ANALYSIS OF TRUSSED-TEE REINFORCED CONCRETE SLAB

by

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INTRODUCTION

A large part of the expense involved in pouring concrete slabs can be attributed to the forms which have to be constructed. Time and labor are also involved in the placing of the reinforcements in the slab.

This project was an investigation of the feasibility of using Trussed-Tees and corrugated sheet metal in the construction of concrete slabs. The Tees and the sheet metal function both as forms and as reinforcement in the finished slab.

This investigation was divided into two parts: The first part involved an analysis of a type of Trussed-Tee. Design procedures based on the working strength of the steel and on deflections between supports were derived.

It was determined that quite a number of temporary supports are necessary while the concrete is curing. Many of these supports can be removed forty-eight hours after the concrete is poured.

The second part of this investigation involved an analysis of the concrete slab. Theoretical calculations were made to determine the efficiency of the Trussed-Tee and corrugated metal as reinforcements. The calculated stresses in the concrete and the Tee were compared with measured stresses. Deflections of the slab were checked and compared with the allowable deflections of the American Concrete Institute's Building Code.

The slab was made continuous over one support, and was finally loaded until it cracked over this support. It was determined that this type of slab only four inches thick could carry loads of more than 100 lb per sq ft with a factor of safety greater than three, and without exceeding the allowable deflection.

Deflection was the main criterion of design throughout this experiment. The maximum deflection was attained before the slab approached a maximum stress in either the concrete or the Trussed-Tee.

PART 1. ANALYSIS OF THE TRUSSED-TEES

The Trussed-Tee consisted of two top bars; two angle strips, and another bar that ran sinuously between the top bars and angle strips. The sinuous bar was welded to the top bars and angle strips at points of intersection. (See Fig. 1.)

For the use of Trussed-Tees in reinforced concrete slab construction it was necessary to make two different investigations. The first investigation was to determine the action of the members when the concrete was poured but before it set.

At this point in the construction the Tees were assumed to have truss action and top bars were unsupported. Since the member was continuous across several supports, the investigation was made under these conditions. The length of the members available was 12 ft, so the investigation was limited to the stresses in two span members. This was the most critical condition encountered.

The Trussed-Tee was by no means longitudinally homogeneous; therefore the first problem for this experiment was to determine the distribution of load among the supports.

Analytically the weld-jointed, Trussed-Tee is quite different from one which is assumed to be connected at its intersections by frictionless pins. A comparison of the actual stresses measured from the Tee with the calculated stresses based on a pin-connected simple truss of the same dimensions revealed some relationship between the stresses which furnished an aid to predict the actual stresses raised from various load conditions.

PROCEDURE

Distribution of Load Among the Supports

The Tee was treated as a pin-connected structure continuous across a middle support. The simulated pin-connected truss consisted of 159 members. The principle of virtual work was employed in the determination of reactions of the middle support. The truss was assumed to be loaded as shown in Fig. 2. Energy stored in each member was calculated

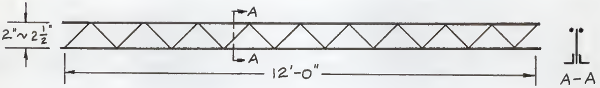


Fig. 1. Schematic view of trussed-tee.

Fig. 2. Truss load for calculation of mid-point deflection.

Fig. 3. Application of reaction at mid-point of truss.

From this calculation the deflection at the middle point (a) of the beam was obtained. A force R_2 was then applied at point (a) to eliminate that deflection. The value of R_2 was obtained by this procedure. Comparison of this value with the corresponding one induced in a homogeneous continuous beam showed that they were almost equal to each other. The value of R_2 is five eights of the total load on the member.

Determination of Maximum Deflections and Stresses Induced in Each Part of the Tee

The dimensions of the first set of Trussed-Tees used in this series of tests were 1-5-16-2 in. which means that the diameter of the top bars was 0.2830 in.; the diameter of the sinuous bar was 0.2250 in.. The strips were of No. 16 gauge (.060 in. thick). The height of the Trussed-Tee was 2 in., and the length used was 11 ft-10 in..

The length of the span was set in such a way that the supports were just underneath the joints of the truss. The plan is shown in Fig. 4.

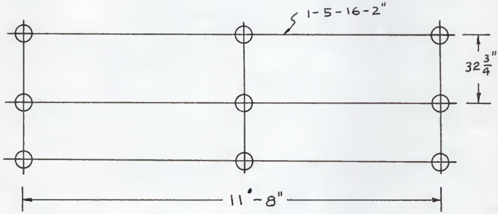


Fig. 4. Plan view of Trussed-Tees.

The Tees rested upon three wooden beams which were placed on nine barrels. In order to have a pinned effect at the supports, small steel cylinders were inserted between the Tees and the beams as bearings.

Bags full of sand were used as loads. A simulated uniform load of 50 lb per running foot along the middle Tee brought the maximum deflection close to the specified limit, (1/360 of the span length). Extensometers were used to measure the strains at various points. These strains were converted into stresses. Each stress obtained was compared with the stress calculated on the basis of a pin-connected simple truss. An over-hung load condition was also covered in the experiments.

Similar measurements of the same set of Trussed-Tees with varied span lengths were made.

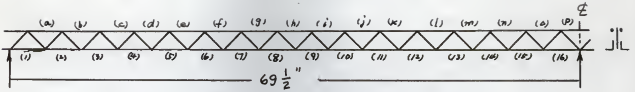
The second set of Trussed-Tees used in the experiment was 0-5-14-2 1/2 in., and each span was 5 ft 9 1/2 in. long. The deflection at the center of the span with these members and a load of 50 lb per running foot was 0.095 in., which is less than 1/360 of span (.193 in.). There are 31 triangular panels through the whole length of the beam. Sixteen readings were taken from the top bars on one-half of the beam, and the same number of readings from the bottom strips. Comparisons were made which showed that at certain points the discrepancies in stresses between the actual beam and the idealized simple beam were still existent. Electrical strain gauges were also applied at several points. At section 9, which was near the center of span, an SR-4 strain gauge was used simultaneously with three extensometers.

Stability tests were carried out by applying a concentrated load to the simply supported Trussed-Tees. The span length covered from 10 in. to 7 ft 0 in.. Results showed that the mode of failure depended upon the intensity of concentration of the applied load. If a load were applied along a sharp edge transverse to the longitudinal axis of the beam, and the span was very short, local buckling would occur at the upper part of the bottom strips. If the load were applied over a considerable area, and the span was larger (4 ft and up) buckling would take place in the top bars. Results also showed that the top bars buckled as a unit which revealed that the welded joints were adequate for stability under dead-load conditions.

Measured and Calculated Stresses

A load of 50 lb per running foot was applied to a 0-5-14-2 1/2 in. truss covering two spans and the load was considered to be applied equally at each connection of the sinuous bar and the bottom angles. The reaction of the center span was calculated by means of virtual work. This gave the middle reaction approximately the same as if the truss had been treated as a beam.

When the reactions were determined, the stresses were calculated treating the members as those of a pin-connected truss. The stresses in each bottom and top chord member were then found experimentally by measuring the strain with extensometers or SR-4 strain gauges. Comparisons of these values are shown in Table 1.



One of two spans — Load = 50 lb per running foot
Note: Deflection of the middle of span = 0.1049 in.

Fig. 5. Identification of member numbers.

Table 1. Measured and Calculated Stresses.

Bar No. Top	Calculated Stress (psi)	Measured Stress (psi)	Bar No. Bottom	Calculated Stress (psi)	Measured Stress (psi)
a	-1350	-1110	1	371	63
b	-2450	-2590	2	675	678
c	-3310	-3200	3	1585	1520
d	-3920	-3820	4	1985	2020
e	-4300	-4170	5	2260	2970*
f	-4420	-4570	6	2385	2390
g	-4300	-4170	7	2400	1005
h	-3920	-4170	8	2355	560
i	-3310	-3690	9	1985	2070*
j	-2450	-3220	10	1580	2390
k	-1350	-1175	11	1043	1005
l	0	0	12	243	- 560
m	1590	1354	13	- 438	- 486
n	3440	3240	14	- 1375	- 807
o	5550	5900	15	- 2450	-2485*
p	7860	6930	16	- 3680	-3510

* Measured with extensometers and SR-4 strain gauge.

Although some fairly large variations in stress existed, the various sections of the chord were not over-stressed, and the only check on the stress that was needed was at the center. Here the calculated stress is larger than the actual stress in the top bars due to the actual moment existing at the support. The stress in adjoining bottom members checked very closely with the stress calculated for a pin-connected truss.

TEST RESULTS

Buckling in a Simple Truss

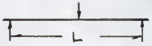


Fig. 6. Transverse load applied to Trussed-Tee.

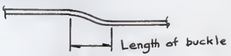


Fig. 7. Buckling curve of top bars.

When the truss was loaded as shown in Fig. 6, buckling occurred in the top bars. It was observed that the buckling took place over only a fraction of the total length of the span, and was in a plane perpendicular to the direction of the applied load. Taking the length of buckling as the column length (Fig. 7), and the moment of inertia as that of the top bars about an axis parallel to the applied load, the Euler formula showed that the critical stress of the buckled portion of the top bars was in the plastic region. The steel in the top bars of the Tee truss has no defined yield point. The point (as shown in Fig. 9) at 0.002 strain was taken as the yield point. This gave a tangent modulus of 5.1×10^6.

The generalized Euler formula $\sigma_{cr} = \frac{\pi^2 E_t}{(\frac{L}{r})^2}$

gave a value for critical stress which was consistent with the experimental results.

Example. For a Tee truss of 1-5-16-2, with a span of 4.5 ft, buckling length of 7 in. (from test).

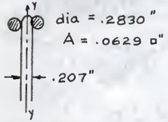


Fig. 8. Transverse section of top bars and sinuous member of Trussed-Tee.

$$I_{yy} = 2\left[\frac{\pi}{4}(0.1415)^4 + 0.0629\,(0.1035 + 0.1415)^2\right] = 0.008176 \text{ in.}^4$$

$$r = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{0.008176}{0.1258}} = 0.255 \text{ in.}$$

$$\frac{L}{r} = \frac{7}{0.255} = 27.5$$

$$S_{cr} = \frac{\pi^2 E_t}{(\frac{L}{r})^2} = \frac{\pi^2 \times 5.1 \times 10^6}{27.5^2} = 66{,}800 \text{ psi}$$

The distance between the centroid of the angles and top bars was found to be 1.75 in.

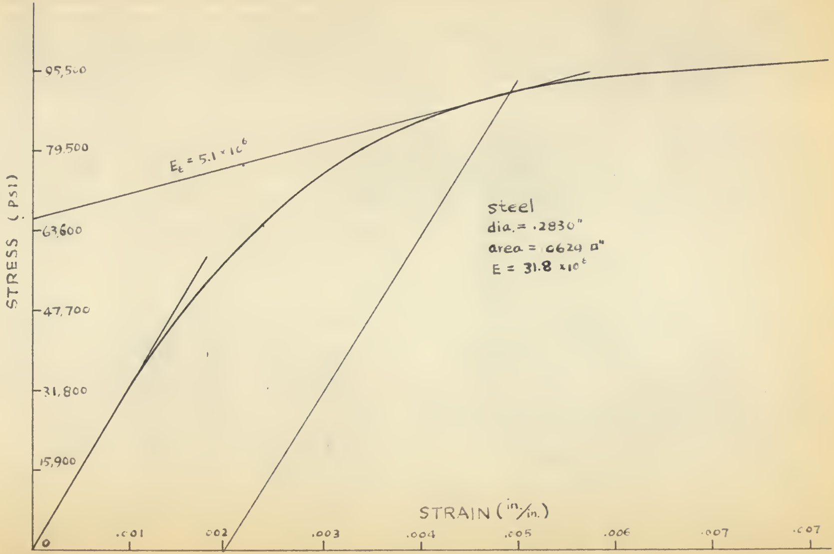


Fig. 9. Stress-strain curve of top bar of Trussed-Tee.

Resisting moment of the member = 66,800 x 1.75 x 0.1258 = 14,700 in-lb.

Moment caused by the concentrated load P acting at the center of span

$$= \frac{PL}{4} = \frac{54\ P}{4} = 13.5\ P$$

Setting the resisting moment equal to the external moment, the calculated failing load is determined as

$$P = 1095\ \text{lb}$$

This is very close to the actual failing load of 1140 lb.

For a Tee truss of 1-5-16-2 1/2 with a span length 5 ft long, the buckling length was 7.8 in.

$$I_{yy} = 0.008176$$

$$r = 0.255\ \text{in.}$$

$$\frac{L}{r} = \frac{7.8}{0.255} = 30.6$$

$$S_{cr} = \frac{\pi^2 E_t}{(\frac{L}{r})^2} = \frac{\pi^2 \times 5.1 \times 10^6}{30.6^2} = 53,000\ \text{psi}$$

moment arm = 2.17 in.

area = 0.1258 sq. in.

resisting moment = 53,800 x 2.17 x 0.1258 = 14,700 in-lb

moment caused by the concentrated load acting at the center of span = $\frac{60\ P}{4}$ = 15P.

The calculated failing load, P, = $\frac{14,700}{15}$ = 980 lb

The predicted and actual failing loads for other members of various span lengths are shown in Table 2.

Table 2. The Predicted and Actual Failing Loads.

Specimen	Span Length	Predicted failing Load	Actual failing Load
1-5-16-2 in.	4' - 6"	1095	1140
1-5-16-2 1/2	5' - 0"	980	980
1-5-16-2	4' - 2"	1177	1233
1-5-16-2 1/2	6' - 6"	755	758
0-5-14-2 1/2	5' - 5"	1200	1250

These results were not pertinent to design procedures in this investigation because the members were subjected to a uniformly distributed load acting downward instead of a concentrated load. They illustrate, however, the remarkable strength consistency of these members.

Buckling of Tee truss with a Uniformly Distributed Load.

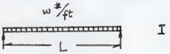


Fig. 10. Buckling under uniform load.

Tests showed that the length of buckling of a simply supported Tee truss with a uniformly distributed load is so short that any type of column formula with E_t substituted for E gave a critical stress which was greater than the yield stress. Therefore, the yield stress of the material was used.

For a 1-5-16-2 1/2 Tee-section with span length 5 ft 10 in.

$S_{cr} = 80,000 \text{ psi} = S_{yp}$

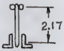


Fig. 11. Transverse section of Tee.

Resisting moment = 2.17 x 0.1258 x 80,000 = 21,800 in-lb

Maximum external moment = $\frac{WL}{8}$

Let the resisting moment equal the external moment, and the total load can be calculated,

$$21,800 = \frac{WL}{8}$$

$$W = \frac{21,800 \times 8}{70} = 2492 \text{ lb}$$

or

$$w = \frac{2492}{70} = 35.6 \text{ lb per in.} = 427 \text{ lb per running foot.}$$

Total uniformly distributed load causing failure was 2560 lb or 438 lb per running foot.

Concentrated Load Applied to Truss with Angles at the Top and Bars at the Bottom

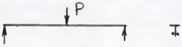


Fig. 12. Buckling of Tee with load applied on angles.

Under this condition, the angles of the member were in compression. In order to keep the truss from tilting, a lateral support was applied to the truss which was comparable to that supplied by the weight of the fresh concrete transferred by the corrugated metal to the angles. Failure was caused by the angles buckling under the applied load, and the length of buckling curve was so short as to justify using the yield stress in determining the resisting moment of the members.

The specimen for test was 0-5-14-2, with a span length of 5 ft-9 in.

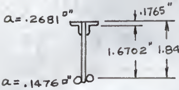


Fig. 13. Transverse section of Tee.

Resisting moment = 38,000 x 0.2681 x 1.6702 = 17,000 in-lb

External moment = $\frac{PL}{4}$. External moment = resisting moment.

$$\frac{PL}{4} = 17,000 \quad L = 69''$$

$$P = \frac{4 \times 17,000}{69} = 986 \text{ lb}$$

The actual failing load = 1025 lb.

The other specimen for test was 0-5-14-2 1/2, which was broken at a concentrated load of 1300 lb.

Span length = 5' 6 3/4"

Center to center distance = 2.17 in.

Resisting moment = 38,000 x 2.17 x 0.2681 = 22,100 lb

External moment = 1/4 PL

$$P = \frac{4 \times 22,100}{66.75} = 1320 \text{ lb.}$$

<u>Example.</u> Consider a two-span continuous Tee truss,

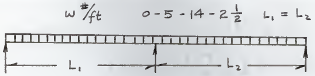


Fig. 14. Two-span continuous Trussed-Tee under uniform load.

Note: It was proved, by means of the method of virtual work, that the reactions would be approximately the same as those of a beam.

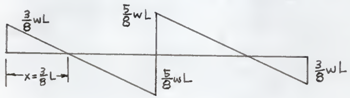


Fig. 15. Shear diagram of two-span beam.

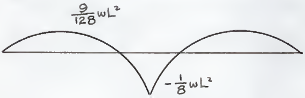


Fig. 16. Moment diagram of two-span beam.

Maximum positive moment = $\frac{9}{128}$ wL^2

For truss details, see figure.

S_w = 40,000 psi

R. M. = 40,000 x 0.1476 x 2.17 = 12,800 in-lb = 1066 ft-lb

$\frac{9}{128} wL^2 = 1066$

$L = \sqrt{\frac{128 \times 1066}{9\ w}} = \sqrt{\frac{15,150}{w}}$

Maximum negative moment $= -\frac{1}{8}wL^2$

$$R.\ M. = 23,000 \times 0.2681 \times 2.17 = 13,350 \text{ in-lb}$$

$$= 1,113 \text{ ft-lb}$$

$$\frac{1}{8}wL^2 = 1,113 \qquad L = \sqrt{\frac{8904}{w}}$$

RECOMMENDATIONS

Design Procedure Based on Allowable Working Stresses

The test results obtained from placing a concentrated load on the top of the Tee had no particular value in design procedure. In the other two tests the methods of calculating the failing load can be applied to determine the allowable working load by substituting for the yield stress, the allowable working stress of the material. Since the desired load will usually be known, the maximum span lengths can be determined. Although these tests have been made on single spans, the same procedure can be applied to any number of spans in the manner shown below.

1. Determine the maximum positive moments and negative moments using as many spans as desired. Determine these moments in terms of wL^2, (w = lb per running foot of the Tee truss).

2. Determine the maximum resisting moment of the top bars of the trussed tee in the manner described by the test of buckling of the simple truss under a uniformly distributed load.

3. Set the resisting moment from 2 equal to the maximum positive moment. If w is known, L can be determined.

4. Determine the maximum resisting moment of the two angles of the trussed tee as described in test of a tee with a concentrated load applied to the bottom of the tee.

5. Set the resisting moment from 4 equal to the maximum negative moment. If w is known, L can be determined.

6. The smaller of the two L's found in 3 and 5 will be the maximum span length based on the allowable working stress of the members.

The resisting moment in this procedure was determined by treating the trussed-Tee as spaced facings.[1] The test results show that this assumption is fairly accurate. The last value of L would govern, since it is smaller. If the trusses were 16 in. apart, and carried a load of 4 in. of fresh concrete,

$$w(\text{approx.}) = \frac{4}{3} \times \frac{1}{3} \times 150 = 66.7 \text{ lb},$$

and

$$\max L = \sqrt{\frac{8904}{66.7}} = 11.55 \text{ ft.}$$

Design Procedure Based on Deflection

In the design procedure based on the working stress of the members satisfactory results were obtained (verified by failure tests) by considering the external moment to be the same as the moment of a beam uniformly loaded, and setting this equal to the resisting moment of the Tee truss, when it is considered as spaced facings.

Considering the deflection, however, the Tee truss acted more like a pin-connected truss than like a beam. The loads appeared to act more as concentrated loads at the intersection of the sinuous member and the angles than as a uniform load. While this affected the moment very little, it had a considerable effect on the deflection.

[1] This means to consider the top bars and the bottom angles with the distance between these members, but to neglect the sinuous member.

When the clear span was large enough to cause the truss members to approach their allowable working stress, the deflection was greater than allowable. Therefore it was assumed that deflection would determine the clear span length in most cases.

A truss was set up and loaded in such a way that at each intersection of the sinuous member and the angles, a concentrated load was attached (See Fig. 17). The total maximum deflection corresponding to a uniform load of 100 lb per running foot was 0.148 in., less the deflections at the supports. The net total deflection was 0.1268 in..

It was determined by calculations that a pin-connected truss of the same dimensions would deflect 0.141 in. (the figures are not included but are available). This is greater than the actual deflection.

Consider a beam with the same I as the spaced facings, and with a system of concentrated loads acting at the joints along the angles equivalent to 100 lb per running foot. This beam would deflect 0.089 in.

Experiments showed that the values obtained in this manner are approximately two-thirds of the actual deflections of a Tee truss. The methods used in calculating truss deflections are very laborious. If the member is treated as a beam, the deflection can be determined in a much shorter period of time. Therefore, beam deflection methods are recommended in determining the deflection of the Tee truss. The deflections obtained in this manner are multiplied by a factor (1.5) to give equivalent truss deflections. (Factor may be reduced to 1.35 when clear span is greater than 6 ft.)

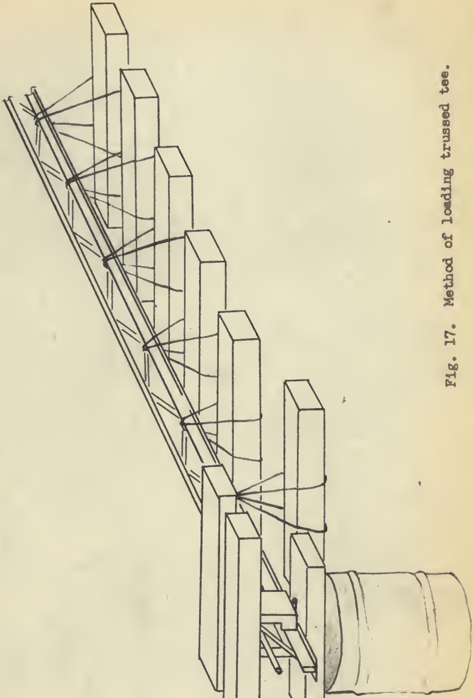


Fig. 17. Method of loading trussed tee.

Fig. 18. Deflection of truss subjected to concentrated loads.

$L = 5' 5''$

$a = 0.361'$

$P = 0.374$ lb

$R_1 = 2.03$ lb

Maximum deflection occurs at $x = 2.116'$

Number of concentrated loads for first span = 15

$$M_x = R_1x - P(x-a)H(x-a) - P(x-2a)H(x-2a) \ldots . -P(x-14a)H(x-14a)$$

$$H(x-na) \begin{cases} = 0 & \text{when } x < na \\ = 1 & \text{when } x \geq na \end{cases} \qquad n = 1, 2, \ldots .. 14$$

$$M_x = R_1x - 6P(x-3.5a)h(x-3.5a) - 9P(x-11a)H(x-11a)$$

$$EI\theta = \frac{1}{2} R_1x^2 - 3P(x-3.5a)^2 H(x-3.5a) - \frac{9P}{2}(x-11a)^2 H(x-11a) + C_1$$

$$EIY = \frac{1}{6} R_1x^3 - P(x-3a)^3 H(x-3.5a) - \frac{9P}{6}(x-11a)^3 H(x-11a) + C_1x + C_2$$

$y = 0$ when $x = 0 \quad \therefore \; C_2 = 0$

$y = 0$ when $x = L$

$$C_1 = -\frac{R_1}{6} L^2 + \frac{P}{L}(L - 3.5a)^3 + \frac{9P}{6L}(L - 11a)^3$$

$$= -\frac{1}{6} \times 2.03 \times 5.41^2 + \frac{0.374}{5.41} \times 4.15^3 + \frac{0.561}{5.41} \times 1.444^3$$

$$= -4.65$$

$$EIY = \frac{1}{6} R_1x^3 - P(x-3.5a)^3 H(x-3.5a) - \frac{9}{6}P(x-11a)^3 H(x-11a) - 4.65x$$

$$EIY_{(x = 2.166)} = \frac{2.03}{6} \times 2.166^3 - .374 \times .902^3 - 4.65 \times 2.166 = -6.97$$

$$y_{(max)} = \frac{6.97 \times 1728}{0.451 \times 30 \times 10^6} = 0.00089'' \text{ (Beam deflection for unit load)}$$

$$y_{(max)} = 0.00089 \times 1.5 \times 100 = 0.133'' \text{ (Truss deflection for load of 100 lb per running ft.)}$$

Note: The actual deflection for an 0-5-14-2 1/2 loaded in this manner is 0.1268 in.. This factor gives a conservative deflection.

Table 3. Allowable clear span length for truss with a load of fresh concrete.

Tee-truss	Load (in. of concrete)	Trusses spaced 12" on center — Max. clear span: based on working stress	Trusses spaced 12" on center — Max. clear span: based on deflection equal to 1/360 of span	Trusses spaced 16" on center — Max. clear span: based on working stress	Trusses spaced 16" on center — Max. clear span: based on deflection equal to 1/360 of span
4-5-16-2"	4	10' 7"	4' 9"		
1-5-16-2	4	10' 7"	5' 1"	9' 1"	4' 8"
	6	8' 7"	4' 6"		
0-5-14-2	4	11' 8"	5' 3"	10' 1"	5' 0"
	6	9' 7"	4' 8"	8' 4"	4' 6"
4-5-16-2 1/2	4	12' 0"	6' 9"	10' 5"	6' 8"
	6	9' 10"	6' 0"	8' 6"	5' 0"
1-5-16-2 1/2	4	12' 0"	7' 2"	10' 4"	6' 8"
	6	9' 9"	6' 8"	8' 5"	5' 8"
0-5-14-2 1/2	4	13' 6"	7' 7"	11' 8"	6' 10"
	6	11' 0"	6' 8"	9' 7"	6'

Note: These are maximum span lengths based on the weight of fresh concrete before it has set enough to give support to the trusses. A span length less than the maximum should be used as there will be additional deflection when the concrete cures and the temporary supports are removed.

It is recommended that the Tee-trusses be increased in depth to three and one half inches, if they are to be used with concrete. It would appear desirable in some manner to decrease the deflection so that permanent supports or shoring could be placed farther apart. With the members used in this investigation, deflection is the criterion. It is further recommended that the present angle of the sinuous member be reduced from 90 degrees to 68 degrees. This would allow the top bars to be supported laterally at about the same space as the ones presently in use.

A simple way to decrease the deflection is to increase the depth of the member. The only increase in area would be in the length of the sinuous bar, and, if necessary, the cross sectional area of this bar could be reduced to compensate for the increased length without great effect on the deflection.

It seems reasonable to assume that a member of this depth bears the same relationships to a beam with the same I, and a pin-connected truss of the same dimensions as existed in the case of the 2 in. and 2 1/2 in. trusses. On the basis of this assumption, an example of the decrease in deflection can easily be illustrated.

<u>Example:</u> Assumed T-truss, 0-5-14-3 1/2

Angle of sinuous members = 68.2°

Load = 100 lb per running ft (equivalent to 6" of concrete with Tees placed 16" on centers)

Clear span length = 8 ft

Max. deflection occurs at $x = 9a$

$R_1 = 3.02$ lb

$I = 1.0568$

$M_x = R_1 x = -P(x-a)H(x-a) - P(x-2a)H(x-2a) \ldots\ldots -P(x-22a)H(x-22a)$

$$H(x-na) \begin{cases} = 0 \quad \text{when } x < na \\ \qquad\qquad\qquad\qquad n = 1, 2, \ldots\ldots 22 \\ = 1 \quad \text{when } x \geq na \end{cases}$$

$M_x = R_1 x - 9P(x-5a)H(x-5a) - 13P(x-16a)H(x-16a)$

$EI\theta = \frac{1}{2} R_1 x^2 - \frac{2P}{2}(x-5a)^2 H(x-5a) - \frac{13P}{2}(x-16a)^2 H(x-16a) + C_1$

$EIY = \frac{1}{6} R_1 x^3 - \frac{9P}{6}(x-5a)^3 H(x-5a) - \frac{13P}{6}(x-16a)^3 H(x-16a) + C_1 + C_2$

when $x = 0$, $y = 0$, $\therefore C_2 = 0$

when $x = L$, $y = 0$ $C_1 = -15.485$

$EIY = \frac{1}{6} R_1 x^3 + \frac{1.5}{8} P(x-5a)^3 H(x-5a) - 15.485\,x$

when $x = 9a$

$EIY = 3.5525 \times 10$

$$\therefore \; Y = \frac{3.5525 \times 10^1 \times 1728}{30 \times 10^6 \times 1.0568} = 0.001935$$

$\delta = 0.001935 \times 1.35 \times 100 = 0.261$

The allowable deflection for an 8 ft clear span is 0.267 in. $(\frac{L}{360})$.

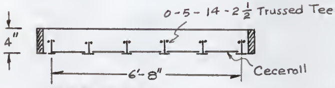
With reference to Table 3, it can be seen that, in this case, the allowable clear span length could be increased two feet, if the Tee truss were one inch deeper.

Part II. ANALYSIS OF THE CONCRETE SLAB

Method of Curing and Loading

The principal parts of the form to hold the concrete consisted of Trussed-Tees with corrugated sheet metal* placed between them. Four-inch boards were placed around the edges of the form. The slab form was 18 ft long and 6 ft 8 in. wide, with permanent supports at each end and in the center. Temporary supports were placed in the middle of each span. The Trussed-Tees were 16 in. apart, with corrugated sheet metal between. Round steel bars 3 4 in. in diameter were fastened to the supports before the Tees were placed on them, in order to give a pinned effect, (Fig. 19).

(a) Side view



(b) End view

Fig. 19. Views of form for concrete slab.

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* The type of corrugated sheet metal used in this investigation was Cecoroll.

The form was filled with readymixed concrete which had a minimum ultimate compressive strength of 3000 psi. The concrete was brought in with wheelbarrows and poured from a height of about one foot above the bottom of the form. The sheet metal was examined, but no bending was evident. The concrete was troweled smooth, and left to cure. About eight hours later it was covered with wet sacks. This sack covering was kept moist for about four days after which the temporary supports were removed.

Two, 6-inch SR-4 strain gauges were sunk in the concrete in the middle of the first span. One gauge, 1/4 in. from the bottom of the slab, ran perpendicular to the Tees; the other, 1/2 in. from the bottom of the slab, ran parallel to the Tees. The gauges were enclosed in copper tubes and pressed flat to be very flexible. Some gauges were placed on the top and bottom of the concrete. (Fig. 20.)

During the first loading, one of the gauges which was sunk in the concrete failed, due to a short circuit. This gauge ran perpendicular to the Tees. The gauge in the concrete which ran parallel to the Tees worked until the second loading. During the second loading it failed. Some of the other SR-4 gauges came loose from the surface of the concrete.

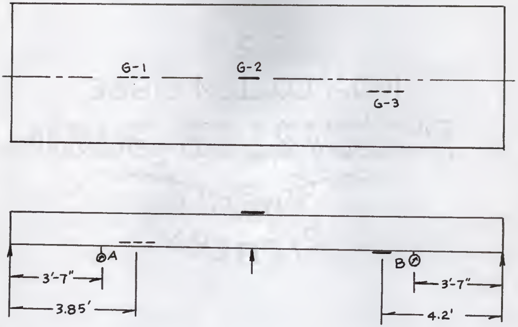

Fig. 20. Location of SR-4 strain gauges and Ames dial gauges on concrete slab.

The load was not applied until the slab had cured for 11 days. Sand was used for the first test load. Four 12- by 1-in. boards were placed along the edges of the slab. These boards were rigidly supported from without. Since the density of sand in use was 94 lb per cu ft, this height of wall would contain an amount of sand corresponding to a uniformly distributed load of 94 lb per sq ft.. The sand was placed in the form to depths of (1) 5 13/16 in., (2) 9 in., and (3) 1 ft. There was an interval of 24 hr after the loading to each depth. During loading, sand was first put into large buckets, transported and elevated to a convenient height, and placed on the slab. Measurements were taken immediately after each loading, and 24 hr after each loading. The sand over the center support was removed after each loading, and the concrete checked carefully for any cracks; none could be detected.

In order to meet the age requirements, according to the American Concrete Institute's Building Code, this load was left on the slab for 60 days (the structure must be 56 days old). At the end of this period there was no indication of failure. The maximum deflection was 0.193 in. Allowable deflection according to the ACI Code is

$$D = \frac{L^2}{12,000\,t} = \frac{(9 \times 12)^2}{12,000 \times 4} = 0.243$$

Also, $\quad .193 < \frac{108}{180} = 0.60.$

After 60 days, the load was removed; twenty-four hours later the deflection was 0.117 in. Twenty-four hours after the load was first applied (average of three loads) the deflection was 0.099 in.

Recoverable deflection $= \frac{0.1915 - 0.117}{0.099} = \frac{0.0745}{0.099} = 0.753$ per cent of deflection after 24-hr load.

As far as can be determined, this structure meets every requirement of the American Concrete Institute's Code except for spacing and covering of reinforcements.

After the load was removed, the box was reformed. Boards transverse to the slab were moved towards the center of each span and two more boards were added to form two boxes on the middle of each span,

(Fig. 21). These boxes were two-feet high, six feet eight inches wide, and four feet long. The excess sand was placed at the center of each box. The deflection under the new load was 0.22 in., which is below the A.C.I. Code allowable deflection. No cracks appeared under this load. This method of loading represents an equivalent uniform load of 175 lb per sq ft..

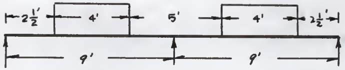

Fig. 21. Form for second load applied to slab.

An additional load consisting of 30 steel forms, each weighing 61.5 lb, was placed on each side, and distributed over the sand. The first hairline crack appeared on the surface of the slab after these forms were added to the load. This crack began at the right edge of the slab over the middle support. It extended inward directly over the middle support for about nine inches.

Characteristics of the Concrete

Two cylinders six inches in diameter were made from the same batch of concrete, and cured under the same conditions as the slab. Under the ultimate load test, the stress-strain curves are shown in Figs. 22 and 23. One of these cylinders failed at a stress of 3010 psi and the other, at a stress of 3190 psi.

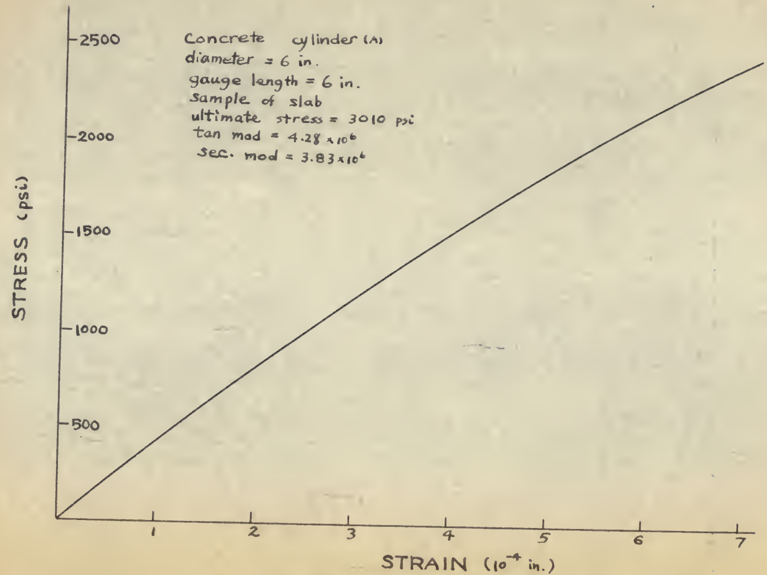

Fig. 22. Stress-strain curve for concrete slab; Cylinder No. 1.

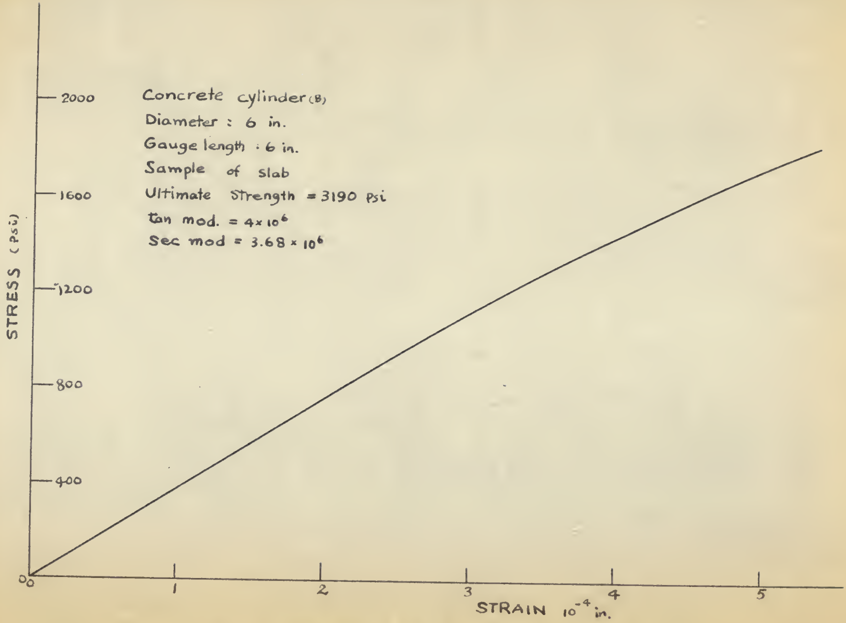

Fig. 23. Stress-strain curve for concrete slab; Cylinder No. 2.

CALCULATIONS

Loading

The slab was subjected to the two types of loading discussed in Part I. A 16-in. section of the slab was considered when the stresses were calculated. (The Trussed-Tees were placed 16 in. apart.) The form was arranged so that the slab was supported only at ends and at the center, (Fig. 24).

The first load consisted of sand placed on the slab to a depth of one foot. Weight of the sand is 94 lb per cu ft..

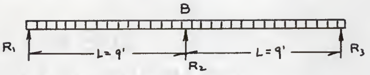

Fig. 24. The concrete slab.

Total weight of sand on each span = $9 \times 6\ 2/3 \times 94 = 5640$ lb.

Total weight on each span per 16 in. strip of slab,

$$W = \frac{5640}{5} = 1128 \text{ lb.}$$

The moment at the center of the support of a slab strip consisting of two symmetrical spans with a uniformly distributed load over the entire strip is given by the formula,

$$M_B = \frac{1}{8}\ WL = \frac{1128 \times 9}{8} = 1275 \text{ ft-lb.}$$

The end reactions are equal to 3/16 of the total load or 3/8 of the span load in the case of equal spans uniformly loaded.

$$R_1 = R_3 = \frac{3}{8} \times 1128 = 424 \text{ lb.}$$

The second method of loading placed the total amount of sand on each span on a four-foot section in the middle of the span, and added 1845 lb to each four-foot section, making a total load of 7485 lb on each span, (Fig. 25).

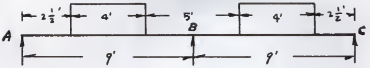

Fig. 25. Method of loading.

If a uniform load is placed in the center of a beam with fixed ends, as shown in Fig. 26, the fixed end moment at A and B is

$\frac{1 + 2a - 2a^2}{12}$ WL.

Fig. 26. Beam with fixed ends; uniform load on center of beam.

After using moment distribution with joints A and C relaxed,

$$M_B = \frac{1 + 2\,\frac{2.5}{9} - 2(\frac{2.5}{9})^2}{8}\ WL = 0.175\ WL$$

$$W = \frac{7485}{5} = 1497 \text{ for a 16-in. section.}$$

$$M_B = 0.175 \times 1497 \times 9 = 2360 \text{ ft-lb.}$$

$$R_1 = R_3 = \frac{1497 \times 4.5 - 2360}{9} = 487 \text{ lb.}$$

The equivalent uniform load over the entire span is found by equating

$$\frac{wl^2}{8} = 2360$$

$$w = \frac{2360 \times 8}{9^2} = 233$$

$$233 \times \frac{12}{16} = 175 \text{ lb per sq ft.}$$

Composite Section

A composite section was used to check the stress in the concrete before any effective crack occurred. The concrete was evidently taking tensile stress, and it was desired to calculate this stress.

$$n = \frac{E_s}{E_c} = \frac{30 \times 10^6}{3.76 \times 10^6} = 8.$$

The areas of the various steel members were multiplied by eight. A section 16 in. wide was used, (Fig. 27). When the sand was unloaded and replaced in the center of each span, this left the strain gauge on top of the center support uncovered, and the gauge acted more effectively.

The position of the centroidal axis and moment of inertia can be calculated as follows:

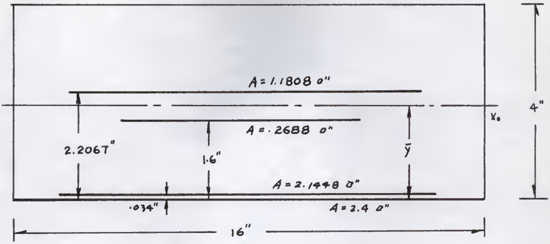

Fig. 27. Sixteen-inch composite section of slab.

$$
\begin{array}{llllll}
2.4 & \times & 0 & = & 0 \\
2.1448 & \times & 0.034 & = & 0.073 \\
0.2688 & \times & 1.6 & = & 0.430 \\
1.1808 & \times & 2.2067 & = & 2.61 \\
63.25 & \times & 2.0 & = & 126.50 \\
\hline
69.24\ \bar{y} & & & = & 129.61 \\
 & & \bar{y} & = & 1.875 \text{ in.}
\end{array}
$$

$$
\begin{array}{lllll}
2.4 & \times & 1.875^2 & = & 8.45 \\
2.1448 & \times & 1.841^2 & = & 7.28 \\
0.2688 & \times & 0.275^2 & = & 0.02 \\
1.1808 & \times & 0.3317^2 & = & 0.13
\end{array}
$$

$$\frac{33.25 \times 4^2}{12} + 63.25 \times .127^2 = 85.39$$

$$I_{x_0} = 101.27$$

Calculated Stresses Versus Measured Stresses

The placement of strain gauges is shown in Fig. 28.

Note: G-1 is in concrete one-half inch from the bottom.

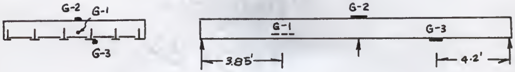

Fig. 28. Location of SR-4 strain gauges on slab.

On a 16-in. section the uniform load is

$$w = 94 \times \frac{16}{12} = 125 \text{ lb per sq ft.}$$

G-1: Calculated stress for uniform load.

$$M_A = 424 \times 3.85 - \frac{1}{2} \times 125 \times 3.85^2 = 703 \text{ ft-lb}$$

$$S_c = \frac{703 \times 12 \times (1.875 - 0.5)}{101.27} = 118.8 \text{ psi}$$

G-1 reading after load applied = 0-7-632

G-1 reading before load applied = 0-7-590

42

$$S_q = 42 \times 10^{-6} \times 3.76 \times 10^6 = 158 \text{ psi}$$

G-1: Calculated stress for second loading, (Fig. 29).

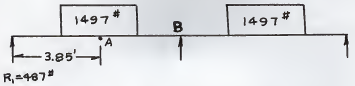

Fig. 29. Location of loads on slab.

$$M_A = 487 \times 3.85 - \frac{1}{2} \frac{1497}{4} (3.85 - 2.5)^2 = 1540 \text{ ft-lb}$$

$$S = \frac{1540 \times 12 \times 1.375}{101.27} = 250 \text{ psi.}$$

The No. 1 gauge ceased to function before the second load was applied, and no reading could be obtained.

G-2: Calculated stress for uniform load.

When gauge No. 2 was covered with sand in the first loading, the gauge was affected by the moisture in the sand, and acted so erratically that no effective reading could be obtained.

Since $M_B = 1275$ ft-lb

$$S = \frac{1275 \times 12(4 - 1.875)}{101.27} = 321 \text{ psi.}$$

G-2: Calculated stress versus measured stress; second loading.

$$M_B = 2360 \text{ ft-lb}$$

$$S = \frac{2360 \times 12 \times 2.125}{101.27} = 594 \text{ psi}$$

G-2 reading with load applied = A - 3 - 255

G-2 reading before load applied = A - 3 - 120

135

$S = 135 \times 3.76 = 508$ psi

G-3: Calculated stress versus measured stress for uniform load.

Note: This gauge was placed on the bottom of the Trussed-Tee.

$R_3 = 424$ lb

$M_c = 424 \times 4.2 - \frac{1}{2} \times 125 \times 4.2^2 = 690$ ft-lb

$S = \frac{nM_c}{I} = \frac{8 \times 680 \times 12 \times 2.02}{101.27} = 1300$ psi

G-3 reading after load applied = A - 1 - 950

G-3 reading before load applied = A - 1 - 905

45

$S = 45 \times 30 = 1350$ psi.

G-3: Calculated stress versus measured stress; second loading.

$R_3 = 487$ lb

$M_c = 487 \times 4.2 - \frac{1.7}{4} \times 1497\ (.85) = 1505$ ft-lb

$S = \frac{8 \times 1505 \times 12 \times 2.02}{101.27} = 2890$ psi

SR-4 reading after load applied = A - 1 - 1218

SR-4 reading before load applied = A - 1 - 1110

108

$S = 108 \times 30 = 3240$ psi.

In analyzing the composite section, it was assumed that the sinuous bar acted as a straight bar with its axis located at its centroid. It was also assumed that the corrugated sheet metal acted as a straight sheet of metal bounded to the base of the concrete.

The calculated stresses for the points on the slab section were based on the assumption that straight-line stress distribution existed through the depth of the slab. This would cause the position of the neutral axis to coincide with the position of the centroidal axis. This assumption was evidently in error, as shown by a comparison of the calculated stresses with the measured stresses.

If the neutral axis were assumed to be 2.1 in. from the bottom, with straight-line stress distribution, the calculated stresses would check more closely with the measured stresses. (See Table 4.)

Table 4. Stresses with neutral axis changed.

Location	Calculated stress, psi	Measured stress, psi
G-1 (1st reading)	136	150
G-2 (2nd reading)	515	505
G-3 (1st reading)	1358	1350
G-3 (2nd reading)	3150	3240

Transformed Sections

After loading by the second method described, the slab did not crack. The center support was then jacked up until a crack occurred. This involved jacking up the center support 3/8 in.. This joint translation increased the negative moment at the center support by 3070 ft-lbs.. Total moment at center support = 2360 + 3070 = 5430 ft-lb.

Equivalent uniformly distributed load,

$$w = \frac{5430 \times 8 \times 12}{9^2 \times 16} = 402 \text{ lb per sq ft.}$$

With the concrete cracked across the center support, it was necessary to use a transformed section for stress calculations. The concrete is assumed to carry no tensile stress under this condition. (See Fig. 30)

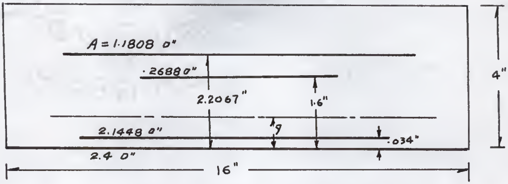

Fig. 30. Sixteen-inch transformed section of concrete slab.

$$1.1808\,(2.2067 - y) + 0.2688\,(1.6 - y) = 16y\,\frac{y}{2} + \frac{7}{8} \times 2.4y + \frac{7}{8}\,2.1448\,(u - 0.034)$$

$$8y^2 + 5.53y - 3.105 = 0$$

$$\bar{y} = 0.367 \text{ in.}$$

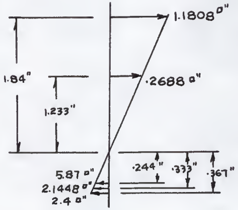

Fig. 31. Location of stressed members in transformed section.

From Fig. 31, the following equation can be deduced:

$$1.1808\,S\,\frac{1.84^2}{1.84} + \frac{1.233^2}{1.84}S \times 0.2688 + 5.87 \times \frac{0.244^2}{1.84}S + 2.1448$$

$$\times \frac{0.333^2}{1.84}S + 2.4\,\frac{0.367^2}{1.84}S = 5430 \times 12$$

$$S_{\text{top bar}} = \frac{5430 \times 12 \times 1.84}{5.315} = 22,500 \times 8 \times 180,000 \text{ psi}$$

$$S_{\text{sin bar}} = 180,000 \times \frac{1.23}{1.84} = 120,000 \text{ psi}$$

$$S_{\text{concrete}} = 180,000 \times \frac{0.244}{1.84} \times \frac{1}{8} = 2990 \text{ psi}$$

$$S_{\text{angle}} = 180,000 \times \frac{0.333}{1.84} = 32,500 \text{ psi}$$

$$S_{\text{Ceccoroll}} = 180,000 \times \frac{0.367}{1.84} = 35,900 \text{ psi}$$

The assumption of straight-line stress distribution must be in error due to the large stress in the top bars.

It is evident that under these stresses the top bars would yield and move the axis lower. As the bottom stress increased, the angles and the Ceccoroll yielded. The member now would work very much like two simply supported beams with a hinge at the center.

The worst condition with complete freedom of rotation at the center is assumed. The concrete is assumed to be cracked on the bottom at the center of the span. The critical condition will be at the center of the slab with compression in the top of the beam.

The transformed section and location of the stressed members for the center of the span are shown in Figs. 32 and 33.

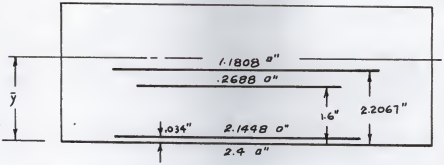

Fig. 32. Transformed section at center of slab.

$$16(4-y)\frac{(4-y)}{2} = 1.1808(y - 2.2067) + 0.2688(y - 1.6) + 2.1448(y - 0.034) + 2.4y$$

$$8y^2 - 69.99y + 131.32 = 0$$

$$\bar{y} = 2.72 \text{ in.}$$

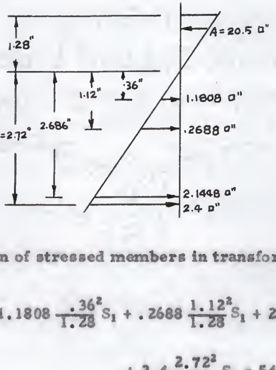

Fig. 33. Location of stressed members in transformed section.

$$20.5 \times \frac{.84^2}{1.28} S_1 + 1.1808 \frac{.36^2}{1.28} S_1 + .2688 \frac{1.12^2}{1.28} S_1 + 2.1448 \times \frac{2.686^2}{1.28} S_1 + 2.4 \frac{2.72^2}{1.28} S_1 = 5430 \times 12$$

$$48.09\, S_1 = 5430 \times 12 \times 1.28$$

$S_1 = 1735$ psi (maximum stress in concrete)

$$S_{\text{top bars}} = \frac{.36}{1.28} \times 1735 \times 8 = 3900 \text{ psi}$$

$$S_{\text{sinuous bars}} = \frac{1.12}{1.28} \times 1735 \times 8 = 12,125 \text{ psi}$$

$$S_{\text{angles}} = \frac{2.686}{1.28} \times 1735 \times 8 = 29,100 \text{ psi}$$

$$S_{\text{Ceccoroll}} = \frac{2.73}{1.28} \times 1735 \times 8 = 30,600 \text{ psi}$$

As can be seen from these figures, the bottom members are stressed more than allowable, but not up to the yield stress of the angle. (Yield stress of Ceccoroll not known.) Under this loading the slab has considerable deflection but is not near the point of collapse. This structure can carry an external load of 150 lb per sq ft with a safety factor greater than three, if failure load is considered in determining the safety factor. The slab can carry a load of over 100 lb per sq ft in addition to its weight for long periods of time without exceeding the allowable deflection.

Relation of Slab Deflection to Loads

A dial gauge was placed at the point of maximum deflection of each span of the slab. The deflections were noted for the various loads. The results are shown in Table 5.

Table 5. Deflections resulting from the various loadings.

Uniform Load	Lb / sq ft			94 lb/sq ft after loading		0 lb/sq ft sand removed	Lb/sq ft 1 week after loading			402 lb/sq ft Slab cracked over middle support
	47	70	94	30 da	50 da.		132	160	175	
Gauge A	.055	.071	.097	.181	.193	.118	.215	.294	.332	.407
Gauge B	.047	.071	.101	.179	.188	.116	.205	.278	.322	.452
Av.			.099		.1915	.117				

$$\frac{.1915 - .117}{.099} = 0.753$$

Slab meets A.I.S.C. specification on 75 per cent recovery of deflection.

OBSERVATIONS

Two SR-4 strain gauges were placed on the slab perpendicular to the direction of the Trussed-Tees. One of the gauges was glued to the top surface, and the other, to the Ceccoroll on the bottom. The effect of the moisture from the slab caused the gauge on the top surface to give erratic readings. No data were taken, but the readings of the gauge on the Ceccoroll became smaller as the load was increased. This is an indication of anticlastic effect at the center of the beam.

One interesting feature of this loading was the action of the slab when the sand was first placed on it. When the strain gauge readings were first taken, it was found that the gauges on top parallel to the Tees showed that compression in the top was decreasing, and those on the bottom showed that the tension in the bottom of the slab was also decreasing. It was also noted that, after the first loading, the deflection of the span was about 0.05 in., but 24 hr later, before any more sand was placed on it, the deflection was only 0.04 in.. In this manner, it was determined that the moisture in the sand actually caused the top of the slab to swell to such an extent as to reduce the deflection. This is an illustration of the very definite effect of moisture on concrete.

RECOMMENDATIONS

This investigation has shown that a slab of this size and depth can carry a load of 100 lb per sq ft with a factor of safety greater than that required by the American Institute of Steel Construction's specifications regarding deflection and deflection recovery. It is recommended that, in the use of slabs of this type, provisions be so made that the slab is not supported on the sides parallel to the direction of the Tees. Some very light wire mesh might be added as temperature reinforcement for slabs of larger span and width. However, it is not necessary for a slab of the size used in this investigation.

It is recommended that consideration be given to increasing the depth of Tees to 3.5 inches. This would increase the strength of continuous slabs over the supports.

CONCLUSIONS

The use of Trussed-Tees and corrugated sheet metal offer a rapid and desirable method of pouring concrete floors. This method eliminates the necessity of building and removing forms. The metal forms that hold the concrete become an integral part of the slab when the concrete sets. Considerable shoring is required, but this is relatively inexpensive in comparison with forms. Some difficulty with deflection may be encountered with a 4-in. slab of this type, unless it is continuous over at least one support. This large deflection would be encountered only with fairly heavy loads. From the results of this experiment, it appears that, in a single span of this size, the maximum load (based on A.C.I. allowable deflection) in addition to weight of the concrete, would be about 85 lb per sq ft.. A heavier load would probably cause a deflection greater than that allowed by the A.C.I. code.

ACKNOWLEDGMENTS

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APPENDIX

EXPLANATION OF PLATE I

Close-up view of failure of truss under concentrated load.

PLATE I

EXPLANATION OF PLATE II

A distant view of failure of truss under concentrated load.

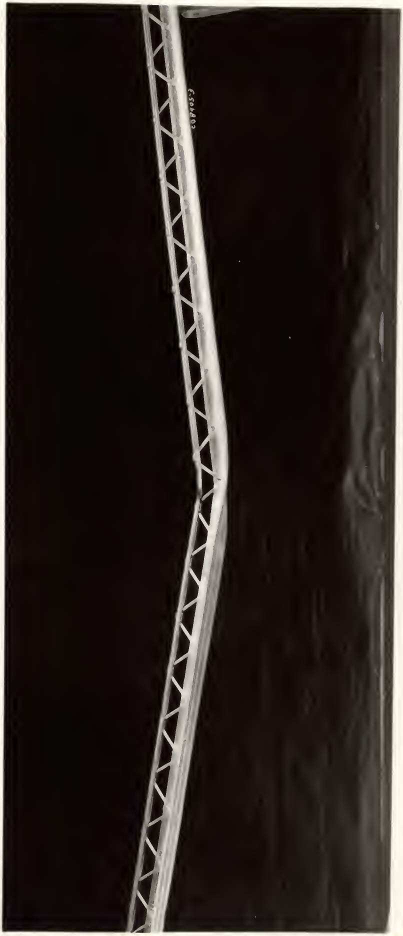
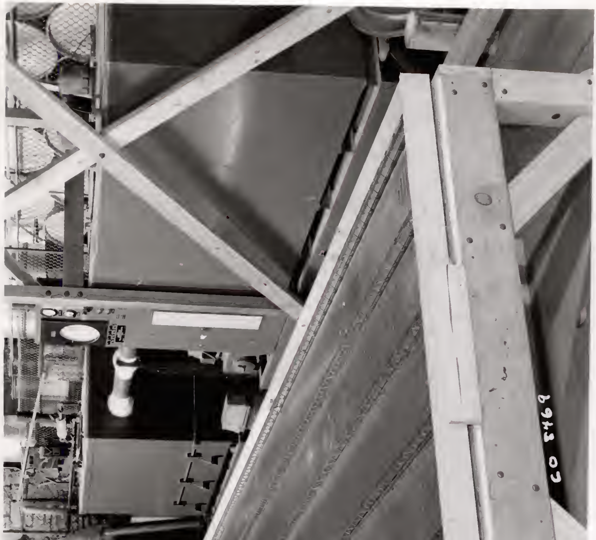


PLATE II

EXPLANATION OF PLATE III

Trussed-Tee and Ceccoroll form for concrete.

PLATE III



ANALYSIS OF TRUSSED-TEE REINFORCED CONCRETE SLAB

by

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B. S., National Southwest Associated University, Kumming, China, 1945

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This project involved an investigation of the feasibility of using Trussed-Tees and corrugated sheet metal in the construction of concrete slabs. The Tees and sheet metal function both as forms and as reinforcements in the finished slab.

The first part of this investigation involved an analysis of certain types of Trussed-Tees. A relationship between the stresses and deflections in these members and those of pin connected trusses of the same size was established. It was determined that while the Tees acted as part of the form for the concrete, a maximum allowable deflection was obtained before the steel in the truss had reached its allowable stress. The calculations indicated that quite a number of temporary supports would be required.

The second part of this investigation involved an analysis of the concrete slab. Theoretical calculations were made to determine the efficiency of the Trussed-Tee and corrugated metal as reinforcements. The calculated stresses in the concrete and the Tees were compared with measured stresses. Deflections of the slab were checked and compared with the allowable deflections of the American Concrete Institute's Building Code.

The slab cracked over the center supports when it was subjected to an equivalent uniform load of 402 lb per sq ft.. It was determined that this type of slab, only four inches thick, could carry loads of more than 100 lb per sq ft in addition to its own weight with a factor of safety greater than three, and without exceeding the allowable deflection.