THE EFFECT OF THE LEARNING PROCESS IN DETERMINING ECONOMIC ORDER QUANTITIES

by

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INTRODUCTION

Whenever supplies are ordered, whether they are for the office, for production, or for research, whether they are purchased or manufactured, there is a cost associated with each order. In the case of purchased items, it is called simply order cost, but in the case of manufactured items it is usually called setup cost. The setup cost embodies such costs as machine setup, associated paperwork, idle production time during change over, and other tangible and intangible costs.

These procurement or setup costs can sometimes be extremely high; therefore, it is natural to place as large an order as possible, but a hidden factor in this reasoning is the resultant cost of carrying and holding this inventory, be it manufactured or purchased. There then must be some means of balancing these two costs, that is, production costs versus inventory costs, and the method developed approximately a half-century ago is called an economic order quantity. The term "economic" used in the order quantity context should carry the connotation of "spending or saving resources or time to the best possible advantage", as defined by Maynard (8).

Over the past fifty years there have been many economic order quantity formulae derived. Many of these formulae were derived for specific cases and were not general formulae. Since it is impossible to include into a single formula all associated costs, only the relevant costs are included. Also there are always some simplifying assumptions made so the formula can readily be used. It is the effect of one of these assumptions that we want to investigate in this thesis.

Man has known since the beginning of time that as he performs certain tasks he becomes more proficient in each succeeding repetition of these tasks, but it was not until recent times that this decrease in production time was predictable. The decrease in production time is called learning and, as noted by Andress (1), there is a "rising productivity." It is the affect of this rising productivity verses linear productivity in determining economic order quantities that is under investigation.

It was discovered just prior to World War II that the rate of improvement in manufacturing is predictable. It was found that as the quantity of units produced is doubled, the time required to produce each of these units will decrease by a constant; thereby, making it possible to predict production time on a future item. The discovery was made in the aircraft industry where the total item cost is large and a slight rise in productivity will produce a noticeable decrease in the total item cost. The decrease in total item cost, including setup cost, production time, direct labor cost, carrying cost, and holding cost, holds true for most types of industry such as aircraft, electronics, textile, metal working, or candy making. As will be shown in subsequent sections, the difference in annual cost obtained by using the learning curve production rate as opposed to the linear production rate is significant, and could have a detrimental effect on predicted future costs and budgeting. This could be especially true if the project is on a bid contract basis. It could mean a loss of business or a projected deficit if the project is contracted. The effect of the learning process in determining economic order quantities will occupy the remaining portion of this thesis.

BACKGROUND

Economic Order Quantities Under Linear Production Rate

Industry has long used the economic order quantity to guide the scheduling of production and inventory control. This is particularly true of continuous production and demand industries, such as electronics, household goods, and appliances. Usually the manufacturer has numerous product lines and it is impossible and impractical to have every different item on a continuous production basis. The most logical decision would then be to stock these items in inventory and schedule production to maintain an adequate inventory level.

What is the proper production and inventory schedule? Certainly it is not economical to produce a full year's demand for each item during each production run. The most feasible solution would be to find the minimum annual cost policy incurred by each item and use that policy. The optimal policy would take into consideration the relevant costs, the production rate, and the usage or demand rate. In order to arrive at an optimal policy a mathematical model will be constructed for the situation.

As in the derivation of any model, the boundaries must be defined by some stipulations and assumptions. The following assumptions will be followed throughout the paper with a few exceptions to be noted later.

1. Assumptions pertaining to the production-inventory mechanism:
   a. Demand is continuous at a linear or constant rate R with dimensions of units per direct labor man-hour. The inventory level will remain positive, making the output rate equal to the demand rate.
   b. Production occurs at a constant rate A, where 1/A is greater than R, and the dimensions of A are dlmhrs per unit.
   c. The amount produced each production cycle (X) is constant.
   d. All time units (t) are measured in direct labor man-hours (dlmhr).
   e. The process is continuous.
2. Assumptions pertaining to the measure of effectiveness (cost):
   a. There is a fixed charge (S) for each setup. This cost includes machine setup, necessary paperwork, idle time necessitated by the setup, and so forth.
   b. There is an inventory holding cost (H), which includes only the cost of physical storage, such as warehouse cost, warehouse labor, warehouse overhead cost, and so forth. The dimensions for H are dollars per unit per dlmhr.
   c. There is also an inventory carrying cost (id) which is the interest (i in per cent per dlmhr) on the original cost of the product (d in dollars per dlmhr); that is, the interest on the idle capital. This cost will be

directly proportional to the direct labor man-hours consumed per unit.

d. The original material costs have been disregarded in this report in order to simplify the resultant equation.

The model described by the above assumptions is shown in graphical form in Fig. 1. The figure shows the inventory level under a linear production input during any production cycle.

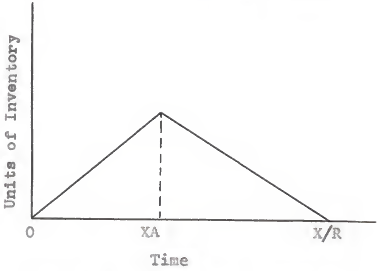


Fig. 1. Inventory level under a linear production input.

From Fig. 1 we see the cycle duration is X/R, and the length of production during the cycle is XA. Since R and X are constants, the cycle duration will also remain a constant. The mathematical model which describes the above situation is given by the total cycle linear cost equation,

$$TCLNC = S + (H + idA)\left(\frac{X^2}{2R} - \frac{X^2A}{2}\right) \qquad (1)$$

The derivation of equation (1) is included in the Appendix. Consequently the total annual cost is simply a summation of each cycle cost during the year. Since each cycle has the same cost then the total annual linear cost is,

$$\text{TALNC} = (\text{TCLNC})(\text{Number of cycles per year}) \qquad (2)$$

This is, then, the classical order quantity model described by Hanssmann (6).

Most inventory models are simplified to a per unit cost, but for comparative reasons, which will be manifested later, the linear annual cost is the desired equation.

The desired solution to equation (2) is the value of X which will produce the minimum annual cost. In the linear case an explicit minimal solution can be found, but a simulation technique will be used instead. By varying the different parameter values

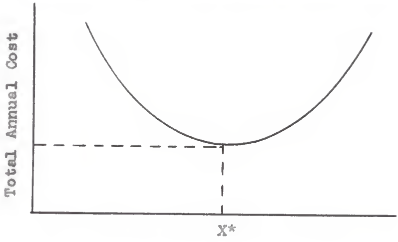


Fig. 2. Total annual cost curve for a linear production input.

of equation (2) a parabolic curve is produced as shown in Fig. 2. It can be seen from Fig. 2 that the optimum X, X*, value can be found that will give the minimum annual cost. Fortunately, a cost curve of the type shown in Fig. 2 is not extremely sensitive to deviations from the optimum X value.

The preceding discussion was presented as a background for the total cost equation under learning to be derived later. A comparison will be presented to show the difference in the total annual cost obtained by using the linear input opposed to a learning input.

Learning and Retention

The psychologist along with the industrial engineer is interested in the subject of the why's and how's of learning. The subject of learning is indeed a complex subject and considerable research has been done on the many facets of learning, but surprisingly enough, the facet of long-term retention of motor skills has not produced its share of technical papers. In fact, the last century has produced only about two papers per year, which is a drastically small number in proportion to the importance of the subject.

The question of how or why does a person learn is not an easy one to answer. In fact, the question has not been explicitly answered. There have been many theories presented, with some of these being contradictory. The answer to the original question is probably some distance in the future, but this does not obscure the fact that learning does take place. Even though it is not known why or how learning takes place, through empirical means it has been established that learning is predictable. For instance, in the industrial environment it has been found that learning takes place in an exponential manner. The rate or slope of this exponential decrease in production time varies with the complexity of the task, but it remains that learning is present and predictable.

It has been found that many things could cause the learning process to deviate from the exponential curve. This is especially true when production on an item is commenced for the first time. The early stage of production is a period of many changes. The methods department is still in the experimental stage and has not decided on a specific jig or fixture. The research and development engineers are still making some last minute revisions, and there are many more factors which could affect the learning process. Whenever one of these changes go into affect, it could introduce a strange and new challenge to the operator and produce a regression in learning. It is changes of this type that produce an "S" shaped curve instead of an exponential curve.

Regression can be produced by many factors. As seen above, production, procedure, and product changes all produce to a varying degree some type of regression. Regression can also be produced by non-reinforcement of the learned skill. This type of regression is found in industry when production is stopped for a finite period of time and then resumed. If the operator has been removed from production and has had no reinforcement on the original production task during the time lapse then there most certainly will be a regression in learning. If the operator is put on an associated task the regression may be very small and insignificant.

If production is continued only during a portion of each cycle on a specific product, will the regression be the same each cycle or will it vary? The answer to this question could well be the subject for an extended research project. Since the regression is of secondary importance in this paper, it has been assumed that the regression during each cycle of production will remain constant. This assumption could very easily be valid since the off production time for each cycle is a constant for a particular item. The effect of the regression on the total annual cost will be shown in a subsequent section.

In this section, we have had a brief look at learning and some of its implications. Also the subject of regression was presented to give a background for a total annual cost equation under learning. In the next section, we will investigate learning in an industrial situation.

THE MANUFACTURING PROGRESS FUNCTION

Development of the Unit Formula

A brief review of learning and retention was presented in the previous section. In this section we will see how learning can be used in manufacturing. Learning, when applied to manufacturing, is usually used in the form of a curve for estimating future production time. The curve used in this instance is called the learning curve; a more descriptive name sometimes used is the manufacturing progress function. The manufacturing progress function is based on the relationship that the time to complete a unit of production will decrease by a constant percentage with each doubled quantity of production. This empirical relationship was first put forth by T. P. Wright in 1936, in a related but somewhat different form. It is well to note at this point that the ideas in this section draw heavily on Torgerson (13).

The progress function states that the second unit will take eighty per cent as long to produce as the first unit, and the fourth unit will take eighty per cent as long to produce as the second, and the eighth unit eighty per cent as long as the fourth, ad infinitum. It will be noted that the eighty per cent, or a twenty per cent reduction, was arbitrarily chosen. The progress function could be seventy per cent, eighty-five per cent, ninety per cent or any other feasible figure.

One might get the first impression that the reduction in production time could not go on indefinitely. However, this is not true. It should be noted that the reduction holds true for doubled quantities only. This means that if the annual production of an item is 1,000,000 units and 10,000,000 units have been produced to date, it will be another ten years before the full percentage improvement is realized. This type of improvement is not unrealistic. Take the automobile industry for an example. They could very easily be in the position described above. If the manufacturer is operating on an eighty per cent progress function, he will have to improve only two per cent per year for the next ten years to realize the full twenty per cent improvement. It would be safe to state that technology is moving much faster than a two per cent improvement per year. This example has been greatly oversimplified, but the idea has been conveyed. One gross simplification is that the world today is far from static and it is doubtful if the same curve could be used for a period of ten years.

The progress function has gained initial acceptance and one reason for that is the ease with which it can be applied. All one needs to know is the initial unit production time, the slope of the progress function, and some graph paper. Assuming an eighty per cent progress curve with unit number one taking 100,000 direct labor man-hours to produce, the graph of the unit direct labor man-hours can be plotted as shown in Fig. 3.

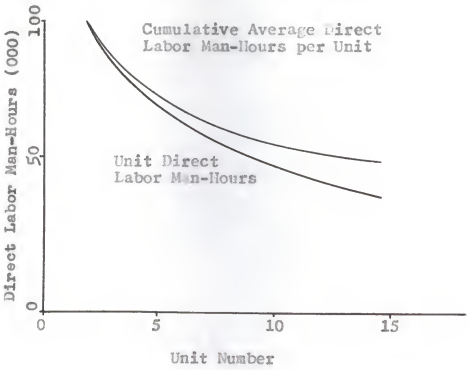


Fig. 3. An eighty per cent progress function with unit number one consuming 100,000 dlmhrs.

From Fig. 3 the direct labor man-hours for any future unit can be found, but to plot Fig. 3 it was necessary to calculate every needed point on the curve. A convenience not noted until now can make the plotting of the curve considerably easier. Since the curve is exponential, by taking the log of every point a straight line can be effected. This relationship is also visible by plotting the points on log-log graph paper. It will be noted that the unit curve plots a straight line, Fig. 4, but the cumulative average curve is curved until one gets past approximately unit number twenty and then it is also straight. The unit curve is of primary importance here.

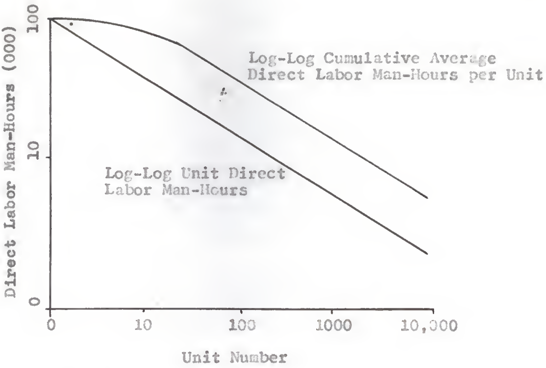


Fig. 4. An eighty per cent progress function with unit number one consuming 100,000 dlmhrs.

The straight line unit curve is very easy to construct and can also be quite helpful to production engineers. By plotting actual consumed time against an estimated unit curve, it is very easy to observe slight deviations from expected values and corrective action can be taken to bring the costs back into line. Since the curve is easily constructed, it is also quite useful for costing and bidding, especially on large equipment such as airplanes.

Since the analytical solution of the learning curve is of prime importance to a total annual cost equation under learning, a complete derivation will be presented.

The empirical relationship, stated before, on which this analytical derivation is based is as follows, as the production quantity of units is doubled, the number of direct labor man-hours required to produce these doubled units will decrease by a constant percentage.

As in any derivation, symbols and definitions are needed. The following terms will be defined as:

X = the number of units produced, counting from the first unit.

Y_x = the number of direct labor man-hours required to produce the Xth unit.

A = the number of direct labor man-hours required to produce the first unit.

N = the per cent improvement expressed as a decimal, for example, for an eighty-five per cent progress function, $N = .85$.

$$n = \frac{\log_{10} N}{\log_{10} 2}$$

A general equation involving X and Y_x can be derived as follows:

$$Y_x = AN^o \qquad \text{where } X = 2^o$$

$$Y_x = AN^1 \qquad \text{where } X = 2^1$$

$$Y_x = AN^2 \qquad \text{where } X = 2^2$$

$$Y_x = AN^3 \qquad \text{where } X = 2^3$$

thus,

$$Y_x = AN^a \qquad \text{where } X = 2^a \tag{3}$$

taking the logarithm of both equations,

$$\log Y_x = a \log N + \log A, \text{ and } \log X = a \log 2$$

and solving both equations for a,

$$a = \frac{\log Y_x - \log A}{\log N}, \text{ and } a = \frac{\log X}{\log 2}$$

equating the two equations,

$$\frac{\log Y_x - \log A}{\log N} = \frac{\log X}{\log 2}$$

solving,

$$\log Y_x - \log A = \frac{\log N}{\log 2} \log X$$

by original definition, $n = \frac{\log N}{\log 2}$, therefore,

$$\log Y_x = n \log X + \log A \qquad (4)$$

taking the antilog of both sides,

$$Y_x = AX^n \qquad (5)$$

Thus, equation (5) is the manufacturing progress unit formula. In a following section several examples in the use of the unit formula will be given.

The Cumulative Formula

In the development of a total cost equation it will also be necessary that the cumulative direct labor man-hours be known; therefore, in this section a cumulative formula will be developed.

Let,

T_x = the cumulative number of direct labor man-hours required to produce X units of production. The dimensions of T_x will be direct labor man-hours.

The cumulative formula is then,

$$T_x = Y_1 + Y_2 + Y_3 + Y_4 + \dots + Y_x = \sum_{1}^{x} Y_x \qquad (6)$$

Since the summation of equation (6) could become quite lengthy for large values of X, an approximation can be made.

$$T_x = \sum_{1}^{x} Y_x \approx \int_{0.5}^{x+0.5} Y_x \, dX$$

$$T_x \approx \left[\frac{A}{n+1} \quad (x + 0.5)^{n+1} - (0.5)^{n+1} \right] \qquad (7)$$

Graphical representation of the approximation of equation (6) can be seen in Fig. 5.

An example will illustrate the relative error experienced by using the integral function. Assuming a progress function of eighty per cent and production time for unit number one to be 100,000 direct labor man-hours, by using equation (7),

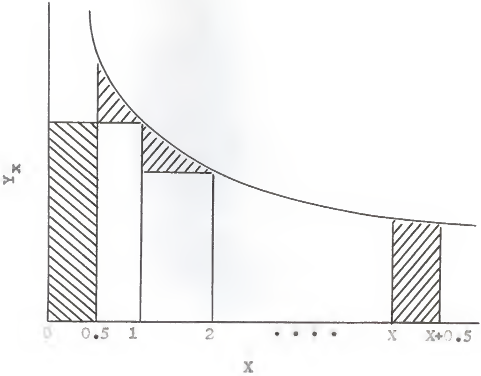


Fig. 5. The function $\sum_{1}^{x} Y_x$ and

the integral $\int_{+0.5}^{x+0.5} Y_x \, dX$.

$$T_4 = \frac{100,000}{.678} \left[(4.5)^{(.678)} - (0.5)^{(.678)} \right]$$

$$= 316,667 \text{ dlmhrs}$$

The correct answer is 314,210 direct labor man-hours, giving in this example an error of +0.78 per cent. This error is indeed not a serious error and for the purposes of this paper will be sufficient.

It will be noted that for limits not including the first unit, the integral will approximate the step function quite closely. The validity of the approximations in this section rest on the postulate that the area under the integral approximates that under the step function.

Example Using the Manufacturing Progress Function

It is sometimes necessary to know how long a time is required for the manufacture of a specific unit. An example will, therefore, be presented illustrating the use of equation (5). Using N = .80, A = 100,000 dlmhrs, and solving for Y_5,

$$Y_x = AX^n$$

$$Y_5 = 10^5 \times 5^{\left(\frac{\log .8}{\log 2}\right)}$$

$$= 10^5 \times 5^{\left(\frac{-.0969}{.3010}\right)}$$

$$= 10^5 \times 5^{-.322}$$

Y_5 = 59,600 direct labor man-hours

This is, then, the time required to produce the fifth unit of production.

The time required to produce the first five units can be found from equation (7).

$$T_5 = \frac{10^5}{.678}\left[(5.5)^{(.678)} - (0.5)^{(.678)}\right]$$

$$= \frac{2.547 \times 10^5}{.678} = 373,000 \text{ direct labor man-hours}$$

This is the time required to produce the first five units of production.

The Uses

In Pricing. As has been pointed out in previous sections, the progress function has realized a great deal of use in the area of pricing and contracting. The Air Force and the Navy use the progress function to predict the cost of a predetermined number of airframes for negotiating prices with prospective contractors. The days of cost plus federal contracting are gone and now the government wants to know exactly what a certain item or items will cost. It therefore behooves industry to utilize the most efficient and expedient means possible in costing production items. This is where the progress function plays an important part. If learning were not taken into consideration, the cost of the frames would be grossly overestimated, leaving someone to pay for something not received. This is but one use for the manufacturing progress function.

Make or Buy. The decision to make or buy an item is a problem most industries face. Certainly the item should be produced where the best cost advantage is available. This is where the progress function again plays its role. If the progress function is applied to the in-house situation, it must also be used by the external supplier. This would seem to have a balancing effect, but what if one party or the other has already produced several production quantities of the item? Then most certainly both parties will not start at the same place on the progress function.

Using a linear cost curve could very easily have produced a wrong decision; one which could be very costly and one which could have been avoided.

In Production. The scheduling of production is one of the most perplexing jobs in a manufacturing organization, and anything which will facilitate the determining of schedules is always welcome. The progress function can be a helpful tool in projecting schedules. Since a rise in productivity is evident over a long period of time with the same size work force, action can be taken at the proper time to increase or decrease the work force to meet the desired output. If this rise in productivity were not visualized, an overstock of inventory could result, which could carry a very high carrying and holding cost and result in needlessly spent capital.

In Financial Planning. There are not many companies today that can afford to disregard financial planning. It is, therefore, of utmost importance that available capital is used to the benefit of the company. Since the preponderance of a company's capital is usually constrained in the form of inventory, both finished and in-process, it is therefore helpful to the treasurer if he knows at what point in time the financial drain will fluctuate, in what direction, and for what duration. The progress function is a device which can help in predicting the rate of drain on the company's capital. This use is closely allied with production scheduling and inventory control.

<u>Other Uses.</u> This is by no means all of the possible uses for the progress function. As is evident, the majority of uses lies in the predicting and planning of costs. Since the progress function is a dynamic tool, not static, it can be used to predict a dynamic situation, such as manufacturing. As the progress function becomes more widely used, it is certain new and interesting uses will be found for such a tool.

In this section the development and uses of the manufacturing progress function have been discussed. The purpose of this section was to derive the progress function so it can be used in the derivation of an order quantity formula to be presented in the next section.

DEVELOPMENT OF A TOTAL ANNUAL COST EQUATION UNDER LEARNING

Some Salient Features

One might ask, why develop another order quantity formula to add to the already bulging library shelves? The answer is very simple. In order for a mathematical model to be useful, it must be realistic, that is, it must be accurate in its conclusions if it is to be used in making a decision in the real world. If the model does not accurately represent the real world it would be very easy to make a wrong decision based on the model; therefore, the model would be useless except as an academic exercise.

It has been seen in many industrial situations that there is opportunity for learning, therefore, any scheduling or costing decision should not disregard that fact in making a decision. In view of the progress function, it has been seen that there is a rise in productivity as production advances. It was also noticed in the development of the classical linear model that this rise in productivity was not included. What is the cost of the exclusion of the rising productivity? The answer to that question is the purpose of this thesis and, more expecially, of this section.

The First Production Cycle

In the derivation of the order quantity formula under learning, the amount of inventory in holding must be obtained so that the cost equation can be formulated.

The assumptions and definitions of the linear derivation will be used with the following additions and exceptions:

1. There will be a rise in productivity as production advances and will follow the manufacturing progress function $Y_x = AX^n$.
2. The quantity produced (X) during each cycle of production will be the same in each succeeding cycle.
3. The time, in direct labor man-hours, required to produce the first unit will be A.
4. There will be a regression in learning during each off-production cycle of a constant number of units (m).
5. The production cycle will be designated by k, where $k = 1,2,\ldots,k$.
6. For simplicity, let $n + 1 = q$.
7. The annual demand rate is represented by B which is in units per year.

The inventory in holding during the first cycle, using an exponential input and a linear output, will be as shown in Fig. 6. The holding inventory for cycle one is then the area under the curve in Fig. 6. The area can be obtained by integrating the curve from t_o to t_2. The equation for the holding inventory

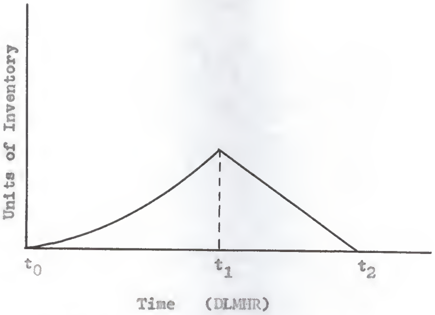


Fig. 6. First cycle inventory in holding.

during cycle one is:

$$I = \int_{t_o}^{t_1} \left(\frac{X}{T_x} - R\right)t \, dt + \int_{t_1}^{t_2} (X - Rt) \, dt \qquad (8)$$

The input quantity during cycle one totals X units, but the rate of input per unit will vary so the cycle average X/T_x is used.

Simplifying equation (8),

$$I = Xt_2 - \frac{Xt_1}{2} - \frac{Rt_2^{\,2}}{2}$$

where,

$$t_1 - t_o = T_x$$

$$t_2 - t_o = \frac{X}{R}$$

then,

$$I = \frac{X^2}{2R} - \frac{XT_x}{2} \qquad (9)$$

where T_x is the time required to produce the production during this cycle only. T_x for the first cycle is simply equation (7), because there is no regression of learning during the first cycle. The inventory in holding during the first cycle is then given by equation (9).

The Second Production Cycle

During the second cycle the first regression in learning, due to the interruption in production during cycle one, is experienced. The quantity produced during the second cycle will be Q units, which will equal X, and the time required to produce these Q units will be T_Q, which is not equal to T_x.

Because of the rise in productivity the holding inventory for the second cycle will be greater than for the first cycle, this is shown in Fig. 7. By virtue of the fact that the quantity produced remains the same and the demand function is constant, the duration of each cycle will remain constant. That is,

$$t_2 - t_o = t_4 - t_2 \qquad (10)$$

The total inventory in holding during the second cycle is then,

$$I = \int_{t_2}^{t_3} \left(\frac{Q}{T_Q} - R\right)t\,dt + \int_{t_3}^{t_4} (Q - Rt)\,dt \qquad (11)$$

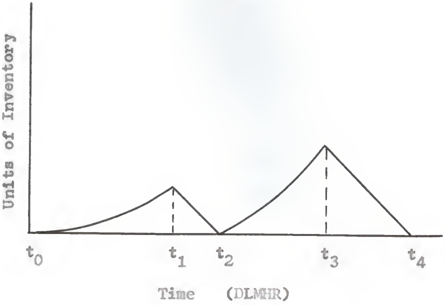


Fig. 7. Second cycle inventory in holding.

simplifying,

$$I = \frac{Qt_2}{2} + Qt_4 - \frac{Qt_3}{2} + \frac{R}{2}(t_2^{\,2} - t_4^{\,2})$$

where,

$$t_4 - t_2 = \frac{Q}{R}$$

$$t_4 - t_3 = \frac{Q}{R} - T_Q$$

$$Q = X$$

resulting in,

$$I = \frac{X^2}{2R} - \frac{XT_Q}{2} \qquad (12)$$

Since there is a regression accounted for during the second cycle, the value of T_Q will have to be adjusted to accomplish this fact. The regression (m) is shown in Fig. 8.

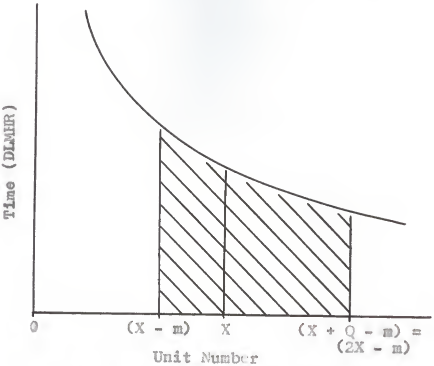


Fig. 8. The shaded area is the value of T_Q including the regression in learning.

T_Q is found by integrating the curve in Fig. 7 from (X - m) to (2X - m). Thus,

$$T_Q = \int_{(X-m)}^{(2X-m)} AX^n \, dX$$

$$= \frac{A}{q} \left[(2X - m)^q - (X - m)^q \right] \qquad (13)$$

The inventory in holding during the second cycle can then be found from equations (12) and (13).

The Third Production Cycle

The third production cycle is simply an extension of the first and second cycles, but is presented here so a general equation can be deduced from the previous cycles.

The holding inventory during the third cycle is shown in Fig. 9. The amount produced during cycle three will be designated P, where P = X = Q. The time required to produce these P units will be designated T_P, where T_P T_Q T_x. The inventory in holding is then,

$$I = \int_{t_4}^{t_5} \left(\frac{P}{T_P} - R\right) t \, dt + \int_{t_5}^{t_6} (P - Rt) \, dt \qquad (14)$$

Simplifying equation (14),

$$I = \frac{Pt_4}{2} - \frac{Pt_5}{2} + Pt_4 + \frac{R}{2}(t_4^2 - t_6^2)$$

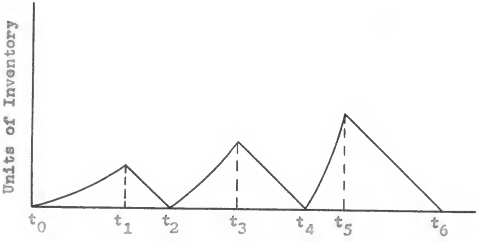


Fig. 9. The third cycle inventory in holding.

where,

$$t_6 - t_4 = \frac{P}{R}$$

$$t_6 - t_5 = \frac{P}{R} - T_p$$

$$P = X$$

resulting in,

$$I = \frac{X^2}{2R} - \frac{XT_p}{2} \tag{15}$$

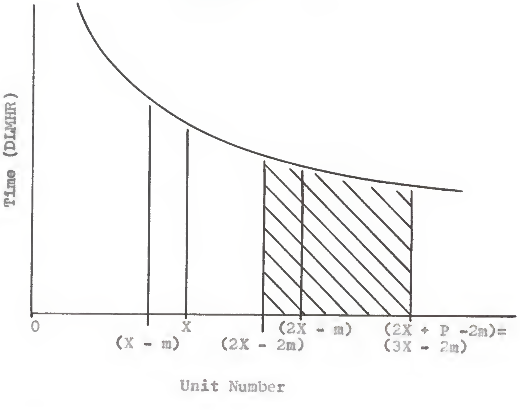


Fig. 10. The shaded area is the value for T_p including the regression from the previous cycles.

The value for T_p will have to be adjusted for another regression because of the interruptions experienced during cycles one and two. The limits of integration can be seen in Fig. 10.

Thus,

$$T_p = \int_{2X-2m}^{3X-2m} AX^n \, dX$$

$$= \frac{A}{q} \left[(3X - 2m)^q - (2X - 2m)^q \right] \qquad (16)$$

The inventory in holding during cycle three is given by equations (15) and (16).

The kth Production Cycle

The kth cycle is used to represent the general case and will be developed by deduction from the previous cycles. The inventory in holding during any cycle is shown in Fig. 11. Using K to represent the quantity produced and T_k the time required to produce these K units, the resulting equation is,

$$I = \int_{t_i}^{t_{i+1}} \left(\frac{K}{T_k} - R\right) t \, dt + \int_{t_{i+1}}^{t_{i+2}} (K - Rt) \, dt \qquad (17)$$

where,

$$t_{i+2} - t_i = \frac{K}{R}$$

$$t_{i+1} - t_i = T_k$$

$$K = X$$

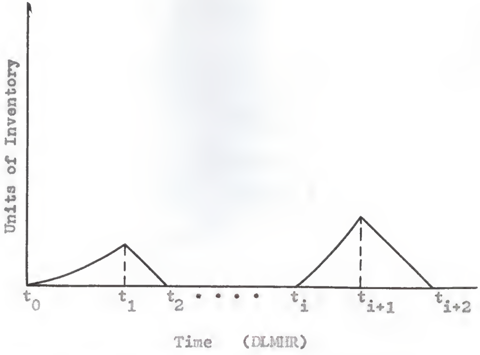


Fig. 11. The inventory in holding for any production cycle k.

then,

$$I = \frac{X^2}{2R} - \frac{XT_k}{2} \qquad (18)$$

The limits of integration for T_k can be deduced from cycles two and three and are shown in Fig. 12.

Thus,

$$T_k = \int_{(k-1)(X-m)}^{(kX - (k-1)m)} AX^n \, dX$$

$$= \frac{A}{q} \left[(kX - (k-1)m)^q - ((k-1)(X-m))^q \right] \qquad (19)$$

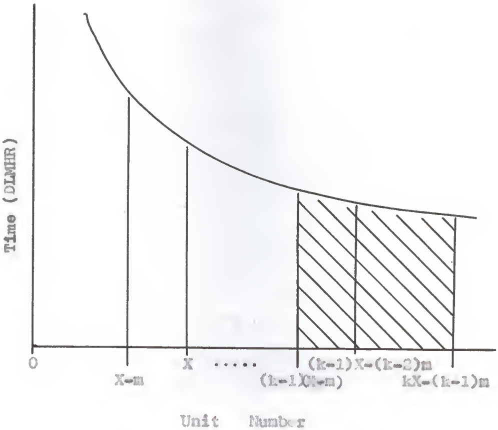


Fig. 12. The shaded area is the value of T_k including preceding regressions.

where,

> k denotes the cycle in which production is presently in progress, for example: $k = 2,3,...,k$.

Thus, the holding inventory for any cycle (k) is given by equations (18) and (19) with the exception of the first cycle. As was noted when the cumulative formula was derived, whenever the limits of integration include the first unit, the approximation, equation (7), will have to be used. Therefore, equations (18) and (19) can be used for $k = 2,3,...,k$; and for $k = 1$ equations (7) and (9) should be used.

A Total Annual Cost Equation

In the derivation of a total cost equation, a cycle cost equation must be derived first. The cycle cost equation under learning will resemble the cycle cost equation presented using a linear input, equation (1), with the exception that the input will now be exponential. The cost factors under consideration will be defined as they were in the linear case. There will be a setup cost, a holding cost proportional to the quantity of goods in storage, and there will also be a carrying cost proportional to the invested cost of each item in inventory.

For the first cycle, the total cost equation will use the holding inventory for the first cycle, equation (9), and the time required to produce the first X units, T_x, from equation (7). Thus, the total cycle cost for cycle one, TCCA, is

$$TCCA = S + (H + id \frac{T_x}{X})(\frac{X^2}{2R} - \frac{XT_x}{2}) \qquad (20)$$

The total cycle cost for any kth cycle, other than the first, will use equations (18) and (19) and will be designated TCCk.

$$TCCk = S + (H + id \frac{T_k}{X})(\frac{X^2}{2R} - \frac{XT_k}{2}) \qquad (21)$$

Thus, the total cycle cost for any cycle can be obtained by using the appropriate equation, either equation (20) or (21).

The total annual cost equation is merely a combination of the cycle cost equations. Thus, a total annual cost equation under learning (TAC) can be written as,

$$TAC = TCCA + \sum_{k=2}^{k} TCCk$$

$$= S + (H + \frac{id\ T_x}{X})(\frac{X^2}{2R} - \frac{XT_x}{2})$$

$$+ \sum_{k=2}^{k} \left[S + (H + \frac{id\ T_k}{X})(\frac{X^2}{2R} - \frac{XT_k}{2}) \right] \qquad (22)$$

where,

$$T_x = \frac{A}{q} \left[(X + 0.5)^q - (0.5)^q \right]$$

$$T_k = \frac{A}{q} \left[(kX + (k-1)m)^q - (\ (k-1)(X-m)\ )^q \right]$$

Equation (22) will give the total annual production and inventory cost for a given set of parameters. The desired solution to an equation of this type is the production quantity which will minimize the total annual cost. A method of solution and some specific examples are discussed in the next section.

TO AN OPTIMAL ORDER QUANTITY
UNDER LEARNING AND ITS IMPLICATIONS

Simulation

Simulation is a term covering a broad expanse of techniques for problem solving. Generally simulation is used when the problem is too large and complex to be described by a few mathematical equations or when after deriving a mathematical model, one finds the model unsolvable by mathematics. The alternate method of solution is then simulation.

Morris (9) defines, generally speaking, three broad classes of simulation, as follows:

1. Iconic schemes -- For example, flight simulator.
2. Analogue schemes -- For example, plant layout models.
3. Symbolic schemes -- For example, mathematical models.

Of course, a simulation could be a combination of the several classes of simulation.

The iconic schemes usually have a human decision-maker incorporated into the system. The human then is expected to make certain decisions based on his environmental parameters presented to him at the time. For example, if the flight simulator showed an altitude of 1,500 feet and the airplane in a dive, the pilot would be expected to take the proper corrective action. Thus, the iconic scheme usually has a human decision-maker incorporated into the system in order to simulate reality.

The analogue schemes usually involve models of reality. It is true that the flight simulator described above is an analogue, but it is included in the iconic schemes because of the human element in the system. The types of analogues are numerous. For example, one might use an analogue computer to simulate a traffic queuing problem, or one could use templates to simulate the layout of a plant in order to effect an efficient layout.

The third class of simulation is symbolic schemes. In this scheme the real world is represented by abstract symbols, not analogues. It is the symbolic class which embraces the world of mathematical models. After derivation of a mathematical model it has to be tested for its proximity to reality, for its sensitivity, and for its use as a decision making tool; thus the need for simulation.

As was noted above, most simulations cross class lines. The flight simulator has to pass through the symbolic stage before it can be built, then because it has a human element in the system it is iconic, and more than likely the flight simulator will have many analogue schemes to accurately simulate flight.

Of course, the class that is of interest at the present time is the symbolic class, into which the production-inventory model derived in the previous section fits. The symbolic scheme of solution will become more evident as it is used in the next section. Simulation is then another of the tools available for solving the many problems of the world today. However as Hanssmann (6) states:

> Unfortunately, it is not needless to say that the only reason for resorting to a simulation is the complexity of the model which prevents one from writing down the desired measure of performance in closed, "Analytical" form.

Solution to the Total Annual Cost Equation

The desired solution to the total annual cost equation, equation (22), is an explicit optimal value for the production quantity (X). Thereby, simply solving for X one could quickly and easily find the optimum value.

Of course, when an economical optimum is sought the optimum is usually the lowest total expenditure. This is usually found by differentiating the total cost equation with respect to the independent variable and solving for that variable. But by differentiating equation (22) with respect to X and setting it equal to zero, it was found that it was impossible to solve for X and obtain a reasonably easy equation to manipulate.

Since an explicit solution could not be found, another method was used. A simulation method, as discussed in the previous section, was used. The simulation method is not a wholly desirable method of solution, because all that can be done is to select parameter values and observe fluctuations and characteristics of the equation using the specific values. The simulation method, in this case, will not give a general equation so specific generalizations cannot be stated. In order to be able to make a generalized statement, all possible combinations of parameter values would have to be investigated, which of course is virtually impossible. But since the simulation method was the only alternative, it was used.

The IBM 1620 digital computer was programmed, the computer program and sample output are given in the Appendix, so that different parameter values could be investigated. After considerable investigation, a set of parameter values were chosen for illustration here because of their possible feasibility and also in order to manifest some of the characteristics of the total annual cost equations.

The parameter values chosen could easily pertain to some fairly complex minature electronic subassembly. This type of work would provide the opportunity for learning and, being a reasonably complex subassembly, would also incur a regression in learning if the operator was removed from the task for a length of time exceeding two or three days. An eighty-five per cent learning curve (L/C = N) was used which provides ample opportunity for learning, and a regression (m) per production cycle of 15 units was also incorporated. The first unit production time (A) was set at 1.50 hours per unit and a demand function (R) of one unit every two hours or 0.50 units per direct labor man-hour was also used. Since the production item is not exceptionally bulky and did not require excess setup time, a setup cost (S) of $9.00 per setup was used. The operator in this case is probably a skilled worker so a direct labor man-hour cost (d), including manufacturing overhead, of $5.00 per direct labor man-hour was decided upon. In view of the size of the item a holding cost (H) of \$7.00 x 10^{-6} per direct labor man-hour per unit, that is approximately \$0.02 per year per unit, was used and an interest rate (i) in the carrying cost was set at 10 per cent per year which is equal to 3.43 x 10^{-5} per cent per direct labor man-hour. Then using the preceding parameters and by varying one variable at a time in order to observe its influence on the TAC, the following results were obtained and are shown in the following plates. The preceding parameter values were used for all plates except Plate I where a specific characteristic is being displayed. Each plate shows the specific parameter values used. The total annual cost under learning is also compared to the total annual linear cost. The parameters for the linear case are the same as for the learning case.

Plate I shows the effect of varying the learning rate using the given parameter values. At first analysis, Plate I shows what would be expected; that is, the linear curve, L/C = 1.00, would incur the higher total annual cost. But the linear curve will not always incur the highest cost. A comparison of the cycle cost equations for the linear and the learning case will point out the reason. It will be noted that the inventory in holding for the learning case will always be larger than in the

$$TCLNC = S + (H + id\ A)\ \frac{X^2}{2R}\ (1 - RA) \tag{23}$$

$$= S + (H + id\ A)\left(\frac{X^2}{2R} - \frac{X^2A}{2}\right) \tag{24}$$

$$TCCk = S + (H + id \frac{T_k}{X})(\frac{X^2}{2R} - \frac{XT_k}{2}) \qquad (25)$$

linear case; thereby, making the parameters H, i, d, A, and T_k determine whether the linear or learning cost is higher. This phenomenon is illustrated in Plate II, where the linear cost is not larger than some of the learning costs. The variation was caused principally by changing R and A. In this case, the linear curve falls between the eighty-five and ninety per cent learning curves.

The primary purpose of this investigation was to find the effect of learning on the optimum production or order quantity, but because of the prominent effect of the various parameters on the relation between the linear and the learning functions an encompassing generalization cannot be stated. It can be shown from Plate I that, depending upon the specific parameter values, the optimum X will become less as the learning curve is increased. In the case of Plate I, a significant change occurred in X*, optimum X, as the learning curve was varied from .80 to 1.00 causing X* to vary from approximately 275 units to 175 units. A similar result is noted for Plate II.

A variable which was expected to be more significant than resulted was the regression per cycle (m). Examination of Plate III shows that as m is varied from 0 to 55 units per cycle, the resulting change in TAC was approximately \$0.25 per an increase of 15 units per cycle. The regression value was varied from zero to more than 10 per cent of the production quantity. This

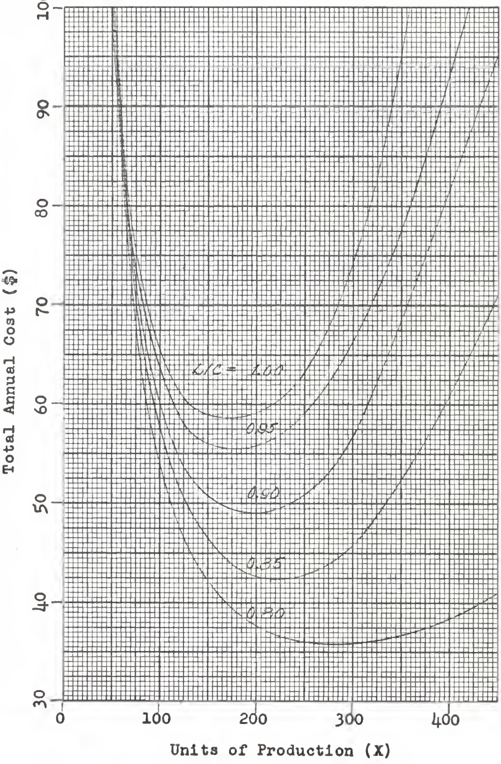
EXPLANATION OF PLATE 1

TAC as a function of X for various learning rates.

These curves compare the effect of various learning rates, from .80 to 1.00, on the TAC and on X*, using the following parameter values:

L/C	=	variable	q	=	variable
A	=	2.00	m	=	10.00
B	=	520.00	R	=	0.25
S	=	9.00	H	=	7.00×10^{-6}
d	=	5.00	i	=	3.43×10^{-5}

PLATE I


100
90
80
70
60
50
40
30
Total Annual Cost ($)
L/C = 1.00
0.95
0.90
0.85
0.80
0
100
200
300
400
Units of Production (X)

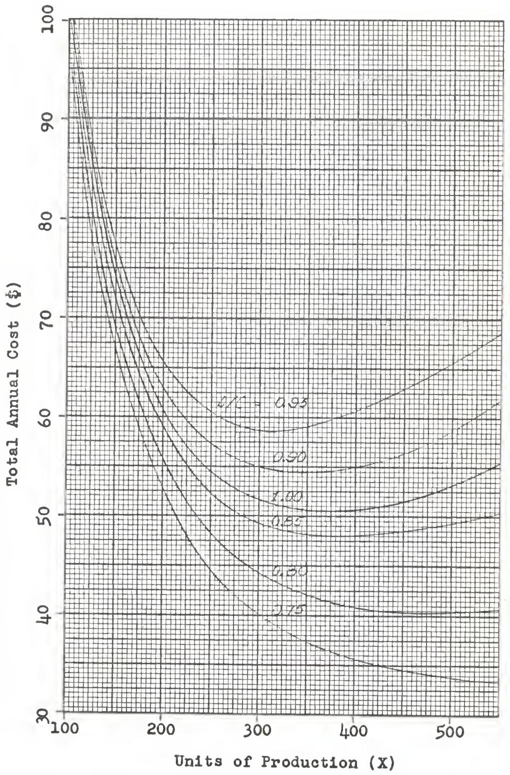
EXPLANATION OF PLATE II

TAC as a function of X for various learning rates.

These curves compare the effect of various learning rates from .75 to 1.00 on the TAC and on the optimum order quantity X, using the following parameter values:

L/C	=	variable	q	=	variable
A	=	1.50	m	=	15.00
B	=	1040.00	R	=	0.50
S	=	9.00	H	=	7.00×10^{-6}
d	=	5.00	i	=	3.43×10^{-5}

PLATE II


100
90
80
70
60
50
40
30
Total Annual Cost ($)
100
200
300
400
500
Units of Production (X)
0.95
0.90
1.00
0.85
0.80
0.75

EXPLANATION OF PLATE III

TAC as a function of X for various regression quantities.

This Plate shows in tabular form the effect of different regression quantities, from $m = 0$ to 55 units/cycle, on the TAC. The parameter values used are as follows:

L/C	=	0.85	q	=	0.765
A	=	1.50	m	=	variable
B	=	1040.00	R	=	0.50
S	=	9.00	H	=	7.00×10^{-6}
d	=	5.00	i	=	3.43×10^{-5}

PLATE III

X	Regression Units/Cycle							Linear
	0	5	15	25	35	45	55	
520.00	49.53	49.55	49.58	49.61	49.64	49.68	49.71	53.73
346.67	47.96	47.98	48.03	48.08	48.13	48.18	48.23	50.82
260.00	51.69	51.72	51.78	51.83	51.89	51.96	52.02	53.86
208.00	57.54	57.57	57.63	57.70	57.77	57.84	57.91	59.29
173.33	64.44	64.47	64.54	64.61	64.69	64.77	64.85	65.91
148.57	71.95	71.98	72.05	72.13	72.21	72.30	72.39	73.21
130.00	79.82	79.86	79.93	80.01	80.10	80.20	80.31	80.93
115.56	87.95	87.99	88.06	88.15	88.24	88.35	88.47	88.94
104.00	96.25	96.29	96.37	96.46	96.56	96.68	96.81	97.15

means, at the maximum regression, that the production rate regressed an equivalent of 55 units each cycle of production.

Since research is very limited in the regression of this type of motor skill, it cannot be said for certain that a constant type of regression accurately represents the actual, but it can be stated that a constant regression up to 10 per cent of the production quantity does not have a significant effect on the total annual cost. It can also be stated that, using similar parameter values, the expense of finding the regression quantity in a specific situation could not be justified by a more accurate total annual cost, as long as the actual regression was not more than 10 per cent of the production quantity. The same conclusion can also be drawn with respect to the optimum X.

A quick glance at equations (24) and (25) will manifest the effect of S on the total annual cost and on X*. It can be seen that as S is increased, the total annual cost approaches a multiple of S. This relationship can be seen in Plate IV. Depending on the specific parameter values whether the linear curve is above or below the learning curve, a change in S will not alter the relationship of the two curves. That is, if the linear is above the learning curve, as in Plate IV, it will remain so as S is varied. It is also seen in Plate IV that as S is increased from $2.00 to $40.00 per setup, X* moves toward larger production quantities. As S moves from $2.00 to $12.00 per setup, X* moves from approximately 175 units to 450 units per cycle. The

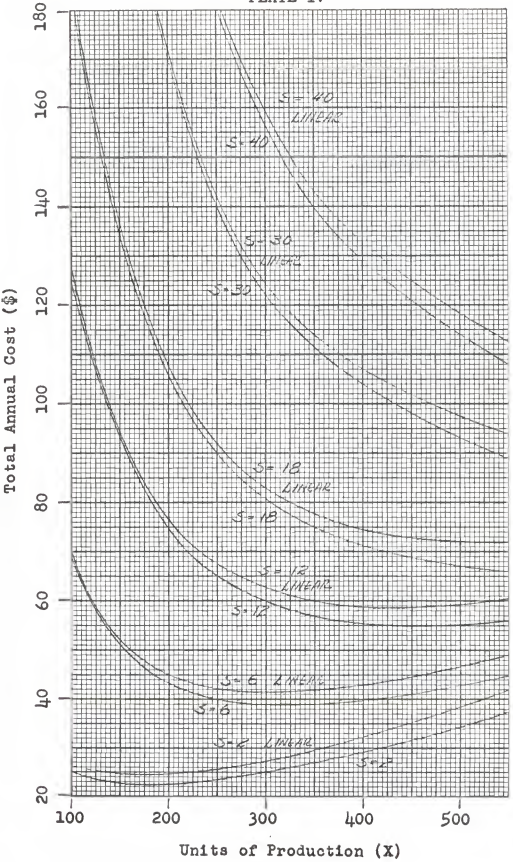
EXPLANATION OF PLATE IV

TAC as a function of X for various values of S.

These curves compare the effect of a range of S values from \$2.00 to \$40.00 per setup on the TAC and on X^*. The parameter values used are as follows:

L/C	=	0.85	q	=	0.765
A	=	1.50	m	=	15.00
B	=	1040.00	R	=	0.50
S	=	variable	H	=	7.00×10^{-6}
d	=	5.00	i	=	3.43×10^{-5}

PLATE IV


Total Annual Cost ($)
180
160
140
120
100
80
60
40
20
100
200
300
400
500
Units of Production (X)
S = 40 LINEAR
S = 40
S = 30 LINEAR
S = 30
S = 18 LINEAR
S = 18
S = 12 LINEAR
S = 12
S = 6 LINEAR
S = 6
S = 2 LINEAR
S = 2

reason for the move to larger production quantities is obvious. As the setup cost dominates the holding and carrying cost it becomes more economical to manufacture larger production quantities.

A decision based on the learning or the linear function would produce almost the same conclusion as to the optimum production quantity, X*. It can then be stated with respect to the setup cost, using similar parameters, that there is not a significant difference between X* found by the linear or learning methods. It must be pointed out that different parameter values could have a pronounced effect on the relationship between the two curves.

The variation of the holding cost (H) exemplifies an interesting characteristic of the total cost equations. The phenomenon is illustrated in Plate V. Since the inventory in holding in the learning case is always larger than in the linear case, it can be seen from equations (24) and (25) that as H begins to dominate the value of (id T_k/X) the learning curve will be above the linear curve and vise versa. Therefore, it is evident that this crossover point will depend entirely on the specific parameter values, but it can be stated that there will be a point at which the two curves will crossover.

It is evident from Plate V, that as H becomes larger, the optimum production quantity, X*, moves toward smaller lot quantities. This is obviously just the reciprocal of the setup case, because as H rises the inventory cost becomes larger than the setup cost and thereby makes it less economical to produce large

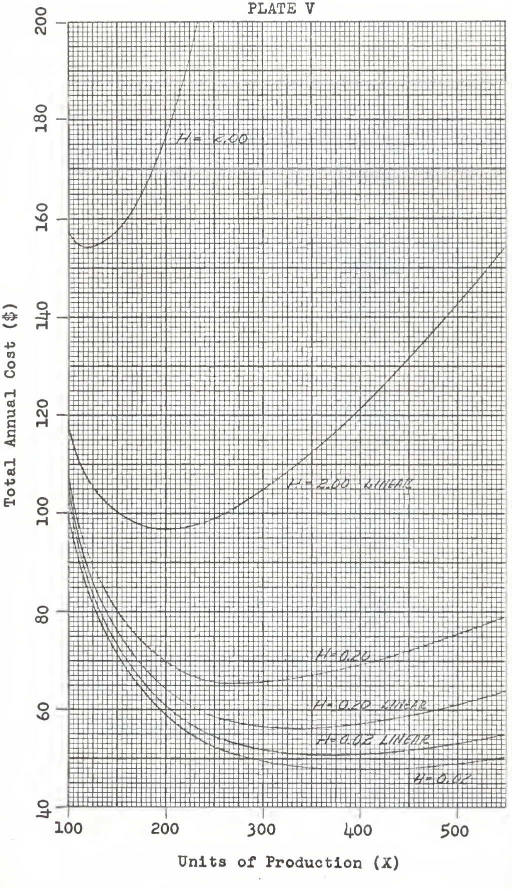
EXPLANATION OF PLATE V

TAC as a function of X for various values of H.

These curves illustrate the effect of varying H from \$0.02 to \$2.00 per year per unit on TAC and on X*. The parameter values used are as follows:

L/C	=	0.85	q	=	0.765
A	=	1.50	m	=	15.00
B	=	1040.00	R	=	0.50
S	=	9.00	H	=	variable
d	=	5.00	i	=	3.43×10^{-5}

PLATE V


Total Annual Cost ($)
200
180
160
140
120
100
80
60
40
100
200
300
400
500
Units of Production (X)
H= K.00
H= 2.00 LINEAR
H=0.20
H=0.20 LINEAR
H=0.02 LINEAR
H=0.02

quantities of production and carry these in inventory, therefore, more setups are made and smaller quantities are produced in order to effect the optimum total annual cost. It is also shown that as H becomes larger, the difference between the decision based on the linear curve and on the learning curve becomes more significant. In the case illustrated in Plate V, the difference between the X* based on H = 2.00 and H = 2.00 linear is approximately 75 units per cycle. This difference is indeed significant when X* is in the 100 to 200 units/cycle range.

The effect of varying d is similar to that of varying i as displayed in Plates VI and VII, respectively. The relationship between the two variables can be clarified by equations (24) and (25). It is seen that i and d appear only as id and in only one position, thereby, making corresponding changes in either variable similar. It is seen that as d or i are varied, they effect a linear and learning curve crossover, much as exemplified by H and for the same reasons. H can be made to dominate by either increasing H or as in this case by decreasing either i or d. Thus, from Plates VI and VII, as d or i are decreased, a crossover is effected. The point of crossover will depend on the parameter values chosen.

A definite trend is noticed in the value of X* as i or d is increased. This is true again because as the inventory costs rise it becomes more economical to make smaller production quantities and, therefore, more setups. The difference between the

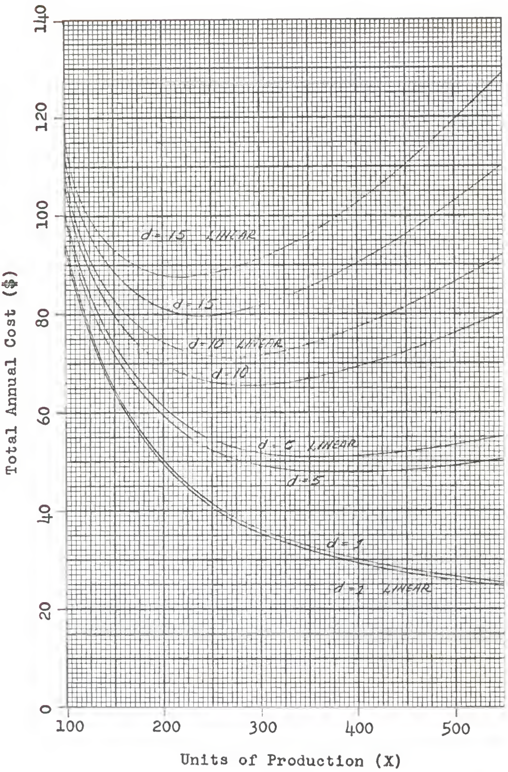
EXPLANATION OF PLATE VI

TAC as a function of X for various values of d.

These curves illustrate the effect of varying d, from $1.00 to $15.00 per direct labor man-hour, on TAC and on X*. The parameter values used are as follows:

L/C	= 0.85	q	= 0.765
A	= 1.50	m	= 15.00
B	= 1040.00	R	= 0.50
S	= 9.00	H	= 7.00×10^{-6}
d	= variable	i	= 3.43×10^{-5}

PLATE VI


Total Annual Cost ($)
0
20
40
60
80
100
120
140
100
200
300
400
500
Units of Production (X)
d=15 LINEAR
d=15
d=10 LINEAR
d=10
d=5 LINEAR
d=5
d=1
d=1 LINEAR

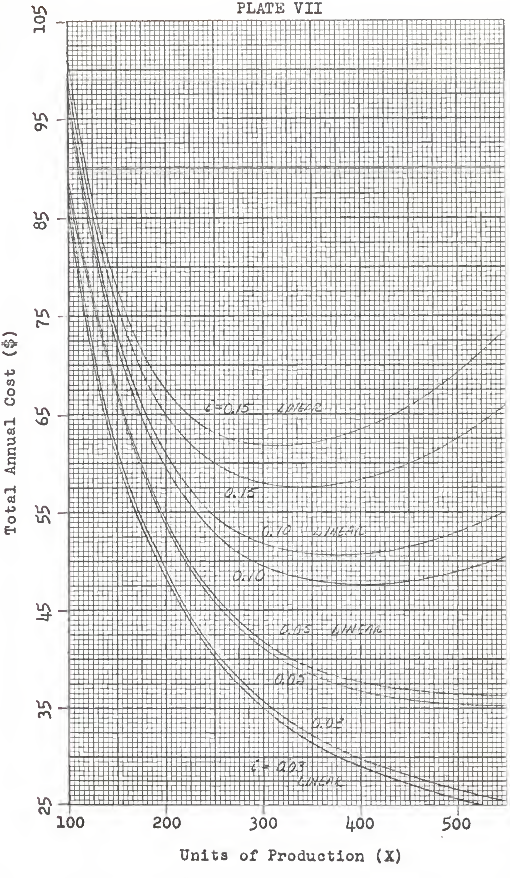
EXPLANATION OF PLATE VII

TAC as a function of X for various values of i.

These curves compare the effects of varying i, from 3 to 15 per cent per year, on TAC and on X*. The parameter values used are as follows:

L/C	=	0.85	q	=	0.765
A	=	1.50	m	=	15.00
B	=	1040.00	R	=	0.50
S	=	9.00	H	=	7.00×10^{-6}
d	=	5.00	i	=	variable

PLATE VII


105
95
85
75
65
55
45
35
25
Total Annual Cost ($)
100
200
300
400
500
Units of Production (X)
i = 0.15 LINEAR
0.15
0.10 LINEAR
0.10
0.05 LINEAR
0.05
0.03
i = 0.03 LINEAR

optimum X, X*, found in the linear case and the one found in the learning case becomes more significantly different as d or i decreases.

Plate VIII illustrates the effect of varying A on the TAC and on X*. As A is varied from .75 to 2.00 direct labor man-hours per unit, the linear-learning relation again effects a crossover. By examining equation (23) it is seen that as A approaches the reciprocal of R, the TALNC will approach a multiple of the setup cost. This relation is shown by A = 2.00, thereby, making the demand rate equal the production rate. Because the learning function still has inventory in holding, its total annual cost will exceed the cost incurred in the linear case. It is also evident from Plate VIII that the difference between X* found by the linear curve and X* found by the learning curve is significant on both sides of the crossover, that is at A = 0.75 and A = 2.00.

Plate IX exemplifies a strange curve made by varying R. In programming the computer it was necessary to include a value for the annual demand (B) in order to make the cycles per year an integer. An X was found in this manner which would provide an integer number of cycles per year. The value B is merely another way of stating R, which is the direct labor man-hour demand function. In most of the parameter variations, a value of K = 0.50 was used which would make B = 1040 dlmhrs/year using 2080 as an annual man-hour figure.

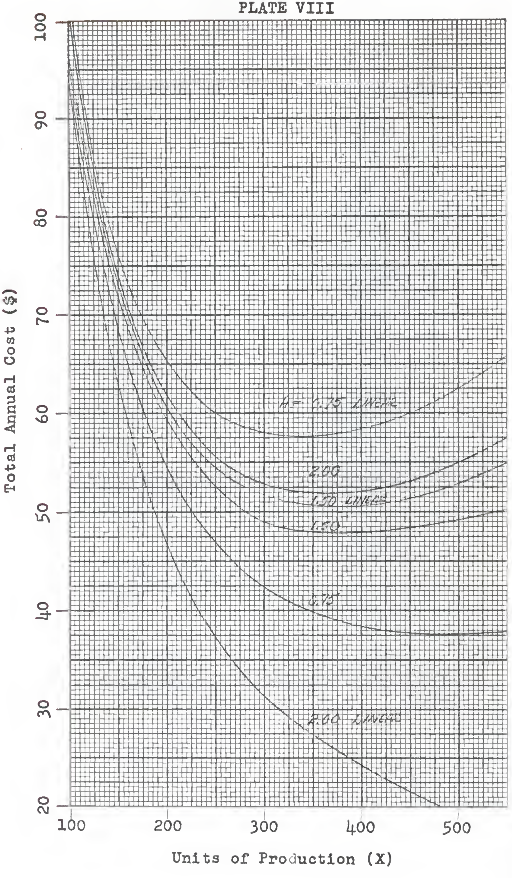
EXPLANATION OF PLATE VIII

TAC as a function of X for various values of A.

These curves illustrate the effect of varying A from .75 to 2.00 direct labor man-hours per unit on TAC and X*. The parameter values used are as follows:

L/C	=	0.85	q	=	0.765
A	=	variable	m	=	15.00
B	=	1040.00	R	=	0.50
S	=	9.00	H	=	7.00×10^{-6}
d	=	5.00	i	=	3.43×10^{-5}

PLATE VIII


Total Annual Cost ($)
100
90
80
70
60
50
40
30
20
100
200
300
400
500
Units of Production (X)

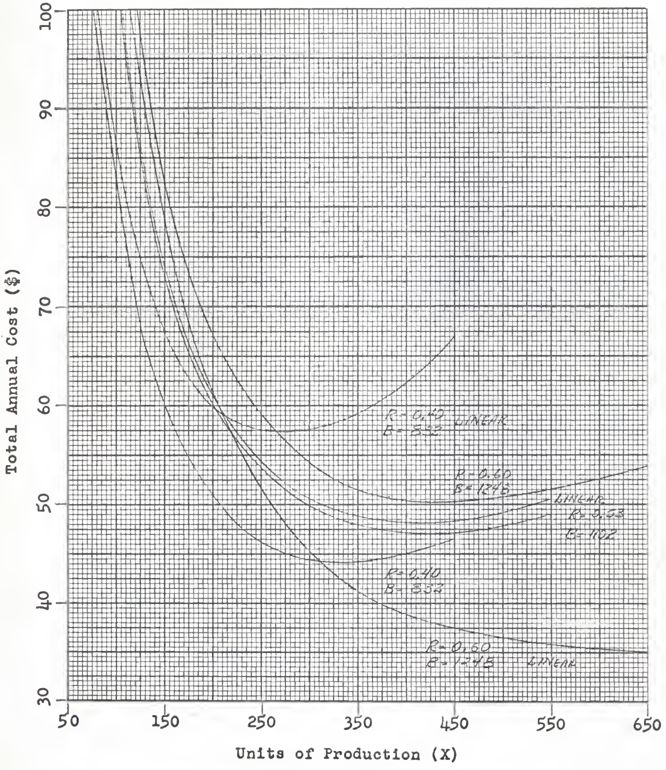
EXPLANATION OF PLATE IX

TAC as a function of X for various values of R.

These curves illustrate the effect of varying R from .40 to .60 units per direct labor man-hours on TAC and on X. The parameter values used are as follows:

L/C	=	0.85	q	=	0.765
A	=	1.50	m	=	15.00
B	=	variable	R	=	variable
S	=	9.00	H	=	7.00×10^{-6}
d	=	5.00	i	=	3.43×10^{-5}

PLATE IX


100
90
80
70
60
50
40
30
Total Annual Cost ($)
R=0.40
B=832
LINEAR
R=0.60
B=1245
LINEAR
R=0.53
B=1102
R=0.40
B=832
R=0.60
B=1245
LINEAR
50
150
250
350
450
550
650
Units of Production (X)

The phenomenon of the linear-learning crossover is again illustrated in Plate IX. It is also evident that on either side of the crossover, the X* linear is significantly different from X* learning.

In this section a presentation of a method of solution to the total annual cost equation was made along with the results of a simulation solution. The conclusions drawn from the data collected and illustrated here will be presented and summarized in the next section.

SUMMARY AND CONCLUSIONS

The results of mathematical analysis depend for their validity and usefulness on the assumptions on which they are based. In an industrial situation it might be felt that one or more of the assumptions made in this paper are not valid for a specific situation and would have to be modified by the analyst. As Morris (9) has stated, "Analytical models of the type discussed here are rarely presented in 'ready-to-wear' form. They must be tailored to fit specific situations." A different type of learning curve might fit a specific situation better or the regression might be altered to give a more accurate representation of a specific situation or maybe another variable needs to be added to increase the validity. Any alteration would be for a specific situation.

From a research point of view, there is no limit to how many variables there might be accounted for in constructing a model. However, as a practical matter a compromise between reality and simplicity must be reached. No model will ever be absolutely "true". Of course, after deriving a model the next step is to verify the model by applying it to the real world and finding its weak points.

In this paper, an attempt has been made to try to make an already existing model more realistic; whether this has been accomplished will remain for another investigation to establish.

Along with the validation of this model there are some questions to be answered. What kind of regression really exists? Could the learning curve be modified to better fit industry? The long list of "things to do" continues.

It is regretful that a general economic order quantity equation under learning could not be explicitly stated, but mathematical complexity sometimes stymies a solution. In spite of the absence of an explicit solution, the simulation method provided some insight into the characteristics and relationships between the learning and linear cost equations.

One concrete conclusion is that at points on either side of the crossover the two curves, linear and learning, are significantly different. Therefore, if the learning situation applies to an industrial concern it would be wise to incorporate the learning curve into the decision making. In some instances, the X* was not significant, but the TAC was significant, and could be the basis for a wrong decision if the curves were used for financial planning.

Another significant phenomenon was that the regression quantity did not provide for a significant difference between different regression amounts. This was an interesting development because before the investigation it was certain the regression quantity would be significant. Based on the data given, it can be stated that, if the actual regression curve follows the assumptions given, it would be uneconomical for a manufacturer to investigate to find out what the actual regression quantity is.

Of course, more investigation is needed to find out the exact pattern of the regression.

The phenomenon of the learning-linear crossover was also surprising, because at first blush it would appear that the linear curve would always incur a larger cost than the learning curve. Such is not the case, though because of variations in parameter values, the linear and the learning curves could be equal, one higher, or vise versa.

The primary purpose of the investigation was to find the effect of the learning process on determining economic order quantities. Even though an explicit formula for X^* cannot be stated, it has been shown that there could be a very significant difference between the learning cost and the linear cost, depending upon the parameter values. If for no other reason, this in itself justifies the investigation.

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REFERENCES

(1) Andress, Frank J.
The learning curve as a production tool. Harvard Business Review. vol. 32, no. 1, January-February, 1954.

(2) Boman, Edward H., and R. B. Fetter.
Analysis for production management. Homewood, Illinois: Richard D. Irwin, Inc., 1959.

(3) Bush, R. R., and F. A. Mosteller.
Mathematical model for simple learning. Psychological Review. vol. 50, no. 5, September, 1951.

(4) Cochran, E. B.
New concepts of the learning curve. Jour. of Industrial Engineering. vol. XI, no. 4, July-August, 1960.

(5) Estes, W. K.
Toward a statistical theory of learning. Psychological Review. vol. 57, no. 2, March, 1950.

(6) Hansmann, Fred.
Operations research in production and inventory control. New York: John Wiley & Sons, Inc., 1962.

(7) Koen, Francis T.
Dynamic evaluation. Factory. vol. 117, no. 9, September, 1959.

(8) Maynard, H. B., editor-in-chief.
Industrial engineering handbook. New York: McGraw-Hill, 1956.

(9) Morris, William T.
Analysis for materials handling management. Homewood, Illinois: Richard D. Irwin, Inc., 1962.

(10) Naylor, James C., and George E. Briggs.
Long-term retention of learned skills: a review of the literature. ASD Technical Report 61-390. Wright-Patterson Air Force Base, Ohio. August, 1961.

(11) Reinfeld, N. V.
Production control. Englewood Cliff, N. J.: Prentice-Hall, Inc., 1959.

(12) Sasieni, Maurice, Arthur Yaspan, and Lawrence Friedman. Operations research--methods and problems. New York: John Wiley & Sons, Inc., 1960.

(13) Torgerson, P. E. The manufacturing progress function. Unpublished manuscript, Oklahoma State University, Stillwater, Oklahoma, 1961.

APPENDIX

Glossary

Special Terms. Some special terms discussed in this paper are defined as follows:

Carrying cost. The cost of having idle capital in storage. The carrying cost will be directly proportional to the direct labor man-hours consumed per unit.

Crossover. The point at which the linear cost curve and the learning cost curve are equal is designated the crossover point.

Holding cost. The cost of physical storage of any item in a warehouse is the holding cost and includes such costs as, warehouse overhead, warehouse direct labor cost, physical storage cost, etc.

Inventory cost. The cost directly incurred during holding in storage is designated inventory cost and includes the holding and carrying costs.

Production cost. The cost directly incurred during the production cycle is designated production cost and includes, in this paper, only the setup cost.

Total annual cost. In the learning context the total annual cost refers to the cost incurred during the year using the learning input rate. When the total annual cost is used in an undesignated context it refers to the yearly cost incurred by either the linear or the learning methods.

Total annual linear cost. When using a linear production input the cost incurred by such a policy is called the total annual linear cost.

Total cycle cost. The cost incurred during any production-inventory cycle is designated the total cycle cost and usually refers to the learning production input, but can apply either to the learning or linear case.

Symbols. The algebraic symbols used in mathematical formulae are defined as follows:

A	=	the first unit production time in the learning case and the constant production rate in the linear case.
B	=	the total annual demand.
d	=	the direct labor man-hour cost.
dlmhr	=	direct labor man-hours.
H	=	holding cost in dollars per direct labor man-hour per unit.
i	=	interest cost in per cent per direct labor man-hour.
k	=	the production cycle under consideration.
K	=	the production quantity in the kth cycle.
L/C	=	rate of learning and is equal to N.
m	=	the regression quantity in units per cycle.
n	=	log N / log 2
N	=	the rate of improvement in the learning situation.

P = the production quantity in the third cycle.

q = n + 1

Q = the production rate in the second cycle.

R = the demand rate in units per direct labor man-hour.

S = setup cost in dollars per setup.

t = time in direct labor man-hours.

TAC = total annual cost.

TALNC = total annual linear cost.

TCC = total cycle cost.

TCLNC = total cycle linear cost.

TCCA = total cycle cost for cycle one under learning.

TCCB = total cycle cost for any cycle after cycle one under learning.

TCCk = total cycle cost for the kth cycle.

T_k = the time in dlmhr required to produce K units in the kth cycle.

T_P = the time in dlmhr required to produce P units in the third cycle.

T_Q = the time in dlmhr required to produce Q units in the second cycle.

T_X = the time in dlmhr required to produce X units in the first cycle.

X = the production quantity per cycle.

X* = optimum X.

Y_X = the time required to produce the Xth unit.

Computer Program

The following computer program was written, for the IBM 1620, to compute the total annual cost for both a linear and a learning production input. In this program, the interest rate is represented by T, the direct labor man-hour cost as D, the learning improvement rate as G, the regression quantity as O, T_x by W, and T_k by Y. Statement 50, 55, 65, and 70 represent equations (7), (20), (19), and (21), respectively. Data cards with appropriate values of the various parameters were prepared to accompany this program.

```
C TOTAL ANNUAL INVENTORY COST UNDER LEARNING-THESIS 1963
 1 FORMAT(12(E6.1))
 5 FORMAT(18H DLMHR/UNIT     A =E18.7,17H DEMAND/DLMHR R =E18.7)
 6 FORMAT(18H SETUP          S =E18.7,17H CARRYING COST H=E18.7)
 7 FORMAT(18H INT PER/DLMHR T =E18.7,17H DOL/DLMHR     D =E18.7)
 8 FORMAT(18H LEARNING CURVE G=E18.7,17H N+1           Q =E18.7)
 9 FORMAT(18H REG UNIT/CYC  O =E18.7,17H RANGE U   TEST =E18.7)
10 FORMAT(18H DEMAND/YEAR    B =E18.7,17H DEMAND/DLMHR R =E18.7)
11 FORMAT(15H CYCLES/YEAR U=E20.7,6H TCCA=E20.7)
12 FORMAT(6H TAC =E18.7,4H X =E18.7,8H TALNC =E18.7)
13 FORMAT(E20.7)
25 READ 1,R,H,T,D,A,B,O,P,Q,S,TEST,G
26 IF(SENSE SWITCH2)27,33
27 PRINT5,A,R
28 PRINT6,S,H
```

```
29 PRINT7,T,D
30 PRINT8,G,Q
31 PRINT9,O,TEST
32 PRINT10,B,R
33 PUNCH5,A,R
34 PUNCH6,SH
35 PUNCH7,T,D
36 PUNCH8,G,Q
37 PUNCH9,O,TEST
38 PUNCH10,B,R
45 M=TEST
46 DO 120 I=2,M
47 U=I
48 X=B/U
50 W=(((X+.5)**Q)-((.5)**Q))*A/Q
55 TCCA=(((X**2.)/(2.*R))-(X*W/2.))*(H+(T*D*W/X))+S
56 TAC=TCCA
57 L=U
60 DO 76 J=2,L
61 C=J
65 Y=(((C*X)-(C-1.)*O)**Q-((C-1.)*(X-O))**Q)*A/Q
70 TCCB=(((X))2.)/(2.*R))-(X*Y/2.))*(H+(T*D*Y/X))+S
75 TAC= TAC+TCCB
76 CONTINUE
80 TCLNC=S+((X**2.)/(2.*R))*(1.-(R*A))*(H+(T*D*A))
85 TALNC=(TCLNC)*(U)
```

```
 89 IF(SENSE SWITCH2)90,95
 90 PRINT12,TAC,X,TALNC
 91 PRINT11,U,TCCA
 95 TYPE13,U
 96 PUNCH12,TAC,X,TALNC
 97 PUNCH11,U,TCCA
120 CONTINUE
121 GO TO 25
140 STOP
    END
```

Sample of Computer Output

The output from the previous computer program appeared in the following form:

```
DLMHR/UNIT      A =      1.5000000E+00 DEMAND/DLMHR R =       5.0000000E-01
SETUP           S =      9.0000000E+00 CARRYING COST H=       7.0000000E-06
INT PER/DLMHR T =        3.4300000E-05 DOL/DLMHR     D =      5.0000000E+00
LEARNING CURVE G=        8.5000000E-01 N+1           Q =      7.6500000E-01
REG UNIT/CYC  O =        1.5000000E+01 RANGE U    TEST =      1.0000000E+01
DEMAND/YEAR     B =      1.0400000E+03 DEMAND/DLMHR R =       5.0000000E-01
TAC =      4.9578618E+01 X =      5.2000000E+02 TALNC =       5.3726576E+01
CYCLES/YEAR U=        2.0000000E+00 TCCA=       2.6617698E+01
TAC =      4.8029828E+01 X =      3.4666666E+02 TALNC =       5.0817720E+01
CYCLES/YEAR U=        3.0000000E+00 TCCA=       1.7293146E+01
```

TAC = 5.1776596E+01 X = 2.6000000E+02 TALNC = 5.3863292E+01

CYCLES/YEAR U= 4.0000000E+00 TCCA= 1.3848941E+01

TAC = 5.7632940E+01 X = 2.0800000E+02 TALNC = 5.9290635E+01

CYCLES/YEAR U= 5.0000000E+00 TCCA= 1.2193517E+01

TAC = 6.4541259E+01 X = 1.7333333E+02 TALNC = 6.5908866E+01

CYCLES/YEAR U= 6.0000000E+00 TCCA= 1.1268013E+01

TAC = 7.2049537E+01 X = 1.4857142E+02 TALNC = 7.3207589E+01

CYCLES/YEAR U= 7.0000000E+00 TCCA= 1.0696912E+01

TAC = 7.9932258E+01 X = 1.3000000E+02 TALNC = 8.0931648E+01

CYCLES/YEAR U= 8.0000000E+00 TCCA= 1.0319067E+01

TAC = 8.8064295E+01 X = 1.1555555E+02 TALNC = 8.8939242E+01

CYCLES/YEAR U= 9.0000000E+00 TCCA= 1.0055757E+01

TAC = 9.6370698E+01 X = 1.0400000E+02 TALNC = 9.7145317E+01

CYCLES/YEAR U= 1.0000000E+01 TCCA= 9.8647355E+00

Mathematical Derivation

Equation (1) expressed the total annual cost under a linear production rate and is derived from Fig. 1. The inventory units in holding during the production cycle is the area under the triangle in Fig. 1.

$$I = \int_{0}^{XA} \left(\frac{1}{A} - R\right) t \, dt + \int_{XA}^{X/R} (X - Rt) \, dt$$

$$= \frac{X^2}{2} \left[\frac{1}{R} - A\right]$$

$$= \frac{X^2}{2R} (1 - RA)$$

Now by combining the carrying cost and the holding cost thus,

$$(H + idA)$$

and multiplying by the inventory in holding, then adding in the setup cost the total cycle cost is obtained.

$$TCLNC = S + (H + idA)\left(\frac{X^2}{2R} - \frac{X^2 A}{2}\right) \qquad (1)$$

THE EFFECT OF THE LEARNING PROCESS IN DETERMINING ECONOMIC ORDER QUANTITIES

by

GEORGE LaVERNE DICKEY

B. S., Kansas State University, 1961

AN ABSTRACT OF A MASTER'S THESIS

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The many economic order quantity formulae that have been derived over the past quarter of a century have one thing in common and that is, they all have made some simplifying assumptions. The effect of the assumptions depends on the specific situation under consideration. One formula cannot hope to be all-encompassing, which is the primary reason for the many formulae which have been derived. However, the one assumption which many of these formulae have made is that the production input to the inventory is linear. It is the effect of this basic assumption opposed to a rising productivity that is under investigation.

In 1939, T. Wright discovered the phenomenon that learning in an industrial situation is predictable. It was found in the aircraft industry that the time required to produce doubled quantities of production decreased by a constant percentage. That is, the time required to produce the eighth unit is eighty per cent of the time required for the fourth unit, and the time required to produce the sixteenth unit is eighty per cent of the time required to produce the eighth unit, and so forth. Of course, the percentage improvement depends upon the complexity of the task being performed.

Just as it is known that man learns, it is also known that he forgets if the learned skill is not reinforced. The exact rate or type of regression is not known. Research on the subject of long-term retention of learned skills is indeed scarce, and much more research needs to be conducted before the forgetting of an industrial operation can be predicted. For the purpose of this investigation, a constant regression per cycle of production was assumed.

Using the exponential learning curve as the production rate and a constant regression per production cycle, a total annual cost equation for a production-inventory mechanism was derived. The total annual cost equation included such relevant costs as:

1. Setup cost
2. Inventory holding costs
3. Inventory carrying costs
4. Regression costs
5. Costs of a rising productivity

After deriving the total annual cost equation under learning, it was found that an explicit optimum equation could not be written for the production quantity per cycle. The simulation method of solution was then used. By choosing different parameter values, the familiar economic order quantity parabolic curve was drawn and the optimum production quantity was found, although, this method of solution is not wholly desirable because all possible combinations of parameters cannot be investigated to find the different idiosyncrasies of the equation.

The total annual cost equation under a linear input was compared to the cost equation under learning and the results analysed. Since an explicit equation could not be found, the results of the analysis only apply to the specific parameter values used, but several general results can be visualized.

The linear optimum value was not always larger than the learning value as at first glance would be expected. The linear curve may or may not be larger than the learning curve, depending upon the specific parameter values. A regression of up to ten per cent of the production quantity did not produce a significant change in the total annual cost or in the optimum production quantity.

A definite conclusion can be drawn from the investigation and that is that if the learning curve applies to a specific industrial situation it should be used in arriving at an economic order quantity decision because, depending upon the specific parameter values, the difference between the learning and the linear equations could be significant. Also, the regression quantity did not seem to produce a significant affect on the total annual cost.