OPTIMIZATION OF ROUTES AND MODES OF NUCLEAR MATERIAL TRANSPORTATION CONSTRAINED BY SAFETY CONSIDERATIONS

by

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1.0 INTRODUCTION

The transportation of nuclear materials involved in the nuclear power industry can be understood by examining the nuclear fuel cycle which is illustrated in Fig. 1. The cycle begins with mining and preparation of fuels and ends with recovery from "spent" fuel elements of unused and newly-generated fuel materials. The central steps of the cycle concerns the fission process employed by the reactor plant to generate electricity. These plants are fueled by one or more of the fissionable materials uranium-235, uranium-233, or plutonium-239.

After sustained operation of the reactor, the fuel elements must be reprocessed since the fissionable material has been used to the extent that a chain reaction can no longer be maintained. During the fission process, the fissionable atoms are split releasing energy, and many fission fragments are often of unstable configuration and decay radioactively by emission of beta, gamma, or alpha rays. Since they are highly radioactive, the fuel elements are left to "cool" as the radioactivity decays exponentially with time.

After a sufficient cooling period, the spent fuel elements are packed in a shipping cask. The cask shielding generally is made of a dense material such as lead to attenuate the radiation. Shielding, heat transfer, impact fire damage and other design criteria demand a heavy and expensive shipping cask. Generally these casks are leased for costly rates.

Fissionable material is recovered from the depleted fuel elements at fuel-reprocessing plants. In this process, highly radioactive liquid and

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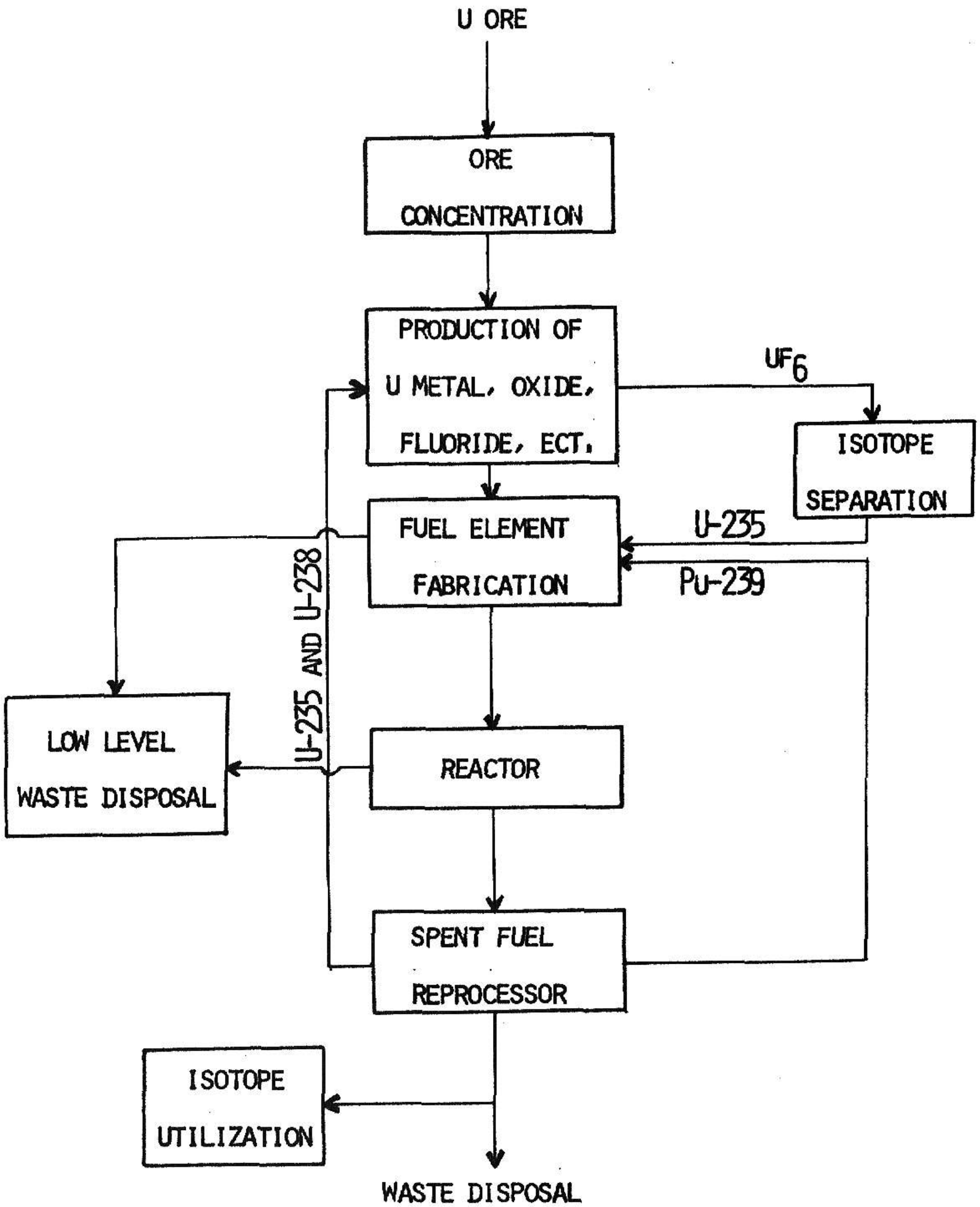

FIGURE 1 URANIUM-PLUTONIUM REACTOR FUEL CYCLE

solid wastes are produced. Currently, these wastes are stored in large underground tanks. Much consideration has been directed towards shipment of these wastes to nuclear waste burial grounds in salt or other geologic formation repositories.

In general, shipping casks are of standardized sizes at a weight allowing conventional modes of transportation shipment. A truck container may weigh between 12 to 15 tons, while a railroad container may range up to 120 tons or more. The heaviness of the casks is due almost completely to the shielding requirements. Considerable effort has been made to optimize shielding while minimizing handling costs.

It is speculated by Tremmel (1) that spent fuel transportation from the reactor to the reprocessing plant for the recovery of valuable uranium and plutonium fuels may reach such a magnitude as to hinder nuclear power industry progress. To understand this growing need for transportation, it is enlightening to examine the nuclear power industry growth. Figure 2 has become a familiar and accepted curve depicting the projected growth of the nuclear power industry (1). Not so familiar is the associated spent fuel discharge which clearly indicates an increasing need for spent fuel transportation within this decade. Figure 2 has been combined with the carrying capacity of different transportation modes to arrive at required cask movements in order to handle the anticipated transportation load, as shown in Figure 3.

Many different sizes of shipping casks may be used for different transportation modes. Presently, four transport modes seem a possiblity (truck, rail, air, barge) to handle this projected load.

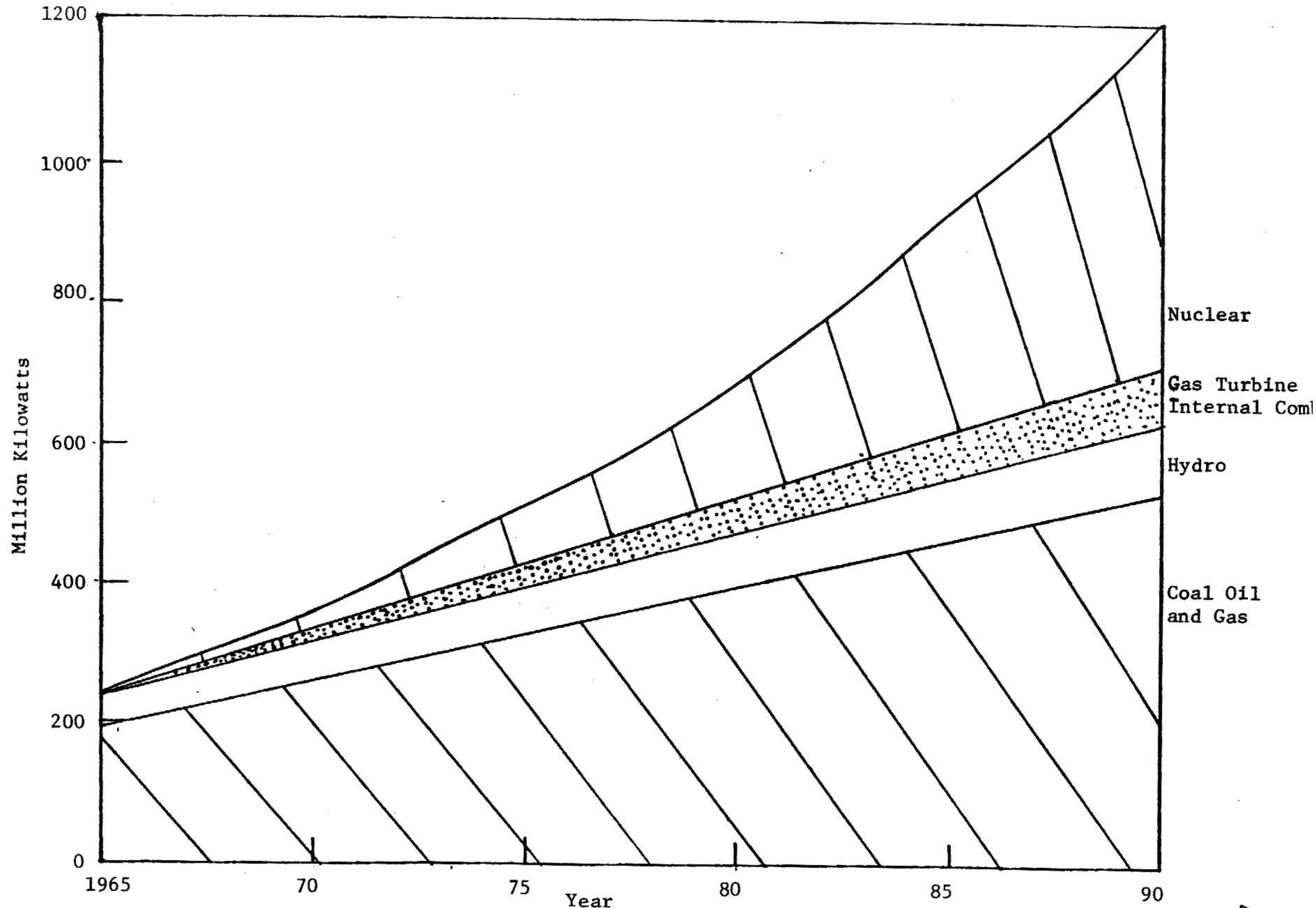

Figure 2 Electric Generating Capacity in the United States (11)

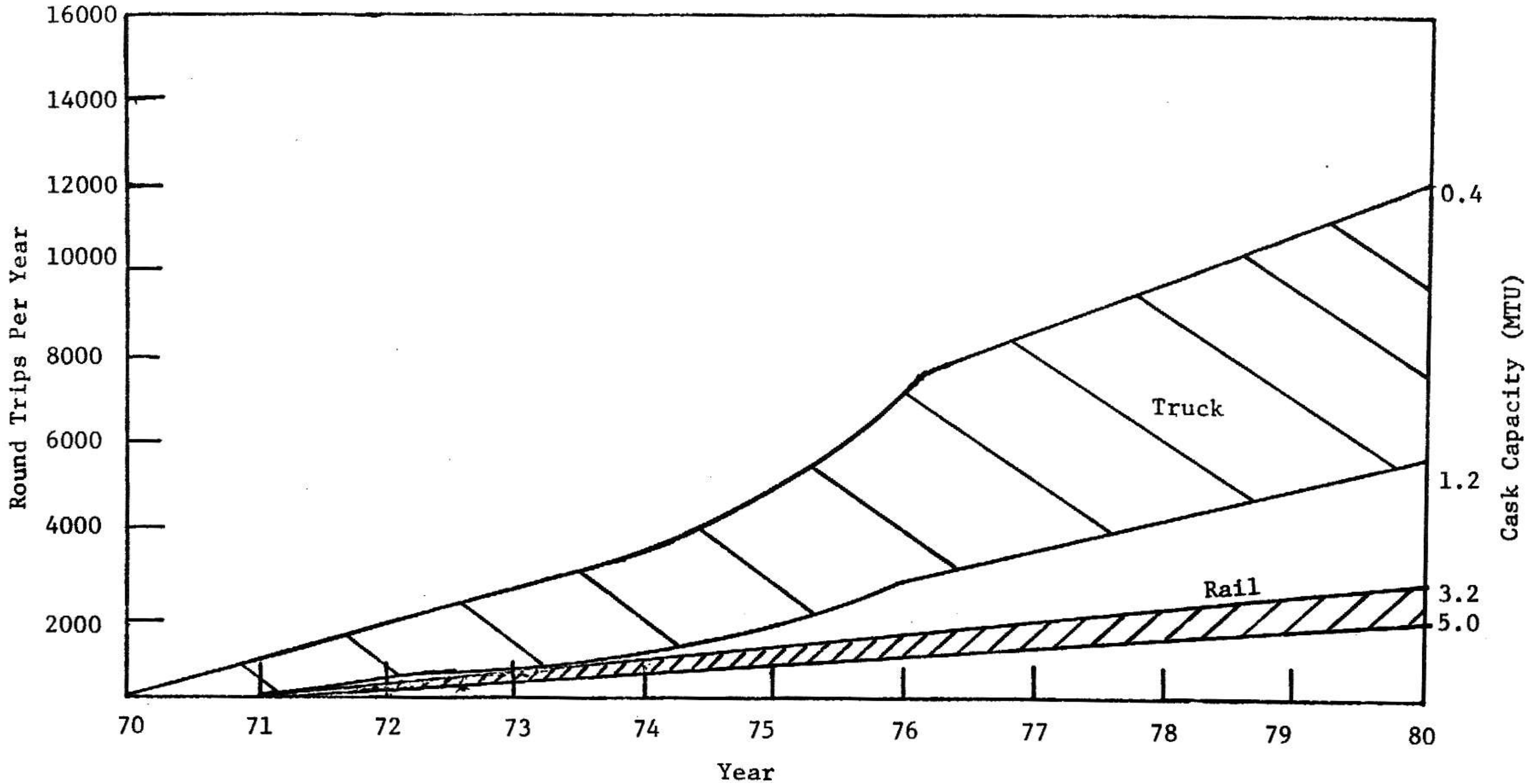

Figure 3 Required Spent Fuel Cask Movements Assuming Single Mode Shipment (1)

2.0 SHIPPING CONSTRAINTS

Associated with each mode of transportation and each reactor site are obvious factors such as accessibility of transportation, cost, weight restrictions and legal restrictions. Other not so obvious factors include safety considerations, transportation safeguards, insurance and indemnity. All these factors represent constraints on the shipment of the spent fuel. In order to understand more completely the effect that each of these constraints may have, they are examined in the following sections.

2.1 Accessibility

Accessibility may be limited at a reactor site due to geographical location of shipping and receiving facilities or mechanical capabilities of the particular reactor. For instance, some reactors do not have rail sidings necessary to ship the casks by rail. Although sidings could be constructed, often the cost of building a bridge or tunnel is prohibitive when compared with truck shipment. In addition, the cooling pond cranes are sometimes not capable of handling the heavier rail shipping casks (10).

Obviously the expense of building a canal would sometimes prohibit immediate access of barge transportation. Air runways can be built at a reasonable cost if VSTOL (Vertical and Short Take-Off and Landing) air craft ever become feasible; however, geographical terrain may eliminate the possibility of a landing strip. Also, closeness to populated districts may cause adverse public and government reaction.

2.2 Cost of Shipment

Of prime importance to the transportation considerations is cost. The cost of transportation depends not only on the mode used, but also on legal restrictions which apply along the transportation route. This problem is described by Dufrane and North (2). Taken from that paper are Figs. 4, 5, and 6 which develop a relative cost comparison for two overweight cask configurations under various highway restrictions. Freight costs have been excluded since the relative results are not affected.

Figure 4 represents the number of trips per week that could be anticipated for a cask traveling a 1000 mile trip between a power utility's reactor and the fuel reprocessor. A cask turnaround of 24 hours was assumed at both the reactor and reprocessor sites along with a 24 hour day, 7 day week operation at each end of the transportation link. Three sets of driving restrictions were considered -- 1) none 2) weekend restrictions, i.e., no travel for 48 hours over the weekend but loading and unloading operations scheduled for any time 3) night and weekend restrictions, i.e., travel 5 days at an average of 12 hours per day, but loading and unloading at any time. It can be seen from the results that at a speed of 25 miles per hour the number of trips completed per week varies between 1 and 1.9; at 36 miles per hour, the number of trips varies between 1.25 and 2.2. In both cases, the number of trips (equivalent to the amount of fuel transported) could vary by almost a factor of two.

To evaluate this effect on transportation, the anticipated cask use charges must be determined first. Figure 5 presents typical data on two casks of interest. A three PWR (pressurized water reactor) fuel assembly

500 MILE ONEWAY DISTANCE
24 HR TURNAROUND AT UTILITY
24 HR TURNAROUND AT REPROCESSOR
24 HR/DAY OPERATION AT UTILITY & REPROCESSOR

Average Travel Speed	Driving Restrictions	Trips/Week
	NONE	1.91
25 MPH	WEEKEND RESTRICTIONS	1.5
	NIGHT & WEEKEND RESTRICTIONS	1.0
	NONE	2.21
36 MPH	WEEKEND RESTRICTIONS	1.66
	NIGHT & WEEKEND RESTRICTIONS	1.25

FIGURE 4

TRUCK FUEL SHIPMENT STANDARD TRIP (2)

Cask Capacity	Weight (lbs)		Cask Cost	Cask Use Charge	
(tonnes)	Loaded vehicle (GVW)	cask	(dollars)	(dollars/week) Probable Range	Selected for Study
1.35 (3 PWR)	105,000	75,000	500,000	3000-5000	4000
0.90	90,000	65,000	430,000	2500-4500	3500

FIGURE 5

CASK USE CHARGE APPROXIMATION (2)

STANDARD TRIP OF 500 MILES
CASK COST APPROXIMATION
EXCLUDES FREIGHT COSTS

Average Transport Speed (MPH)	Cask Capacity (MTU)	Transportation Costs (dollars/MTU) Driving Restrictions: None	Weekends	Nights & Weekends
25	1.35	1550	1980	2960
	0.90	2040	2600	3900
36	1.35	1340	1780	2370
	0.90	1760	2340	3110

FIGURE 6

SPENT FUEL TRANSPORT COST (2)

cask could carry about 1.35 tons of uranium in a 75,000 lb. cask at a gross vehicle weight of 105,000 lbs. The cask, constructed with depleted uranium to save weight, would cost approximately one-half million dollars. A smaller cask, carrying 2 PWR fuel assemblies, would weigh about 65,000 lbs providing a gross vehicle weight of 90,000 lbs. Factors such as cask capital cost, desired return on investment, estimated cask life, cask utilization factor, insurance cost, cask maintenance cost, and fleet management costs make it difficult to estimate the cask use cost. A crude estimate has been made by Dufrane and North of $3,000 per week to $5,000 per week for the 2 PWR fuel assembly cask. By using an average cask usage cost of $4,000 per week and $3,500 per week with the number of trips per week from Fig. 4, the cost per ton of spent fuel shipped can be calculated. As detailed in Fig. 6, for an average transport speed of 25 MPH, driving restrictions placed upon the 3 or 2 PWR assembly cask would cause a variation on cask transportation costs of $1,410 and $1,860 per ton respectively. A combination of both weight and driving restrictions would provide a total differential cost of approximately $2,350 per ton. It is therefore, evident that, in general, shipping cost is far from fixed for a particular transportation mode.

2.3 Safety

The cost of transportation is related directly to safety. In general, the effect on cost, which would result from a transportation accident involving spent fuel, is recognized by personnel of the nuclear power industry. Aside from direct costs, indirect costs due to unfavorable public reaction are almost certain. Clearly, safety considerations have a major role in the selection of routes and modes of spent fuel transportation.

The risk involved in the shipment of radioactive materials is of growing concern. As yet this risk has not been quantified. To do so, one must recognize the many factors involved. This can be illustrated best by a simplified picture as shown in Fig. 7.

From Fig. 7 it can be seen that the risk is a function of several variables. Data for analysis of many of these variables is difficult to obtain, e.g., accident probability. Certainly one of the major parameters in risk quantification, the probability of a transportation accident may be estimated on an average basis. A detailed study of accident analysis on a cost basis was performed by Limekohler (3). Limekohler's analysis gives evidences that the probability of accident is indeed a function of the parameters indicated in Fig. 7. Table I is from the Atomic Energy Commission accident analysis report showing the number of accidents occurring by mode over a 15 year period (5). The probability of accident may be approximated by the number of accidents per mile if the number of miles traveled in those 15 years can be estimated. A rough estimate of these milages has been obtained from Dufrane and North (2). The mileages and the calculated average probability per mile appear in Table II.

The seriousness of the accident depends on the fuel material, amount of fuel, and the mode of transport. For instance, plutonium, a highly toxic and dispersable low level alpha emmiter, would present a much greater potential hazard than liquid U-235 during transport. A barge accident, for example, could cause a very difficult cleanup operation, but the probability of a barge accident is quite low as shown in Table II. A table of relative seriousness of different accidents is also shown in Table II.

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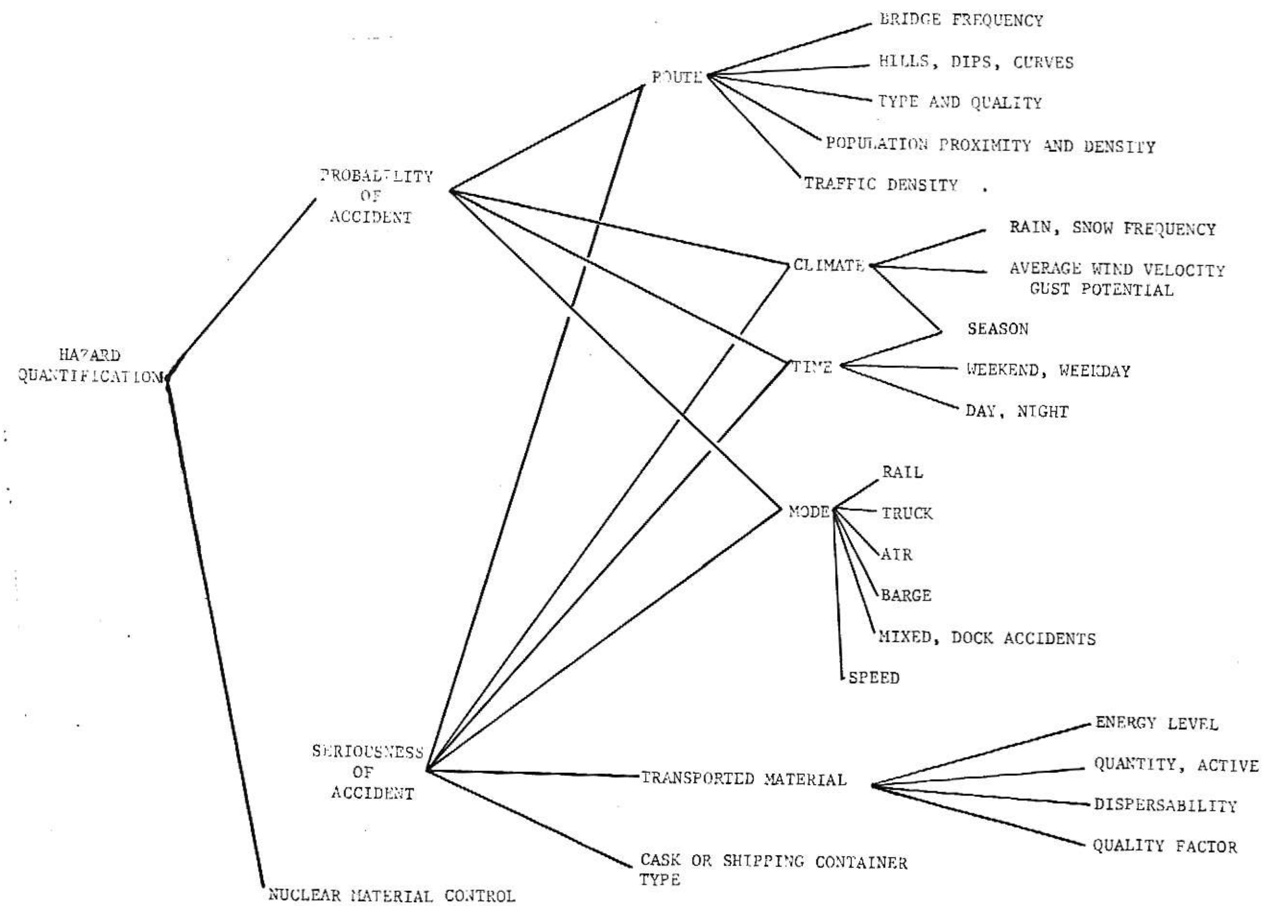

FIGURE 7 FACTORS PERTAINING TO HAZARD QUANTIFICATION

TABLE I

INCIDENT EXPERIENCE INVOLVING NUCLEAR MATERIALS BY MODE OR LOCATION (5)

Mode	No. of Incidents			
	1963-1964	1962	1957-1961	1949-1956
Truck Incidents	17	8	21	9
Rail Incidents	5	5	11	2
Air Incidents	0	0	0	1
Terminal Incidents	2	1	15	1
Total	24	14	47	13

TABLE II

MODE, MILES TRAVELED AND ACCIDENT CHARACTERISTICS

	TOTAL MILES* TRAVELED 1949-1964	PROBABILITY OF** ACCIDENT, MILE^{-1}	RELATIVE SERIOUSNESS OF ACCIDENT, TON^{-1}	
TRUCK	1,400,000	4.6×10^{-6}	1000 (LWT)	2000 (OWT)
RAIL	880,000	7.2×10^{-6}	1000	
AIR	64,000	5.1×10^{-7}	3000 (VSTOL)	2000 (L.A.)
BARGE	11,000	1.0×10^{-9}	4000	

*Dufrane and North (2).

**TID-16764 (5).

For the purpose of this study the relative seriousness of an accident is estimated on the basis of mode only. A more sophisticated analysis would demand data concerning the seriousness of accidents as a function of the several parameters indicated in Fig. 7.

⊗	1	Reactor 1
	2	Reactor 2
	3	Reactor 3
	4	Reactor 4
☆	5	Fuel Reprocessor

0 25 50 100 150 miles

Figure 8 Major truck routes for spent fuel shipment selected by the author

⊗ 1 Reactor 1
2 Reactor 2
3 Reactor 3
4 Reactor 4

☆ 5 Fuel Reprocessor

0 25 50 100 150 miles

Figure 9 Major railroads for spent fuel shipment selected by the author

3.0 MODEL DESCRIPTION

The model developed in this work considers 4 different reactor locations and one spent fuel reprocessor. These plants and the reprocessor are shown along with the major highways and railroads in Figs. 8 and 9, respectively. Four modes are considered -- truck, rail, air, and barge. The possibility of mixed mode is also considered in the route candidates. The objective of the problem was to minimize the cost of transporting a representative amount of spent fuel from hypothetical reactor plants, Reactor 1, Reactor 2, Reactor 3, and Reactor 4 to the fuel reprocessor at a risk below a specified hazard level. The amount of spent fuel shipment for Point Beach 2, Dresden 2, Zion and Palisades reactors, for 1973, was estimated by Tremmel and Berte (1):

Point Beach 2	16 Tons of spent fuel
Dresden 2	42 Tons
Zion	28 Tons
Palisades	28 Tons

These amounts are used as representative for hypothetical reactors 1, 2, 3, and 4 respectively since their geographical locations are similar. The optimization of this model can be handled by linear programming methods as shown in the following sections.

3.1 Linear Programming

In general, a linear programming (LP) problem can be defined as a system of linear equations which represents constraints on an objective function. This objective function is to be maximized or minimized (7). In mathematical terms the LP problem may be stated as follows:

Find values of the variables $x_1, x_2, \ldots, x_n$ which satisfy the inequalities

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1N}x_N \; [\geq \text{ or } \leq] \; b_1$$
$$\vdots \qquad\qquad\qquad\qquad \vdots$$
$$a_{p1}x_1 + a_{p2}x_2 + \ldots + a_{pN}x_N \; [\geq \text{ or } \leq] \; b_N$$

where $x_1, x_2, \ldots, x_N$ all are greater than or equal to zero, such that

$$z = c_1x_1 + c_2x_2 + \ldots + c_Nx_N$$

is a maximum or minimum. The a_{ij}'s, b's, and c's are known.

An example of a simple linear programming problem will help to clarify the technique. Consider a nuclear reactor which must ship its spent fuel to a reprocessor. There are two modes of transportation available for the shipment under consideration ... truck and rail. Necessary information is:

1. 12 tons of spent fuel to be shipped
2. Costs of shipment are \$38 and \$45 per ton for truck and rail respectively
3. Relative risk indices are 0.2 and 0.4 per ton for truck and rail respectively
4. An upper limit of relative risk index is set at 0.3 per ton

The problem is to minimize the cost of shipment of 12 tons of spent fuel while keeping the relative risk index below 0.3 per ton.

This model may be formulated as follows.

Let x_1 denote the number of tons of spent fuel shipped by truck
x_2 denote the number of tons of spent fuel shipped by rail

Then the objective is to minimize the cost

$$z = 38x_1 + 45x_2$$

subject to

$$x_1 + x_2 \geq 12 \qquad \text{Demand shipment constraint} \qquad (1)$$

$$\frac{0.2x_1 + 0.4x_2}{x_1 + x_2} \leq 0.3 \qquad \text{Risk index constraint} \qquad (2)$$

It was assumed that a "mixed" risk index may be represented as a weighted average of risk indices for x_1 and x_2. If Eq. (2) is multiplied by (x_1+x_2) and the x's are collected on the left hand side, Eq. (2) can be rewritten as

$$-0.1\ x_1 + 0.1\ x_2 \leq 0 \qquad (2.1)$$

Graphical representation of the problem leads to a simple solution as illustrated in Fig. 10. The shaded regions represent the feasible regions allowed by constraints (1) and (2.1). Line 2.1 and Line 1 correspond to Eq. (2.1) and Eq. (1). Thus the doubly shaded region is the feasibility region for the solution. Suppose z=17, then a line may be constructed which contains all points x_1 and x_2 which yield this cost value. Notice that all cost lines are parallel to this line since only z will change, not the variable coefficients. Then by moving this "equi-cost" line outward toward the feasibility region, the minimum value of z which will satisfy both constraints exists at the intersection of lines (1) and the x_1-axis. This point is

$$x_1 = 12,\ x_2 = 0$$

and the total cost is

$$z = 38(12) + 45(0) \text{ or } \$456 \quad .$$

More complex LP problems can be solved by using computer programs such as IBM's Mathematical Programming System (MPS)/360 (8).

MPS/360 is composed of a set of procedures, a subset of which deals with LP. These LP procedures use the bounded variable/product form of the inverse revised simplex method (8). The simplex method is based on the fact that if there are m constraints (or rows) in the constraint matrix which are linearly dependent, then there is a set of m columns (variables or

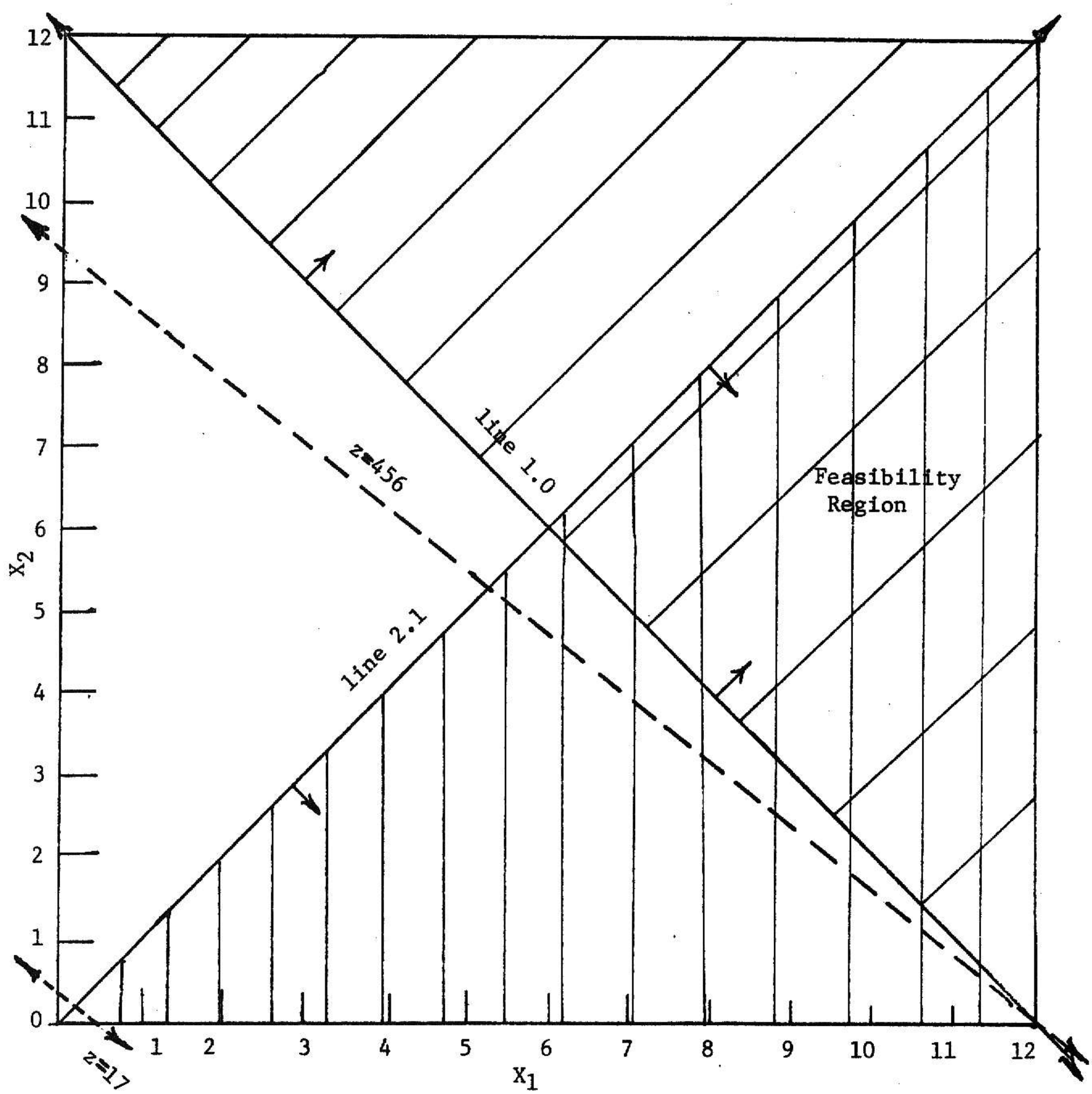

Figure 10 Linear Programming Solution to Example Problem

vectors) which are linearly independent and form a so called basic solution. That is, the right hand side (RHS) of the problem formation can be expressed as a linear combination of the m columns in the basis. The MPS/360 program systematically exchanges the column vectors according to the revised simplex method until an optimal solution is reached (8). Short notes which explain the control cards needed follow (9).

SHORT NOTES ON MPS COMPUTER PROGRAM

PROGRAM. PROGRAM is a mandatory statement at the beginning of each program.

INITIALIZE. INITIALIZE is used to establish initial settings for tolerance, frequencies and demands. This is a system macro instruction.

TITLE ('EXAMPLE'). This statement is used to suitably name the problem.

MOVE (XDATA, 'EXAMPLE') and MOVE (XPBNAME, 'PBFILE'). The first statement moves input data set name EXAMPLE into the cell XDATA. The second moves the problem file name, PBFILE, into the cell XPBNAME.

CONVERT. This routine is the master control card of the convert procedure, which is used to convert external input data into a binary form and transfers these data to PBFILE.

SETUP ('MAX'). This routine is the master control card of the setup procedure, which is used to set up the problem name in XPBNAME by

1. searching for the problem,
2. opening the matrix, eta, and scratch files,
3. making the storage allocation,
4. building the work matrix,
5. setting the logical basis and structural bounded variables at the upper bound, and
6. building the inverse of the logical basis.

For minimization 'MAX' should be changed to 'MIN'.

BCDOUT. This converts the specified binary problem into the external input data format. The output may be listed and/or punched out and is in the order of the input data sections, ROWS, COLUMNS, RHS, RANGES, and BOUNDS. The NAME and ENDATA cards may also be produced.

MOVE (XOBJ, 'PROFIT') and MOVE (XRHS, 'LIMITS'). These statements are for identifying the objective function and the right hand side limits. For minimizing, 'PROFIT' becomes 'COST'. Objective function and right hand side have to be named because a program can work with many objective functions and limits.

PICTURE. The PICTURE procedure produces a "picture" of the current work matrix in condensed format. Up to 45 rows and 55 columns for an output page are given. The pages are numbered using matrix notation for ease of identification. The output of PICTURE gives a quick visual check to see if the structure of the matrix is correct and if any coefficients are missing.

The range on the problem right hand side, and bounds on the variables are indicated if they exist.

PRIMAL. Primal optimizes the current problem using a composite primal algorithm. It can work with a composite objective function and/or a composite right hand side. It can work with a composite restraint row if neither the objective function nor the right hand side is composite.

SOLUTION. Solution is the output of a summary of the solution corresponding to the current basis. This output may be either printed or filed depending on a parameter in the procedure card.

EXIT. Exit is a procedure linked by the executor before return to the operating system. It is mainly used to close data sets and print the time of the executor job step.

PEND. This is equivalent to the END statement in FORTRAN.

3.2 Mathematical Formulation

To formulate the model in mathematical terms, it is necessary to define the following quantities:

x_{ij} the number of ton-miles of spent fuel sent by mode i on route j

c_i the relative cost per ton-mile of shipment by mode i

d_k the demand of tons of spent fuel needed to be shipped from reactor k

P_{ij} the associated hazard of transportation of spent fuel by mode i along route j per ton-mile

The objective is to minimize the cost of transport which may be written as

$$z = \sum_{ij} c_i x_{ij} \qquad i = 1, \ldots, 6 \qquad j = 1, \ldots, 32$$

where c_i is speculated to correspond to the following values for i = 1 through 6:

c_1	Legal Weight Truck	1.0 unit cost per ton-mile
c_2	Over Weight Truck	1.9 unit cost per ton-mile
c_3	Barge	2.7 unit cost per ton-mile
c_4	VSTOL Aircraft	2.5 unit cost per ton-mile
c_5	Large Aircraft	2.2 unit cost per ton-mile
c_6	Railroad	1.2 unit cost per ton-mile

Table III lists the routes and modes.

TABLE III. VARIABLE DESCRIPTION

x_{ij}	Plant	Mode	Route	Total Miles
$x_{1,1}$	Reactor 1	LWT	41-141-94-80-75-70	620
$x_{2,1}$		OWT	41-141-94-80-75-70	620
$x_{4,2}$		VSTOL	-----	410
$x_{5,2}$		L.A.	-----	410
$x_{3,3}$		BARGE	To Grand Rapids - Truck	250
$x_{1,3}$		LWT	Grand Rapids -96-75-70	390
$x_{2,3}$		OWT	Grand Rapids -96-75-70	390
$x_{3,4}$		BARGE	Grand Rapids - Truck	250
$x_{1,4}$		LWT	Grand Rapids -196-94-23-75-70	420
$x_{2,4}$		OWT	Grand Rapids -196-94-23-75-70	420
$x_{3,5}$		BARGE	Grand Rapids - Truck	250
$x_{1,5}$		LWT	Grand Rapids -96-33-75-70	350
$x_{2,5}$		OWT	Grand Rapids -96-33-75-70	350
$x_{1,6}$		LWT	Grand Rapids -196-94-23-70	330
$x_{2,6}$		OWT	Grand Rapids -196-94-23-70	330
$x_{3,6}$		BARGE	Grand Rapids - Truck	250
$x_{6,22}$		RAIL	1-2-6-13-20-21-24	500
$x_{6,23}$		RAIL	1-2-6-7-8-20-21-24	520
$x_{6,24}$		RAIL	Barge -11-14-24	480
$x_{3,24}$		BARGE	Rail	120
$x_{6,25}$		RAIL	1-2-3-7-8-20-21-24	500
$x_{1,7}$	Reactor 2	LWT	36-94-70	400
$x_{2,7}$		OWT	36-94-70	400
$x_{1,8}$		LWT	36-Barge-75-70	400
$x_{2,8}$		OWT	36-Barge-75-70	400
$x_{3,8}$		BARGE	Truck-Barge-Truck	440
$x_{4,9}$		VSTOL	-----	300
$x_{5,9}$		L.A.	-----	300
$x_{6,26}$		RAIL	5-9-10-17-24	400
$x_{6,27}$		RAIL	5-4-8-20-21-24	400
$x_{6,28}$		RAIL	5-24-Barge-26-34	230
$x_{3,28}$		BARGE	Rail-Barge-Rail	300

x_{ij}	Plant	Mode	Route	Total Miles
$x_{1,10}$	Reactor 3	LWT	80-75-70	420
$x_{2,10}$		OWT	80-75-70	420
$x_{1,11}$		LWT	8-57-74-70	400
$x_{2,11}$		OWT	8-57-74-70	400
$x_{1,12}$		LWT	80-69-70	450
$x_{2,12}$		LWT	80-69-70	450
$x_{1,13}$		LWT	80-24-74-70	360
$x_{2,13}$		OWT	80-24-74-70	360
$x_{1,14}$		LWT	24-70-57-Barge-75-70	350
$x_{2,14}$		OWT	Truck-Barge-Truck	350
$x_{3,14}$		BARGE	Truck-Barge-Truck	250
$x_{1,15}$		LWT	80-57-Barge-75-70	300
$x_{2,15}$		OWT	80-57-Barge-75-70	300
$x_{3,15}$		BARGE	Truck-Barge-Truck	250
$x_{4,40}$		VSTOL	-----	210
$x_{5,40}$		L.A.	-----	210
$x_{6,29}$		RAIL	3-7-8-20-21-24	410
$x_{6,30}$		RAIL	3-5-9-10-17-24	450
$x_{1,16}$	Reactor 4	LWT	96-23-75-70	390
$x_{2,16}$		OWT	96-23-75-70	390
$x_{1,17}$		LWT	196-23-15-70	420
$x_{2,17}$		OWT	196-23-75-70	420
$x_{1,18}$		LWT	96-23-75-70	350
$x_{2,18}$		OWT	96-23-75-70	350
$x_{1,19}$		LWT	196-94-23-70	330
$x_{2,19}$		OWT	196-94-23-70	330
$x_{1,20}$		LWT	Barge-80-75-70	290
$x_{2,20}$		OWT	Barge-80-75-70	290
$x_{3,20}$		BARGE	Barge-Truck	150
$x_{6,31}$		RAIL	12-14-15-24	250
$x_{6,32}$		RAIL	12-13-20-21-24	380

The amount of spent fuel transported from each reactor must be at least as large as the previously mentioned representative demand. This may be formulated as follows:

$$\sum^{j(k)} x_{ij}/(\text{total miles})_j \geq d_k$$

where corresponds to

k = 1	Reactor 1	j = 1 to 10
2	Reactor 2	11 to 16
3	Reactor 3	17 to 25
4	Reactor 4	26 to 32

When a mixed transport mode situation exists for a given route j, the quantity of spent fuel which is shipped on both modes is constant. For instance, route 4 involves shipment by barge for 250 miles and then truck for 420 miles. Therefore, in this case, the amount shipped by OWT and LWT must equal the amount shipped by barge, or

$$x_{1,4}/420 + x_{2,4}/420 = x_{3,4}/250 \quad .$$

Similar cases exist for other bi-modal routes.

In order to formulate restrictions on transportation due to risk consideration, it is necessary to discuss the parameters involved in quantification of risk. As shown in Fig. 7 risk depends on many factors. Of major importance among all these factors are probability of accident, severity of accident, and the population density near an accident. The major risk involved is that of overexposing a person to radioactive material. Although such variables as type of radioactive material, climate, time of day, etc. are important, for the purposes of this work the risk evaluation was limited to consideration only of the three factors: probability of accident, severity of accident, and potential population near an accident. For the purposes of this work, risk was assumed to be modeled by the following equation:

$$P_{ij} = (\text{Risk})_{ij} = \sum_{\substack{\text{All} \\ \text{Cities}}} \left(\frac{(\text{Population of City})}{(\text{Total Distance})(\text{Distance from Route to City})^2} (SA)(PA) + (0.001)(SA) \right) \quad \text{for each route and mode}$$

where

SA is the severity of accident per ton of fuel, and

PA is probability of accident per mile for mode i.

Again, the probability of accident shown in Table II has been calculated from accident reports taken from (5) and are obtained by averaging the number of accidents per mile calculated from Table I. The relative severity of accident is an estimate of the characteristics of each mode arrived at using general considerations such as average speed traveled, damage upon typical accident, and amount of cargo transported. Clearly, a more detailed analysis of seriousness of accident would be required for a more sophisticated model.

An example of the calculation of risk of route 1, legal weight truck from Reactor 1 to the fuel reprocessor along route 41-141-94-80-75-70, appears below. A list of the cities exposed to this route, their population, and the closest distance of that city to route 1 is tabulated in Table IV. This information was used as input to the risk formula along with the seriousness of accident and probability of accident by truck from Table II. The result is a risk index of 0.736 per ton mile. Similarly, the risks of all routes concerned with Reactor 1 were calculated. Thus the risk associated with each route was multiplied by the number of ton-miles (x_{ij}) attributed to this route and the weighted average was restricted to be less than some predetermined upper limit of risk. The upper limit criterion can be set arbitrarily or by the best available information. For the purposes of this work, it was arrived at by assuming the following allowable values of the parameters:

Table IV

Route 1 HAZARD QUANTIFICATION DATA

CITY	POPULATION	(CLOSEST DISTANCE)^2
MENOMINEE, WISC.	8,600	1
GREENBAY, WISC.	82,500	1.25
MANITOWOC, WISC.	32,200	4
MILWAUKEE, WISC.	765,000	1
CHICAGO, ILL.	3,520,000	2.25
KENTLAND, IND.	1,780	625
SOUTH BEND, INC.	135,000	25
PLYMOUTH, IND.	7,300	900
BRYAN, OHIO	7,800	4
TOLEDO, OHIO	354,000	625
MANSFIELD, OHIO	49,000	400
SPRINGFIELD, OHIO	82,500	2
DAYTON, OHIO	260,000	400

1 city exposed to the route	
population	1,000,000
closest distance	1 mile
total distance	400 miles
seriousness of accident	2000
probability of accident	0.0000001

Therefore;

upper limit = UL

$$= \frac{1,000,000}{400} (2000(10^{-7})+2(10^{-5}))$$

$$= 0.5 \text{ ton}^{-1} \text{ mile}^{-1}$$

Then the risk constraint for a particular reactor can be written as:

$$\frac{\sum_{i=1}^{6} \sum_{j=k_1}^{k_2} P_{ij} X_{ij}}{\sum_{i=1}^{6} \sum_{j=k_1}^{k_2} X_{ij}} \leq UL$$

where k_1 is the first numbered route servicing reactor k

k_2 is the last numbered route servicing reactor k

The four risk constraints, the four demand constraints, and the mixed mode constraints are summarized in Fig. 11. This matrix serves as the input matrix to be optimized by the revised simplex LP method. The MPS/360 was used to find the optimal solution and the printout is displayed in appendix A.

3.3 Results and Conclusions

The optimal solution is displayed on page 11 of the computer output, Appendix A. As indicated, the lowest cost possible incurred in shipping the demanded fuel and satisfying the risk criterion is 48152.00720 cost units. The unit cost corresponds to that of a legal weight truck. It is estimated that the cost per ton-mile for a legal weight truck is $0.38.

ROW		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	COLUMN	$X_{1,1}$	$X_{2,1}$	$X_{4,2}$	$X_{5,2}$	$X_{3,3}$	$X_{1,3}$	$X_{2,3}$	$X_{3,4}$	$X_{1,4}$	$X_{3,4}$	$X_{3,5}$	$X_{1,5}$	$X_{2,5}$	$X_{1,6}$	$X_{2,6}$
	COST	1.0	1.9	2.5	2.2	2.7	1.0	1.9	2.7	1.0	1.9	2.7	1.9	1.0	1.0	1.9
1	REACTOR 2 DEMAND	1	1	1.51	1.51		1.59	1.59		1.48	1.48		1.77	1.77	1.87	1.87
2	MIXED MODE ROUTE 3					-2.48	1.59	1.59								
3	MIXED MODE ROUTE 4								-2.48	1.48	1.48					
4	MIXED MODE ROUTE 5											-2.48	1.77	1.77		
5	MIXED MODE ROUTE 6														1.87	1.87
6	MIXED MODE ROUTE 24															
7	REACTOR 2 DEMAND															
8	MIXED MODE ROUTE 8															
9	MIXED MODE ROUTE 28															
10	REACTOR 3 DEMAND															
11	MIXED MODE ROUTE 14															
12	MIXED MODE ROUTE 15															
13	REACTOR 4 DEMAND															
14	MIXED MODE ROUTE 20															
15	RISK REACTOR 1	.236	.972	-.476	-.464	-.462	0.7	1.9	-.462	-.2	.1	-.462	.196	-1.56	-.182	.136
16	RISK REACTOR 2															
17	RISK REACTOR 3															
18	RISK REACTOR 4															

FIGURE 11 LINEAR PROGRAMMING CONSTRAINT MATRIX

ROW	16 $X_{3,6}$ 2.7	17 $X_{6,22}$ 1.2	18 $X_{6,23}$ 1.2	19 $X_{6,24}$ 1.2	20 $X_{6,25}$ 1.2	21 $X_{3,24}$ 2.7	22 $X_{1,7}$ 1.0	23 $X_{1,7}$ 1.9	24 $X_{1,8}$ 1.0	25 $X_{2,8}$ 1.9	26 $X_{3,8}$ 2.7	27 $X_{5,9}$ 2.2	28 $X_{4,9}$ 2.5	29 $X_{6,26}$ 1.2	30 $X_{6,27}$ 1.2	31 $X_{6,28}$ 1.2	32 $X_{3,28}$ 2.7	33 $X_{1,10}$ 1.0
1		1.24	1.19	1.24														
2																		
3																		
4																		
5	-2.48																	
6				1.29		-2.7												
7							1.0	1.0	2.22	2.22		1.33	1.33	1.0	1.0	1.79		
8									2.22	2.22	-.91							
9																-1.74	1.35	
10																		1.0
11																		
12																		
13																		
14																		
15	-.462	.87	.92	-.45	-.282	-.324												
16							-.288	-.08	.116	.834	-.49	-.47	-.48	.305	-.37	-.45	-.49	
17																		.89
18																		

FIGURE 11 (CONT'D)

	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
	$X_{2,10}$	$X_{1,11}$	$X_{2,12}$	$X_{2,12}$	$X_{1,12}$	$X_{2,13}$	$X_{1,13}$	$X_{1,14}$	$X_{2,14}$	$X_{3,14}$	$X_{1,15}$	$X_{2,15}$	$X_{3,15}$	$X_{4,40}$	$X_{5,40}$	$X_{6,29}$	$X_{6,30}$	$X_{1,16}$
ROW	1.9	1.0	1.9	1.9	1.0	1.9	1.0	1.0	1.9	2.7	1.0	1.9	2.7	2.5	2.2	1.2	1.2	1.0
1																		
2																		
3																		
4																		
5																		
6																		
7																		
8																		
9																		
10	1.0	1.25	1.05	.935	.935	1.17	1.17	1.2	1.2		1.4	1.4		1.4	1.4	1.02	.935	
11								1.2	1.2	-1.3								
12											1.4	1.4	-1.68					
13																		1.0
14																		
15																		
16																		
17	2.2	.15	2.8	.94	2.5	.16	.82	-.16	.19	-.49	1.5	3.5	-.49	-.43	-.43	-.37	-.33	
18																		-.46

FIGURE 11 (CONT'D)

ROW	52	53	54	55	56	57	58	59	60	61	62	63	
	$X_{2,16}$	$X_{1,17}$	$X_{2,17}$	$X_{2,18}$	$X_{1,18}$	$X_{1,19}$	$X_{1,10}$	$X_{1,20}$	$X_{2,20}$	$X_{3,20}$	$X_{6,31}$	$X_{6,32}$	
	1.9	1.0	1.9	1.0	1.9	1.0	1.9	1.0	1.9	2.7	1.2	1.2	
1													$\geq$ 9,900
2													= 0
3													= 0
4													= 0
5													= 0
6													= 0
7													$\geq$ 16,800
8													= 0
9													= 0
10													$\geq$ 11,750
11													= 0
12													= 0
13	1.0	.93	.93	1.1	1.1	1.18	1.18	1.34	1.34		1.21	1.11	$\geq$ 10,900
14								1.34	1.34	-.28			= 0
15													$\leq$ 0
16													$\leq$ 0
17													$\leq$ 0
18	-.43	-.47	-.45	-.46	-.43	-.46	-.42	1.8	4.1	-.49	-.48	-.47	$\leq$ 0

FIGURE 11 (CONT'D)

The cost in dollars is calculated as (48152.00720)(.38) and is approximately equal to $18,297.00. This cost does not include the cost of shipping cask indemnity or insurance. It was assumed that consideration of these additional costs would simply add on additional cost in the final analysis and would not affect the choice of optimal routes. For a more detailed examination of the problem, these additional costs could be included.

The amount that should be shipped on a particular route is obtained from pages 13 and 14 of the computer output, appendix A. The values listed are the number of ton-miles attributed to shipment on a given route by a given mode. The amount of spent fuel shipped along a route is calculated by dividing the ton-miles by the distance that the spent fuel is shipped on the route by a given mode. For instance, Column 20 corresponds to $x_{6,25}$. The computer output indicates 1983.87 ton-miles associated with shipment on route 25 by train. The total distance traveled by the train on route 25 is 500 miles; therefore, the amount of spent fuel shipped is 7893.187/500 = 16 tons. A summary of all the amounts shipped by various routes and modes appears in Table V.

As mentioned previously, many of the quantities used in the calculations of risk, seriousness of accident, and cost require more detailed study. In addition, some of these quantities are not fixed. For instance, cost of transport is regionally and seasonally dependent. Cost is also subject to negotiation and is by no means fixed. It was, however, hypothesized that a given reactor facility will be able to arrive at many of these quantities accurately and then may apply this procedure realistically. In addition to economic aspects of nuclear transport, the personnel of the nuclear

Table V

Shipping Routes and Quantities for the Model

REACTOR	VARIABLE	AMOUNT TON-MILES	ROUTE TOTAL MILES	AMOUNT TONS	MODE	ROUTE
Reactor 1	$X_{6,25}$	7,983	500	16	RAIL	1-2-3-7-8-20-21-24
Reactor 2	$X_{1,7}$	16,800	400	42	LWT	36-94-70
Reactor 3	$X_{1,13}$	3,425	360	9.5	LWT	80-24-74-70
Reactor 3	$X_{6,29}$	7,590	410	18.5	RAIL	3-7-8-20-21-24
Reactor 4	$X_{1,18}$	4,681	350	13.5	LWT	96-23-75-70
Reactor 4	$X_{1,19}$	4,793	330	14.5	LWT	196-94-23-70

industry are beginning to feel the ever-growing pressure of the Environmental Protection Agency to choose a safe method of transportation of nuclear material. From consideration of the magnitude of the predicted spent fuel shipments, it seems imperative that some criterion of hazard limitation be implemented at a realistic expense.

Comparison of the LP method to a common current method of route and mode selection suggests the potential value of the LP method. As an example, the shipment of spent fuel from Reactor 3 was examined employing a selection process as follows:

SINGLE MODE CHOICE
↓
LEAST EXPENSIVE?
↓
SATISFY HAZARD LIMITATION?
↓
CHOICE OF ROUTE AND MODE

Following the above outlined logic, $x_{6,29}$ is the resulting choice (see Fig. 11); that is, shipment by rail over route 3-7-8-20-21-24. To satisfy the demand for spent fuel shipped,

$$\frac{x_{6,29} \text{ ton-miles}}{410 \text{ miles}} = 28 \text{ tons} \quad \text{or} \quad x_{6,29} = 10,490 \text{ ton-miles}$$

which corresponds to a cost of

$$(10,490 \text{ ton-miles})\ (1.2 \text{ unit cost/ton-miles}) = 12,600 \text{ unit cost}$$

As shown in Table V, the LP solution is

$x_{1,13} = 3,425$ ton-miles

$x_{6,29} = 7,590$ ton-miles

which corresponds to a cost of

(3,425 ton-miles)(1.0 unit cost/ton-mile)

+ (7,590 ton-miles)(1.2 unit cost/ton-mile) = 12,525 unit cost

A saving of 75 cost units is realized by the LP method even in this simplified model. The full value of the LP method clearly becomes more obvious as the model becomes more complicated. One could imagine the use of this method by a shipping agency which is constrained by such factors as the following:

1. Limited number of each type of vehicle and cask,
2. Limited maintenance available depending on the vehicle,
3. Legal restrictions imposed by state or government agencies, and
4. Others,

while having many options available such as

1. Several existing modes,
2. Possible new modes, and
3. Mixed modes.

LP also offers the capacity for ready updating of information. This seems imperative considering the variable nature of route characteristics, cost, etc. This aspect is discussed in detail in Sections 4.0 - 4.4.

4.0 POSTOPTIMAL ANALYSIS

After an optimal solution has been attained, there is information to be gained from the MPS/360 output concerning the optimal solution. As mentioned previously, the cost of transportation by various modes is by no means fixed. MPS/360 allows the user to examine the range for which a cost coefficient may be varied while maintaining the same basic solution (although the total cost may change). Similarly, the range of the RHS (demands) and risk coefficients may be examined without changing the basis.

An extension of this analysis would allow the cost coefficients, RHS, row coefficients, or column coefficients to be varied to determine their effect on the solution. For instance, the cost coefficient of over weight truck shipment may be varied over any range, say 1.9 through 0.5, thus generating a series of related LP problems. Clearly, the basis most likely would change in this instance introducing a new solution. This procedure is referred to as parameteric programming.

4.1 Range Analysis

Range analysis is used postoptimally to generate an output analysis of the current basis. This analysis includes:

1. The effects of cost changes on optimum activities,
2. The cost of changing an activity from the optimum level and the activity range for which this cost is valid,
3. The value of changing the row activity (RHS) and the interval for which this change is valid.

Although the parameters may be changed from their optimum value, a better total optimum is possible. In general, a cost increase forces an activity decrease, and conversely a cost decrease promotes an activity increase.

The computer output of the RANGE analysis for the model problem is displayed in Appendix A. Sections 1 and 3 are row sections in which section 1 contains economic information for rows at their lower or upper limit; section 3 contains such information at an intermediate level. Parameters which may appear are defined as follows:

Number	The internal serial number of this row
Row	User's name for row
AT	A two-character code denoting the status that the row activity has in the solution. Codes and their meanings are: BS Row activity at intermediate level EQ Row activity at fixed level UL Row activity at upper limit LL Row activity at lower limit
Activity	The value of the row activity in the solution
Slack Activity	The value of the associated slack variable

The upper line for each row shows the activity-cost relationship for activity decrease/cost increase and employs the following parameters:

Lower Limit	The input lower limit for this row
Lower Activity	The level to which the row activity may be decreased at a cost per unit of decrease given by Unit Cost
Unit Cost	The change of the objective function per unit of decrease of activity. The problem can be modified to decrease the row activity as far as Lower Activity.
Limiting Process	The name of the row or column that would change its status if the activity level of the row were decreased below Lower Activity. In section 1, Limiting Process will leave the basis; in section 3, Limiting will enter the basis.
AT	The status associated with Limiting Process LL leaves or enters basis at lower limit UL leaves or enters basis at upper limit

The lower line for each row shows the activity-cost relationship for activity increase/cost decrease.

Upper Limit — The upper limit for the row

Upper Activity — The level to which the row activity may be increased at a cost per unit increase of Unit Cost

Unit Cost — The change in the objective function per unit increase in row activity. The problem can be modified to increase the row activity level as far as Upper Activity, at this cost per unit increase.

Limiting Process — The name of the row or column which would change its status if the activity level of the row were increased above Upper Activity. In section 1, Limiting Process will leave the basis; in section 2, Limiting Process will enter the basis

AT — The status associated with Limiting Process
- LL Leaves or enters the basis at lower limit
- UL Leaves or enters the basis at upper limit

Sections 2 and 4 are column sections. Section 2 contains the economic information for the variables at their lower limit; section 4, for those at intermediate level. The following parameters may appear:

Number — The internal serial number of the column

Column — User's name for the column

AT — Column activity status
- BS In the basis
- FR Non basic or free
- EQ Non basic, artificial
- UL Non basic at upper level
- LL Non basic at lower level

Activity — The value of column activity in the solution

Similar to sections 1 and 3, the upper line displays the activity-cost relationship for activity decrease/cost increase and the lower line displays the activity-cost relationships for activity increase/cost decrease.

Input Cost refers to the unit cost of the variable as specified by the user. In sections 2 and 4 Input Cost is varied to Upper Cost and Lower Cost, respectively. The other parameters are Lower Limit, Lower Activity, Unit Cost, Limiting Process, AT, Upper Activity, Upper Limit, and Lower Cost. Their meanings are analogous to those in sections 1 and 3.

An example will more clearly illustrate the use of Range. Referring to page 19 of the Range output in the Appendix A, consider Number 64. The output indicates this is Column 45 at its lower limit (LL) with a solution value (activity) of 0. The Input cost is listed as 1.9, and the lower limit is 0. The upper limit is listed as infinity; that is, no upper limit exists. Reading the top line, Lower Activity reveals the activity level of -4117.51953 would result if Input Cost were increased to an Upper Cost of infinity. The change in the objective function per unit decrease in Column 45 down to -4117.51953 is shown as 1.50506. Limiting Process indicates that Column 49 would leave the basis if the activity level of this column (Column 45) were decreased below Lower Activity. AT demonstrates that Column 49 will leave the basis at its Lower Limit (LL). For more detailed information on Range see Mathematical Programming Systems Handbook (8).

4.2 Cost Coefficient Analysis

Due to the variable nature of transportation mode cost coefficients, it may be informative to examine the effect of varying certain cost coefficients. This procedure can be performed by a MPS/360 subroutine PARAOBJ.

PARAOBJ is used postoptimally to perform parametric programming on the objective function. The original LP problem is modified by replacing the original objective function by the sum of the original objective function and a multiple of a "change row". This multiple is called Param. In other words the cost coefficients may be altered to examine their effect on the solution.

PARAOBJ was used to examine the effect of changing the cost coefficient of 1) over weight truck and 2) Large and VSTOL aircraft. These are realistic goals since the cost of OWT may be considerably decreased once the Interstate system is completed and appropriate legal legislation is enacted. Much research is being done concerning VSTOL aircraft. Their tremendous lift coefficients and short runway requirements are making these aircraft potentially exciting. The computer results of analysis of situation 1) appears in Appendix B and are summarized below.

As Param is gradually varied, the OWT cost is decreased by an amount equal to (0.1)(Param). The table on page 1 of the computer output in Appendix B displays an iteration table showing the vector leaving the basis and the vector entering the basis for the corresponding value of Param as it is gradually changed. The associated objective function value is indicated as Function Value. These results are summarized in Table VI as the cost coefficient of OWT is varied from 1.9 to 0.4. It is seen that no change in the basic solution occurs until the cost of OWT shipment is decreased to 1.1 unit cost per ton-mile. At this point shipment by $x_{1,13}$ left the basis and was replaced by $x_{2,13}$ at 7293.23939 ton-miles, while $x_{6,29}$ decreased from 7590.79808 to 3153.83325 ton-miles, i.e., OWT shipment along route 13 (36-94-70) has replaced LWT shipment and a part of the rail shipment along route 29. This appears on pages 36-39 of the PARAOBJ computer output, Appendix B.

Similarly, Table VII represents the effect of varying the cost coefficient of VSTOL and Large Aircraft (LA). Notice that air transport enters into use in all reactors except Reactor 4 where it has been hypothesized to be geographically prohibitive to construct an airstrip.

Table VI

EFFECT OF VARYING OVER WEIGHT TRUCK COST

PARAM	OWT COST	VARIABLE	TON-MILES	OBJECTIVE FUNCTION
0	1.9	$x_{6,25}$	7983.87097	48152.00720
		$x_{1,7}$	16800.0	
		$x_{1,13}$	3425.11621	
		$x_{6,29}$	7590.79808	
		$x_{1,19}$	9237.28814	
1	1.8	no change		
2	1.7	no change		
3	1.6	no change		
4	1.5	no change		
5	1.4	no change		
6	1.3	no change		
7	1.2	no change		
8	1.1	$x_{6,25}$	7983.87097	47425.09652
		$x_{1,7}$	16800.0	
		$x_{2,13}$	7293.23939	
		$x_{6,29}$	3153.83325	
		$x_{1,19}$	9237.28814	
9	1.0	$x_{6,25}$	7983.87097	46695.77259
		$x_{1,7}$	16800.0	
		$x_{2,13}$	7293.23939	
		$x_{6,29}$	3153.83325	
		$x_{1,19}$	9237.28814	
10	0.9	$x_{6,25}$	7983.87097	43576.97806
		$x_{2,7}$	16800.0	
		$x_{2,13}$	7293.23939	
		$x_{6,29}$	3153.83325	
		$x_{2,18}$	4681.93405	
		$x_{2,19}$	4793.41379	
11	0.8	no change		40220.11884
12	0.7	no change		36863.25962
13	0.6	no change		33506.40039
14	0.5	no change		30149.54117
15	0.4	no change		26792.68195

Table VII

EFFECT OF VARYING VSTOL AND LARGE AIRCRAFT COST

PARAM	VSTOL COST	LA COST	VARIABLE	TON-MILES	TOTAL COST
0	2.5	2.2	$x_{6,25}$	7983.87097	48152.0072
			$x_{1,7}$	16800.0	
			$x_{1,13}$	3425.1162	
			$x_{6,29}$	7590.79808	
			$x_{1,19}$	9237.28814	
2	2.1	1.8	-- NO CHANGE --		
4	1.7	1.4	$x_{5,2}$	6556.29139	46445.88229
			$x_{1,7}$	16800.0	
			$x_{1,13}$	3060.08116	
			$x_{5,40}$	5835.5036	
			$x_{1,19}$	9237.28814	
6	1.3	1.0	$x_{5,2}$	6556.29139	36818.01562
			$x_{5,9}$	12631.57895	
			$x_{5,40}$	8392.85714	
			$x_{1,19}$	9237.28814	
8	0.9	0.6	-- NO CHANGE --		25785.72462
10	0.5	0.2	-- NO CHANGE --		14753.43363

4.3 Risk Coefficient Analysis

The risk coefficient derived in section 2.3 is based on parameters which may be subject to change. For example, the seriousness of accident index (SA) may be greatly affected by improved construction of vehicle, careful route choices or better maintenance. It is then of interest to examine the effect that a change in the risk coefficient of a particular mode has on the model. This objective is attained by the use of the MPS/360 subroutine PARAROW.

PARAROW is used postoptimally to perform parametric programming on a constraint row. For any LP problem, a series of related problems may be generated by replacing the coefficients of a chosen row by new coefficients that are obtained by adding the original coefficients to Param times corresponding coefficients of a change row. Param is gradually changed thus changing the coefficients of the original constraint row.

PARAROW was applied to the risk coefficients of LWT for Reactor 2. As the Param value gradually changed the solution remained at the optimal activity until Param equalled 2.82. At this point the solution changed. Originally all the spent fuel from Reactor 2 was shipped by LWT over route 36-94-70. The current risk coefficient equals 0.218+(0.1)(2.82) or 0.5. This forces some of the spent fuel to be shipped by rail to satisfy the risk constraint. A further increase in risk coefficient does not change the basis, but it does affect the amount shipped by truck or rail. It is noted that the higher the LWT risk coefficient becomes, the more ton-miles are attributed to rail shipment by route 5-48-20-21-24. For instance when Param equals 10, the risk coefficient LWT by route 36-94-70 equals 0.218+10(0.10) or 1.218. At this point, 5713.23529 ton-miles are attributed to LWT and 11086.76471 are attributed to rail.

A further example of the effect of changing the risk was performed by decreasing the risk coefficient of OWT for all reactors. This had no effect on the solution which is to be expected since there is no shipment by OWT in the original solution.

4.4 Conclusions

The shipping model formulated for this work determines the most economical mode and route of transportation of spent fuel. Even in the simplified model used here it is evident that the complexity of the problem makes an optimal solution far from intuitively obvious. MPS/360 appears to be an inexpensive and speedy method of determining an optimal shipping campaign. The computer solution (Primal) for this model costs $1.55 and had an execution time of 0.24 minutes.

It has been noted that much of the data used is subject to change, but this is readily accomplished with the computer procedures described. The shipping model described is quite flexible with regard to sensitivity of the optimal results to changes in input data. The model can easily be modified to accept any corrections or updating. It is easy to imagine an agency responsible for shipping spent fuel utilizing a model of this nature and continually updating the matrix to accompany changes such as:

1) Cost rise or fall due to economic trends or negotiations
2) Route characteristic variance, e.g., by season
3) Changes in existing transportation modes
4) Introduction of new modes
5) Risk Limits become more or less restrictive
6) Any other model fluctuations

Reviewing the results of the present model, it is predicted that the four reactors in question will ship approximately two thirds of their spent fuel by LWT and the rest by rail. Presently, it is expected that in the near future about one half of the spent fuel will be shipped by truck and slightly less than that will be shipped by rail (10). However, the future result may be quite different depending upon the updating of existing cost or risk coefficients. For instance, air shipment could become economically competitive if its cost could be decreased to 1.4 as shown in Table VII. Further, air transport would completely dominate Reactors 1, 2, and 3 if the cost could be decreased to 1.0. Reactor 4 is hypothesized to be geographically prohibited from building an air strip, but certainly a mixed mode transportation campaign could be considered. It is evident that this type of model may find use as a predictive aid in anticipating future transport mode posibilities or routes of shipment as well as analyzing existing modes and routes.

5.0 SUGGESTIONS FOR FURTHER STUDY

It is suggested that to make the model more accurate a more detailed analysis of quantification of hazard be performed. This analysis would involve a detailed study of accident probability, seriousness of accident, and the relative importance of these factors to hazard quantification. In addition, a realistic evaluation of the risk upper limit and the risk coefficient demands that an absolute rather than relative evaluation of risk be performed. That is, an actual population dose for a given set of circumstances should be calculated. The maximum permissible dose would then determine the upper limit of risk. This would require a study of the radionuclides released upon accident and their dispersal characteristics. This implies a study of the relation between severity of accident and the consequential release of radionuclides.

Further parametric programming seems warranted. In particular, postoptimal analysis is possible on the RHS of the matrix (the risk upper limit) through a MPS/360 subroutine called PARARHS. It may also be of interest to change values in the objective function and RHS simultaneously. This can be done through a MPS/360 subroutine called PARARIM. These procedures enable the existing model to be readily analyzed concerning the effect of changing the risk upper limit or the effect of simultaneously varying the cost coefficients and the risk upper limit.

6.0 Acknowledgement

The author expresses his gratitude to Dr. Walter Meyer and Dr. N. D. Eckhoff for their guidance and encouragement throughout this work. Financial support from the National Defense and Education Act Fellowship is gratefully acknowledged. A note of thanks is extended to Janet Gaines for her perceptive typing and to the entire Nuclear Engineering faculty and staff for successfully integrating the author into a new university and curriculum.

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APPENDICES

APPENDIX A

MPS/360 Computer Listing and Output for Optimization and Range Analysis

This computer program arrives at an optimal primal solution and performs RANGE analysis on the solution. MPS programming is explained in Section 3.1; Range analysis is explained in Section 4.1. Execution times for PRIMAL and RANGE were 0.45 minutes and 0.17 minutes, respectively.

CONTRCL PROGRAM COMPILER - MPS/360 V2-M10 PAGE 1 - 72/194

```
0001                PROGRAM
0002                INITIALZ
0065                MOVE(XDATA,'TRANS')
0066                MOVE(XPBNAME,'PBFILE')
0067                CONVERT('SUMMARY')
0068                SETUP('MIN')
0069                PICTURE
0070                BCDOUT
0071                MOVE(XOBJ,'COST')
0072                MOVE(XRHS,'LIMITS')
0073                PRIMAL
0074                SOLUTION
0075                RANGE
0076                EXIT
0077                PEND
```

THIS BOOK CONTAINS NUMEROUS PAGES WITH THE ORIGINAL PRINTING BEING SKEWED DIFFERENTLY FROM THE TOP OF THE PAGE TO THE BOTTOM.

THIS IS AS RECEIVED FROM THE CUSTOMER.

```
          EXECUTOR.    MPS/360 V2-M10                                        PAGE    1 - 72/194
CONVERT TRANS    TO PBFILE
TIME =  0.03
    SUMMARY

1- ROWS SECTION.

  0 MINOR ERROR(S) -   0 MAJOR ERROR(S).
2- COLUMNS SECTION.

  0 MINOR ERROR(S) -   0 MAJOR ERROR(S).
3- RHS'S SECTION.
  LIMITS
  0 MINOR ERROR(S) -   0 MAJOR ERROR(S).
THE FOLLOWING ROWS ARE NOT USED.
 15
```

```
          EXECUTOR.    MPS/360 V2-M10                                        PAGE    2 - 72/194
NUMBER OF ELEMENTS BY COLUMN ORDER
 20  COL1  .....3  COL2  .....3  COL3  .....3  COL4  .....3  COL5  .....3  COL6  .....4  COL7  .....4
 27  COL8  .....3  COL9  .....4  COL10 .....4  COL11 .....3  COL12 .....4  COL13 .....4  COL14 .....4
 34  COL15 .....4  COL16 .....3  COL17 .....3  COL18 .....3  COL19 .....4  COL20 .....2  COL21 .....3
 41  COL22 .....3  COL23 .....3  COL24 .....4  COL25 .....4  COL26 .....3  COL27 .....3  COL28 .....3
 48  COL29 .....3  COL30 .....3  COL31 .....4  COL32 .....3  COL33 .....3  COL34 .....3  COL35 .....3
 55  COL36 .....3  COL37 .....3  COL38 .....3  COL39 .....3  COL40 .....3  COL41 .....4  COL42 .....4
 62  COL43 .....3  COL44 .....4  COL45 .....4  COL46 .....3  COL47 .....3  COL48 .....3  COL49 .....3
 69  COL50 .....3  COL51 .....3  COL52 .....3  COL53 .....3  COL54 .....3  COL55 .....3  COL56 .....3
 76  COL57 .....3  COL58 .....3  COL59 .....4  COL60 .....4  COL61 .....3  COL62 .....3  COL63 .....3
```

```
          EXECUTOR.    MPS/360 V2-M10                                        PAGE    3 - 72/194
NUMBER OF ELEMENTS BY ROW ORDER, EXCLUDING RHS'S, INCLUDING SLACK ELEMENT
  1 N COST  ....64  G ROW1  ....17  E ROW2  .....4  E ROW3  .....4  E ROW4  .....4  E ROW5  .....4  E ROW6  .....3
  8 G ROW7  ....10  E ROW8  .....3  E ROW9  .....4  G ROW10 ....17  E ROW11 .....4  E ROW12 .....4  G ROW13 ....13
 15 E ROW14 .....4  L ROW15 ....21  L ROW16 ....12  L ROW17 ....19  L ROW18 .....4
PROBLEM STATISTICS -    19 ROWS,     82 VARIABLES,     225 ELEMENTS, DENSITY =  14.44
THESE STATISTICS INCLUDE ONE SLACK VARIABLE FOR EACH ROW.
   0 MINOR ERRORS,     0 MAJOR ERRORS.
```

```
          EXECUTOR.    MPS/360 V2-M10                                        PAGE    4 - 72/194
SETUP  PBFILE
TIME =  0.14
  MIN

MATRIX1  ASSIGNED TO MATRIX1

ETA1     ASSIGNED TO ETA1

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE    7

NO CYCLING

    POOLS          NUMBER    SIZE     CORE
H.REG-BITS MAP                          88
WORK   REGIONS       9        176     1584
MATRIX BUFFERS       3       1680     5040
ETA    BUFFERS       4       2788     9152

                          TOTAL   NORMAL   .FREE.   FIXED BOUNDED
ROWS    (LOG.VAR.)          19       8        1       10      0
COLUMNS (STR.VAR.)          63      63        0        0      0

     225 ELEMENTS - DENSITY =  14.44 -     2 MATRIX RECORDS (WITHOUT RHS'S)

PICTURE - USING PBFILE

TIME =  0.17
```

1, 1

		COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8	COL9	COL10	COL11	COL12	COL13	COL14	COL15	COL16	COL17	COL18	COL19	COL20	COL21	COL22	COL23	COL24	COL25	COL26	COL27
COST	N	1	A	A	A	A	1	A	A	1	A	A	A	1	1	A	A	A	A	A	A	A	1	A	1	A	A	A
ROW1	G	1	1	A	A		A	A		A	A		A	A	A	A		A	A	A	A							
ROW2	E					-A	A	A																				
ROW3	E								-A	A	A																	
ROW4	E											-A	A	A														
ROW5	E														A	A	-A											
ROW6	E																			A		-A						
ROW7	G																						1	1	A	A		A
ROW8	E																											
ROW9	E																								A	A	-T	
ROW10	G																											
ROW11	E																											
ROW12	E																											
ROW13	G																											
ROW14	E																											
ROW15	L	T	T	-T	-T	-T	T	A	-T	-T	U	-T	T	-T	-T	T	-T	T	T	-T		-T						
ROW16	L																						-T	-U	T	T	-T	-T
ROW17	L																											
ROW18	L																											

	COL28	COL29	COL30	COL31	COL32	COL33	COL34	COL35	COL36	COL37	COL38	COL39	COL40	COL41	COL42	COL43	COL44	COL45	COL46	COL47	COL48	COL49	COL50	COL51	COL52	COL53	COL54	COL55
COST	A	A	A	A	A	1	A	1	A	A	1	A	1	1	A	A	1	A	A	A	A	A	A	1	A	1	A	1
ROW1																												
ROW2																												
ROW3																												
ROW4																												
ROW5																												
ROW6																												
ROW7	A	1	1	A																								
ROW8				-A	A																							
ROW9																												
ROW10						1	1	A	A	T	T	A	A	A	A		A	A		A	A	A	T					
ROW11														A	A	-A												
ROW12																	A	A	-A									
ROW13																								1	1	T	T	A
ROW14																												
ROW15																												
ROW16	-T	T	-T	-T	-T																							
ROW17						T	A	A	A	T	A	T	T	-T	T	-T	A	A	-T	-T	-T	-T	-T					
ROW18																								-T	-T	-T	-T	-T

1, 2

		COL56	COL57	COL58	COL59	COL60	COL61	COL62	COL63	LIMITS
COST	N	A	1	A	1	A	A	A	A	
ROW1	G									D
ROW2	E									
ROW3	E									
ROW4	E									
ROW5	E									
ROW6	E									
ROW7	G									E
ROW8	E									
ROW9	E									
ROW10	G									E
ROW11	E									
ROW12	E									
ROW13	G	A	A	A	A	A		A	A	E
ROW14	E				A	A	-A			
ROW15	L									
ROW16	L									
ROW17	L									
ROW18	L	-T	-T	T	A	A	-T	-T	-T	

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SUMMARY OF MATRIX

SYMBOL	RANGE			COUNT (INCL.RHS)
Z	LESS THAN		.000001	
Y	.000001	THRU	.000009	
X	.000010		.000099	
W	.000100		.000999	
V	.001000		.009999	
U	.010000		.099999	2
T	.100000		.999999	57
1	1.000000		1.000000	28
A	1.000001		10.000000	129
B	10.000001		100.000000	
C	100.000001		1,000.000000	
D	1,000.000001		10,000.000000	1
E	10,000.000001		100,000.000000	3
F	100,000.000001		1,000,000.000000	
G	GREATER THAN		1,000,000.000000	

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RCDOUT - USING PFILE

TIME = 0.19

NAME TRANS
ROWS
N COST
G ROW1
E ROW2
E ROW3
E ROW4
E ROW5
E ROW6
G ROW7
E ROW8
E ROW9
G ROW10
E ROW11
E ROW12
G ROW13
E ROW14
L ROW15
L ROW16
L ROW17
L ROW18
COLUMNS

COL1	COST		1.00000	ROW1		1.00000
COL1	ROW15		.23600			
COL2	COST		1.90000	ROW1		1.00000
COL2	ROW15		.97200			
COL3	COST		2.50000	ROW1		1.51000
COL3	ROW15	-	.47600			
COL4	COST		2.20000	ROW1		1.51000
COL4	ROW15	-	.46400			
COL5	COST		2.70000	ROW2	-	2.48000
COL5	ROW15	-	.46700			
COL6	COST		1.00000	ROW1		1.59000
COL6	ROW2		1.59000	ROW15		.70000
COL7	COST		1.90000	ROW1		1.59000
COL7	ROW2		1.59000	ROW15		.90000
COL8	COST		2.70000	ROW3	-	2.48000
COL8	ROW15	-	.46200			
COL9	COST		1.00000	ROW1		1.48000
COL9	ROW3		1.48000	ROW15	-	.20000
COL10	COST		1.90000	ROW1		1.48000
COL10	ROW3		1.48000	ROW15		.10000
COL11	COST		2.70000	ROW4	-	2.48000
COL11	ROW15	-	.46200			
COL12	COST		1.90000	ROW1		1.70000
COL12	ROW4		1.77000	ROW15		.19600
COL13	COST		1.00000	ROW1		1.77000
COL13	ROW4		1.77000	ROW15	-	.15200
COL14	COST		1.00000	ROW1		1.87000
COL14	ROW5		1.87000	ROW15	-	.18200
COL15	COST		1.90000	ROW1		1.87000
COL15	ROW5		1.87000	ROW15		.13600
COL16	COST		2.70000	ROW5	-	2.48000
COL16	ROW15	-	.46200			

COL17	COST		1.20000	ROW1		1.24000
COL17	ROW15		.87000			
COL18	COST		1.20000	ROW1		1.19000
COL18	ROW15		.92000			
COL19	COST		1.20000	ROW1		1.29000
COL19	ROW6		1.29000	ROW15	-	.31500
COL20	COST		1.20000	ROW1		1.24000
COL21	COST		2.70000	ROW6	-	2.70000
COL21	ROW15	-	.32400			
COL22	COST		1.00000	ROW7		1.00000
COL22	ROW16	-	.28200			
COL23	COST		1.90000	ROW7		1.00000
COL23	ROW16	-	.08000			
COL24	COST		1.00000	ROW7		2.22000
COL24	ROW9		2.20000	ROW16		.11600
COL25	COST		1.90000	ROW7		2.22000
COL25	ROW9		2.20000	ROW16		.83400
COL26	COST		2.70000	ROW9	-	.91000
COL26	ROW16	-	.49000			
COL27	COST		2.20000	ROW7		1.33000
COL27	ROW16	-	.47000			
COL28	COST		2.50000	ROW7		1.33000
COL28	ROW16	-	.48000			
COL29	COST		1.20000	ROW7		1.00000
COL29	ROW16		.30500			
COL30	COST		1.20000	ROW7		1.00000
COL30	ROW16	-	.37000			
COL31	COST		1.20000	ROW7		1.74000
COL31	ROW8	-	1.74000	ROW16	-	.42000
COL32	COST		2.70000	ROW8		1.35000
COL32	ROW16	-	.44000			
COL33	COST		1.00000	ROW10		1.00000
COL33	ROW17		.89000			
COL34	COST		1.90000	ROW10		1.00000
COL34	ROW17		2.20000			
COL35	COST		1.00000	ROW10		1.05000
COL35	ROW17		1.15000			
COL36	COST		1.90000	ROW10		1.05000
COL36	ROW17		2.40000			
COL37	COST		1.90000	ROW10		.91500
COL37	ROW17		.99000			
COL38	COST		1.00000	ROW10		.91500
COL38	ROW17		2.90000			
COL39	COST		1.90000	ROW10		1.17000
COL39	ROW17		.16000			
COL40	COST		1.00000	ROW10		1.17000
COL40	ROW17		.82000			
COL41	COST		1.00000	ROW10		1.22000
COL41	ROW11		1.20000	ROW17	-	.15000
COL42	COST		1.90000	ROW10		1.22000
COL42	ROW11		1.20000	ROW17		.19300
COL43	COST		2.70000	ROW11	-	1.30000
COL43	ROW17	-	.49000			
COL44	COST		1.00000	ROW10		1.40000

```
          EXECUTOR.      MPS/360 V2-M10                                                    PAGE    9  -  72/194

    COL44      ROW12              1.40000   ROW17              1.50000
    COL45      COST               1.90000   ROW10              1.40000
    COL45      ROW12              1.40000   ROW17              3.50000
    COL46      COST               2.70000   ROW12       -      1.68000
    COL46      ROW17       -       .49000
    COL47      COST               2.50000   ROW10              1.40000
    COL47      ROW17       -       .43000
    COL48      COST               2.20000   ROW10              1.40000
    COL48      ROW17       -       .43000
    COL49      COST               1.20000   ROW10              1.02000
    COL49      ROW17       -       .37000
    COL50      COST               1.20000   ROW10               .93500
    COL50      ROW17       -       .33000
    COL51      COST               1.00000   ROW13              1.00000
    COL51      ROW18       -       .46000
    COL52      COST               1.90000   ROW13              1.00000
    COL52      ROW18       -       .43000
    COL53      COST               1.00000   ROW13               .93000
    COL53      ROW18       -       .47000
    COL54      COST               1.90000   ROW13               .93000
    COL54      ROW18       -       .45000
    COL55      COST               1.00000   ROW13              1.12000
    COL55      ROW18       -       .46000
    COL56      COST               1.90000   ROW13              1.12000
    COL56      ROW18       -       .43000
    COL57      COST               1.00000   ROW13              1.18000
    COL57      ROW18       -       .46000
    COL58      COST               1.90000   ROW13              1.18000
    COL58      ROW18               .42000
    COL59      COST               1.00000   ROW13              1.34000
    COL59      ROW14              1.34000   ROW18              1.80000
    COL60      COST               1.90000   ROW13              1.34000
    COL60      ROW14              1.34000   ROW18              4.10000
    COL61      COST               2.70000   ROW14       -      2.80000
    COL61      ROW18       -       .49000
    COL62      COST               1.20000   ROW13              1.20000
    COL62      ROW18       -       .48000
    COL63      COST               1.20000   ROW13              1.11000
    COL63      ROW18       -       .47000
RHS
    LIMITS     ROW1            9900.00000   ROW7           16800.00000
    LIMITS     ROW10          11750.00000   ROW13          10900.00000
ENDATA
```

```
          EXECUTOR.      MPS/360 V2-M10                                                    PAGE   10  -  72/194

PRIMAL        OBJ = COST         RHS = LIMITS

TIME =    0.24  MINS.        PRICING    7

 INVERT CALLED      TIME   0.24   CURRENT INVERSE ---- ETA-VECTORS ......4   ELEMENTS ......4   RECORDS ......1   ITERATION .......3
  BASIS ---- NO.OF ROWS ....19    LOGICALS ........19  STRUCTURALS ......0   ELEMENTS .....19
  INVERSE -- NUCLEUS .........0   TRANSFORMED ......0  ETA-VECTORS ......4   ELEMENTS ......4   RECORDS ......1   TIME TAKEN   0.00

PRIMAL        OBJ = COST         RHS = LIMITS

TIME =    0.25  MINS.        PRICING    7
 SCALE =     .

   ITER    NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER   INFEAS    OUT      IN     COST     INFEAS
M    1        3        2      23   1.51000-  39450.0
M    2        2       11      66   1.40000-  27700.0
M    3        1       14      76   1.18000-  16800.0
M    4        0        8      46   1.33000-     .
 FEASIBLE SOLUTION

PRIMAL        OBJ = COST         RHS = LIMITS

TIME =    0.27  MINS.        PRICING    7
 SCALE =     .
 SCALE RESET TO    1.00000

   ITER    NUMBER  VECTOR  VECTOR  REDUCED    FUNCTION
  NUMBER   NONOPT    OUT      IN     COST       VALUE
M    5       32       17      43   2.67218-   72432.7
     6                 4      31   1.72450-   72432.7
     7                 9      50   1.67819-   72432.7
     8                 5      32   1.57881-   72432.7
     9                13      63   1.50000-   72432.7
    10                 1      25   1.31656-   72432.7
M   11       21       18      59   1.08928-   69099.4
    12                 4      28   1.15629-   69099.4
    13                12      60    .95021-   69099.4
    14                 7      38    .67947-   69099.4
M   15       11       46      41    .65414-   58110.0
    16                66      68    .67381-   52995.2
    17                23      39    .60662-   48152.0
M   18        1       15      78    .13559-   48152.0
 OPTIMAL SOLUTION
```

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SOLUTION (OPTIMAL)

TIME = 0.29 MINS. ITERATION NUMBER = 18

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL	48152.00720	COST
RESTRAINTS		LIMITS

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SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	48152.00720	48152.00720-	NONE	NONE	1.00000
2	ROW1	LL	9900.00000	.	9900.00000	NONE	.96774-
3	ROW2	EC	.	.	.	.	.33881
4	ROW3	EC	.	.	.	.	.29207
5	ROW4	EC	.	.	.	.	.40277
6	ROW5	EC	.	.	.	.	.43298
7	ROW6	EC	.	.	.	.	.03751
8	ROW7	LL	16800.00000	.	16800.00000	NONE	1.00000-
9	ROW8	EC	.	.	.	.	.31034-
10	ROW9	EC	.	.	.	.	.55455
11	ROW10	LL	11750.00000	.	11750.00000	NONE	1.06673-
12	ROW11	EC	.	.	.	.	.27373
13	ROW12	EC	.	.	.	.	.02831
14	ROW13	LL	10900.00000	.	10900.00000	NONE	.84746-
15	ROW14	EC	.	.	.	.	.10119
16	ROW15	BS	.	.	NONE	.	.
17	ROW16	BS	4737.60000-	4737.60000	NONE	.	.
18	ROW17	UL	.	.	NONE	.	.30253
19	ROW18	BS	4249.15254-	4249.15254	NONE	.	.

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SECTION 2 - COLUMNS

	NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
	20	COL1	LL	.	1.00000	.	NONE	.03226
	21	COL2	LL	.	1.90000	.	NONE	.93226
	22	COL3	LL	.	2.50000	.	NONE	1.03871
	23	COL4	LL	.	2.20000	.	NONE	.73871
	24	COL5	LL	.	2.70000	.	NONE	1.85975
	25	COL6	BS	.	1.00000	.	NONE	.
	26	COL7	LL	.	1.90000	.	NONE	.90000
	27	COL8	LL	.	2.70000	.	NONE	1.97568
	28	COL9	BS	.	1.00000	.	NONE	.
	29	COL10	LL	.	1.90000	.	NONE	.90000
	30	COL11	LL	.	2.70000	.	NONE	1.70113
	31	COL12	LL	.	1.90000	.	NONE	.96774
	32	COL13	BS	.	1.00000	.	NONE	.
	33	COL14	BS	.	1.00000	.	NONE	.
	34	COL15	LL	.	1.90000	.	NONE	.90000
	35	COL16	LL	.	2.70000	.	NONE	1.62620
A	36	COL17	LL	.	1.20000	.	NONE	.
	37	COL18	LL	.	1.20000	.	NONE	.04839
	38	COL19	BS	.	1.20000	.	NONE	.
	39	COL20	BS	7983.87097	1.20000	.	NONE	.
	40	COL21	LL	.	2.70000	.	NONE	2.59872
	41	COL22	BS	16800.00000	1.00000	.	NONE	.
	42	COL23	LL	.	1.90000	.	NONE	.90000
	43	COL24	BS	.	1.00000	.	NONE	.
	44	COL25	LL	.	1.90000	.	NONE	.90000
	45	COL26	LL	.	2.70000	.	NONE	2.19536
	46	COL27	LL	.	2.20000	.	NONE	.87000
	47	COL28	LL	.	2.50000	.	NONE	1.17000
	48	COL29	LL	.	1.20000	.	NONE	.20000
	49	COL30	LL	.	1.20000	.	NONE	.20000
	50	COL31	BS	.	1.20000	.	NONE	.
	51	COL32	LL	.	2.70000	.	NONE	2.28103
	52	COL33	LL	.	1.00000	.	NONE	.23252
	53	COL34	LL	.	1.90000	.	NONE	1.49883
	54	COL35	LL	.	1.00000	.	NONE	.22784
	55	COL36	LL	.	1.90000	.	NONE	1.62701
	56	COL37	LL	.	1.90000	.	NONE	1.20211
	57	COL38	LL	.	1.00000	.	NONE	.75893
	58	COL39	LL	.	1.90000	.	NONE	.70033
	59	COL40	BS	3425.11621	1.00000	.	NONE	.
	60	COL41	BS	.	1.00000	.	NONE	.
	61	COL42	LL	.	1.90000	.	NONE	1.00589
	62	COL43	LL	.	2.70000	.	NONE	2.19591
	63	COL44	BS	.	1.00000	.	NONE	.
	64	COL45	LL	.	1.90000	.	NONE	1.50506
	65	COL46	LL	.	2.70000	.	NONE	2.50421
	66	COL47	LL	.	2.50000	.	NONE	.87649
	67	COL48	LL	.	2.20000	.	NONE	.57649
	68	COL49	BS	1590.79808	1.20000	.	NONE	.

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NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
69	COL50	LL	.	1.20000	.	NONE	.10277
70	COL51	LL	.	1.00000	.	NONE	.15254
71	COL52	LL	.	1.90000	.	NONE	1.05254
72	COL53	LL	.	1.00000	.	NONE	.21186
73	COL54	LL	.	1.90000	.	NONE	1.11186
74	COL55	LL	.	1.00000	.	NONE	.05085
75	COL56	LL	.	1.90000	.	NONE	.95085
76	COL57	BS	9237.28814	1.00000	.	NONE	.
77	COL58	LL	.	1.90000	.	NONE	.90000
78	COL59	BS	.	1.00000	.	NONE	.
79	COL60	LL	.	1.90000	.	NONE	.90000
80	COL61	LL	.	2.70000	.	NONE	2.41667
81	COL62	LL	.	1.20000	.	NONE	.18305
82	COL63	LL	.	1.20000	.	NONE	.25932

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RANGE

TIME = 0.33 MINS. ITERATION NUMBER = 18

```
INVERT CALLED      TIME   0.33  CURRENT INVERSE ---- ETA-VECTORS ....22  ELEMENTS ....68  RECORDS .....1  ITERATION .....18
 BASIS ---- NO.OF ROWS ....19   LOGICALS ........4    STRUCTURALS ....15  ELEMENTS ....58
 INVERSE -- NUCLEUS ........2   TRANSFORMED .....1    ETA-VECTORS ....17  ELEMENTS ....56  RECORDS .....1  TIME TAKEN   0.03
```

EXECUTOR. MPS/360 V2-M10 PAGE 16 - 72/194

RANGE

TIME = 0.34 MINS. ITERATION NUMBER = 18

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL	48152.00391	COST
RESTRAINTS		LIMITS

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SECTION 1 - ROWS AT LIMIT LEVEL

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT. ..UPPER LIMIT.	LOWER ACTIVITY UPPER ACTIVITY	...UNIT COST.. ...UNIT COST..	..UPPER COST.. ..LOWER COST..	LIMITING PROCESS.	AT AT
2	ROW1	LL	9900.00000	.	9900.00000 NONE	. INFINITY	.96774- .96774		COL20 NONE	LL
3	ROW2	EQ	.	.	. .	. .	.33881 .33881-		COL6 ROW19	LL UL
4	ROW3	EQ	.	.	. .	. 9899.99609	.29207 .29207-		COL9 COL20	LL LL
5	ROW4	EQ	.	.	. .	. 9899.99609	.40277 .40277-		COL13 COL20	LL LL
6	ROW5	EQ	.	.	. .	. 9899.99609	.43298 .43298-		COL14 COL20	LL LL
7	ROW6	EQ	.	.	. .	. 9899.99609	.03751 .03751-		COL19 COL20	LL LL
8	ROW7	LL	16800.00000	.	16800.00000 NONE	. INFINITY	1.00000- 1.00000		COL22 NONE	LL
9	ROW8	EQ	.	.	. .	16799.99609- .	.31034- .31034		COL22 COL31	LL LL
10	ROW9	EQ	.	.	. .	. 14046.03125	.55455 .55455-		COL24 ROW16	LL UL
11	ROW10	LL	11750.00000	.	11750.00000 NONE	. INFINITY	1.06673- 1.06673		COL40 NONE	LL
12	ROW11	EQ	.	.	. .	. 9871.92578	.27373 .27373-		COL41 COL49	LL LL
13	ROW12	EQ	.	.	. .	. 2971.92358	.02831 .02831-		COL44 COL40	LL LL
14	ROW13	LL	10900.00000	.	10900.00000 NONE	. INFINITY	.84746- .84746		COL57 NONE	LL
15	ROW14	EQ	.	.	. .	. 2451.74512	.10119 .10119-		COL59 ROW18	LL UL
18	ROW17	UL	.	.	NONE .	4262.25391- 8235.04297	.30253 .30253-		COL40 COL49	LL LL

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SECTION 2 - COLUMNS AT LIMIT LEVEL

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT. ..UPPER LIMIT.	LOWER ACTIVITY UPPER ACTIVITY	...UNIT COST.. ...UNIT COST..	..UPPER COST.. ..LOWER COST..	LIMITING PROCESS.	AT AT
20	COL1	LL	.	1.00000	. NONE	INFINITY- .	.03226- .03226	INFINITY .96774	NONE ROW15	UL
21	COL2	LL	.	1.90000	. NONE	INFINITY- .	.93226- .93226	INFINITY .96774	NONE ROW15	UL
22	COL3	LL	.	2.50000	. NONE	. 6556.28516	1.03871- 1.03871	INFINITY 1.46129	ROW15 COL20	UL LL
23	COL4	LL	.	2.20000	. NONE	. 6556.28516	.73871- .73871	INFINITY 1.46129	ROW15 COL20	UL LL
24	COL5	LL	.	2.70000	. NONE	. .	1.85975- 1.85975	INFINITY .84025	COL6 ROW15	LL UL
26	COL7	LL	.	1.90000	. NONE	INFINITY- .	.90000- .90000	INFINITY 1.00000	NONE ROW15	UL
27	COL8	LL	.	2.70000	. NONE	. 3991.93359	1.97568- 1.97568	INFINITY .72432	COL9 COL20	LL LL
29	COL10	LL	.	1.90000	. NONE	INFINITY- .	.90000- .90000	INFINITY 1.00000	NONE COL9	LL
30	COL11	LL	.	2.70000	. NONE	. 3991.93359	1.70113- 1.70113	INFINITY .99887	COL13 COL20	LL LL
31	COL12	LL	.	1.90000	. NONE	141428.50000- .	.96774- .96774	INFINITY .93226	COL20 COL13	LL LL
34	COL15	LL	.	1.90000	. NONE	INFINITY- .	.90000- .90000	INFINITY 1.00000	NONE COL14	LL
35	COL16	LL	.	2.70000	. NONE	. 3991.93359	1.62620- 1.62620	INFINITY 1.07380	COL14 COL20	LL LL
36	COL17	LL	.	1.20000	. NONE	INFINITY- .	. .	INFINITY 1.20000	NONE ROW15	UL
37	COL18	LL	.	1.20000	. NONE	INFINITY- .	.04839- .04839	INFINITY 1.15161	NONE ROW15	UL
40	COL21	LL	.	2.70000	. NONE	. 3666.66479	2.59872- 2.59872	INFINITY .10128	COL19 COL20	LL LL
42	COL23	LL	.	1.90000	. NONE	INFINITY- 16799.99609	.90000- .90000	INFINITY 1.00000	NONE COL22	LL

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NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT. ..UPPER LIMIT.	LOWER ACTIVITY UPPER ACTIVITY	...UNIT COST.. ...UNIT COST..	..UPPER COST.. ..LOWER COST..	LIMITING PROCESS.	AT AT
44	COL25	LL	.	1.90000	. NONE	INFINITY- .	.90000- .90000	INFINITY 1.00000	NONE OL24	 LL
45	COL26	LL	.	2.70000	. NONE	. 18295.21094	2.19536- 2.19536	INFINITY .50464	COL24 COL22	LL LL
46	COL27	LL	.	2.20000	. NONE	49900.96484- 12631.57422	.87000- .87000	INFINITY 1.33000	ROW16 COL22	UL LL
47	CCL28	LL	.	2.50000	. NONE	45145.77344- 12631.57422	1.17000- 1.17000	INFINITY 1.33000	ROW16 COL22	UL LL
48	COL29	LL	.	1.20000	. NONE	INFINITY- 8070.86328	.20000- .20000	INFINITY 1.00000	NONE ROW16	 UL
49	COL30	LL	.	1.20000	. NONE	53836.33594- 16799.99609	.20000- .20000	INFINITY 1.00000	ROW16 COL22	UL LL
51	COL32	LL	.	2.70000	. NONE	. 12444.44141	2.28103- 2.28103	INFINITY .41897	COL31 COL22	LL LL
52	COL33	LL	.	1.00000	. NONE	43538.18750- 3402.33179	.20252- .20252	INFINITY .79748	COL49 COL40	LL LL
53	COL34	LL	.	1.90000	. NONE	5493.15625- 1663.15967	1.49883- 1.49883	INFINITY .40117	COL49 COL40	LL LL
54	COL35	LL	.	1.00000	. NONE	19886.48047- 2784.18164	.22784- .22784	INFINITY .77216	COL49 COL40	LL LL
55	COL36	LL	.	1.90000	. NONE	3989.64819- 1339.95972	1.62701- 1.62701	INFINITY .27299	COL49 COL40	LL LL
56	CCL37	LL	.	1.90000	. NONE	24604.18750- 9206.71191	1.20211- 1.20211	INFINITY .69789	COL49 COL40	LL LL
57	COL38	LL	.	1.00000	. NONE	4464.16016- 1501.23438	.75893- .75893	INFINITY .24107	COL49 COL40	LL LL
58	COL39	LL	.	1.90000	. NONE	INFINITY- 7293.23828	.70033- .70033	INFINITY 1.19967	NONE COL40	 LL
61	COL42	LL	.	1.90000	. NONE	23528.68750- .	1.00589- 1.00589	INFINITY .89411	COL49 COL41	LL LL
62	COL43	LL	.	2.70000	. NONE	. 5230.44141	2.19591- 2.19591	INFINITY .50409	COL41 COL49	LL LL
64	COL45	LL	.	1.90000	. NONE	4117.51953- .	1.50506- 1.50506	INFINITY .39494	COL49 COL44	LL LL

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NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT. ..UPPER LIMIT.	LOWER ACTIVITY UPPER ACTIVITY	...UNIT COST.. ...UNIT COST..	..UPPER COST.. ..LOWER COST..	LIMITING PROCESS.	AT AT
65	COL46	LL	.	2.70000	. NONE	. 2220.60449	2.50421- 2.50421	INFINITY .19579	COL44 COL40	LL LL
66	COL47	LL	.	2.50000	. NONE	INFINITY- 5835.50000	.87649- .87649	INFINITY 1.62351	NONE COL49	 LL
67	COL48	LL	.	2.20000	. NONE	INFINITY- 5835.50000	.57649- .57649	INFINITY 1.62351	NONE COL49	 LL
69	COL50	LL	.	1.20000	. NONE	INFINITY- 8357.90625	.10277- .10277	INFINITY 1.09723	NONE COL49	 LL
70	COL51	LL	.	1.00000	. NONE	60555.57813- 10899.99609	.15254- .15254	INFINITY .84746	ROW18 COL57	UL LL
71	COL52	LL	.	1.90000	. NONE	105780.62500- 10899.99609	1.05254- 1.05254	INFINITY .84746	ROW18 COL57	UL LL
72	COL53	LL	.	1.00000	. NONE	39542.60156- 11720.42578	.21186- .21186	INFINITY .78814	ROW18 COL57	UL LL
73	COL54	LL	.	1.90000	. NONE	48585.28906- 11720.42578	1.11186- 1.11186	INFINITY .78814	ROW18 COL57	UL LL
74	COL55	LL	.	1.00000	. NONE	181666.68750- 9732.13672	.05085- .05085	INFINITY .94915	ROW18 COL57	UL LL
75	COL56	LL		1.90000	. NONE	INFINITY- 9732.13672	.95085- .95085	INFINITY .94915	NONE COL57	 LL
77	COL58	LL	.	1.90000	. NONE	INFINITY- 4828.58203	.90000- .90000	INFINITY 1.00000	NONE ROW18	 UL
79	COL60	LL	.	1.90000	. NONE	INFINITY- .	.90000- .90000	INFINITY 1.00000	NONE COL59	 LL
80	COL61	LL		2.70000	. NONE	. 973.96851	2.41667- 2.41667	INFINITY .28333	COL59 ROW18	LL UL
1	COL62	LL	.	1.20000	. NONE	348194.37500- 9083.32813	.18305- .18305	INFINITY 1.01695	ROW18 COL57	UL LL
82	COL63	LL	.	1.20000	. NONE	113954.50000- 9819.81641	.25932- .25932	INFINITY .94068	ROW18 COL57	UL LL

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SECTION 3 - ROWS AT INTERMEDIATE LEVEL

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT. ..UPPER LIMIT.	LOWER ACTIVITY UPPER ACTIVITY	...UNIT COST.. ...UNIT COST..	..UPPER COST.. ..LOWER COST..	LIMITING PROCESS.	AT AT
16	ROW15	BS	.	.	NONE .	3042.11621- 6945.96094	1.59205 .		COL4 COL17	LL LL
17	ROW16	BS	4737.59766-	4737.59766	NONE .	6215.99707- 5123.99609	2.27273 .34072		COL30 COL29	LL LL
19	ROW18	BS	4249.15234-	4249.15234	NONE .	5508.60083- 4249.15234-	1.97161 .39130		COL53 COL60	LL LL

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SECTION 4 - COLUMNS AT INTERMEDIATE LEVEL

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT. ..UPPER LIMIT.	LOWER ACTIVITY UPPER ACTIVITY	...UNIT COST.. ...UNIT COST..	..UPPER COST.. ..LOWER COST..	LIMITING PROCESS.	AT AT
25	COL6	BS	.	1.00000	. NONE	. .	.90030 1.19234	1.90000 .19234-	COL7 COL5	LL LL
28	COL9	BS	.	1.00000	. NONE	. 6689.18359	.90030 1.17903	1.90000 .17903-	COL10 COL8	LL LL
32	COL13	BS	.	1.00000	. NONE	. 9593.21484	.96774 1.21411	1.96774 .21411-	COL12 COL11	LL LL
33	COL14	BS	.	1.00000	. NONE	. 5294.10938	.90030 1.22621	1.90000 .22621-	COL15 COL16	LL LL
38	COL19	BS	.	1.20000	. NONE	. 7674.41016	INFINITY 1.24161	INFINITY .04161-	NONE COL21	 LL
39	COL20	BS	7983.86719	1.20000	. NONE	7983.86719 INFINITY	. 1.20000	1.20000 .	COL17 ROW1	LL LL
41	COL22	BS	16799.99609	1.00000	. NONE	8729.13672 INFINITY	.20030 1.00000	1.20000 .	COL29 ROW7	LL LL
43	COL24	BS	.	1.00000	. NONE	6598.32031- 7567.56250	.90000 5.33747	1.90000 4.30747-	COL25 COL26	LL LL
50	COL31	BS	.	1.20000	. NONE	. 9655.16797	INFINITY 2.94000	INFINITY 1.74000-	NONE COL32	 LL
59	COL40	BS	3425.11597	1.00000	. NONE	INFINITY- INFINITY	.18521 3.65946	1.18521 2.65946-	COL35 ROW10	LL LL
60	COL41	BS	.	1.00000	. NONE	12177.86328- 5666.30859	1.00589 2.02699	2.00589 1.02699-	COL42 COL43	LL LL
63	COL44	BS	.	1.00000	. NONE	2131.12695- 2664.72485	1.53506 2.08684	2.50506 1.08684-	COL45 COL46	LL LL
68	COL49	BS	7590.79688	1.20000	. NONE	414705.76563- 11519.60547	.11316 .32821	1.31316 .87179	COL50 ROW17	LL UL
76	COL57	BS	9237.28516	1.00000	. NONE	INFINITY- INFINITY	.05357 1.00000	1.05357 .	COL55 ROW13	LL LL
78	COL59	BS	.	1.00000	. NONE	1847.45728- 2035.15771	.90000 1.15655	1.90000 .15655-	COL60 COL61	LL LL

APPENDIX B

MPS/360 PARAOBJ Computer Listing and Output for Analysis of Overweight Truck Cost

As detailed in Section 4.2, the cost of over weight truck shipment was varied in order to examine the effect this cost variation would have on the solution. The computer results are summarized in Table VI. Only the PARAOBJ output corresponding to basis changes is contained in the following output. Pages 1 through 9 are also deleted as they are identical to pages 1 through 9 of Appendix A.

```
CONTROL PROGRAM COMPILER - MPS/360 V2-M10                                PAGE    1 - 72/194

0001                     PROGRAM
0002                     INITIALZ
0065                     MOVE(XDATA,'TRANS')
0066                     MOVE(XPBNAME,'PBFILE')
0067                     CONVERT('SUMMARY')
0068                     SETUP('MIN')
0069                     PICTURE
0070                     BCDOUT
0071                     MOVE(XOBJ,'COST')
0072                     MOVE(XRHS,'LIMITS')
0073                     PRIMAL
0074          *          PARAOBJ
0075          TESTOBJ    TITLE('PARAOBJ')
0076                     MOVE (XCHROW,'VARY')
0077                     XPARAM=0.0
0078                     XPARMAX=15.0
0079                     XPARDELT=1.0
0080                     PARAOBJ
0081                     SOLUTION
0082                     EXIT
0083                     PEND
```

```
              EXECUTOR.    MPS/360 V2-M10                                                              PAGE   10 - 72/194

PRIMAL        OBJ = COST         RHS = LIMITS

TIME =    0.80  MINS.       PRICING    7

 INVERT CALLED      TIME   0.81   CURRENT INVERSE ---- ETA-VECTORS .....4   ELEMENTS .....4   RECORDS .....1   ITERATION ......0
  BASIS ---- NO.OF ROWS ....20    LOGICALS ........20   STRUCTURALS .....0   ELEMENTS ....20
  INVERSE -- NUCLEUS ........0    TRANSFORMED .....0    ETA-VECTORS .....4   ELEMENTS .....4   RECORDS .....1   TIME TAKEN   0.00

PRIMAL        OBJ = COST         RHS = LIMITS

TIME =    0.83  MINS.       PRICING    7
 SCALE =     .

   ITER    NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER   INFEAS    OUT     IN     COST      INFEAS
M    1        3       2      24   1.51000-   39450.0
M    2        2      11      67   1.40000-   27700.0
M    3        1      14      77   1.18000-   16800.C
M    4        0       8      47   1.33000-      .
* FEASIBLE SOLUTION

PRIMAL        OBJ = COST         RHS = LIMITS

TIME =    0.86  MINS.       PRICING    7
 SCALE =     .
 SCALE RESET TO    1.00000

   ITER    NUMBER  VECTOR  VECTOR  REDUCED    FUNCTION
  NUMBER   NONOPT    OUT     IN     COST       VALUE
M    5       32      10      44   2.67218-   72432.7
     6                6      34   1.72450-   72432.7
     7                9      51   1.67819-   72432.7
     8                5      33   1.57881-   72432.7
     9               13      64   1.50000-   72432.7
    10                3      26   1.31656-   72432.7
M   11       21      18      60   1.08428-   69099.4
    12                4      29   1.15629-   69099.4
    13               12      61    .95021-   69099.4
    14                7      39    .67947-   69099.4
M   15       11      47      42    .65414-   58110.C
    16               67      69    .67381-   52995.2
    17               24      40    .60662-   48152.C
M   18        1      15      79    .13559-   48152.0
 OPTIMAL SOLUTION
```

PARACEJ PAGE 1 - 72/194

PARAOBJ OBJ = CCST RHS = LIPITS CHROW = VARY PARAM =

TIME = 0.96 MINS.

PARACBJ PAGE 2 - 72/194

SOLUTION (OPTIMAL)

TIME = 0.97 MINS. ITERATION NUMBER = 18

...NAME...	...ACTIVITY...	DEFINED AS.....................			
FUNCTIONAL	48152.C0720	COST	+	1.CC000	VARY
RESTRAINTS		LIMITS			

PARAOBJ PAGE 3 - 72/194

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	48152.C0720	48152.C0720-	NONE	NONE	1.00000
2	ROW1	LL	9900.COC00	.	99C0.CCCCO	NCNE	.96774-
3	ROW2	EC	.	.	.	.	.33881
4	ROW3	EC	.	.	.	.	.29207
5	RCW4	EC	.	.	.	.	.40277
6	ROW5	EC	.	.	.	.	.43298
7	ROW6	EC	.	.	.	.	.03751
8	ROW7	LL	16800.00C00	.	16800.CCCCO	NONE	1.C00C0-
9	ROW8	EC	.	.	.	.	.31034-
10	ROW9	EC	.	.	.	.	.55459
11	ROW10	LL	11750.CCC00	.	11750.CC0C0	NONE	1.06673-
12	ROW11	EC	.	.	.	.	.27373
13	ROW12	EC	.	.	.	.	.02831
14	ROW13	LL	10900.CCC00	.	109C0.CCCCO	NONE	.84746-
15	ROW14	EC	.	.	.	.	.10119
16	ROW15	BS	.	.	NCNE	.	.
17	ROW16	BS	4737.6CC00-	4737.6CCCO	NONE	.	.
18	ROW17	LL	.	.	NONE	.	.30253
19	ROW18	BS	4249.15254-	4249.15254	NCNE	.	.
20	VARY	BS	.	.	NONE	NCNE	1.00000

PARACEJ PAGE 4 - 72/194

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
21	COL1	LL	.	1.COCCO	.	NONE	.03226
22	COL2	LL	.	1.80CCO	.	NONE	.83226
23	COL3	LL	.	2.5CCCO	.	NCNE	1.03871
24	COL4	LL	.	2.20CCO	.	NONE	.73871
25	COL5	LL	.	2.7CCCO	.	NONE	1.85975
26	COL6	BS	.	1.CCCCC	.	NCNE	.
27	CCL7	LL	.	1.80CCO	.	NONE	.8COCO
28	COL8	LL	.	2.70CCC	.	NCNE	1.97568
29	COL9	BS	.	1.CCCCO	.	NCNE	.
30	CCL10	LL	.	1.8CCCO	.	NONE	.80000
31	COL11	LL	.	2.7COCO	.	NONE	1.70113
32	COL12	LL	.	1.8CCCO	.	NCNE	.86774
33	CCL13	BS	.	1.00CCO	.	NONE	.
34	COL14	BS	.	1.CCC00	.	NONE	.
35	COL15	LL	.	1.80CCC	.	NCNE	.80000
36	CCL16	LL	.	2.7CCCC	.	NCNE	1.62620
37	COL17	LL	.	1.2CCCO	.	NONE	.
38	COL18	LL	.	1.2CCCO	.	NONE	.04839
39	COL19	BS	.	1.2CCCC	.	NCNE	.
40	COL20	BS	7983.87097	1.2CCCO	.	NCNE	.
41	COL21	LL	.	2.7CCCC	.	NONE	2.59872
42	CCL22	BS	16800.0CCCO	1.CCCCC	.	NCNE	.
43	CCL23	LL	.	1.8CCCO	.	NONE	.80000
44	COL24	BS	.	1.CCCCO	.	NONE	.
45	COL25	LL	.	1.8CCCO	.	NONE	.80000
46	CCL26	LL	.	2.70CCO	.	NONE	2.19536
47	COL27	LL	.	2.20CCO	.	NONE	.87CCO
48	COL28	LL	.	2.5CCCO	.	NONE	1.17000
49	CCL29	LL	.	1.2CCCC	.	NONE	.20000
50	COL30	LL	.	1.2CCCO	.	NONE	.2CC00
51	COL31	BS	.	1.2CCCO	.	NONE	.
52	CCL32	LL	.	2.7CCCC	.	NCNE	2.28103
53	COL33	LL	.	1.00CCO	.	NONE	.20252
54	COL34	LL	.	1.80CCO	.	NCNE	1.39803
55	COL35	LL	.	1.COCCC	.	NCNE	.22784
56	CCL36	LL	.	1.80CCO	.	NONE	1.527CL
57	COL37	LL	.	1.80CCO	.	NONE	1.10211
58	COL38	LL	.	1.CCCCC	.	NCNE	.75893
59	CCL39	LL	.	1.80CCO	.	NONE	.60033
60	COL40	BS	3425.11671	1.COCCO	.	NONE	.
61	COL41	BS	.	1.CCCCO	.	NONE	.
62	CCL42	LL	.	1.8CCCC	.	NCNE	.90589
63	COL43	LL	.	2.7CC00	.	NONE	2.19591
64	COL44	BS	.	1.CGCCO	.	NONE	.
65	COL45	LL	.	1.8CCCO	.	NONE	1.40506
66	COL46	LL	.	2.7CCCO	.	NONE	2.50421
67	COL47	LL	.	2.5CCCO	.	NONE	.87649
68	COL48	LL	.	2.2CCCO	.	NCNE	.57649
69	CCL49	BS	7590.79808	1.20000	.	NONE	.

PARACBJ PAGE 5 - 72/194

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
70	COL50	LL	.	1.20CCO	.	NONE	.10277
71	COL51	LL	.	1.COCCO	.	NONE	.15254
72	COL52	LL	.	1.8OCCO	.	NCNE	.95254
73	CCL53	LL	.	1.COCCO	.	NONE	.21186
74	COL54	LL	.	1.8OCCO	.	NONE	1.01186
75	COL55	LL	.	1.COCCO	.	NCNE	.05085
76	COL56	LL	.	1.80CCO	.	NONE	.85085
77	COL57	BS	9237.28814	1.COCCO	.	NONE	.
78	COL58	LL	.	1.80CCO	.	NONE	.80000
79	COL59	BS	.	1.CCCCO	.	NCNE	.
80	COL60	LL	.	1.8CCCO	.	NONE	.8C000
81	COL61	LL	.	2.7CCCC	.	NONE	2.41667
82	COL62	LL	.	1.2CCCO	.	NCNE	.18305
83	COL63	LL	.	1.20CCO	.	NONE	.25932

```
         PARAOBJ                                                                          PAGE   36  -  72/194

PARAOBJ      OBJ = COST        RHS = LIMITS      CHROW = VARY        PARAM =   7.00000

TIME =    1.49  MINS.

  ITER   NUMBER  VECTOR  VECTOR  REDUCED   FUNCTION   PARAM
 NUMBER  NONOPT    OUT      IN     COST      VALUE     VALUE
M   19       0      60      59   .00019   48152.0   7.00331
 INVERT DEMANDED AFTER  17 MAJOR/  19 MINOR ITERATIONS - CLOCK CONTROL

 INVERT CALLED      TIME   1.50   CURRENT INVERSE ---- ETA-VECTORS ....23   ELEMENTS ....72   RECORDS .....1   ITERATION .....19
  BASIS ---- NO.OF ROWS ....20   LOGICALS ........5   STRUCTURALS ....15   ELEMENTS ....60
  INVERSE -- NUCLEUS ........2   TRANSFORMED .....1   ETA-VECTORS ....17   ELEMENTS ....97   RECORDS .....1   TIME TAKEN   0.00

PARAOBJ      OBJ = COST        RHS = LIMITS      CHROW = VARY        PARAM =   7.00331

TIME =    1.50  MINS.
```

```
         PARAOBJ                                                                          PAGE   37  -  72/194

SOLUTION   (OPTIMAL)

TIME =   1.52 MINS. ITERATION NUMBER =    19

      ...NAME...    ...ACTIVITY...      DEFINED AS......................

      FUNCTIONAL      41429.09692       COST      +      8.00000  VARY
      RESTRAINTS                        LIMITS
```

```
         PARAOBJ                                                                          PAGE   38  -  72/194

SECTION 1 - ROWS
```

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	53259.68804	53259.68804-	NONE	NONE	1.00000
2	ROW1	LL	9900.00000	.	9900.00000	NONE	.96774-
3	ROW2	EC	.	.	.	.	.33881
4	ROW3	EC	.	.	.	.	.29207
5	ROW4	EC	.	.	.	.	.40277
6	ROW5	EC	.	.	.	.	.43298
7	ROW6	EC	.	.	.	.	.03751
8	ROW7	LL	16800.00000	.	16800.00000	NONE	1.00000-
9	ROW8	EC	.	.	.	.	.31034-
10	ROW9	EC	.	.	.	.	.59455
11	ROW10	LL	11750.00000	.	11750.00000	NONE	1.00486-
12	ROW11	EC	.	.	.	.	.23461
13	ROW12	EC	.	.	.	.	.21629-
14	ROW13	LL	10900.00000	.	10900.00000	NONE	.84746-
15	ROW14	EC	.	.	.	.	.10119
16	ROW15	BS	.	.	NONE	.	.
17	ROW16	BS	4737.60000-	4737.60000	NONE	.	.
18	ROW17	LL	.	.	NONE	.	.47307
19	ROW18	BS	4249.15254-	4249.15254	NONE	.	.
20	VARY	BS	729.32394-	729.32394	NONE	NONE	8.00000

PARACBJ PAGE 39 - 72/194

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
21	COL1	LL	.	1.00000	.	NONE	.03226
22	COL2	LL	.	1.10000	.	NONE	.13226
23	COL3	LL	.	2.50000	.	NONE	1.03871
24	COL4	LL	.	2.20000	.	NONE	.73871
25	COL5	LL	.	2.70000	.	NONE	1.85975
26	COL6	BS	.	1.00000	.	NONE	.
27	COL7	LL	.	1.10000	.	NONE	.10000
28	COL8	LL	.	2.70000	.	NONE	1.97568
29	COL9	BS	.	1.00000	.	NONE	.
30	COL10	LL	.	1.10000	.	NONE	.10000
31	COL11	LL	.	2.70000	.	NONE	1.70113
32	COL12	LL	.	1.10000	.	NONE	.16774
33	COL13	BS	.	1.00000	.	NONE	.
34	COL14	BS	.	1.00000	.	NONE	.
35	COL15	LL	.	1.10000	.	NONE	.10000
36	COL16	LL	.	2.70000	.	NONE	1.62620
37	COL17	LL	.	1.20000	.	NONE	.
38	COL18	LL	.	1.20000	.	NONE	.04839
39	COL19	BS	.	1.20000	.	NONE	.
40	COL20	BS	7983.87097	1.20000	.	NONE	.
41	COL21	LL	.	2.70000	.	NONE	2.59872
42	COL22	BS	16800.00000	1.00000	.	NONE	.
43	COL23	LL	.	1.10000	.	NONE	.10000
44	COL24	BS	.	1.00000	.	NONE	.
45	COL25	LL	.	1.10000	.	NONE	.10000
46	COL26	LL	.	2.70000	.	NONE	2.19536
47	COL27	LL	.	2.20000	.	NONE	.87000
48	COL28	LL	.	2.50000	.	NONE	1.17000
49	COL29	LL	.	1.20000	.	NONE	.20000
50	COL30	LL	.	1.20000	.	NONE	.20000
51	COL31	BS	.	1.20000	.	NONE	.
52	COL32	LL	.	2.70000	.	NONE	2.28103
53	COL33	LL	.	1.00000	.	NONE	.41617
54	COL34	LL	.	1.10000	.	NONE	1.13590
55	COL35	LL	.	1.00000	.	NONE	.46893
56	COL36	LL	.	1.10000	.	NONE	1.36950
57	COL37	LL	.	1.10000	.	NONE	.62880
58	COL38	LL	.	1.00000	.	NONE	1.24314
59	COL39	BS	7293.23939	1.10000	.	NONE	.
60	COL40	LL	.	1.00000	.	NONE	.21223
61	COL41	BS	.	1.00000	.	NONE	.
62	COL42	LL	.	1.10000	.	NONE	.26558
63	COL43	LL	.	2.70000	.	NONE	2.16320
64	COL44	BS	.	1.00000	.	NONE	.
65	COL45	LL	.	1.10000	.	NONE	1.04615
66	COL46	LL	.	2.70000	.	NONE	2.83196
67	COL47	LL	.	2.50000	.	NONE	.80977
68	COL48	LL	.	2.20000	.	NONE	.58977
69	COL49	BS	3153.83325	1.20000	.	NONE	.

PARACBJ PAGE 40 - 72/194

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
70	COL50	LL	.	1.20000	.	NONE	.10434
71	COL51	LL	.	1.00000	.	NONE	.15254
72	COL52	LL	.	1.10000	.	NONE	.25254
73	COL53	LL	.	1.60000	.	NONE	.21186
74	COL54	LL	.	1.10000	.	NONE	.31186
75	COL55	LL	.	1.00000	.	NONE	.05085
76	COL56	LL	.	1.10000	.	NONE	.15085
77	COL57	BS	9237.20814	1.00000	.	NONE	.
78	COL58	LL	.	1.10000	.	NONE	.10000
79	COL59	BS	.	1.00000	.	NONE	.
80	COL60	LL	.	1.10000	.	NONE	.10000
81	COL61	LL	.	2.70000	.	NONE	2.41667
82	COL62	LL	.	1.20000	.	NONE	.18305
83	COL63	LL	.	1.20000	.	NONE	.25932

PARAOBJ — PAGE 46 - 72/194

PARAOBJ OBJ = CCST RHS = LIMITS CHROW = VARY PARAM = 9.C0000

TIME = 1.62 MINS.

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE	PARAM VALUE
M	20	0	16	27	.	46695.8	9.0C000
M	21	0	42	43	.	46695.8	9.CCC00
M	22	0	44	45	.	46695.8	9.CCCC0
M	23	0	19	78	.	46695.8	9.00C00
M	24	0	29	30	.	46695.8	9.0CC00
M	25	0	34	35	.	46695.8	9.CCCC0
M	26	0	77	76	.05085	45214.0	9.51232
M	27	0	33	32	.03137	43731.C	9.95411

PARACEJ — PAGE 47 - 72/194

SOLUTION (OPTIMAL)

TIME = 1.64 MINS. ITERATION NUMBER = 27

...NAME...	...ACTIVITY...	DEFINED AS.........................
FUNCTIONAL	43576.97806	CCST + 10.0C000 VARY
RESTRAINTS		LIMITS

PARACEJ — PAGE 48 - 72/194

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	77145.57028	77145.57028-	NONE	NONE	1.C00C0
2	ROW1	LL	9900.CCC00	.	9900.CC000	NONE	.96774-
3	ROW2	EC	.	.	.	.	.30212
4	ROW3	EC	.	.	.	.	.354C0
5	ROW4	EC	.	.	.	.	.41177
6	ROW5	EC	.	.	.	.	.48040
7	ROW6	EC	.	.	.	.	.05786
8	ROW7	LL	16800.CCC00	.	16800.00000	NONE	.9C0C0-
9	ROW8	EC	.	.	.	.	.21034-
10	ROW9	EC	.	.	.	.	.49909
11	ROW10	LL	11750.CCCC0	.	11750.CC000	NONE	.88072-
12	ROW11	EC	.	.	.	.	.15610
13	ROW12	EC	.	.	.	.	.70710-
14	ROW13	LL	10900.C0C00	.	10900.0C000	NONE	.78237-
15	ROW14	EC	.	.	.	.	.03808-
16	ROW15	UL	.	.	NONE	.	.08333
17	ROW16	BS	1344.00C00-	1344.00000	NONE	.	.
18	ROW17	UL	.	.	NONE	.	.81530
19	ROW18	UL	.	.	NONE	.	.05523
20	VARY	BS	3356.85922-	3356.85922	NONE	NONE	10.C0000

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
21	COL1	LL	.	1.00000	.	NONE	.09192
22	COL2	LL	.	.90000	.	NONE	.01926
23	COL3	LL	.	2.50000	.	NONE	.99904
24	COL4	LL	.	2.20000	.	NONE	.70004
25	COL5	LL	.	2.70000	.	NONE	1.91223
26	COL6	BS	.	1.00000	.	NONE	.
27	COL7	BS	.	.90000	.	NONE	.
28	COL8	LL	.	2.70000	.	NONE	1.78357
29	COL9	LL	.	1.00000	.	NONE	.07500
30	COL10	BS	.	.90000	.	NONE	.
31	COL11	LL	.	2.70000	.	NONE	1.64032
32	COL12	BS	.	.90000	.	NONE	.
33	COL13	LL	.	1.00000	.	NONE	.00326
34	COL14	LL	.	1.00000	.	NONE	.07350
35	COL15	BS	.	.90000	.	NONE	.
36	COL16	LL	.	2.70000	.	NONE	1.47011
37	COL17	LL	.	1.20000	.	NONE	.07250
38	COL18	LL	.	1.20000	.	NONE	.12505
39	COL19	BS	.	1.20000	.	NONE	.
40	COL20	BS	7983.87097	1.20000	.	NONE	.
41	COL21	LL	.	2.70000	.	NONE	2.51678
42	COL22	LL	.	1.00000	.	NONE	.10000
43	COL23	BS	16800.00000	.90000	.	NONE	.
44	COL24	LL	.	1.00000	.	NONE	.10000
45	COL25	BS	.	.90000	.	NONE	.
46	COL26	LL	.	2.70000	.	NONE	2.24583
47	COL27	LL	.	2.20000	.	NONE	1.00300
48	COL28	LL	.	2.50000	.	NONE	1.30300
49	COL29	LL	.	1.20000	.	NONE	.30000
50	COL30	LL	.	1.20000	.	NONE	.30000
51	COL31	BS	.	1.20000	.	NONE	.
52	COL32	LL	.	2.70000	.	NONE	2.41603
53	COL33	LL	.	1.00000	.	NONE	.84489
54	COL34	LL	.	.90000	.	NONE	1.81293
55	COL35	LL	.	1.00000	.	NONE	1.01283
56	COL36	LL	.	.90000	.	NONE	2.25008
57	COL37	LL	.	.90000	.	NONE	.88367
58	COL38	LL	.	1.00000	.	NONE	2.21477
59	COL39	BS	7293.23939	.90000	.	NONE	.
60	COL40	LL	.	1.00000	.	NONE	.63810
61	COL41	BS	.	1.00000	.	NONE	.
62	COL42	LL	.	.90000	.	NONE	.18535
63	COL43	LL	.	2.70000	.	NONE	2.09758
64	COL44	BS	.	1.00000	.	NONE	.
65	COL45	LL	.	.90000	.	NONE	1.53060
66	COL46	LL	.	2.70000	.	NONE	3.48842
67	COL47	LL	.	2.50000	.	NONE	.91641
68	COL48	LL	.	2.20000	.	NONE	.81641
69	COL49	BS	3153.83325	1.20000	.	NONE	.

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
70	COL50	LL	.	1.20000	.	NONE	.10747
71	COL51	LL	.	1.00000	.	NONE	.19223
72	COL52	LL	.	.90000	.	NONE	.09388
73	COL53	LL	.	1.00000	.	NONE	.24644
74	COL54	LL	.	.90000	.	NONE	.14755
75	COL55	LL	.	1.00000	.	NONE	.09834
76	COL56	BS	4681.93905	.90000	.	NONE	.
77	COL57	LL	.	1.00000	.	NONE	.05140
78	COL58	BS	4795.41379	.90000	.	NONE	.
79	COL59	BS	.	1.00000	.	NONE	.
80	COL60	LL	.	.90000	.	NONE	.02702
81	COL61	LL	.	2.70000	.	NONE	2.77958
82	COL62	LL	.	1.20000	.	NONE	.23465
83	COL63	LL	.	1.20000	.	NONE	.30561

PARAOBJ PAGE 51 - 72/194

PARAOBJ OBJ = COST RHS = LIMITS CHROW = VARY PARAM = 10.00000

TIME = 1.68 MINS.

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE	PARAM VALUE
M	28	0	79	80	.02702	42782.1	10.2368
M	29	0	27	22	.00876	41234.6	10.6978

PARAOBJ PAGE 52 - 72/194

SOLUTION (OPTIMAL)

TIME = 1.69 MINS. ITERATION NUMBER = 29

...NAME...	...ACTIVITY...	DEFINED AS......................
FUNCTIONAL	40220.11884	COST + 11.00000 VARY
RESTRAINTS		LIMITS

PARAOBJ PAGE 53 - 72/194

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	77145.57028	77145.57028-	NONE	NONE	1.00000
2	ROW1	LL	9900.00000	.	9900.00000	NONE	.96774-
3	ROW2	EQ	.	.	.	.	.26284
4	ROW3	EQ	.	.	.	.	.41554
5	ROW4	EQ	.	.	.	.	.45838
6	ROW5	EQ	.	.	.	.	.52738
7	ROW6	EQ	.	.	.	.	.07965
8	ROW7	LL	16800.00000	.	16800.00000	NONE	.80000-
9	ROW8	EQ	.	.	.	.	.11034-
10	ROW9	EQ	.	.	.	.	.44364
11	ROW10	LL	11750.00000	.	11750.00000	NONE	.01865-
12	ROW11	EQ	.	.	.	.	.11684
13	ROW12	EQ	.	.	.	.	.95250-
14	ROW13	LL	10900.00000	.	10900.00000	NONE	.69544-
15	ROW14	EQ	.	.	.	.	.05178-
16	ROW15	LL	.	.	NONE	.	.17257
17	ROW16	BS	1344.00000-	1344.00000	NONE	.	.
18	ROW17	UL	.	.	NONE	.	.98641
19	ROW18	LL	.	.	NONE	.	.04909
20	VARY	BS	1356.85922-	1356.85922	NONE	NONE	11.00000

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
21	COL1	LL	.	1.00000	.	NONE	.07299
22	COL2	BS	.	.80000	.	NONE	.
23	COL3	LL	.	2.50000	.	NONE	.99656
24	COL4	LL	.	2.20000	.	NONE	.69864
25	COL5	LL	.	2.70000	.	NONE	1.96844
26	COL6	BS	.	1.00000	.	NONE	.
27	COL7	LL	.	.80000	.	NONE	.00709
28	COL8	LL	.	2.70000	.	NONE	1.58973
29	COL9	LL	.	1.00000	.	NONE	.14823
30	COL10	BS	.	.80000	.	NONE	.
31	COL11	LL	.	2.70000	.	NONE	1.40548
32	COL12	BS	.	.80000	.	NONE	.
33	COL13	LL	.	1.00000	.	NONE	.07220
34	COL14	LL	.	1.00000	.	NONE	.14512
35	COL15	BS	.	.80000	.	NONE	.
36	COL16	LL	.	2.70000	.	NONE	1.31236
37	COL17	LL	.	1.20000	.	NONE	.15014
38	COL18	LL	.	1.20000	.	NONE	.20716
39	COL19	BS	.	1.20000	.	NONE	.
40	COL20	BS	7983.87097	1.20000	.	NONE	.
41	COL21	LL	.	2.70000	.	NONE	2.42903
42	COL22	LL	.	1.00000	.	NONE	.20000
43	COL23	BS	16800.00000	.80000	.	NONE	.
44	COL24	LL	.	1.00000	.	NONE	.20000
45	COL25	BS	.	.80000	.	NONE	.
46	COL26	LL	.	2.70000	.	NONE	2.29629
47	COL27	LL	.	2.20000	.	NONE	1.13600
48	COL28	LL	.	2.50000	.	NONE	1.43600
49	COL29	LL	.	1.20000	.	NONE	.40000
50	COL30	LL	.	1.20000	.	NONE	.40000
51	COL31	BS	.	1.20000	.	NONE	.
52	COL32	LL	.	2.70000	.	NONE	2.55103
53	COL33	LL	.	1.00000	.	NONE	1.05925
54	COL34	LL	.	.80000	.	NONE	2.15145
55	COL35	LL	.	1.00000	.	NONE	1.27479
56	COL36	LL	.	.80000	.	NONE	2.70237
57	COL37	LL	.	.80000	.	NONE	1.01111
58	COL38	LL	.	1.00000	.	NONE	2.70059
59	COL39	BS	7293.23939	.80000	.	NONE	.
60	COL40	LL	.	1.00000	.	NONE	.85103
61	COL41	BS	.	1.00000	.	NONE	.
62	COL42	LL	.	.80000	.	NONE	.14524
63	COL43	LL	.	2.70000	.	NONE	2.06476
64	COL44	BS	.	1.00000	.	NONE	.
65	COL45	LL	.	.80000	.	NONE	1.77282
66	COL46	LL	.	2.70000	.	NONE	3.81686
67	COL47	LL	.	2.50000	.	NONE	.92973
68	COL48	LL	.	2.20000	.	NONE	.62973
69	COL49	BS	3153.83325	1.20000	.	NONE	.

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
70	COL50	LL	.	1.20000	.	NONE	.10904
71	COL51	LL	.	1.00000	.	NONE	.28190
72	COL52	LL	.	.80000	.	NONE	.08345
73	COL53	LL	.	1.00000	.	NONE	.33017
74	COL54	LL	.	.80000	.	NONE	.13115
75	COL55	LL	.	1.00000	.	NONE	.19853
76	COL56	BS	4681.93905	.80000	.	NONE	.
77	COL57	LL	.	1.00000	.	NONE	.15680
78	COL58	BS	4791.41179	.80000	.	NONE	.
79	COL59	LL	.	1.00000	.	NONE	.08709
80	COL60	BS	.	.80000	.	NONE	.
81	COL61	LL	.	2.70000	.	NONE	2.82092
82	COL62	LL	.	1.20000	.	NONE	.34191
83	COL63	LL	.	1.20000	.	NONE	.40499

OPTIMIZATION OF ROUTES AND MODES OF NUCLEAR MATERIAL TRANSPORTATION CONSTRAINED BY SAFETY CONSIDERATIONS

by

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ABSTRACT

The amount of spent fuel from reactors which is to be shipped in the near future may grow to a level sufficient enough to limit the development of the nuclear power industry. Specifically, the complexity of determining the most economic route and mode while satisfying safety restrictions will most likely lead to increasingly non-optimal or unsafe shipments as the amount of spent fuel to be shipped increases.

The purpose of this work was to develop and analyze a model which characterizes the choice of route and mode for spent fuel shipment from one of several reactors constrained by safety considerations. A model was established which represents the choice of route and mode constrained by a safety factor. The safety factor employed is a function of probability of accident and severity of accident.

The complexity of analyzing such a model implies a computer approach. This model is analyzed with linear programming through a computer package, Mathematical Programming System (MPS). MPS efficiently handles the model formulated in this work and can evidently handle more complex situations. That is the least expensive route and mode was found for the model, while satisfying the safety requirements. In addition, the flexibility of MPS is demonstrated through the use of parametric programming which modifies specified parameters of the model.