#### POST-TENSIONED RIBBED MAT FOUNDATIONS ON HIGHLY EXPANSIVE SOILS

by

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#### **Abstract**

Highly expansive soils can severely damage the foundations which they support. These damages create unnecessary maintenance cost to the owner and can be detrimental to the building superstructure. Post-tensioned ribbed mat foundations are commonly used in light commercial construction in areas in the United States that have highly expansive soils. Mildreinforced ribbed mat foundations are rarely used in these areas. This report investigates why post-tensioned ribbed mat foundations are more common in these areas than mild-reinforced ribbed mat foundations. The approach to this investigation is a design example which designs and compares the two foundation types. The design example is a typical 2-story office building located in Dallas, Texas, which is an area that has highly expansive soils. First, a post-tensioned ribbed mat foundation is designed for the office building. Next, a mild-reinforced ribbed mat foundation is designed for the same building. A comparison is done between the two foundations based on serviceability, strength requirements and construction costs. The findings in the comparison is that post-tensioning is a more economical and constructible method. Using mild-reinforcement requires the use of shear reinforcement in the ribs which is not typical in foundation design and construction and is less economical, and additional reinforcement in the slab is needed to resist bending stresses which is also less economical. The finding of the report is that of the two foundation types, the post-tensioned ribbed mat foundation is the better design based on the three areas of interest listed above. The use of a mild-reinforced mat foundation would require construction procedures that are not typical and would be less economical.

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# **Dedication**

I dedicate this report to my parents, my two brothers, my grandparents, and to the memory of my great grandfather.

#### **CHAPTER 1 - Introduction**

Post-tensioning is a method of prestressing concrete to relieve tension stresses within the concrete. This method is used for several types of reinforced concrete structures and can have several advantages such as reduced section size, reduced cracking, and longer spans when compared to the typical mild reinforcement used in reinforced concrete. This report focuses on one application in which post-tensioning is advantageous and determines why this method is more advantageous than mild reinforcement.

The application under consideration is ribbed mat foundations on highly expansive soils. Highly expansive soils, such as clays, expand when exposed to a large amount of moisture and contract when they are exposed to very dry conditions. Moreover, highly expansive soils can severely damage foundations and structures which they support. Specifically, these soils tend to create two conditions: center lift and edge lift. The way in which the soil expands or contracts can either lift the center of the foundation upward creating high tensile stresses in the top of the foundation, or lift the edges of the foundation upward creating high tensile stresses in both the top and bottom of the foundation. The results of these two conditions will be determined in two design examples executed in Chapter 7 and Chapter 8. General background information regarding mat foundations, post-tensioning, and mild-reinforcement will be discussed prior to the design examples.

Chapter 2 of this report is an introduction to mat foundations in general. Mat foundations are defined, compared to other foundations, advantages and disadvantages are listed, methods of analysis are given, and design procedures as given by the American Concrete Institute (ACI) are presented. Chapter 2 is also an introduction to the edge lift and center lift conditions within ribbed mat foundations that are created by expansive soils. Chapter 3 is a brief introduction to typical mild reinforcement. This chapter also introduces the elements and characteristics of concrete along with the characteristics of reinforcing steel. Chapter 4 is a discussion of post-tensioning in general including all applications and advantages, not just those related to foundations. Chapter 4 is also an introduction to the components used in typical post-tensioning construction and ends by reviewing ACI Code requirements for post-tensioned slabs-on-ground. Chapter 2, Chapter 3, and Chapter 4 give the background information needed to lead into a

comparison of post-tensioned ribbed mat foundations and mild-reinforced ribbed mat foundations.

Chapter 5 introduces the design steps for both a post-tensioned ribbed mat foundation and a mild-reinforced ribbed mat foundation. Additionally, the design steps list the soil properties that are needed to complete the design and also the appropriate analysis procedures used for both methods.

Chapter 6 introduces a design problem to compare both a post-tensioned ribbed mat foundation and a mild-reinforced ribbed mat foundation. This chapter defines the design loads and calculates the required thickness of the foundation slab. The design problem is a typical 2-story office building located in Dallas, Texas. The soil conditions at the construction site are such that the soil is considered to be highly expansive. Due to the inability to obtain an actual soils report for site conditions, the following soil properties were assigned to create a soil profile that would be considered highly expansive: plastic limit, liquid limit, allowable soil bearing pressure, percentage of soil passing No. 200 sieve, percentage of soil finer than 2 microns, soil unit weight, and modulus of elasticity of soil.

Chapter 7 is a design of a post-tensioned ribbed mat foundation for the design problem using the design steps in Chapter 5 and the design information in Chapter 6. The section properties of this design are a ribbed mat foundation consisting of a 6" slab with ribs extending 21" past the slab at specified locations. For comparison, Chapter 8 is a design of a mild-reinforced ribbed mat foundation using the same section properties as found in Chapter 7. The difference between the two designs is the type and amount of reinforcement used in the section. In Chapter 9, a comparison of the two sections and related costs and constructability of each section is shown. In conclusion, Chapter 10 evaluates the findings from the design problem.

#### **CHAPTER 2 - Ribbed Mat Foundations**

A ribbed mat foundation can be one of the more complicated support systems for building superstructures. To further understand this support system, this Chapter is a discussion of what mat foundations are, how they are different from other foundations, their advantages and applications, different methods of analysis, the edge moisture variation distance  $(e_m)$  and the differential soil movement  $(y_m)$ , and ACI code recommendations.

#### **DEFINITION**

Every building structure rests on the earth, which means all vertical and horizontal loads are transferred through the superstructure down into the earth. The element of the structural system that supports the superstructure is the foundation, which is supported in some way by the soil or rock which lies underneath it. (Bowles, 1977) The way in which the foundation is supported by the soil or rock depends on the type of foundation used.

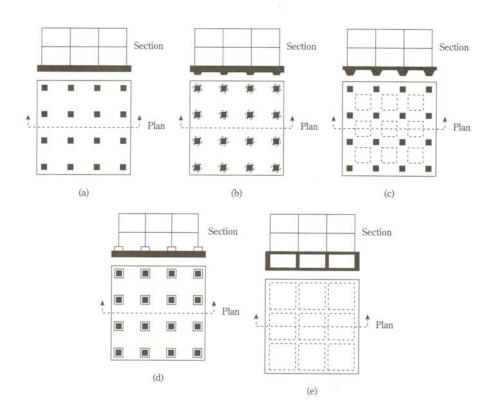
Foundations have several requirements that need to be satisfied to adequately support the superstructure above it. These requirements take into account stability and deformation issues. First, the depth of the foundation should be below the frost line of the soil to protect against freeze and thaw cycles within the soil. This is important because the freeze and thaw cycles can either push the foundation upwards or leave the foundation unsupported in certain areas. Also sliding, overturning, rotation, and shear requirements need to be satisfied. Furthermore, because foundations consist of concrete and steel reinforcing and are exposed to moisture and other harmful materials, corrosion and deterioration also need to be addressed. In addition, the foundation should be able to perform as intended in the event of changes to the surrounding site conditions such as new excavations. Finally, settlement and differential movements should be limited so as to not detract from the performance of the superstructure and the foundation. These requirements are addressed in Chapter 7 & Chapter 8. (Bowles, 1977)

Along with the requirements that a foundation needs to satisfy, each type of foundation has a specific function or purpose for its use. A ribbed mat foundation (sometimes referred to as a raft foundation or stiffened slab) is a type of combined footing. Combined footings are used when at least two or more columns are supported on the same foundation because a standard

spread footing is not adequate for each column. (Das, 2004) A ribbed mat foundation is a large concrete slab with ribs that rests on the ground and transmits the loads from columns and walls into the ground. The slab is generally flat and 4" to 6" thick for typical low-rise office and residential buildings supported on ribbed mat foundations and can be 10" or thicker for mat foundations of a constant thickness or for mat foundations used in high-rise buildings. (Bowles, 1977) The slab can be made thicker if needed to adequately resist design moments and shears in the mat. The soil under the mat is generally of poor type, not typically rocky, and has low bearing capacities. Therefore, the purpose of a ribbed mat foundation is to attempt to transfer all loads uniformly into the soil to reduce differential settlement. Should some columns carry vastly higher column loads than other columns, a mat foundation allows the higher column loads to be distributed over a larger area of the soil instead of into an isolated area of the soil such as with individual spread footings. This will promote even settlement throughout the foundation. When used over expansive soils, ribbed mat foundations also prevent cracking in the mat. Expansive soils experience expansion when exposed to moisture and also shrinkage when exposed to moisture loss (Jones, 1973). Consequently, cracking occurs because swelling of the soil pushes the slab up and induces negative moment (tension stresses in the top of the slab) into the slab. The tension stresses in the top of the slab can increase the slab thickness and can also cause cracking which is not desired. Expansive soils can also contract away from the foundation, which can in turn cause negative moments throughout other sections of the mat. (Chen, 1975)

Several types of mat foundations are popular including ribbed mat foundations. The type of foundation used depends on the intensity of the loading from columns and walls, geometry of the slab, and soil properties. Figure 2-1 on the next page shows a section and plan view of some of the more common types. Figure 2-1a is a flat slab of uniform thickness. Figure 2-1b is a flat slab with the slab thickened underneath the columns. Figure 2-1c is a slab combined with beams that run both directions where the columns are located at the intersection of the beams, also considered a ribbed mat foundation. Figure 2-1d is a flat slab with the columns located on pedestals. Finally, Figure 2-1e is a slab with basement walls used as part of the mat. (Das, 2004)

Figure 2-1 Section and Plan Views of Common Mat Foundation Types (Reproduced from Das, 2004)



## **Difference Between Mat and Other Foundation Types**

A shallow foundation is defined as one whose depth is less than its width (Bowles, 1977). Thus, continuous wall footings and column spread footings are shallow concrete foundations that isolate the loads from the building into a designated area of soil underneath. The load capacity of the footing is based upon the allowable bearing capacity of the soil and also upon the allowable settlement of the footing. Consequently, each individual spread footing and each continuous wall footing can be designed for its individual load case. (Das, 2004)

A deep foundation is one whose depth is greater than its width. Piles and caissons are examples of deep foundations. Pile foundations can be made of timber, concrete, or steel. The load capacity of a pile is based upon two different elements that affect the piles' bearing capacity, surface friction and point bearing. Surface friction takes into account the interaction between the surface area of the pile and the surrounding soil. For some piles, the majority of capacity comes from surface friction. Point bearing is based on the area of the pile that is

bearing on soil or typically rock. To bear on rock, the pile depth can be 75 feet or even deeper. Piles can be used to support single columns or to support shallow foundations bearing on the piles. (Das, 2004) Generally, piles are used in combination with other piles to support the structure above them. (Bowles, 1977)

Ribbed mat foundations are similar to spread footings and continuous wall footings in how they distribute the loads to the soil. This is mainly because a ribbed mat foundation is also a shallow foundation. The difference between ribbed mat foundations and the others lies within settlement considerations. Column spread footings are isolated and their settlement can affect parts of the structure connected to the column but will not affect parts of the structure not connected to the column. Continuous wall footings can have problems with differential settlement but will not affect parts of the structure that are not connected to the continuous footing. Conversely, ribbed mat foundations are connected to multiple parts if not every part of the superstructure, making differential settlement the main concern with ribbed mat foundations. Because ribbed mat foundations cover a larger area than spread and continuous wall footings, the settlement of any part of the foundation will directly affect all other parts of the superstructure. On the other hand, piles and caissons differ from ribbed mat foundations in both load bearing and settlement considerations. The deep foundations use surface friction as well as point bearing to obtain their bearing capacity. Therefore, the settlements of piles and caissons are isolated and can affect parts of the superstructure, but an isolated settlement won't affect the entire superstructure. When settlements are isolated into a single foundation element, the part of the superstructure connected to that foundation element will settle but the rest of the superstructure will only settle as much as the isolated foundation element it is connected to. However, because a ribbed mat foundation is one single large foundation element, any differential settlement of the foundation affects the entire superstructure because the entire superstructure is connected to the foundation. Consequently, for a ribbed mat foundation to be effective, the designer needs to meet all serviceability requirements to minimize any differential settlements.

## **Advantages and Applications of Ribbed Mat Foundations**

The advantages of ribbed mat foundations are directly related to the applications in which mats are used. One of these advantages is the ability of the foundation to support high column loads. When a building has several columns that support high loading conditions, placing a

ribbed mat foundation can be more economical than placing several spread footings. Generally, when more than 50% of the building plan area is covered by footings, a ribbed mat foundation can be the most cost-effective solution. (Bowles, 1977) This is taking into account the costs of labor and formwork. For some commercial structures and high rise structures with several columns to support high loading conditions, a ribbed mat foundation could be an economical and functional support system.

A second advantage of ribbed mat foundations is their ability to evenly distribute building loads onto the soil. This allows for an even settlement of the building structure as long as differential settlements are small. Even settlement is important because it can help mitigate cracking in the mat. For several structures, such as warehouses, mitigating cracking and differential settlement is important because of the operation of forklifts and other machinery. These machines can be sensitive to lips or bumps caused by cracking in the slab.

Another advantage of ribbed mat foundations is their ability to resist expansive soils. Expansive soils can cause several problems for foundations. Mat foundations are applicable for locations which contain these soils. Expansive soils may cause considerable differential movement in a foundation. Ribbed mat foundations can be used effectively to transfer the moments caused by differential settlement induced by the expansive soil. (Desai 1977)

Another application for ribbed mat foundations is their use in residential construction. Properly designed mats can mitigate cracking in foundation walls and slabs. Cracks can allow moisture into the building and are not aesthetically pleasing.

## **Methods of Analyzing Ribbed Mat Foundations**

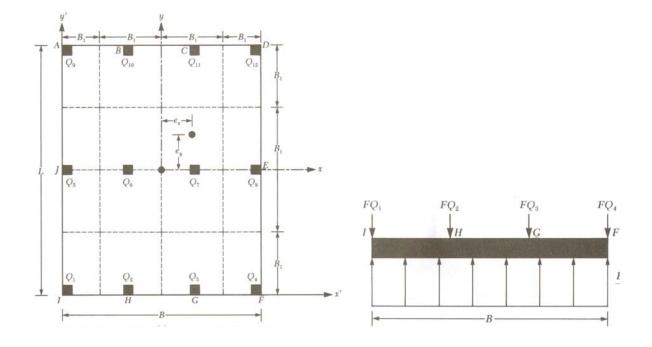
The structural analysis of mat foundations can be carried out by several different traditional analysis methods. Such methods discussed in this section will include the rigid method, the approximate flexible method, the finite difference method, the finite grid method, and the finite-element method.

#### Rigid Method

The rigid method involves calculating the total column loads of the building structure and then determining the pressure acting on the soil based on the area of the ribbed mat foundation. The determined pressure acting on the soil is compared with the allowable soil pressure to make

sure that the allowable soil pressure is not exceeded. The actual analysis of the foundation starts by dividing the mat into several strips in both directions. Then each individual strip is analyzed based on total column loads acting on the strip along with the soil reaction acting on the strip. Figure 2-2 on this page illustrates this analysis. The column loads and soil reactions are modified to account for the shear between adjacent strips. The modification is a weighted average between the individual column loads and the average soil pressure the foundation resists. The modified column loads and soil reactions are used to create shear and moment diagrams for each strip. The next step is to determine the effective depth of the mat. Diagonal tension and shear near the columns are used as the criteria for determining the effective depth. After the moment and shear diagrams have been created for each individual strip, the maximum positive and negative moments for each direction are determined. These moments are used to determine the top and bottom steel used in each direction. In this method, the mat is assumed to be infinitely rigid, which means that every element of the foundation is fixed to each element connected to it, and the soil pressure is distributed in a straight line with the centroid coincident with the line of action of the resultant column loads. (Das, 2004)

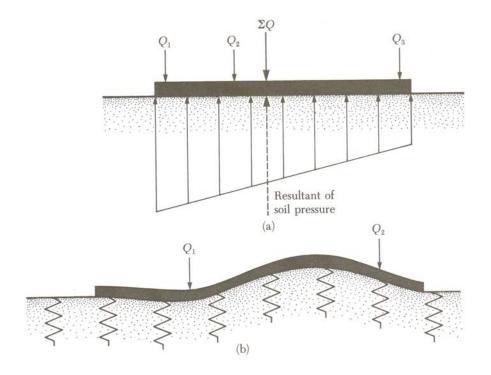
Figure 2-2 Illustration of Rigid Method Analysis (Reproduced from Das, 2004)



#### Approximate Flexible Method

The approximate flexible method assumes the soil is equivalent to an infinite number of elastic springs, sometimes referred to as the Winkler foundation. The Winkler foundation is a method used to model the foundation as individual plates that are supported by elastic springs. The springs are given an elastic constant referred to as the coefficient of subgrade reaction. The value of the coefficient of subgrade reaction depends on the length and width of the foundation, and also the depth of embedment of the foundation. Typically, the coefficient of subgrade reaction can be calculated based on load tests carried out in the field by geotechnical engineers. After determining the coefficient of subgrade reaction, engineers can begin the design procedure. Figure 2-3 on this page illustrates the approximate flexible method analysis. (Das, 2004)

Figure 2-3 Illustration of Approximate Flexible Method (Reproduced from Das, 2004)



The approximate flexible method is based on the theory of plates, which allows the moment, shear, and deflection of each concentrated column load to be evaluated. The first step is to assume a thickness for the mat and then determine the flexural rigidity of the mat based on the assumed thickness. The assumed mat thickness is based on calculations for punching shear

and one-way shear within the mat. Using the flexural rigidity and the coefficient of subgrade reaction, the radius of effective stiffness, L', is determined by Equation 2-1. The next step is to determine the moment in polar coordinates (to simplify calculations in the next step) caused by a column load. The moment is then converted into Cartesian coordinates (Mx and My). The shear force caused by the column load is then determined for a unit width of the mat. Finally the last step is determining the deflection at certain points in the mat by using appropriate equations or analysis techniques. (Das, 2004)

$$L' = \sqrt[4]{\frac{R}{k}}$$
 [Equation 2-1]

where

k = coefficient of subgrade reaction

$$R = \frac{E_F h^3}{12(1 - \mu^2_F)}$$

where

 $E_F$  = modulus of elasticity of foundation material

 $\mu_F$  = Poisson's ratio of foundation material

h = assumed mat thickness

## Finite Element Method, Finite Difference Method, & Finite Grid Method

The following information covering the finite element method has been produced from Edward J. Ulrich's article "Mat foundation design; An historical perspective" from the journal titled "Vertical and horizontal deformations of foundations and embankments; proceedings of Settlement '94." The finite element method (FEM) is usually implemented into a computer program and provides a complete structural analysis of mat foundations. The FEM is capable of offering two-way bending considerations, comprehensive bearing stratum interaction using the beam-on-elastic foundation concept. It is capable of analyzing unusual and complex mat shapes, mats with significant thickness differences, mats transferring large moments and axial forces from laterally loaded shear walls or frames, and mats in which structure rigidity affects mat behavior and stress distribution. The FEM analysis is based on the theory of flat-plate bending with the mat supported by soil. For analysis, the soil is modeled as springs and the mat is

modeled as a mesh of discrete elements interconnected at the node points. The springs are used as the soil response model at each node.

The finite difference method, the finite grid method, and the finite element method all use computer programs that break the mat down into plate elements, which have certain boundary criteria for each method. Differential equations are applied to each separate plate element and used to determine moments in both the x and y directions of the plate. The analysis and programming used for these methods can be very intense are beyond the scope and focus of this report because expansive soils are being considered and so more time can be focused on design of the foundation. The design problem in Chapter 8 of this report will use an estimation of design forces following the guidelines set in the PTI design method for slabs on ground. For a more detailed explanation of each traditional method please consult the following article and textbook references in the back of this report:

**Bowles**, 1986

Bowles 1977

Ulrich, 1994

# Edge Moisture Variation Distance (e<sub>m</sub>) & Differential Soil Movement (y<sub>m</sub>)

The edge moisture variation distance  $(e_m)$  is the distance beneath the edge of a shallow foundation within which moisture will change due to wetting or drying influences around the perimeter of the foundation. For the edge lift condition, the moisture in the soil is higher at the edges than in the center. For the center lift condition, the moisture in the soil is higher in the center than in the edges. The major factor in determining the edge moisture variation distance is the unsaturated diffusion coefficient,  $\alpha$ . This coefficient depends on the level of suction, the permeability, and the cracks in the soil. For the same unsaturated diffusion coefficient, the  $e_m$  value will be larger for the center lift case in which the moisture is drawn from soil around the perimeter of the foundation. The  $e_m$  value will be smaller for the edge lift condition in which moisture is drawn beneath the building into drier soil. Additionally, conditions such as roots, layers, fractures, or joints in the soil will increase the unsaturated diffusion coefficient value and increase the  $e_m$  value for both center lift and edge lift conditions. Representative values based on laboratory test results in each layer of soil are used to calculate the edge moisture variation

distance. The representative values of each layer of soil needed to perform the calculations are listed on the next page: (Post-Tensioning Institute, 2004)

- Liquid Limit, LL
- Plastic Limit, PL
- Plasticity Index, PI
- Percentage of soil passing No. 200 sieve, (%-#200)
- Percentage of soil finer than 2 microns,  $(\%-2\mu)$

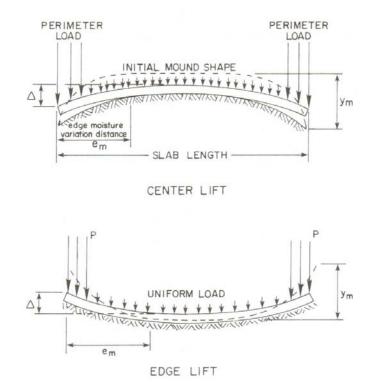
The above soil properties are important because they determine the soil type, how the soil reacts when exposed to different environments, and the structural integrity of the soil. A detailed calculation for determining  $e_m$  is given in Chapter 7 and Chapter 8 of this report.

The differential soil movement  $(y_m)$  is estimated using the change in soil surface elevation at two locations separated by the edge moisture variation distance within which the differential movement will occur. An initial and a final suction profile should be used at the edge of the foundation to determine differential movement. The initial profile may be equilibrium suction or a wet or dry profile, depending on conditions that are believed to be present at time of construction. The final suction profile at each location should be determined from controlling suction conditions at the surface. These suction profiles are used to determine the  $y_m$  value. (Post-Tensioning Institute, 2004)

Soil suction quantifies the energy level in the soil-moisture system. An imbalance of total suction between either the environment or adjacent soil tends to drive moisture towards the higher value of soil suction. Soil suction is expressed as (pF), which is the logarithm to the base 10 of a column of water in centimeters that could be theoretically supported by the energy level described, or as direct measurement of the height of a column of water in centimeters or as a negative pressure in pounds per square foot. Soil suction can be measured in a sample of soil by the filter paper method (ASTM 5298) or by various types of phychrometers, pressure membranes or ceramic pressure plate systems. Soil suction field values range from high values of 4.5pF in the vicinity of trees at the wilting point up to 6.0pF for bare sun-baked ground. On the wetter side, soil suction can range down to 2.5pF, which is about as wet as soil practically gets in field conditions. (Post-Tensioning Institute, 2004)

A detailed calculation for determining  $y_m$  is given in Chapter 7 and Chapter 8 of this report. Figure 2-4 illustrates the center and edge lift conditions.

Figure 2-4 Illustration of Center and Edge Lift Conditions (Reproduced from Post-Tensioning Institute, 2004)



# ACI 336.2R Suggested Analysis and Design Procedures for Combined Footings and Mats

ACI 336.2R gives suggested analysis and design procedures for combined footings and mats. A mat foundation is defined in Section 1.3.1 as a continuous footing supporting an array of columns in several rows in each direction, having a slab like shape with or without depressions or openings, covering an area of at least 75 percent of the total area within the outer limits of the assembly. Section 2.1 states that the mat transmits loads from columns and walls into the soil. The way the mat responds is a complex interaction of the mat foundation itself, the superstructure above it, and the soil below it. Unfortunately, the accurate determination of the contact pressures and associated subgrade response can not be evaluated because too many variables and uncertainties are unknown with soil. Consequently, assumptions must be made to simplify the design of mats. The assumptions are made based on previous experience and

several variables concerning the mat foundation. These variables include the following: soil type below, soil type at great depths below the footing, size, shape, and stiffness of the footing, eccentricity of all the loads, superstructure stiffness, and the modulus of subgrade reaction. All of these variables are discussed in detail in Section 2.2 of ACI 336.2R.

Chapter 6 in ACI 336.2R discusses mat foundations exclusively. In Section 6.1.2, the suggested design procedure is given as follows:

- 1.) Proportion the mat plan using unfactored loads and any overturning moments. The proportioned mat is used to determine the actual soil contact pressure based on the eccentricities within the mat. A total resultant load of the structure, P, is applied based on eccentricities in both the x and y directions. The unfactored loads are used to compare the applied pressure from the mat to the allowable soil pressure.
- 2.) Compute the minimum mat thickness based on punching shear at critical columns, which are columns carrying the highest gravity loads. The mat thickness is usually made such that shear reinforcement is not needed.
- 3.) Design the reinforcing steel for bending by treating the mat as a rigid body and considering strips in both directions.
- 4.) Perform an approximate analysis based on the method suggested by ACI 336.2R-66 or a computer analysis of the mat and revise the rigid body design as necessary. When using a simplified design method, the subgrade response values are determined by a geotechnical engineer. Computer analysis methods, which are the finite difference (FD), the finite grid method (FGM), and the finite element method (FEM) are the methods suggested by ACI. All three methods use the modulus of subgrade reaction (given the symbol, k) as the method in which the soil reacts or contributes to the structural model.

The subgrade reaction, k, is used to model the soil reaction behavior to the foundation more accurately than using a linear distribution soil reaction. Using a spring as the soil reaction at various locations helps to provide a more realistic prediction of interaction between the mat and the soil. This is because most soil conditions in which a mat is used are poor and can be variable. Therefore, the spring concept allows the mat foundation to be subjected to positive and

negative bending moments based on the variable soil interactions at different parameters of the
mat.

# **CHAPTER 3 - Typical Reinforced Concrete**

Concrete and steel reinforcing compliment each other very well in reinforced concrete structures, because the advantages of each material compensate for the disadvantages of the other. Concretes disadvantage is low tensile strength while high tensile strength is one of the advantages of reinforcing steel. Consequently, when bonded together, the two materials create a structural element that has strengths high in compression and high in tension. This chapter will briefly review the elements and characteristics of concrete, give information about mild steel reinforcement, and explain the design procedures for the strength of reinforced concrete members.

#### **Elements and Characteristics of Concrete**

The three basic ingredients in a concrete mixture are portland cement, water, and aggregates. Portland cement is the type of cement commonly used in concrete, and it is a closely controlled chemical combination of calcium, silicon, aluminum, iron, and small amounts of other ingredients to which gypsum is added in the final grinding process to regulate the setting time of the concrete. Water is needed to chemically react with the cement, a process called hydration, and to provide workability with the concrete. Thus, the water and the cement form a paste that coats the aggregate and sand in the mix. When the paste hardens, it binds the aggregates and sand together. The amount of water in the mix is compared with the amount of cement in the mix and is called the water/cement ratio (w/c ratio). A low w/c ratio creates a stronger concrete and a high w/c ratio creates a concrete which is easier to work with. The aggregates can be any combination of sand (fine aggregates) and gravel or crushed stone (coarse aggregates). All three ingredients together form concrete.

Admixtures can be used to improve certain characteristics of the concrete. Common types of admixtures are; accelerating admixtures, retarding admixtures, fly ash, air entraining admixtures, and water reducing admixtures. Accelerating admixtures reduce the amount of time required for the concrete to set. Retarding admixtures increase the amount of time required for the concrete to set. Fly ash helps improve the workability of the concrete and makes it easier to finish. Air entraining admixtures help protect the concrete from freeze and thaw cycles.

Specific characteristics of concrete include compressive strength and tensile strength. In particular, the compressive strength of concrete is typically 3000-7000psi, while the tensile strength of concrete varies from about 8%-15% of its compressive strength. Often, the tensile strength of concrete is neglected in design considerations because of its relatively small contribution. (McCormac, 2006)

#### Mild Steel Reinforcement

The reinforcing steel used for concrete structures may be in the form of bars or welded-wire reinforcement. Reinforcing bars are referred to as plain or deformed, although deformed bars are more popular because they have ribbed projections rolled onto their surfaces to provide better bonding between the concrete and the steel. In fact, deformed bars are used for almost all applications and welded-wire reinforcement is used typically for slabs, pavements, and shells. (McCormac, 2006)

Reinforcing bars are available in different grades of steel: Grade 50, Grade 60, and Grade 75. Grade 60 steel is commonly used in reinforced concrete and has a yield strength of 60,000psi. Different types of reinforcing bars are designated by an ASTM standard. These types are listed below. The majority of reinforcing bars used in reinforced concrete structures conform to ASTM A615. (McCormac, 2006)

- 1. ASTM A615: Deformed and plain billet steel bars. These bars, which must be marked with the letter S (for type of steel), are the most widely used reinforcing bars in the United States.
- 2. ASTM A706: Low alloy deformed and plain bars. These bars, which must be marked with the letter W (for type of steel), are to be used where controlled tensile properties and/or specially controlled chemical composition is required for welding purposes.
- 3. ASTM A996: Deformed rail steel or axle steel bars. They must be marked with the letter R (for type of steel).

# **Design Procedures for Reinforced Concrete**

The entire cross section of a concrete member will resist bending stresses when the tensile stresses in the member are smaller than the modulus of rupture of the concrete. The modulus of rupture is the bending tensile stress at which the concrete begins to crack. The

moment that produces the bending stresses large enough to surpass the modulus of rupture is called the cracking moment,  $M_{cr}$ . Once the concrete section has been subjected to a moment higher than  $M_{cr}$ , the steel located in the area where the tensile stresses occur will begin to resist the tensile stress. This is where the concrete section benefits from having reinforcing steel in it. To illustrate, Figure 3-1 shows a simple span beam subjected to bending stresses. The top portion of the beam will resist the compression stresses while the bottom portion of the beam will resist the tensile stresses. When steel is placed in the bottom portion of the beam, the steel will resist the tensile stresses. To determine if concrete section is adequate, the area of concrete which is responsible for resisting the compression stresses needs to be determined. This is done by determining the depth of the compression block in the member, a. The depth of the compression block is determined by setting the ultimate tensile capacity of the section ( $T = A_s f_y$ ) equal to the ultimate compressive capacity of the section ( $T = A_s f_y$ ) and solving for a. Once the depth of the compression block is determined, the nominal bending strength of the section can be determined. Figure 3-2 illustrates the distribution of forces in the member section.

Figure 3-1 Compression and Tension Forces

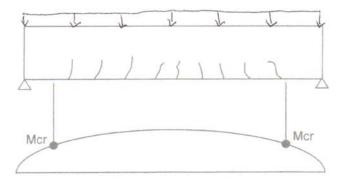
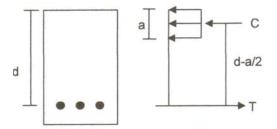


Figure 3-2 Compression and Tension Forces



Mild reinforced concrete members are used often in concrete design and construction. In fact, the majority of applications for reinforced concrete members can be designed using mild reinforcement. However, situations occur where using mild reinforcement in concrete is not economical or even practical. A solution to some of these situations is post-tensioning, which is another method of relieving tension stresses in concrete. Post-tensioning is explained next in Chapter 4.

## **CHAPTER 4 - Post-Tensioning**

Post-tensioning is a method of prestressing concrete to relieve tension stresses in the concrete. This section explains post-tensioning, its applications, and its components. ACI design requirements for post-tensioned slabs on ground, which includes ribbed mat foundations, are also covered.

#### **Post-Tensioning in General**

Post-tensioning is a method of prestressing concrete, which is designed to relieve tension stresses in the concrete. Prestressed concrete is defined in ACI 440.4R Prestressing Concrete Structures with FRP Tendons (Secured) as concrete in which the internal stresses have been initially introduced so that the subsequent stresses resulting from dead load and superimposed loads are counteracted to a desired degree. Prestressing is accomplished by two methods, pretensioning and post-tensioning. Both methods use prestressing steel strands to provide the internal stresses introduced to the concrete. The difference between the two methods occurs when the strands are tensioned (or stressed). In pre-tensioning the strands are tensioned before the concrete is placed around the steel, whereas in post-tensioning the strands are tensioned after the concrete has reached its required strength, usually between 3500 psi and 7000 psi. The required strength depends on the design strength of the concrete. This chapter will focus on post-tensioning because it is the focus of this report. (Post-Tensioning Institute, 2006)

Three types of post-tensioning systems exist; unbonded post-tensioning systems, which are common in building construction, bonded post-tensioning systems which are common in bridge construction, and external post-tensioning systems, which are common in retrofit of building structures. Unbonded and bonded post-tensioning discussed next, are considered internal post-tensioned systems. (Post-Tensioning Institute, 2006)

#### **Unbonded Post-Tensioning Systems**

Unbonded post-tensioning systems, which are used almost exclusively in the United States, consist of tendons that are single strands coated with a corrosion inhibitor such as P/T coating shown in Figure 4-4. These strands are also protected by an extruded plastic sheathing. The sheathing allows the strand to move inside of it and prevents water from contacting the strand. The purpose of allowing the strand to move inside the sheathing is to keep the strand unbonded from the surrounding concrete. The strands are anchored to the concrete by ductile iron anchors and hardened steel wedges. (Post-Tensioning Institute, 2006) The benefits of unbonded post-tensioning include maximum possible tendon eccentricities which is beneficial in thin slabs, simpler and quicker installation, low losses of prestressing forces due to friction, and more economical installation (Ritz, 1985). The tendon is supported by chairs and bolsters to maintain the desired shape and height of the tendon. When the tendon is placed in aggressive environments, where chlorides and other harmful agents are present, it can be encapsulated. An encapsulated tendon is defined in ACI 423.6-01 Specification for Unbonded Single Strand Tendons and Commentary as a tendon that is completely enclosed in a watertight covering from end to end, including a protective cap over the tendon tail at each end. Figure 4-1 on this page illustrates the difference between an encapsulated tendon and a standard tendon. The standard tendon on the left is shown with no extra protection around the tendon. The encapsulated tendon on the right shows encapsulation of the tendon to provide extra protection around the tendon. (Post-Tensioning Institute, 2006)

Figure 4-1 Illustration of Standard Tendon and Encapsulated Tendon (Reproduced from Post-Tensioning Institute, 2006)



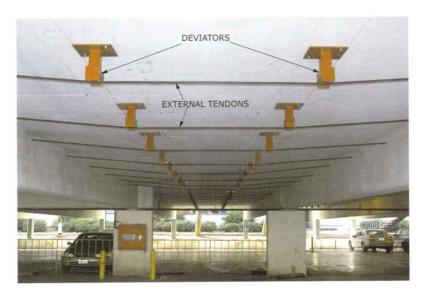
#### **Bonded Post-Tensioning Systems**

Bonded post-tensioning systems utilize tendons that consist of multiple strands. These strands are placed in corrugated galvanized steel, high density polyethylene (hard plastic made from petroleum similar to laundry detergent containers), or polypropylene (smooth plastic similar to plastic used in margarine tubs or straws) ducts. The strands can be installed before the concrete is placed, or sometimes the steel ducts are installed without the strands inside them. When the steel ducts are installed without the strands are pushed or pulled through the ducts after the concrete has been placed. In both situations, after the concrete has reached the required strength, the tendons are stressed and the ducts are filled with grout. The grout is used to both provide protection for the strands from corrosion and to bond the strands to the surrounding concrete. (Post-Tensioning Institute, 2006) The benefits of bonded post-tensioning systems are larger ultimate moment capacity and limited effects to the structure due to local failure of a tendon (Ritz, 1985).

#### **External Post-Tensioning Systems**

In external post-tensioning systems, the tendons are installed outside of the structural concrete member except at anchorages or deviation points. External tendons are used primarily for bridges, retrofit, and repair applications. The prestressing steel is either greased and sheathed, as in a typical single strand unbonded tendon, or enclosed in a duct which is filled with grout. In both applications, the system is considered to be an unbonded system because relative movement is allowed between the tendon and the member the tendon is attached to. Figure 4-2 gives an illustration of external post-tensioning. The prestressing tendons are seen installed outside of the structural floor above them. The influence from the tendons is applied to the floor above them through deviators as seen in Figure 4-2 on the next page. (Post-Tensioning Institute, 2006)

Figure 4-2 Illustration of External Post-Tensioning System (Reproduced from Post-Tensioning Institute, 2006)



#### **Applications of Post-Tensioning Systems**

Post-tensioning offers many benefits for a wide range of new construction and retrofit applications. Its primary benefit is its ability to balance the individual strengths of concrete and prestressing steel to utilize the total cross section of the structural component. Notably, concrete is strong in compression and weak in tension in terms of strength compared to steel. Further, prestressing steel has a very high tensile strength of 270,000 psi per strand, which is more than 4 times that of common reinforcing bars (60,000 psi). Therefore, the two materials combined are able to efficiently resist both compressive and tensile forces. So ultimately, post-tensioning can be a very efficient applications for buildings (both commercial and residential), parking structures, bridges, storage structures, stadiums, rock and soil anchors, and slabs on ground. However, the primary focus of this report is the application of post-tensioning in buildings and primarily ribbed mat foundations (slabs on ground). (Post-Tensioning Institute, 2006)

Commercial buildings, residential apartments, high-rise condominiums, office buildings, parking structures, and several other types of facilities can structurally utilize post-tensioning to obtain several benefits. The first benefit is the reduction of concrete and steel required to obtain the required strength of the structural member as compared to typical reinforcing. A reduced

amount of materials can lead to a reduced amount in the cost of the building. The second benefit concerns the depth of the structure. Post-tensioned members tend to be less deep than nonprestressed members. This is because post-tensioned members are able to use their gross section instead of their cracked section. A more shallow structure depth results in lower building heights and reduced weight of the structure. Another benefit is the ability of the structure to span greater lengths, which reduces the number of columns used to transfer loads to the ground. Some other benefits are reduction in deflection, better crack control, and a reduction in overall building mass. Reduction in deflection provides better serviceability for the materials used in construction such as glass or brick. Better crack control provides protection from corrosion causing agents and is often desired in slabs. Reduction in the overall building mass allows the building to be lighter in weight and allows for smaller design forces when considering seismic design. Some construction benefits of post-tensioning are faster floor construction cycles, lower floor weight, lower floor-to-floor height, larger spans between columns, and reduced foundations. The floor construction cycles are faster because a smaller amount of mild reinforcement is placed and the floor thickness is smaller.( Post-Tensioning Institute, 2006)

Residential post-tensioned ribbed (mat) foundations are used in areas with problems with expansive soils where soil movements can induce tension stresses in the foundation. The compressive stresses created from the post-tensioning help in resisting the induced tension stresses. Post-tensioning also allows for reduced costs of the foundation because of the reduced quantities of concrete, steel, and excavation. Other benefits of post-tensioned ribbed mat foundations are reduced cracking and reduced control joints. Cracking and control joints are reduced because the post-tensioning provides greater tensile strength in the slab, which increases the slabs' ability to resist cracking. (Post-Tensioning Institute, 2006)

Post-tensioned mat foundations use the total available area beneath a foundation for bearing, which is different from piles and individual spread footings. This is advantageous for foundations resting on soils with low allowable bearing capacity because it helps reduce deflections and allows for a uniform bearing pressure, which reduces differential settlements and bearing pressures. Post-tensioned mats also increase the speed of construction by eliminating all pile caps, reducing grade beams, and reducing the amount of excavation needed. (Cronin 1980)

A typical post-tensioned foundation on expansive soil consists of a monolithic "ribbed" foundation with 4-5 inch thick slab, a perimeter beam and interior beams spaced in both direction

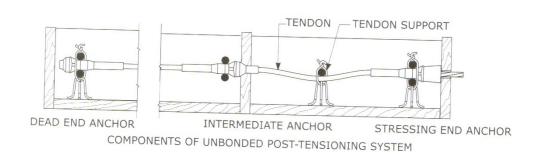
at 10-15 feet and ½" single-strand tendons distributed in both directions in both the slab and ribs. The ½" tendons are initially stressed to 33,000 pounds which is 80% of its tensile strength to provide a residual compressive stress ranging from 50 to 100 psi depending on the area of concrete being stressed. (Post-Tensioning Institute, 2006)

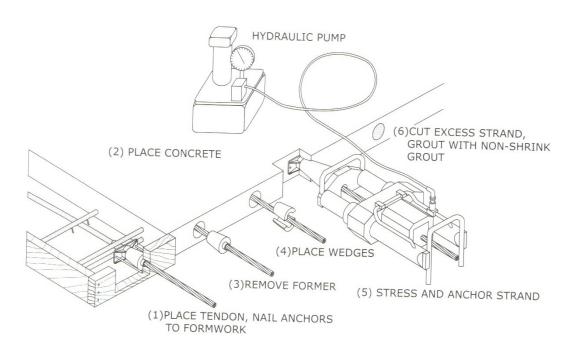
#### **Components of Post-Tensioning**

A post-tensioned tendon consists of the assembly of prestressing steel, end anchorage, and the duct. In a single-strand system, the tendon consists of a single 7-wire steel strand, P/T coating (for protection against corrosion), fixed end anchorage, stressing end anchorage, and intermediate anchorage for long tendons. Also a pocket former is used to create a pocket in the concrete for fitting the stressing jack and embedding the anchors. After the tendon is stressed, the pocket is grouted with a high strength non-shrink grout to prevent moisture penetration. (Post-Tensioning Institute, 2006)

Figure 4-3 on the next page illustrates the components and construction sequence of an unbonded system. First, the tendon is placed, and the anchors are nailed to the formwork. Second, the concrete is placed and set. Third, the formwork is removed after the concrete has reached the required strength, and then the wedges are placed. The fourth step is using the hydraulic pump to stress and anchor the strand. The final step is to cut the excess strand and fill with the grout.

Figure 4-3 Construction Sequence for Unbonded Post-Tensioned Slab (Reproduced from Post-Tensioning Institute, 2006)





CONSTRUCTION SEQUENCE FOR UNBONDED POST-TENSIONED SLAB

In a mutlistrand system, multiple strands are installed in a single duct. The strands are anchored by a multistrand anchor specially designed by the post-tensioning supplier. They are used to anchor the multiple strands to the concrete and are designed to accommodate the concentrated forces produced in the anchorage zone by individual strands. The strands in a bonded system are installed inside the duct without any P/T coating. After the strands are

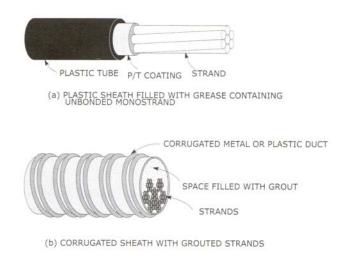
stressed, the ducts are grouted to bond the strands to the surrounding concrete, and the grout provides protection to the strands from corrosion. (Post-Tensioning Institute, 2006)

Bar tendons consist of either single or multiple bars. The tendons consist of highstrength steel rods, anchorages and ducts. Prestressing bars typically have an ultimate strength of 150 ksi and diameters ranging from 0.625 inch to 2.5 inch. (Post-Tensioning Institute, 2006)

The 7-wire strands in a tendon consist of 6 wires helically wrapped around a central straight wire. The diameter of the strands range from 0.375" to 0.6", but is typically 0.5". The strands are typically low relaxation steel with an ultimate tensile strength of 270ksi. The low relaxation properties are achieved by a process called stabilizing, which stretches the steel strand and then heats it. Stabilizing increases the strands' resistance to relaxation. Most of the steel strands produced in North America are low relaxation strands. (Post-Tensioning Institute, 2006)

Figure 4-4 illustrates the assemblies of a bonded and unbonded system with 7-wire strands. The top illustration shows a single unbonded strand. The bottom illustration shows a bonded multi-strand system.

Figure 4-4 Illustration of Bonded and Unbonded Systems (Reproduced from Post-Tensioning Institute, 2006)



Anchorages are used to transfer the tendon force to the concrete. Anchorages include stressing end anchorages, fixed end anchorages, and intermediate anchorages. Stressing end

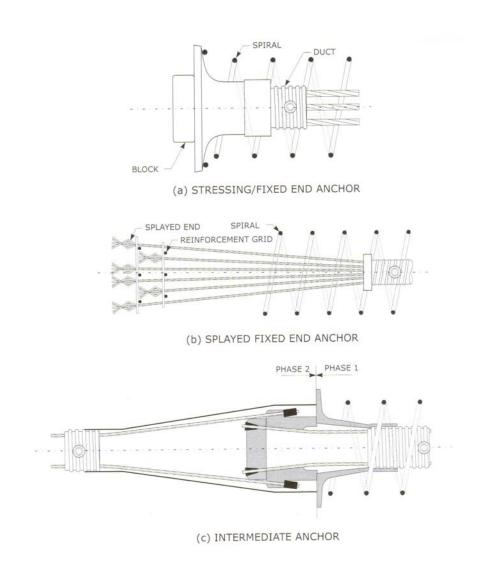
anchorages are used to stress the strand on site. A pocket former is typically used during forming and casting operations to embed the stressing anchors in the concrete as shown in Figure 4-3 previously. After stressing the tendons, the workers cut the tendon nails and fill the pocket with grout. (Post-Tensioning Institute, 2006)

Fixed-end anchorages for unbonded systems are typically attached to the tendon at the fabrication facility before being sent to the project site. The tendons are stressed to a specific load to seat the wedges securely in the anchor. This helps ensure that no slippage at the fixed-end will occur during stressing operations on site. Fixed end anchorages are only used when the tendon is stressed from only one end. Intermediate anchors are used when the tendon is very long or when a construction joint is provided along the length of the tendon. The anchorage is required to stress the strand at the construction joint. Anchorage is placed on the edge of the slab or at construction joints. Single strands can be combined to form multistrands to increases the spacing between the groups of strands as long as the spacing between the groups is less than the maximum spacing allowed. Figure 4-5 on the next page illustrates several assemblies of different anchorages for both single strand and multiple strand bonded and unbonded tendons.

The P/T coating used in unbonded construction is a corrosion inhibiting material that is made of a special grease. It acts as a barrier for ingress of water, inhibits corrosion of the steel, and lubricates the strand so it can move independently of the surrounding concrete. (Post-Tensioning Institute, 2006) The special grease used in the P/T coating must conform to 8 tests outlined in ACI 423.6-01. These tests are used to ensure the corrosion-inhibiting ability of the grease.

Ducts are used in bonded construction to provide a void between the concrete and strands after the concrete has been placed and hardened. The ducts are rigid or semi-rigid and are made from ferrous metal (either HDPE or PP). The shapes of the ducts can be oval, round, or flat depending on whether or not single or multistrand tendons are required. The ducts are corrugated to help transfer the force between the tendon and the concrete. Also the metal ducts are usually galvanized to provide corrosion protection both before and after construction. When ducts need to be joined together, they are done so with fittings and sleeves that reduce grout leakage and water ingress. (Post-Tensioning Institute, 2006)

Figure 4-5 Illustration of Anchorages (Reproduced from Post-Tensioning Institute, 2006)



In bonded construction, grout is used to fill the ducts, and it is placed as soon as practical after the stressing of the tendons. The grout has three main functions. First, it bonds the strand to the duct, which is bonded to the surrounding concrete; this is very important in bonded construction. The strand is placed inside the duct, which is rigid and bonded to the concrete after the concrete is placed and hardened. Therefore, the grout is the link between the strand and the duct that creates the bond between the strand and the concrete. Second, the grout slows the ingress of water and other corrosion causing agents. Third, the alkalinity of the grout further helps prevent corrosion, thus to be effective, the grout must fill all the voids in the tendon, and to

do that, the grout must be fluid enough to be pumped a long distance. (Post-Tensioning Institute, 2006)

Figures 4-6, 4-7, 4-8, & 4-9 on the following pages list the commonly used standard specifications and recommendations for the post-tensioning components.

Figure 4-6 Flowchart of Standards and Specifications for Materials and Components of Unbonded Tendons (Reproduced from Post-Tensioning Institute, 2006)

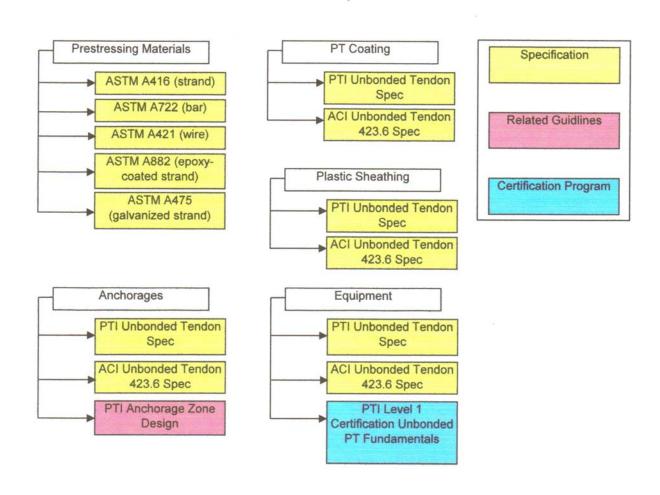


Figure 4-7 Flowchart of Standards and Specifications for Fabrication and Construction of Unbonded Tendons (Reproduced from Post-Tensioning Institute, 2006)

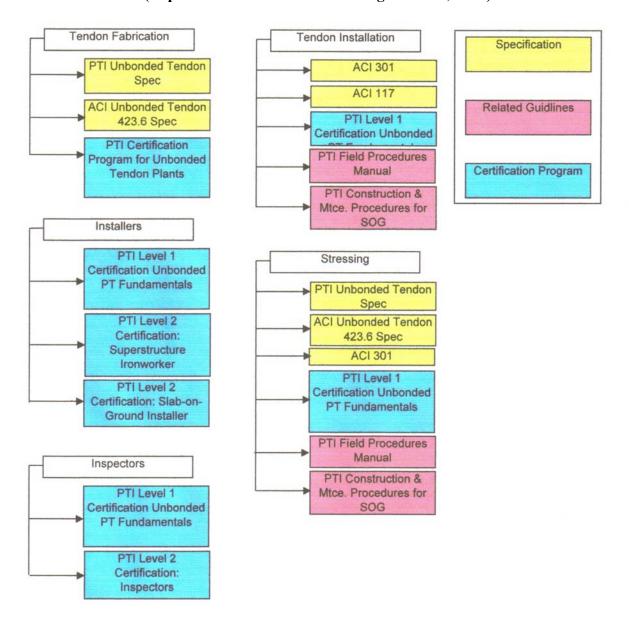


Figure 4-8 Flowchart of Standards and Specifications for Materials and Components of Bonded Tendons (Reproduced from Post-Tensioning Institute, 2006)

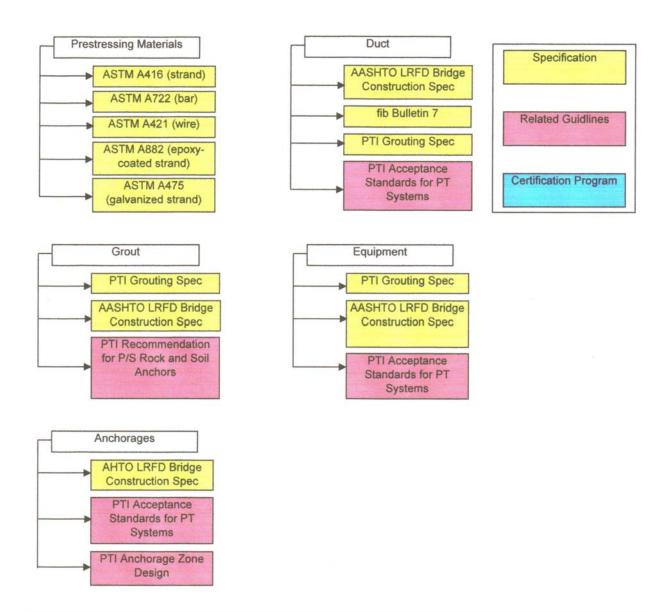
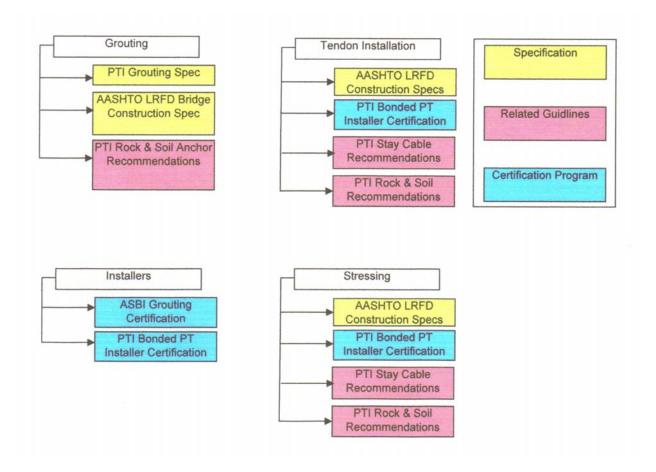


Figure 4-9 Flowchart of Standards and Specifications for Fabrication and Construction of Bonded Tendons (Reproduced from Post-Tensioning Institute, 2006)



# ACI Standard's and Post-Tensioning Slabs-on-Ground

ACI 360R Design of Slabs-on-Ground Chapter 8 outlines the design of post-tensioned slabs on grade (slabs on ground), it states that slabs on grade may be prestressed using unbonded tendons that are post-tensioned and anchored after the concrete has obtained sufficient strength to withstand the force at the anchorage (Equations 4-1 & 4-2 in this report, also referenced in Appendix A of this report). Some areas of caution to consider when designing post-tensioned slabs on ground are to make sure the anchors have sufficient size and holding capacity, that the tendons are placed, stressed, and anchored properly, and that any slab penetrations are located to avoid severing the tendons. Penetrations can be made after stressing the tendons as long as the engineer of record gives approval.

At service Load

$$f_{bp} = 0.6 f'_{c} \sqrt{\frac{A'_{b}}{A_{b}}} \le f'_{c}$$
 [Eq 4-1]

At Transfer

$$f_{bp} = 0.8 f'_{ci} \sqrt{\frac{A'_b}{A_b} - 0.2} \le 1.25 f'_{ci}$$
 [Eq 4-2]

Section 8.4.2 of ACI 360R recommends using the Post-Tensioning Institute (PTI) method for designing post-tensioned slabs on ground. Equations are given for determining the moment, deflection, and shear requirements for slabs cast on expansive or compressible soils. These equations take into consideration both center lift and edge lift conditions of the foundation.

Section 8.5 of ACI 360R describes the data needed to properly design the slabs. For instance, the soil properties needed are the allowable soil bearing pressure ( $q_{allow}$ , psf), the edge moisture variation distance ( $e_m$ , ft), the differential soil movement ( $y_m$ , in), and the slab-subgrade friction coefficient ( $\mu$ ). The structural data needed are slab length (L), spacing of stiffening beams (S), depth of beam (d), and perimeter loading criteria (P). The materials properties needed are the 28-day compressive strength of concrete ( $f_c$ ), type, grade, and strength of prestressing steel, type and grade of mild reinforcement, and prestress losses.

Section 8.6 of ACI 360R gives design equations for slabs on expansive soils. Slabs designed using the PTI method must meet the requirements of the design equations. Either mild reinforcement, post-tensioned reinforcement, or a combination of both may be used to meet these requirements. Many of the design equations found in Section 8.6 of ACI 360R are applied in Chapter 5 and Chapter 7 of this report. Section 8.7 of ACI 360R gives design equations for slabs on compressible soils, those soils whose allowable bearing capacity is 1500 psf or less. Section 8.8 of ACI 360R gives an equation for the maximum spacing of post-tensioning tendons in normal weight concrete.

# **CHAPTER 5 - Design Steps**

This Chapter provides a design guide for designing ribbed mat foundations on highly expansive soils. Highly expansive soils are soils that vastly expand and contract when exposed to wet and dry conditions. Both mild-reinforced concrete foundation and post-tensioned concrete foundation design steps are given. Each variable used in the equations is defined in Appendix A of this report. References for the design equations are also found in Appendix A of this report and also after each equation designation. Before any section properties of the foundation can be determined for either mild-reinforcement or post-tensioning, a soils investigation for the construction site needs to be performed in order to determine the soil properties and perform the calculations.

# **Post-Tensioning**

The following design steps for a post-tensioned ribbed mat foundation on highly expansive soils were taken from the Design of Post-Tensioned Slabs-on-Ground manual 3<sup>rd</sup> edition from the Post-Tensioning Institute (PTI). These design steps were utilized for the design example in Chapter 7 and no further research has been done to verify the formulas obtained from the design manual.

#### Step 1.

Obtain and organize all known design data including:

- Soil Properties
  - o Plastic Limit, PL, %
  - o Liquid Limit, LL, %
  - o Allowable soil bearing pressure, q<sub>allow</sub>, psf
  - o Percentage of soil passing No. 200 sieve, %-#200, %
  - o Percentage of soil finer than 2 microns, %-2μ, %
  - o Soil unit weight, pcf
  - o Modulus of elasticity of the soil, E<sub>soil</sub>, psi
  - Subgrade friction coefficient, μ

• Determine edge moisture variation distance, e<sub>m</sub>, ft (equations used in this step can be referenced in Section 3.6 of the Post-Tensioning Institutes Design of Post-Tensioned Slabs-on-Ground 3<sup>rd</sup> edition manual).

Step 1. Calculate the Plasticity Index (PI)

$$PI = LL-PL$$

Step 2. Calculate % fine clay (%fc)

$$% \text{fc} = \frac{\% - 2u}{\% - \#200}$$

Step 3. Determine Zone using the Mineral Classification Chart (Figure B-1 in Appendix B)

Step 4. Calculate the Activity Ratio (PI / %fc)

Step 5. Calculate LL / %fc

Steps 1-5 may be found in soils report.

Step 6. Determine  $\gamma_0$  using the Zone Chart (Figure B-2 in Appendix B)

Step 7. Calculate Suction Compression Index (γ<sub>h</sub>)

$$\gamma_{\text{h swell}} = \gamma_{\text{o}} e^{\gamma_{\text{o}}} (\% \text{fc} / 100)$$

$$\gamma_{h \text{ shrink}} = \gamma_{o} e^{-\gamma_{o}} (\% fc / 100)$$

Step 8. Calculate S

$$S = -20.29 + 0.1555(LL) - 0.117(PI) + 0.0684(\% - #200)$$

Step 9. Calculate Unsaturated Diffusion Coefficient (α)

$$\alpha_{swell} = 0.0029 \text{-} 0.000162(S) \text{-} 0.0122(\gamma_{h \text{ swell}})$$

$$\alpha_{\text{shrink}} = 0.0029 - 0.000162(S) - 0.0122(\gamma_{\text{h shrinkl}})$$

Step 10. Fabric Factor (F<sub>f</sub>) (Figure B-3 in Appendix B)

Step 11. Calculate Modified Unsaturated Diffusion Coefficient (α')

$$\alpha'_{\text{swell}} = \alpha_{\text{swell}} (F_f)$$

$$\alpha'_{\text{shirnk}} = \alpha_{\text{shink}} (F_f)$$

Step 12. Determine Thornthwaite Moisture Index  $(I_m)$  (Figure B-4 in Appendix B)

- Step 13. Determine  $e_m$  based on  $I_m$  for center and edge lift (Figure B-5 in Appendix B)
- Step 14. Determine  $e_m$  based on  $\alpha$ ' for center and edge lift (Figure 5 in Appendix B)
- Determine soil movement, y<sub>m</sub>, in (equations used in this step can be referenced in Section 3.6 of the Post-Tensioning Institutes Design of Post-Tensioned Slabs-on-Ground 3<sup>rd</sup> edition manual)
  - Step 1. Determine Measured Suction at Depth (Figure B-6 in Appendix B)
  - Step 2. Determine Dryest Suction
  - Step 3. Determine Wettest Suction
  - Step 4. Determine Stress Change Factors (SCF) for center and edge lift (Figure B-7 in Appendix B)
  - Step 5. Calculate y<sub>m</sub> for center and edge lift

$$y_m \text{ edge} = (SCF\text{-edge})(\gamma_h \text{ swell mod})$$
  
 $y_m \text{ center} = (SCF\text{-center})(\gamma_h \text{ shrink mod})$ 

- Structural data and material properties
  - o Foundation length, L, ft (both directions)
  - o Perimeter loading, P, plf
  - o Average stiffening rib spacing, S, ft (both directions)
  - o 28-day design compressive strength of the concrete, f'c, psi
  - O Allowable flexural tensile stress in the concrete,  $f_t$ , psi, the value is used to justify using the gross concrete section in calculations, this is why  $f_t$  is taken as a value less than the modulus of rupture of concrete.

$$f_t = 6\sqrt{f'c}$$
 [Equation 5-1] *PTI Eq 6-5, ACI360R 8.5.3*

o Allowable flexural compressive stress in the concrete, f<sub>c</sub>, psi

$$f_c = 0.45(f'c)$$
 [Equation 5-2] PTI Eq 6-6, ACI360R 8.5.3

- o Type, grade, and strength of prestressing steel
- o Stiffness coefficient,  $C_{\Delta}$  (Figure B-8 in Appendix B)

#### Step 2.

If the foundation plan is an irregular shape (nonrectangular), divide the plan into overlapping rectangles to design each rectangle individually. Verify the shape factor (SF), is less than or equal to 24 in order to apply the overlapping rectangle procedure. The design equations used for this method are based on the assumption that the SF is less than or equal to 24. Design rectangles whose SF is larger than 24 tend to have problems resisting bending stresses due to torsion effects based on previous experiences. The SF is also used to prevent the foundation from taking a long and thin rectangular shape which is not desirable. If SF exceeds 24, a finite element procedure should be used to determine design forces. Design each rectangular portion of the plan separately.

$$SF = \frac{(FoundationPerimeter)^2}{(FoundationArea)}$$
 [Equation 5-3] PTI Eq 4-1

#### Step 3.

Assume a trial section for a ribbed foundation in both the long and short directions of the design rectangle. The assumed initial rib depth (h), is determined using equation 5-4 and the stiffness coefficient ( $C_{\Delta}$ ) along with an assumed rib spacing (S). The assumed rib spacing is determined from the layout of the foundation. The initial rib depth is taken as the deeper value from each direction based on the following equation:

h(center lift) = 
$$\left[\frac{((y_m)(L))^{0.205}(S)^{1.059}(P)^{0.523}(e_m)^{1.296}(C_\Delta)}{4560(z)}\right]^{0.824}$$
 [Equation 5-4] *PTI Eq 6-1* where z = the smaller of L or 6 $\beta$  where  $\beta$  = assumed relative stiffness length

If different rib depths are used in the analysis, the ratio of the depth of the deepest and shallowest rib shall not exceed 1.2. The limits of this ratio are assumed in the design equations. If the ratio exceeds 1.2, a finite element procedure should be used to determine the design stresses because the difference in rib depth will change the way the stresses are distributed throughout the ribbed mat and are not applicable to this design method. The rib depth shall be 11" or greater and the rib must extend at least 7" below the bottom of the flat slab which is based according to Mr. Ken Bondy (Chairman of the

PTI Slab-on-Ground Committee) on the collective judgment of the PTI Slab-on-Ground Committee.

#### Step 4.

Check the soil bearing capacity. This is done by determining all dead loads and live loads of the structure. Be sure to include the slab and rib weight along with the perimeter wall load. After the weight of the building has been determined, determine the rib bearing area of the foundation. It can be assumed that portions of the slab will help the ribs in bearing the soil pressure. To find the area of the slab that helps in bearing, it can be assumed according to the PTI Design of Post-Tensioned Slabs-on-Ground manual 3<sup>rd</sup> edition that the bearing width of edge ribs is the width of the rib plus 6 times the slab thickness and the bearing width of interior ribs is the width of the rib plus 16 times the slab thickness. Check that the actual soil pressure is less than the allowable soil pressure.

## Step 5.

Determine the following properties of the foundation for both directions:

- o distance from top of slab to center of mass, y<sub>t</sub>, in
- o gross concrete moment of inertia, I, in<sup>4</sup>
- $\circ$  section modulus with respect to the top fiber,  $S_t$ , in<sup>3</sup>
- o section modulus with respect to the bottom fiber, S<sub>b</sub>, in<sup>3</sup>

#### Step 6.

Determine the number of tendons required for the slab and the design prestress force. First, determine the number of tendons required for the minimum required force. This is done by applying the following equations:

Stress in tendons immediately after anchoring

$$f_{pi} = 0.7 f_{pu}$$
 [Equation 5-5]  $PTI Eq 6-9$ 

Stress in tendons after losses, it is standard practice to assume 15 ksi for prestress losses for low-relaxation strand

$$f_e = f_{pi} - 15$$
 [Equation 5-6] *PTI Sect 6.6*

Minimum prestress force required for ribbed foundations,  $P_e = 0.05A$ 

Number of tendons = 
$$N_t = \frac{0.05A}{f_e A_{ps}}$$
 [Equation 5-7] *PTI Sect A.3.2.1*

Second, determine the number of tendons required to overcome slab-sub-grade friction, which is the frictional resistance to movement of the foundation on the subgrade during stressing.

$$N_{t} = \frac{\mu W_{slab}}{2000 f_{e} A_{ps}}$$
 [Equation 5-8] *PTI Eq 6-12a*

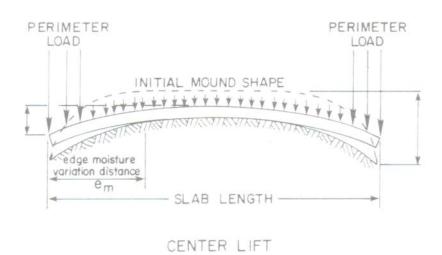
The total number of tendons needed is the sum of the tendons calculated in equations 5-7 and 5-8. Determine the design prestress force using the following equation:

$$P_r = N_t(f_e \times A_{ps}) - \mu W_{slab}/2000$$
 [Equation 5-9] *PTI Eq 6-12b*

## <u>Step 7.</u>

Design the foundation based on center lift design criteria. These criteria include: design moments, service load stresses, foundation stiffness, and shear calculations. Figure 5-1 on this page illustrates the center lift of the foundation.

**Figure 5-1 Center Lift on Foundation** 



#### A) Design Moments

1. Long Direction

$$M_L = A_o[B(e_m)^{1.238} + C]$$

[Equation 5-10] *PTI Eq 6-13* 

where

$$A_{o} = \frac{(L)^{0.013}(S)^{0.306}(h)^{0.688}(P)^{0.534}(y_{m})^{0.193}}{727}$$
 [Equation 5-11] PTI Eq 6-14 and for 
$$0 \le e_{m} \le 5 \quad B = 1, C = 0$$
 [Equation 5-12] PTI Eq 6-15a 
$$5 < e_{m} \quad B = \frac{y_{m} - 1}{3} \le 1.0$$
 [Equation 5-13] PTI Eq 6-15b 
$$C = \left[8 - \frac{P - 613}{255}\right] \left[\frac{4 - y_{m}}{3}\right] \ge 0$$
 [Equation 5-14] PTI Eq 6-15c

 $M_L$  should be calculated also at  $e_m = 5$  when  $e_m > 5$  because there is a discontinuity in the equations for long direction center lift moments at  $e_m = 5$ . The higher value of the two should be used for the design moment.

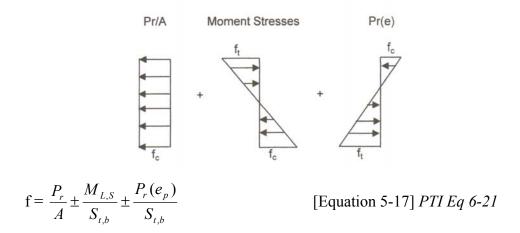
#### 2. Short Direction

For 
$$L_L / L_S < 1.1$$
  $M_S = M_L$  [Equation 5-15]  $PTI Eq 6-17$   
For  $L_L / L_S \ge 1.1$   $M_S = \left[\frac{58 + e_m}{60}\right] M_L$  [Equation 5-16]  $PTI Eq 6-16$ 

 $M_S$  should be calculated also at  $e_m = 5$  when  $e_m > 5$ . The higher value of the two should be used for the design moment.

B) Compare actual and allowable service stresses in both the long and short directions (allowable stresses were calculated in Step 1.) Compare the tension in the top fiber with the allowable concrete flexural tension stress and compression in the bottom fiber with the allowable concrete flexural compression stress. Use equation 5-17 to determine the actual service stress. Figure 5-2 on the next page illustrates the distribution of stresses to concrete section. If it is determined that the section is not adequate for the service stresses, the engineer can increase the depth of the rib, increase the number of ribs, or add tendons to each rib in order to decrease the service stresses.

Figure 5-2 Distribution of Stresses in Center Lift Design



C) Calculate the minimum foundation stiffness and compare with the actual foundation stiffness using the required and actual moment of inertias. Use Equation 5-18 for the long direction minimum stiffness and Equation 5-19 for short direction minimum foundation stiffness. If it is determined that the section is not adequate, the designer can increase the rib height or increase the number of ribs. Rib height should be increased first before trying to increase the number of ribs. Increase the rib height creates fewer changes to the overall foundation properties regarding rib location within the mat.

$$I_{L} \ge \frac{18000(M_{L})(L_{S})(C_{\Delta})(z_{L})}{1500000}$$
[Equation 5-18] *PTI Eq 6-22*

$$I_{S} \ge \frac{18000(M_{S})(L_{L})(C_{\Delta})(z_{S})}{1500000}$$
[Equation 5-19] *PTI Eq 6-22*

where  $z = \text{smaller value of } L \text{ or } 6\beta \text{ for the given direction.}$ 

where 
$$\beta = \frac{1}{12} \sqrt[4]{\frac{E_{cr}I}{E_{soil}}}$$
 [Equation 5-20] *PTI Sect A.1*

D) Calculate the expected service shear using Equation 5-21 and Equation 5-22. Calculate the permissible service shear using Equation 5-23. Calculate the applied service load shear stress using Equation 5-24. Compare the values from equation 5-24 with the values from equation 5-23. If it is determined that the section is not adequate enough, the designer can increase the rib height, increase the number of ribs, or add stirrups.

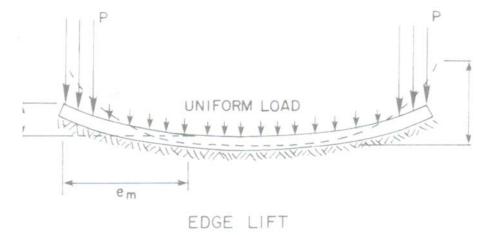
# Step 8.

Design the foundation based on edge lift design criteria. These criteria include; design moments, service load stresses, foundation stiffness, and shear calculations. Figure 5-3 illustrates the edge lift of the foundation.

## A) Design Moments

$$M_{L} = \frac{(S)^{0.1} (he_{m})^{0.78} (y_{m})^{0.66}}{7.2(L)^{0.0065} (P)^{0.04}}$$
 [Equation 5-25] *PTI Eq 6-18*  
For  $L_{L} / L_{S} < 1.1$   $M_{S} = M_{L}$  [Equation 5-26] *PTI Eq 6-20*  
For  $L_{L} / L_{S} \ge 1.1$   $M_{S} = h^{0.35} [(19 + e_{m})/57.75] M_{L}$  [Equation 5-27] *PTI Eq 6-19*

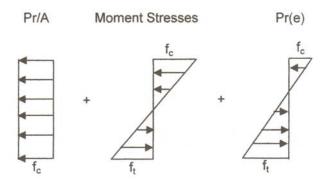
Figure 5-3 Illustration of Edge Lift on Foundation



B) Compare actual and allowable service stresses in both the long and short directions. (Allowable stresses were calculated in Step 1. Eq 5-1 and Eq 5-2)

Compare the tension in the bottom fiber with the allowable concrete flexural tension stress and compression in the top fiber with the allowable concrete flexural compression stress. Use equation 5-17 to determine the actual service stress. If it is determined that the section is not adequate for the service stresses, the engineer can increase the depth of the rib, increase the number of ribs, or add tendons to each rib in order to decrease the service stresses. Figure 5-4 illustrates the stress distribution in the concrete section for center edge lift design.

Figure 5-4 Stress Distribution for Edge Lift



- C) Calculate the minimum foundation stiffness and compare with the actual foundation stiffness using the required and actual moment of inertias. Use Equation 5-18 for the long direction minimum stiffness and Equation 5-19 for short direction minimum foundation stiffness. If it is determined that the section is not adequate, the designer can increase the rib height or increase the number of ribs.
- D) Calculate the expected service shear using Equation 5-28. Calculate the permissible service shear using Equation 5-29. Calculate the applied service load shear stress using Equation 5-30. Compare the values from equation 5-30 with the values from Equation 5-29. If it is determined that the section is not adequate enough, the designer can increase the rib height, increase the number of ribs, or add stirrups.

$$v_{S} = v_{L} = \frac{(L)^{0.07} (h)^{0.4} (P)^{0.03} (e_{m})^{0.16} (y_{m})^{0.67}}{3S^{0.015}}$$
 [Equation 5-28] *PTI Eq 6-25*  

$$v_{c} = 1.7 \sqrt{f'_{c}} + 0.2 (f_{p})$$
 [Equation 5-29] *PTI Sect A.3.2.2*  
where  $f_{p} = P_{r}/A$   

$$v = \frac{vW}{nbh}$$
 [Equation 5-30] *PTI Eq 6-28*

#### Step 9.

Determine if the cracked section moment capacity of the foundation in both directions is adequate to support the design moment that is to be developed with the cracked section for both center and edge lift conditions. The design equations used are listed below. If the cracked section capacities are not adequate, the engineer can increase rib depth, increase the number of ribs, or add appropriate reinforcement.

Total Long Direction Moment for Center Lift

$$M_{LTOTAL} = M_L(L_S)$$
 [Equation 5-31] *PTI Sect 4.5.7*

To be Developed with Cracked Section

Slab Tendon Force

$$N_{t(Long)}(f_eA_{ps})$$
 [Equation 5-33] PTI Sect A.3.2.4

Capacity with Tendons Alone

Depth of Compression Block, a

$$a = \frac{N_{t(Long)}(f_e A_{ps})}{0.85(f'_e)(b)(n)}$$
 [Equation 5-34] *PTI Sect A.3.2.4*

**Cracked Section Capacity** 

$$M_{cr} = \frac{N_{t(Long)} f_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right)$$
 [Equation 5-35] *PTI Sect A.3.2.4*

Total Short Direction Moment for Center Lift

$$M_{STOTAL} = M_S(L_L)$$
 [Equation 5-36] *PTI Sect 4.5.7*

To be Developed with Cracked Section

$$0.9 M_{\rm S}$$
 [Equation 5-37] PTI Sect 4.5.7

Slab Tendon Force

$$N_{t(Short)}(f_eA_{ps})$$

[Equation 5-38] PTI Sect A.3.2.4

Capacity with Tendons Alone

Depth of Compression Block, a

$$a = \frac{N_{t(Short)}(f_e A_{ps})}{0.85(f'_c)(b)(n)}$$

[Equation 5-39] PTI Sect A.3.2.4

Cracked Section Capacity

$$\mathbf{M}_{cr} = \frac{N_{t(Short)} f_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right)$$

[Equation 5-40] *PTI Sect A.3.2.4* 

Total Long Direction Moment for Edge Lift

$$M_{LTOTAL} = M_L(L_S)$$

[Equation 5-41] *PTI Sect 4.5.7* 

To be Developed with Cracked Section

$$0.9\ M_L$$

[Equation 5-42] *PTI Sect 4.5.7* 

Rib Tendon Force

$$n(f_eA_{ps})$$

[Equation 5-43] *PTI Sect A.3.2.4* 

Capacity with Tendons Alone

Depth of Compression Block

$$a = \frac{n(f_e A_{ps})}{0.85(f'_c)(L_S)(12)}$$

[Equation 5-44] *PTI Sect A.3.2.4* 

Cracked Section Capacity

$$M_{cr} = \frac{nf_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right)$$

[Equation 5-45] *PTI Sect A.3.2.4* 

Total Short Direction Moment for Edge Lift

$$M_{STOTAL} = M_S (L_L)$$

[Equation 5-46] *PTI Sect 4.5.7* 

To be Developed with Cracked Section

$$0.9\ M_{S}$$

[Equation 5-47] *PTI Sect 4.5.7* 

Rib Tendon Force

$$n(f_eA_{ps})$$

[Equation 5-48] PTI Sect A.3.2.4

Capacity with Tendons Alone

Depth of Compression Block

$$a = \frac{n(f_e A_{ps})}{0.85(f'_c)(L_L)(12)}$$
 [Equation 5-49] *PTI Sect A.3.2.4*

Cracked Section Capacity

$$M_{cr} = \frac{nf_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right)$$
 [Equation 5-50] *PTI Sect A.3.2.4*

## Step 10.

Repeat steps 1-9 for each rectangular section of nonrectangular foundations. Increase section properties as needed.

#### Mild Reinforced

The following design steps are for a mild-reinforced ribbed mat foundation on expansive soils.

#### Step 1.

Obtain and organize all known design data. This step can be referenced in Step 1 of Post-Tensioning.

#### Step 2.

Check the soil bearing capacity. This step can be referenced in Step 4 of Post-Tensioning. Step 3.

Determine the following properties of the foundation for both directions:

- o distance from top of slab to center of mass,  $y_t$ , in
- o gross concrete moment of inertia, I, in<sup>4</sup>
- o section modulus with respect to the top fiber,  $S_t$ , in<sup>3</sup>
- o section modulus with respect to the bottom fiber,  $S_b$ , in<sup>3</sup>

#### Step 4.

#### Center Lift Design

Design the foundation based on center lift design criteria. These criteria include; design moments, service load stresses, foundation stiffness, and shear calculations. The

maximum moments and shears in the foundation in both directions can be determined by using one of the three general element formulations according to chapter 6 of ACI 336.2R. These three general discrete element formulations are finite difference (FD), finite grid method (FGM), and finite element method (FEM). Computer analysis software can be used to appropriately model the foundation to determine the design values. For the purposes of this paper, the design moments, service load stresses, foundation stiffness, and shear calculations will be determined using the same formulas as used in the Post-Tensioning Design Steps.

#### A) Design Moment

The design moment equations can be referenced in Step 7 of Post-Tensioning.

# B) Flexure Design of Ribs [ACI 318-05]

$$M_{u(L,S)} = M_{L,S}(L_{S,L})(1.6)$$
 [Equation 5-51]
$$R_{u} = \frac{Mu}{\phi(b)(d)^{2}}$$
 [Equation 5-52]
$$m = \frac{f_{y}}{0.85(f'_{c})}$$
 [Equation 5-53]
$$\rho_{req} = \frac{1}{m} \left[ 1 - \sqrt{1 - \frac{2(R_{u})(m)}{f_{y}}} \right]$$
 [Equation 5-54]
$$\rho_{min} = \frac{200}{f_{y}}$$
 [Equation 5-55] ACI 318-05 Eq 10-3
$$\rho_{max} = \frac{0.319(B_{1})(f'_{c})}{f_{y}}$$
 [Equation 5-56] ACI 318-05 10.2.3
$$A_{st} = \rho \text{ (b)(d)}$$
 [Equation 5-57]

#### C) Shear Calculations

**Expected Service Shear** 

$$v_{L} = \frac{(L)^{0.09}(S)^{0.71}(h)^{0.43}(P)^{0.44}(y_{m})^{0.16}(e_{m})^{0.93}}{1940}$$
 [Equation 5-21] *PTI Eq 6-24*  
$$v_{S} = \frac{(L)^{0.19}(S)^{0.45}(h)^{0.20}(P)^{0.54}(y_{m})^{0.04}(e_{m})^{0.97}}{1350}$$
 [Equation 5-22] *PTI Eq 6-23*

$$V_u = v_{L,S}(L_{S,L})(1.6)$$
 [Equation 5-58]  
 $\phi V_c = \phi(2) \sqrt{f'_c}(b)(d)$  [Equation 5-59] ACI 318-05 Eq 11-3

If  $\phi V_c < V_u$ , then shear reinforcement is needed

Shear Stirrups Need to Resist 
$$V_s = \frac{V_u - \phi V_c}{\phi}$$
 [Equation 5-60]

Spacing of Stirrups

$$S_{req} = \frac{nA_v f_y d}{V_s}$$
 [Equation 5-61] ACI 318-05 Eq 11-15  
 $S_{max} = \frac{A_v f_y}{\phi \sqrt{f'_c}(b)}$  [Equation 5-62]  
or  $S_{max} = \frac{A_v f_y}{50b}$  [Equation 5-63]  
or  $S_{max} = \frac{d}{2} \le 24$ " [Equation 5-64] ACI 318-05 11.5.5

D) Calculate the minimum foundation stiffness and compare with the actual effective foundation stiffness using the required and effective moment of inertias. Use Equation 5-18 for the long direction minimum stiffness and Equation 5-19 for short direction minimum foundation stiffness. If it is determined that the section is not adequate, the designer can increase the rib height or increase the number of ribs.

$$I_{L} \ge \frac{18000(M_{L})(L_{S})(C_{\Delta})(z_{L})}{1500000}$$
 [Equation 5-18] *PTI Eq 6-22* 
$$I_{S} \ge \frac{18000(M_{S})(L_{L})(C_{\Delta})(z_{S})}{1500000}$$
 [Equation 5-19] *PTI Eq 6-22*

where  $z = \text{smaller value of } L \text{ or } 6\beta \text{ for the given direction.}$ 

where 
$$\beta = \frac{1}{12} \sqrt[4]{\frac{E_{cr}I}{E_{soil}}}$$
 [Equation 5-20] *PTI Sect A.1*

Effective I Calculations

Mer = 
$$\frac{f_r(I_g)}{y}$$
  $f_r = 7.5\sqrt{f'_c}$  [Equation 5-65] *ACI 318-05 Eq 9-9*

$$nAs = \frac{E_s}{E_c} A_s$$
 [Equation 5-66]

Find x, the distance from the neutral axis of the transformed section

$$I_{cr} = \frac{bx^3}{3} + nA_s(d-x)^2$$
 [Equation 5-67]

$$I_{E} = \left(\frac{Mcr}{M_{A}}\right)^{3} (I_{g}) + \left[1 - \left(\frac{Mcr}{M_{A}}\right)^{3}\right] (I_{cr})$$
 [Equation 5-68]

#### Step 5.

#### Edge Lift Design

Design the foundation based on edge lift design criteria. These criteria include; design moments, service load stresses, foundation stiffness, and shear calculations. The maximum moments and shears in the foundation in both directions can be determined by using one of the three general element formulations according to chapter 6 of ACI 336.2R. These three general discrete element formulations are finite difference (FD), finite grid method (FGM), and finite element method (FEM). Computer analysis software can be used to appropriately model the foundation in order to determine the design values. For the purposes of this report, the design moments, service load stresses, foundation stiffness, and shear calculations will be determined using the same formulas as used in the Post-Tensioning Design Steps.

#### A) Design Moment

The design moment equation can be referenced in Step 8 of Post-Tensioning.

#### B) Flexure Design of Ribs [ACI 318-05]

$$M_{u(L,S)} = M_{L,S}(L_{S,L})(1.6)$$
 [Equation 5-51]

$$R_{\rm u} = \frac{Mu}{\phi(b)(d)^2}$$
 [Equation 5-52]

$$m = \frac{f_y}{0.85(f'_{-})}$$
 [Equation 5-53]

$$\rho_{\text{req}} = \frac{1}{m} \left[ 1 - \sqrt{1 - \frac{2(R_u)(m)}{f_y}} \right]$$
[Equation 5-54]
$$\rho_{\text{min}} = \frac{200}{f_y}$$
[Equation 5-55] *ACI 318-05 Eq 10-3*

$$\rho_{\text{max}} = \frac{0.319(B_1)(f'_c)}{f_y}$$
[Equation 5-56] *ACI 318-05 10.2.3*

$$A_{\text{st}} = \rho \text{ (b)(d)}$$
[Equation 5-57]

#### C) Shear Calculations

**Expected Service Shear** 

$$v_{S} = v_{L} = \frac{(L)^{0.07} (h)^{0.4} (P)^{0.03} (e_{m})^{0.16} (y_{m})^{0.67}}{3S^{0.015}}$$
 [Equation 5-28] *PTI Eq 6-25*  

$$V_{u} = v_{L,S}(L_{S,L})(1.6)$$
 [Equation 5-58] *PTI Eq 6-25*  

$$\phi V_{c} = \phi(2) \sqrt{f'_{c}}(b)(d)$$
 [Equation 5-59] *ACI 318-05 Eq 11-3*

If  $\phi V_c < V_u$ , then shear reinforcement is needed

Shear Stirrups Need to Resist 
$$V_s = \frac{V_u - \phi V_c}{\phi}$$
 [Equation 5-60]

Spacing of Stirrups

$$S_{req} = \frac{nA_v f_y d}{V_s}$$
 [Equation 5-61] ACI 318-05 Eq 11-15  
 $S_{max} = \frac{A_v f_y}{\phi \sqrt{f'_c}(b)}$  [Equation 5-62]  
or  $S_{max} = \frac{A_v f_y}{50b}$  [Equation 5-63]  
or  $S_{max} = \frac{d}{2} \le 24$ " [Equation 5-64] ACI 318-05 11.5.5

D) Calculate the minimum foundation stiffness and compare with the actual effective foundation stiffness using the required and effective moment of inertias. Use Equation 5-18 for the long direction minimum stiffness and Equation 5-19 for short direction

minimum foundation stiffness. If it is determined that the section is not adequate, the designer can increase the rib height or increase the number of ribs.

$$I_{L} \ge \frac{18000(M_{L})(L_{S})(C_{\Delta})(z_{L})}{1500000}$$
 [Equation 5-18] *PTI Eq 6-22*

$$I_{S} \ge \frac{18000(M_{S})(L_{L})(C_{\Delta})(z_{S})}{1500000}$$
 [Equation 5-19] *PTI Eq 6-22*

where  $z = \text{smaller value of } L \text{ or } 6\beta \text{ for the given direction.}$ 

where 
$$\beta = \frac{1}{12} \sqrt[4]{\frac{E_{cr}I}{E_{soil}}}$$
 [Equation 5-20] *PTI Sect A.1*

**Effective I Calculations** 

Find x, the distance from the neutral axis of the transformed section

$$Icr = \frac{bx^3}{3} + nA_s(d-x)^2$$
 [Equation 5-67]

$$I_{E} = \left(\frac{Mcr}{M_{A}}\right)^{3} (I_{g}) + \left[1 - \left(\frac{Mcr}{M_{A}}\right)^{3}\right] (I_{cr})$$
 [Equation 5-68]

# **CHAPTER 6 - Design Loads and Slab Thickness Calculations**

This Chapter is used to define the design loads and the general building layout used in the design examples in the next two Chapters of this report. The slab-on-ground thickness is based off the design of the continuous span of the slab in both directions. The slab thickness and reinforcement needed will be used for both Chapter 7 and Chapter 8 of this report.

The building is a typical 2-story office building located in Dallas, Texas. The floor to floor heights are 13 feet with a 4 foot parapet. The exterior non-load bearing walls are metal studs with a brick veneer. The foundation plan, 2<sup>nd</sup> floor framing plan, and roof framing plan are shown in Figures 6-1, 6-2, and 6-3 on the following pages.

# **Design Loads**

The following design loads are obtained from ASCE 7-05 and are listed below:

Exterior Stud Walls w/ Brick Veneer = 50psf			[Table C3-1]
Roof Dead Load			
	Single Ply Roof Membrane	=0.7psf	[Table C3-1]
	4" Rigid Insulation	=6psf	[Table C3-1]
	20 ga Roof Deck	= 2.5 psf	[Table C3-1]
	Ceiling	=2psf	[Table C3-1]
	Mechanical	=4psf	[Table C3-1]
	Misc.	=4.8psf	
	Total	=20psf	
2 <sup>nd</sup> Floor Dead Load			
	5" Composite Slab	=63psf	[Table C3-2]
	Misc.	=7psf	
	Total	= 70psf	
at .			

1st Floor Dead Load

To be calculated after slab-on-grade thickness has been determined.

Roof Live Load = 20psf [Table 4-1]

Snow Load + Rain = 10psf [Section 7.10]  

$$2^{\text{nd}}$$
 Floor Live Load = 80psf [Table 4-1]  
 $1^{\text{st}}$  Floor Live Load = 100psf [Table 4-1]

## **Slab Thickness Calculations (ACI 318-05)**

The ultimate and service load moment and shear forces for the slab were determined using Risa 3D, a structural analysis software, for the design examples in Chapter 7 and Chapter 8. The 4000 psi concrete slab was divided into 12 inch strips of slab and analyzed as a continuous slab strip in each direction supported by the ribs shown in Figure 6-1. The dead load was applied to the entire continuous slab and the live load was applied using skip loading over the different spans of the continuous slab. The design loads were a live load of 100 psf, a soil pressure load of 225 psf, and a dead load of an assumed 6 inch slab of 75 psf. Risa 3D gave the following output for the design forces:

$$M_u(+) = 4.7 \text{k-ft} \qquad \qquad M_{\text{service}} \ (+) = 1.41 \text{k-ft}$$
 
$$M_u(-) = 5.1 \text{k-ft} \qquad \qquad M_{\text{service}} \ (-) = 2.82 \text{k-ft}$$
 
$$V_u = 2.1 \text{k}$$

#### Moment Design

The value d will be assumed to be 3" for both positive and negative moment. The highest moment of 5.1 k-ft will be used for design.

$$\begin{split} M_u &= 5.1 \text{k-ft} \\ R_u &= \frac{5.1(12)}{0.9(12)(3)^2} = 0.630 \\ m &= \frac{60}{0.85(4)} = 17.65 \\ \rho_{req} &= \frac{1}{17.65} \Bigg[ 1 - \sqrt{1 - \frac{2(17.65)(0.630)}{60}} \Bigg] = 0.0118 \\ \rho_{min} &= \frac{200}{60000} = 0.00333, \quad use \; \rho_{req} = 0.0118 \\ \rho_{max} &= \frac{0.319(0.85)(4000)}{60000} = 0.0181 \\ A_{st} &= 0.0118(12)(3) = 0.43 \; in^2/\text{ft} \end{split}$$

USE 6" thick slab w/ #6 bars at 1'-0" on center both ways.

Shear Design

$$\phi V_c = 0.75(2)\sqrt{4000}(12)(4) = 4.55k$$

4.55k > 2.1k, therefore no shear reinforcement is needed.

#### Deflection

Table 9.5(a) of ACI 318-05 states that deflections need not be considered if the slab thickness meets the criteria of the table. For ribbed one-way slabs with both ends continuous the minimum thickness of the slab is (span/21). It is assumed for the purpose of this report that the span =  $10^{\circ}$ , although for two bays the span =  $15^{\circ}$ .

$$h = \frac{10'(12'')}{21} = 5.72"$$

Therefore a 6" slab meets the deflection criteria.

Temperature and Shrinkage Steel

 $\rho_{\text{temp\&shrink}} = 0.0018 < 0.0118$ , #6's at 1'-0" on center are OK [ACI 318-05 7.12.2.1]

# **Summary**

The following information is used for both Chapter 7 and Chapter 8.

Design Loads

Roof DL = 
$$20psf$$

Roof 
$$LL = 20psf$$

$$2^{nd}$$
 Floor DL =  $70$ psf

$$2^{nd}$$
 Floor LL =  $80$ psf

$$1^{st}$$
 Floor LL = 100psf

$$1^{st}$$
 Floor DL = 6" Slab = 75psf

Exterior Stud Wall Load = 50psf

Slab-On-Ground

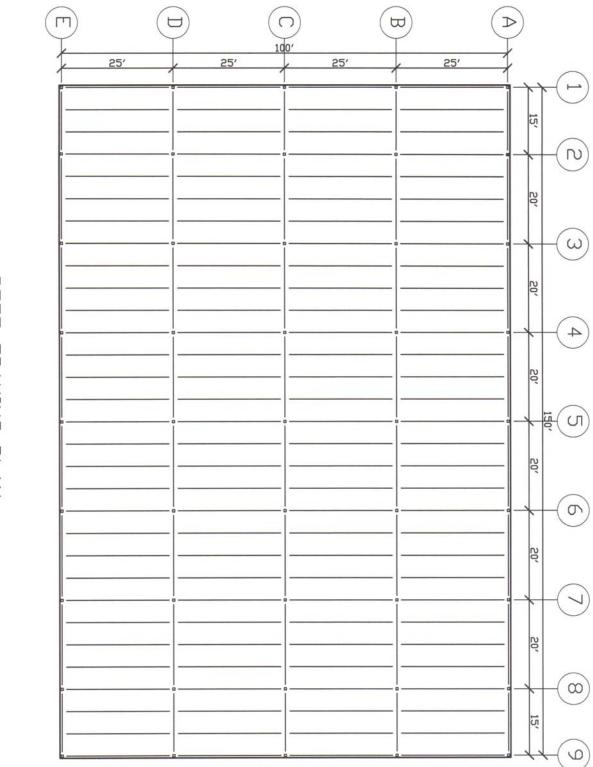
6" thick SOG w/#6's spaced at 1'-0" on center both ways.

FOUNDATION PLAN

() m) 25′ 25′ 25' 25' FLOOR FRAMING PLAN  $\infty$ 

Figure 6-2 2nd Floor Framing Plan

Figure 6-3 Roof Framing Plan



# CHAPTER 7 - Post-Tensioned Ribbed Mat Foundation Design Example

This chapter is a design example of a post-tensioned ribbed mat foundation. The design information including framing plans are given in Chapter 6. The design steps are explained in Chapter 5 of the Design Steps for Post-Tensioned Ribbed Mat Foundations.

## Step 1.

Obtain and organize all known design data including:

Soil Properties

$$PL = 45$$

$$LL = 60$$

$$q_u = 2500 \text{ psf}$$

$$\%$$
-#200 = 40

$$\%-2\mu = 10$$

soil unit weight = 105 pcf

$$E_s = 1500 \text{ psi}$$

Subgrade friction coefficient, µ

$$\mu = 0.75$$

Edge moisture variation distance, em, ft

Step 1. Calculate the Plasticity Index (PI)

$$PI = LL-PL = 60-45 = 15$$

Step 2. Calculate % fine clay (%fc)

$$\%$$
 fc =  $\frac{\% - 2u}{\% - \#200} = \frac{10}{40} = 25\%$ 

Step 3. Determine Zone using the Mineral Classification Chart (Figure B-1 in

Appendix B)

Zone VI

Step 4. Calculate the Activity Ratio (PI / %fc)

$$PI / %fc = 15/25 = 0.6$$

Step 5. Calculate LL / %fc

$$LL / %fc = 60/25 = 2.4$$

Step 6. Determine 
$$\gamma_0$$
 using the Zone Chart (Figure B-2 in Appendix B)  $\gamma_0 = 0.15$ 

Step 7. Calculate Suction Compression Index  $(\gamma_h)$ 

$$\begin{split} \gamma_{h \text{ swell}} &= \gamma_o e^{\gamma_0} (\% \text{fc} / 100) = 0.15 e^{0.15} (25/100) = 0.044 \\ \gamma_{h \text{ shrink}} &= \gamma_o e^{-\gamma_0} (\% \text{fc} / 100) = 0.15 e^{-0.15} (25/100) = 0.032 \end{split}$$

Step 8. Calculate S

Step 9. Calculate Unsaturated Diffusion Coefficient (α)

$$\begin{split} &\alpha_{swell} = 0.0029\text{-}0.000162(S)\text{-}0.0122(\gamma_{h \ swell}) \\ &\alpha_{swell} = 0.0029\text{-}0.000162(\text{-}10)\text{-}0.0122(0.044) = 0.0040 \\ &\alpha_{shrink} = 0.0029\text{-}0.000162(S)\text{-}0.0122(\gamma_{h \ shrink}) \\ &\alpha_{shrink} = 0.0029\text{-}0.000162(\text{-}10)\text{-}0.0122(0.032) = 0.0041 \end{split}$$

Step 10. Fabric Factor ( $F_f$ ) (Figure B-3 in Appendix B)  $F_f = 1.0$ 

Step 11. Calculate Modified Unsaturated Diffusion Coefficient (α')

$$\alpha'_{swell} = \alpha_{swell} (F_f) = 0.0040(1.0) = 0.0040$$
  
 $\alpha'_{shirnk} = \alpha_{shink} (F_f) = 0.0041(1.0) = 0.0041$ 

Step 12. Determine Thornthwaite Moisture Index ( $I_m$ ) (Figure B-4 in Appendix B)  $I_m = 0$ 

Step 13. Determine  $e_m$  based on  $I_m$  for center and edge lift (Figure B-5 in Appendix B)

$$e_m$$
center $(I_m) = 4.2$  ft  
 $e_m$ edge $(I_m) = 3.5$  ft

Step 14. Determine  $e_m$  based on  $\alpha^\prime$  for center and edge lift (Figure B-5 in

Appendix B)

$$e_m center(\alpha'_{shirnk}) = 8.0 ft$$

$$e_m edge(\alpha'_{swell}) = 4.1 ft$$

USE 
$$e_m$$
center = 8.0 ft

USE 
$$e_m edge = 4.1 ft$$

Differential soil movement, y<sub>m</sub>, in

Step 1. Determine Measured Suction at Depth (Figure B-6 in Appendix B)

Suction at Depth = 3.6 pF

Step 2. Determine Dryest Suction

Dryest Suction = 4.2 pF

Step 3. Determine Wettest Suction

Wettest Suction = 3.2 pF

Step 4. Determine Stress Change Factors (SCF) for center and edge lift (Figure B-

7 in Appendix B)

$$SCF$$
-edge =  $7.9$ 

$$SCF$$
-center = -9.4

Step 5. Calculate y<sub>m</sub> for center and edge lift

$$y_m \text{ edge} = (SCF\text{-edge})(\gamma_h \text{ swell mod})$$

$$y_m \text{ edge} = (7.9)(0.044) = 0.35$$
"

$$y_m$$
 center = (SCF-center)( $\gamma_h$  shrink mod)

$$y_m$$
 center =  $(-9.4)(0.032) = -0.30$ ", use +  $0.30$ "

Structural Data and Material Properties

L(long direction) = 151'

L(short direction) = 101'

P = (20' of exterior wall)(50psf) = 1000plf DL

S(long direction) = 15' & 10', average spacing = [(12x10')+(2x15')]/14 = 10.7'=11'

S(short direction) = 10'

 $f_c^2 = 4000 \text{psi}$ 

$$f_t = 6\sqrt{f'_c} = 6\sqrt{4000} = 379 \text{ psi}$$
 [Eq 5-1]

$$f_c = 0.45(f_c) = 0.45(4000) = 1800 \text{ psi}$$
 [Eq 5-2]

Prestressing Steel:  $\frac{1}{2}$ "  $\phi$  270ksi 7 wire low-relaxation strand

 $C_{\Delta}$  (center lift) = 360 [Figure B-8 in Appendix B]

 $C_{\Delta}$  (edge lift) = 720 [Figure B-8 in Appendix B]

The  $C_{\Delta}$  values were chosen with the assumption that effective jointing details will be used to minimize cracking.

## Step 2.

Check that Shape Factor is smaller than 24

$$SF = \frac{(FoundationPerimeter)^2}{(FoundationArea)} = \frac{(504)^2}{15251} = 16.66 < 24$$
 **O.K.** [Eq 5-3]

#### Step 3.

Initial Rib Depth

1. Long Direction, use a first approximation of  $\beta = 15$  ft

L=151ft > 
$$6\beta = 6(15) = 90$$
 ft, therefore use z = 90 ft

h(center lift) = 
$$\left[ \frac{((y_m)(L))^{0.205} (S)^{1.059} (P)^{0.523} (e_m)^{1.296} (C_{\Delta})}{4560(z)} \right]^{0.824}$$
 [Eq 5-4]

h(center lift) = 
$$\left[\frac{((0.3)(151))^{0.205}(11!)^{1.059}(1000)^{0.523}(8.0)^{1.296}(360)}{4560(90)}\right]^{0.824} = 8.45$$

2. Short Direction, L=101ft,  $6\beta = 90$  ft, therefore use z = 90 ft

h(center lift) = 
$$\left[ \frac{((y_m)(L))^{0.205} (S)^{1.059} (P)^{0.523} (e_m)^{1.296} (C_{\Delta})}{4560(z)} \right]^{0.824}$$
 [Eq 5-4]

h(center lift) = 
$$\left[ \frac{((0.30)(101))^{0.205}(10')^{1.059}(1000)^{0.523}(8.0)^{1.296}(360)}{4560(90)} \right]^{0.824} = 7.26"$$

try 
$$h = 27$$
"

#### Step 4.

**Check Soil Bearing Capacity** 

1. Allowable Soil Pressure,  $q_u = 2500 \text{ psf}$ 

#### 2. Applied Loading

Roof and Ceiling DL = 
$$20 \text{ psf x } 150 \text{ ft x } 100 \text{ ft} = 300 \text{ K}$$
  
Roof LL =  $20 \text{ psf x } 150 \text{ ft x } 100 \text{ ft} = 300 \text{ K}$   
 $2^{\text{nd}}$  Floor DL =  $70 \text{ psf x } 150 \text{ ft x } 100 \text{ ft} = 1050 \text{ K}$   
 $2^{\text{nd}}$  Floor LL =  $80 \text{ psf x } 150 \text{ ft x } 100 \text{ ft} = 1200 \text{ K}$   
Slab Weight =  $(6^{\circ\prime}/12) \text{ x } 150 \text{ pcf x } 150 \text{ ft x } 100 \text{ ft} = 1125 \text{ K}$   
Rib Weight =  $(21^{\circ\prime\prime} \text{ x } 12^{\circ\prime\prime})/144 \text{ x } 150 \text{ pcf x } 3150 \text{ ft} = 827 \text{ K}$   
 $1^{\text{st}}$  Floor LL =  $100 \text{ psf x } 150 \text{ ft x } 100 \text{ ft} = 1500 \text{ K}$   
Perimeter Wall Load =  $50 \text{ psf x } 2(100 \text{ ft} + 150 \text{ ft}) \text{ x } 30 \text{ ft} = 750 \text{ K}$   
Total Load =  $7052 \text{ K}$ 

#### 3. Rib Bearing Area

bearing width of edge ribs = 
$$12$$
" +  $6(6$ ") =  $48$ " =  $4$ '  
bearing width of interior ribs =  $12$ " +  $16(6$ ") =  $108$ " =  $9$ '  
bearing area =  $2(101)(4) + 2(151)(4) + 13(101)(9) + 9(151)(9) = 26064$  sf

#### 4. Soil Pressure

$$q_{act} = 7052 \text{ K}/26064 \text{ sf} = 271 \text{ psf} < 2500 \text{ psf}$$
 **O.K.**

Step 5.
Section Properties

	Long Direction	Short Direction
Rib Depth h (in)	27"	27"
Rib Width (in)	12"	12"
Number of Ribs	11	15
Total Rib Width (in)	132"	180"
Slab Thickness (in)	6"	6"

## Long Direction

Section	Area (in <sup>2</sup> )	y(in)	Ay(in <sup>3</sup> )	$Ay^2 (in^4)$	$I_o$ (in <sup>4</sup> )
Slab	7272	-3	-21816	65448	21816
(101' x 12" x 6")					
Ribs	2772	-16.5	-45738	754677	101871
(132" x 21")					
	10044		-67554	820125	123687
				+123687	
				943812	

$$\begin{split} y_t &= \Sigma Ay \: / \: \Sigma A = 67554 \: / \: 10044 = 6.73 \: in \\ I &= (\Sigma Ay^2 + \Sigma I_o) \: - Ay_t^2 = 943812 - 10044(6.73)^2 = 488890 \: in^4 \\ S_t &= I \: / \: y_t = 488890 \: / \: 6.73 = 72643 \: in^3 \\ S_b &= I \: / \: y_b = 488890 \: / \: (27\text{-}6.73) = 24119 \: in^3 \end{split}$$

#### **Short Direction**

Section	Area (in <sup>2</sup> )	y(in)	Ay(in <sup>3</sup> )	$Ay^2$ (in <sup>4</sup> )	$I_o$ (in <sup>4</sup> )
Slab	10872	-3	-32616	97848	32616
(151' x 12" x 6")					
Ribs	3780	-16.5	-62370	1029105	138915
(180" x 21")					
	14652		-94986	1126953	171531
				<u>+171531</u>	
				1298484	

$$\begin{split} y_t &= \Sigma Ay \, / \, \Sigma A = 94986 \, / \, 14652 = 6.48 \text{ in} \\ I &= (\Sigma Ay^2 + \Sigma I_o) \, - Ay_t^2 = 1298484 - 14652(6.48)^2 = 683241 \text{ in}^4 \\ S_t &= I \, / \, y_t = 683241 \, / \, 6.48 = 105438 \text{ in}^3 \\ S_b &= I \, / \, y_b = 683241 \, / \, (27\text{-}6.48) = 33296 \text{ in}^3 \end{split}$$

## Step 6.

## Prestressing Steel

1. Number of tendons required for minimum required force

a. Stress in tendons immediately after anchoring

$$f_{pi} = 0.7 f_{pu} = 0.7 (270 \text{ksi}) = 189 \text{ ksi}$$
 [Eq 5-5]

 Stress in tendons after losses, assume 15ksi for prestress losses for lowrelaxation strand

$$f_e = f_{pi} - 15 = 189 - 15 = 174 \text{ ksi}$$
 [Eq 5-6]

Minimum required force =  $0.05A_{long} = 0.05(10044) = 503K$ 

$$N_{t} \text{ (Long)} = \frac{0.05A}{f_{e}A_{ps}}$$
 [Eq 5-7]

$$N_t \text{ (Long)} = \frac{503K}{174ksi(0.153in^2 / tendon)} = 18.9$$

$$N_t$$
 (Short) =  $\frac{0.05(14652)}{174ksi(0.153in^2/tendon)}$  = 27.6

2. Number of tendons required to overcome slab-sub-grade friction

Weight of Ribs and Slab = 827 K + 1125 K = 1952 K

$$N_{t} = \frac{\mu W_{slab}}{2000 f_{e} A_{ps}} = \frac{0.75(1952000)}{2000(174)(0.153)} = 27.5$$
 [Eq 5-8]

3. Total number of tendons to provide minimum required force

Long 
$$N_T = 18.9 + 27.5 = 46.4$$
 USE 47

Short 
$$N_T = 27.6 + 27.5 = 55.1$$
 USE 56

4. Design Prestress Force

Force per tendon = 
$$f_e \times A_{ps} = 174 \times 0.153 = 26.6 \text{ K}$$

$$P_r = N_t(f_e \times A_{ps}) - \mu W_{slab}/2000$$
 [Eq 5-9]

Long 
$$P_r = 47(26.6) - .75(1952000)/2000 = 519 \text{ K}$$

Short 
$$P_r = 56(26.6) - .75(1952000)/2000 = 758 \text{ K}$$

Summary:	Long Direction	Short Direction
Cross Sectional Area A (in <sup>2</sup> )	10044	14652
Centroid of Tendons (in. From Top)	-3.0	-3.0
Top Depth to Section Centroid, yt (in	-6.73	-6.48
Prestress Eccentricity, e (in)	3.73	3.48
Allowable Concrete Tensile Stress (k	si) 0.379	0.379
Allowable Concrete Compressive Str	ess (ksi) 1.8	1.8

#### Step 7.

Center Lift Design ( $e_m = 8.0 \text{ ft}$ ,  $y_m = 0.30 \text{ in}$ )

#### A) Design Moments

1. Long Direction

$$M_L = A_o[B(e_m)^{1.238} + C]$$
 [Eq 5-10]

where 
$$A_0 = \frac{(L)^{0.013}(S)^{0.306}(h)^{0.688}(P)^{0.534}(y_m)^{0.193}}{727}$$
 [Eq 5-11]

$$A_o = \frac{(151)^{0.013}(11)^{0.306}(27)^{0.688}(1000)^{0.534}(0.30)^{0.193}}{727} = 0.936$$

$$5 < e_{\text{m}}$$
  $B = \frac{y_m - 1}{3} \le 1.0 = \frac{0.30 - 1}{3} = -0.233 < 1.0$  [Eq 5-13]

$$C = \left[ 8 - \frac{P - 613}{255} \right] \left[ \frac{4 - y_m}{3} \right] \ge 0 = \left[ 8 - \frac{(1000 - 613)}{255} \right] \left[ \frac{(4 - 0.30)}{3} \right] = 8.0 \text{ [Eq 5-14]}$$

$$M_L = 0.936[-0.233(8.0)^{1.238} + 8.0] = 4.63 \text{ k-ft/ft}$$
 [Eq 5-10]

Check  $M_L$  at  $e_m = 5$ ft (per section 4.3.2 PTI Design Manual)

$$M_L = 0.936[1(8.0)^{1.238} + 0] = 12.28 \text{ k-ft/ft}$$
 [Eq 5-10 & Eq 5-12]

USE  $M_L = 12.28 \text{ k-ft/ft}$ 

2. Short Direction

$$L_I/L_S = 151/101 = 1.5 > 1.1$$

$$M_S = \left[ \frac{58 + e_m}{60} \right] M_L = \frac{(58 + 8.0)(4.63)}{60} = 5.09 \text{ k-ft/ft}$$
 [Eq 5-16]

Check  $M_S$  at  $e_m = 5$  ft (per section 4.3.2 PTI Design Manual)

$$M_S = \frac{(58+5.0)(12.28)}{60} = 12.90 \text{ k-ft/ft}$$
 [Eq 5-16]

USE  $M_S = 12.90 \text{ k-ft/ft}$ 

#### B) Compare Actual and Allowable Service Load Stresses

$$f = \frac{P_r}{A} \pm \frac{M_{L,S}}{S_{t,b}} \pm \frac{P_r(e_p)}{S_{t,b}}$$
 [Eq 5-17]

#### 1. Long Direction

Tension in top fiber (tension(-), compression(+))

$$f = \frac{519}{10044} - \frac{12.28(101)(12)}{72643} + \frac{519(3.73)}{72643} = -0.127 \text{ ksi}$$
 [Eq 5-17]

-0.127 ksi < -0.379 ksi **O.K.** 

Compression in bottom fiber

$$f = \frac{519}{10044} + \frac{12.28(101)(12)}{24119} - \frac{519(3.73)}{24119} = 0.589 \text{ ksi}$$
 [Eq 5-17]

0.589 ksi < 1.8 ksi **O.K.** 

#### 2. Short Direction

Tension in top fiber

$$f = \frac{758}{14652} - \frac{12.90(151)(12)}{105438} + \frac{758(3.48)}{105438} = -0.145 \text{ ksi}$$
 [Eq 5-17]

-0.145 ksi < -0.379 ksi **O.K.** 

Compression in bottom fiber

$$f = \frac{758}{14652} + \frac{12.90(151)(12)}{33296} - \frac{758(3.48)}{33296} = 0.675 \text{ ksi}$$
 [Eq 5-17]

0.675 ksi < 1.8 ksi **O.K.** 

#### C) Minimum Foundation Stiffness

1. Long Direction

$$\beta = \frac{1}{12} \sqrt[4]{\frac{E_{cr}I}{E_{soil}}} = \frac{1}{12} \sqrt[4]{\frac{(1,500,000)(488890)}{1500}} = 12.4 \text{ ft}$$
 [Eq 5-20]

$$6\beta = 6(12.4) = 74.4$$
 ft < 151 ft, therefore  $z_L = 74.40$  ft

$$I_{L} \ge \frac{18000(M_{L})(L_{S})(C_{\Delta})(z_{L})}{1500000} \ge \frac{18000(12.28)(101)(360)(74.4)}{1500000} = 398636 \text{ in}^{4}$$

[Eq 5-18]

 $I_L = 488890 \text{ in}^4 > 398636 \text{ in}^4 \text{ O.K.}$ 

2. Short Direction

$$\beta = \frac{1}{12} \sqrt[4]{\frac{E_{cr}I}{E_{soil}}} = \frac{1}{12} \sqrt[4]{\frac{(1,500,000)(683241)}{1500}} = 13.47 \text{ ft}$$
 [Eq 5-20]

$$6\beta = 6(13.47) = 80.82$$
 ft < 101 ft, therefore  $z_s = 80.82$  ft

$$I_{S} \ge \frac{18000(M_{S})(L_{L})(C_{\Delta})(z_{S})}{1500000}$$
 [Eq 5-19]

$$I_{\rm S} \ge \frac{18000(12.90)(151)(360)(80.82)}{1500000} = 680094 \text{ in}^4$$
 [Eq 5-19]

$$I_S = 683241 \text{ in}^4 > 680094 \text{ in}^4 \text{ O.K.}$$

## D) Shear Calculations

#### 1. Long Direction

**Expected Service Shear** 

$$v_{L} = \frac{(L)^{0.09} (S)^{0.71} (h)^{0.45} (P)^{0.44} (y_{m})^{0.16} (e_{m})^{0.93}}{1940}$$
 [Eq 5-21]

$$v_{L} = \frac{(151)^{0.09}(11)^{0.71}(27)^{0.45}(1000)^{0.44}(0.3)^{0.16}(8.0)^{0.93}}{1940} = 2.33 \text{ klf}$$
 [Eq5-21]

Permissible Shear Stress

$$f_p = P_r/A = \frac{519}{10044} = 0.051 \text{ ksi} = 51 \text{ psi}$$

$$v_c = 1.7\sqrt{f'_c} + 0.2(f_p) = 1.7\sqrt{4000} + 0.2(51) = 118 \text{ psi}$$
 [Eq 5-23]

Applied Service Load Shear Stress

$$v = {VW \over nbh} = {2.33(101)(1000) \over 11(12)(27)} = 66.1 \text{ psi}$$
 [Eq 5-24]

66.1 psi < 118 psi **O.K.** 

#### 2. Short Direction

**Expected Service Shear** 

$$\mathbf{v}_{s} = \frac{(L)^{0.19} (S)^{0.45} (h)^{0.20} (P)^{0.54} (y_{m})^{0.04} (e_{m})^{0.97}}{1350}$$
 [Eq 5-22]

$$v_S = \frac{(101)^{0.19} (10)^{0.45} (27)^{0.20} (1000)^{0.54} (0.3)^{0.04} (8.0)^{0.97}}{1350} = 2.90 \text{ klf} \qquad \text{[Eq 5-22]}$$

Permissible Shear Stress

$$f_p = P_r/A = \frac{758}{14652} = 0.052 \text{ ksi} = 52 \text{ psi}$$
  
 $v_c = 1.7\sqrt{f'_c} + 0.2(f_p) = 1.7\sqrt{4000} + 0.2(52) = 118 \text{ psi}$  [Eq 5-23]

Applied Service Load Shear Stress

$$v = {VW \over nbh} = {2.90(151)(1000) \over 15(12)(27)} = 90.1 \text{ psi}$$
 [Eq 5-24]

90.1 psi < 118 psi **O.K.** 

#### Step 8.

Edge Lift Design ( $e_m = 4.1$  ft,  $y_m = 0.34$  in )

#### A) Design moments

1. Long Direction

$$M_{L} = \frac{(S)^{0.1} (he_{m})^{0.78} (y_{m})^{0.66}}{7.2(L)^{0.0065} (P)^{0.04}} = \frac{(11)^{0.1} (27x4.1)^{0.78} (0.34)^{0.66}}{7.2(151)^{0.0065} (1000)^{0.04}} = 2.5 \text{ k-ft/ft [Eq 5-25]}$$

2. Short Direction

$$M_{\rm S} = h^{0.35} [(19 + e_m) / 57.75] M_L = (27)^{0.35} [(19 + 4.1) / 57.75] (2.5) = 3.17 \text{ k-ft/ft}$$
[Eq 5-27]

B) Compare Actual and Allowable Service Load Stresses

$$f = \frac{P_r}{A} \pm \frac{M_{L,S}}{S_{t,b}} \pm \frac{P_r(e_p)}{S_{t,b}}$$
 [Eq 5-17]

1. Long Direction

Tension in Bottom Fiber

$$f = \frac{519}{10044} - \frac{2.5(101)(12)}{24119} - \frac{519(3.73)}{24119} = -0.154 \text{ ksi}$$
 [Eq 5-17]

-0.154 ksi < -0.379 ksi **O.K.** 

Compression in Top Fiber

$$f = \frac{519}{10044} + \frac{2.5(101)(12)}{72643} + \frac{519(3.73)}{72643} = 0.120 \text{ ksi}$$
 [Eq 5-17]

0.120 ksi < 1.8 ksi **O.K.** 

2. Short Direction

Tension in Bottom Fiber

$$f = \frac{758}{14652} - \frac{3.17(151)(12)}{33296} - \frac{758(3.48)}{33296} = -0.200 \text{ ksi}$$
 [Eq 5-17]

-0.200 ksi < 1.8 ksi **O.K.** 

Compression in Top Fiber

$$f = \frac{758}{14652} + \frac{3.17(151)(12)}{105438} + \frac{758(3.48)}{105438} = 0.131 \text{ ksi}$$
 [Eq 5-17]

0.131 ksi < 1.8 ksi **O.K.** 

#### C) Minimum Foundation Stiffness

1. Long Direction

$$I_{L} \ge \frac{18000(M_{L})(L_{S})(C_{\Delta})(z_{L})}{1500000} = \frac{18000(2.5)(101)(720)(74.4)}{1500000} = 162311 \text{ in}^{4}$$
[Eq 5-18]

 $I_L = 488890 \text{ in}^4 > 162311 \text{ in}^4 \text{ O.K.}$ 

2. Short Direction

$$I_{S} \ge \frac{18000(M_{S})(L_{L})(C_{\Delta})(z_{S})}{1500000} = \frac{18000(3.17)(151)(720)(80.82)}{1500000} = 334248 \text{ in}^{4}$$

[Eq 5-19]

 $I_S = 683241 \text{ in}^4 > 334248 \text{ in}^4 \text{ O.K.}$ 

### D) Shear Calculations

## 1. Long Direction

**Expected Service Shear** 

$$\mathbf{v}_{S} = \mathbf{v}_{L} = \frac{(L)^{0.07} (h)^{0.4} (P)^{0.03} (e_{m})^{0.16} (y_{m})^{0.67}}{3S^{0.015}}$$
 [Eq 5-28]

$$v_L = \frac{(151)^{0.07} (27)^{0.4} (1000)^{0.03} (4.1)^{0.16} (0.34)^{0.67}}{3(11)^{0.015}} = 1.27 \text{ klf}$$
 [Eq 5-28]

Permissible Shear Stress

$$v_c = 1.7\sqrt{f'_c} + 0.2(f_p) = 1.7\sqrt{4000} + 0.2(51) = 118 \text{ ksi}$$
 [Eq 5-29]

Design (Actual) Shear Stress

$$v = \frac{vW}{nbh} = \frac{1.27(101)(1000)}{11(12)(27)} = 36 \text{ ksi}$$
 [Eq 5-30]

36 ksi < 118 ksi **O.K.** 

**Expected Service Shear** 

$$\mathbf{v}_{S} = \mathbf{v}_{L} = \frac{(L)^{0.07} (h)^{0.4} (P)^{0.03} (e_{m})^{0.16} (y_{m})^{0.67}}{3S^{0.015}}$$
 [Eq 5-28]

$$v_s = \frac{(101)^{0.07} (27)^{0.4} (1000)^{0.03} (4.1)^{0.16} (0.34)^{0.67}}{3(15)^{0.015}} = 1.24 \text{ klf}$$
 [Eq 5-28]

Permissible Shear Stress

$$v_c = 1.7\sqrt{f'_c} + 0.2(f_p) = 1.7\sqrt{4000} + 0.2(52) = 118 \text{ ksi}$$
 [Eq 5-29]

Design (Actual) Shear Stress

$$v = {vW \over nbh} = {1.27(151)(1000) \over 15(12)(27)} = 39.5 \text{ ksi}$$
 [Eq 5-30]

39.5 ksi < 118 ksi **O.K.** 

#### Step 9. Equivalent Cracked Sections

#### A) Center Lift

#### 1. Long Direction

**Total Long Direction Moment** 

$$M_{LTOTAL} = M_L(L_S) = 12.28(101) = 1241 \text{ k-ft}$$
 [Eq 5-31]

To be Developed with Cracked Section

$$0.9 \text{ M}_{L} = 0.9(1241) = 1117 \text{ k-ft}$$
 [Eq 5-32]

Slab Tendon Force

$$N_{t(Long)}(f_eA_{ps}) = 47x26.6 = 1250 \text{ K}$$
 [Eq 5-33]

Capacity with Tendons Alone

Depth of Compression Block, a

$$a = \frac{N_{t(Long)}(f_e A_{ps})}{0.85(f'_e)(b)(n)} = \frac{1250}{0.85(4)(12)(11)} = 2.79$$
 [Eq 5-34]

**Cracked Section Capacity** 

$$M_{cr} = \frac{N_{t(Long)} f_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right) = \frac{1250}{12} \left( 27 - 3 - \frac{2.79}{2} \right) = 2354 \text{ k-ft [Eq 5-35]}$$

2354 k-ft > 1117 k-ft **O.K.** 

**Total Short Direction Moment** 

$$M_{STOTAL} = M_S(L_L) = 12.9(151) = 1948 \text{ k-ft}$$
 [Eq 5-36]

To be Developed with Cracked Section

$$0.9 \text{ M}_{\text{S}} = 0.9(1948) = 1753 \text{ k-ft}$$
 [Eq 5-37]

Slab Tendon Force

$$N_{t(Short)}(f_eA_{ps}) = 56(26.6) = 1490 \text{ K}$$
 [Eq 5-38]

Capacity with Tendons Alone

Depth of Compression Block, a

$$a = \frac{N_{t(Short)}(f_e A_{ps})}{0.85(f'_c)(b)(n)} = \frac{1490}{0.85(4)(12)(15)} = 2.43$$
 [Eq 5-39]

Cracked Section Capacity

$$M_{cr} = \frac{N_{t(Short)} f_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right) = \frac{1490}{12} \left( 27 - 3 - \frac{2.43}{2} \right) = 2829 \text{ k-ft [Eq 5-40]}$$

2829 k-ft > 1753 k-ft **O.K.** 

### B) Edge Lift

#### 1. Long Direction

**Total Long Direction Moment** 

$$M_{LTOTAL} = M_L(L_S) = 2.5 (101) = 253 \text{ k-ft}$$
 [Eq 5-41]

To be Developed with Cracked Section

$$0.9 \text{ M}_{L} = 0.9(253) = 228 \text{ k-ft}$$
 [Eq 5-42]

Rib Tendon Force

$$n(f_eA_{ps}) = 11(26.6) = 293 \text{ K}$$
 [Eq 5-43]

Capacity with Tendons Alone

Depth of Compression Block

$$a = \frac{n(f_e A_{ps})}{0.85(f'_e)(L_s)(12)} = \frac{293}{0.85(4)(101)(12)} = 0.071$$
" [Eq 5-44]

**Cracked Section Capacity** 

$$M_{cr} = \frac{nf_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right) = \frac{293}{12} \left( 27 - 3 - \frac{0.071}{2} \right) = 585 \text{ k-ft}$$
 [Eq 5-45]

585 k-ft > 228 k-ft **O.K.** 

**Total Short Direction Moment** 

$$M_{STOTAL} = M_S (L_L) = 3.17 (151) = 479 \text{ k-ft}$$
 [Eq 5-46]

To be Developed with Cracked Section

$$0.9 \text{ M}_{\text{S}} = 0.9(479) = 431 \text{ k-ft}$$
 [Eq 5-47]

Rib Tendon Force

$$(f_e A_{DS}) = 15(26.6) = 399 \text{ K}$$
 [Eq 5-48]

Capacity with Tendons Alone

Depth of Compression Block

$$a = \frac{n(f_e A_{ps})}{0.85(f'_e)(L_L)(12)} = \frac{399}{0.85(4)(151)(12)} = 0.065$$
" [Eq 5-49]

**Cracked Section Capacity** 

$$M_{cr} = \frac{nf_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right) = \frac{399}{12} \left( 27 - 3 - \frac{0.065}{2} \right) = 797 \text{ k-ft}$$
 [Eq 5-50]

797 k-ft > 431 k-ft **O.K.** 

#### Summary of Design

#### A) Long Direction

Use 27" deep ribs, 12" wide, spaced as shown on the foundation plan in Figure 6-1,  $(47) \frac{1}{2}$ "  $\phi$  270ksi low-relaxation tendons in the slab with centroids 3" below the top of slab and at most 26" on center. Figure 7-1 on the following page shows the section requirements for the long direction.

#### B) Short Direction

Use 27" deep ribs, 12" wide, spaced as shown on the foundation plan in Figure 6-1, (56)  $\frac{1}{2}$ "  $\phi$  270ksi low-relaxation tendons in the slab with centroids 3" below the top of slab and at most 32" on center. Figure 7-1 on the next page shows the section requirements for the short direction.

C) Check Prestressing Tendons ability to resist tension stresses from slab moment

$$M_{\text{service}} = 2.82 \text{ k-ft [Chapter 6]}$$

$$\frac{P}{A} + \frac{Mc}{I} \le f_t = -0.379$$
ksi

where, P = prestress force per tendon

A = area per tendon

M = service moment in slab

c = distance to centroid of tendon

I = moment of inertia

$$\frac{26.6}{b(6)} - \frac{2.82(12)(3)(b/12)}{b(6^3)/12} \le -0.379$$

$$b \le 48.72$$
"

therefore, tendon spacing at 26" and 32" are okay

Temperature and Shrinkage Steel required

$$\rho = 0.0018$$
,  $A_{st} = 0.0018(12")(6") = 0.1269$  [ACI 318-05 7.12.2.1]

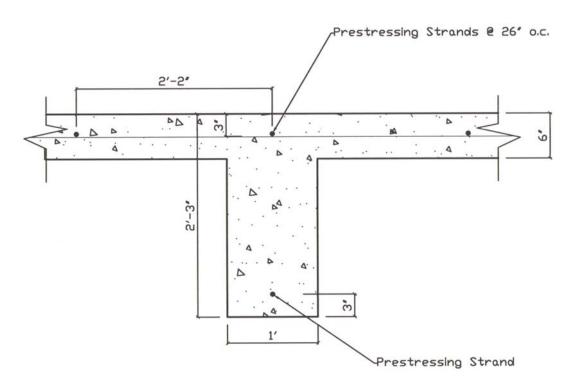
Check ACI 318-05 7.12.3.1

$$\frac{26.6K}{32''(6'')}$$
 = 138 psi > 100 psi, therefore prestressing tendons can be used for

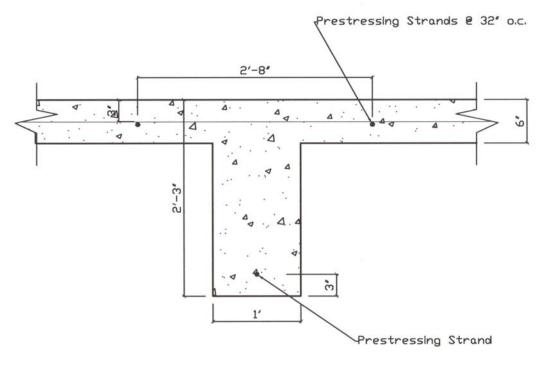
temperature and shrinkage steel

The tendon layout is shown in Figure 7-2.

**Figure 7-1 Section Requirements** 

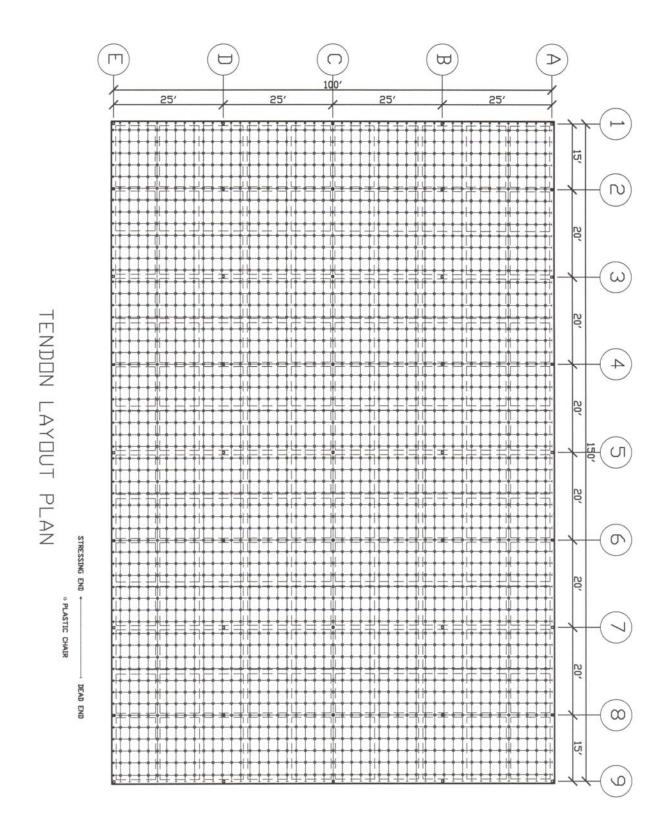


LONG DIRECTION



SHORT DIRECTION

Figure 7-2 Tendon Layout Plan



# **CHAPTER 8 - Mild Reinforced Mat Foundation Design**

This Chapter is a design of a mild reinforced ribbed mat foundation. The design information including framing plans are given in Chapter 6 of this report. The design steps are explained in Chapter 5 Design Steps for Mild Reinforced Ribbed Mat Foundations. This Chapter will use the concrete section designed in Chapter 7 to compare reinforcing requirements for both design examples. Appendix C of this report shows another design of a ribbed mat foundation for mild reinforcement in the attempt to eliminate shear reinforcement in the ribs. Step 1.

Obtain and organize all known design data including:

Found in Step 1 in Chapter 7.

#### Step 2.

Check Soil Bearing Capacity

Found in Step 4 in Chapter 7.

#### Step 3.

**Section Properties** 

Found in Step 5 in Chapter 7.

#### Step 4.

Center Lift Design ( $e_m = 8.0 \text{ ft}, y_m = 0.30$ ")

- A) Design Moments
  - 1. Long Direction

USE  $M_L = 12.28 \text{ k-ft/ft}$ , found in Step 7 in Chapter 7.

2. Short Direction

 $L_I/L_S = 151/101 = 1.5 > 1.1$ 

USE  $M_S = 12.90 \text{ k-ft/ft}$ , found in Step 7 in Chapter 7.

#### B) Flexure Design of Ribs

#### 1. Long Direction

$$M_{u(L,S)} = M_{L,S}(L_{S,L})(1.6) = 12.28(101')(1.6) = 1986 \text{ k-ft}$$
 [Eq 5-51]

$$R_{\rm u} = \frac{Mu}{\phi(b)(d)^2} = \frac{1986(12)}{0.9(12x15)(24)^2} = 0.255$$
 [Eq 5-52]

$$m = \frac{f_y}{0.85(f'_c)} = \frac{60}{0.85(4)} = 17.65$$
 [Eq 5-53]

$$\rho_{\text{req}} = \frac{1}{m} \left[ 1 - \sqrt{1 - \frac{2(R_u)(m)}{f_y}} \right] = \frac{1}{17.65} \left[ 1 - \sqrt{1 - \frac{2(17.65)(0.255)}{60}} \right] = 0.0045 \text{ [Eq 5-54]}$$

$$\rho_{\min} = \frac{200}{f_{\nu}} = \frac{200}{60000} = 0.00333, \text{ use } \rho_{\text{req}} = 0.0045$$
 [Eq 5-55]

$$\rho_{\text{max}} = \frac{0.319(B_1)(f'_c)}{f_y} = \frac{0.319(0.85)(4000)}{60000} = 0.0181$$
 [Eq 5-56]

$$A_{st} = \rho$$
 (b)(d) = 0.0045(24)(12) = 1.30 in<sup>2</sup>/rib [Eq 5-57] use 3 # 6's in each rib,  $A_{st} = 1.32$  in<sup>2</sup>

#### 2. Short Direction

$$M_{u(L,S)} = M_{L,S}(L_{S,L})(1.6) = 12.9(151')(1.6) = 3117 \text{ k-ft}$$
 [Eq 5-51]

$$R_{\rm u} = \frac{Mu}{\phi(b)(d)^2} = \frac{3117(12)}{0.9(12x11)(24)^2} = 0.547$$
 [Eq 5-52]

$$m = \frac{f_y}{0.85(f'_c)} = \frac{60}{0.85(4)} = 17.65$$
 [Eq 5-53]

$$\rho_{\text{req}} = \frac{1}{m} \left[ 1 - \sqrt{1 - \frac{2(R_u)(m)}{f_y}} \right] = \frac{1}{17.65} \left[ 1 - \sqrt{1 - \frac{2(17.65)(0.547)}{60}} \right] = 0.010 \text{ [Eq 5-54]}$$

$$\rho_{\min} = \frac{200}{f_v} = \frac{200}{60000} = 0.00333, \text{ use } \rho_{\text{req}} = 0.010$$
 [Eq 5-55]

$$\rho_{\text{max}} = \frac{0.319(B_1)(f'_c)}{f_v} = \frac{0.319(0.85)(4000)}{60000} = 0.0181$$
 [Eq 5-56]

$$A_{st} = \rho (b)(d) = 0.010(24)(12) = 2.88 \text{ in}^2/\text{rib}$$
 [Eq 5-57]

use 3 # 9's in each rib,  $A_{st} = 3.00 \text{ in}^2$ 

#### C) Shear Calculations

## 1. Long Direction

Expected Service Shear

$$v_L = 2.33$$
 klf, found in Step 7 of Chapter 7

$$V_u = v_{L,S}(L_{S,L})(1.6) = 2.33(101)(1.6) = 377 \text{ K}$$
 [Eq 5-58]

$$\phi V_c = \phi(2) \sqrt{f'_c}(b)(d) = 0.75(2) \sqrt{4000}(12x11)(24) = 300k$$
 [Eq 5-59]

 $\phi V_c \le V_u$ , therefore need shear reinforcement

Shear Stirrups Need to Resist

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{377 - 300}{0.75} = 103 \text{ K}$$
 [Eq 5-60]

Stirrup Spacing Required using #4 stirrups

$$S_{\text{req}} = \frac{nA_{\nu}f_{\nu}d}{V_{s}} = \frac{11(2)(0.2)(60000)(24)}{103000} = 61.6^{\circ}$$
 [Eq 5-61]

Maximum spacing of stirrups, using #4 stirrups

$$s_{\text{max}} = \frac{A_{y} f_{y}}{\phi \sqrt{f'_{c}}(b)} = \frac{2(0.2)(60,000)}{0.75\sqrt{4000}(12)} = 42.2$$
 [Eq5-62]

or 
$$s_{\text{max}} = \frac{A_v f_y}{50b} = \frac{2(0.2)(60,000)}{50(12)} = 40$$
" [Eq 5-63]

or 
$$s_{\text{max}} = \frac{d}{2} \le 24$$
" =  $\frac{24}{2} = 12$ "  $\le 24$ " [Eq 5-64]

 $s_{max} = 12$ ", use #4 stirrups at 12" on center

#### 2. Short Direction

**Expected Service Shear** 

 $v_S = 2.90$  klf, found in Step 7 of Chapter 7

$$V_u = v_{L,S}(L_{S,L})(1.6) = 2.90(151)(1.6) = 701 \text{ K}$$
 [Eq 5-58]

$$\phi V_c = \phi(2) \sqrt{f'_c}(b)(d) = 0.75(2) \sqrt{4000}(12x15)(24) = 409 \text{ K}$$
 [Eq 5-59]

 $\phi V_c = 409 \text{ K} < 701 \text{ K}$ , therefore need shear reinforcement

Shear Stirrups Need to Resist

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{701 - 409}{0.75} = 390 \text{ K}$$
 [Eq 5-60]

Stirrup Spacing Required using #4 stirrups

$$S_{\text{req}} = \frac{nA_{v}f_{y}d}{V_{s}} = \frac{15(2)(0.2)(60000)(24)}{390000} = 21$$
" [Eq 5-61]

 $s_{max} = 12$ ", use #4 stirrups at 12" on center

#### D) Foundation Stiffness

#### 1. Long Direction

 $I_L \ge 398636 \text{ in}^4$ , found in Step 7 in Chapter 7

Effective I

$$Mcr = \frac{f_r(I_g)}{y} f_r = 7.5\sqrt{f'_c}, Mcr = \frac{7.5\sqrt{4000}(488890)}{6.73} = 2871 \text{ k-ft [Eq 5-65]}$$

$$nAs = \frac{29000000}{57000\sqrt{4000}}(1.32) = 10.62 \text{ in}^2$$
 [Eq 5-66]

Find Distance to Neutral Axis of Transformed Section

$$12(x)\frac{x}{2} = 10.62(24 - x) \qquad x = 5.7$$
"

Icr = 
$$\frac{bx^3}{3} + nA_s(d-x)^2 = \frac{12(5.7)^3}{3} + 10.62(24-5.7)^2 = 4297 \text{ in}^4$$
 [Eq 5-67]

$$I_{E} = \left(\frac{Mcr}{M_{A}}\right)^{3} (I_{g}) + \left[1 - \left(\frac{Mcr}{M_{A}}\right)^{3}\right] (I_{cr})$$
 [Eq 5-68]

$$I_E = \left(\frac{2871}{1241}\right)^3 (488890) + \left[1 - \left(\frac{2871}{1241}\right)^3\right] (4297)(15) = 5319730 \text{ in}^4$$

 $I_E < I_g$ , therefore  $I_E = 488890 \text{ in}^4$ 

 $488890 \text{ in}^4 > 398636 \text{ in}^4 \text{ O.K.}$ 

#### 2. Short Direction

 $I_S \ge 680094 \text{ in}^4$ , found in Step 7 in Chapter 7

Effective I

Mcr = 
$$\frac{f_r(I_g)}{y}$$
 f<sub>r</sub> = 7.5 $\sqrt{f'_c}$ , Mcr =  $\frac{7.5\sqrt{4000}(683241)}{6.48}$  = 4168 k-ft [Eq 5-65]

$$nAs = \frac{29000000}{57000\sqrt{4000}}(3.00) = 24.2 \text{ in}^2$$
 [Eq 5-66]

Find Distance to Neutral Axis of Transformed Section

$$12(x)\frac{x}{2} = 24.2(24 - x) \qquad x = 8.0$$
"

Icr = 
$$\frac{bx^3}{3} + nA_s(d-x)^2 = \frac{12(8.0)^3}{3} + 24.2(24-8.0)^2 = 8243 \text{ in}^4$$
 [Eq 5-67]

$$I_{E} = \left(\frac{Mcr}{M_{A}}\right)^{3} (I_{g}) + \left[1 - \left(\frac{Mcr}{M_{A}}\right)^{3}\right] (I_{cr})$$
 [Eq 5-68]

$$I_E = \left(\frac{4168}{1948}\right)^3 (683241) + \left[1 - \left(\frac{4168}{1948}\right)^3\right] (8243)(11) = 5895030 \text{ in}^4$$

 $I_E < I_g$ , therefore  $I_E = 683241 \text{ in}^4$ 

 $683241 \text{ in}^4 > 398636 \text{ in}^4 \text{ O.K.}$ 

### Step 5.

Edge Lift Design ( $e_m = 4.1 \text{ ft}$ ,  $y_m = 0.34 \text{ in}$ )

- A) Design moments
  - 1. Long Direction

 $M_L = 2.5 \text{ k-ft/ft}$ , found in Step 8 in Chapter 7

2. Short Direction

 $M_S = 3.17 \text{ k-ft/ft}$ , found in Step 8 in Chapter 7

- B) Flexure Design of Ribs
  - 1. Long Direction

$$M_{u(L,S)} = M_{L,S}(L_{S,L})(1.6) = 2.5(101')(1.6) = 404 \text{ k-ft}$$
 [Eq 5-51]

$$R_{\rm u} = \frac{Mu}{\phi(b)(d)^2} = \frac{404(12)}{0.9(12x15)(24)^2} = 0.052$$
 [Eq 5-52]

$$m = \frac{f_y}{0.85(f_a^{\prime})} = \frac{60}{0.85(4)} = 17.65$$
 [Eq 5-53]

$$\rho_{\text{req}} = \frac{1}{m} \left[ 1 - \sqrt{1 - \frac{2(R_u)(m)}{f_v}} \right] = \frac{1}{17.65} \left[ 1 - \sqrt{1 - \frac{2(17.65)(0.052)}{60}} \right] = 0.0009 \text{ [Eq 5-54]}$$

$$\rho_{\min} = \frac{200}{f_v} = \frac{200}{60000} = 0.0033, \text{ use } \rho = 0.0033$$
 [Eq 5-55]

$$\rho_{\text{max}} = \frac{0.319(B_1)(f'_c)}{f_v} = \frac{0.319(0.85)(4000)}{60000} = 0.0181$$
 [Eq 5-56]

$$A_{st} = \rho (b)(d) = 0.0033(24)(12) = 0.95 \text{ in}^2/\text{rib}$$
 [Eq 5-57]

use 3 # 6's in each rib,  $A_{st} = 1.32 \text{ in}^2$ 

#### 2. Short Direction

$$M_{u(L,S)} = M_{L,S}(L_{S,L})(1.6) = 3.17(151')(1.6) = 767 \text{ k-ft}$$
 [Eq 5-51]

$$R_{\rm u} = \frac{Mu}{\phi(b)(d)^2} = \frac{767(12)}{0.9(12x11)(24)^2} = 0.135$$
 [Eq 5-52]

$$m = \frac{f_y}{0.85(f'_c)} = \frac{60}{0.85(4)} = 17.65$$
 [Eq 5-53]

$$\rho_{\text{req}} = \frac{1}{m} \left[ 1 - \sqrt{1 - \frac{2(R_u)(m)}{f_y}} \right] = \frac{1}{17.65} \left[ 1 - \sqrt{1 - \frac{2(17.65)(0.135)}{60}} \right] = 0.0023 \text{ [Eq 5-54]}$$

$$\rho_{\min} = \frac{200}{f_{\nu}} = \frac{200}{60000} = 0.00333, \text{ use } \rho_{\text{req}} = 0.00333$$
 [Eq 5-55]

$$\rho_{\text{max}} = \frac{0.319(B_1)(f'_c)}{f_v} = \frac{0.319(0.85)(4000)}{60000} = 0.0181$$
 [Eq 5-56]

$$A_{st} = \rho$$
 (b)(d) = 0.00333(24)(12) = 0.96 in<sup>2</sup>/rib [Eq 5-57] use 3 # 6's in each rib,  $A_{st} = 1.32$  in<sup>2</sup>

#### C) Shear Calculations

#### 1. Long Direction

**Expected Service Shear** 

 $v_L = 1.27$  klf, found in Step 8 of Chapter 7

$$V_u = v_{L,S}(L_{S,L})(1.6) = 1.27(101)(1.6) = 205 \text{ K}$$
 [Eq 5-58]

$$\phi V_c = \phi(2) \sqrt{f'_c}(b)(d) = 0.75(2) \sqrt{4000}(12x15)(24) = 409 \text{ K}$$
 [Eq 5-59]

 $\phi V_c > V_u$ , therefore no shear reinforcement is needed

#### 2. Short Direction

**Expected Service Shear** 

 $v_s = 1.24$  klf, found in Step 8 of Chapter 7

$$V_u = v_{L,S}(L_{S,L})(1.6) = 1.24(151)(1.6) = 300 \text{ K}$$
 [Eq 5-58]

$$\phi V_c = \phi(2) \sqrt{f'_c}(b)(d) = 0.75(2) \sqrt{4000}(12x11)(24) = 300 \text{ K}$$
 [Eq 5-59]

 $\phi V_c = V_u$ , therefore no shear reinforcement is needed

#### D) Foundation Stiffness

## 1. Long Direction

 $I_L \ge 162311 \text{ in}^4$ , found in Step 8 of Chapter 7

Effective I

Mcr = 
$$\frac{f_r(I_g)}{v}$$
 f<sub>r</sub> = 7.5 $\sqrt{f'_c}$ , Mcr =  $\frac{7.5\sqrt{4000}(488890)}{6.73}$  = 2871 k-ft [Eq 5-65]

$$nAs = \frac{29000000}{57000\sqrt{4000}}(1.32) = 10.62 \text{ in}^2$$
 [Eq 5-66]

Find Distance to Neutral Axis of Transformed Section

$$12(x)\frac{x}{2} = 10.62(24 - x) \qquad x = 5.7$$
"

Icr = 
$$\frac{bx^3}{3} + nA_s(d-x)^2 = \frac{12(5.7)^3}{3} + 10.62(24-5.7)^2 = 4297 \text{ in}^4$$
 [Eq 5-67]

$$I_{E} = \left(\frac{Mcr}{M_{A}}\right)^{3} (I_{g}) + \left[1 - \left(\frac{Mcr}{M_{A}}\right)^{3}\right] (I_{cr})$$
 [Eq 5-68]

$$I_E = \left(\frac{2871}{1241}\right)^3 (488890) + \left[1 - \left(\frac{2871}{1241}\right)^3\right] (4297)(15) = 5319730 \text{ in}^4$$

 $I_E < I_g$ , therefore  $I_E = 488890 \text{ in}^4$ 

 $488890 \text{ in}^4 > 162311 \text{ in}^4 \text{ O.K.}$ 

 $I_S \ge 334248 \text{ in}^4$ , found in Step 8 of Chapter 7

Effective I

Mcr = 
$$\frac{f_r(I_g)}{y}$$
 f<sub>r</sub> = 7.5 $\sqrt{f'_c}$ , Mcr =  $\frac{7.5\sqrt{4000}(683241)}{6.48}$  = 4168 k-ft [Eq 5-65]

$$nAs = \frac{29000000}{57000\sqrt{4000}}(3.00) = 24.2 \text{ in}^2$$
 [Eq 5-66]

Find Distance to Neutral Axis of Transformed Section

$$12(x)\frac{x}{2} = 24.2(24 - x) \qquad x = 8.0$$
"

Icr = 
$$\frac{bx^3}{3} + nA_s(d-x)^2 = \frac{12(8.0)^3}{3} + 24.2(24-8.0)^2 = 8243 \text{ in}^4$$
 [Eq 5-67]

$$I_{E} = \left(\frac{Mcr}{M_{A}}\right)^{3} (I_{g}) + \left[1 - \left(\frac{Mcr}{M_{A}}\right)^{3}\right] (I_{cr})$$
 [Eq 5-68]

$$I_E = \left(\frac{4168}{1948}\right)^3 (683241) + \left[1 - \left(\frac{4168}{1948}\right)^3\right] (8243)(11) = 5895030 \text{ in}^4$$

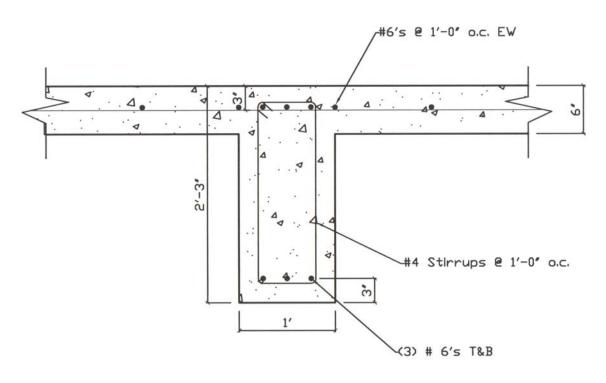
 $I_E < I_g$ , therefore  $I_E = 683241$  in<sup>4</sup>

 $683241 \text{ in}^4 > 334248 \text{ in}^4 \quad \textbf{O.K.}$ 

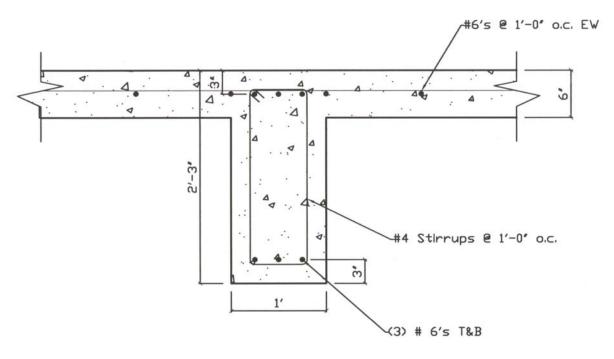
## Summary of Design

The design sections used for both the long and short directions of the foundation are shown in Figure 8-1. Because of the shear reinforcement requirements for the ribs require #4 stirrups at 12" on center, an increased depth is calculated to try to obtain a more constructible section for design which does not require shear reinforcement. This design is shown in Appendix C of this report.

**Figure 8-1 Design Sections for Mat Foundation** 



LONG DIRECTION



SHORT DIRECTION

## **CHAPTER 9 - Comparison of Design Problems**

The main difference between the foundation section designed in Chapter 7 using post-tensioning and the foundation section designed in Chapter 8 using mild-reinforcement is the post-tensioned ribbed mat foundation's ability to better resist shear stresses and its ability to eliminate slab reinforcement. Because the cracking moment of the foundation section was greater than the actual moment the section had to resist, the entire gross section of the mild-reinforced foundation section was able to be used to resist deflection. Therefore, there was no real difference between the two design examples ability to resist deflection, because they were both able to use the entire gross section.

Considering the flexural capacities of each case study, the mild-reinforcement design section was a constructible section that would not necessarily show any disadvantage when compared to the post-tensioned design section. The highest amount of steel that would be needed for the mild-reinforced section was 3 #6 bars at the top of each rib and 3 #6 bars at the bottom of each rib and this steel was placed only to meet minimum steel requirements.

Ultimately, however, the shear strength of the post-tensioned foundation is what makes it the better design for a ribbed mat foundation on expansive soils. For the mild-reinforced foundation to provide the same shear strength as the post-tensioned foundation, stirrups had to be added in the ribs at 12" on center in the short direction and in the long direction. Placing stirrups every 12" on center is constructible but not typical in construction of foundations. Appendix C of this paper was added to try and find a more constructible section more typical of a foundation. The findings of Appendix C was that a 50" deep rib was required in order to eliminate the need for shear reinforcement in the ribs. The fact that mild-reinforcement required 50" deep ribs while post-tensioning required only 27" deep ribs makes it is easy to see that the post-tensioned ribbed mat foundation is still the better solution. Typically, any structural foundation element is designed such that there is no need to provide shear reinforcement. Consequently, a mild-reinforced mat foundation would only be used for construction if the ribs were 50" deep.

Table 9-1 and Table 9-2 are construction cost analyses of the two design problems. The unit costs include labor and materials, and the cost information was taken from the RSMeans Building Construction Cost Data 2006 catalog.

Table 9-1 Post-Tensioning Foundation Cost Analysis Using Design Sections Shown in Figure 7-1.

Material	Amount	Unit Cost	Amount
4ksi Concrete	$489yd^3$	$$91/yd^3$	\$44500
Post-Tensioning Tendons	10960lbs	\$1.57/lb	\$17206
15929ft(0.688lbs/ft)			
Total			\$61706

Table 9-2 Mild-Reinforcement Foundation Cost Analysis Using Design Sections Shown in Figure 8-1.

Material	Amount	Unit Cost	Amount
4ksi Concrete	489yd <sup>3</sup>	\$91/yd <sup>3</sup>	\$44500
Slab Reinforcement (#6's)	22.8tons	\$1410/ton	\$32148
30250ft(1.5021lbs/ft)			
Rib Reinforcement (#6's)	14.3tons	\$1410/ton	\$20163
19056ft(1.502lbs/ft)			
#4 Stirrups	5.9tons	\$1645/ton	\$9706
3176 stirrups			
x(2*21"+2*12")/12			
x(0.668lbs/ton)			
Total			\$106517

## **CHAPTER 10 - Conclusion**

Post-tensioning and mild-reinforcement are both effective in helping concrete to resist tensile stresses. Each method is used regularly in general construction and each has advantages over the other. Specifically, focusing the use of reinforced concrete on foundations and ribbed mat foundations in particular, both methods are typically used in general construction. When the mat foundation is supported by highly expansive soils, post-tensioning is typically used. To determine if in fact post-tensioning is the better method for designing mat foundations on very expansive soils and if so, why, the two design problems of the 2-story office building in Dallas, Texas in Chapter's 7 and 8 were conducted. The findings were that the post-tensioned ribbed mat foundation was in fact the better foundation.

After the foundation sections for the post-tensioned ribbed mat foundation were designed, the design sections in Figure 7-1 were determined. With the same concrete section determined in Chapter 7 used to design the mild-reinforced ribbed mat foundation in Chapter 8, it was clear that the post-tensioned mat foundation was the better design. Comparing Figure 8-1 with Figure 7-1, the obvious difference between the two designs is the ability of the post-tensioning to effectively resist the design shear stresses and its ability to eliminate the need for slab reinforcement. Thus, the design problem using post-tensioning showed the foundation was able to resist the shear stresses without the aid of shear reinforcing being placed in the ribs. However, the design problem using mild-reinforcing needed the aid of shear reinforcing to resist the shear stresses (#4 stirrups at 12" on center in both the long and short directions) and also needed additional slab reinforcement. Because typical foundations are designed to resist the shear stresses based on the concrete shear strength alone, using shear reinforcement is not a viable design. Appendix C of this paper presents information to determine if it was possible to design a mild-reinforced foundation without the use of shear reinforcement or at least with the use of a smaller amount of shear reinforcement. The findings in Appendix C were that the ribs depths had to be increased to 50" in order to eliminate the need for shear reinforcement in the mildreinforced foundation. While this design is constructible, it requires a rib 23" deeper than the post-tensioning required. The extra shear strength in the post-tensioned ribbed mat foundation comes directly from the prestress force applied to the slab from the prestressing tendons.

Given the very basic cost analysis in Chapter 9, it is still clear that the post-tensioned foundation is a better choice over the mild-reinforced mat foundation: the post-tensioned mat foundation cost is roughly 58% of that of the mild-reinforced mat foundation.

The fact that the mild-reinforced foundation required stirrups to aid in shear strength makes the post-tensioned mat foundation the better solution. Even after the attempt was made in Appendix C to eliminate the need for shear reinforcement, post-tensioning is clearly still the best method. Considering the strength, constructability, and costs of each foundation, the post-tensioned ribbed mat foundation is the better solution for the 2-story office building located in Dallas, Texas. Indeed, it would be safe to assume that for any typical light-commercial or residential building that is to be built on a mat foundation over highly expansive soils, post-tensioning will be the better solution.

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## Appendix A - Definition of Symbols & Equation References

## **Definition of Symbols**

A = Area of gross concrete cross-section, in<sup>2</sup>

a = Depth of equivalent rectangular stress block, in

A'<sub>b</sub> = Maximum area of the portion of the supporting surface that is geometrically similar to and concentric with the tendon anchor, in<sup>2</sup>

 $A_b$  = Bearing area beneath a tendon anchor, in<sup>2</sup>

 $A_0$  = Coefficient used in Equation 5-10

 $A_{ps}$  = Area of prestressing steel, in<sup>2</sup>

As = Area of reinforcing steel, in<sup>2</sup>

B = Constant used in Equation 5-10

b = Width of an individual stiffening rib, in

C = Constant used in Equation 5-10

 $C_{\Delta}$  = Coefficient used to establish minimum foundation stiffness, (see Figure 8 Appendix B)

d = Distance from extreme compression fiber to centriod of longitudinal reinforcement, in

 $E_{cr}$  = Long-term or creep modulus of elasticity of concrete, psi. May be taken as 1,500,000psi

 $e_m$  = Edge moisture variation distance, ft

e<sub>p</sub> = Eccentricity of post-tensioning force (perpendicular distance between the center of gravity of prestressing force and the geometric centroid of gross concrete section), in

 $E_{soil}$  = Modulus of elasticity of the soil, psi

f = Applied flexural concrete stress (tension or compression), psi

 $f_{bp}$  = Allowable bearing stress under tendon anchorages, psi

 $f_c$  = Allowable concrete compressive flexural stress, psi

f'<sub>c</sub> = 28-day concrete compressive strength, psi

f'ci = Concrete compressive strength at time of stressing tendons, psi

f<sub>e</sub> = Effective tendon stress after losses due to elastic shortening, creep and shrinkage of concrete, and steel relaxation, ksi

F<sub>f</sub> = Fabric factor used to modify unsaturated diffusion coefficient, α, for presence of roots, layers, fractures, and joints (see Figure 3 in Appendix B)LL= Liquid limit of soil, %

f<sub>p</sub> = Minimum average residual prestress compressive stress, psi

f<sub>pi</sub> = Allowable tendon stress immediately after stressing, ksi

f<sub>pu</sub> = Specified maximum tendon tensile stress, ksi

 $f_r$  = modulus of rupture of concrete, psi

f<sub>t</sub> = Allowable concrete flexural tension stress, psi

f<sub>y</sub> = Specified yield strength of reinforcement, psi

h = Total depth of stiffening rib, measured from top of slab to bottom of rib, in

I = Gross concrete moment of inertia, in<sup>4</sup>

Icr = Moment of inertia of cracked section transformed to concrete, in<sup>4</sup>

I<sub>g</sub> = Moment of inertia of gross concrete section, in<sup>4</sup>

I<sub>m</sub> = Thornthwaite Moisture Index (see Figure 4 in Appendix B)

 $L_L$  = Long length of the design rectangle, ft

 $L_S$  = Short length of the design rectangle, ft

m = Relationship between steel strength and concrete compressive strength

M<sub>A</sub> = Maximum unfactored moment in member, k-ft

Mcr = Cracked section moment capacity, k-ft

 $M_L$  = Maximum applied service load moment in the long direction (causing bending stresses on the short cross section) from either center lift or edge lift swelling conditions, ft-k/ft

M<sub>S</sub> = Maximum applied service load moment in the short direction (causing bending stresses on the long cross-section) from either center lift or edge lift swelling conditions, k-ft/ft

M<sub>u</sub> = Ultimate factored moment, k-ft

n = Number of stiffening ribs in a cross-section of width W

n = Ratio of modulus of elasticity of reinforcement to modulus of elasticity of concrete

 $N_t$  = Number of tendons

P = A uniform unfactored service line load acting the entire length of the perimeter stiffening ribs representing the weight of the exterior building material and that portion of the superstructure dead and live loads which frame into the exterior wall, does not include any portion of the foundation concrete, plf

PI = Plasticity Index of the soil, psi

PL = Plastic Limit of soil, %

P<sub>r</sub> = Effective prestress force considering subgrade friction, k

q<sub>allow</sub> = Allowable soil bearing pressure, psf

R<sub>u</sub> = Relationship between the ultimate factored moment and the size of the section, ksi

S = Interior stiffening rib spacing, ft

s = Spacing of shear reinforcement, in

 $S_b$  = Section modulus with respect to the bottom fiber, in<sup>3</sup>

SCF = Stress change factor (see Figure 7 in Appendix B)

SF = Shape factor comparing foundation perimeter to foundation area

 $S_s$  = Slope of suction vs. volumetric water content curve

 $S_t$  = Section modulus with respect to the top fiber, in<sup>3</sup>

 $S_{ten}$  = Maximum spacing of post-tensioning tendons in normal weight concrete, in

t = Slab thickness, in

v = Service load shear stress, psi

v<sub>c</sub> = Allowable concrete shear stress, psi

V<sub>c</sub> = Nominal shear strength provided by concrete, lb

V<sub>s</sub> = Nominal shear strength provided by shear reinforcement, lb

 $V_{ij}$  = Factored shear force, lb

W = Foundation width (or width of design rectangle) in the direction being considered, ft

 $W_{slab}$  = Foundation weight, lbs

x = Distance from the top of the concrete section to the neutral axis of the transformed section, in

y<sub>m</sub> = Maximum unrestrained differential soil movement or swell, in

y<sub>t</sub> = distance from top of slab to center of mass, in

 $z = \text{smaller of } L \text{ or } 6\beta \text{ in direction being considered, ft}$ 

α = Unsaturated diffusion coefficient, a measure of moisture movement in unsaturated soils

 $\alpha'$  = Unsaturated diffusion coefficient modified by soil fabric factor

β = Relative stiffness length, approximate distance from edge of foundation to point of maximum moment, ft

 $\beta_1$  = Factor relating depth of equivalent rectangular compressive stress block to neutral axis depth

 $\gamma_h$  = Change of soil volume for a change in suction corrected for actual % fine clay. Also referred to as matrix suction compression index

 $\gamma_0$  = Change of soil volume for a change in suction for 100% fine clay

%-#200= Percentage of soil passing No. 200 sieve, %

%-2 $\mu$  = Percentage of soil finer than 2 microns, %

 $\phi$  = Appropriate strength reduction factor

 $\rho$  = Ratio of area of steel to the width of concrete (b) and depth of steel (d)

μ = Coefficient of friction between foundation and subgrade

## **Equation References**

All references with ACI designation refer to ACI 318-05 Building Code Requirements for Structural Concrete unless noted otherwise. All references with PTI designation refer to Design of Post-Tensioned Slabs-on-Ground manual 3<sup>rd</sup> edition.

	Design Equation	Reference
$f_{bp} = 0.6f'_c \sqrt{\frac{A'_b}{A_b}} \le f'_c$	[Eq 4-1]	PTI Eq 6-5
$f_{bp} = 0.8 f'_{ci} \sqrt{\frac{A'_b}{A_b} - 0.2} \le 1.25 f'_{ci}$	[Eq 4-2]	PTI Eq 6-6
$f_t = 6\sqrt{f'c}$	[Eq 5-1]	PTI Eq 6-3
$f_c = 0.45(f^*c)$	[Eq 5-2]	ACI360R 8.5.3 PTI Eq 6-4 ACI360R 8.5.3
$SF = \frac{(FoundationPerimeter)^2}{(FoundationArea)}$	[Eq 5-3]	PTI Eq 4-1
h(center lift) = $ \frac{((y_m)(L))^{0.205}(S)^{1.059}(P)^{0.523}(e_m)^{1.296}(P)^{0.523}(e_m)^{0.523}($	$\frac{(C_{\Delta})}{\left[\text{Eq 5-4}\right]}$	PTI Eq 6-1
$f_{pi} = 0.7 f_{pu}$	[Eq 5-5]	PTI Eq 6-9

	Design Equation	Reference
$f_e = f_{pi} - 15$	[Eq 5-6]	PTI Sect 6.6
$N_{t} = \frac{0.05A}{f_e A_{ps}}$	[Eq 5-7]	PTI Sect A.3.2.1
$N_{\rm t} = \frac{\mu W_{slab}}{2000 f_e A_{ps}}$	[Eq 5-8]	PTI Eq 6-12a
$P_r = N_t(f_e \ x \ A_{ps}) - \mu W_{slab}/2000$	[Eq 5-9]	PTI Eq 6-12b
$M_L = A_o[B(e_m)^{1.238} + C]$	[Eq 5-10]	PTI Eq 6-13
$A_{o} = \frac{(L)^{0.013} (S)^{0.306} (h)^{0.688} (P)^{0.534} (y_{m})^{0.193}}{727}$	[Eq 5-11]	PTI Eq 6-14
$0 \le e_m \le 5$ $B = 1, C = 0$	[Eq 5-12]	PTI Eq 6-15a
$< e_{\rm m}$ $B = \frac{y_m - 1}{3} \le 1.0$	[Eq 5-13]	PTI Eq 6-15b
$C = \left[8 - \frac{P - 613}{255}\right] \left[\frac{4 - y_m}{3}\right] \ge 0$	[Eq 5-14]	PTI Eq 6-15c
For $L_L / L_S < 1.1$ $M_S = M_L$	[Eq 5-15]	PTI Eq 6-17
For $L_L / L_S \ge 1.1$ $M_S = \left[ \frac{58 + e_m}{60} \right] M_L$	[Eq 5-16]	PTI Eq 6-16
$f = \frac{P_r}{A} \pm \frac{M_{L,S}}{S_{t,b}} \pm \frac{P_r(e_p)}{S_{t,b}}$	[Eq 5-17]	PTI Eq 6-21
$I_{L} \ge \frac{18000(M_{L})(L_{S})(C_{\Delta})(z_{L})}{1500000}$	[Eq 5-18]	PTI Eq 6-22
$I_{S} \ge \frac{18000(M_{S})(L_{L})(C_{\Delta})(z_{S})}{1500000}$	[Eq 5-19]	PTI Eq 6-22
$\beta = \frac{1}{12} \sqrt[4]{\frac{E_{cr}I}{E_{soil}}}$	[Eq 5-20]	PTI Section A.1
$v_{L} = \frac{(L)^{0.09} (S)^{0.71} (h)^{0.45} (P)^{0.44} (y_{m})^{0.16} (e_{m})^{0.93}}{1940}$	[Eq 5-21]	PTI Eq 6-24
$v_{S} = \frac{(L)^{0.19} (S)^{0.45} (h)^{0.20} (P)^{0.54} (y_{m})^{0.04} (e_{m})^{0.97}}{1350}$	[Eq 5-22]	PTI Eq 6-23
$v_c = 1.7\sqrt{f'_c} + 0.2(f_p)$	[Eq 5-23]	PTI Sect A.3.2.2

	Design Equation	Reference
$\mathbf{v} = \frac{v_{L,S}W}{nbh}$	[Eq 5-24]	PTI Eq 6-28
$M_{L} = \frac{(S)^{0.1} (he_{m})^{0.78} (y_{m})^{0.66}}{7.2(L)^{0.0065} (P)^{0.04}}$	[Eq 5-25]	PTI Eq 6-18
For $L_L / L_S < 1.1$ $M_S = M_L$	[Eq 5-26]	PTI Eq 6-20
For $L_L / L_S \ge 1.1$ $M_S = h^{0.35} [(19 + e_m) / 57.75] M_S$	$I_L$ [Eq 5-27]	PTI Eq 6-19
$v_S = v_L = \frac{(L)^{0.07} (h)^{0.4} (P)^{0.03} (e_m)^{0.16} (y_m)^{0.67}}{3S^{0.015}}$	[Eq 5-28]	PTI Eq 6-25
$v_c = 1.7\sqrt{f'_c} + 0.2(f_p)$	[Eq 5-29]	PTI Sect A.3.2.2
$\mathbf{v} = \frac{vW}{nbh}$	[Eq 5-30]	PTI Eq 6-28
$M_{LTOTAL} = M_L(L_S)$	[Eq 5-31]	PTI Sect 4.5.7
$0.9~\mathrm{M_L}$	[Eq 5-32]	PTI Sect 4.5.7
$N_{t(Long)}(f_eA_{ps})$	[Eq 5-33]	PTI Sect A.3.2.4
$a = \frac{N_{t(Long)}(f_e A_{ps})}{0.85(f'_c)(b)(n)}$	[Eq 5-34]	PTI Sect A.3.2.4
$M_{cr} = \frac{N_{t(Long)} f_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right)$	[Eq 5-35]	PTI Sect A.3.2.4
$M_{STOTAL} = M_S(L_L)$	[Eq 5-36]	PTI Sect 4.5.7
$0.9~\mathrm{M_S}$	[Eq 5-37]	PTI Sect 4.5.7
$N_{t(Short)}(f_eA_{ps})$	[Eq 5-38]	PTI Sect A.3.2.4
$a = \frac{N_{t(Short)}(f_e A_{ps})}{0.85(f'_c)(b)(n)}$	[Eq 5-39]	PTI Sect A.3.2.4
$M_{cr} = \frac{N_{t(Short)} f_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right)$	[Eq 5-40]	PTI Sect A.3.2.4
$M_{LTOTAL} = M_L(L_S)$	[Eq 5-41]	PTI Sect 4.5.7
$0.9~\mathrm{M_L}$	[Eq 5-42]	PTI Sect 4.5.7
$n(f_eA_{ps})$	[Eq 5-43]	PTI Sect A.3.2.4
$a = \frac{n(f_e A_{ps})}{0.85(f'_c)(L_S)(12)}$	[Eq 5-44]	PTI Sect A.3.2.4

	Design Equation	Reference
$M_{cr} = \frac{nf_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right)$	[Eq 5-45]	PTI Sect A.3.2.4
$M_{STOTAL} = M_S (L_L)$	[Eq 5-46]	PTI Sect 4.5.7
$0.9~\mathrm{M_S}$	[Eq 5-47]	PTI Sect 4.5.7
$(f_{\rm e}A_{ m ps})$	[Eq 5-48]	PTI Sect A.3.2.4
$a = \frac{n(f_e A_{ps})}{0.85(f'_c)(L_L)(12)}$	[Eq 5-49]	PTI Sect A.3.2.4
$M_{cr} = \frac{nf_e A_{ps}}{12} \left( h - y - \frac{a}{2} \right)$	[Eq 5-50]	PTI Sect A.3.2.4
$\rho_{\min} = \frac{200}{f_y}$	[Eq 5-55]	ACI Eq 10-3
$\rho_{\text{max}} = \frac{0.319(B_1)(f'_c)}{f_y}$	[Eq 5-56]	ACI Sect 10.2.3
$\phi V_{\rm c} = \phi(2) \sqrt{f'_{\rm c}}(b)(d)$	[Eq 5-59]	ACI Eq 11-3
$S_{\text{req}} = \frac{nA_{v}f_{y}d}{V_{s}}$	[Eq 5-61]	ACI Eq 11-15
$Mcr = \frac{f_r(I_g)}{y} \qquad f_r = 7.5\sqrt{f'_c}$	[Eq 5-65]	ACI Eq 9-9

# **Appendix B - Design Tables and Charts**

Figure B-1 Mineral Classification Chart. (Reproduced from Post-Tensioning Institute, 2004)

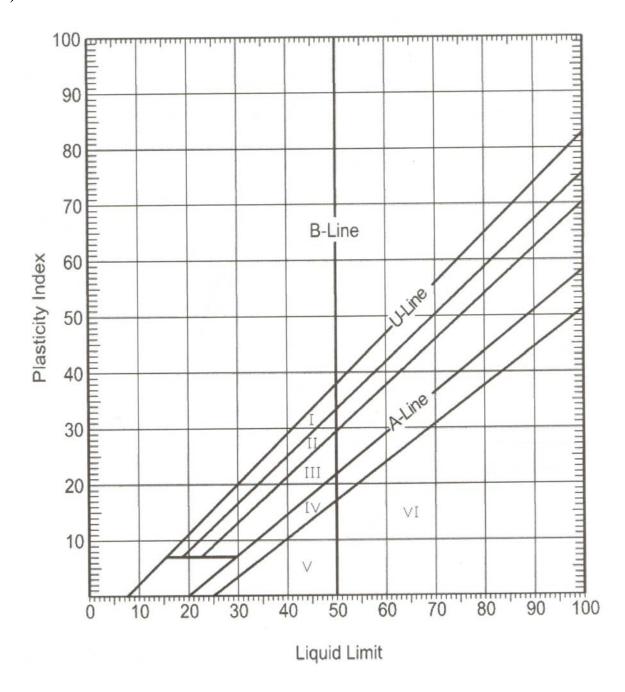


Figure B-2 Zone Chart for Determining Change of Soil Volume Coefficient for Zone VI. (Reproduced from Post-Tensioning Institute, 2004)

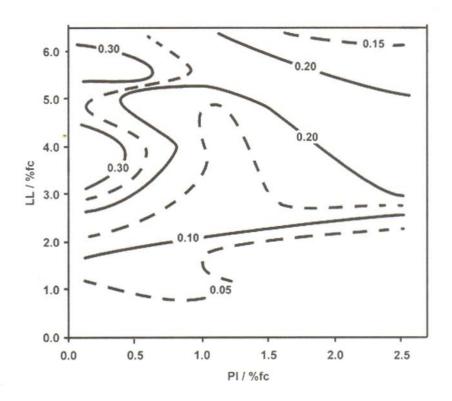


Figure B-3 Soil Fabric Factor (Reproduced from Post-Tensioning Institute, 2004)

Condition	$F_{f}$
Soil profiles contain few roots, layers, fractures or joints (No more than 1 per vertical foot)	1.0
Soil profiles contain <b>some</b> roots, layers, fractures or joints (2 to 4 per vertical foot)	1.3
Soil profiles contain many roots, layers, fractures or joints (5 or more per vertical foot)	1.4

Figure B-4 Thornthwaite Index for Texas (20 year average, 1955-1974) (Reproduced from Post-Tensioning Institute, 2004)

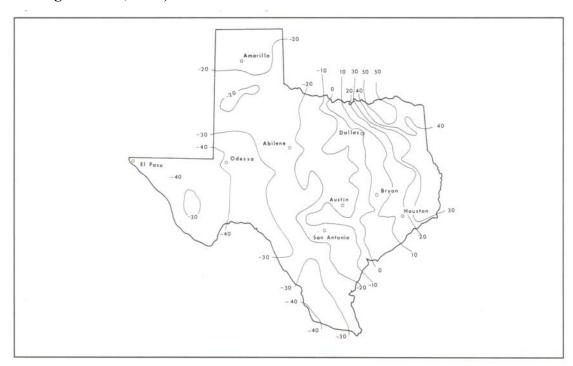


Figure B-5 Edge Moisture Variation Distance,  $e_m$ , Selection Chart. (Reproduced from Post-Tensioning Institute, 2004)

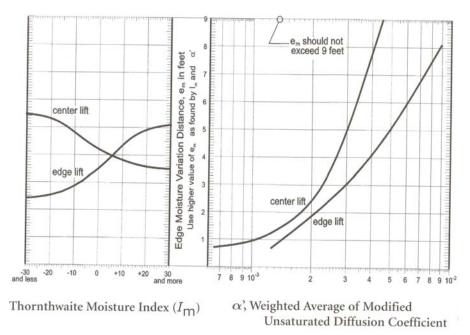


Figure B-6 Thornthwaite Index-Equilibrium Suction Correlation. (Reproduced from Post-Tensioning Institute, 2004)

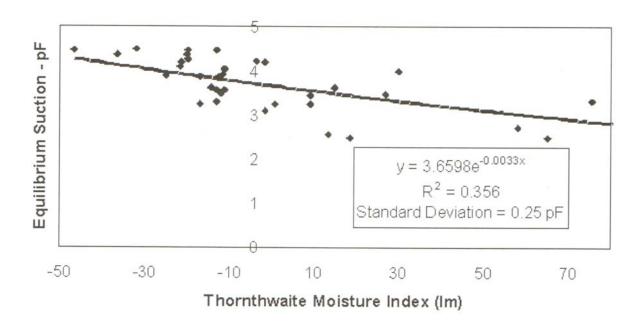


Figure B-7 Stress Change Factor (SCF) for Use in Determining  $y_m$ . (Reproduced from Post-Tensioning Institute, 2004)

Measured Suction ( <i>pF</i> ) at	F	inal Co	ntrollin	g Suction	n At Sui	face, p	F
Depth	2.5	2.7	3.0	3.5	4.0	4.2	4.5
2.7	+3.2	0	-4.1	-13.6	-25.7	-31.3	-40.0
3.0	+9.6	+5.1	0	-7.5	-18.2	-23.1	-31.3
3.3	+17.7	+12.1	+5.1	-2.6	-11.5	-15.8	-23.1
3.6	+27.1	+20.7	+12.1	+1.6	-5.7	-9.4	-15.8
3.9	+38.1	+30.8	+20.7	+7.3	-1.3	-4.1	-9.4
4.2	+50.4	+42.1	+30.8	+14.8	+3.2	0	-4.1
4.5	+63.6	+54.7	+42.1	+23.9	+9.6	+5.1	0

Figure B-8 Recommended Values of Stiffness Coefficient  $C_\Delta$ . (Reproduced from Post-Tensioning Institute, 2004)

Material	Center Lift	Edge Lift
Wood Frame	240	480
Stucco or Plaster	360	720
Brick Veneer	480	960
Concrete Masonry Units	960	1920
Prefab Roof Trusses*	1000	2000

# Appendix C - Mild Reinforced Mat Foundation Design

This section is a design of a mild reinforced ribbed mat foundation. The section properties of the foundation have been changed drastically in order to provide a foundation section that does not need the aid of shear reinforcement. The design information including framing plans are given in Chapter 6 of this paper. The design steps are explained in Chapter 5 Design Steps for Mild Reinforced Mat Foundations. This design example will focus only on shear forces and stresses.

#### Step 4.

Center Lift Design ( $e_m = 8.0 \text{ft}$ ,  $y_m = 0.30$ ")

C) Shear Calculations

1. Long Direction

**Expected Service Shear** 

$$v_{L} = \frac{(L)^{0.09} (S)^{0.71} (h)^{0.45} (P)^{0.44} (y_{m})^{0.16} (e_{m})^{0.93}}{1940}$$
 [Eq 5-21]

$$V_u = v_{L,S}(L_{S,L})(1.6)$$
 [Eq 5-58]

$$\phi V_c = \phi(2) \sqrt{f'_c}(b)(d)$$
 [Eq 5-59]

 $\phi V_c \ge V_u$  in order to eliminate the need for shear reinforcement in the ribs

$$\frac{(L)^{0.09}(S)^{0.71}(h)^{0.45}(P)^{0.44}(y_m)^{0.16}(e_m)^{0.93}}{1940} (L_{S,L})(1.6) \le \phi(2) \sqrt{f'_c}(b)(d)$$

$$\frac{(151)^{0.09}(11)^{0.71}(d+3)^{0.45}(1000)^{0.44}(0.3)^{0.16}(8.0)^{0.93}}{1940} (101)(1.6) \le 0.75(2) \sqrt{4000}(12x11)(d)$$

$$85.58(d+3)^{0.45} \le 12523d$$

$$d = 36$$

$$h = 39$$

**Expected Service Shear** 

$$v_{S} = \frac{(L)^{0.19} (S)^{0.45} (h)^{0.20} (P)^{0.54} (y_{m})^{0.04} (e_{m})^{0.97}}{1350}$$
 [Eq 5-22]

$$V_u = v_{L,S}(L_{S,L})(1.6)$$
 [Eq 5-58]

$$\phi V_{c} = \phi(2) \sqrt{f'_{c}}(b)(d)$$
 [Eq 5-59]

 $\phi V_c \ge V_u$  in order to eliminate the need for shear reinforcement in the ribs

$$\frac{(L)^{0.19}(S)^{0.45}(h)^{0.20}(P)^{0.54}(y_m)^{0.04}(e_m)^{0.97}}{1350}(L_{S,L})(1.6) \le \phi(2)\sqrt{f'_c}(b)(d)$$

$$\frac{(101)^{0.19}(10)^{0.45}(d+3)^{0.20}(1000)^{0.54}(0.3)^{0.04}(8.0)^{0.97}}{1350}(151)(1.6) \le 0.75(2)\sqrt{4000}(12x15)(d)$$

$$362(d+3)^{0.20} \le 17077d$$

$$d = 47$$

$$h = 50$$

The rest of the design for this design example is not needed in order to see that the 50" deep ribs required to eliminate shear reinforcement are deeper than the 27" deep ribs required using post-tensioning. The best solution for the design of this foundation is still the post-tensioned foundation designed in Chapter 7.