-1} \rfloor}^{2^{z_1-1}-1} \sum_{u_2=\lfloor 2^{z_2-1} \rfloor}^{2^{z_2-1}-1} F^2(u_1, u_2) \quad (2-9)$$

where

$$z_i = 0, 1, \dots, k_i; \quad n_i = \log_2 N_i, \quad i=1, 2.$$

and

$$\lfloor 2^{z_i-1} \rfloor \text{ is the integer part of } 2^{z_i-1}.$$

From (2-9) it follows that the 2-BT power spectrum can be expressed in matrix form to obtain

$$[\underline{P}(k_1, k_2)] = \begin{bmatrix} P(0,0) & P(0,1) & \dots & P(0,n_2) \\ P(1,0) & P(1,1) & \dots & P(1,n_2) \\ \dots & \dots & \dots & \dots \\ P(n_1,1) & P(n_1,2) & \dots & P(n_1,n_2) \end{bmatrix} \quad (2-10)$$

Inspection of (2-10) reveals that the number of 2-BT power spectrum points is given by

$$\sum = (1+n_1)(1+n_2); \quad n_i = \log_2 N_i, \quad i=1, 2 \quad (2-11)$$

2.3 A Numerical Example

Before proceeding further, it is instructive to consider a simple numerical example which helps clarify the material discussed above. Suppose the 2-BT and 2-BT power spectrum of the pattern shown in Figure 2-1 are desired.

0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Figure 2-1. An (8x8) pattern

From Figure 2-1 it follows that $N_1=N_2=8$. Thus (2-5) yields

$$\left[\underline{F}(u_1, u_2) \right] = \frac{1}{64} \left[H(3) \right] \left[\underline{f}(x_1, x_2) \right] \left[H(3) \right] \quad (2-12)$$

Using (2-6) to evaluate $[H(3)]$ and subsequently substituting in (2-12) results in the following matrix equation.

$$[F(u_1, u_2)] = \frac{1}{64} [H(3)] \quad [H(3)] \quad (2-13)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$[H(3)] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ \hline 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \quad (2-14)$$

Evaluation of (2-13) yields the transform matrix

$$\left[\underline{F}(u_1, u_2) \right] = \frac{1}{64} \begin{bmatrix} 16 & 0 & 0 & 4 & 0 & -4 & -16 & 0 \\ 0 & 0 & 0 & -4 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & -4 & 0 \\ -4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 12 & 0 & 0 & 0 & 0 & 0 & -12 & 0 \\ -4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix} \quad (2-15)$$

Substitution of the values of $\underline{F}(u_1, u_2)$ obtained from (2-15) into (2-9) results in the following 2-BT power spectrum in matrix form.

$$\underline{P}(k_1, k_2) = \frac{1}{256} \begin{bmatrix} 16 & 0 & 1 & 17 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 12 & 0 & 0 & 12 \end{bmatrix} \quad (2-16)$$

2.4. Motivation for Using the 2-BT Power Spectrum.

In this study, we attempt to characterize the size and shape of grain kernels by the $(1+\log_2 N_1)$ $(1+\log_2 N_2)$ power spectrum points given by the matrix $\left[\underline{P}(k_1, k_2) \right]$ in (2-10). There are basically three reasons which provide the motivation for doing so. These are discussed in what follows.

(1). The power spectrum represents the distribution of power in a given two-dimensional pattern. This is best illustrated by a simple example with $N_1=2, N_2=4$ and

$$\left[\underline{f}(x_1, x_2) \right] = \begin{bmatrix} 3 & 0 & 3 & 4 \\ 3 & 8 & 7 & 8 \end{bmatrix} \quad (2-17)$$

Applying (2-5) and (2-9) to (2-12) results in the following 2-BT power spectrum

$$\begin{aligned} P(0,0) &= 81/4, & P(0,1) &= 1/4, & P(0,2) &= 1 \\ P(1,0) &= 4, & P(1,1) &= 1, & P(1,2) &= 1. \end{aligned} \quad (2-18)$$

Now, it can be shown that $[\underline{f}(x_1, x_2)]$ in (2-17) can be decomposed into the following mutually orthogonal sub patterns:

$$[\underline{f}_{00}(x_1, x_2)] = \begin{bmatrix} 4.5 & 4.5 & 4.5 & 4.5 \\ 4.5 & 4.5 & 4.5 & 4.5 \end{bmatrix}$$

$$[\underline{f}_{01}(x_1, x_2)] = \begin{bmatrix} 0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix}$$

$$[\underline{f}_{02}(x_1, x_2)] = \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$[\underline{f}_{10}(x_1, x_2)] = \begin{bmatrix} -2 & -2 & -2 & -2 \\ 2 & 2 & 2 & 2 \end{bmatrix}$$

$$[\underline{f}_{11}(x_1, x_2)] = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

and

$$[\underline{f}_{12}(x_1, x_2)] = \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \end{bmatrix} \quad (2-19)$$

It is straight forward to verify that

$$[\underline{f}(x_1, x_2)] = \sum_{i=0}^2 [\underline{f}_{0i}(x_1, x_2)] + \sum_{i=0}^2 [\underline{f}_{1i}(x_1, x_2)] \quad (2-20)$$

Inspection of the sub-patterns in (2-19) reveals that $\left[\underline{f}_{ij}(x_1, x_2) \right]$ is 2^i - periodic with respect to the first dimension (i. e. the columns) and 2^j - periodic in the second dimension (i. e. the rows). If

$\sum_{i,j} \left[\underline{f}_{ij}(x_1, x_2) \right]^2$ denotes the sum of the squares of the elements in the sub-pattern $\left[\underline{f}_{i,j}(x_1, x_2) \right]$, divided by $N_1 N_2$, then it can be verified that

$$\begin{aligned} \sum_{i,j} \left[\underline{f}_{00}(x_1, x_2) \right]^2 &= 81/4 & \sum_{i,j} \left[\underline{f}_{01}(x_1, x_2) \right]^2 &= 1/4 \\ \sum_{i,j} \left[\underline{f}_{02}(x_1, x_2) \right]^2 &= 1 & \sum_{i,j} \left[\underline{f}_{10}(x_1, x_2) \right]^2 &= 4 \\ \sum_{i,j} \left[\underline{f}_{11}(x_1, x_2) \right]^2 &= 1 & \sum_{i,j} \left[\underline{f}_{12}(x_1, x_2) \right]^2 &= 1 \end{aligned} \quad (2-21)$$

Comparing (2-18) and (2-21) it is clear that each of the 2-BT power spectrum points represents the average power in one of the mutually orthogonal sub-patterns $\left[\underline{f}_{ij}(x_1, x_2) \right]$. Thus the 2-BT power spectrum represents the distribution of power in a given two-dimensional pattern. In the problem at hand such two-dimensional patterns are coded images (see Figure 1-3) which characterize the sizes and shapes of grain kernels.

(2). From the periodicities present in the sub-patterns $\left[\underline{f}_{ij}(x_1, x_2) \right]$, the analogy between the 2-BT power spectrum and the familiar discrete Fourier power spectrum is apparent. Again, the property that the Fourier power spectrum is invariant with respect to cyclic shifts without rotation of a pattern as illustrated in Figure 2-2 is also valid for the BIFORE spectrum. However, the 2-BT power spectrum yields considerable data compression since it consists of $(1+\log_2 N_1)(1+\log_2 N_2)$ spectrum points in contrast to $(N_1/2+1)(N_2/2+1)$ independent discrete Fourier spectrum points.

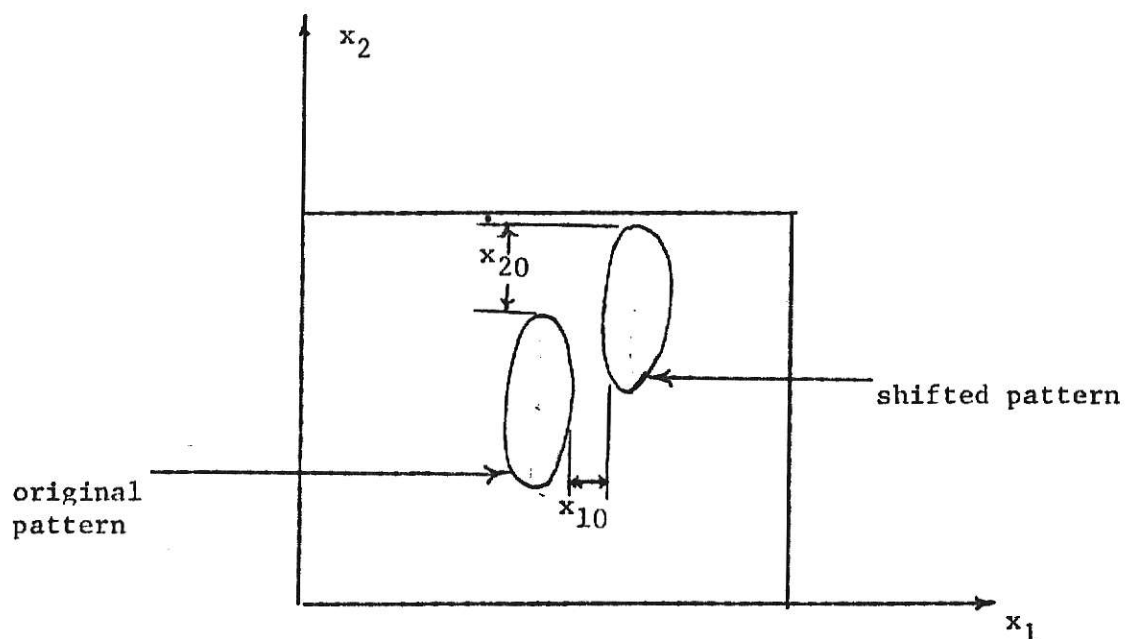


Figure 2-2 Illustration of the shift-invariance property.

(3). The 2-BT power spectrum can be computed rapidly using an algorithm called the fast BIFORE transform (FBT). The corresponding computer program is included in Appendix 3-2. Since only real arithmetic operations are involved, the corresponding implementation is simpler relative to that of the Fourier spectrum which requires complex arithmetic operations.

CHAPTER III

EXPERIMENTAL RESULTS

3-1. Data Collection

A block diagram of the set up used to obtain the data is shown in Figure 3-1. Each kernel was placed at the base of a microfilm reader and projected on its screen to which a 32x32 grid was attached. Each square of the grid which was occupied by the kernel was coded by a "1" and by a "0" otherwise. A typical output obtained from this stage of the data processing is shown in Figure 3-2. Before such data was recorded, the reader was calibrated using a standard circular pattern which is shown below in Figure 3-3. The image coded pattern corresponding to this



Figure 3-3. Calibration pattern for microfilm reader

calibration pattern is shown in Figure 3-4.

The above output of the film reader which was in the form of a 32x32 array of ones and zeros was punched on IBM cards and subsequently fed to an IBM 360 computer. The computer program listed in Appendix 3-2 was used to compute the 2-BT power spectra of several grain kernels placed in various orientations for each kernel. The corresponding 2-BT power spectra that resulted are summarized in Appendix 3-1. For convenience, each spectrum point in this table has been multiplied by a scale factor of 10^3 .

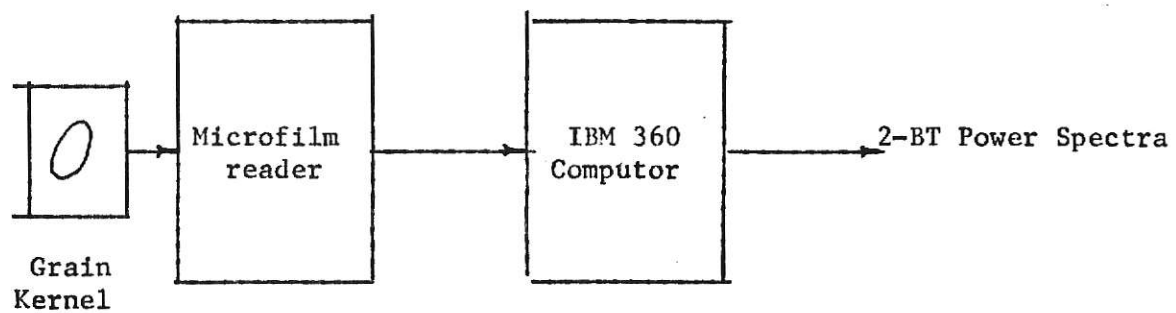


Figure 3-1. Block diagram of set up to gather data

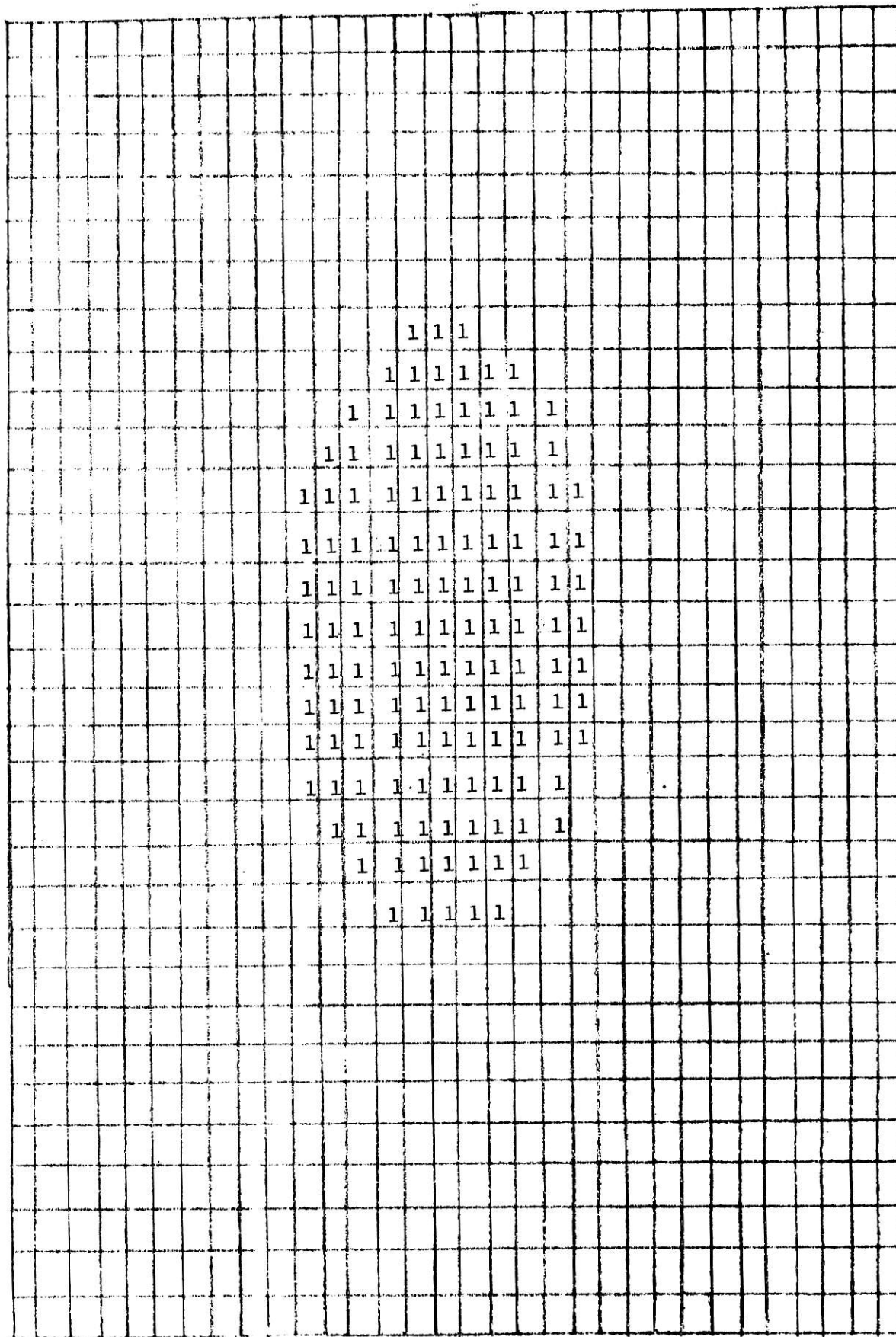


Figure 3-2. Typical output from microfilm reader.

(Blank elements of the array consist of zeros).

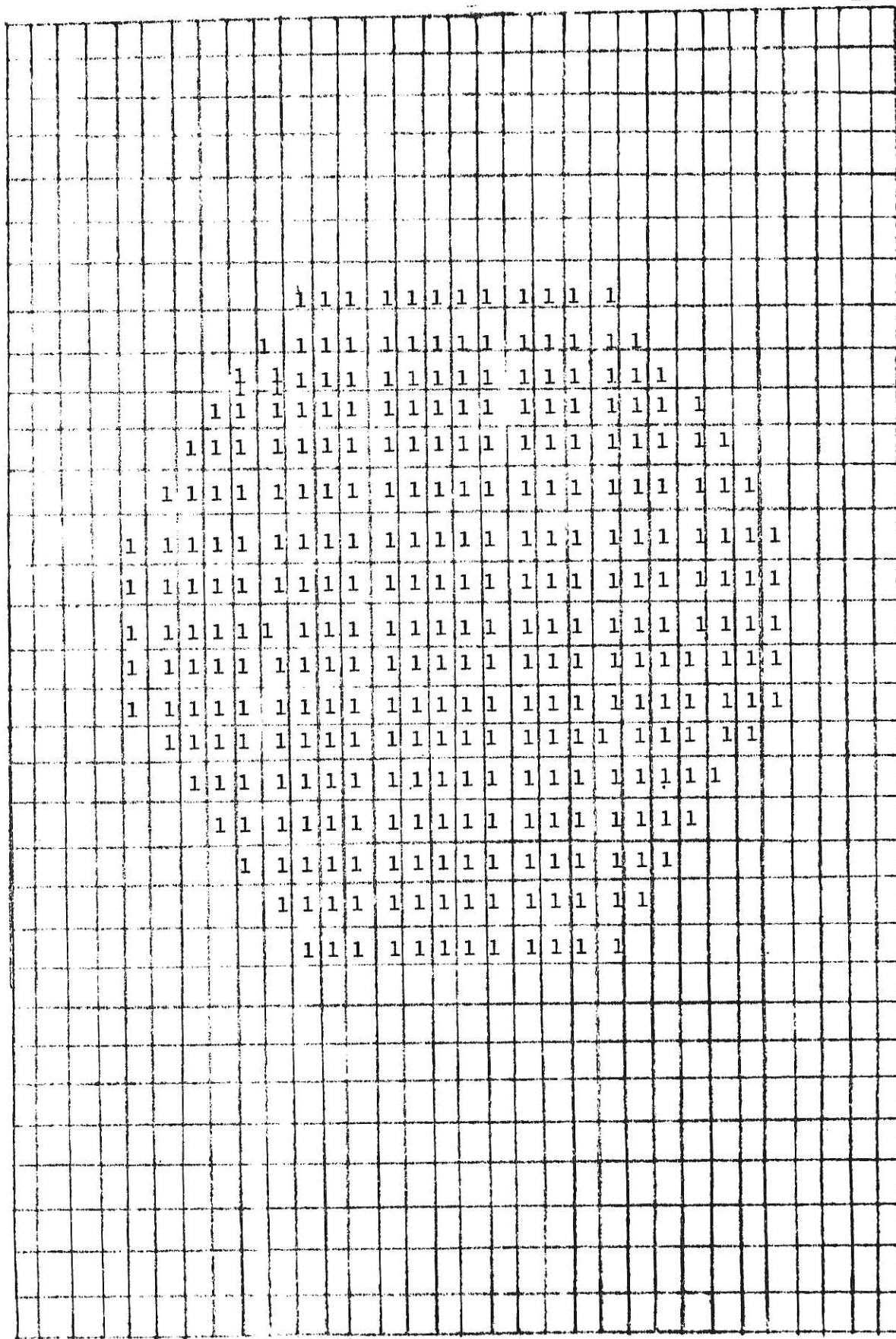


Figure 3-3. Image of calibration pattern.

(Blank elements of the array consist of zeros)

APPENDIX 3-1

In this appendix, the 2-BT power spectra of various kernels of corn, wheat, barley, oats and milo are included. The spectrum points are denoted by $P(i,j)$, $i,j=0,1,\dots,5$.

	P(0,1)	P(1,1)	P(2,1)	P(3,1)	P(4,1)	P(5,1)	P(0,2)	P(1,2)	P(2,2)	P(3,2)	P(4,2)	P(5,2)	P(0,3)	P(1,3)	P(2,3)	P(3,3)	P(4,3)	P(5,3)
corn 3	0	0.02	0	0.07	0.15	0.24	0.08	0.07	0.02	0.23	0.34	0.85	1.12	0.08	0.23	0.76	2.44	3.17
	0	0.02	0.02	0.02	0.12	0.24	0.06	0.02	0.14	0.32	0.31	0.98	0.54	0.07	0.31	0.31	1.95	4.39
	0	0.05	0	0.03	0.11	0.26	0.09	0.02	0.04	0.05	0.75	0.95	0.15	0.01	0.16	0.50	1.68	2.99
	0	0.03	0.05	0.02	0.08	0.24	0.06	0	0.21	0.24	0.46	1.10	0.68	0.07	0.32	0.76	2.81	3.17
	0.05	0	0.09	0.06	0.14	0.23	0.09	0.03	0.16	0.16	0.32	1.13	0.78	0.11	0.16	0.50	1.19	3.97
5	0	0.02	0	0.01	0.04	0.20	0.16	0.03	0.07	0.16	0.66	0.82	0.09	0.06	0.27	0.35	2.11	1.89
	0	0.02	0.01	0.05	0.08	0.15	0.04	0.01	0.12	0.14	0.31	0.85	0.43	0.07	0.41	0.73	1.40	2.32
	0	0.02	0.01	0	0.07	0.23	0.03	0	0.03	0.19	0.14	0.52	0.21	0.06	0.36	0.56	0.95	1.15
	0.03	0	0.01	0.03	0.05	0.12	0	0.02	0.05	0.12	0.24	0.79	0.43	0.05	0.24	0.73	1.46	3.91
	0.01	0.01	0.03	0.03	0.08	0.05	0.22	0	0.03	0.07	0.38	0.82	0.29	0.04	0.27	0.59	1.80	1.53
9	0.10	0.01	0	0.01	0.06	0.24	0.2	0.04	0.05	0.32	0.43	0.79	1.47	0.01	0.08	0.27	0.73	1.83
	0.02	0	0.05	0.01	0.08	0.17	0.05	0	0.22	0.14	0.29	0.58	0.58	0.10	0.19	0.87	1.37	2.14
	0	0	0.01	0.06	0.07	0.20	0.02	0.7	0.12	0.21	0.47	0.64	0.32	0.05	0.40	0.78	1.07	2.87

	$P(0,1)$	$P(1,1)$	$P(2,1)$	$P(3,1)$	$P(4,1)$	$P(5,1)$	$P(0,2)$	$P(1,2)$	$P(2,2)$	$P(3,2)$	$P(4,2)$	$P(5,2)$	$P(0,3)$	$P(1,3)$	$P(2,3)$	$P(3,3)$	$P(4,3)$	$P(5,3)$
corn 9	0.05	0	0.06	0.01	0.08	0.32	0.24	0.01	0.02	0.21	0.35	0.82	1.29	0.06	0.07	0.75	0.95	2.62
	0.01	0.01	0.02	0.04	0.14	0.17	0.12	0.3	0.05	0.11	0.44	1.01	0.32	0.04	0.08	0.69	2.11	3.97
12	0	0	0.02	0.07	0.06	0.21	0.04	0.06	0.04	0.2	0.34	0.55	0.43	0.01	0.14	0.76	1.10	3.17
	0.01	0	0.03	0.03	0.04	0.17	0.02	0.02	0.09	0.19	0.26	0.58	0.43	0	0.04	0.32	0.89	2.01
	0.01	0	0.05	0.04	0.07	0.23	0.05	0.02	0.16	0.21	0.44	0.64	0.10	0.03	0.10	0.56	1.19	1.89
	0.01	0	0	0.04	0.02	0.08	0.12	0.02	0.03	0.11	0.29	0.58	0.77	0.03	0.07	0.44	0.81	2.14
	0	0	0.04	0.03	0.14	0.15	0.02	0	0.02	0.14	0.24	0.55	0.16	0.02	0.05	0.34	1.10	2.44
13	0	0.05	0.03	0.04	0.12	0.23	0.05	0	0.03	0.11	0.56	0.52	0.19	0.19	0.26	1.16	0.94	2.75
	0.01	0.08	0.1	0.05	0.11	0.20	0.01	0.05	0.04	0.10	0.26	0.70	0.36	0.03	0.21	0.53	0.64	1.77
	0.11	0.01	0	0.02	0.21	0.11	0.21	0.03	0.03	0.08	0.17	1.01	1.15	0.04	0.18	0.26	1.13	2.49
	0.03	0	0.01	0.02	0.09	0.15	0.20	0.04	0.05	0.21	0.46	0.98	2.49	0.02	0.15	0.27	0.85	3.30
	0	0.02	0	0.02	0.08	0.18	0.06	0.06	0.07	0.11	0.61	0.79	0.48	0.05	0.27	0.52	1.34	2.93

$P(0,0)$	$P(1,0)$	$P(2,0)$	$P(3,0)$	$P(4,0)$	$P(5,0)$	$P(0,4)$	$P(1,4)$	$P(2,4)$	$P(3,4)$	$P(4,4)$	$P(5,4)$	$P(0,5)$	$P(1,5)$	$P(2,5)$	$P(3,5)$	$P(4,5)$	$P(5,5)$
corn 3 383.34	0	0.05	0.77	15.50	75.32	11.80	0.17	0.49	2.32	3.30	11.72	49.88	0.21	1.16	2.93	17.82	32.71
366.59	0	0.02	0.57	15.32	70.07	4.65	0.17	0.55	1.34	4.03	24.90	37.99	0.15	1.28	3.78	19.29	44.92
351.38	0.01	0.12	1.94	15.34	79.80	0.806	0.08	0.38	1.07	4.70	15.50	89.98	0.23	0.22	3.36	9.40	22.29
365.41	0	0.02	0.46	3.17	34.59	13.55	0.08	0.32	1.80	3.85	19.9	77.01	0.17	1.01	5.9	23.32	47.51
378.51	0	0.06	1.78	10.33	58.47	9.51	0.11	0.40	1.95	4.15	18.55	66.73	.05	0.67	3.78	13.92	35.66
5 226.18	0	0.19	2.57	7.64	105.36	11.77	0.04	0.09	1.25	2.14	15.93	54.12	0.17	0.82	2.87	15.50	21.73
228.49	0	0.04	1.46	0.78	136.32	2.84	0.06	0.46	3.11	5.25	11.72	32.01	0.40	0.67	3.54	20.02	23.44
224.30	0	0.5	0.31	1.01	188.89	1.92	0.14	0.60	0.82	2.75	5.25	9.32	0.23	0.3	4.58	13.79	13.43
221.56	0	0.13	0.82	2.52	120.87	3.92	0.11	0.37	0.85	3.05	18.55	44.71	0.31	0.79	5.00	22.46	18.01
202.46	0.06	0.06	3.57	6.68	117.37	13.73	0.04	0.37	0.56	1.64	13.70	44.04	0.16	0.52	3.85	12.63	22.34
9 203.56	0.06	0.05	0.92	19.38	74.28	7.75	0.12	0.24	1.28	2.81	19.04	81.42	0.06	0.67	2.08	14.83	24.90
207.99	0.05	0.08	0.04	8.69	70.97	7.59	0.08	0.35	1.19	4.09	14.77	80.18	0.17	0.52	2.99	17.46	3.98
204.44	0.01	0.03	0.55	0.87	143.39	1.11	0.13	0.06	3.45	7.39	6.47	16.3	0.20	0.19	2.14	13.79	42.40

$P(0,0)$	$P(1,0)$	$P(2,0)$	$P(3,0)$	$P(4,0)$	$P(5,0)$	$P(0,4)$	$P(1,4)$	$P(2,4)$	$P(3,4)$	$P(4,4)$	$P(5,4)$	$P(0,5)$	$P(1,5)$	$P(2,5)$	$P(3,5)$	$P(4,5)$	$P(5,5)$
corn 9																	
220.64	0.08	0.06	0.53	11.01	70.08	9.24	0.05	0.06	1.62	2.14	18.68	87.05	0.20	0.82	1.65	15.50	22.20
190.55	0	0.01	0.35	8.6	61.87	6.19	0.14	0.35	1.19	6.9	12.82	78.66	0.11	0.76	1.65	11.35	45.65
12 165.04	0.03	0.02	0.37	0.21	121.25	2.88	0.14	0.25	2.14	6.35	13.18	39.43	0.18	1.04	4.03	15.87	26.27
175.82	0.01	0.10	0.16	1.81	151.28	2.85	0.10	0.69	3.27	3.23	9.16	2.59	0.23	0.90	4.23	14.53	25.5
167.4	0.01	0.16	1.63	1.41	111.81	5.99	0.04	0.35	1.37	3.36	14.04	46.55	0.17	0.70	4.46	14.28	28.23
170.64	0.02	0.19	0.41	3.07	174.33	5.30	0.05	0.17	0.64	1.53	1.69	13.75	0.14	0.46	1.65	5.78	28.73
171.45	0.0	0.06	2.43	2.91	108.33	9.03	0.03	0.21	0.73	1.71	15.14	51.09	0.12	0.55	4.66	16.40	25.88
13 210.04	0.02	0.05	1.35	1.21	121.02	17.0	0.2	0.74	1.93	9.4	11.94	22.52	0.28	0.98	4.08	13.75	50.4
193.28	0.12	0.02	0.81	3.93	93.86	3.96	0.18	0.66	2.57	9.58	13.55	41.21	0.20	0.76	4.09	12.08	53.7
201.18	0.01	0.04	0.83	11.1	70.27	6.77	0.08	0.26	1.2	2.24	19.47	84.92	0.05	0.64	1.41	15.43	25.01
186.31	0	0.07	0.53	10.7	61.70	5.0	0.09	0.46	1.53	2.4	17.3	9.24	0.34	0.61	2.09	14.40	28.86
181.29	0	0.03	0.21	7.61	66.80	6.48	0.05	0.27	1.59	7.2	12.7	72.60	0.01	0.92	2.69	10.0	47.36

$P(0,0)$	$P(1,0)$	$P(2,0)$	$P(3,0)$	$P(4,0)$	$P(5,0)$	$P(0,4)$	$P(1,4)$	$P(2,4)$	$P(3,4)$	$P(4,4)$	$P(5,4)$	$P(0,5)$	$P(1,5)$	$P(2,5)$	$P(3,5)$	$P(4,5)$	$P(5,5)$
wheat 1																	
**17.9	0.01	0.08	0.64	18.62	37.25	0.25	0.07	0.26	1.07	1.65	3.30	11.25	0.14	0.27	0.67	12.33	24.66
** 17.64	0.0	0.19	0.24	10.41	28.47	0.18	0.03	0.09	0.31	2.81	3.42	14.32	0.15	0.67	2.08	15.38	33.20
*15.14	0.0	0.02	0.54	1.66	17.36	9.32	0.08	0.12	0.49	0.98	10.99	24.96	0.12	0.31	1.22	4.15	30.76
*13.28	0.0	0.01	0.56	3.30	17.15	2.70	0.05	0.24	0.43	6.0	9.52	16.36	0.12	0.09	1.59	10.25	28.81
*14.19	0.0	0.02	0.56	4.91	19.68	2.56	0.06	0.31	0.37	4.52	7.81	17.15	0.12	0.03	1.59	10.50	29.79
5 *16.12	0.0	0.02	0.59	3.88	20.68	1.89	0.09	0.34	0.49	5.79	8.54	18.34	0.15	0.55	1.95	10.74	31.74
*14.90	0.0	0.01	0.31	5.03	20.22	0.80	0.07	0.32	0.21	6.29	7.69	15.88	0.14	0.52	1.16	12.82	30.52
*16.12	0.0	0.03	0.22	1.05	17.43	10.65	0	0.15	0.55	0.61	11.69	27.16	0	0.31	1.10	3.17	31.74
**16.87	0.01	0.02	0.10	14.87	31.88	0.22	0.10	0.32	0.40	1.77	2.81	9.54	0.20	0.70	3.36	15.26	29.05
**13.28	0	0.86	3.82	17.96	35.92	0.61	0.03	0.09	0.61	1.34	2.69	5.92	0.12	1.28	1.71	9.03	18.07
6 * 9.73	0	0.01	0.39	3.14	13.26	6.75	0.07	0.17	1.01	1.05	9.64	16.65	0.11	0.27	1.65	5.98	24.66
* 9.92	0	0.02	0.21	4.44	14.59	4.33	0.03	0.21	0.67	2.81	8.06	14.56	0.09	0.49	1.71	8.06	24.90
** 9.54	0	0.06	0.02	6.70	16.33	1.65	0.06	0.12	0.49	3.78	6.10	11.54	0.12	0.31	0.98	11.47	24.41

$P(0,0)$	$P(1,0)$	$P(2,0)$	$P(3,0)$	$P(4,0)$	$P(5,0)$	$P(0,4)$	$P(1,4)$	$P(2,4)$	$P(3,4)$	$P(4,4)$	$P(5,4)$	$P(0,5)$	$P(1,5)$	$P(2,5)$	$P(3,5)$	$P(4,5)$	$P(5,5)$
wheat 6																	
** 9.35	0.02	0.30	1.74	11.41	22.81	0.11	0.08	0.26	0.46	0.92	1.83	8.87	0.14	0.70	1.65	11.35	22.71
** 10.31	0	0.02	0.02	8.32	18.68	0.87	0.02	0.09	0.49	3.17	4.64	11.51	0.03	0.18	0.98	12.70	25.39
8 ** 17.38	0	0.35	2.06	19.78	39.57	0.3	0.05	0.35	1.68	2.38	4.76	8.47	0.11	0.76	0.79	10.13	20.26
* 19.50	0	0	0.22	0.98	20.71	10.19	0.08	0.14	0.27	0.67	11.35	30.26	0.11	0.21	1.04	3.3	34.91
* 14.43	0.01	0.07	0.24	1.50	16.25	7.68	0.05	0.11	0.75	2.99	11.60	22.54	0.08	0.27	1.65	5.49	30.03
18.96	0	0.01	0.74	6.58	26.29	0.25	0.10	0.41	0.52	4.46	5.74	16.62	0.14	0.64	1.53	15.50	34.42
19.50	0.02	0.02	0.54	5.81	25.89	0.36	0.05	0.23	0.64	5.43	6.71	17.96	0.14	0.64	1.10	14.77	34.91
10 8.43	0	0.10	0.16	5.51	14.19	1.5	0.08	0.15	0.79	4.76	6.84	9.64	0.12	0.37	1.34	11.47	22.95
7.9	0	0	0.23	2.63	10.76	6.8	0.02	0.11	0.58	2.14	9.64	14.88	0.05	0.27	1.28	5.74	22.22
** 8.25	0.02	0.25	2.86	11.39	22.78	0.02	0.02	0.47	0.40	0.92	1.83	6.94	0.05	0.89	2.01	9.89	19.78
** 8.43	0	0.05	0.08	6.81	15.38	0.93	0.05	0.31	0.79	3.74	5.86	9.55	0.09	0.49	1.34	11.47	22.95
8.25	0	0	0.16	2.53	10.94	5.42	0.02	0.26	0.56	3.6	9.89	13.87	0.08	0.40	1.40	6.96	22.71

	P(0,1)	P(1,1)	P(2,1)	P(3,1)	P(4,1)	P(5,1)	P(0,2)	P(1,2)	P(2,2)	P(3,2)	P(4,2)	P(5,2)	P(0,3)	P(1,3)	P(2,3)	P(3,3)	P(4,3)	P(5,3)
wheat 1	** 0	0.01	0.02	0.01	0.04	0.08	0	0	0.03	0.07	0.11	0.21	0.17	0.05	0.13	0.35	0.70	1.40
	** 0	0	0	0.02	0.05	0.09	0.04	0	0.01	0.05	0.15	0.24	0.05	0.05	0.14	0.24	0.49	0.98
	* 0	0	0	0.01	0.02	0.03	0.25	0	0.05	0.03	0.34	0.67	0.25	0.04	0.11	0.15	1.16	0.71
	* 0	0	0.04	0.02	0.06	0.12	0.04	0.01	0.05	0.15	0.18	0.43	0.34	0.06	0.15	0.43	0.61	0.59
	* 0	0.02	0	0.01	0.06	0.08	0.1	0.02	0.02	0.08	0.15	0.37	0.29	0.03	0.08	0.58	0.85	1.83
5	*0.02	0	0	0.04	0.03	0.09	0.05	0	0.02	0.05	0.18	0.31	0.27	0.05	0.17	0.79	0.92	2.20
	*0.0	0.01	0.02	0.05	0.05	0.14	0.03	0.05	0.03	0.08	0.20	0.40	0.15	0.01	0.13	0.49	1.25	2.01
	* 0	0	0.02	0.05	0.02	0.09	0.22	0	0.05	0.06	0.34	0.67	0.17	0	0.05	0.21	1.16	1.59
	** 0	0	0.01	0.03	0.04	0.08	0.02	0.05	0.10	0.04	0.14	0.34	0.12	0.03	0.14	0.11	0.40	0.79
	** 0	0	0	0.01	0.02	0.03	0.01	0	0.04	0.05	0.09	0.16	0.14	0.03	0.05	0.15	0.37	0.73
6	* 0	0	0	0.02	0.05	0.08	0.12	0.02	0.04	0.07	0.26	0.52	0.04	0.01	0.05	0.07	0.26	0.52
	* 0	0	0.02	0.02	0.08	0.12	0.02	0	0.05	0.23	0.12	0.43	0.28	0.05	0.18	0.58	0.61	0.71
	** 0	0.03	0	0.01	0.05	0.09	0.02	0.01	0.04	0.03	0.21	0.31	0.34	0.02	0.08	0.43	0.73	0.59

	P(0,1)	P(1,1)	P(2,1)	P(3,1)	P(4,1)	P(5,1)	P(0,2)	P(1,2)	P(2,2)	P(3,2)	P(4,2)	P(5,2)	P(0,3)	P(1,3)	P(2,3)	P(3,3)	P(4,3)	P(5,3)
wheat 6	**	0	0.01	0	0.01	0.02	0.05	0	0.01	0.09	0.06	0.14	0.27	0.01	0.01	0.18	0.14	0.67
	**	0	0	0	0.01	0.05	0.06	0.02	0	0.04	0.06	0.18	0.31	0.30	0.01	0.03	0.40	1.71
8	**	0	0.01	0	0.03	0.04	0.08	0.02	0.01	0.04	0.10	0.17	0.34	0.29	0.03	0.02	0.11	0.92
	*	0	0	0	0.02	0.02	0.05	0.03	0.02	0.09	0.11	0.02	0.46	0.06	0.01	0.13	0.87	1.89
	*	0	0	0.01	0.03	0.07	0.14	0.12	0.01	0.03	0.10	0.32	0.58	0.02	0.04	0.01	0.41	2.14
	0.02	0.01	0.01	0.01	0.04	0.04	0.08	0.17	0	0.05	0.16	0.20	0.58	0.25	0	0.04	0.47	1.53
	0.01	0.01	0	0	0.04	0.08	0.08	0.03	0.01	0.04	0.18	0.29	0.46	0.06	0.10	0.22	0.63	1.77
10	0.02	0.02	0	0.02	0.05	0.09	0.09	0.02	0.01	0.04	0.06	0.24	0.37	0.14	0.02	0.08	0.31	1.46
	0	0	0	0	0.01	0.02	0.02	0	0.01	0.01	0.02	0.05	0.09	0.01	0.01	0.24	0.20	0.92
	**	0	0.02	0.01	0.03	0.06	0.06	0.05	0	0.04	0.09	0.12	0.31	0.14	0.02	0.08	0.37	1.34
	**	0	0.02	0.03	0.07	0.14	0.14	0.03	0	0.10	0.13	0.14	0.41	0.13	0.02	0.05	0.32	1.28
	0.02	0	0.01	0.07	0.05	0.14	0.14	0.07	0.02	0.06	0.11	0.02	0.46	0.13	0.03	0.02	0.47	1.28

	$P(0,0)$	$P(1,0)$	$P(2,0)$	$P(3,0)$	$P(4,0)$	$P(5,0)$	$P(0,4)$	$P(1,4)$	$P(2,4)$	$P(3,4)$	$P(4,4)$	$P(5,4)$	$P(0,5)$	$P(1,5)$	$P(2,5)$	$P(3,5)$	$P(4,5)$	$P(5,5)$
oats 1																		
*30.56	0.01	0.09	0.62	1.06	21.65	13.70	0.02	0.2	0.7	2.75	8.67	44.91	0.05	0.4	1.89	5.74	34.42	
*35.2	0	0.02	0.08	2.72	29.2	10.21	0.02	0.37	0.88	1.75	7.27	27.30	0.05	0.38	2.72	8.71	27.20	
*29.3	0	0.02	0.57	12.32	39.2	0.55	0.2	0.37	2.19	3.27	5.32	20.21	0.15	0.21	3.14	8.73	35.24	
*30.56	0	0.02	0.27	0.87	8.29	27.32	0.01	0.21	0.58	2.31	8.82	58.33	0.82	0.20	0.87	3.27	30.31	
*28.21	0.03	0.55	2.56	31.36	62.71	0.72	0.05	0.40	1.04	2.20	4.39	5.22	0.34	0.15	2.08	7.81	75.63	
2 *20.33	0.06	0.23	2.45	23.07	46.14	2.32	0.08	0.21	0.92	2.08	4.15	6.10	0.18	1.04	2.44	9.77	19.53	
*29.40	0.02	0.32	3.12	29.70	58.32	0.57	0.05	0.32	0.87	2.13	7.15	9.27	0.20	1.22	3.40	5.28	25.37	
*37.13	0.00	0.82	3.21	3.37	28.2	0.59	0.01	0.20	0.83	3.12	9.23	7.37	0.20	1.02	1.28	8.27	23.57	
*26.83	0	0.02	0.07	0.88	35.3	13.23	0.02	0.33	0.63	2.20	10.22	34.73	0.02	0.53	2.28	9.72	39.83	
*27.89	0	0.05	0.09	0.47	24.70	11.30	0.01	0.23	0.52	1.69	9.64	40.02	0.05	0.52	1.28	4.27	37.35	

	$P(0,1)$	$P(1,1)$	$P(2,1)$	$P(3,1)$	$P(4,1)$	$P(5,1)$	$P(0,2)$	$P(1,2)$	$P(2,2)$	$P(3,2)$	$P(4,2)$	$P(5,2)$	$P(0,3)$	$P(1,3)$	$P(2,3)$	$P(3,3)$	$P(4,3)$	$P(5,3)$
oats 1	*	0	0	0.02	0.03	0.05	0.02	0.01	0.03	0.13	0.14	0.82	0.54	0	0.05	0.41	1.74	3.23
	*	0	0	0	0.03	0.05	0.01	0	0.02	0.27	0.29	0.52	0.57	0.21	2.1	0.27	2.22	4.72
	*	0	0.01	0.01	0.03	0.04	0.01	0.02	0.03	0.03	0.52	0.87	0.08	0.12	0.21	0.37	0.42	2.10
	*	0	0	0	0.01	0.03	0.58	0	0.02	0.06	0.38	0.93	3.12	0.01	0.05	0.21	2.12	3.13
	*	0	0	0	0.01	0.02	0.02	0	0.04	0.03	0.12	0.24	0.31	0.01	0.05	0.12	0.49	0.98
2	*	0	0	0	0.02	0.03	0.01	0	0.02	0.12	0.15	0.31	0.03	0.05	0.08	0.40	0.55	1.10
	*	0	0	0.02	0.01	0.02	0.01	0.02	0.04	0.24	0.28	0.59	0.03	0.05	0.08	0.27	0.92	2.10
	*	0	0	0	0.02	0.07	0.01	0	0.02	0.21	0.28	0.52	0.03	0.03	0.08	0.37	0.47	2.10
	*	0	0	0.02	0.05	0.02	0.01	0.01	0.03	0.05	0.07	0.18	0.82	0.01	0.27	0.58	2.32	10.83
	*	0	0.01	0.05	0.01	0.04	0.02	0.01	0.03	0.19	0.14	0.27	0.72	0.02	0.16	0.47	1.74	2.62

P(0,0)	P(1,0)	P(2,0)	P(3,0)	P(4,0)	P(5,0)	P(0,4)	P(1,4)	P(2,4)	P(3,4)	P(4,4)	P(5,4)	P(0,5)	P(1,5)	P(2,5)	P(3,5)	P(4,5)	P(5,5)
barley 1																	
*30.90	0.14	0.28	0.42	23.96	55.69	1.79	0.05	0.12	0.37	2.32	4.64	12.48	0.21	0.48	1.22	14.47	25.88
*37.42	0	0.04	0.50	2.92	19.67	28.73	0.05	0.18	0.38	3.20	5.32	43.82	0.23	0.37	0.28	14.20	28.72
*34.70	0	0.09	0.32	2.53	11.92	26.63	0.05	0.14	0.46	2.87	8.67	62.55	0.11	0.27	0.22	5.74	23.68
*36.64	0	0.02	0.30	0.31	25.73	0.31	0.06	0.31	1.53	12.70	10.01	21.76	0.09	0.73	2.81	14.16	56.15
*43.67	0	0.03	1.67	16.40	61.58	0.28	0.26	0.34	0.83	3.91	5.86	2.78	0.05	0.79	2.40	29.30	35.16
2 *40.86	0	0.02	0.10	2.17	14.24	24.93	0.02	0.17	0.58	3.23	9.96	67.18	0.05	0.34	0.02	5.98	26.61
*54.93	0.06	0.31	1.68	21.91	78.89	2.21	0.11	0.31	0.67	4.03	7.32	17.3	0.09	0.67	3.87	5.39	28.81
*37.77	0	0.02	0.10	1.55	14.05	22.48	0.02	0.17	0.46	2.62	11.11	61.81	0.03	0.34	1.16	5.25	28.56
*42.88	0.06	0.07	0.74	2.11	20.32	17.87	0.05	0.15	0.88	3.17	7.57	64.00	0.08	0.31	1.93	5.86	31.25
*29.21	0.02	0.15	0.46	0.86	19.55	0.10	0.16	0.20	0.07	14.87	13.08	19.79	0.2	0.07	3.36	11.35	50.05
Milo 1																	
* 7.72	0.02	0	0.25	6.73	14.53	2.70	0.05	0.18	0.49	2.20	5.62	10.68	0.12	0.43	0.98	9.77	21.97
2 * 9.35	0.02	0.01	0.10	7.90	17.44	2.11	0.07	0.16	0.76	1.89	5.00	11.86	0.14	0.40	1.16	10.82	24.17

	$P(0,1)$	$P(1,1)$	$P(2,1)$	$P(3,1)$	$P(4,1)$	$P(5,1)$	$P(0,2)$	$P(1,2)$	$P(2,2)$	$P(3,2)$	$P(4,2)$	$P(5,2)$	$P(0,3)$	$P(1,3)$	$P(2,3)$	$P(3,3)$	$P(4,3)$	$P(5,3)$	
barley 1	*	0	0	0.02	0.04	0.03	0.09	0.14	0	0.01	0.03	0.18	0.37	0.40	0.02	0.06	0.12	0.61	1.22
	*	0	0	0.04	0	0.03	0.62	0.38	0.01	0.02	0.04	0.27	0.93	0.48	0.02	0.05	0.18	0.68	1.77
	*0.16	0	0.01	0	0.02	0.38	0.39	0.02	0.01	0.05	0.11	0.52	0.57	0.03	0.02	0.08	0.21	1.89	
	*0.02	0	0.02	0.02	0.19	0.19	0.03	0.04	0.07	0.2	0.21	0.43	0.27	0.11	0.08	1.50	0.34	3.30	
	*0.02	0	0.03	0.02	0.05	0.12	0	0.06	0.02	0.06	0.27	0.43	0.08	0.01	0.24	0.15	0.85	1.34	
2	*	0	0	0.02	0.01	0.04	0.08	0.6	0.01	0.08	0.05	0.14	1.50	0.79	0.01	0.04	0.17	0.40	3.85
	*	0	0	0	0.03	0.06	0.09	0.01	0.04	0.11	0.09	0.24	0.49	0.15	0.01	0.08	0.70	0.61	1.59
	*	0	0.01	0.01	0.02	0.04	0.09	0.7	0.01	0.01	0.14	0.20	0.70	0.86	0	0.13	0.44	0.52	2.50
	*0.03	0.02	0.01	0.07	0.05	0.18	0.65	0.01	0.03	0.05	0.12	0.61	2.56	0.02	0.05	0.12	0.31	2.56	
	*0.01	0.01	0.03	0.02	0.07	0.08	0.01	0.03	0.32	0.08	0.50	0.95	0.05	0.03	0.11	1.24	0.82	1.77	
milo 1	*	0	0	0	0.01	0.05	0.06	0	0.02	0.04	0.06	0.18	0.25	0.04	0.20	0.37	0.73	1.59	
2	*	0	0.01	0	0.01	0.02	0.05	0	0.01	0.07	0.14	0.17	0.40	0.36	0.03	0.11	0.14	0.64	1.28

APPENDIX 3-2

A listing of the computer program used to compute the 2-BT power spectra listed in Appendix 3-1 is presented in this appendix. For the purposes of illustration, the computation of the 2-BT power spectrum for an oats kernel is included in the listing which follows.

ILLEGIBLE DOCUMENT

**THE FOLLOWING
DOCUMENT(S) IS OF
POOR LEGIBILITY IN
THE ORIGINAL**

**THIS IS THE BEST
COPY AVAILABLE**

```

1 001 SUBROUTINE OF CORR
2  DIMENSION A(32,32),P(6)
3  READ(5,END=4) N
4  DO 20 I=1,M
5  20 READ(5,30) (A(I,J),J=1,N)
6  30 FORMAT(4F2.1)
7  35 FORMAT(20X,32F3.0)
8  DO 40 I=1,M
9  40 WRITE(6,35) (A(I,J),J=1,N)
10 CALL ADJUST (M,N,A)
11 60 FORMAT(20X,8F10.3)
12 LL=0
13 MS=1
14 120 K=0
15 L=0
16 NS=1
17 140 PSP=C.C
18 DO 150 I=1,MS
19 IF(I.LE.LL) GO TO 150
20 DO 145 J=1,NS
21 IF (J.LE.L) GO TO 145
22 PSP=PSP+A(I,J)*2
23 CONTINUE
24 150 CONTINUE
25 K=K+1
26 S(K)=PSP
27 L=IS
28 NS=2*NS
29 IF(NS.LE.N) GO TO 140
30 WRITE(6,165)(D(I),I=1,K)
31 165 FORMAT(///20X,6F10.5)
32 LL=NS
33 MS=2*MS
34 IF(NS.LE.M) GO TO 120
35 STOP
36 END
**WARNING** FORMAT STATEMENT 60 IS UNREFERENCED

37 SUBROUTINE ADJUST (M,N,A)
38 DIMENSION A(M,N),X(32)
39 DO 30 J=1,N
40 DO 20 I=1,M
41 X(I)=A(I,J)
42 CALL BFT (M,X)
43 DO 30 I=1,M
44 A(I,J)=X(I)
45 DO 40 I=1,M
46 DO 50 J=1,N
47 X(J)=A(I,J)
48 CALL BFT (N,X)
49 DO 60 J=1,N
50 A(I,J)=X(J)
51 RETURN
52 END

53 SUBROUTINE BFT (NUM,X)
54 DIMENSION IPOWER(10),X(32),Y(32)
55 25 ITER=0

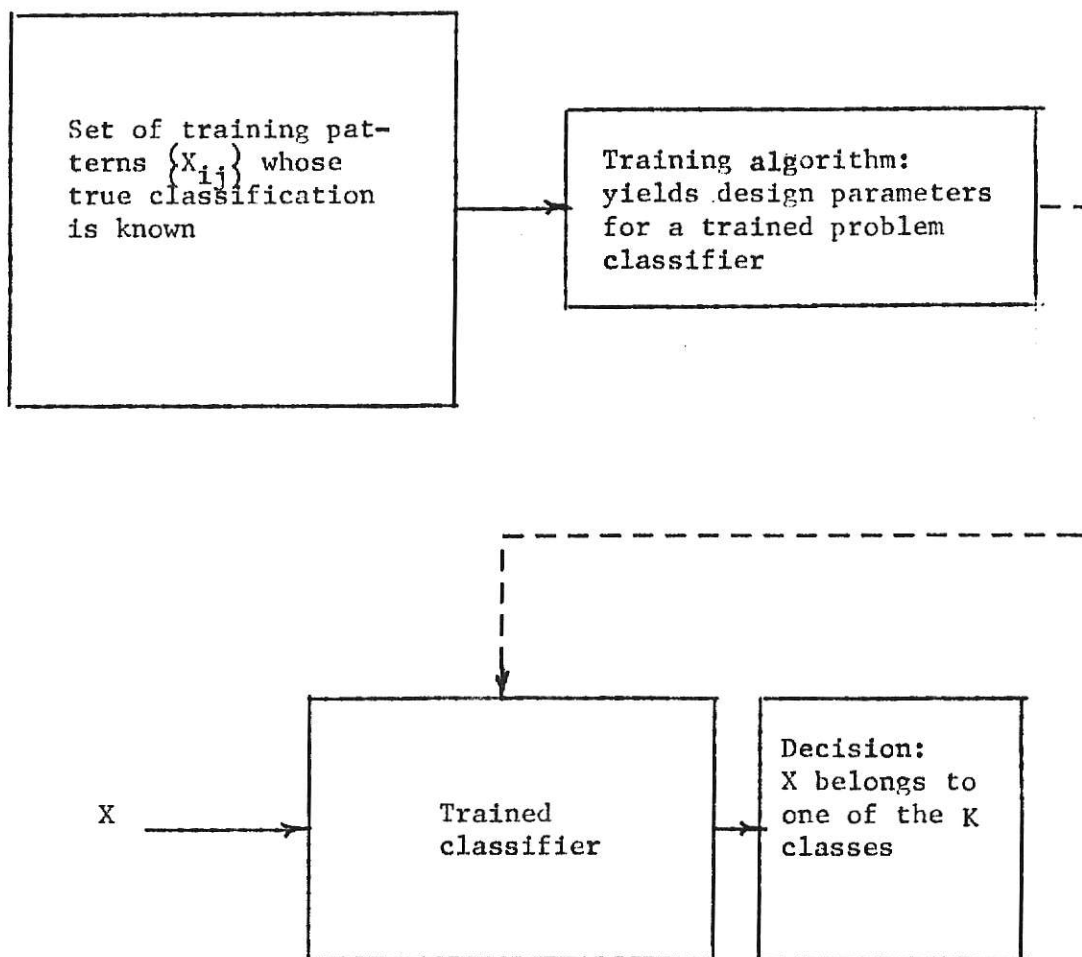
```


CHAPTER IV

GRAIN CLASSIFICATION RESULTS

4.1. Elements of Pattern Classification

Some basic concepts of pattern classification are best introduced by referring to Figure 4-1. Let X_{ij} represent the j -th pattern belonging to class or category $i, i=1,2,\dots,K$, where K is the total number of classes. Generally X_{ij} is in the form of a vector - however it can also be in the form of a multi-dimensional array. The set of patterns $X_{ij}, j=1,2,\dots,N_i$ is denoted by $\{X_{ij}\}$ and consists of a total of N samples, N_i of which belong to class $i, i=1,2,\dots,K$. This set of patterns $\{X_{ij}\}$ is called the training set since they are used to "train" or "teach" the classifier to classify a pattern X (whose classification is unknown) as belonging to a particular class. That is, once a classifier has been trained, it is capable of classifying incoming patterns on its own. The overall performance of a classifier is generally measured in terms of the number of errors (i.e. misclassifications) it makes. Many different types of training algorithms are available [8]. The choice of a particular algorithm is generally dictated by the problem which has to be solved.



X: A pattern whose classification is not known.

Figure 4-1: A pattern classification scheme

4.2. The Training Algorithm

As mentioned earlier, there are various types of training algorithms [8]. The specific algorithm used in the present grain classification study uses the "least squares mapping" approach. The basic idea in this approach is to derive a linear transformation A which maps in the least-squares sense the training samples belonging to class i , into the unit vector V_i . We recall that V_i is a vector, whose elements are all zero, except for the i -th element which is unity. Once the classifier is trained, the transformation A is obtained in the form of a matrix. To classify an incoming pattern X whose classification is unknown, the following steps are used:

- (1) Compute $Z=AX$. Then Z is the mapping of X into the unit vector space.
- (2) Find the distance of Z from the unit vectors $V_i, i=1,2,\dots,K$.
If Z falls closest to V_{i_0} , then X it is decided that X belongs to class i_0 .

For a detailed discussion of the above training algorithm, the reader should consult references [4] and [5]. A listing of the computer program associated with this algorithm is included in Appendix 4-1.

4.3. Classification of Corn, Wheat, Barley, Oats and Milo.

The 2-BT power spectrum points tabulated in Appendix 3-1 are used to obtain the training set. The following 10 of the 36 power spectrum points are used.

$$\begin{aligned} &P(0,0), P(4,0), P(5,0), P(5,4), P(0,5) \\ &P(4,5), P(5,5), P(3,3), P(4,3), P(5,3) \end{aligned}$$

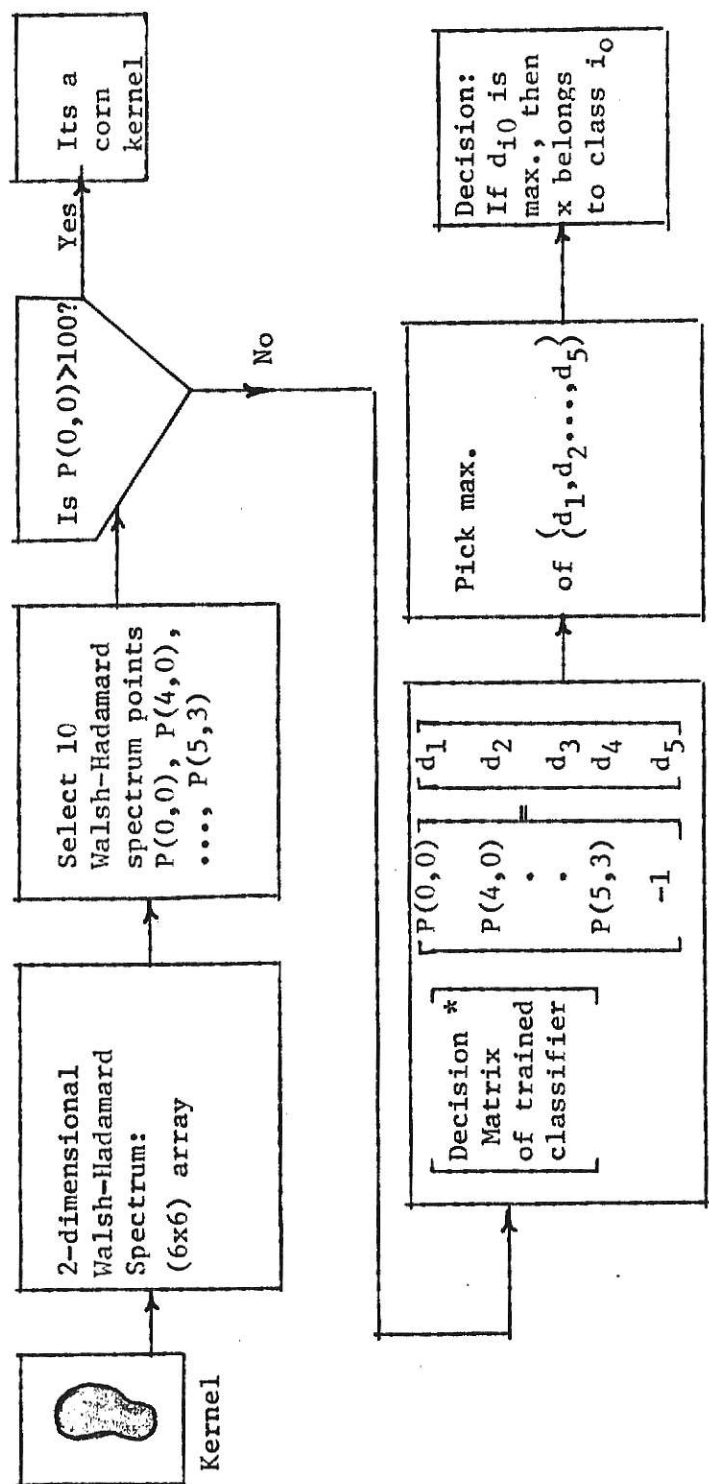
Thus each X_{ij} in Figure 4-1 is a (10×1) vector in this application.

Examination of the table of 2-BT power spectrum points in Appendix 3-1 reveals that $P(0,0)$ for each corn sample is much greater than that for any sample of wheat, barley, oats or milo. Thus the classification of corn becomes quite simple as shown in Figure 4-2. If a $P(0,0)$ exceeds a convenient threshold such as 100, it is decided that the corresponding kernel is corn. On the other hand, if $P(0,0)$ is less than 100, it is then decided whether the corresponding kernel is wheat, barley, oats or milo, (see Figure 4-2). The decision matrix referred to in Figure 4-2 is directly related to the transformation A referred to in Section 4.2. It is obtained using the training algorithm as evident from Appendix 4-1 (see page 49).

4.4. Classification Results

During the training process it was detected that the 10 components used for training purposes (see Section 4-3) formed a bi-modal distribution in the case of wheat. Thus, two separate classes of wheat were considered while training to obtain the decision matrix in Figure 4-2 (See page 49 of Appendix 4-1.) That is, class 1 and class 5 both represented wheat. The corresponding training samples used as belonging to classes 1 and 5 are denoted by the asterisk and double asterisk respectively.

After the decision matrix was obtained, the training samples were classified using the trained classifier as shown in Figure 4-2. The overall results may be summarized by means of a confusion matrix (see Appendix 4-1, page 49).



* See Appendix 4-1, page 49

Figure 4-2 Summary of the classification of corn, wheat, barley,

oats and milo.

$$F = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 1 & 7 & 1 & 0 & 1 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix} \quad (4-1)$$

where, class 1 through class 5 are wheat, oats, barley, milo and wheat respectively.

With respect to F in (4-1) we make the following observations:

- (1) Of the 20 samples of wheat, 2 have been erroneously classified as being milo.
- (2) Of the 10 samples of oats a total of three are erroneously classified as being wheat, barley and milo.
- (3) All samples of barley and milo (10 and 5 respectively) are classified correctly.

The above observations lead to the following % correct classification scores:

Type of grain	% correct classification score
Wheat	90%
Oats	70%
Barley	100%
Milo	100%

We recall that corn can also be attributed with a correct recognition score of 100% since for each sample of corn, $P(00) > 100$ (see Appendix 4-1).

APPENDIX 4-1

This appendix provides a listing of a computer program for a training algorithm which uses the least - squares mapping principle. Details pertaining to the algorithm are available in references [4] and [5].

```

*JOB
C      AS, RUN=CHECK, TIME=10, PAGES=50
C      # OF CLASSES: MAX. 20
C      # OF SAMPLES IN EACH CLASS: MAX. 20
C      DIMENSION OF SAMPLES & UNIT VECTORS: MAX. 10
C      FIRST CARD IN THE MAIN PROGRAM SHOULD BE CHANGED DEPENDING ON THE
C      COMPILER USED
C      FIRST DATA CARD: # OF CLASSES, DIMENSIONS OF SAMPLES & UNIT VECTORS IN 315
C      SECOND CARD: # OF SAMPLES IN EACH CLASS IN FORMATS OF 15
C      THIRD CARD: PROBABILITY OF EACH CLASS IN FORMATS OF F8.4
C      READ IN UNIT VECTORS, ONE PER CARD, EACH COMPONENT IN F8.4
C      SAMPLES, ONE PER CARD, WITH COMPONENTS IN F8.4
C
C      DATA NR, NR/5, 6/
C      DIMENSION NSAMP(10), PROB(10), V(10,10), X(11,20), P(220,10), Y(11,11)
C      DIMENSION VX(11,11), A(10,11), VX(10,11), E(10,11), O(10,20)
C      DIMENSION ICF(10,10)
C      READ(NR,1) NCLAS, NCOMP, LV
C      FORMAT(I6,15)
C      WRITE(NR,2) NCLAS, NCOMP
C      FORMAT(10X, 'THIS PROBLEM HAS', 15, ' CLASSES & EACH CLASS IS', 15,
C      1 ' 15', F10.4)
C      N=NCLAS
C      READ(NR,1) (NSAMP(I), I=1, N)
C      READ(NR,2) (PROB(I), I=1, N)
C      3 FORMAT(////)
C      DO 4 I=1, N
C      4 WRITE(NR,5) I, NSAMP(I), PROB(I)
C      5 FORMAT(10X, 'CLASS', 15, ' HAS', 15, ' SAMPLES & ITS PROBABILITY
C      115', F10.4)
C      L=NCOMP
C      DO 10 J=1, N
C      10 READ (NR,20) (V(I,J), I=1, LV)
C      20 FORMAT(10F8.4)
C      WRITE(NR,3)
C      WRITE(NR,25)
C      25 FORMAT(10X, ' V MATRIX', ////)
C      DO 30 I=1, LV
C      30 WRITE(NR,35) (V(I,J), J=1, N)
C      35 FORMAT(10X, 11F10.3)
C      LI=L+1
C      DO 40 K=1, N
C      NI=NSAMP(K)
C      FI=ELUCATION
C      PI=PRCH(K)
C      DO 40 J=1, NI
C      40 READ(NR,20) (X(I,J), I=1, L)
C      X(LI,J)=-1.0
C      NI=1
C      DO 41 J=1, NI
C      DO 41 I=1, LI
C      P(N,K)=X(I,J)
C      M=M+1
C      WRITE(NR,3)
C      WRITE(NR,45) K
C      45 FORMAT(10X, ' AUGMENTED PATTERNS OF CLASS', 15, ////)
C      DO 50 I=1, LI
C      50 WRITE(NR,35) (X(I,J), J=1, NI)
C      DO 60 I=1, LI
C      DO 60 I2=1, LI
C      Y(I,I2)=0.0
C      46

```

```

47      DO 55 J=1,NI
48      Y(I1,I2)=Y(I1,I2)*X(I1,J)*X(I2,J)
49      Y(I1,I2)=Y(I1,I2)/FI
50      IF(K-50-1)*X(I1,I2)=0.0
51      X(I1,I2)=X(I1,I2)+Y(I1,I2)
52      DO 65 I=1,LI
53      DO 65 J=2,NI
54      X(I,I)=X(I,I)+X(I,J)
55      DO 70 I=1,LV
56      DO 70 J=1,LI
57      A(I,J)=PI*V(I,K)*X(I,J,1)/FI
58      IF(K-50-1)*X(I,J)=0.0
59      V(I,J)=V(I,J)+A(I,J)
60      CONTINUE
61      CALL INVER3(X,X,LI)
62      DO 120 I=1,LV
63      DO 120 J=1,LI
64      A(I,J)=0.0
65      DO 120 K=1,LI
66      A(I,J)=A(I,J)+VX(I,K)*XX(K,J)
67      WRITE(NW,3)
68      WRITE(NW,125)
69      FORMAT(10X,'TRANSFORMATION MATRIX A'//)
70      DO 130 I=1,LV
71      DO 130 J=1,LI
72      DO 135 I=1,N
73      DO 135 J=1,LI
74      E(I,J)=0.0
75      DO 135 K=1,LV
76      E(I,J)=E(I,J)+V(K,I)*A(K,J)
77      WRITE(NW,3)
78      WRITE(NW,140)
79      FORMAT(10X,'DECISION MATRIX'//)
80      DO 142 I=1,N
81      DO 142 J=1,N
82      DO 145 I=1,N
83      DO 145 J=1,N
84      IC(I,J)=0
85      DO 185 K=1,N
86      NI=NSAMP(K)
87      M=1
88      DO 150 J=1,NI
89      DO 150 I=1,LI
90      X(I,J)=P(M,K)
91      M=M+1
92      DO 160 I=1,N
93      DO 160 J=1,NI
94      O(I,J)=0.0
95      DO 160 K=1,LI
96      O(I,J)=O(I,J)+E(I,M)*X(M,J)
97      WRITE(NW,3)
98      DO 180 J=1,NI
99      LS=1
100      C=0(I,J)
101      DO 170 I=1,N
102      IF(C.LT.O(I,J))GO TO 165
103      GO TO 170
104      LS=1
105      C=0(I,J)
106      CONTINUE
165
170

```

```

107 ICF(K,LS)=ICF(K,LS)+1
108 CONTINUE
109 WRITE(NR,3)
110 WRITE(NR,190)
111 FORMAT(10X,'CONFUSION MATRIX',///)
112 DO 125 I=1,N
113 WRITE(NR,200)(ICF(I,J),J=1,N)
114 FORMAT(10X,10I5)
115 STOP
116 END

117 SUBROUTINE INVERS(L,N)
118 DIMENSION U(11,11),V(11,11),Y(12,11),Z(11,11)
119 DO 5 I=1,N
120 DO 5 J=1,N
121 U(I,J)=0.0
122 IF (1.0-EG.J)U(I,J)=1.0
123 L=1
124 K=L
125 Q=ABS(Z(K,L))
126 IF(Q-C.000001)20,20,30
127 Z(K,L)=0.0
128 K=K+1
129 IF(K-N)11,11,90
130 DO 31 M=1,N
131 Y(K,M)=Z(K,M)
132 Z(K,M)=Z(L,M)
133 Z(L,M)=Y(K,M)
134 V(K,M)=U(K,M)
135 U(K,M)=U(L,M)
136 U(L,M)=V(K,M)
137 V(L,L)=Z(L,L)
138 DO 41 M=1,N
139 Z(L,M)=Z(L,M)/Y(L,L)
140 U(L,M)=U(L,M)/Y(L,L)
141 K=1
142 IF(K-L)52,51,52
143 K=K+1
144 IF(K-N)50,50,70
145 Q=ABS(Z(K,L))
146 IF(Q-C.000001)53,53,54
147 Z(K,L)=0.0
148 K=K+1
149 IF(K-N)50,50,60
150 Y(K,L)=Z(K,L)
151 DO 55 M=1,N
152 Z(K,M)=Z(K,M)-Z(L,M)*Y(K,L)
153 U(K,M)=U(K,M)-U(L,M)*Y(K,L)
154 K=K+1
155 IF(K-N)50,50,60
156 L=L+1
157 IF(L-N)10,10,70
158 DO 80 I=1,N
159 DO 80 J=1,N
160 Z(I,J)=U(I,J)
161 RETURN
162 END

```

SENTRY
THIS PROBLEM HAS 5 CLASSES & EACH CLASS IS 10 DIMENSIONAL

CLASS	1	HAS	10	SAMPLES &	ITS PROBABILITY	IS	0.2000
CLASS	2	HAS	10	SAMPLES &	ITS PROBABILITY	IS	0.2000
CLASS	3	HAS	10	SAMPLES &	ITS PROBABILITY	IS	0.2000
CLASS	4	HAS	5	SAMPLES &	ITS PROBABILITY	IS	0.2000
CLASS	5	HAS	10	SAMPLES &	ITS PROBABILITY	IS	0.2000

V MATRIX

1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	1.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	1.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	1.000	0.000	0.000

TRANSFORMATION MATRIX A

-0.041	-0.044	0.037	0.100	0.012	0.002	-0.000	-0.065	-0.101	-0.058	0.258
0.028	0.029	-0.019	-0.012	-0.019	-0.043	0.007	0.329	0.108	-0.308	-0.101
0.022	0.012	-0.006	0.020	0.011	0.035	-0.004	-0.242	-0.025	0.291	0.640
0.006	0.008	-0.020	-0.020	-0.014	-0.005	-0.000	-0.078	0.040	-0.246	-1.046
-0.015	-0.006	0.008	-0.069	0.010	0.012	-0.003	0.056	-0.043	0.324	-0.540

DECISION MATRIX

-0.041	-0.044	0.037	0.100	0.012	0.002	-0.000	-0.065	-0.101	-0.058	0.258
0.028	0.029	-0.019	-0.012	-0.019	-0.043	0.007	0.329	0.108	-0.308	-0.101
0.022	0.012	-0.006	0.020	0.011	0.035	-0.004	-0.242	-0.025	0.291	0.640
0.006	0.008	-0.020	-0.020	-0.014	-0.005	-0.000	-0.078	0.040	-0.246	-1.046
-0.015	-0.006	0.008	-0.069	0.010	0.012	-0.003	0.056	-0.043	0.324	-0.540

CONFUSION MATRIX

10	0	0	0	0	0
1	7	1	0	1	
0	0	10	0	0	
0	0	0	5	0	
0	0	0	2	8	

CORE USAGE OBJECT CODE= 12640 BYTES,ARRAY AREA= 15100 BYTES,TOTAL AREA AVAILABLE= 49320 BYTES
 DILCNOSTICS NUMBER OF ERRORS= 0, NUMBER OF WARNINGS= 0, NUMBER OF EXTENSIONS= 0
 COMPILE TIME= 4.93 SEC,EXECUTION TIME= 21.19 SEC, WATFIV - VERSION 1 LEVEL 3 MARCH 1971 DATE= 72/040

CHAPTER V
CONCLUSIONS AND RECOMMENDATIONS FOR
FUTURE WORK

5.1. Conclusions

The results of this initial study show that it is plausible that the two-dimensional BIFORE or Walsh-Hadamard transform power spectrum can be used to distinguish several types of grain on the basis of their shapes and sizes. The variation in the shapes and sizes of different types of kernels is reflected in a corresponding variation in the spectrum points $P(0,0)$, $P(4,0)$, $P(5,0)$, $P(5,4)$, $P(0,5)$, $P(4,5)$, $P(5,5)$, $P(3,3)$, $P(4,3)$, and $P(5,3)$. Subsequently, such variations in this set of spectrum points can be "learned" by means of training algorithms. One such algorithm which uses a least-squares mapping approach seems to be adequate as evident from the % correct classification scores listed in Section 4-4. However, considering the small sample sizes, one must appreciate that these figures are merely estimates. Larger sample sites must be considered to come to some definite conclusions.

5.2. Recommendations For Future Work.

On the basis of the results obtained from the present study, the following recommendations are made for future work:

- (1) Obtain % correct classification scores using a larger training set with the objective of realizing better estimates for the classification scores reported in this study.
- (2) Incorporate soybeans and/or rye with the types of grain considered in this study and hence obtain the overall performance of the classification scheme initiated in this study.
- (3) Compare the results obtained in (2) with those obtained by an alternate approach to the automatic grain classification problem [6] , [7] .

REFERENCES

- 1 . N. Ahmed, R. M. Bates and K. R. Rao: "Multi-dimensional Binary Fourier Representation", Proc. of the Second Southeastern Symposium, Univ. of Florida, Gainesville, Florida, March 16 - 17, 1970, pp. J3-0 to J3-10.
- 2 . W. K. Pratt, et. al.: "Hadamard Transform Image Coding", Proc IEEE, vol. 57, Jan. 1969, pp. 58-68.
- 3 . N. Ahmed and R. M. Bates: "A Power Spectrum and Related Physical Interpretation of the Multi-dimensional BIFORE Transform", IEEE Trans. on Electromagnetic Compatability (Special Issue), August 1971,
- 4 . N. Ahmed and K. R. Rao: "Principles of Basic Digital Signal Processing Techniques", Manuscript (Kansas State University), Chapter VII.
- 5 . W. G. Chaplin and V. S. Levadi: "A Generalization of the Linear Threshold Decision Algorithm to N-Dimensions", The Second Symposium on Computers and Information Sciences, Columbus, Ohio, August 1968.
- 6 . A. R. Edison and W. L. Brogan: "Feasibility Study for Automatic Grading Samples", Contract No. 12-14-100-11, 001(51), Quarterly Progress Report, September 1971.
- 7 . A. R. Edison and W. L. Brogan: "Feasibility Study for Automatic Grading Samples," Contract No. 12-14-100-11, 001(51), Quarterly Program Report, December 1971.
- 8 . W.G.We: "A Survey of Pattern Recognition", IEEE Proc. of the Seventh Symposium on Adaptive Processings December 1968, pp. 2E-13.

ACKNOWLEDGEMENTS

The author wishes to express his deep gratitude and sincere appreciation to his advisor Dr. N. Ahmed for his advice, encouragement and patience during the course of this report.

A PATTERN RECOGNITION APPROACH TO GRAIN SAMPLE ANALYSIS

by

ANIL HARIKANT VYAS

B. E., M. S. University, Baroda, India, 1970

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1973

ABSTRACT

This report concerns an initial feasibility study of grain sample analysis. Specifically, the problem of automatically separating various types of grains is considered. The approach entertained in this study is based on pattern recognition techniques.

The shape and size of a kernel are used as criteria to distinguish it from a kernel belonging to a different type of grain. Each kernel is coded in the form of a (32x32) array of zeros and ones. The portion of the array occupied by the kernel is represented by a "1" while that which is not is represented by a "0". The two-dimensional BIFORE (Binary Fourier Representation) or Walsh-Hadamard transform is used to carry out a spectral analysis of the coded representations of several types of kernels in various orientations. The variations in the shapes and sizes of the kernels were thus obtained in terms of a set of BIFORE or Walsh-Hadamard power spectrum points.

A subset of the above power spectrum points were used to train a specific pattern classifier which uses the least-squares mapping principle. The final percent correct classification scores for the types of grain considered were as follows: (1) corn, 100%, (2) wheat, 90%, (3) barley, 100%, (4) oats, 70%, and (5) milo, 100%.

The results of this initial study demonstrate that the two-dimensional BIFORE power spectrum can be used as an effective tool to characterize the shape and size of given kernel. Since the classification results cited above were obtained from the analysis of a small number of grain samples, it is recommended that the techniques introduced in this study be applied

to a relatively larger number of grain samples. It is further recommended that grains such as rye and soybeans also be included in future studies since the same were not considered in this research effort.