A STUDY OF THE ELASTIC STRESSES
AROUND HOLES IN A WIDE-FLANGE BEAM
WITH A CONCENTRATED LOAD

by 500

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TABLE OF CONTENTS

I. Introduction.

A. Statement of Problem, Purpose and Scope.

The scope of building structures is changing rapidly. Building heights are growing up and up, and volumes are getting larger and larger, but the space reserved for structural members is getting smaller. So new problems involved in the design of building frames are being developed, and old problems are taking on more importance. One of these problems concerns openings in the webs of steel beams or girders, for example, for the passage of pipes, electric cables and ventilation ducts to minimize the space or for architectural reasons. For this reason, the determination of the stress distribution around holes in the webs of wide-flange beams is an important problem for structural engineers.

The purpose of this paper is to introduce a method for calculating the elastic stresses around a hole in a web of a wide-flange beam with a concentrated load (Fig. 1). As far as the method is concerned, a hole of general shape can be treated. A numerical example is presented for the case of a simply supported beam with a concentrated load, 2P, at midspan, which has a span length of 2L and a circular hole with a radius "a" at a distance "u" from the support. The hole is centered on the neutral axis of the beam.

The equations for the stresses in the beam with a hole at different locations, u = 80", 60", 40", 20", are calculated using a digital computer.

B. Brief Review of the Literature.

Theoretical and experimental solutions for the stresses around a hole in a plate subjected to bending moment were obtained by Tuzi (1)* early in this century. The Airy stress function which he used was based on the results of photoelastic experiments, and a theoretical solution was worked out using the stress function and assuming an infinite plate.

Subsequently, a theoretical solution using conformal mapping and theory of complex variables was initiated by Greenspan (2). The mapping function assumed was approximate, so the results were also approximate. The procedure was very similar to Tuzi's, but a different stress function was employed.

The basic method for determining the stress distribution around holes was given by Savin (3), who treated the problem in general form. However, his solution was restricted to plates subjected to pure tensile or compressive stresses.

Recently, Joseph and Brock (4) have studied the stresses around a general small opening in a beam subjected to pure bending, using the complex variable method associated with Muskhelisvili (5). An exact mapping was employed for the first time, but details of the solution were not presented. Numerical examples were obtained using Greenspan's approximate function.

The stresses around a small opening in a beam subjected to bending with shear were determined by Heller (6), who super-imposed a stress equation considering bending with shear with

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* The numbers in parentheses refer to references in Bibliography.

the equation solved by Joseph and Brock (4). This solution was also based on Greenspan's mapping function. Heller continued to study various mapping functions with Brock and Bart (7). Finally, a general mapping function was developed along with a method of choosing the coefficients which are needed for the solution.

Another method, slightly different, was developed by Bower (8). The procedure was different, which is that the stresses around an opening in a web of a wide-flange beam subjected to bending with shear can be solved by taking the sum of the basic stresses and the perturbated stresses. This approach is discussed in the following section.

II. Assumptions.

A. The stresses in a wide-flange beam with a web hole are computed from the following equations:

$$\sigma_\theta = \sigma_{\theta b} + \sigma_{\theta p}$$
$$\sigma_r = \sigma_{rb} + \sigma_{rp} \tag{1}$$
$$\tau_{r\theta} = \tau_{r\theta b} + \tau_{r\theta p}$$

where σ_θ and σ_r are the total normal stresses on the cross section perpendicular to the θ and r axes, and $\tau_{r\theta}$ is the total shear stress on that cross section. $\sigma_{\theta b}$ and σ_{rb} are the basic normal stresses which occur in a beam when there is no opening and act on the cross section perpendicular to the θ and r axes, while $\tau_{r\theta b}$ is the basic shear stress on that cross section. $\sigma_{\theta p}$, σ_{rp} and $\tau_{r\theta p}$ are the perturbated stresses which occur in a beam as a result of forces applied to the boundary of the hole.

B. The forces applied to the boundary of the hole are applied in such a manner that the resulting perturbated stresses and the basic stresses satisfy a required boundary condition at the hole. The boundary of the hole is assumed to be free of force; therefore, the shear stress and stress normal to the boundary of the hole are zero at the boundary (8).

C. The perturbated stresses must attenuate according to Saint Venant's principle as distance from the hole increases, because the hole causes only localized redistribution of stress (8).

D. The ratio of the hole diameter to the web depth does not exceed a maximum value. This assumption will be discussed

later.

E. The solution is based on the usual assumptions of plane elasticity: homogeneous, isotropic material within the elastic limit.

III. Method of Solution.

A. Basic Stresses and Stress Functions.

To calculate the stresses in a wide-flange beam with a hole, the stress equations which were given by Timoshenko (9) may be used. A simply supported beam with a concentrated load at the center can be treated as a cantilever. Then the stress equations are:

$$\sigma_x = - \frac{Pxy}{I} \tag{2}$$

$$\sigma_y = -0 \tag{3}$$

$$\tau_{xy} = - \frac{P}{2I} (c^{*2} - y^2) \tag{4}$$

in which σ_x is the normal stress on a cross section perpendicular to the longitudinal direction of the beam, σ_y is the normal stress on a cross section parallel to the longitudinal axis of the beam and τ_{xy} is the shear stress in the web on the above cross section. c^* is a modified halfdepth of the beam given by (5)

$$c^{*2} = d^2(1 + \frac{2hp}{td}) \tag{5}$$

These symbols are defined in Fig. 1. The bracketed quantity in Eq. 5 accounts for the difference in shear stress distribution in a wide-flange beam as compared with that of a beam with a rectangular cross beam.

Eqs. 2 through 4 give the stresses as a function of the loading and beam geometry at locations defined by the x and y coordinates. For subsequent work, it is convenient to give the

stresses in terms of complex stress functions so that the
boundary of the hole can eventually be expressed as a continuous
function of a single complex coordinate rather than as a
function of the two coordinates x and y. The complex stress
functions of Muskhelishvili are related to the normal and shear
stresses given in Eqs. 2 through 4 by the formulas (5)

$$\sigma_x + \sigma_y = 4\underline{\text{Re}}\,\phi' \tag{6}$$

$$\sigma_y - \sigma_x + 2i\tau_{xy} = 2(\bar{z}\phi'' + \psi') \tag{7}$$

and to the boundary forces by the formula

$$g_1 + ig_2 = \phi + 2\bar{\phi}' + \bar{\psi} \tag{8}$$

in which ϕ and ψ are complex stress functions of the single
coordinate z, which is defined as $x + iy$, $\bar{\phi}$ and $\bar{\psi}$ are the complex
conjugate of ϕ and ψ, the symbol $\underline{\text{Re}}$ denotes the real part of the
quantity following the symbol, the primes denote differentiation
with respect to z, and i is the imaginary part. The quantities g_1
and g_2 are functions of the boundary forces (5).

Because the basic normal and shear stresses in the beam are
known in terms of the loading conditions and beam geometry, the
stress function can be expressed in terms of the loading and
beam geometry by combining Eqs. 2, 3, 4, 5, 6, and 7 (8).

B. Mapping Functions.

To simplify the subsequent computation of stresses in
the neighborhood of a hole, it is convenient to transform the
complex coordinate system used in Eqs. 6 through 8 to a different

complex coordinate system in which the future hole boundary is defined by radial polar coordinate r (8). This is accomplished by conformally mapping the area outside the hole into the interior of a unit circle so that the boundary of the hole becomes the perimeter of the circle. The following general transformation equation performs this mapping,

$$z = W(w) = \frac{A}{w} + Bw + Cw^3 + Dw^5 + Ew^7 \tag{9}$$

in which w is the coordinate in the transformed complex plane, and A, B, C, D, and E are real coefficients that vary according to the shape of the hole being transformed. This mapping function was obtained from the following Schwarz - Christoffel integral (4, 5)

$$z = z(w) = a \int_1^w (t^4 - \tfrac{1}{2}(1 - K^2)t^2 + \frac{1}{16}(1 + K^2)^2)^{\frac{1}{2}} \frac{dt}{t^2}$$
$$+ \text{ const.} \tag{10}$$

The coefficients can be evaluated using Newton's Approximate Method. Typical examples are given in Ref. 7.

Because the new plane is the interior of a unit circle, w is defined in terms of polar coordinates r and θ. Specifically

$$w = r \exp i\theta \tag{11}$$

The hole boundary lies on the circle where r is unity, and the exterior boundary of a beam are not explicitly defined by the transformation (8).

C. Transformation of σ_x and σ_y into σ_r and σ_θ.

To provide equations relating these complex stress functions to the normal and shear stresses and to the boundary forces, Eqs. 6, 7, and 8 are also transformed by using Eq. 9.(4).

$$\sigma_x + \sigma_y = \sigma_r + \sigma_\theta$$

$$(\sigma_y - \sigma_x + 2i\tau_{xy})e^{2i\theta} = \sigma_\theta - \sigma_r + 2i\tau_{r\theta} \tag{12}$$

The transformation defined in Eq. 9 describes the relations between an increment dz in the z-plane and an increment dw in the w-plane. Therefore,

$$dz = e^{i\alpha}|dz| \quad , \qquad dw = e^{i\theta}|dw|$$

Therefore,

$$e^{i\alpha} = \frac{dz}{|dz|} = \frac{W'(w)}{|W'(w)|}\frac{dw}{|dw|} = \frac{W'(w)}{|W'(w)|}e^{i\theta} = \frac{w}{r}\frac{W'(w)}{|W'(w)|} \tag{13}$$

and

$$e^{i2\alpha} = \frac{w^2}{r^2}\frac{W'(w)}{W'(w)} \tag{14}$$

and

$$z = W(w)$$

$$\phi(z) = \phi(W(w)) = \phi_1(w)$$

$$\phi_1'(w) = \phi'(z) = \frac{d\phi}{dz} = \frac{d\phi}{dw}\frac{dw}{dz} = \frac{d\phi}{dw} \Big/ \frac{dz}{dw} = \frac{\phi'(w)}{W'(w)}$$

Therefore,

$$\phi'(z) = \frac{\phi'(w)}{W'(w)} \tag{15}$$

The same procedure yields

$$\psi'(z) = \frac{\psi'(w)}{W'(w)}$$ (5. pp. 183.)

By substituting Eqs. 14 and 15 into Eq. 12

$$\sigma_r + \sigma_\theta = 4\underline{Re}\,\frac{\phi'}{W'}$$ (16)

$$\sigma_\theta - \sigma_r + 2i\zeta_{r\theta} = \frac{2w^2}{r^2\,\overline{W'}}\left(\overline{W}(\frac{\phi'}{W'})' + \psi'\right)$$ (17)

$$\phi + W\frac{\overline{\phi'}}{\overline{W'}} + \overline{\psi} = g_1 + g_2 i$$ (18)

in which ϕ and ψ are now functions of w, and primes indicate differentiation with respect to w. σ_r and σ_θ are stresses normal to the cross section perpendicular to the axis defined by the subscript, and $\zeta_{r\theta}$ is the shear stress on their cross section.

D. Perturbated stresses and stress functions.

"Stress functions for perturbated stresses are defined by the power series" (8),

$$\phi_0 = e_1 w + e_2 w^2 + - - - - - + e_n w^n$$ (19)

$$\psi_0 = f_0 + f_1 w + f_2 w^2 + f_3 w^3 + - - - + f_n w^n$$ (20)

in which ϕ_0 and ψ_0 are the perturbated stress functions in terms of w, and e_n and f_n are complex coefficients to be determined by satisfying boundary conditions around the hole.

The perturbated stress coefficients e_n and f_n are calculated from the condition that the sum of the boundary condition forces

related to the basic stresses and the perturbated stresses must be zero. Eq. 18 yields

$$\phi_0(w) + W(w)\frac{\overline{\phi_0}(w)}{\overline{W}'(w)} + \overline{\psi}_0(w) = g_1 + ig_2 \tag{21}$$

and complex conjugate of Eq. 18 becomes

$$\overline{\phi}_0(w) + \overline{W}(w)\frac{\phi_0(w)}{W'(w)} + \psi_0(w) = g_1 - ig_2 \tag{22}$$

Integrating, Eq. 21 yields

$$\frac{1}{2\pi i}\int_C \frac{\phi_0(w_0)}{w_0 - w}\,dw_0 + \frac{1}{2\pi i}\int_C W(w_0)\frac{\overline{\phi_0}'(w_0)\,dw_0}{\overline{W}'(w_0)\,w_0 - w}$$

$$+ \frac{1}{2\pi i}\int_C \frac{\overline{\psi}_0(w_0)}{w_0 - w}\,dw_0 = \frac{1}{2\pi i}\int_C \frac{(g_1 + ig_2)}{w_0 - w}\,dw_0 \tag{23}$$

and by Harnack's theorem (Appendix 1.) and a given form of $\phi_0(w)$

$$\frac{1}{2\pi i}\int_C W(w_0)\frac{\overline{\phi_0}'(w_0)}{\overline{W}'(w_0)}\frac{dw_0}{w_0 - w} = 0$$

By setting

$$\frac{1}{2\pi i}\int_C \frac{\phi_0(w_0)}{w_0 - w}\,dw_0 = \phi_0(w)$$

and

$$\frac{1}{2\pi i}\int_C \frac{\overline{\psi}_0(w_0)\,d(w_0)}{w_0 - w} = \psi_0(0) = \text{Const.} \qquad (5.\ pp.\ 291.)$$

yields

$$\phi_0(w) = \frac{1}{2\pi i} \int_C \frac{g_1 + ig_2}{(w_0 - w)} \, dw_0 \tag{24}$$

By integration, Eq. 22 yields

$$\frac{1}{2\pi i} \int_C \frac{\overline{\phi_0}(w_0)}{w_0 - w} \, dw_0 + \frac{1}{2\pi i} \int_C \overline{W}(w_0) \frac{\phi_0{}'(w_0)}{\overline{W'}(w_0)} \frac{dw_0}{w_0 - w}$$

$$\frac{1}{2\pi i} \int_C \frac{\psi_0(w_0)}{w_0 - w} dw_0 = \frac{1}{2\pi i} \int_C \frac{g_1 - ig_2}{w_0 - w} \, dw_0$$

By setting

$$\frac{1}{2\pi i} \int_C \frac{\overline{\phi_0}(w_0)}{w_0 - w} \, dw_0 = \overline{\phi}_0(0) = 0 \qquad (4, \text{ pp. } 291)$$

and

$$\frac{1}{2\pi i} \int_C \frac{\psi_0(w_0)}{w_0 - w} \, dw_0 = \psi_0(w)$$

then,

$$\psi_0(w) = \frac{1}{2\pi i} \int_C \frac{g_1 - ig_2}{w_0 - w} \, dw - \frac{1}{2\pi i} \int_C \frac{\overline{W}(w_0)}{\overline{W'}(w)} \phi'_0(w_0) \frac{dw_0}{w_0 - w} \tag{25}$$

in which w_0 is the boundary value of w, C denotes the contour of the unit circle over which integration is performed, and $\psi_0(0)$ indicates that ψ_0 is to be evaluated when w equals zero.

IV.   Numerical Example.

In the following example, the stresses around a circular hole in the web of simply supported 12WF 45 steel beam with a concentrated load 2P applied at the center (Fig. 1) are calculated.

A.   Basic Stresses.

From Eqs. 2, 3, and 6,

$$\frac{P(x + u)y}{I} = 4\underline{Re}\,\phi'\qquad\qquad(26)$$

where u is the longitudinal distance from the reaction, or point of zero moment, to the center of the hole.

By integrating with respect to z through the axis ox, then dz = dx.   Therefore,

$$\underline{Re}\,\phi' = \frac{P}{4I}(x + u)y$$

$$\int \underline{Re}\,\phi'dz = \frac{P}{4I} \int (x + u)y\;dz\qquad\qquad(27)$$

$$\underline{Re}\,\phi = \frac{P}{4I}(\frac{x^2 y}{2} + uxy) + C(y)\qquad\qquad(28)$$

By setting $\underline{Re} = P$

$$\phi = P + iQ\qquad\qquad(29)$$

Q is the quantity of an imaginary part and determined by the Cauchy-Riemann equation,

$$\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y},\qquad\qquad -\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}\qquad\qquad(30)$$

Therefore,

$$\frac{\partial P}{\partial x}\,\partial y = \partial Q$$

$$Q = \int \frac{\partial P}{\partial x} \, dy \qquad (31)$$

$$Q = \frac{P}{4I} \left(\frac{xy^2}{2} + \frac{uy^2}{2} \right) + C(x) \qquad (32)$$

Using the second part of Eq. 30 yields

$$C(x) = -\frac{P}{4I} \left(\frac{x^3}{6} + \frac{ux^2}{2} \right)$$

$$C(y) = \frac{P}{4I} \left(\frac{y^3}{6} \right)$$

Therefore,

$$P = \frac{P}{4I} \left(\frac{x^2 y}{2} + uxy + \frac{y^3}{6} \right) \qquad (33)$$

$$Q = \frac{P}{4I} \left(\frac{xy^2}{2} + \frac{uy^2}{2} - \frac{x^3}{6} - \frac{ux^2}{2} \right) \qquad (34)$$

By substituting Eqs. 33 and 34 into Eq. 29,

$$\phi = \frac{P}{8I} \left(x^2 y + 2uxy - \frac{y^3}{3} \right) + \frac{Pi}{8I} \left(xy^2 + uy^2 - \frac{x^3}{3} - ux^2 \right) \qquad (35)$$

Applying $-i^2 = 1$ to the first term of Eq. 35 yields

$$\phi = \frac{Pi}{8I} \left(-ix^2 y - 2iuxy + \frac{iy^3}{3} + xy^2 + uy^2 - \frac{x^3}{3} - ux^2 \right)$$

Using $z = x + iy$

$$\phi = \frac{Pi}{8I} \left(-\frac{z^3}{3} - uz^2 \right) \qquad (36)$$

By substituting Eqs. 2, 3, and 4 into Eq. 7, and using the same procedure, ψ is obtained. By differentiating with respect to z from Eq. 35

$$\phi' = \frac{Pi}{8I}(-x^2 - 2ixy + y^2) + \frac{Pi}{8I}(-2ux - 2uyi) \qquad (37)$$

and

$$\phi'' = \frac{Pi}{8I}(-2x - 2iy - 2u) \qquad (38)$$

Therefore, from Eq. 7

$$\psi' = \frac{P}{2I}(-xy - uy + i(c^{*2} - y^2)) - \frac{Pi}{8I}(-2x^2 - 2ixy - 2ux$$
$$+ 2ixy - 2y^2 - 2uyi) \qquad (39)$$

Integrating in terms of z yields

$$\psi = \frac{P}{8I}(-2x^2y - 4uxy + 4ic^{*2}x - 4ixy^2) - \frac{Pi}{8I}(-\frac{2}{3}x^3 - ux^2$$
$$- 2xy^2 + 2iuxy) + C_1(y) + iC_2(y) \qquad (40)$$

The constants C_1 and C_2 are obtained by the Cauchy-Riemann Equations. Then,

$$\psi = \frac{P}{8I}(-2x^2y - 2uxy - 4c^{*2}y - \frac{2}{3}y^3) + \frac{Pi}{8I}(4c^{*2}x - 2xy^2$$
$$+ \frac{2}{3}x^3 + ux^2 - uy^2) \qquad (41)$$

Applying $-i^2 = 1$ and using $z = x + iy$,

$$\psi = \frac{Pi}{8I}(\frac{2}{3}z^3 + uz^2 + 4c^{*2}z) \qquad (42)$$

B. Mapping function.

The Mapping function for the circular hole in the form of Eq. 9 is

$$z = W(w) = \frac{a}{w} \tag{43}$$

where "a" is the radius of the circle.

The transformed equations for ϕ and ψ in terms of w are:

$$\phi(w) = -\frac{Pi}{8I} \left(\frac{1}{3}\left(\frac{a}{w}\right)^3 + u\left(\frac{a}{w}\right)^2\right) \tag{44}$$

$$\psi(w) = \frac{Pi}{8I} \left(\frac{2}{3}\left(\frac{a}{w}\right)^3 + u\left(\frac{a}{w}\right)^2 + 4c^{*2}\left(\frac{a}{w}\right)\right) \tag{45}$$

C. Perturbated Stresses.

With a circular hole and the previously defined stress functions the perturbated stress functions are calculated as shown below. From Eq. 18

$$g_1 + ig_2 = -\frac{Pi}{8I}\left(\frac{1}{3}\left(\frac{a}{w}\right)^3 + u\left(\frac{a}{w}\right)^2\right) + \frac{Pi}{8I}\frac{a}{w}\left(a^2w^2 + 2uaw\right)$$
$$ - \frac{Pi}{8I}\left(\frac{2}{3}(aw)^3 + ua^2w^2 + 4c^{*2}aw\right) \tag{46}$$

By substituting Eq. 46 into Eq. 24

$$\phi_0(w) = -\frac{1}{2\pi i}\int_C \frac{g_1 + ig_2}{w_0 - w}dw_0$$
$$= \frac{Pi}{8I}\left(\frac{1}{3}\left(\frac{a}{w}\right)^3 + u\left(\frac{a}{w}\right)^2 - a^3w - 2a^2u + \frac{2}{3}a^3w^3\right.$$
$$\left. + ua^2w^3 + 4c^{*2}aw\right)$$

Integration using Harnack's theorem yields

$$\phi_0(w) = \frac{Pia^3}{8I}\left(\frac{2}{3}w^3 + \frac{u}{a}w^2 + \left(\frac{4c^{*2}}{a^2} - 1\right)w - 2\frac{u}{a}\right) \tag{47}$$

Then,

$$e_1 w + e_2 w^2 + - - - + e_n w^n = \frac{Pia^3}{8I}(\frac{2}{3} w^3 + \frac{u}{a} w^2$$

$$+ (\frac{4c^{*2}}{a^2} - 1)w - 2\frac{u}{a})$$

This yields

$$\left. \begin{array}{l} e_1 = \frac{4c^{*2}}{a^2} - 1 \\[2ex] e_2 = \frac{u}{a} \\[2ex] e_3 = \frac{2}{3} \\[2ex] e_4 - - - e_n = 0 \end{array} \right| \quad x \; \frac{Pia^3}{8I} \qquad (48)$$

From Eq. 25

$$\psi_0(w) = -\frac{1}{2\pi i} \int_C \frac{g_1 - ig_2}{w_0 - w} dw_0 - \frac{1}{2\pi i} \int_C \frac{\overline{W}(w_0)}{W'(w)}\phi'_0(w_0) \frac{dw_0}{w_0 - w}$$

$$(25)$$

The first term of Eq. 25 becomes

$$-\frac{1}{2\pi i} \int_C \frac{g_1 - ig_2}{w_0 - w} dw_0 = -\frac{Pia^3}{8I}(\frac{w^3}{3} + \frac{u}{a}w^2) - \frac{Pia^3}{8I}(\frac{2u}{a})$$

and second term of Eq. 25 becomes

$$-\frac{1}{2\pi i} \int_C \frac{\overline{W}(w_0)}{W'(w_0)}\phi_0'(w_0)\frac{dw_0}{w_0 - w} = -\frac{Pia^3}{8I}(2w^5 + \frac{2u}{a}w^4 + (\frac{4c^{*2}}{a^2} - 1)w^3)$$

Therefore,

$$\psi_0(w) = \frac{Pia^3}{8I}(-2w^5 - \frac{2u}{a}w^4 - (\frac{4c^{*2}}{a^2} - \frac{2}{3})w^3 - \frac{u}{a}w^2 + \frac{2u}{a}) \quad (49)$$

$$f_0 = \frac{2u}{a}$$

$$f_1 = 0$$

$$f_2 = -\frac{u}{a}$$

$$f_3 = -(\frac{4c^{*2}}{a^2} - \frac{2}{3}) \qquad\qquad x \; \frac{Pia^3}{8I} \qquad\qquad (50)$$

$$f_4 = -\frac{2u}{a}$$

$$f_5 = -2$$

$$f_6 = - - - - - = f_n = 0$$

Therefore, the perturbated stress functions become

$$\phi_0(w) = \frac{Pia^3}{8I} (\frac{2}{3} w^3 + \frac{u}{a} w^2 + (\frac{4c^{*2}}{a^2} - 1)w) \qquad (51)$$

$$\psi_0(w) = -\frac{Pia^3}{8I}(2w^5 + \frac{2u}{a}w^4 + (\frac{4c^{*2}}{a^2} - \frac{2}{3})w^3 + \frac{u}{a}w^2 - \frac{2u}{a}) \quad (52)$$

D.   Basic stresses in terms of σ_r, σ_θ, and $\tau_{r\theta}$.
Adding Eqs. 16 and 17 yields

$$2\sigma_\theta + 2i\tau_{r\theta} = 4\underline{Re} \frac{\phi'}{W'} + \frac{2w^2}{r^2 \overline{W'}} (\overline{W}(\frac{\phi'}{W'})' + \psi') \qquad (53)$$

From Eq. 15 the first term of Eq. 53 becomes

$$\underline{Re} \frac{\phi'}{W'} = \underline{Re} \; \phi'(z) = -\frac{Pi}{8I} (z^2 + 2uz)$$

Therefore,

$$4\underline{Re} \frac{\phi'}{W'} = -\frac{Pi}{2I}(\frac{a^2}{w^2} + 2u\frac{a}{w}) = -\frac{Pi}{2I}(\frac{a^2}{r^2} e^{-2i\theta} + \frac{2ua}{r} e^{-i\theta})$$

From the equation $e^{i\theta} = \cos\theta + i\sin\theta$,

$$4\underline{Re}\,\frac{\phi'}{W'} = -\frac{P}{2I}\left(\frac{a^2}{r^2}\sin 2\theta + \frac{2ua}{r}\sin\theta\right)$$

The second term becomes

$$\frac{2w^2}{r^2\overline{W}'}\left(\overline{W}\left(\frac{\phi'}{W'}\right)' + \psi'\right)$$

$$= \frac{2w^2}{r^2 a}\left(aw\left(-\frac{Pi}{8I}\left(\frac{a^2}{w^2} + 2u\frac{a}{w}\right)\right)' + \frac{Pi}{8I}\left(-2\frac{a^3}{w^4} - 2u\frac{a^2}{w^3} - 4c^{*2}\frac{a}{w^2}\right)\right)$$

$$= \frac{2}{r^2}\left(\frac{Pi}{8I}\right)\left(2a^2 + 2uaw - \frac{2a^2}{w^2} - 4c^{*2} - \frac{2ua}{w}\right)$$

$$= -\frac{2Pi}{8r^2 I}\,\frac{2a^2}{r^2}(\cos 2\theta - i\sin 2\theta) + \frac{2ua}{r}(\cos\theta - i\sin\theta)$$

$$+ 2(2c^{*2} - a^2) - 2uar(\cos\theta + i\sin\theta))$$

Therefore,

$$2\sigma_\theta = -\frac{P}{2I}\left(\frac{a^2}{r^2}\sin 2\theta + \frac{2ua}{r}\sin\theta\right)$$

$$-\frac{2P}{8r^2 I}\left(\frac{2a^2}{r^2}\sin 2\theta + \frac{2ua}{r}\sin\theta + 2uar\sin\theta\right)$$

$$\sigma_\theta = -\frac{p}{4I}\left(\frac{a^2}{r^2}\sin 2\theta + \frac{2ua}{r}\sin\theta\right)$$

$$-\frac{2P}{16r^2 I}\left(\frac{2a^2}{r^2}\sin 2\theta + \frac{2ua}{r}\sin\theta + 2ua\sin\theta\right) \qquad (54)$$

Eq. 16 yields

$$\sigma_r = -\frac{P}{2I}\left(\frac{a^2}{r^2}\sin 2\theta + \frac{2ua}{r}\sin\theta\right) + \frac{P}{4I}\left(\frac{a^2}{r^2}\sin 2\theta + \frac{2ua}{r}\sin\theta\right)$$

$$+ \frac{P}{8Ir^2}\left(\frac{2a^2}{r^2}\sin 2\theta + \frac{2ua}{r}\sin\theta + 2uar\sin\theta\right) \qquad (55)$$

Eq. 17 yields

$$2\tau_{r\theta} = -\frac{P}{8r^2 I}\left(\frac{2a^2}{r^2}\cos 2\theta + \frac{2ua}{r}\cos\theta + 2(2c^{*2} - a)\right.$$
$$\left. - 2uar\cos\theta\right) \tag{56}$$

At the boundary of the hole, $r = 1$, and σ_r and $\tau_{r\theta}$ are zero. Therefore, from Eq. 16

$$\sigma_\theta = -\frac{Pa^2}{2I}\left(\sin 2\theta + \frac{u}{a}\sin\theta\right) \tag{57}$$

E. Perturbated Stresses in Terms of σ_r, σ_θ, and $\tau_{r\theta}$.
Eqs. 16, 17, 51, and 52 yield the perturbated stresses:

$$\frac{\Phi_0'}{W'} = -\frac{Pia^3}{8I}\frac{w^2}{a}\left(\left(\frac{4c^{*2}}{a2} - 1\right) + \frac{2u}{a}w + 2w^2\right)$$

$$= -\frac{Pia}{8I}\left(\left(\frac{4c^{*2}}{a^2} - 1\right)w^2 + \frac{2u}{a}w^3 + 2w^4\right)$$

$$= -\frac{Pia^2}{8I}\left(\left(\frac{4c^{*2}}{a^2} - 1\right)r^2(\cos 2\theta + i\sin 2\theta) + \frac{2ur^3}{a}(\cos 3\theta\right.$$

$$\left. + i\sin 3\theta) + 2 r^4(\cos 4\theta + i\sin 4\theta)\right)$$

Therefore,

$$4\mathrm{Re}\frac{\Phi_0'}{W'} = \frac{Pa^2}{2I}\left(\left(\frac{4c^{*2}}{a^2} - 1\right)r^2\sin 2\theta + \frac{2ur^3}{a}\sin 3\theta + 2r^4\sin 4\theta\right)$$

and from Eq. 17

$$\frac{2w^2}{r^2\overline{W}'}\left(\overline{W}\left(\frac{\Phi_0'}{W'}\right)' + \psi_0'\right)$$

$$= \frac{2w^2}{r^2 a}\left(aw\left(\frac{Pia^2}{8I}\left(\left(\frac{4c^{*2}}{a^2} - 1\right)2w + \frac{6u}{a}w^2 + 8w^3\right)\right)\right.$$

$$- \frac{Pia}{8I}\left(\frac{2u}{a}w + 3\left(\frac{4c}{a^2} - \frac{2}{3}\right)w^2 + \frac{8u}{a}w^3 + 10\,w^4\right))$$

$$= - \frac{Pia^2}{8I}\left(w^6 + \frac{u}{a}w^5 + \frac{c^{*2}}{a^2}w^4 + \frac{u}{a}w^3\right)$$

Eq. 53 yields

$$2\sigma_\theta + 2i\tau_{r\theta} = \frac{Pa^2}{2I}\left(\left(\frac{4c^{*2}}{a^2} - 1\right)r^2\sin 2\theta + \frac{2ur^3}{a}\sin 3\theta + 2r^4\sin 4\theta\right)$$

$$- \frac{Pia^2}{2Ir^2}\left((\cos 6\theta + i\sin 6\theta) + \frac{u}{a}(\cos 5\theta + i\sin 5\theta)\right.$$

$$\left.+ \frac{2c^{*2}}{a^2}(\cos 4\theta + i\sin 4\theta) + \frac{u}{a}(\cos 3\theta + i\sin 3\theta)\right)$$

Therefore,

$$\sigma_\theta = \frac{Pa^2}{4I}\left(\left(\frac{4c^{*2}}{a^2} - 1\right)r^2\sin 2\theta + \frac{2ur^3}{a}\sin 3\theta + 2r^4\sin 4\theta\right)$$

$$+ \frac{Pa^2}{4Ir^2}\left(\sin 6\theta + \frac{u}{a}\sin 5\theta + \frac{2c^{*2}}{a^2}\sin 4\theta + \frac{u}{a}\sin 3\theta\right) \qquad (58)$$

From Eq. 16,

$$\sigma_r = \frac{Pa^2}{4I}\left(\left(\frac{4c^{*2}}{a^2} - 1\right)r^2\sin 2\theta + \frac{2ur^3}{a}\sin 3\theta + 2r^4\sin 4\theta\right)$$

$$- \frac{Pa^2}{4Ir^2}\left(\sin 6\theta + \frac{u}{a}\sin 5\theta + \frac{2c^{*2}}{a^2}\sin 4\theta + \frac{u}{a}\sin 3\theta\right) \qquad (59)$$

$$\tau_{r\theta} = - \frac{Pa^2}{4Ir^2}\left(\cos 6\theta + \frac{u}{a}\cos 5\theta + \frac{2c^{*2}}{a^2}\cos 4\theta + \frac{u}{a}\cos 3\theta\right) \qquad (60)$$

At the boundary of the hole σ_r and $\tau_{r\theta}$ are equal to zero.

Therefore, from Eq. 16,

$$\sigma_\theta = \frac{Pa^2}{2I}((\frac{4c^{*2}}{a^2} - 1) \sin 2\theta + \frac{2u}{a}\sin 3\theta + 2 \sin 4\theta) \qquad (61)$$

F. Total Stress at the Boundary of the Hole

The total stress at the boundary of the hole is the sum of Eqs. 57 and 61,

$$\sigma_\theta = \frac{Pa^2}{2I}(2\sin 4\theta + \frac{2u}{a}\sin 3\theta + (\frac{4c^{*2}}{a^2} - 2)\sin 2\theta - \frac{u}{a} \sin \theta) \qquad (62)$$

G. Stresses in terms of σ_x and τ_{xy}.

From Eqs. 12 and 14

$$\sigma_x + \sigma_y = \sigma_r + \sigma_\theta$$

$$\sigma_y - \sigma_x + 2i\tau_{xy} = (\sigma_\theta - \sigma_r + 2i\tau_{r\theta})e^{-2i\theta}$$

$$= (\sigma_\theta - \sigma_r + 2i\tau_{r\theta}) \frac{r^2 \overline{W}(w)'}{w^2 W'(w)}$$

By subtracting these two equations,

$$2\sigma_x - 2i\tau_{xy} = \sigma_r(1 + \frac{r^2 \overline{W}'(w)}{w^2 W'(w)}) + \sigma_\theta(1 - \frac{r^2 \overline{W}'(w)}{w^2 W'(w)})$$
$$- 2i\tau_{xy} (\frac{r^2 \overline{W}'(w)}{w^2 W'(w)})$$

Therefore,

$$\sigma_x = \frac{\sigma_r}{2}(1 + \frac{r^2 \overline{W}'(w)}{w^2 W'(w)}) + \frac{\sigma_\theta}{2} (1 - \frac{r^2 \overline{W}'(w)}{w^2 W'(w)}) \qquad (63)$$

$$\tau_{xy} = \tau_{r\theta} (\frac{r^2 \overline{W}'(w)}{w^2 W'(w)}) \qquad (64)$$

For a circular opening, Eq. 14 becomes

$$e^{-2i\theta} = \frac{r^2\ W'(w)}{w^2\ \overline{W}'(w)}$$

and

$$W(w) = \frac{a}{u}, \qquad W'(w) = -\frac{a}{w^2}, \qquad \overline{W}'(w) = a$$

Therefore,

$$e^{2i\theta} = \frac{-w^2 a}{r^2 w^2 a} = -\frac{1}{r^2}$$

$$e^{-2i\theta} = -r^2 \tag{65}$$

Substituting Eq. 65 into Eqs. 63 and 64 yields

$$\sigma_x = \frac{\sigma_r}{2}(1 - r^2) + \frac{\sigma_\theta}{2}(1 + r^2) \tag{66}$$

$$\tau_{xy} = \tau_{r\theta}(-r^2) \tag{67}$$

At the boundary of hole $(r = 1)$,

$$\sigma_x = \sigma_\theta \tag{68}$$

$$\tau_{xy} = -\tau_{r\theta} \tag{69}$$

Therefore, at the boundary of the hole σ_x and τ_{xy} become

$$\sigma_x = \frac{Pa^2}{2I}(2\sin 4\theta + \frac{2u}{a}\sin 3\theta + (\frac{4c^{*2}}{a^2} - 2)\sin 2\theta - \frac{2u}{a}\sin\theta)$$

$$\tau_{xy} = \frac{Pa^2}{4I}(\cos 6\theta + \frac{u}{a}\cos 5\theta + \frac{2c^{*2}}{a^2}\cos 4\theta + \frac{u}{a}\cos 3\theta + \cos 2\theta$$
$$+ (2c^{*2} - a^2))$$

The general equations for σ_x and ζ_{xy} are obtained as follows:
from Eqs. 55 and 59

$$\sigma_r = \frac{Pa^2}{4Ir^2}(-\sin 6\theta - \frac{u}{a}\sin 5\theta - 2(\frac{c^{*2}}{a^2} - r^6)\sin 4\theta - \frac{u}{a}(1 - 2r^5)$$

$$\cdot \sin 3\theta + ((\frac{4c^{*2}}{a^2} - 1)r^4 + (\frac{1}{r^2} - 1))\sin 2\theta + \frac{u}{a}$$

$$\cdot (\frac{1}{r} - r)\sin \theta)$$

and from Eqs. 54 and 58

$$\sigma_\theta = \frac{Pa^2}{4Ir^2}(\sin 6\theta + \frac{u}{a}\sin 5\theta + 2(\frac{c^{*2}}{a^2} + r^6)\sin 4\theta$$

$$+ \frac{u}{a}(2r^5 + 1)\sin 3\theta + ((\frac{4c^{*2}}{a^2} - 1)r^4 - \frac{1}{r^2} - 1)\sin 2\theta$$

$$- \frac{u}{a}(3r + \frac{1}{r})\sin \theta)$$

Equations 56 and 60 yield

$$\zeta_{r\theta} = - \frac{Pa^2}{4Ir^2}(\cos 6\theta + \frac{u}{a}\cos 5\theta + \frac{2c^{*2}}{a^2}\cos 4\theta + \frac{u}{a}\cos 3\theta$$

$$+ \frac{1}{r^2}\cos 2\theta + \frac{u}{a}(\frac{1}{r} - r)\cos \theta + (\frac{2c^{*2}}{a^2} - 1))$$

Therefore, from Eq. 66

$$\sigma_x = \frac{Pa^2}{8Ir^2}((-\sin 6\theta - \frac{u}{a}\sin 5\theta - 2(\frac{c^{*2}}{a^2} - r^6)\sin 4\theta$$

$$- \frac{u}{a}(1 - 2r^5)\sin 3\theta + ((\frac{4c^{*2}}{a^2} - 1)r + (\frac{1}{r^2} - 1))$$

$$\cdot \sin 2\theta + \frac{u}{a}(\frac{1}{r} - 1)\sin\theta)(1 - r^2)$$

$$+ (\sin 6\theta + \frac{u}{a}\sin 5\theta + 2(\frac{c^{*2}}{a^2} + r^6)\sin 4\theta$$

$$+ \frac{u}{a}(2r^5 + 1)\sin 3\theta + ((\frac{4c^{*2}}{a^2} - 1)r^4 - \frac{1}{r^2} - 1)\sin 2\theta$$

$$- \frac{u}{a}(3r + \frac{1}{r}) \sin \theta)(1 + r^2)) \tag{70}$$

$$\zeta_{xy} = \frac{Pa^2}{4I}(\cos 6\theta + \frac{u}{a} \cos 5\theta + \frac{2c^{*2}}{a^2} \cos 4\theta + \frac{u}{a} \cos 3\theta$$

$$+ \frac{1}{r^2} \cos 2\theta + \frac{u}{a}(\frac{1}{r} - r)\cos \theta + (\frac{2c^{*2}}{a^2} - 1)) \tag{71}$$

H. Graphical presentation of Results.

The tangential stresses, the bending stresses and the shear stresses around a hole in a beam have been calculated using a digital computer. The programs for these calculations are given in Appendix II.

Circular holes with three different radii are treated, a = 2.0", 2.5", 3.0". For each radius the stresses have been calculated for the locations u = 80", 60", 40", 20".

In Figs. 2 through 4, the tangential stress parameters are presented. Figs. 5 to 7 show the normal bending stresses corresponding to the above tangential stresses on transverse sections at $\theta = 0°$, $90°$, $180°$. $\theta = 0°$ corresponds to the low moment edge of the hole and $\theta = 180°$ to the high moment edge of the hole.

The shear stresses at locations u = 80", 60", 40", 20", and at the transverse sections corresponding to $\theta = 0°$, $90°$, $180°$ are drawn in Fig. 8 for a beam with a circular hole of 2.0" radius.

V. Discussion.

For any shape or size of opening or for any loading
condition, the total stresses can be obtained numerically.
However, the applicability of elasticity theory, as far as the
size of opening is concerned, is limited to beams in which the
ratio of the hole diameter to the web depth does not exceed a
maximum value determined by examining moment and shear force
equality. On any transverse cross section in the beam, "the
internal moment computed from the total stresses must equal the
applied moment, and the internal shear force computed from the
total stresses must equal the applied shear force" (8). Because
stresses defined by Eqs. 19 and 20 attenuate as the distance from
the hole increases, the stresses at the top and bottom of the
beam would equal the basic stresses if the ratio of the hole
diameter to web depth were small. For such small holes, the total
stresses obtained by using Eqs. 19 and 20 satisfy the moment and
shear force equalities on all cross sections as well as the
boundary conditions at the hole. However, for larger holes the
total stresses would be greater than the basic stresses at the
top and bottom of the beam, and the total stresses obtained by
using Eqs. 19 and 20 may not satisfy the moment and shear
equalities. This limits the applicability of this analysis to
beams in which the ratio of the hole diameter to web depth does
not exceed a maximum value that would be examined at each opening
(8). For practical purposes the analysis should be satisfactory
for beams with a depth to diameter ratio as low as 2.0.

Second, the mapping functions must be chosen to suit the

shape of the opening. As given before, the general mapping function is:

$$z = W(w) = \frac{A}{w} + Bw + Cw^3 + Dw^5 + Ew^7 \qquad (79)$$
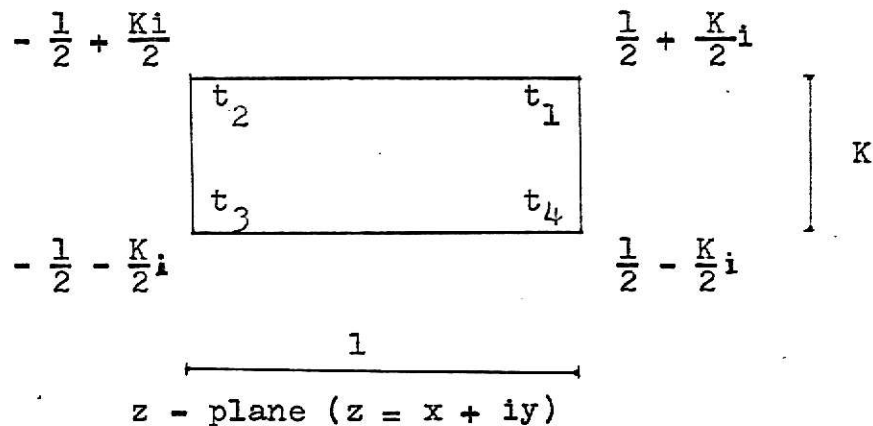
where A, B, C, D, and E are real coefficients. For a rectangular opening this equation can be developed from the following Schwarz-Christoffel integral (7, 3),

$$z = z(w) = A \int_1^w (t^4 - \frac{1}{2}(1 - K^2)t^2 + \frac{1}{16}(1 + K^2)^2)^{\frac{1}{2}}\frac{dt}{t^2}$$

$$+ \text{ const.} \qquad (80)$$

where

$$t = \frac{1}{2}(\pm 1 \pm Ki)$$

K : ratio of vertical side length to horizontal side one.

z - plane (z = x + iy)

By expanding the bracket in a descending power series and choosing the arbitrary constant such that no constant term appears in the integral, Eq. 79 becomes

$$z = A(w + \frac{1 - K^2}{4}\frac{1}{w} - \frac{K^2}{24w^3} - \frac{K^2(1 - K^2)}{16^0 w^5} - \frac{K^2(1 - 3K + K^4)}{896\ w^7}$$

$$+ ---) \qquad (81)$$

This expression is the required mapping function in expanded form. This infinite series maps a rectangle with perfectly straight sides in the z-plane onto the unit circle in the w plane. The sides of the rectangle meet at 90 degrees and the radius of curvature at the vertex is not defined (7). Approximate rectangles with rounded corners may be mapped by retaining only a finite number of terms of Eq. 81. Openings with different radii of curvature at the corners may be obtained by changing the number of terms which are retained. But a more attractive alternative is to keep a specified number of terms in the series and to permit changes in the coefficients. Thus the form of the mapping function finally is given by Eq. 79.

The specified mapping functions for particular openings are:

circle : $z = W(w) = \dfrac{A}{w}$

ellipse: $z = W(w) = \dfrac{A}{w} + Bw$

square : $z = W(w) = \dfrac{A}{w} + Cw^3$

The coefficients of these mapping functions could be obtained by Newton's Approximate Method (7). The stress functions ϕ and ψ for a rectangular opening in a simply supported beam with a concentrated load at the center of the beam are given in Appendix (III).

Third, the elastic analysis is always the same, whatever the loading condition. But the stress equations for σ_x, σ_y, and τ_{xy} in two dimensional elasticity depend on the loading

condition. For a cantilever beam with a concentrated load at the end the stress equations are expressed in Eqs. 2, 3, and 4 and for a simply supported beam with a uniformly distributed load:

$$\sigma_x = -\frac{q}{2I}\left((L^2 - x^2)y + \frac{2}{3}y^3 - \frac{2}{5}c^{*2}y\right)$$

$$\sigma_y = \frac{q}{2I}\left(\frac{y^3}{3} - c^{*2}y + \frac{2}{3}c^{*3}\right)$$

$$\tau_{xy} = \frac{q}{2I}\left((c^{*2} - y^2)x\right)$$

If a small eccentricity of the center of the hole in the y-direction occurs, the stress equations would be changed as indicated below, but the calculation procedure is the same. For a simply supported beam with a concentrated load

$$\sigma_x = \frac{P(L - x)(y + e)}{I}$$

$$\sigma_y = 0$$

$$\tau_{xy} = \frac{P(y + e)^2}{2I}$$

where

"e" is the eccentricity from the center of the hole in the y direction.

VI. Summary and Conclusions.

An elastic analysis for calculating stresses around a hole in the web of a wide-flange beam has been studied. The applicability of the analysis depends on the size of the web hole. For circular holes, boundary conditions at the hole and a moment equality are satisfied when the ratio of beam depth to hole diameter is 2.0 or greater.

From the study of a wide-flange beam with a circular hole the following conclusions are drawn:

1. For a simply supported beam with a concentrated load at midspan, the predicted maximum stress at the edge of a circular hole occurs at $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$, when the hole is located at mid-depth of the beam. When the opening moves away from middepth, the location of the predicted maximum stress changes slightly, and the maximum stresses occur at the side of the hole which is located closer to the center of the span.

2. The magnitude of the stresses occuring at the edge of the hole near mid-depth increase as the hole is moved away from the center of the beam.

Acknowledgement

The author wishes to express appreciation to Dr. Peter B. Cooper, his major Professor, for his guidance and advice throughout this study.

Appendix I.

Theorem of Harnack (6, 11)

1.   If $f(w)$ is continuous in the closed region $w > 1$ and analytic in the region exterior to r, with the possible exception of the point $w = \infty$, where $f(w)$ has the structure

$$f(w) = A_0 + A_1 w + A_2 w^2 + \ - \ - \ - \ - \ + \ A_n w^n + \sum_{n=1}^{\infty} \frac{B_k}{w^k}$$

then

$$\frac{1}{2\pi i} \int_C \frac{f(\sigma)}{\sigma - w} d\sigma = -f(w) + A_0 + A_1 + \ - \ - \ - \ + \ A_n w^n \quad \text{for } |w| > 1$$

2.   Let $f(w)$ be continuous in the closed region $w > 1$ and analytic in the interior with the possible exception of the point $w = 0$, where $f(w)$ has the structure

$$f(w) = \frac{A_1}{w} + \frac{A_2}{w^2} + \ - \ - \ - \ + \ \frac{A_n}{w^n} + g(w)$$

and where $g(w)$ is analytic; then

$$\frac{1}{2\pi i} \int_C \frac{f(\sigma)}{\sigma - w} d\sigma = -\frac{A_1}{w} - \frac{A_2}{w^2} - \ - \ - \ - \ - \ \frac{A_n}{w^n} \quad \text{for } |w| > 1$$

APPENDIX II.

```
C   C   COMPUTE THE TANGENTIAL STRESSES AROUND A CIRCULAR HOLE
        A(I) = RADIUS OF A CIRCULAR HOLE
        U(J) = DISTANCE FROM THE SURPPORT TO THE CENTER OF A HOLE
        D(K) = ANGLE AROUND A CIRCULAR HOLE
        R(N) = RADIUS IN POLAR COORDINATE
        E = DEPTH OF A 12 WF 45 BEAM
        H = WIDTH OF A 12 WF 45 BEAM
        P = FLANGE THICKNESS OF A 12 WF 45
        T = WEB THICKNESS OF A 12 WF 45
        DIMENSION A(3),U(4),D(50)
     1  FORMAT (3F10.2)
     2  FORMAT (4F10.2)
     3  FORMAT (5F10.2)
     4  FORMAT (5F10.2)
        READ 1,(A(I),I=1,3)
        READ 2,(U(J),J=1,4)
        READ 3,(D(K),K=1,19)
        F=12.06
        H=8.042
        P=0.576
        T=0.336
        C=E**2.*(1.+(2.*H*P)/(T*E))
        DO 5 I=1,3
        PUNCH 1,A(I)
        DO 5 J=1,4
        PUNCH 2,U(J)
        DO 5 K=1,19
        PUNCH 3,D(K)
        V=4.*3.14159*D(K)/180.
        Y=3.*3.14159*D(K)/180.
        Z=2.*3.14159*D(K)/180.
        W=3.14159*D(K)/180.
        XA=(2.*SIN(V))+(2.*U(J)*SIN(Y)/A(I))+((4.*C)/(A(I)**2.)-2.)*SIN(Z
        XB=(2.*U(J)*SIN(W))/(A(I)**2.)
        X=XA-XB
     5  PUNCH 4,X
        STOP
        END
```

2.0	2.5	3.0		
80.	60.	40.	20.	
0.	10.	20.	30.	40.
50.	60.	70.	80.	90.
100.	110.	120.	130.	140.
150.	160.	170.	180.	

```
C  C   COMPUTE THE BENDING STRESSES AROUND A CIRCULAR HOLE  Y. C. LEE
        A(I) = RADIUS OF A CIRCULAR HOLE
        U(J) = DISTANCE FROM THE SURPPORT TO THE CENTER OF A HOLE
        D(K) = ANGLE AROUND A CIRCULAR HOLE
        R(N) = RADIUS IN POLAR COORDINATE
        E = DEPTH OF A 12 WF 45 BEAM
        H = WIDTH OF A 12 WF 45 BEAM
        P = FLANGE THICKNESS OF A 12 WF 45
        T = WEB THICKNESS OF A 12 WF 45
        DIMENSION A(3),U(4),D(20),R(10)
  1 FORMAT (3F10.2)
  2 FORMAT (4F10.2)
  3 FORMAT (5F10.2)
  4 FORMAT (F14.5)
        READ 1,(A(I),I=1,3)
        READ 2,(U(J),J=1,4)
        READ 3,(D(K),K=1,19)
        READ 3,(R(N),N=1,8)
        E=12.06
        H=8.042
        P=0.576
        T=0.336
        C=E**2.*(1.+(2.*H*P)/(T*E))
        DO 5 I=1,3
        PUNCH 1,A(I)
        DO 5 J=1,4
        PUNCH 2,U(J)
        DO 5 K=1,19
        PUNCH 3,D(K)
        DO 5 N=1,8
        PUNCH 3,R(N)
        B=6.*3.14159*D(K)/180.
        S=5.*3.14159*D(K)/180.
        V=4.*3.14159*D(K)/180.
        Y=3.*3.14159*D(K)/180.
        Z=2.*3.14159*D(K)/180.
        W=3.14159*D(K)/180.
        XA=(1./R(N)**2.-1.)*(-SIN(B)-U(J)*SIN(S)/A(I))
        XB=(-2.)*(1./R(N)**2.-1.)*(C/A(I)**2.-R(N)**6.)*SIN(V)
        XC=(1./R(N)**2.-1.)*(4.*C/A(I)**2.-1.)*R(N)**4.
        XD=(1./R(N)**2.-1.)*(1./R(N)**2.-1.)*SIN(Z)
        XE=(1./R(N)**2.-1.)*(U(J)*(1./R(N)-1.)*SIN(W)/A(I))
        XF=(1./R(N)**2.+1.)*(SIN(B)+U(J)*SIN(S)/A(I))
        XG=(1./R(N)**2.+1.)*2.*(C/A(I)**2.+R(N)**6.)*SIN(V)
        XH=(1./R(N)**2.+1.)*(U(J)*(2.*R(N)**5.+1.)*SIN(Y))
        XO=(1./R(N)**2.+1.)*(4.*C/A(I)**2.-1.)*R(N)**4.*SIN(Z)
        XP=(1./R(N)**2.+1.)*(-1./R(N)**2.-1.)*SIN(Z)
        XQ=-(1./R(N)**2.+1.)*U(J)*(3.*R(N)+1./R(N))*SIN(W)
        X=XA+XB+XC+XD+XE+XF+XG+XH+XO+XP+XQ
  5 PUNCH 4,X
        STOP
        END
```

```
2.0        2.5        3.0
80.        60.        40.        20.
0.         10.        20.        30.        40.
50.        60.        70.        80.        90.
100.       110.       120.       130.       140.
150.       160.       170.       180.
1.0        1.2        1.4        1.6        1.8
2.0        2.5        3.0
```

```
C  C   COMPUTE THE SHEAR STRESSES AROUND A CIRCULAR HOLE
        A(I) = RADIUS OF A CIRCULAR HOLE
        U(J) = DISTANCE FROM THE SURPPORT TO THE CENTER OF A HOLE
        D(K) = ANGLE AROUND A CIRCULAR HOLE
        R(N) = RADIUS IN POLAR COORDINATE
        E = DEPTH OF A 12 WF 45 BEAM
        H = WIDTH OF A 12 WF 45 BEAM
        P = FLANGE THICKNESS OF A 12 WF 45
        T = WEB THICKNESS OF A 12 WF 45
        DIMENSION A(3),U(4),D(20),R(10)
      1 FORMAT (3F10.2)
      2 FORMAT (4F10.2)
      3 FORMAT (5F10.2)
      4 FORMAT (F14.5)
        READ 1,(A(I),I=1,3)
        READ 2,(U(J),J=1,4)
        READ 3,(D(K),K=1,19)
        READ 3,(R(N),N=1,8)
        E=12.06
        H=8.042
        P=0.576
       .T=0.336
        C=E**2.*(1.+(2.*H*P)/(T*E))
        DO 5 I=1,3
        PUNCH 1,A(I)
        DO 5 J=1,4
        PUNCH 2,U(J)
        DO 5 K=1,19
        PUNCH 3,D(K)
        DO 5 N=1,8
        PUNCH 3,R(N)
        B=6.*3.14159*D(K)/180.
        S=5.*3.14159*D(K)/180.
        V=4.*3.14159*D(K)/180.
        Y=3.*3.14159*D(K)/180.
        Z=2.*3.14159*D(K)/180.
        W=3.14159*D(K)/180.
        XA=COS(B)+U(J)*COS(S)/A(I)+2.*C*C**(1./2.)*COS(V)/A(I)**2.
        XB=U(J)*COS(Y)/A(I)+COS(Z)/R(N)**2.
        XC=U(J)*(1./R(N)-R(N))*COS(W)/A(I)+(2.*C/A(I)**2.-1.)
        X=XA+XB+XC
      5 PUNCH 4,X
        STOP
        END
2.0           2.5          3.0
80.           60.          40.         20.
0.            10.          20.         30.         40.
50.           60.          70.         80.         90.
100.          110.         120.        130.        140.
150.          160.         170.        180.
1.0           1.2          1.4         1.6          1.8
2.0           2.5          3.0
```

Appendix III.

The stress functions ϕ and ψ for a rectangular web opening in a simply supported wide flange beam with a concentrated load at the center of the beam are given below.

A. Basic stresses.

$$\phi(w) = -\frac{Pi}{8I}(\frac{1}{3}(\frac{v_1}{w^3} + \frac{v_2}{w} + v_3w + v_4w^3 + v_5w^5 + v_6w^7 + v_7w^9$$

$$+ v_8w^{11} + v_9w^{13} + v_{10}w^{15} + v_{11}w^{17} + v_{12}w^{19}$$

$$+ v_{13}w^{21}) + (v_{14}w^{-2} + v_{15} + v_{16}w^2 + v_{17}w^4$$

$$+ v_{18}w^6 + v_{19}w^8 + v_{20}w^{10} + v_{21}w^{12} + v_{22}w^{14})u)$$

$$\psi(w) = \frac{Pi}{8I}(\frac{2}{3}(\frac{v_1}{w^3} + \frac{v_2}{w} + v_3w + v_4w^3 + v_5w^5 + v_6w^7 + v_7w^9$$

$$+ v_8w^{11} + v_9w^{13} + v_{10}w^{15} + v_{11}w^{17} + v_{12}w^{19}$$

$$+ v_{13}w^{21}) + u(v_{14}w^{-2} + v_{15} + v_{16}w^2 + v_{17}w^4$$

$$+ v_{18}w^6 + v_{19}w^8 + v_{20}w^{10} + v_{21}w^{12} + v_{22}w^{14})$$

$$+ 4c^{*2}(v_{23}w^{-1} + v_{24}w + v_{25}w^3 + v_{26}w^5 + v_{27}w^7))$$

where

$$v_1 = A^3$$

$$v_2 = 3A^2B$$

$$v_3 = 3(A^2C + AB^2)$$

$$v_4 = 3A^2D + 6ABC + B^3$$

$$v_5 = 3A^2E + 4ABC + 3AC^2 + 2A^2D + 2ABC + CB^2$$

$$v_6 = 6ADC + 5ABE + AE + 3B^2D + 3BC^2$$

$$v_7 = 5AEC + 3AD^2 + 6BCD + 3B^2E + BE + 4ACE + C^3$$

$$v_8 = 6ADE + 5BCE + 3BD^2 + 3C^2D + CE + DE^2$$

$$v_9 = 3AE^2 + 5BDE + 3C^2E + 3CD^2 + DE$$

$$v_{10} = 2BE^2 + 6CDE + D^3 + E^3$$

$$v_{11} = 3CE^2 + 2D^2E$$

$$v_{12} = 3DE^2$$

$$v_{13} = E^3$$

$$v_{14} = A^2$$

$$v_{15} = 2AB$$

$$v_{16} = 2AC + B^2$$

$$v_{17} = 2(AD + BC)$$

$$v_{18} = 2AD + 2BD + C^2$$

$$v_{19} = 2CD + BE + E$$

$$v_{20} = 2CE + D^2$$

$$v_{21} = 2DE$$

$$v_{22} = E^2$$

$$v_{23} = A$$

$$v_{24} = B$$

$$v_{25} = C$$

$$v_{26} = D$$

$$v_{27} = E$$

B. Perturbated Stress functions.

$$\phi_{10}(w) = \frac{Pi}{8I}(e_{21}w^{21} + e_{19}w^{19} + e_{17}w^{17} + e_{15}w^{15} + e_{14}w^{14}$$

$$+ e_{13}w^{13} + e_{12}w^{12} + e_{11}w^{11} + e_{10}w^{10} + e_9w^9$$

$$+ e_8w^8 + e_7w^7 + e_6w^6 + e_5w^5 + e_4w^4 + e_3w^3$$

$$+ e_2w^2 + e_1w + e_0)$$

where

$$e_0 = uv_{15} - h_{10} + uv_{15}$$

$$e_1 = \frac{v_3}{3} - h_9 + \frac{2}{3}v_2$$

$$e_2 = uv_{14} + uv_{16} - h_8$$

$$e_3 = \frac{v_4}{3} - h_7 + \frac{2}{3}v_1$$

$$e_4 = uv_{17} - h_6$$

$$e_5 = \frac{v_5}{3} - h_5$$

$$e_6 = uv_{18} - h_4$$

$$e_7 = \frac{v_6}{3} - h_3$$

$$e_8 = uv_{19} - h_2$$

$$e_9 = \frac{v_7}{3} - h_1$$

$$e_{10} = uv_{20}$$

$$e_{11} = \frac{v_8}{3}$$

$$e_{12} = uv_{21}$$

$$e_{13} = \frac{v_9}{3}$$

$$e_{14} = uv_{22}$$

$$e_{15} = \frac{v_{10}}{3}$$

$$e_{16} = 0$$

$$e_{17} = \frac{v_{11}}{3}$$

$$e_{19} = \frac{v_{12}}{3}$$

$$e_{21} = \frac{v_{13}}{3}$$

$$e_{18} = e_{20} = e_{22} = - - - - = e_n = 0$$

in which

$$h_1 = v_{14}v_{27}$$

$$h_2 = 2u(v_{23}v_{27})$$

$$h_3 = v_{14}v_{26} + v_{15}v_{27}$$

$$h_4 = 2u(v_{23}v_{26} + v_{24}v_{27})$$

$$h_5 = v_{14}v_{25} + v_{15}v_{26} + v_{16}v_{27}$$

$$h_6 = 2u(v_{23}v_{25} + v_{24}v_{26} + v_{25}v_{27})$$

$$h_7 = v_{14}v_{24} + v_{15}v_{25} + v_{16}v_{26} + v_{17}v_{27}$$

$$h_8 = 2u(v_{23}v_{24} + v_{24}v_{25} + v_{25}v_{26} + v_{26}v_{27})$$

$$h_9 = v_{14}v_{23} + v_{15}v_{24} + v_{16}v_{25} + v_{17}v_{26} + v_{18}v_{27}$$

$$h_{10} = 2u(v_{23}^2 + v_{24}^2 + v_{25}^2 + v_{26}^2 + v_{27}^2)$$

$$h_{11} = v_{15}v_{23} + v_{16}v_{24} + v_{17}v_{25} + v_{18}v_{26} + v_{19}v_{27}$$

$$h_{12} = 2u(v_{23}v_{24} + v_{24}v_{25} + v_{25}v_{26} + v_{26}v_{27})$$

$$h_{13} = v_{16}v_{23} + v_{17}v_{24} + v_{18}v_{25} + v_{19}v_{26} + v_{20}v_{27}$$

$$h_{14} = 2u(v_{23}v_{24} + v_{24}v_{25} + v_{25}v_{26} + v_{26}v_{27})$$

$$h_{15} = v_{17}v_{23} + v_{18}v_{24} + v_{19}v_{25} + v_{20}v_{26} + v_{21}v_{27}$$

$$h_{16} = 2u(v_{23}v_{26} + v_{24}v_{27})$$

$$h_{17} = v_{18}v_{23} + v_{19}v_{24} + v_{20}v_{25} + v_{21}v_{26} + v_{22}v_{27}$$

$$h_{18} = 2u(v_{23}v_{27})$$

$$h_{19} = v_{19}v_{23} + v_{20}v_{24} + v_{21}v_{26} + v_{22}v_{27}$$

$$h_{20} = v_{20}v_{23} + v_{21}v_{24} + v_{22}v_{25}$$

$$h_{21} = v_{21}v_{23} + v_{22}v_{24}$$

$$h_{22} = v_{22}v_{23}$$

$$\psi_0(w) = -\frac{Pi}{81}(n_{29}w^{29} + n_{27}w^{27} + n_{25}w^{25} + n_{23}w^{23} + n_{21}w^{21}$$

$$+ m_{23}w^{23} + m_{21}w^{21} + (n_{19} + m_{19})w^{19} + (n_{17}$$

$$+ m_{17})w^{17} + (n_{15} + m_{15})w^{15} + (n_{16} + m_{16})w^{16}$$

$$+ (n_{14} + m_{14})w^{14} + (n_{13} + m_{13})w^{13} + (n_{12}$$

$$+ m_{12})w^{12} + (n_{11} + m_{11})w^{11} + (n_{10} + m_{10})w^{10}$$

$$+ (n_9 + m_9)w^9 + (n_8 + n_8)w^8 + (n_7 + m_7)w^7$$

$$+ (n_6 + m_6)w^6 + (n_5 + m_5)w^5 + (n_4 + m_4)w^4$$

$$+ (n_3 + m_3)w + (n_2 + m_2)w + (n_1 + m_1)w$$

$$+ (n_0 + m_0)/(7v_{27}w^8 + 5v_{26}w^6 + 3v_{25}w^4$$

$$+ v_{24}w^2 - v_{23})$$

where

$$m_0 = 2e_2 + 4e_4 + 6e_6$$

$$m_1 = e_1 v_{24} + 3e_3 v_{25} + 5e_5 v_{26} + 7e_7 v_{27}$$

$$m_2 = 2e_2 v_{24} + 4e_4 v_{25} + 6e_6 v_{26} + 8e_8 v_{27}$$

$$m_3 = e_1 v_{23} + 3e_3 v_{24} + 5e_5 v_{25} + 7e_7 v_{26} + 9e_9 v_{27}$$

$$m_4 = 2e_2 v_{23} + 4e_4 v_{24} + 6e_6 v_{25} + 8e_8 v_{26} + 10e_{10} v_{27}$$

$$m_5 = 3e_3 v_{23} + 5e_5 v_{24} + 7e_7 v_{25} + 9e_9 v_{26} + 11e_{11} v_{27}$$

$$m_6 = 4e_4 v_{23} + 6e_6 v_{24} + 8e_8 v_{25} + 10e_{10} v_{26} + 12e_{12} v_{27}$$

$$m_7 = 5e_5 v_{23} + 7e_7 v_{24} + 9e_9 v_{25} + 11e_{11} v_{26} + 13e_{13} v_{27}$$

$$m_8 = 6e_6 v_{23} + 8e_8 v_{24} + 10e_{10} v_{25} + 12e_{12} v_{26} + 14e_{14} v_{27}$$

$$m_9 = 7e_7 v_{23} + 9e_9 v_{24} + 11e_{11} v_{25} + 13e_{13} v_{26} + 15e_{15} v_{27}$$

$$m_{10} = 8e_8 v_{23} + 10e_{10} v_{24} + 12e_{12} v_{25} + 14e_{14} v_{26}$$

$$m_{11} = 9e_9 v_{23} + 11e_{11} v_{24} + 13e_{13} v_{25} + 15e_{15} v_{26} + 17e_{17} v_{27}$$

$$m_{12} = 10e_{10}v_{23} + 12e_{12}v_{24} + 14e_{14}v_{25}$$

$$m_{13} = 11e_{11}v_{23} + 13e_{13}v_{24} + 15e_{15}v_{25} + 17e_{17}v_{26} + 19e_{19}v_{27}$$

$$m_{14} = 12e_{12}v_{23} + 14e_{14}v_{24}$$

$$m_{15} = 13e_{13}v_{23} + 15e_{15}v_{24} + 17e_{17}v_{25} + 19e_{19}v_{26} + 21e_{21}v_{27}$$

$$m_{16} = 14e_{14}v_{23}$$

$$m_{17} = 15e_{15}v_{23} + 17e_{17}v_{24} + 19e_{19}v_{25} + 21e_{21}v_{26}$$

$$m_{18} = 0$$

$$m_{19} = 17e_{17}v_{23} + 19e_{19}v_{24} + 21e_{21}v_{25}$$

$$m_{20} = 0$$

$$m_{21} = 19e_{19}v_{23} + 21e_{21}v_{24}$$

$$m_{22} = 0$$

$$m_{23} = 21e_{21}v_{23}$$

$$n_0 = -v_{23}t_0$$

$$n_1 = -v_{23}t_1$$

$$n_2 = v_{24}t_0 - v_{23}t_1$$

$$n_3 = v_{24}t_1 - v_{23}t_3$$

$$n_4 = 3v_{25}t_0 + v_{24}t_2 - v_{23}t_4$$

$$n_5 = 3v_{25}t_1 + v_{24}t_3 - v_{23}t_5$$

$$n_6 = 5v_{26}t_0 + 3v_{25}t_2 + v_{24}t_4 - v_{23}t_6$$

$$n_7 = 5v_{26}t_1 + 3v_{25}t_3 + v_{24}t_5 - v_{23}t_7$$

$$n_8 = 7v_{27}t_0 + 5v_{26}t_2 + 3v_{25}t_4 + v_{24}t_6 - v_{23}t_8$$

$$n_9 = 7v_{27}t_1 + tv_{26}t_3 + 3v_{25}t_5 + v_{24}t_7 - v_{23}t_9$$

$$n_{10} = 7v_{27}t_2 + 5v_{26}t_4 + 3v_{25}t_6 + v_{24}t_8$$

$$n_{11} = 7v_{27}t_3 + 5v_{26}t_5 + 3v_{25}t_7 + v_{24}t_9 - v_{23}t_{11}$$

$$n_{12} = 7v_{27}t_4 + 5v_{26}t_6 + 3v_{25}t_8$$

$$n_{13} = 7v_{27}t_5 + 5v_{26}t_7 + 3v_{25}t_9 + v_{24}t_{11} - v_{23}t_{13}$$

$$n_{14} = 7v_{27}t_6 + 5v_{26}t_8$$

$$n_{15} = 7v_{27}t_7 + 5v_{26}t_9 + 3v_{25}t_{11} + v_{24}t_{13} - v_{23}t_{15}$$

$$n_{16} = 7v_{27}t_8$$

$$n_{17} = 7v_{27}t_9 + 5v_{26}t_{11} + 3v_{25}t_{13} + v_{24}t_{15} - v_{23}t_{17}$$

$$n_{18} = 0$$

$$n_{19} = 7v_{27}t_{11} + 5v_{26}t_{13} + 3v_{25}t_{15} + v_{24}t_{17} - v_{23}t_{19}$$

$$n_{20} = 0$$

$$n_{21} = 7v_{27}t_{13} + 5v_{26}t_{15} + 3v_{25}t_{17} + v_{24}t_{19} - v_{23}t_{21}$$

$$n_{22} = 0$$

$$n_{23} = 7v_{27}t_{15} + 5v_{26}t_{17} + 3v_{25}t_{19} + v_{24}t_{21}$$

$$n_{24} = 0$$

$$n_{25} = 7v_{27}t_{17} + 5v_{26}t_{19} + 3v_{25}t_{21}$$

$$n_{26} = 0$$

$$n_{27} = 7v_{27}t_{19} + 5v_{26}t_{21}$$

$$n_{28} = 0$$

$$n_{29} = 7v_{27}t_{21}$$

in which

$$t_0 = uv_{15} - h_{10}$$

$$t_1 = \frac{2}{3}v_3 + \frac{1}{3}v_1 - h_{11}$$

$$t_2 = uv_{14} - h_{12}$$

$$t_3 = \frac{2}{3}v_4 - h_{13} + \frac{1}{3}v_1$$

$$t_4 = -h_{14}$$

$$t_5 = \frac{2}{3}v_5 - h_{15}$$

$$t_6 = -h_{16}$$

$$t_7 = \frac{2}{3}v_6 - h_{17}$$

$$t_8 = -h_{18}$$

$$t_9 = \frac{2}{3}v_7 - h_{19}$$

$$t_{10} = 0$$

$$t_{11} = \frac{2}{3} v_8 - h_{20}$$

$$t_{12} = 0$$

$$t_{13} = \frac{2}{3} v_9 - h_{21}$$

$$t_{14} = 0$$

$$t_{15} = \frac{2}{3} v_{10} - h_{22}$$

$$t_{16} = 0$$

$$t_{17} = \frac{2}{3} v_{11}$$

$$t_{18} = 0$$

$$t_{19} = \frac{2}{3} v_{12}$$

$$t_{20} = 0$$

$$t_{21} = \frac{2}{3} v_{13}$$

Appendix IV. Notation.

$A, B, - - E$ = coefficients in mapping function;

a = radius of circular hole;

c^* = modified half-depth of a web of a wide flange beam;

C = contour of a unit circle;

d = half depth of beam;

$e_1, - - - e_n$ = complex coefficients used in perturbated stress functions;

$f_0, - - - f_n$ = complex coefficients used in perturbated stress functions;

g_1, g_2 = functions representing forces on beam boundaries;

h = flange width of a wide flange beam;

I = moment of intia about the strong axis of a beam;

i = imaginary unit;

L = half length of a wide flange beam;

M = moment;

n = summation index;

p = flange thickness of a wide flange beam;

q = uniform load per unit length;

r = radial polar coordinate;

t = web thickness of a wide flange beam;

u = distance from a reaction or center of beam to center of hole;

V = shear force;

W = transformation function;

w = nondimensional distance in a complex plane;

x, y = rectangular cartesian coordinates;

z = distance in a complex plane, $z = x + iy$;

θ = angular polar coordinate;

$\sigma_x, \sigma_y, \sigma_r, \sigma_\theta$ = normal stress on the cross section perpendicular to the axis defined by the subscript;

$\tau_{xy}, \tau_{r\theta}$ = shear stress on the above mentioned cross section;

φ, ψ = complex stress functions;
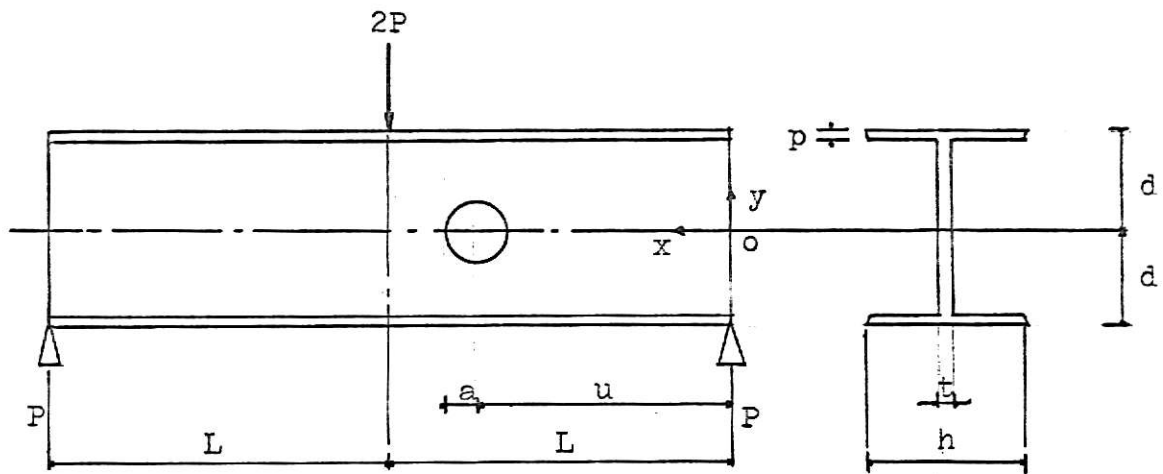
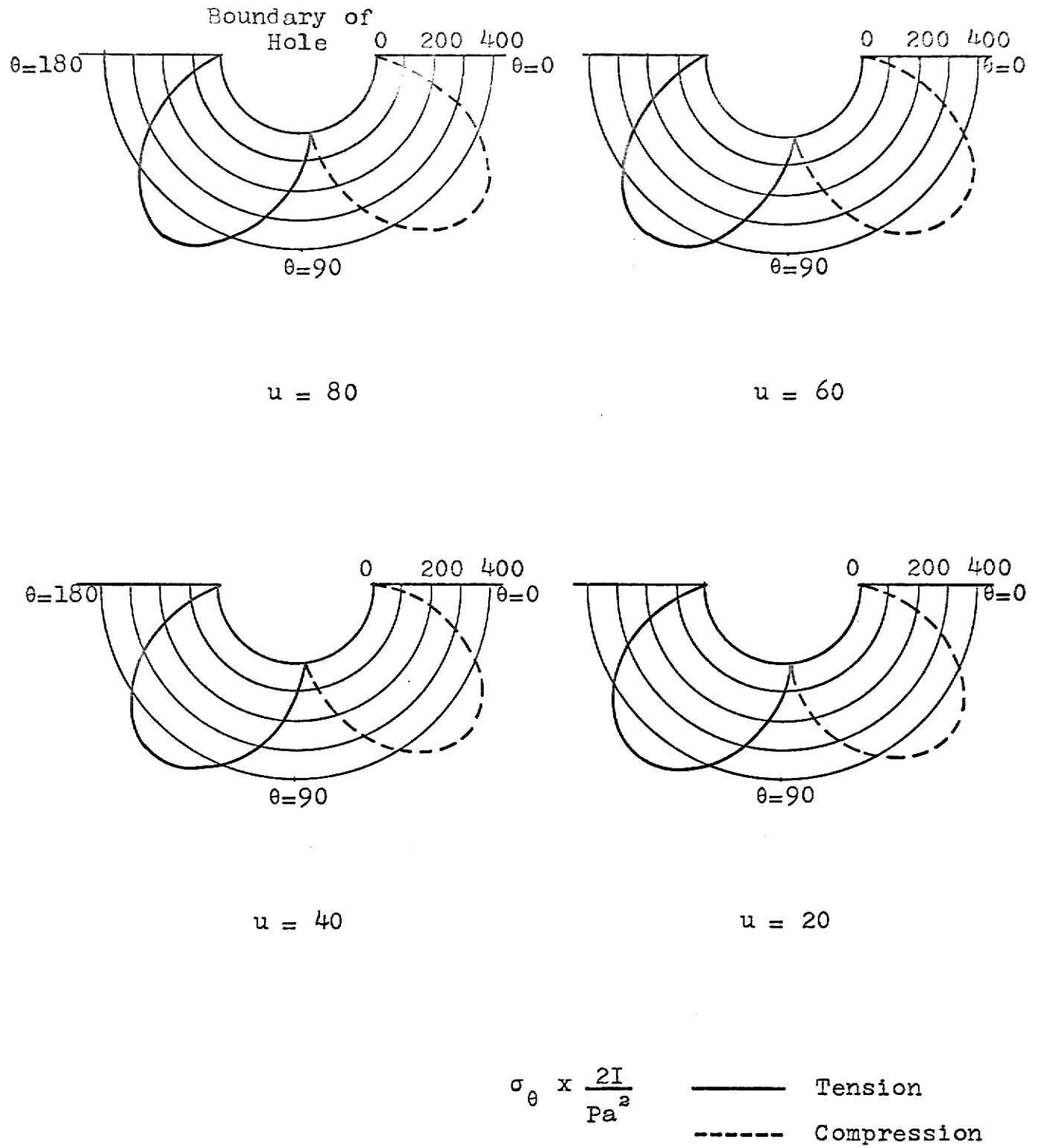Fig. 1. A simply supported beam with a concentrated load.

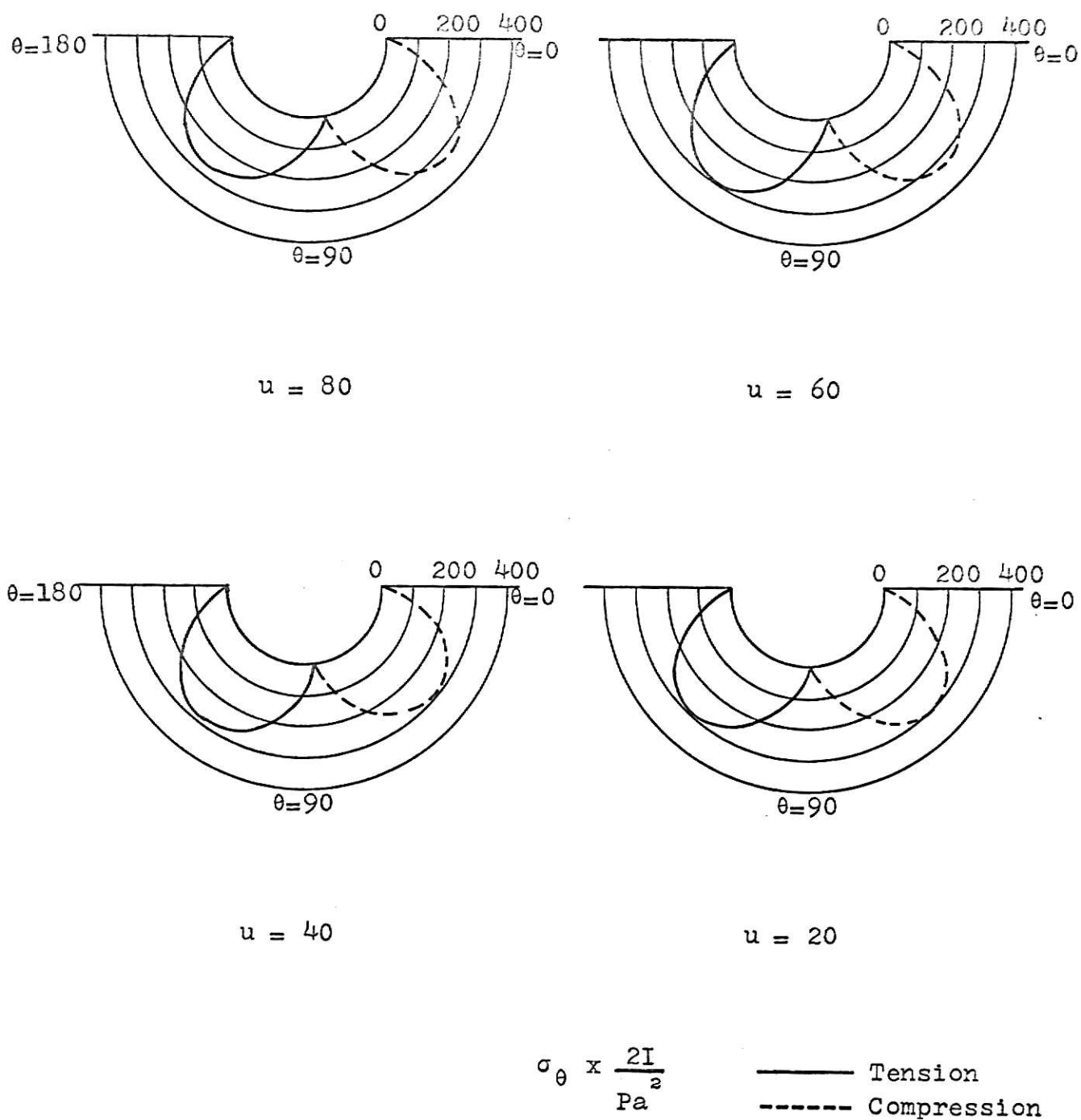Fig. 2. Example Case 1. a = 2.0
Tangential Stress Parameters.

$$\sigma_\theta \ \text{x} \ \frac{2I}{Pa^2}$$

——— Tension

------ Compression

Fig. 3.   Example Case 2, a = 2.5
Tangential Stress Parameters.

u = 80

u = 60

u = 40

u = 20

$$\sigma_\theta \times \frac{2I}{Pa^2}$$

——— Tension

------ Compression
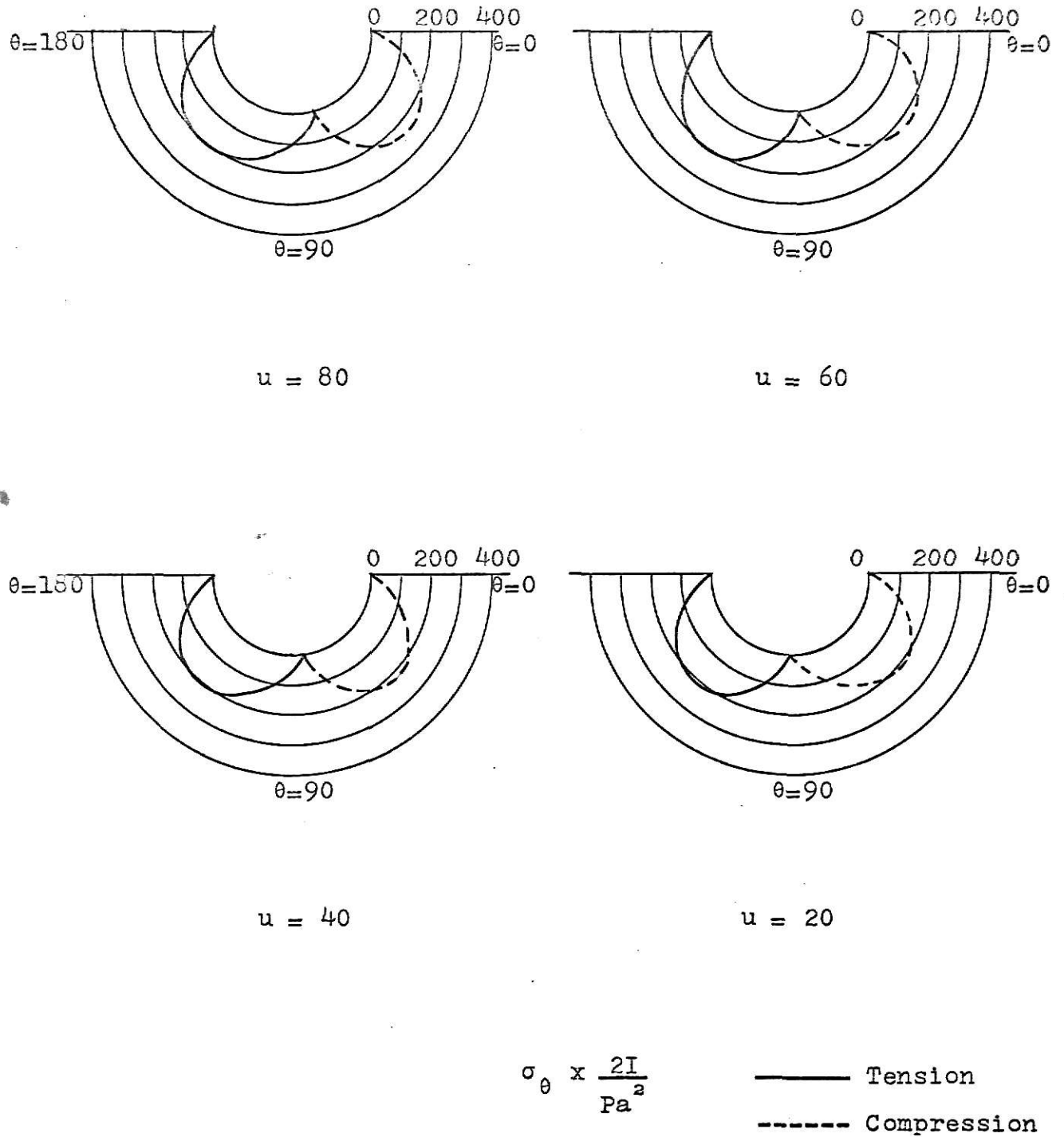
Fig. 4.   Example Case 3, a = 3.0
Tangential Stress Parameters.

180°   Boundary of Hole   0°

90°

a = 2.0

u = 80

Web Flange inter-
face

200   0   200   0   200   0

180°   Boundary of Hole   0°

90°

a = 2.0

u = 60

Web Flange inter-
face

200   0   200   0   200   0

Fig. 5. Bending stress $\sigma_x \times \dfrac{2I}{Pa^2}$.

180°   Boundary of Hole   0°

90°

a = 2.0

u = 40

Web Flange inter-
face

100   0   100   0   100   0

180°   Boundary of Hole   0°

90°

a = 2.0

u = 20

Web Flange inter-
face

50   0   50   0   50   0
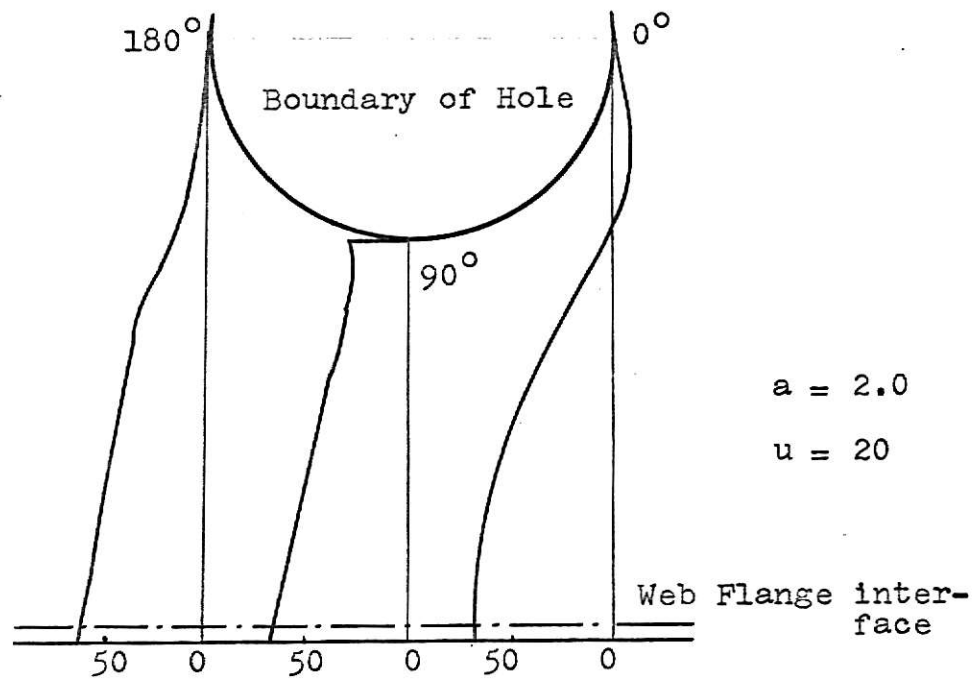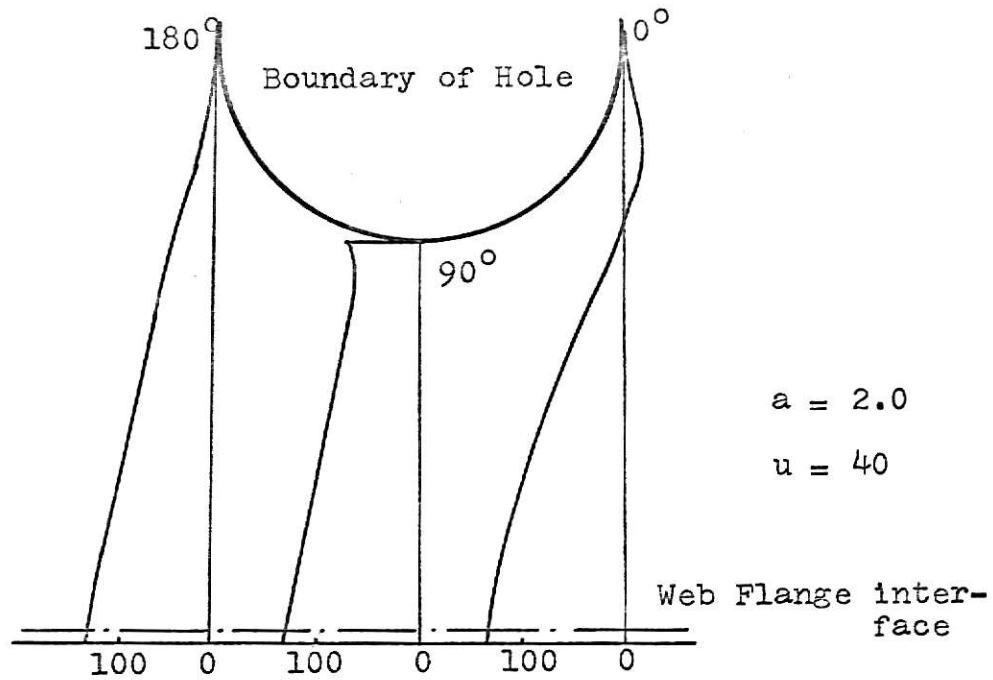
Fig. 5.   Bending stress $\sigma_x \times \frac{2I}{Pa^2}$ (cont.)

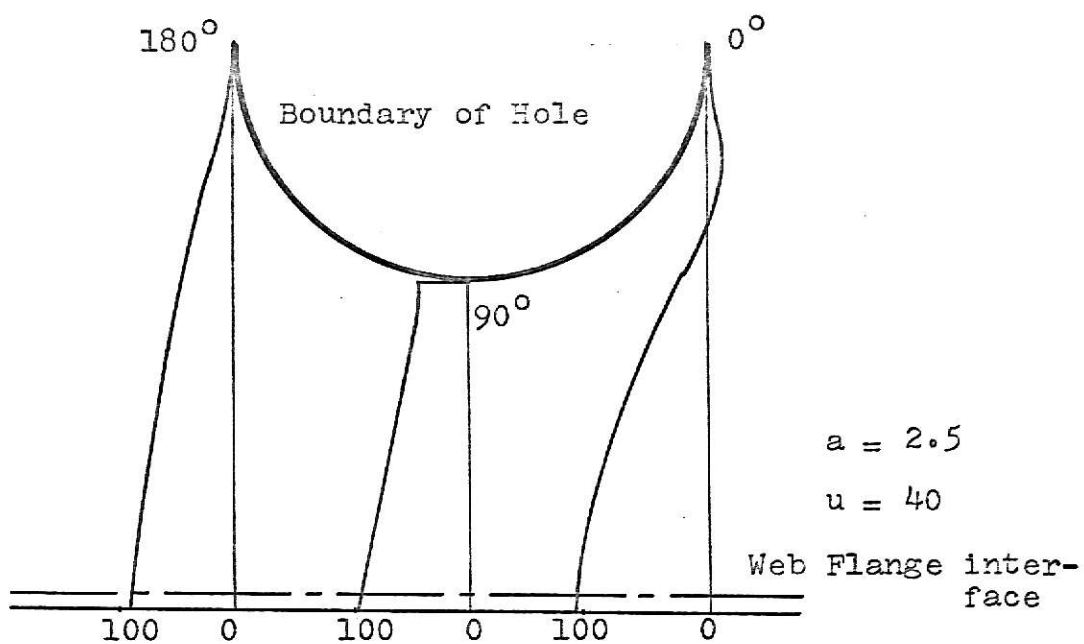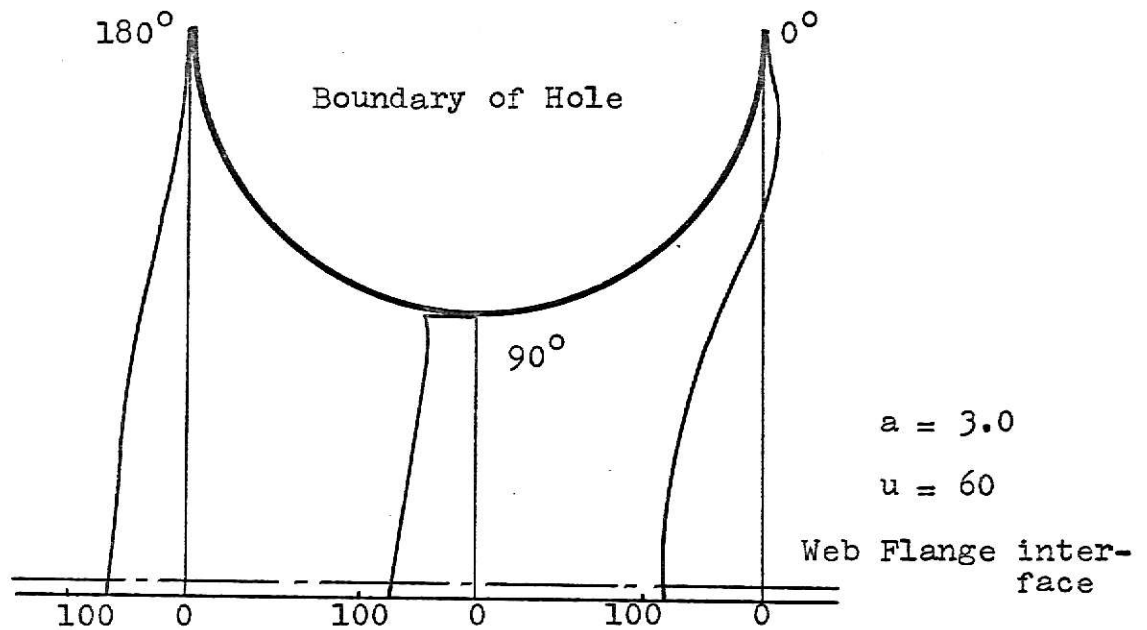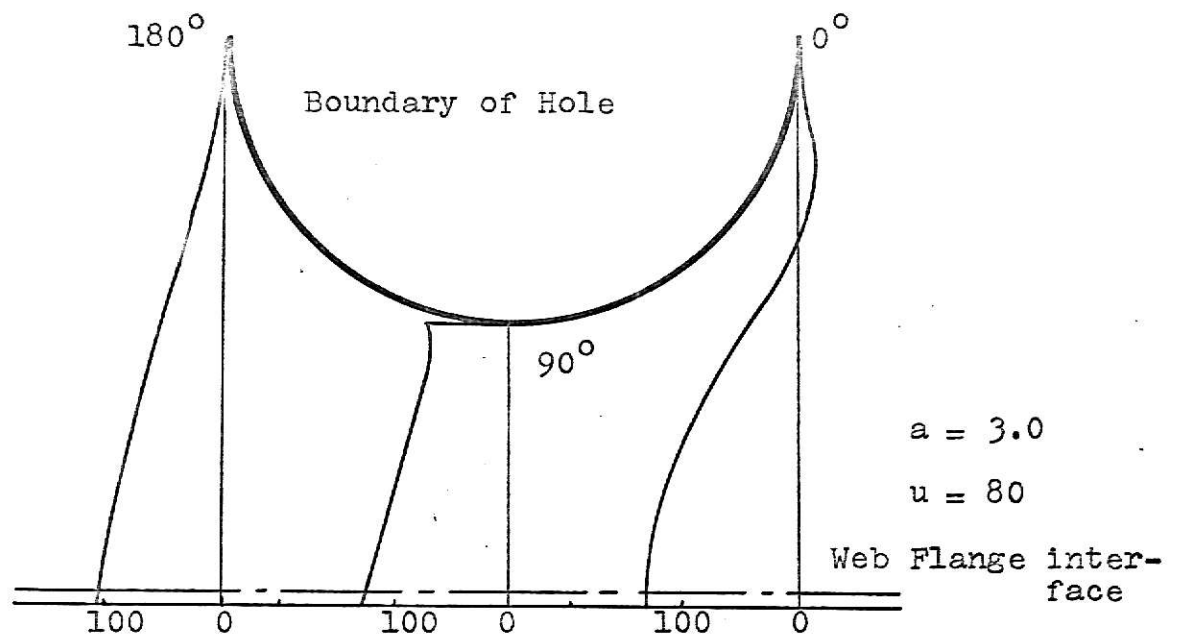Fig. 6. Bending stress $\sigma_x \times \dfrac{2I}{Pa^2}$.

180°   0°

Boundary of Hole

90°

a = 2.5

u = 40

Web Flange inter-
face

100   0   100   0   100   0

180°   0°

Boundary of Hole

90°

a = 2.5

u = 20

Web Flange inter-
face

100   0   100   0   100   0
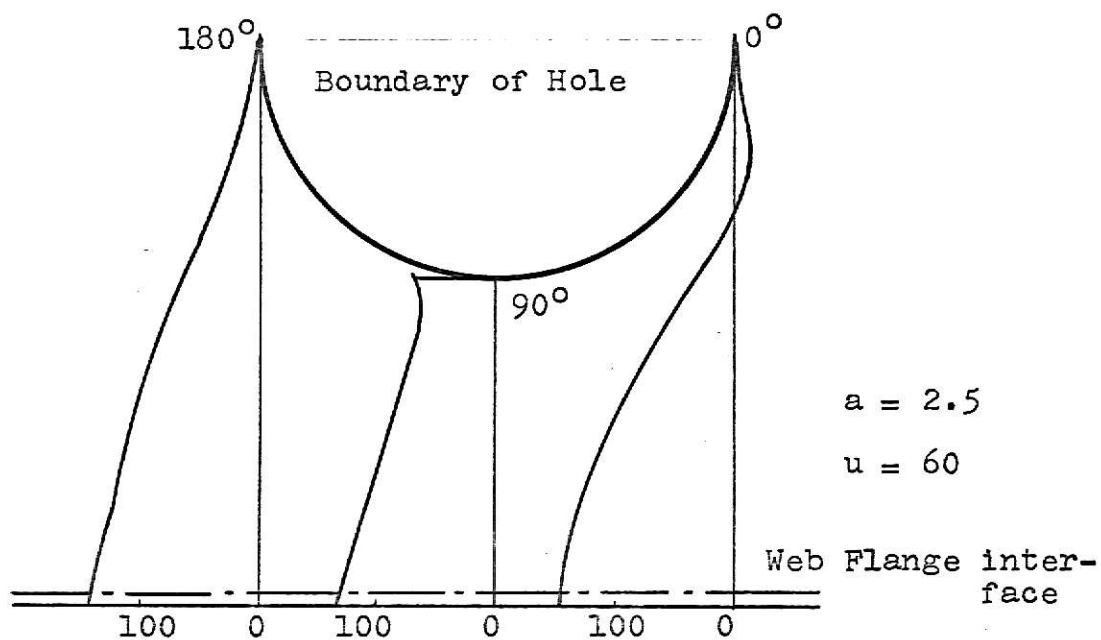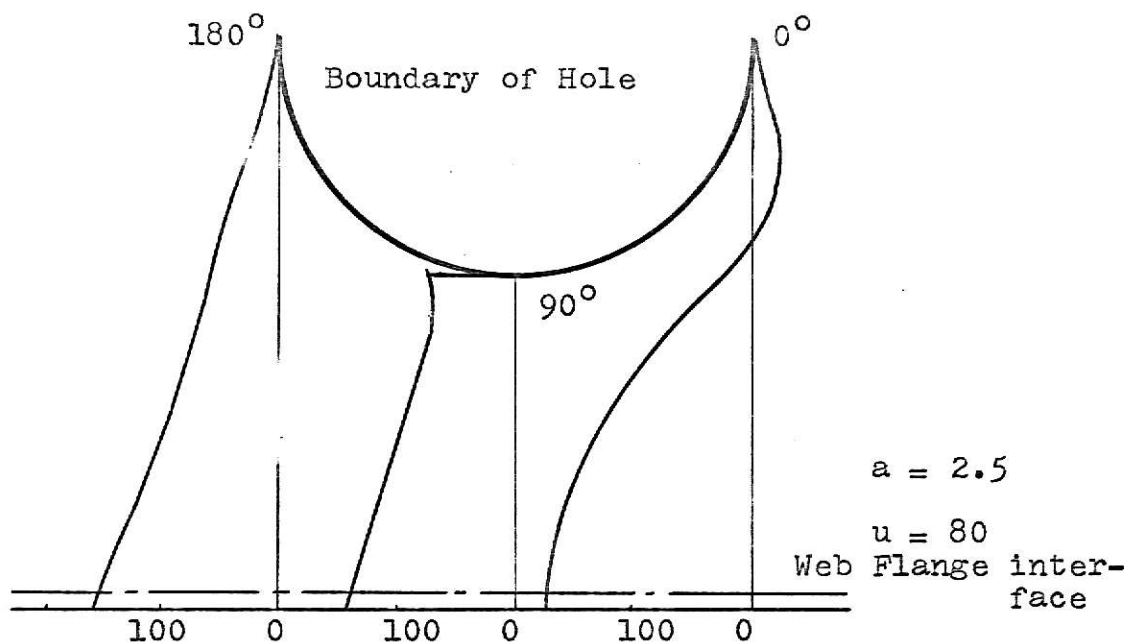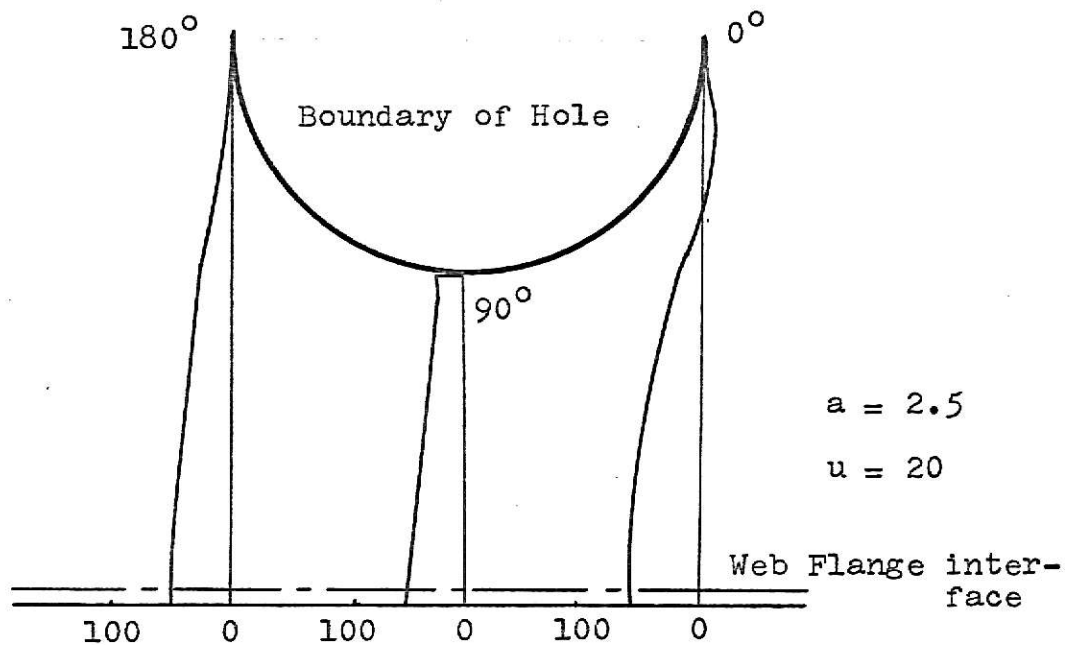
Fig. 6. Bending stress σ_x x $\frac{2I}{Pa^2}$ (cont.)
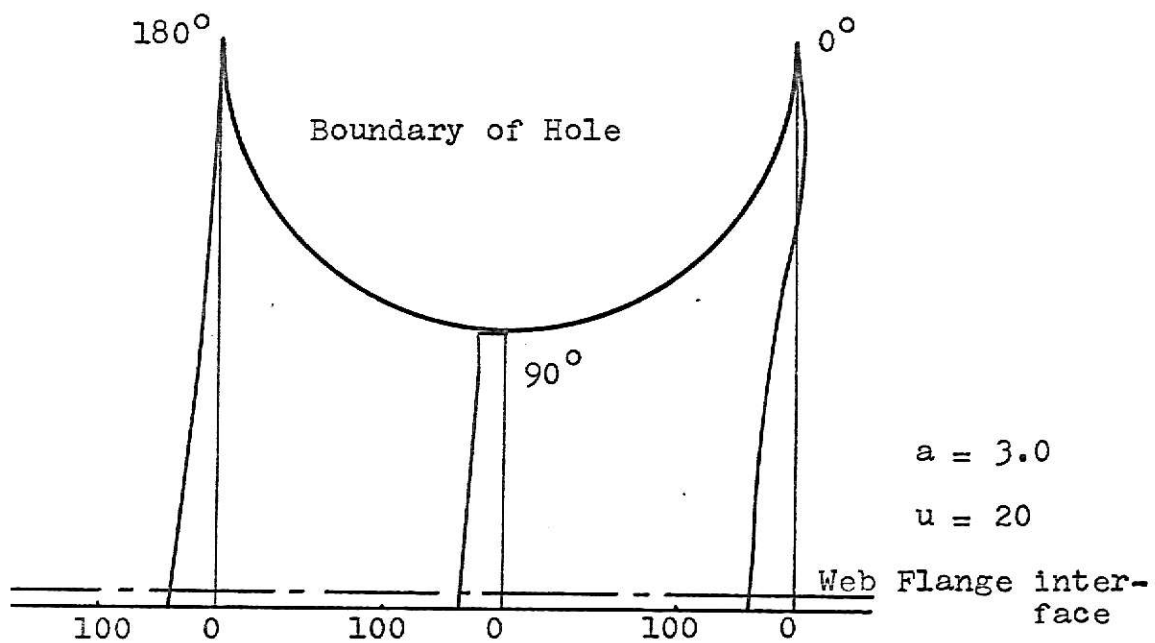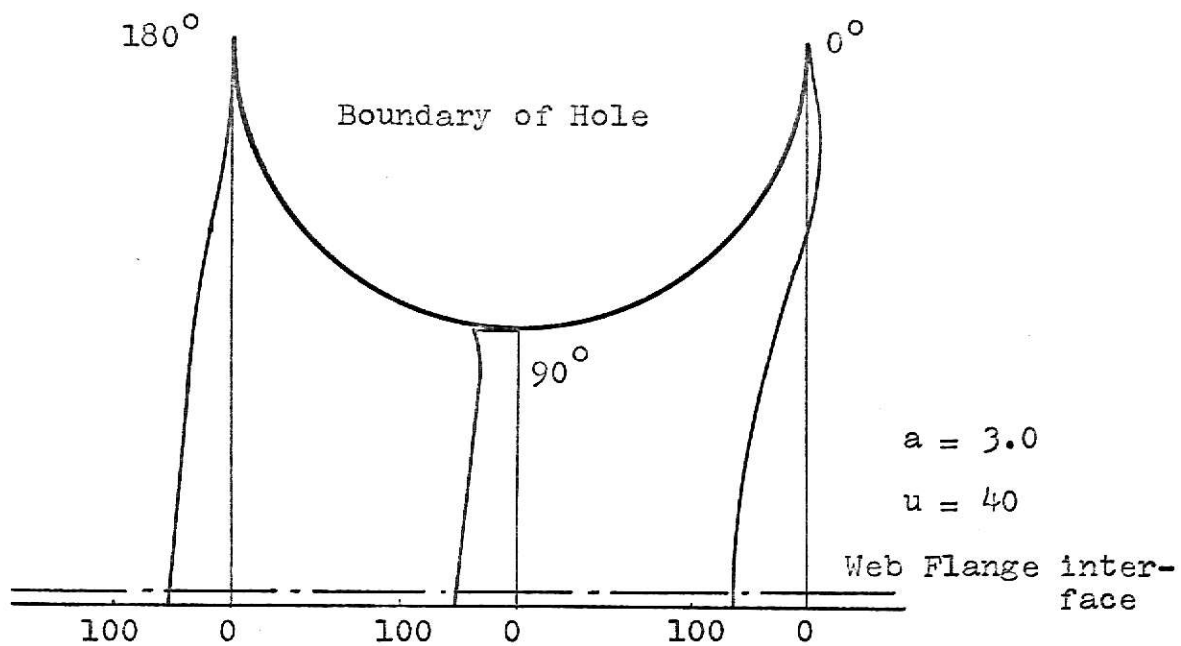
Fig. 7. Bending stress $\sigma_x \times \dfrac{2I}{Pa^2}$.

Fig. 7. Bending stress $\sigma_x \times \dfrac{2I}{Pa^2}$ (cont.)

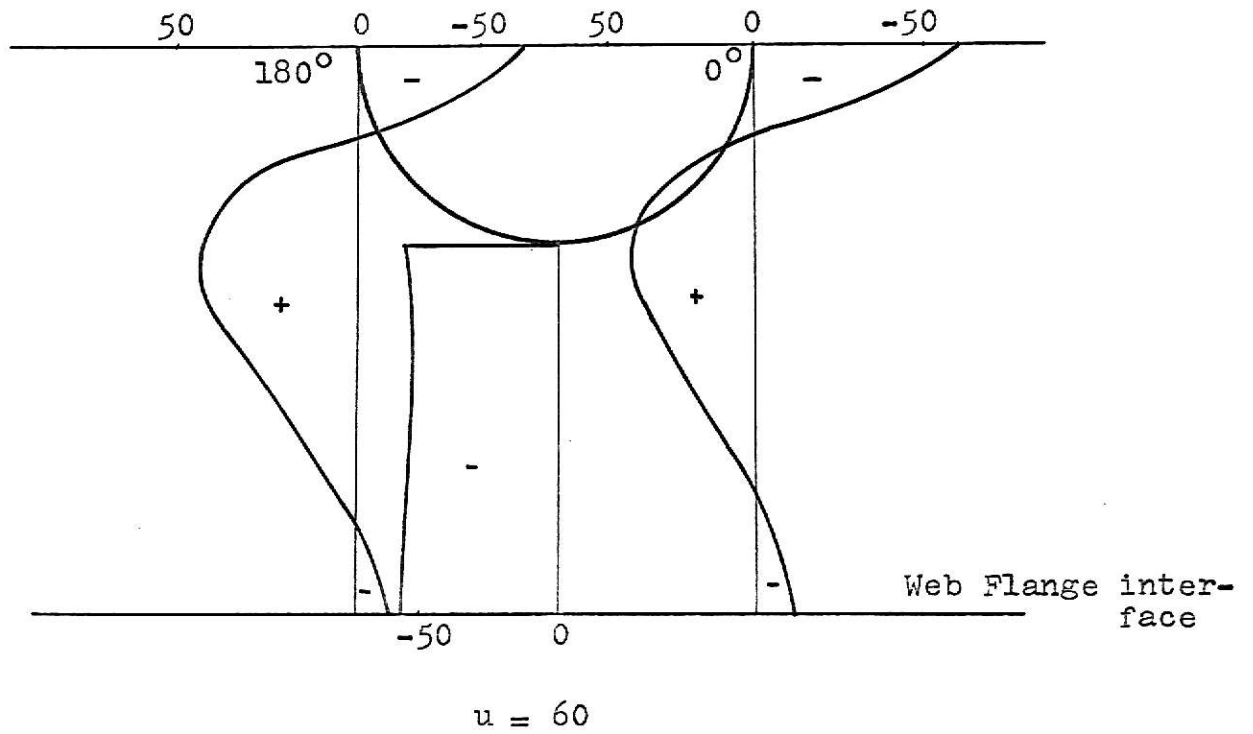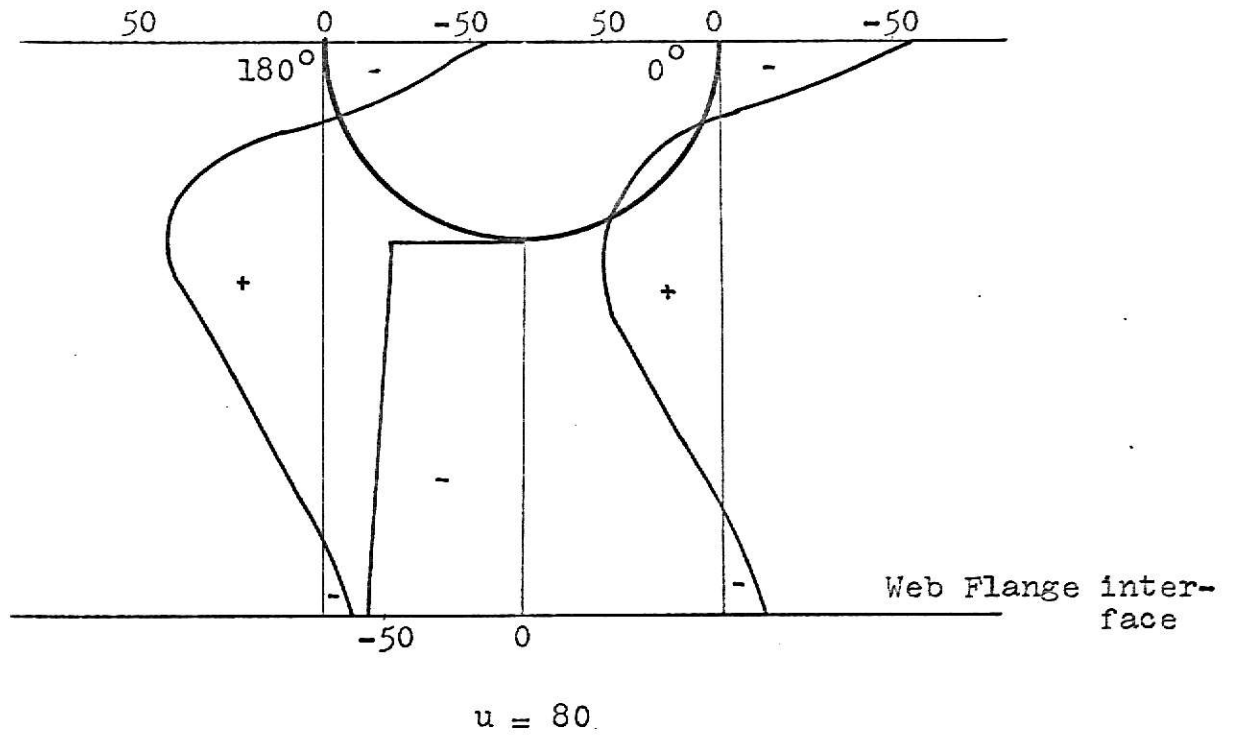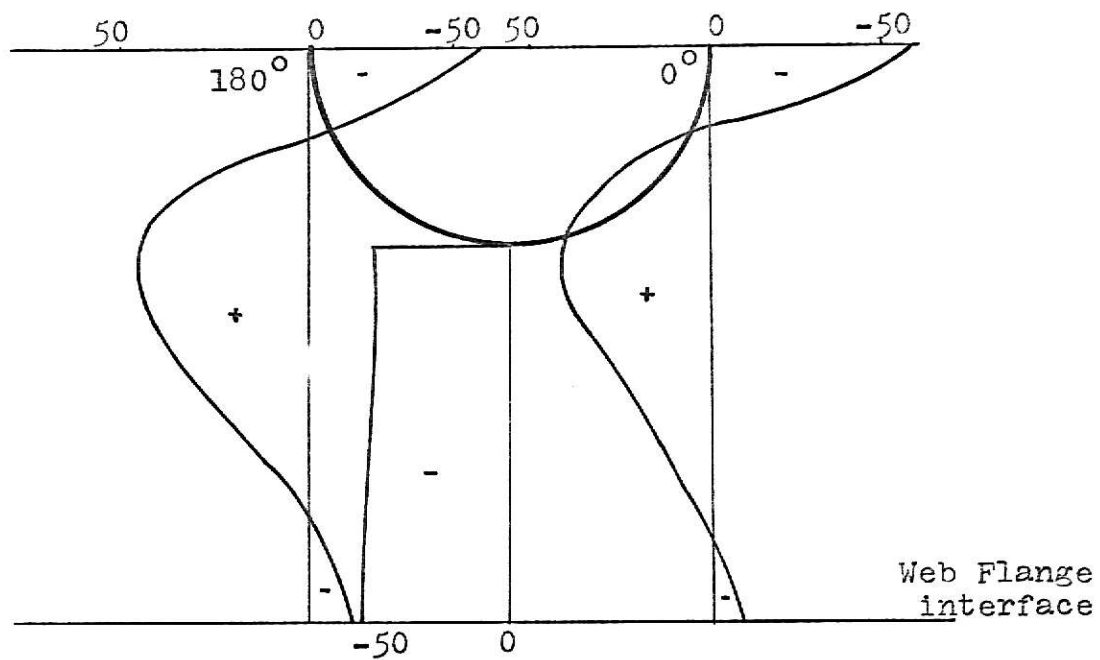Fig. 8. Shear stress τ_{xy} x $\dfrac{I}{25Pa^2}$, a = 2.0.

50   0   -50 50   0   -50

180°   −   0°   −

+   +

−

−   −

Web Flange
interface

-50   0

u = 40

50   0   -50   50   0   -50

180°   −   0°   −

+   +

−

−   −

Web Flange
interface

-50   0

u = 20

Fig. 8. Shear stress $\zeta_{xy} \times \dfrac{I}{25Pa^2}$, a = 2.0 (cont.)

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A STUDY OF THE ELASTIC STRESSES
AROUND HOLES IN A WIDE-FLANGE BEAM
WITH A CONCENTRATED LOAD

by

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ABSTRACT

The elastic analysis of a beam with a web hole is a very
interesting and practical problem for structural engineers.
The solution of this problem is developed on the basis of various
assumptions. One of the assumptions, which is studied in this
paper, is that the total stress around a hole in a wide-flange
beam can be computed as the sum of the stresses occuring in the
beam when there is no hole, called basic stresses, and the
stresses occuring in the beam as a result of forces applied to
the boundary of the hole, called perturbated stresses.

To calculate the basic stresses, the stress equations ex-
pressed in x and y coordinates are transformed to z coordinates
in order to express them as a single complex coordinate. Complex
stress functions are calculated by Muslhekishvili's equations.
To simplify the computation of the stresses around a hole, the
complex stress functions are transformed by confirmally mapping
the outside of the hole into the interior of a unit circle.

The perturbated stresses are computed from the forces which
are applied to the boundary of the hole in such a manner that
the resulting perturbated stresses and the basic stresses sat-
isfy the required boundary conditions at the hole.

A numerical example is presented for the calculation of the
total stresses around a circular hole in the web of a 12 WF 45
loaded with a concentrated load 2P at the center of the beam and
simply supported at the ends.

For any shape or size of opening, or for any loading

condition the total stresses can be calculated numerically. However, applicability of the elastic theory is limited to beams in which the ratio of the hole diameter to web depth does not exceed a maximum value determined by examining moment and shear equality. For circular holes, the analysis is applicable to a minimum beam-depth-to-hole diameter ratio of about 2.0.

For a simply supported beam with a concentrated load at midspan, the predicted maximum stress at the edge of a circular hole occurs at $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$. The magnitude of the stresses occurring at the edge of the hole near mid-depth increase as the hole is moved away from the center of the beam.