A COMPARISON OF ADJUSTMENTS ON TWO TRAVERSES BY THE TRANSIT RULE,COMPASS RULE AND METHOD OF LEAST SQUARES

by

STEVEN JAMES COHORST

B. S., Kansas State University, 1970

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1971

Approved by:

Jack B. Blackburn
Major Professor

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

During the last decade there have been more technological advances in the surveying industry than during the preceding 90 years. A century ago angles were being measured with compasses and distances were measured with the Gunter chain. Fifty years ago transits and engineers chains were being used. Even in the past decade the transit and engineers tape were still being used. Today, surveyors are using compact theodolites to measure angles and electronic distance measuring devices to measure distances with more accuracy and precision than in the decade just past.

The increased accuracy and precision mentioned above has led to much smaller closure errors and to a desire for better techniques for adjustment of this error in achieving closure for either level or traverse circuits. The electronic computer is a major factor in the improvement of error adjustment procedures. Through the use of the computer, methods which were previously impractical or difficult to use are now easily applied.

The transit rule, the compass rule and the method of least squares, are methods of adjustment of traverse closure errors which will be compared in this study. Each method has its own advantages and disadvantages which are of interest. The reason for this study is to satisfy the curiosity of the author regarding the manner in which the error of closure is adjusted by each of these methods when applied to a given traverse.

PURPOSE AND SCOPE

Many articles have been written on the subject of traverse adjustments using the transit rule, the compass rule and the method of least squares. Each of these methods of adjustment partitions the closure error in a unique manner which results in the allocation of different portions of the closure error to given traverse segments by each method.

The purpose of this report is to apply the different methods of traverse adjustments to a set of field data to compare the results obtained. The field data were obtained from two traverses run on the KSU Campus, one North-South and one East-West, for which highly accurate information was available from earlier traverse work in which a precision of 1/150,000 was obtained.

REVIEW OF LITERATURE

In the adjustment of traverses, the errors are adjusted. Rainsford (9) (chapter 1) discusses four types of errors: blunders, constant errors, systematic errors, and accidental or random errors.

"Blunders (or mistakes) are a definite mis-reading of whatever scale is being used." Examples would be the reading of 60° 45' for an angle which actually is 61° 45' or reading 100.75 feet instead of 99.75 feet.

"Constant errors are those which do not vary throughout the particular work concerned. They always have the same sign." An example would be an uncalibrated tape 100.02 feet in length which would be assumed to be 100.00 feet.

"Systematic errors are those which follow some fixed law (possibly unknown) dependent on local circumstances." Examples would be failure to apply temperature and sag corrections to tapes.

"Accidental or random errors are the remaining small errors after all the others just mentioned have been eliminated. They are due to the imperfection of the instrument used, the fallibility of the observer, and the changing conditions, all of which affect the quality of the observation to a greater or lesser degree." These accidental of random errors are the only errors which are adjusted. Wolf (14) and Vreeland (13) discuss the source of these errors as either in distance or in angle.

The adjustment of the errors depends upon the type and source of the errors. Blunders are not acceptable whether in angle or distance and must be remeasured. The source of constant errors must be identified and corrections or adjustments made in instrumentation or technique so that this type of error may be eliminated. Systematic errors follow definite laws (2) which are used to compute their values in order that they may be eliminated. The accidental or random errors conform to the normal distribution (9) which, for a given set of data, have unique values of the mean, $\bar{X}$, and standard deviation, s.

These accidental or random errors are the errors which are to be adjusted. Several procedures have been devised to adjust these errors (10). Each of these procedures adjusts the observations or measured quantities to fulfill the conditions of angular closure, latitude closure, and departure closure. The procedures include the transit rule, the compass rule and the method of least squares.

The transit rule assumes that the angles are measured more precisely than the distances. "The corrections to be applied to the latitude/departure of any course is to the total error in latitude/departure as the latitude/departure of the course is to the sum of all latitude/departure (without regard to algebraic sign)."(3)

Goussinsky (5) explains that the adjustment by the compass rule (also called the Bowditch Method) assumes that the error in the angle is as great as the error in the distance. This was true 100 years ago when the compass and the Gunter chain were common surveying equipment, hence the name compass rule. "The corrections to be applied by this rule to the latitude/departure of any course is to the total error in latitude/departure as the length of the course is to the perimeter."(3) Richardus (10) explains that in applying this procedure, the bearings are adjusted before computing the latitudes and departures for the adjustment.

The method of least squares which makes no assumptions about the configuration of the traverse applies corrections to the angles and distances such that the cumulative sum of the squares of the corrections is a minimum. The method, although developed over 150 years ago by Gauss and Legendre, was not used extensively until recently and has gained general acceptance since the era of the computer. Adams (1) in 1924 presented the basic procedures for obtaining the least squares solution through the use of several simple examples.

Gale (4) indicates that least squares adjustments can be divided into three categories;

1. The observation equation or parametric method. In survey adjustments, this method is sometimes referred to as variation of coordinates and is discussed by Wolf (14). He states "Two observation equations (equations relating the observed quantities and their inherent random error to the most probable X and Y coordinates of the points involved), one for distance, and one for direction, may be written for each side of a traverse. Therefore the number of observation equations for any traverse is 2r, where r is the number of sides in the traverse. Each traverse point introduces two unknowns, an X and a Y coordinate, except that the initial and final traverse point coordinates are either known or assumed; hence the number of unknowns is 2r - 2. Normally, therefore, there are two redundant observation equations in the traverse adjustment."

Schmid (11), Gale (4) and Madkour (7) present the idea of writing the observation equations in the following matrix form.

$$ {}_{M}A_{N} \; {}_{N}X_{1} = {}_{M}L_{1} \qquad (M>N) $$

where

${}_{N}X_{1}$ = A column vector of parameters

${}_{M}A_{N}$ = A matrix containing the coefficients of ${}_{N}X_{1}$

${}_{M}L_{1}$ = A column vector of the correlated observations

M = The number of rows in the matrix

N = The number of columns in the matrix

No adjustment is necessary if the equations are consistent. If they are inconsistent an adjustment vector, ${}_M V_1$, is added to the correlated observations thus obtaining

$${}_M A_N \, {}_N X_1 = {}_M L_1 + {}_M V_1$$

To apply the method of least squares to this problem, the Gaussian Function, Φ, must be minimized. This minimization is accomplished by setting the matrix first derivative equal to zero.

$$\Phi = {}_1 V^*_M \, {}_M P_M \, {}_M V_1$$

where

${}_1 V^*_M$ = The transpose of the adjustment vector

${}_M P_M$ = The observation weight matrix (discussed in Appendix A)

2. The condition equation or correlate method. Vreeland (13) presents a traverse adjustment by the condition equation method using the summation of latitude and departure as conditions. Gale (4) and Madkour (7) present their discussions of the condition equation method through the use of this matrix equation;

$${}_M B^*_N \, {}_N X_1 + {}_M C_1 = 0$$

where

${}_M B^*_N$ = The transpose of a matrix containing the partial derivatives of the unknowns ${}_N X_1$

${}_N X_1$ = A column vector of the true values of the unknowns

${}_M C_1$ = A column vector of constants necessary to complete the condition equations

Then by introducing the undetermined Lagrangian Multipliers, ${}_1K_M^*$ (Adams (1) presents a simple example of Lagranian Multipliers), and minimizing the Gaussian Function,

$$\Phi = {}_1V_N^* \, {}_NP_N \, {}_NV_1 - 2 \, {}_1K_M^* \left({}_MB_N \, {}_NX_1 + {}_MC_1 \right)$$

the least squares solution is obtained. Further development of the algebraic formulas may be found in Appendix A.

3. The general or combined methods. Schmid (11) discusses the general category of least squares. In his discussion are presented the difficulties met when trying to write a general least squares computer program. This category is not discussed here because it is not used to adjust a traverse.

Since all of the least squares procedures allow for the use of weights, Meyer (8) discusses the use of the arithmetic mean of several observations to obtain a better value of the observation. He expands his discussion to include the case where an angle might be read 2 times with a theodolite and 3 times with a 30" transit. Obviously the angles read by the theodolite are more accurate than those read with the 30" transit. He therefore applies a weight to the angles to obtain a weighted arithmetic mean.

Vreeland (13) presents the precisions of the various surveying instruments used, but doesn't discuss weights. Wolf (14) on the other hand discusses methods which may be used to obtain the relative weights of the angles and distances (See Appendix A for more detail.).

Rainsford (9) and Meyer (8) discuss the use of the inverse of the variance as a weight. Rainsford presents a derivation which shows the weight of the observation is equal to 1 ÷ variance.

PROCEDURES AND APPARATUS

Recently work was begun to place the Kansas State University campus on a coordinate system. The initial phase of the project was to fix control points whose coordinates were accurately obtained on the perimeter of the campus. This was done by completing a loop traverse (see Fig. 1) using a Wild T-2 theodolite and the Tellurometer MRA-4 electronic distance measuring device. A precision of 1/150,000 was obtained from this traverse. (The U.S. Coast and Geodetic Survey considers 1/25,000 as first-order accuracy.)

In order to further subdivide the campus, field parties were sent out with Wild T-16 theodolites and 100-ft. engineers tapes to obtain the two traverses under study here. The two traverses--one in a North-South direction along Mid-Campus Drive (Fig. 2) and the other in an East-West direction along College Heights and Vattier Drive (Fig. 3)--yielded data which had angles estimated to the nearest 6 seconds and distances obtained to the nearest 0.01 foot.

The North-South traverse was completed by a field party from the KSU Campus Planning Department and the East-West traverse was completed by an advanced surveying class from the Department of Civil Engineering. Since the traverse is the particular case in surveying in which both angles and distances are measured, they naturally fall into these four categories depending on the type of closure:

1. No Closure
2. Angular Closure (i.e., the angles must meet a specified condition)
3. Coordinate Closure (i.e., the latitude and departure must meet specified conditions)
4. Coordinate and Angular Closure (i.e., the angles, latitude and departures must meet specified conditions)

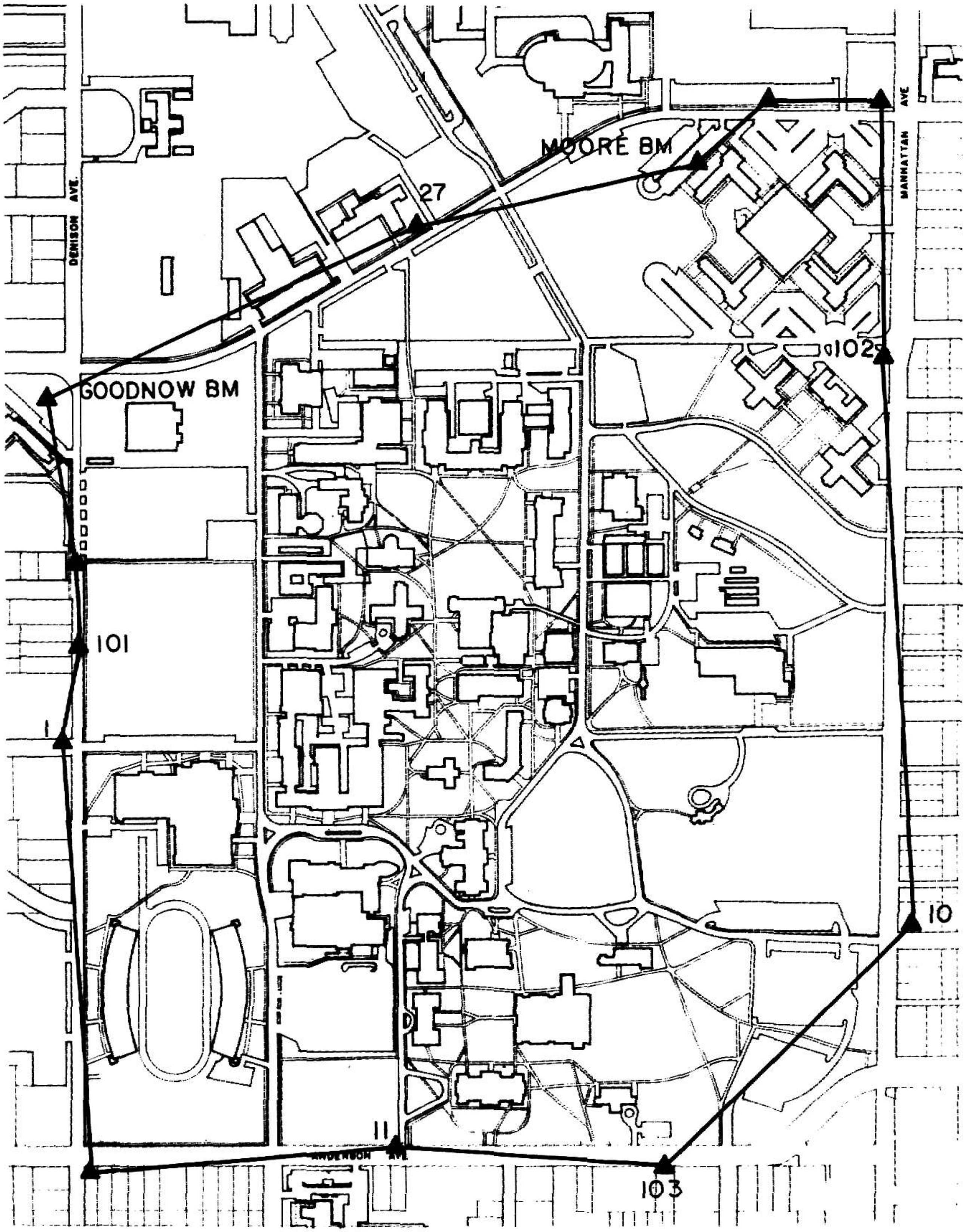

Fig. 1. A map of the KSU Campus indicating the location of the control traverse.

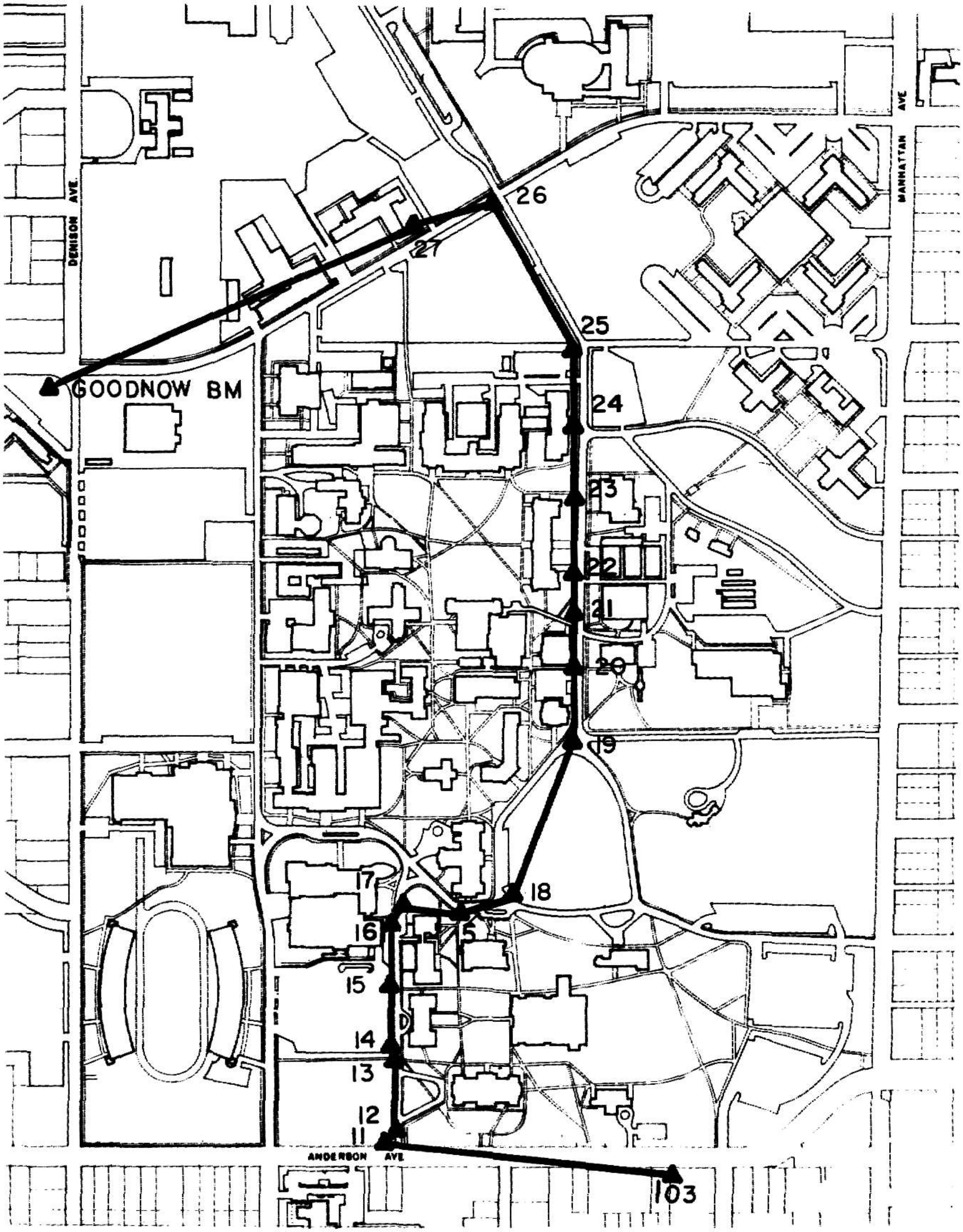

Fig. 2. A map of the KSU Campus indicating the location of the North-South traverse.

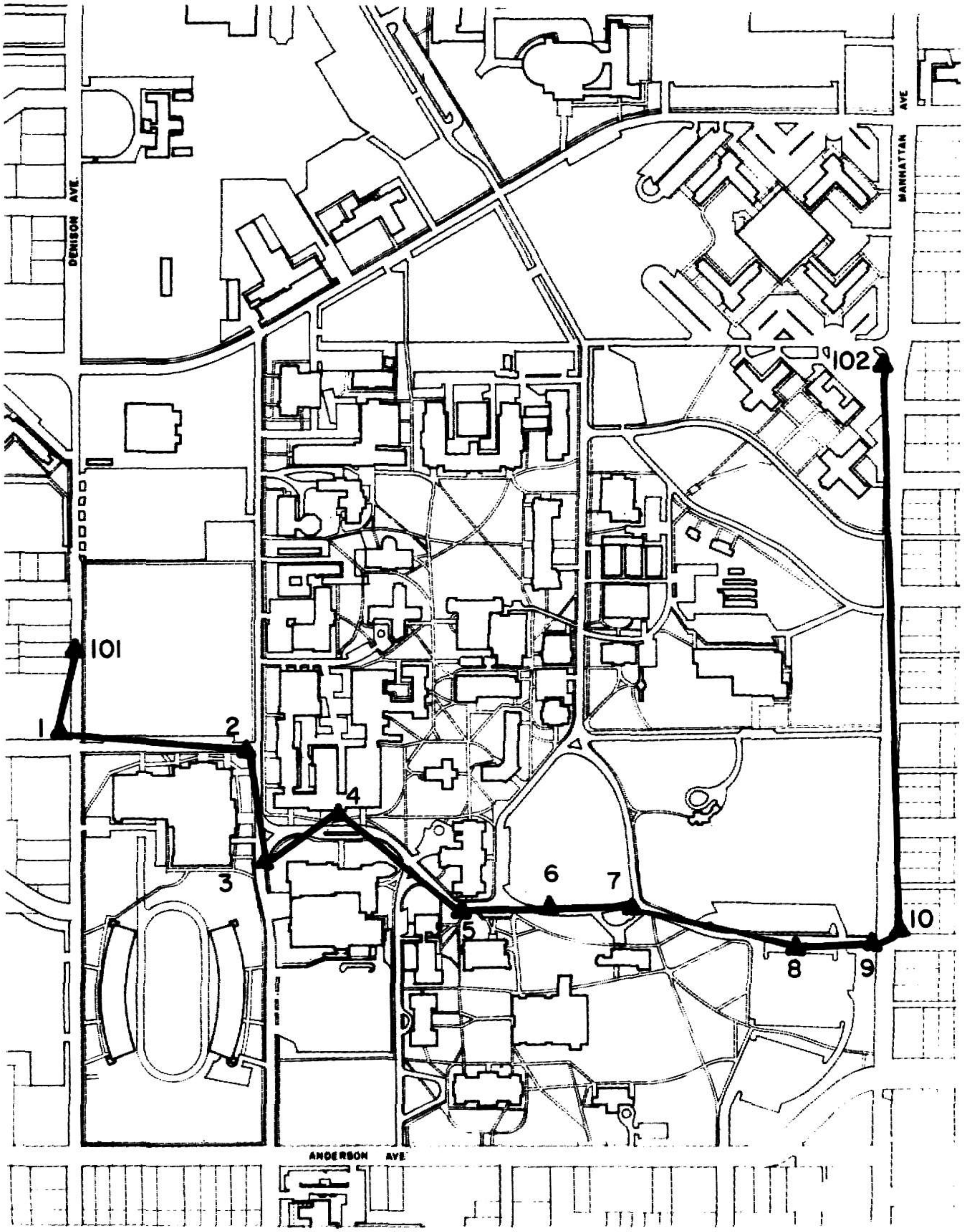

Fig. 3. A map of the KSU Campus indicating the location of the East-West traverse.

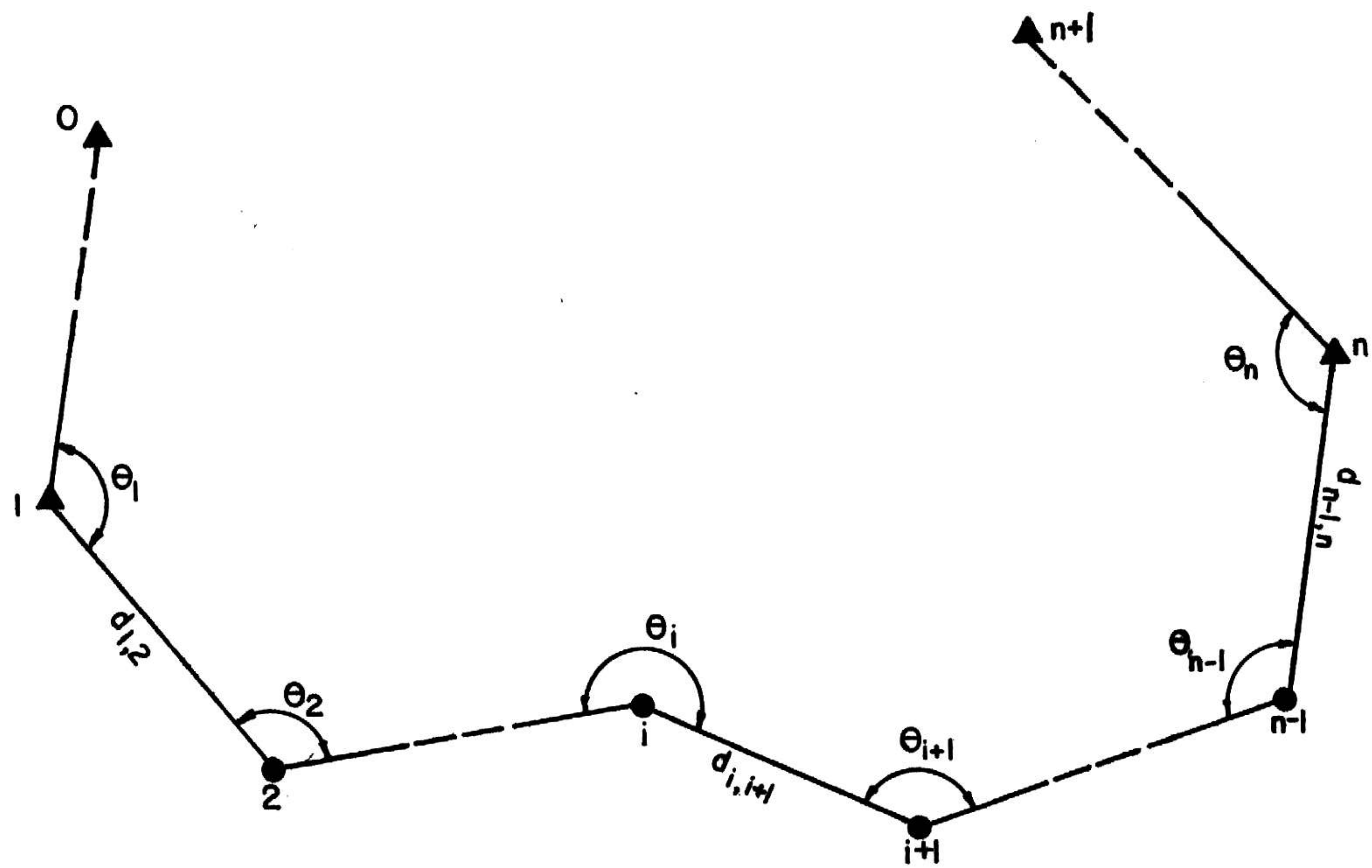

Fig. 4. A Generalized Traverse Showing the Notation Used in This Study.

The two traverses had both angle and coordinate closure and therefore fell into the fourth category.

The conditions referred to above are described below with reference made to Fig. 4. Stations 1 and n are the terminals. The observed angle on station 1 is θ_1, or in general θ_i on station i. The measured distance between i and i+1 is $d_{i,i+1}$ and the azimuth of the line is $Az_{i,i+1}$.

Stations 0 and n+1 are the orientation stations, the orienting azimuths are $Az_{0,1}$ and $Az_{n,n+1}$. If stations 0 and n, and n+1 and 1 are the same stations the traverse is a loop.

1. The Angular Condition. The sum of the first orientation azimuth $Az_{0,1}$ and the observed angles, θ_i, must equal the terminal azimuth $Az_{n,n+1} \pm n\ 180°$ which may be written as

$$Az_{0,1} + \sum_{i=1}^{n}\theta_i = Az_{n,n+1} \pm n\ 180° \qquad 1)$$

The relationship of any azimuth to any other azimuth should also be noted

$$Az_{i,i+1} = Az_{0,1} + \sum_{j=1}^{i}\theta_j \pm n\ 180° \qquad 1a)$$

2. The Departure Condition. The sum of the products of the distances and the sines of their azimuths should be equal to the differences in the abscissas of the terminal stations.

$$\sum_{i=1}^{n} d_{i,i+1}\ \mathrm{Sin}(Az_{i,i+1}) = X_n - X_1 \qquad 2)$$

3. The Latitude Condition. The sum of the products of the distances and the cosines of their azimuths must equal the differences in the ordinates of the terminal stations.

$$\sum_{i=1}^{n} d_{i,i+1}\ \mathrm{Cos}(Az_{i,i+1}) = Y_n - Y_1 \qquad 3)$$

The three methods of adjustment to be used in this study--the transit rule, the compass rule and the method of least squares--apportion the closure errors of these conditions differently. The transit rule and compass rule adjust the angles prior to computing the course latitudes and departures by adding the amount (angular closure error ÷ number of angles) to each angle whereas the method of least squares uses the unadjusted angles to compute the course latitudes and departures and adjusts the angles simultaneously.

The transit rule adjustment may be stated as follows: the correction to be applied to the latitude/departure of any course is to the total error in latitude/departure as the latitude/departure of that course is to the sum of all the latitude/departure (without respect to sign) (14). The compass rule adjustment of the latitude and departure of a traverse is simply stated as follows: the corrections to be applied to the latitude/departure of any course is to the total error in latitude/departure as the length is to the perimeter (14).

The method of least squares adjustment applies corrections which have been minimized to the observed values. The method uses the matrix condition

$$B\,X + C = 0$$

where
B = The partial derivatives of the conditions
X = The true values of the variables
C = The constants of the conditions

And its Gaussian Function, Φ, solved for a minimum.

$$\Phi = V^{*}\,P\,V - 2\,K^{*}\,(B\,X + C)$$

where
V = The corrections to the observed values
P = The weight of the variable
K = A Lagrangian Multiplier
(The * refers to a transposed matrix)

A more detailed algebraic explanation of the condition equation method of least squares is in Appendix A.

The various adjustments were computed through the use of the MIT ICES COGO--Geometric processor. This Processor did not perform the least squares adjustment using the unadjusted angles as described in Appendix A, but rather used the adjusted angles of the transit and compass rule. Although not correct, since the angles have been adjusted, this procedure is useful in that the latitude and departure errors are equal for all three thereby producing an easy comparison of error adjustment by the three methods.

As the data were being prepared for use in the ICES COGO--Geometric Processor it was learned that another error adjustment procedure, the Crandall Rule, was contained in the program package. Since the Crandall Rule could be examined with very little additional effort it was decided to apply this procedure of error adjustment to determine its applicability when compared with the three procedures originally included in the study. This rule holds the course bearing as correct and adjusts only the course lengths. The Crandall Rule is discussed in more detail in Appendix B.

PRESENTATION AND DISCUSSION OF DATA

The presentation of these data is divided into two sections--one for the North-South traverse and one for the East-West traverse.

The North-South traverse.

The initial point (Pt. 27) of the North-South traverse (Fig. 2) had coordinates (N 6687.05, E 2243.70) and an initial azimuth of 244° 46' 50" when sighting to the Civil Engineering Dept. Bench Mark in front of Goodnow Hall. The coordinates of the final point (Pt. 11) were (N 3756.26, E 2188.87) and an azimuth of 93° 16' 51" to Pt. 103. The computation of the traverse produced the following closure errors--N -0.012 ft. and E -0.434 ft. Tables 1, 2, and 3 show the computation of the latitude and departure adjustments and Table 4 shows the angle and distance adjustments produced by the three adjustment methods.

The Angle Adjustment. The first half of Table 4 shows adjustments which were made to the angles. A comparison of the arithmetic totals of the angle adjustments indicated that the method of least squares produced the smallest corrections (89 sec. total) followed by the compass rule (116 sec. total) and then the transit rule (375 sec.).

Comparing the individual adjustments, the method of least squares yielded many adjustments which were within one estimation unit (6 sec.) of the T16 theodolite and all within 2 units (12 sec.). The compass rule had 4 adjustments larger than 2 units (12 sec.) but smaller than 5 units (0.5 minute). The transit rule had 11 adjustments larger than 2 units with 7 of these larger than 5 units (0.5 minute).

The Distance Adjustment. The last half of Table 4 indicates the adjustments to the distance. A comparison of the arithmetic totals indicated a wide variation in adjustments. The method of least squares had 0.011 ft. adjustment, followed by the compass rule with 0.127 ft. adjustment and then the transit rule with 0.306 ft. adjustment.

A comparison of the individual adjustments indicate the method of least squares makes no adjustment to distance (The largest adjustment was 0.005 ft.) The compass rule yielded 5 adjustments which were more than 0.01 ft. with 4 of these adjustments between 0.025 ft. and 0.030 ft. These four adjustments were in the four courses with large departure components. The transit rule also yielded 5 adjustments more than 0.01 ft. These adjustments were in the same courses as the compass rule but were much larger--all between 0.036 ft. and 0.091 ft.

The Latitude Adjustment. No comparison was made due to the extremely small closure error of 0.012 ft.

The Departure Adjustment. The comparison of the departure adjustment (Fig. 5) yielded a transit rule adjustment with most of the error in the four courses with large departure components. The compass rule yielded the expected straight line (This was expected since the magnitude of adjustment is in direct proportion to the total length.) The method of least squares yielded adjustments which increased to -0.485 ft. for the next to the last point on the traverse and dropped to -0.434 ft. for the last point (This drop is not possible with the transit and compass rules.).

The Crandall Rule Adjustment. A discussion of the Crandall Rule Adjustment is in Appendix B.

Table 1. Transit Rule Adjustment of the North-South traverse.

Point on Traverse	Adjusted Angle ° ' "	Course Length (feet)	Course Latitude (feet)	Per Cent of Total Latitude	Latitude Adjustment (feet)	Adjusted Latitude (feet)	Course Departure (feet)	Per Cent of Total Departure	Departure Adjustment (feet)	Adjusted Departure (feet)
27	189 23 9	256.16	69.867	2.2	0.000	69.867	246.448	22.1	-0.095	246.353
26	256 9 1	481.76	-418.501	13.5	-0.002	-418.503	238.640	21.3	-0.091	238.549
25	209 1 2	290.02	-290.000	9.3	-0.001	-290.001	3.373	0.3	-0.002	3.371
24	179 16 1	250.05	-249.975	8.1	-0.001	-249.976	6.108	0.5	-0.002	6.106
23	179 59 54	250.01	-249.935	8.1	-0.001	-249.936	6.115	0.5	-0.003	6.112
22	179 59 54	100.00	-99.970	3.2	0.000	-99.970	2.449	0.2	-0.001	2.448
21	179 59 54	149.98	-149.935	4.8	-0.001	-149.935	3.677	0.3	-0.001	3.676
20	179 59 54	249.98	-249.905	8.1	-0.001	-249.906	6.137	0.5	-0.003	6.134
19	205 24 57	541.75	-494.907	15.9	-0.002	-494.909	-220.363	19.7	-0.085	-220.448
18	215 39 17	124.69	-62.994	2.0	0.000	-62.994	-107.607	9.6	-0.041	-107.648
5	215 17 49	217.45	18.811	0.6	0.000	18.811	-216.635	19.5	-0.083	-216.718
17	114 10 40	71.87	-62.780	2.0	0.000	-62.780	-34.985	3.1	-0.014	-34.999
16	149 38 4	199.08	-199.034	6.4	-0.001	-199.035	4.263	0.4	-0.001	4.262
15	180 2 36	213.96	-213.914	6.9	-0.001	-213.915	4.420	0.4	-0.001	4.419
14	158 59 32	17.88	-16.555	0.5	0.000	-16.555	6.755	0.5	-0.002	6.753
13	201 5 41	248.12	-248.075	8.0	-0.001	-248.075	4.745	0.4	-0.003	4.742
12	212 33 15	15.21	-12.976	0.4	0.000	-12.976	-7.936	0.7	-0.006	-7.942
11	69 49 19									
Algebraic Totals			-2930.778		-0.012	-2930.790	-54.396		-0.434	-54.830
Arithmetic Totals		3677.97	3108.134	100.0			1120.656	100.0		

Table 2. Compass Rule Adjustment of the North-South traverse.

Point on Traverse	Adjusted Angle ° ' "	Course Length (feet)	Per Cent of Total Length	Course Latitude (feet)	Latitude Adjustment (feet)	Adjusted Latitude (feet)	Course Departure (feet)	Departure Adjustment (feet)	Adjusted Departure (feet)
27	189 23 24	256.16	7.0	69.867	-0.001	69.866	246.448	-0.030	246.418
26	256 8 33	481.76	13.1	-418.501	-0.002	-418.503	238.640	-0.056	238.584
25	209 1 39	290.02	7.9	-290.000	-0.001	-290.001	3.373	-0.034	3.339
24	179 16 00	250.05	6.8	-249.975	-0.001	-249.976	6.108	-0.029	6.079
23	179 59 54	250.01	6.8	-249.935	-0.001	-249.936	6.115	-0.029	6.086
22	179 59 54	100.00	2.7	-99.970	0.000	-99.970	2.449	-0.012	2.437
21	179 59 54	149.98	4.1	-149.935	0.000	-149.935	3.677	-0.017	3.660
20	179 59 54	249.98	6.8	-249.905	-0.001	-249.906	6.137	-0.029	6.108
19	205 24 27	541.75	14.7	-494.907	-0.002	-494.909	-220.363	-0.063	-220.426
18	215 39 2	124.69	3.4	-62.994	-0.001	-62.995	-107.607	-0.015	-107.622
5	215 18 15	217.45	5.9	18.811	-0.001	18.810	-216.635	-0.025	-216.660
17	114 10 23	71.87	1.9	-62.780	0.000	-62.780	-34.985	-0.009	-34.994
16	149 38 39	199.08	5.5	-199.034	0.000	-199.034	4.263	-0.023	4.240
15	180 2 36	213.96	5.8	-213.914	0.000	-213.914	4.420	-0.025	4.395
14	158 59 4	17.88	0.5	-16.555	0.000	-16.555	6.755	-0.002	6.753
13	201 6 13	248.12	6.7	-248.075	-0.001	-248.076	4.745	-0.030	4.715
12	212 32 38	15.21	0.4	-12.976	0.000	-12.976	-7.936	-0.006	-7.942
11	61 49 33								
Algebraic Totals				-2930.778	-0.012	-2930.790	-54.396	-0.434	-54.830
Arithmetic Totals		3677.97	100.0						

Table 3. Least Squares Distance Adjustment of the North-South traverse.

Point on Traverse	Adjusted Angle ° ' "	Course Length (feet)	Course Latitude (feet)	Latitude Adjustment (feet)	Adjusted Latitude (feet)	Course Departure (feet)	Departure Adjustment (feet)	Adjusted Departure (feet)
27	189 23 40	256.16	69.867	-0.013	69.854	246.448	0.001	246.449
26	256 8 16	481.76	-418.501	-0.022	-418.523	238.640	-0.043	238.597
25	209 1 43	290.02	-290.000	-0.001	-290.001	3.373	-0.040	3.333
24	179 16 6	250.05	-249.975	-0.002	-249.977	6.108	-0.041	6.067
23	179 59 58	250.01	-249.935	-0.002	-249.937	6.115	-0.047	6.068
22	179 59 57	100.00	-99.970	-0.001	-99.971	2.449	-0.020	2.429
21	179 59 56	149.98	-149.935	-0.001	-149.936	3.677	-0.031	3.646
20	179 59 55	249.98	-249.905	-0.001	-249.906	6.137	-0.055	6.082
19	205 24 30	541.75	-494.907	0.043	-494.864	-220.363	-0.109	-220.472
18	215 39 9	124.69	-62.994	0.021	-62.973	-107.607	-0.013	-107.620
5	215 18 26	217.45	18.811	0.042	18.853	-216.635	0.002	-216.633
17	114 9 57	71.87	-62.780	0.006	-62.774	-34.985	-0.011	-34.996
16	149 38 32	199.08	-199.034	-0.001	-199.035	4.263	-0.030	4.233
15	180 2 31	213.96	-213.914	-0.001	-213.915	4.420	-0.028	4.392
14	158 59 00	17.88	-16.555	0.000	-16.555	6.755	-0.001	6.754
13	201 6 6	248.12	-248.075	0.000	-248.075	4.745	-0.019	4.726
12	212 32 34	15.21	-12.976	-0.079	-13.055	-7.936	0.051	-7.885
11	61 49 46							
Algebraic Totals			-2930.778	-0.012	-2930.790	-54.396	-0.434	-54.830

Table 4. Angle and Distance Adjustments of the North-South traverse.

Point on Traverse	Angle °	'	"	Angle Adjustment (sec.) Transit Rule	Compass Rule	Least Squares	Course Length (feet)	Distance Adjustment (feet) Transit Rule	Compass Rule	Least Squares
27	189	23	30	-21	-6	10	256.16	-0.091	-0.029	0.002
26	256	8	6	55	27	10	481.76	-0.044	-0.027	0.003
25	209	1	36	-33	3	8	290.02	0.001	0.000	0.000
24	179	59	54	1	0	6	250.05	0.001	0.000	0.000
23	179	59	54	0	0	4	250.01	0.001	0.000	0.000
22	179	59	54	0	0	3	100.00	0.000	0.000	0.000
21	179	59	54	0	0	2	149.98	0.000	0.000	0.000
20	179	59	54	0	0	1	249.98	0.001	0.000	0.000
19	205	24	30	27	-2	0	541.75	0.036	0.027	0.005
18	215	39	12	5	-10	-3	124.69	0.036	0.013	0.000
5	215	18	30	-41	-14	-3	217.45	0.083	0.025	0.001
17	114	10	00	40	23	-3	71.87	0.007	0.004	0.000
16	149	38	36	-32	3	-3	199.08	0.000	0.000	0.000
15	180	2	36	0	0	-5	213.96	0.001	0.000	0.000
14	158	59	6	26	-2	-6	17.88	-0.001	-0.001	0.000
13	201	6	12	-26	2	-6	248.12	0.001	0.000	0.000
12	212	32	42	33	-4	-8	15.21	0.002	0.001	0.000
11	61	49	54	-35	-20	-8				
Arithmetic Totals				375	116	89		0.306	0.127	0.011

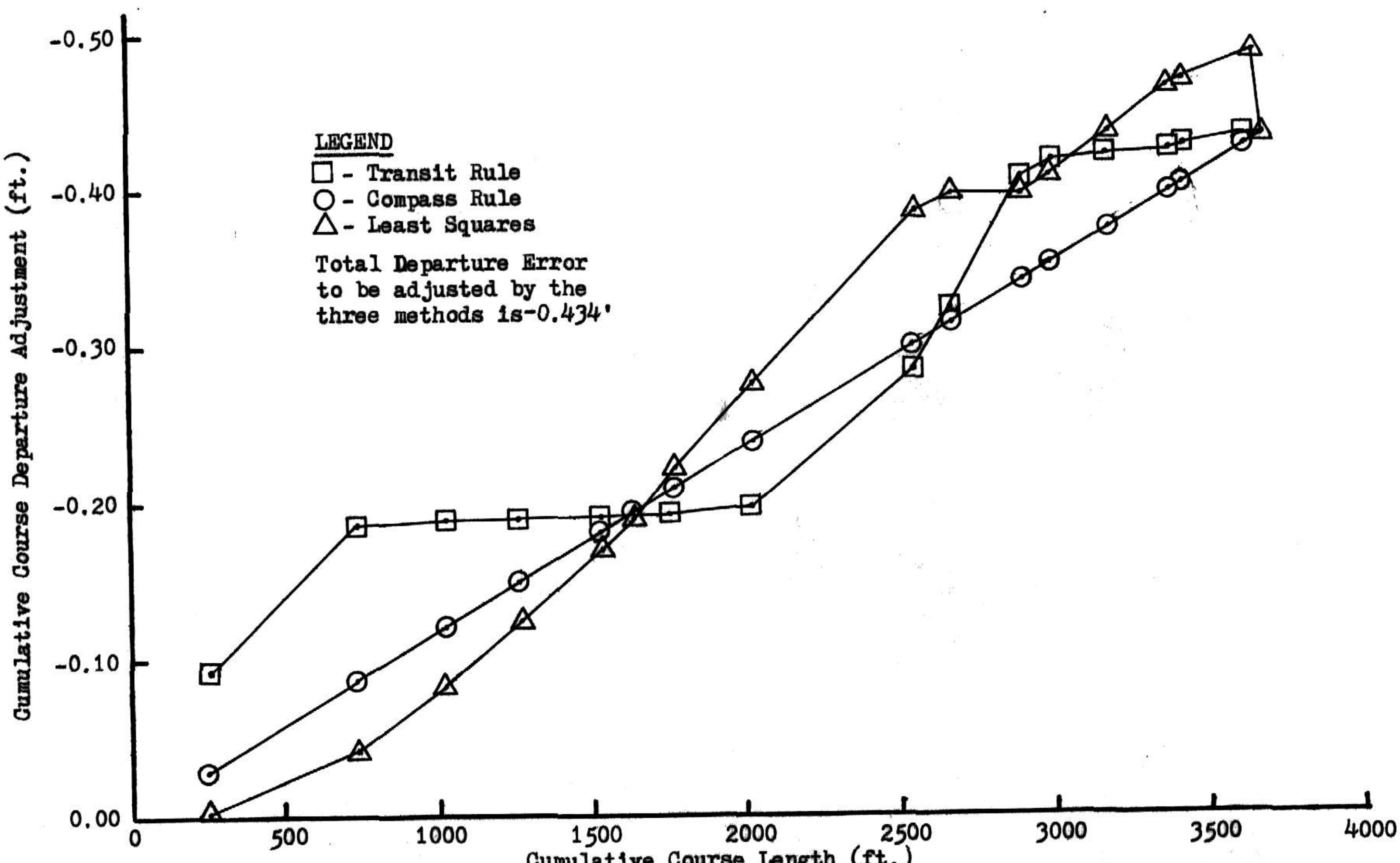

Fig. 5. Cumulative Course Length vs. Cumulative Course Departure Adjustment for the North-South traverse.

The East-West traverse.

The East-West traverse (Fig. 3) is like the North-South traverse in that it had both angular and coordinate closure. The initial point on the traverse (Pt.1) had coordinates (N 5041.86, E 1117.35) and an initial azimuth of 187° 2' 43" to point 101. The final point (Pt. 10) had coordinates (N 4479.63, E 3737.13) and an azimuth of 358° 22' 31" to point 102. The closure errors obtained were--N -0.538 ft. and E 0.269 ft. Tables 5, 6, and 7 present the computations for the latitude and departure adjustments and Table 8 summarizes the angle and distance adjustments.

The Angle Adjustment. The first half of Table 8 shows the adjustments which were made to the angles. A comparison of the arithmetic totals of the angle adjustments indicated that the method of least squares produced the smallest correction (108 sec.) followed by the compass rule (214 sec.) and then the transit rule (274 sec.).

Comparing the individual adjustments, the method of least squares yielded all but 1 adjustment within 3 estimation units of the T16 theodolite (one unit is 6 sec.). The compass rule had 5 adjustments within 3 units and 4 of the remaining 5 adjustments are larger than 0.5 minutes. The transit rule had 4 adjustments wit 3 units (18 sec.), 4 adjustments between 3 and 5 units (18-30 sec.) and 2 adjustments of more than 1 minute.

The Distance Adjustment. The last half of Table 8 indicates the adjustments to the distance. A comparison of the arithmetic totals indicated a wide variation in adjustments. The method of least squares had 0.031 ft. followed by the compass rule with 0.346 ft. and then the transit rule with 0.512 ft.

Comparison of the individual course adjustments indicated that the method of least squares produced small adjustments (the largest was 0.013 ft.). The compass rule has 5 adjustments less than 0.03 ft., 3 between 0.06-0.07 ft.

and one adjustment of 0.093 ft. The transit rule also has 5 adjustments less than 0.03 ft., 2 adjustments between 0.06-0.07 ft. and 2 adjustments of over 0.10 ft. The last two adjustments of over 0.10 ft. are possible but unlikely. Such an adjustment is more likely attributable to field measurement error and would require remeasuring part, or all, of the traverse in order to correct a likely error which would cause such a large adjustment.

The Latitude Adjustment. The comparison of the latitude adjustments (Fig. 6) yielded a transit rule adjustment with most of the error being adjusted in the first half of the traverse where the large course latitude was located. The compass rule yielded the expected straight line and the method of least squares yielded small adjustments in the first two courses and an almost linear adjustment for the rest of the traverse.

The Departure Adjustment. The comparison of the departure adjustment (Fig. 7) yielded an almost linear trend for the transit rule and a linear trend for the compass rule. These trends were expected since the transit rule is apportioned using the cumulative course departure (2620 ft.) which is approximately equal to the cumulative course length (2987 ft.) used by the compass rule. The method of least squares yielded a much different type of adjustment than the other two methods. It adjusted the error in a positive direction rather than a negative direction like the transit and compass rules and placed a -0.376 ft. adjustment in the last course closing the traverse.

The Crandall Rule Adjustment. A discussion of the Crandall Rule Adjustment is presented in Appendix B.

Table 5. Transit Rule Adjustment of the East-West traverse.

Point on Traverse	Adjusted Angle ° ' "	Course Length (feet)	Course Latitude (feet)	Per Cent of Total Latitude	Latitude Adjustment (feet)	Adjusted Latitude (feet)	Course Departure (feet)	Per Cent of Total Departure	Departure Adjustment (feet)	Adjusted Departure (feet)
1	87 50 00	592.17	-50.385	5.2	0.029	-50.356	590.022	22.5	-0.060	589.962
2	246 8 41	303.36	-286.886	30.0	0.162	-286.724	98.607	3.8	-0.010	98.597
3	82 41 43	199.66	88.357	9.2	0.050	88.407	179.044	6.8	-0.018	179.026
4	240 34 6	536.57	-302.368	31.6	0.170	-302.198	443.262	16.9	-0.046	443.216
5	139 24 37	240.38	26.369	2.7	0.015	26.384	238.929	9.1	-0.025	238.904
6	187 6 58	280.40	-3.984	0.4	0.001	-3.983	280.371	10.7	-0.029	280.342
7	191 45 56	539.24	-117.493	12.2	0.066	-117.427	526.285	20.1	-0.054	526.231
8	165 50 31	181.98	5.010	0.5	0.001	5.011	181.911	7.0	-0.019	181.892
9	137 38 4	113.32	78.612	8.2	0.044	78.656	81.618	3.1	-0.008	81.610
10	132 19 10									
Algebraic Totals			-562.768		0.538	-562.230	2620.049		-0.269	2619.780
Arithmetic Totals		2987.08	959.464	100.0			2620.049	100.0		

Table 6. Compass Rule Adjustment of the East-West traverse.

Point on Traverse	Adjusted Angle ° ' "	Course Length (feet)	Per Cent of Total Length	Course Latitude (feet)	Latitude Adjustment (feet)	Adjusted Latitude (feet)	Course Departure (feet)	Departure Adjustment (feet)	Adjusted Departure (feet)
1	87 49 33								
		592.17	19.8	-50.385	0.106	-50.279	590.022	-0.053	589.969
2	246 9 43								
		303.36	10.1	-286.886	0.054	-286.832	98.607	-0.026	98.581
3	82 41 22								
		199.66	6.7	88.357	0.036	88.393	179.044	-0.018	179.026
4	240 34 17								
		536.57	18.0	-302.368	0.097	-302.271	443.262	-0.049	443.213
5	139 23 50								
		240.38	8.0	26.369	0.043	26.412	238.929	-0.022	238.907
6	187 6 46								
		280.40	9.4	-3.984	0.051	-3.933	280.371	-0.025	280.346
7	191 46 19								
		539.24	18.1	-117.493	0.098	-117.395	526.285	-0.049	526.236
8	165 50 9								
		181.98	6.1	5.010	0.033	5.043	181.911	-0.017	181.894
9	137 39 7								
		113.32	3.8	78.612	0.020	78.632	81.618	-0.010	81.608
10	132 18 42								
Algebraic Totals				-562.768	0.538	-562.230	2620.049	-0.269	2619.780
Arithmetic Totals		2987.08	100.0						

Table 7. Least Squares Distance Adjustment of the East-West traverse.

Point on Traverse	Adjusted Angle ° ' "	Course Length (feet)	Course Latitude (feet)	Latitude Adjustment (feet)	Adjusted Latitude (feet)	Course Departure (feet)	Departure Adjustment (feet)	Adjusted Departure (feet)
1	87 49 48							
		592.17	-50.385	0.059	-50.326	590.022	0.002	590.024
2	246 8 50							
		303.36	-286.886	0.021	-286.865	98.607	0.043	98.650
3	82 41 58							
		199.66	88.357	0.039	88.396	179.044	-0.017	179.027
4	240 33 48							
		536.57	-302.368	0.118	-302.250	443.262	0.065	443.327
5	139 24 6							
		240.38	26.369	0.063	26.432	238.929	-0.006	238.923
6	187 6 46							
		280.40	-3.984	0.070	-3.914	280.371	0.001	280.372
7	191 46 20							
		539.24	-117.493	0.120	-117.373	526.285	0.020	526.305
8	165 50 28							
		181.98	5.010	0.030	5.040	181.911	-0.001	181.910
9	137 39 24							
		113.32	78.612	0.018	78.630	81.618	-0.376	81.242
10	132 18 20							
Algebraic Totals			-562.768	0.538	-562.230	2620.049	-0.269	2619.780

Table 8. Angle and Distance Adjustments of the East-West traverse.

Point on Traverse	Angle °	'	"	Angle Adjustment (sec.) Transit Rule	Compass Rule	Least Squares	Course Length (feet)	Distance Adjustment (feet) Transit Rule	Compass Rule	Least Squares
1	87	50	8	-8	-35	-21				
							592.17	-0.062	-0.061	-0.004
2	246	9	2	-21	40	-12				
							303.36	-0.154	-0.060	-0.006
3	82	42	8	-25	-47	-11				
							199.66	0.006	0.000	0.001
4	240	33	56	10	21	-8				
							536.57	-0.132	-0.093	-0.013
5	139	24	8	29	-18	-2				
							240.38	-0.022	-0.016	0.000
6	187	6	44	14	2	2				
							280.40	-0.028	-0.026	0.000
7	191	46	14	-19	5	6				
							539.24	-0.066	-0.068	-0.006
8	165	50	14	16	-5	13				
							181.98	-0.018	-0.015	0.000
9	137	39	8	-64	-2	16				
							113.32	0.024	0.007	0.001
10	132	18	2	68	39	17				
Arithmetic Totals				274	214	108		0.512	0.346	0.031

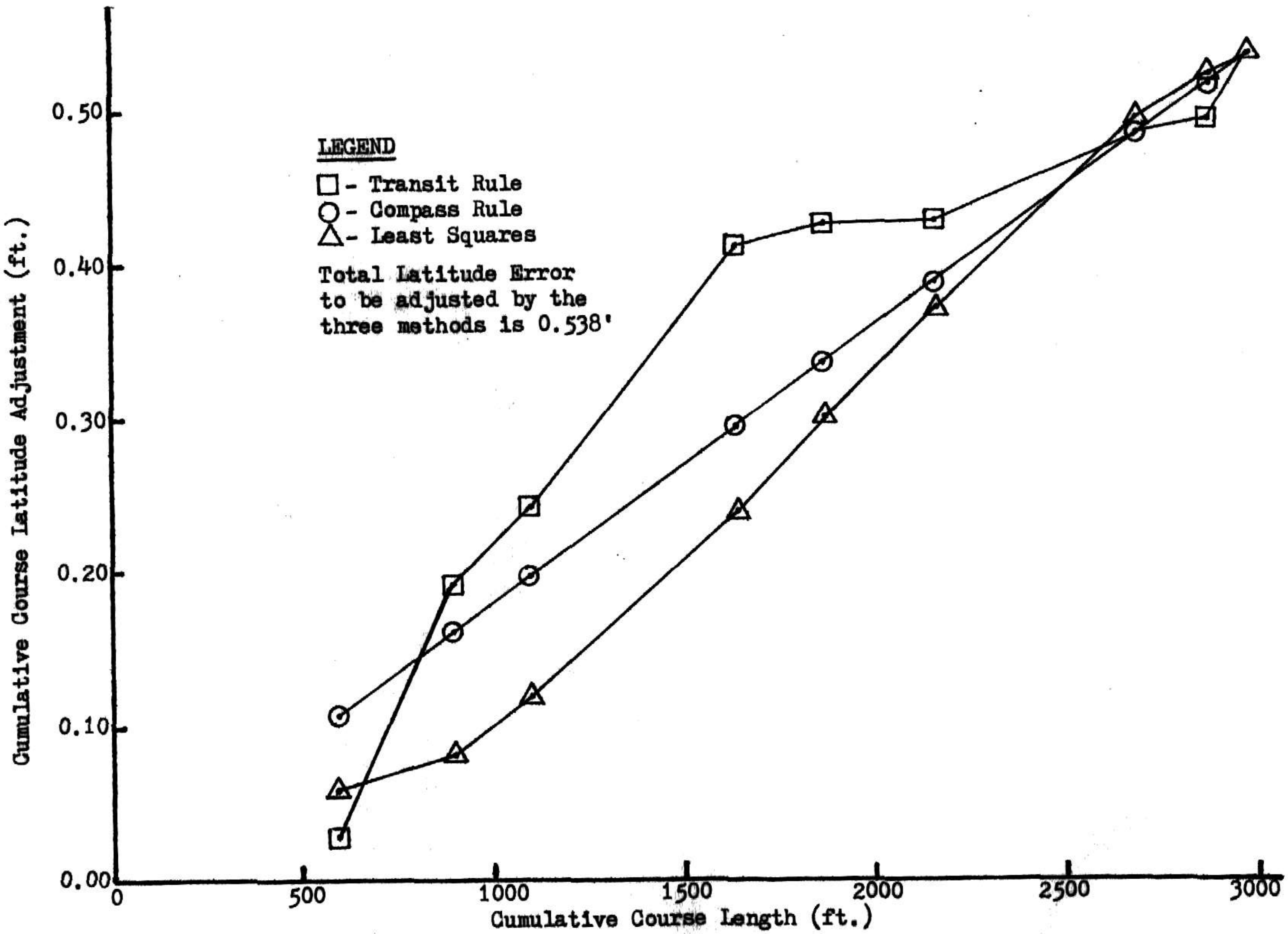

Fig. 6. Cumulative Course Length vs. Cumulative Course Latitude Adjustment for the East-West traverse.

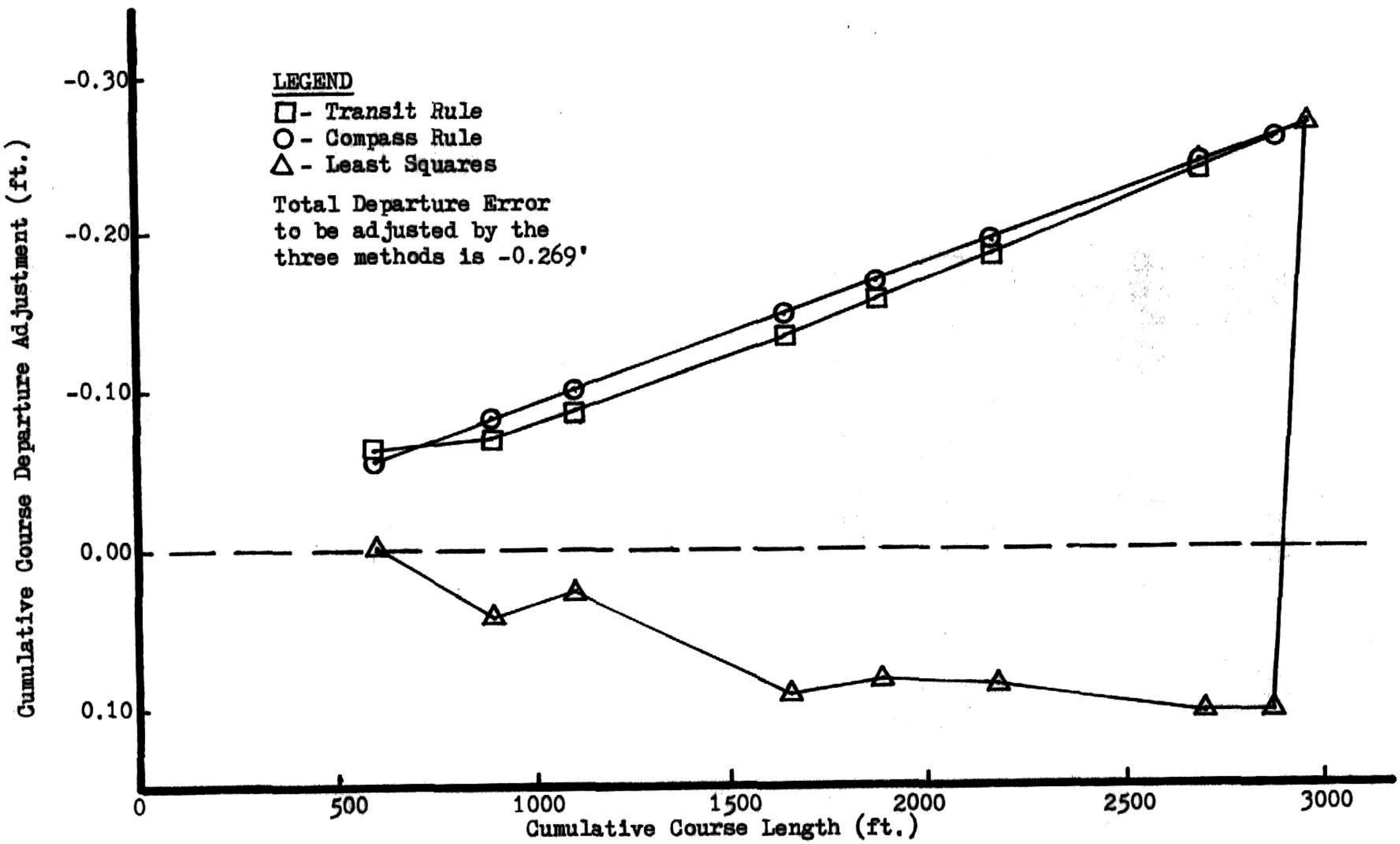

Fig. 7. Cumulative Course Length vs. Cumulative Course Departure Adjustment for the East-West traverse.

CONCLUSIONS

After analyzing the distance and angular adjustments caused by the latitude and departure adjustments, adjusted by the three methods, the method of least squares showed the smallest adjustments, both distance and angular, followed by the compass rule and the transit rule.

The various distance and angle adjustments could be expected after studying the adjustment rules;

The transit rule, which assumes angles are more accurate than distances, would yield large distance adjustments and relatively small angular adjustments. The angular adjustment is more erratic than the compass rule because the compass rule partitions equal proportions of latitude and departure to the same course. This is not true for the transit rule.

The compass rule, which assumes distances and angles to be of equal weight, produces distance and angle adjustments which lie between those obtained with the transit rule or with least squares.

The method of least squares, which makes no assumption about the angles and distances, yields small distance corrections and small angle corrections.

Several conclusions were obtained concerning the ease of computation of adjustments by the various methods. The transit rule and the compass rule adjustments are easily computed with a desk calculator. The adjustment by the method of least squares, being much more complicated, should be used only if an electronic computer is available.

The adjustment procedure to be used should depend on the use of the traverse. If the traverse is to be used as a control traverse, or if the difference in the reliability in the angles and distances is great, the method of least squares would be the best method.

FURTHER RESEARCH

After the analyses of the results obtained from the two traverses, questions were raised about the results that would be obtained for a traverse oriented diagonally with respect to the coordinate system. Unlike the two traverses studied which were extreme cases--one had a very large latitude and small departure and the other had a large departure and small latititude--the proposed research would study the case with relatively equal latitudes and departures. Two possible approaches which could be used to study this problem are; 1) a design similar to the one used in this report and 2) a design similar to the one described in the following paragraph. The second design would allow the experimenter to compare his results with a "true" known value.

An experimental design which would be utilized in a large number of experiments would involve establishing many points with accurately known coordinates. The coordinates could be used to obtain the accepted or "true" value of the variable studied which allows a comparison of the true value versus the observed value. Six experiments are listed below which could use this experimental design.

1. Study whether or not the angular corrections applied to the compass and transit rules are actually necessary. The adjustment of latitude and departure alone may yield angles and distances which are closer to the true angles and distances than the procedure which applies angular adjustments prior to performing the latitude and departure adjustment.

2. Study the differences obtained by using various least squares traverse adjustment techniques. The techniques which could be studied would include both observation equation techniques and condition equation techniques.

3. Study the adjustment of a traverse net, i.e. several interconnecting traverses, as compared to the adjustment of the various traverses individually. The traverse net adjustment is most easily handled by the method of least squares. The approach would be to compare both the least squares traverse net adjustment and the individual traverse adjustments with the true known adjustments.

4. Study the effect of a net traverse adjustment on traverses which were previously adjusted by simple traverse adjustment procedures. Although this procedure is seldom used, it is used when adjusting the net in hopes of forming a homogeneous unit through the use of the additional condition equations of the net adjustment.

5. Study the relationship between the weights of angles and distances. The relationship of the weight is well-known for the angles and for the distances, but the relationship between the angles and distances which are used in traversing is still only vaguely known. The approach would be to observe the variables by various techniques and then to divide the total error into components which are parallel and perpendicular to the line of sight and which represent the errors due to distance and angle respectively. A comparison and correlation of these data could then be performed.

6. Study whether using a weighted compass or transit rule adjustment would give adjustments which are closer to the true values.

REFERENCES

1. Adams, Oscar S., "Some Elementary Examples of Least Squares," Serial No. 250, U.S. Dept. of Commerce, U.S. Coast and Geodetic Survey, 1924.

2. Bouchard, Harry, and Moffitt, Francis H., Surveying, International Textbook Com., Scanton, Pennsylvania, 1962.

3. Breed, Charles B. and Hosmer, George L., Elementary Surveying, John Wiley and Sons, Inc., New York, 1958.

4. Gale, L.A. "Theory of Adjustment by Least Squares," The Canadian Surveyor, Vol. 19, 1965, pp.42-63.

5. Goussinsky, B., "Bowditch Method of Adjusting Transit Traverses--A Justification," Journal, American Congress on Surveying and Mapping, Dec., 1948.

6. Linnik, Yu, Method of Least-Squares and Principles of the Theory of Observations (translated from Russian), Pergamon Press, New York 1961.

7. Madkour, M.F., "Precision of Adjusted Variables by Least Squares," Journal, Surveying and Mapping Division, ASCE, Sept., 1968.

8. Meyer, H. Arthur, "Application of Statistical Methods to Surveying Problems," Journal, ACSM, Dec., 1954.

9. Rainsford, Hume F., Survey Adjustments and Least Squares, Constable and Com., Ltd., London, 1957.

10. Richardus, P., Project Surveying, North Holland Publishing Com., Amsterdam, 1966

11. Schmid, Hellmut and Schmid, Erwin, "A Generalized Least Squares Solution for Hybrid Measuring Systems," The Canadian Surveyor, Vol. 19, 1965, pp. 22-41.

12. Thomas, George B., Calculus and Analytic Geometry, Addison- Wesley Publishing Com., Inc., Reading, Mass., 1960.

13. Vreeland, R. P., "Adjustment of Traverses," Journal, Surveying and Mapping Division, ASCE, Oct., 1969.

14. Wolf, Paul R., "Horizontal Position Adjustment," Journal, ACSM, Dec., 1969.

APPENDIX A

DEVELOPMENT OF THE LEAST SQUARES EQUATIONS

Considering the earlier discussion of conditions, this discussion of the method of least squares covers the condition equation or correlate method as presented in the class, Adjustment of Survey. This method forms adjustments in a manner such that the adjustments meet the conditions

$$Az_{0,1} + \sum_{i=1}^{n}\theta_i = Az_{n,n+1} \pm n\ 180° \qquad 1)$$

$$\sum_{i=1}^{n-1} d_{i,i+1}\ \mathrm{Sin}(Az_{i,i+1}) = X_n - X_1 \qquad 2)$$

$$\sum_{i=1}^{n-1} d_{i,i+1}\ \mathrm{Cos}(Az_{i,i+1}) = Y_n - Y_1 \qquad 3)$$

while maintaining the condition that the sum of the adjustments squared be a minimum.

As states in the review of the literature Gale (4) and Madkour (7) present discussions concerning the condition equation method using this notation for the condition:

$${}_M B^*_Q\ {}_Q X_1 + {}_M C_1 = 0 \qquad 4)$$

where

X = A column vector of the unknowns which is equal to the sum of the column vector of the observed values (L) and a column vector of the corrections (V)

B = A matrix containing the partial derivatives of the unknowns (X)

C = A column vector of constants

M = Number of conditions (normally three)

Q = Number of variables (i.e. the number of angles and distances measured)

Introducing Lagrangian Multipliers, $2\ {}_1K^*_M$, the Gaussian Function may be written as

$$\Phi = {}_1V^*_Q\ {}_QP_Q\ {}_QV_1 - 2\ {}_1K^*_M\ ({}_MB^*_Q\ {}_QX_1 + {}_MC_1) \qquad 5)$$

where

V = A column vector of the adjustments

P = The weight matrix (discussed later)

Differentiating 5) with respect to X to obtain a minimum yields

$$\frac{d\Phi}{dX} = 2\ {}_1V^*_Q\ {}_QP_Q - 2\ {}_1K^*_M\ {}_MB^*_Q \overset{\text{set}}{=} 0 \qquad 6)$$

Solving the normal equations, 4) and 6), simultaneously

$${}_MK_1 = -({}_MB^*_Q\ {}_QP^{-1}_Q\ {}_QB_M)^{-1}\ ({}_MB^*_Q\ {}_QL_1 + {}_MC_1) \qquad 7)$$

$${}_QX_1 = {}_QL_1 + {}_QV_1 = {}_QL_1 + {}_QP^{-1}_Q\ {}_QB_M\ {}_MK_1 \qquad 8)$$

where

$^{-1}$ = The inverse of the matrix

The vectors X, L, and V are all the same size, Q x 1. X is a column vector of the adjusted values of each of the variables $d_{1,2}$, $d_{2,3}$, . . . , $d_{n-1,n}$, θ_1, θ_2, . . . , and θ_n. It is equal to the observed values l_1, l_2, . . . , and l_{2n-1}, which are referred to as the L vector, plus the adjustments to the observed values v_1, v_2, . . . , and v_{2n-1}, which is referred to as the V vector. Written in algebraic form they appear as

$$\begin{array}{lclcl}
d_{1,2} & = & l_1 & + & v_1 \\
d_{2,3} & = & l_2 & + & v_2 \\
\cdot & & \cdot & & \cdot \\
d_{n-1,n} & = & l_{n-1} & + & v_{n-1} \\
\theta_1 & = & l_n & + & v_n \\
\cdot & & \cdot & & \cdot \\
\theta_{2n-1} & = & l_{2n-1} & + & v_{2n-1}
\end{array} \qquad 9)$$

The B matrix is composed of the partial derivatives of the variables for each of the three traverse conditions--the total sum of which for any one condition, is equal to the total differential (12).

$$_3B_{2n-1} = \begin{bmatrix} \frac{\partial(1)}{\partial d_{1,2}} & \frac{\partial(1)}{\partial d_{2,3}} & \cdots & \frac{\partial(1)}{\partial d_{n-1,n}} & \frac{\partial(1)}{\partial \theta_1} & \cdots & \frac{\partial(1)}{\partial \theta_n} \\ \frac{\partial(2)}{\partial d_{1,2}} & \frac{\partial(2)}{\partial d_{2,3}} & \cdots & \frac{\partial(2)}{\partial d_{n-1,n}} & \frac{\partial(2)}{\partial \theta_1} & \cdots & \frac{\partial(2)}{\partial \theta_n} \\ \frac{\partial(3)}{\partial d_{1,2}} & \frac{\partial(3)}{\partial d_{2,3}} & \cdots & \frac{\partial(3)}{\partial d_{n-1,n}} & \frac{\partial(3)}{\partial \theta_1} & \cdots & \frac{\partial(3)}{\partial \theta_n} \end{bmatrix} \qquad 10)$$

In the case of the traverse, the formulation of the B matrix becomes quite complicated. For this reason, some space will be devoted to describing its formulation. The partial derivatives will be taken with respect to the 2n-1 variables $d_{1,2}$, $d_{2,3}$, . . . , $d_{n-1,n}$, θ_1, θ_2, . . . , and θ_n.

Referring to the angular condition, 1), the partial derivatives are as follows;

$$\frac{\partial(1)}{\partial d_{1,2}} = 0$$

$$\cdots \qquad 11)$$

$$\frac{\partial(1)}{\partial d_{n-1,n}} = 0$$

$$\frac{\partial(1)}{\partial \theta_1} = 0$$

$$\cdots \qquad 12)$$

$$\frac{\partial(1)}{\partial \theta_n} = 0$$

The partial derivatives of the departure condition, 2), with respect to the distances $d_{1,2}$, $d_{2,3}$, . . . , and $d_{n-1,n}$ are

$$\frac{\partial(2)}{\partial d_{1,2}} = \mathrm{Sin}(Az_{1,2})$$

$$\frac{\partial(2)}{\partial d_{2,3}} = \mathrm{Sin}(Az_{2,3}) \qquad 13)$$

$$\cdot \quad \cdot \quad \cdot$$

$$\frac{\partial(2)}{\partial d_{n-1,n}} = \mathrm{Sin}(Az_{n-1,n})$$

The partial derivatives of this condition with respect to the angles θ_1, θ_2, . . . , and θ_n are more difficult to obtain since the angles have been converted into azimuths. For this reason the azimuth will be changed to show the true function to be differentiated.

$$X_n - X_1 = d_{1,2}\mathrm{Sin}(Az_{0,1} + \theta_1 \pm 180°) + d_{2,3}\mathrm{Sin}(Az_{0,1} + \theta_1 + \theta_2 \pm 360°)$$
$$+ \ . \ . \ . + d_{n-1,n}\mathrm{Sin}(Az_{0,1} + \theta_1 + \theta_2 + \ . \ . \ . + \theta_n \pm n\ 180°)$$

Differentiating 14) obtains

$$\frac{\partial(2)}{\partial \theta_1} = d_{1,2}\mathrm{Cos}(Az_{0,1} + \theta_1 \pm 180°) + d_{2,3}\mathrm{Cos}(Az_{0,1} + \theta_1 + \theta_2 \pm 360°)$$
$$+ \ . \ . \ . + d_{n-1,n}\mathrm{Cos}(Az_{0,1} + \theta_1 + \theta_2 + \ . \ . \ . + \theta_{n-1} \pm n\ 180°)$$
$$= \sum_{i=1}^{n-1} d_{i,i+1}\mathrm{Cos}(Az_{i,i+1})$$

$$\frac{\partial(2)}{\partial \theta_2} = d_{2,3}\mathrm{Cos}(Az_{0,1} + \theta_1 + \theta_2 \pm 360°) + d_{3,4}\mathrm{Cos}(Az_{0,1} + \theta_1 + \theta_2 + \theta_3 \pm 540°)$$
$$+ \ . \ . \ . + d_{n-1,n}\mathrm{Cos}(Az_{0,1} + \theta_1 + \theta_2 + \ . \ . \ . + \theta_{n-1} \pm n\ 180°)$$
$$= \sum_{i=2}^{n-1} d_{i,i+1}\mathrm{Cos}(Az_{i,i+1}) \qquad 15)$$

$$\frac{\partial(2)}{\partial \theta_{n-1}} = d_{n-1,n}\text{Cos}(Az_{0,1} + \theta_1 + \theta_2 + \ldots + \theta_{n-1} \pm n\ 180^\circ)$$

$$= \sum_{i=n-1}^{n-1} d_{i,i+1}\text{Cos}(Az_{i,i+1})$$

$$\frac{\partial(2)}{\partial \theta_n} = 0$$

Applying the same procedure to the latitude condition, 3) as to the departure condition, 2), the following partial derivatives are obtained.

$$\frac{\partial(3)}{\partial d_{1,2}} = \text{Cos}(Az_{1,2})$$

$$\cdot \quad \cdot \quad \cdot \qquad 16)$$

$$\frac{\partial(3)}{\partial d_{n-1,n}} = \text{Cos}(Az_{n-1,n})$$

$$\frac{\partial(3)}{\partial \theta_1} = -\sum_{i=1}^{n-1} d_{i,i+1}\text{Sin}(Az_{i,i+1})$$

$$\frac{\partial(3)}{\partial \theta_2} = -\sum_{i=2}^{n-1} d_{i,i+1}\text{Sin}(Az_{i,i+1})$$

$$\cdot \quad \cdot \quad \cdot \qquad 17)$$

$$\frac{\partial(3)}{\partial \theta_{n-1}} = -\sum_{i=n-1}^{n-1} d_{i,i+1}\text{Sin}(Az_{i,i+1})$$

$$\frac{\partial(3)}{\partial \theta_n} = 0$$

The vector C is composed of the constants required to make each of the three conditions consistent. The constant for the angular condition would be $(Az_{n,n+1} - Az_{0,1} \pm n\ 180^\circ)$ expressed in radians, whereas, the constants for the latitude and departure conditions would be $(Y_n - Y_1)$ and $(X_n - X_1)$, respectively.

Thus far the X, L, V, and C vectors and the B matrix have been discussed. The real problem with the traverse adjustment by least squares is the weighting of the different variables. Unlike the compass rule and transit rule which adjust the angular error separately from the latitude and departure errors, the method of least squares adjusts all of these errors simultaneously. This simultaneous solution must somehow distinguish between traverses obtained by instrumentation and techniques which secure the same relative precision in the angular measurements and distance measurements as compared to those which secure different precisions.

The differences in precision of angular and distance measurements have presented a major problem to professional surveyors for years. Vreeland (13) presents the following list of instrumentation and techniques together with the range of precision which one may expect from their use.

Angles	Range of Precision
Compass	15' to 1°
Transit	5" to 1'
Optical Theodolite	0.1" to 5"
Distances	
Taping	1/100,000 to 1/1,000
Stadia	1/ 1,500 to 1/ 100
Subtense	1/ 10,000 to 1/1,000
Electronic	1/300,000 to 1/5,000

The professional surveyor formulated the concept of relative weights to correlate these differences. The reasoning was than angle obtained by a 30" transit had approximately the same precision as that of an angle repeated twice with a 1' transit, hence the weight of 2 was given to the 30" transit.

The same concept was applied to the distances. It was not until the method of least squares came into prominent use that the weight relationship between angles and distances became important.

Initially the "old timer" was to assign the relative weights as he felt his experience dictated. Today, the concept of weights has taken on a more statistical approach. Surveyors now agree that the observation relative weights p_1, p_2, . . . , through p_n are related to the inverse of the observation variances s_1^2, s_2^2, . . . , and s_n^2. Rainsford (9) presents a mathematical discussion proving this relationship while Wolf (14) presents whese formulas to obtain the relative precisions (weights) for use in the diagonal matrix, P. Since relative weights are being obtained, each formula is multiplied by the common factor of 1000 in order to obtain reative weights closer to 1.0.

A. For Taped courses

$$N = 1000\left[\sqrt{n}\,(\pm e_t)\right]^2 \qquad 18)$$

where

n = The number of tape lengths in the course

e_t = The appropiate standard error per tape length

B. For Electronically measured courses

$$N = 1000\left[(\pm L \div 200{,}000) \pm 0.04\right]^2$$

where

L = The course length in feet

C. For Direction observations

$$N = 1000\left[\ L(\pm e_a) \div 206{,}265\right]^2$$

where

e_a = The appropiate standard error of the angle

The appropiate values of e_t for taped distances and e_a for angles may be estimated from previous work of a certain order for the various field conditions and for various equipment-operator combinations. Wolf presents the nomogragh for relative weight inverses shown in Fig. 8 in an effort to simplify the process of obtaining the relative weights.

"The use of the nomograph is illustrated in the following example: Assume that a course 2600 feet long is taped using procedures estimated to produce an e_t of ±0.01 foot. Enter the nomograph at 2600 feet on the abscissa, move vertically to intersect the line of e_t = ±0.01 foot, then horizontally to the ordinate, read N = 2.6. Inverse weight numbers from the nomograph are the inverses of the elements of the P matrix." (14)

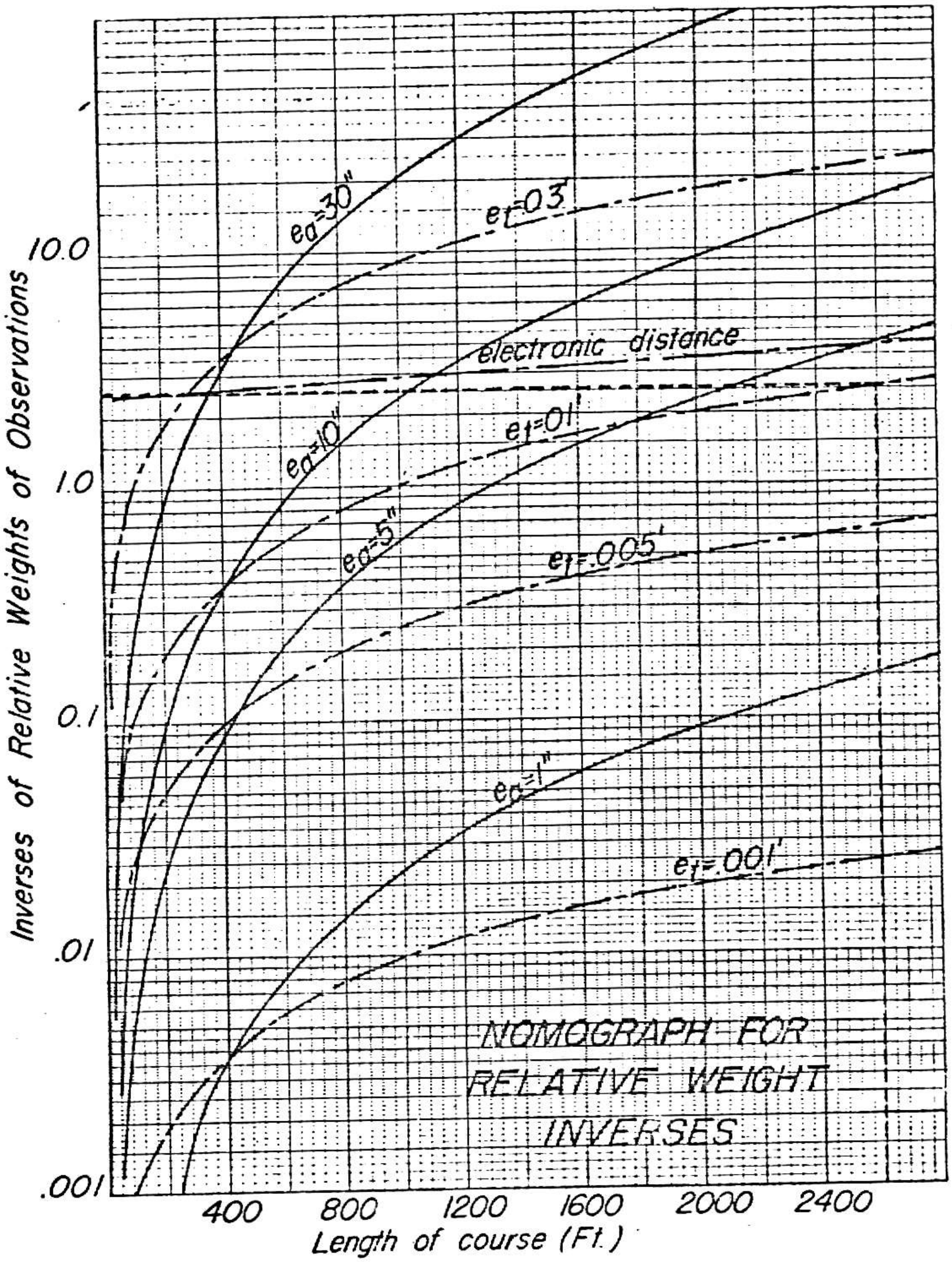
10.0
1.0
0.1
.01
.001
Inverses of Relative Weights of Observations
400
800
1200
1600
2000
2400
Length of course (Ft.)
ea=30"
et=.03'
electronic distance
ea=10"
et=.01'
ea=5"
et=.005'
ea=1"
et=.001'
NOMOGRAPH FOR
RELATIVE WEIGHT
INVERSES

Figure 8

APPENDIX B

CRANDALL RULE ADJUSTMENT

As the data were being prepared for use in the ICES COGO--Geometric Processor it was learned that another error adjustment procedure, the Crandall Rule, was contained in the program package. This method may be used on electronic computers where the computer memory bank is too small to use the method of least squares. The Crandall rule, which requires the use of adjusted angles, holds the bearing of the course as correct and adjusts only the distances by solving the latitude and departure conditions simultaneously.

Table 9 shows the computations of the latitude and departure adjustments for the north-South traverse. There were no angle adjustments and the distance adjustments are tabulated in Table 10. The distance adjustments to courses 18-19 and 25-26 are 0.217 ft. and -0.226 ft., respectively, These adjustments are larger than most surveyors and engineers would accept as possible. Such an adjustment is likely attributable to field measurement error and would require remeasuring part, or all, of the traverse in order to correct a likely error which would cause such a large adjustment. Fig. 9 shows the graphs of the departure adjustment which indicates that the Crandall rule strongly resembles the transit rule.

Table 11 shows the computations of the latitude and departure adjustments for the East-West traverse. The distance adjustments are tabulated in Table 12 and again several of these distance adjustments, -0.254 ft. and -0.399 ft., are highly unlikely. From Fig. 10 which shows the graphs of the latitude adjustment one can easily see the resemblance between the Crandall rule and the transit rule but when studying Fig. 11 which shows the graphs of the departure adjustment no resemblance may be seen to the other three rules.

The Crandall rule should only be used when the angles have been measured much more accurately than the distances. Otherwise it appears to be an adjustment rule to be used by an unknowing surveyor in order to say the traverse was adjusted on an electronic computer.

Table 9. Crandall Rule Adjustment of the North-South traverse.

Point on Traverse	Adjusted Angle ° ' "	Course Length (feet)	Course Latitude (feet)	Latitude Adjustment (feet)	Adjusted Latitude (feet)	Course Departure (feet)	Departure Adjustment (feet)	Adjusted Departure (feet)
27	189 23 30	256.16	69.867	-0.032	69.835	246.448	-0.115	246.333
26	256 8 6	481.76	-418.501	0.196	-418.305	238.640	-0.112	238.528
25	209 1 36	290.02	-290.000	0.005	-289.995	3.373	0.000	3.373
24	179 16 00	250.05	-249.975	0.005	-249.970	6.100	0.000	6.108
23	179 59 54	250.01	-249.935	0.005	-249.930	6.115	0.000	6.115
22	179 59 54	100.00	-99.970	0.001	-99.969	2.449	0.000	2.449
21	179 59 54	149.98	-149.935	0.002	-149.933	3.677	0.000	3.677
20	179 59 54	249.98	-249.905	0.006	-249.899	6.137	0.000	6.137
19	205 24 30	541.75	-494.907	-0.198	-495.105	-220.363	-0.088	-220.451
18	215 39 12	124.69	-62.994	-0.013	-63.007	-107.607	-0.022	-107.629
5	215 18 30	217.45	18.811	0.009	18.820	-216.635	-0.089	-216.724
17	114 10 00	71.87	-62.780	-0.004	-62.784	-34.985	-0.003	-34.988
16	149 38 36	199.08	-199.034	0.003	-199.031	4.263	0.000	4.263
15	180 2 36	213.96	-213.914	0.003	-213.911	4.420	0.000	4.420
14	158 59 6	17.88	-16.555	0.000	-16.555	6.755	0.000	6.755
13	201 6 12	248.12	-248.075	0.005	-248.070	4.745	-0.001	4.744
12	212 32 42	15.21	-12.976	-0.005	-12.981	-7.936	-0.004	-7.940
11	61 49 54							
Algebraic Totals			-2930.778	-0.012	-2930.790	-54.396	-0.434	-54.830

Table 10. Computation of the Distance Adjustment for the North-South traverse.

Point on Traverse	Measured Distance (feet)	Distance Adjustment (feet)	Adjusted Distance (feet)
27			
	256.16	-0.119	256.041
26			
	481.76	-0.226	481.534
25			
	290.02	-0.005	290.015
24			
	250.05	-0.005	250.045
23			
	250.01	-0.005	250.005
22			
	100.00	-0.001	99.999
21			
	144.98	-0.002	149.978
20			
	249.98	-0.005	249.975
19			
	541.75	0.217	541.967
18			
	124.69	0.025	124.715
5			
	217.45	0.090	217.540
17			
	71.87	0.005	71.875
16			
	199.08	-0.003	199.077
15			
	213.96	-0.004	213.956
14			
	17.88	0.000	17.880
13			
	248.12	-0.005	248.115
12			
	15.21	0.000	15.210
11			
Arithmetic Total		0.717	

Table 11. Crandall Rule Adjustment of the East-West traverse.

Point on Traverse	Adjusted Angle ° ' "	Course Length (feet)	Course Latitude (feet)	Latitude Adjustment (feet)	Adjusted Latitude (feet)	Course Departure (feet)	Departure Adjustment (feet)	Adjusted Departure (feet)
1	87 50 8							
		592.17	-50.385	-0.004	-50.389	590.022	0.052	590.074
2	246 9 9							
		303.36	-286.886	0.239	-286.647	98.607	-0.083	98.524
3	82 42 8							
		199.66	88.357	0.031	88.388	179.044	0.062	179.106
4	240 33 56							
		536.57	-302.368	0.224	-302.144	443.262	-0.330	442.932
5	139 24 8							
		240.38	26.369	0.005	26.374	238.929	0.043	238.972
6	187 6 44							
		280.40	-3.984	0.000	-3.984	280.371	0.029	280.400
7	191 46 14							
		539.24	-117.493	0.017	-117.476	526.285	-0.076	526.209
8	165 50 14							
		181.98	5.010	0.000	5.010	181.911	0.016	181.927
9	137 39 8							
		113.32	78.612	0.026	78.638	81.618	0.018	81.636
10	132 18 2							
Algebraic Totals			-562.768	0.538	-562.230	2620.049	-0.269	2619.780

Table 12. Computation of the Distance Adjustment for the East-West traverse.

Point on Traverse	Measured Distance (feet)	Distance Adjustment (feet)	Adjusted Distance (feet)
1			
	592.17	0.052	592.222
2			
	303.36	-0.254	303.106
3			
	199.66	0.068	199.728
4			
	536.57	-0.399	536.171
5			
	240.38	0.043	240.423
6			
	280.40	0.029	280.429
7			
	539.24	-0.078	539.162
8			
	181.98	0.016	181.996
9			
	113.32	0.031	113.351
10			
Arithmetic Total		0.970	

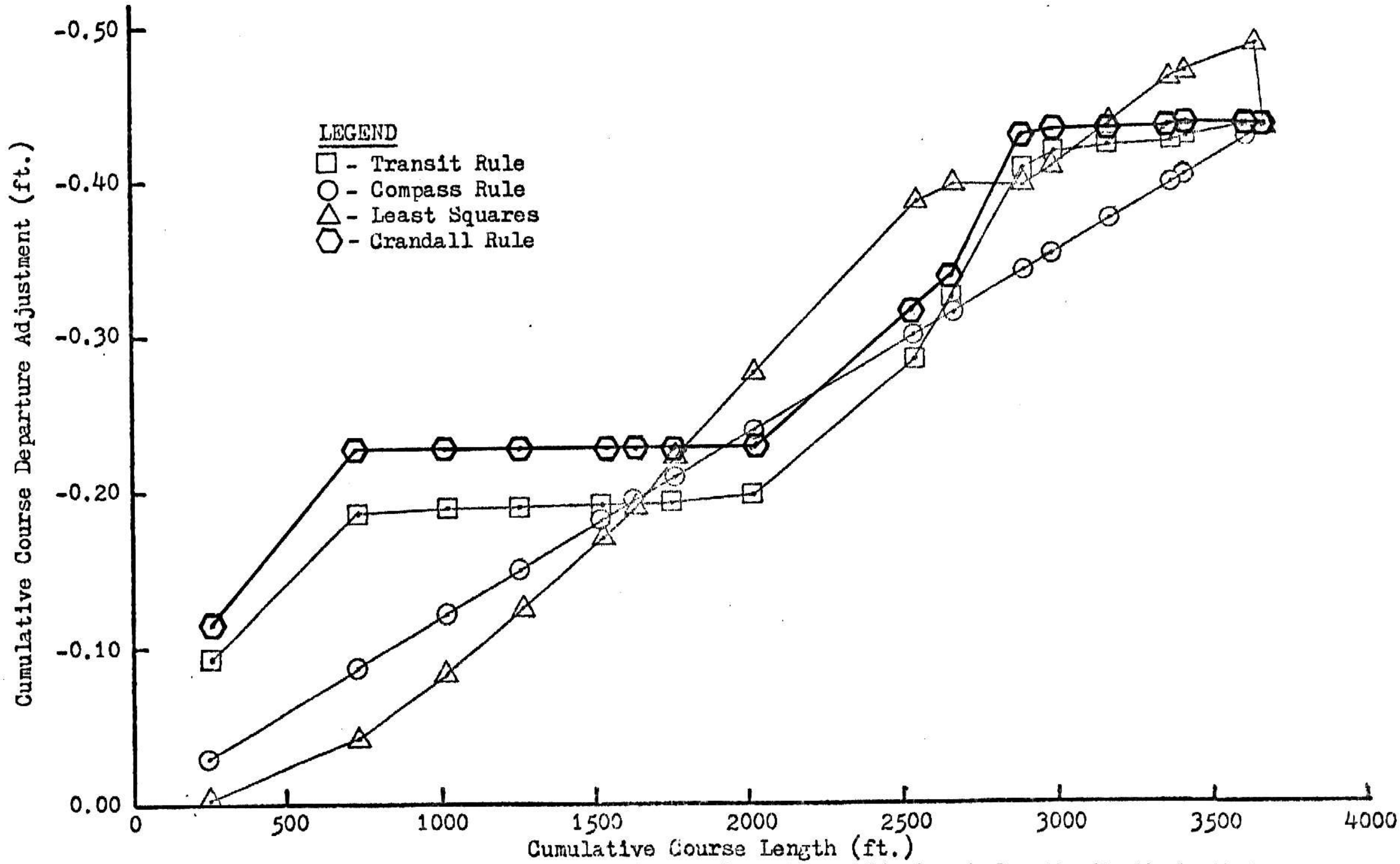

Fig. 9. Cumulative Course Length vs. Cumulative Course Departure Adjustment for the North-South traverse.

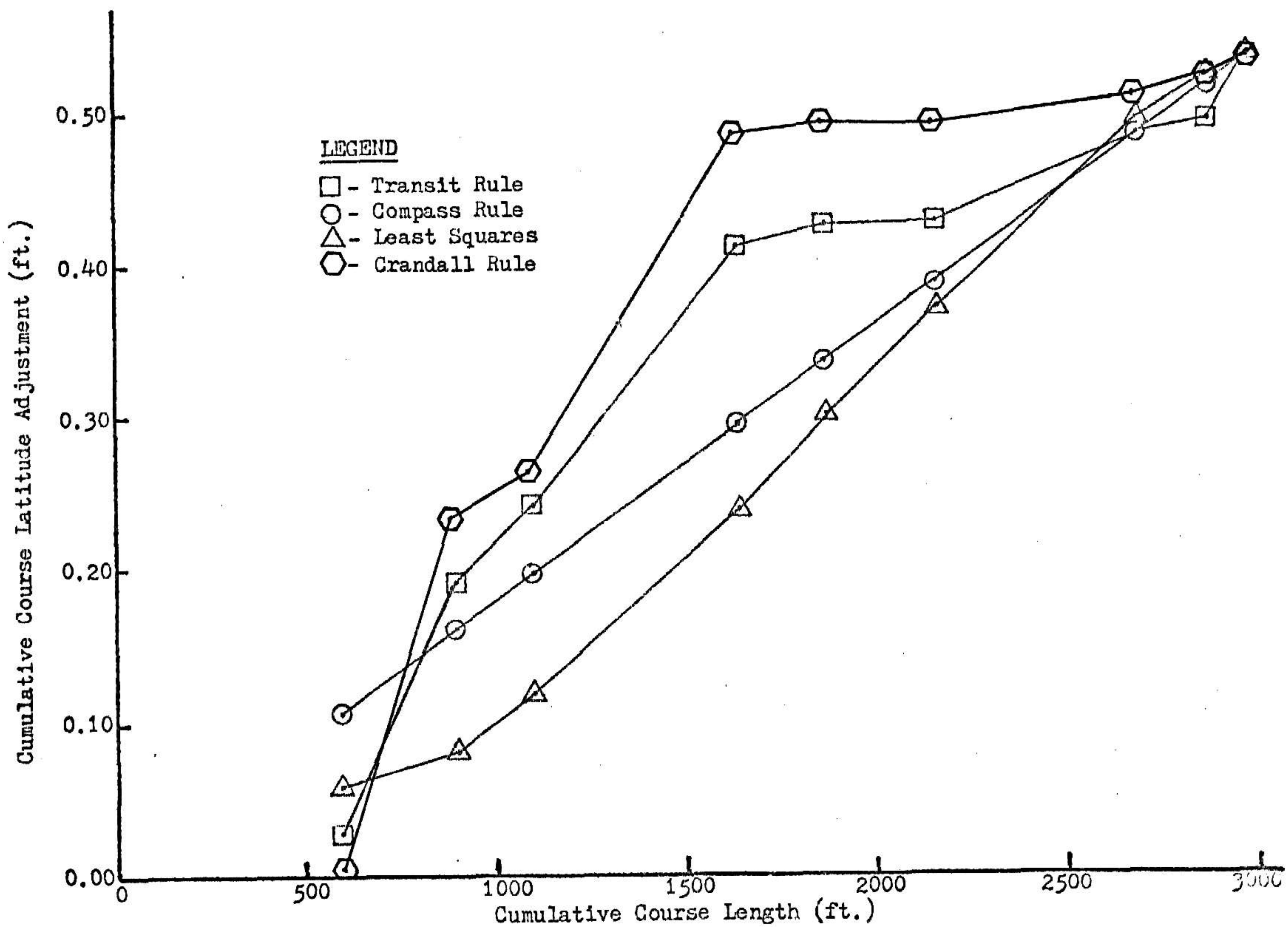

Fig. 10. Cumulative Course Length vs. Cumulative Course Latitude Adjustment for the East-West traverse.

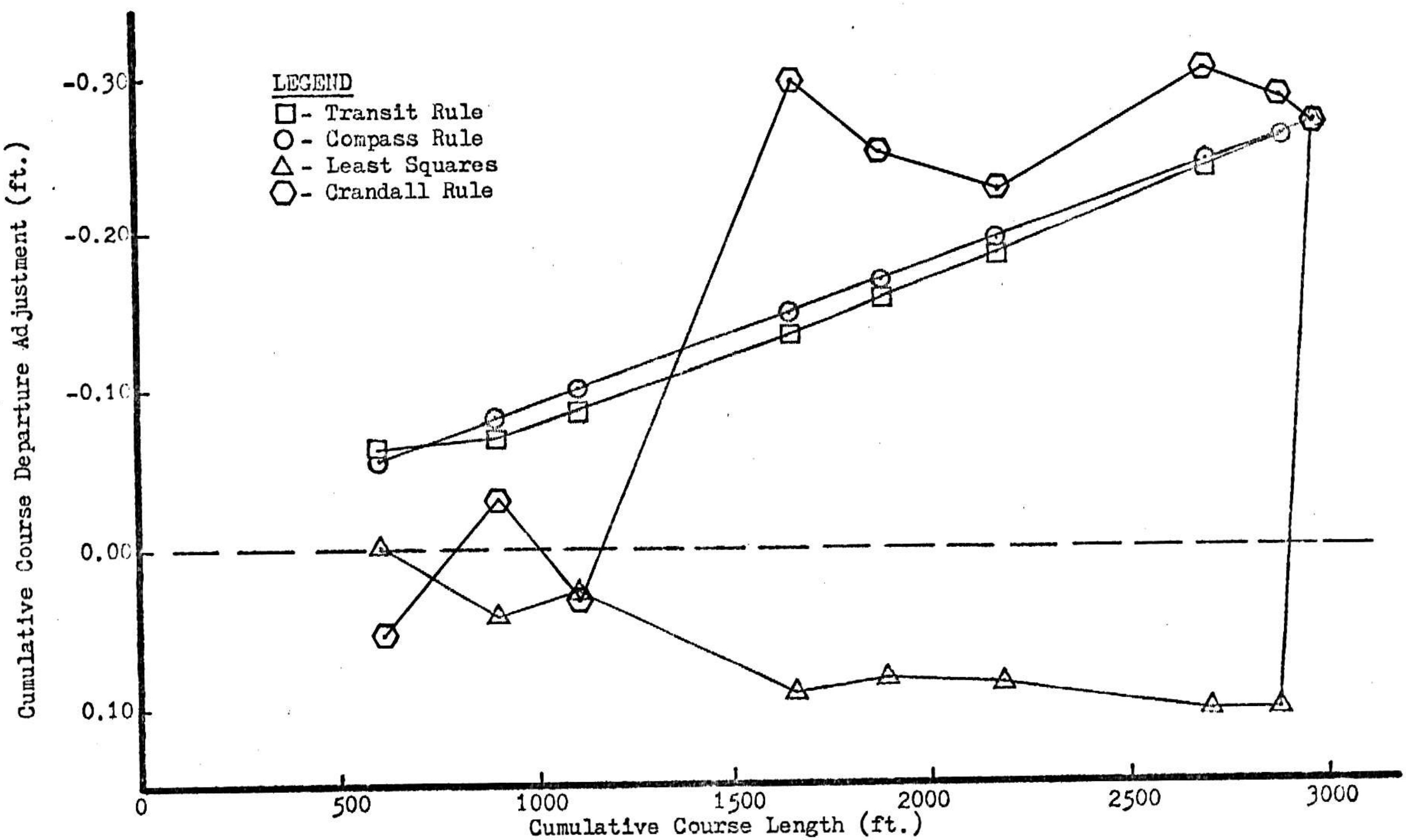

Fig. 11. Cumulative Course Length vs. Cumulative Course Departure Adjustment for the East-West traverse.

A COMPARISON OF ADJUSTMENTS ON TWO TRAVERSES BY THE TRANSIT RULE, COMPASS RULE AND METHOD OF LEAST SQUARES

by

STEVEN JAMES COHORST

B. S., Kansas State University, 1970

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1971

ABSTRACT

The purpose of this paper is to present an overview of multiprogramming with an emphasis on those components and methodologies found in contemporary multiprogramming systems. The properties, advantages, and disadvantages are presented, and the conditions multiprogramming must satisfy to be acceptable are discussed. The concepts and methodologies presented are the operating system, queueing, core allocation, and protection.

Some possible improvements in multiprogramming technology are noted. At this time multiprogramming seems to be the most reasonable way for general purpose computing systems to produce work efficiently.