

K-SHELL AUGER AND X-RAY
RATES, TRANSITION ENERGIES,
AND FLUORESCENCE YIELDS
FOR MULTIPLY IONIZED NEON,
OXYGEN, AND NITROGEN

by

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CHAPTER I

INTRODUCTION

The de-excitation of an atom with an inner shell vacancy can occur mainly by two independent competing processes: (a) the emission of x-rays and (b) the ejection of an Auger electron. The fluorescence yield for a particular vacancy is defined as:

$$\omega_i = \frac{\Gamma_x}{\Gamma_x + \Gamma_A}$$

where Γ_x and Γ_A are the total x-ray rate and the total Auger rate respectively. The theoretical values of Γ_x and Γ_A (and therefore ω_i) depend upon the electronic configuration of the atom with an inner shell vacancy.

When the inner shell vacancy is produced in an atom without changing the electronic configuration in other shells, the theoretical results, which have been already reported by McGuire¹, and by Bhalla and co-workers²⁻³ can be used. Calculations in Ref. 1-3 are with the nonrelativistic Hartree-Fock-Slater (HFS) model and are for the K-shell, L-subshells and M-subshells. Such calculations were first performed by Rubenstein⁴. The theoretical values obtained by using the hydrogenic wavefunctions with (and without) screening parameters also appear in the literature⁵⁻⁸.

The relativistic calculations of the Auger rates with the statistical Thomas-Fermi-Model¹⁰, with the nonrelativistic HFS potential⁹ and with the relativistic HFS model¹¹⁻¹³ have been performed for the

K-shell vacancy. The x-ray rates for the K-shell and the L-subshells were calculated by Scofield¹⁴, and by Rosner and Bhalla¹⁵ where the effects of retardation were included and the self consistent relativistic HFS wave-functions were used. Bhalla¹⁶ extended these calculations for the M-subshells.

It should be noted that the calculations, referred to above whether nonrelativistic or relativistic in nature, are not appropriate when an inner shell vacancy is produced in a heavy ion collision. The reason is that several additional electrons are stripped from the atom (in addition to the creation of an inner shell vacancy) in violent heavy ion collisions. The relevant experimental data are:

- a) experimental observations of the charge state distributions¹⁷
- b) increase in the x-ray transition energies¹⁸⁻²⁴ as compared to the normal x-ray energies²⁵
- c) decrease in the Auger electron energies²⁶
- d) observations of both the satellite^{19,20} and hypersatellite²⁷ K_{α} lines.
- e) significant deviations of the experimental fluorescence yields^{28,29} from the normal values³⁰.

In order to identify the possible electronic configurations which can lead to the shifts in the x-ray and Auger energies, the relative intensities of x-ray (for example K_{α}/K_{β}) and the satellite and hypersatellite structure, explicit calculations with a realistic atomic model are needed. The importance of such calculations become more evident by another set of experiments, where the cross section of a

particular x-ray production, $\sigma_x(E)$, is measured as a function of the bombarding energy, E of the heavy ion. Several groups³¹ have reported such measurements of σ_x for a range of incident energies and combinations of projectiles and target materials. The theoretical models³²⁻³⁶ predict the cross section of creating a vacancy, $\sigma_I(E)$. The comparison of the theoretical $\sigma_I(E)$ can be made with $\sigma_x(E)/\omega_I$, where ω_I is the appropriate fluorescence yield which depends on the relevant electronic configuration before the de-excitation of the vacancy occurs.

Recently Larkins³⁷, has estimated the effects on ω_K and ω_{2p} of argon, due to alterations of the electronic configuration. This was done by using scaling factors to the x-ray and the Auger rates as calculated for the normal argon configuration with only one vacancy. Fortner et al.³⁸ have reported similar calculations for copper. The above-mentioned procedure, which is extremely simple to use, ignores the effects which arise from the change in the bound state wave-functions, as electrons are removed from the normal configuration. Bhalla and Walters³⁹ have explicitly calculated for several configurations of argon, the x-ray rates, the Auger rates and the fluorescence yields. Significant deviations between the statistical scaling procedure³⁷ and the explicit calculations³⁹ were found for several cases in argon.

It is the purpose of this thesis to present theoretical x-ray rates, Auger rates and K-shell fluorescence yields for multiply ionized neon, oxygen and nitrogen, and to critically examine the approximate statistical scaling procedure for these elements.

Chapter II contains a description of the theoretical model used in the present calculations, followed by the numerical results in

Chapter III. A comparison with the statistical scaling procedure and the experimental data appears in Chapter IV. A discussion and conclusions are presented in Chapter VI.

CHAPTER II

THEORY

The one-electron bound-state wave-functions and continuum wave-functions are defined below. These are used in calculation of the x-ray and Auger transition rates. Atomic units ($\hbar = m_e = e = 1$) are used throughout.

A. Bound-State and Continuum Wave-Functions

The one-electron wave-functions are solutions of the Schrodinger equation,

$$\left[-\frac{1}{2} \nabla^2 + V(r) - \epsilon \right] \phi(\vec{r}) = 0 \quad (1)$$

The spherically symmetric potential, $V(r)$, experienced by the electron of interest, is specified for an atom of proton number Z and electron occupation numbers $N(n, l)$. The total number of electrons in a particular configuration is $Z_0 = \sum_{n, l} N(n, l)$. This potential is defined as,

$$V(r) \equiv \begin{cases} V'(r) & , -V'(r) > (Z - Z_0 + 1)/r \\ -(Z - Z_0 + 1)/r & , -V'(r) \leq (Z - Z_0 + 1)/r \end{cases}$$

where

$$V'(r) \equiv V_d(r) + V_x(r)$$

the direct term being

$$V_d(r) \equiv -Z/r + \int d^3r' \frac{\rho(r')}{|\vec{r} - \vec{r}'|} ,$$

and the exchange term being

$$V_x(r) \equiv -2 \left[\frac{3\rho(r)}{8\pi} \right]^{1/3} [1 + \tanh(G)]$$

where G is an inhomogeneity term given by,

$$G \equiv \beta \left[2 \left(\frac{\nabla \rho}{\rho} \right)^2 - 3 \nabla^2 \rho / \rho \right] / \rho^{2/3} .$$

The optimum choice for β , suggested by Herman and Schwarz⁴⁰, is

$\beta = 0.0028$. All of the above equations depend on a density

ρ , which is defined as

$$4\pi r^2 \rho(r) \equiv \sum_{n,l} N(n,l) P_{nl}^2(r)$$

wherein the wave function, $\phi(\vec{r})$, is separated as

$$\phi_{nlm}(\vec{r}) = P_{nl}(r) Y_{lm}(\hat{r}) / r .$$

B. Transition Rates

Consider one inner shell vacancy designated by n, l , in an atom with proton number Z and N electrons. Ignoring the spin-orbit coupling term, the Hamiltonian can be written as,

$$H = H_0 + H'$$

where

$$H_0 = \sum_{i=1}^N \left[-\frac{1}{2} \nabla_i^2 - \frac{Z}{r_i} + V(r_i) \right]$$

$$H' = \sum_{i,j} \frac{1}{r_{ij}} - \sum_i V(r_i)$$

The total Auger transition rate from an initial state ϕ_i to a

final state ϕ_f is

$$\Gamma_A = 2\pi \mathcal{S} |\langle \phi_f(1,2,\dots,N) | \sum_{i,j} \frac{1}{r_{ij}} | \phi_i(1,2,\dots,N) \rangle|^2$$

where $\mathcal{S}$ means average over the initial states and sum over the final states.

It is more convenient to express this total Auger rate, Γ_A , as a sum of various possible Auger group rates, T_A . The initial state is described by two electrons with quantum labels $n_1 l_1$ and $n_2 l_2$, and in the final state, one of these electrons has filled the vacancy $n_3 l_3$, and the other, denoted by $E l_4$, has been ejected into the continuum. Then,

$$T_A(n_1 l_1, n_2 l_2 \rightarrow n_3 l_3, E l_4) = 2\pi N_{12} \sum |\mathcal{M}|^2 \quad (2a)$$

with

$$\mathcal{M} = \langle \phi(n_3 l_3, E l_4) | \frac{1}{r_{12}} | \phi(n_1 l_1, n_2 l_2) \rangle \quad (2b)$$

The ϕ 's above are two-particle anti-symmetrized wave functions described by a LSJM or any other coupling scheme. The weighting factors, N_{12} , are given by,

$$N_{12} = \begin{cases} \frac{N_1 N_2}{(4l_1+2)(4l_2+2)} & \text{for inequivalent electrons,} \\ \frac{N_1(N_1-1)}{(4l_1+2)(4l_1+1)} & \text{for equivalent electrons} \end{cases}$$

($N_1 = N_2$, $n_1 = n_2$, $l_1 = l_2$) .

The matrix element, $\mathcal{M}$, evaluated in the LSJM coupling scheme⁴³, is

$$\begin{aligned}\mathcal{M} &= \langle \phi(n_3 l_3, E l_4, L' S' J' M' | \frac{1}{r_{12}} | \phi(n_1 l_1, n_2 l_2, L S J M) \rangle \\ &= \mathcal{T} (-1)^{L+S+L_4} \sum_{\kappa} [R^{\kappa}(1,2,3,4) \langle l_3 \| C^{\kappa} \| l_1 \rangle \langle l_4 \| C^{\kappa} \| l_2 \rangle \left\{ \begin{matrix} l_3 & l_4 & L \\ l_2 & l_1 & \kappa \end{matrix} \right\} \\ &\quad + (-1)^{L+S} R^{\kappa}(2,1,3,4) \langle l_3 \| C^{\kappa} \| l_2 \rangle \langle l_4 \| C^{\kappa} \| l_1 \rangle \left\{ \begin{matrix} l_3 & l_4 & L \\ l_1 & l_2 & \kappa \end{matrix} \right\} \delta_{MM'} \delta_{SS'} \delta_{JJ'} \delta_{LL'}] \end{aligned} \quad (4a)$$

with

$$\langle l \| C^{\kappa} \| l' \rangle = (-1)^L [(2l+1)(2l'+1)]^{\frac{1}{2}} \left\{ \begin{matrix} l' & \kappa & l \\ 0 & 0 & 0 \end{matrix} \right\} \quad (4b)$$

and

$$R^{\kappa}(1,2,3,4) \equiv \int_0^{\infty} \int_0^{\infty} P_{n_1 l_1}(r_1) P_{n_2 l_2}(r_2) \frac{r_2^{\kappa}}{r_1^{\kappa+1}} P_{n_3 l_3}(r_1) P_{E l_4}(r_2) dr_1 dr_2 \quad (4c)$$

The factor $\mathcal{T}$ is $\sqrt{1/2}$ if $n_1 l_1$ and $n_2 l_2$ are equivalent electrons; $\mathcal{T} = 1$ otherwise. The Auger group rate is now expressed as,

$$T_A(n_1 l_1, n_2 l_2 \rightarrow n_3 l_3, E l_4) = 2\pi N_{12} \sum_j \sum_{l_4} \frac{(2j+1)}{2(2l_3+1)} |\mathcal{M}|^2 \quad (5)$$

with $\mathcal{M}$ given in (4a). The total Auger rate, Γ_A , is now the sum of all possible group rates.

The x-ray transition rate for filling a vacancy $n_3 l_3$ and creating a vacancy $n_f l_f$ is given in the electric dipole approximation as,

$$T_x(n_3 l_3 \rightarrow n_f l_f) = N_{3f} \frac{4}{3} k^3 \frac{l_3}{(2l_3+1)} \left| \int_0^{\infty} P_{n_f l_f} P_{n_3 l_3} r dr \right|^2 \quad (6)$$

The value of k is equal to the x-ray transition energy divided by c .

N_{3f} is the reduction factor given by

$$N_{3f} = \frac{N_f}{2(2l_f + 1)} \quad \text{where } N_f \text{ is the actual}$$

occupation number of the $n_f l_f$ shell.

The total x-ray rate, Γ_x , is then the sum of all possible individual transition rates given in (6). (For the elements studied here, there is only one possible transition: $2p \rightarrow 1d$).

The fluorescence yield for an initial vacancy $n_3 l_3$, is defined as,

$$\omega_{n_3 l_3} = \frac{\Gamma_x}{\Gamma_x + \Gamma_A} \quad (7)$$

C. Transition Energies

The x-ray transition energies and the Auger electron energies for the initial configuration $(1d)^l (2d)^m (2p)^n$ were calculated by taking appropriate differences between the total energies^a of the initial and final configurations:

$$E_x(2p \rightarrow 1d) = E_{HF}(1d^l 2d^m 2p^n) - E_{HF}(1d^{l+1} 2d^m 2p^{n-1})$$

$$E_A(1d-2d-2d) = E_{HF}(1d^l 2d^m 2p^n) - E_{HF}(1d^{l+1} 2d^{m-2} 2p^n)$$

$$E_A(1d-2d-2p) = E_{HF}(1d^l 2d^m 2p^n) - E_{HF}(1d^{l+1} 2d^{m-1} 2p^{n-1})$$

$$E_A(1d-2p-2p) = E_{HF}(1d^l 2d^m 2p^n) - E_{HF}(1d^{l+1} 2d^m 2p^{n-2})$$

^aSee Appendix A for calculation of total energies.

CHAPTER III

NUMERICAL RESULTS

As described in Chapter II, the bound state wave-functions were calculated for all electronic configurations $(1s)^\ell(2s)^m(2p)^n$; $\ell=0,1$, $m+n > 0$ of neon with the HFS model using the HVDO exchange. The continuum wave-functions were also calculated by numerical integration of the Schrodinger equation for the appropriate Auger energies and orbital angular momentum values.

The transition energies for the $2p \rightarrow 1s$ x-ray, and the Auger energies were calculated by the adiabatic procedure, and the energy shifts from the "normal" configuration are listed in Table I. The calculated individual Auger rates, $2p \rightarrow 1s$ x-ray rate and the K-shell fluorescence yield are given in Table II for a single K-shell vacancy and in Table III for a double K-shell vacancy. The numerical results in these tables are for neon with configurations $(1s)^\ell(2s)^m(2p)^n$; $\ell=0,1$, $m+n > 0$.

Similar theoretical results for oxygen and nitrogen are given in Tables IV through IX. The total energies for all electronic configurations $(1s)^\ell(2s)^m(2p)^n$ are given in Appendix A. The indices ℓ and m assume values of 0, 1, and 2; the index n assumes values of 0 to 6 for neon, 0 to 4 for oxygen and 0 to 3 for nitrogen. Configurations from which no transition can occur are not listed.

Table I. X-ray and Auger energy shifts in eV from the normal configuration^a of neon ($1s^1 2s^2 2p^6$), calculated with the adiabatic method, for various configurations of neon ($1s^l 2s^m 2p^n$).

Configuration			ΔE_x	ΔE_A	ΔE_A	ΔE_A
l	m	n	($2p \rightarrow 1s$)	($2s-2s$)	($2s-2p$)	($2p-2p$)
1	2	6	0.0	0.0	0.0	0.0
1	2	5	5.2	-15.2	-17.2	-19.5
1	2	4	12.2	-30.4	-35.1	-40.4
1	2	3	20.5	-45.7	-53.8	-62.7
1	2	2	30.5	-60.9	-73.0	-86.3
1	2	1	42.2	-75.7	-92.6	--
1	1	6	6.1	--	-19.4	-16.3
1	1	5	12.4	--	-37.4	-37.0
1	1	4	20.4	--	-56.2	-59.1
1	1	3	30.0	--	-75.5	-55.0
1	1	2	41.2	--	-95.2	-106.9
1	1	1	54.1	--	-116.4	--

Table I (continued)

1	0	6	13.2	--	--	-33.6
1	0	5	20.7	--	--	-55.5
1	0	4	29.8	--	--	-78.5
1	0	3	40.5	--	--	-102.8
1	0	2	53.1	--	--	-129.2
1	0	1	65.9	--	--	--
0	2	6	96.9	68.6	67.3	65.4
0	2	5	103.8	52.6	48.3	42.9
0	2	4	112.2	36.7	28.5	19.0
0	2	3	122.2	20.8	8.1	-6.2
0	2	2	133.8	5.0	-12.9	-32.7
0	2	1	146.8	-10.6	-34.5	--
0	1	6	103.9	--	45.8	46.5
0	1	5	111.9	--	25.9	22.8
0	1	4	121.4	--	5.3	-2.3
0	1	3	132.4	--	-15.8	-28.6
0	1	2	145.0	--	-37.4	-56.2
0	1	1	159.1	--	-59.8	--

Table I (continued)

0	0	6	111.8	--	--	26.5
0	0	5	120.8	--	--	1.7
0	0	4	131.4	--	--	-24.4
0	0	3	143.5	--	--	-51.7
0	0	2	164.0	--	--	-80.5

$$^a E_X (2p \rightarrow 1s) = 848.5\text{eV};$$

$$E_A (2s - 2s) = 747.3\text{eV};$$

$$E_A (2s - 2p) = 780.0\text{eV};$$

$$E_A (2p - 2p) = 807.7\text{eV}.$$

Table II Single hole K-shell Auger rates $\times 10^4/\text{a.u.}$, X-ray rates $\times 10^4/\text{a.u.}$, and fluorescence yields for various configurations of neon ($1s^1 2s^m 2p^n$).

Configuration n	m	2s - 2s	2s - 2p	2p - 2p	Total Auger rate	Total X-ray rate	ω_K
6	2	8.334	24.448	55.284	88.066	1.436	0.0160
5	2	9.393	24.002	45.358	78.753	1.339	0.0167
4	2	10.701	22.507	33.070	66.278	1.194	0.0177
3	2	12.220	19.682	19.731	51.633	0.994	0.0189
2	2	13.827	15.163	7.698	36.687	0.732	0.0196
1	2	15.388	8.606	--	23.994	0.401	0.0164
6	1	--	14.150	66.278	80.428	1.596	0.0195
5	1	--	13.773	53.548	67.321	1.479	0.0215
4	1	--	12.832	38.330	51.162	1.312	0.0250
3	1	--	11.129	22.495	33.624	1.087	0.0313
2	1	--	8.442	8.670	17.113	0.796	0.0445
1	1	--	4.716	--	4.716	0.434	0.0843

Table II (continued)

6	0	--	--	77.901	77.901	1.762	0.0221
5	0	--	--	61.854	61.854	1.625	0.0256
4	0	--	--	43.571	43.571	1.435	0.0319
3	0	--	--	25.306	25.306	1.183	0.0447
2	0	--	--	9.682	9.682	0.868	0.0823
1	0	--	--	--	--	0.509	--

Table III Double hole K-shell Auger rates $\times 10^4/\text{a.u.}$, X-ray rates $\times 10^4/\text{a.u.}$, and fluorescence yields for various configurations of neon ($1s^0 2s^m 2p^n$).

Configuration n	m	2s-2s	2s-2p	2p-2p	Total Auger rate	Total X-ray rate	ω_K
6	2	10.239	33.699	85.925	129.860	2.363	0.0179
5	2	11.698	32.558	67.900	112.160	2.162	0.0189
4	2	13.390	30.020	47.598	91.009	1.893	0.0204
3	2	15.246	25.753	27.310	68.309	1.547	0.0221
2	2	17.160	19.398	10.233	46.790	1.117	0.0233
1	2	18.930	10.772	---	29.702	0.599	0.0198
6	1	---	19.192	99.936	119.128	2.611	0.0214
5	1	---	18.446	78.032	96.478	2.381	0.0241
4	1	---	16.928	54.054	70.982	2.078	0.0284
3	1	---	14.415	30.757	45.172	1.693	0.0361
2	1	---	10.726	11.475	22.201	1.219	0.0520
1	1	---	5.872	---	5.872	0.651	0.0998

Table III (continued)

6	0	---	---	116.220	116.220	2.936	0.0246
5	0	---	---	89.805	89.805	2.675	0.0289
4	0	---	---	61.849	61.849	2.336	0.0364
3	0	---	---	35.184	35.184	1.907	0.0514
2	0	---	---	13.161	13.161	1.415	0.0971
1	0	---	---	---	---	0.758	---

Table IV X-ray and Auger energy shifts in eV from the normal configuration^a of oxygen ($1s^1 2s^2 2p^4$), calculated with the adiabatic method, for various configurations of oxygen ($1s^{\ell} 2s^m 2p^n$).

Configuration			ΔX	ΔE_A	ΔE_A	ΔE_A
ℓ	m	n	($2p \rightarrow 1s$)	($2s-2s$)	($2s-2p$)	($2p-2p$)
1	2	4	0.0	0.0	0.0	0.0
1	2	3	4.9	-12.1	-13.4	-16.1
1	2	2	11.5	-24.1	-27.9	-33.4
1	2	1	19.8	-35.7	-42.7	--
1	1	4	5.3	--	-15.7	-13.5
1	1	3	11.4	--	-30.4	-30.7
1	1	2	19.2	--	-45.3	-48.8
1	1	1	28.8	--	-61.7	--
1	0	4	11.8	--	--	-27.8
1	0	3	19.1	--	--	-45.7
1	0	2	28.3	--	--	-65.9
1	0	1	37.9	--	--	--

Table IV (continued)

0	2	4	78.7	54.8	53.3	51.1
0	2	3	85.2	42.1	37.5	32.0
0	2	2	93.4	29.5	21.2	11.8
0	2	1	103.1	17.1	4.4	--
0	1	4	84.8	--	35.4	35.0
0	1	3	92.5	--	19.0	14.8
0	1	2	101.5	--	2.1	-6.5
0	1	1	112.5	--	-15.6	--
0	0	4	91.9	--	--	18.0
0	0	3	100.7	--	--	-3.0
0	0	2	111.1	--	--	-25.6
0	0	1	122.5	--	--	--

$$^a E_x(2p \rightarrow 1s) = 530.5 \text{ eV};$$

$$E_A(2s-2s) = 463.1 \text{ eV};$$

$$E_{\Lambda}(2s-2p) = 482.8 \text{ eV};$$

$$E_A(2p-2p) = 498.4 \text{ eV}.$$

Table V Single hole K-shell Auger rates $\times 10^4/\text{a.u.}$ and X-ray rates $\times 10^4/\text{a.u.}$ and fluorescence yields for various configurations of oxygen ($1s^1 2s^m 2p^n$).

n	m	2s - 2s	2s - 2p	2p - 2p	Total Auger rate	Total X-ray rate	ω_K
4	2	8.470	16.553	22.470	47.493	0.365	0.0076
3	2	9.862	15.138	14.428	39.428	0.315	0.0079
2	2	11.538	12.228	5.980	29.746	0.240	0.0080
1	2	13.249	7.249	--	20.497	0.135	0.0065
0	2	14.821	--	--	14.821	--	--
4	1	--	9.892	28.117	38.009	0.418	0.0109
3	1	--	8.968	17.471	26.439	0.357	0.0133
2	1	--	7.100	7.070	14.170	0.269	0.0186
1	1	--	4.106	--	4.106	0.151	0.0355
4	0	--	--	33.803	33.803	0.472	0.0138
3	0	--	--	20.576	20.576	0.401	0.0191
2	0	--	--	8.242	8.242	0.302	0.0353
1	0	--	--	--	--	0.187	--

Table VI Double hole K-shell Auger rates $\times 10^4/\text{a.u.}$ and X-ray rates $\times 10^4/\text{a.u.}$ and fluorescence yields for various configurations of oxygen ($1s^2 2s^m 2p^n$).

n	m	2s - 2s	2s - 2p	2p - 2p	Total Auger rate	Total X-ray rate	ω_K
4	2	11.281	24.711	38.284	74.276	0.684	0.0091
3	2	13.288	22.037	23.011	58.336	0.572	0.0097
2	2	15.465	17.226	8.942	41.633	0.422	0.0100
1	2	17.530	9.877	--	27.407	0.230	0.0083
0	2	20.126	--	--	20.126	--	--
4	1	--	14.457	45.636	60.093	0.771	0.0127
3	1	--	12.788	26.985	39.773	0.642	0.0159
2	1	--	9.832	10.394	20.226	0.471	0.0228
1	1	--	5.518	--	5.518	0.256	0.0443

Table VI (continued)

4	0	---	--	54.467	54.467	0.895	0.0162
3	0	--	--	32.121	32.121	0.747	0.0227
2	0	--	--	12.414	12.414	0.555	0.0428
1	0	--	--	--	--	0.311	--

Table VII X-ray and Auger energy shifts in eV from the normal configuration^a of nitrogen ($1s^1 2s^2 2p^3$), calculated with the adiabatic method, for various configurations of nitrogen ($1s^l 2s^m 2p^n$).

Configuration			ΔX	ΔE_A	ΔE_A	ΔE_A
l	m	n	($2p \rightarrow 1s$)	($2s-2s$)	($2s-2p$)	($2p-2p$)
1	2	3	0.0	0.0	0.0	0.0
1	2	2	4.8	-10.4	-12.2	-14.3
1	2	1	11.4	-20.4	-24.7	--
1	1	3	5.0	--	-13.8	-12.0
1	1	2	11.0	--	-26.3	-27.0
1	1	1	19.0	--	-40.4	--
1	0	3	11.2	--	--	-24.4
1	0	2	18.7	--	--	-41.5
1	0	1	26.6	--	--	--
0	2	3	69.5	48.0	46.3	43.9
0	2	2	76.0	37.0	32.3	26.8
0	2	1	84.1	26.2	7.8	--

Table VII (continued)

0	1	3	75.3	--	30.3	29.3
0	1	2	82.9	--	15.9	11.1
0	1	1	92.1	--	0.6	--
0	0	3	82.1	--	--	14.0
0	0	2	90.9	--	--	-5.4
0	0	1	100.7	--	--	--

^a

$$E_x(2p \rightarrow 1s) = 399.3 \text{ eV};$$

$$E_A(2s-2s) = 346.2 \text{ eV};$$

$$E_A(2s-2p) = 360.6 \text{ eV};$$

$$E_A(2p-2p) = 371.4 \text{ eV}.$$

Table VIII Single hole K-shell Auger rates $\times 10^4/\text{a.u.}$ and X-ray rates $\times 10^4/\text{a.u.}$ and fluorescence yields for various configurations of nitrogen ($1s^2 2s^m 2p^n$).

n	m	2s - 2s	2s - 2p	2p - 2p	Total Auger rate	Total X-ray rate	ω_K
3	2	8.490	12.389	11.033	31.912	0.152	0.0047
2	2	10.086	10.341	4.850	25.277	0.118	0.0046
1	2	11.836	6.345	--	18.181	0.069	0.0038
0	2	13.504	--	--	13.504	--	--
3	1	--	7.611	14.232	21.843	0.177	0.0080
2	1	--	6.220	6.003	12.223	0.136	0.0110
1	1	--	3.692	--	3.692	0.078	0.0207
3	0	--	--	17.479	17.479	0.204	0.0115
2	0	--	--	7.266	7.266	0.157	0.0212
1	0	--	--	--	--	0.101	--

Table IX Double hole K-shell Auger rates $\times 10^4/\text{a.u.}$ and X-ray rates $\times 10^4/\text{a.u.}$ and fluorescence yields for various configurations of nitrogen ($1s^0 2s^m 2p^n$).

n	m	2s - 2s	2s - 2p	2p - 2p	Total Auger rate	Total X-ray rate	ω_K
3	2	12.046	19.641	20.225	51.912	0.311	0.0060
2	2	14.348	15.790	8.088	38.226	0.233	0.0061
1	2	16.587	9.272	--	25.858	0.129	0.0050
0	2	19.459	--	--	19.459	--	--
3	1	--	11.710	24.512	36.222	0.356	0.0097
2	1	--	9.224	9.667	18.891	0.265	0.0138
1	1	--	5.274	--	5.274	0.146	0.0269
3	0	--	--	30.071	30.071	0.424	0.0139
2	0	--	--	11.901	11.901	0.320	0.0262
1	0	--	--	--	--	0.182	--

CHAPTER IV

COMPARISON WITH STATISTICAL PROCEDURE

Larkins proposed a simple approximate statistical scaling procedure to obtain the fluorescence yields for multiply ionized atoms when the theoretical x-ray and the Auger rates are available for atoms with only one inner shell vacancy. The technique, use by Larkins, and Fortner et al.³⁸, is to scale the individual Auger group rates and the x-ray rates, calculated only for a single inner shell vacancy, with the weighting factors (N_{ij} in eq.2) which depend upon the number of electrons in each subshell. The fluorescence yields can then be calculated for different defect configurations. This simple procedure ignores the effects on the x-ray and Auger transition energies and the wave-functions which result from the multiply ionized configurations. We present in Fig. 1 the K-shell fluorescence yield obtained from the statistical model and that of our explicit HFS calculations, for various configurations of neon. Figures 2 and 3 show the percent deviation of the statistical model from our HFS calculations, for the K_{α} x-ray rate and the total Auger rate respectively in neon. Figures 4 and 5 show the resulting percent deviation in the fluorescence yield for single and double K-shell vacancies for neon respectively. Figure 6 shows a similar deviation in the K_{α} x-ray rate for selected configurations of oxygen; Fig. 7 shows an analogous deviation in the total Auger rate for selected configurations of oxygen. The large errors, introduced by the statistical approximation are evident for most configurations. Figure 8 displays the HFS fluores-

cence yield normalized to the fluorescence yield for the "normal" configuration $(1s)^1(2s)^2(2p)^4$ of oxygen, as a function of the 2p-shell occupancy.

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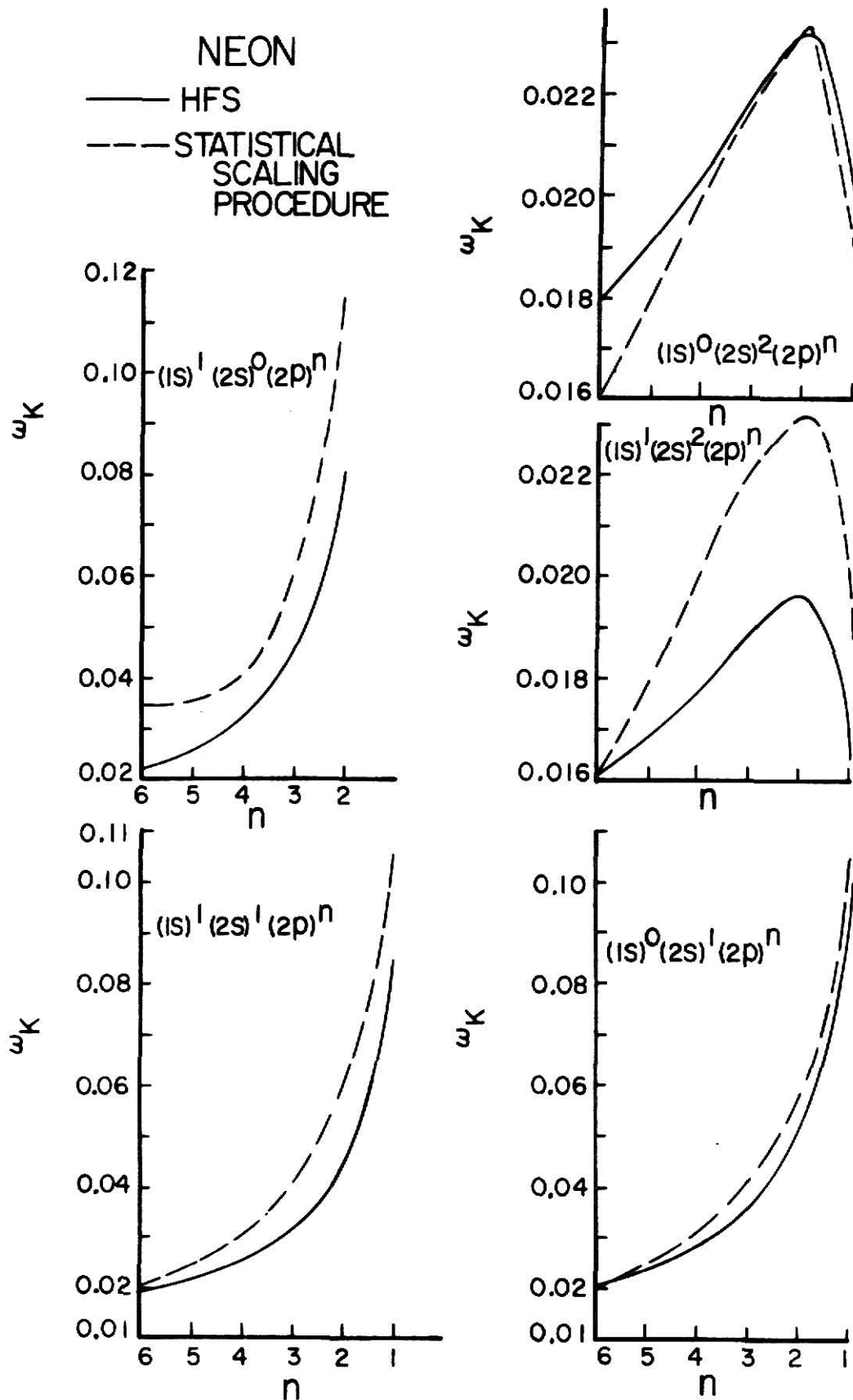


Fig. 1 K-shell fluorescence yields, calculated with the HFS model (solid lines), and K-shell fluorescence yields, calculated with the statistical model (dashed lines), versus the number of 2p-shell electrons for various configurations of neon.

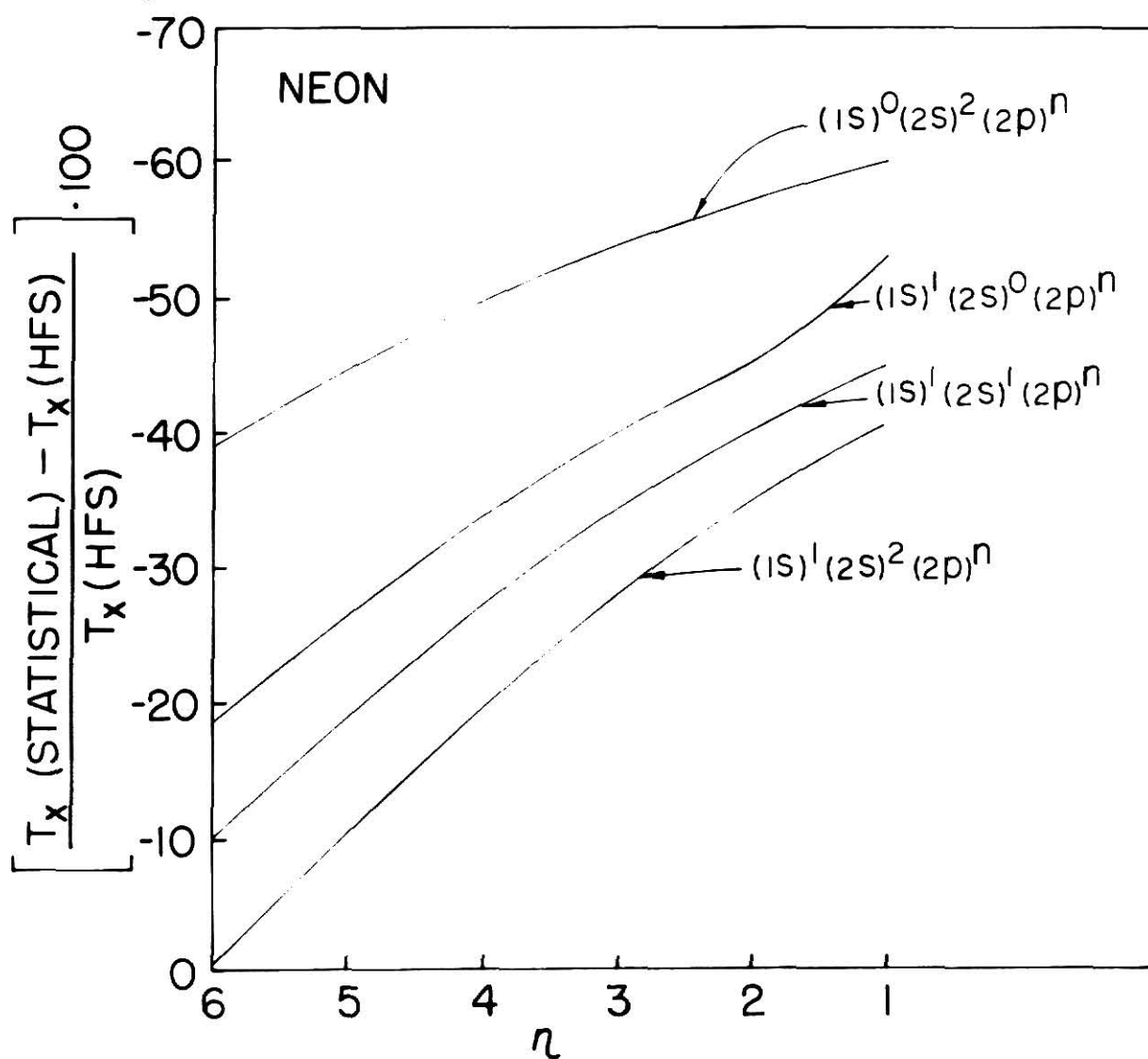


Fig. 2 Percent deviation between the K_{α} x-ray rates, calculated with the HFS model, and the K_{α} x-ray rates, calculated with the statistical model, versus the number of 2p-shell electrons for various configurations of neon.

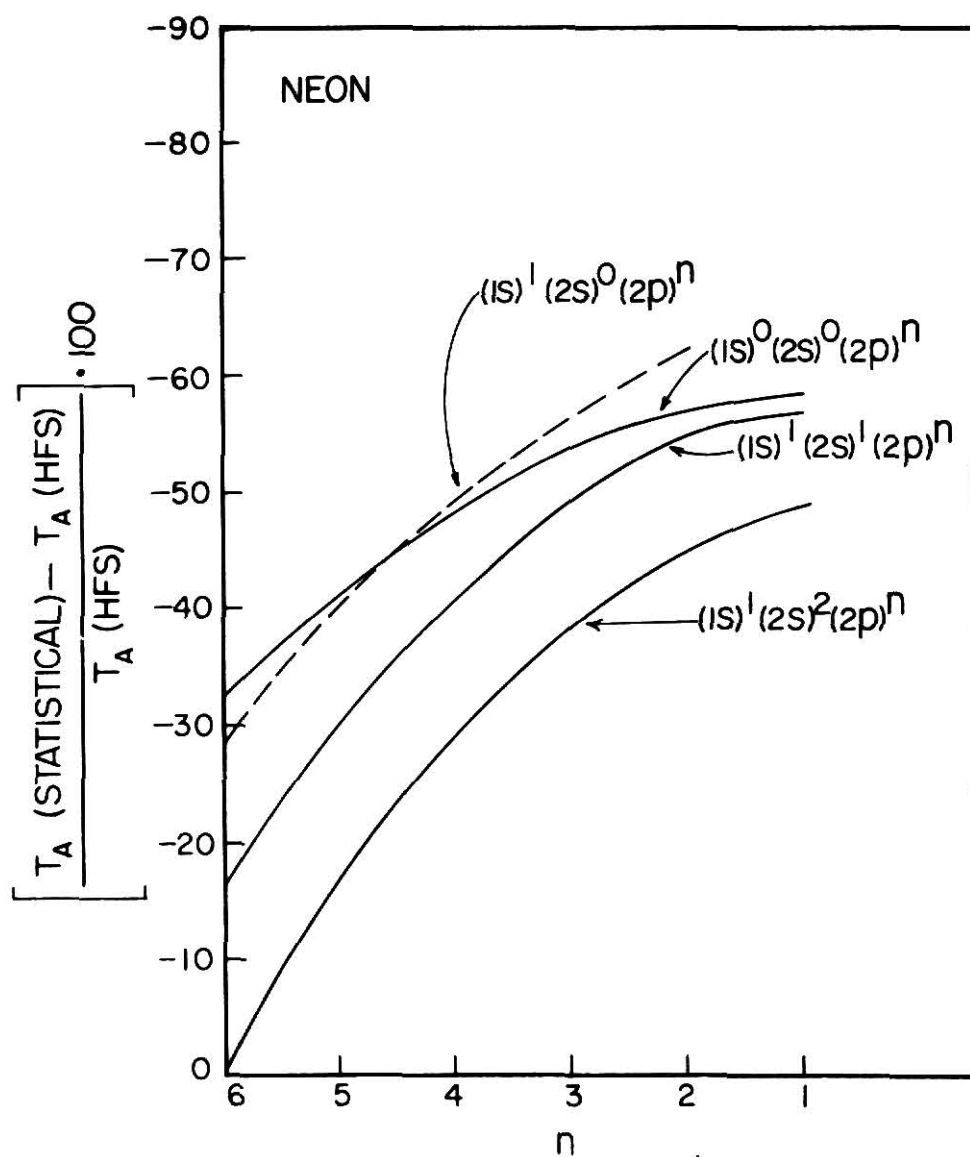


Fig. 3 Percent deviation between the total Auger rates, calculated with the HFS model, and the total Auger rates, calculated with the statistical model, versus the number of 2p-shell electrons for various configurations of neon.

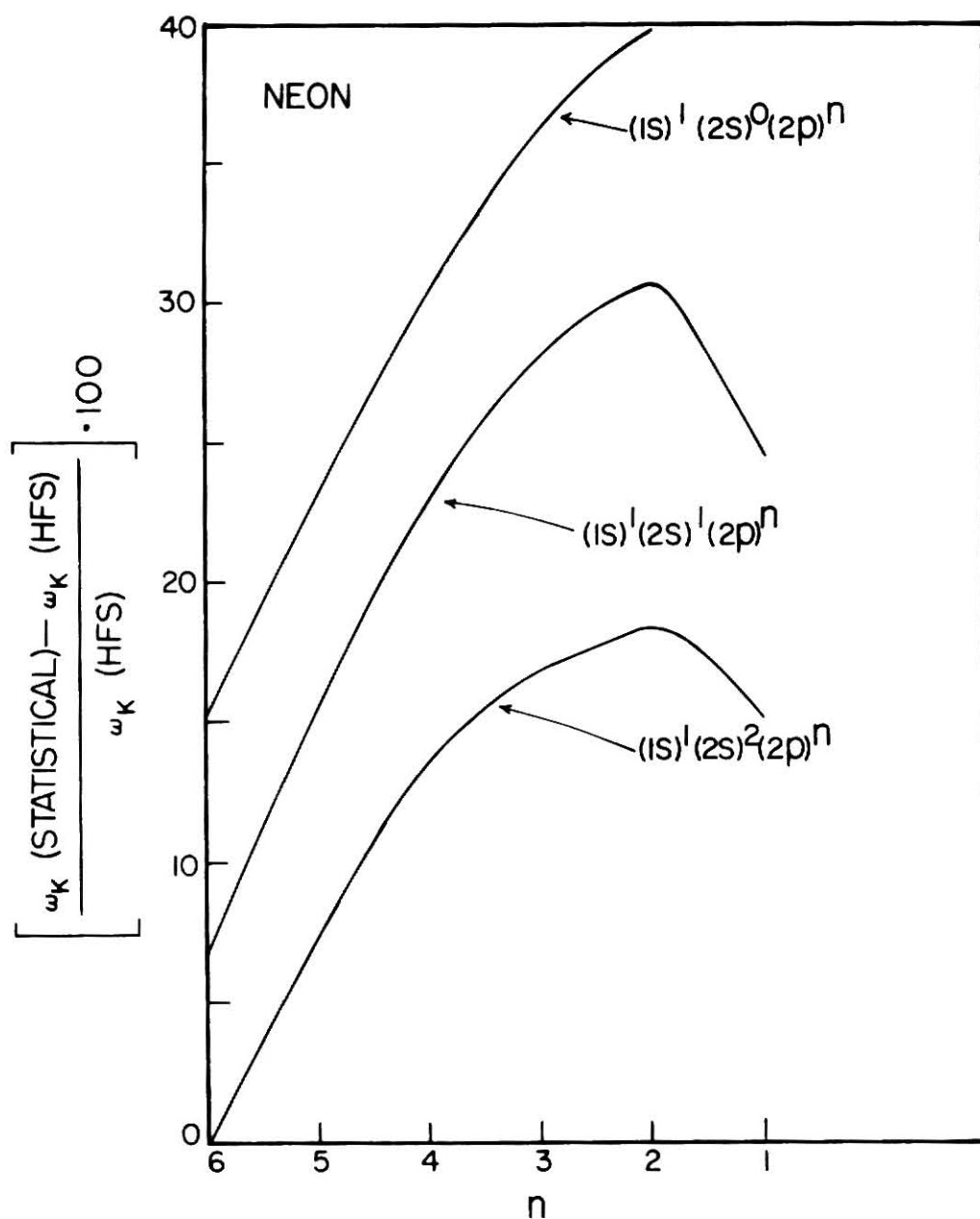


Fig. 4 Percent deviation between the K-shell fluorescence yield, calculated with the HFS model, and the K-shell fluorescence yield, calculated with the statistical model, versus the number of 2p-shell electrons for all configurations of neon with a single K-shell vacancy.

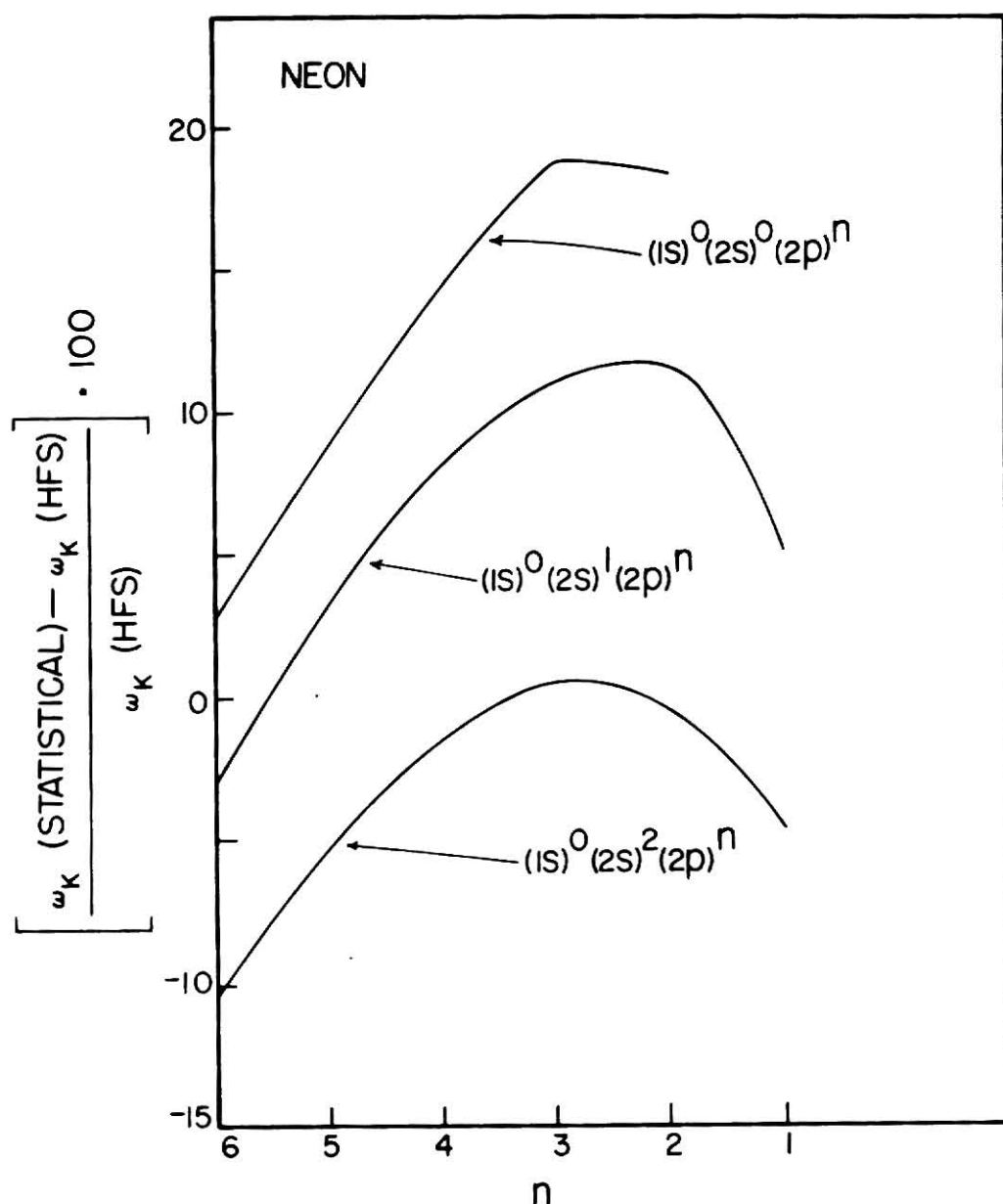


Fig. 5 Percent deviation between the K-shell fluorescence yield, calculated with the HFS model, and the K-shell fluorescence yield, calculated with the statistical model, versus the number of 2p-shell electrons for all configurations of neon with a double K-shell vacancy.

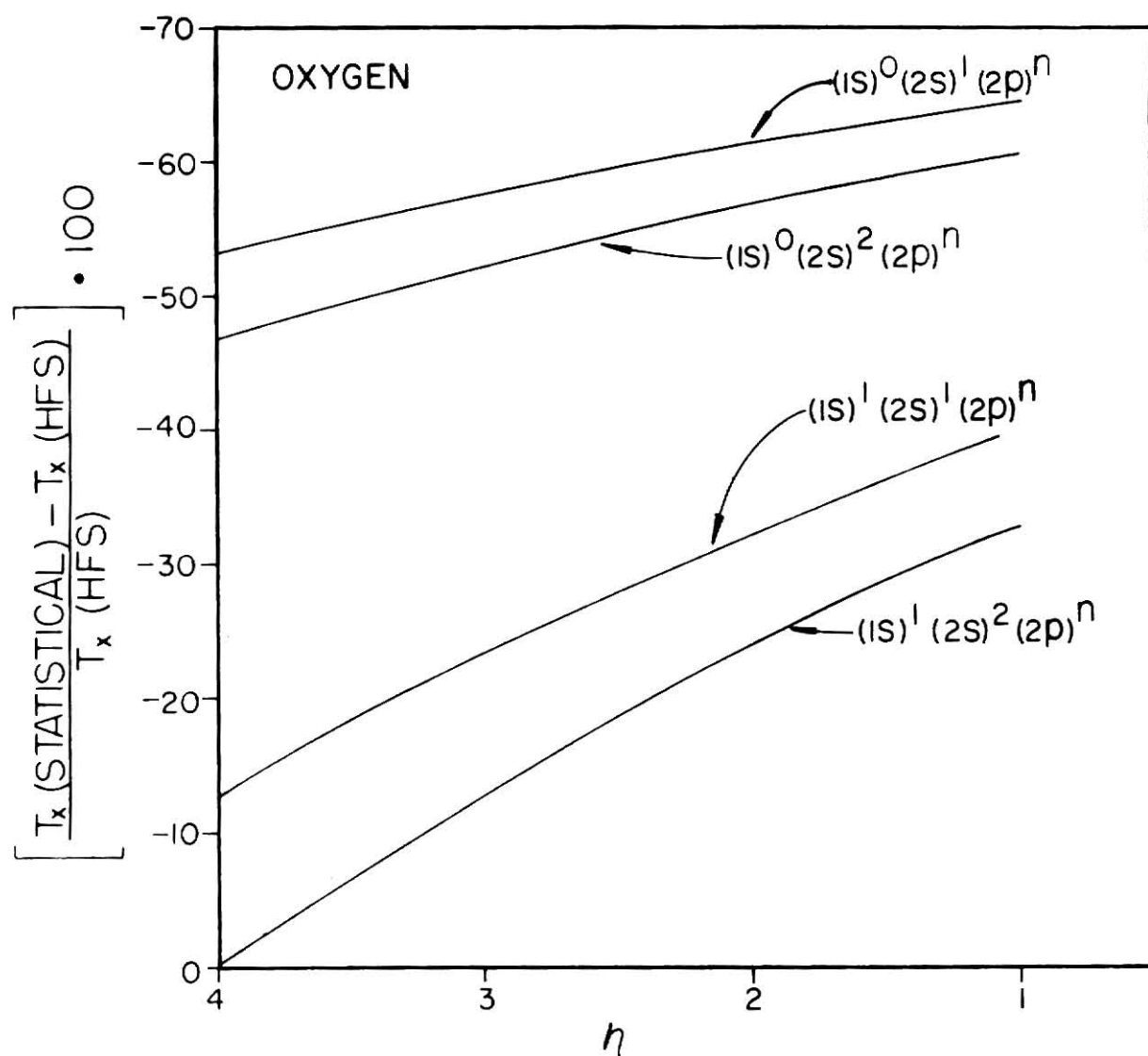


Fig. 6 Percent deviation between the $K\alpha$ x-ray rates, calculated with the HFS model, and the $K\alpha$ x-ray rates, calculated with statistical model, versus the number of 2p-shell electrons for various configurations of oxygen.

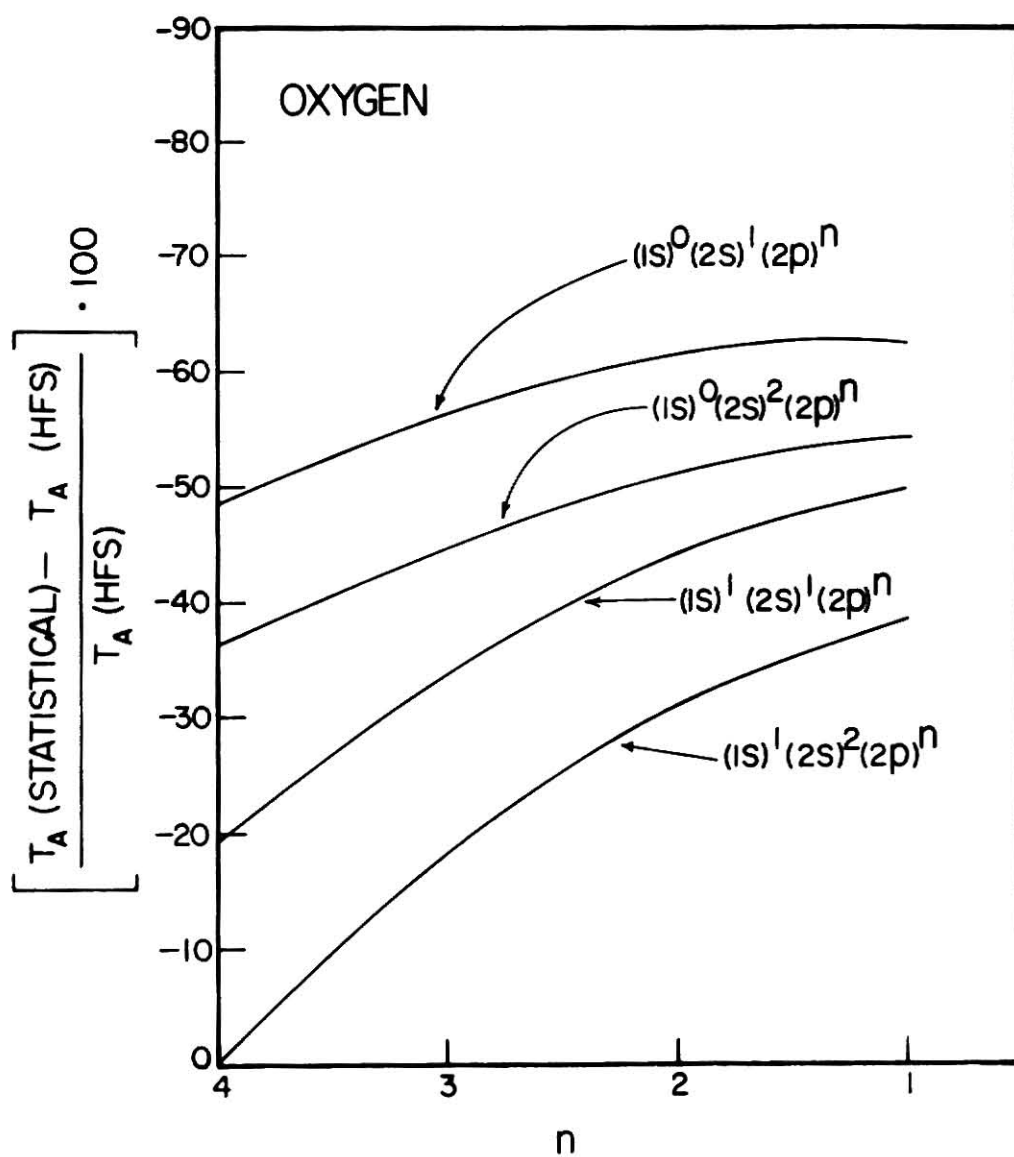


Fig. 7 Percent deviation between the total Auger rates, calculated with the HFS model, and the total Auger rates, calculated with the statistical model, versus the number of 2p-shell electrons for various configurations of oxygen.

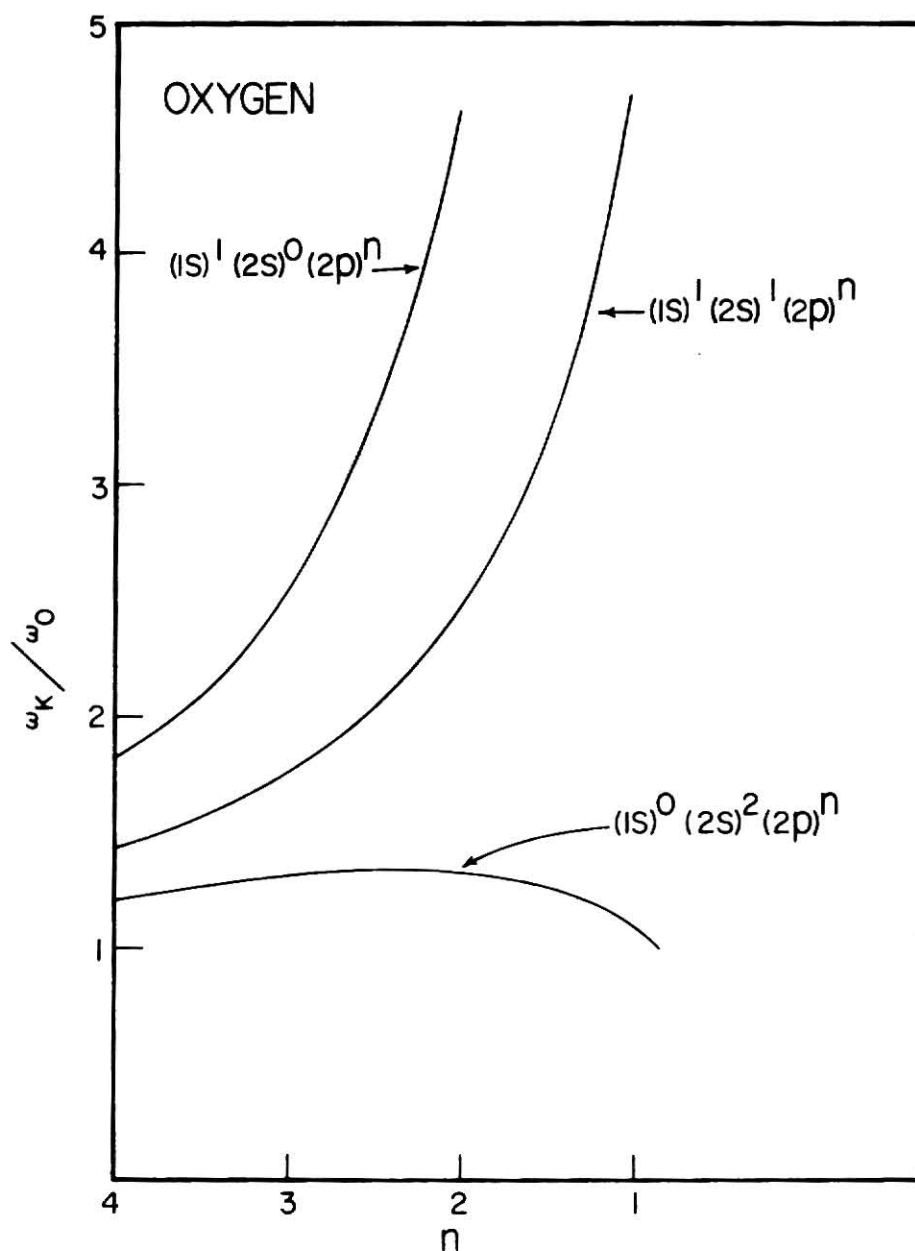


Fig. 8 Normalized K-shell fluorescence yields, calculated with the HFS model, versus the number of 2p-shell electrons for various configurations of oxygen. ω_0 is the value of ω_K for the configuration $(1s)^1(2s)^2(2p)^4$.

CHAPTER V

COMPARISON WITH EXPERIMENT

The experimental value⁴⁷ of the K-shell fluorescence yield for the electronic configuration $(1s)^1(2s)^2(2p)^6$ is 0.018 ± 0.04 , which is to be compared with our calculated value of 0.016. The calculated Auger electron energies are also in excellent agreement with the high-energy resolution measurements⁴⁸. The experimental Auger intensities relative to the $1s-2s-2s$ (1S_0) Auger transition for low Z values do not agree with the values calculated in the L-S coupling scheme. The configuration interaction between the final states (1S_0) arising from the $1s-2s-2s$ and the $1s-2p-2p$ Auger transitions has been shown by Asaad⁴⁹, and Mehlorn and Asaad⁵⁰ to be important. The detailed calculations of Bhalla⁵¹, with the inclusion of the above-mentioned configuration interaction agree reasonably well with the high-energy resolution data of the relative Auger intensities⁴⁶. It should be noted that the total Auger rates are independent of the choice of the coupling scheme.

As mentioned earlier, the K-shell vacancy produced in heavy ion-atom collision is usually accompanied by multiple inner shell ionization. This is in contrast to the proton-atom or electron-atom collisions. Burch et al.²⁹ have measured the ratios of the yields for the 30 MeV oxygen-neon and the 5 MeV proton-neon collisions. A relative fluorescence yield, $\omega_K(\text{oxygen})/\omega_K(\text{proton}) = 2.4 \pm 0.5$ was obtained. Their measurements also included the x-ray transition energies ($2p \rightarrow 1s$) and the Auger electron energies. The population of the various defect

configurations of neon, produced in the oxygen-neon collision cannot be inferred by a comparison of the theoretical energy shifts with the low-resolution experimental data.⁵² However, the following configurations: $(1s)^1(2s)^1(2p)^3$, $(1s)^1(2s)^1(2p)^2$, $(1s)^1(2s)^0(2p)^4$, and $(1s)^1(2s)^0(2p)^3$ are consistent with the relative fluorescence yield and the observed shifts in the x-ray and Auger transition energies.

CHAPTER VI

DISCUSSION AND CONCLUSIONS

We have presented in this thesis the x-ray rates, the Auger groups rates and the K-shell fluorescence yield for multiply ionized configurations of neon, oxygen and nitrogen. The calculations are performed with the nonrelativistic Hartree-Fock-Slater model and the exchange contributions are included by the HVDO approximation. The shifts in the x-ray transition energies and Auger electron energies are also presented.

The electronic configurations, which are consistent with $\Delta E_x, \Delta E_A$ and the relative K-shell fluorescence yield in the 30 MeV oxygen-neon collision, were found to be $(1s)^1(2s)^1(2p)^3$, $(1s)^1(2s)^1(2p)^2$, $(1s)^1(2s)^0(2p)^4$, and $(1s)^1(2s)^0(2p)^3$. The relative population of the defect configuration cannot be ascertained until high energy resolution data become available.

The statistical scaling procedure, which is in common use, is found to be in error for most defect configurations of neon, oxygen and nitrogen. This approximate procedure, therefore, should be used with caution for low Z .

In conclusion, the theoretical results for defect configurations of neon, oxygen and nitrogen are presented, and these are essential in the correct interpretation of the experimental data pertaining to heavy ion-gas collisions.

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APPENDIX A

CALCULATION OF TOTAL ENERGY

A numerical subroutine was employed to calculate the total energy and its constituent parts for the atomic systems investigated in this thesis. The results of these calculations are presented in Tables I through XI. To obtain the total energy of an atomic system one can calculate the expectation value of the HF Hamiltonian, using the HF bound state wave-functions.

The HF Hamiltonian is,

$$H = \sum_i \left(\frac{p_i^2}{2m} - \frac{Ze^2}{r_i} \right) + \frac{1}{2} \sum_{i \neq j} \frac{e^2}{r_{ij}} \quad (1)$$

The total energy is then given by,

$$E_{HF} = \langle \psi_{HF} | H | \psi_{HF} \rangle$$

$$\begin{aligned} E_{HF} = & \sum_i \langle \phi_i | \frac{p_i^2}{2m} | \phi_i \rangle - \sum_i \langle \phi_i | \frac{Ze^2}{r_i} | \phi_i \rangle \\ & + \frac{1}{2} \sum_{i,j} \langle \phi_i(1) \phi_j(2) | \frac{e^2}{r_{12}} | \phi_i(1) \phi_j(2) \rangle \\ & - \frac{1}{2} \sum_{i,j} \langle \phi_i(1) \phi_j(2) | \frac{e^2}{r_{12}} | \phi_i(2) \phi_j(1) \rangle \end{aligned}$$

or,

$$E_{HF} = KE + NIE + DEIE + XEIE, \quad (2)$$

where KE is the kinetic energy, NIE is the Coulomb nuclear interaction energy, DEIE is the direct electron-electron interaction energy, and XEIE is the exchange electron-electron interaction energy.

To scale to atomic units we take $\frac{\hbar^2}{2ma_0^2} = \frac{e^2}{2a_0} \equiv 1 R_y$.

For further convenience take $\phi_i(\vec{r}) = R_{nl}(r) Y_{lm}(\hat{r}) \alpha_\sigma$,

and define $P_{nl}(r) \equiv r R_{nl}(r)$, where the $R_{nl}(r)$ are the radial

functions, $Y_{\ell m}(\hat{r})$ are the angular functions, and α_σ are the spin functions.

Consider the terms of equation (2) separately:

KE $\equiv$

$$\begin{aligned} \langle \phi_i | \frac{p^2}{2m} | \phi_i \rangle &= 1 R_y \langle R_{n\ell}(r) Y_{\ell m}(\hat{r}) | -\nabla^2 | R_{n\ell}(r) Y_{\ell m}(\hat{r}) \rangle \\ &= 1 R_y \int_0^\infty \left\{ \left(\frac{d}{dr} P_{n\ell} \right)^2 + \frac{\ell(\ell+1)}{r^2} P_{n\ell}^2 \right\} dr . \end{aligned}$$

NIE $\equiv$

$$\langle \phi_i | -\frac{Ze^2}{r} | \phi_i \rangle = -2Z R_y \int_0^\infty \frac{P_{n\ell}^2}{r} dr .$$

DEIE $\equiv$

$$\begin{aligned} \sum_{i,j} \langle \phi_i(1) \phi_j(2) | \frac{e^2}{r_{12}} | \phi_i(1) \phi_j(2) \rangle &= \\ 2 R_y \sum_{n,\ell,m,\sigma} \sum_{n',\ell',m',\sigma'} \int d^3r_1 R_{n\ell}(r_1) |Y_{\ell m}(\hat{r}_1)|^2 \int d^3r_2 R_{n'\ell'}(r_2) |Y_{\ell'm'}(\hat{r}_2)|^2 \frac{1}{|\vec{r}_1 - \vec{r}_2|} . \end{aligned}$$

For closed shells $4\pi \sum_{m,\sigma} |Y_{\ell m}(\hat{r})|^2 = 2(2\ell+1)$

To account for non-closed shell, we take $4\pi \sum_{m,\sigma} |Y_{\ell m}(\hat{r})|^2 = N(n,\ell)$.

Expanding $\frac{1}{|\vec{r}_1 - \vec{r}_2|} = \sum_{\kappa} \frac{r_<^\kappa}{r_>^{\kappa+1}} P_\kappa(\hat{r}_1 \cdot \hat{r}_2)$, and performing the angular integration, only the $\kappa=0$ term contributes, so we get

$$DEIE = 1 R_y \sum_{n,\ell} N(n,\ell) \sum_{n',\ell'} N'(n',\ell') \int_0^\infty P_{n\ell}^2(r_1) dr_1 \int_0^\infty P_{n'\ell'}^2(r_2) dr_2 \frac{1}{r_>} .$$

XEIE $\equiv$

$$\begin{aligned} \sum_{i,j} \langle \phi_i(1) \phi_j(2) | \frac{e^2}{r_{12}} | \phi_i(2) \phi_j(1) \rangle &= \\ 2 R_y \sum_{n,\ell,m,\sigma} \sum_{n',\ell',m',\sigma'} \int d^3r_1 R_{n\ell}(r_1) Y_{\ell m}^*(\hat{r}_1) R_{n'\ell'}(r_1) Y_{\ell'm'}(\hat{r}_1) \alpha_\sigma^* \cdot \alpha_{\sigma'} \times \\ \int d^3r_2 R_{n'\ell'}(r_2) Y_{\ell'm'}^*(\hat{r}_2) R_{n\ell}(r_2) Y_{\ell m}(\hat{r}_2) \alpha_{\sigma'}^* \cdot \alpha_\sigma \frac{1}{|\vec{r}_1 - \vec{r}_2|} . \end{aligned}$$

Making the assumption that the spins are paired, the integral over the cross terms cancel, and the integral over the direct terms contribute a factor of 2, so,

$$XEIE = 2R_y \times 2 \sum_{n,\ell} \sum_{n',\ell'} \int_0^\infty r_1^2 dr_1 R_{n\ell}(r_1) R_{n'\ell'}(r_1) \int_0^\infty r_2^2 dr_2 R_{n\ell}(r_2) R_{n'\ell'}(r_2) \times \\ \iint d\Omega_1 d\Omega_2 \sum_{m,\ell} \sum_{m',\ell'} \sum_{k,\ell''} \frac{4\pi}{(2k+1)} \frac{r_{\ell''}^k}{r_{\ell''}^{k+1}} Y_{\ell m}^*(\hat{r}_1) Y_{\ell' m'}(\hat{r}_1) Y_{k m''}(\hat{r}_1) Y_{\ell m}(\hat{r}_2) Y_{\ell' m'}^*(\hat{r}_2) Y_{k m''}^*(\hat{r}_2)$$

Now express the three sums over azimuthal indices of the product spherical harmonics as Legendre functions.

Note that we have already taken into account the spin so the factor corresponding to $\frac{(2\ell+1)}{4\pi}$ is $\frac{N(n,\ell)}{2}$. Performing the above operations

we get,

$$XEIE = -\frac{1}{2} R_y \sum_{n,\ell} N(n,\ell) \sum_{n',\ell'} N'(n',\ell') \int_0^\infty dr_1 P_{n\ell}(r_1) P_{n'\ell'}(r_1) \times \\ \int_0^\infty dr_2 P_{n\ell}(r_2) P_{n'\ell'}(r_2) \sum_{k,\ell''} \frac{r_{\ell''}^k}{r_{\ell''}^{k+1}} \iint d\Omega_1 d\Omega_2 P_\ell(\hat{r}_1 \cdot \hat{r}_2) P_{\ell'}(\hat{r}_1 \cdot \hat{r}_2) P_k(\hat{r}_1 \cdot \hat{r}_2) / (4\pi)^2$$

or,

$$XEIE = -\frac{1}{2} R_y \sum_{n,\ell} N(n,\ell) \sum_{n',\ell'} N'(n',\ell') \int_0^\infty dr_1 P_{n\ell}(r_1) P_{n'\ell'}(r_1) \int_0^\infty dr_2 P_{n\ell}(r_2) P_{n'\ell'}(r_2) \times f(\ell, \ell', r, r').$$

$$f(\ell, \ell', r, r') \equiv \sum_{k,\ell''} \frac{r_{\ell''}^k}{r_{\ell''}^{k+1}} A_k(\ell, \ell') ;$$

$$A_k(\ell, \ell') \equiv \iint d\Omega_1 d\Omega_2 P_\ell(\hat{r}_1 \cdot \hat{r}_2) P_{\ell'}(\hat{r}_1 \cdot \hat{r}_2) P_k(\hat{r}_1 \cdot \hat{r}_2) / (4\pi)^2 .$$

Further,

$$A_k(\ell, \ell') = \frac{1}{2} \int_{-1}^1 dx P_\ell(x) P_{\ell'}(x) P_k(x)$$

or in terms of $3j$ symbols,¹

$$A_k(\ell, \ell') = \begin{pmatrix} \ell & \ell' & k \\ 0 & 0 & 0 \end{pmatrix}^2 .$$

The total energy is found by summing up the four separate parts, as in equation (2).

By specializing the notation to that employed previously, and restricting the shells to those up to and including the 2p-shell, we can simplify the total energy expression.

As before, we define

$$R^{\kappa}(1,2,3,4) \equiv \int_0^{\infty} \int_0^{\infty} P_{n_1 \ell_1}(r_i) P_{n_2 \ell_2}(r_j) \frac{r_i^{\kappa}}{r_j^{\kappa+1}} P_{n_3 \ell_3}(r_i) P_{n_4 \ell_4}(r_j) dr_i dr_j \quad .$$

Then,

$$DEIE = \frac{1}{2} \sum_{n, \ell} N(n, \ell) \sum_{n', \ell'} N'(n', \ell') R^0(n\ell, n'\ell', n\ell, n'\ell') \quad ;$$

$$XEIE = -\frac{1}{4} \sum_{n, \ell} N(n, \ell) \sum_{n', \ell'} N'(n', \ell') \sum_{\kappa} R^{\kappa}(n\ell, n\ell, n'\ell', n'\ell') A_{\kappa}(\ell, \ell')$$

where $A_{\kappa} = \begin{pmatrix} \ell & \ell' & \kappa \\ 0 & 0 & 0 \end{pmatrix}^2$.

In general $N(n, \ell) N'(n', \ell')$ is the product of the full-shell occupation numbers, multiplied by the existing number of possible pairs of electrons, and divided by the full-shell number of pairs of electrons. This procedure guarantees that each term is averaged correctly. Let m_i and n_i be the actual occupation numbers and m_o and n_o be the full-shell occupation numbers of two distinct shells.

Then

$$N(n, \ell) N'(n', \ell') = \begin{cases} m_i n_i & n' \neq n \text{ and } \ell' \neq \ell \\ \frac{n_i(n_i-1) n_o^2}{n_o(n_o-1)} & n' = n \text{ and } \ell' = \ell \end{cases} .$$

So for the general configuration $(1s)^2(2s)^m(2p)^n$,

we obtain,

$$\begin{aligned} DEIE = \frac{1}{2} [& 2\ell(\ell-1) R^0(1s, 1s, 1s, 1s) + 2m(m-1) R^0(2s, 2s, 2s, 2s) \\ & + \frac{6}{5} n(n-1) R^0(2p, 2p, 2p, 2p) + 2\ell m R^0(1s, 2s, 1s, 2s) \\ & + 2\ell n R^0(1s, 2p, 1s, 2p) + 2mn R^0(2s, 2p, 2s, 2p)] \end{aligned}$$

and,

$$\begin{aligned} XEIE = & -\frac{1}{4} \left\{ 2\ell(\ell-1) R^0(1\downarrow, 1\downarrow, 1\downarrow, 1\downarrow) + 2m(m-1) R^0(2\downarrow, 2\downarrow, 2\downarrow, 2\downarrow) \right. \\ & + \frac{6}{5} n(n-1) \left[\frac{1}{3} R^0(2p, 2p, 2p, 2p) + \frac{2}{15} R^2(2p, 2p, 2p, 2p) \right] \\ & + 2\ell m R^0(1\downarrow, 1\downarrow, 2\downarrow, 2\downarrow) + \frac{2}{3} \ell n R^1(1\downarrow, 1\downarrow, 2p, 2p) \\ & \left. + \frac{2}{3} mn R^1(2\downarrow, 2\downarrow, 2p, 2p) \right\} ; \end{aligned}$$

$$E_{HF} = KE + NIE + DEIE + XEIE .$$

This total energy is consistent with the results given by Slater².

In Slater's notation $F^0(n\ell, n'\ell') \equiv R^0(n\ell, n'\ell', n\ell, n'\ell') = F^0(n'\ell', n\ell)$

and $G^k(n\ell, n'\ell') \equiv R^k(n\ell, n\ell, n'\ell', n'\ell') = G^k(n'\ell', n\ell)$, and the total energy

in Rydbergs is given by,

$$\begin{aligned} E_{HF} = & KE + NIE + \frac{\ell(\ell-1)}{2} F^0(1\downarrow, 1\downarrow) + \frac{m(m-1)}{2} F^0(2\downarrow, 2\downarrow) \\ & + \frac{n(n-1)}{2} [F^0(2p, 2p) - \frac{2}{25} F^2(2p, 2p)] + \ell m [F^0(1\downarrow, 2\downarrow) - \frac{1}{2} G^0(1\downarrow, 2\downarrow)] \\ & + \ell n [F^0(1\downarrow, 2p) - \frac{1}{6} G^1(1\downarrow, 2p)] + mn [F^0(2\downarrow, 2p) - \frac{1}{6} G^1(2\downarrow, 2p)] . \end{aligned}$$

An analogous treatment can be made to obtain the HF binding energy, ϵ_j .

$$H\phi(i) = \left(\frac{p_i^2}{2m} - \frac{Ze^2}{r_i} \right) \phi(i) + \sum_i \int d^3r_2 \phi_i^*(2) \frac{e^2}{|r_1 - r_2|} (\phi_i(2)\phi(i) - \phi_i(i)\phi(2)) .$$

$$\epsilon_j = \int d^3r_i \phi_j^*(i) H\phi_j(i) = KE_j + E_{dj} + E_{xj} ,$$

where KE_j , $E_{d,j}$, and $E_{x,j}$ are the kinetic, direct electron-electron, and exchange electron-electron contributions respectively. j now denotes the set of quantum labels of the j th. electron.

$$KE_j = 1/R_y \int_0^\infty dr \left\{ \left(\frac{d}{dr} P_j \right)^2 + \frac{\ell(\ell+1)}{r^2} P_j^2 \right\} .$$

$$E_{d,j} = 1/R_y \int_0^\infty dr P_j(r) V_d(r)$$

$$\text{where } V_d(r) \equiv -\frac{2Z}{r} + 2 \int_0^\infty dr_2 \frac{Q(r_2)}{r_2 r}$$

$$\text{and } Q(r) \equiv \sum_{n,\ell} N(n,\ell) P_{n\ell}^2 .$$

$$E_{x,j} = -1/R_y \sum_{n,\ell} \frac{N(n,\ell)}{2} \int_0^\infty dr_1 P_j(r_1) P_{n\ell}(r_1) \int_0^\infty dr_2 P_j(r_2) P_{n\ell}(r_2) f(\ell, \ell_j, r, r') ,$$

$$\text{where } f(\ell, \ell_j, r, r') \equiv \sum_k \frac{r_2^k}{r_1^{k+1}} A_k(\ell, \ell_j)$$

and

$$A_k(\ell, \ell_j) \equiv \iint d\Omega_1 d\Omega_2 P_\ell(\hat{r}_1 \cdot \hat{r}_2) P_{\ell_j}(\hat{r}_1 \cdot \hat{r}_2) P_k(\hat{r}_1 \cdot \hat{r}_2) / (4\pi)^2 .$$

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TABLE IV (cont.)

$1s^1 2s^2 2p^3$	1s	-82.2818	97.6374	-197.6099	30.5676	-12.8768	0.0808
$1s^1 2s^2 2p^3$	2s	-11.3702	15.4893	-39.5058	15.9828	-3.3365	0.3927
$1s^1 2s^2 2p^3$	2p	-10.0611	15.0720	-38.6819	17.0697	-3.5210	0.3561
		Total Energy = -177.0203					
$1s^1 2s^2 2p^2$	1s	-85.4932	97.9053	-197.8831	27.3264	-12.8418	0.0807
$1s^1 2s^2 2p^2$	2s	-13.5620	16.7004	-40.9888	13.9275	-3.2011	0.3786
$1s^1 2s^2 2p^2$	2p	-12.5928	16.3128	-40.3033	14.9867	-3.5890	0.3381
		Total Energy = -167.1691					
$1s^1 2s^2 2p^1$	1s	-89.0125	98.2924	-198.2764	23.7689	-12.7974	0.0805
$1s^1 2s^2 2p^1$	2s	-15.9102	17.9247	-42.4384	11.6494	-3.0459	0.3657
$1s^1 2s^2 2p^1$	2p	-15.3224	17.5135	-41.8002	12.5967	-3.6324	0.3230
		Total Energy = -154.7267					
$1s^1 2s^2 2p^0$	1s	-92.8122	98.8601	-198.8512	19.9268	-12.7480	0.0801
$1s^1 2s^2 2p^0$	2s	-18.4015	19.0700	-43.7576	9.1561	-2.8700	0.3542
		Total Energy = -139.4907					

TABLE V (cont.)

$1s^1 1s^1 2p^3$	1s	-85.1287	97.7769	-197.7520	27.6200	-12.7736	0.0808
$1s^1 2s^1 2p^3$	2s	-13.7097	16.4844	-40.7275	13.9722	-3.4388	0.3809
$1s^1 2s^1 2p^3$	2p	-12.2645	16.1219	-40.0595	15.0856	-3.4124	0.3405
$1s^1 1s^1 2p^2$	1s	-88.5829	Total Energy = -165.8315 98.0894	-198.0702	24.1220	-12.7241	0.0806
$1s^1 2s^1 2p^2$	2s	-16.0403	17.7183	-42.1945	11.7296	-3.2937	0.3678
$1s^1 2s^1 2p^2$	2p	-14.9551	17.3490	-41.5987	12.7523	-3.4577	0.3249
$1s^1 2s^1 2p^1$	1s	-92.3220	Total Energy = -153.7448 98.5169	-198.5044	20.3310	-12.6656	0.0803
$1s^1 2s^1 2p^1$	2s	-18.5198	18.9989	-43.6688	9.2852	-3.1351	0.3553
$1s^1 2s^1 2p^1$	2p	-17.8322	18.5293	-43.0185	10.1398	-3.4828	0.3115
$1s^1 2s^1 2p^0$	1s	-96.2314	Total Energy = -138.9103 98.1198	-198.1024	16.2906	-12.5394	0.0805
$1s^1 2s^1 2p^0$	2s	-21.1418	21.6675	-46.5605	6.7759	-3.0247	0.3377
			Total Energy = -121.1245				

TABLE III Calculated H.F. binding energy, kinetic energy, nuclear interaction energy, electron-electron interaction energy in Rydbergs, and $\langle r \rangle$ in Angstroms for neon with configuration $1s^2 2s^2 2p^6$.

Configuration	Shell	HF, B.E.	K.E.	N.I.E.	E.E.I.E.	EX.E.	$\langle r \rangle$ in Å
$0_1 1^6$							
$1s 2s 2p$	1s	-86.4209	99.3552	-199.3466	26.7474	-13.1769	0.0800
$0_1 1^6$							
$1s 2s 2p$	2s	-10.0963	15.5085	-39.4858	17.7862	-3.9052	0.3959
$0_1 1^6$							
$1s 2s 2p$	2p	-8.1083	15.9139	-39.6570	19.1545	-3.5197	0.3525
			Total Energy = -112.5913				
$0_1 1^5$							
$1s 2s 2p$	1s	-89.3771	99.6664	-199.6587	23.7678	-13.1526	0.0799
$0_1 1^5$							
$1s 2s 2p$	2s	-12.1919	16.7654	-41.0298	15.8666	-3.7941	0.3810
$0_1 1^5$							
$1s 2s 2p$	2p	-10.5043	17.1957	-41.3151	17.2106	-3.5954	0.3339
			Total Energy = -104.7873				
$0_1 1^4$							
$1s 2s 2p$	1s	-92.6581	100.0640	-200.0574	20.4528	-13.1175	0.0797
$0_1 1^4$							
$1s 2s 2p$	2s	-14.4527	18.1197	-42.6281	13.7191	-3.6634	0.3672
$0_1 1^4$							
$1s 2s 2p$	2p	-13.1129	18.5023	-42.9207	14.9566	-3.6511	0.3179
			Total Energy = -94.5430				

TABLE IX (cont.)

$1s^0 2s^0 2p^2$	1s	-103.4554	97.7909	-197.7773	9.1832	-12.6522	0.0803
$1s^0 2s^0 2p^2$	2p	-21.6366	22.2637	-47.1760	6.8128	-3.5372	0.2824
			Total Energy = -46.5489				
$1s^0 2s^0 2p^1$	1s	-107.8146	99.9999	-199.9998	4.8560	-12.6706	0.0793
$1s^0 2s^0 2p^1$	2p	-24.9996	24.9999	-49.9995	3.6328	-3.6328	0.2645
			Total Energy = -24.9996				

Table X Total energy in Rydbergs and expectation value of r in Angstroms of all shells for various configurations of oxygen ($1s^2 2s^m 2p^n$).

Configuration			Total Energy	$\langle r \rangle_{1s}$	$\langle r \rangle_{2s}$	$\langle r \rangle_{2p}$
l	m	n				
2	2	4	-149.4644	0.1053	0.6100	0.7194
2	2	3	-148.4101	0.1052	0.5873	0.6220
2	2	2	-146.0491	0.1051	0.5605	0.5592
2	2	1	-142.1495	0.1049	0.5344	0.5134
2	2	0	-136.4913	0.1046	0.5103	---
2	1	4	-147.1555	0.1052	0.5868	0.6203
2	1	3	-144.9019	0.1051	0.5616	0.5607
2	1	2	-141.1593	0.1050	0.5364	0.5158
2	1	1	-135.7077	0.1047	0.5116	0.4799
2	1	0	-128.3482	0.1051	0.4739	---

Table x (Cont.)

2	0	4	-143.4562	0.1052	---	0.5624
2	0	3	-139.8485	0.1050	---	0.5185
2	0	2	-134.5820	0.1048	---	0.4813
2	0	1	-127.4625	0.1052	---	0.4310
2	0	0	-118.1856	0.1061	---	---
1	2	4	-109.4180	0.1014	0.5470	0.5419
1	2	3	-106.6988	0.1013	0.5217	0.4940
1	2	2	-102.3136	0.1011	0.4977	0.4578
1	2	1	- 96.0443	0.1008	0.4759	0.4295
1	2	0	- 87.6870	0.1003	0.4564	0.4077
1	1	4	-105.5180	0.1014	0.5244	0.4975
1	1	3	-101.3276	0.1012	0.5009	0.4613
1	1	2	- 95.3033	0.1010	0.4790	0.4323
1	1	1	- 87.2365	0.1006	0.4582	0.4085
1	1	0	- 76.9210	0.1010	0.4291	---

Table X (Cont.)

1	0	4	- 99.9914	0.1013	---	0.4653
1	0	3	- 94.1873	0.1011	---	0.4358
1	0	2	- 86.3927	0.1008	---	0.4100
1	0	1	- 76.4092	0.1013	---	0.3739
1	0	0	- 63.9997	0.0992	---	---
0	2	4	- 61.9246	---	0.4949	0.4454
0	2	3	- 57.0569	---	0.4715	0.4162
0	2	2	- 50.1885	---	0.4503	0.3930
0	2	1	- 41.1129	---	0.4310	0.3740
0	2	0	- 29.6370	---	0.4118	---
0	1	4	- 56.0982	---	0.4777	0.4198
0	1	3	- 49.5127	---	0.4562	0.3955
0	1	2	- 40.7668	---	0.4365	0.3756
0	1	1	- 29.6559	---	0.4177	0.3596

Table x (Cont.)

0 0 4	- 48.4372	---	---	0.3992
0 0 3	- 39.9987	---	---	0.3780
0 0 2	- 29.2478	---	---	0.3590
0 0 1	- 15.9991	---	---	0.3306

Table XI Total energy in Rydbergs and expectation value of r in Angstroms of all shells for various configurations of nitrogen ($1s^2 2s^m 2p^n$).

Configuration			Total Energy	$\langle r \rangle_{1s}$	$\langle r \rangle_{2s}$	$\langle r \rangle_{2p}$
l	m	n				
2	2	3	-108.5268	0.1211	0.7108	0.8626
2	2	2	-107.6443	0.1210	0.6805	0.7280
2	2	1	-105.5907	0.1208	0.6447	0.6463
2	2	0	-102.1404	0.1204	0.6104	----
2	1	3	-106.7495	0.1210	0.6793	0.7242
2	1	2	-104.7967	0.1208	0.6460	0.6473
2	1	1	-101.4955	0.1206	0.6117	0.5902
2	1	0	- 96.6465	0.1212	0.5579	----

Table XI (Cont.)

2	0	3	-103.7421	0.1209	---	0.6488
2	0	2	-100.5709	0.1206	---	0.5913
2	0	1	- 95.9064	0.1213	---	0.5158
2	0	0	- 89.4353	0.1225	---	---
1	2	3	- 78.2943	0.1159	0.6232	0.6175
1	2	2	- 75.8891	0.1157	0.5906	0.5582
1	2	1	- 71.9562	0.1154	0.5604	0.5151
1	2	0	- 66.2863	0.1148	0.5336	---
1	1	3	- 75.0794	0.1159	0.5942	0.5622
1	1	2	-71.3357	0.1156	0.5643	0.5185
1	1	1	-65.9014	0.1152	0.5359	0.4841
1	1	0	- 58.5702	0.1157	0.4963	---

Table XI (Cont.)

1	0	3	- 70.4012	0.1157	---	0.5228
1	0	2	- 65.1838	0.1154	---	0.4858
1	0	1	- 58.1299	0.1161	---	0.4356
1	0	0	- 48.9998	0.1134	---	---
0	2	3	- 41.4299	---	0.5539	0.4961
0	2	2	- 37.0212	---	0.5248	0.4624
0	2	1	- 30.7561	---	0.4987	0.4358
0	2	0	- 22.4383	---	0.4733	---
0	1	3	- 36.4488	---	0.5328	0.4657
0	1	2	- 30.4582	---	0.5061	0.4380
0	1	1	- 22.4518	---	0.4810	0.4161

Table XI (Cont.)

0	0	3	- 24.7995	---	---	0.4411
0	0	2	- 22.0981	---	---	0.4154
0	0	1	- 12.2493	---	---	0.3778

APPENDIX B

CASCADE EFFECTS ON THE FLUORESCENCE YIELD

In this work we have presented explicit calculations to obtain the K-shell fluorescence yield, however, similar calculations could be performed for higher shells. For the K-shell, the experimental x-ray cross section, σ_K^x , is related to the ionization cross section, σ_K^i , by $\sigma_K^x = \omega_K \sigma_K^i$, where ω_K is the K-shell fluorescence yield. Extension of this relation to higher shells requires modification of the definition of the fluorescence yield. To illustrate, consider the L-shell. Coster-Kronig transitions are now possible. These are Auger transitions between the various L subshells. Explicitly, for the L-shell, these are the Auger transitions $L_1 - L_2 - X_\alpha$, $L_1 - L_3 - Y_\beta$, and $L_2 - L_3 - Z_\gamma$, where X_α , Y_β , and Z_γ represent any Auger electrons which are energetically possible. It is clear that these Coster-Kronig transitions can shift a primary vacancy in the L_1 or L_2 subshells to a vacancy in the L_3 subshell, with the simultaneous ejection of an M subshell Auger electron.

The extension of the relation between the x-ray cross section and the ionization cross section can be formulated most easily by defining a modified fluorescence yield, ω_{L_1} , equal to the ratio of the number of L x-rays produced to the number of primary vacancies in the L_1 subshell. Further, we define the value of $f_{L_j L_k}$ as the probability of an L_j subshell vacancy being filled by an L_k subshell electron.

Consider the specific case of one vacancy in the L_1 subshell. Then, the number of L_1 x-rays is equal to ω_{L_1} ; the number of L_2 x-rays is equal to $f_{L_1 L_2} \omega_{L_2}$; and the number of L_3 x-rays is equal to $f_{L_1 L_3} \omega_{L_3} + f_{L_1 L_2} f_{L_2 L_3} \omega_{L_3}$. The total number of x-rays for the L-shell is the sum of the number of x-rays for the individual L subshells, so the total number of L x-rays produced per L_1 vacancy is,

$$\nu_{L_1} = \omega_{L_1} + f_{L_1 L_2} \omega_{L_2} + (f_{L_1 L_3} + f_{L_1 L_2} f_{L_2 L_3}) \omega_{L_3}. \quad (1)$$

Similarly, for an initial L_2 subshell vacancy, we have

$$\nu_{L_2} = \omega_{L_2} + f_{L_2 L_3} \omega_{L_3}. \quad (2)$$

And, for an initial L_3 subshell vacancy, we have simply

$$\nu_{L_3} = \omega_{L_3}. \quad (3)$$

The relation between the x-ray production cross section and the inner shell ionization cross sections is

$$\sigma_L^x = \sigma_{L_1}^i \nu_{L_1} + \sigma_{L_2}^i \nu_{L_2} + \sigma_{L_3}^i \nu_{L_3}$$

where $\sigma_{L_1}^i$, $\sigma_{L_2}^i$, and $\sigma_{L_3}^i$ are the inner-shell ionization cross sections for the L_1 , L_2 , and L_3 shells respectively.

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K-SHELL AUGER AND X-RAY RATES, TRANSITION ENERGIES, AND
FLUORESCENCE YIELDS FOR MULTIPLY IONIZED NEON, OXYGEN, AND NITROGEN

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ABSTRACT

The K-shell Auger rates, x-ray rates, fluorescence yields, and transition energies for multiply ionized configurations of neon, oxygen, and nitrogen are presented. Numerical calculations have been performed using a non-relativistic Hartree-Fock-Slater model with the Herman-Van Dyke-Ortenburger exchange approximation. Comparisons of these explicit calculations with calculations based on the statistical model are reported. Differences up to 40% are shown for the fluorescence yield. Much higher differences, up to 80%, are found for the transition rates.