### THREE ESSAYS ON THE ECONOMICS OF CONFLICT AND CONTEST

by

### SHANE SANDERS

B.A., Indiana University, 2002

### AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

#### DOCTOR OF PHILOSOPHY

Department of Economics College of Arts and Sciences

KANSAS STATE UNIVERSITY Manhattan, Kansas

2007

### Abstract

The first essay develops a simple sequential-move game to characterize the *endogeneity* of third-party intervention in conflict. We show how a third party's "intervention technology" interacts with the canonical "conflict technologies" of two rival parties in affecting the sub-game perfect Nash equilibrium outcome. From the perspective of *deterrence strategy*, we find that it is more costly for a third party to support an ally to deter a challenger from attacking (i.e., to maintain peace), as compared to the alternative case when the third party supports the ally to gain a disputed territory by attacking (i.e., to create war), ceteris paribus. However, an optimally intervening third party can be either "peace-making," "peace-breaking," or neither depending on the characteristics of the conflict and the third party's stake with each of the rival parties.

The second essay develops a simple model to characterize the role that an intervening third party plays in raising the cost of rebellion in an intrastate conflict. Extending the Gershenson-Grossman (2000) framework of conflict in a two-stage game to the case involving outside intervention in a three-stage game, we examine conditions under which an outside party optimally intervenes such that (i) the strength of the rebel group is diminished or (ii) the rebellion is deterred altogether. We also find conditions in which a third party optimally intervenes at a level *insufficient* to deter rebellion. Such behavior, which improves the incumbent government's potential to succeed in conflict, is often overlooked in conflict studies evaluating the effectiveness of intervention. One policy implication of the model is that an increase in the strength of inter-governmental trade partnerships increases the likelihood that third-party intervention acts to deter rebellion.

In the final essay, a simple model of a college basketball season is constructed to examine the existence of conference bias in college basketball's Ratings Percentage Index. Given the nature of the RPI formula and the hierarchical structure of college basketball's 31 conferences, we expect the RPI to be biased against teams playing a difficult conference schedule. The model verifies that, even in a perfect world where teams play to expectation and can be transitively compared based on revealed performance level, the RPI does not necessarily provide an ordinal mapping from revealed team ability level to the real number line. This result has important implications on NCAA tournament selection and seeding.

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### Acknowledgements

I would like to thank Dr. Yang-Ming Chang for invaluable help and guidance. Having him as an "academic parent" these past couple years, I've found his beta value to be exceptionally high (See Chang and Weisman (2005, *S.E.J.*) if you have no idea what this means.). Fortunately, I was an only child for much of this time and enjoyed a (somewhat contestable) monopoly on his altruism. It seems that Dr. Chang is as excited about research today as he was during his own graduate school days, and such a mindset is contagious. At some point, however, his research will have exhausted the sub-fields of microeconomics.

Thanks to Dr. Dennis L. Weisman and Dr. Dong Li for their constant help, guidance, and encouragement in coursework and in research. They possess and teach skills that are powerful and truly on the frontier of microeconomic research. Dr. Weisman is a figurative giant, albeit a humble one, in terms of his ability to think and his capacity to engage in research. I'll always remember the day Dr. Weisman was absent from a departmental seminar. Despite a nearly-full room, the end chair remained vacant in his honor! Dr. Li is as good as any teacher I've had in making a student think in the classroom. His knowledge runs so deep that he is able to use the Socratic method effectively, questioning a student's understanding until he or she realizes its limitations or contradictions. In addition to the material I've learned from him, his creative teaching style has given me useful ideas for my own teaching.

I'd also like to thank Bhavneet Walia, who pressed me to continue in the Ph.D. program during our first year. If not for her, I'd probably be in a job right now for which I was ill-suited (rather than headed to one for which I'm ill-suited). It was really nice to go through the Ph.D. courses with her. She has an eagerness to learn and is very good at breaking down what steps are necessary to finish a homework or to study for an exam. I really admire those qualities and believe they will help her a great deal in her research career.

I'd like to thank my classmate friends for engaging discussions and interesting experiences. Thanks to Bandar Aba-Alkhail, Casey Abington, Mohaned Al-Hamdi, Mofleh Al-Shogeathri, Yaseen Alhaj-Yaseen, Dave Brown, Emanuel Castro de Oliveira, Alexandra Gregory, Hana Janoudova, Shin-Jae Kang, Eddery Lam, Canh Le, Ramil Mehdiyev, Ebrahim Merza, Lanier Nalley, Boaz Nandwa, Burak Onemli (a true hamburger connoisseur), Andrew Ojede, and Bernard Oteng. Thanks to Joel Potter for four interesting years and two enlightening

West Coast trips, in which we stayed in a youth hostel, were mistaken as German tourists, discussed price discrimination on municipal buses (They don't return change.), saw a Padres day game with \$8 tickets and a second Padres game that evening with \$90 tickets (guess which tickets we didn't pay for), and drove 4,600 miles in an uninsured rental car. Lastly, thanks to Kara Ross, Kyle Ross, Ruben Sargsyan, Daigyo Seo, Jaime Stamatson, and Renfeng Xiao.

# Dedication

To my parents, who were always patient in allowing me to develop my interests and encouraged me despite slow progress in early education.

# Essay 1

# War and Peace: Third-Party Intervention in Conflict

## 1. Introduction

Understanding the role of third parties in conflict is necessary to better comprehend armed confrontation in general. At the forefront of this issue are the assumptions made as to why third parties intervene. For example, Regan (2002) assumes that third parties act in an attempt to limit hostilities. Thus, he takes the role of the third party as that of a "conflict manager." Siqueira (2003) similarly assumes that the short run goal of the intervener is to reduce and suppress the existing level of conflict. The view of intervention posited by the above researchers can be described as the liberal or idealist perspective. This view is embodied in the belief that aversion to humanitarian tragedies is the primary reason outside parties become involved in conflict. But is this view of third-party intervention a realistic one? Do third parties care only about creating peace?

Intuitively, the idealist perspective appears to give an incomplete description of thirdparty intervention. During the Cold War, for example, the Soviet Union intervened militarily on behalf of Afghanistan's ruling Marxist government not to promote peace in the region but to protect its own national security against anti-Soviet forces.<sup>1</sup> Furthermore, empirical research does not complement the view of the idealist perspective. In an empirical investigation that contradicts his main assumption, Regan (2002) found that, on average, third-party intervention tends to increase the duration over which fighting takes place. Given the fundamental assumption of the idealist perspective, this result indicates that an intervening third party would

<sup>&</sup>lt;sup>1</sup> This assessment is based on a top secret communication between Soviet officials dated December 31, 1979. To view the correspondence, visit the Soviet Archives database at http://psi.ece.jhu.edu.

better achieve its objective by ignoring the conflict altogether! Obviously, a broader explanation is necessary to better understand the general nature of third-party effect.

Many studies, such as those by Morgenthau (1967), Bull (1984), and Feste (1992), conclude that parties choose to intervene when national interests are at stake. Regan (1996, 1998) describes this view as the "paradigm of realism" and identifies it as the dominant philosophy in international politics. Complementary to realism is the view that ethical issues and domestic politics play a crucial role in third-party decisions to intervene, a perspective supported by Blechman (1995), Carment and James (1995), and Dowty and Loescher (1996). Regan (1998) discusses the United States intervention in Bosnia as an example of domestic politics swaying a country's decision to intervene. He asserts that public outcry in the United States over failure to take action in Bosnia influenced the Clinton administration's policy. Similar examples exist in which an outside party does not intervene due to the high political cost of doing so. A strength of the realist perspective, taken in union with complementary views, is its recognition that national interest can derive from many disparate sources. In a paper addressing the history and nature of third-party intervention, Morgenthau (1967, p.430) states, "All nations will continue to be guided in their decisions to intervene...by what they regard as their respective national interests." Thus, it is clear that realism views the interests of the third party as selfdefined and potentially broad. In other words, success in a territorial conflict on the part of an "ally" can benefit the third party in a number of ways. Potential future benefits to the third party include enhanced access to natural resources and trade, improved national security, ethical fulfillment, and geo-strategic advantage (Moseley, 2006).

In this paper, we consider a scenario in which a third party's welfare depends on the outcome of a territorial conflict between two rival parties. Specifically, the third party receives a

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greater level of expected payoff when its "ally" gains (or maintains) possession of a disputed territory. As indicated by Vasquez (1993), territorial disputes have been shown to be more salient and more likely to lead to war than conflicts that derive from other issues.<sup>2</sup> Although the specific roots of conflict over territory vary from one land to another, they are directly related to a territory's economic value, nationalist value, or both (Huth, 1996; Wiegand, 2004). We therefore focus our analysis on territorial dispute.

In our model, we do not take the third party as valuing peace between the two rival parties in and of itself.<sup>3</sup> Rather, we take the third party as having a "derived demand" for peace between the rival parties in some cases (and a "derived demand" for fighting between the rival parties in other cases), depending upon how its own direct national interests will be affected. Note that none of the aforementioned potential benefits require that the third party place a positive value on peace between a pair of outside parties, in and of itself. Further, none of these motivations require that the third party intervene to increase the likelihood of peace. After all, even a third party seeking ethical fulfillment may need a change in the status quo to achieve its overall goals. Interestingly, our analysis reveals that under certain conditions, even a third party that does not directly value outward peace will cause such a peace in order to maximize its expected payoff function. This peace creation is simply a bi-product of other third-party goals.

<sup>&</sup>lt;sup>2</sup> Social scientists have observed that territorial disputes are the primary cause of war (see, e.g., Goetz and Diehl, 1992; Vasquez, 1993; Kocs, 1995; Forsberg, 1996; Huth, 1996).

<sup>&</sup>lt;sup>3</sup> We use the terms "peace" and "peaceful outcome" interchangeably in this paper to indicate an absence of fighting. In other words, the defending party is able to effectively deter the challenging party from attacking. This definition is consistent with the notion of "acquiescence" or "deterrence" in sequential-move games of conflict as discussed in Grossman (1999), Gershenson and Grossman (2000), and Gershenson (2002), a "nonaggressive equilibrium" in Grossman and Kim (1995), and "peace" in Chang, Potter, and Sanders (2006). The term "war," on the other hand, indicates a presence of fighting (i.e., an attack by the challenging party). This definition is consistent with the notion of "armed confrontation" in conflict analysis as discussed in Gershenson and Grossman (2000), "engagement" in Gershenson (2002), and "war" in Chang, Potter, and Sanders (2006). <sup>2</sup> Social scientists have observed that territorial disputes are the primary cause of war (see, e.g., Goetz and Diehl, 1992; Vasquez, 1993; Kocs, 1995; Forsberg, 1996; Huth, 1996).

The assumption that third parties do not directly value outward peace acts to restrict the possible nature of the third party. However, the assumption that all or most third parties directly and primarily value peace is also quite restrictive. Perhaps our assumption can shed additional light on the *general* effect of third-party intervention, as shown by Regan (2002).

Having described the assumptions that incorporate the costs and benefits of intervening, we are able to consider the tradeoffs a third party faces when deciding whether to become involved in a conflict. One interesting and prevalent type of third-party intervention, considered in Siqueira's (2003) model, is the military subsidy. As subsidies increase, the likelihood that the ally gains or maintains possession of the territory increases as well. Additionally, we assume the cost of supporting an ally is influenced by the degree of military subsidy. In the Siqueira (2003) model of third-party intervention, the third party is treated exogenously and thus does not act as an economic agent in any general sense when choosing a level of intervention. The third party acts strictly as peacemaker, regardless of the stakes involved in a specific conflict. Additionally, Gershenson (2002) studies the effect of third-party sanctions in the case of *civil* conflict. Though an important contribution to our understanding of civil conflict intervention, Gershenson's scope also precludes an examination concerning the motivations and optimizing behavior of the third party.

We show that modeling a territorial dispute within a *three*-stage game framework allows us to *endogenize* the intervention decision of a third party and, in so doing, to understand the nature and potential effects of third-party intervention in a more comprehensive manner. The timing of the game is as follows. The third party moves first to support its ally, taking into account the impact of its actions in the subsequent leader-follower sub-games played between two rival parties (1 and 2) over a disputed territory. We examine two alternative scenarios for the second and third stages of the three-stage game. In the first scenario, Party 1, as the territorial defender, moves at the second stage to decide on its defensive allocation of military goods, while Party 2, as the challenger, moves at the third and final stage of the overall game played among the three parties. The second scenario just reverses the order of moves between 1 and 2 in the last two stages of the overall game. In both scenarios, the third party considers supporting Party 1, its ally.<sup>4</sup>

Our study complements a recent contribution by Amegashie and Kutsoati (2007), who examine the endogeneity of third-party intervention in a civil conflict.<sup>5</sup> They find, among other things, that a third party is likely to intervene and help the stronger faction when success in the conflict is sensitive to effort or when two warring factions are sufficiently close in ability. They show that benefits from "making the playing field unequal" may exceed the cost of intervention. Methodologically, our work differs from theirs in some important aspects. First, we incorporate third-party intervention into the Gershenson-Grossman (2000) framework of conflict in which two rival parties play a sequential-move game, whereas Amegashie and Kutsoati analyze thirdparty intervention in a setting in which two warring factions play a simultaneous-move game. Second, Amegashie and Kutsoati assume that the third party is a "benevolent social planner" in that it maximizes a weighted sum of the welfare of the warring factions and the non-combatant population when deciding an optimal level of intervention. In our setting, however, the third party is not a social planner but a "selfish" agent who seeks its own interest by maximizing a weighted sum of strategic values associated with a disputed territory, which may be in the "wrong" hands of a non-ally party. Third, we show how "intervention technology" in the form

<sup>&</sup>lt;sup>4</sup> The motivation for this structure is addressed within Section 2.

<sup>&</sup>lt;sup>5</sup> We thank the editor, Arye L. Hillman, and an anonymous referee for drawing our attention to the forthcoming paper by Amegashie and Kutsoati (2007).

of military assistance (Siqueira, 2003) interacts with the canonical "conflict technologies" of two rival parties in affecting the outcomes of the sequential-move game. This three-stage game framework permits us to examine the role of a third party in supporting its ally, viewed from the perspective of deterrence.

Our model demonstrates that the potential of third-party intervention to maintain peace (i.e., to effectively deter the non-ally from attacking a disputed territory) *or* create war (i.e., to help the ally launch a war to gain the territory) crucially depends on the characteristics of the primary parties in conflict, the value (strategic or intrinsic) held by the third party, and the efficacy of military support provided to the ally. In the analysis, we compare third-party intervention over alternative scenarios in order to examine the relative ability of a third party to create peace as compared to war. From the perspective of *deterrence strategy*, we find that it is more costly for the third party to militarily support its ally to defend than to attack, ceteris paribus. That is, for the intervener, it is more costly to create peace than to create a war. However, an optimally intervening third party can be either "peace-making," "peace-breaking," or neither depending on the nature of the conflict and the relationship of the third party with each of the two rival parties.

The remainder of the paper is structured as follows. Section 2 develops a conflict model of third-party intervention in a three-stage game. We examine two alternative scenarios in terms of whether Party 1 or 2 is initially a defender or challenger of a disputed territory. Section 3 presents a comparison between the two scenarios and discusses issues related to relative military costs of creating peace or war. Section 4 summarizes and concludes.

### 2. Third-Party Intervention in a Three-Stage Game

Before characterizing the endogeneity of third-party intervention in a conflict between two rival parties (1 and 2), it is necessary to discuss the term "intervention technology." This term reflects the extent to which a third party can affect the capability of an allied party and, in so doing, affect the overall outcome of the conflict. We assume that Party 3 supports its ally, Party 1, through military subsidy transfers (*M*), which serve to enhance Party 1's military efficiency by reducing its unit cost of arming. Denote such a cost-reduction function as s = s(M), where s'(M) = ds/dM < 0 and  $s''(M) = d^2s/dM^2 > 0$ . That is, an increase in *M* lowers the average cost of arming for Party 1, but the cost-reducing effect is subject to diminishing returns. We will examine how Party 3's intervention technology interacts with the respective conflict technologies of the contending parties to determine a conflict's outcome.

As in the conflict literature, we use a canonical "contest success function" to capture the technology of conflict. That is, the probabilities that Party 1 and Party 2 will succeed in armed confrontation are given respectively by

$$p_1 = \frac{G_1}{G_1 + \gamma G_2}$$
 and  $p_2 = \frac{\gamma G_2}{G_1 + \gamma G_2}$ , (1)

where  $G_1(>0)$  is the amount of military goods that Party 1 allocates to defend the territory,  $G_2(\ge 0)$  is the amount of military goods that Party 2 allocates to challenge for the territory, and  $\gamma$  represents the relative effectiveness of a unit of Party 2's military goods to a unit of Party 1's.<sup>6</sup>

<sup>&</sup>lt;sup>6</sup> For alternative forms of contest success functions, see, e.g., Tullock (1980), Hirshleifer (1989), Skaperdas (1996), and Garfinkel and Skaperdas (2006).

The probabilities of success specified above are in a simple additive form of conflict technologies. According to Garfinkel and Skaperdas (2006), a wide class of contest success functions (CSFs) in an additive form has been utilized in many fields of economics. They further indicate one important characterization associated with these CSFs, which is referred to as the *Independence of Irrelevant Alternatives* property. Specifically, Garfinkel and Skaperdas (2006, p. 4) remark that: "In the context of conflict, this property requires that the outcome of conflict between any two parties depend only on the amount of guns held by these two parties and not on the amount of guns held by third parties to the conflict." This property suggests that third parties have no role in a two-party conflict. It is easy to verify that this statement is valid for the case in which the two conflicting parties determine their optimal amounts of guns in a simultaneous-move game. Interestingly, in the multiple-stage sequential-move game we consider, an intervening third party has an important role in affecting the equilibrium outcome of the two conflicting parties, despite the additive form of conflict technologies in (1). This leads us to examine the endogeneity of third-party intervention.

To endogenously characterize Party 3's choice of intervention level, we adopt a threestage game in our analysis. Party 3 moves first by optimally choosing a level of military subsidy transfers that maximizes its own objective function. In the second and third stages of the game, Parties 1 and 2 move sequentially to determine optimal levels of military goods allocation for the conflict, with the first mover being the territorial possessor. We consider two generic scenarios. In the first case, Party 1 occupies the territory and thus assumes the role of Stackelberg leader during the game's second stage. Party 2, as challenger, then moves in the third and last stage of the game. In the second case, Party 2, as the land's possessor, moves in the second stage while Party 1, as challenger, moves in the third and last stage of the game. In the game's second and third stages, we follow Grossman and Kim (1995) and others after them in utilizing a Stackelberg framework in which the defender leads in determining its defensive allocation of military goods. Gershenson (2002) defends this structure by assuming that the incumbent's institutional framework is relatively rigid; therefore, defensive allocations constitute a commitment on the part of the incumbent. The advantage of this assumption is that it allows for the analysis of a deterrence strategy on the part of the defender. Chang, Potter, and Sanders (2006) develop a model to characterize possible outcomes of a land dispute between two rival parties in a Stackelberg game.<sup>7</sup>

Given that Party 3 provides military subsidy transfers (*M*) to Party 1, we assume for analytical simplicity that the cost-reduction function is  $s = 1/(1+M)^{\theta}$ , where  $\theta$  measures the degree of effectiveness with which a dollar of subsidy reduces Party 1's unit cost of arming and  $0 < \theta < 1$ .

Since Party 3 commits M in stage one, the payoff functions for Parties 1 and 2 in the subsequent stages of the game are given respectively by

$$Y_{1} = \left(\frac{G_{1}}{G_{1} + \gamma G_{2}}\right) V_{1} - \frac{1}{\left(1 + M\right)^{\theta}} G_{1},$$
(2a)

$$Y_2 = \left(\frac{\gamma G_2}{G_1 + \gamma G_2}\right) V_2 - G_2,$$
(2b)

where  $M(\ge 0)$  is the level of military subsidies transferred from Party 3 to Party 1;  $\theta$  represents effectiveness with which a dollar of subsidy reduces Party 1's unit cost of arming;  $V_i$  is total value Party i(i = 1, 2) attaches to holding the territory in the next period, where a party can value

<sup>&</sup>lt;sup>7</sup> More generally, and perhaps more fundamentally, Leininger (1993) shows in an interesting rent-seeking model that players are expected to engage in a sequential-move game. Morgan (2003) further uses a sequential-move game to examine the possibility of asymmetric contests for uncertain realizations of values to rival competitors.

a piece of land for economic and deep intrinsic reasons. Note that the specification in (2a) implies that a third-party intervention is tactically "indirect" in that Party 3's military support does not directly affect the contest success function of Party 1.<sup>8</sup> The incorporation of  $\gamma(>0)$  reflects asymmetry in the technology of conflict and has been adopted by several studies in the literature (see, e.g., Gershenson and Grossman, 2000; Grossman and Mendoza, 2003; Grossman, 2004).<sup>9</sup> Note also that a unit of military goods is somewhat of an abstraction. We might think of it as a "composite good" which includes some amount of weapons, trained soldiers, and strategic information.

### 2.1 Case I: Party 1, the ally, defends a disputed territory.

We examine the first scenario, in which Party 1, the defender in the territorial dispute, moves first in the second stage to determine its defensive allocation of military goods and Party 2, the challenger, moves in the third and last stage of the three-stage game.<sup>10</sup> Consistent with backward induction in game theory, we begin with the game's last stage to analyze Party 2's optimization problem in military goods allocation.

<sup>&</sup>lt;sup>8</sup> When there is no third-party intervention such that M = 0, the three-country, three-stage model reduces to a twocountry, two-stage model as those examined in Gershenson and Grossman (2000), Grossman (2004), and Chang, Potter, and Sanders (2006). Following Hillman and Riley (1989) and Gershenson and Grossman (2000), we consider asymmetric valuations associated with a contested prize which is a disputed territory in our analysis.

<sup>&</sup>lt;sup>9</sup> To show the independent effect of Party 3's military assistance on Party 1's probability of success, we separate the third party's military assistance (*M*) from Party 2's military effectiveness ( $\gamma$ ). It proves intractable, within the framework of our model, to consider an endogenous third party that simultaneously affects  $\gamma$  and provides military subsidy (*M*).

<sup>&</sup>lt;sup>10</sup> We thank an anonymous referee who links this approach to the case of market competition in which an established firm and a potential entrant compete in a Stackelberg fashion. As indicated by the referee, in a typical market entry or barriers to entry game, the incumbent, the one who faces loss of market share is usually modeled as the leader and the entrant, the follower. Please recall footnote 9 and its related discussions where we indicate the adoption of this sequential-move approach by several studies in the conflict or rent-seeking literature.

Given Party 2's payoff function in (2b), if  $\frac{\partial Y_2}{\partial G_2} > 0$  where  $G_2 = 0$ , then  $G_2 > 0$ . In this

case, Party 2 challenges for the territory by choosing an optimal level of arming that satisfies the following Kuhn-Tucker conditions:

$$\frac{\partial Y_2}{\partial G_2} = \left[\frac{\gamma G_1}{\left(G_1 + \gamma G_2\right)^2}\right] V_2 - 1 \le 0; \quad \frac{\partial Y_2}{\partial G_2} < 0 \text{ if } G_2 = 0.$$
(3)

From (3) it follows that

$$G_{2} = \frac{G_{1}^{\frac{1}{2}}}{\gamma} \left[ (\gamma V_{2})^{\frac{1}{2}} - G_{1}^{\frac{1}{2}} \right] \ge 0 \text{ if } 0 \le G_{1} \le G_{1}^{c}, \tag{4}$$

where  $G_1^c$  is Party 1's *deterrent* level of arming.<sup>11</sup> Using (4), we find that  $G_2 = 0$  when

$$G_1^c = \gamma V_2 \,. \tag{5}$$

Equation (4) also defines Party 2's best-response function whose slope is

$$\frac{dG_2}{dG_1} = \frac{V_2^{\frac{1}{2}}}{2\gamma^{\frac{1}{2}}G_1^{\frac{1}{2}}} - \frac{1}{\gamma}.$$
(6)

If  $\frac{\partial Y_2}{\partial G_2} < 0$  where  $G_2 = 0$ , then  $G_2 = 0$ . In this case, Party 2 finds it optimal to refrain from

arming for attack. That is,  $G_2 = 0$  if  $G_1 \ge G_1^c$ .

When Party 2 chooses a positive amount of arming, the Kuhn-Tucker conditions in (3)

imply that  $G_1 + \gamma G_2 = (\gamma G_1 V_2)^{\frac{1}{2}}$ . Substituting this expression into the payoff function of Party 1 yields

<sup>&</sup>lt;sup>11</sup> In this scenario, Party 1 has allocated enough defensive arms to conflict such that Party 2 is deterred from attacking. Within the Stackelberg game between Parties 1 and 2, this equilibrium is characterized by "acquiescence" in that there is an absence of fighting (see, e.g., Gershenson and Grossman, 2000)).

$$Y_{1} = \left[\frac{G_{1}}{\left(\gamma G_{1} V_{2}\right)^{\frac{1}{2}}}\right] V_{1} - \frac{1}{\left(1 + M\right)^{\theta}} G_{1}.$$
(7)

Given that Party 3 determines M in the first stage, Party 1's optimal level of arming must satisfy the following first-order condition:

$$\frac{\partial Y_1}{\partial G_1} = \frac{V_1}{\left(4\gamma G_1 V_2\right)^{\frac{1}{2}}} - \frac{1}{\left(1+M\right)^{\theta}} = 0.$$
(8)

Solving equation (8) for Party 1's optimal defense level of military goods allocation yields

$$G_1^* = \frac{V_1^2 (1+M)^{2\theta}}{4\gamma V_2}.$$
(9)

It is easy to verify that  $\partial G_1^* / \partial M > 0$ , which indicates that an increase in Party 3's military support raises Party 1's allocation of arming. This shows that Party 1's military goods and Party 3's military assistance are "complements," rather than independent of one another.

Substituting  $G_1^*$  into the best-response function of Party 2 in (4), we have Party 2's optimal level of arming as follows:

$$G_{2}^{*} = \frac{V_{1}}{\gamma} \left[ \frac{\left(1+M\right)^{\theta}}{2} - \frac{V_{1}\left(1+M\right)^{2\theta}}{4\gamma V_{2}} \right].$$
 (10)

Substituting  $G_1^*$  from (9) into the slope of Party 2's best-response function in (6) yields the following:

$$\frac{dG_2}{dG_1} = \frac{V_2}{V_1(1+M)^{\theta}} - \frac{1}{\gamma}.$$
(11)

This will be a useful equation when interpreting comparative-static derivatives.

Considering equations (5), (9) and (10), we can say that Party 2 strategically reacts to Party 1 in the following manner:

(i) If Party 1 chooses the critical level of arming such that  $G_1^* = G_1^c = \gamma V_2$ , then  $G_1^* = \gamma V_2$  and  $G_2^* = 0$ .<sup>12</sup> It is then clear that  $p_1^* = 1$  and  $p_2^* = 0$ .

As in Gershenson and Grossman (2000), Party 1 has deterred Party 2 from attacking, and no fighting occurs in this scenario (i.e., a peaceful outcome).

(ii) If Party 1's optimal level of arming is less than the deterrent level of arming such that  $G_1^* < G_1^c = \gamma V_2$ , then  $G_2^* > 0$ .

As also in Gershenson and Grossman (2000), Party 1 has failed to deter Party 2 from attacking, and fighting occurs in this scenario (i.e., war or armed confrontation).<sup>13</sup>

Using the CSFs in (1) and the equilibrium levels of arming  $\{G_1^*, G_2^*\}$ , we calculate the probabilities that Party 1 and Party 2 will succeed in armed confrontation as follows:

$$p_1^* = \frac{V_1}{2\gamma V_2} (1+M)^{\theta} \text{ and } p_2^* = 1 - \frac{V_1}{2\gamma V_2} (1+M)^{\theta}.$$
 (12)

<sup>&</sup>lt;sup>12</sup> The expression  $G_1^c = \gamma V_2$  is derived in the three-stage game when Party 2, as Stackelberg follower, moves in the third and last stage of the game. Using the backward induction approach to solve for the subgame-perfect equilibrium, we begin with Party 2's choice of arming. Because Party 2 does not receive military assistance from Party 3, *M* does not enter into the objective function of Party 2. This explains why *M* does not appear in the expression  $G_1^c = \gamma V_2$ . The term  $G_1^c$  shows the *minimum* level of arming that Party 1 should have for deterring Party 2. As indicated by the best-response function in (6), Party 2's choice of arming depends on that of Party 1 which, in turn, depends on Party 3's military assistance *M*.

<sup>&</sup>lt;sup>13</sup> The justification of such terminology lies in the purpose of defensive arming for a conflict (security over a territory one already possesses), as compared to the purpose of offensive arming for a conflict (forcible acquisition of a territory). We can envision the assumption within a sequential move game. The defender allocates armed soldiers to defend the border of the disputed territory. The potential challenger assesses this defensive allocation. If there are too few soldiers, they attack and fighting commences. If there are an adequate number of soldiers, they find it in their interests not to attack and fighting does not commence.

It also follows from  $\{G_1^*, G_2^*\}$  in (9) and (10) that Party 1 effectively deters Party 2 from challenging for the territory if  $G_2^* = 0$  or

$$\frac{V_1}{2\gamma V_2} - \frac{1}{(1+M)^{\theta}} \ge 0.$$
(13)

Other things being equal, the "effective deterrence" condition (13) is more likely to hold when M rises,  $V_1$  rises,  $\theta$  rises,  $V_2$  falls, or  $\gamma$  falls. Conversely, if  $G_2^* > 0$ , i.e.,

$$\frac{V_1}{2\gamma V_2} - \frac{1}{\left(1+M\right)^{\theta}} < 0, \tag{14}$$

then the "deterrence strategy" is incomplete and Party 1 fails to prevent Party 2 from challenging. In this scenario, the probabilities that Party 1 and Party 2 will succeed in conflict are given by  $p_1^*$  and  $p_2^*$  in (12). From equations (7), (9), and (12), we have the following comparative-static results:

$$\frac{\partial p_1^*}{\partial \gamma} < 0, \quad \frac{\partial p_1^*}{\partial M} > 0, \quad \frac{\partial p_1^*}{\partial V_1} > 0, \quad \frac{\partial p_1^*}{\partial V_2} < 0, \quad \frac{\partial p_1^*}{\partial \theta} > 0, \quad \frac{\partial Y_1^*}{\partial M} > 0, \text{ and } \quad \frac{\partial Y_2^*}{\partial M} < 0. \tag{15}$$

Thus, the probability that Party 1 maintains the land increases in the value of military subsidies transferred from Party 3 to Party 1, decreases in the relative military effectiveness of Party 2 compared to Party 1, increases in the value Party 1 places on the land relative to Party 2, and increases in the effectiveness with which a dollar of military transfers reduces Party 1's unit cost of military goods. Furthermore, Party 1's expected payoff increases, and Party 2's expected payoff decreases, as the value of military subsidies transferred to Party 1 rises.

The above analysis has an interesting implication. Intervention by Party 3 to help the defender may create peace between the two primary parties when armed confrontation would otherwise have occurred. Using the effective deterrence condition (13), we find that Party 3 has

the effect of *preventing war* if its choice of military subsidy level, M, satisfies the following condition:

$$M \ge M^c > 0$$
, where  $M^c \equiv \left(\frac{2\gamma V_2}{V_1}\right)^{\frac{1}{\theta}} -1.$  (16)

Note that  $M^c (\ge 0)$  defines the *critical* level of military subsidies such that Party 1 effectively deters Party 2. For the special case in which  $M^c = 0$ , Party 1 deters Party 2 from attacking in the absence of third-party intervention. If Party 3's military subsidy is such that  $M = M^c (\ge 0)$ , we can be sure that Party 2 is deterred from attacking. Additionally,  $M^c > 0$  on the right hand side of the expression assures that, had Party 3 not intervened (M = 0), war would have occurred.

Next, we proceed to the first stage of the three-stage game to examine the optimal subsidy allocation problem of Party 3. There are potential benefits to an intervening third party should its ally possesses the land. Denote  $S_i$  as the benefit or strategic value Party 3 will derive from the land should Party i(i=1,2) hold possession. It is postulated that  $S_1 > S_2 \ge 0$ , i.e., Party 3 will be better off if Party 1, its ally, holds the land.<sup>14</sup> We assume that the objective of Party 3 is to maximize the expected benefits or strategic value associated with the disputed territory net of its military subsidies to the allied Party 1. Specifically, this objective function is taken as

<sup>&</sup>lt;sup>14</sup> Given this assumption, it can be shown that Party 3 would never subsidize Party 2 if "allowed" the opportunity within the framework of the model. This is due to the fact that victory in the conflict by Party 2 constitutes the less preferred outcome from Party 3's perspective.

$$U_3 = p_1 S_1 + p_2 S_2 - M, (17)$$

where  $p_1$  and  $p_2$  are the probabilities that Party 1 and Party 2 will succeed in armed confrontation as given in (12).

Party 3 provides military subsidies only when they are able to *increase* the probability that Party 1 will hold the land. In other words, they stop intervening either before Party 1 becomes deterrent or at the point in which Party 1 becomes deterrent. Also, Party 3 never intervenes when Party 1 will achieve deterrence independently. Hence, the range  $[0, M^c]$  constitutes its relevant subsidy choice set. The Kuhn-Tucker conditions for Party 3's optimal choice of military subsidies are:

$$\frac{\partial U_3}{\partial M} = \frac{\theta \left(S_1 - S_2\right)}{2\gamma} \frac{V_1}{V_2} \left(1 + M\right)^{\theta - 1} - 1 \le 0; \quad \frac{\partial U_3}{\partial M} < 0 \text{ if } M = 0.^{15}$$

$$\tag{18}$$

It follows from (18) that

$$\frac{\partial U_3}{\partial M} < 0 \text{ if and only if } 0 < S_1 < \frac{2\gamma V_2}{\theta V_1} (1+M)^{1-\theta} + S_2.$$

This result indicates that Party 3's military subsidies to Party 1 will be zero when the strategic value of the disputed land to the third party,  $S_1$ , is critically low. To examine implications of third-party intervention, we assume that  $S_1$  is sufficiently high in value such that the necessary condition for expected payoff maximization,  $\partial U_3/\partial M = 0$ , has an interior solution. This condition implies that, in equilibrium, the expected marginal benefit  $(mb_1)$  of allocating one dollar to military subsidies,

<sup>&</sup>lt;sup>15</sup> We thank an anonymous referee for suggesting that we use Kuhn-Tucker conditions to characterize the solution of Party 3's optimization problem. This approach allows us to examine the conditions under which outsiders may or may not be involved in a two-party conflict.

$$mb_{1} \equiv \frac{\theta(S_{1}-S_{2})}{2\gamma} \frac{V_{1}}{V_{2}} (1+M)^{\theta-1},$$

is equal to marginal cost (i.e., one dollar).<sup>16</sup> Solving for the sub-game perfect equilibrium subsidy yields<sup>17</sup>

$$M^{*} = \left[\frac{\theta(S_{1} - S_{2})}{2\gamma} \frac{V_{1}}{V_{2}}\right]^{\frac{1}{1-\theta}} - 1.$$
(19)

It is easy to verify the following comparative-static derivatives:

$$\frac{\partial M^*}{\partial S_1} > 0, \ \frac{\partial M^*}{\partial S_2} < 0, \ \frac{\partial M^*}{\partial \gamma} < 0, \ \frac{\partial M^*}{\partial V_1} > 0, \ \text{and} \ \frac{\partial M^*}{\partial V_2} < 0.$$

Thus, Party 3's optimal military assistance to Party 1 increases with the strategic value  $S_1$ , decreases with the strategic value  $S_2$ , decreases with the relative effectiveness  $\gamma$  of Party 2's military goods to that of Party 1's, and increases with the intrinsic value Party 1 places on the land relative to Party 2,  $V_1/V_2$ .

For clarity, let us focus on the latter three comparative-static derivatives, which are not immediately intuitive. As Party 2 becomes relatively stronger than Party 1 (as  $\gamma$  rises,  $V_2$  rises, or  $V_1$  falls), Party 2 reacts more heavily, as follower, to each additional military good that Party 1, as defender, allocates to defense. It follows from (11) that

$$\frac{\partial}{\partial \gamma} \left( \frac{dG_2}{dG_1} \right) > 0, \ \frac{\partial}{\partial V_2} \left( \frac{dG_2}{dG_1} \right) > 0, \text{ and } \frac{\partial}{\partial V_1} \left( \frac{dG_2}{dG_1} \right) < 0.$$

This, in turn, implies that the subsidy becomes less marginally effective in increasing the probability that Party 1 wins the conflict as Party 2 becomes relatively stronger. That is, equation (11) implies that

<sup>&</sup>lt;sup>16</sup> For the special case in which  $S_1 = S_2$ , we have a solution where the optimal third-party military subsidy is zero.

<sup>&</sup>lt;sup>17</sup> See Appendix A for a detailed derivation of the optimal military subsidy.

$$\frac{\partial}{\partial \gamma} \left( \frac{dp_1}{dM} \right) < 0, \ \frac{\partial}{\partial V_2} \left( \frac{dp_1}{dM} \right) < 0, \text{ and } \frac{\partial}{\partial V_1} \left( \frac{dp_1}{dM} \right) > 0.$$

Thus, Party 3 derives less expected marginal benefit  $(mb_1)$  with each dollar of military transfer as Party 2 becomes relatively stronger. That is,

$$\frac{\partial(mb_1)}{\partial \gamma} < 0, \ \frac{\partial(mb_1)}{\partial V_2} < 0, \ \text{and} \ \frac{\partial(mb_1)}{\partial V_1} > 0.$$

It then follows that

$$\frac{\partial M^*}{\partial \gamma} < 0, \ \frac{\partial M^*}{\partial V_2} < 0, \text{ and } \frac{\partial M^*}{\partial V_1} > 0.$$

Using (16) and (19), we find, in terms of the exogenous parameters, the necessary and sufficient condition under which Party 3 creates peace when war would otherwise have occurred,

$$\left[\frac{\theta\left(S_1-S_2\right)V_1}{2\gamma V_2}\right]^{\frac{\theta}{1-\theta}} \ge \frac{2\gamma V_2}{V_1} > 1.$$
(20)

The first inequality relation in (20) is more likely to hold as a dollar of subsidy becomes more effective in reducing Party 1's cost of arming or as Party 3 places more value on the land not changing hands (i.e.,  $(S_1 - S_2)$  increases). The second inequality relation in (20) requires that  $2\gamma V_2 > V_1$ .<sup>18</sup> In other words, Party 1 should not be able to deter Party 2 in the absence of intervention if Party 3 is to create peace.

The findings of the analysis lead us to establish the following proposition:

**Proposition 1**: Given that the outcome of the conflict (whether peaceful or otherwise) can be altered through a third party's intervention, Party 3 will not support Party 1 (its ally) in defending a disputed territory unless the territory's strategic value to the intervening party is

<sup>&</sup>lt;sup>18</sup> In view of equation (16) that  $M^c > 0$ , we have  $(2\gamma V_2 / V_1)^{1/\theta} > 1$  which implies that  $2\gamma V_2 > V_1$ .

sufficiently high. After having decided to intervene by supplying military subsidies to the ally, the third party is more likely to create <u>peace</u> in this case (i) as its alliance with Party 1 becomes stronger (i.e.,  $(S_1 - S_2)$  is sufficiently large), (ii) as Party 2 becomes relatively weaker in terms of military effectiveness, and (iii) as Party 2 becomes weaker in terms of relative land valuation.

### Case II: Party 1, the ally, challenges for gaining the disputed territory.

Next, we examine an alternative scenario in which the disputed territory is initially in the "wrong" hands of Party 2, viewed from the standpoint of the intervening Party 3. In this scenario, Party 2 becomes the territorial defender (i.e., an incumbent) whereas Party 1, hoping to gain the territory, is the challenger. In terms of the timing of the sequential game, Party 2 moves in the second stage to decide its defensive allocation of military goods and Party 1 moves in the third and final stage of the three-stage game. We continue to examine possible military subsidy allocations (M) from Party 3 to its ally, Party 1, in the first stage of the three-stage game. We use backward induction to solve for the subgame-perfect Nash equilibrium. Given Party 3's commitment in military assistance in stage one, we begin with the game's third stage to analyze Party 1's optimization problem in military goods allocation.

Given Party 1's payoff function (see (2a)), if  $\frac{\partial Y_1}{\partial G_1} > 0$  where  $G_1 = 0$ , then  $G_1 > 0$ . With military subsidies *M* from Party 3, Party 1's optimal choice of military goods allocation satisfies the following Kuhn-Tucker conditions:

$$\frac{\partial Y_1}{\partial G_1} = \left[\frac{\gamma G_2}{\left(G_1 + \gamma G_2\right)^2}\right] V_1 - \frac{1}{\left(1 + M\right)^{\theta}} \le 0; \quad \frac{\partial Y_1}{\partial G_1} < 0 \text{ if } G_1 = 0.$$
(21)

It follows from (21) that

$$G_{1} = \left(\gamma G_{2} V_{1}\right)^{\frac{1}{2}} \left(1 + M\right)^{\frac{\theta}{2}} - \gamma G_{2} \ge 0 \quad \text{if } G_{2}^{c} \ge G_{2} > 0,$$
(22)

where  $G_2^c$  is Party 2's deterrent level of arming and is given as  $G_2^c = \frac{V_1(1+M)^{\theta}}{\gamma}$ . That is,

 $\frac{\partial Y_1}{\partial G_1} < 0$  when Party 2's arming is set at the critically high level of  $G_2^c$ . In this case, Party 1's best decision is to not challenge, i.e.,  $G_1 = 0$ . Equation (22) defines the best-response function of Party 1's allocation in military goods to Party 2's arming. The deterrent level of arming,  $G_2^c$ , is higher in the presence of third-party intervention (M > 0) than in its absence (M = 0). This finding implies that third-party intervention to support Party 1 (the challenger) makes it more costly for Party 2 (the defender) to achieve a deterrent strategy.

Next, we examine Party 2's optimization problem in stage two. Substituting  $G_1$  from (22) into Party 2's payoff function yields

$$Y_{2} = \left[\frac{\gamma G_{2}}{(\gamma G_{2}V_{1})^{\frac{1}{2}}(1+M)^{\frac{\theta}{2}}}\right]V_{2} - G_{2}$$

The objective of Party 2 in stage two is to maximize  $Y_2$  by choosing its optimal defensive level of military goods allocation, which is given as follows:

$$G_2^{**} = \frac{\gamma V_2^2}{4V_1 (1+M)^{\theta}}.$$
(23)

It is straightforward that  $\partial G_2^{**}/\partial M < 0$ , which indicates the effect of third-party intervention through military support in lowering Party 2's defensive allocation of arming.

Substituting  $G_2^{**}$  from (23) into (22) yields Party 1's optimal level of arming:

$$G_{1}^{**} = \frac{\gamma^{\frac{1}{2}}V_{2}}{2} - \frac{(\gamma V_{2})^{2}}{4V_{1}(1+M)^{\theta}}.$$
(24)

It is clear that  $\partial G_1^{**}/\partial M > 0$ , which indicates the effect of third-party intervention through military support in raising Party 1's offensive allocation of arming.

Using equations (23) and (24), we determine the probabilities that Party 1 and Party 2 will succeed in armed confrontation as follows:

$$p_2^{**} = \frac{\gamma^{\frac{3}{2}}V_2}{2V_1(1+M)^{\theta}} \text{ and } p_1^{**} = 1 - \frac{\gamma^{\frac{3}{2}}V_2}{2V_1(1+M)^{\theta}}.$$
 (25)

We thus have the following comparative-static derivatives:

$$\frac{\partial p_1^{**}}{\partial \gamma} < 0, \ \frac{\partial p_1^{**}}{\partial M} > 0, \ \frac{\partial p_1^{**}}{\partial V_1} > 0, \ \frac{\partial p_1^{**}}{\partial V_2} < 0, \ \frac{\partial p_1^{**}}{\partial \theta} > 0, \ \frac{\partial Y_1^{**}}{\partial M} > 0, \ \text{and} \ \frac{\partial Y_2^{**}}{\partial M} < 0.$$

In Case II, as in Case I, we find that parameters affect Party 1's optimal probability of success in terms of qualitative results.

It is instructive to discuss deterrent conditions for the sequential-move game of conflict.

(i) If  $\frac{\partial Y_1}{\partial G_1} \le 0$  where  $G_1 = 0$ , then  $G_1^{**} = 0$ . In this case, Party 2 deters Party 1 from challenging.

In view of the Kuhn-Tucker conditions in (21), we have  $G_1^{**} = 0$  if  $G_2 = G_2^c = \frac{V_1(1+M)^{\theta}}{\gamma}$ .<sup>19</sup>

(ii) If  $\frac{\partial Y_2}{\partial G_2} \ge 0$  where  $G_2 = G_2^c$ , then  $G_1^{**} = 0$ . Party 2 effectively deters Party 1 in this case. It

follows from (24) that  $G_1^{**} = 0$  when

$$\frac{\gamma V_2}{2V_1 (1+M)^{\theta}} - 1 \ge 0.$$
(26)

Thus it is more likely that Party 2 will deter Party 1 when  $V_2$  rises,  $V_1$  falls, M decreases, and  $\gamma$  increases.

Conversely, Party 2 fails to deter Party 1 from challenging when the following condition is satisfied:

$$\frac{\gamma V_2}{2V_1 (1+M)^{\theta}} - 1 < 0.$$
<sup>(27)</sup>

We use the above inequality to find the condition under which Party 3's support causes Party 1, the challenger, to attack Party 2 when peace would otherwise have occurred (i.e. Party 3 creates war). Party 3 acts as a peace-breaker when its choice of military subsidy level is such that

<sup>&</sup>lt;sup>19</sup> The expression  $G_2^c = V_1(1+M)^{\theta}/\gamma$  is derived in the three-stage game when Party 2, as the Stackelberg leader, moves before Party 1. Using the backward induction approach to solve for the subgame-perfect equilibrium, we begin with Party 1's choice of arming. Because Party 1 receives military assistance from Party 3, *M* directly enters into the objective function of Party 1. This explains why *M* directly appears in the expression  $G_2^c = V_1(1+M)^{\theta}/\gamma$ , where  $G_2^c$  is the *minimum* level of arming that Party 2 should have for deterring Party 1.

$$M > M^{cc}$$
, where  $M^{cc} \equiv \left(\frac{\gamma V_2}{2V_1}\right)^{\frac{1}{\theta}} - 1 \ge 0.$  (28)

The inequality assures that Party 3's military subsidy level is sufficient to induce Party 1 to attack when they would not have otherwise done so.

Finally, we examine the first stage of the three-stage game, in which Party 3 chooses its optimal level of intervention to support its ally. As defined previously, Party 3's payoff function is  $U_3 = p_1S_1 + p_2S_2 - M$ , but  $p_1$  and  $p_2$  are given by the probabilities of success in (25) for Case II. Substituting these probabilities of success into the payoff function, the Kuhn-Tucker conditions for Party 3's optimal choice of military subsidy are

$$\frac{\partial U_3}{\partial M} = \left[\gamma^{3/2} \frac{\theta(S_1 - S_2)}{2} \frac{V_2}{V_1}\right] (1 + M)^{-(1+\theta)} - 1 \le 0; \quad \frac{\partial U_3}{\partial M} < 0 \text{ if } M = 0.$$

$$\tag{29}$$

It follows from (29) that

$$\frac{\partial U_3}{\partial M} < 0 \text{ if and only if } 0 < S_1 < \frac{2V_1 (1+M)^{(1+\theta)}}{\gamma^{3/2} \theta V_2} + S_2$$

This result indicates that Party 3's military subsidies to Party 1 will be zero when the strategic value of the disputed land to the third party,  $S_1$ , is critically low. To derive the implications of third-party intervention for territorial conflict, we assume that the value of  $S_1$  is sufficiently high such that the necessary condition for expected payoff maximization,  $\partial U_3/\partial M = 0$ , has an interior solution. Solving for the sub-game perfect equilibrium subsidy yields

$$M^{**} = \left[\gamma^{\frac{3}{2}} \frac{\theta(S_1 - S_2)}{2} \frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}} - 1.^{20}$$
(30)

It is easy to verify the following comparative-static results:

$$\frac{\partial M^{**}}{\partial S_1} > 0, \ \frac{\partial M^{**}}{\partial S_2} < 0, \ \frac{\partial M^{**}}{\partial \gamma} > 0, \ \frac{\partial M^{**}}{\partial V_1} < 0, \ \text{and} \ \frac{\partial M^{**}}{\partial V_2} > 0.$$

Notice that the last three derivatives have changed signs from Case I to Case II. We will explain the signs of these derivatives as, again, they may not be immediately intuitive. As Party 1 becomes relatively stronger than Party 2 ( $\gamma$  decreases,  $V_1$  increases), Party 1 as follower naturally allocates more military goods to attack Party 2 ( $G_1$  increases). That is,

$$\frac{\partial G_1}{\partial \gamma} < 0, \ \frac{\partial G_1}{\partial V_1} > 0, \text{ and } \frac{\partial G_1}{\partial V_2} < 0.$$

As  $G_1$  increases, Party 1's marginal expected benefit from an additional unit of  $G_1$ 

declines.  $\frac{\partial}{\partial G_1} \left( \frac{dU_1}{dG_1} \right) < 0$ . Hence, as Party 1 becomes stronger, a dollar of military subsidy

becomes less effective in increasing the probability that Party 1 will take the land. That is,

$$\frac{\partial}{\partial \gamma} \left( \frac{\partial p_1}{\partial M} \right) > 0, \ \frac{\partial}{\partial V_1} \left( \frac{\partial p_1}{\partial M} \right) < 0, \ \text{and} \ \frac{\partial}{\partial V_2} \left( \frac{\partial p_1}{\partial M} \right) > 0.$$

Therefore, Party 3 derives less expected marginal benefit from providing a dollar of subsidy as Party 1 becomes relatively stronger. That is,

$$\frac{\partial(mb_2)}{\partial \gamma} > 0, \ \frac{\partial(mb_2)}{\partial V_1} < 0, \text{ and } \frac{\partial(mb_2)}{\partial V_2} > 0, \text{ where } mb_2 \equiv \frac{\gamma^2}{2\theta} \left(S_1 - S_2\right) V_2}{2V_1 \left(1 + M\right)^{1+\theta}}.$$

This explains why we have  $\frac{\partial M^{**}}{\partial \gamma} > 0$ ,  $\frac{\partial M^{**}}{\partial V_1} < 0$ , and  $\frac{\partial M^{**}}{\partial V_2} > 0$  in Case II.

<sup>&</sup>lt;sup>20</sup> See Appendix B for a detailed derivation of the optimal military subsidy.

Lastly, we find the necessary and sufficient condition under which Party 3 creates war (i.e., Party 3's support causes Party 1 to attack Party 2 when peace would otherwise have occurred). From equations (28) and (30), the "peace-breaking" condition is:

$$\left[\gamma^{\frac{3}{2}}\frac{\theta(S_1 - S_2)V_2}{2V_1}\right]^{\frac{\theta}{1+\theta}} > \frac{\gamma V_2}{2V_1}.$$
(31)

This inequality becomes more likely to hold (i) as Party 3 places more value on the land changing hands in the next period or (ii) as a dollar of military subsidy becomes more effective in reducing Party 1's cost of arming.<sup>21</sup>

Based on the above analyses, we have

**Proposition 2**: Given that the outcome of the conflict (whether peaceful or otherwise) can be altered through a third party's intervention, Party 3 will not support Party 1 (its ally) to gain a disputed territory unless the additional strategic value associated with such a change of possession  $(S_1 - S_2)$  is significantly high. The third party is more likely to support the ally to launch a <u>war</u> in this case (i) as its alliance with Party 1 becomes stronger (i.e.,  $(S_1 - S_2)$  is sufficiently large), (ii) as Party 2 becomes relatively stronger in terms of military effectiveness,

and (iii) as Party 2 becomes stronger in terms of relative land valuation.

<sup>&</sup>lt;sup>21</sup> In view of equation (28) that  $M^{cc} \ge 0$ , we have  $(\gamma V_2 / 2V_1)^{1/\theta} \ge 1$  which implies that  $\gamma V_2 \ge 2V_1$ .

### **3.** A Comparison Between the Two Cases

In this section, we compare Case I, in which the ally is a territorial defender, to Case II, in which the ally is a territorial challenger. As shown in the previous section, the conflicting nature of a territorial dispute, whether it is peaceful or not, can strategically and militarily be altered through third-party intervention. We wish to understand whether it is more costly (requires more resources) for Party 3 to support Party 1 to deter (Case I) or for Party 3 to support Party 1 to launch an attack (Case II), *ceteris paribus*. In other words, is it more expensive for Party 3 to help its ally maintain peace defensively or create war offensively?

To answer the question, note that  $M^c$  in equation (16) is the critical level of military subsidies that creates peace when Party 3's ally is the defender. Furthermore,  $(M^{cc} + \varepsilon)$ , or a value marginally above  $M^{cc}$ , is the critical subsidy level that creates war when Party 3's ally is the challenger.<sup>22</sup> A comparison between  $M^c$  and  $(M^{cc} + \varepsilon)$  reveals that

$$M^c > (M^{cc} + \varepsilon).^{23}$$

We thus have

**Proposition 3**: From the perspective of deterrence strategy, it is always more costly for Party 3 to create peace when a conflict would otherwise result in war (Case I) than to create war when a conflict would otherwise result in peace (Case II), ceteris paribus.

The crucial factor for the findings in Proposition 3 is the difficulty or cost with which the deterrence condition is achieved, from the standpoint of intervention. It turns out to be much

<sup>&</sup>lt;sup>22</sup> To re-examine the term  $M^{cc}$ , please see expression (28). The term epsilon ( $\varepsilon$ ) represents an arbitrarily small, positive number and is added to our Case II critical value due to the strictness of the left-hand side inequality in (28). <sup>23</sup> See Appendix C for a derivation of this result.
easier to break a deterrence (i.e., cause an ally to attack in Case II) than to create a scenario of deterrence (i.e. cause an ally to deter its rival in Case I). The reason for this is that, technically, it requires an increase in  $G_1$  of  $\varepsilon$  to cause Party 1 to attack in Case II. In other words, in a Gershenson-Grossman style sequential game of armed confrontation, the state of attack is a spectrum of which the challenging party is on the brink. On the other hand, in Case I, it requires a  $(G_1^c - G_1^{**})$  increase in  $G_1$ , where  $G_1^{**}$  represents Party 1's level of arming if no outside intervention were to occur, to cause Party 1 to become deterrent. The latter increase is sufficiently greater than the former to assure that creating an attack in Case II is always less costly than creating a state of deterrence in Case I.

In an alternative approach to war or peace, Cai (2003) examines a two-stage game of conflict in which two players allocate resources between arms and domestic production in stage one and engage in peace negotiations trying to avoid war in stage two. He finds conditions under which the two players will build up more arms in the peace equilibrium than in the war equilibrium. Although Cai's analysis does not allow for third-party intervention, his finding suggests that it is more costly to create peace than to create war.

Next, we compare the optimal intervention level of Case I, when Party 3 supports the defender, to that of Case II, when Party 3 supports the challenger. Given that  $M^*$  in equation (19) is the optimal subsidy level for Case I and  $M^{**}$  in (30) is the optimal subsidy level for Case II, we have  $M^* < M^{**}$  when

$$\left(S_1 - S_2\right) < \gamma^{\frac{5-\theta}{4\theta}} \frac{2}{\theta} \left(\frac{V_2}{V_1}\right)^{\frac{1}{\theta}}.$$
(32)

Condition (32) implies that Party 3 is providing more military subsidies to its ally in Case II than in Case I, *ceteris paribus*. Furthermore, Proposition 3 indicates that it is less costly, in terms of military subsidy level, to create war in Case II than to create peace in Case I. Therefore, when condition (32) holds, it is clear that Party 3 is more likely to cause war in Case II than to maintain peace in Case I, *ceteris paribus*.

Conversely, we have  $M^* > M^{**}$  when

$$\left(S_{1}-S_{2}\right) > \gamma^{\frac{5-\theta}{4\theta}} \frac{2}{\theta} \left(\frac{V_{2}}{V_{1}}\right)^{\frac{1}{\theta}}.$$
(33)

In this scenario, Party 3 provides more military subsidies in Case I than in Case II. However, as Proposition 3 indicates, it always requires a greater level of military subsidies for Party 3 to support its ally to create peace in Case I than create war in Case II. Therefore, all else being equal, Party 3's relative likelihood of creating peace in Case I and creating war in Case II cannot be determined unambiguously when condition (33) holds.

The above findings allow us to establish the following proposition.

**Proposition 4**: When Parties 3 and 1 are sufficiently weak allies (i.e.,  $(S_1 - S_2)$  is sufficiently small such that (32) holds), Party 3 optimally chooses a greater subsidy in Case II than in Case I. Furthermore, in this scenario, Party 3 is more likely to create war in Case II than to create peace in Case I, other things equal. In an opposite scenario where Parties 3 and 1 are sufficiently strong allies (i.e.,  $(S_1 - S_2)$  is sufficiently large such that (33) holds), Party 3 chooses a greater subsidy in Case I. However, in this latter scenario, Party 3's relative effectiveness in peace-making (Case I) and peace-breaking (Case II) is ambiguous.

The ambiguity in Party 3's relative effectiveness as peace-maker or peace-breaker arises from the fact that the ranking of the optimal subsidy choice across cases is ambiguous and subject to the parameters of the conflict. Figure 1 presents a graphical ordering of possible subsidy allocations on a number line. Three intervals on the line are defined as follows:  $A = [0, M^{CC}], B = (M^{CC}, M^{C}), and E = [M^{C}, \infty)$ . For the case in which  $M^{*} \le M^{**}$ , the following possibilities are of interest. (i) If  $M^{**} \in A$ , Party 3 does not create war in Case II. Since  $M^{*} \le M^{**}$ , Party 3 does not create peace in Case I either. (ii) If  $M^{**} \in B$ , Party 3 creates war in Case II but does not create peace in Case I. (iii) If  $M^{**} \in E$ , Party 3 creates war in Case II. Since  $M^{*} \le M^{**}$ , Party 3 may or may not create peace in Case I. Thus, if Party 3 creates peace in Case I, then it also creates war in Case II. The converse of the statement, however, is not true. These results suggest that, ceteris paribus, Party 3 is more likely to be peace-breaking when  $M^{*} \le M^{**}$ .

For the case in which  $M^* > M^{**}$ , there are three possibilities of interest. (i) If  $M^* \in A$ , Party 3 does not create peace in Case I. And since  $M^* > M^{**}$ , Party 3 does not create war in Case II either. (ii) If  $M^* \in B$ , Party 3 does not create peace in Case I. Given that  $M^* > M^{**}$ , Party 3 may or may not create war in Case II. (iii) If  $M^* \in E$ , Party 3 creates peace in Case I. However, Party 3 may or may not create war in Case II. The result is that, when  $M^* > M^{**}$ , Party 3's relative effectiveness as peace-maker or peace-breaker cannot be determined a priori.

### 4. Concluding remarks

In this paper, we develop a simple three-stage sequential-move game to characterize explicitly the *endogeneity* of third-party intervention in a territorial conflict. In the first stage of the game, a third party determines its mode and level of intervention (referred to as an "intervention technology") with the purpose of increasing its ally's (Party 1's) military goods production efficiency. In the second and third stages of the game, the aligned party and its opponent move sequentially to determine optimal allocations of military goods to maximize their

respective payoffs in conflict. We examine how the third party's "intervention technology" interacts strategically with the canonical "conflict technology" of the two primary parties in determining the sub-game perfect equilibrium outcome. In contrast to the *Independence of Irrelevant Alternatives* property in the conflict literature, which suggests that third parties have no role in affecting the outcome of a two-party conflict for an additive form of contest success functions, we find conditions under which third-party intervention is relevant.

The model shows that an expected-payoff maximizing third party can intervene to create peace *or* to upset an existing peace, depending on the nature of the conflict and the values held by the third party. Therefore, according to our analysis, third parties can be either "peace making" or "peace-breaking." This finding contradicts the liberal/idealist perspective that the goal of the intervener is always to reduce the existing level of conflict. In general, our findings suggest that there is a theoretically ambiguous relationship between third-party intervention and outcome of conflict (whether peaceful or violent). Thus, there is a valid theoretical explanation for Regan's (2002) empirical finding that third-party intervention, on average, does not induce peace. Obviously, a more detailed empirical study, which accounts for party characteristics for a particular conflict, is needed to comprehensibly understand third-party intervention and its effect.

One caveat should be mentioned: This paper does not intend to be in any way prescriptive. Our contribution should be regarded from a purely positive perspective concerning endogenous effects of third-party intervention on the outcome of a two-party conflict. Some other comments are in order. First, our three-stage game is one shot in that we do not examine third-party intervention within the framework of a dynamic or repeated game. Second, our paper does not model direct conflict or fighting between a third party and its non-allying party involved in territorial dispute. Although it would complicate the analysis of third-party intervention, such a conflict with the third party may also affect the intervention decision as well as the outcome of a disputed territory.<sup>24</sup> One possible extension of the three-stage model is to consider a type of third party that places positive value on the realization of a peaceful outcome. Further research might explain how a peace-valuing, unbiased third party affects the theoretical conclusions of this paper. Additionally, other third-party mechanisms that alter conflict outcomes can be explored. For example, as in Siqueira (2003), the third party could provide negative incentives to their enemy, perhaps by raising the cost of the enemy's military goods

<sup>&</sup>lt;sup>24</sup> We thank an anonymous referee for this point. As in Siqueira (2003) and Rowlands and Carment (2004), we do not consider possible effects on an intervening third party. In our analysis, we assume that the "battlefield" is on a disputed land directly related to the two primary conflicting parties.

Figure 1. Possible ranges of third-party military subsidy for the two cases



Figure 1. The possible ranges of third-party military subsidy for the two cases

## Essay 2

# **Raising the Cost of Rebellion: The Role of Third Party Intervention in Intrastate Conflict**

## 1. Introduction

Outside intervention in intrastate conflict has been oft analyzed in the political science and economics literature. Several studies (Collier and Hoeffler, 1998; Balch-Lindsay and Enterline, 2000; Murdoch and Sandler, 2002) discuss the social losses borne out of insurrection, which include human death, injury, and displacement, destruction of physical capital and natural resources within the conflict state, disintegration of property rights, possible creation of rogue lands that come to serve as a terrorist resource, disruption of economic activity, and loss of productive labor to the rebellion. Collier and Hoeffler (2005) estimate the average global economic loss from a single intrastate conflict to be more than \$64 billion.<sup>25</sup>

Third-party intervention to suppress rebellion has been discussed as an effective means of decreasing the social losses associated with insurrection. Given the empirical evidence, Azam, Collier, and Hoeffler (2001, p.1) conclude, "International policies for conflict reduction should therefore be aimed at increasing the cost of rebellion and at reducing the revenues from it." Siqueira (2003) explores the efficacy of third-party interventions that seek to reduce the level of fighting in an intrastate conflict. Among other modes of intervention, he analyzes outside efforts to raise the marginal cost of rebellion. Further, Gershenson (2002) examines the effect of sanctions on intrastate conflict.

<sup>&</sup>lt;sup>25</sup> The majority of conflicts after World War II have been intrastate conflicts. Balch-Lindsay and Enterline (2000) report that civil wars constitute 80 of the 104 post-World War II conflicts. Further, Murdoch and Sandler (2002) observe that the majority of civil wars take place in developing countries. Collier et al. (2003) present a systematic survey of studies on civil wars.

However, to fully understand the role and scope of rebel-suppressing third-party intervention in cases of potential or realized intrastate conflict, we must consider both underlying third-party interests and the efficacy with which those interests are served through intervention efforts. According to the paradigm of realism in political science, the supply of rebel-suppressing third-party intervention is predicated upon the direct stakes that an outside party holds with each of the rival parties. While capturing a part of third-party motivation, Regan (1998) finds that the paradigm of realism is too narrow to describe intervention efforts in general. For instance, he discovers that intervention is more likely to occur in the presence of a humanitarian crisis, *ceteris paribus*. This result suggests that a representative third party acts partly from moral imperative.

Incorporating Regan's findings with respect to third-party motivation, this paper considers the supply and effect of third-party intervention on behalf of an incumbent government by *endogenizing* the third party within a three-stage game-theoretic model of conflict. Further, we use comparative static analysis to discuss the effect of international policy on optimal third-party intervention. One policy implication of the model is that an increase in the strength of inter-governmental trade partnerships would increase the likelihood that third-party intervention acts to deter rebellion, *ceteris paribus*.

In our analysis, we define the term "intrastate conflict" as an armed confrontation between interest groups in a state (Gershenson and Grossman, 2000). Our model considers a potential or realized intrastate conflict between two primary parties- an incumbent government and a rebel group. For a given decision period, the situation can end in one of two ways.<sup>26</sup> In the

<sup>&</sup>lt;sup>26</sup> As in Gershenson and Grossman (2000) and Chang, Potter, and Sanders (2007a), we take each of the rival parties as allocating some amount of military spending in a given "decision period." In other words, a decision period is a length of time over which military spending decisions are committed for each party.

<sup>&</sup>lt;sup>27</sup> Rowlands and Carment (2006) remark that recent third-party interventions have seldom been impartial in nature.

first possible outcome, government military spending in defense of the state is sufficient to deter rebellion, and armed confrontation does not ensue. Otherwise, government military spending in defense of the state is insufficient to deter a rebellion, and an armed confrontation between government and rebel group ensues. To understand the nature of "biased" third-party intervention on behalf of an incumbent government,<sup>27</sup> we assume the presence of a third party whose preferred outcome is that the incumbent government retains power over the state. The reasons for this preference may be enhanced access to trade and natural resources, improved national security, ethical fulfillment, and geo-strategic advantage (Moseley, 2006).<sup>28</sup> The behavior of the third party is examined to find (i) when there is rebel-suppressing third-party intervention, (ii) the marginal effect of said intervention, and (iii) conditions under which a third party has the effect of deterring a rebellion that would otherwise have occurred. A key finding of the paper is that the third party treats an allied government's relative military effectiveness and relative value for political dominance as *complementary* to its own intervention efforts. It does so because intervention efforts are more marginally effective in restraining a rebellion that is relatively ineffective militarily or one that is relatively unmotivated, ceteris paribus. Additionally, we find conditions in which a third party optimally intervenes at a level *insufficient* to change incumbent government policy. It may do so simply to improve the incumbent government's potential to succeed (i.e., to maintain power) in conflict. This result leads us to question the criterion by which many political science studies evaluate the effectiveness of thirdparty intervention.

<sup>&</sup>lt;sup>28</sup> Collier and Hoeffler (1998, 2004) show that economic factors, such as the value of state natural resources, have a strong effect on group decisions regarding civil conflict.

Gershenson and Grossman (2000) develop a rational-choice model to identify the determinants of intrastate conflict. In explaining the onset and persistence of intrastate conflict, their model focuses on the values, intrinsic and economic, that rival parties place on political dominance. We wish to broaden the Gershenson-Grossman framework in this study by considering a model of intrastate conflict that features an endogenous third party. In particular, we analyze the scenario in which a third party considers supporting the incumbent government by means of raising the marginal cost of rebellion. For instance, a third party might impose and enforce targeted arms trade sanctions upon the rebel group. Such sanctions potentially deny the rebel group lowest-cost sources of military goods and have been implemented by the United Nations, for instance, to address conflicts in Angola, Sierra Leone, Guinea, and Liberia (Fleshman, 2001).

Our paper complements a recent contribution by Gershenson (2002) on sanctions and civil conflict. Gershenson systematically examines the effect of sanctions on civil conflict when two rival parties compete for control of economic rents. In our analysis, we examine outside intervention intended to raise the cost of rebellion in a target state. In terms of modeling outside intervention, our analysis departs from Gershenson's in some important aspects. Foremost, in our setting the third party acts to maximize its expected payoff with respect to the target state. As previously emphasized, a third party's expected payoff may incorporate humanitarian interests in addition to strategic and economic considerations. Hence, in characterizing the endogeneity of outside intervention, our paper addresses both the role and scope of biased third-party intervention. However, the two studies share an important analytical feature. By adopting a Stackelberg or sequential game approach in characterizing the outcome of civil conflict, both studies are capable of analyzing explicitly issues on engagement and deterrence.

The remainder of the paper is structured as follows. Section 2 develops a sequential game framework of intrastate conflict, taking into account the presence of third-party intervention in raising the cost of rebellion. Section 3 examines the optimizing behavior of the intervening third party to analyze the endogeneity of rebel-suppressing third-party intervention. In the section, we further present policy implications of the model. Section 4 summarizes and concludes.

### 2. The Model

### 2.1 Basic Assumptions

We consider a scenario in which two rival parties, an incumbent government and rebel group, are in a situation of potential or realized intrastate conflict.<sup>29</sup> In other words, there exists an incumbent government and a rebel group, each of whom value control of the state or a sub-region of the state. However, the rebel group can achieve control of the target region only by wresting it from the incumbent government.

As in the conflict literature, we use a canonical "contest success function" to capture the technology of conflict. That is, the probabilities that the government and rebel group will succeed in armed confrontation are given respectively by

$$p_1 = \frac{G}{G + \mu R}$$
 and  $p_2 = \frac{\mu R}{G + \mu R}$ , (1)

<sup>&</sup>lt;sup>29</sup> By the term "potential intrastate conflict," it is meant that a state lies under the shadow of conflict or armed confrontation. Several theoretical models describing interstate conflict, including Gershenson and Grossman (2000), Garfinkel and Skaperdas (2003), and Chang, Potter, and Sanders (2007a), adopt the same starting point. Gershenson (2003) and Chang, Potter, and Sanders (2007b) study third-party efforts to reduce an ally's unit cost of arming under the shadow of conflict.

where  $p_1$  represents the likelihood that the government remains politically dominant over the decision period,  $p_2$  represents the likelihood that the rebel group becomes politically dominant during the period, *G* represents the amount of military defense spending the incumbent government allocates at the beginning of the period; *R* represents amount of military spending the rebel allocates to challenge for the state or for a sub-region of the state at the beginning of the period;  $\mu$  represents the relative effectiveness of a unit of rebel military spending to a unit of government military spending.<sup>30</sup>

The probabilities of success specified above are in a simple additive form of conflict technologies. According to Garfinkel and Skaperdas (2006), a wide class of additive form contest success functions (CSFs) has been utilized in many fields of economics. They further indicate one important characterization associated with these CSFs, which is referred to as the *Independence of Irrelevant Alternatives* property. Specifically, Garfinkel and Skaperdas (2006, p. 4) remark that: "In the context of conflict, this property requires that the outcome of conflict between any two parties depend only on the amount of guns held by these two parties and not on the amount of guns held by third parties to the conflict." This property suggests that third parties have no role in a two-party conflict. It is easy to verify that this statement is valid for the case in which the two conflicting parties determine their optimal amounts of guns in a simultaneous-move game. Interestingly, in the three-stage sequential game we consider, an intervening third party has an important role in affecting the equilibrium outcome of the two conflicting parties, despite the additive form of conflict technologies in (1).

<sup>&</sup>lt;sup>30</sup> For alternative forms of contest success functions, see, e.g., Tullock (1980), Hirshleifer (1989), Skaperdas (1996), and Garfinkel and Skaperdas (2006).

The game we consider constitutes a single decision period in a potential or realized intrastate conflict.<sup>31</sup> The timing of the game is as follows. The third party moves first in expending effort to raise the cost of rebellion, taking into account the impact of its actions in the subsequent sub-games played between the government and rebel group. In the second and third stages of the three-stage game, the government, as defender, moves at the second stage to decide on its defensive military spending allocation. The rebel group, as challenger, moves at the third and final stage of the overall game played among the three parties. The methodological advantage of this game is twofold. First, it extends the Gershenson-Grossman (2000) framework of conflict in a two-stage game to the case involving an intervening third party in a three-stage game.<sup>32</sup> As a result, we are able to characterize explicitly the endogeneity of third-party intervention in raising the cost of rebellion. Second, this sequential game approach allows for the analysis of a deterrence strategy on the part of the defender.

### 2.2 A Third Party's "Intervention Technology"

Before characterizing the role of third-party intervention in an intrastate conflict between a government and rebel group, we first discuss the term "intervention technology." This term reflects the extent to which a third party can affect the capability of a rebel group and, in so doing, affect the overall outcome of the potential or realized conflict. Given its preference for

<sup>&</sup>lt;sup>31</sup> A decision period may begin under the shadow of conflict (potential intrastate conflict) or amidst ongoing conflict (realized intrastate conflict).

 $<sup>^{32}</sup>$  In the game's second and third stages, we also follow Grossman and Kim (1995) and others after them in utilizing a Stackelberg framework in which the defender leads in determining its defensive allocation of military goods. Gershenson (2002) defends this structure by assuming that the incumbent's institutional framework is relatively rigid; therefore, defensive allocations constitute a commitment on the part of the incumbent. Chang, Potter, and Sanders (2007a) develop a model to characterize possible outcomes of a land dispute between two rival parties in a Stackelberg game. More generally, and perhaps more fundamentally, Leininger (1993) shows in an interesting rentseeking model that players are expected to engage in a sequential-move game. Morgan (2003) further uses a sequential-move game to examine the possibility of asymmetric contests for uncertain realizations of values to rival competitors.

the status quo (i.e., that the incumbent government maintains state control), the third party we examine considers an intervention effort on behalf of the incumbent government.<sup>33</sup> Should the third party decide to intervene, we assume that it does so indirectly by expending effort to raise the costs of insurrection. Denote such a cost-raising effort as C = C(M), where

$$C'(M) = \frac{dC}{dM} > 0$$
 and  $C''(M) = \frac{d^2C}{dM^2} < 0$ .

That is, an increase in M raises the average cost of arming for rebellion, and this costraising effect is subject to diminishing returns. We will examine how the third-party's intervention technology interacts with the respective conflict technologies of the contending parties to determine whether rebellion ensues and, if so, to what degree.

Given that the third party expends effort to raise the cost of rebellion, we assume for analytical simplicity that the cost-raising effect is  $C = (1+M)^{\theta}$ , where  $\theta$  measures the intervention technology's degree of effectiveness in raising the costs of rebellion and  $0 < \theta < 1$ . Alternative functional forms may exist, but this function has some interesting features. First, the value of *C* equals one when M = 0 such that the rebel group's military cost is unaffected in the absence of intervention. Second, third party effort has a negative effect on the rebellion, but this effect is subject to diminishing returns.

#### 2.3 Intrastate Conflict and Rebel-suppressing Third-Party Intervention

As the third party invests M during stage one toward raising the cost of rebellion, the payoff functions for the incumbent government and rebel group in the subsequent stages of the game are given respectively by

<sup>&</sup>lt;sup>33</sup> Within the model's framework, we find this preference to be a necessary but not sufficient condition for such an intervention to take place.

$$Y_1 = \left(\frac{G}{G + \mu R}\right) V_1 - G \tag{2a}$$

and

$$Y_2 = \left(\frac{\mu R}{G + \mu R}\right) V_2 - \left(1 + M\right)^{\theta} R , \qquad (2b)$$

where  $M \ge 0$  represents Party 3's level of investment in policies that act to raise the cost of rebellion (i.e., enforcement of targeted arms sanctions);  $\theta$  reflects effectiveness with which a unit of intervention investment raises the unit cost of rebellion;  $V_1$  and  $V_2$  are respectively the total values that the government and rebel group attach to political dominance for a period, where a party can value political dominance for economic and deep intrinsic reasons.<sup>34</sup>

Within the model, each of the primary parties chooses a level of military spending to maximize its expected payoff. Consistent with backward induction in game theory, we begin with the game's last stage to analyze the rebel group's optimization problem. Namely, the rebel group chooses its investment in military goods to challenge for control of the disputed state or sub-state through armed confrontation.

Given the rebel group's payoff function in (2b), the Kuhn-Tucker conditions for its optimal expenditure on military goods are:

$$\frac{\partial Y_2}{\partial R} = \left[\frac{\mu G}{\left(G + \mu R\right)^2}\right] V_2 - \left(1 + M\right)^{\theta} \le 0; \quad \frac{\partial Y_2}{\partial R} < 0 \text{ if } G = 0.$$
(3)

From (3), we solve for the rebel's best-response function in terms of G and the parameters:

<sup>&</sup>lt;sup>34</sup> When there is no outside intervention such that M = 0, the three-country, three-stage model reduces to a twocountry, two-stage model as those examined in Gershenson and Grossman (2000), Grossman (2004), and Chang, Potter, and Sanders (2007a).

$$R = \frac{G^{\frac{1}{2}}}{\mu} \left[ \frac{\mu^{\frac{1}{2}} V_2^{\frac{1}{2}}}{\left(1+M\right)^{\frac{\theta}{2}}} - G^{\frac{1}{2}} \right] \ge 0 \text{ if } 0 \le G \le \tilde{G},$$
(4)

where  $\tilde{G}$  represents the incumbent government's minimum level of expenditure on arming for effective deterrence. That is,

$$R = 0$$
 when  $G \ge \tilde{G}$ , where  $\tilde{G} = \frac{\mu V_2}{\left(1 + M\right)^{\theta}}$ . (5)

In this case, the rebel group finds it optimal to refrain from arming for attack, and there is no armed confrontation between the two parties in the period.<sup>35</sup>

For the case in which  $G < \tilde{G}$ , the rebel group chooses a positive amount of arming, and armed confrontation ensues. If follows from (4) that the slope of the rebel's best-response function is

$$\frac{dR}{dG} = \frac{V_2^{\frac{1}{2}}}{2G^{\frac{1}{2}}\mu^{\frac{1}{2}}(1+M)^{\frac{\theta}{2}}} - \frac{1}{\mu}.$$
(6)

For  $G < \tilde{G}$ , we have from the Kuhn-Tucker conditions that  $G + \mu R = G^{\frac{1}{2}} \mu^{\frac{1}{2}} V_2^{\frac{1}{2}} (1+M)^{-\frac{\theta}{2}}$ .

Substituting this expression into the payoff function of the government in (2a) yields

$$Y_{1} = \left[\frac{G(1+M)^{\theta}}{\mu V_{2}}\right]^{\frac{1}{2}} V_{1} - G.$$

The first-order condition with respect to G is:

$$\frac{\partial Y_1}{\partial G} = \left[\frac{\left(1+M\right)^{\theta}}{2G\mu V_2}\right]^{\frac{1}{2}} V_1 - 1 = 0.$$

<sup>&</sup>lt;sup>35</sup> Several Stackelberg models of civil conflict utilize this deterrence condition. See, for example, Gershenson and Grossman (2000).

Solving for the government's optimal allocation of defensive military spending yields

$$G^* = \frac{\left(1+M\right)^{\theta} V_1^2}{4\mu V_2}.$$
(7)

Substituting  $G^*$  into R in equation (4) gives us

$$R^* = \frac{V_1}{2\mu} \left[ 1 - \frac{\left(1 + M\right)^{\theta} V_1}{2V_2} \right].$$
 (8)

It is easy to verify the following comparative static derivatives:

$$\frac{\partial G^*}{\partial M} > 0; \quad \frac{\partial R^*}{\partial M} < 0.$$

These findings allow us to establish the following proposition:

**Proposition 1**: In an intrastate conflict involving an incumbent government and a rebel group, the involvement of a third party in raising the cost of rebellion enhances the level of military defense on the part of the incumbent government, other things being equal. Moreover, this costraising intervention unambiguously reduces the scale of military challenge by the rebel group, ceteris paribus.

If the government's optimal expenditure on military goods at least equals the deterrent level, i.e.,  $G^* \ge \tilde{G}$ , the rebel's level of arming for attack becomes  $R^* = 0$ . This is the case of "perfect deterrence," which occurs when the following sufficient condition is satisfied.

$$\frac{\left(1+M\right)^{\theta}V_{1}}{2\mu V_{2}} - \frac{1}{\left(1+M\right)^{\theta}} \ge 0.$$
(9)

As in the literature, we characterize this equilibrium as one in which there is no armed confrontation between government and rebel group. The incumbent government maintains state control without challenge. If, however, the sufficient condition is violated, then the rebel group arms to confront the incumbent government. This outcome occurs when

$$\frac{(1+M)^{\theta}V_1}{2V_2\mu} - \frac{1}{(1+M)^{\theta}} < 0.$$
<sup>(10)</sup>

It is clear that if  $M \ge \hat{M} > 0$ , where  $\hat{M} = \left(\frac{2V_2\mu}{V_1}\right)^{\frac{1}{\theta}}$ , the third party deters a rebellion that

would otherwise have occurred. The left hand side inequality, which follows from (9), says that the third party meets the critical level of intervention to deter rebellion. The right hand side says that this critical level is positive. In other words, the latter inequality ensures that armed confrontation would have ensued in the absence of intervention.

However, if  $\hat{M} > M > 0$ , the third party intervenes at a sub-deterrent level. That is, the third party intervenes without the intent of deterring rebellion when such an opportunity exists. The motivation for sub-deterrent intervention is simply to improve the incumbent government's potential to succeed (i.e., to maintain power) in conflict.

Using  $G^*$ ,  $R^*$ , and the CSFs, we calculate the probabilities of success for the government and rebel group in equilibrium as follows:

$$p_1^* = \frac{(1+M)^{\theta} V_1}{2\mu V_2} \text{ and } p_2^* = 1 - \frac{(1+M)^{\theta} V_1}{2\mu V_2}.$$
 (11)

Thus, comparative statics for  $p_1^*$  and  $p_2^*$  are:

$$\frac{\partial p_1^*}{\partial M} > 0; \quad \frac{\partial p_1^*}{\partial V_1} > 0; \quad \frac{\partial p_1^*}{\partial V_2} < 0; \quad \frac{\partial p_1^*}{\partial \mu} < 0; \quad \frac{\partial p_1^*}{\partial \theta} > 0;$$
$$\frac{\partial p_2^*}{\partial M} < 0; \quad \frac{\partial p_2^*}{\partial V_1} < 0; \quad \frac{\partial p_2^*}{\partial V_2} > 0; \quad \frac{\partial p_2^*}{\partial \mu} > 0; \quad \frac{\partial p_2^*}{\partial \theta} < 0; \quad \frac{\partial Y_1^*}{\partial M} > 0; \quad \frac{\partial Y_2^*}{\partial M} < 0.$$

The findings of the analysis lead to the following proposition:

**Proposition 2**: In an intrastate conflict involving an incumbent government and a rebel group, the involvement of a third party in raising the cost of rebellion increases the probability that the government will succeed in the intrastate conflict. The government's probability of success in retaining political dominance increases in its value for this status  $(V_1)$ , decreases in the rebel group's value for political dominance  $(V_2)$ , decreases in the rebel group's relative military spending effectiveness  $(\mu)$ , and increases in the third party's effectiveness in raising the cost of rebellion  $(\theta)$ , ceteris paribus. It follows that these parameters have an opposite effect on the probability of success for the rebel group.

Within our model, intervention is unambiguously effective in raising the likelihood of a favorable conflict result, as viewed from the perspective of the third party. One important issue remains concerning the conditions under which a third party intervenes in an intrastate conflict. This leads us to examine the incentives of third-party intervention.

## 3. The Endogeneity of Third-Party Intervention

In this section, we proceed to the first stage of the three-stage game to examine optimal intervention by the third party. There are potential benefits to a third party should the government retain its political dominance. Denote  $S_1$  as the value the third party will derive should the government remain politically dominant over the decision period. As stated in the Introduction of the paper, this value may derive from enhanced access to trade and natural resources, improved national security, ethical fulfillment, and geo-strategic advantage. Let  $S_2$  represent the value the third party will obtain should the rebel group achieve political dominance in the decision period. We assume that  $S_1 > S_2 \ge 0$ , i.e., the third party will be better off if the

government maintains political dominance. As Werner (2000) states, "One important reason for involvement is often the third party's perception that the attacking country poses a significant threat to the status quo." Within our model, this motivational threat is represented by the term  $(S_1 - S_2)$ .

It is postulated that the objective of the third party is to maximize its expected benefit with respect to the disputed state, net of its effort in raising the cost of rebellion. Specifically, this objective function is taken as  $Y_3 = p_1S_1 + p_2S_2 - M$ . Substituting the probabilities of success in (9) into the objective function yields

$$Y_{3} = \left[\frac{(1+M)^{\theta} V_{1}}{2\mu V_{2}}\right]S_{1} + \left[1 - \frac{(1+M)^{\theta} V_{1}}{2\mu V_{2}}\right]S_{2} - M.$$

The Kuhn-Tucker conditions for the third party's intervention are:

$$\frac{\partial Y_3}{\partial M} = \theta \left( 1 + M \right)^{\theta - 1} \frac{V_1 \left( S_1 - S_2 \right)}{2\mu V_2} - 1 \le 0; \tag{12a}$$

$$\frac{\partial Y_3}{\partial M} < 0 \text{ if } M = 0.$$
(12b)

It follows from the Kuhn-Tucker conditions that

$$\frac{\partial Y_3}{\partial M} < 0 \text{ if } 0 < S_1 < \tilde{H} + S_2, \tag{13}$$

where  $\tilde{H} = \frac{2\mu V_2 (1+M)^{1-\theta}}{V_1 \theta}$ . In this case,  $M^* = 0$ . Given this result, we have

**Proposition 3**: The third party finds it optimal not to intervene when the additional value it derives from the incumbent government holding power is "critically low." In other words, the inequality condition in (13) becomes less likely as  $S_1$  increases or  $S_2$  decreases. This non-intervention inequality becomes more likely to hold as the incumbent government's relative value

for political dominance decreases or as the rebel group's relative military spending effectiveness declines. Thus, in its intervention decision, the third party treats the incumbent government's relative military effectiveness and relative value for political dominance as complementary to its own efforts.

#### Proof: See Appendix D

A third party will intervene to raise the marginal cost of rebellion only when it places sufficient value on political dominance by the state's incumbent government, as compared to political dominance by the state's rebel group. The complementarity discussed in Proposition 3 above exists because intervention efforts are more marginally effective in restraining a rebellion that is relatively ineffective militarily or one that is relatively unmotivated, *ceteris paribus*. That is, the third party's intervention technology endogenously interacts with the respective conflict technologies indirectly such that intervention is more marginally effective in reducing  $p_2^*$  for such a rebellion.

To examine implications of third-party intervention, we assume that  $S_1$  is sufficiently high in value such that the necessary condition for maximizing the expected payoff,  $\frac{\partial Y_3}{\partial M} = 0$ , has an interior solution. It is easy to verify that the optimal level of intervention

$$M^{*} = \left[\frac{\theta V_{1}(S_{1}-S_{2})}{2\mu V_{2}}\right]^{\frac{1}{1-\theta}} - 1.^{36}$$

A comparative static analysis indicates that

$$\frac{\partial M^*}{\partial S_1} > 0; \ \frac{\partial M^*}{\partial S_2} < 0; \ \frac{\partial M^*}{\partial V_1} > 0; \ \frac{\partial M^*}{\partial V_2} < 0; \ \frac{\partial M^*}{\partial \mu} < 0.$$

<sup>&</sup>lt;sup>36</sup> See Appendix E for a detailed derivation of the optimal intervention level.

Based on these findings, we have

**Proposition 4**: In an intrastate conflict between an incumbent government and a rebel group, the optimal level of third party intervention in raising the cost of rebellion increases with the strategic value to the third party when the government retains its power  $(S_1)$ , decreases with the strategic value to the third party when the government retains its power  $(S_2)$ , increases with the government's intrinsic value  $(V_1)$ , decreases with the rebel's intrinsic  $(V_2)$ , decreases with the rebel group's military effectiveness  $(\gamma)$ , ceteris paribus.

Proposition 4 leads us to conclude that an increase in the strength of inter-governmental trade partnerships increases the level of third-party intervention and thus the likelihood that intervention acts to deter a rebellion. This finding derives from the fact that  $S_1$  increases as an incumbent government provides better access to trade, *ceteris paribus*.

There is a vast literature, primarily within the political science paradigm, that questions the effectiveness of economic sanctions and other forms of intervention. Such papers generally define an intervention effort as effective in the event that it creates a policy change that the intervener favors. Morgan and Schwebach (1997, p.28) state, "Most political science studies conclude that sanctions do not 'work'... in the sense of bringing about a desired change in the policy of the target country." However, our model shows that this policy change criterion appears to be invalid in measuring the success of third-party intervention. The third party we have specified could potentially bring about one type of policy change. In a given decision period, the third party may cause an incumbent government to effectively deter an active or mounting rebellion. In section 2, we find conditions in which the third party optimally chooses to intervene at a sub-deterrent level. In other words, the third party may purposefully intervene at a level insufficient to change incumbent government policy. It may do so simply to improve

the incumbent government's potential to succeed (i.e., to maintain power) in conflict. Such optimal third-party behavior calls into question the criterion by which the effectiveness of intervention is generally measured and is therefore relevant to any paper studying the effectiveness of sanctions or of intervention in general.

## 4. Concluding Remarks

In this paper, we use a standard game-theoretic model to analyze potential or realized conflict between an incumbent government and rebel party. Many important studies contribute to our understanding of intrastate conflict but do not for any form of outside intervention. There are a few models that do take into account third-party intervention or various forms of sanctions imposed by a third party. Nevertheless, these models consider the third party as exogenous in determining its level of intervention.

We incorporate third-party intervention explicitly into the Gershenson-Grossman (2000) model of intrastate conflict and find that raising the costs of rebel movement reduces the level of rebellion in intrastate conflict. Further, the model reveals that raising the cost of rebellion reduces the likelihood that a rebellion is successful in wresting control from the incumbent government. The magnitude of these effects depends on the effectiveness of intervention technology, the degree to which the third party values the status quo, and on the relative military spending effectiveness of the primary parties. Within the analysis, we find conditions in which third-party intervention is sufficient to deter an insurrection that would otherwise have occurred. However, it turns out that a third party in favor of the status quo in a state may optimally intervene at a level *insufficient* to deter rebellion. A third party may act in such a way simply to increase the likelihood that an incumbent government succeeds in conflict (i.e., maintains state

control). Such optimal third-party behavior calls into question the criterion by which the effectiveness of intervention is often measured. In terms of third-party objective, a successful intervention does not necessarily bring about policy change (i.e., deterrence of rebellion).

In characterizing biased third-party intervention, we also find that the third party treats an allied government's relative military effectiveness and relative value for political dominance as *complementary* to its own intervention efforts. It does so because intervention efforts are more marginally effective in restraining a rebellion that is relatively ineffective militarily or one that is relatively unmotivated, *ceteris paribus*. Lastly, given that access to trade affects third-party stakes in a conflict, we find that an increase in the strength of inter-governmental trade partnerships improves the likelihood that third-party intervention deters rebellion.

In closing, some caveats should be mentioned. To consider intervention and its effect on the duration of intrastate conflict, our simple sequential game framework could be modified to allow for a dynamic or repeated game. Another possible extension is to consider alternative intervention mechanisms implemented to suppress rebellion. Although our analysis has interesting implications concerning the effect of inter-governmental trade partnerships on the possibility of outside intervention to suppress rebellion, we do not model endogenously the effect of international trade on *intra*state conflict. This research topic, which parallels increasingly important studies concerning the effect of international trade on *intra*state conflict.

<sup>&</sup>lt;sup>37</sup> For studies that examine trade and interstate conflict see, e.g., Polachek (1980, 1997), Reuveny and Maxwell (1998). Skaperdas and Syropoulos (1996, 2001), and Haaparanta and Kuisma (2005).

## Essay 3

# A Cheap Ticket to the Dance: Conference Bias in College Basketball's Ratings Percentage Index

## 1. Introduction

The Ratings Percentage Index (RPI) is the prominent measure of team ability level in NCAA Division I Men's Basketball. The RPI essentially uses information from a team's revealed performance to numerically describe that team's ability. Since 1981, the RPI has aided the NCAA Division I Men's Basketball Committee in selecting and seeding NCAA tournament teams. Thus, it is generally valued for its ability to comprehensively and unambiguously rank college basketball teams. Given the enormous monetary and emotional value of a marginal NCAA tournament berth, it is important to understand the methodology of the RPI and determine any biases it might contain.

Important methodological distinctions exist between the RPI and team ranking systems in other sports. Therefore, we cannot develop a satisfactory characterization of the RPI based on past studies of other ranking systems. The RPI as a subject of study has heretofore been the domain of college basketball analysts and a few statisticians. With varying degrees of underlying thought, college basketball analysts have devoted many words toward characterizing the RPI. We will invoke some of their observations to motivate the model. In a statistical study, Harville (2003) finds that a least-squares approach performs better than the RPI in predicting post-season tournament outcomes. In this paper, we seek to shed additional light on Harville's analysis by exploring a potential source of significant RPI bias.

(.25), the average winning proportion of the team's opponents (.5), and the average winning

proportion of the team's opponents' opponents (.25).<sup>38</sup> By measuring the latter two winning proportions, the RPI is commonly believed to control for a team's strength of schedule. Noted college basketball analyst Jerry Palm states, "(The RPI) is a measure of strength of schedule and how a team does against that schedule" (Keri, 2007). However, the measure is problematic in that a large proportion of Division I college basketball games take place between conference opponents.<sup>39</sup> As conferences are essentially hierarchical groups within college basketball, we expect a strong positive correlation between a team's ability level, the ability level of a team's opponent, and the ability level of a team's opponent's opponents.<sup>40</sup> Thus, it likely requires more ability for a team in a major conference to achieve a particular season winning percentage than a team in a mid-major conference. Similarly, we expect that it requires more ability, on average, for a major conference team's opponents to achieve a particular season winning percentage than a mid-major conference team's opponents. Lastly, given the effect of the conference season, we expect that it requires more ability for a major conference team's opponents' opponents to achieve a particular season winning percentage than a mid-major conference team's opponents' opponents. Analyst Jon Scott (2007) addresses this point,

"The major reason the RPI is a poor model for determining team strength is because it is too simplistic to reliably differentiate teams and relies completely on the assumption that winning percentage is a valid indicator of how strong a team is. Comparing only the won-loss percentage of the last place team of a power conference with the won-loss percentage of a low-level conference champion, with no regard for the schedule each school actually played, one would be completely misled as to which team was the stronger."

<sup>&</sup>lt;sup>38</sup> The NCAA Division I Men's Basketball Committee has recently adjusted the RPI such that away wins are more valuable than home wins (NCAA). A detailed description and history of the RPI formula can be found at http://www.ncaasports.com/basketball/mens/story/9033549

<sup>&</sup>lt;sup>39</sup> Many teams play more than half of their schedule against conference opponents.

<sup>&</sup>lt;sup>40</sup> There are two conference tiers in college basketball that are relevant to the NCAA tournament discussion- major (i.e., first tier) and mid-major (i.e., second tier). Though the distinction is unofficial, it is commonly accepted that six of the 31 Division I conferences are major conferences, while the remainder are mid-majors or low-majors. Despite their relative scarcity, major conference teams have earned 97 of 108 NCAA Tournament Final Four spots during the RPI era.

If the RPI is conference biased, we might expect a mid-major conference team that is strong relative to its conference opponents to sometimes achieve a higher RPI than a major conference team that is mediocre or weak relative to its conference opponents, *even when the major conference team has more ability and has exhibited this in games against strong mid-major conference teams*. The possibility of a positive "mid-major bias" in the RPI has implications upon the selection of the NCAA tournament pool, a process for which the RPI was created by the NCAA. Though mid-major teams are slightly outnumbered in the NCAA Tournament, the NCAA Division I Men's Basketball Committee may be overstocking the tournament with relatively strong mid-major conference teams due to a measurement system that belies such a team's true strength of schedule.<sup>41</sup>

In this paper, we construct a simple model of a college basketball season. We assume there are four team-types within college basketball, where a team-type is defined as a group of teams sharing the same ability level. A team's expected likelihood of victory in a game is determined by its ability level relative to the opponent's ability level. That is, if a team of type iwere to play a team of type j, the probabilities of victory would be given respectively by

$$p_i = \frac{t_i}{t_i + t_j}$$
 and  $p_j = \frac{t_j}{t_i + t_j}$ ,

where  $t_{i(j)}$  = ability or talent level for team-type i(j).

We focus on the ranking of two college basketball teams, each of a distinct type. Specifically, we wish to ascertain whether a more talented team might earn a *lower* RPI due to the nature of its conference schedule. If every team type plays to its expected level and all team types can be transitively compared during the course of a season, an unbiased measure of team

<sup>&</sup>lt;sup>41</sup> The Men's Basketball Committee selects 34 of the 65 tournament teams. The remaining 31 teams enter through automatic berths. The Committee also has the responsibility of seeding all tournament teams.

ability would always come up with a correct ordinal ranking of team-types. However, we find that, even if all teams play at their expected ability levels and can be transitively compared, the RPI does not necessarily represent an ordinal mapping from revealed team performance to the real number line. In other words, the RPI is flawed even in a perfect world because it does not properly value a team's schedule. Rather than serving as a pure measure of team ability, the RPI value may, in a sense, reward top teams in low-ability conferences at the expense of mediocreto-bottom teams in high-ability conferences.

## 2. The Model

In this simple model of a college basketball season, we assume that there are four types of teams, where a team-type is a set of teams sharing the same ability level. We consider the ranking of two teams, each of a distinct type. Specifically, Team  $i \in (2,3)$  is of type *i*. The two teams inhabit distinct conferences. Team 2 inhabits a conference in which all teams are of type 1 or 2. Team 3 resides in a conference in which all teams are of type 3 or 4.

Given a cursory examination as to the relative magnitude of conference schedules, we assume that half of a team's games are against conference opponents, and remaining games are inter-conference in nature. Thus, a team from one type might play an opponent of the same type or an opponent of a different type. For simplicity, we assume that each team within a type plays the same types of opponents with the same relative frequency.

Let the ability level differ across types such that

 $t_1 > t_2 > t_3 > t_4$ .

Further, assume that teams of type 1 play two-thirds of games against other type 1 teams and remaining games against type 2 teams. Also, type 2 teams play type 1, 2, and 3 opponents

with equal frequency such that their conference schedule is identical to that of a type 1 team. Similarly, type 3 teams play type 2, 3, and 4 opponents with equal frequency. Lastly, type 4 teams play two-thirds of games against other type 4 teams and remaining games against type 3 teams such that their conference schedule is identical to that of a type 3 team.

Thus, our model allows for the existence of heterogeneous schedules across teams, conference play, and inter-conference play. The implication of these distributional assumptions is that conference play causes a team to remain in the neighborhood of its own type. This is not the only scheduling distribution one could consider but is useful and sufficiently plausible for a general evaluation of the RPI. Given these assumptions, we can tractably examine the RPI's ability to interpret the heterogeneous experience of each team type in a setting where teams perform to expectation and can be transitively compared.

Given our scheduling assumptions, the expected winning proportion for each team-type is as follows:

$$E(w_1) = \frac{\left[2\left(\frac{t_1}{t_1 + t_1}\right) + \frac{t_1}{t_1 + t_2}\right]}{3} = \frac{\left[1 + \frac{t_1}{t_1 + t_2}\right]}{3}$$

$$E(w_2) = \frac{\left[\frac{t_2}{t_1 + t_2} + \frac{t_2}{t_2 + t_2} + \frac{t_2}{t_2 + t_3}\right]}{3} = \frac{\left[\frac{1}{2} + \frac{t_2}{t_1 + t_2} + \frac{t_2}{t_2 + t_3}\right]}{3}$$

$$E(w_3) = \frac{\left[\frac{t_3}{t_2 + t_3} + \frac{t_3}{t_3 + t_3} + \frac{t_3}{t_3 + t_4}\right]}{3} = \frac{\left[\frac{1}{2} + \frac{t_3}{t_2 + t_3} + \frac{t_3}{t_3 + t_4}\right]}{3}$$

$$E(w_4) = \frac{\left[\frac{t_4}{t_3 + t_4} + 2\left(\frac{t_4}{t_4 + t_4}\right)\right]}{3} = \frac{\left[1 + \frac{t_4}{t_3 + t_4}\right]}{3}$$

From these expected winning proportions, we can calculate the expected RPI for Teams 2 and 3, respectively, as follows

$$E(RPI_2) = 0.25\left(\frac{E(w_2)}{3}\right) + 0.5\left(\frac{E(w_1) + E(w_2) + E(w_3)}{9}\right) + 0.25\left(\frac{3E(w_1) + 3E(w_2) + 2E(w_3) + E(w_4)}{27}\right)$$

$$E(RPI_2) = \frac{E(w_1)}{12} + \frac{E(w_2)}{6} + \frac{2E(w_3)}{27} + \frac{E(w_4)}{108}$$

and

$$E(RPI_3) = 0.25\left(\frac{E(w_3)}{3}\right) + 0.5\left(\frac{E(w_2) + E(w_3) + E(w_4)}{9}\right) + 0.25\left(\frac{E(w_1) + 2E(w_2) + 3E(w_3) + 3E(w_4)}{27}\right)$$

$$E(RPI_3) = \frac{E(w_1)}{108} + \frac{2E(w_2)}{27} + \frac{E(w_3)}{6} + \frac{E(w_4)}{12}$$

Subtracting Team 3's expected RPI from Team 2's, we can determine whether Team 2's expected RPI is unambiguously greater.

$$\left[E(RPI_2) - E(RPI_3)\right] = \frac{8\left(1 + \frac{t_1}{t_1 + t_2}\right) + 10\left(\frac{1}{2} + \frac{t_2}{t_1 + t_2} + \frac{t_2}{t_2 + t_3}\right) - 10\left(\frac{1}{2} + \frac{t_3}{t_2 + t_3} + \frac{t_3}{t_3 + t_4}\right) - 8\left(1 + \frac{t_4}{t_3 + t_4}\right)}{108}$$

$$\left[E(RPI_{2}) - E(RPI_{3})\right] = \frac{\left[\frac{8t_{1}}{t_{1} + t_{2}} + \frac{10t_{2}}{t_{1} + t_{2}} + \frac{10t_{2}}{t_{2} + t_{3}} - \frac{10t_{3}}{t_{2} + t_{3}} - \frac{10t_{3}}{t_{3} + t_{4}} - \frac{8t_{4}}{t_{3} + t_{4}}\right]}{108}$$

By virtue of each team's defined talent level, we know the following:

$$\frac{8t_1}{t_1+t_2} > \frac{8t_4}{t_3+t_4}; \quad \frac{10t_2}{t_2+t_3} > \frac{10t_3}{t_2+t_3}; \text{ but } \frac{10t_2}{t_1+t_2} < \frac{10t_3}{t_3+t_4}.$$

For  $E(RPI_2) > E(RPI_3)$ , the first two inequalities must dominate the third. However, we cannot be certain that this is the case. This brings us to the following proposition:

**Proposition**: It is indeterminate, a priori, that  $E(RPI_2) > E(RPI_3)$  (i.e., team 2 outranks team 3 in RPI), despite the fact that (i)  $t_2 > t_3$ , (ii) teams play to expectation, and (iii) information exists concerning relative performance in the intersecting portion of these two teams' schedules.

## 3. Simulation

This section presents a counter-example.

Let  $t_1 = 11x$ ,  $t_2 = 8x$ ,  $t_3 = 7.5x$ ,  $t_4 = 5x$ .

Given our additive form contest success function, the implications of these attributed talent levels are as follows:

$$\frac{t_1}{t_1 + t_2} = \frac{11}{19} \approx .579 \qquad \frac{t_1}{t_1 + t_3} = \frac{11}{18.5} \approx .595 \qquad \frac{t_1}{t_1 + t_4} = \frac{11}{16} \approx .688$$
$$\frac{t_2}{t_2 + t_3} = \frac{8}{15.5} \approx .516 \qquad \frac{t_2}{t_2 + t_4} = \frac{8}{13} \approx .615 \qquad \frac{t_3}{t_3 + t_4} = \frac{7.5}{12.5} = .6$$

In other words, these values appear collectively plausible within our simple model of a college basketball season.

Then,  $E(RPI_2) \approx .5014$  and  $E(RPI_3) \approx .5018$ .

In this case,  $(E(RPI_2) < E(RPI_3))$ .

## 4. Conclusion

Even in a perfect world in which teams play to expectation and can be transitively compared based on revealed performance level, we have found that the RPI does not necessarily provide an ordinal mapping from revealed team ability level to the real number line. Indeed, a team's RPI ranking may be significantly dependent upon the ability level of conference opponents. Rather than creating random noise in the NCAA tournament selection process, such a ranking error might repeatedly hurt the same deserving team or team-type. By considering outcomes from inter-conference games (i.e., interactions between conferences) within the RPI, the NCAA might discontinue the potential re-distributive effects of the conference season in college basketball. As a cause for further study, one might empirically test for the existence of an RPI conference bias.

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### Appendix A - The Optimal Military Subsidy in Case I

Substituting the contest success functions of Party 1 and Party 2 from (12) into Party 3's objective function in (17), we have

$$U_{3} = p_{1}S_{1} + p_{2}S_{2} - M$$

$$= \left[\frac{V_{1}(1+M)^{\theta}}{2\gamma V_{2}}\right]S_{1} + \left[1 - \frac{V_{1}(1+M)^{\theta}}{2\gamma V_{2}}\right]S_{2} - M$$

$$= \frac{V_{1}}{2\gamma V_{2}}(1+M)^{\theta}(S_{1} - S_{2}) + S_{2} - M.$$

Assuming that the value of  $S_1$  is sufficiently high such that there is an interior solution for M, the partial derivative of  $U_3$  with respect to M is

$$\frac{\partial U_3}{\partial M} = \frac{\theta \left(S_1 - S_2\right)}{2\gamma} \frac{V_1}{V_2} \left(1 + M\right)^{\theta - 1} - 1 = 0,$$

which implies that  $(1+M^*)^{1-\theta} = \frac{\theta(S_1-S_2)}{2\gamma} \frac{V_1}{V_2}$  or that  $(1+M^*) = \left[\frac{\theta(S_1-S_2)}{2\gamma} \frac{V_1}{V_2}\right]^{\frac{1}{1-\theta}}$ . Solving

for the optimal subsidy yields

$$M^* = \left[\frac{\theta(S_1 - S_2)}{2\gamma} \frac{V_1}{V_2}\right]^{\frac{1}{1-\theta}} - 1$$

The second-order condition for expected payoff maximization is satisfied at the optimal solution because

$$\frac{\partial^2 U_3}{\partial M^2} = -\frac{(1-\theta)\theta(S_1 - S_2)}{2\gamma V_2} (1+M^*)^{\theta-2} < 0,$$

given that  $0 < \theta < 1$  and  $S_1 > S_2 \ge 0$ .

### Appendix B - The Optimal Military Subsidy in Case II

Substituting the contest success functions of Party 1 and Party 2 from (25) into Party 3's objective function, we have

$$U_{3} = p_{1}S_{1} + p_{2}S_{2} - M$$

$$= \left[1 - \gamma^{\frac{3}{2}} \frac{V_{2}}{2V_{1}(1+M)^{\theta}}\right]S_{1} + \left[\gamma^{\frac{3}{2}} \frac{V_{2}}{2V_{1}(1+M)^{\theta}}\right]S_{2} - M$$

$$= S_{1} - \left[\gamma^{\frac{3}{2}} \frac{(S_{1} - S_{2})}{2} \frac{V_{2}}{V_{1}}\right](1+M)^{-\theta} - M.$$

Assuming that the value of  $S_1$  is sufficiently high such that there is an interior solution for M, the partial derivative of  $U_3$  with respect to M is

$$\frac{\partial U_3}{\partial M} = \left[ \gamma^{\frac{3}{2}} \frac{\theta(S_1 - S_2)}{2} \frac{V_2}{V_1} \right] (1 + M)^{-(1+\theta)} - 1 = 0,$$

which implies that  $(1+M^{**})^{1+\theta} = \left[\gamma^{\frac{3}{2}} \frac{\theta(S_1-S_2)}{2} \frac{V_2}{V_1}\right]$  or that  $(1+M^{**}) = \left[\gamma^{\frac{3}{2}} \frac{\theta(S_1-S_2)}{2} \frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}}$ .

Solving for the optimal military subsidy yields

$$M^{**} = \left[\gamma^{\frac{3}{2}} \frac{\theta(S_1 - S_2)}{2} \frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}} - 1.$$

The second-order condition for expected payoff maximization is satisfied at the optimal solution because

$$\frac{\partial^2 U_3}{\partial M^2} = -\frac{\gamma^{3/2} (1+\theta) \theta (S_1 - S_2) V_2}{2 V_1 (1+M^{**})^{\theta+2}} < 0.$$

## Appendix C - Cost of Peace-making versus that of Peace-breaking.

Let  $M^c > M^{cc} + \varepsilon$ . Show that this must hold. If  $M^c > M^{cc} + \varepsilon$ , then

$$\left(\frac{2\gamma V_2}{V_1}\right)^{\frac{1}{\theta}} - 1 > \left(\frac{\gamma V_2}{2V_1}\right)^{\frac{1}{\theta}} - 1 + \varepsilon .$$

Thus,  $\left[1 - \left(\frac{1}{4}\right)^{\frac{1}{\theta}}\right] \left[\frac{2\gamma V_2}{V_1}\right]^{\frac{1}{\theta}} > \varepsilon$ , where  $0 < \theta < 1$ . That is, there is always a sufficiently small,

positive value for epsilon such that this is true. Hence,  $M^c > M^{cc} + \varepsilon$ .

# Appendix D - Complementarity between Government Defensive Spending and Third-Party Intervention

As the incumbent government's relative valuation for political dominance rises, other things being equal, the third party's marginal value of intervention rises. This is due to the fact that an increase in  $(V_1/V_2)$  will make a unit of M more effective in decreasing the likelihood of successful rebellion. That is,  $\frac{\partial p_2^*}{\partial M}$  becomes more negative when  $(V_1/V_2)$  increases as illustrated by the following derivative:

$$\frac{\partial}{\partial \left(V_1/V_2\right)} \left[\frac{\partial p_2^*}{\partial M}\right] = \frac{-\left(1+M\right)^{\theta}}{2\mu} < 0.$$
(a.1)

At the meantime, the increase in the incumbent government's relative valuation lowers the critical value of  $\tilde{H}$  as shown by the following expression:

$$\frac{\partial \tilde{H}}{\partial \left(V_1/V_2\right)} = -\frac{2\mu V_2^2 \left(1+M\right)^{1-\theta}}{V_1^2 \theta} < 0.$$
(a.2)

It becomes more likely that the third party will decide to intervene. The results in (a.1) and (a.2) thus imply that an intervening third party treats the incumbent government's value for political dominance as complementary to its own intervention efforts.

Similarly, as the incumbent government's relative military spending effectiveness rises (i.e.,  $\mu$  decreases), a unit of M becomes more effective in decreasing the likelihood of successful rebellion. This is due to the fact that a decrease in  $\mu$  causes  $\frac{\partial p_2^*}{\partial M}$  to become more negative as illustrated by the following derivative:

$$\frac{\partial}{\partial \mu} \left[ \frac{\partial p_2^*}{\partial M} \right] = \frac{\left(1 + M\right)^{\theta} V_1}{2\mu^2 V_2} > 0.$$
(a.3)

At the meantime, the increase in the incumbent government's relative military spending effectiveness (i.e., the decrease in  $\mu$ ) lowers the value of  $\tilde{H}$ . This result can easily be verified by the following expression:

$$\frac{\partial \tilde{H}}{\partial \mu} = \frac{2V_2 \left(1+M\right)^{1-\theta}}{V_1 \theta} > 0.$$
 (a.4)

It becomes more likely that the third party will decide to intervene. The results in (a.3) and (a.4) thus imply that an intervening third party treats the incumbent government's relative military spending effectiveness as complementary to its own intervention efforts.

## **Appendix E - The Optimal Intervention Level**

Substituting the contest success functions of Party 1 and Party 2 from (11) into Party 3's objective function, we have

$$U_{3} = p_{1}S_{1} + p_{2}S_{2} - M$$

$$= \left[\frac{(1+M)^{\theta}V_{1}}{2\mu V_{2}}\right]S_{1} + \left[1 - \frac{(1+M)^{\theta}V_{1}}{2\mu V_{2}}\right]S_{2} - M$$

$$= \frac{(1+M)^{\theta}V_{1}}{2\mu V_{2}}(S_{1} - S_{2}) + S_{2} - M.$$

Assuming that the value of  $S_1$  is sufficiently high such that there is an interior solution for M, the partial derivative of  $U_3$  with respect to M is

$$\frac{\partial U_3}{\partial M} = \theta \left( 1 + M \right)^{\theta - 1} \frac{V_1 (S_1 - S_2)}{2\mu V_2} - 1 = 0,$$

which implies that 
$$(1+M^*)^{1-\theta} = \left[\frac{\theta V(S_1-S_2)}{2\mu V_2}\right]$$
 or that  $(1+M^*) = \left[\frac{\theta V(S_1-S_2)}{2\mu V_2}\right]^{\frac{1}{1-\theta}}$ .

Solving for the optimal intervention level yields

$$M^* = \left[\frac{\theta V(S_1 - S_2)}{2\mu V_2}\right]^{\frac{1}{1-\theta}} - 1.$$

The second-order condition for expected payoff maximization is satisfied at the optimal solution because

$$\frac{\partial^2 U_3}{\partial M^2} = \theta (\theta - 1) (1 + M^*)^{\theta - 2} \frac{V_1 (S_1 - S_2)}{2 \mu V_2} < 0.$$