A SIMULATION MODEL FOR
DEVELOPING THE TRIP LENGTH FREQUENCY DISTRIBUTION

by

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INTRODUCTION

A. Problem Statement

This paper deals with a simulation technique for developing the trip length frequency distribution (abbreviated TLFD).

The trip length frequency distribution is related to the development of F-Factors (the traveltime factors) which express the effect that spatial separation exerts on trip interchange[1]. In the gravity model calibration procedure[2], the development of a well accepted set of F-Factors involves an iterative process of comparison between the actual and calculated TLFD[3]. The calculated TLFD is an output of the gravity model calibration process[4]. The actual TLFD is derived from the trip table using the Origin-Destination (O-D) survey data[5]. Without this actual TLFD (i.e., without the O-D survey), the development of F-Factors can not be done.

There are two typical situations where the gravity model calibration for trip distribution is conducted with no O-D information for F-Factor development. One is the calibration of future trip interchanges and the other is the calibration for some small urban areas where the comprehensive O-D survey is too costly to obtain.[6] For the former case, the existing F-Factors at the base year are used for the calibration of future trip interchanges. This assumes that F-Factors remain constant over time and will not be affected by the future improvement of transportation service, the increasing travel demand, or the changing patterns of land use development[7]. For the latter case, the entire travel study is based upon the synthesis of other cities' information, a method called the synthetic travel study technique[8]. The basis of synthesizing the F-Factors is that cities having similar physical patterns have similar spatial separation effects[9].

However, neither of these two assumptions stated above is considered adequate. The constancy of F-Factors over time is contrary to the basic transportation planning principles.[10] The current synthetic method for F-Factors is naive and lacks theoretical support[11]. Consequently, it is technically necessary to simulate a proper TLFD which is capable of reflecting the existing or future travel patterns for the development of Factors.

B. The Objective and Scope of the Study

The objective of this paper is to develop a systematic procedure for simulating the TLFD which is capable of reflecting the changing impact of urban future and existing structure upon travel patterns. In this paper, analyses are restricted to the home-based work trip. Other trip purposes such as shopping, social, and non-home based trips are beyond the scope of study.

The applicability of the simulation model developed in this paper, which is primarily based on Voorhees' research on work trip length, is designed for small urban areas. The sample data are collected from those cities with population under 100,000.

C. Contents

Part One presents a background review of (a) the gravity model theory and definitions of its parameters, (b) the iterative process of F-Factor development in the model calibration procedure, and (c) the definition of TLFD and its characteristics.

Part Two is the model development. Through an analysis of Voorhees' research on work trip length, a model to simulate the small city's TLFD is developed. This simulation model is developed on the basis of the following hypotheses:

(a) The variation of TLFD is highly associated with the level of urban transportation service, the city size and the land-use locational pattern.

(b) The TLFD can be described in terms of its mean (average trip length) and its variance. These two parameters can be estimated, according to Voorhees regression analysis, by the size of city population and the average network speed.

(c) The gamma distribution was introduced in Voorhees' paper to be the mathematical model for simulating the entire distribuiton of trip length. By taking the estimated mean and variance as the input parameters of the gamma density function, the calculated gamma distribution was found to fit the actual TLFD very well.

Part Three tests the simulation model developed in Part Two. The model test consists of the following works:

(a) Testing the statistical credibility of using the gamma distribution as the mathematical model for TLFD simulation.

(b) Equation generation for approximating the parameters of the gamma density function through regression analysis.

The simulation model is demonstrated for the city of Lawrence, Kansas. Using the generated equations, the mean and variance for the Lawrence area are approximated. These two estimated values are then used as the parameters in the calculation of the gamma density function.

PART ONE: BACKGROUND REVIEW

A. The Gravity Model Theory

The gravity model applies the Newtonian gravitational principle for the problem of urban trip distribution. Essentially, this model theorizes that the frequency of trip movement from zone i to zone j is directly proportional to the relative attractiveness of zone i and inversely proportional to some function of distance (or travel time) separating zone i from zone j[12]. This statement can be written in mathematical form as follows[13]:

$$T_{ij} = \frac{P_i A_j / t_{ij}^{x}}{\sum_{j=1}^{n} A_j / t_{ij}^{x}} \qquad (1.1)$$

where T_{ij} = trip movement from zone i to zone j;

P_i = trips produced by zone i;

A_j = trips attracted by zone j;

t_{ij} = travel time (or distance) from zone i to zone j;

x = a constant exponent; and

$1/t_{ij}^{x}$ = the friction factor or the traveltime function.

According to the gravity model theory, a trip produced at one zone will be pulled to the other zone when outside attractiveness exists. If such outside attractiveness comes from more than one zone, the probability of making a trip to a particular zone is determined by the combined effect of spatial friction between zones and the relative attraction of that destination zone competing with other possible zones. By multiplying $1/P_i$ through both sides of (1.1), the probability of trip making from zone i is derived as follows:

$$P(T_{ij}) = \frac{T_{ij}}{P_i} = \frac{A_j/t_{ij}^x}{\sum_{j=1}^{n} A_j/t_{ij}^x} \qquad (1.2)$$

where $P(T_{ij})$ = the probability of making a trip from zone i to zone j.

Note

$$\sum_{j=1}^{n} A_j/t_{ij}^x = \text{constant; and}$$

$$\sum_{j=1}^{n} T_{ij} = P_i$$

Hence

$$P(T_{ij}) = f(A_j\,,\ t_{ij}) \begin{cases} 0 & \text{as } A_j \rightarrow 0 \\ 1 & \text{as } A_j/t_{ij}^x \longrightarrow \sum_{j=1}^{n} A_j/t_{ij}^x \end{cases} \qquad (1.3)$$

Several assumptions are made under the gravity model theory[14].

They are:

(1) The gravity model is universally applicable to every zone in the urban area regardless of the socio-economic conditions which vary among zones.

(2) Traveltime functions are assumed to be applicable for each pair of zones regardless of the special geographical circumstances (i.e., circumferential or corridor distribution).

(3) Trip length distributions (by trip purposes) remain constant throughout the urban area (i.e., the exponent is independent of geographic location).

(4) Distance (or travel time) between zones remains constant and can be accurately determined for the particular time (day, year, etc.) period chosen.

B. Parameters of the Gravity Model Formula

There are 4 parameters in the gravity model formula.

(1) P_i and A_j

These two variables are measures of trip production and attraction on the zonal level. Usually the number of trips produced or the number of trips attracted by each traffic zone are related to the use of the land and to the socio-economic characteristics of the people who make trips.[15]

The gravity model distributes trips from production zone to attraction zone, while the other travel models in use distribute trips from origin zone to destination zone.[16] To distinguish their differences, it is first necessary to class all trips as home based or nonhome based. Home based trips always have one end at the residence of the trip maker. Nonhome based trip have neither end at the residence of the trip maker. Home based trips are always produced by the zone of residence of the trip maker whether the trip begins or ends in that zone. Home based trips are always attracted at the non-residential end of the trip. While nonhome based trips are always produced by the zone of origin and attracted by the zone of destination.

(2) Inter-zonal Travel Time, t_{ij}

In the early development of gravity model, straight line distance between zonal centroids was used for measuring the spatial friction between two points. However, it was later found that the impact of the spatial friction would be much accurately reflected if measured by travel time. This is because the friction between two points in an urban area may vary depending upon the modes, topographical situation, and the level of transportation service involved.

(4) Traveltime function

In the past, many arguments had been brought up about the true value of the constant x in the traveltime function. The historical evolution of this constant can be generally divided into three stages. In the early nineteen sixties, Voorhees contended that the exponent varies with trip purposes[17]. In general, the more important the activity, the smaller the value of the exponent. That is to say, the travel time would be a less restrictive factor for an activity which people are compelled to do. For example, a person has relatively less freedom to choose the location of a job than the location of the grocery store - one would go farther for work than for shopping. This phenomenon is reflected in the traveltime function by varying the exponent. For instance, to describe the traveltime function of a given trip purpose which is less restrictive, a smaller value of the exponent may be assigned so the value of $1/t_{ij}^{x}$ becomes larger.

A few years later, Voorhees concluded another fact that the exponent may not be constant as the travel time increases[18]. This is especially true when terminal time (additional traveltime such as the walking time from parking lot to office) is not taken into account. In addition, research has revealed that the effect of spatial separation upon tripmaking has a certain connection with the topographical barriers in an urban area[19]. The real reason(s) which cause the fluctuation of the traveltime function are still unknown. However, it is widely recognized that the simple traveltime function, $1/t^x$, does not represent the real effect of spatial separation upon trip-making which varies from city to city.

At the present time, the traditional traveltime function is replaced by some empirical values called the F-Factors. By substituting $1/t_{ij}^x$ with F_{ij}, Equation (1.1) becomes to the following form:

$$T_{ij} = \frac{P_i A_j F_{ij}}{\sum_{j=1}^{n} A_j F_{ij}} \qquad (1.4)$$

where $F_{ij} \approx f(t_{ij})$, expresses the effect that spatial separation exerts on trip interchange.

These friction factors are derived through an iterative process of comparison between the actual and the calculated TLFD which will be discussed next.

C. The Iterative Process of F-Factor Development

The procedure for gravity model calibration is an iterative process[20]. The current result at any stage is the basis of adjustment for the next trial. Iteration continues until all the results meet some acceptance criterion. As stated earlier, F-Factors are a set of empirical values used for expressing the effect of spatial friction upon trip interchange. These empirical factors are developed, in the gravity model calibration procedure, through the following iterative process:

First, let all the friction factors (F-Factors in absolute value) be equal to 1.0[21]. Then input these initial F-Factors along with other parameters into the gravity model calibration program for the first trial. The results of such a trial are: (1) an estimated trip table (i.e., an N x N matrix representing the trip interchanges of every pair of zones in the area) and (2) a calculated trip length frequency distribution (TLFD). TLFD expresses the distribution of the relative trip-making frequency on various trip lengths (in increments of one-minute travel time). It will be further discussed below.

The second step of F-Factor development is to compare the calculated TLFD with the actual TLFD (i.e., the relative frequency distribution based on the O-D surveyed data). This comparison would provide a mean to measure the difference between the estimated and the actual probability of trip-making of various trip length. Thus, it is also an indicator for the adjustment of F-Factors. The mathematical expression of such adjustment is as follows[22]:

$$F_{adj,m} = F_{used,m} \times \frac{TLFD_{cal,m}}{TLFD_{act,m}} \qquad (1.5)$$

where $F_{adj,m}$ = the F-Factor adjusted for the trip length of m minutes;

$F_{used,m}$ = the F-Factor used in the preceding trial;

$TLFD_{cal,m}$ = the TLFD calculated; and

$TLFD_{act,m}$ = the actual TLFD.

The new set of F-Factors is then used in the next calibration of the gravity model program. Other parameters such as zonal trip production, trip attraction and the minimum path traveltime (t_{ij}) between all zones remain as previously described for use in the second calibration. This second calibration results in another estimate of trip interchanges and a new TLFD. Then it begins another comparison between the calculated and the actual TLFD for the adjustment of F-Factors.

This iterative process continues until the comparison of TLFD meets the acceptance criterion. In general, the acceptance criterion requires that the discrepancy between the actual and the calculated TLFD should not exceed $\pm$ 3%.[23]

The final, accepted F-Factors are used in the final calibration of the gravity model formula.

C. Definition and Characteristics of TLFD

By definition, the TLFD expresses the probability (or relative frequency) distribution of the trips of various lengths in an area.[24] That is

$$\text{TLFD}_{t=m} \approx P(T)_{t=m} \approx f(t) \qquad (1.5)$$

where $\text{TLFD}_{t=m}$ = the relative frequency of the trips lasting m minutes; and

$P(T)_{t=m}$ = the probability of making a trip lasting m minutes.

TLFD can be graphed as a histogram with one-minute intervals of travel time on the abscissa and the probability of trips of each length on the ordinate. It can be tabulated by sorting the trip table into several groups, in which the interzonal traveltime (t_{ij}) of all trip movements in a given group are the same[25]. The following example may facilitate the understanding of TLFD tabulation procedure.

Let $\theta T = \sum_{i=1}^{n} \sum_{j=1}^{n} T_{ij}$ be the sum of the elements of the n x n trip table, T.

$$T = \begin{vmatrix} T_{11} & . & . & . & T_{1n} \\ T_{21} & . & . & . & T_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ T_{n1} & . & . & . & T_{nn} \end{vmatrix}$$

Also let $\Delta T_{ij,m}$ denote the sum of those trip interchanges all having the same trip length, m minutes. For instance

if $t_{12} = t_{23} = t_{56} = t_{76} = m$,

then $\Delta T_{ij,m} = T_{12} + T_{23} + T_{56} + T_{76}$

If there are q groups of $\Delta T_{ij,m}$ (i.e., m = 1,2,...,q) in the whole area, the sum of all q groups should equal the total number of trips. That is

$$\sum_{m=1}^{q} \Delta T_{ij,m} = \sum_{i=1}^{n} \sum_{j=1}^{n} T_{ij} = \theta T \qquad (1.6)$$

By definition, the probability of areawide trip-making at the m_{th} interval of trip length is as follows:

$$P(T)_{t=m} = \Delta T_{ij,m} / \theta T = TLFD_{t=m} \qquad (1.7)$$

The mean of the distribution defined in (1.7) is the arithmatic mean as follows[26]:

$$\overline{TL} = \sum_{m=1}^{q} (m \cdot \Delta T_{ij,m}) \;/\; \theta\, T \qquad (1.8)$$

where $\overline{TL}$ = the mean of TLFD, termed as the average trip length in minutes.

The second parameter of the TLFD is the variance. By definition, variance is a measure of the dispersion of the frequency distribution from the mean[27]. The mathematical expression of the variance of TLFD is stated as follows:

$$VAR = \sum_{m=1}^{q} (m - \overline{TL})^2 \times (\Delta T_{ij,m}) \;/\; \theta\, T \qquad (1.9)$$

The average trip length and the variance are the parameters of TLFD. The distribution of TLFD is usually described in terms of these two quantities. In Figure 1, sample distributions of TLFDs are shown for selected means and variances. Through wide observation, TLFD appears to be an unsymmetric, bell-shaped curve.[28]

The actual TLFD which is usually generated from the O-D trip table represents the actual urban trip-making pattern. The adjustment procedure of F-Factor development discussed previously can not be made without it. It is the purpose of this paper to develop a mathematical model to simulate an actual TLFD for small urban area.

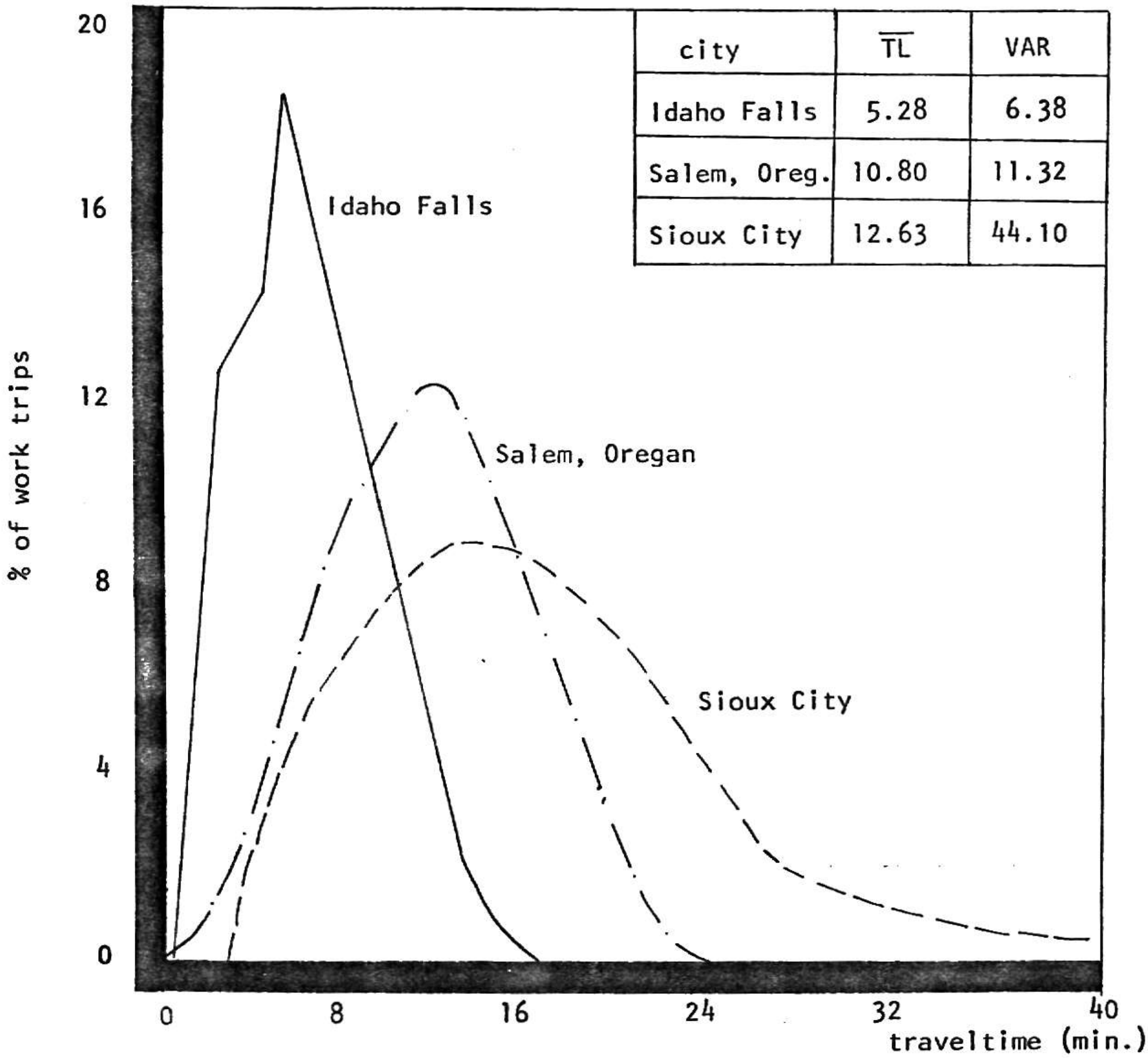

FIGURE 1 Selected TLFDs

PART TWO: MODEL DEVELOPMENT

To develop a mathematical model for TLFD simulation, it is essential to understand the factors which cause the variation of trip length and to provide some quantifiable means to measure the variation.

A. The Factors Associated in the Variation of TLFD

Generally speaking, the frequency distribution of trips with various lengths may depend on the effects of the following factors: (1) level of transportation service, (2) city size, and (3) land-use patterns. The frequencies of trips of a given length may be determined by the level of transportation service which is usually characterized by the average network speed. It is conceivable that a city will have a shorter average trip length than a similar city with inferior transportation service because the latter needs longer travel times to move people for the same distance than the former. The average trip length may also be affected by the city size which is usually indicated by the city population or by the travel demand volume. Furthermore, the shape of the trip length distribution curve may vary from one city to another depending upon the land-use patterns. For instance, the work trip length may be decreased by developing a large job-oriented center which is closer to some denser residential areas.

It is possible that the final variation of TLFD is determined by the combined effects of these three factors. For example, while the average trip length is decreased by increasing the network speed, the decentralized housing development may cause an increase of the average trip length. Therefore, it is necessary to measure these joint effects upon the variation of TLFD with some quantifiable means.

B. The Functional Relationships Between TLFD and Urban Factors

The following analyses are the reviews of Voorhees' research on work trip length[29]. The variation of TLFD, in Voorhees' study, was measured in terms of its mean and variance.

(1) Urban Structure and TLFD

In Voorhees' research, urban structure is measured by the work trip opportunity distribution[30]. To some extent, the meaning of urban structure defined by Voorhees is related to the locational patterns of urban land use. The work opportunity distribution is the frequency distribution of separations (travel times) between homes and jobs[31]. The advantages of using the work opportunity distribution to measure the urban structure are two-fold. First, the changes in the city structure can be dynamically determined. Figure 2 shows the change in work opportunity distributions in Washington, D.C. between 1948 and 1955 with no substantial change in the speed of the transportation network. Second, the work opportunity distribution was highly associated with the TLFD. Thus, it provides the chance to measure the relationship between urban structure and TLFD based on their parametric relationships with work opportunity distribution.

To measure the relationship between work trip length and city structure, Voorhees first demonstrated the direct relationship between zonal work opportunity distribution and the typical declining residential density pattern from the CBD. Voorhees stated the simple fact that the zone close to the job place has shorter average length of work-trip opportunity distribution (thus greater opportunity to work) than those zones farther away.[32] This is shown in Figure 3 in which comparisons of zonal job opportunity distribution were made among three zones (Zone A, B and C) having different travel distances from the CBD of the Washington, D.C. metropolitan area.

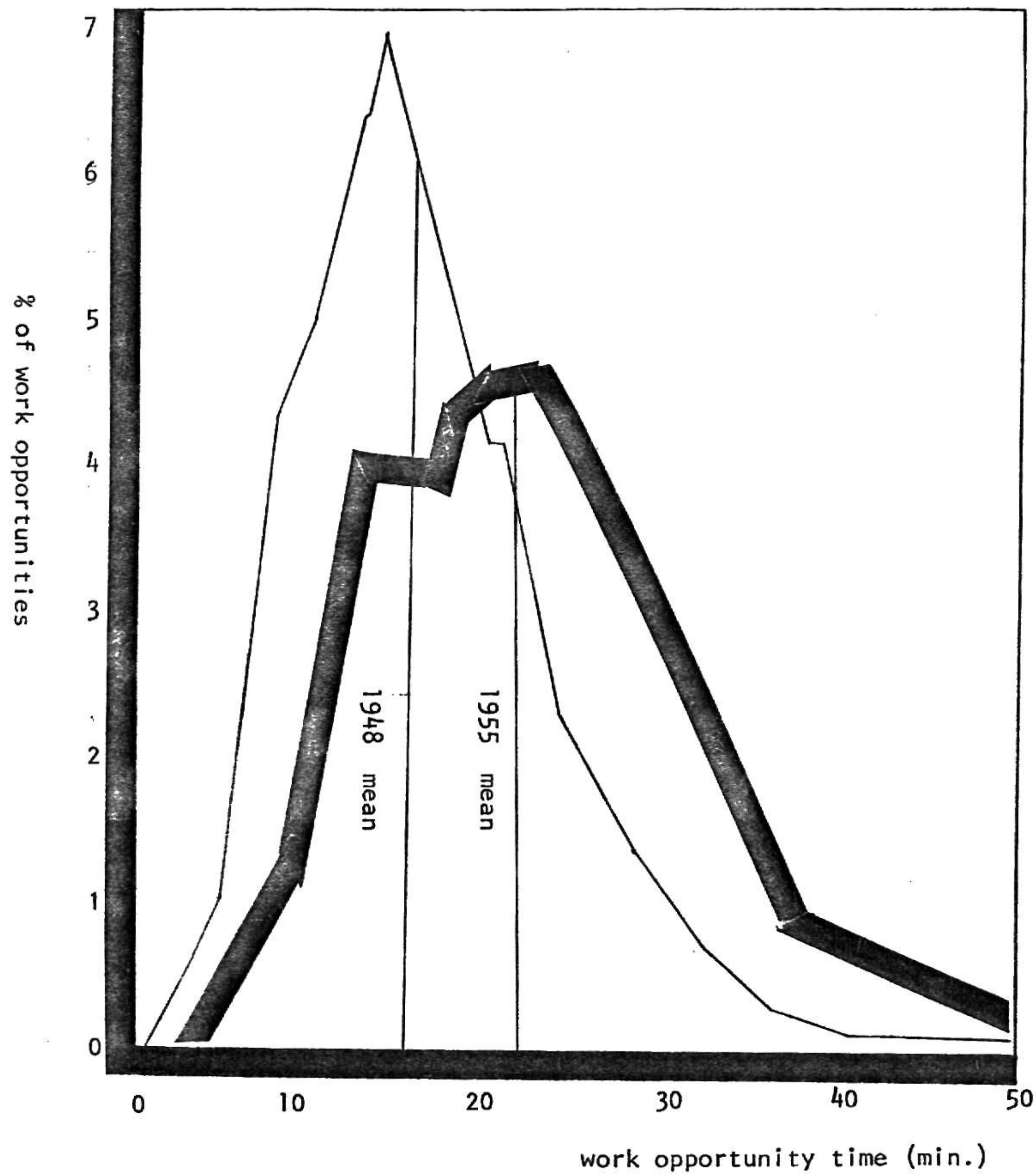

FIGURE 2 Work Opportunity Distribution In Washington, D.C. 1948/1955

(From Alan M. Voorhees' Factors In Work Trip Lengths, HRB Record 141, pp. 24-26)

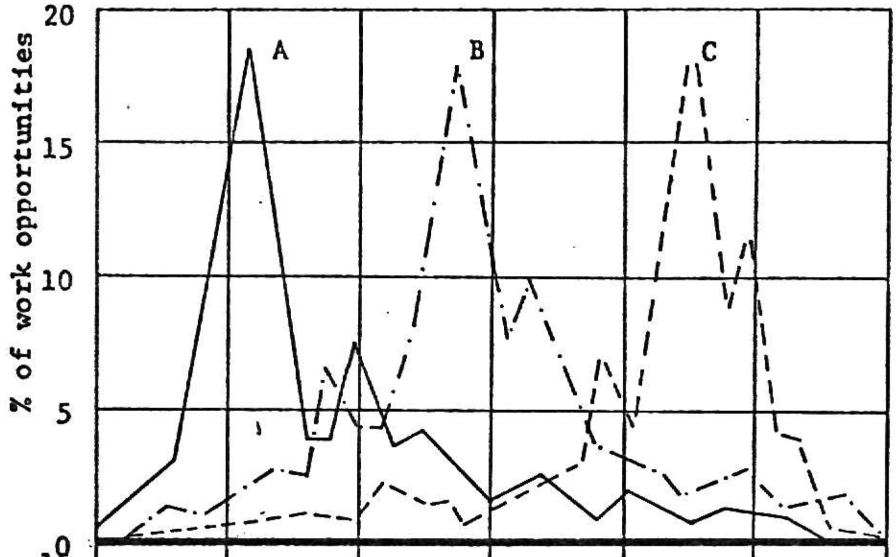

FIGURE 3

JOB-OPPORTUNITY DISTRIBUTION FOR SELECTED ZONES IN WASHINGTON, D.C.

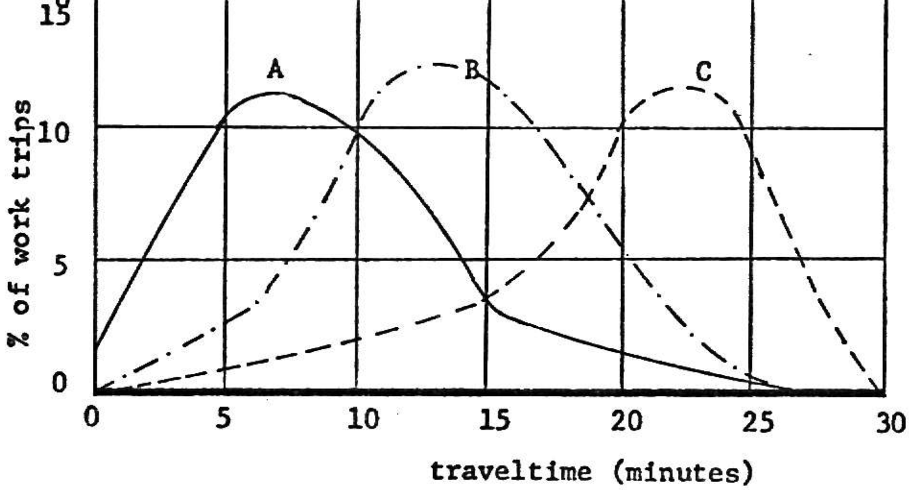

FIGURE 4

TRIP LENGTH FREQUENCY DISTRIBUTION FOR THE SAME ZONES IN WASHINGTON, D.C.

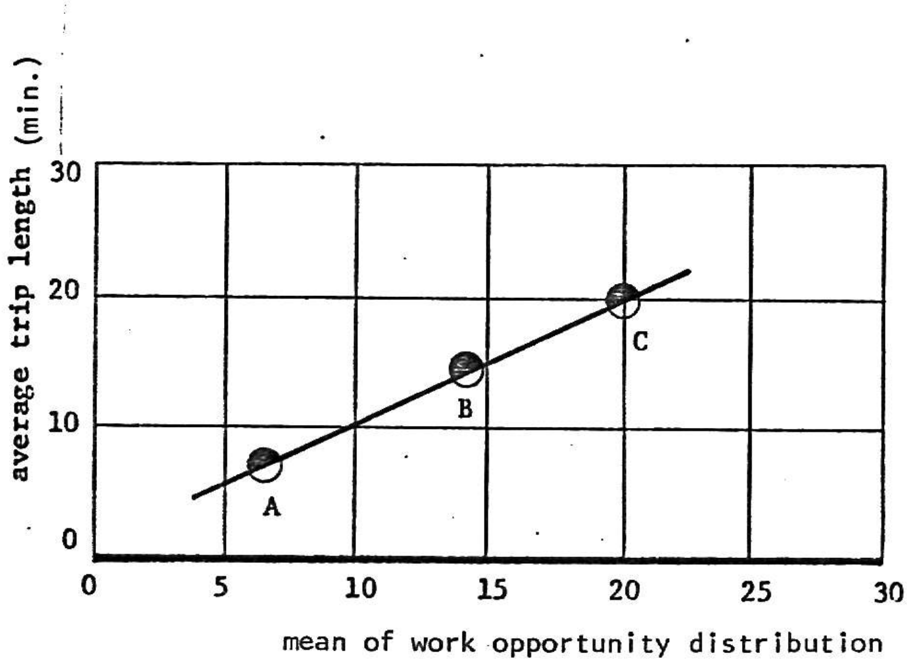

FIGURE 5

LINEAR RELATIONSHIP BETWEEN AVERAGE TRIP LENGTH AND MEAN OF JOB-OPPORTUNITY DISTRIBUTION

Comparing these three zones' TLFDs, Figure 4 shows a close match between TLFD and work opportunity distribution with respect to their curve shape and dispersion from the mean. Voorhees found that the average trip length increases as the average work opportunity length increases[33]. This conclusion is demonstrated in Figure 5 as the result of the analysis.

The total trip pattern that is produced in a metropolitan area is the composite of all of the opportunity distributions in various sections of the urban area[34]. Therefore, it is not surprising that some kind of relationship between TLFD and work opportunity distribution may be found on the urban areawide level. Figure 6 shows the overall work opportunity distributions for three cities that have quite marked differences in their physical and socio-economic structures. In Erie, the mean job-home travel time is 20 minutes; in Detroit, these work-trip opportunities vary largely around 40 minutes; and in Seattle-Tacoma, the work-trip opportunity distribution seems to be almost flat. It is not surprising, therefore, that we may find the cities of Seattle and Tacoma with the longest actual work trip length and Erie with the shortest $\overline{TL}$ because of their difference in struture. Figure 7 shows that the average work trip length increases as the mean of the work opportunity distribution increases in several cities. Combined with a result of Voorhees' simulation study, this relationship is stated in the mathematical form as follows:

$$\overline{TL} = k\overline{o}^{\,0.5} \qquad (2.1)$$

where $\overline{TL}$ = the average trip length of TLFD, in minute;

$\overline{o}$ = the mean of the work opportunity distribution; and

k = constant.

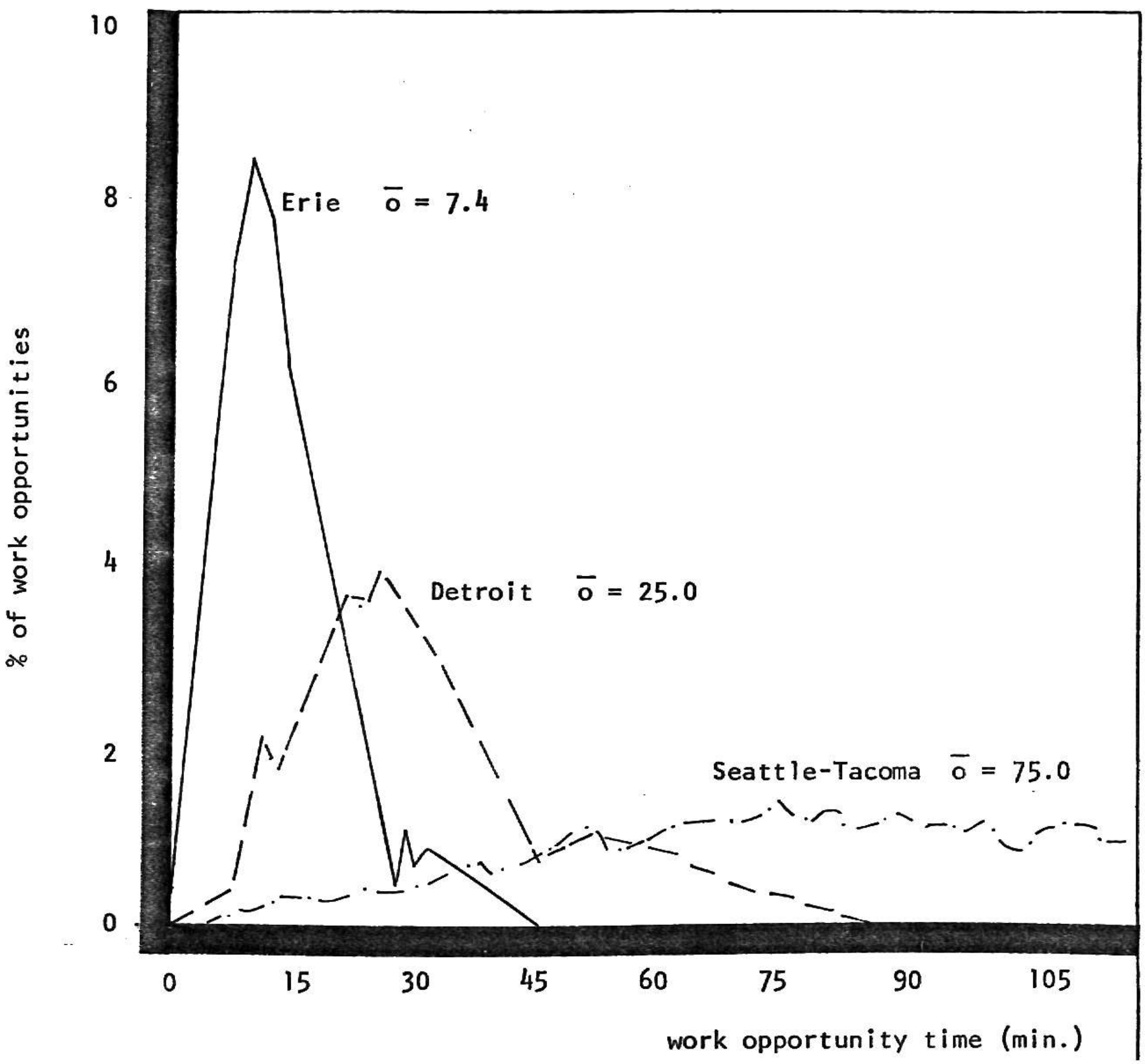

FIGURE 6 Selected Work Opportunity Distributions

$\overline{o}$ = the mean of work opportunity distribution

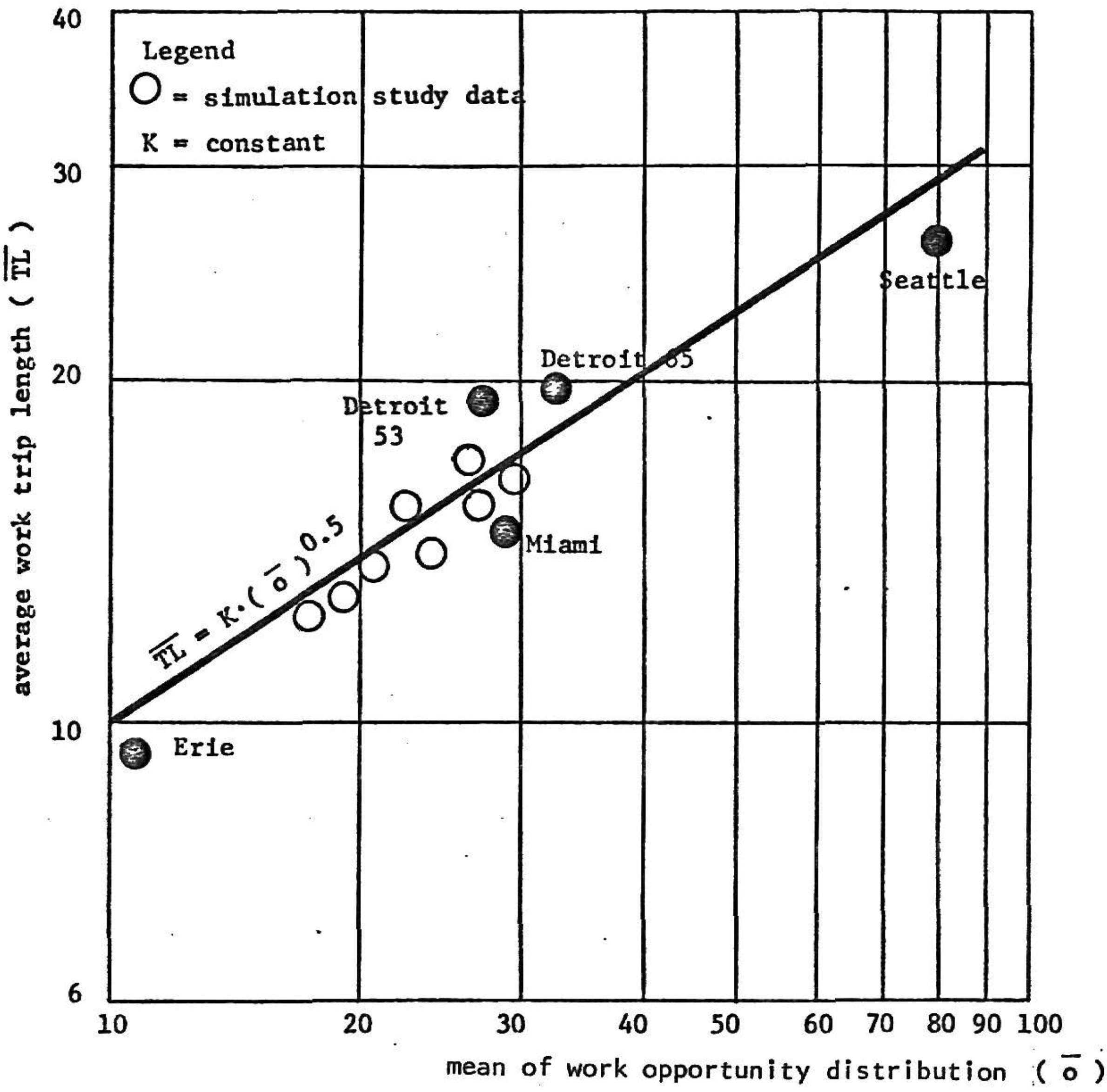

FIGURE 7 The Functional Relationship Between Average Trip Length and the Mean of Work Opportunity Distribution

Source: Alan M. Voorhees and Salvatore J. Bellomo, Factors and Trends in Trip Lengths, National Cooperative Highway Research Project 7-4, 1969.

(2) Network Speed and Trip Length

An important finding in Voorhees' study was that trip length in miles (e.g. the actual length of a trip measured by distance) for a constant urban form is proportional to approximately the 0.75 power of the change in network speed[35]. This conclusion was from a simulation analysis on some hypothetical cities obtained by holding other factors as constant.

However, this simulated result was somewhat inconsistent with the real travel phenomena. The historical data available on trip length in several cities indicated that the trip length in miles increased according to the 1.5 power of the change in network speed. The difference between the 0.75 power and the 1.5 power must reflect the amount of change that was a result of the change in the urban structure, where the cities tended to spread out. In addition, while the average work-trip length in minutes for Washington, D.C. metropolitan area increased since 1948, there was no substantial concurrent change in network speed. Therefore, the trip-length change was caused primarily by the change of urban structure.

(3) Population Size and Trip Length

Usually the travel demand volume is measured in proportion to the size of urban population. In Voorhees' study, regression analysis was performed to examine the effect of population size upon the average trip length (in minute) in data obtained for 23 cities[36]. The developed equation was as follows:

$$\text{Log}_e \overline{TL} = -0.025 + 0.19 \text{ Log}_e POP \qquad (2.2)$$

or

$$\overline{TL} = e^{-0.025} \, POP^{0.19} \qquad (2.3)$$

where $\overline{TL}$ = the average trip length of TLFD in minute;

POP = city population; and

e = base of natural logarithms, e=2.8171...

Equation (2.3) shows that $\overline{TL}$ is approximately related to the fifth root of the city population. About 90% of Voorhees' observations covered the urban population size of 100,000 to 10,000,000. The standard error of the regression coefficient was 0.026, and the coefficient of determination, R^2, was 0.71. Voorhees was not satisfied with the statistics shown, and contended that the size of city population did not contribute significantly to the explanation of variation in trip length in minutes.

In Voorhees' analysis, changes in population alone may not always affect the average trip length. The main reason is that increases in trip length associated with higher populations would be offset if some of the population growth occured at higher densities[37]. In other words, growth did not extend the urban area, and instead the growing population filled in previously unused land.

However, it is not to be concluded that population is a bad indicator of changes of trip length. It is a matter of how the sample data are collected. Inconsistency would appear in the case that, with the same rate of population growth, a city with a centralized land-use development would have an average trip length inevitably smaller than the average trip length of the city with decentralized development. The inconsistency can be eliminated if the population data are collected from those cities which have the same pattern of urban land-use development.

C. Gamma Function: A Mathematical Model For TLFD Simulation

Previous analyses have identified the relationships between TLFD and some urban factors and measured these relationships in terms of the changes of average trip length. However, to simulate the whole relative frequency distribution of trip-making of various lengths, it is important to consider not only the mean but the dispersion from the mean (standard deviation). Furthermore, it is also important to look for a mathematical model which can accurately describe the complete picture of TLFD, i.e.: it is essential to find out if TLFD is exponentially distributed, normally distributed, or distributed according to other probability laws.

One of the most important accomplishments of Voorhees' research in work trip length was the application of the gamma density function for TLFD simulation. In Voorhees study, the probability of making trips of a given length, t, is stated in the following equation:[38]

$$f(t) = K \left(t^{(\overline{TL}^2 - VAR)/VAR}\right) \cdot \left(e^{-(\overline{TL}/VAR)t}\right) \qquad (2.4)$$

Where $f(t)$ = the relative frequency (or probability) of trip-making in each incremental 1-minute travel time, t;

K = a normalizing constant which makes the area under the curve equal to unity (or 100 percent);

e = the base of natural logarithms;

$\overline{TL}$ = the average trip length, or the mean of TLFD; and

VAR = the variance of TLFD.

The gamma distribution expressed in Equation (2.4) which was used in Voorhees' simulation model is different from the common form. The following is a brief review of the gamma distribution and mathematical derivation from the common form into Voorhees' expression.

The gamma distribution is a well-known probability distribution. The gamma function, denoted Γ, is a mapping of the interval $(0, \infty)$ into itself[39] and is defined by

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \, dt \qquad (2.5)$$

One of the important characteristics of gamma function is its recursive property which follows from (2.5) by integration by parts[40]. That is

$$\Gamma(x + 1) = x \; \Gamma(x) \qquad \text{for every } x \geq 0 \qquad (2.6)$$

By definition,

$$\Gamma(1) = \int_0^\infty e^{-t} \, dt = 1$$

Hence $\Gamma(2) = 1 \cdot \Gamma(1) = 1$

$\Gamma(3) = 2 \cdot \Gamma(2) = 2 \cdot 1$

.

.

and $\Gamma(k) = (k - 1) \cdot \Gamma(k - 1) = (k - 1)!$ (2.7)

Furthermore, using the change-of-variable technique, the following integration can be verified[41]. Let $a \geqslant 0$ and $b \geqslant 0$ be real numbers. Then, letting $z = t/b$ we get

$$\int_0^\infty t^{a-1} e^{-t/b} \, dt = b^a \int_0^\infty z^{a-1} e^{-z} \, dz = b^a \, \Gamma(a) \qquad (2.8)$$

A two-parameter family of density function can be defined as follows for $a, b > 0$

$$f(t) = \begin{cases} \dfrac{t^{a-1} e^{-t/b}}{b^a \Gamma(a)} & t \geqslant 0 \\ 0 & t < 0 \end{cases} \qquad (2.9)$$

Let us consider (2.9) when $t \geqslant 0$. It is known that the mean and variance of the gamma distribution are related to the parameters a and b by the following relationships:

$$ab = \overline{TL}$$

and $$ab^2 = VAR$$

Hence

$$a - 1 = \frac{ab^2}{ab^2}(a - 1) = \frac{a^2b^2 - ab^2}{ab^2} = \frac{\overline{TL}^2 - VAR}{VAR}$$

$$-t/b = \frac{ab}{ab}(-t/b) = -(\frac{ab}{ab^2}) \cdot t = -(\frac{\overline{TL}}{VAR}) \cdot t$$

Substituting into (2.9), a revised form of gamma density function is stated as follows:

$$f(t) = \frac{1}{b^a \Gamma(a)} t^{a-1} e^{-t/b}$$

$$= K \, (t^{(\overline{TL}^2 - VAR)/VAR}) \cdot (e^{-(\overline{TL}/VAR)t}) \qquad (2.10)$$

D. The TLFD Simulation Model

Based on previous analyses, the TLFD for small urban areas is simulated by the flow diagram shown in Figure 7. The simulation model is a simple, straight forward process. After the necessary data are gathered, the first step is to simulate the parameters of TLFD, the mean and variance. This can be done by utilizing their functional relationships with factors such as population size and network speed. Through regression analysis, a best fitting curve (or straight line) can be derived[42] The mean and variance are then approximated by these equations with the necessary input from the study area. These approximated figures are then input into the gamma density function for calibrating the simulated TLFD.

In the development of those equations for approximating the mean and variance, it is necessary to be careful about the selection of the independent variable(s). In Voorhees' study, the average trip length and variance were estimated by their functional relationships with the city population size and the mean of TLFD respectively.[43] To some extent, Voorhees was not satisfied with the equations developed. The hypothesis that trip length increases as the total area population grows was not fully accepted in his regression analysis.

However, population size is still selected in this paper as the only independent variable for approximating the average trip length in small urban areas. This decision is made on the basis of the following assumptions:

(1) Voorhees' observation regarding the population growth and tendency to higher density use (discussed in page 22) will not have significant effect in small urban areas, and

(2) By taking the sampling strategy that population data are collected from those cities which have the similar pattern of land use,

the inconsistency would be eliminated.

In view of these considerations, it is very important that each city should develop its own equations for estimating the average trip length and the variance. The accuracy of this simulation model is largely dependent upon the data sampling strategy and the statistical analysis in equation development.

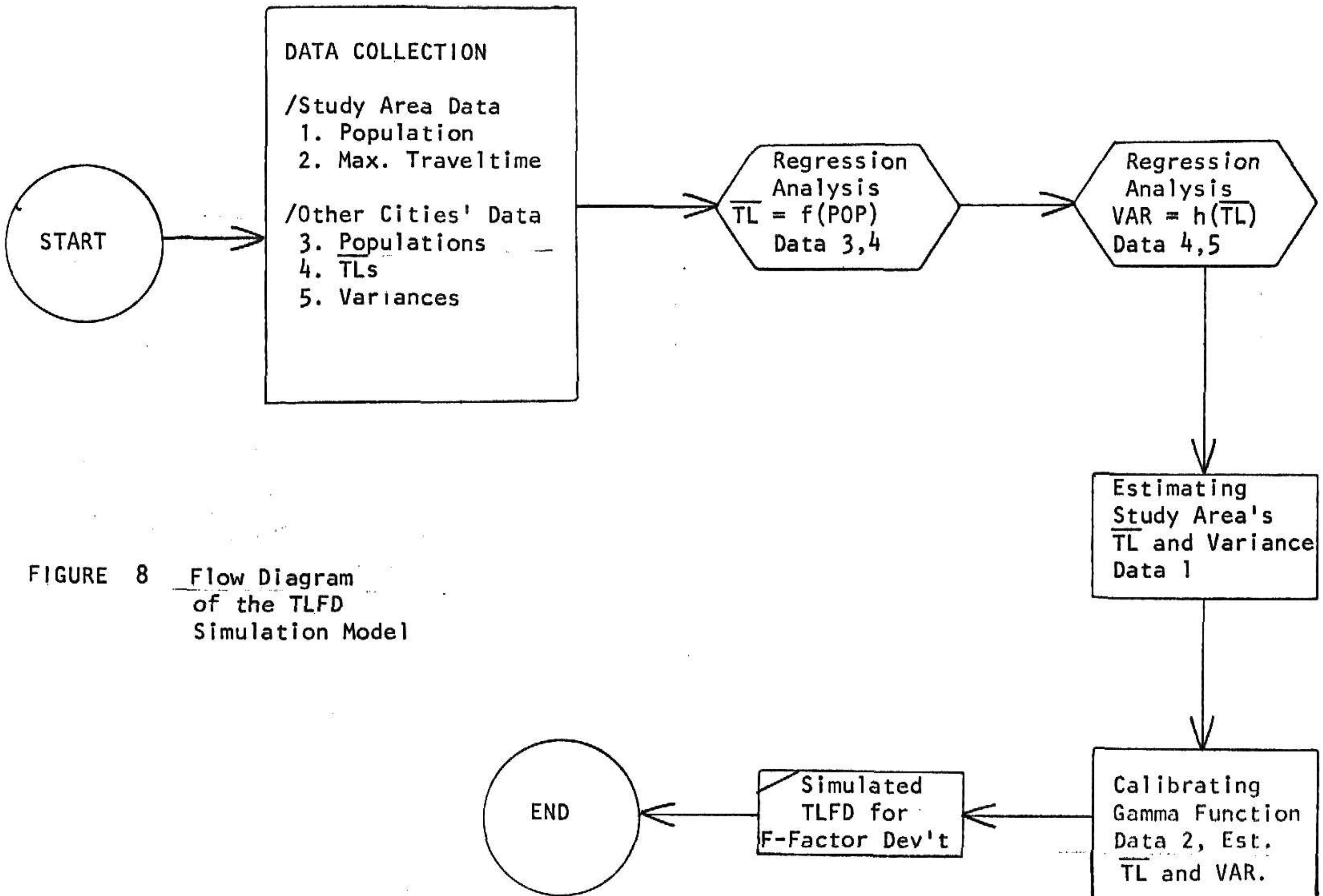

FIGURE 8 Flow Diagram of the TLFD Simulation Model

PART THREE: MODEL TESTING

To test the simulation model developed in Part Two, some analyses of real data have been conducted and consist of the following works:

(1) Testing the statistical validity of the gamma distribution as the model for simulating the TLFD.

(2) Regression analysis for the functional relationship between average trip length and small city's population.

(3) Regression analysis for the functional relationship between variance and average trip length.

Simulation of the 1970 city of Lawrence's home-based work TLFD by the gamma density function was conducted for demonstration purpose. According to the 1970 census, Lawrence, Kansas had a population of 48,700, and had 92 internal traffic zones with a maximum inter-zonal traveltime of 25 minutes.

(1) Testing the Statistical Validity of the Gamma Distribution as the Model for Simulating the TLFD

Before the simulation is carried out, it is important to make sure the gamma distribution is the right mathematical model. To examine the statistical validity, a comparison between the actual TLFD and the simulated TLFD by this model was made. This is done by comparing the observed TLFD from seven small cities with the calculated gamma distribution using the actual mean and variance from each original TLFD as the input parameters of the gamma density function.

As shown in a series of histograms (Figure 8 through Figure 15), all seven cities' TLFDs are closely matched with the calculated gamma distribution. 90% of the observations have only $\pm$ 1.5% of discrepancies

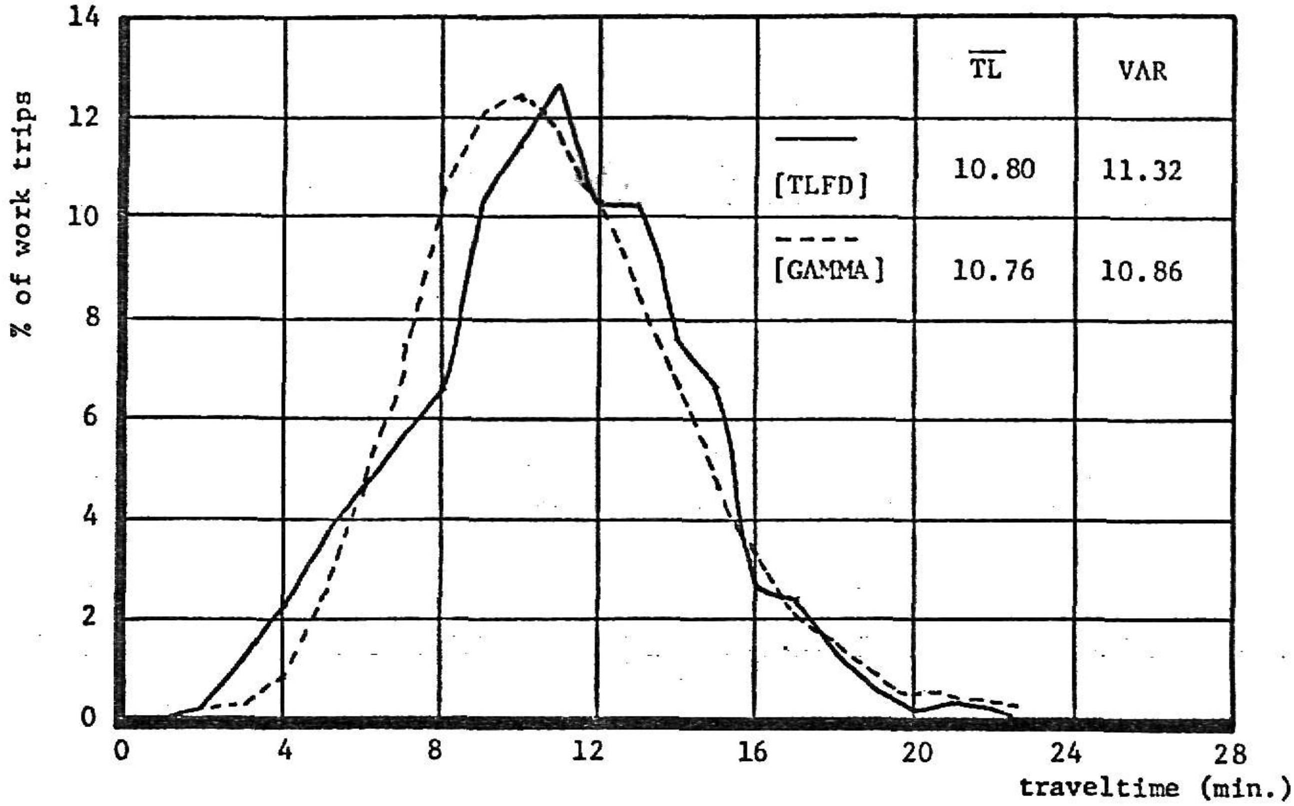

FIGURE 9 Gamma Distribution Vs. TLFD

Data Source: Salem, Oregan

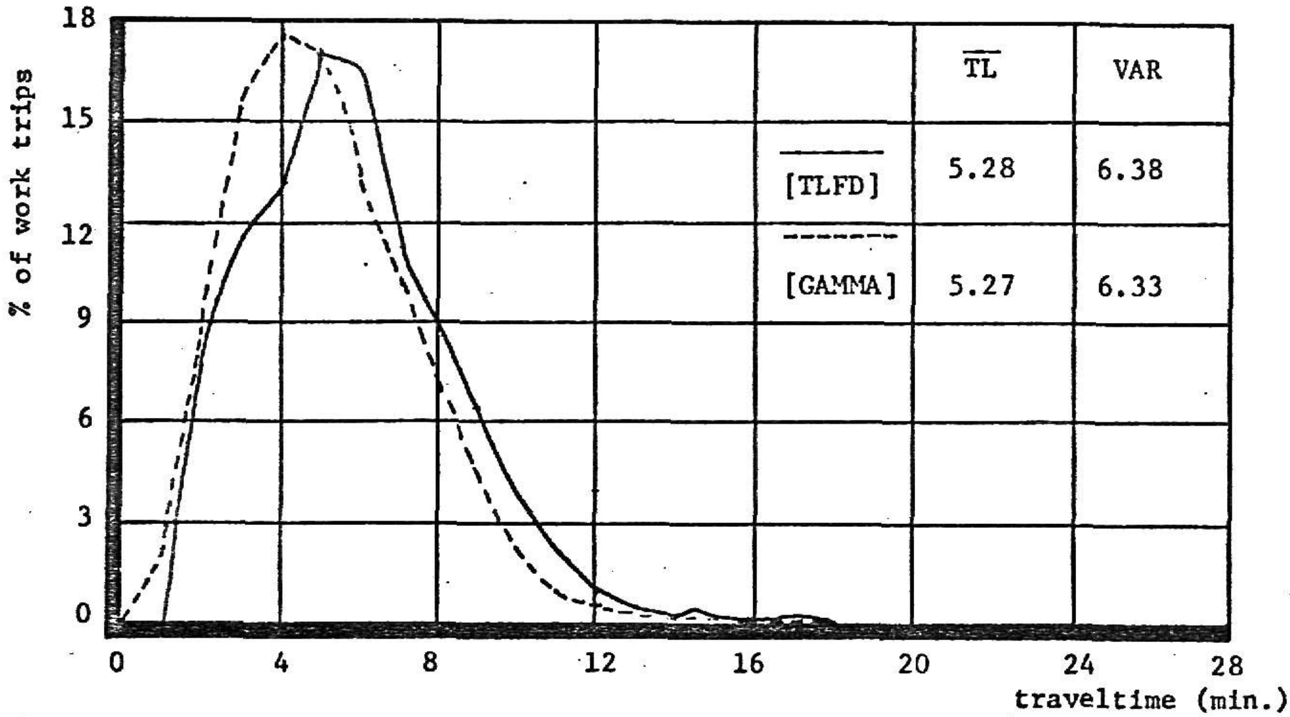

FIGURE 10 Gamma Distribution Vs. TLFD

Data Source: Idaho Falls, Idaho

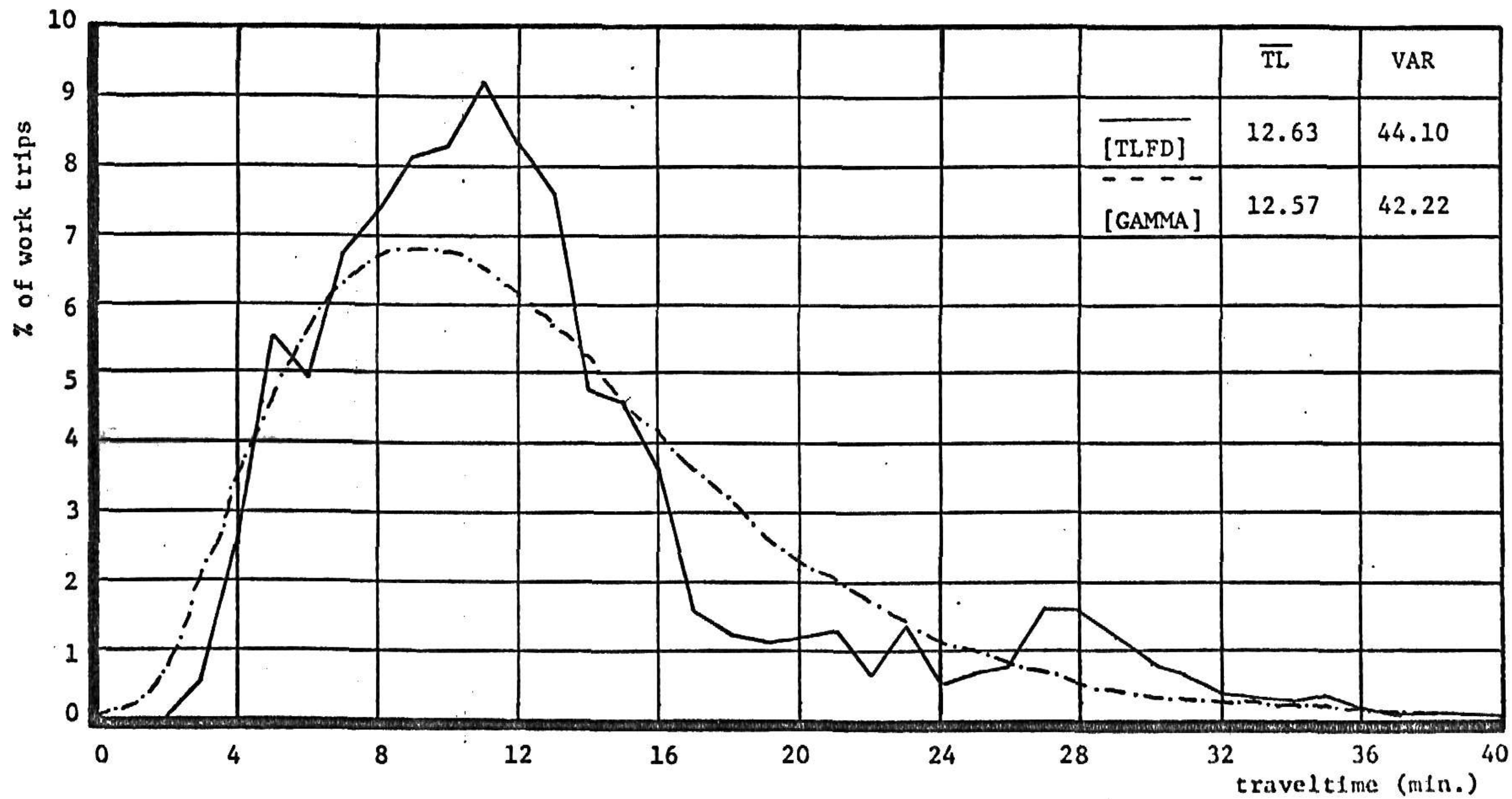

FIGURE 11 Gamma Distribution Vs. TLFD

Data Soruce: Sioux City

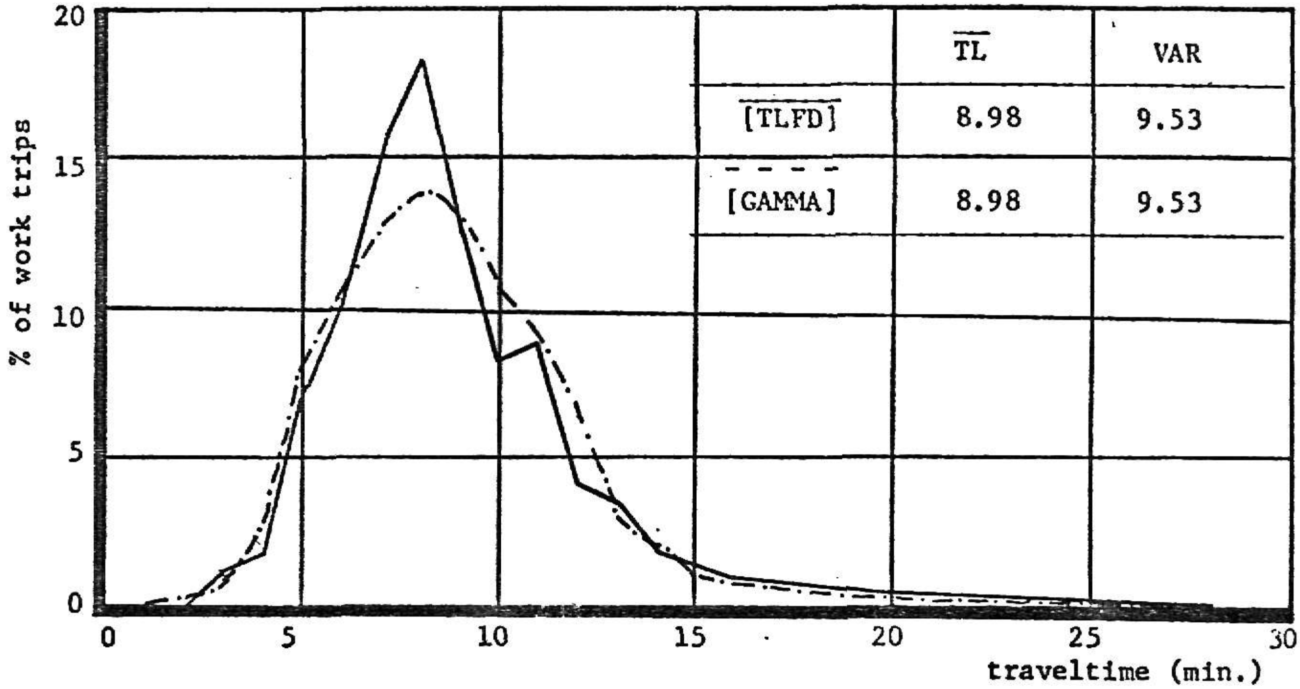

FIGURE 12 Gamma Distribution Vs. TLFD

Data Source: Hutchinson, Kansas

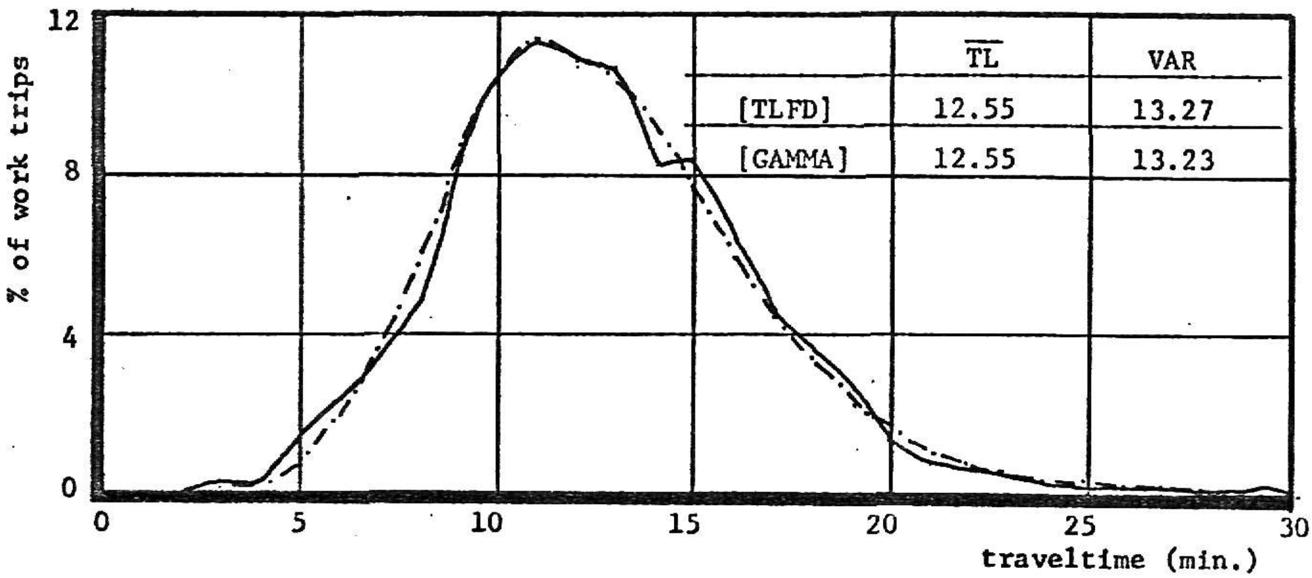

FIGURE 13 Gamma Distribution Vs. TLFD

Data Source: Champaign-Urbana, Illinois

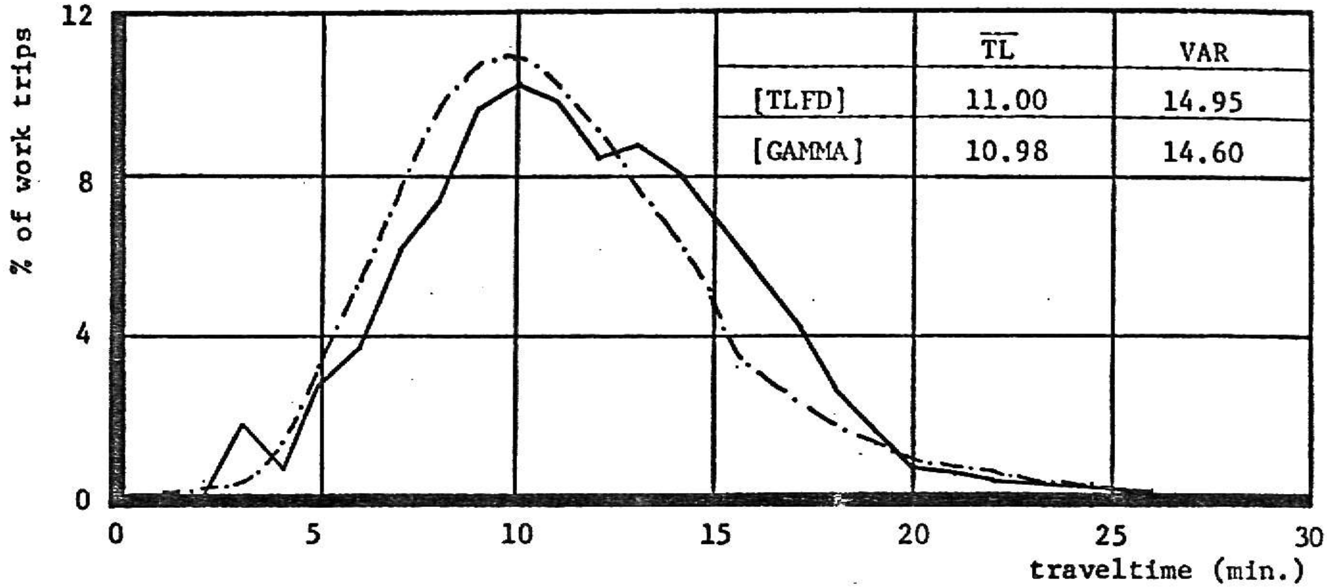

FIGURE 14 Gamma Distribution Vs. TLFD

Data Source: Boise, Idaho

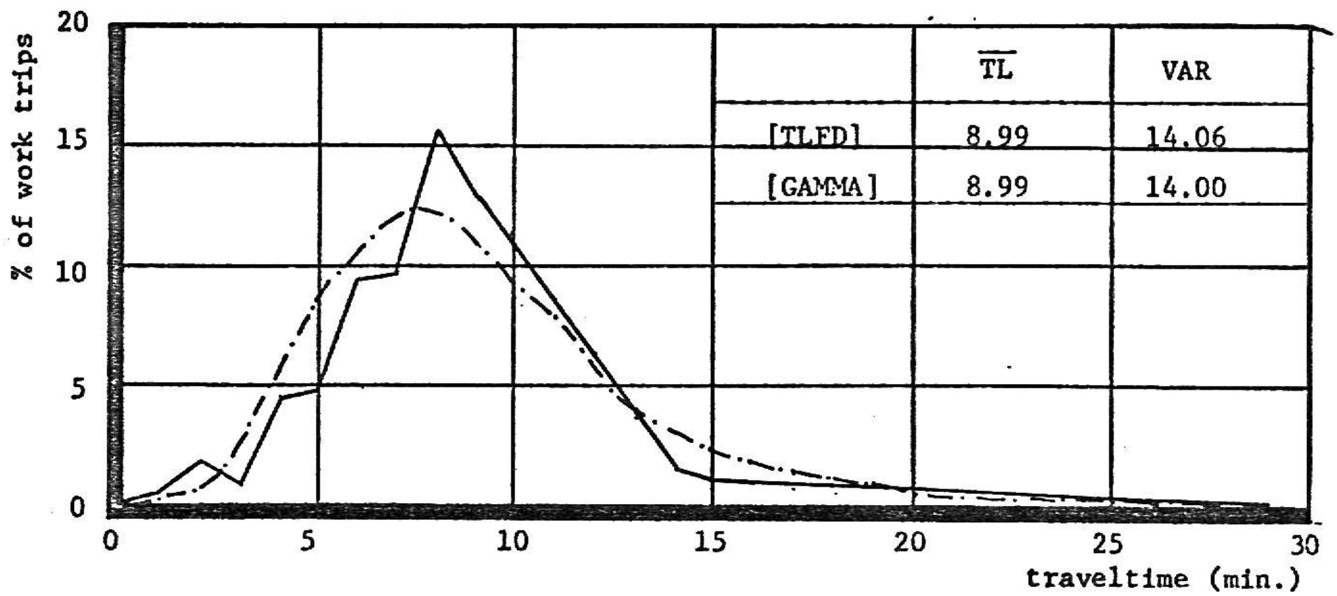

FIGURE 15 Gamma Distribution Vs. TLFD

Data Source: Fayetteville-Springdale, Arkansas

TABLE 1

Gamma Distribution Vs. TLFD

Data Source: Salem, Oregan

Travel Time	Gamma %	TLFD %	Difference
1	0.000	0.000	0.000
2	0.008	0.072	0.064
3	0.134	1.065	0.931
4	0.751	2.206	1.455
5	2.307	3.859	1.552
6	4.847	4.555	-0.292
7	7.834	5.543	-2.291
8	10.452	6.495	-3.957
9	12.044	10.057	-1.987
10	12.364	11.630	-0.734
11	11.559	12.775	1.216
12	10.003	10.057	0.054
13	8.114	10.147	2.033
14	6.228	7.708	1.480
15	4.558	6.747	2.189
16	3.200	2.650	-0.550
17	2.167	2.484	0.317
18	1.420	1.141	-0.279
19	0.905	0.517	-0.388
20	0.562	0.072	-0.490
21	0.341	0.148	-0.193
22	0.202	0.072	-0.130
Mean	10.76	10.80	
Variance	10.86	11.32	

TABLE 2

Gamma Distribution Vs. TLFD

Data Source: Idaho Falls, Idaho

Travel time	Gamma %	TLFD %	Difference
1	1.966	0.000	-1.966
2	8.881	7.716	-1.165
3	15.220	11.851	-3.369
4	17.538	13.183	-4.355
5	16.260	16.872	0.612
6	13.138	15.813	2.675
7	9.654	11.677	2.023
8	6.617	8.830	2.213
9	4.302	6.097	1.795
10	2.682	3.847	1.165
11	1.616	2.032	0.416
12	0.947	1.011	0.064
13	0.542	0.636	0.094
14	0.304	0.147	-0.157
15	0.168	0.152	-0.016
16	0.091	0.087	-0.004
17	0.049	0.043	-0.006
18	0.026	0.005	-0.021
Mean	5.27	5.28	
Variance	6.33	6.38	

TABLE 3

Gamma Distribution Vs. TLFD

Data Source: Sioux City, Iowa

Travel time	Gamma %	TLFD %	Difference
1	0.216	0.000	-0.216
2	0.993	0.000	-0.993
3	2.155	0.520	-1.635
4	3.436	2.550	-0.886
5	4.627	5.550	0.923
6	5.599	4.950	-0.649
7	6.295	6.820	0.525
8	6.705	7.380	0.675
9	6.853	8.080	1.227
10	6.780	8.150	1.370
11	6.534	9.190	2.656
12	6.162	8.170	2.008
13	5.706	7.610	1.904
14	5.202	4.840	-0.362
15	4.680	4.680	0.000
16	4.161	3.620	-0.540
17	3.662	1.620	-2.042
18	3.194	1.210	-1.984
19	2.763	1.090	-1.673
20	2.373	1.140	-1.233
21	2.025	1.270	-0.755
22	1.717	0.710	-1.007
23	1.449	1.330	-0.119
24	1.216	0.540	-0.676
25	1.016	0.770	-0.246
26	0.846	0.830	-0.016
27	0.701	1.590	0.889
28	0.579	1.560	0.981
29	0.477	1.120	0.643
30	0.391	0.810	0.419
31	0.320	0.740	0.420
32	0.261	0.410	0.149
33	0.213	0.370	0.157
34	0.173	0.260	0.087
35	0.140	0.290	0.150
36	0.113	0.120	0.007
37	0.091	0.020	-0.071
38	0.073	0.040	-0.033
39	0.059	0.020	-0.039
40	0.047	0.010	-0.037
Mean	12.57	12.63	
Variance	42.22	44.10	

TABLE 4

Gamma Distribution Vs. TLFD

Data Source: Hutchinson, Ks.

Travel time	Gamma %	TLFD %	Difference
1	0.002	0.000	-0.002
2	0.125	0.000	-0.125
3	1.003	1.187	0.183
4	3.345	1.429	-1.916
5	6.891	6.134	-0.757
6	10.468	10.095	-0.373
7	12.888	15.959	3.071
8	13.604	18.114	4.510
9	12.768	12.641	-0.127
10	10.922	7.889	-3.033
11	8.669	8.401	-0.268
12	6.467	4.873	-1.594
13	4.580	4.747	0.167
14	3.103	2.606	-0.497
15	2.024	1.955	-0.069
16	1.277	1.354	0.077
17	0.782	0.987	0.205
18	0.467	0.754	0.287
19	0.272	0.419	0.147
20	0.156	0.135	-0.021
21	0.087	0.140	0.053
22	0.048	0.074	0.026
23	0.026	0.042	0.016
24	0.014	0.037	0.023
25	0.007	0.014	0.007
26	0.004	0.005	0.001
27	0.002	0.005	0.003
28	0.001	0.005	0.004
Mean	8.98	8.98	
Variance	9.53	9.53	

TABLE 5

Gamma Distribution Vs. TLFD

Data Source: Champaign-Urbana, Illinois

Travel time	Gamma %	TLFD %	Difference
1	0.000	0.000	0.000
2	0.001	0.000	-0.001
3	0.016	0.123	0.107
4	0.142	0.055	-0.087
5	0.621	1.375	0.754
6	1.751	2.258	0.507
7	3.633	3.449	-0.184
8	6.023	4.928	-1.095
9	8.415	8.288	-0.127
10	10.273	10.238	-0.035
11	11.242	11.032	-0.210
12	11.242	10.740	-0.502
13	10.422	10.552	0.130
14	9.058	8.072	-0.986
15	7.447	8.349	0.902
16	5.833	6.331	0.498
17	4.379	4.470	0.091
18	3.165	3.655	0.490
19	2.213	2.587	0.374
20	1.501	1.338	-0.163
21	0.991	0.824	-0.167
22	0.638	0.594	-0.044
23	0.402	0.418	0.016
24	0.248	0.209	-0.039
25	0.150	0.071	-0.079
26	0.089	0.031	-0.058
27	0.052	0.028	-0.024
28	0.030	0.015	-0.015
29	0.017	0.000	-0.017
30	0.010	0.009	-0.001
Mean	12.55	12.55	
Variance	13.23	13.27	

TABLE 6

Gamma Distribution Vs. TLFD

Data Source: Fayetteville-Springdale, Ark.

Travel time	Gamma %	TLFD %	Difference
1	0.051	0.900	0.849
2	0.728	2.200	1.472
3	2.635	0.958	-1.677
4	5.450	4.893	-0.577
5	8.296	4.915	-3.381
6	10.403	9.453	-0.950
7	11.411	9.632	-1.779
8	11.350	15.615	4.265
9	10.476	12.771	2.295
10	9.115	11.626	2.511
11	7.562	8.514	0.952
12	6.031	6.194	0.163
13	4.653	4.020	-0.633
14	3.490	1.868	-1.622
15	2.555	1.157	-1.398
16	1.832	1.117	-0.715
17	1.289	0.761	-0.528
18	0.892	0.611	-0.279
19	0.608	0.543	-0.065
20	0.409	0.652	0.243
21	0.272	0.357	0.085
22	0.179	0.462	0.283
23	0.117	0.319	0.202
24	0.075	0.225	0.150
25	0.048	0.124	0.076
26	0.031	0.064	0.033
27	0.019	0.078	0.059
28	0.012	0.013	0.001
29	0.008	0.007	-0.001
Mean	8.99	8.99	
Variance	14.00	14.06	

TABLE 7

Gamma Distribution Vs. TLFD

Data Source: Boise, Idaho

Travel time	Gamma %	TLFD %	Difference
1	0.001	0.000	-0.001
2	0.043	0.000	-0.043
3	0.366	1.769	1.403
4	1.349	0.659	-0.690
5	3.147	2.731	-0.416
6	5.496	3.854	-1.642
7	7.859	6.075	-1.784
8	9.710	7.446	-2.264
9	10.728	9.639	-1.089
10	10.853	10.140	-0.713
11	10.224	9.704	-0.520
12	9.081	8.415	-0.666
13	7.677	8.718	1.041
14	6.222	8.055	1.833
15	4.863	6.945	2.072
16	3.683	5.705	2.022
17	2.713	4.152	1.439
18	1.950	2.608	0.659
19	1.371	1.524	0.153
20	0.945	0.741	-0.204
21	0.640	0.545	-0.095
22	0.427	0.290	-0.137
23	0.280	0.134	-0.146
24	0.182	0.093	-0.089
25	0.116	0.050	-0.066
26	0.074	0.017	-0.057
Mean	10.98	11.00	
Variance	14.60	14.95	

between the relative trip frequencies and the gamma function values (Table 1 through Table 7). The largest difference was 4.51 which corresponds to only 1% of the total observations.

The possibility of using a statistical method such as the chi-square test of goodness of fit for testing the model had been considered. However, the chi-square test requires a frequency distribution rather than a probability distribution (i.e. TLFD is shown by percentage form in most cases)[44]. Therefore, it can not be used in this type of comparison.

(2) The Functional Relationship Between $\overline{TL}$ and Population Size

To estimate the relationship between the size of urban population and the average trip length, regression analyses were conducted with some observed data from 13 selected cities[45]. Two mathematical functions, linear and logarithmic, were tried in the regression analysis to find the best-fitted line.

In linear form, the developed equation was as follows:

$$\overline{TL} = 3.494 + 0.0001 \text{ POP} \qquad (3.1)$$

The developed equation which used the logarithm form was as follows:

$$\text{Log}_e \overline{TL} = -5.355 + 0.6889 \text{ Log}_e \text{POP}$$

or

$$\overline{TL} = e^{-5.355} \cdot \text{POP}^{0.6889} \qquad (3.2)$$

These two equations are represented graphically by the regression lines shown in Figure 16 and Figure 17.

Through a series of statistical analysis and evaluation, Equation (3.2) is considered better than (3.1) for approximating the average trip length. These statistical analyses are presented as follows:

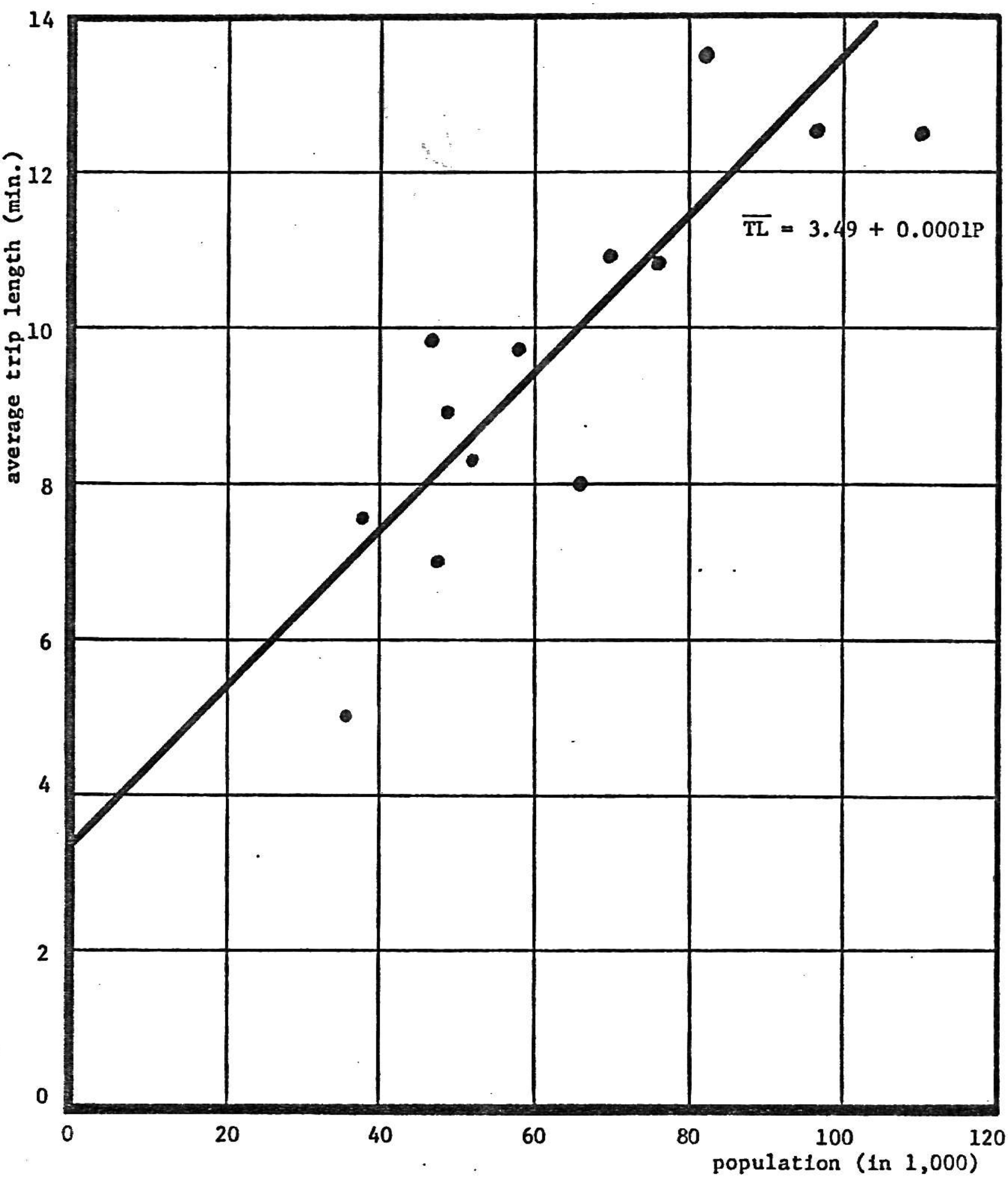

FIGURE 16 Average Trip Length Vs. Population

(Linear Relationship)

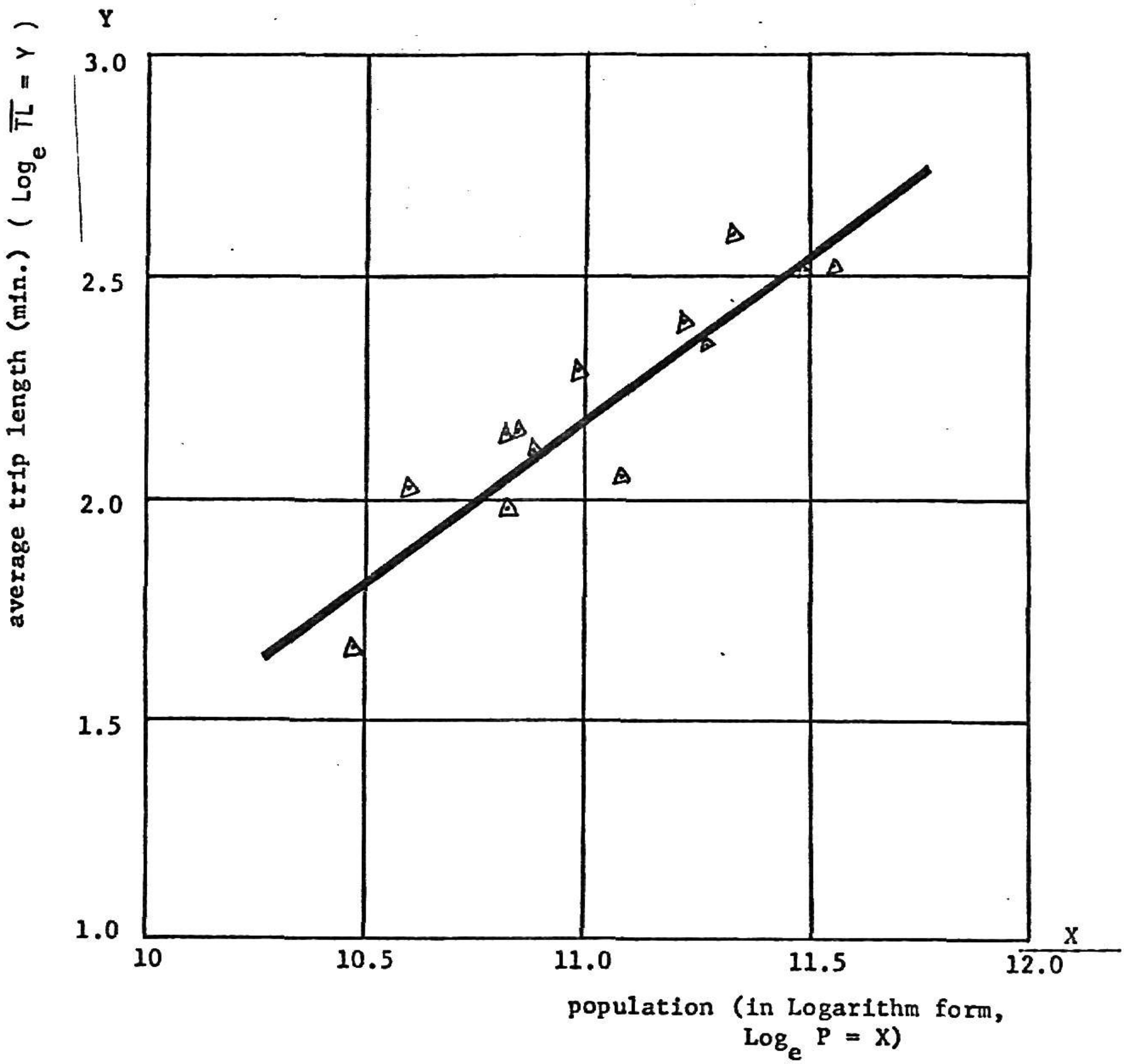

FIGURE 17 Average Trip Length Vs. Population

(Non-linear Relationship)

Let all the statistics in the linear equation be denoted with a superscript "*", and let " Ø " denote all the statistics in the logarithm equation.

(a) Standard error of estimate, $S_{y.x}$

$S_{y.x}$ is the statistic measuring the dispersion of the observed Y from the regression line[46]. As result of each regression analysis, they are

$$S^{*}_{y.x} = 1.2526 \quad ; \qquad S^{Ø}_{y.x} = 0.1311$$

$$(\text{Log}_e\ S^{Ø}_{y.x} = 0.1311)$$

It is known that approximately 68.3% of observations would be expected to fall within one $S_{y.x}$ from the regression line; 95% within two $S_{y.x}$; and 99% within three $S_{y.x}$. To obtain an uniform comparison of these statistics from different forms of equations (e.g. linear vs. non-linear), the coefficients of variation are calculated by dividing $S_{y.x}$ with the observed mean. They are

$$S^{*}_{y.x} / \overline{Y}^{*} = 0.1297 \; ; \qquad S^{Ø}_{y.x} / \overline{Y}^{Ø} = 0.0586$$

It is obvious that smaller error of estimating the dependent variable $\overline{TL}$ is expected by Equation (3.2) than by (3.1).

(b) Standard error of regression coefficient, S_b

This statistic indicates the range of the independent variable's coefficient of the developed equation from the true regression coefficient[47].

$$S^{*}_{b} = 0.00002 \quad ; \qquad S^{Ø}_{b} = 0.11245$$

Associated with the coefficients shown in Equation (3.1) and (3.2), an error of $\pm 20\%$ (the ratio between S_b and the coefficient of the independent variable) from the true regression coefficient is expected in the linear equation and about $\pm 16\%$ error is expected in the logarithm form.

(c) Coefficient of determination, R^2

This statistic represents the measurement of the degree of association between the dependent and independent variable[48]. The value of this statistic ranges from zero to +1.0. The larger the absolute value of R, the higher degree of association.

$$\overset{*}{R}{}^2 = 0.8689 \quad ; \quad \overset{\emptyset}{R}{}^2 = 0.8794$$

R^2 is relatively high in both equations, and $\overset{\emptyset}{R}{}^2$ is somewhat higher.

(d) t-test

The t-value is a statistic used to test the significance of the estimated coefficient from the true regression coefficient[49]. Based on the Student's t distribution, the t-test is done by comparing the derived t value with a table value at n-1 degrees of freedom and with a given level of significance. The decision rule is to reject the null hypothesis (B' = 0 where B' is the hypothesized true coefficient) if the observed t-value is equal to or greater than the table value in a two-tailed test[50]. The calculated t-value of each regression analysis is

$$t^* = 5.8217 \quad ; \quad t^{\emptyset} = 6.1267$$

while the table value is found to be 3.106 at 12 degrees of freedom with 95% confidence interval. Hence, the alternative hypothesis that regression coefficient is statistically significant is accepted.

In Voorhees' research, the $\overline{TL}$ in large metropolitan areas were estimated approximately by the fifth root of population size. The developed equation was not considered to be significant because the offset effect of population growth in inner vacant land instead of outward spatial expansion. However, through consistent data sampling, it is proved that the average trip length in small urban areas can be estimated by population alone.

(3) The Functional Relationship Between $\overline{TL}$ and Variance

A regression analysis for estimating the relationship between the average trip length and the variance of TLFD was also made. The logarithmatic equation which generated the best-fitted curve between $\overline{TL}$ and variance is shown below:

$$\text{Log}_e \text{ VAR} = -0.6494 + 1.4341 \text{ Log}_e \overline{TL}$$

or

$$\text{VAR} = e^{-0.6494} \cdot \overline{TL}^{1.4341} \qquad (3.3)$$

This equation is represented graphically by the regression line shown in Figure 18.

The statistical evaluation was generally acceptable. The standard error of estimate was 0.4487. About 68.3% of observations would fall within one $S_{\hat{y}.x}$ above or below the observed mean. The standard error of regression coefficient was 0.6078; about 39% error from the true regression coefficient is expected. The coefficient of determination, R^2, was 0.7258, showing a fairly high degree of association between the dependent and independent variables. In the t-test, the alternative hypothesis was accepted at alpha level of 0.1.

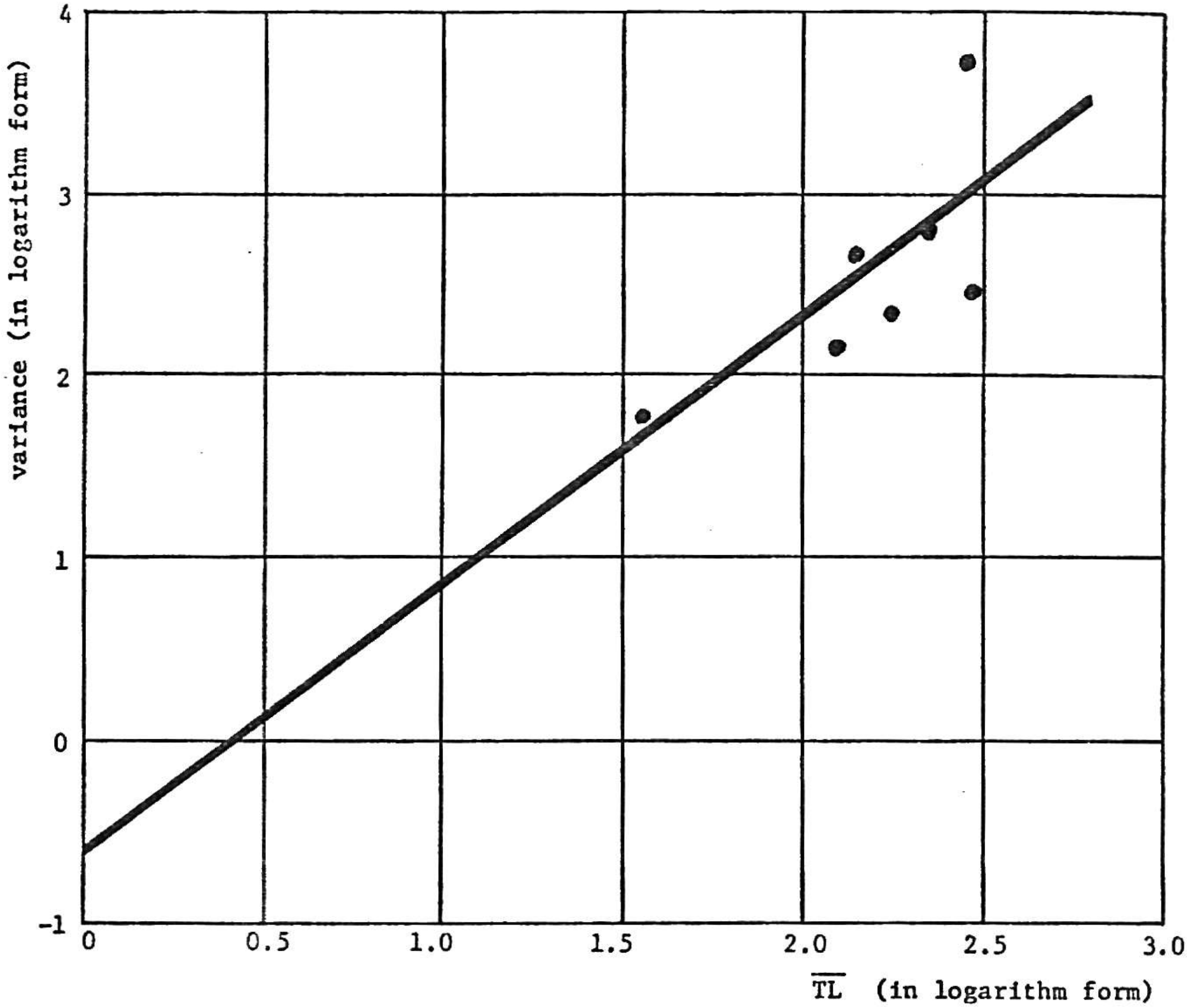

FIGURE 18 Variance of TLFD Vs. Average Trip Length

(In Logarithm Form)

Some additional tasks were done to demonstrate how the simulation model works after the equations for approximating the mean and variance have been developed.

The average trip length and the variance of the TLFD for the home-based work trips in the Lawerence, Kansas study area were estimated by Equation (3.2) and (3.3) respectively. The estimated $\overline{TL}$ and variance are

$$\overline{TL} = 8.267 \qquad \text{(minutes)},$$

$$VAR = 10.8$$

Taking these two estimated values as the *input parameters in* Equation (2.4), the gamma distribution was calculated as follows:

$$f(t) = K\ (t^{(8.267^2-10.8)/10.8}) \cdot (e^{(-8.267/10.8)t})$$

$$0 < t \leqslant 25$$

The simulated TLFD is shown in Table 19. Although comparison with the actual TLFD was not conducted because lack of the 1970 Lawerence O-D information, it is believed that the simulated TLFD is reasonable.

TABLE 8

Gamma Distribution
as the
Simulated TLFD

Lawerence, Kansas

Traveltime	Gamma Function Values **	Gamma %
1	0.5	0.040
2	8.7	0.754
3	35.1	3.044
4	75.6	6.558
5	115.4	10.017
6	141.8	12.311
7	150.0	13.021
8	142.2	12.339
9	123.9	10.753
10	101.0	8.770
11	78.1	6.780
12	57.8	5.015
13	41.2	3.574
14	28.4	2.468
15	19.1	1.658
16	12.5	1.088
17	8.1	0.699
18	5.1	0.441
19	3.2	0.274
20	1.9	0.167
21	1.2	0.101
22	0.7	0.060
23	0.4	0.036
24	0.2	0.021
25	0.1	0.012
Total	1152.2	100.0
Mean		8.267
Variance		10.752

** Constant K = 1.0

CONCLUSION

The simulation model developed in this paper provides two findings. One is that the population size can be used as the independent variable for estimating the average trip length for small urban areas. The other is that the gamma distribution fits the TLFD very well and thus can be used as the model for simulating the TLFD.

For further research, the following considerations may be necessary:

(1) To simulate the TLFD of other trip purposes (e.g. non-home based trips), the relationship between the average trip length and some other variables such as labor force and retailing floor area may be generated for better approximation.

(2) To a great extent, the accuracy of this simulation model depends upon the proper approximation of the mean and the variance of TLFD. More checking devices in addition to what have been provided in Part Three may be necessary.

From a broader view, simulation of the TLFD is not merely for the purpose of developing the F-Factors in the gravity model calibration process. The relationship between TLFD and the urban structure would provide, more importantly, a mechanism for decision-making in transportation planning. It may provide the answer to the problem of how and where the development of housing, shopping centers and other job-oriented land uses should be so that the average trip length for work can be minimized. It may also answer the problem of how to optimize the locational distribution of urban travel demand so that people would travel faster and more efficiently. These can not be answered until a comprehensive understanding of the urban growth impact upon this travel phenomenon is reached.

FOOTNOTES

1. Traffic Assignment and Distribution for Small Urban Areas, U.S. Department of Commerce, Bureau of Public Roads, Office of Planning, Urban Planning Division, Washington, D.C., p. V-4.

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4. ibid.

5. ibid.

6. Origin and Destination: Methods and Evaluation - 10 reports. Highway Research Record 114, 1965.

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31. Factors in Work Trip Lengths, op. cit.

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(1) Sioux Metropolitan Area, Iowa
(2) Champaign-Urbana, Illinois
(3) Salem, Oregan
(4) Idaho Falls, Idaho
(5) Hutchinson, Kansas
(6) Boise, Idaho
(7) Fayetteville-Springdale, Arkansas
(8) Sioux Falls, South Dakota
(9) Fargo-Moorehead, Minnesota
(10) Mankato, Minnesota
(11) Tallahassee, Florida
(12) Rochester, Minnesota
(13) St. Cloud, Minnesota

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A SIMULATION MODEL FOR

DEVELOPING THE TRIP LENGTH FREQUENCY DISTRIBUTION

by

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ABSTRACT

This paper deals with a simulation technique for developing the trip length frequency distribution. Part One reviews the general background of the gravity model in which definition of trip length frequency distribution and the iterative procedure of developing the traveltime function are discussed. Part Two is the model development. Factors associated in the variation of TLFD such as the level of transportation service, city size and land-use patterns are analyzed with some quantifiable means. Voorhees' research on work trip length was reviewed, by which the gamma distribution was introduced and used as the mathematical model for simulating the trip length frequency distribution. Part Three tested the statistical validity of the model which included the test of comparing the actual TLFD with the simulated TLFD for seven small cities and regression analysis for developing the mathematical equation to generate the input parameters of the gamma density function, mean and variance.

The simulation model developed in this paper provides two findings. One is that the population size can be used as the independent variable for estimating the average trip length (the mean of TLFD) for small urban areas. The other is that the gamma distribution fits the TLFD very well and thus can be used as the mathematical model for simulating the TLFD.