

DESIGN OF BLAST RESISTANT STRUCTURES

by

GANPAT MAL SINGHVI

B. E., University of Rajasthan, India, 1961

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1963

Approved by:

Kahkan N. Jabbour
Major Professor

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GANPAT MAL SINGHVI¹

SYNOPSIS

The principal objective in the design of a blast-resistant structure is to protect the structure itself including its equipment, contents, and occupants from the various effects of atomic weapons. The design consists of the determination of the load acting on the structure as a function of time. The structural resistance required to limit deflections of the individual members within a prescribed maximum value is calculated by dynamic analysis of the system. The limits of allowable deflections depend on economical factors.

A numerical example presents the design of the important elements of a windowless, one-story, reinforced concrete frame building. The blast loads on the frame are calculated as suggested by the United States Atomic Energy Commission and the United States Army Corps of Engineers. Preliminary design of members is done using an idealized straight line load-time curve and is checked by numerical integration using the calculated load-time data.

¹Graduate student, Department of Civil Engineering, Kansas State University, Manhattan, Kansas.

INTRODUCTION

The three most important phenomena associated with an above-ground explosion of an atomic bomb¹ are air blast, thermal radiation, and nuclear radiation. In designing protective construction, the dynamic loading caused by air blast pressure is of primary concern², though the radiation phenomena must be considered in the design of structures intended to offer protection for personnel.

It is not possible to protect³ a surface structure from a direct hit of any size of atomic or hydrogen bomb; but, there are large areas surrounding an explosion in which conventional structures would collapse or suffer severe damage while a blast resistant structure would suffer little or no damage. The cost of construction that will provide this protection will depend on the type of bomb assumed, the assumed distance from the ground-zero (the point on the ground directly below the exploding bomb), and the degree of damage that can be tolerated in the structure⁴.

Several types of construction may be used for protection³.

¹"The Effects of Atomic Weapons", Los Alamos Scientific Laboratory, U. S. Government Printing Office, Revised Edition, 1962.

²"Windowless Structures, A Study in Blast-Resistant Design", Federal Civil Defense Administration, U. S. Government Printing Office, 1952.

³Norris, Hansen, et.al, "Structural Design for Dynamic Loads" McGraw-Hill, 1959.

⁴C. S. Whitney, B. G. Anderson and E. Cohen, "Design of Blast Resistant Construction for Atomic Explosion", Proceedings, American Concrete Institute, Vol. 51, 1954-55, p. 589.

Presumably, the greatest level of protection can be secured in buried or semi-buried structures. When it is not possible to construct a buried or semi-buried structure, one of the several forms of a surface structure may be considered, such as shear wall construction, arch and dome construction, or rigid frame construction. Windowless construction is preferable for blast-resistant structures, because it eliminates the danger of personal injuries from nuclear and thermal radiation and from broken glass.

In the design of a structure capable of resisting blast forces which are large in magnitude and dynamic in character, members and joints are permitted to deflect and strain much further than is allowed for usual static loads. If the structure is designed to resist the dynamic blast forces with stresses in all structural members remaining within the elastic range, the resulting structure will be more expensive than a structure in which plastic yielding of a reasonable amount is permitted. The amount of plastic distortion permitted³ must be kept small enough to provide a margin of safety against collapse and to limit damage of building services.

The dynamic character of loading coupled with the plastic yielding of members requires that the design procedure be based on dynamic analysis.⁵ Under the rapid rates of strain that occur during blast loading, materials develop higher yield stresses

³Norris, op. cit., p. 236.

⁵"Structural Elements Subjected to Dynamic Loads", U. S. Army Corps of Engineers, U. S. Government Printing Office, March, 1957.

than they do in the case of statically loaded members.⁶ The increased dynamic yield stresses are used to determine plastic strength of dynamically loaded members.

WEAPON-EFFECT DATA

An explosion, in general, results from the very rapid release of a large amount of energy within a limited space. The sudden release of energy causes¹ a considerable increase of temperature and pressure, so that all the materials present are converted into hot compressed gases. Since these gases are at very high temperatures and pressures, they expand rapidly and thus initiate a pressure wave called a shock or blast wave.

Blast Wave in an Infinite Homogeneous Atmosphere

It is necessary to evaluate various aspects of air-blast phenomena associated with detonation in air of an atomic bomb in order to determine air blast loading on a structure.

Immediately after the occurrence of the detonation, hot gases initiate a pressure wave⁷ in the surrounding air. Fig. 1 (a) shows the general nature of variation^{3,7} of the air overpressure, i.e., excess pressure over the atmospheric pressure. As the pressure wave is propagating away from the center of the explosion, the following (or inner) part moves through a region

⁶"Strength of Materials and Structural Elements", U. S. Army Corps of Engineers, U. S. Gov't. Printing Office, Mar., 1957.

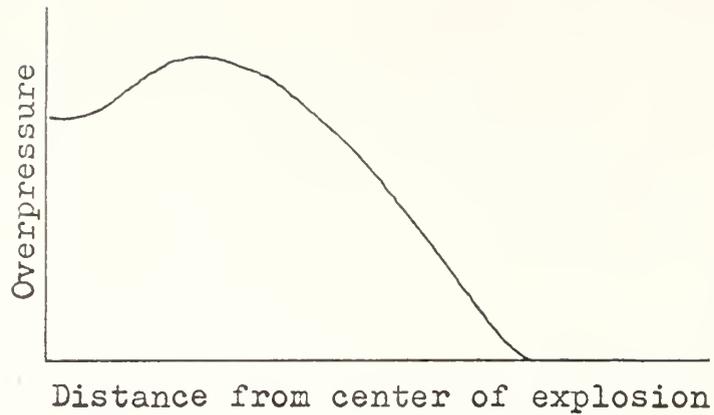
⁷"Weapon Effect Data", U.S. Army Corps of Engineers, U.S. Government Printing Office, July, 1959, paragraph 3.

which has been previously compressed and heated by the leading (or outer) parts of the wave. Due to the increase in the temperature and pressure of the air through which the wave is moving, the inner part of the wave moves more rapidly and catches up with the outer part as shown in Fig. 1 (b). Thus, the wave front gets steeper and steeper and within a very short period it becomes abrupt as shown in Fig. 1 (c). At the advancing front of the wave, there is a very sudden increase of pressure from normal atmospheric to the peak shock pressure. The shock front thus behaves like a moving wall of highly compressed air.

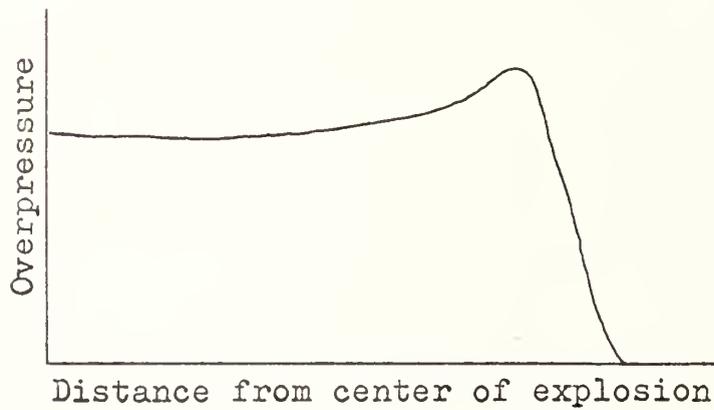
As the blast-wave travels in the air away from its source, the overpressure at the front steadily decreases, and pressure behind the front falls off in a regular manner. After a short time, when the shock front has travelled a certain distance from the fireball, the pressure behind the front drops below that of the surrounding atmosphere forming the negative phase of blast-wave as shown in Fig. 2. The symbol P_{s0} denotes the peak overpressure in pounds per square inch, and U_0 is the velocity of shock front in feet per second.

Figure 3 shows⁷ the variation of overpressure versus time of travel. The symbol t_0 denotes the duration of the positive phase. The overpressure, P_s , at any time t after the arrival of the shock front is given by^{3,7} the expression:

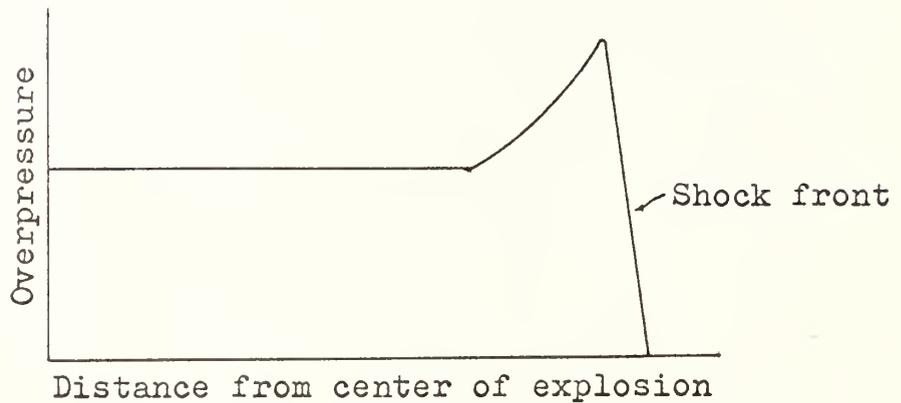
$$P_s = P_{s0} (1 - t/t_0) e^{-t/t_0} \quad (1)$$



(a)



(b)



(c)

FIG. 1. OVERPRESSURE DISTRIBUTION IN EARLY STAGES OF SHOCK FORMATION.

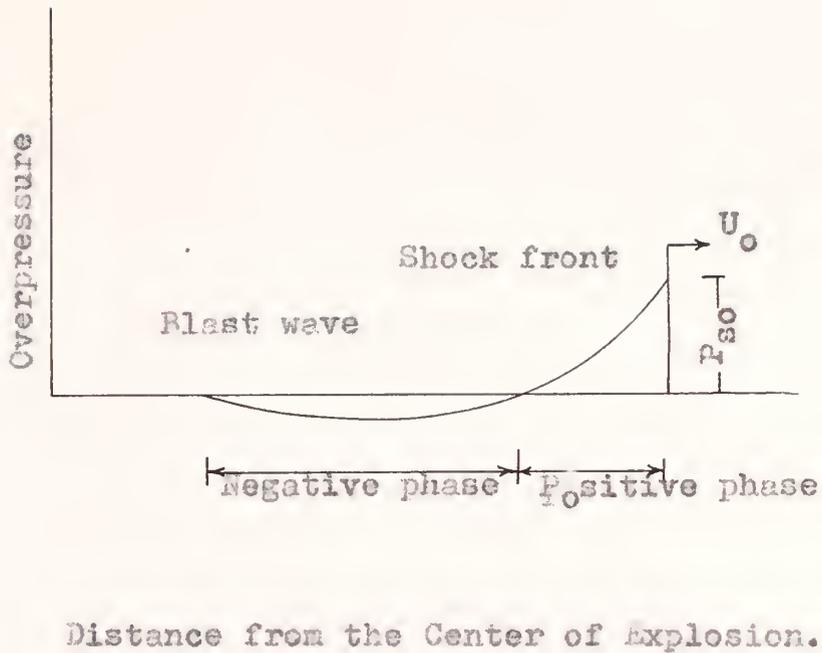


FIG. 2. VARIATION OF OVERPRESSURE WITH DISTANCE AT A GIVEN TIME. (Ref. 3, p. 242.)

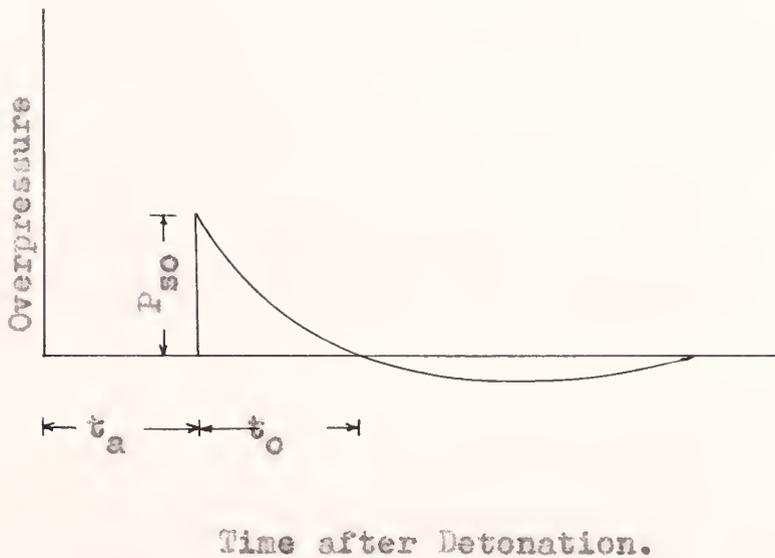


FIG. 3. VARIATION OF OVERPRESSURE WITH TIME AT A GIVEN LOCATION. (Ref. 3, p. 243.)

Scaling the Blast Phenomena

It has been found^{3,7} that air-blast phenomena, such as the pressure and duration at different distances, are related for different strength bombs according to the ratio of the cube root of the equivalent weights of TNT. These relationships are referred to as the scaling laws.

These scaling laws state that if a given peak overpressure is experienced at a distance r_1 from an explosion of bomb of total energy yield W_1 , the same peak overpressure will be experienced at a r_2 from the explosion of a bomb of total energy yield W_2 , where⁷

$$\frac{r_2}{r_1} = \left(\frac{W_2}{W_1}\right)^{1/3} \quad (2)$$

The same scaling laws also state that while the peak pressure from the two bombs are equal at the two radii r_1 and r_2 , the durations of the blast pressure wave at two points are different. If, for example, the duration of the positive phase of the pressure wave from the first bomb is t_{o1} at a distance r_1 , the duration of the positive phase of pressure wave from the second bomb at distance r_2 will be⁷,

$$t_{o2} = t_{o1} \left(W_2/W_1\right)^{1/3} \quad (3)$$

Loading on Structures

The manner⁷ in which the blast wave loads a structure is a function of distance of the structure from ground zero, the

height of burst of the weapon, and the weapon size.

Blast loading on above ground structures may be divided into two parts¹ as follows:

a. Diffraction Loading. When the front of an air-blast wave strikes the face of a building, reflection occurs. As a result, the overpressure builds up rapidly, generally several times greater than the incident wave front. As the wave front moves forward, the reflected overpressure on the face drops rapidly to that produced by the blast wave without reflection plus an added drag force due to wind pressure. At the same time, the air pressure wave bends or diffracts around the structure, so that the structure is eventually engulfed by the blast, and approximately the same pressure is exerted on the side walls and the roof. However, the front wall is still subjected to wind pressure, although the back wall is shielded from it. Figure 4 shows the plan of a building which is being struck by an air blast wave. In Fig. 4 (a) the wave has just reached its front face, producing a high overpressure. In Fig. 4 (b) the blast wave has proceeded about half way along the building and in Fig. 4 (c), it has reached the back. The pressure on the front face has dropped to some extent and it is building up on the sides as the blast wave diffracts around the structure. Finally, when the shock front has passed as shown in Fig. 4 (c), approximately equal air pressures are exerted on the walls of the structure.

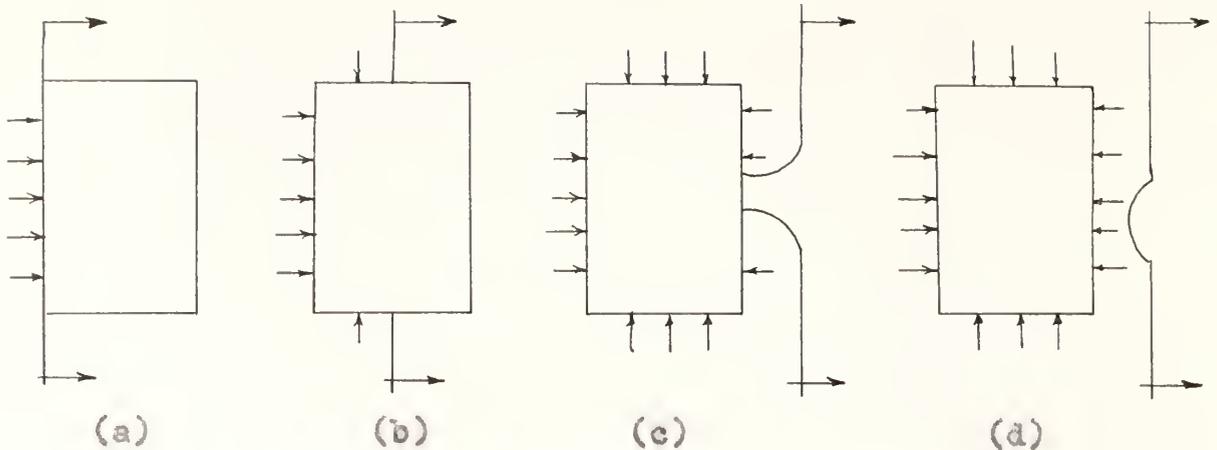


FIG. 4. STAGES IN THE DIFFRACTION OF A BLAST WAVE BY A STRUCTURE.

Under the condition that the blast wave has not yet completely surrounded the structure, there will be a considerable pressure difference between the front and back faces. Such a pressure difference will produce a lateral force, tending to cause the structure to move bodily in the same direction as the blast wave. This force is known as the "diffraction loading".

b. Drag Loading. Drag loading is the term given to the forces on a structure resulting from the high velocity of the air particles in the air blast acting as a high velocity wind. This type of loading is most important on truss-type structures such as bridges, and buildings in which the wall panels fail, leaving the structural frame exposed to the air blast.

ANALYSIS FOR DYNAMIC LOADS

Two different methods are used⁸ either separately or

⁸"Principles of Dynamic Analysis and Design", U.S. Army Corps of Engineers, U.S. Gov't. Printing Office, March, 1957.

concurrently in the analysis of structures under dynamic load. They are based on the equilibrium principle and on the work done and energy considerations. The equation of dynamic equilibrium takes the form of Newton's equation of motion, which is given by the following equation:

$$(\text{Mass})(\text{acceleration}) = (\text{external force} - \text{internal force}) \quad (4)$$

According to the principle of conservation of energy, the following equation is obtained:

$$\text{Work done} = \text{kinetic energy} + \text{strain energy} \quad (5)$$

The strain energy in equation (5) includes both the reversible elastic strain energy and the irreversible plastic strain energy. The difference between the behavior of structures under static and dynamic load is in the presence of inertial force, i.e., (mass times acceleration) in the equation of dynamic equilibrium and in the presence of kinetic energy in the equation of energy conservation. Both of these terms are related to the mass of the structure. Hence, the mass of structure becomes a very important consideration in the dynamic analysis.

Dynamic Equivalent System

In dynamic analysis^{3,3} there are three quantities to be considered; the work done, the strain energy, and the kinetic energy. To evaluate the work done, the displacement at any point of the structure under external load must be known. The strain energy is equal to the summation of strain energies in all the structural elements, due to bending, compression, or shear. The kinetic energy involves the energy of translation

and rotation of all the masses of the structure. The actual evaluation of these quantities for a given structure under dynamic load is very complex. For this reason, it is necessary in many problems to idealize both the structure and the loading. The distributed masses of the given structure are lumped together into a number of concentrated masses, and the strain energy is assumed to be stored in a number of weightless springs.

When idealizing dynamic load on structures, two simplifications are generally required. The first one involves the geometric distribution of the load over the structure. If for the purpose of analysis the mass of the system is concentrated at certain points, the load must be applied at the same points. This requires modification of the magnitude of the loads. The second simplification involves idealizing the load-time curve. If a numerical method of analysis is used, it is not necessary to idealize this function because any variation can be handled.

Hence, the equivalent system consists merely of a number of concentrated masses joined together by weightless springs and subjected to concentrated loads which vary with time. Figure 5 shows two simple structures together with corresponding idealized or equivalent dynamic systems.

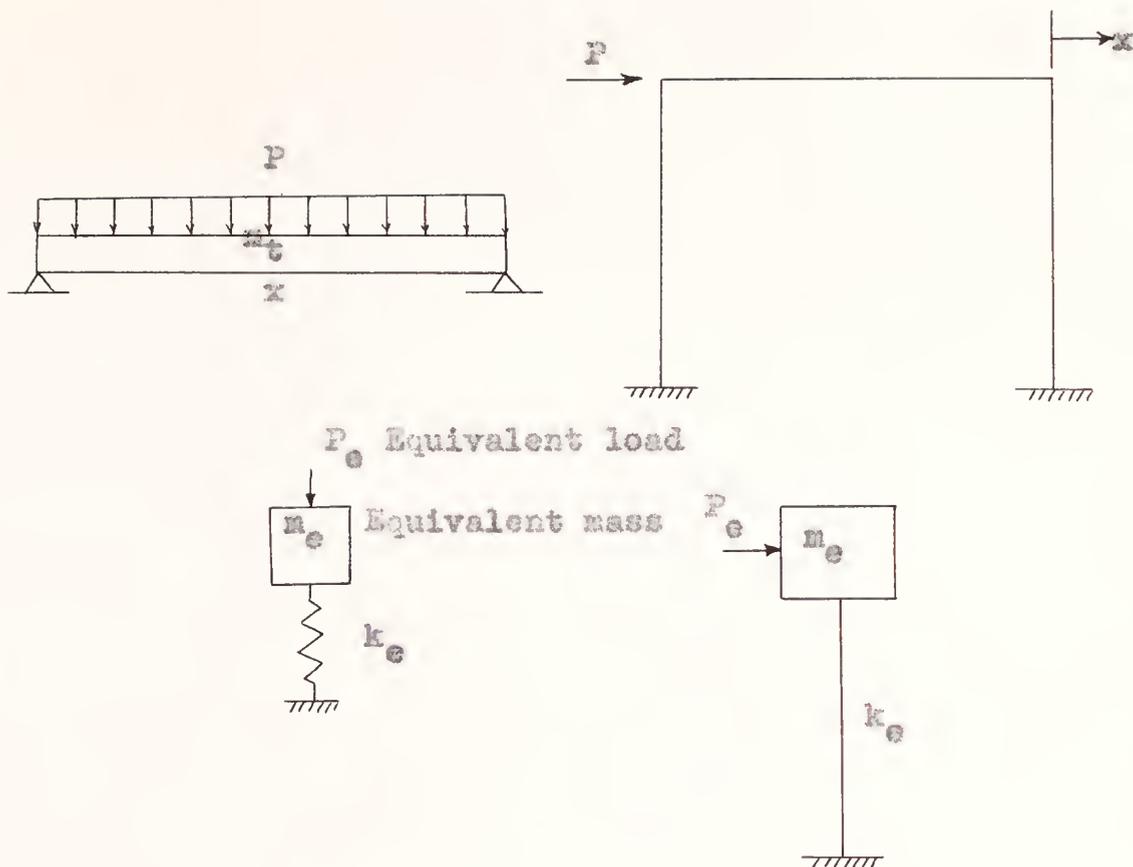


FIG. 5. IDEALIZED SYSTEMS.

Basic Dynamic System

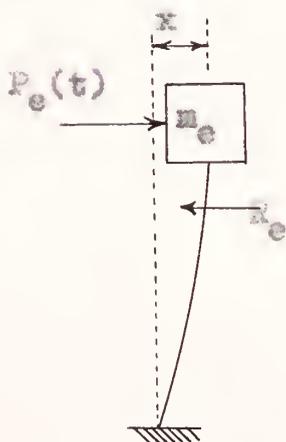


FIG. 6. BASIC DYNAMIC SYSTEM.

The equivalent system shown in Fig. 6 is considered^{8,3}, in which $P_e(t)$ denotes the external load as a function of time, m_e , the mass of equivalent system, $R_e(x)$, the internal resistance as a function of deflection and x is the deflection.

Application of Newton's equation of motion yields:

$$m_e \frac{d^2x}{dt^2} = P_e(t) - R_e(x) \quad (6)$$

where $\frac{d^2x}{dt^2}$ denotes the acceleration

Equation (5) gives:

$$\int_0^x P_e(t) dx = \frac{1}{2} m_e \left(\frac{dx}{dt}\right)^2 + \int_0^x R_e(x) dx \quad (7)$$

in which $\frac{dx}{dt}$ denotes velocity

At the point of maximum deflection, i.e., zero velocity, equation (7) yields:

$$\int_0^{x_m} P_e(t) dx = \int_0^{x_m} R_e(x) dx \quad (8)$$

where x_m = maximum deflection

External Work Done:

A typical dynamic load is shown in Fig. 7⁸, where T is defined as the duration of the load. If this load is applied to the dynamic system shown in Fig. 6, $W_e(t)$, the external work done up to any time t , is given by:

$$\begin{aligned} W_e(t) &= \int_0^{x(t)} P_e(t) dx \\ &= \int_0^t P_e(t) \frac{dx}{dt} dt \end{aligned} \quad (9)$$

Integration of equation (6) yields:

$$\frac{dx}{dt} = \frac{1}{m_e} \int_0^t [P_e(t) - R_e(x)] dt \quad (10)$$

Thus, the expression for the work done becomes

$$W_e(t) = \int_0^t P_e(t) \left[\frac{1}{m_e} \int_0^t (P_e(t) - R_e(x)) dt \right] dt$$

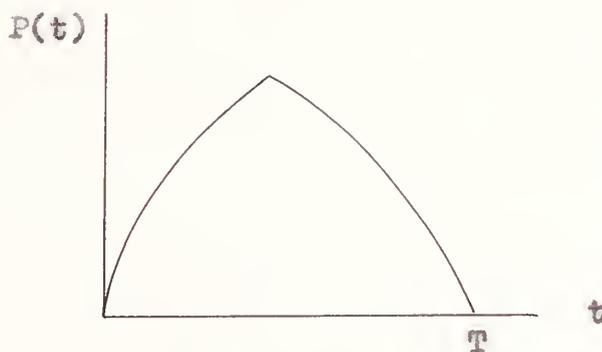


FIG. 7. TYPICAL DYNAMIC LOAD

At time t_m corresponding to the maximum displacement of the mass, the maximum work done, W_{me} , is given by:

$$W_{me} = \int_0^{t_m} P_e(t) \left[\frac{1}{m_e} \int_0^t (P_e(t) - R_e(x)) dt \right] dt \quad (11)$$

If t_m is much greater than T , the resistance $R_e(x)$ is small during the application of load and may be neglected.

Thus, the expression for work done becomes:

$$W_{pe} = \int_0^T P_e(t) \left[\frac{1}{m_e} \int_0^t P_e(t) dt \right] dt \quad (12)$$

Where W_{pe} denotes the work done ignoring the contribution of the resistance.

Integration of equation (12) yields:

$$W_{pe} = \frac{H^2}{2 m_e} \quad (13)$$

in which

$$H_{me} = \int_0^T P_e(t)$$

H_{me} is the total impulse of the external load and is equal to the area under the load-time curve.

After the application of the load, the mass has acquired a kinetic energy equal to the work done:

$$\frac{H^2}{2 m_e} = \frac{1}{2} m_e v^2$$

Therefore, the initial velocity, V , is given by

$$V = \frac{H_{me}}{m_e} \quad (14)$$

In many cases, the internal resisting force, $R_e(x)$, cannot be neglected in the time interval between 0 and T . Therefore, the work given by equation (13) may be considered to be the absolute maximum work which could be done by a given load on a dynamic system. The ratio, $\frac{W_{me}}{W_{pe}}$, of the actual work done divided by the maximum work is called the work-done ratio.

Curves giving the work-done ratio, C_w , for two simple load-time functions and for one-degree of freedom are plotted⁸ for design purposes. These are given in terms of ratios

⁸Ibid., Fig. 5.24 to 5.27, p. 50.

$$C_R = \frac{R_{me}}{B_e} \quad \text{and} \quad C_T = \frac{T}{T_n} \quad (14a)$$

where R_{me} denotes the maximum or plastic resistance of the system, B_e the peak value of external loads, T the duration of load, and $T_n = 2 \sqrt{\frac{me}{K_e}}$ is the natural period of the system.

Transformation Factors

The application of transformation factors to the dynamic parameters of a structure transform the system to an idealized system.³ They are:

(a) Load Factor K_L . The concentrated dynamic load on the equivalent system is obtained by multiplying the total load on the actual structure by the load factor.

$$K_L = \frac{P_e(t)}{P(t)} \quad (15)$$

K_L is determined by equating the external work done by P_e on the equivalent system to that done by the real system.

(b) Mass Factor K_M . When the total mass of the structure is multiplied by the mass factor, the concentrated mass of equivalent system is obtained.

$$K_M = \frac{m_e}{m_t} \quad (16)$$

This factor is obtained by equating kinetic energy of the real system and the equivalent system.

³Norris, op. cit., p. 149.

(c) Resistance Factor K_R . The resistance of an element is the internal force tending to restore the element to its equilibrium position. At a given deflection, the resistance is defined as being numerically equal to the static load required to produce the same deflection. The product of the resistance factor, K_R , and the resistance of the real element gives the resistance of the spring in the equivalent system. Equating the internal strain energies of the two systems yields:

$$K_R = \frac{R_e}{R} \quad (17)$$

(d) Maximum Resistance, R_{me} and Spring Constant R_e . The maximum resistance of a real element is defined as the maximum total load which can be carried by this element. The product of the resistance factor, K_R and the maximum resistance give the maximum resistance of the equivalent system.

The spring constant, R of the real system is defined as the total static load to cause a unit deflection. Since the deflection of the two systems should be the same, the spring constant of the equivalent system is obtained by applying the resistance factor thus,

$$\begin{aligned} R_{me} &= K_R R_n \\ k_e &= K_R k \end{aligned} \quad (18)$$

(e) Load Mass Factor K_{LM} . The ratio of mass factor to load factor is defined as the load mass factor.

$$K_{LM} = \frac{K_m}{K_L} \quad (19)$$

(f) Dynamic Reaction V. Dynamic reactions are needed to determine the shear at the supports and are obtained by consideration of the resistance of the equivalent system and actual applied loads.

Transformation Factors for a Simply Supported,
Uniformly Loaded Beam.

As an example, transformation factors are calculated below for a simply supported beam^{5,3} having uniform mass and subjected to a uniformly distributed load. (Fig. 8.)

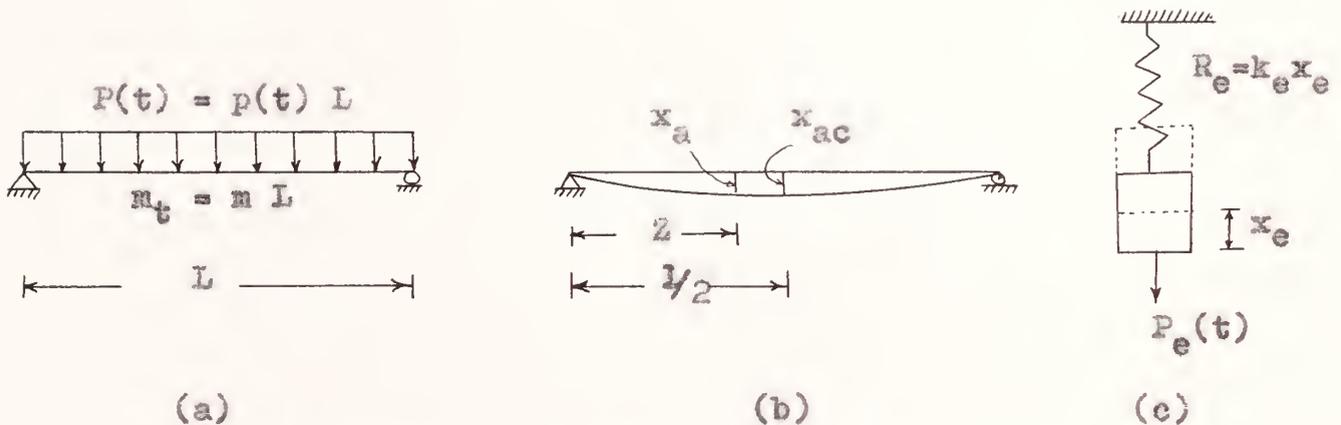


FIG. 8. DETERMINATION OF THE EQUIVALENT SYSTEM IN THE ELASTIC RANGE. (a) UNIFORMLY DISTRIBUTED LOAD ON SIMPLY SUPPORTED BEAM. (b) ASSUMED DEFLECTION SHAPE. (c) EQUIVALENT SINGLE DEGREE SYSTEM.

⁵"Structural Elements Subjected to Dynamic Loads", U. S. Army Corps of Engineers, U. S. Government Printing Office, March, 1957, p. 37.

(a) Load Factor. The elastic curve of a simply supported beam subjected to a uniform static load $p(t)$ lbs/ft is given by:

$$x_a = \frac{p^2}{24 EI} (L^3 - 2Lz^2 + z^3) \quad (20)$$

The deflection at the mid span is given by

$$x_{ac} = \frac{5pL^4}{384 EI} \quad (21)$$

Thus,

$$x_a = \frac{16}{5L^4} (L^3z - 2Lz^3 + z^4) x_{ac} \quad (22)$$

The total work done by the load is equal to:

$$\begin{aligned} W_a &= \int_0^L \frac{P x_a}{2} dz \\ &= \frac{16}{25} \frac{P x_{ac}}{2} \end{aligned} \quad (23)$$

where P is the total load

In the equivalent system, the work done by the external load is,

$$W_e = \frac{P_e x_e}{2}$$

Making x_e equal to x_{ac} and equating the work done in both cases, gives:

$$P_e = \frac{16}{25} P$$

Thus, the elastic load factor is,

$$K_1 = \frac{P_e}{P} = \frac{16}{25} \quad (24)$$

After the formation of plastic hinge at mid-span, the deflected shape is assumed to be as shown in Fig. 9.

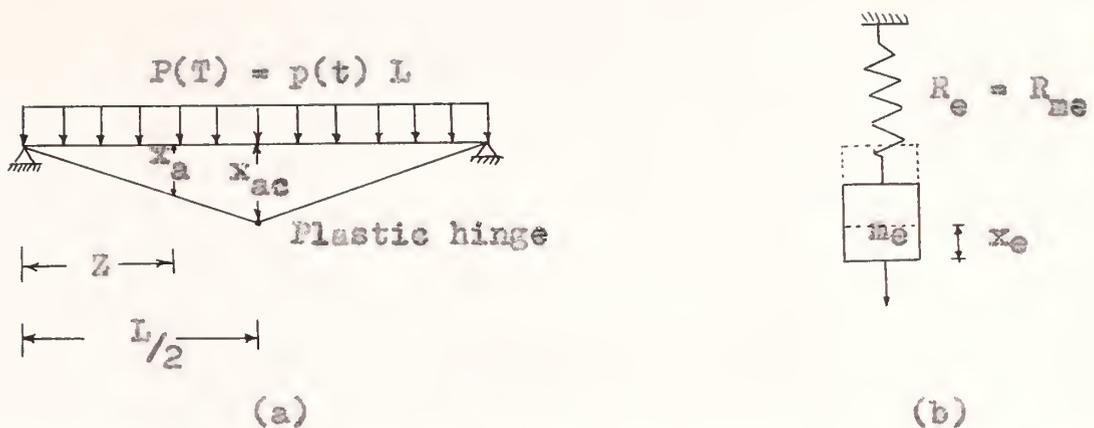


FIG. 9. DETERMINATION OF THE EQUIVALENT SYSTEM IN THE PLASTIC RANGE. (a) ASSUMED DEFLECTION SHAPE. (b) EQUIVALENT SINGLE-DEGREE SYSTEM.

$$\text{In this case } K_L = \frac{1}{2} \quad (25)$$

(b) Mass Factor. The mass factor is obtained by equating the total kinetic energy of the beam to that of an equivalent system. In the simple harmonic motion of the type assumed here, the maximum velocity at any point along the beam is proportional to the ordinate of the deflection curve at the same point.

Thus, in the elastic range,

$$V_a = \frac{16}{5L^4} (L^3Z - 2LZ^3 + Z^4) V_{ac} \quad (26)$$

where V_{ac} is the velocity at mid-span and V_a is the velocity at a distance Z from the support.

The kinetic energy of the real system is given by:

$$KE_a = \int_0^L \frac{1}{2} m V_a^2 dZ \quad (27)$$

where m is the mass per unit length.

Integrating equation (27), yields:

$$KE_a = 0.25 m_t v_{ac}^2 \quad (28)$$

where m_t is the total mass of the beam.

The kinetic energy of the equivalent system is given by:

$$KE_e = \frac{1}{2} m_e v_e^2 \quad (29)$$

Equating the kinetic energies of the real system and the equivalent system, yields the elastic mass factor,

$$K_M = \frac{m_e}{m_t} = 0.5 \quad (30)$$

In the plastic range, the deflected shape is shown in Fig. 9 (a) and the mass factor is given by,

$$K_M = 0.33 \quad (31)$$

(c) Maximum Resistance and Spring Constant. The maximum moment for a simply supported beam at mid-span is given by

$$M_c = \frac{PL}{8}$$

where P is the sum of the uniform load. The maximum resistance, R_m is obtained by equating the bending moment to the resisting moment. Thus,

$$R_m = \frac{8 M_p}{L} \quad (32)$$

In the equivalent system, the maximum elastic resistance is given by:

$$R_{me} = K_R R_m = \frac{16}{25} \frac{8 M_p}{L} \quad (33)$$

This is the limiting resistance in the elastic range. However,

in the plastic range, the resistance of the equivalent system is,

$$R_{me} = \frac{1}{2} \frac{8 M_p}{L} \quad (34)$$

The stiffness or the spring constant of the real beam in the elastic range is,

$$k = \frac{384 EI}{L^3} \quad (35)$$

and the spring constant of the equivalent system is:

$$k_e = K_L k = \frac{16}{25} \frac{384 EI}{L^3} \quad (36)$$

(d) Dynamic Reactions. In order to determine the dynamic reactions on the actual beam, the inertia forces distributed along the beam must be considered. In Fig. 10, it is assumed that the inertia forces are at all points proportional to the ordinates of the deflected shape of the beam.

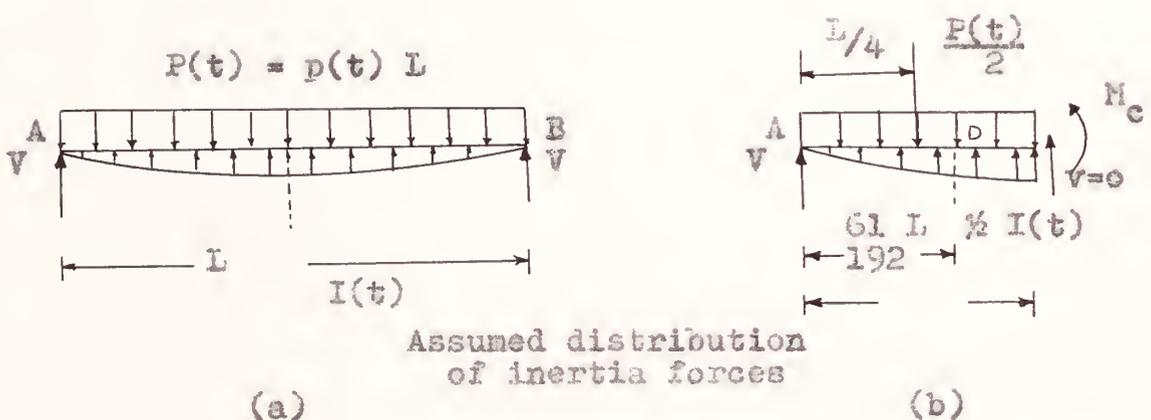


FIG. 10. DETERMINATION OF DYNAMIC REACTION IN THE ELASTIC RANGE. (a) LOAD AND INERTIAL-FORCE DISTRIBUTION; (b) FORCES ON ONE-HALF OF BEAM.

Considering one-half of the beam and taking moments about point D, the centroid of the inertial forces, gives:

$$V \frac{61}{192} L - M_c - \frac{1}{2} P \left(\frac{61}{192} L - L/4 \right) = 0 \quad (37)$$

where M_c is the bending moment at mid-span.

Assuming the resistance is equal to $\frac{8 M_c}{L}$, and substituting for M_c in equation, yields:

$$V = 0.39 R + \frac{1}{8} P \quad (38)$$

Using the same procedure in the plastic range along with the assumed deflected shape for this range, Fig. 9 (a), yields:

$$V = \frac{3}{8} R + \frac{1}{8} P \quad (39)$$

ANALYSIS BY NUMERICAL METHODS

The analysis of a dynamic system of a single degree of freedom consists of the evaluation of displacement using Newton's equation of motion (Eq. 4). Two approaches are generally used in solving this type of differential equation. The first one is the exact method in which the solution is obtained directly from the differential equation. This approach is applicable only when both the load and resistance function can be expressed in simple mathematical forms.⁸ The other approach is the numerical integration which is generally applicable to any type of load and resistance function.

⁸"Principles of Dynamic Analysis and Design", U. S. Army Corps of Engineers, U. S. Government Printing Office, March, 1957.

Principles of Numerical Analysis

The numerical method of evaluating the displacement from Equation 4 is called the method of numerical integration.^{3,8.}

The load, resistance, acceleration, velocity, and displacements are plotted versus time in Fig. 11 for a typical case.

Assume that $t_0, t_1, t_2, \dots, t_{n-1}, t_n,$ and t_{n+1} constitute a time sequence, and that Δt_n denotes the time interval from t_n to t_{n+1} . The dynamic load is assumed to be initiated at $t = t_0$. The acceleration, velocity, and displacement at t_n are denoted by $a_n, v_n,$ and x_n , respectively. If the acceleration in the time interval Δt_n is represented by $a(t)$, the velocity and displacement at t_{n+1} are given by the following equations:

$$v_{n+1} = v_n + \int_{t_n}^{t_{n+1}} a(t) dt \quad (40)$$

and

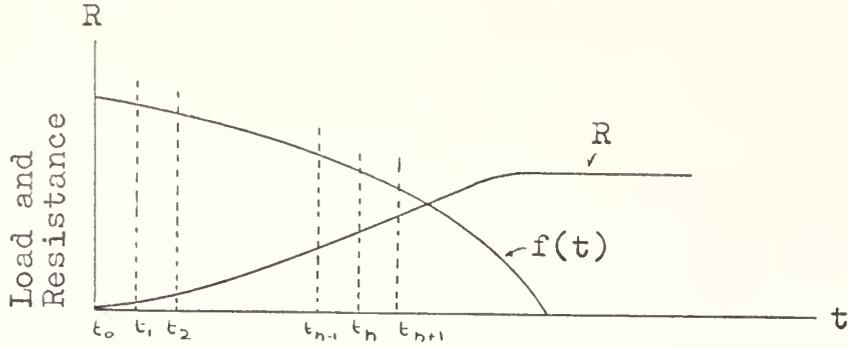
$$x_{n+1} = x_n + v_n (\Delta t_n) + \int_{t_n}^{t_{n+1}} [a(t)] dt \cdot dt \quad (41)$$

These equations indicate that the velocity and the displacement at t_{n+1} can be obtained by extrapolation from the corresponding values at t_n , once the acceleration in the time interval Δt_n is known.

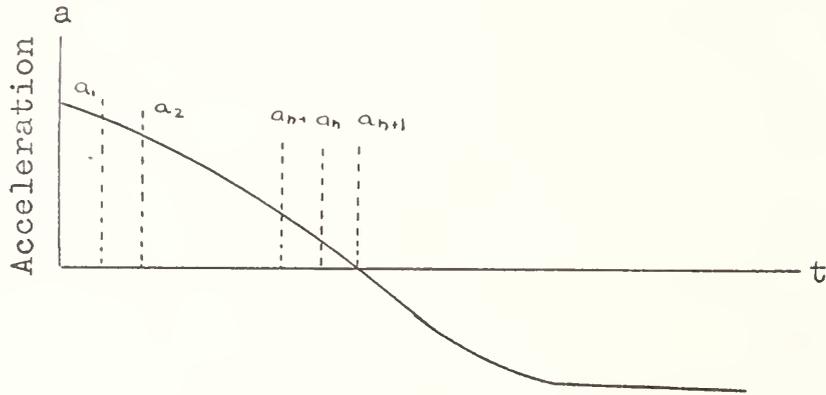
³Norris, op.cit., p. 183, chapter 8.

⁸"Principles of Dynamic Analysis and Design", U. S. Army Corps of Engineers, U. S. Government Printing Office, March, 1957.

$f(t)$ and

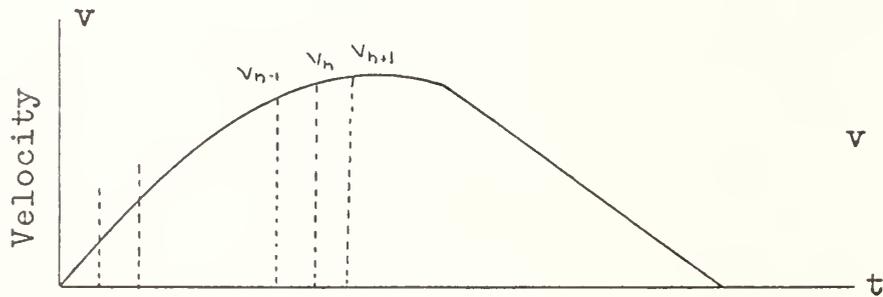


(a) Load and Resistance



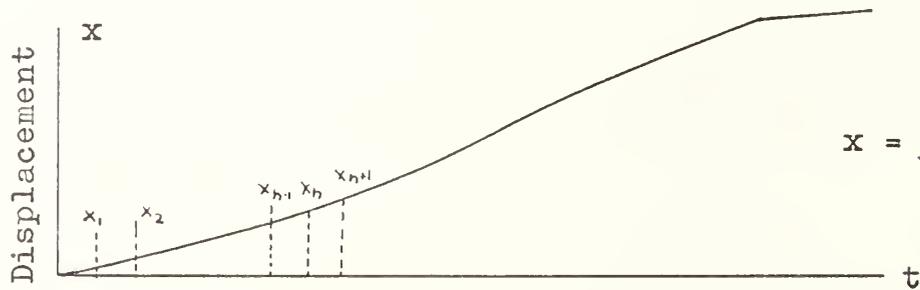
$$a = \frac{f(t) - R}{m}$$

(b) Acceleration



$$v = \int a dt$$

(c) Velocity



$$x = \int v dt$$

(d) Displacement

FIG. 11. LOAD, RESISTANCE, ACCELERATION, VELOCITY, AND DISPLACEMENT VERSUS TIME.

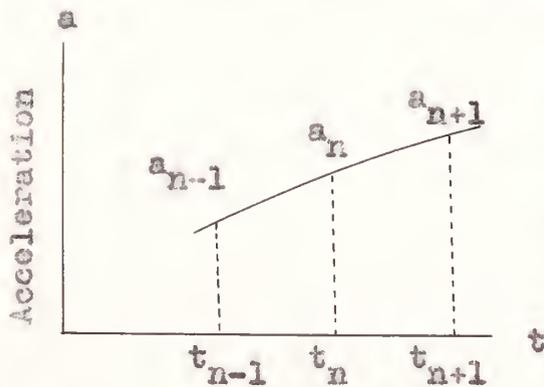
In the analysis of structures under dynamic load, the velocity and the displacement at the time $t = 0$ are assumed to be equal to zero, that is,

$$V_0 = 0 \quad \text{and} \quad x_0 = 0$$

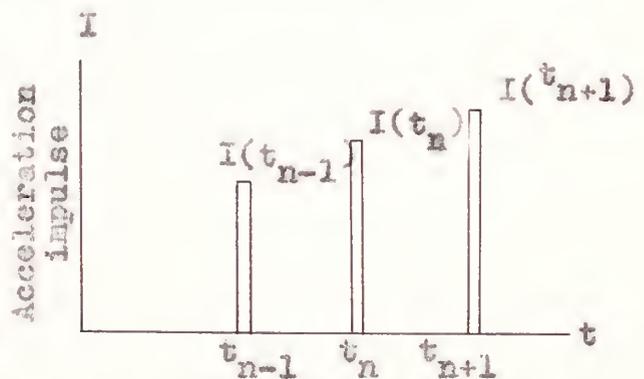
In applying Equations (40) and (41), the values of v_1 and x_1 can be obtained provided $a(t)$ is known in the time interval from t_0 to t_1 . This process can be continued until the values of v_n and x_n for any value of n are obtained.

Many extrapolation formulas have been derived³ for the solution of differential equation by assuming a simple acceleration-time relation in any time interval t_n . An extrapolation formula based on acceleration impulse^{3,8} is described below.

Acceleration Impulse Extrapolation Method



(a) Actual acceleration curve.



(b) Acceleration pulses to replace the given acceleration curve.

³Norris, op. cit., p. 186.

³Ibid., p. 191.

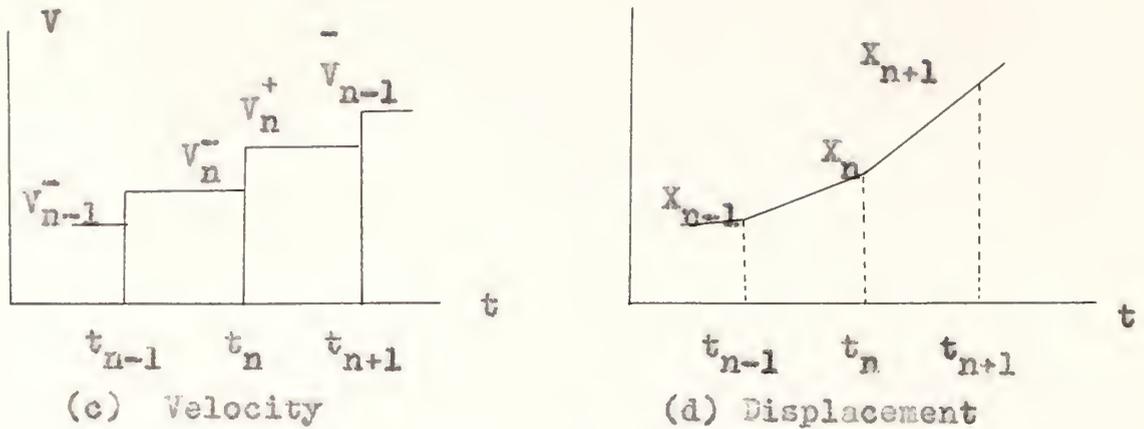


FIG. 12. ACCELERATION IMPULSE EXTRAPOLATION METHOD.

In this method, the actual acceleration curve shown in Fig. 12(a) is replaced by a train of equally spaced impulses occurring at $t_0, t_1, t_2, \dots, t_n$. The magnitude of the acceleration impulse at t_n is given by:

$$I(t_n) = a_n (\Delta t) \quad (42)$$

where

$$\Delta(t) = t_1 - t_0 = t_2 - t_1 = t_3 - t_2 = \dots = t_n - t_{n-1}$$

Since an impulse is applied at t_n , there is discontinuity in the value of velocity at t_n . In the time interval from t_n to t_{n+1} , the velocity is constant and displacement varies linearly with time. t_n^- and t_n^+ denote the time immediately before and after the application of the impulse at t_n , and V_n^- and V_n^+ denote the corresponding velocities which are related by:

$$V_n^+ = V_n^- + a_n (\Delta t) \quad (43)$$

The relation between x_{n-1} and x_n , and between x_n and x_{n+1} are given by:

$$x_n - x_{n-1} = V_n^- (\Delta t)$$

$$x_{n+1} - x_n = V_n^+ (\Delta t) \quad (44)$$

Combining Equations (43) and (44) yields the basic recurrence formula for the acceleration impulse extrapolation method

$$x_{n+1} = 2 x_n - x_{n-1} + a_n (\Delta t)^2 \quad (45)$$

Once the values of x at t_{n-1} and t_n are known, the value of x at t_{n+1} can be thus computed.

DESIGN OF A WINDOWLESS STRUCTURE

As an example, the frame as shown in Fig. 14 is designed to resist the effect of a 20 K. T. bomb. This building is assumed to be 3700 ft. from ground zero.

The building is assumed to be of reinforced concrete construction.

The concrete compressive strength is specified at 3000 psi and intermediate grade reinforcing steel is used. The following stresses are used in the design:⁹

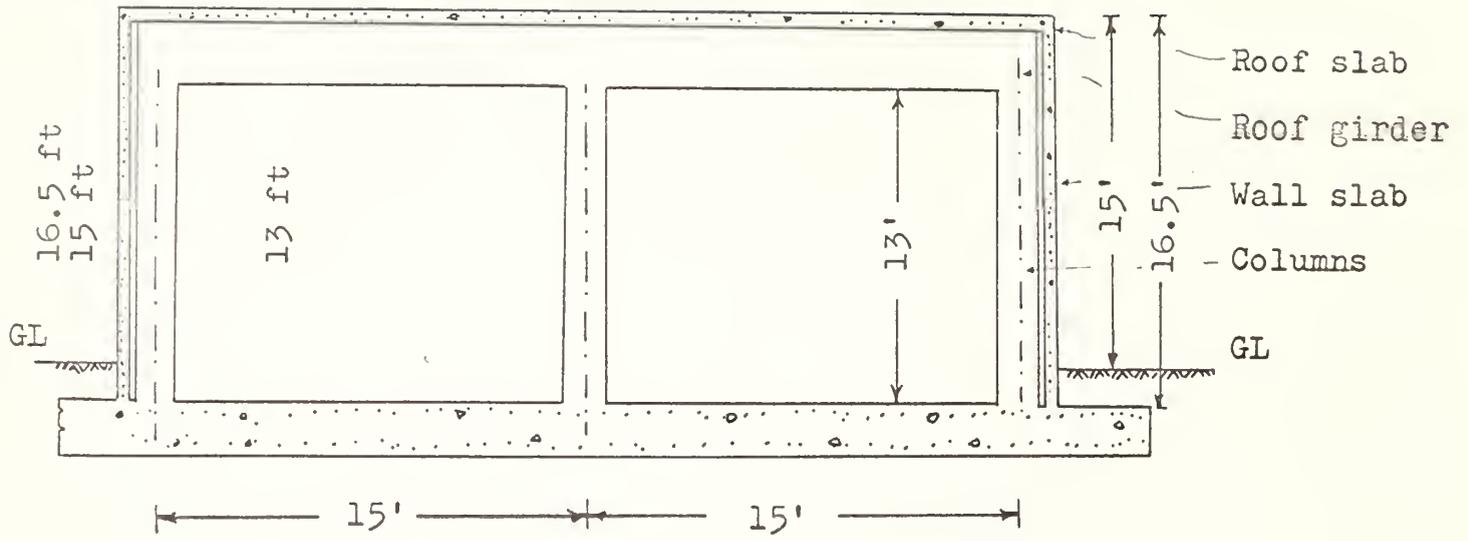
$$f'_c = 3000 \text{ psi}$$

$$\text{Dynamic strength of concrete, } f'_{dc} = 3900 \text{ psi}$$

$$\text{Modulus of elasticity of concrete, } E_c = 3(10^6) \text{ psi}$$

Ratio of modulus of elasticity of steel to modulus of elasticity of concrete, $n = 10$

⁹"Strength of Materials and Structural Elements", U. S. Army Corps of Engineers, U. S. Government Printing Office, July, 1959.



Section XY

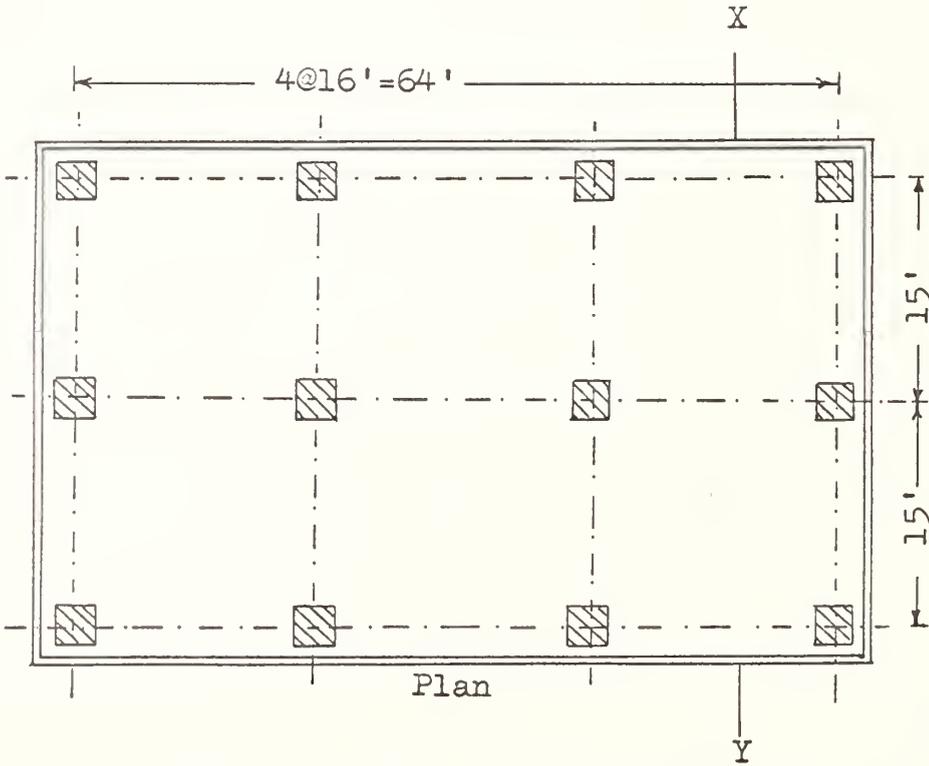


FIG. 14. PLAN AND THE SECTION OF THE BUILDING.

Reinforcing steel:

Static yield point stress, $f_y = 40,000$ psi

Dynamic yield strength of steel, $f_{dy} = 52,000$ psi

Load Determination

The computations for the various pressure-time curves are based on the formulas given by the Corps of Engineers, Reference 7.

The peak overpressure, P_{so} , is obtained from Fig. 1, Appendix B

$$P_{so} = 10 \text{ psi}$$

The duration of positive phase, t_o , is obtained from Fig. 2, Appendix B,

$$t_o = 0.71 \text{ secs}$$

(a) Determination of Front Face Overpressure Versus Time Curve.

Velocity of sound, C_{refl} , in the region of the reflected overpressure is obtained from Reference 3, Fig. 11 - 21,

$$C_{refl} = 1290 \text{ psi}$$

The time required to clear the front face of the structure is given by Equation (11.4) Reference 3.

$$t_c = \frac{3 h'}{C_{refl}}$$

$$= 0.0349 \text{ secs.}$$

For $P_{so} = 10$ psi, reflected overpressure, P_{refl} , is given by Fig. 11.20, Reference 3,

$$P_{\text{refl}} = 25.3 \text{ psi}$$

For $P_{s0} = 10 \text{ psi}$, dynamic pressure, q is given by,

$$q = 2.23 \text{ psi}$$

Average overpressure on the front face is given by the equation

$$P_{\text{front}} = P_s + 0.85 q$$

Table 1. Determination of front face overpressure versus time.

| t secs. | $\frac{t}{t_0}$ | $\frac{q}{q_0}$ (From Fig. 3 Appendix B) | q psi | $\frac{P_s}{P_{s0}}$ (From Fig. 4 Appendix B) | P_s psi | $P_{\text{front}} =$ $P_s + 0.85q$ psi |
|--------------|-----------------|--|------------|---|--------------|--|
| 0 | 0 | 1.00 | 2.230 | 1.000 | 10.00 | 12.12 |
| 0.1 | 0.141 | 0.525 | 1.170 | 0.746 | 7.46 | 8.46 |
| 0.2 | 0.282 | 0.267 | 0.595 | 0.543 | 5.43 | 5.94 |
| 0.3 | 0.423 | 0.118 | 0.264 | 0.360 | 3.60 | 3.83 |
| 0.4 | 0.564 | 0.060 | 0.134 | 0.247 | 2.47 | 2.58 |
| 0.5 | 0.705 | 0.025 | 0.056 | 0.146 | 1.46 | 0.68 |
| 0.6 | 0.846 | 0.008 | 0.018 | 0.066 | 0.66 | 0.05 |
| 0.71 | 1.00 | 0.00 | 0.00 | 0.005 | 0.05 | 0.00 |

Fig. 17 shows front face overpressure versus time curve.

(b) Average Back wall Overpressure Versus Time.

For $P_{s0} = 10 \text{ psi}$, the velocity of incident shock front, U_0 is given by Fig. 11.21, Reference 3,

$$U_o = 1403 \text{ psi}$$

The time displacement factor, t_d , is given by:

$$t_d = \frac{L}{U_o} \quad (46)$$

where, L is the length of the structure.

Equation (46) yields:

$$t_d = 0.024 \text{ sec.}$$

Time required for overpressure on the rear face to rise from zero to its maximum value is given by:³

$$t_b = \frac{4h}{C_o} \quad (47)$$

where, C_o is the velocity of sound in air and is equal to 1115 fps.

From Appendix B, Fig. 4, for $\frac{t - t_d}{t_o} = 0.0756$, the value of $\frac{P_s}{P_{so}}$ is found to be:

$$\frac{P_s}{P_{so}} = 0.859$$

or

$$P_s = 8.59 \text{ psi}$$

The peak value of average overpressure on the back wall is given by Equation (11.9) Reference 3:

$$(\bar{P}_{back})_{max} = \frac{P_{sb} [1 + (1-\beta) e^{-\beta}]}{2} \quad (48)$$

where

P_{sb} = incident blast wave overpressure on back face at time $t - t_c = t_b$

$$\beta = \frac{0.5 P_{so}}{14.7}$$

$(\bar{P}_{back})_{max}$ = peak value of average overpressure on back wall which occurs at time $t = t_d + t_b$

Equation (48) yields:

$$(\bar{P}_{back})_{max} = 6.30 \text{ psi}$$

For time in excess of $t - t_d = t_b$, overpressure on back wall is given by Equation (11.10) Reference 3.

$$\frac{P_{back}}{P_s} = \frac{(\bar{P}_{back})_{max}}{P_{sb}} + \left[1 + \frac{(\bar{P}_{back})_{max}}{P_{sb}} \right] \left[\frac{t - (t_d + t_b)}{t_o - t_b} \right]^2 \quad (49)$$

where

t_o is the duration of positive phase.

Table 2. Determination of the back wall overpressure versus time.

| t secs. | t-t _d secs. | $\frac{t-t_d}{t_o}$ | $\frac{P_s}{P_{so}}$ (Fig. 4 Appendix B) | P _s psi | $\frac{P_{back}}{P_s}$ | P _{back} psi |
|------------|---------------------------|---------------------|--|-----------------------|------------------------|--------------------------|
| 0.078 | 0.054 | 0.076 | 0.859 | 8.59 | 0.734 | 6.30 |
| 0.095 | 0.071 | 0.10 | 0.814 | 8.14 | 0.734 | 5.96 |
| 0.166 | 0.142 | 0.20 | 0.655 | 6.55 | 0.739 | 4.84 |
| 0.308 | 0.284 | 0.40 | 0.402 | 4.02 | 0.766 | 3.08 |
| 0.450 | 0.426 | 0.60 | 0.220 | 2.20 | 0.820 | 1.80 |
| 0.598 | 0.568 | 0.80 | 0.090 | 0.90 | 0.901 | 0.81 |
| 0.734 | 0.710 | 1.00 | 0.00 | 0.0 | 1.00 | 0 |

Sample calculation of Table 2.

$$\text{For } \frac{t - t_d}{t_o} = 0.40$$

$$\begin{aligned} t &= (0.40) t_o + t_d \\ &= 0.308 \text{ sec.} \end{aligned}$$

Substituting the values of $(\bar{P}_{\text{back}})_{\text{max}} = 6.30$, $P_{\text{sb}} = 8.59$,

$t = 0.308$, $t_d = 0.024$, and $t_o = 0.0538$ in Equation (49) yields:

$$\frac{(\bar{P}_{\text{back}})}{P_s} = 0.766$$

or

$$\begin{aligned} P_{\text{back}} &= 766 (4.02) \\ &= 3.08 \text{ psi} \end{aligned}$$

Fig. 18 shows the back overpressure versus time curve.

(c) Net Lateral Overpressure Versus Time. At any time, t net lateral overpressure is given by:

$$\bar{P}_{\text{net}} = \bar{P}_{\text{front}} - \bar{P}_{\text{back}} \quad (50)$$

Figure 19 shows net lateral overpressure versus time curve.

(d) Average Roof Overpressure Versus Time. Figure 15 shows the relation between the overpressure ratio, $\frac{\bar{P}_{\text{roof}}}{P_s}$ and the time t in secs.⁷

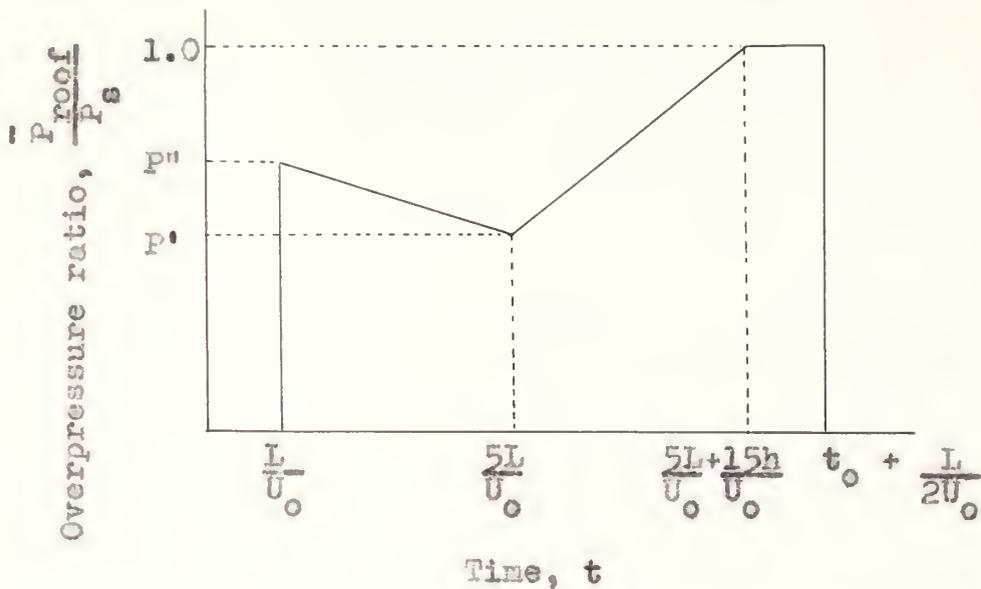


FIG. 15. AVERAGE ROOF OVERPRESSURE RATIO VERSUS TIME FOR A CLOSED RECTANGULAR STRUCTURE.

where

$$P'' = 0.9 + 0.1 \left(1 - \frac{P_{so}}{14.7}\right)^2 \quad (51)$$

P'' should always be smaller than 1.

The value of P' should be taken by the equations (52), whichever gives a smaller value.

$$P' = 2 - \left(\frac{P_{so}}{14.7} + 1\right) \left(\frac{h}{L}\right)^{1/3} \quad (52)$$

or

$$P' = 0.5 + 0.125 \left(2 - \frac{P_{so}}{14.7}\right)^2$$

The time displacement factor, $t_d = \frac{L}{2U_0}$

$$= 0.012 \text{ sec.}$$

At time $t = t_d + \frac{L}{2U_0}$

= 0.024 sec., Equation (51) yields:

$$\frac{\bar{P}_{\text{roof}}}{P_s} = 0.91$$

$$\text{At time } t = \frac{5L}{U_0}$$

= 0.120, Equations (52) yield:

$$\frac{\bar{P}_{\text{roof}}}{P_s} = 0.720$$

or

$$\frac{\bar{P}_{\text{roof}}}{P_s} = 0.717$$

therefore

$$P' = \frac{\bar{P}_{\text{roof}}}{P_s} = 0.717$$

$$\text{At time } t = \frac{5L}{U_0} + \frac{15h}{U_0} = 0.28 \text{ sec.},$$

$$\frac{\bar{P}_{\text{roof}}}{P_s} = 1.0$$

Table 3. Determination of the average roof overpressure versus time.

| t secs. | t-t _d secs. | $\frac{t-t_d}{t_0}$ | $\frac{P_s}{P_{so}}$ (Figure 4 Appendix B) | P _s psi | $\frac{\bar{P}_{roof}}{P_s}$ | \bar{P}_{roof} psi |
|------------|---------------------------|---------------------|--|-----------------------|------------------------------|-------------------------|
| 0.024 | 0.012 | 0.017 | 0.966 | 9.66 | 0.910 | 8.75 |
| 0.083 | 0.071 | 0.10 | 0.814 | 8.14 | 0.791 | 6.45 |
| 0.120 | 0.108 | 0.154 | 0.726 | 7.26 | 0.717 | 5.20 |
| 0.156 | 0.144 | 0.20 | 0.655 | 6.55 | 0.779 | 5.10 |
| 0.225 | 0.213 | 0.30 | 0.519 | 5.19 | 0.901 | 4.68 |
| 0.296 | 0.284 | 0.40 | 0.402 | 4.02 | 1.00 | 4.02 |
| 0.367 | 0.355 | 0.50 | 0.303 | 3.03 | 1.00 | 3.03 |
| 0.438 | 0.426 | 0.60 | 0.220 | 2.20 | 1.00 | 2.20 |
| 0.509 | 0.497 | 0.70 | 0.149 | 1.49 | 1.00 | 1.49 |
| 0.580 | 0.568 | 0.80 | 0.090 | 0.90 | 1.00 | 0.90 |
| 0.651 | 0.639 | 0.90 | 0.041 | 0.41 | 1.00 | 0.41 |
| 0.722 | 0.710 | 1.00 | 0.0 | 0 | 1.00 | 0 |

(Fig. 20 shows average roof overpressure versus time curve.)

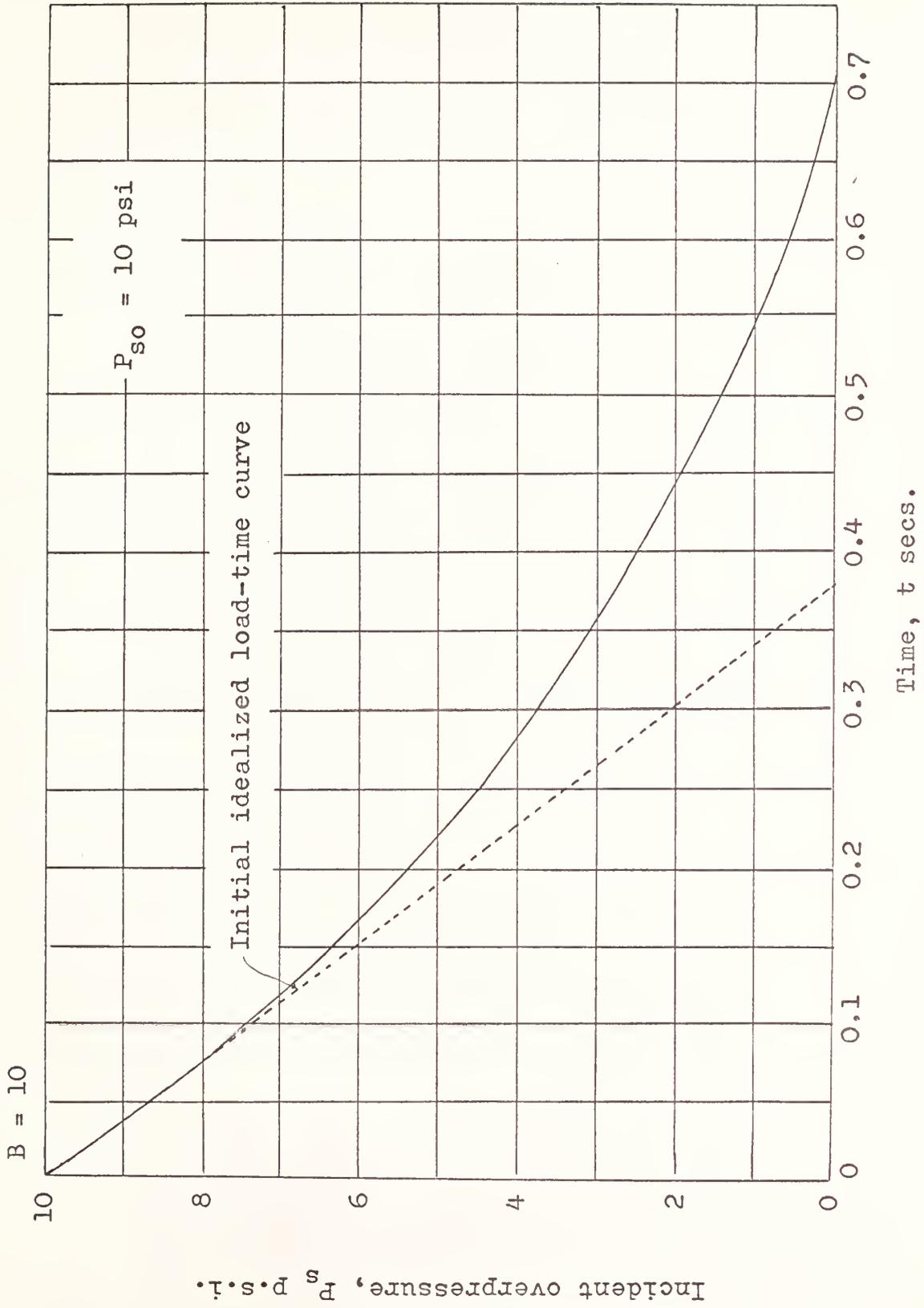


FIG. 16. INCIDENT OVERPRESSURE VERSUS TIME.

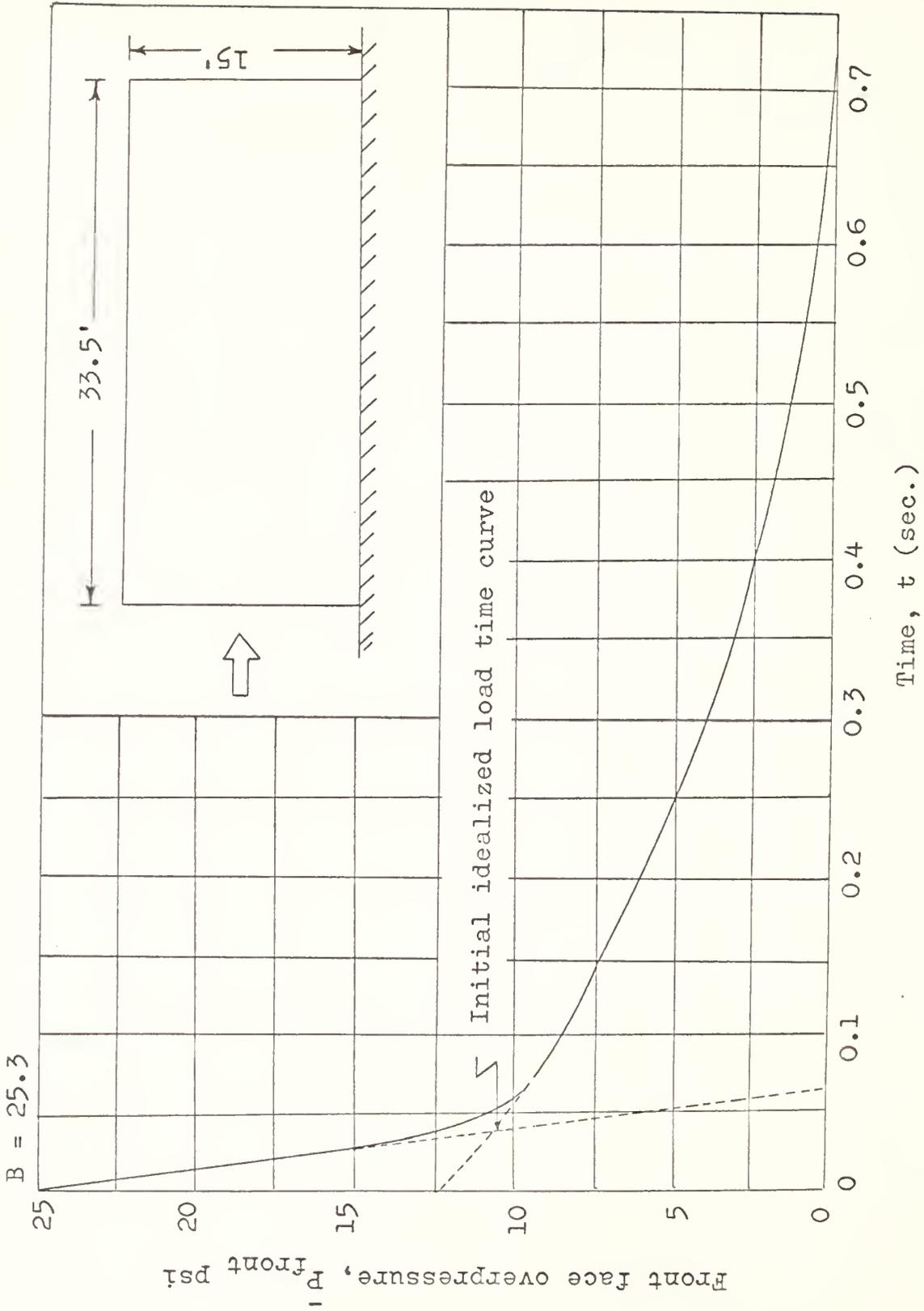


FIG. 17. FRONT FACE OVERPRESSURE VERSUS TIME.

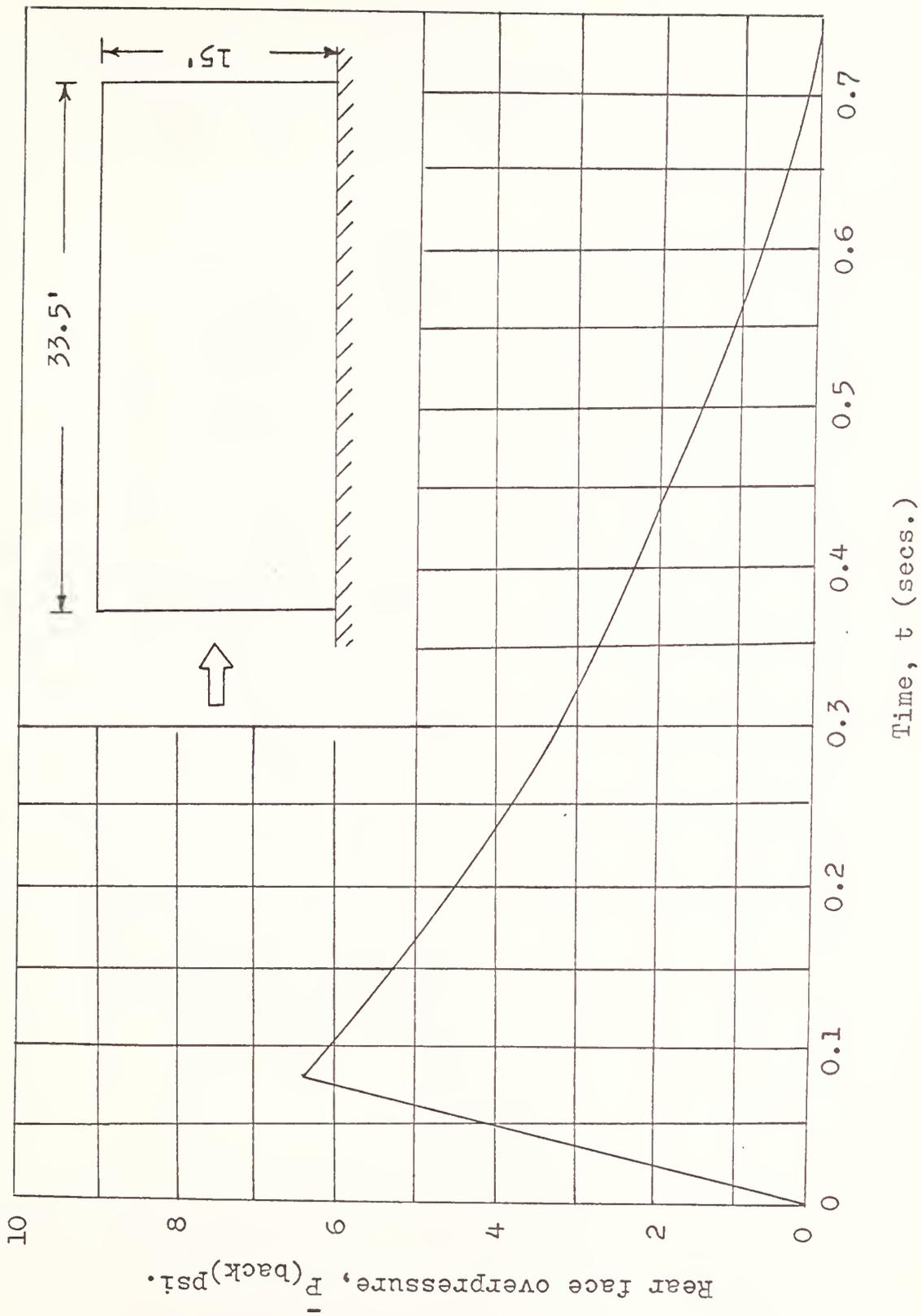


FIG. 18. REAR FACE OVERPRESSURE VERSUS TIME.

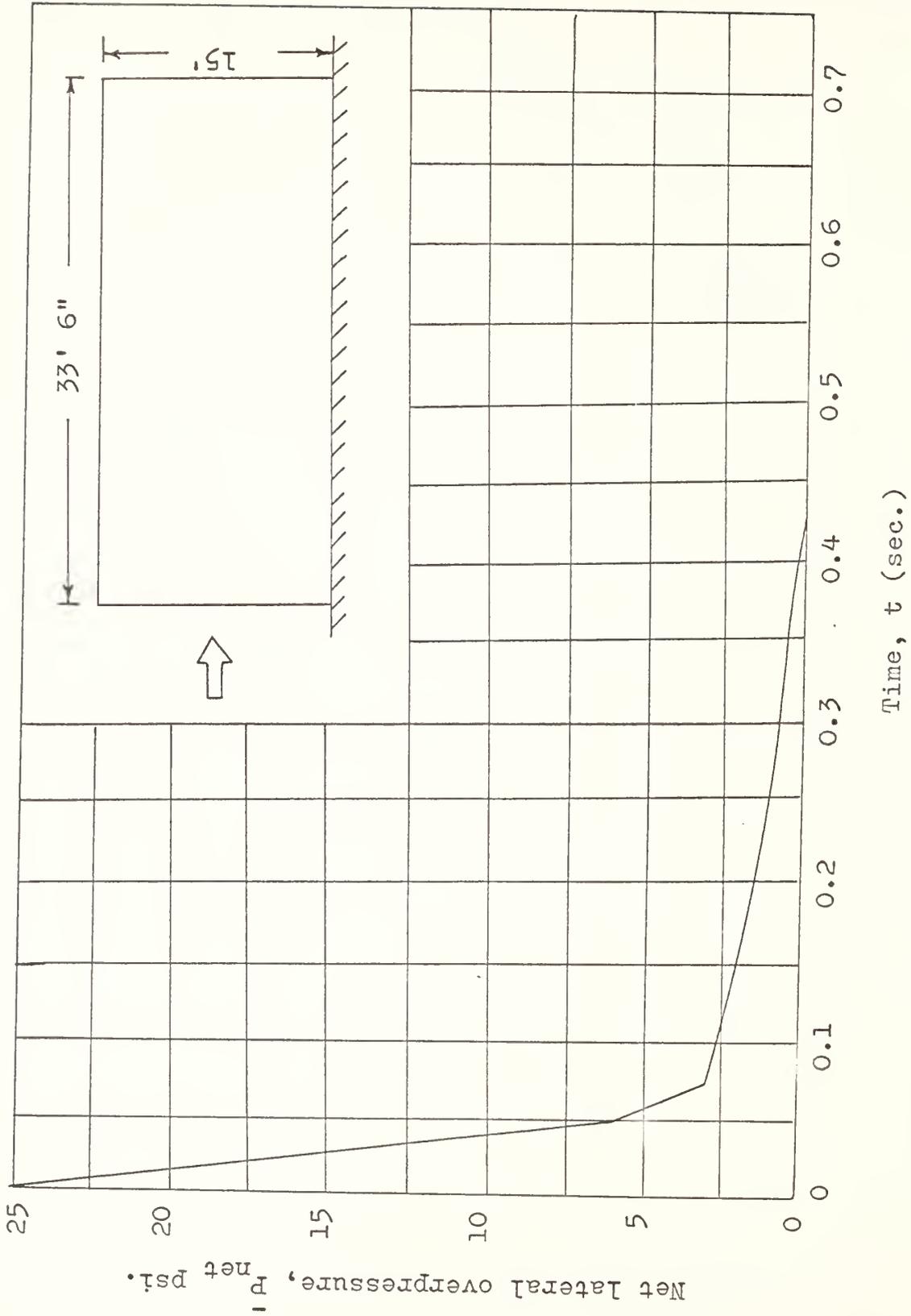


FIG. 19. NET LATERAL OVERPRESSURE VERSUS TIME.

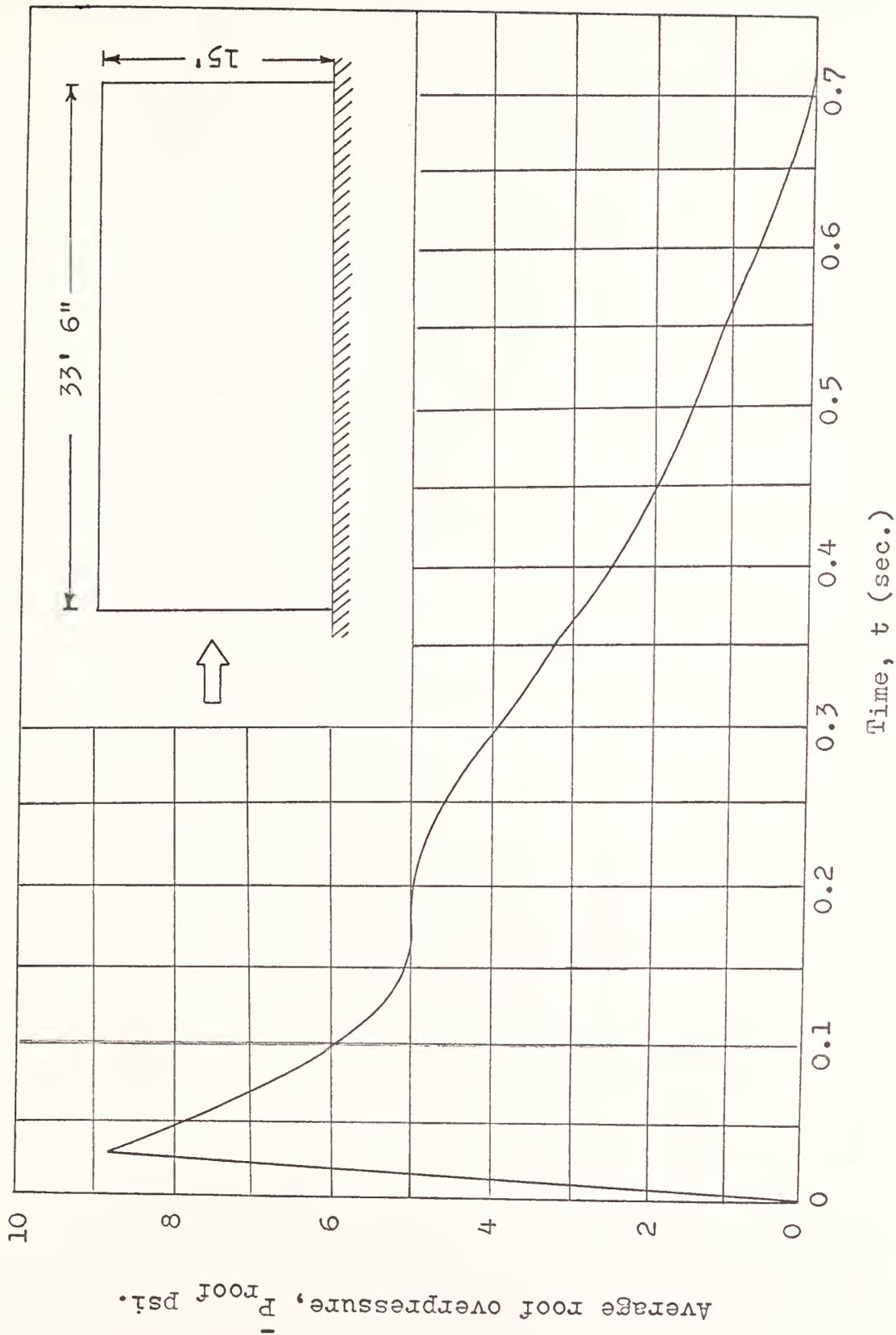


FIG. 20. AVERAGE ROOF OVERPRESSURE VERSUS TIME.

Design of Wall Slab

The wall is designed as a one-way reinforced concrete slab spanning from fixed support at foundation to a pinned support at the roof slab.¹⁰ The slab is permitted to deform in plastic region by developing plastic hinges at the foundation and near mid-height. The span of slab is equal to clear height of wall.

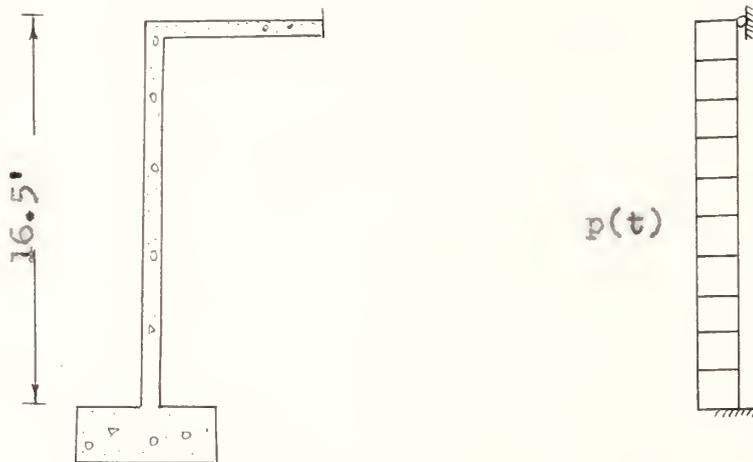


FIG. 21. DESIGN OF WALL SLAB.

(a) Design Loading. Design loads as idealized from the computed loading shown in Fig. 17 is defined by:

B, peak value of load is equal to:

$$\frac{(25.3) 144 (16.5)}{1000}$$

= 60 kip

¹⁰"Single Story Frame Buildings", U. S. Army Corps of Engineers, U. S. Government Printing Office, Jan. 1958.

Duration of external load, $T = 0.062$ sec.

Pulse Loading, H is given by, $H = B T/2$

$$= 1.86 \text{ kip/sec.}$$

(b) The Dynamic Design Factors. The dynamic design factors for the slab shown in Fig. 21 are obtained from Appendix B, Fig. 5, as follows:

(i) Elastic range:

$$K_L = 0.58, \quad K_M = 0.45, \quad K_{LM} = 0.78$$

$$R_{1m} = \frac{8 M_{Ps}}{L}, \quad K_1 = \frac{185 EI}{L^3} \quad (53)$$

$$V_1 = 0.26R + 0.12 P, \quad V_2 = 0.43R + 0.19 P$$

(ii) Elasto-plastic range:

$$K_L = 0.64, \quad K_M = 0.50$$

$$R_m = \frac{4}{L} (M_{Ps} + 2 M_{Pm}), \quad K_{ep} = \frac{384 EI}{5L^3} \quad (54)$$

$$V = 0.39 R + 0.11 P$$

(iii) Plastic range:

$$K_L = 0.50, \quad K_M = 0.33, \quad K_{LM} = 0.78$$

$$R_m = \frac{4}{L} (4 M_{Ps} + 2 M_{Pm}) \quad (55)$$

$$V = 0.38 R + 0.12 P$$

(iv) Average values of the dynamic design factors in elasto-plastic and plastic range are given as follows:

$$K_L = \frac{0.64 + 0.50}{2}$$

$$= 0.57$$

$$K_M = \frac{0.50 + 0.33}{2}$$

$$= 0.42$$

(56)

$$R_M = \frac{4}{L} (M_{Ps} + 2 M_{Pm})$$

$$K_E = \frac{160 RI}{L^3}$$

(c) First Trial - Actual Properties. Assuming the ratio of tensile reinforcement to concrete area, $p = 0.015$.

Assuming $\mu = 5$

where $\mu =$ Design load ductility reduction factor and
 $\mu =$ Ductility factor.

Assuming C_R , the ratio of maximum resistance to the peak load be 0.7

$$R_m = C_R B$$

$$= 0.75 \times 60$$

$$= 45 \text{ kips}$$

The plastic moment resistance of beams with tension steel only is given by Equation 4.16, Ref. 5,

$$M_P = A_s f_{dy} d \left(1 - \frac{p f_{dy}}{1.7 f'_{dc}} \right) \quad (57)$$

By substituting the values of f_{dy} , A_s , and f'_{dc} in Equation 57, yields:

$$M_P = 0.688 d^2$$

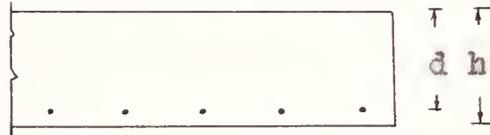
Assuming $M_p = M_{ps} = M_{pm}$, Equation (55) gives:

$$R_m = \frac{12 M_p}{L}$$

Substituting the values of M_p , L and R_m gives:

$$d = 9.5 \text{ in}$$

Trying $h = 10.75 \text{ in}$ and $d = 9.5 \text{ in}$



For a value of $d = 9.5 \text{ in}$, Equation (57) yields:

$$M_p = 61.92 \text{ kip ft}$$

Gross moment of inertia of the section is given by:

$$I_g = \frac{bh^3}{12}$$

$$= 1160 \text{ in}^4$$

Net moment of inertia is given by:

$$I_{net} = bd^3 \left[\frac{k^3}{3} + np(1-k)^2 \right]$$

where

k_d is depth of neutral axis.

Value of k is equal to ¹¹ 0.42

therefore,

$$I_{net} = 756 \text{ in}^4$$

Average moment of inertia of section is given by:

$$I_a = \frac{1}{2} (I_g + I_{net})$$

$$= 908 \text{ in}^4$$

¹¹"Reinforced Concrete Design Hand Book", American Concrete Institute, Second edition, p. 54.

By Equation (56)

$$\begin{aligned} K_E &= \frac{160 EI}{L^3} \\ &= \frac{160 (3 \times 10^6) 908}{(16.5)^3 (144)} \\ &= 680 \text{ kip/ft} \end{aligned}$$

The elastic deflection Y_e is given by:

$$\begin{aligned} Y_e &= \frac{R_m}{K_E} \\ &= 0.0655 \end{aligned}$$

The maximum allowable displacement, y_m is given by:

$$\begin{aligned} y_m &= \alpha \beta Y_e \\ &= 0.338 \text{ ft.} \end{aligned}$$

Weight of the section is equal to $\frac{10.75 (150)(16.5)}{12 (1000)}$

or

$$2.2 \text{ kips}$$

The mass of the section is given by:

$$\begin{aligned} m &= \frac{2.2}{32.2} \\ &= 0.0690 \frac{\text{kip} \cdot \text{sec}^2}{\text{ft}} \end{aligned}$$

(d) First Trial - Equivalent System Properties. The equivalent properties of the system are as follows:

$$R_{ne} = K_L R_m = 25.6 \text{ kip}$$

$$H_e = K_L H = 1.06 \text{ kip/sec}$$

$$M_e = K_M M = 0.029 \frac{\text{kip} \cdot \text{sec}^2}{\text{ft}}$$

Equation (13) gives:

$$W_{Pe} = \frac{(H_e)^2}{2 m_e}$$

$$= 19.30 \text{ kip ft}$$

The natural period of oscillation is given by the Equation

14(a) :

$$T = 2 \sqrt{\frac{m_e}{K_E}}$$

$$= 0.056 \text{ secs.}$$

(e) First Trial - Work Done Versus Energy Absorption Capacity.

Equation 14 (a) gives:

$$C_T = T/T_h = \frac{0.062}{0.056}$$

$$= 1.11$$

and

$$C_R = \frac{R_m}{B} = \frac{45}{60}$$

$$= 0.75$$

From Fig. 7, Appendix B, value of $\frac{t_m}{T}$ is obtained as:

$$\frac{t_m}{T} = 0.71 \text{ secs.}$$

The time for maximum displacement, t_m is:

$$t_m = 0.71 (T)$$

$$= 0.044 \text{ secs.}$$

The work done ratio, C_w is given by Fig. 6, Appendix B as:

$$C_w = 0.32$$

W_m , the maximum work done on the equivalent system is:

$$\begin{aligned}
 W_m &= C_W W_P \\
 &= 6.16 \text{ ft kips}
 \end{aligned}$$

The energy absorbed by the equivalent system is given by Equation 6.18, Reference 5,

$$\begin{aligned}
 E &= R_{me} (Y_m - 0.5 Y_e) \\
 &= 7.8 \text{ ft kips}
 \end{aligned}$$

Since, E is greater than W, the selected proportions are satisfactory as a preliminary design.⁵

(f) Preliminary Design for Bond Stress. At the fixed-end of the wall cover requirements results in a smaller value of $d = 8.75$ in than at mid span $d = 9.75$ in.

At the fixed end:

The estimated maximum dynamic reaction is given by¹⁰

$$\begin{aligned}
 V_{\max} &= 0.5 R_m \\
 &= 22.5 \text{ kips}
 \end{aligned}$$

The value of the allowable bond stress, u , is given by

$$\begin{aligned}
 u &= 0.15 f'_c \\
 &= 450 \text{ psi}
 \end{aligned}$$

The bond stress, u is given by

$$u = \frac{V}{\sum o_j d} \quad (58)$$

where

V is the shear force at the section considered, $\sum o$ is the total perimeter of the steel bars and j is the lever arm.

Equation (58) yields:

$$\begin{aligned}\sum o &= \frac{V}{u_j d} \\ &= \frac{22500}{450(7/8)875} \\ &= 6.6 \text{ in.}\end{aligned}$$

$$\begin{aligned}A_s &= pbd \\ &= 0.15 (12)(8.75) \\ &= 1.68 \text{ in.}^2\end{aligned}$$

Trying No. 8 round bars at 5 in. spacing gives

$$A_s = 1.90 \text{ in.}^2 \text{ and } \sum o = 7.5 \text{ in.}$$

$$\begin{aligned}\text{Therefore, } p &= \frac{A_s}{bd} \\ &= 0.0182\end{aligned}$$

At pinned end.

The estimated maximum dynamic reaction, V_{\max} , is given by¹⁰

$$\begin{aligned}V_{\max} &= \frac{1}{3} R_m \\ &= 15.0 \text{ kips}\end{aligned}$$

Equation (58) yields:

$$\begin{aligned}\sum o &= 4.0 \text{ in.} \\ A_s &= pbd \\ &= 0.15(12)(9.5) \\ &= 1.82 \text{ in.}^2\end{aligned}$$

Trying No. 8 round bars at 5 in. spacing gives

$$A_s = 1.90 \text{ in.}^2 \text{ and } \sum o = 7.5 \text{ in.}$$

$$\text{or } p = 0.0167$$

(g) Determination of Maximum Deflection and Dynamic Reaction by Numerical Integration Method.

Substituting the values of A_s , b , and d in Equation (57) yields:

$$M_{FM} = 67.6 \text{ kip ft.}$$

and

$$M_{PS} = 62.4 \text{ kip ft.}$$

Net moment of inertia is given by

$$\begin{aligned} I_t &= bd^3 \left[\frac{k^3}{3} + np(1-k)^2 \right] \\ &= 12 \times (9.5)^3 \left[\frac{0.43^3}{3} + 0.167(1-0.43)^2 \right] \\ &= 870 \text{ in.}^4 \end{aligned}$$

Therefore

$$I_a = 0.5 (I_g + I_t) = 1015 \text{ in.}^4$$

Equations (54) and (55) yield the dynamic design factors as follows:

Elastic range:

$$\begin{aligned} R_{1m} &= 8 \frac{M_{PS}}{I} \\ &= 30.2 \text{ kips} \end{aligned}$$

$$\begin{aligned} K_1 &= \frac{185 EI}{L^3} \\ &= 870 \text{ kip/ft} \end{aligned}$$

$$\begin{aligned} Y_E &= \frac{R_{1m}}{K_1} \\ &= 0.0347 \text{ ft} \end{aligned}$$

Elasto-plastic range:

$$R_m = 4 (M_{Ps} + 2 P_M)$$

$$= 48 \text{ kips}$$

$$K_{ep} = \frac{384 EI}{5 L^3}$$

$$= 360 \text{ kip/ft}$$

$$Y_{ep} = Y_e + \frac{R_m - R_{lm}}{K_{ep}}$$

$$= 0.084 \text{ ft.}$$

Plastic range:

$$R_m = 48.0 \text{ kips}$$

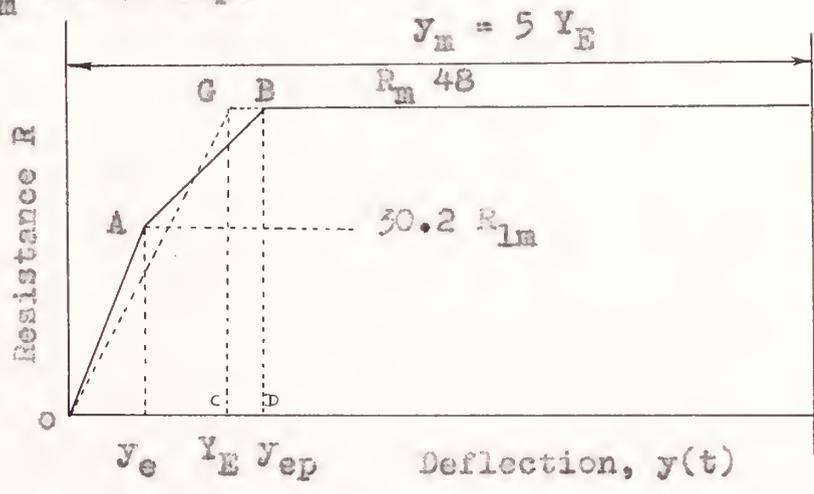


FIG. 22. RESISTANCE FUNCTION FOR A 10-3/4" SLAB.

As plastic deformation is permitted, the plastic K_E is determined by limiting the resistance to the computed maximum value, R_m and equating the areas under resistance deflection curve.

$$\begin{aligned} \text{Area under OABD} &= \frac{1}{2} \times 30.2 \times .0347 + \frac{(30.2 + 48)}{2} \\ &\quad \times (0.084 - 0.0347) \\ &= 2.435 \end{aligned}$$

$$\text{Area OGBD} = \frac{1}{2} \times 48 \times OC + (0.084 - OC) \times 48$$

$$\text{Therefore, } OC = Y_E = 0.0662 \text{ ft}$$

$$K_E = \frac{R_M}{Y_E} = \frac{48}{0.0662} = 725 \text{ kip/ft}$$

By Equation 14 (a) T_n is equal to

$$\begin{aligned} 2 \sqrt{\frac{K_{LM} (m)}{K_E}} \\ = 0.0544 \text{ sec.} \end{aligned}$$

The maximum allowable deflection, y_m , is given by:

$$\begin{aligned} y_m &= \alpha \beta y_e \\ &= 5 (0.0662) \\ &= 0.331 \text{ ft.} \end{aligned}$$

The basic equation for numerical integration is given by Equation (45):

$$Y_{n+1} = y_n (\Delta t)^2 + 2 y_n - y_{n-1}$$

where

$$Y_n (\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{LM}(m)}$$

$$\text{Assuming the time interval} = \frac{T}{10}$$

$$= 0.005 \text{ secs.}$$

$$Y_n (\Delta t)^2 = \frac{(P_n - R_n) 25 \times 10^{-6}}{0.78 \times 0.069}$$

$$= 4.65(10^{-4})(P_n - R_n) \text{ ft. elastic range}$$

$$Y_n (\Delta t)^2 = \frac{(P_n - R_n) 25 \times 10^{-6}}{0.78 (0.069)}$$

$$= 4.65 (10)^{-4} (P_n - R_n) \text{ ft. for elasto-plastic range}$$

$$Y_n (\Delta t)^2 = \frac{(P_n - R_n) (25 \times 10^{-6})}{0.66 (0.069)}$$

$$= 5.5 10^{-4} (P_n - R_n) \text{ ft. for plastic range}$$

Table 4. Determination of maximum deflection and dynamic reactions.

| t secs. | P _n kips | R _n kips | P _n -R _n kips | Y _n (t) ² ft | Y _n ft | V ₁ kips | V ₂ kips |
|------------|------------------------|------------------------|--|---------------------------------------|----------------------|------------------------|------------------------|
| 0 | 60.0 | 0 | 30.0 | 0.01359 | 0 | 7.20 | 11.40 |
| 0.005 | 55.0 | 11.8 | 43.2 | 0.02020 | 0.01359 | 9.66 | 15.60 |
| 0.010 | 50.4 | 34.3 | 16.1 | 0.00750 | 0.04738 | 18.90 | 18.90 |
| 0.015 | 47.5 | 48 | -0.5 | -0.00023 | 0.08867 | 23.70 | 23.70 |
| 0.020 | 40.6 | 48 | -7.4 | -0.00406 | 0.12973 | 23.00 | 23.00 |
| 0.025 | 35.9 | 48 | -11.1 | -0.00610 | 0.16673 | 22.40 | 22.40 |
| 0.030 | 31.0 | 48 | -17.0 | -0.00935 | 0.19763 | 21.82 | 21.82 |
| 0.035 | 27.2 | 48 | -20.8 | -0.01142 | 0.21918 | 21.36 | 21.36 |
| 0.040 | 21.4 | 48 | -26.6 | -0.01460 | 0.22931 | 20.67 | 20.67 |
| 0.045 | 16.6 | 48 | -31.4 | -0.01722 | 0.21475 | 20.0 | 20.0 |
| 0.050 | 11.7 | 48 | -36.3 | -0.02010 | 0.18297 | 19.50 | 19.50 |

$$(Y_n)_{\max} = 0.22931 = 0.23 \text{ ft.}$$

Sample calculations for Table 4:

for $t = 0.025$ and $n = 5$, Y_5 is given by:

$$\begin{aligned} Y_5 &= Y_4 (\Delta t)^2 + 2 Y_4 - Y_3 \\ &= -(0.00406) + 2 (0.12973) - 0.08867 \\ &= 0.16673 \text{ ft.} \end{aligned}$$

Since $Y_5 = 0.16673 > 0.0662$, $R_n = 48$

The dynamic reaction is given by Equation (55) and is equal to

$$\begin{aligned} V &= 0.38 R + 0.12 P \\ &= 0.38 (48) + 0.12 (35.9) = 22.40 \text{ kip} \end{aligned}$$

By Table 4,

Maximum deflection = 0.2293 ft. which is less than allowable

$$Y_m = 0.331 \text{ ft.}$$

(h) Shear and Bond Strength. By Table 4, $V_{\max} = 23.70$ kip

(i) for no shear reinforcement, allowable shear strength is given by

$$\begin{aligned} v_y^1 &= 0.04 f'_c + 5000 p \\ &= 0.04 (3000) + 5000 (0.0167) \\ &= 120 + 83.5 = 203.5 \text{ psi} \end{aligned}$$

$$\text{Shear stress } v = \frac{V}{b_j d} = \frac{8(23.700)}{7(12)(8.75)} = 258.0 \text{ psi}$$

As the shear stress is more than allowable, shear reinforcement is required for $(258 - 203.5) = 54.5$ psi.

Contribution of shear reinforcement to allowable shear stress is given by:

$$r = \frac{f}{f_y} = \frac{54.5}{40,000} = 0.00136$$

Trying one No. 3 round bar, $A_s = 0.11 \text{ in.}^2$

$$r = \frac{A_s}{bs} = \frac{0.11}{10(s)} = 0.00136$$

$s = 8.10 \text{ in.}$, using #3 bars at 8 in. spacing

At the top of the wall:

$$V_{\max} = 23.70 \text{ kip}$$

$$V_y = 203.50 \text{ psi}$$

$$v = \frac{V}{b_j d} = \frac{23700}{7/8(12)9.5}$$

$$= 248 \text{ psi}$$

Using No. 3 round bar at 8 in. c/c

Summary: Slab 10.75 in. thick, $p = 0.0163$ shear reinforcement for bottom 5 ft. and top 5 ft. 1 No. 3 at 8 in. spacing.

Design of Roof Slab

The roof slab is designed as a one-way reinforced concrete slab. The slab is permitted to deform into plastic range developing plastic hinges at the supports. The critical roof-slab loading is the incident overpressure versus time curve shown in the Fig. 16. This loading results from the blast wave moving parallel to the long axes of the building.

The blast loading curve of the Fig. 16 is idealized to a triangular load of 10 psi peak value and the duration of 0.38 secs. Following the dynamic design procedure, the thickness of slab was determined to be 7.25 in. with $p = 0.016$. Numerical

integration method was used to check the design and to find the dynamic reactions.

Preliminary Column Design

In determining the spring constant, the column height is taken equal to 14.75 ft. from center line of girder to the top of the footing. The resistance computations are based on the clear height of 13 feet.

(a) Design Loading. In computing the total concentrated load on the frame, it is assumed that the wall slab transmit the blast equally to the roof slab and the foundations.

The design load as idealized from the computed loading as shown in Fig. 17 is defined by:

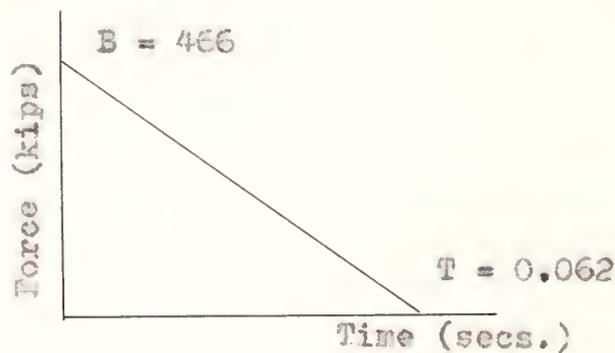


FIG. 23. IDEALIZED LOAD TIME CURVE.

$$B = \frac{(25.3)(144)(16)(16/2)}{1000}$$

$$= 466 \text{ kips}$$

$$T = 0.062 \text{ secs.}$$

$$H = \frac{BT}{2}$$

$$= 14.6 \text{ kip sec}$$

(b) Mass Computation.

$$\begin{aligned} \text{Weight of the roof slab} &= \left(\frac{7.25 \times 150}{12} \right) \left(\frac{16 \times 33.5}{1000} \right) \\ &= 48.8 \text{ kips} \end{aligned}$$

$$\begin{aligned} \text{Weight of the girder (assumed)} &= \frac{18.(32) 32.(150)}{12(12) 1000} \\ &= 19.2 \text{ kips} \end{aligned}$$

$$\begin{aligned} \text{Weight of the three columns (assumed)} &= \frac{12(24) 13(150) 3}{12(12) 1000} \\ &= 11.7 \text{ kips} \end{aligned}$$

$$\begin{aligned} \text{Weight of the two wall slabs} &= \frac{95 (15) 16 (150) 2}{12 (1000)} \\ &= 57.0 \text{ kip} \end{aligned}$$

The mass, m , of the single degree freedom system is given by¹⁰
 m , is equal to the total mass of (roof + the girder) + 1/3 mass
of the (columns plus wall slabs),

therefore

$$m = 2.83 \frac{\text{kip sec}^2}{\text{ft}}$$

(c) First Trial Actual Properties.

Assuming $p = 0.015$,

$$= 6,$$

$$\text{and } C_R = 0.50$$

$$R_M = C_R B$$

$$= 233 \text{ kips}$$

The maximum design moment, M_D , under axial load is given
by Equation (7.15) Reference 10.

$$M_D = \frac{R_m h}{2n} \quad (59)$$

where

n = number of columns

Equation (59) yields:

$$M_D = 500 \text{ kip ft.}$$

Average roof overpressure, from Fig. 20 = 6.5 psi.

Average blast load per column, due to roof overpressure is equal to

$$\frac{33.50 (16) 6.5 (144)}{3 (1000)}$$

$$= 167 \text{ kips.}$$

Dead load per column is given by

$$1/3 (48.8 + 19.2)$$

$$= 23 \text{ kips}$$

$$\text{Column design load } P_D = 167 + 23$$

$$= 190 \text{ kips}$$

The moment for failure in a symmetrically reinforced column is given by the Equation (4.32) Reference 9.

$$M_D = A_s f_{dy} d' + P_D \left(\frac{t}{2} - \frac{P_D}{1.7 b f'_{dc}} \right) \quad (60)$$

where

P_D = load on the column

t = thickness of the column

d' = distance between centroids of the compression and the tensile steel

Assuming $b = 12$ in., $p = 0.01$ and substituting the values of P_D , M_D , f'_{dc} , and f_{dy} in the Equation (60), and solving for t yields:

$$t = 23.3 \text{ in.}$$

Trying $t = 24$ in.

Equation (60), for $t = 24$ in. yields:

$$M_D = 516 \text{ kip ft.}$$

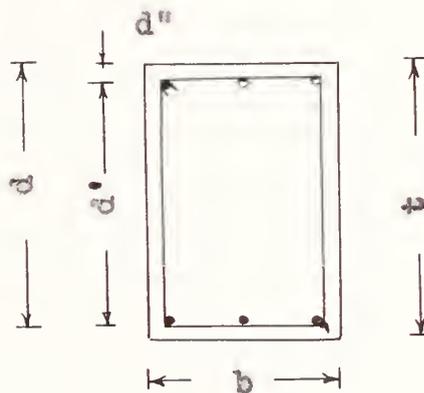


FIG. 24. SECTION OF A COLUMN

$$\text{For } \frac{d''}{d} = \frac{2.25}{21.75} = 0.103,$$

$$\text{and } p = p' = 0.015$$

k is given ' equal to 11 0.30

Net moment of inertia of a doubly reinforced beam is given

by:

$$I_t = bd^3 \left[\frac{k^3}{3} + (n - 1) \left[k - \frac{d''}{d} \right]^2 + np (1 - k)^2 \right] \quad (61)$$

Equation (61) yields:

$$I_t = 10900 \text{ in.}^4$$

Gross moment of inertia is given by

$$I_g = \frac{bt^3}{12}$$

$$= 13824 \text{ in.}^4$$

Average moment of inertia is obtained as

$$I_a = 12562 \text{ in.}^4$$

The spring constant of the column is given by Equation (7.8)

Reference 10,

$$k = \frac{n \cdot 12 \cdot EI \cdot a}{h^3} \quad (62)$$

Equation (62) yields:

$$k = 2880 \text{ kip/ft.}$$

For $M_D = 516 \text{ ft kip}$, Equation (59) yields:

$$R_x = 238 \text{ kip/ft.}$$

The elastic deflection, x_e is given by,

$$x_e = \frac{R_x}{k}$$

$$= 0.083 \text{ ft.}$$

and the maximum allowable deflection is given by:

$$x_m = \lambda \beta x_e$$

$$= 0.498 \text{ ft.}$$

Natural frequency is given by:

$$T = 2 \sqrt{\frac{m}{k}}$$

$$= 0.197 \text{ secs.}$$

(d) First Trial - Work Done Versus Energy AbsorptionCapacity.

The ratios $\frac{T}{T_n}$ and C_R are given by

$$\frac{T}{T_n} = \frac{0.062}{0.197}$$

$$= 0.314,$$

$$\text{and } C_R = \frac{R_m}{E}$$

$$= 0.51$$

For this ratio, value of $\frac{t_m}{T}$ is obtained from Fig. 7, Appendix B, and is equal to

$$\frac{t_m}{T} = 1.04$$

or

$$t_m = 0.0645 \text{ sec.}$$

The original assumed load-time curve shown in Fig. 23 is revised to obtain a closer approximation up to the time t_m . The impulse up to $t = 0.10$ is $H = 1.167$ (from Fig. 17.)

Therefore

$$T = \frac{2H}{B}$$

$$= \frac{2(1.167)}{25.3}$$

$$= 0.0923 \text{ sec.}$$

Therefore

$$\frac{T}{T_n} = \frac{0.0923}{0.197}$$

$$= 0.47$$

For $\frac{T}{T_n} = 0.47$ and $C_R = 0.51$,

$$\frac{t_m}{T} = 1.025 \text{ (from Fig. 7, Appendix B)}$$

or

$$t_m = 0.095 \text{ secs., which is O.K.}$$

The work done ratio, C_W , is obtained from the Fig. 6, Appendix B and is equal to

$$C_W = 0.7$$

W_p is given by

$$W_p = \frac{H^2}{2m}$$

$$= 86.0 \text{ ft kips}$$

The energy absorbed by the equivalent system is given by the Equation 6.18, Reference 5 as follows:

$$E = R_m (x_m - 0.5 x_e)$$

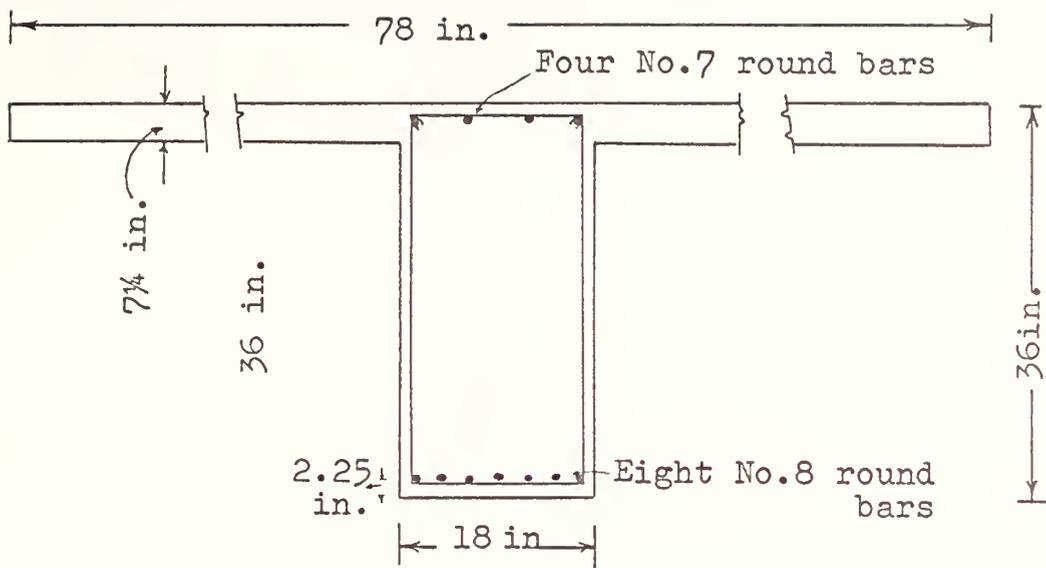
$$= 107 \text{ ft kips}$$

$E > W$, hence design is satisfactory as a preliminary design.

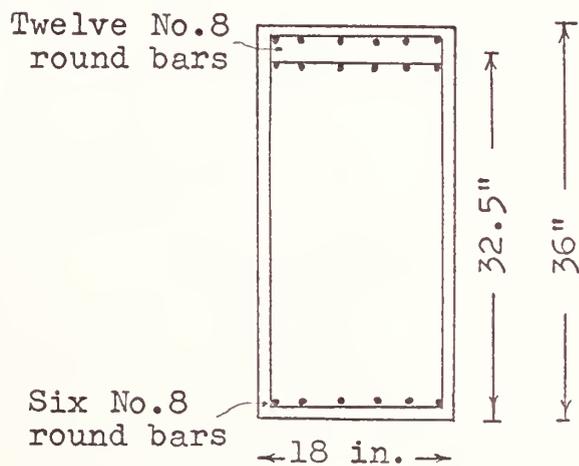
The design was also found to be adequate after checking by the numerical integration method.

Design of Roof Girder

The frame of this building consists of three rectangular columns supporting a rectangular girder which forms a tee beam with the roof slab. The roof girder is designed to resist the



(a) Tee-section at mid-span.



(b) Section at support.

FIG. 25. SECTIONS OF THE GIRDER AT THE MID-SPAN AND AT THE SUPPORT.

combined vertical loads on the roof and lateral loads on the frame. Although the other structural elements of this building are permitted to deflect plastically, the girder is designed to behave elastically so that proper restraint is maintained for the column throughout the deflection of the frame.

The design bending moment of the girder is the sum¹¹ of moments due to: (a) the roof slab dynamic reactions, (b) the static loads, and (c) the frame action. The moment due to frame action is equal to one-half the column plastic moment.

The girder is designed as a tee-beam in the region of positive moment. In the region of negative moment near the interior support, the girder is designed as rectangular section.

The sections (Fig. 25) are found to be adequate for this building after checking them by the numerical integration method.

CONCLUSIONS

Structures designed to resist the blast loads are subjected to completely different types of loads than those considered in conventional designs. Due to the large magnitude and dynamic character of loading, the designs are based on dynamic analysis. In order to simplify the analysis, a given structure is replaced by a dynamically equivalent system.

The design example presented in this study is analyzed by the numerical integration method. Due to economical reasons, the members of the structure are allowed to deflect plastically

and the dynamic yield stresses are used for design. It is felt that concrete members of greater mass are more suitable for blast resistant construction than steel members of smaller masses due to the inertia of members.

ACKNOWLEDGMENTS

The author wishes to express his gratitude to his major professor, Dr. K. N. Jabbour, for his help and guidance throughout this study.

APPENDIX A - BIBLIOGRAPHY

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APPENDIX B - DESIGN CURVES

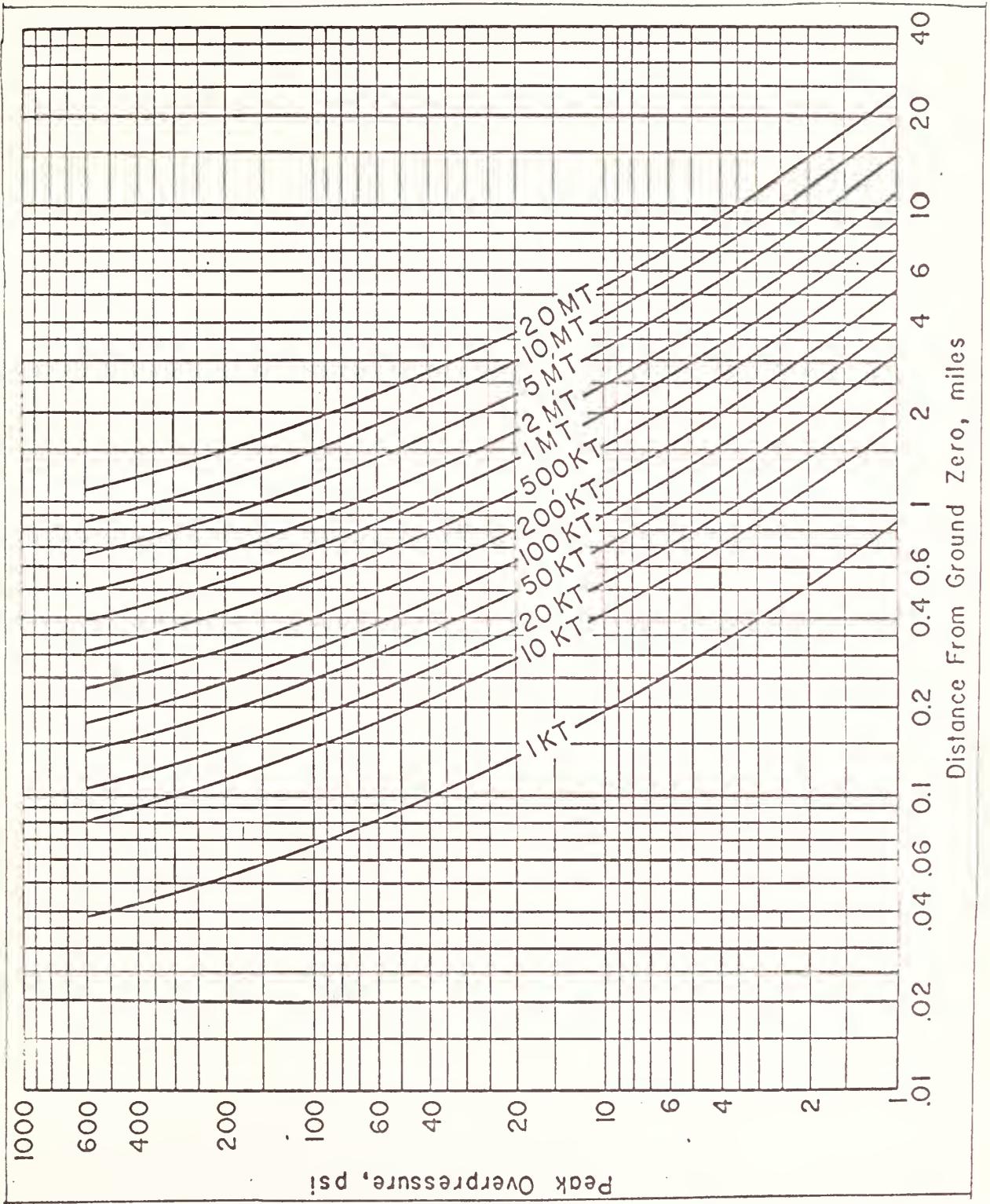


FIG. 1. PEAK OVERPRESSURE VERSUS DISTANCE FROM GROUND ZERO. (P. 18, Ref. 7.)

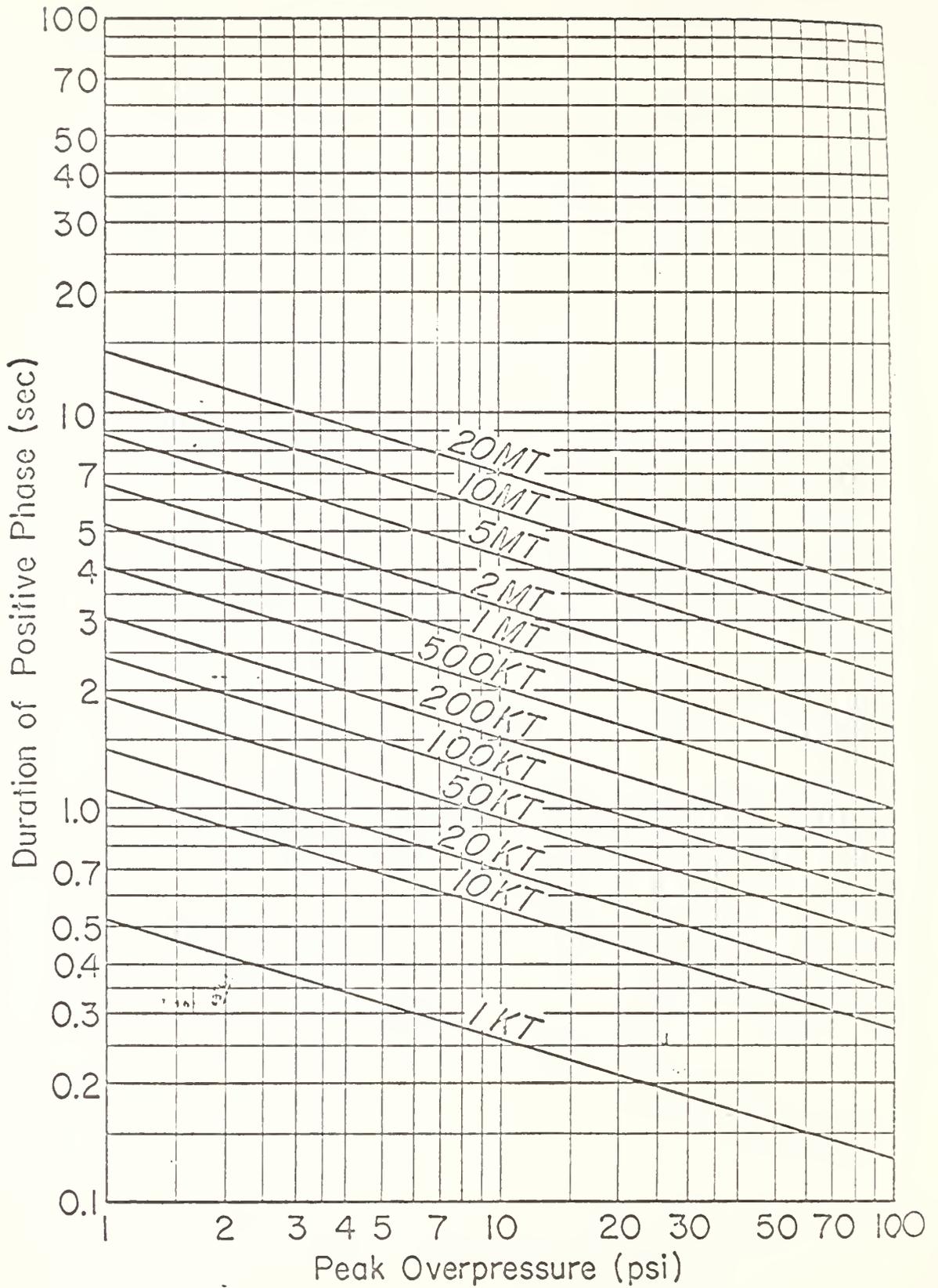


FIG. 2. DURATION OF POSITIVE PHASE. (P. 14, Ref. 7.)

| $(t - t_d)t_0$ | a/a_0 | $(t - t_d)t_0$ | a/a_0 | $(t - t_d)t_0$ | a/a_0 |
|----------------|---------|----------------|---------|----------------|---------|
| 0.00 | 1.000 | 0.30 | 0.245 | 0.60 | 0.049 |
| 0.01 | 0.956 | 0.31 | 0.233 | 0.61 | 0.046 |
| 0.02 | 0.914 | 0.32 | 0.222 | 0.62 | 0.043 |
| 0.03 | 0.873 | 0.33 | 0.211 | 0.63 | 0.041 |
| 0.04 | 0.835 | 0.34 | 0.201 | 0.64 | 0.038 |
| 0.05 | 0.798 | 0.35 | 0.191 | 0.65 | 0.036 |
| 0.06 | 0.762 | 0.36 | 0.182 | 0.66 | 0.034 |
| 0.07 | 0.728 | 0.37 | 0.173 | 0.67 | 0.032 |
| 0.08 | 0.695 | 0.38 | 0.164 | 0.68 | 0.030 |
| 0.09 | 0.664 | 0.39 | 0.156 | 0.69 | 0.028 |
| 0.10 | 0.634 | 0.40 | 0.148 | 0.70 | 0.026 |
| 0.11 | 0.606 | 0.41 | 0.141 | 0.71 | 0.024 |
| 0.12 | 0.578 | 0.42 | 0.133 | 0.72 | 0.022 |
| 0.13 | 0.552 | 0.43 | 0.127 | 0.73 | 0.021 |
| 0.14 | 0.527 | 0.44 | 0.120 | 0.74 | 0.020 |
| 0.15 | 0.503 | 0.45 | 0.114 | 0.75 | 0.018 |
| 0.16 | 0.480 | 0.46 | 0.108 | 0.76 | 0.017 |
| 0.17 | 0.458 | 0.47 | 0.102 | 0.77 | 0.016 |
| 0.18 | 0.437 | 0.48 | 0.097 | 0.78 | 0.014 |
| 0.19 | 0.417 | 0.49 | 0.092 | 0.79 | 0.013 |
| 0.20 | 0.397 | 0.50 | 0.087 | 0.80 | 0.012 |
| 0.21 | 0.379 | 0.51 | 0.082 | 0.81 | 0.011 |
| 0.22 | 0.361 | 0.52 | 0.078 | 0.82 | 0.010 |
| 0.23 | 0.344 | 0.53 | 0.074 | 0.83 | 0.009 |
| 0.24 | 0.328 | 0.54 | 0.070 | 0.84 | 0.008 |
| 0.25 | 0.313 | 0.55 | 0.066 | 0.85 | 0.008 |
| 0.26 | 0.298 | 0.56 | 0.062 | 0.86 | 0.007 |
| 0.27 | 0.284 | 0.57 | 0.059 | 0.87 | 0.006 |
| 0.28 | 0.270 | 0.58 | 0.055 | 0.88 | 0.006 |
| 0.29 | 0.257 | 0.59 | 0.052 | 0.89 | 0.005 |
| | | | | 0.90 | 0.004 |
| | | | | 0.91 | 0.004 |
| | | | | 0.92 | 0.003 |
| | | | | 0.93 | 0.003 |
| | | | | 0.94 | 0.002 |
| | | | | 0.95 | 0.002 |
| | | | | 0.96 | 0.001 |
| | | | | 0.97 | 0.001 |
| | | | | 0.98 | 0.001 |
| | | | | 0.99 | 0.000 |
| | | | | 1.00 | 0.000 |

$$\frac{a}{a_0} = \left[1 - \left(\frac{t - t_d}{t_0} \right) \right] e^{-3.5(t - t_d)/t_0}$$

FIG. 3. DYNAMIC PRESSURE RATIO VERSUS TIME RATIO.
(P. 35, Ref. 7.)

| $(t - t_d)/t_o$ | P_s/P_{so} | $(t - t_d)/t_o$ | P_s/P_{so} | $(t - t_d)/t_o$ | P_s/P_{so} |
|-----------------|--------------|-----------------|--------------|-----------------|--------------|
| 0.00 | 1.000 | 0.30 | 0.519 | 0.60 | 0.220 |
| 0.01 | 0.980 | 0.31 | 0.506 | 0.61 | 0.212 |
| 0.02 | 0.961 | 0.32 | 0.494 | 0.62 | 0.204 |
| 0.03 | 0.941 | 0.33 | 0.482 | 0.63 | 0.197 |
| 0.04 | 0.922 | 0.34 | 0.470 | 0.64 | 0.190 |
| 0.05 | 0.904 | 0.35 | 0.458 | 0.65 | 0.183 |
| 0.06 | 0.885 | 0.36 | 0.447 | 0.66 | 0.176 |
| 0.07 | 0.867 | 0.37 | 0.435 | 0.67 | 0.169 |
| 0.08 | 0.849 | 0.38 | 0.424 | 0.68 | 0.162 |
| 0.09 | 0.832 | 0.39 | 0.413 | 0.69 | 0.155 |
| 0.10 | 0.814 | 0.40 | 0.402 | 0.70 | 0.149 |
| 0.11 | 0.797 | 0.41 | 0.392 | 0.71 | 0.143 |
| 0.12 | 0.780 | 0.42 | 0.381 | 0.72 | 0.136 |
| 0.13 | 0.764 | 0.43 | 0.371 | 0.73 | 0.130 |
| 0.14 | 0.748 | 0.44 | 0.361 | 0.74 | 0.124 |
| 0.15 | 0.732 | 0.45 | 0.351 | 0.75 | 0.118 |
| 0.16 | 0.716 | 0.46 | 0.341 | 0.76 | 0.112 |
| 0.17 | 0.700 | 0.47 | 0.331 | 0.77 | 0.106 |
| 0.18 | 0.685 | 0.48 | 0.322 | 0.78 | 0.101 |
| 0.19 | 0.670 | 0.49 | 0.312 | 0.79 | 0.095 |
| 0.20 | 0.655 | 0.50 | 0.303 | 0.80 | 0.090 |
| 0.21 | 0.640 | 0.51 | 0.294 | 0.81 | 0.085 |
| 0.22 | 0.626 | 0.52 | 0.285 | 0.82 | 0.079 |
| 0.23 | 0.612 | 0.53 | 0.277 | 0.83 | 0.074 |
| 0.24 | 0.598 | 0.54 | 0.268 | 0.84 | 0.069 |
| 0.25 | 0.584 | 0.55 | 0.260 | 0.85 | 0.064 |
| 0.26 | 0.571 | 0.56 | 0.251 | 0.86 | 0.059 |
| 0.27 | 0.557 | 0.57 | 0.243 | 0.87 | 0.054 |
| 0.28 | 0.544 | 0.58 | 0.235 | 0.88 | 0.050 |
| 0.29 | 0.531 | 0.59 | 0.227 | 0.89 | 0.045 |
| | | | | 0.90 | 0.041 |
| | | | | 0.91 | 0.036 |
| | | | | 0.92 | 0.032 |
| | | | | 0.93 | 0.028 |
| | | | | 0.94 | 0.023 |
| | | | | 0.95 | 0.019 |
| | | | | 0.96 | 0.015 |
| | | | | 0.97 | 0.011 |
| | | | | 0.98 | 0.008 |
| | | | | 0.99 | 0.004 |
| | | | | 1.00 | 0.000 |

$$\frac{P_s}{P_{so}} = \left[1 - \frac{(t - t_d)}{t_o} \right] e^{-(t - t_d)/t_o}$$

FIG. 4. BLAST WAVE OVERPRESSURE RATIO VERSUS TIME RATIO.
(P. 31, Ref. 7.)

| DYNAMIC DESIGN FACTORS BEAMS AND ONE-WAY SLABS | | | | | | | | | | | |
|---|----------------|-------------------|--------------------|--------------|---------------------------|--------------|---------------------------------|----------------------|---------------------------------|------------------------------|--|
| Loading Diagram | Strain Range | Load Factor K_L | Mass Factor K_M | | Load-Mass Factor K_{LM} | | Maximum Resistance R_m | Spring Constant k | Effective Spring Constant k_E | | Dynamic Reaction V |
| | | | Concentrated Mass* | Uniform Mass | Concentrated Mass* | Uniform Mass | | | Elastic | Plastic | |
| | Elastic | 0.58 | | 0.45 | | 0.78 | $8M_{Ps}/L$ | $185EI/L^3$ | $\frac{153EI}{L^3}$ | | $V_1 = 0.26R + 0.12P$ $V_2 = 0.43R + 0.19P$ |
| | Elasto-Plastic | 0.64 | | 0.50 | | 0.78 | $\frac{4}{L}(M_{Ps} + 2M_{Pm})$ | $\frac{384EI}{5L^3}$ | $\frac{160EI}{L^3}$ | | $V_1 - V_2 = 0.39R + 0.11P$ |
| | Plastic | 0.50 | | 0.33 | | 0.66 | $\frac{4}{L}(M_{Ps} + 2M_{Pm})$ | 0 | $14.6M_F$ | $R_{mf} = \frac{6.3M_F}{L}$ | $V_1 - V_2 = 0.38R_m + 0.12P$ |
| | Elastic | 1.0 | 1.0 | 0.43 | 1.0 | 0.43 | $16M_{Ps}/3L$ | $107EI/L^3$ | $\frac{104EI}{L^3}$ | | $V_1 = 0.54R + 0.14P$ $V_2 = 0.25R + 0.07P$ |
| | Elasto-Plastic | 1 | 1.0 | 0.49 | 1.0 | 0.49 | $\frac{2}{L}(M_{Ps} + 2M_{Pm})$ | $48EI/L^3$ | $\frac{160EI}{L^3}$ | | $V_1 - V_2 = 0.78R - 0.28P$ |
| | Plastic | 1 | 1.0 | 0.33 | 1.0 | 0.33 | $\frac{2}{L}(M_{Ps} + 2M_{Pm})$ | 0 | $6.63M_F$ | $R_{mf} = \frac{6.63M_F}{L}$ | $V_1 - V_2 = 0.75R_m - 0.25P$ |
| | Elastic | 0.81 | 0.67 | 0.45 | 0.83 | 0.55 | $6M_{Ps}/L$ | $132EI/L^3$ | $\frac{117.5EI}{L^3}$ | | $V_1 = 0.17R + 0.17P$ $V_2 = 0.33R + 0.33P$ |
| | Elasto-Plastic | 0.87 | 0.76 | 0.52 | 0.87 | 0.60 | $\frac{2}{L}(M_{Ps} + 3M_{Pm})$ | $\frac{56EI}{L^3}$ | $\frac{9.52M_F}{L}$ | | $V_1 - V_2 = 0.62R - 0.12P$ |
| | Plastic | 1 | 1.0 | 0.56 | 1.0 | 0.56 | $\frac{2}{L}(M_{Ps} + 3M_{Pm})$ | | | | $V_1 = 0.56R - 0.25P$ $V_2 = 0.56R_m + 0.13P$ |

* Equal parts of the concentrated mass are lumped at each concentrated load.

FIG. 5. DYNAMIC DESIGN FACTORS, BEAMS AND ONE-WAY SLABS. (P. 12, Ref. 3.)

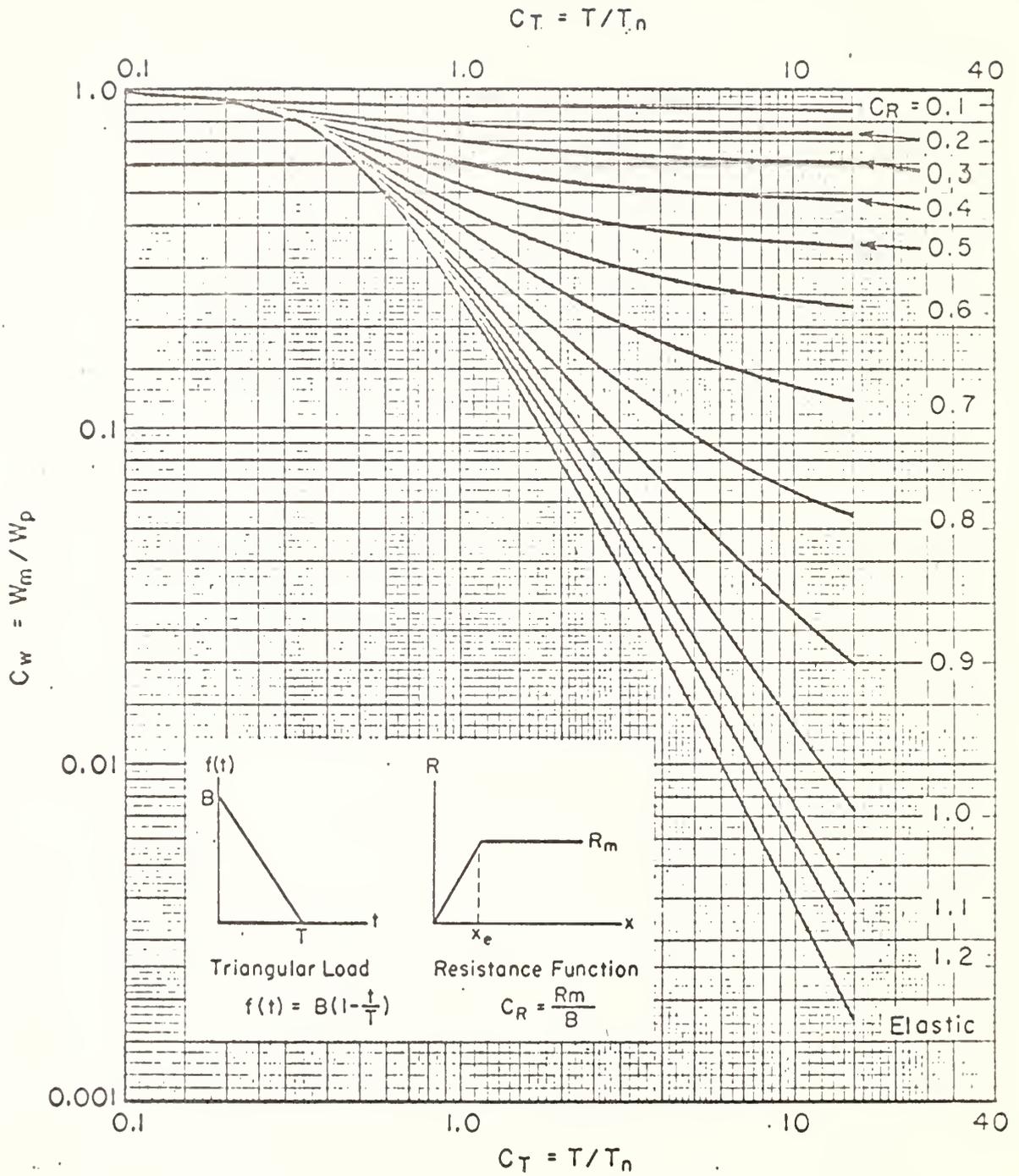


FIG. 6. WORK-DONE RATIOS FOR TRIANGULAR LOADS AND ONE-DEGREE SYSTEM. (P. 53, Ref. 8.)

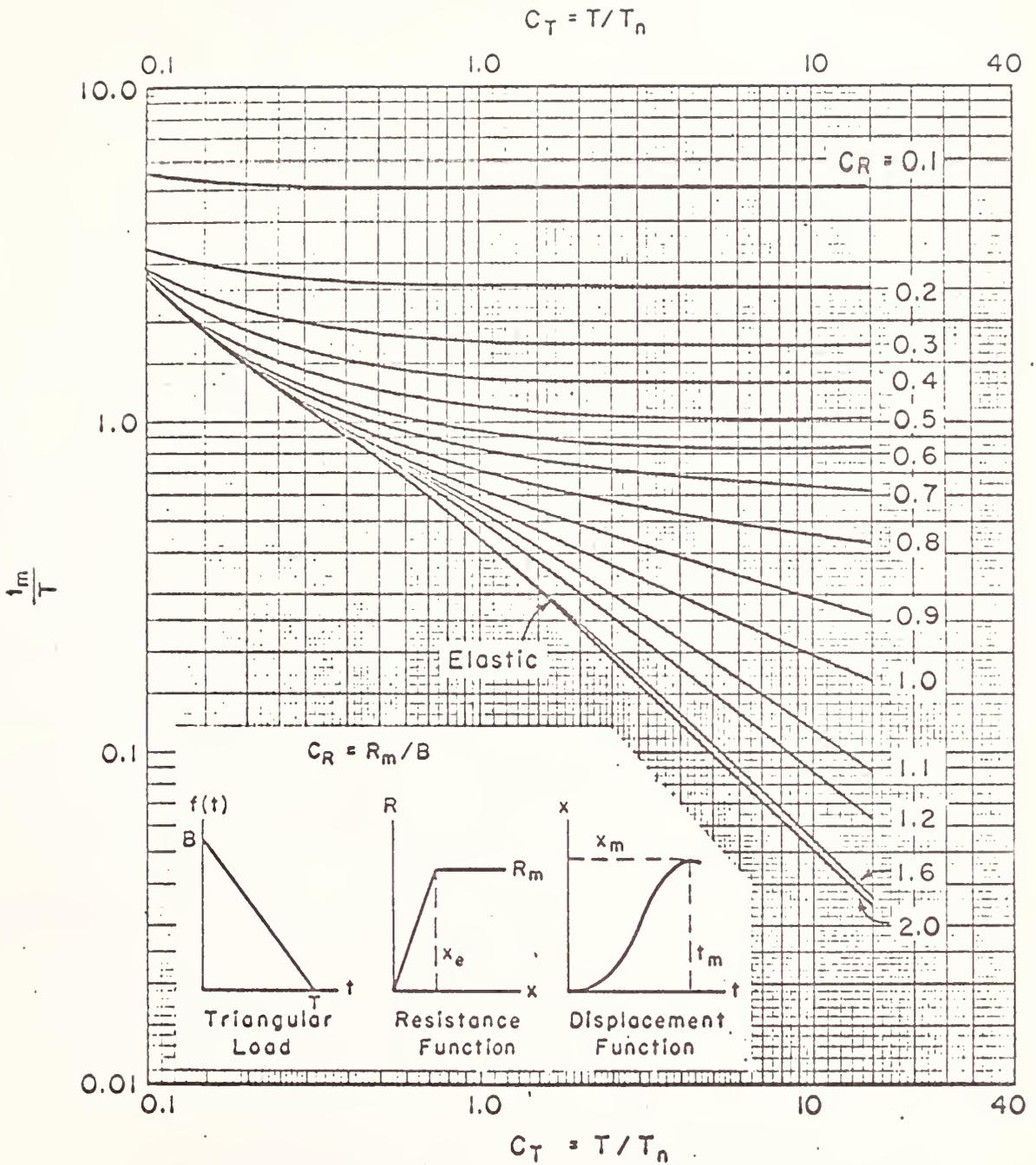


FIG. 7. TIME OF MAXIMUM DEFLECTION FOR TRIANGULAR LOADS AND ONE-DEGREE SYSTEMS. (P. 55, Ref. 8.)

APPENDIX C - SYMBOLS

| | |
|-----------|---|
| A_s | Area of tension steel in reinforced concrete member. |
| B | Peak value of externally applied load. |
| C_R | Ratio of maximum resistance to peak load, $C_R = \frac{R_m}{B}$ |
| C_T | Ratio of load duration to natural period of oscillation, $C_T = \frac{T}{T_n}$ |
| C_W | Ratio of maximum work done to absolute maximum work done, $C_W = \frac{W_m}{W_p}$ |
| DLF | Dynamic load factor = $\frac{X_m}{X_s}$ |
| E | Energy absorbed by the equivalent system, modulus of elasticity. |
| f_{dy} | Dynamic yield strength of steel (psi). |
| f'_{dc} | Dynamic ultimate compressive strength of concrete. |
| H | Impulse per unit area, impulse per foot of length. |
| H_e | Equivalent impulse acting on equivalent system. |
| I_a | Average of the gross and transformed moment of inertia. |
| I_g | Moment of inertia of gross section. |
| I_t | Moment of inertia of transformed section. |
| K_L | Load factor |
| K_{LM} | Load mass factor. |
| K_M | Mass factor. |

| | |
|----------|---|
| K_R | Resistance factor. |
| k | Spring constant. |
| k_E | Effective spring constant. |
| k_e | Equivalent spring constant. |
| k_{ep} | Spring constant in elasto plastic range. |
| M_D | Maximum design moment in a member under axial load P_D . |
| M_P | Plastic bending moment under bending. |
| M_{Pm} | Plastic bending moment at center line of beam or slab. |
| M_{Ps} | Plastic bending moment at support. |
| m | Mass per unit length (kip - sec ² /ft ²). |
| m_e | Mass of equivalent system (kip - sec ² /ft ²). |
| P_D | Maximum axial load on column. |
| P_S | Overpressure existing in incident shock wave for any value of $t - t_d$. |
| P_{so} | Initial peak incident overpressure. |
| $p(t)$ | Actual load on a structural element as a function of time. |
| p | Ratio of tensile reinforcement to concrete area $\frac{A_s}{bd}$ |
| R | Total resistance of structural element. |
| R_m | Maximum resistance developed by a structural system. |
| R_{ne} | Maximum resistance in equivalent system. |
| T | Duration of external loads. |
| T_n | Natural period of oscillation. |

- t_m Time required for maximum displacement of element.
- t_o Duration of the positive phase of incident shock wave.
- t Time interval used in numerical analysis.
- V Dynamic reaction.
- v Shear stress.
- W_m Maximum work done on equivalent system by equivalent load.
- W_p Fictitious maximum work done on equivalent system.
- x_m Maximum displacement.
- y_e Deflection of equivalent system, limiting elastic deflection.
- y_{ep} Limiting deflection in elasto plastic range.
- y_m Maximum displacement.
- y_n Displacement at time, t_n .
- α Design load ductility reduction factor.
- β Ductility ratio.

DESIGN OF BLAST RESISTANT STRUCTURES

by

GANPAT MAL SINGHVI

B. E., University of Rajasthan, India, 1961

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1963

The three most important phenomena associated with an above-ground explosion of an atomic bomb are air blast, thermal radiation, and nuclear radiation. In designing protective construction, the dynamic loading caused by air blast pressure is of a primary concern.

The principal objective in the design of a blast-resistant structure is to protect the structure itself including its equipment and occupants. To resist the blast-forces which are large in magnitude and dynamic in character, members and joints are allowed to deflect plastically. The amount of plastic distortion permitted is kept small enough to provide a margin of safety against collapse and to limit the damage of building services. Due to dynamic character of loading, the design procedure is based on dynamic analysis.

Important elements of a windowless, one story, reinforced concrete frame building are designed to resist the effect of a 20 K. T. atomic bomb. The blast loads on the frame are calculated as suggested by the United States Atomic Energy Commission and the United States Army Corps of Engineers. Preliminary design of members is done using an idealized straight line load-time curve and is checked by numerical integration using the calculated load time data.

