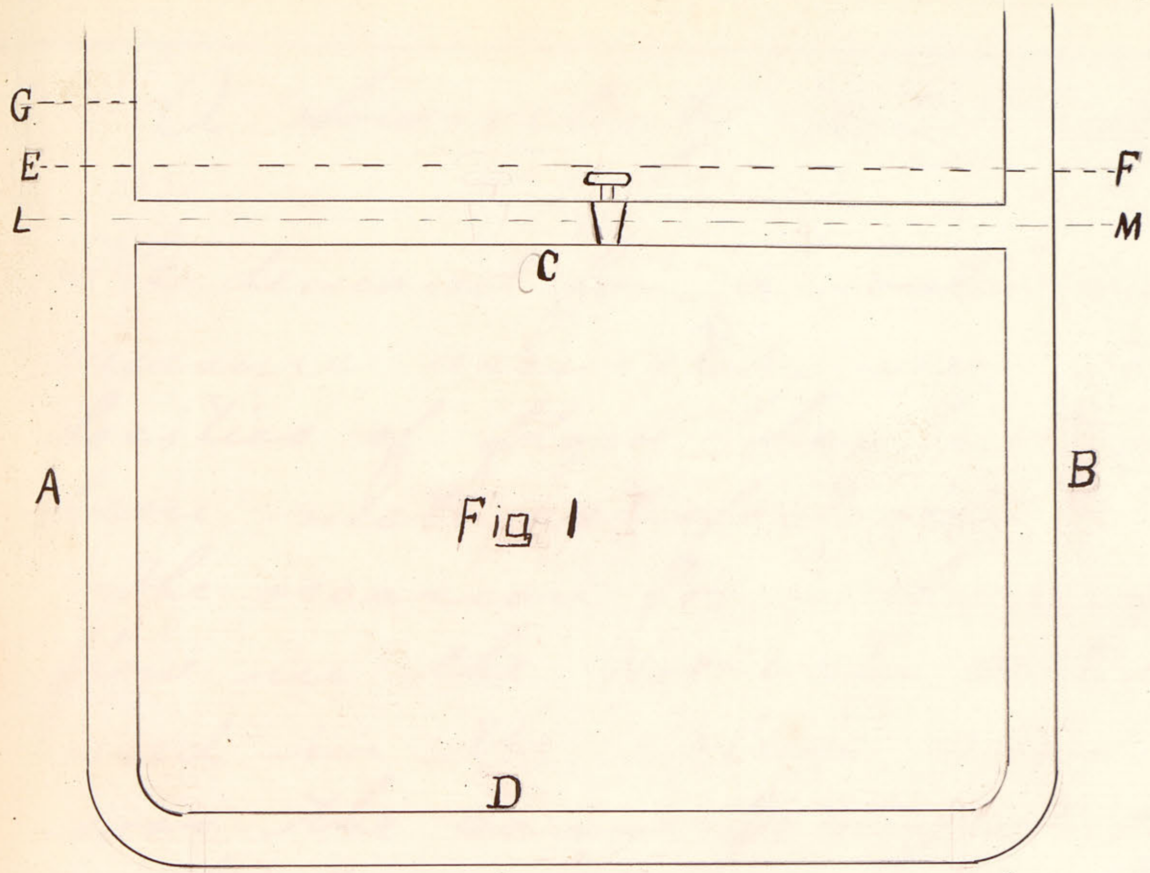


Fig 1

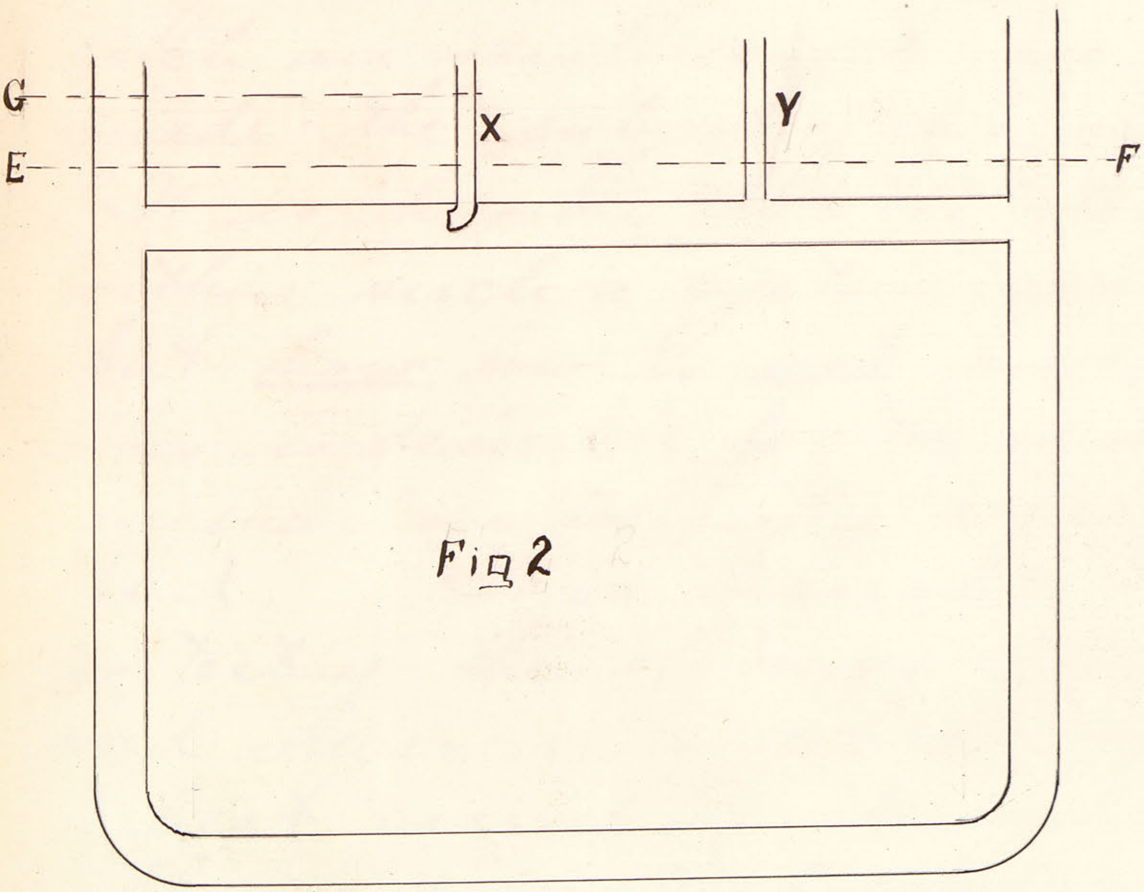


Fig 2

Fig 3

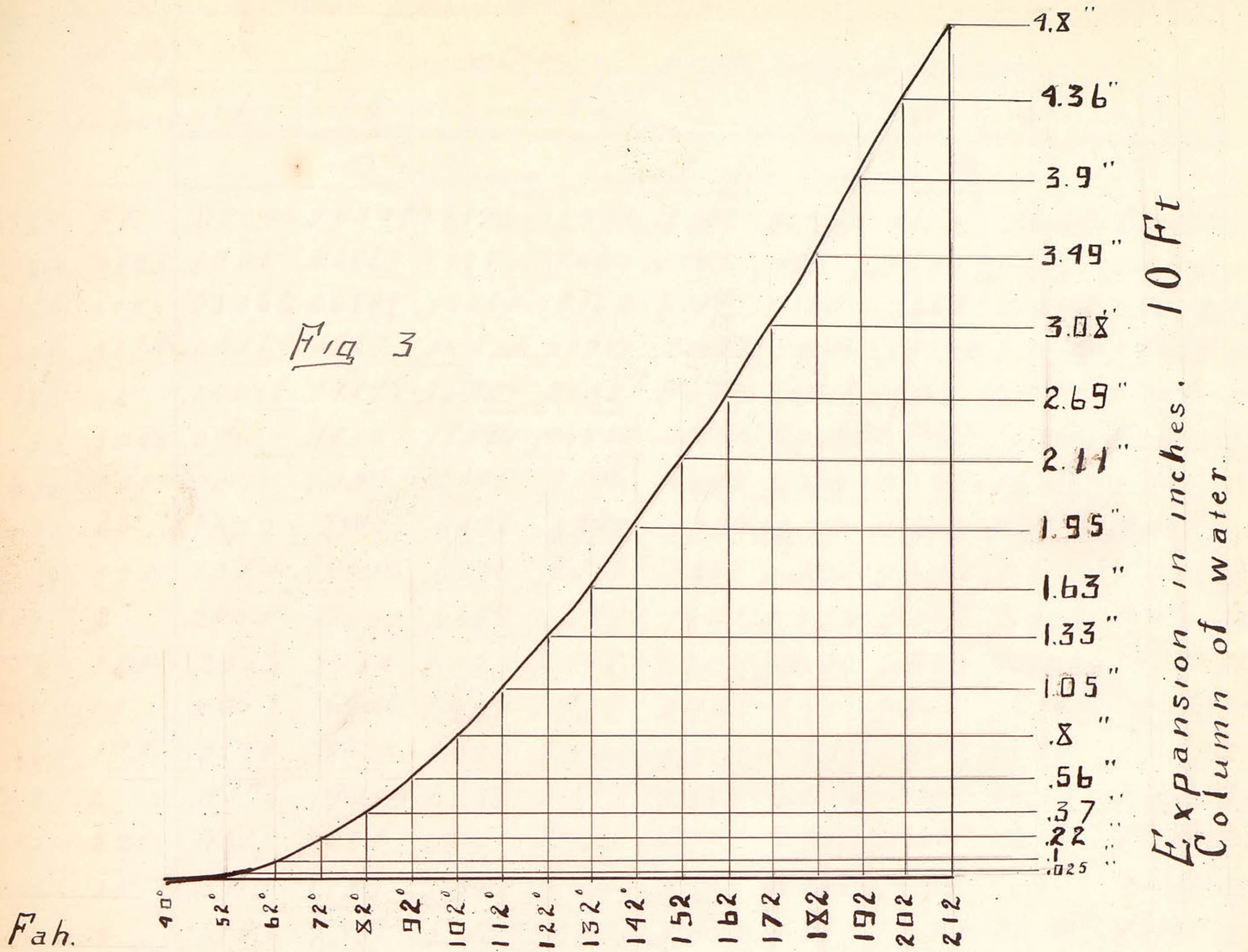

4.8"
4.36"
3.9"
3.49"
3.08"
2.69"
2.11"
1.95"
1.63"
1.33"
1.05"
.8"
.56"
.37"
.22"
.1"
.025"
10 Ft
Expansion in inches.
Column of water.
Fah.
40°
52°
62°
72°
82°
92°
102°
112°
122°
132°
142°
152
162
172
182
192
202
212

Fig 4.

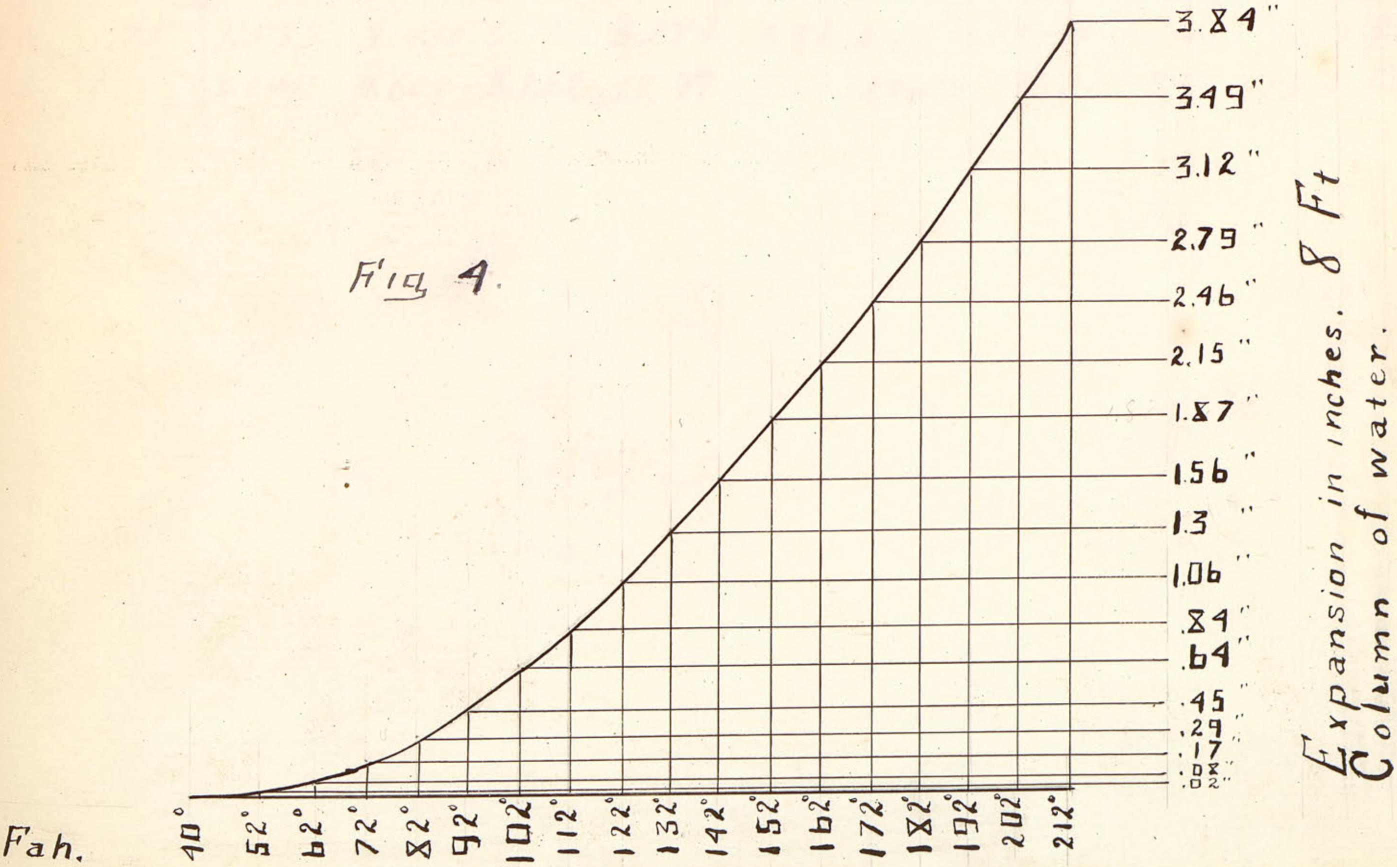

3.84"
3.49"
3.12"
2.79"
2.46"
2.15"
1.87"
1.56"
1.3"
1.06"
.84"
.64"
.45"
.29"
.17"
.08"
.02"
8 Ft
Expansion in inches.
Column of water.
Fah.
40°
52°
62°
72°
82°
92°
102°
112°
122°
132°
142°
152°
162°
172°
182°
192°
202°
212°

Fig. 5 Water passed by pipes. (From Thomas Box) (Revised)

$\frac{H \times d}{L}$	Velocity in feet per second	Diameter of pipe in inches. 1	1½	2	2½	3	4	5	6	7	8
		Gallons passed per minute.									
00000887	.01	.02044	.04599	.08176	.12775	.18396	.32764	.5112	.73584	.90156	1.30816
00001123	.0125	.0305	.06863	.1220	.19062	.2745	.488	.7625	1.098	1.4945	1.952
00001366	.015	.03066	06898	.12264	.19162	2759	.4905	.7665	1.1037	1.5023	1.9622
00001616	.0175	.03569	.0803	.14277	.22308	.32123	.57108	.89232	1.2849	1.7489	2.2843
00001872	.02	.04088	.09198	.16352	.2555	.36792	.65408	1.022	1.4716	2.004	26263
00002133	.0225	.046	.10347	.18396	.28744	.41391	.73584	1.1497	1.6556	2.2535	2.9433
00002402	.025	.0511	.1150	.2045	.3196	.4602	.8180	1.278	1.841	2.504	3.2704
00005437	.05	.1022	.2301	.4091	.6392	.9204	1.636	2.556	3.682	5.008	6.544
00009108	.075	.1534	.3450	.6136	.9588	1.381	2.454	3.834	5.523	7.512	9.816
0001341	.1	.2045	.4602	.8182	1.278	1.841	3.273	5.113	7.363	10.02	13.09
0001836	.125	.2556	.5750	1.023	1.598	2.301	4.090	6.390	9.205	12.52	16.36
0002394	.15	.3067	.6900	1.227	1.917	2.761	4.908	7.668	11.05	15.02	19.63
0003016	.175	.3578	.8053	1.432	2.237	3.221	5.728	8.947	12.88	17.53	22.95
0003702	.2	.4096	.9204	1.636	2.557	3.682	6.546	10.23	14.73	20.03	26.18
0004452	.225	.4601	1.035	1.841	2.876	4.142	7.363	11.50	16.57	22.54	29.45
0005266	.25	.5112	1.150	2.045	3.196	4.602	8.180	12.78	18.41	25.04	32.72
0007080	.3	6133	1.381	2.454	3.835	5.522	9.819	15.34	22.09	30.05	39.27
0011480	.4	.8180	1.841	3.273	5.113	7.363	13.09	20.45	29.45	40.06	52.36
001698	.5	1.023	2.301	4.091	6.392	9.204	16.37	25.57	36.82	50.08	65.45
003490	.75	1.533	3.450	6.136	9.588	13.81	24.54	38.34	55.23	75.12	98.16
005928	1.	2.045	4.602	8.132	12.78	18.41	32.73	51.13	73.63	100.2	130.9

Fig 6

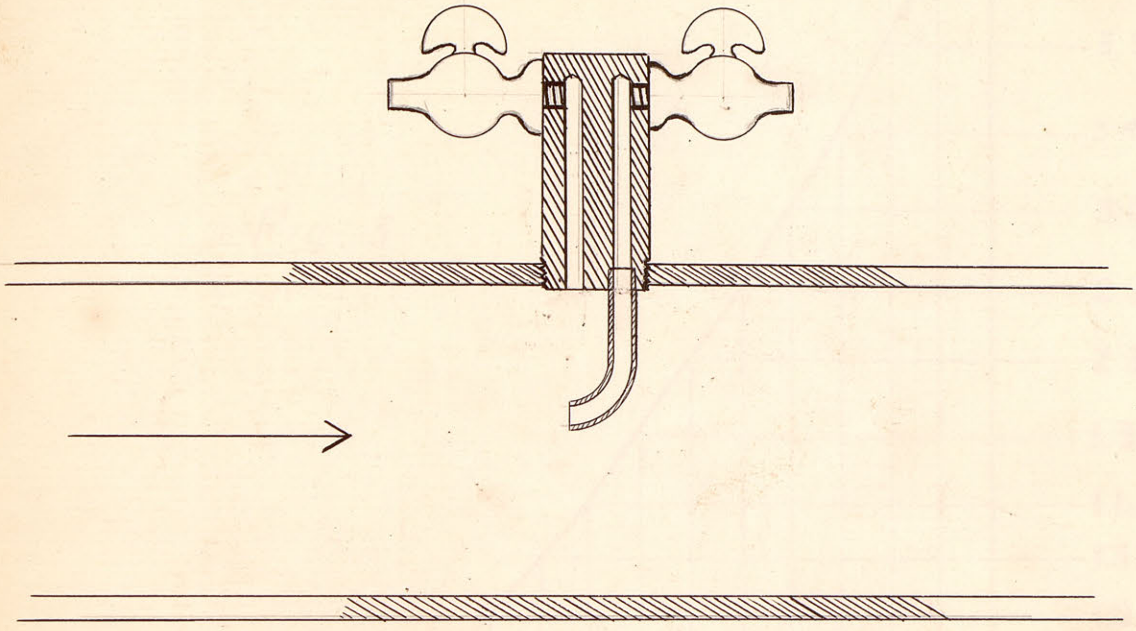


Fig 7.

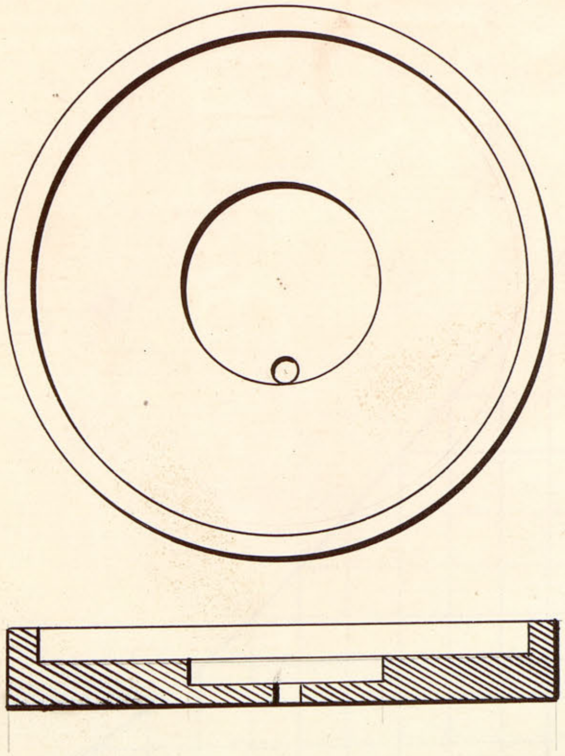
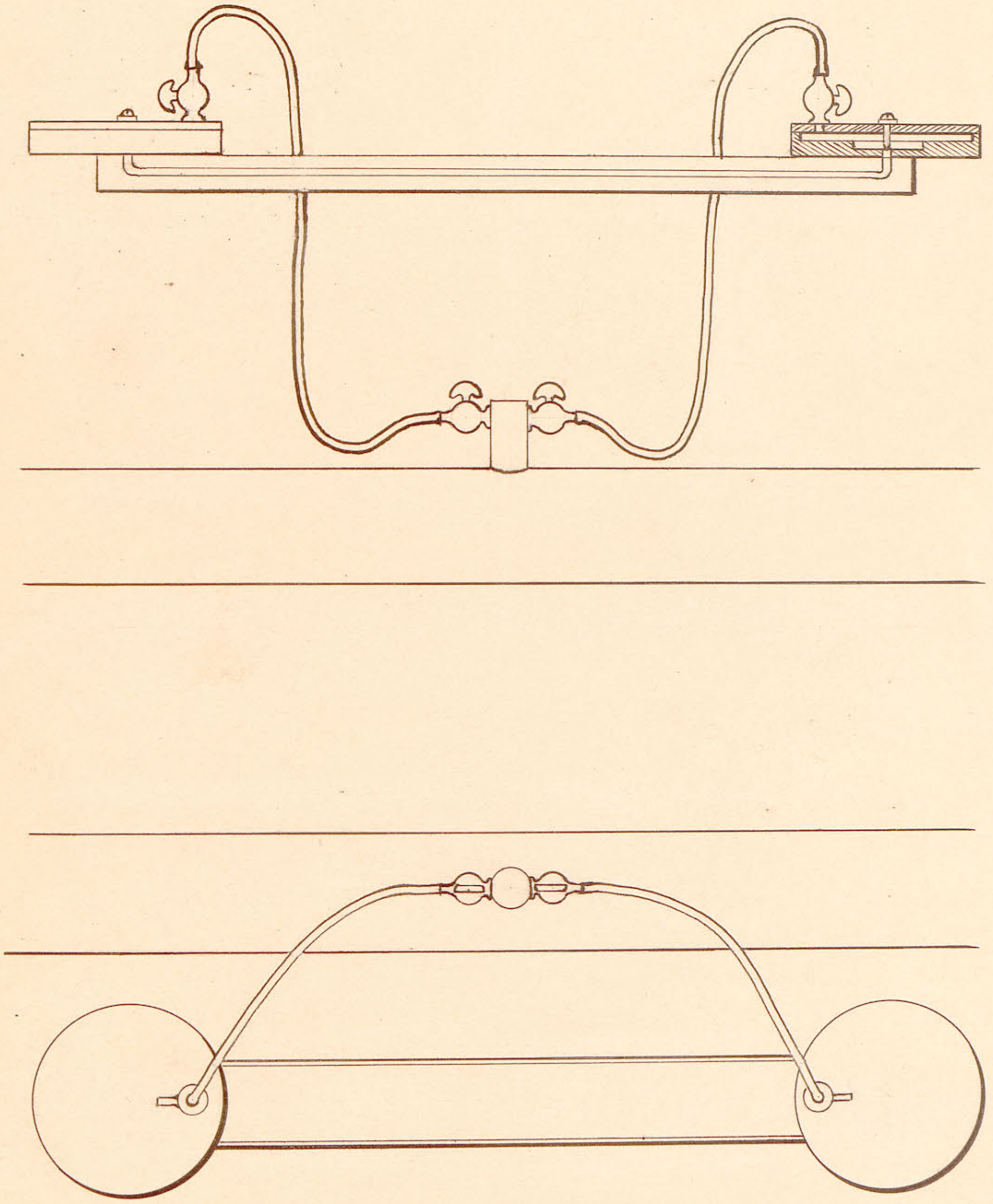


Fig X



A LOW-VELOCITY WATER-METER.

K. S. A. C.

JUNE 13. 1894.

CHAS. R. HUTCHINGS.

A Low-velocity Water-meter.

The demand for a water-meter to measure accurately very low velocities of flow has not as yet been met satisfactorily.

The occasion for measuring the flow in the Hot-water Heating System used in the College greenhouses, was the cause for my study of this subject and the construction of such an instrument has been made the subject of this production.

It is known that the velocities within such a system are very low but how low is not known. This is desirable for the sanitary engineer in designing a plant of this kind. It is desirable too, for testing the efficiency of a plant and measuring accurately the amount of heat carried by its circulation, that all variations and velocities be measured with accuracy. The meter, then, to answer these requirements must respond readily to any variation in flow, must be sensitive enough to indicate all appreciable flows and must not offer any considerable resistance or obstruction to the movement of the water.

The hundreds of commercial meters, on the market and off the market, which have been patented, are by Walter G. Kent divided into two classes — "Positive or those which actually measure the water passed through them," and "Inferential or those which indicate the velocity" of flow, from the records and readings of which, the amount of water passing can be calculated.

The objection to all that I have found described is that the friction of moving parts is so great and being a constant quantity under known conditions but an unknown variable for the slightest variation of conditions, that for very low velocities they are not to be depended upon. Neither are they alike accurate for low and high velocities nor pressures of great variations. The demand then is for one in which the friction is of known value at all times or better still one in which friction would not affect the record or reading at all. The friction of a gas or liquid in motion through a conductor is proportional to the velocity with which it moves, while the friction of solids is independent of velocity. Therefore if a gas or a liquid be used and made to move at a low velocity the friction which influences its movement is reduced until at a velocity of zero its friction will be zero. If then a gas or a liquid tends to assume a position on account of pressure it will ultimately assume that position regardless of friction.

But a gas is liable to greater variation on account of temperatures than liquids and its use is not indicated on account of the errors it may introduce from that source. Liquids are not subject to so great variation on account of changes in temperature, and by their use the errors due to this cause are avoided.

Having determined this much of the character of the instrument we wish to determine its capacity or the limits within which it is to be used. To do this it is necessary to know the causes of circulation and conditions which influence its velocity.

If an apparatus as shown in fig. 1. be considered we shall find an explanation of the primary cause of circulation in a hot-water heating system. Let the apparatus be filled with water to the level E.F. If the temperature is uniform throughout the density is the same and the whole is in equilibrium. If however heat be applied to A the water in that column will begin to expand as its temperature rises and its surface will tend to rise above E. If C were closed so that no water could pass through, the surface of the column in A would rise abobe E to some height as G. The columns being of equal weight, the pressure at H and K would be equal and the same throughout the lower pipe D. The pressures at L and M however are not equal for the reason that the weight of water in column B due to greater density must balance the weight due to greater height of the column A and the greater density of part of the column B or MF can not balance the part of column A or LG. Therefore the pressure at L is greater than at M by the amount due to the head EG multiplied by its density as compared with column B. If now the pipe C be opened a movement will take place from L toward M to restore the equilibrium of the upper parts of the columns. This lowers the surface from G and unbalances the the pressure in D by the decreased weight over H. A movement through

D from K will tend to restore the equilibrium of the whole column. If heat be continually applied to A this movement will continue until the water in both arms is at the same temperature and density or until the boiling point is reached, provided no radiation of heat took place. But if heat be continually given off from B it will remain at a lower temperature than A and the circulation will continue as long as heat is applied.

If A be made in the form of a boiler and B made up of a number of folds of pipe some hundreds of feet in length the conditions of balanced and unbalanced water columns are not changed. The vertical height of the warm ascending and colder descending columns is the same as in the apparatus described and by the laws of Hydraulics the movement will continue.

It will be understood then that the greater difference there is between the weights of these unbalanced columns of water the greater will be the tendency to restore equilibrium and the greater amount of energy necessary to restore it and the greater the amount of water will pass in a given time. According to the laws of falling bodies the velocity at any time is equal to the square root of twice the acceleration due to gravity into the height or distance fallen.

$$v = \sqrt{2gh} .$$

Now in figure 1 the column of water EG when unbalanced by pressure in B may be regarded as a falling body in its tendency to restore equilibrium. The velocity it would gain then in falling the distance EG would be equal to $\sqrt{2g \times EG}$.

As liquids transmit pressures and movements perfectly this velocity would be imparted to the water in C as it tended to rise above

F in the column B. Therefore the velocity of flow in C equals $\sqrt{2g \times EG}$ or $\sqrt{2gh}$ if $h = EG$. If then a constant difference in height of A and B can be maintained a constant velocity will be obtained. But any value for EG may be called h and the velocity obtained in any case will be represented by $\sqrt{2gh}$. or as $g = 32$, $\sqrt{2gh} = 8\sqrt{h}$.

To find the amount of water passing in a given time it is only necessary to know the velocity during that time and the capacity of the pipe through which it is flowing.

Now to measure the velocity with which it flows, advantage must be taken of some effect of its velocity. This is found in the measurement of its energy or a portion of its energy.

The energy of a moving body is its capacity for doing work. From Mechanics we learn that it is expressed by the formula. $E = \frac{m}{2} v^2$ when m = mass, v = velocity and E = Energy

If this energy $\frac{m}{2}v^2$ be the energy of the moving water, its effect can be noted by giving it a chance to expend its energy in work by supporting a column of water.

In the apparatus Fig. 1 we may insert two pipes X and Y as shown in Fig 2. The water will rise in both to the common level EF. by hydrostatic pressure. But the energy of the moving water will press on the contents of the bent tube X and its effect transmitted in the line of least resistance, upward through the tube, will cause the surface to rise above the line EF. The energy then is expended in sustaining this column above the common level. To ascertain the height of this column we will supply in the equation $E = \frac{m}{2}v^2$, the values of m in terms of weight and gravity $\frac{w}{g}$; and for E its value in terms of work or weight raised to a height Wh.

This gives $Wh = \frac{W}{2g}v^2$, or $W2gh = Wv^2$.

Cancelling the common factor W and solving for h we find it to be. $h = \frac{v^2}{2g}$.
Then the energy possessed by a particle of water moving with a velocity V will sustain a like particle at a height equal to the height from which the first must fall to obtain its velocity.
We saw that the water in Y rose to the common level E F due to the hydrostatic pressure and as the movement of the water in the pipe C is at right angles to the opening of Y there is no effect from its energy upon the column above.
The difference in height then between the columns in X and Y is due to the dynamic or kinetic energy of the water in motion.
According to table Fig 3 based on Mr Baldwins Hot-water Heating and fitting we see that the expansion per foot of vertical water column per degree of increased temperature, is very slight, being only .48" for 212 degrees increase.

Assuming then a constant difference of temperature amounting to 30° (taking the temperatures of flow at 192° and return at 162°) we see by the table that the expansion per foot of vertical water column is .121 inch. Taking the height of our system between the lower part of the boiler where heat is applied, and the highest point of direct circulation, to be eight feet we may obtain the head of water in the heated column above the surface in the colder column.

The expansion per foot of water column per 30° difference in temperature being .121 inch, for eight feet it will be .121 x 8 or .968" or .08066'.

This then is the element which governs the theoretical flow in our system. If $v = \sqrt{2gh}$, the velocity of circulation under theoretical conditions and at the temperatures given is found by

$$v = \sqrt{2g \times .08066} \quad \text{or} \quad v = 8\sqrt{.08066}$$

$$v = 2.272 \text{ ft per second.}$$

But friction, the disturbing element

here as well as elsewhere, opposes the velocity which this difference in temperatures would give. It is so variable a quantity that its value must be determined empirically. Thomas Box in his work on Practical Hydraulics has indirectly done this by giving a table of actual discharges of water from pipes under given heads in connection with length of pipe and diameter of same.

A portion of this table is given here Fig 5 with additions to include very small velocities. The original took no account of flows lower than .025 ft per second. It has been extended to include flows of low as .01 ft per second.

The formula he uses for lower velocities than 1.5 ft per second measured by actual discharge is

$$\frac{(v + .0816)^2 - .00665}{196.24} = \frac{H \times d}{L}$$

In the above formula

V = velocity of dischage in ft per second.
H = head of water in inches
d = diameter of pipe in inches
L = length of pipe in inches.

Bends or turns in the pipe also consume some energy by the friction due to change of direction. Mr. Baldwin states as a practical rule based on a number of experiments that ordinary bends made by the common elbows in use will offer as much resistance as a length of pipe equal to 38 times its diameter. That is an ordinary 3" elbow will offer a resistance to flow equal to 114 inches of 3" pipe.

The form and character of openings through which water must pass from the boiler to the pipes is a great influence too upon the velocity with which it passes out. Estimates as to the amount of this resistance may be found in several works on hot-water heating and Hydraulics.

By inspection of the system in our greenhouses we find that the shortest length of any circulation which we wish to measure and the one in which the circulation is quickest, is, including its bends, equivalent to 100 feet of plain pipe 3" in diameter. We wish to find the velocity corresponding to this length and diameter of pipe in order to determine the highest velocity we shall be obliged to measure.

Assuming then a head of water due to a difference of temperature of 60° (212°-152°) (the greatest difference likely to occur in any part of the system and referring to our table of expansions we find it to be .269 inch for each foot of vertical water column. Multiplying this by the height of the system in feet we have the total head expressed by .269×8= 2.152 inches. Supplying this in

the formula $\frac{H \times d}{L}$ we have
$\frac{2.152 \times 3}{1200} = .00538$. Referring to our table we find the nearest number to this for the value of $\frac{H \times d}{L}$ is .005928 Opposite this we find the actual velocity for this head of water, diameter and length of pipe, to be 1 ft per second This then represents the highest velocity which it will be necessary for our instrument to indicate. Therefore the limits of velocity to be measured are a maximum of 1 foot per second and a minimum as low as any appreciable velocity occurs. This we assumed to be .01 ft per second.

The velocities obtained are, on account of friction of entry into, length of and bends in, the pipes, considerably lower than the difference in temperature would indicate.

We saw that the energy of this flow at any velocity was equal to the expression $\frac{m}{2} v^2$ and that it would sustain a column of water at a height h. Theoretically this h was equal to the head produced by expansion of the heated column but the element of friction having entered, the velocity has been retarded and this height is not equal to that produced by expansion of the heated column, but is due to the energy of $\frac{m}{2}v^2$. Now if we could read directly this height in a tube inserted as x in Fig 2 we would have an index to the velocity, so far considered as a literal quantity $v = 8\sqrt{h}$. or $h = \frac{v^2}{2g}$. We saw that the probable velocity was never greater than 1 ft per second. Then $h = \frac{1}{64}$. $\frac{1}{64}$ ft $= \frac{3}{16}$ inch. The value of h then is so small that an accurate reading of slight variations can not be made directly. Some method of multiplying the reading must be used.

After consideration of several different methods the following was decided upon.

Two cups of mercury were supported at exactly the same level by being attached to a rigid frame fitted with leveling screws. The connection with the interior of the system is made through openings in a plug tapped into the side of a pipe as shown in fig. 6. To one of these openings is attached the bent tube, corresponding to X in the experimental apparatus, and turned with its opening toward the current. The other is merely a straight opening into the pipe and into which the water flows by hydrostatic pressure alone. These connections are fitted with valves and a hose connection made with similar valves on the mercury cups. The valves on the plug when closed allow the removal of the hose and instrument at will while the valves on the mercury cups confine their contents securely while moving the instrument.

If while the instrument is

connected, the pressure be admitted to the mercury cups from the openings in the pipe, the conditions will be these. In the cup connected with the plain opening, the pressure will be wholly due to the Hydrostatic pressure of the system but in the other cup which is connected with the bent tube inserted in the pipe the pressure is partly due to the hydrostatic pressure of the system and there is the added pressure due to $\frac{m}{2}v^2$ which would sustain the column of water h in the experimental apparatus. Now if an equal quantity of mercury be in each cup and they be connected by a glass tube the excess of pressure in one cup will cause a movement of the mercury from this cup to the other through the tube connecting them. But any movement of this kind lowers the surface level of the first and raises it in the second, thus producing a vertical column of mercury to be sustained by the excess of pressure in the first. The specific gravity of mercury being 13.5 this column when compared with the height of water column h will be represented by $\frac{h}{13.5}$. The small movement of the water column then is still further reduced by using mercury.

But to multiply the movement of the mercury in the cups, the tube may be made very small compared to the surface on which the pressure acts. By doing this and enclosing a bubble of air in the tube between the ends of the mercury columns a movement of the mercury can be readily observed by noting the position of this bubble. This movement in the glass tube as indicated by the air bubble within it we will call the reading or R. Its value can be calculated by letting the movement in the cup which we saw was due to the excess of pressure, be represented by m. Let S = the surface in the cup on which the pressure is acting, and a the cross sectional area of the glass tube in which the movement is observed. Then $R : m :: S : a$. The movement in the tube is to the movement in the cup as the inverse ratio of their areas.

$$Ra = mS.$$
$$R = \frac{mS}{a}.$$

But we saw that the movement m in the cup was $\frac{h}{13.5}$. Then

$$R = \frac{\frac{h}{13.5}S}{a} \quad \text{or} \quad 13.5Ra = hS.$$

But h is equal to $\frac{v^2}{2g}$. Then

$$13.5Ra = \frac{v^2}{2g}S. \quad \text{or} \quad R = \frac{v^2 S}{13.5 a\, 2g} \quad ×$$

Selecting a tube small enough that the mercury will not unite in it if once separated by a bubble of air, and measuring its area it was found to be .002 sq inch. Then

$$R = \frac{v^2 S}{13.5 × .002'' × 64'} = \frac{v^2 S}{20.898''}.$$

The quantity in the denominator of this term being made up of constant factors is a constant and may be represented always by c or 20.736".

Then $R = \frac{v^2 S}{c}$, or

$$S = \frac{Rc}{v^2} = \frac{R\ 20.736''}{v^2}$$

It is desirable that R shall be an appreciable quantity for the lowest velocity it is intended to measure.

For this minimum velocity we will assume .1 inch per second and for the reading to indicate this velocity we will assume one hundredth ($\frac{1}{100}$) inch.

Supplying these values in our formula we have $S = \frac{.01 \times c}{(.1)^2}$.

$$S = \frac{.01 \times 20.736}{.01}$$

$$S = 20.736 \text{ sq inches}$$

By making the mercury cups circular and 5.1" in diameter this surface is approximately obtained.

Fixing this area then at 20.736, the lowest limit of the instrument capacity is fixed at one tenth inch per second.

After fixing this lowest limit we wish to find its highest limit or rather the reading for its highest limit which we calculated to be 1 foot per second. In the formula $R = \frac{v^2 S}{c}$, we supply the values determined for v, S and c.

$$R = \frac{(12)^2 \times 20.736}{20.736}$$

$R =$ 144 inches or 12 ft.

This would indicate that the glass tube connecting the mercury cups be 12 ft long in order to measure all velocities between .1 inch and 12 inches per second. This is inconvenient and may be avoided by making a second value for S which we will call S'.

Now the tube selected for use as of convenient ~~length~~ length and size is 22 inches long, therefore all readings of velocities by means of the air bubble must be within that length or distance.

To establish S' supply in the formula $S = \frac{Rc}{v^2}$ the maximum value of R and ~~of~~ v and solve for S'. $S = \frac{22 \times 20.736}{144} = 3.168$ or a circular area of about 2" diameter. The two values S and S' may be

practically used by means of a cup shown in Fig. 7
Then by filling it with mercury until the surface is bounded by the upper part of the cup, the value of S is used and low velocities may be read in the displacement of the air bubble in the tube.
Then by draining out enough mercury to lower its surface into the smaller part of the cup, the value of S' is obtained and higher velocities may be read in the same displacement. It will be seen that the same movement in the tube when S' is used instead of S, signifies a greater energy due to a greater velocity of flow. For the movement m in one case is multiplied by the ratio of .002 : 20.736 while in the other case it is multiplied by the ratio of .002 : 3.168.
The instrument as completed is shown in Fig 8 with one of the cups shown in section.
The calibration of this instru-

ment of course, to be accurate must be done experimentally. This takes account of any variations in the bore of the tube used, and also of friction in the various parts of the instrument, or of inaccuracies of construction. If it be attached to an experimental pipe and a known quantity of water passed through at a known constant velocity the energy of the moving current is manifested by a certain displacement of the air bubble from a marked point or zero point which marks its position when no current is flowing.

As the energy of this flow varies with the square of the velocity, each velocity will cause its own displacement. The marking to indicate different flows will be made on the upper scale when S is used as the surface exposed to pressure, and on the lower scale when S' is exposed to pressure.

By the use of this instrument

lower velocities can be measured than with most commercial meters. But its principal use will be on a closed system where circulation must not be retarded by any considerable amount and where accurate indications are wanted as in testing the quantity of heat carried by the system under all working conditions.