A STUDY OF HYDROMAGNETIC WAVES

by

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TABLE OF CONTENTS

INTRODUCTION

The hydromagnetic wave was first predicted in 1942 by H. Alfvén (2), Professor of Electronics, Royal Institute of Technology, Stockholm. It is, in its simplest form, a propagation of some disturbance along the direction of a magnetic field, in a conducting fluid, as a result of a coupling between the fluid and the field due to a motion of the fluid perpendicular to the field (1).

The possible control of a fusion reactor seems, at this time, to require as much knowledge as possible concerning the interactions between a plasma and a magnetic field. The hydromagnetic wave concerns such an interaction and is, therefore, of prime interest at this time.

The design of a suitable experiment depends greatly upon the equations describing a disturbance of the desired form. For this reason, a development of such equations is given here.

Consider the case of plane waves in an incompressible fluid with conductivity and density . In c.g.s. units, Maxwell's field equations are as follows:

$$\nabla \times \underline{H} = \frac{1}{c}\left(4\pi \underline{i} + \frac{\partial \underline{D}}{\partial t}\right) \tag{1}$$

$$\nabla \times \underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t} \tag{2}$$

$$\nabla \cdot \underline{B} = 0 \tag{3}$$

$$\nabla \cdot \underline{D} = \rho_c \tag{4}$$

where

$$\underset{\sim}{B} = \mu \underset{\sim}{H} \qquad \text{and} \qquad \underset{\sim}{i} = \sigma \left[\underset{\sim}{E} + \frac{\underset{\sim}{v} \times \underset{\sim}{B}}{c} \right] \tag{5}$$

The hydrodynamic equation of motion for a unit mass of the fluid is

$$\frac{d\underset{\sim}{v}}{dt} = \underset{\sim}{G} + \frac{1}{\rho} \left[\frac{\underset{\sim}{i} \times \underset{\sim}{B}}{c} - \nabla p \right] \tag{6}$$

where $\frac{d}{dt} = (\underset{\sim}{v} \cdot \nabla) + \frac{\partial}{\partial t}$, $\underset{\sim}{G}$ is gravitational acceleration, and p is the pressure.

In order to simplify the problem of finding a useful wave equation from these fundamental relations, it is assumed that

$$\underset{\sim}{G} = 0 , \quad \frac{\partial \underset{\sim}{D}}{\partial t} << 4\pi \underset{\sim}{i} , \quad \text{and} \quad \underset{\sim}{H} = H_0 \hat{k} + \underset{\sim}{h} \tag{7}$$

$\underline{h}$ is due to induced currents in the fluid caused by motion of the fluid in the constant field $H_o\hat{k}$, $\hat{k}$ being the unit vector in the z direction. It is also assumed that all vector quantities are functions of z and t only.

From equations (1) and (7), we have, in component form,

$$i_x = -\frac{c}{4\pi} \frac{\partial h_y}{\partial z} , \quad i_y = 0 , \quad i_z = 0 , \quad h_x = 0 , \quad h_y = h_y , \quad h_z = 0 \tag{8}$$

Equations (6) and (7) lead to

$$\frac{\partial v_y}{\partial t} = -\frac{\mu H_0}{\rho c} i_x \tag{9}$$

Combining (8) and (9) we have

$$\frac{\partial v_y}{\partial t} = -\frac{\mu H_0}{4\pi\rho} \frac{\partial h_y}{\partial z} \tag{10}$$

From equations (2) and (5) we obtain

$$\frac{\partial h_y}{\partial t} = - \frac{c}{\mu\sigma}\frac{\partial i_x}{\partial z} + H_0 \frac{\partial v_y}{\partial z} \tag{11}$$

Differentiating equation (11) with respect to time and combining it with (8) and (10), we have the wave equation in h or h_y (1) given by

$$\frac{\partial^2 h_y}{\partial t^2} = \frac{c^2}{4\pi\mu\sigma}\frac{\partial^3 h_y}{\partial t \partial z^2} + \frac{\mu H_0^2}{4\pi\rho}\frac{\partial^2 h_y}{\partial z^2} \tag{12}$$

The conductivity of a plasma is very large and, to a good approximation, $1/\sigma \cong 0$. The equation then takes on the simple form

$$\frac{\partial^2 h_y}{\partial t^2} = \frac{\mu H_0^2}{4\pi\rho}\frac{\partial^2 h_y}{\partial z^2} \tag{13}$$

Equation (13) is seen to be of the same form as that for a vibrating string given by

$$\frac{\partial^2 y}{\partial t^2} = \frac{S}{m}\frac{\partial^2 y}{\partial z^2} \tag{14}$$

where the signal velocity along the string is given by $V = \sqrt{(S/m)}$, S being the tension in the string and m the mass per unit length. It is easily shown that $H_0\sqrt{(\mu/4\pi\rho)}$ does have the dimensions of tension divided by mass per unit line length, (1), (2) thus supporting the analogy of treating the hydromagnetic wave as simply a vibration in the lines of force.

According to the string analogy, an experiment to generate and detect a hydromagnetic wave would simply be a way of "plucking" the lines of force as they pass through a highly conducting medium and detecting the transmitted vibration some distance along the "plucked" lines.

Work was done in 1959 at the Lawrence Radiation Laboratory, (3) University of California, Berkeley, California, with an experiment to detect hydromagnetic waves in a hydrogen plasma. The wave tube consisted of a conducting cylinder 34 inches long and 5 3/4 inches in diameter, filled with hydrogen gas, and mounted in a uniform axial magnetic field. A coaxial electrode, 2 inches in diameter and 2 inches long, was placed at each end of the tube.

The gas was ionized by discharging a 45 ufd. condenser bank between the two central electrodes. After this discharge current reached a maximum (50 kiloamperes), a smaller condenser bank of 1.2 ufd. was discharged between the center electrode and the outer electrode of one of the coaxial electrodes. This discharge current gave rise to a torsional field. (See Plate I.) An oscilloscope was used at the opposite coaxial electrode to measure the induced electric field as a result of a propagated hydromagnetic wave.

The data obtained for an axial magnetic field between 6 and 14 kilogauss and hydrogen gas pressure of 100 microns gave positive indication of hydromagnetic wave transmission. The measured velocities were on the order of 3×10^7 cm/sec., approximately 20% above the theoretical value given by $V = H_0 \sqrt{(\mu / 4\pi\rho)}$. This discrepancy between measured and predicted values was believed to be due to incomplete preionization of the gas. The amplitude of the transmitted signal was found to be reduced by approximately 50% in traveling through the tube.

The purpose of this work was to design and perform an experiment for the detection of a hydromagnetic wave in argon, using the

EXPLANATION OF PLATE I

Fig. 1. Magnetic field and current configuration for experiment at Berkeley.
Fig. 2. Magnetic field and current configuration for experiment performed at Kansas State University.

PLATE I

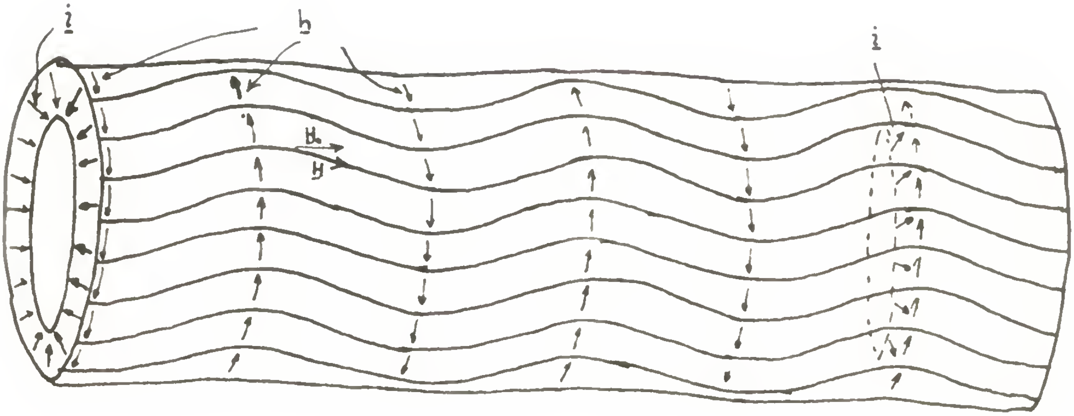


Fig. 1

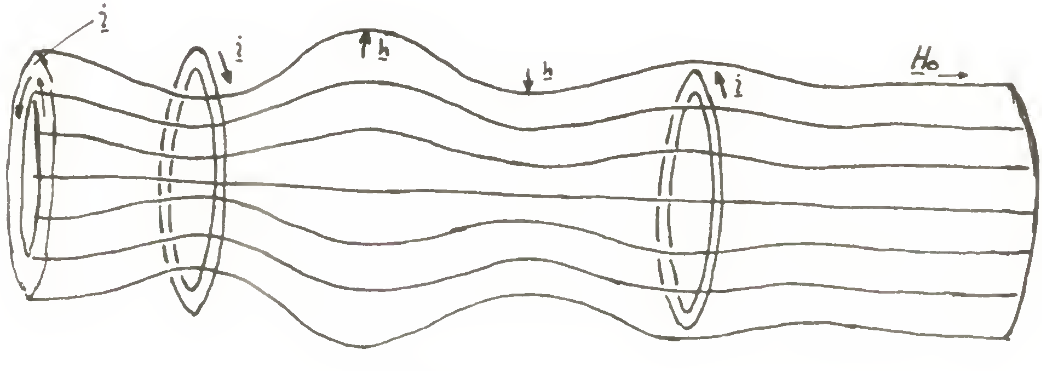


Fig. 2

equipment available at Kansas State University. However, emphasis was placed on "plucking" the lines of force by means of an external current and observing the change in the magnetic field along the axial field by measuring the current generated in an external conductor. (See Plate I.) Whereas, in the experiment at Berkeley, currents in the plasma were used to generate the wave, and the changes in the electric field in the plasma were detected as the result of wave transmission.

The following design was an attempt to utilize the same condenser bank for exciting the plasma and impressing the signal upon the constant magnetic field.

APPARATUS AND PROCEDURE

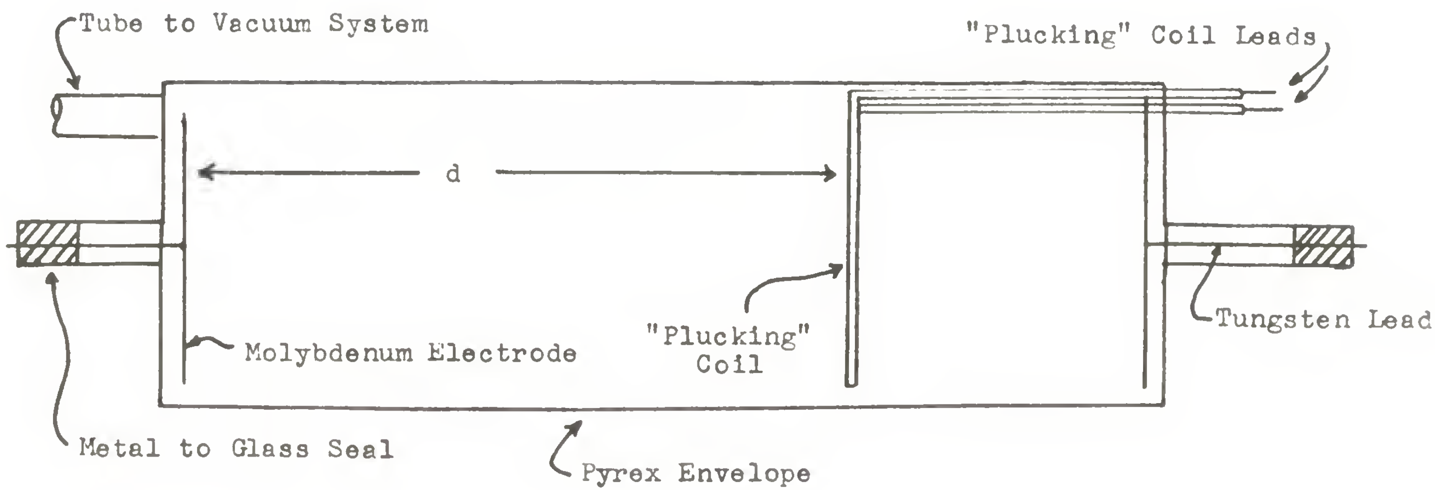
The wave tube, as seen in Plate II, was constructed by using 5 mm. diameter pyrex tubing as an envelope. The electrodes were of molybdenum and were slightly smaller in diameter than the envelope. They were also slotted diagonally to cut down on damping of the signal as it passed through the electrodes. The electrode leads were made of tungsten, silver-soldered to the electrodes, and making a tungsten to uranium glass to pyrex seal with the larger tube. The "plucking" coil was encased in a 2 mm. diameter pyrex tube.

The condenser bank consisted of three 1.8 ufd., 25 KV capacitors connected in parallel. These were supplied with power by a 0-40 KV supply capable of delivering a maximum current of one milliampere. The oscilloscope used was a Model 551 Techtronics, dual beam oscilloscope.

EXPLANATION OF PLATE II

Wave tube for experiment with argon.

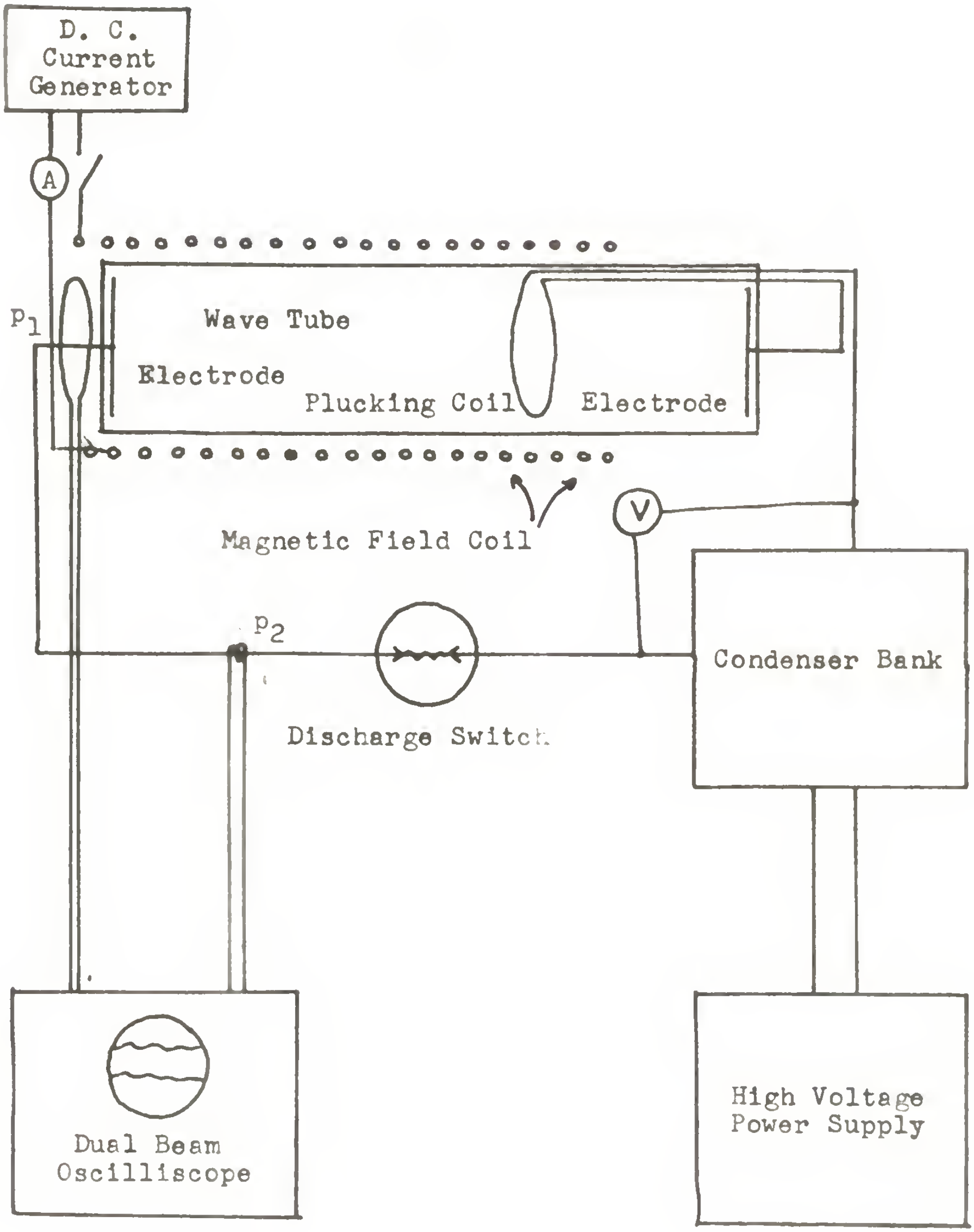
PLATE II


Tube to Vacuum System
"Plucking" Coil Leads
d
Tungsten Lead
Molybdenum Electrode
"Plucking" Coil
Metal to Glass Seal
Pyrex Envelope

EXPLANATION OF PLATE III

Circuit diagram of experiment.

PLATE III


D. C.
Current
Generator
A
P1
Wave Tube
Electrode
Plucking Coil
Electrode
V
Magnetic Field Coil
P2
Condenser Bank
Discharge Switch
Dual Beam
Oscilliscope
High Voltage
Power Supply

The discharge was initiated by bringing two brass electrodes together until the intervening air gap was small enough to break down and conduct.

Current for the coils creating the axial magnetic field was supplied by a 150 volt, direct-current generator. Resistance in the line from the generator and in the coil limited this current to 15 amperes, giving rise to a maximum H_o of 2,000 gauss.

First, the gas in the tube was brought to the desired pressure. With the external field equal to zero, the voltage was allowed to rise to the desired value. (See Plate III.) At this voltage, the switch 3 was triggered and the condenser bank discharged through the wave tube and the "plucking" coil in series. This procedure was then repeated with the exception that H_o was given a constant value. The pick-up loops, p_1 and p_2, leading from the oscilloscope, as shown in Plate III, impressed a signal upon the scope for each firing. (See Plate IV.)

The signals from p_1 and p_2 were damped sine waves and showed a definite phase difference between the two signals. These wave forms were recorded on film so that any change in this phase difference, as a result of the magnetic field, could be measured.

This expected change of phase may be explained in the following manner. Since the coil within the plasma and the external circuit are in series, their currents should be of the same frequency. Hence, there should be a signal from p_1 as a result of the changing magnetic field of the same frequency as the signal from p_2. However, the signal arriving at p_1, due to the changing magnetic field, must have traveled from the "plucking" coil which requires a time given by d/v, where d is the distance traveled and v is the velocity.

For a vacuum, or where there is no conducting material in the tube, the displacement current, $\frac{\partial D}{\partial t}$, may not be neglected in deriving the expression for velocity. It has been found that for this case $v = \left[\frac{4\pi\rho}{H_0^2} + \frac{1}{c^2}\right]^{-1/2}$. For $\rho \cong 0$, we see that $v = c$, the speed of light. For $\rho \neq 0$, but $\frac{4\pi\rho}{H_0^2} >> \frac{1}{c^2}$, $v = H_0\sqrt{(\mu/4\pi\rho)}$. For $H_0\sqrt{(\mu/4\pi\rho)} << c$, we see that $d/H_0\sqrt{(\mu/4\pi\rho)} > \frac{d}{c}$ and there should be a corresponding change in phase difference for the signals when transmitting the signal through a plasma as compared to a vacuum. For $H_0 = 0$, there should be no transmission of the type signal discussed. The signal should travel at the speed of light between the electrode and p_1. The effect of having p_1 outside of the tube should then be negligible, and the time for the signal to travel through the plasma should then be very nearly equal to the time required to travel from the "plucking" coil to p_1.

A measurement of this phase shift from the film should then yield the time of travel over the distance, d, and enable one to compare the measured experimental value of velocity, d/t, with the theoretical, $v = H_0\sqrt{(\mu/4\pi\rho)}$.

RESULTS

Typical data are shown in Plate IV, Fig. 1, for the case of no plasma. (a) shows the wave form with no magnetic field;

EXPLANATION OF PLATE IV

Fig. 1. Data for d = 5 cm. with no plasma, discharge-voltage = 15 K.V., sweeptime = 20 μsec/cm.

(a) Wave forms from p_1 (upper trace) and p_2 (lower trace) with H_o = 0 gauss.
(b) Wave forms from p_1 and p_2 with H_o = 950 gauss showing extension of signal in time.

Fig. 2. Data for d = 10 cm. with no plasma, discharge-voltage = 15 K.V., sweeptime = 20 μsec/cm.

(a) Wave form from p_1 with H_o = 0 gauss.
(b) Wave form from p_1 with H_o = 2,000 gauss showing extension of signal in time.

PLATE IV

(b)

(a)

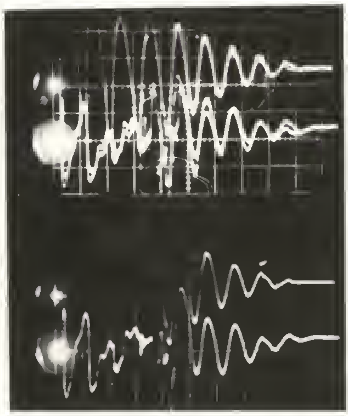


Fig. 1

(a)

(b)

(a)

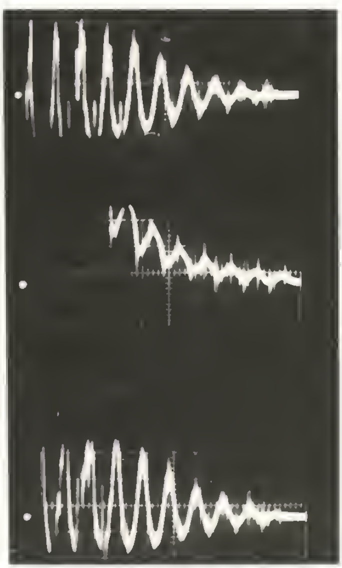


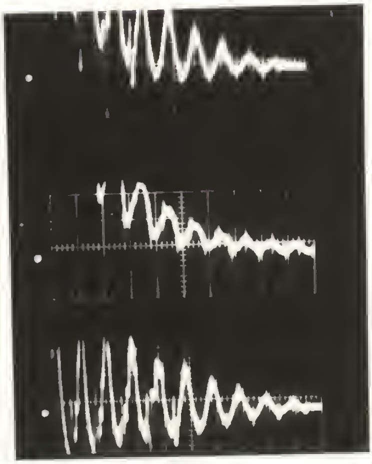
Fig. 2

EXPLANATION OF PLATE V

Data for d = 10 cm. with no plasma, discharge-voltage = 15 K.V., sweeptime = 20 μsec/cm.

(a) Wave form from p_1 with H_o = 0 gauss.
(b) Wave form from p_1 with H_o = 2,000 gauss showing extension of signal in time.

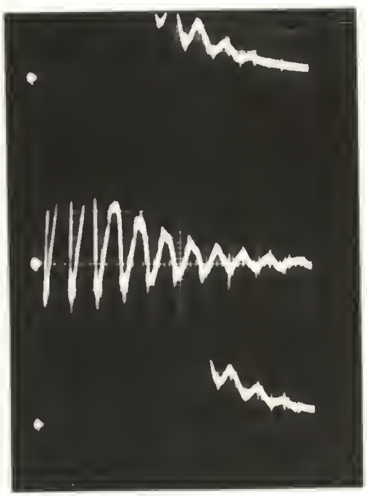
PLATE V



(a)

(b)

(a)



(a)

(b)

(a)

(b) shows the superimposed wave forms with and without the magnetic field. The upper trace corresponds to p_1 and the lower corresponds to p_2. For these data, d = 5 cm., V = 15 KV, and H_o = 950 gauss.

These data at (b) show that, as a result of the field, the amplitude of the signal is increased. This suggests that a hydromagnetic wave is being transmitted, but the size of the increase indicates that the wave amplitude is very small. These data are of such a nature that no quantitative measurements could be made.

For the wave tube containing argon, the data did not yield any information other than to suggest that the wave is rapidly attenuated.

DISCUSSION AND CONCLUSIONS

The absence of a positive indication of a transmitted wave in argon did not show conclusively that such waves did not exist for these conditions, as the reproducibility of the data was poor. The data did suggest that a wave was not transmitted. Using the string analogy, it is easily seen experimentally that for certain strings and experimental conditions, it is very difficult to transmit vibrations down the string. This is caused by the imperfections of the string i.e., friction within the string and between the string and the surrounding media. The same should be true for a magnetic field in a medium where there are many collisions between ions resulting from the compression due to wave motion.

The restrictions on the experiment were (a) the size of the condenser bank, (b) upper limit of the high voltage power supply, and (c) the strength of the axial magnetic field. Raising these restrictions so that one could vary the parameters and explore transmission conditions at the higher values of H_o would yield more useful data.

The experiment would also be greatly improved by using a separate condenser bank, which could be triggered at any prescribed time, to initiate the signal.

One of the results of this experiment might then be to obtain information regarding the effect of compressibility of the conducting fluid on a hydromagnetic wave. For values of h of the same order of magnitude as H_o, in this experiment, compressibility is essential to wave generation and transmission. This would enable one to make comparisons of measured quantities with those derived from more general considerations.

ACKNOWLEDGMENTS

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It was predicted by Alfvén, in 1942, that for a conducting fluid in a homogeneous magnetic field any motion of the fluid perpendicular to the field gives rise to a wave motion propagated in the direction of the field. The plasma is a very good conductor and should exhibit this wave motion.

Using Maxwell's equations, the hydrodynamic equation for the force on a unit mass of the fluid, and the current relation based upon the Lorentz force expression, an equation may be found for ideal conditions in the variable h, where h is the magnetic field component due to induced currents caused by motion of the fluid. This equation is seen to have the same form as the wave equation for a vibrating string. The hydromagnetic wave motion may then be thought of as being vibrations in the lines of force of the axial field.

The experiment was designed to pluck these lines of force and observe any transmission in the direction of the field.

The above conditions were approximated by a cylinder of plasma which was permeated by a homogeneous axial magnetic field, the lines of which were plucked radially by a current-carrying coil which alternately increased and decreased the field strength at some point in the plasma. Another coil was placed similarly at some distance, d, away from the plucking coil along the z axis.

The experiment was performed for values of the axial field less than 2,000 gauss and hydrogen gas pressure less than 80 mm. Hg. The experimental results for these values were inconclusive, but suggested no transmission of a wave in argon for d = 10 cm. but transmission for d having values of 5 and 10 cm. with no plasma present.

These results suggest that it would be fruitful to extend the experiment to higher values of field strength and more energetic plasmas.