ACCEPTANCE SAMPLING: A FOCUS ON ATTRIBUTES VERSUS VARIABLES SAMPLING

by

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TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

The characteristics of all objects (existing in nature as well as manufactured by man) are subject to inherent variations. Not only are the hetrogenities in nature beyond human control but so also are many of the variations in the different processes involved in manufacturing. These basic variations make it unrealistic to hope for a product's strict conformity to the requirements. In real life one is usually dealing with statistical populations (discrete or continuous), with unknown parameters.

In practice, the specifications for a product are determined by the designer. The problem of the manufacturer is to ensure that a batch (lot) adheres to these standards. This assurance is needed well in advance, to avoid unwelcome damages caused by the defective unit. The implications are twofold (i) quality evaluation (ii) quality control. Both of these aspects are interrelated and usually go hand in hand but the techniques used for each, differ.

Restricting the discussion to quality evaluation with the aim of eliminating defective items, there are two possibilities:

1. One hundred percent inspection.
2. Sampling inspection.

1.1 ADVANTAGES OF SAMPLING INSPECTION

Unless it is imperative to remove all defective items, item by item inspection will be avoided as a rule because of the following advantages offered by sampling inspection.

1. The cost of inspecting each item may be prohibitive. Sampling leads to the inspection of just a fraction of the total items while giving (hopefully) sufficient quality assurance.

2. Sometimes the time required for inspecting all the items may be so large that it may be infeasible to do so.

3. Trained inspectors are not easy to obtain. Sampling inspection obviously needs fewer inspectors.

4. Even if inspection of all items could be arranged it would often fail to detect all of the defective items. This failure on part of the inspectors is enhanced when (i) number of defectives is small or (ii) the lots are large, or (iii) inspection is tiring. In some cases it has been found that sampling inspection actually resulted in better quality of the outgoing product than hundred percent inspection. [4]

5. With hundred percent inspection, only the defective items are rejected. Compared to this, sampling inspection would lead to the rejection of the complete lot if the quality is not satisfactory. This acts as a great incentive for a producer to tighten his quality control. The tendency to depend on the customer to weed out the defectives is eliminated by sampling inspection. [30]

6. Damage from handling during inspection is minimized.

7. When production volume is decreased or increased the corresponding increase or decrease in sampling inspection is proportionately quite small and may easily be accommodated. For hundred percent inspection, changes in production level may well lead to bottlenecks or undesirable lay offs.

8. The knowledge that his sampling work results in the rejection or acceptance of an entire lot, gives a sense of pride and responsibility to the inspector. He, in turn, works more efficiently.

9. In case of destructive testing there is no alternative but to use sampling.[1]

1.2 SAMPLING PLANS

As soon as one opts for sampling inspection, risks are introduced. Both producer and consumer have to accept them. The trade off between reliability and effort required to get the reliability is primarily based on economics. It becomes now important to minimize these risks with the help of statistical as well as optimization techniques.

Much effort has already been expended to facilitate the aforesaid optimization. A number of standard plans have been published and are readily available. The well known Dodge & Romig sampling plans, [17] MIL-STD-414 and ABC-STD-105 sampling plans are so well established that their mutual acceptance by the producer & the consumer is almost a routine.

1.3 SCOPE OF THE PLANS

Enough sampling plans already exist to cover effectively almost the whole spectrum of processes and possibilities. The plans can be used at any stage

a) Raw materials

b) In-process goods

c) Finished products

Custom tailored plans are not uncommon in situations where it is worthwhile. The possibilities are as diversified as the requirements.

Recent work is not only aiming at improving the old solutions but also finding and evaluating new ones.

1.4 TYPES OF PLANS

Basically all sampling plans can be classified as

1. Attribute plans
2. Variable plans

1.4.1 Attribute Plans

The measurements made for the quality evaluation are of a discrete nature. Theoretically discrete distributions are viable in this case (commonly the hypergeometric, binomial and Poisson distributions are used). Usually a distinction is made between the plans based on counting the number of defectives and the ones based on the counting of number of defects per unit area or item. For the former, go-no go gauges may be used and items are classified either as defective or good. For the latter, thorough inspection of an item is needed to

ascertain the number of defects - an item with some defects may still be classified as good.

1.4.2 Variable Plans

For these plans the measurements made for quality evaluation are on a continuous scale, consequently the underlying probability distributions are continuous (the normal distribution is most often assumed).

1.4.3 Types of Sampling

For both attribute as well as variable plans there can be further classification based on the number of samples asked for in the plan.

1.4.3.1 Single Sampling. Just one random sample is sufficint to make the decision for rejecting or accepting the lot, according to the single sampling plans.

1.4.3.2 Double Sampling. With these plans, although a lot may be accepted or rejected on the basis of just one sample (if the sample is very good or very bad) yet for the more common intermediate zone, a second sample is taken to make the final decision.

1.4.3.3 Sequential Sampling. With these plans there is no limit on the number of samples to be taken. The acceptance/rejection criterion is the crossing of acceptance or rejection limits. Although a decision is possible after each item is inspected yet it is not uncommon in practice to make the decision after sampling a small group of items.

1.4.3.4 Multiple Sampling. With these plans the basic principle is the same as sequential plans, the difference being that in this case a decision is forced after a small finite number (three or more) of samples.

1.5 OTHER RELEVENT FACETS OF ACCEPTANCE SAMPLING

In addition to deciding which sampling plan is to be used, every sampling program will involve such basic decisions as -

1. What is to be inspected for? Depending on the application, it may be decided to inspect for more than one characteristic of each item.

2. How will the lot quality be described? For example will it be (i) percent defective or (ii) average number of defects per unit. or (iii) arithmetic mean of some characteristic.

3. One quality at a time or more? Depending on the correlation between different characteristics to be inspected, the convenience and practicability, it will be decided whether to inspect just one characteristic or more at an inspection station.[2]

4. Where will this inspection be done?

5. Who will do it?

6. What will be the lot size?

7. What sampling technique will be used to eliminate bias? Some sampling plans, for example in bulk sampling, explicitly include the standard procedure for taking a sample, while others take it for granted that the user will ensure randomness of the sample.

8. What inspection records will be kept and how will they be maintained?

All of these questions are pertinent[3] but the aim of the report is to make an overview of various sampling plans while stressing the comparison of attribute and variable sampling plans, specifically the matching of a variable plan with an existing attribute plan.

CHAPTER II

HISTORY

When was the idea of sampling inspection introduced? It is difficult to say exactly. The modern concept comes from the merging of two great streams, first the testing of materials and the second is inductive logical inference. [1]

The statistical foundation was probably laid by Bayes (1763)[4] and elaborated by Laplace. The work of "Student" and Fischer aroused the interest needed. W. A. Shewhart and his colleagues at Bell Telephone Laboratories in 1926 finally set up the stage for starting sampling inspection as it is known today. The nineteen twenties were the golden age of sampling inspection research and the credit for the great accomplishment goes, without any doubt, to Bell Telephone Systems and its affiliate the Western Electric Company. The works of G. A. Campbell (1923); Molina & Crowell (1924); W. A. Shewart, Frances Thorndyke (1926); Paul Coggins[5] and H. F. Dodge (1928) all reached its climax in October 1929[6] with the publication of "A Method of Sampling Inspection" by H. F. Dodge and H. G. Romig in The Bell System Technical Journal. This was a landmark. The concepts introduced in it are still used, more or less in the same form.

Despite the great advancement in statistical technology it was left to the second world war to accelerate the work in sampling inspection once again. It was necessitated by the wartime activities. Various departments of Armed Forces consulted more and more statisticians

to make their decisions. The army in conjunction with experts from Bell Telephone Laboratories compiled and printed the Dodge and Romig sampling plans. The Navy on the other hand organized the Statistical Research Group, Columbia University. The members of this group were later responsible for publishing the two monographs "Sampling Inspection" and "Sampling Inspection by Variables".

The Office of Production Research and Development took the ideas to the industries and won a place for them.

Today extensive and comprehensive tables exist which provide ready made plans for different applications.[7] Literature is replete with papers criticizing the old work and giving ideas for improvements and for designing custom made plans for specific applications. Though there is enough scope for improvement yet undoubtedly sampling inspection is now on firm standing.

CHAPTER III

TERMINOLOGY, DEFINITIONS AND DESCRIPTION

3.1 ACCEPTANCE SAMPLING

"The art or science that deals with procedures in which decisions to accept or reject lots or processes are based on the examinations of samples". [20]

3.2 SAMPLING PLAN

"A specific plan which states (a) the sample sizes (b) the criteria for accepting/rejecting a lot or taking another sample". [20]

3.3 INSPECTION LOT[8]

"A specific quantity of similar material, or a collection of similar[9] units, offered for inspection and acceptance at one time". [20]

3.4 LOT SIZE

Number of items (units) or quantity in a lot.

3.5 SAMPLE

"A portion of material or a group of units, taken from a lot, the inspection of which provides information for reaching a decision regarding acceptance". [20]

3.6 SAMPLE SIZE

The number of items (units) or quantity in the sample.

3.7 PROCESS AVERAGE QUALITY (PROCESS AVERAGE)

"The expected quality of product from a given process, usually estimated from first sample inspection results from the past." [20]

3.8 PROBABILITY OF ACCEPTANCE (P_a)

The probability that a lot is accepted.

3.9 OPERATING CHARACTERISTIC CURVE (O.C.)

The graphical representation of the response of a given plan. It is the plot of probability of acceptance versus lot quality or process quality, whichever is applicable (Figure 1). The shape of O.C. curve decides the discerning power (between a good lot and a bad lot) of the plan.[10]

3.10 ACCEPTABLE QUALITY LEVEL (AQL:p_1')

That percent defective or number of defects per hundred units which is acceptable as the process average. Materials of this quality will have only a small (specified) risk of rejection.[11]

3.11 LOT TOLERANCE PERCENT DEFECTIVE (LTPD:p_2')

That percent defective which is not acceptable to the consumer. Materials of this quality have a very small (specified) risk of acceptance.[12] This is expressed as Lot Tolerance Fraction Defective (LTFD) sometimes.

3.12 INDIFFERENCE QUALITY

That percent defective which has fifty-fifty chance of being accepted or rejected.

3.13 PRODUCERS RISK (α)

The risk (probability) of rejecting a lot with a quality equal to AQL. This is also known as the probability of type I error.

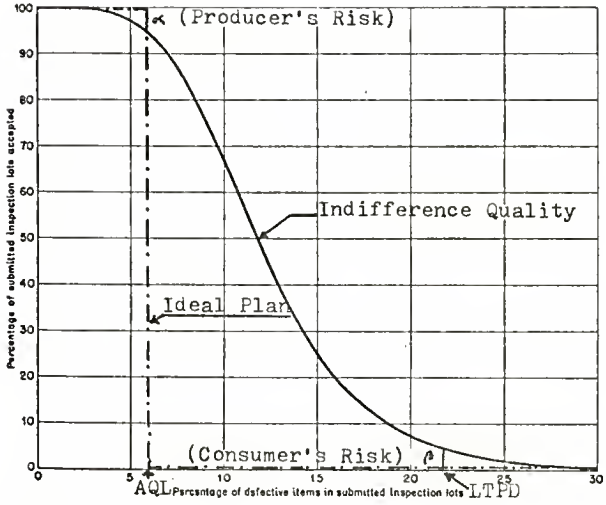


Figure 1. A typical operating characteristic curve

Source: M. Friedman et al., Sampling Inspection.[25]

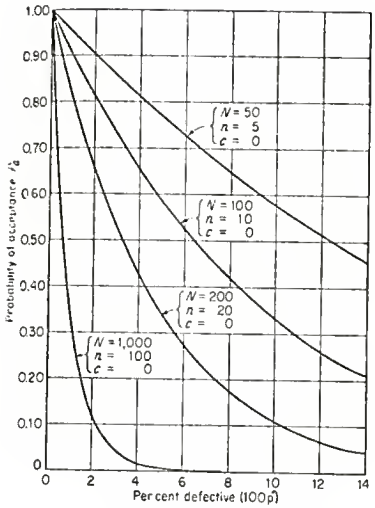


Figure 2. Comparision of operating characteristic curves for four sampling plans involving 10% samples

Source: E.L. Grant and R.S. Leavenworth, Statistical Quality Control. [30]

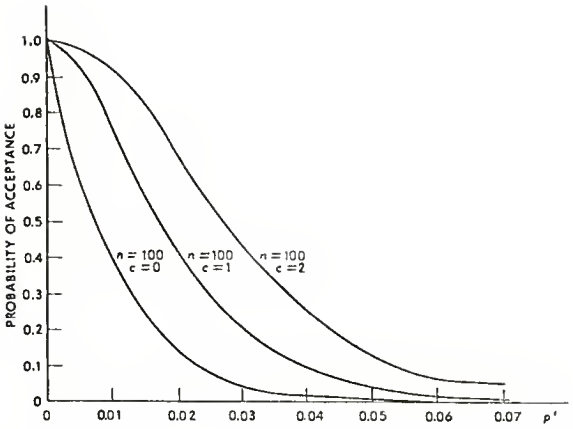


Figure 3. Operating characteristic curves for different acceptance numbers

Source: A.J. Duncan, Quality Control and Industrial Statistics.[19]

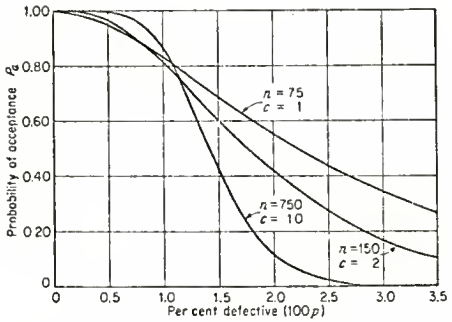


Figure 4. Comparision of OC curves with different sample sizes, all permitting the same fraction of the sample to be defective

Source: E.L. Grant and R.S. Leavenworth, Statistical Quality Control. [30]

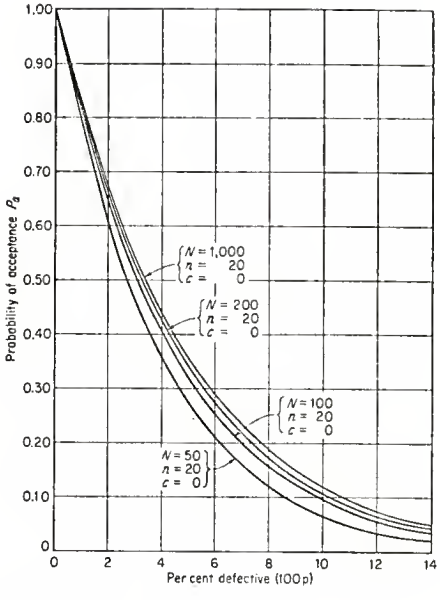


Figure 5. Comparisons of OC curves for four sampling plans involving samples of 20, each with acceptance number of 0.

Source: E. L. Grant and R. S. Leavenworth, Statistical Quality Control. [30]

3.14 CONSUMERS RISK (β)

The risk of accepting a lot having quality level equal to LTPD. This is also known as the probability of type II error.

3.15 AVERAGE SAMPLE SIZE (ASN)

"The average number of sample units inspected per lot in reaching the decision to accept or reject". [20]

3.16 AVERAGE TOTAL INSPECTION (ATI)

"The average number of sample units inspected per lot including all units in rejected lots (i.e. with screening)". [20]

3.17 NORMAL INSPECTION

"Inspection in accordance with a plan that is used under ordinary circumstances". [20].

3.18 REDUCED INSPECTION

Inspection with a smaller sample size (lax compared to normal inspection). Generally done when past record shows lot quality consistently better than AQL.

3.19 TIGHTENED INSPECTION

Inspection in accordance to a plan that has more strict acceptance criterion than normal. Generally used when past record shows consistent poorer quality compared to AQL.

3.20 CURTAILED INSPECTION

Sampling Inspection in which, as soon as a decision is reached, the inspection is stopped. As soon as the number of defectives exceed

the maximum allowable limit no more are inspected. However, it is common to inspect the first sample completely for maintaining the control charts or at least to determine the process average.

3.21 ACCEPTANCE NUMBER

The maximum number of defectives (or defects) allowed in a sample without causing the rejection of the lot.

3.22 REJECTION NUMBER

The smallest number of defectives (or defects) which will cause the lot to be rejected.

3.23 AVERAGE OUTGOING QUALITY (AOQ)

The average quality of the outgoing product after all defectives in the rejected lots have been replaced with good ones.

3.24 AVERAGE OUTGOING QUALITY LIMIT (AOQL)

The maximum AOQ an acceptance plan will result in, irrespective of the incoming quality (Figure 6).

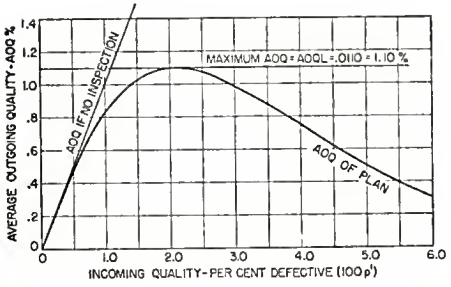


Figure 6. AOQ curve and AOQL for a typical sampling plan

Source: J. M. Juran, Quality Control Handbook. [39]

CHAPTER IV

FORMULATION OF DIFFERENT TYPES OF SAMPLING PLANS

The characteristics of a good sampling plans as per Cowden [14] are as follows.

(i) Protect the producer against rejection when his process is under statistical control and satisfactory as to level and uniformity.

(ii) Protect the consumer from accepting bad lots.

(iii) Give long run protection to consumer.

(iv) Act as an incentive to producer to keep his process under control.

(v) Minimize the cost of sampling, inspection and administration.

(vi) Provide information about the incoming quality.

As already described basically sampling plans are of two types,

1. Attribute (Discrete)
2. Variable

depending on how inspection is carried out.

4.1 ATTRIBUTE PLANS

Before we can proceed with the formulation of these plans we have to make a distinction between type A and type B O.C. curves which is due to Dodge and Romig [17,19,30].

4.1.1 Type A O.C. Curves

These curves give the probabilities of acceptance for different incoming fraction defectives from parent lots of finite sizes. The hypergeometric distribution thus would be the correct one to use for computations of various probabilities. Note also that such curves would be discontinuous in principle though it is a common practice to draw them as continuous.

4.1.2 Type B O.C. Curves

These curves give the probabilities of acceptance against fraction defectives from infinite lots (from a continuous, statistically stable process). The binomial distribution would be the correct one to be used here. Note that it is common to use the Poisson approximation in both cases, especially when lots are large and sample sizes small. For practical purposes when the sample size is less than one-tenth of the lot size both type A and type B curves are identical for practical purposes. [30]

It may also be noted here that type A curve always falls below the type B curve (this becomes obvious when you consider the underlying distributions) and hence type B curves always give a higher value for consumers risk.

4.1.3 Single Sampling Attribute Plan

It is identified by n (sample size) and c (acceptance number). It works as follows. Take a random sample of size n and count tne defectives (or number of defects) if these are less than or equal to c then accept the lot, otherwise reject it.

4.1.4 Derivation of a Single-Sampling Fraction Defective Plan with Specified p_1', p_2', α and β.

We can either wish the O.C. curve to pass through two designated points or pass through one point plus satisfy some other criterion (such as keep sample size as preassigned fixed number).

4.1.4.1 Type A O.C. Curve. Using the following notation

Let N = Lot size

M = number of defectives in the lot

p' (fraction defective in lot) = $\frac{M}{N}$

n = sample size

m = number of defectives in a sample (it is a random variable)

Probability of finding m defectives in a random sample is given by the hypergeometric probability.

$$P(m|n,N,M) = \frac{C_{n-m}^{N-M} C_m^M}{C_n^N} \quad m = 0,1,2, \ldots, \min(n,M) \qquad (1)$$

$$= \frac{(N-M)!\ M!n!(N-n!)}{(n-m)!(N-M-n+m)!(M-m)!m!N!}$$

If c is the acceptance number then probability of acceptance is given by

$$P_a = \sum_{m=0}^{c} \frac{C_{n-m}^{N-M} C_m^M}{C_n^N}$$

substitutiong p'N = M we can obtain two equations for the two points as follows.

$$1 - \alpha = \sum_{m=0}^{c} \frac{C_{n-m}^{N-p_1'N} \; C_m^{p_1'N}}{C_n^N} \tag{2}$$

and

$$\beta = \sum_{m=0}^{c} \frac{C_{n-m}^{N-p_2'N} \; C_m^{p_2'N}}{C_n^N} \tag{3}$$

It can be seen that solving for c and n from equations (2) and (3) is not easy, it can be done by trial and error only.

There are alternatives of course.

1. Use G. J. Leiberman and D. B. Owen's Tables of Hypergeometric Probability Distribution. [42] Tabulated values are for N = 2 to 100 and n = 1 to 50 plus some other specific values of N and n.

2. Use one of the approximations.

(a) When both N and n are large and neither M nor N-M is very small: use normal approximation. The P_a is given by area under standard normal curve with

$$Z = \left(\frac{c + 0.5}{n} - p'\right) \Bigg/ \sqrt{\frac{p'(1-p')}{n} \left(\frac{N-n}{N-1}\right)} \tag{4}$$

where p' is $\frac{M}{N}$

(b) When N,M and N-M are large relative to n and m then it can be approximated by binomial with p' = M/N and n (binomial) = n (Hypergeometric).[13]

$$\sum_{m=0}^{c} C_{n-m}^{N-M} C_m^M \Big/ C_n^N = \sum_{m=0}^{c} C_m^n p'^x (1-p')^{n-x} \qquad (5)$$

(c) When N and n are large but m or N-m is relatively small then it is approximated by a binomial distribution with p' = n/N and n (binomial) = m (hypergeometric).[14]

$$\sum_{m=0}^{c} C_{n-m}^{N-M} C_n^M \Big/ C_n^N = \sum_{m=0}^{c} C_m^M p'^x (1-p')^{n-x} \qquad (6)$$

The binomial distribution has been extensively tabulated. However again it is to be noted that trial and error is still the only practical way to calculate the required values for n and c.

(d) In case p' for binomial approximation is small and n or M are not too small, it is possible to use Poisson distribution as an approximation.[15]

$$\sum_{m=0}^{c} C_{n-m}^{N-M} C_m^M \Big/ C_n^N \doteq \sum_{m=0}^{c} \frac{(\frac{nM}{N})^m e^{-\frac{nM}{N}}}{m!} \qquad (7)$$

This cummulative Poisson distribution can be read from the tables or graphs provided by Dodge and Romig [32] (Figure 7).

A perfect solution is usually never found because of the discrete nature of N and n etc. and we have to make adjustments to get whole numbers.

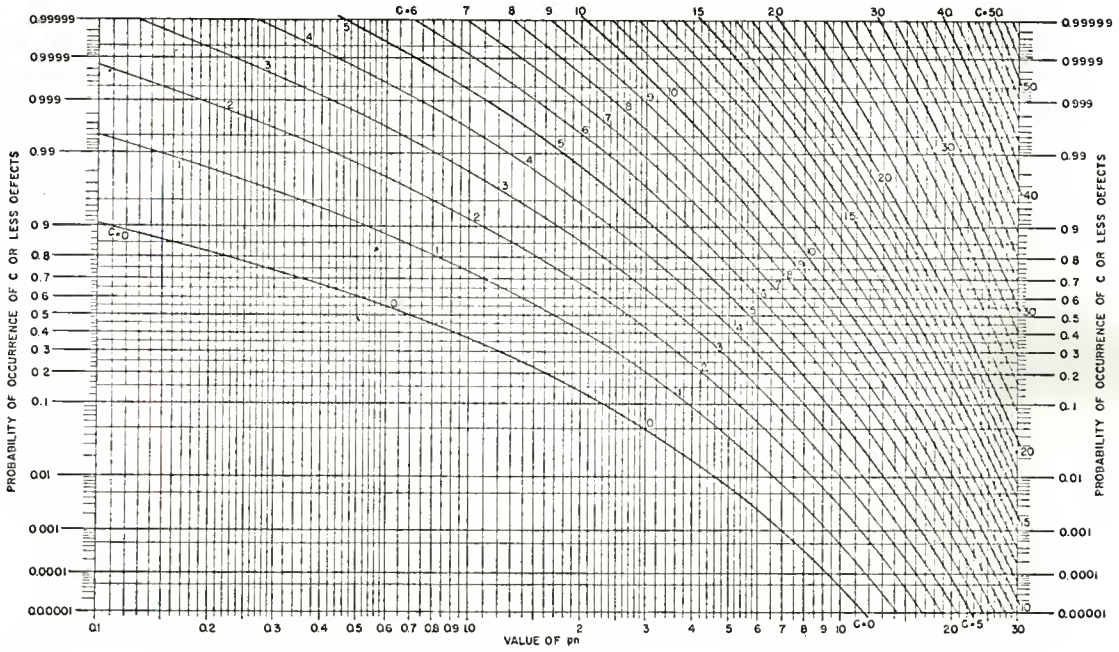


Figure 7. Cumulative probability curves of the Poisson exponential distribution

Source: H. F. Dodge and H. G. Romig, Sampling Inspection Tables.[17]

Examples.

Say $p_1' = 0.01$

$p_2' = 0.08$

$\alpha = 0.05$

$\beta = 0.10$

and lot size N = 100 [19].

Using the Lieberman-Owen tables we see the best solution is n = 40, c = 1 which gives $\alpha = 0$ and $\beta = 0.097$.

When one has to work out of the range of these tables then Binomial approximation is used for better results.

For example if N = 200 and other criteria are the same as in the above example viz.

$p_1' = 0.01$

$p_2' = 0.08$

$\alpha = 0.05$

$\beta = 0.10$

then using National Bureau of Standards Tables for Binomial Distribution[16] the solution after some trial and error comes as n = 44, c = 1 which gives $\alpha = 0.048$ and $\beta = 0.097$ (using equation 5).

4.1.4.2 Type B O.C. Curve. These are the usual ones that are developed in practice. Here the Binomial distribution is valid and Normal approximation or Poisson approximation may be used.

(a) The procedure can be simplified if Figure 7 is used in conjunction with the method described by A. R. Burgers.[17]

Since the p'n scale in Figure 7 is logarithmic, equal distances anywhere along the axis represent the same ratio. Cut an L shaped piece of paper and proceed as follows. "Starting at 1.0, mark off the p'n scale along the (inner) bottom edge of the 'L'. To find a single-sampling plan with a specified p_1', p_2', α and β compute ratio p_2'/p_1'. Place the (inner) bottom edge of the 'L' on the horizontal line of Figure 7 for which $P_a = \beta$ and mark off the point on the (inner) upper part of the L through which passes the horizontal line $P_a = 1-\alpha$ then move 'L' to the left or right until an X curve is found that passes through the $1-\alpha$ mark on the upper part of L and at the same passes through, or close to, the p_2'/p_1' ratio on the lower part of 'L'. The X will be the c for the sampling plan. To find n read off p'n at this point and divide it by p_1'. The plan so found will have desired α but its β will be higher than that desired if the X line passes to the right of the ratio mark or lower if it passes to the left of the ratio mark. If β is needed to be held constant then we can make X curve pass through ratio mark and make adjustments on the other end." [19]

The advantage here is that we can work with any α and β.

(b) Another method is to use the tables given by A. H. Bowker and G. J. Lieberman [7]. This table directly gives the design of single sampling plan. The sample entries are shown in Table 1.

TABLE 1*

Table for two-point design of a single-sampling with α approximately 0.05 and β approximately 0.10.

c	$p'n_{0.95}$	$p'n_{0.10}$	$p'n_{0.10}/p'n_{0.95} = p'_{0.10}/p'_{0.95}$
0	0.051	2.30	45.10
1	0.355	3.89	10.96
2	0.818	5.32	6.50
3	1.366	6.68	4.89

*A.J. Duncan [19]

Example

Using the same values $p'_1 = 0.01$

$p'_2 = 0.08$

$\alpha = 0.05$

$\beta = 0.10$

calculate $\frac{p'_2}{p'_1} = 8.$

This lies between the ratios corresponding to c = 1 (10.96) and c = 2 (6.50). A choice has to be made now, between being conservative (c = 1) or liberal (c = 2).

Say c = 1 is picked then again it is to be decided whether α is to be held constant or β. If α is held constant then $p'n_{0.95}$ is read off as 0.355 which gives $n = 0.355/0.01 = 35.5 \doteq 36$.[18]

The final plan is thus n = 36, c = 1 α = .05 and β = 0.23. A better choice can be had by keeping β constant and thus getting n = 49, c = 1, α = 0.08 and β = 0.10.

(c) If one wants to be more exact then Binomial Tables can be used as described earlier while discussing the type A O.C. curve plans. For a fuller explanation and description reader is referred to Annals of Mathematical Statistics Vol XX, page 242-56.[19]

4.1.5 Single-Sample Defects per Unit Acceptance Sampling Plans

This kind of plan is needed when the items subjected to sampling inspection are like cloth, linoleum or large items consisting of a big number of components eg. T.V. set, referigerator etc.

The number of defects per unit may follow the Poisson distribution and it can be proved (any text book on statistics will verify it) that total number of defects in n units will also follow a Poisson distribution with a parameter (mean) equalling n times the population parameter.

And now it can be seen that defects per unit plans can be formed just like the fraction defective plans. Sample size n and acceptance number c will be similar. The O.C. curves will be approximately the same (exactly the same if the Poisson approximation has been used) as a fraction defective plan with same n and c. Only the abscissa will now read 'number of defects per unit' instead of 'fraction defective'. In practice the same sampling plans are used interchangeably.

4.1.6 Double Sampling Attribute Plans

These are designated as n_1 (1st sample size), n_2 (2nd sample size), c_1 (1st acceptance number), c_2 (1st rejection number), c_3 (2nd acceptance number). The plan works as follows: take a sample of size n_1, check for defectives (or number of defects); if these are less than or equal to c_1, accept the lot; if more than or equal to c_2, reject the lot; if more than c_1 but less than c_2 then take another sample of size n_2. If the combined number of defectives are less than or equal to c_3 then accept the lot, otherwise reject it. Note that it is common to have $c_2 = c_3$.

4.1.7 Double Sampling Plans with Specified p_1', p_2', α and β

The basic principles behind calculations of the various probabilities remain the same as before. The probability of acceptance for the combined sample is the sum of two probabilities, the probability of acceptance at the first sample and the probability of acceptance at the second sample.[20]

Once again both type A and type B curves are possible. Now that the difference is clear, attention will be confined to type B curves only, as they are more common.

The formulation of the plan is now more complicated. To begin with, just knowing p_1', p_2', α and β is not sufficient to give a unique plan [19]. Thus some additional criterion has to be used. One such popular criterion (which leads to good plans) is to use a fixed relationship between n_1 and n_2.

Assuming p' is small (which is reasonable in practice) the Poisson approximation may be used. This in conjunction with the fixed ratio between n_1 and n_2 would mean that for a given set of values for c_1 and c_2, the probabilities of acceptance would only depend on n_1p'. Hence it means that plans with fixed p_2' and p_1' ratio can be made to have identical O.C. curves by simply varying n_1. [19] In other words if p_1' and p_2' are both multiplied by a common factor 'a' (maintaining their ratio) and n_1 is divided by 'a' then values of P_1 and P_2 (probabilities of acceptance) will be the same because $n_1'p'$ has been held constant. Bearing this in mind tables similar to Table 1 (for single sampling plans) may be constructed. A sample portion is given in Table 2.

TABLE 2*

Values useful in deriving a double-sampling plan with a specified p_1' and p_2'. ($n_1 = n_2$, $\alpha = 0.05$ and $\beta = 0.10$)

Plan No	$\frac{p_2'}{p_1'}$	Acceptance numbers		Approximate values of $p_1' n_1$ for		
		C_1	C_2	p = 0.95	p = 0.50	p = 0.10
1	11.90	0	1	0.21	1.00	2.50
2	7.54	1	2	0.52	1.82	3.92
3	6.79	0	2	0.43	1.42	2.96
4	5.39	1	3	0.76	2.11	4.11

*For extensive tables see Chemical Corps Engineering Agency, Manual Number 2, Master Sampling Plans for Single, Duplicate, Double and Multiple Sampling (Army Chemical Center, Md., 1953).

The above procedure leads to an extensive table of double sampling plans; however all the plans will not be good. The plans that are intuitively bad may be rejected. For example if we take n_2 equal to 2.5 n_1 then it is intuitively appealing to have c_2 at least 3.5 c_1. Observe that in the sample table given (Table 2) $n_2 = n_1$ and c_2 is at least twice c_1.

It may be once again stressed that here the probabilities are calculated by the Poisson approximation. For more accurate results, one must again use the binomial or hypergeometric distributions and calculate n and c by trial and error. The probability of accepting an inspection lot is given by

$$P_a = P''(c_1;n_1) + \sum_{K=c_1+1}^{c_2} P(K;n_1)\, P''(c_2-K,n_2)$$

where $P(K,n)$ denotes probability of getting K defectives in n items and $P''(c,n)$ denotes probability of c or fewer defectives.
It may be mentioned here that the probability calculations can be made with the use of Incomplete Beta Function. For details reader may refer W. E. Deming. [16] Pearson's Tables of Incomplete Beta Function give the tabulated values.

4.1.8 The Average Sample Size Curve (ASN Curve)

Now that the formulation of double sampling plans has been discussed, it is appropriate to discuss another important characteristic of the sampling plans, the average sample size (A.S.N.). This is very

important from the economic point of view. For a good comparison between two plans we need to compare not only the O.C. curves (the level of protection) but also the average sample sizes expected (the inspection effort required). The average number to be inspected will be constant for single sampling but will vary for double plans with incoming quality because the second sample is drawn only on the basis of the number of defectives observed in the first sample which in turn is dependent on incoming quality (p').

The plot of the average sample size (A.S.N.) against p' is known as the ASN curve. For a comparison between single, double and multiple (sequential sampling) see Figure 8. It is important to note that two different curves are possible

(a) with complete inspection of all samples

(b) complete inspection of first sample and then termination of inspection at any other stage as soon as a decision is reached.

4.1.8.1 The ASN Curve with Complete Inspection. The general formula for ASN is given as

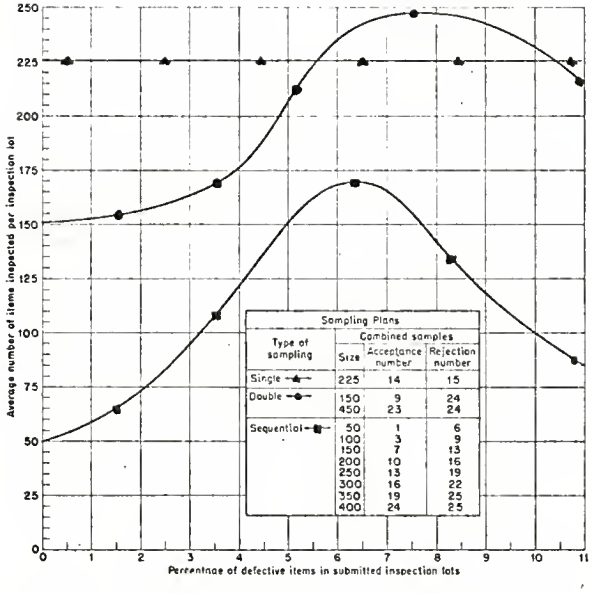
$$ASN = n_1 P_1 + (n_1+n_2)(1-P_1)$$

$$= n_1 + n_2 (1-P_1)$$

where P_1 = probability of decision after first sample

= probability of acceptance after first sample plus probability of rejection after first sample.

And this probability can be calculated from the binomial distribution or the Poisson approximation as discussed earlier.



Sampling Plans

Type of sampling	Combined samples		
	Size	Acceptance number	Rejection number
Single	225	14	15
Double	150	9	24
	450	23	24
Sequential	50	1	6
	100	3	9
	150	7	13
	200	10	16
	250	13	19
	300	16	22
	350	19	25
	400	24	25

Figure 8. Average amount of inspection for single, double and sequential sampling plans

Note: For single sampling, entire sample inspected; for double and sequential sampling, entire first sample inspected, inspection of later samples curtailed as soon as decision can be reached.

Source: M. Friedman et al., Sampling Inspection. [25]

4.1.8.2 The ASN Curve with Curtailed Inspection. Here the inspection is curtailed just at the point where number of defectives exceed c_2 (acceptance number for combined samples).

The formula for ASN is given in this case as [21]

$$ASN = n_1 + \sum_{K=c_1+1}^{c_2} P_{n_1:K} \left[n_2 P''_{n_2:c_2-K} + \frac{c_2-K+1}{p'} P'_{n_2+1:c_2-K+2} \right]$$

where

n_1 = size of first sample

n_2 = size of second sample

c_1 = acceptance number for first sample

c_2 = acceptance number for combined samples

$P_{n:x}$ = probability of exactly x defectives out of n

$P'_{n:x}$ = probability of x or more defectives out of n

$P''_{n:x}$ = probability of x or less defectives out of n.

4.1.9 Item by Item Sequential Plans

In order to further reduce the amount of inspection, it may be decided to take a sequence of single observations and then base the sample size entirely on the sampling result. Inspect one more item until a decision can be reached as described below.

To expound the basis for these plans it is worthwhile to quote Acheson J. Duncan. [19] "Item-by-item sequential sampling is based fundamentally upon the notion of the "random walk". Suppose, for example, two gamblers, A and B, each have a capital of $10 each. They agree to play the following game. A perfectly unbiased coin is to be tossed in a random manner. If the coin turns up heads, A pays B $1.00. If the coin turns up tails, B pays A $1.00. They agree to play until either one has lost all his money to the other. If the coin were biased, the game could still be played, but the outcomes, A's ruin or B's ruin, would not now be equally likely."

In sampling inspection the probability of a defective is comparitively small but still the principle holds, and acceptance and rejection limits (A's ruin / B's ruin) can be set over the whole range of sampling. See Figure 9. The better known sequential plans in the USA were developed by A. Wald (member, Statistical Research Group Columbia University during W.W. II). Another one was developed independently in England by G. A. Barnard. [19]

4.1.9.1 Formulation of an Item-by-Item Sequential Ratio Plan With Specified p_1', p_2', α and β. (a) Wald's Sequential Probability Ratio Plan: To satisfy the given conditions and keep a score to check whether the rejection or acceptance limit is exceeded, Wald uses SPR, which is defined as the ratio of the probability of getting cumulative result at any stage assuming that population fraction defective is p_1' to the probability of getting the same results under the assumption that true population fraction defectives is p_2'. That is,

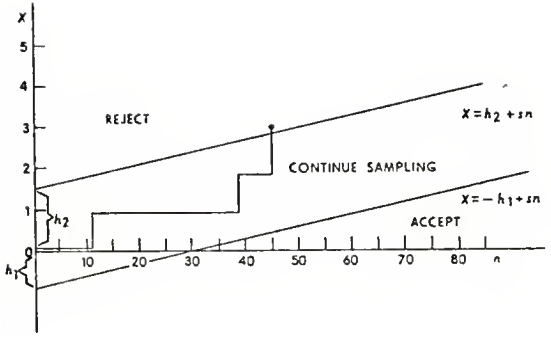


Figure 9. Sequential-Sampling Chart

Source: A. J. Duncan, Quality Control and Industrial Statistics. [19]

$$SPR = \frac{P(X|p_1')}{P(X|p_2')}$$

When this ratio becomes "too large" accept the lot, "too small" then reject the lot. Otherwise keep on sampling. How large is "too large" and how small is "too small" can be strictly determined by level of protection needed (given parameters).

The SPR need not be calculated in practice. Wald showed that the procedure can be simplified by using a chart like Figure 9.

The equations for the two limit lines are as follows.[22]

$$x = -h_1 + sn$$

$$x = h_2 + sn$$

where $$h_1 = \log \frac{1-\alpha}{\beta} \Big/ \log \left(\frac{p_2'(1-p_1')}{p_1'(1-p_2')}\right)$$

$$h_2 = \log \frac{1-\beta}{\alpha} \Big/ \log \left(\frac{p_2'(1-p_1')}{p_1'(1-p_2')}\right)$$

$$s = \frac{\log \left(\frac{1-p_1'}{1-p_2'}\right)}{\log \left(\frac{p_2'(1-p_1')}{p_1'(1-p_2')}\right)}$$

Tables have been developed by the Statistical Research Group, Columbia University and included in the book "Sequential Analysis of Data: Application." These tables list values of h_1, h_2 and s for $\alpha = 0.05$, $\beta = 0.10$ and 0.50, $p_1' = .0002$ to 0.10 and $p_2' = 0.002$ to 0.35.

(b) The O.C. curve for SPR Sampling Plan: The O.C. curve for such a plan can be sketched by designating three points. [19] It has been proved that s lies between p_1' and p_2'[23] and the probability of acceptance for a lot with fraction defective equal to s is $h_2/(h_1+h_2)$.[24]

Furthermore using a parameter θ such that when $\theta = 1$, $p' = p_1'$, when $\theta = -1$, $p' = p_2'$ and when $\theta = 0$, $p' = s$, other points can be obtained from the following equation. [74]

Product or Lot Quality $$p' = \frac{1 - \left(\frac{1-p_2'}{1-p_1'}\right)^{\theta}}{\left(\frac{p_2'}{p_1'}\right)^{\theta} - \left(\frac{1-p_2'}{1-p_1'}\right)^{\theta}}$$

Probability of Acceptance $$P_a = \frac{\left(\frac{1-\beta}{\alpha}\right)^{\theta} - 1}{\left(\frac{1-\beta}{\alpha}\right)^{\theta} - \left(\frac{\beta}{1-\alpha}\right)^{\theta}}$$

(c) The ASN curve for SPR plan: Different points on the ASN curve can be located by using these relationships

At $p' = 0$, $ASN = h_1/s$

$p' = 1$, $ASN = h_2/(1-s)$

$$p' = p_1', \quad ASN = \frac{(1-\alpha)\, h_1 - \alpha h_2}{s - p_1'}$$

$$p' = p_1', \quad ASN = \frac{(1-\beta) h_2 - \beta h_1}{p_2' - s}$$

$$p' = s, \quad ASN = \frac{h_1 h_2}{s(1-s)} \qquad [24]$$

It was proved by Wald [74] that

$$ASN = \frac{P_a \log \frac{\beta}{1-\alpha} + (1-P_a) \log \frac{1-\beta}{\alpha}}{p' \log (p_2'/p_1') + (1-p') \log [(1-p_2')/(1-p_1')]}$$

Statistical Research Group, Columbia University lists the above five points for all their plans.

The ASN curve has its maximum somewhere between p_1' and p_2', and as indicated by the relationships above, the ASN is extremely low at the extremes and this can result in appreciable cost savings.

It has actually been shown that no other attribute plan with two given points on the O.C. curve can have ASN lower than SPR plan at those points.[25]

However for rest of the points it is possible (but not likely) for single and double sampling plans to have lesser ASN than SPR plan. [19]

(d) Barnard's Approach to Sequential Sampling: His approach was to start from the acceptance/rejection limits and show how O.C. curve could be obtained from these boundaries.[26]

Here the scoring system is 1 for each good item and -b for each bad item. When the score exceeds or equals a certain value H then the lot is accepted, when the score falls below or equals another value -H' then the lot is rejected. Some feel that administratively this is better than Wald's approach.

The parameters in the two approaches are related as follows.

$$b = \frac{1-s}{s} \rightarrow s = \frac{1}{b+1}$$

$$H = h_1/s \rightarrow h_1 = \frac{H}{b+1}$$

$$H' = h_2/s \rightarrow h_2 = \frac{H'}{b+1} \qquad [19]$$

Following this approach F.J. Anscombe has worked out tables of sequential sampling plans indexed simply on h_1 and h_2.

4.1.10 Group Sequential and Multiple Sampling Plans

In practical situations it is sometimes better to inspect a group of items rather than one item at a time and then base the decision on the result of the group. The acceptance-rejection limits are kept the same as in item by item sequential sampling but the step size on the plot, in this case, is equal to the group size. Both O.C. curve and ASN are expected to change under this condition. The change in the O.C. curve is not appreciable, but the change in ASN is usually greater as one has to wait for the inspection of the whole group containing the Kth item which under sequential sampling would have given the decision.

Recalling that s is the slope of the acceptance and rejection limit lines (i.e. representing the increase in n corresponding to unit increase in x) and assuming that h_1 and h_2 are integers, then effectively no decision can be made till $\frac{1}{s}$ items have been inspected. Hence $\frac{1}{s}$ is preferred as the group size.

Group sequential plans are usually truncated, that is after a certain number of samples it is stipulated that either the lot will be accepted or rejected. These are what are known as Multiple Sampling Plans.

4.1.11 Formulation of a Multiple Sampling Plan with Specified p_1', p_2', α and β

It is easy to see that problem is now more tedious than even double sampling. In addition to the two points (AQL and LTPD) specified above, one has to specify some other criteria such as minimizing sampling inspection to get a unique plan.

(a) An algebraic expression can be formed for calculating the probability of acceptance and then trial and error can be used to get an acceptable plan.

In addition to the formulae already stated we can use the following recursive relationships for calculations

$$P_i = P_i' - P_{i+1}'$$

$$P_i = P_i'' - P_{i-1}''$$

$$P_i' = 1 - P_{i-1}''$$

$$P_i'' = 1 - P_{i+1}'$$

$$P_i'' = P_0 + P_1 + \ldots P_i$$

where as usual P_i = probability of exactly i defectives in a sample of n

P'_i = probability of i or more defectives

P''_i = probability of i or fewer defectives

Either Poisson approximation or Incomplete Beta Function tables can be used to cut down on calculations.

(b) Another approach is to develop and use tables similar to the one described in section 4.1.7. (Table 2). Such tables can be found in Army Chemical Corps Master Sampling Plan for Single, Duplicate, Double and Multiple Sampling and in Enters and Hamaker 'Multiple Sampling in Theory and Practice'.[27] A sample portion is given in Table 3.

TABLE 3*

Values useful in designing a multiple-sampling plan $\alpha = 0.05$, $\beta = 0.10$

No.	p'_1/p'_2	Acceptance and Rejection numbers	Approximate value of $p'n_j$ for $P_a = 0.95$	0.50	0.10
1	18.46	Ac * * 0 0 1 2 3 Re 2 2 2 2 3 4 4	.048	.38	.89
2	12.15	Ac * * * 0 0 1 2 Re 2 2 2 2 2 3 3	.065	.31	.79
3	9.95	Ac * * 0 0 1 2 4 Re 2 2 2 3 3 4 5	.10	.43	1.00
4	8.91	Ac * * 0 0 0 0 0 2 Re 2 2 2 2 2 3 3 3	.088	.34	.78
5	8.06	Ac * * 0 0 0 0 0 1 2 Re 2 2 2 3 3 3 3 3 3	.093	.36	.75
6	7.04	Ac * 0 0 1 1 1 2 3 Re 2 3 3 3 4 4 4 4	.18	.62	1.27

*A. J. Duncan [19]

The use of the table is similar to Table 1 and Table 2 and can best be explained by the previous example with $p_1' = 0.01$ $p_2' = 0.08$ $\alpha = 0.05$ and $\beta = 0.10$.

Then $p_1'/p_2' = 0.08/0.01 = 8$

which lies somewhere between number 5 and 6. The closest is number 5 (8.06), so let this be selected. Then keeping 2 constant the value of $p_1'n_i$ is given (in the column under P = 0.95) as 0.093. So group size n = 0.093/.01 = 9.3. To be conservative make it equal to ten. Now recall that with this kind of approach, a constant relationship is kept between samples (in this case it is $n_1 = n_2 = \ldots n_i$). The required plan can be stated as follows.

Cumulated sample	Acceptance numbers	Rejection numbers
10	*	2
20	*	2
30	0	2
40	0	3
50	0	3
60	0	3
70	0	3
80	1	3
90	2	3

It is to be noted that at the last stage the rejection number is just one more than the acceptance number, thus ensuring a definite decision at the last sample.

(c) The third (and a very sensible) approach is to try and make the multiple sampling plan conform to the sequential ratio plans' acceptance rejection limit, which will ensure proper α and β values. This approach was used by the Statistical Research Group, Columbia University [25] to formulate their plans. The book 'Sampling Inspection' [25] gives the details for computations. The basic idea is to first construct the proper SPR as described in 4.1.9.1 and select a group size n (usually $\frac{1}{s}$) and then read off Acceptance and rejection numbers from the SPR plan plot for corresponding value of n (rounding off to the next higher integer for rejection number and lower integer for acceptance Ac number, to be a conservative side).

4.1.11.1 The ASN Curve for Multiple Sampling Plan. In general when inspection is not curtailed (i.e. stopped in midsample) then

$$ASN = P_1 n_1 + P_2(n_1+n_2) + \ldots P_k(n_1+n_2 + \ldots n_k)$$

where P_i = Probability of decision at the ith stage

4.2 VARIABLE SAMPLING PLANS

The fundamental assumption here is that the quality characteristic of interest is distributed normally (later on, the implications of this assumption being erroneous will be briefly discussed). On this basis the proportion of defectives (p') in a population with respect to a specification limit (L) can be determined by constructing a standard normal deviate

$$z = \frac{L-\mu}{\sigma} .$$

The probability of an item falling below L is thus given by $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\, dt$, which has been extensively tabulated as the Standard Normal Distribution.

Basically a single variable plan will be designated as n,K, where n is the sample size and K the specification limit. There are several approaches to explain the working of such a plan but the following two are common and readily understood. [19]

Assuming that there is a single lower limit (L) the first approach is as follows:

1. Take a random sample of size n and calculate its mean $\bar{X}$.
2. Calculate standard normal deviate $Z_L = \frac{\bar{X}-L}{\sigma'}$ (assume σ' known for the time being).
3. Estimate the fraction defective from Z_L.
4. If this exceeds a maximum specified limit K reject the lot, otherwise accept it.

The 2nd approach is as follows

1. Take a random sample of size n and calculate the mean $\bar{X}$.
2. Compute $Z_L = \frac{\bar{X}-L}{\sigma'}$.
3. If this Z_L is less than or equal to K, accept, otherwise reject the lot.

In this case K is the critical limit for standard normal deviate and not fraction defectives. It is to be noted that this procedure is equivalent to the procedure which rejects a lot when $\bar{X} - K\sigma' < L$.

Both approaches are equivalent and the O.C. curve is the same but is easily constructed by following the latter. [19] Hence only the

second procedure will be followed here. The interested reader may refer to Acheson J. Duncan [19] for the details of first procedure under different circumstances.

When μ or σ' are unknown (either or both of them) different probability distributions have to be used and thus their formulations are different. In practice, we can classify the variable plans under three headings.

(a) When μ is unknown but σ' is known

(b) When μ is known but σ' is unknown

(c) When both μ and σ' are unknown

The treatment will differ when there is:

(a) Single specification (either lower or higher) limit

(b) Double specification limits

Also the possibility of a) single sampling b) double sampling and c) sequential and multiple sampling exists with each combination.

Only the basic formulations will be discussed here, the reader can find detailed formulations in any other specific combination he is interested in, in the references included in the bibliography.

4.2.1 Variable Sampling Plans when σ' is Known

The most fundamental, a single sampling plan with σ' known and with single specification limit will he dealt with first.

4.2.1.1 Formulation of a Single Sampling Variable Plan with σ' Known and with Specified α, β, p_1' and p_2' (Single Specification Limit).

Say the lower limit (L) is specified. Then under the procedure decided on, the lot is accepted

$$\text{if} \quad \frac{\bar{X}-L}{\sigma'} \geq K$$

adding and subtracting $\frac{\mu'}{\sigma'}$ to left hand side

$$\text{if} \quad \frac{\bar{X}-\mu}{\sigma'} + \frac{\mu-L}{\sigma'} \geq K$$

$$\text{or if} \quad \frac{\bar{X}-\mu}{\sigma'} \geq K - \frac{\mu-L}{\sigma'}$$

Multiplying both sides by $\sqrt{n}$ leads to the condition

$$\text{if} \quad \frac{\bar{X}-\mu}{\sigma'/\sqrt{n}} \geq (K - \frac{\mu-L}{\sigma'}) \sqrt{n} \tag{8}$$

Now let those values of μ which will yield p_1' and p_2' quality be designated as μ_1 and μ_2.

If Z_1 and Z_2 are designated as

$$Z_1 = \frac{\mu_1-L}{\sigma'/\sqrt{n}}$$

$$Z_2 = \frac{\mu_2-L}{\sigma'/\sqrt{n}}$$

then equation (8) in conjunction with the concept of α and β will yield the following probability statements:

$$\Pr\left(\frac{\bar{X}-\mu}{\sigma'/\sqrt{n}} \geq (K-Z_1) \sqrt{n}\right) = 1-\alpha \tag{9}$$

$$\Pr\left(\frac{\bar{X}-\mu}{\sigma'/\sqrt{n}} \geq (K-Z_2) \sqrt{n}\right) = \beta \tag{10}$$

We know that when

$$X \sim N(\mu, \sigma'^2)$$

then $\frac{\bar{X}-\mu}{\sigma'/\sqrt{n}} \sim N(0,1)$

which means eq. (9) and (10) yield

$$(K-Z_1)\sqrt{n} = Z_{1-\alpha} \tag{11}$$

$$(K-Z_2)\sqrt{n} = Z_{\beta} \tag{12}$$

where $Z_{1-\alpha}$ and Z_{β} are the standard normal probability points corresponding to $1-\alpha$ and β respectively. Note that $Z_{1-\alpha} = -Z_{\alpha}$. The reader is urged to refer to Figure 10 which illustrates all these relationships graphically.

The equations (11) and (12) can now be solved for n and K giving

$$n = \left(\frac{Z_{\alpha} + Z_{\beta}}{Z_1 - Z_2}\right)^2 \tag{13}$$

$$K = Z_1 - \frac{Z_{\alpha}}{\sqrt{n}} = Z_2 + \frac{Z_{\beta}}{\sqrt{n}} = \frac{Z_{\alpha}Z_2 + Z_{\beta}Z_1}{Z_{\alpha} + Z_{\beta}} \tag{14}$$

Example

Taking the previously used values. [19]

$p_1' = 0.01$

$p_2' = 0.08$

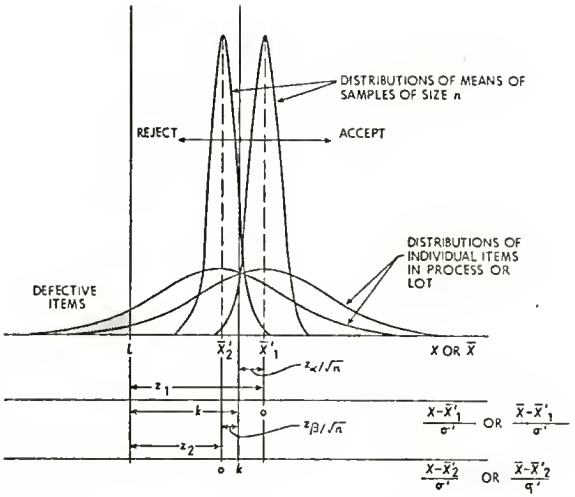


Figure 10. Illustrating the relationship between the z's involved in the design of a variables sampling plan (variance known)

Source: A. J. Duncan, Quality Control and Industrial Statistics.[19]

$\alpha = 0.05$

$\beta = 0.10$

Then the normal distribution tables give

$Z_1 = 2.3263 \qquad Z_2 = 1.4053$

$Z_\alpha = 1.6449 \qquad Z_\beta = 1.2816$

and eq (13) gives

$$n = \left[\frac{1.6449 + 1.2816}{2.3263 - 1.4053}\right]^2 = 10.1 \doteq \underline{10} \text{ (being a little lax)}$$

and

$$K = Z_1 - \frac{Z\alpha}{\sqrt{n}} = 2.3263 - \frac{1.6449}{\sqrt{10}} = \underline{1.806}$$

Note that this value of K will yield α exactly = 0.05; if β is to be maintained then use

$$K = Z_2 + \frac{Z\beta}{\sqrt{n}} = 1.4053 + \frac{1.2816}{\sqrt{10}} = \underline{1.811}$$

The complete O.C. curve can be obtained by using the following relationships

$$\mu_{p'} = L + Z_{p'}(\sigma') \tag{15}$$

$$P_a \text{ (Probability of acceptance)} = P\left[\frac{\bar{X}-\mu_{p'}}{\sigma'/\sqrt{n}} \geq (K-Z_{p'})\sqrt{n}\right] \tag{16}$$

Example

For the above plan the point corresponding to p' = .03 is determined as follows:

$$Z_{.03} \text{ from tables} = 1.881$$

which yields

$$(K-Z_{p'})\sqrt{n} = (1.809 - 1.881)\sqrt{10} = -0.2277$$

Using equation (16) with p' = 0.03, one finds that

$$P_a = P(Z > -0.2277) = \underline{0.59}$$

4.2.1.2 Formulation of a Single Sampling Variable Plan with σ' Known and with Specified α, β, p_1', p_2' (Double Specification Limits).

The first thing to note is that with the normal distribution, as with any other symmetric distribution, the minimum fraction defective for a process (or lot) will be obtained when μ is centered exactly at the middle of the specification limits, and if this minimum expected fraction defective is more than the maximum allowed then the lot may be rejected wthout any further consideration. In otherwords sampling is justified only when $(U-L)/2\sigma'$ is so large that if process were centered at the middle, there would be practically no defective material.

For widely spread specification limits ($U-L \geq 6\sigma'$ is a good criterion for this) the solution is simpler. A pair of single plans, each working at one of the specification limits, is all that is needed. The pair consists basically of one plan with two different rejection/acceptance criteria.[28] With specified values of p_1', p_2', α and β the corresponding single specification limit plan is the solution.

Example

Say U and L are wide enough to use the above approach and

$p'_1 = .01$

$p'_2 = .08$

$\alpha = .05$

$\beta = .10$

Then a single plan is devised as in section 4.1.2.1 giving $n = 10$, $K = 1.809$ which works as follows. Take a sample of size 10;

accept if $\frac{\bar{X}-L}{\sigma'} \geq 1.809$ and $\frac{U-\bar{X}}{\sigma'} \geq 1.809$, otherwise reject.

When two limits are not relatively far apart, say $U-L < 6\sigma'$, at the same time they are not so close that minimum defective possible exceeds the acceptable quality level.[29] In this case the fraction defective of the population will occur on both sides (below the lower limit as well as above the upper limit) and the population can be shifted and centered about such a mean as to give the total sum of proportion defectives as stipulated. In this case, as a rule more fraction defective will occur in one tail than the other.

Two single limit sampling plans can now be set up in such a manner that at one end (lower or upper limit), the probability of acceptance for an incoming fraction defective p'_{1t} (corresponding to this tail area) is equal to $1-\alpha$, while the probability of acceptance for a fraction defective of p'_2 (at the same end) is equal to β. Value of K may be determined by taking the average of the two values calculated at the upper and lower limits.[30]

Example [19]

Say $U = 0.880$ and $p_1' = 0.01$

$L = 0.8773$ $p_2' = 0.08$

$\sigma' = .005$ $\alpha = 0.05$

$\beta = 0.10$

From the normal tables it can be found that $p' = 0.01$ corresponds to $\bar{X}' = 0.878485$ or $\bar{X}' = 0.878815$. The distribution in both cases is such that fraction defective in one tail is 0.0089 while in other 0.0011.

Now find a single sampling plan so that

$p_1' = .0089$ $p_2' = 0.08$

$\alpha = .05$ $\beta = 0.10$

$$n = \left(\frac{Z_{0.05} + Z_{0.10}}{Z_{0.0089} - Z_{0.0800}}\right)^2 = \left(\frac{1.6449 + 1.2816}{2.3698 - 1.4053}\right)^2 = 9.2 \doteq \underline{9}$$

K is given by the average of

$$\frac{Z_{.10}}{\sqrt{n}} + Z_{0.08} \quad \text{and} \quad Z_{0.0089} - \frac{Z_{.05}}{\sqrt{n}} \quad \text{(from equation (14))}$$

i.e. $K = \frac{1}{2}\left(\frac{1.2816}{\sqrt{9}} + 1.4053 + 2.3698 - \frac{1.6449}{\sqrt{9}}\right)$

i.e. $K = \frac{1}{2}(1.8325 + 1.8215)$

i.e. $K = \underline{1.827}$

and the acceptance/rejection criteria becomes:

accept the lot if $\frac{\bar{X} - 0.8773}{\sigma'} \geq 1.827$ and

$\frac{0.8800 - \bar{X}}{\sigma'} \geq 1.827$ otherwise reject.

It is important to see that if procedure two was used then the fraction defectives expected in both tails could be added to give just one unified plan. This becomes a more satisfactory approach when σ' is unknown. [19][31]

The O.C. curve can be completely plotted just like in section 4.2.1.1.

4.2.2 Variable Sampling Plans with Known μ and Unknown σ'

The sample statistic used in this case is $t = \sqrt{n}(\bar{x}-\mu)/s$ where s (unbiased estimate of σ') is equal to $\sqrt{\sum(x-\bar{x})^2/(n-1)}$. This statistic follows the Student's t-distribution with n-1 degrees of freedom. This kind of plan is viable when quality assurance is needed for the population mean only.[32]

To begin with, some value for σ' has to be assumed (based on past performance or a small sample). Later on if the true value of σ' is larger than what was assumed for devising the plan then the plan will be more liberal in accepting low quality product (β will be more than the stipulated figure). [19]

The second difficulty is faced because of the fact that the t statistic depends on n which is also unknown. This is overcome by using the plot between P_a and $\lambda = (\mu_1' - \mu_2')/\sigma'$.[33] With specific values of n the plot can be easily made by using t statistic instead of Z for the sample mean. The same plot, once it is completed, can be used

to estimate n for different values of P_a and λ (μ_1 and μ_2).

To see a worked out example reader may refer to page 281-282. [19][34]

4.2.3 Formulation of a Variable Sampling Plan with Specified α, β, p_1', p_2'; Both μ and σ' Unknown (Single Specification Limit)

(a) Analytical approach - One good approximation is obtained by observing that if

$$X \sim N(\mu, \sigma'^2)$$

then $$\bar{X} \sim N(\mu, \sigma'^2/n)$$

and unless the sample size n is very small, say less than 5, it will be approximately true that

$$S \sim N\left(a\sigma', \frac{b^2\sigma^2}{2(n-1)}\right)$$

where $$a = \frac{\sqrt{\frac{2}{n-1}}\,\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$$

and $$b = \sqrt{2(n-1)(1-a)^2} \qquad [21]$$

Factors a and b are less than 1 and they tend to be equal to 1 as n increases. For this purpose one makes the approximation that $a = b = 1$ and $s \sim N\left(\sigma', \frac{\sigma'^2}{2(n-1)}\right)$. It can be proved that

$$E(ax + by) = aE(x) + bE(y)$$

and $$Var(ax + by) = a^2\,Var(X) + b^2\,Var(y)$$ when x and y are independent.

It has also been proved that if two variables follow some normal distributions then their sum too follows a normal distribution. With the help of the above facts the probability distribution of $\bar{X} + Ks$ is indicated as follows:

$$\bar{X} + Ks \sim N\left(\mu + K\sigma', \sigma^2 \left(\frac{1}{n} + \frac{K^2}{2(n-1)}\right)\right) \qquad [21]$$

The probability of acceptance

P_a = Probability that $\bar{x} + Ks$ in a sample will be less than U (upper limit)

$$= P\left(Z = \frac{U - (\mu + K\sigma')}{\sigma'\sqrt{\frac{1}{n} + \frac{K^2}{2(n-1)}}} \leq -Z_\varepsilon\right) \qquad (17)$$

Where ε is the probability of being less than Z. Equation (17) can be written as an equality

$$\frac{U-\mu}{\sigma'} - K = -Z_\varepsilon\sqrt{\frac{1}{n} + \frac{K^2}{2(n-1)}} \qquad (18)$$

Note that $(U-\mu)/\sigma$ is a standard normal deviate which will be exceeded with a specific probability giving p' proportion of defectives thus equation (18) can be written as

$$K - Z_{p'} = Z_\varepsilon\sqrt{\frac{1}{n} + \frac{K^2}{2(n-1)}} \qquad (19)$$

Squaring both sides and rearranging

$$\left(1 - \frac{Z_\varepsilon^2}{2(n-1)}\right) K^2 - 2 Z_{p'}K + \left(Z_{p'}^2 - \frac{Z_\varepsilon^2}{n}\right) = 0 \qquad (20)$$

Solving this quadratic for K it gives

$$K = \frac{Z_{p'} \pm \sqrt{Z_{p'}^2 - \left(1 - \frac{Z_\varepsilon^2}{2(n-1)}\right)\left(Z_{p'}^2 - \frac{Z_\varepsilon^2}{n}\right)}}{1 - \frac{Z_\varepsilon^2}{2(n-1)}} \tag{21}$$

When α, β, p_1 and p_2 are specified it is easier to work with equation (19). Using the usual notations it leads to the following equations.

$$K - Z_1 = Z_{1-\alpha}\sqrt{\frac{1}{n} + \frac{K^2}{2(n-1)}} \tag{22}$$

$$K - Z_2 = Z_\beta\sqrt{\frac{1}{n} + \frac{K^2}{2(n-1)}} \tag{23}$$

Dividing (22) by (23) yields

$$\frac{K-Z_1}{K-Z_2} = \frac{Z_{1-\alpha}}{Z_\beta} = -\frac{Z_\alpha}{Z_\beta}$$

solving for K this gives

$$K = \frac{Z_\alpha Z_2 + Z_\beta Z_1}{Z_\alpha + Z_\beta} \tag{24}$$

Note this is the same weighted average as the one obtained when σ' was known (see equation (14)).

To simplify the solution for n, assume that n is large enough so that n-1 can be replaced by n and thus equation (22) can be written as[35]

$$K - Z_1 = \frac{-Z_\alpha}{\sqrt{n}}\sqrt{1 + \frac{K^2}{2}}$$

i.e. $$n = \frac{Z_\alpha^2}{(K-Z_1)^2}\left(1 + \frac{K^2}{2}\right) \qquad (25)$$

Similarly (23) leads to

$$n = \frac{Z_\beta^2}{(K-Z_2)^2}\left(1 + \frac{K^2}{2}\right) \qquad (26)$$

Once n and K are determined the probability of acceptance for any p' can be calculated, using equation (19) and thus the O.C. curve can be fully determined.[36]

(b) Graphical Approach: A graphical solution to the above equations can be obtained by constructing corresponding nomograms. One such set of nomographs has been constructed by Leo J. Jacobson and given in his article "Nomograph for Determination of Variable Inspection Plan for Fraction Defectives". Industrial Quality Control, Nov. 1949.

The use of these nomograms is simple but they seem to give results which have upward bias in n and downward bias in K. [21][37]

For double and sequential plans reader may refer to the literature. References are included.

One important point to be noted is that with variable sampling, group sequential sampling or multiple sampling hardly offers any advantage over sequential sampling because, unlike attribute plans, a decision is possible after each measurement. Hence no multiple sampling plans with variable sampling exist.

CHAPTER V

SELECTED ASPECTS OF SAMPLING

This chapter is intended to throw some light on those important aspects of sampling which are not to be treated in such a detail as to deserve separate handling.

5.1 INDEXING AND CLASSIFICATION CRITERIA FOR SAMPLING PLANS

An indexing scheme is essential for any standardized set of plans. It is not only necessary for reference and search for the most suitable plan but also helpful in maintaining an optimal level of some useful criterion such as AOQL etc. The common criteria for indexing are listed below.

5.1.1 Points of Interest on the O.C. Curve

One point on O.C. curve is sufficient for indexing. Three points on the O.C. curve most commonly used are:

(a) AQL: Producers Risk - As previously defined, this is the point a producer is interested in. It is common to designate this point as $p_{.95}$ (i.e. the probability of acceptance at AQL is commonly 0.95).

Example: Plans developed by Statistical Research Group, Columbia University (The basis for MIL standards).

(b) LTPD: Consumers Risk - This is the point consumer is more interested in. Often designated as $p_{.10}$ it implies that 0.10 is a common value for consumer's risk.

Example: Dodge and Romig (1 set).

(c) Indifference Quality - Some times referred as 'Point of Control', it is designated as $p_{0.50}$. This is the point both consumer and producer can use.

Example: Philips Standard Sampling System. [30]

5.1.2 Average Outgoing Quality Limit (AOQL)

The most logical choice for some experts, especially in view of the fact that this gives the overall effectiveness of a plan independent of incoming quality.

Example: Dodge and Romig (1 set)

5.1.3 The Ratio LTPD/AQL (p_2'/p_1')

When used for indexing has one distinct advantage that the plans can be tabulated in a very concise form.

Example: Army Chemical Corps' Master Sampling Plans for Single, Duplicate, Double and Multiple Sampling. [21]

The term classification is sometimes used for indexing. There are several ways for classifying plans. They may be classified on the basis of their indexing i.e. (i) AQL, (ii) LTPD, (iii) AOQL, etc. Another way to look at all plans is on the basis of the basic nature of measurement used i.e. (i) Variable (ii) Attribute.

Another way to classify is on the basis of number of samples involved i.e. (i) Single Sampling (ii) Double Sampling (iii) Multiple Sampling (iv) Sequential Sampling.

Yet another way to classify the plans is on the basis of the kind of quality control the plans are supposed to provide (the purpose of the

plans) i.e. (i) Fraction defective assurance plans (ii) Mean or Standard Deviation assurance plans (iii) Rectifying Inspection Plans.

There are still more ways of classification e.g. Lot by Lot/ Continuous Production plans. The aim here is to let the reader see the possibilities.

5.2 COMPARISON OF SINGLE SAMPLING, DOUBLE SAMPLING, AND SEQUENTIAL/ MULTIPLE SAMPLING

The comparison is made in Table 4. The 'law of diminishing returns' starts operating when we go for multiple sampling. Double sampling probably offers the best compromise between administration and inspection costs. This has been proved in practice over the years. [17]

5.3 COST OF SAMPLING

This is basically two fold (i) Administrative Cost: Including training and establishment of personnel, equipment, overheads. Usually these costs are fixed. (ii) Inspection Cost: The cost of inspecting all the items to be inspected under a plan. The cost of getting a sample is some times included in this category and some times in the previous one depending on a particular situation.

These costs usually depend on

(a) Average amount of inspection

(b) Number of samples inspected

(c) Maximum amount of inspection

(d) Variability in amount of inspection. [25]

Sampling inspection of course introduces consumer's as well as producer's risks. Consumer's risk is two fold

TABLE 4

Comparison of single, double, and sequential/multiple sampling.

	Single sampling	Double sampling	Sequential/ multiple sampling
Amount of inspection:	For the same OC curve, these plans usually entail maximum inspection.	Amount of inspection is much less than single sampling. * Usually 10-50% less, compared to single sampling.	Usually amount of sampling is least here. (See Figure 8). Usually 30% less than double sampling.
Sampling:	Sample is taken just once. Fixed costs for sampling are incurred once only.	Not only is the fixed cost of sampling repeated but also it may be difficult to go back (in some instances) for a second sample. *	These plans pose maximum difficulty in obtaining samples.
Administration:	These are the best; easiest for administration.	In between.	Worst for administration purpose. Explanation and operation is much more complicated.
Cost:	Administration cost is lowest while inspection cost is higher.	Frequently, the total cost - inspection plus administration is optimal (least) in this case.	The increased administration costs may outweigh the savings in inspection for these plans.
Psychological advantage:	These are simple and easily understood but give just one chance for the lot to be accepted.	These plans never reject with just one defective, at least two defectives are needed for rejection. Moreover, frequently a second sample is drawn when first turns out to be a poor one. This makes the producer happy despite the fact that the degree of protection for consumer remains same.	Here the consumer may start feeling that he is giving chance after chance to the producer and therefore he (consumer) is more liable to receive bad lots. These plans are also open to criticism as being indecisive.
Control chart information (information about prevailing level of quality)	These plans give maximum information for control charting.	These plans need curtailment of inspection in the second sample, if the advantage of double sampling is to be fully reaped. Less data is thus available for control charting. Moreover, the data is biased because lots with inferior quality face curtailment of inspection more often than lots with better quality.	These plans obviously give the least amount of data for control charting.

*The deterioration of submitted (incoming) quality causes a very rapid increase in the number of items inspected per lot, often to a number well above than that for single sampling. This may cause a peak load well beyond the effort of regular inspectors thus causing a bottleneck [6].

(a) That a bad lot will be sumbitted

(b) The inspector in conjunction with the plan will pass it.

The former can not be so easily reckoned with while the latter can be. [17]

The sampling plan selection and its related costs will depend on the amount of these risks involved.

This in turn depends on the seriousness of the defect. What kind of loss is expected if a defective item is encountered (consumer) and how much can the rejection of a good item be tolerated (producer). How much is the discrimination power (steepness of O.C. curve) worth? A more discriminating plan will also exert greater pressure on the producer to maintain adequate standard.

One important point to note here is that discriminating power basically depends only on number of items in a sample while the inspection cost depends on percentage of items inspected. Larger lots can therefore be submitted to more discriminating plans without increasing sampling costs. This is intuitively appealing as well. Making a wrong decision for a larger lot is more serious.

Optimization techniques can be used; the current literature has an account of the approaches used. We can mathematically formulate all these considerations as shown by Freeman et al. [25], but to be of any practical importance we should be able to incorporate the actual costs and their distributions into the mathematical model. It is not easy to compute costs for expected risks.

Individual cases merit separate consideration. Both theoretical and practical aspects are to be considered. Overall cost and application under hurried shop conditions is of prime importance. "The most advantageous plan selection will often be determined on the proving ground of experience". [17]

5.4 GUIDLINES FOR SELECTING AND INSTALLING A SAMPLING SYSTEM

The following are the factors and steps usually to be considered. The list is not all inclusive; specific situations would demand special attention but this should be handy as a guideline.

(i) Decide what characteristics are important and should be inspected for.

(ii) Decide whether to include them at one or more inspection stations.

(iii) Decide whether attribute and/or variable measurements can be made for inspection.

(iv) Determine if production will be submitted in lots or as a continuous stream.

(v) Determine if 100 percent inspection of rejected lots is feasible or not.

(vi) Decide the type of protection desired e.g. AOQL or AQL or LTPD etc.

(vii) Determine the level of protection required.

(viii) Make a tentative selection of the desired plan. Try and pick out more than one type of plan, if possible. Check the O.C. curves to see that the plan is satisfactory for the level of protection required.

(ix) Try to compare and evaluate the prospective plans by estimating the administrative and sampling costs involved. Make the final selection.

(x) Follow the recommended procedure.

(xi) Keep a running check on the quality of incoming product and the performance of the plan.

(xii) Make adjustments whenever necessary.

The informed reader of course knows that it is more easily said than done. Much work has been done to facilitate the decisions (the bibilography lists some good references) yet much subjective judgement is involved. Only common sense, expertise and experience together can give the best practical solution.

A comparison of common published plans is given in the appendix for quick reference.

5.5 COMMON PITFALLS AND PRECAUTIONS

5.5.1 Sampling Inspection as a Substitute of Process Control

The most common mistake is to use the sampling inspection as a substitute for operating process control. While sampling inspection does give some information about the quality of the material, it can never be a substitute for process control and engineering.

5.5.2 Arbitrary Modification of Sampling Plans

Another common mistake is to modify the sampling plans without understanding the principles behind them. In order to get the proper protection, the specified procedure should be faithfully carried out. If a situation warrants, custom tailoring can be done but only with the help of the proper techniques.

5.5.3 Estimation Based on Invalid Data

Some sampling plans explicitly require an estimate of process average (incoming average lot quality) others incorporate this more implicitly. This knowledge is always helpful and the usual source is from the past data. The precaution here is that no average or distribution is helpful for any kind of estimate or prediction unless the process giving rise to the data is under statistical control. The correct way to this is by a control chart. Engineers frequently fail to make this check. [14]

5.5.4 Representative Sample

Not much has been said about it. Some plans explicitly state how the sample shall be picked up while others presume that sample will be picked up at random, which means that each item in the lot is given an equal chance of being selected for inspection. How exactly a random sample shall be selected? There are quite a few approaches and controversies too. Use of a random number table is as good as any.

Practical aspects such as selection of a sample from a huge lot, cost and effort may act as crippling restraints. Instead of a completely random sample a stratified sample may be taken. Whatever is done, the implications should be clearly understood. How much of a bias and increased risks can be afforded to save sampling costs? Many quality conscious companies specify detailed random sampling techniques to avoid personal biases. [14]

5.5.5 Assumptions Underlying Various Plans

For simplification and for the purpose of getting practical solutions, assumptions are made at almost every step of sampling. These include the presence of infinite populations, random sampling, probability distributions etc. Some of these assumptions are better justified and do not affect the results as much as others. Quite a few attempts have been made and solutions have been suggested to get better results under different situations. Specifically it should be mentioned here that the normality assumption underlying the variable sampling plans has been open to the maximum of criticism. Some feel that the plans are robust and adequate [61] while others point out the vast difference when normal distribution is far from the real situation, possible solutions have also been suggested [15,28,53,64]. Tests have been suggested to check for normal distribution, randomness, and homogenity. [4,6] The need for care and expertise for operating a good sampling procedure can not be overstressed.

CHAPTER VI

VARIABLE VERSUS ATTRIBUTE PLANS

Attribute plans were the first to be developed and thus used extensively but variable plans are worthy of much more attention than has been given to them in the past. Following is a brief comparative discussion of the advantages/disadvantages and applications of the two types.

6.1 ADVANTAGES OF VARIABLE PLANS

6.1.1 Smaller Sample Size

In variable sampling the full measurement of the quality characteristic is used for the decision while in an attribute plan an item is classified either as defective or nondefective. Intuitively it is obvious that instead of just knowing that a part is defective, it is far more informative to know in quantitative terms, how badly it is defective. Frequently the observations are first collected as continuous variates (go or no go gauges are not so frequently feasible as one may think) and then converted into a discrete kind. As per the Information Theory the compression of a two parameter distribution, such as normal distribution, into one parameter distributions, such as binomial and poisson distribution, results in considerable loss of information and moreover this process is irreversible [50].

For the same sample size, a variable plan will give a higher degree of protection; in other words for the same degree of protection variable

plans will need a smaller sample. Following is a comparison of average sample size for the different kind of sampling plans having approximately the same O.C. curve (the values for α, β, p_1' and p_2' are same as the ones, which have been used till now in the text). [19]

$p_1' = 0.01$

$p_2' = 0.08$

α 0.05

$\beta = 0.10$

Type of Sampling Plan	Average Sample Size
1. Single sampling by attributes	$n = 67$ ($\alpha = .03$)
2. Double sampling by attributes	ASN $\doteq$ 45 (at p_1')
3. Multiple sampling by attributes	ASN $\doteq$ 41 (at p_1')
4. Item by Item SPR sampling by attributes	ASN $\doteq$ 38 (at p_1')
5. Single sampling attribute plan using compressed limits, σ' known	$n = 31$
6. Single sampling by variables, unknown σ', average range method	$n = 35$
7. Single sampling by variables, unknown σ', standard deviation method	$n = 27$
8. Single sampling by variables, σ' known	$n = 10$

When σ' is known, the savings can be as high as 90 percent. [50] The minimum saving is about 30 percent. [6] When the inspection is very expensive or complicated and especially when destructive testing is used this advantage would override all the objections inherent in variable sampling.

6.1.2 More Useful Data for the Control Charts

Any kind of sampling would provide some useful data to keep a running check on the incoming quality but variable sampling data results in $\bar{X}$ and σ control charts compared to p charts for attribute sampling data. The computations are already done and unquestionably $\bar{X}$ and σ charts are more effective. The overall picture is clearer.

6.1.3 Elimination of Personal Bias

This minor advantage may actually prove to be quite useful in proving the fairness of a sampling program and thus maintaining good consumer-producer relationships without compromising the quality in any way. In attribute sampling the acceptance and rejection numbers are small. Sometimes the acceptance or rejection of an entire lot may well depend on just one marginal (defective or non defective) item. The inspector would be under a stress and may well accept the marginal defective item to avoid any controversy (also it is usually not possible to make distinct qualitative judgements). With variable plans such a situation does not exist.

6.2 DISADVANTAGES OF VARIABLE PLANS

All is not rosy. There are limitations. To make the right decision one needs to weigh the pros and cons discussed below.

6.2.1 Variable Plans Need Better Administration

Generally variable inspection needs more skill (expertise), time and expensive equipment. Training for the staff has to be more extensive. Variable inspection plans need more computations and

record keeping. Moreover the selection and installation of a variable plan usually needs more effort. [21] It may however be noted that the arithmetic may be cut down by using graphs and tables. [6]

6.2.2 Call for More Stringent Assumptions

A probability distribution (usually normal) has to be assumed. Perhaps most of the Industrial Quality characteristics do follow normal or quasinormal distributions, but still there can be some bad exceptions and care is needed. On the other hand it has been shown that when $n \geq 4$ the normality assumption is not seriously affected. [53]

It should be noted here that although a quality characteristic follows a continuous normal distribution, individual observations are essentially discrete, and have to be grouped together into discrete classes. If these intervals are less than ten in number, the effect of discontinuity may impair the usefulness of variable plans. [6] The increased number of intervals need better precision in measurement and higher cost. Twenty is a reasonable number of intervals. [6]

6.2.3 Separate Plan for Each Characteristic

When a product needs to be inspected for more than one quality characteristic, just one attribute plan is sufficient because the product is to be judged as defective or non defective and the criteria for such judgement has nothing to do with the plan. Not so for the variable plan. One can not add the measurement of two quality characteristic, and come up with a sensible parameter. Every such characteristic would need a separate plan and separate control limits.

This not only is costly and cumbersome but also infeasible in some situations.

6.3 SCOPE AND APPLICATIONS

There are situations where no variable measurement can be made, for example one cannot measure the degree of unworthiness of pinhole (uncoated spot) on a tin plate and thus present no possibility for variable plans. There are quite frequently situations, as mentioned earlier, where a defective can not be judged directly without first making a measurement on the continuous scale, for example the resistance of an element, and to forgo the use of variable plans in such cases would plainly mean the wastage of a excellent opportunity to cut down costs.

Like the attribute plans these can be used at any stage of production a) raw materials b) in-process goods c) finished products. The outgoing or incoming products all can be subjected to these plans.

The field of application is wide; quality characteristics such as mechanical properties - tensile strength, impact strength and ductility etc., weight, electrical characteristics, chemical properties, dimensions are the excellent qualifiers.

Variable plans can be more effective when the lot quality is expressed in terms of the standard deviation or the mean but percent defective is more acceptable (attribute plans are developed on this basis only) and as such good variable plans now exist to control percent defectives (MIL-STD-414).

The choice should finally be made primarily on the economics of the plans. The reader may find the work of K. Stange, "Comparison of Costs of Inspection by Variables and Attributes" [61], useful.

CHAPTER VII

MATCHING OF AN ATTRIBUTE SAMPLING PLAN WITH A VARIABLE SAMPLING PLAN

One of the two aims of this report is to encourage the practitioner to use variable plans whenever it is possible and result in considerable savings. Thus a good comparison is needed. Acheson J. Duncan says "Meaningful comparisons are only made between plans that have essentially the same O.C. curve". [19] In practice this means that when two plans offer the same protection, it is possible to evaluate them comparatively by comparing their respective costs. It is essential to facilitate a good practical decision. It is difficult enough to specify costs of sampling but to specify costs for different risks involved has to be subjective and erratic, to say the least.

Now, what is meant by the O.C. curves being "essentially the same"? By now one is well aware of the fact that mathematical functions of different types of plans are not the same and the two curves will not coincide at every point. Several reasonable approaches have been suggested.

(a) Match curves at the p_1' (AQL) and p_2' (LTPD) and then assume that other parts of the curve are not so different.

(b) Match the $p_{.50}$ (indifference quality) point and the relative slopes of the curves at this common point.

(c) Match the points of inflection and slopes at these points. [19]

Basically all three approaches intend to make sure that at least the matching is good at the most important points. For a good match

at p_1' point and the classification of certain type of plans, the last two methods are very useful,[38] but the former is probably the simplest and most common method used.

With this method we have two choices (i) match at just one point, either p_1' or p_2' and adhere to some pre-assigned sample size (n), (ii) match at both p_1' and p_2' as well as possible. Obviously much better matching is achieved by matching at two points instead of one.

It may be recalled that due to the discrete nature of sample size, acceptance and rejection number, a perfect match is not possible, as a rule, even at one point.

To facilitate this matching, the formulation of all plans in Chapter V was done by indicating preassigned values of p_1', p_2', α and β. The procedures described were mostly based on some simplifying assumptions. In view of bigger sampling errors, other inaccuracies and faster solutions, all of those procedure can be used effectively.

In addition, this chapter gives brief description of more accurate methods for the more conscientious.

Again there are three possible practical approaches:

a) Visual trial and error search

b) Analytical (mathematical) Method

c) Graphical Method

For the sake of completeness all of these are described briefly.

7.1 VISUAL TRIAL AND ERROR SEARCH

This is simply the technique of trying to match an attribute plan to another existing variable plan by trying to match their O.C. curves. If the O.C. curves are drawn on the same scale one can use a transparent copy of the desired O.C. curve and superimpose it on the prospective curves. Usually, while trying to match at just one or two points, the values can be read at those points and the best match can thus be found. Obviously one has to spend time in search and sometimes it may be futile. It may be used where accuracy desired is not great.

It may be mentioned here that quite a few of the published attribute plans already have matching variable plans, specifically MIL-STD-105A and MIL-STD-414. MIL-STD-414 are in the process of being revised to match more closely to the latest D version of MIL-STD-105. [5]

7.2 ANALYTICAL METHOD

The methods described in Chapter V for formulating a variable plan when both μ and σ' are unknown, used some sort of normal approximations. The exact valid distribution is the non-central t-distribution. This gives the best accuracy but the computations are much more involved.

The acceptance criterion

$$\bar{X} + KS \leq U$$

can be written as

$$\frac{U-\bar{X}}{S} \geq K$$

For determining the proper value of K, knowledge is needed about the

probability distribution of $(U-\bar{X})/S = t$ (say), which is in the form of a non-central t-distribution. The functional form is given as:

$$p(t|f,z_p) = \frac{f!\, f^{f/2}}{2^{f-\frac{1}{2}}\, \Gamma(f|2)} \frac{1}{(f+t^2)^{(f+1)/2}} e^{-[f(f+1)z_p^2]/[2(f+t^2)]}$$

$$\cdot Hh_f\left[-\frac{t\sqrt{f+1}\; z_p}{\sqrt{f+t^2}}\right]$$

where $Hh_f(x) = \int_0^\infty e^{-\frac{1}{2}(v+x)^2} \frac{v^f}{f!}\, dv$

and $f = n-1$ and z_p is the standard normal deviate. [6] Extensive tables have been developed for this distribution by Johnson and Welch. [6] Non-central 't' requires a triple entry because the probability that t exceeds a given value t_0 depends on f (number of degrees of freedom), δ (eccentricity) = $\sqrt{n}\, z_p$, and t_0. This probability is denoted as $P(f,\delta,t_0)$. $t(f,\delta,\varepsilon)$ is used to denote that value of t_0 for which $P(t,\delta,t_0) = \varepsilon$. $\delta(f,t_0,\varepsilon)$ is that value of δ which makes $P(f,\delta,t_0) = \varepsilon$.

Detailed instructions for using the tables are included and computations are also explained by Eisenhart et al. [21]

The steps needed to use these tables for the problems encountered during the formulation of a plan are given below.

Problem I

Given: n, p' (p_1' and p_2'), $\varepsilon(1-\alpha$ or $\beta)$

Required: K

Step 1. Determine z'_p, the normal standard deviate exceeded with probability p' from the normal distribution tables.

Step 2. Calculate f = n-1

$$\delta = \sqrt{n}\, z_{p'}$$

Step 3. Calculate t (f,δ,ε) from the non-central t-distribution tables.

Step 4. Calculate $K = t(f,\delta,\varepsilon)/\sqrt{n}$

Problem II

Given: n,K,t ($P_a \rightarrow$ Probability of acceptance)

Required: p'

Step 1. Calculate

$$t_0 = \sqrt{n}\, K$$

$$f = n-1$$

Step 2. Calculate $\delta(f,t_0,\varepsilon)$ from tables.

Step 3. Calculate $Z = \delta(f,t_0,\varepsilon)/\sqrt{n}$

Step 4. Obtain the required p' from normal distribution table corresponding to the Z calculated above.

Problem III

Given: p'_1, p'_2, α and β

Required: n and K

Step 1. Determine $Z_1, Z_2, Z_\alpha, Z_\beta$. $(Z_1 = Z_{p'_1},\ Z_2 = Z_{p'_2})$

Step 2. Calculate first approximation to K from

$$K = \frac{Z_\alpha Z_2 + Z_\beta Z_1}{Z_\alpha + Z_\beta}$$

Step 3. Approximate n from

$$n = \frac{K^2+2}{2}\left(\frac{Z_\alpha+Z_\beta}{Z_\alpha-Z_\beta}\right)^2$$

Step 4. Determine the fraction defectives $p'_{1-\alpha}$ and p'_β i.e. the fraction defective whose probability of acceptance is $1-\alpha$ and β respectively.

If these are sufficiently close to p_1 and p_2 then k and n calculated in step 2 and 3 give the required plan. Otherwise go on to step 5.

Step 5. If the discrepency in step 4 is large then determine a better plan by adjusting K or n as per the following rules.

a) If $p'_{1-\alpha}$ is greater than p'_1 and p'_β is less than p'_2, take the next lower integer for n and the same K, repeat step 4.

b) If $p'_{1-\alpha}$ is less than p'_1 and p'_β is greater than p'_2, take the next higher digit for n and the same K and repeat step 4.

c) If $p'_{1-\alpha}$ is less than p'_1 and p'_β is less than p'_2, use same K and a smaller value of K, a better value for which can be calculated as

$$K = \frac{\lambda_1 Z_2 + \lambda_2 Z_1}{\lambda_1 + \lambda_2}$$

where λ is such that

$$\delta(f,t_0,\epsilon) = t_0 - \lambda\sqrt{1+\frac{t_0^2}{2f}}$$

and λ_1 and λ_2 are values calculated in step 4 while calculating $p'_{1-\alpha}$ and p'_β respectively. Repeat Step 6.

d) If $p'_{1-\alpha}$ is greater than p'_1 and p'_β is greater than p'_2 use the same n and a larger value of K, which can again be calculated just as in rule c, and repeat step 6.

Step 6. For the required values of n and K take that pair of values for which $p'_{1-\alpha}$ and p'_β are closest to p'_1 and p'_2 respectively.

Tables have been formed with the help of this procedure, giving values of n and K for various combinations of p'_1 and p'_2 for $\alpha = 0.05$ and $\beta = 0.10$. [21]

For the more initiated the computer solution can provide the ease, flexibility and accuracy needed.

Presumably the programs have been developed for the complete procedure. [74]

A computer subroutine for evaluating the non-central t-distribution is readily available [13]; the rest of the program can be developed by following the steps outlined above.

7.3 GRAPHICAL METHOD

The graphical solution of the equations has already been once briefly described in Chapter V. The aim of such nomograms is to provide a fast solution; the accuracy however is limited by the graphical accuracy and the approximations used. Probably the best nomograms available have been developed by P. Th. Wilrich. [79,80] The equations used for nomograms have been developed from the non-central t-distribution. For ready reference the three nomograms (i) known σ' (ii) unknown σ' - based on s (iii) unknown σ' - based on $\bar{R}$, have been included in the appendix. Basically the nomogram used here is a binary field (n,K)

in the middle of two probability scales (left hand scale — percentage defective and right hand scale for probability of acceptance). The usage is simple. The four points p_1', p_2', α and β give two straight lines (with relevent pairing) on the plot. Their intersection gives the plan (n,K). Refer to the figures given in the bottom right hand corner of the nomograms, in case of doubt. The OC curve for the plan (once determined) can be read off the same plot easily.

These nomograms presumably give quite accurate results. In the opinion of P. Th. Wilrich, these nomograms offer the best kind of solution, very fast, and accurate enough for most practical purposes, so that a man on the shop floor can afford to have a custom fit variables sampling plan.

Hopefully more extensive nomograms will be developed in future to offer still more choices.

FOOTNOTES

[1]For comprehensiveness, the philosophy of 'Zero Defect' should be mentioned here. [44] In view of the space age technology and critical defense requirements, this has its own place.

[2]The advantages of including a few characteristics (those subject to same inspection operations) outweigh the disadvantages usually. Procedures are simplified, each characteristic gets more attention and can be better controlled. [17]

[3]The selected bibliography at the end of this report includes references which will help in answering all such pertinent questions.

[4]There is lot of controversy about the usefulness of Bayesian work. The subjective nature of the analysis has been intensively challenged.

[5]Based on Bayes Theorem.

[6]Most of the history can be followed in the Bell Telephone Technical Journal.

[7]A summary of published plans is given in the appendix.

[8]Note that it is not essentially a shipping lot.

[9]How does one determine that units are similar? General Simon states that small sub groups of sample items will respond to Shewart criterion of control when the items are "essentially alike". [30]

[10]The salient facts to be noted are as follows.

A steeper curve means more discerning power. A hypothetical ideal curve would be a Z curve, dipping at AQL (Figure 1).

Same proportion of sample size in different plans gives a different level of quality protection (Figure 4). Absolute sample size determines the level of protection (Figure 5).

A larger sample size means steeper slope and thus more discerning power (Figure 2).

The O.C. curve for a discrete sampling plan becomes steeper with decreasing of acceptance number (Figure 3).

The level of quality protection is very much dependent on incoming quality.

No sampling plan can give complete protection against acceptance of bad lots or rejection of good lots.

A defect free sample does not mean a defect free lot.

[11]See producers risk.

[12]See consumers risk.

[13]For proof, see appendixes of [24].

[14]Also see finite population co rection [16].

[15]To decide whether Normal or Poisson approximation is appropriate, reader may like to look at the chart on Page 60 of [17] in addition to the textbooks in statistics.

[16]Reader is cautioned to understand the notations before using any reference.

[17]A. R. Burgers, "A Graphical Method of Determining a Single Sampling Plan," Industrial Quality Control, May 1948.

[18]Note this roundoff to the next higher integer, this is always done so as to ensure better protection than what one was initially trying for.

[19]The reader may refer to Herman Burstein Attribute Sampling (Tables and explanation) [10] for short cut tables.

[20]One should be aware of the difference in O.C. curve for double sampling. The principal O.C. curve is what we will be talking about. See fig. 8.1 [19] or fig. 12-5 [30].

[21]For proof see Irving W. Burr, "Average Sample Number Under Curtailed or Truncated Sampling", Industrial Quality Control, February, 1957 1957, pp 5-7.

[22]For proof see appendix I (25) of [19]. For full details including the development of SPRT and its uses see Abraham Wald "Sequential Analysis", John Wiley & Sons, New York, 1947. [74]

[23]See appendix I (26) of [24] for proof.

[24]See Statistical Research Group, Columbia University, Sequential Analysis of Statistical Data: Applications, New York: Columbia University Press, 1945), p. 2.48.

[25]A. Wald & J. Wolfowitz, "Optimum Character of Sequential Probability Ratio Test", Annals of Mathematical Statistics, Vol XIX (1948) pp 326-39.

[26]See G. A. Barnard, "Sequential Tests in Industrial Statistics," Journal of the Royal Statistical Society, Ser. B. Vol VIII (1946), pp 1-21.

[27]A paper from Royal Statistical Society Statistical Method in Industrial Production (Printed for private circulation, 1951).

[28]The modification depends on whether the plan is to be used at the upper limit or the lower limit.

[29]It is important to note that the maximum fraction defective, in this context, is AQL.

[30]For complete understanding see pp. 232-33 of [19].

[31]Acheson J. Duncan mentions that a special attribute plan can save considerable cost when σ' is known. (p. 235 [19]). This alternative may also be considered while making the decision.

[32]See A. J. Duncan [19] for the difference in plans devised for assurance of (a) fraction defectives (b) mean values (c) standard deviation of the lot.

For proportion defective there are just two cases, namely (i) σ' known (ii) σ' unknown.

[33]The use of λ is similar to standard normal deviate z. The transformation makes λ unitless, independent of any particular value of $\bar{X}$ or σ' and hence one set of curves can be used for all possible distributions. For further detials see, J. Neyman and B. Tobarska, "Errors of the Second Kind in testing "Student's' Hypothesis," Journal of the American Statistical Association, Vol. 31, pp. 318-26.

[34]Whereever standard deviation (s) is used, the average range ($\bar{R}$) can be used effectively. It offers practical advantages with sufficient accuracy.

$$\hat{\sigma}' = \frac{\bar{R}}{d_2}$$

The factor d_2 has been well tabulated. [30]

Some sampling plans have been devised on the basis of $\bar{R}$. [19,79,80]

[35]If this approximation is not made then

$$n = \frac{b + \sqrt{b^2 - 16a}}{4a}$$

where $a = \left(\frac{z_1 - z_2}{z_\alpha - z_\beta}\right)^2$

and $b = K^2 + 2(a+1)$ [21].

[36]Statistical Research Group [21] gives tables for values of K and n for various p_1' (from 0.001 to 0.05) and p_2' (from 0.0015 to 0.40)

[37]In the last chapter the exact method with non-central distribution is given for matching (attribute and variables) purposes.

Much better nomographs are now available and are such set is included for reference, in the appendix.

[38]For details and relative advantages and disadvantages refer to H. C. Hamaker and Army Chemical Corps, plans.

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APPENDIX

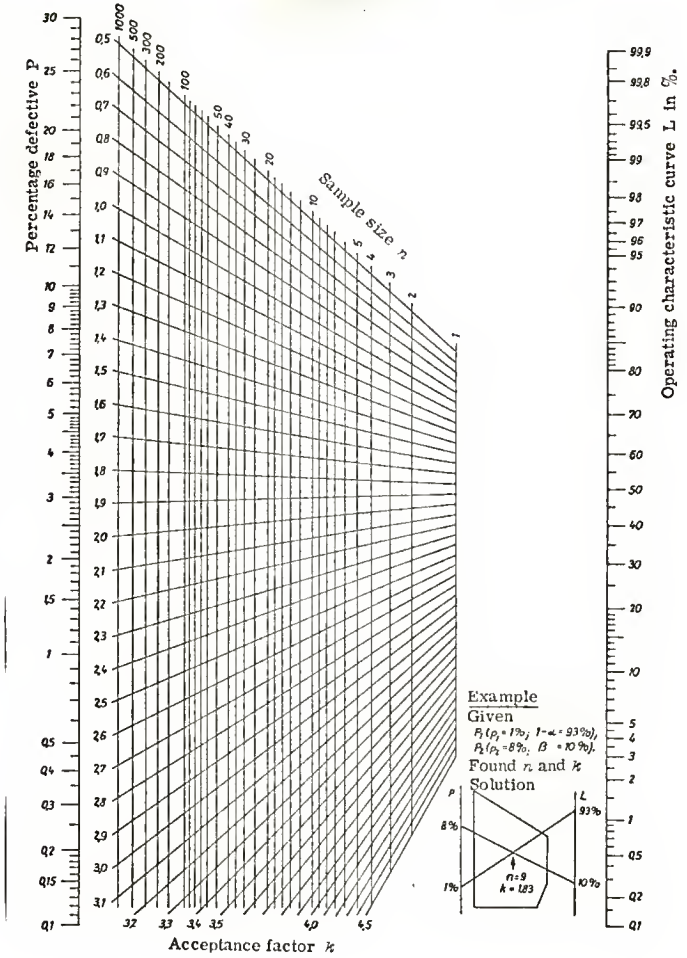
APPENDIX

SUMMARY OF PUBLISHED SAMPLING PLANS*

Name of plan	Type of plan	Type of sampling	Type of application	Key features
Dodge-Romig..	Attribute	Single and double	General application where rejected lots can be 100% inspected	One type of plan has a consumer's risk of 0.10 of accepting bad quality. A second type of plan limits the average quality level in the long run. Protection is provided with minimum inspection per lot
MIL-STD-105B	Attribute	Single, double, and multiple	General application where rejected lots cannot be 100% inspected	Maintains average quality at a specified level or better. Aims to minimize rejection of good lots. Tables and mechanics of operation are simplified to minimize training required to use plan
Chain sampling	Attribute	Single	Particularly useful when inspection involves destructive or costly tests	Aims to minimize rejection of good lots. Occurrence of a single defective does not necessarily cause rejection of lot
Discovery sampling	Attribute	Single	General application where probability of occurrence of defective lots can be estimated	Smaller sample size required as compared with standard attribute plans
Single level continuous sampling plans (H107)	Attribute	Continuous	Particularly useful where production is continuous and lots are not formed	Sampling inspection is instituted when a specified number of consecutive units are found acceptable. Sampling at a defined rate is continued until a specified number of defectives are found within a specified number of units; 100% inspection is then instituted and the cycle repeated
Multilevel continuous sampling plans (H108)	Attribute	Continuous	For continuous production but where the actual quality level is much better than the specified level	Provisions are included for reducing the sampling rate when inspection results warrant
MIL-STD-414 variables sampling plan	Variable	Single	Particularly useful when a measurement is to be made on one or only a few product characteristics and when distribution of characteristic is known	Sample size is smaller as compared with attribute plan having the same risks

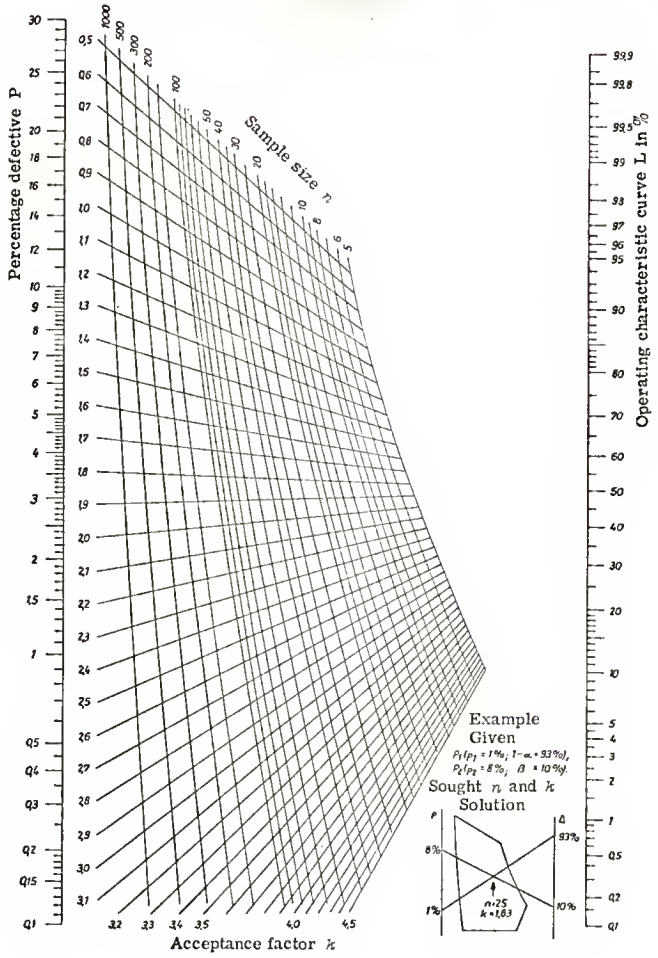
*Reproduced from J. M. Juran (Ed.), Quality Control Handbook. [39]

NOMOGRAM FOR DETERMINING ACCEPTANCE SAMPLING PLANS BY VARIABLES



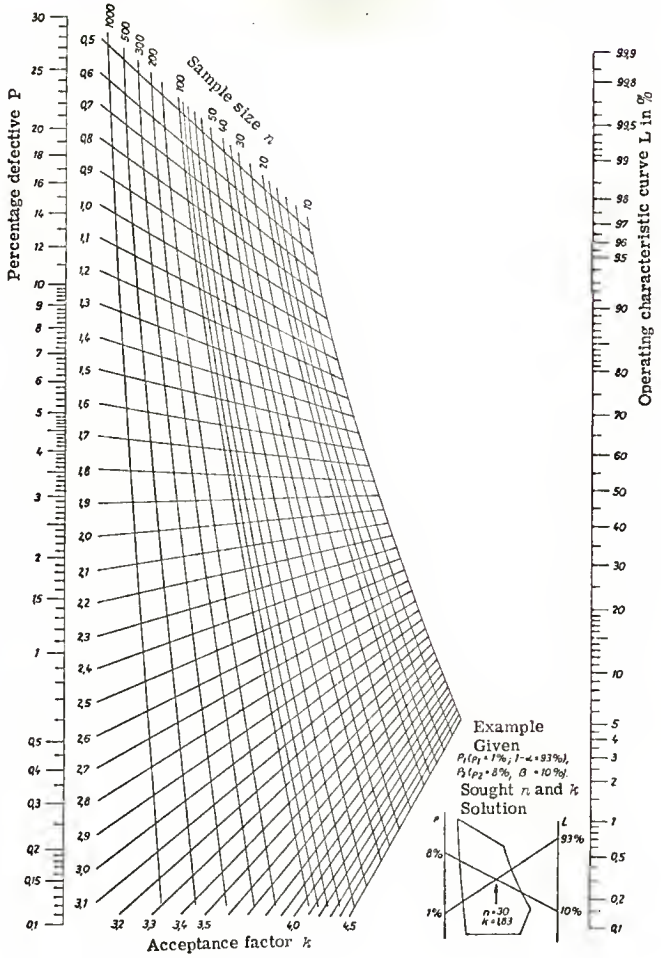
Source: P. Th. Wilrich, Qualitat und Zuverlässigkeit. [79]

NOMOGRAM FOR DETERMINING ($\bar{x}$,s) - ACCEPTANCE SAMPLING PLANS BY VARIABLES



Source: P. Th. Wilrich, Qualitat und Zuverlässigkeit. [80]

NOMOGRAM FOR DETERMINING ($\bar{x}$,$\bar{R}$) - ACCEPTANCE SAMPLING PLANS BY VARIABLES



Source: P. Th. Wilrich, Qualitat und Zuverlässigkeit. [80]

ACCEPTANCE SAMPLING: A FOCUS ON ATTRIBUTES VERSUS VARIABLES SAMPLING

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AN ABSTRACT OF A MASTER'S REPORT

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1976

ABSTRACT

The aim of this report is to provide a comprehensive and a simple guide to the practitioner of acceptance sampling.

The attempt has been to make an overview and cover briefly all the aspects of acceptance sampling including the design, selection and implementation of a sampling program. Special emphasis has been placed on the comparison between attributes and variables sampling, and the matching of a variables plan with an existing attribute plan, thus providing the practioner with an opportunity to use the oft neglected variables sampling and result in considerable savings whenever it is feasible.

An extensive literature survey has been made and sufficient references have been included for the practitioner as well as an interested student to pursue any aspect of acceptance sampling in details.