Balancing the Moments Produced by a Variably Loaded Crank.

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A variably loaded crank is one that is loaded and relieved at certain intervals of its path of rotation. A good example of one is a hay press, or common pump crank.

A moment is the measure of a tendency of a force to rotate about a fixed point. It is the product of the force into the perpendicular distance from the point of rotation to the line in the direction of the acting force.

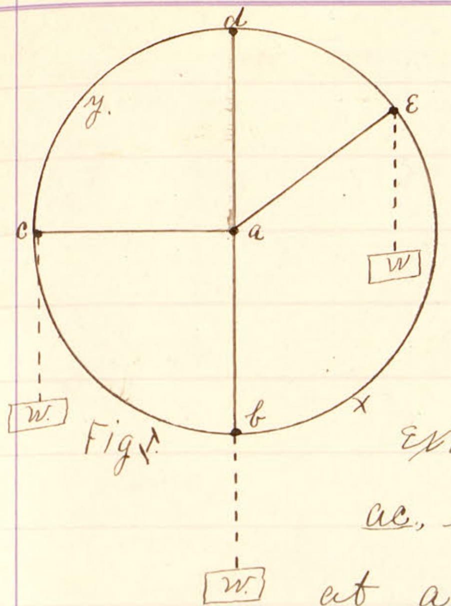


Fig I.

For a diagramatic example of a uniformly loaded crank, let <u>xy</u> in Fig I be the path of the crank and <u>W.</u> the load to be lifted.

In the first position <u>ab</u> the crank exerts no moment. In a new position, say <u>ac</u>, it exerts a maximum moment, while at <u>ad</u> it is relieved of the load and its moments. On the downward stroke instead of being retarded the crank is pulled forward untill ~~untill~~ it reaches its original position at <u>b</u> where it has no moment. and is at rest. From <u>b</u> to <u>d</u> the load is lifted and is positive work, while from <u>d</u> to <u>b</u> the load is retarded and is negative work. In the one energy is spent while in the other it is stored up.

It is these moments that I desire to balance and make more uniform.

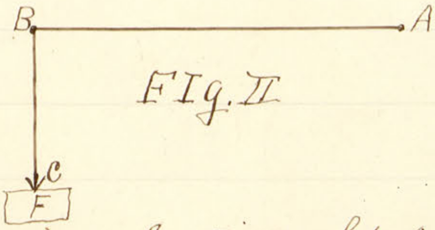
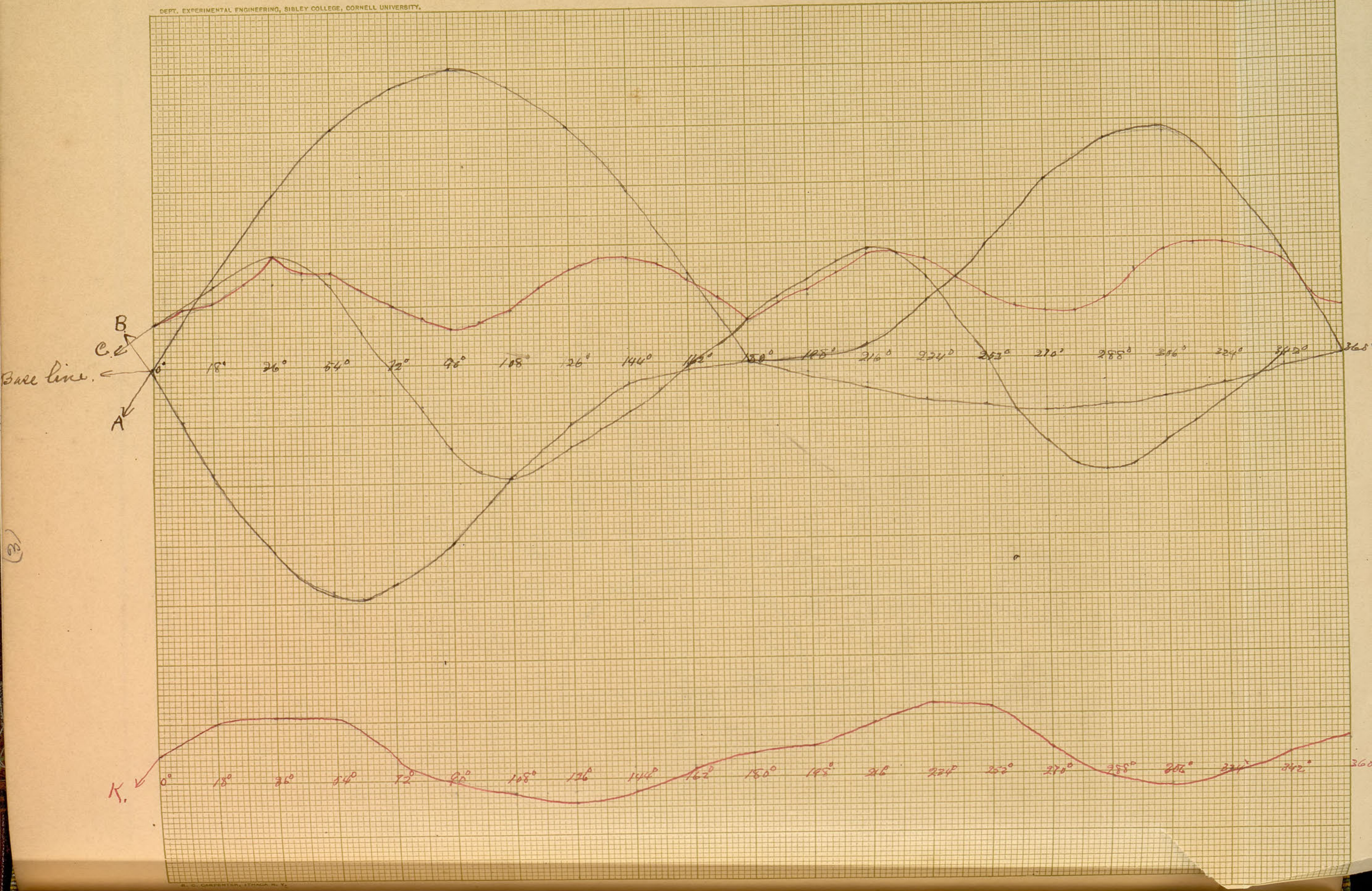


FIG. II

In FIG II let A.B., called the lever arm, be the perpendicular distance from <u>a</u>, the point of rotation, to <u>bc</u> the line in the direction of the acting force F. Then <u>M</u> the moment, equals F X <u>ab</u>.

With the above principles and formula I will determine, by calculation, the moments produced by a variably


B.
C.
A
Base line.
K.
0°
18°
36°
54°
72°
90°
108°
126°
144°
162°
180°
198°
216°
234°
252°
270°
288°
306°
324°
342°
360°

(3)

loaded crank. The results are shown on the plot page 3. The distances above the base line of the plot, represent the moments at various places, those along the base line represent the angles along the cranks path. The lines connecting these various points determine the moments of all the various positions of the crank in its rotation.

The first operation is that of calculating and plotting the moments of the loaded crank its'elf, and is represented by the line lettered <u>A</u> in the plot.

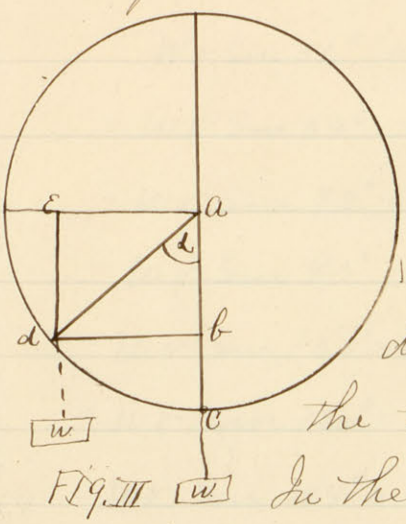


FIG. III

In FIG. III let <u>ac</u> represent a crank centred at <u>a</u> and loaded with a load <u>W</u>. Let α equal the angle of rotation from the lower radius; <u>aε</u> the perpendicular distance from the point of rotation, <u>a</u>, to the line <u>εd</u> in the direction of the force. In the first position, <u>ac</u> there is no lever arm and therefore no moment. In the second position <u>ad</u> there is a tendency to rotate back to the original position <u>ac</u>. In the formula, P × <u>AB</u> = M. P. = W., and <u>AB</u> = <u>aε</u>, <u>ad</u> is the radius of the circle (<u>r</u>), aε = r sin α. Therefore M. = W r sin α.

Assuming that the load W on the upward stroke is 5 and is reduced to 5/6 on the downward stroke, then in the downward stroke from 180° to 360° the tendancy towards rotation is 1/6 less that it is on the upward stroke. Then 1/6 of 5, the assumed weight, = .833

By multiplying r sin 2. by .833 I get the results for the downward stroke. With the above formula I am able to compute the moments of the crank throughout its entire circuit.

For convenience in plotting I will compute the moments for every 18°, and the following are the results, which are shown by line A. on the plot.

In the formula r is assumed to equal 1.

$M = Wr \sin 18° = .309 \times 5 = 1.54$

$= Wr \sin 36° = 5 \times .587 = 2.93$

$= Wr \sin 54° = 5 \times .809 = 4.05$

$= Wr \sin 72° = 5 \times .951 = 4.75$

$= Wr \sin 90° = 5 \times 1 = 5.$

$= Wr \sin 108° = 5 \times .951 = 4.75$

$= Wr \sin 126° = 5 \times .809 = 4.05$

$= Wr \sin 144° = 5 \times .587 = 2.93$

$= Wr \sin 162° = 5 \times .309 = 1.54$

$= Wr \sin 180° = 5 \times 0 = 0.$

At this point the load is reduced to 5/6 then:-

$M = \frac{W}{6} r \sin 198° = .833 \times .309 = .25$

$= \frac{W}{6} r \sin 216° = .833 \times .587 = .47$

$= \frac{W}{6} r \sin 234° = .833 \times .809 = .67$

$= \frac{W}{6} r \sin 252° = .833 \times .951 = .79$

$= \frac{W}{6} r \sin 270° = .833 \times 1 = .833$

$= \frac{W}{6} r \sin 288° = .833 \times .951 = .79$

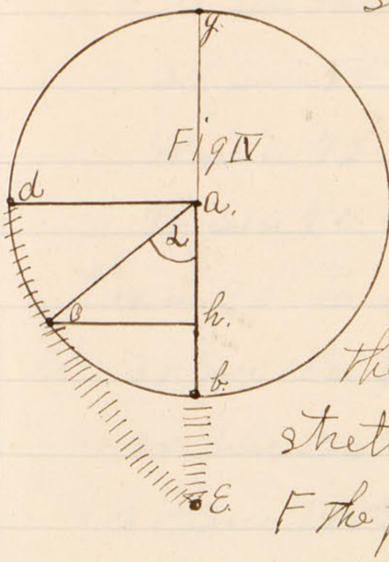
$= \frac{W}{6} r \sin 306° = .833 \times .809 = .67$

$M = \frac{W}{6} r \sin 324° = .833 \times .587 = .47$

$= \frac{W}{6} r \sin 342° = .833 \times .309 = .25$

$= \frac{W}{6} r \sin 360° = .833 \times 0 = 0.$

By the plot it can be seen that the moments vary greatly. At 90° the moment is a maximum while at 0° and 180° it is zero. It is this irregularity that I propose to remedy. To do so I will use a series of springs which will introduce other moments on the crank, at different points in its rotation.



Suppose a spring as shown in Fig IV be fastened at the point E and attached to the crank ab. Let 6 = the tension of the spring when stretched from b to E, let α = the angle of rotation. Then the stretch of the spring at c, which is bh is to the stretch at E, which is bE, as bh is to bE.

F the force at h : F at E = bh : bE.

$\underline{ah} = r \cos \alpha.$

$bh = r - r\cos\alpha.$

Stretch $= r(1 - \cos\alpha) = bh.$

$F : 6 = r(1 - \cos\alpha) : 2r.$

$F 2r = 6r(1 - \cos\alpha).$

$F = 3(1 - \cos\alpha)$

Moment $= F \times \underline{ch}.$

$\underline{ch} = r \sin\alpha.$

$\therefore M = 3(1 - \cos\alpha)\, r \sin\alpha$

With this formula I am able to compute and plot the moments of the spring.

For convenience in construction and operation I will fasten the spring from above instead of below. In this case it will have no tension at 180° and a maximum at 0°. It will store up energy on the downward stroke and give it out on the upward. For from 0° to 90° $M = 3r \sin\alpha (1 + \cos\alpha)$

$M = 3r\sin 18° (1+\cos 18°) = 3 \times .309 \times 1.951 = 1.8$

$= 3r \sin 36° (1+\cos 36°) = 3 \times .587 \times 1.809 = 3.1$

$= 3r \sin 54° (1+\cos 54°) = 3 \times .809 \times 1.587 = 3.8$

$= 3r \sin 72° (1+\cos 72°) = 3 \times .951 \times 1.309 = 3.7$

$= 3r \sin 90 (1+\cos 90) = 3 \times 1 \times 0 = 3.$

From 90° to 270° $M = 3r \sin\alpha (1+\cos\alpha)$

$M = 3r \sin 108° (1-\cos 108°) = 3 \times .951 \times .691 = 1.9$

$= 3r \sin 126° (1-\cos 126°) = 3 \times .809 \times .412 = 1.$

$= 3r \sin 144° (1-\cos 144°) = 3 \times .587 \times .191 = .3$

$= 3r \sin 162° (1-\cos 162°) = 3 \times .309 \times .049 = .1$

$= 3r \sin 180° (1-\cos 180°) = 3 \times 0 \times 1 = 0$

$= 3r \sin 198° (1-\cos 198°) = 3 \times .309 \times .049 = .1$

$= 3r \sin 216° (1-\cos 216°) = 3 \times .587 \times .191 = .3$

$= 3r \sin 234° (1-\cos 234°) = 3 \times .809 \times .412 = 1$

$= 3r \sin 252° (1-\cos 252°) = 3 \times .951 \times .691 = 1.9$

$= 3r \sin 270° (1-\cos 270°) = 3 \times 1 \times 0 = 3.$

From 270° to 360° $M = 3r \sin\alpha (1+\cos\alpha)$

$M = 3r \sin 288° (1+\cos 288°) = 3 \times .951 \times 1.309 = 3.7$

$M. = 3r \sin 306° (1 + \cos 306°) = 3 \times .809 \times 1.587 = 3.8$

$= 3r \sin 324° (1 + \cos 324°) = 3 \times .587 \times 1.809 = 3.1$

$= 3r \sin 342° (1 + \cos 342°) = 3 \times .309 \times 1.951 = 1.8$

$= 3r \sin 360° (1 + \cos 360°) = 3 \times 0 \times 1 = 0.$

The above table gives me the line B. on the plot and represents the moments of the first spring.

Still the moments are not evenly enough balanced yet, because the different changes occur in regular intervals with those of the force. We can see that both cross the line at 180° when the moments are zero. This is in opposition to balancing and something has to be done to overcome this.

To do this I will introduce a second spring which is smaller than the first. Its maximum tension to be only ½ that of the first, and with an initial tension of 1.

Let I = the initial tension of the spring.

Stretch = $R - R \cos 2. = R(1 - \cos 2)$.

Lever arm = $R \sin 2$.

Total Moment = $R \sin 2 (I + KR - KR \cos 2.)$

If the crank be uniformly loaded the moments represent a sinusoid and the maximum moment is at 90°. If the crank has a spring with no tension the maximum moment is at 120°. Therefore is some initial tension be given, the maximum will be between 90° and 120° or about 105°.

If at 105° and the maximum moment is 1.9, and $R = 1$, then $.965(I + 1.258\,K) = 1.9$, $I = 1.968 - 1.258\,K$.

If K is a very small quantity the moment curve approaches a sinusoid.

If $I = 1$, then $1 - 1.968 = -1.258\,K$, and $K = .77$.

Then for any angle other than 105° with $K = .77$ and $I = 1$. $M = 1.77 \sin \alpha - .77 \sin \alpha \cos \alpha$.

$M = 1.77 \sin 18^\circ - .77 \sin 18^\circ \cos 18^\circ = 1.77 \times .309 - .77 \times .309 \times .951 = .3$

$M = 1.77 \sin 36^\circ - .77 \sin 36^\circ \cos 36^\circ = 1.77 \times .587 - .77 \times .587 \times .809 = .8$

$= 1.77 \sin 54^\circ - .77 \sin 54^\circ \cos 54^\circ = 1.77 \times .809 - .77 \times .809 \times .587 = 1.1$

$= 1.77 \sin 72^\circ - .77 \sin 72^\circ \cos 72^\circ = 1.77 \times .951 - .77 \times .951 \times .309 = 1.4$

$= 1.77 \sin 90^\circ - .77 \sin 90^\circ \cos 90^\circ = 1.77 \times 1 - .77 \times 1 \times 0 = 1.7$

$= 1.77 \sin 108^\circ + .77 \sin 108^\circ \cos 108^\circ = 1.77 \times .951 + .77 \times .951 \times .309 = 1.9$

$= 1.77 \sin 126^\circ + .77 \sin 126^\circ \cos 126^\circ = 1.77 \times .809 + .77 \times .809 \times .587 = 1.7$

$= 1.77 \sin 144^\circ + .77 \sin 144^\circ \cos 144 = 1.77 \times .587 + .77 \times .587 \times .809 = 1.4$

$= 1.77 \sin 162^\circ + .77 \sin 162^\circ \cos 162^\circ = 1.77 \times .309 + .77 \times .309 \times .951 = .7$

$= 1.77 \sin 180^\circ + .77 \sin 180^\circ \cos 180^\circ = 1.77 \times 0 + .77 \times 0 \times 1 = 0$

$= 1.77 \sin 198^\circ + .77 \sin 198^\circ \cos 198^\circ = 1.77 \times .309 + .77 \times .309 \times .951 = .7$

$= 1.77 \sin 216^\circ + .77 \sin 261^\circ \cos 216^\circ = 1.77 \times .587 + .77 \times .587 \times .809 = 1.4$

$= 1.77 \sin 234^\circ + .77 \sin 234^\circ \cos 234^\circ = 1.77 \times .809 + .77 \times .809 \times .587 = 1.7$

$= 1.77 \sin 252^\circ + .77 \sin 252^\circ \cos 252^\circ = 1.77 \times .951 + .77 \times .951 \times .309 = 1.9$

$= 1.77 \sin 270^\circ + .77 \sin 270^\circ \cos 270^\circ = 1.77 \times 1 + .77 \times 1 \times 0 = 1.7$

$= 1.77 \sin 288^\circ - .77 \sin 288^\circ \cos 288^\circ = 1.77 \times .951 + .77 \times .951 \times .309 = 1.4$

$= 1.77 \sin 306^\circ - .77 \sin 306^\circ \cos 306^\circ = 1.77 \times .809 - .77 \times .809 \times .587 = 1.1$

$= 1.77 \sin 324^\circ - .77 \sin 324^\circ \cos 324^\circ = 1.77 \times .587 - .77 \times .587 \times .809 = .8$

$= 1.77 \sin 342^\circ - .77 \sin 342^\circ \cos 342^\circ = 1.77 \times .309 - .77 \times .309 \times .951 = .3$

The above gives me the moment lines C on the plot.
I now have three forces acting against and with each other. From 0° to 72° the W and and the second spring C act togather against the first spring B. From 72° to 162° both B and C work togather against W.
From 162° to 180° B works against W and C. From 180° to 252° B and C work togather against W. From 252° to 360 W and C work against B.
Taking the resultant of these three forces I get the following results as shown by the red line on the plot.

18° = 1.1	108° = .9	198° = 1.2	288° = 1
36° = 1.9	126° = 1.6	216° = 1.7	306° = 1.8
54° = 1.6	144° = 1.8	234° = 1.7	324° = 1.9
72° = 1.	162° = 1.4	252° = 1.1	342° = 1.6
90° = .6	180° = .7	270° = .9	360 = 1.4

I now have a resultant line showing the balanced moments but yet the work is not done.
I will endeavor to prove my figures by an actual test.
For the test I will use a machine as shown in the following figure page. The figure is 1/4 the size of the original and is discribed as follows.
A, small wheel 2 3/4" in diameter, with 55 teeth.
B. large wheel 5 1/2" in diameter with 110 teeth.
a. and b. are the cranks of the wheels, 1" from the center.
E. is a pulley wheel rotating on the same axle and

with B. and has a diameter of 1 + the diameter of the cord. C has an initial tension of 1. and is attached to the small wheel. D is attached to the large wheel W. weight attached to the crank of of B.

A. is ⅕ of a revolution in advance of B.

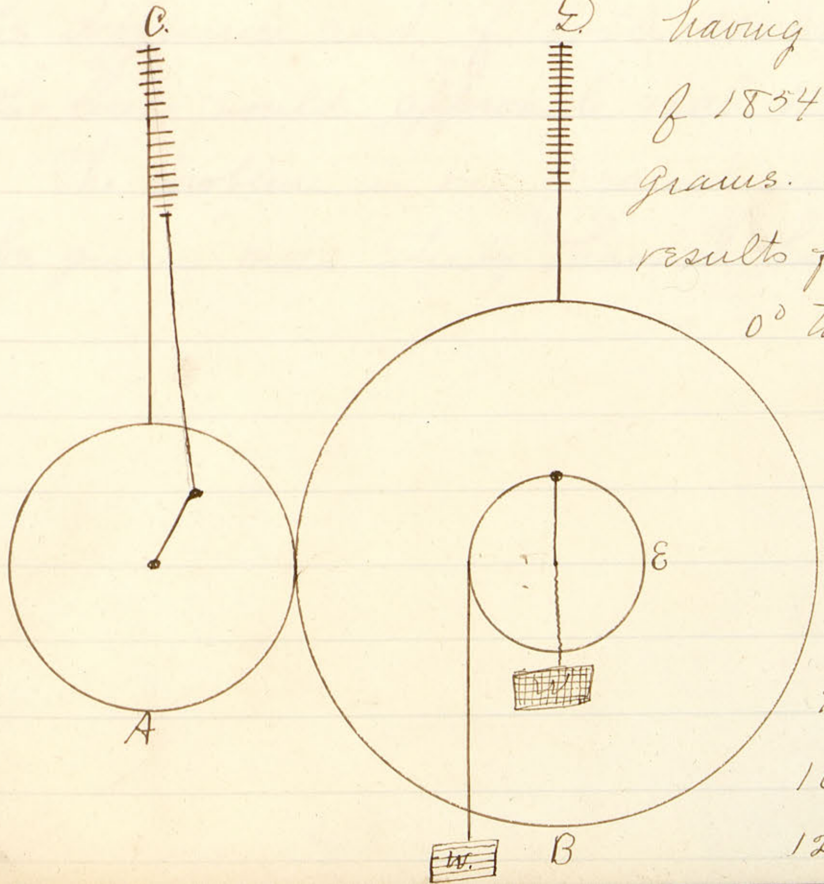
In D. there are 20 coils, in C. 123.

C. was stretched 3" and fastened to the crank of B.

To D. was fastened a weight of 1645 grams for the upward stroke and ⅙ of this or 274 grams for the downward stroke.

Around the pulley wheel E was wound a cord and from it was suspended the different weights which gave the following results.

With the 1645 gram weight on the crank of B. D. having a maximum tension of 1854 grams and C. 927. grams. I found the following results for the angles from 0° to 180°.



0° = +530 grams
18° = +1005 "
36° = +1185 "
54° = +1100 "
72° = +325 "
108° = −295
126° = −483
144° = −235 gr.
162° = +275 "
180° = +475 "

In the downward stroke from 180° to 360°, the 274 gram weight was used on B. and the following was the result.

198° = + 500 grams.	288° = − 105 grams.
216° = + 980 "	306° = − 270 "
234° = + 1275 "	324 = 0 "
252° = + 1155 "	342 = + 340 "
270° = + 460 "	360 = + 500 "

The above table was carefully taken and the result is shown by the red line K of plot.

In comparison of K. and C. it is found that one extends below the line while the other does not.

In explanation of this I will state that friction and difference in spring structure go to make up the difference and if both could be eliminated the lines would approach each other more.

The problem is one of much interest and should be given more study than I have time to give it.