ECONOMICS OF FERTILIZER USE ON WHEAT PRODUCTION

by

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INTRODUCTION

General

From times immemorial, soil had been exploited of its nutrients by raising crops, vegetables, fruits, trees, and ranching livestock. It was not very long ago when civilized world realized that exploitative system of farming cannot be continued indefinitely. Thus more and more researches are being developed for determining soil deficiency, nutrient requirements and the method of applying fertilizers for crop response. In the past, in all advanced countries as well as at present, also in almost all underdeveloped countries, raping the earth by different types of erosion--physical, chemical and biological--created many a situation under which soil surface was left more or less barren and unfit for continued crop production.

Crop rotation, manuring, better breeding, and soil management are methods designed to keep soil productive in the future. Of the methods used for improving or at least conserving the soil, fertilization is very simple and easy. In this connection, chemical research is useful to determine the availability of nutrients in the soil and also the contents and composition of fertilizers; agronomical and biological researches are directed toward plant growing techniques and plant life related with soil nutrients; whereas economic research is needed for determining the optimum and economic use of plant nutrients.

Research dealing with plant nutrients and the effects on production of crops has been extensive; however, for more accurate prediction of crop responses to fertilizer use, still more data are needed.

Inferences drawn for a particular location and crop cannot be of much use to a different crop produced under different climatic and soil conditions.

Allocation of scarce resources is one of the most important problems facing individual farmers as well as regions and nations. Although the present food situation in the United States does not call for this type of research, the growing population in the country and her international status demand research work sufficient enough to make adequate plans for the future.

According to Heady,

> The need for research on fertilizers and fertilization at a time when the nation's warehouses are filled with store food items and when production controls are in use, may be questioned. However, the ultimate economic goals of a society are never reached by placing restraints on imagination and ingenuity in research.[1]

Heady continues,

> The early work of Mitcherlich and Spillman serves as a landmark on fertilizer response curves. It appears strange that Spillman's work was not extended by any significant research on the fertilizer response economics until recently.[2]

Heady notes that some of the reasons for the lack of economic interpretations of agronomic data are lack of training in mathematics, statistics, and econometrics, over-specialization of agriculture and isolation from other allied branches.

He stresses that interdepartmental co-operation would encourage better research, especially in the use of fertilizer use in which economics, chemistry, soil science, agronomy, botany, horticulture,

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[1] Baum, Heady and Blackmore, Economic Analysis of Fertilizer Use Data, p. vii.

[2] Ibid., p. viii.

statistics, mathematics, econometrics and other departments would co-operate. In the light of the present situation, Heady's conclusion concerning such research on economics of fertilizer is very much appropriate.

The agronomic data used here are obtained from an experiment conducted and analyzed by Dr. Smith, Agronomist, Kansas Experiment Station.[3] His analysis covered agronomical aspects, the present work covers economic aspects of the problem of fertilizer use.

If we look into the history of American agriculture,[4] we find that revolutionary changes have occurred during the last 25 years. By 1955, the agricultural production had increased to 154 per cent of 1930. The number of farm workers during the same period declined to 63 per cent of the 1930 level. There have been other changes leading toward improvements of agriculture. Conservation methods that have been developed, use of improved seeds, improved breeds, control in insects, pests and diseases thoroughly changed its original outlook. But in spite of all these changes, one important thing may be pointed out. The attempts were more concentrated in labor saving devices than in intensification of farming devices which would include greater use of fertilizers. Population pressure in the United States compared to many other countries does not exist because the present U. S. surpluses of food, the use of fertilizers seems less important here than in some other parts of the world.

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[3] W. Floyd Smith, The Effect of Time, Rate and Method of Application of Fertilizer on the Yield and Quality of Hard Red Winter Wheat, Soil Sc. Soc. of America Proceedings, 1947, Vol. 12, 1948, pp. 262-265.

[4] Fortune, June 1955, "The Magnificent Decline of U. S. Farming," p. 99.

There are many underdeveloped countries, particularly in the Far East and Middle East, where no systematic research work has been started in production economics and wherever research has been started, technology is wanting. Millions of hungry people put heavy pressure on land. There is no alternative but to increase their food products from the same amount of land by any means of production. Better use of fertilizers with knowledge of the economics of fertilizer use would be one of the ways for improvements of agriculture. This work may help the author to do similar works in Pakistan. The author of this work is a government employee in agricultural economics in Pakistan. He is sponsored by the Food and Agricultural Organization of the United Nations.

Trends and Situations of Wheat

A crop like wheat which is important in many countries of the world, and can provide staple food to a huge section of the world population, definitely should get more attention than minor crops. The history of civilization indicates that staple food is the most vital need. In the normal budget of poor families, a lion's share of the income is spent for wheat or similar food. In the middle and rich classes of families, wheat or a substitute of it is a necessity. For economic efficiency allocation of scarce resources must be made with due consideration to the production of the necessities of life. Production of wheat may not be a serious economic problem for the U. S. A. because of its surplus food and high standard of living, but we must not forget the rest of the world. For obvious reasons, we cannot neglect other countries of the world in days of tense international relations. It is known to all, and will be evident from the statistics[5] that wheat is the staple food for a huge population of the world. These statistics would indicate that any economy with large production of wheat might have far reaching indirect impacts on peace, happiness, and welfare of the mankind.

Trend in the Use of Fertilizer on Wheat[6]

Statistics about fertilizer use on wheat are not available in proper form and whenever available are insufficient for explaining the trend. Wheat is a very important crop which represents about 15 per cent of all the crops in the world. It represents 25 per cent of all grain crops grown in all countries of the world. The trend of fertilizers used for all crops will, therefore, be of interest in connection with wheat. Soil requirements for wheat cultivation are mainly nitrogen (N_2) and phosphorus (P_2O_5). Potassium (K_2O) is also necessary for wheat or for producing other crops in the rotation. Trends in the use of these three nutrients are discussed below.

Between 1938 and 1955, use of nitrogen fertilizer increased to 176 per cent in Europe, 629 per cent in North and Central America, 400 per cent in South America, 192 per cent in Asia, 200 per cent in Africa, 167 per cent in Oceania. By 1955, the over-all total had increased to only 244 per cent of 1938.

The same period shows an increase in the use of phosphorus: in Europe to 164 per cent; in North and Central America to 335 per cent;

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[5] Yearbook of Food and Agricultural Statistics, F. A. O. Production-Production, Vol. X, Part I, 1956, p. 31.

[6] Ibid., pp. 229-231. (Figures exclude U.S.S.R.)

in South America to 500 per cent; and in Asia to only 110 per cent. The increase in the use of phosphorus on other continents amounts to 275 per cent in Africa and 212 per cent in Oceania. The over-all increase for the whole of the world is 203 per cent between 1938 and 1955-56.

During the same period, the consumption of potassium fertilizers has also increased in a similar manner: in Europe to 164 per cent; in North and Central America to 565 per cent; South America shows an increase to 700 per cent; Africa to 314 per cent; and Oceania shows an increase of potassium fertilizer to 300 per cent.

In 1955, the total consumption of nitrogen and potassium fertilizer was 63 million metric tons, 61 million metric tons, respectively. The consumption of iron, magnesium, boron and other trace elements used as soil nutrients is not shown; but it is estimated that there has been a great rise in trace nutrients also. The use of lime in acid soils and the use of other soil amendments has increased along with nitrogen (N_2), phosphorus (P_2O_5) and potassium (K_2O) nutrients, also.

Fertilizer Use in U.S.A.[7] The U.S.A. is a large consumer of fertilizers. Of the total world consumption the United States consumed 33 per cent of nitrogen, 30 per cent of phosphorus, and 32 per cent of potassium in 1955. Fertilizers used in North and Central America are largely used by U.S.A. In 1955, U.S.A. consumed 94 per cent of nitrogen, 73 per cent of phosphorus, and 95 per cent of potassium of the total consumption in North and Central America. In spite of all these facts, it is found that yield per acre in the United States is not as

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[7] Op. cit., pp. 229-231.

high as it is in other countries. U.S.A. is a vast country. The requirements of fertilizers are still higher than the quantities used now-a-days. U.S.A. consumed[8] about 10 million tons of fertilizers in 1942 and about 20 million tons in 1952. It is expected that the consumption fertilizer will still increase many-fold in the future. Consumption of fertilizers varies with 6 to 10 million tons in 1950, although 1952 broke records. For practical purpose one may take the current consumption as 6 million.

Fertilizer Use in Kansas. Fertilizer use in Kansas in 1926 was 8,000 tons but it came down to as low as 2,000 tons in 1933. In 1936, it was 7,000 tons. In 1940, it increased to 18,000 tons. In 1945 it was 38,000 tons and in 1950 the fertilizer use in Kansas was 170,000 tons, thus showing a more rapid increase than that found either in U.S.A. or in the rest of the world.

Of the total agricultural farms, 76 per cent of them grew wheat in 1951. Taking the wheat farms as a total, 30.9 per cent of them used fertilizers. Fertilizers used on wheat farms averaged 94 pounds in the fall of 1950, and 96 pounds per acre in the spring of 1951. Other crops show a variation in average fertilizer use from 97 pounds to 100 pounds of fertilizers.

The reasons for increased use of fertilizers are: (1) better education and knowledge that conservation system is sometimes better than exploitation system, (2) more research in economics, agronomy and chemistry of soil condition, plant growth, crop response, etc., (3) production and availability of fertilizers at a lower cost comparative to

[8] "Profitable Use of Fertilizer in Midwest," Bulletin No. 508, p. 8.

agricultural products, (4) need for efficiency of production in farming, and (5) increased knowledge directs research with accurate optima of fertilizer application.

It is clear from the above statistics that neither in Kansas, nor in the United States nor in other parts of the world the consumption of fertilizer has reached maximum stage. There are various reasons. The reasons why increase is not as high as it should be are as follows:

1. Reluctance of farm operators to change old methods.
2. Land holding and renting procedures.
3. Farmers want to have high current income.
4. Risk and uncertainty.
5. Lack of capital.
6. Time lag between cash outlays and return from land.
7. Lack of education and availability of research facts for fertilizer use.

PROBLEM AND OBJECTIVE

1. Research work done on crop response for wheat is limited and due to the importance of wheat as staple food, more researches are necessary. Food requirements of different crops are different in different places and thus a specific study on wheat crop is of special importance.

2. Too, research work dealing with a crop response in some parts of the country may not help in making proper use of the same for wheat crop in other areas because of differences in soil condition, climate, etc.

3. Any research work done on wheat, even if on the same soil type may not be useful for obtaining optimum yield due to variations in time element. The relative markets, change of habits, custom and taste cause variations in demand for a commodity. It might be that research done in certain time aimed at a certain situation has in time partially or fully changed. Even if the change is slight, optimum level of nutrients will vary.

4. Research in production economics is not an isolated case. The study of a specific problem gives certain findings under a particular set of circumstances. Results of research obtained for particular state of national economy might be different from that of the other. Hence, research work already done is not a sufficient reason for discontinuation of the same, especially in a dynamic economy.

Objectives of the Study

The general objective of the study is to derive wheat response function and to determine the optimum level of nitrogen and phosphorus. The other objectives are:

a. To predict total products and additional product of wheat at different levels of nitrogen, phosphorus and combinations of both associated with each additional 10 pounds of the nutrients.

b. To derive marginal yield of wheat by partial derivatives with respect to per pound of (i) nitrogen and (ii) phosphorus.

c. To derive demand schedules for nitrogen and phosphorus.

d. To develop isoquants for different combinations of nitrogen and phosphorus.

e. To determine least-cost combinations of nitrogen and phosphorus for given yields.

f. To obtain optimum rate of nitrogen, when phosphorus is at zero level.

g. To determine optimum rate of phosphorus when nitrogen is at zero level.

h. To determine optimum rate of nitrogen and phosphorus in combination.

Application of Findings

Results of the study may be of use to farmers in allocation of their resources. This study will give them a better background for fertilizing land devoted to production of wheat.

It may be of use to wheat farmers in redistributing the areas under different crops in relation to this crop. Some may increase and others may decrease the areas now in wheat and thus improve cropping methods, crop rotations, etc.

Optimum use of fertilizer has some indirect effects:

(i) It raises the standard of living on farms.

(ii) Farmers who are generally short of capital will have more capital and thus make them better and abler farmers.

(iii) Consumers get things at a comparatively lower price and can enjoy an increase of income effect.

(iv) Better food position will have more surplus for meeting international obligations and particularly to hungry population of the world. This will enhance the international status of U.S.A.

This work is mostly methodological in nature which has its academic importance. The results and findings of each research may be of use to future researchers. It may be also of use to extension, and to those making plans and programs for fertilizer use.

As a diagnosis of one patient is not sufficient for prescribing medicine for another patient in the same or different area, so also works done with the same or other crops under different conditions may not help in prescribing the accurate quantity of nutrients necessary for particular crop. In this connection a quotation from F. Orazem and F. W. Smith[9] may be used,

> This study was designed to improve basic knowledge of fertilizer crop relationship and to specify more accurately the approach which should be used by farmers in order to maximize returns from fertilizers.

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[9]F. Orazem and F. W. Smith, An Economic Approach of the Use of Fertilizer, May, 1958, p. 1.

BASIC LOGIC OF FERTILIZER INVESTIGATION

The present work dealing with application of two nutrients (nitrogen and phosphorus) and the response of wheat to these nutrients may involve the following methodological backgrounds for the empirical results.

Factor-Product Relationship

This is also known as input-output relationship or production function or crop response. All factors involved in production of a crop are transformed into output. Thus all factors come under one category or may be termed as one in their behavior and services in transformation. In this respect land, labor, and capital may be considered as agents of production of uniform character. The total transformation is reflected as output irrespective of whether one resource is land and the other is labor or fertilizer or machinery, etc. One may measure the productivity of all these factors graphically by showing the resources on the horizontal axis of a coordinate graph and the output on the vertical axis. The over-all effect of factors thus considered on crop yield is known as crop response. The method as to how different factors are combined together is detailed under factor-factor relationship explained below.

Factor-Factor Relationship

As long as one is concerned with one factor responsible for crop response, the problem of factor-factor relationship does not arise. When there is more than one factor under consideration, we cannot apply them arbitrarily for getting satisfactory crop response. It is common experience that labor may not be as efficient as machinery, land may be more important than labor machinery or seeds or fertilizers. Again in the use of fertilizers nitrogen may be more important than phosphorus and potash. In the relative importance also there are cases where one may be a perfect substitute for the other, for example, ammonium nitrogen and nitrate nitrogen. There are other instances when it is observed that these resources are applied as complements where there is no chance of substitution or replacement. In chemical combination this fixed proportion is observed more frequently, for example, composition of water (H_2O), two atoms of hydrogen combined with one atom of oxygen make one molecule of water. Such fixed propositions are also observed in the use of fertilizers, for example, 88 pounds of P_2O_5 (phosphorus) and 10 pounds of K_2O (potassium) for 3.2 ton yield of alfalfa.[10] This type of fixed combination is possible only at an optimum level. The most commonly observed combination of nutrients shows a diminishing rate of substitution owing to the fact that more and more of one nutrient is substituted for the other, unit of one kept constant. The present work involves two nutrients—Nitrogen (N_2) and phosphorus (P_2O_5) which are distinctly different in chemical composition. It may be reasonably expected that they shall neither be perfect substitutes nor perfect compliments in their production of crops. A diminishing rate of substitution is the most probable case.

Crop production is a very complex biological process accompanied with synthesis of plant food matters from fertilizer nutrients, water, carbon

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[10]Pesek, Johnson and E. O. Heady, Two Nutrient Response Functions with Determining Optima for the Rate and Growth of Fertilizer for Alfalfa. Soil Sc. Proc., Vol. 20, April 1956, p. 244.

dioxide, etc. in presence of sunlight. In the whole process of synthesis number of factors involved are many and it is very difficult to make the proper assessment of all of them except in some general ways. Nutrients of crops may be classified into (a) biological, (b) chemical. (a) Biological, manures are originated from plants, animals, birds, fish and man from their excreta and decomposed body. Green manures have become a class of manures of similar constituents as compost and dung. (b) Chemical fertilizers and soil amendments are a class in itself of which nitrogen, phosphorus and potassium deserve special mention. Lime is a very important amendment for acid soil. Thus all these factors that are responsible for production of crops may be expressed in a production function showing some factors as specified and others unspecified.

The whole idea may be digested in the following production function with one dependent variable-yield (Y) and all other independent variables responsible for yield.

$Y = f\ (A, L, S, M, W, F_1, F_2, F_3, F_4, F_5,\ X_1 \ldots X_n)$

when y = yield of crop

f = function of

A = acreage of land

L = quantity of labor in hours

S = quantity of seed in bushels

M = number of hours of the services of machinery

W = water from natural sources or irrigation in inches

F_1 = nitrogen in pounds

F_2 = phosphorus in pounds

F_3 = potassium in pounds

F_4 = green manure in cwt.

F_5 = lime in pounds

X_1 through X_n - refers to other unspecified factors of production.

A linear homogeneous production function, for example, would mean that if it is possible to get 10 bushels of wheat with one unit of each of the factors, it is possible to get 20 bushels of wheat when all factors in the right hand side are doubled. It also follows that if all the variable factors are increased by 3, 4, or 5 times showing output of 30, 40, or 50 bushels, etc.

Now whenever one deals with so many variable factors, it becomes almost an impossible task to conduct an experiment. It is possible to make due considerations and thus keep most of the factors constant. One may make use of this concept of the time element and eliminate some factors by considering them as fixed. Some factors not actually fixed may be considered as fixed and difference of time may be accounted for by the discounted value of marginal product and marginal cost. In practice, it has been experienced that land, labor, machinery, irrigated water supply may be kept constant within certain limit. If one takes one acre of land and keep it fixed, then one factor remains constant. With increased use of fertilizers, it becomes difficult to keep the amount of labor and machinery constant because each extra amount of fertilizers will have to be carried and spread over the land with extra labor and machinery. It may now be considered whether it is possible to keep the labor and machinery constant. (a) More labor and machinery may be engaged for initial cost of production. The same labor and machinery will thus be in a position to handle the extra fertilizers, and also for intercultural operation, and harvesting of the extra yield obtained from fertilization. This method is not reasonable and logical because in the initial stage there is wastage of labor and machinery. (b) It is possible to engage extra labor and machinery for the use of fertilizers and connected expenditure up to the stage of harvesting. In such a case, this extra labor and machinery cost is to be considered as a cost for the items of fertilization. This also is not a perfect method because cost of labor and machinery is taken as cost of fertilization. However, considering the limitations the second method is better. As for the water supply, it is also possible to keep it constant within certain limitations of soil type, evaporation, soil moisture, water holding capacity of soil and especially when the entire water is from artificial source or irrigation. In case there is rain and irrigation it is also possible to sum up both and keep the water content at a constant level. It is very easy to keep seed factor constant. Seed rate per acre is normally fixed and whenever it is necessary, a measured quantity of seeds may be sown.

Under situation described above the production function will be as follows:

$Y = f(F_1, F_2, F_3, F_4, F_5, X_1, X_2 \ldots X_n / A, L, S, M, W,)$ in this production function independent variables on the right hand side of the vertical bar are constant and those in the left hand side of the vertical bar are variable.

Now it is also possible to consider situations when only two factors—two fertilizers, nitrogen and phosphorus—are variables and all other factors are fixed. Attempts are made to fulfill the conditions as much as possible, and this is achieved by method of randomization under statistical design of the experiments, and statistical technique of the theory of probability brings down the errors to a minimum. Thus, we may assume that within certain limitations all factors kept fixed fulfill our required situation. Thus when two variables are involved in a crop response function with other factors fixed, we can express the situation as follows: $Y = f\ (F_1, F_2/F_3, F_4, F_5, X_1, X_2, X_3 . . . X_n, A, L, S, M, W)$[11] The simple form of the above relationships may be written as a production function of only two independent variables, $Y = f\ (F_1, F_2)$. In this case F_1 and F_2 are the two nutrients, nitrogen and phosphorus, respectively.

Geometric Form of Fertilizer-Crop Response Relationship

In this section, attempt is made to give an outline of the geometrical form of fertilizer crop response relationship, when independent factor, yield, is kept constant at a particular level. There are three possibilities of the substitution of two fertilizers F_1 and F_2.

Perfect Substitutes. In our present problem, two nutrients are distinctly different. One is nitrogen and another is phosphorus. There is no liklihood of perfect substitution between the two nutrients.

Perfect Complements. This is a situation under which substitution is zero. There is no possibility of any substitution under such a condition. This situation occurs when isoquant becomes a point. Although there is no chance for coming across perfect complementarity, it is unavoidably essential for finding out the point of maximum output. This type of combination is considered to be rare in our study.

[11] Heady, E. O., J. T. Pesek, and W. G. Brown, Crop Response Surfaces and Economic Optima in Fertilizer Use, Bul. 424, March 1955, p. 294.

Diminishing Rate of Substitution. This is the third type of substitution model that is used in factor-factor relationship. There are diverge adaptations and variations of this model according to the rate of substitution of two nutrients or inputs. This model is widely applicable. The contour lines or isoquants in Figure 1 indicate 40, 42, 44, bushels of wheat under possible combinations of the two nutrients. It is clear from the figure that the two nutrients neither replace each other at a constant rate (constant rate is indicated by straight line isoquant), nor are they perfect complements (perfect complements are indicated by points or right angles). They show diminishing rates of substitution.[12] This, however, means that the same yield can be attained by replacing a fixed quantity of one nutrient with more of the other nutrient. Yield remaining the same, one factor becomes less and less effective substitute of the other. The range of diminishing rates of substitution may be extended up to the two sides, ordinate and abscissa. For various limitations the isoquant lines may become horizontal on one side and vertical on the other. The range of diminishing rates of substitution may be limited after a stage. This happens due to available nutrients in the soil indicating that there is no further possibility of effective application of nutrients.

In figure 2, the curve shows nutrients on the horizontal axis and output or yield in the vertical axis. The marginal yields shown are gradually smaller and smaller. This type of situation is very commonly

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[12] Heady, E. O., J. T. Pesek and W. G. Brown, Crop Response Surface and Economic Optima in Fertilizer Use, Res. Bul 424, March 1955, Ames, Iowa, p. 297.

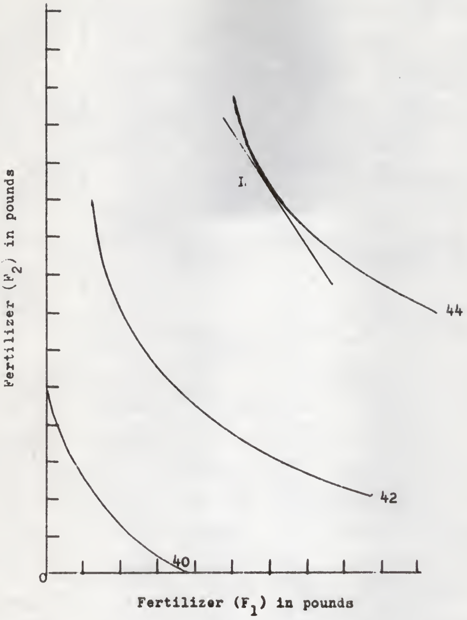

Fig. 1. Hypothetical isoquants with different marginal rates of substitution and inverse price ratio line, indicating least cost combination at the point of tangency (point L).

encountered in agricultural production. Any tangent drawn at any point on this total product curve indicates the marginal product at that point by its slope. The concept of marginal product is of great use in determining the optimum level of output.

The Optimum Level of a Given Element or a Given Combination of Elements [13]

For given combination of elements, the input-output curve is of the type shown in Figure 2. For a farmer with unlimited capital, the optimum level of fertilization is reached when the following equation is satisfied (1a) $\frac{\Delta Y}{\Delta F} = \frac{Pf}{Py}$, where ΔY refers to the change in yield and ΔF refers to the change in the input of fertilizers. Py is the price per unit of crop and Pf is the price for each unit of fertilizers, including other cost items involved in fertilization. $\frac{\Delta Y}{\Delta F}$ is the transformation ratio, crop response ratio or input-output ratio. It is the marginal product at the ΔF unit of fertilizer that yield over the previous total output. $\frac{Pf}{Py}$ is the price ratio.

From equation (1a) one may derive another equation which is $\Delta y.Py = \Delta F.Pf$. . . (1b). This shows that a small increase in fertilizers multiplied by its price will be equal to the corresponding small increase in the output multiplied by its price, or in other words, value of the small increase in input will be equal to the value of the small increase in output. This situation shows the optimum level of output that is possible with the application of nutrients under the given price and physical input-output conditions.

[13] Ibid, p. 229.

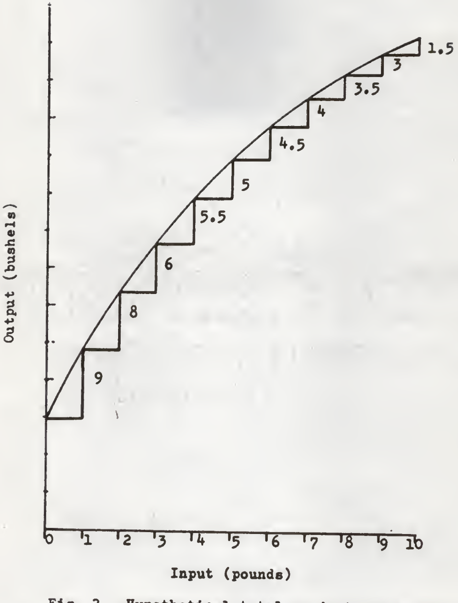

Fig. 2. Hypothetical total product curve with diminishing marginal products.

There are two other possible situations which need explanation in this connection. They also express the relation between the price ratio of the input and the marginal product as shown under the following equations:

(2a) $\frac{\Delta Y}{\Delta F} > \frac{Pf}{Py}$ (2b) $\Delta y\ Py > F\ Pf$

(3a) $\frac{\Delta Y}{\Delta F} < \frac{Pf}{Py}$ (3b) $\Delta y\ Py < F\ Pf$

Equation (2a) shows a situation when marginal product or transformation ratio is greater than the price ratio or, in other words, value of output is greater than the value of input of fertilizer. This is a stage when it is possible to apply more and more fertilizers so as to increase the output and bring it to an optimum level as shown under equation (1a) and (1b).

Equations (3a) and (3b) show that the price ratio or the cost of the factor is relatively greater than the corresponding transformation ratio or the value of output, respectively. It would be possible to continue production at the same level if the price of the output is increased or the cost of input is decreased. If none of these conditions is fulfilled, the intensity of production either diminishes or ceases.

Thus it is evident from the above equations that the only satisfactory equilibrium condition is reached when transformation ratio is exactly equal to the inverse factor-product price ratio. The optimum fertilization rate is attained and the profits are at a maximum when the marginal (added) cost of fertilizer is just equal to the marginal (added) return of crop.

The relationship explained in above equations may be geometrically represented as shown in Figure 3. It has been pointed out that marginal product can be determined at any point on the curve by the tangency at that point. TT' is a tangent at the point P showing the maximum profits with physical output curve OP. OA is the quantity of fertilizer applied and the yield obtained is the AP. Thus the tangent TT' shows the marginal product $\frac{\Delta Y}{\Delta F}$ or $\frac{dY}{dF}$

Least-Cost Combination of Nutrients

In discussion of factor-factor relationships it was implied that the economic problems exist mainly in the areas denoting diminishing rate of substitution. Let us now consider two nutrients and find the model for least-cost combination. According to Heady[14]

> If two or more factors were employed in the production of a single product, cost is at a minimum when the ratio of factor price is inversely equal to the marginal rate of substitution of the factors.

This may be expressed algebraically by the equations:[15]

(4a) $\frac{\Delta F_1}{\Delta F_2} = \frac{P_{F_2}}{P_{F_1}}$ and (4b) $\Delta F_1 \cdot P_{F_1} = \Delta F_2 \cdot P_{F_2}$

(5a) $\frac{\Delta F_1}{\Delta F_2} > \frac{P_{F_2}}{P_{F_1}}$ (5b) $\Delta F_1 \cdot P_{F_1} > \Delta F_2 \cdot P_{F_2}$

(6a) $\frac{\Delta F_1}{\Delta F_2} < \frac{P_{F_2}}{P_{F_1}}$ (6b) $\Delta F_1 \cdot P_{F_1} < \Delta F_2 \cdot P_{F_2}$

[14] Heady, E. O., Economics of Agricultural Production and Resource Use, pp. 172-173.

[15] Heady, E. O., J. T. Pesek, W. G. Brown, op. cit., p. 301.

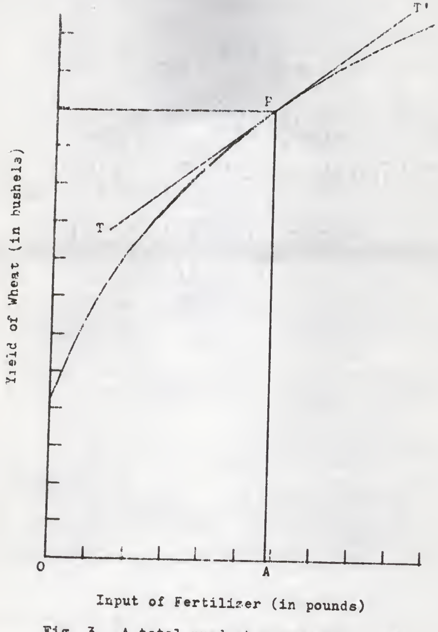

Fig. 3. A total product curve with tangent (inverse price ratio line) indicating optimum level of production at point P.

P_{F_1} is the price of fertilizer F_1. P_{F_2} is the price of fertilizer F_2. ΔF_1 is the small increase in the input of N_2 fertilizer. ΔF_2 is the corresponding small increase in the input of P_2O_5. $\frac{\Delta F_1}{\Delta F_2}$ is replacement ratio, $\frac{P_{F_2}}{P_{F_1}}$ is the price ratio of the fertilizers. $\frac{\Delta F_1}{\Delta F_2}$ the replacement ratio may be represented by tangency at any point on the isoquant. Now if the conditions of the equation number (4a) or (4b) are fulfilled then the least-cost combination of the two nutrients has been achieved. In case the situation is similar to the equation number (5a) or (5b), replacement ratio is greater than the inverse price ratio; this indicates that the added quantity of F_2 is cheaper than the added quantity of F_1. F_1 should be reduced and F_2 increased until the condition specified in equation number (4a) and (4b) is fulfilled.

In case the situation is similar to equation number (6a) and (6b) inverse price ratio is greater than the replacement ratio. The movement in substitution is reverse in this case to that of equation number (5a) and (5b). In this case, the added quantity of F_1 is cheaper than the added quantity of F_2. F_2 should be reduced and F_1 increased until the inverse price ratio equals the replacement ratio.

Border or Ridge Lines

The border or ridge lines indicate the areas of substitution possibilities. A and B in Fig. 4 are two isoclines which trace the points of equal rates of substitution of F_1 and F_2 at different isoquants showing different combinations of the two nutrients. It is interesting to note that the two nutrients do not substitute at a diminishing rate beyond the range of the two isoclines A and B. This situation occurs

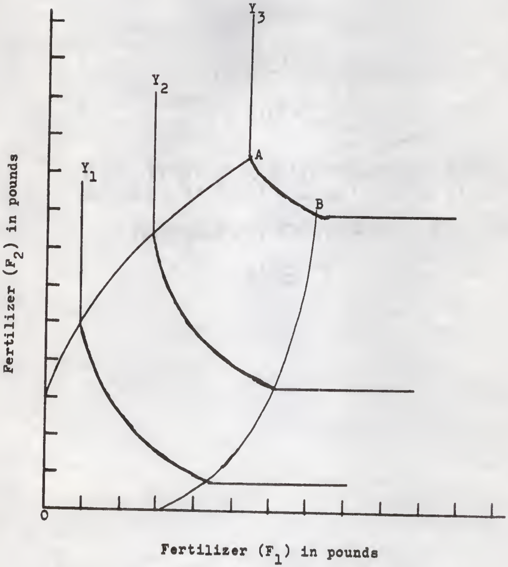

Fig. 4. Three hypothetical isoquants Y_1, Y_2, Y_3 are shown with two ridge or border lines A and B. Substitution of F_1 and F_2 is limited within the border lines.

where two nutrients replace each other only within certain limits and it is only within this range where least-cost combination is a relevant economic problem. Beyond this range, isoquants become horizontal or vertical, and a fixed quantity of one nutrient will produce the same yield whether the other nutrient is increased or not. According to Heady[16]

> Two isoclines can be called ridge lines. They denote zero substitution or replacement rates if (1) the ridge lines are not far apart, (2) the isoclines within their boundary are fairly straight and (3) the yield isoquants for a particular yield have only a slight curvature with a slope not far different from the nutrient price ratio, within the boundaries of the ridge to lines will give costs which are only slightly different, although only one isocline will denote exactly the least cost combination.

If (1) the ridge lines are "sprung far apart," (2) isoclines bend sharply and (3) yield isoquants "curve sharply" away from the price ratios, changing nutrient ratios along an isocline will be considerable.

It cannot be said precisely what situation exists under different conditions. The principles laid out above will be useful in drawing inferences under the specific soil climate, crop, and other conditions.

The principles laid out above apply to situations where farmers have unlimited capital. Inferences made for unlimited capital is also useful for those situations where farmers want to maximize their profit with limited capital. Only slight adaptations of the same rules are required.

[16] Heady, Earl O., Pesek, J. T., Brown, W. G., Crop Response Surfaces and Economic Optima in Fertilizer Use. Res. Bul. 424, Ames, Iowa, 1955, p. 301.

EXPERIMENTAL DATA AND THEIR LIMITATIONS

General

The fertilizer experiment upon which this study is based was conducted in Geary silt loam soil at the Kansas State Agronomy Farm at Manhattan, Kansas.[17] Land was prepared properly for germination and growth of wheat. Fertilizer materials used in this experiment were super-phosphate and ammonium nitrate in addition to other nutrients. Fertilizers were applied at the plow sole and with seed in each sub-plot. The length of each plot was 125 feet. Random method was used in different treatment of the nutrients. Each treatment was replicated four times. There were 100 sub-plots with different levels of nitrogenous, phosphatic, and potashic nutrients on plow sole, broadcast on stubble, with seed as spring top dressing, and also with no treatment. Sixty sub-plots with no treatment, on plow sole, and with seed, were selected for the present economic analysis. These sub-plots include four with no fertilizer, twenty-four sub-plots with nitrogen alone at 25, 50, and 100 pound levels, 12 sub-plot at plow sole and 12 with seed, 16 sub-plots with only phosphorus at two levels of 25 and 50 pounds each of which 8 were at the plow sole and 8 with seed, 16 sub-plots were fertilized with both nitrogen and phosphorus at 25 and 50 pounds, 8 sub-plots at the plow sole and 8 with seed.

From layout it seemed that these two sub-plots would represent a good amount of materials for fertilizer response on wheat. Replications with

[17] F. W. Smith, The Effect of Time Rate and Method of Application of Fertilizer on yield and Quality of Hard red Winter Wheat. Soil. Sc. Soc. of America Proceedings, Vol. 12, 1958, pp. 262-265.

Potassium (K_2O) and other treatments seemed to be comparatively insufficient to achieve a good result in economic analysis. It was also desired to simplify the analysis by avoiding some of the observations conducted with potassium spring top-dressing and broadcast on stubble.

Analysis of variance was conducted for two sets of data with Nitrogen (N_2) and phosphorus (P_2O_5), and an F test was run to see whether there was any significant difference between the plow sole and the with-seed preparation treatments.

The crop response function was derived and different relationships were studied in accordance with the model already set up under basic logic. To facilitate computations and analysis, the original data were coded as shown in appendix 4.

Limitations of Data

1. Plant nutrients--nitrogen, phosphorus, etc. do not directly serve as substitutes in the chemical functions of the plant. The fact that similar yield increases can be attained with different combinations of nutrients causes them to serve as substitutes in the decision-making framework of the farmers. Thus the terms substitution and replacement may not represent an entirely accurate physiological concept.

2. Response of fertilizer is affected by the residual or carry-over response of last year.

3. The experiment is based on one single year's result. Crop response might not be exactly the same under different years, even under the same soil conditions.

4. Production of a crop is a complex biological process. The crop response is sure to vary under different temperature, precipitation,

bright sunshine, evaporation, and other conditions.

5. An agro-economic experiment should be provided with a large area with uniform soil and climatic conditions. However, it is difficult to find an area with such uniformity. Statistical method of randomization is applied to minimize error, but still the results cannot be applicable to an area beyond the specific type of conditions under which the experiment was conducted.

6. It is more costly to make a representative experiment, and such an experiment is limited within the scope of existing facilities.

7. The analysis of the data is made on the assumption that capital invested and result of fruiting was possible without time lag.

Weather and Soil[18]

Weather conditions under which this experiment was undertaken were favorable for germination of wheat and there was no damage either during the germination of the wheat or during the growing period.

The soil is Geary silt loam in which wheat was grown in the previous year and was fertilized with phosphate at the rate of 100 pounds per acre at the time of planting.

The soil on which the experiment was conducted contained the following nutrients:

Element	Lb/acre Plow Layer
Total nitrogen	3,330
Available phosphorus	89
Exchangeable pottassium	405

[18] F. W. Smith, A Time, Rate and Method of Application of Fertilizer on Yield, and Quality of Hard Red Winter Wheat, (Contribution No. 391 from the Department of Agronomy, Kansas Agricultural Experiment Station.) Soil Sc. Soc. of America Proceedings Vol. 12, 1948

METHOD OF ANALYSIS

Profit Maximization and Least-Cost Combination

The following considerations, assumptions and informations are necessary in determining the most profitable level of fertilization.

Price Per Unit of Output. It is essential to know the price per unit of the output. If the price of output (wheat) is relatively high, using more of the inputs and thus increasing the total quantity of the output becomes profitable. If the price of the output is relatively high, a farmer may produce more of this crop. If the price of the product is relatively low, he may reduce or give up production of this crop and find alternative enterprises for investment of his capital.

It is necessary to know what alternative crops may be grown and their expected returns. If it is found that there are alternative substitution possibilities, a farmer will not grow a crop that pays less. Product-product relationship showing supplementary, complementary and competitive enterprises guides the farmer in deciding what he will produce. In our present problem, wheat was grown experimentally at Manhattan, Kansas. The area is in a wheat region and it is assumed that wheat is, presumably, the best crop for the particular soil.

Price of Fertilizers and Other Resources. Fertilizer is used for raising crops whenever the cost of fertilizer and fertilization is lower than the value obtained from the transformed quantity of output. If the cost of fertilizer increases, other things being equal, quantity of fertilizer applied will be reduced or even discontinued. In this case, farmers will invest their money in some alternative resource or resources that give larger returns. They may allocate their funds for labor, machinery, and other inputs. In the present case, one may assume that farmers are ready to use fertilizers and if they know the optimum level of fertilizers under specific market prices.

Marginal Rates of Replacement. Under factor-factor relationship we already considered the marginal rates of substitution. Marginal rates of replacement of nutrients will be of use in determining the optimum level of fertilization and maximization of profit.

Marginal Rate of Transformation. The model for crop response curve and for the marginal product have been discussed. The marginal product indicates the marginal rate of transformation. When one knows with certainty the replacement ratio of nutrients, one needs the concept of marginal rate of transformation for arriving at the optimum level of production.

For a problem of profit maximization, assumption of perfect knowledge and production without a time lag will simplify the problem.

It is assumed that there is approximate idea of the rates of application of fertilizers that will have a good crop response.

When one knows all the informations noted above, one can find the marginal rate of substitution of fertilizers to determine the least-cost combination.

The least-cost combination and most profitable combination of inputs are as follows:

> The Least-Cost Combination. Given the variation in input per unit of output for all the cost elements as input changes, all that is necessary to determine that combination which produces at least cost per unit is to apply prevailing cost rates to the inputs in the various combinations and locate the least cost combination. There will be definite limitations in many cases. For example, one man may be limited by labor, another by capital, another by feed available, and

another by acreage. There will probably be a different least cost combination with each different limitation or combination of limitations.[19]

The Most Profitable Combination of Inputs. The least cost combination is not necessarily the combination which will yield the largest profit. Total profit is the product of the profit per unit of output multiplied by the number of units produced; the number of units produced at a higher cost combination may be enough larger than at the least cost combination to more than offset the lower profit per unit.[20]

Derivation of Wheat Response Curve and Statistical Analysis

The basic logic of wheat response curve when all factors are variable or some are fixed and others are variable has been discussed. The limitations of the experiment were enumerated. At present one may assume that the wheat response is due to the application of nitrogen and phosphorus. When some part of the wheat response curve indicates no treatment response, that part is due to all other resources not accounted for in wheat response.

The wheat response curve may be derived by:

1. Making an arithmetic table of the data for different levels of nutrients.

2. One may make scatter diagrams as (i) free hand curves, (ii) lines through averages, or (iii) least squares regression line, and thus determine the geometric form of the relationship.

3. One may use the least-square method for algebraic form and establish the functional relationship by means of equation. This method

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[19]H. R. Tolly, J. D. Black and M. J. B. Ezekiel, Input As Related to Output in Farm Organization and Cost of Production, U.S.D.A., Bul. 1277, September 1924, p. 15

[20]Ibid.

is perhaps the most accurate and reliable in finding the algebraic form. In this functional relationship, two types of wheat response curves were derived:

1. Linear function;
2. Cobb-Douglas or logarithmic function.

The two functions may be expressed as:

1. $Y = a + b_1x_1 + b_2x_2$

2. $Y' = a\, x_1^{b_1}\, x_2^{b_2}$

Where Y = the predicted total yield of wheat, a is yield obtained without fertilizer and b_1 and b_2 are the regression coefficients showing the increase or decrease associated with the fertilizer applied in the soil. Y' = predicted total yield of wheat. Yield of wheat obtained under different levels of nutrients are shown in Table 1. Analysis of variance for nitrogen (N_2) and phosphorus (P_2O_5) which was performed to see whether two methods with plow sole and with seed are different are shown in Table 2 and Table 3, respectively. It may be seen that statistical tests indicate that the two methods do not show any significant differences.

Table 1. Summary of wheat yields (bushels per acre) for different methods and levels of fertilizer application in agronomic experiment

Treatment	Pound of Nitrogen N_2	Pounds of Phosphorus P_2O_5	Block I	Block II	Block III	Block IV
with seed	0	0	33.3	41.0	42.3	41.4
on plow sole	25	0	29.1	38.1	42.9	38.2
with seed	25	0	41.4	36.7	41.1	43.0
on plow sole	50	0	40.7	39.2	40.5	49.5
with seed	50	0	39.1	40.7	41.1	39.7
on plow sole	100	0	40.4	41.1	42.9	42.8
with seed	100	0	40.1	39.9	38.0	41.5
on plow sole	0	25	38.3	43.1	44.3	42.3
with seed	0	25	38.6	45.5	41.9	37.1
on plow sole	0	50	35.4	43.2	44.3	47.0
with seed	0	50	35.4	38.3	46.5	42.6
on plow sole	25	25	35.5	44.0	43.2	42.2
with seed	25	25	45.5	46.0	43.8	43.2
on plow sole	50	50	44.2	45.0	45.2	45.4
with seed	50	50	42.9	43.9	43.1	43.2

Table 2. Analysis of variance to test the difference between two preparations at plow sole and with seed for nitrogen use*

Source of Variation	Degrees of Freedom	Sum of Square	Mean Square	F
Blocks	3	58.08	19.06	1.94
Preparation	1	0.42	0.42	0.43
Nitrogen	2	28.33	14.17	1.42
Prep x nitrogen	2	42.49	21.25	2.13
Remainder	15	145.99	9.73	-

(1) Difference for preparation is not significant at 5% level of probability.

(2) Difference for blocks, nitrogen (N_2) and prep. nitrogen are also not significant at 5% level of probability.

Table 3. Analysis of variance to test the difference between two preparations at plow sole and with seed for phosphorus use.

Source of Variation	Degrees of Freedom	Sum of Square	Mean Square	F
Blocks	3	117.41	39.13	3.82
Preparation	1	9.03	9.03	0.88
Phosphorus	1	0.16	0.16	0.02
Prep x phosphorus	1	0.34	0.34	0.03
Remainder	9	92.03	10.22	-

(1) Difference for preparation is not significant at 5% level of probability.

(2) Difference for blocks, phosphorus (P_2O_5) and interaction between prep and phosphorus are not significant at 5% level.

* Snedecor, G. W., Statistical Methods, pp. 329-391, Iowa State College Press, Ames, Iowa 1957.

Observations obtained under plow sole treatment and with seed treatment were combined as if the treatments were the same.

The linear function derived was:

1. $Y = 37.88078 + .44499\ X_1 + 1.38652\ X_2$

where Y = yield of wheat in bushels

X_1 = quantity nitrogen applied to the soil (in pounds) = N

X_2 = quantity phosphorus applied to the soil (in pounds) = P

Here Y is dependent and X_1 and X_2 are independent variables.

The logarithmic or Cobb-Douglas function derived was as follows:

2. $Y' = 33.695\ X_1^{.01833}\ X_2^{.05664}$

Here also X_1 = N, X_2 = P

Where Y' is dependent variable, X_1 and X_2 are independent variables.

Table 4. Measure for goodness of fit in statistical analysis

Name of Functions	Correlation determination R^2	Standard error of the estimate S_E	Regression Coefficients		Standard error of the estimate		t values for regression coefficients for	
			by 1.2	by 2.1	$s_{b_{y1.2}}$	$s_{b_{y2.1}}$	by 1.2	by 2.1
Linear function	.90	0.365	0.44499	1.38652	0.04	0.06	11.1**	23.1**
Cobb-Douglas function	.10	0.0155	0.01833	0.05664	0.003	0.0115	6.1**	4.89**

**Value of t is highly significant at 1% level of probability.

Linear function shows a coefficient of determination of 90 percent. This functional relationship is associated with 90 percent of the variation in the dependent variable. Both regression coefficients show high significance in their t values. The standard errors of the regression coefficients are small. The standard error of estimate is 0.365.

The logarithmic, or Cobb-Douglas function shows a unique situation where there is practically no multiple correlation. It accounts for a variability of only 10 percent, however, both regression coefficients are highly significant. Standard errors of regression coefficients are relatively small. The standard error of estimate for the Cobb-Douglas function is 0.0155.

Cobb-Douglas function is mainly concerned with the interaction of the two nutrients. In our present experiment, there were many observations which have only one of the two nutrients, nitrogen or phosphorus. It is in only eight observations that there is a possible interaction of the nutrients. Therefore, the design of the experiment is not suitable for studying Cobb-Douglas functional relations.

From the analysis shown under Table 4 linear function gives a better fit of the data and for some of our conclusions this may be used with success. However, for considering the replacement ratio and for finding out the optimum production, the Cobb-Douglas function may be used to explain how such problems may be solved when a good functional relationship is achieved.

INTERPRETATION OF FINDINGS

Although linear function shows a high multiple correlation, the Cobb-douglas function was used for determining the least-cost combination and optimum level of production. Linear function shows a straight line relationship which cannot be satisfactorily used for finding out the point of tangency. If the inverse price ratio is overlapping, then all points are least-cost points in isoquants showing factor-factor relationship.

In determining the optimum level of production, it is generally necessary to find out the point of tangency of the inverse price ratio of factor and product with the marginal product, but this cannot be done satisfactorily with a linear function. The Cobb-douglas function, derived in the previous chapter, has an exponent of less than one. This function, therefore, lies in the rational zone of the classical production function. If most of our economic problems lie within the rational zone, it would be definately worth while in interpreting the production function based on Cobb-douglas function. Linear function was not, however, ignored. Total yields and marginal yields of wheat are predicted with the help of both functions. Determination of economic efficiency, however, was based on Cobb-douglas function.

Predicted Total Yields of Wheat

Table 5 shows the predicted total yields of wheat for different levels of nitrogen when phosphorus is kept at zero, 10, 20, 30, 40, and 50 pounds of fertilizer. Thus if we look at Table 5, in the first column showing the predicted total yield, we find two production functions. The first 11 figures showthe Cobb-douglas function and the second figures show the linear function when phosphorus is kept at zero level of fertilizer use. If we observe the trend of the total yields in other columns, we find 5 Cobb-douglas functions and 5 linear functions.

The linear function shows a straight line fertilizer-wheat relationship in all the 17 situations as shown under different rows and different columns with second figures. The Cobb-douglas function shows an increase in the yields of wheat at decreasing rate. This will be explained better with the help of Tables 6 and 7.

Table 5.* Predicted total yield of wheat per acre for specified nutrient combinations applied on geary silt loan soil in Manhattan, Kansas

Pounds of Nitrogen (N_2)	Pounds of Phosphorus (P_2O_5)					
	0	10	20	30	40	50
0	40.04314 39.71229	40.80936 40.26690	41.39599 40.82151	41.86907 41.37612	42.26970 41.93073	42.61440 42.48534
10	40.28911 39.89029	41.06679 40.44490	41.65396 40.99951	42.12616 41.55412	42.52916 42.10873	42.87588 42.66334
20	40.47713 40.06829	41.25852 40.62290	41.84844 41.17751	42.32260 41.73212	42.72762 43.28673	43.07602 42.84134
30	40.62673 40.24629	41.4116 40.80090	42.00323 41.35551	42.47929 41.91012	42.88565 42.46473	43.23540 43.01934
40	40.74972 40.42429	41.53650 40.97890	42.13023 41.53351	42.60766 42.08812	43.01537 42.64273	43.36614 42.19734
50	40.85721 40.60229	41.64601 41.15690	42.24137 41.71151	42.72021 42.26612	43.12893 42.82073	43.48036 43.37534
60	40.95324 40.78029	41.74372 41.33490	42.34059 41.88951	42.82062 42.44412	43.23035 42.99873	43.58246 43.55334
70	41.03748 40.95829	41.82998 41.51290	42.42791 42.06751	42.90890 42.62212	43.31930 43.17673	43.67243 43.73134
80	41.11060 41.3629	41.90446 41.69090	42.50332 42.24551	42.98505 42.80012	43.39646 43.35473	43.74992 43.90934
90	41.17967 41.31429	41.97787 41.86890	42.57476 42.42351	43.0572 42.97812	43.46925 43.53273	43.82372 44.08734
100	41.24100 41.49229	42.03721 42.04690	42.63826 42.60151	43.12151 43.15612	43.53428 43.71073	43.88909 44.26534

* First figures from Cobb-douglas functions, second figures from Linear functions.

(1) $Y' = 33.695\ N^{.01833}\ P^{.05664}$

(2) $Y = 37.88078 + .44499\ N + 1.38652P$

Total and Marginal Yield of Wheat in Response to Phosphorus

Table 6 is obtained from Table 5 to explain the wheat response to phosphorus. The production function is of the nature of

(i) $y' = a\ P^{b_2}$, Cobb-douglas function

(ii) $y = a\ b_2\ P$, Linear function

There are 11 rows in the table with the first figures showing function (i) and the second figures showing function (ii). Because of the technical situation as indicated at different levels of nitrogen, there are 11 functional relationships for the Cobb-douglas function and 11 functional relationships for the linear function. One of the 11 set of functions is selected and presented in this table. Wheat response as observed in this table may be discussed under part (a) and (b).

Table 6. Average marginal product of wheat with each additional 10 pounds of phosphorus (P_2O_5), nitrogen (N_2) at zero level of fertilizer use

	Cobb-Douglas Function(1)			Linear Function(2)		
Phosphorus (P_2O_5)	Total Product	Increment of product	Average Marginal Product	Total Product	Increment of product	Average Marginal Product
0	40.04314	----	----	39.71229	----	----
10	40.80936	.76622	.19156	40.26690	.55461	.05546
20	41.39599	.58663	.14666	40.82151	.55461	.05546
30	41.86907	.47308	.11827	41.37612	.55461	.05546
40	42.26970	.40063	.10016	41.93073	.55461	.05546
50	42.61440	.34470	.08618	42.48534	.55461	.05546

Based on (1) Cobb-Douglas Function (2) Linear Function

(1) $Y' = 35.14389\ P^{.05664}$

(2) $Y = 38.32577 + 1.38652\ P$

Part (a) of the table shows an increasing total yield from 40.04314 to 42.61440 bushels of wheat for zero to 50 pounds of phosphorus in fertilizer use. The average marginal product per pound of phosphorus shows a downward trend from .19156 bushels to .08618 bushels of wheat. This function is shown in graph AB in figure 5.

Part (b) of the table shows an increasing total yield from 39.71229 bushels to 42.48534 bushels of wheat for zero to 50 pounds of phosphorus in fertilizer use. The average marginal product is equal at all levels and is .05546 bushels of wheat per pound of additional use of phosphorus. The linear function is shown in graph CD in figure 5.

Total and Marginal Products of Wheat in Response to Nitrogen Fertilizer

Table 7 is also obtained from Table 5. This is presented only to explain the response of nitrogen when phosphorus is kept constant at any level of fertilizer use.

Table 7. Average marginal product of wheat associated with each 10 pounds nitrogen

	Cobb-Douglas Function(1)			Linear Function(2)		
	Total Yield	Increment Yield	Average Marginal Yield	Total Yield	Increment Yield	Average Marginal Yield
0	40.04314			39.71229		
10	40.28911	.24597	.06015	39.89029	.178	.0178
20	40.47713	.18802	.04700	40.06829	.178	.0178
30	40.62673	.14960	.03780	40.24629	.178	.0178
40	40.74972	.12299	.03075	40.42429	.178	.0178
50	40.85721	.10749	.02688	40.60229	.178	.0178
60	40.95324	.09603	.02400	40.78029	.178	.0178
70	41.03748	.08424	.02105	40.95829	.178	.0178
80	41.11060	.07312	.01828	41.13629	.178	.0178
90	41.17967	.06907	.01728	41.31429	.178	.0178
100	41.24100	.06133	.01533	41.49229	.178	.0178

P_2O_5 at zero level

(1) $Y = 38.38871\ N^{.01833}$

(2) $Y = 39.26730 + .44499\ N$

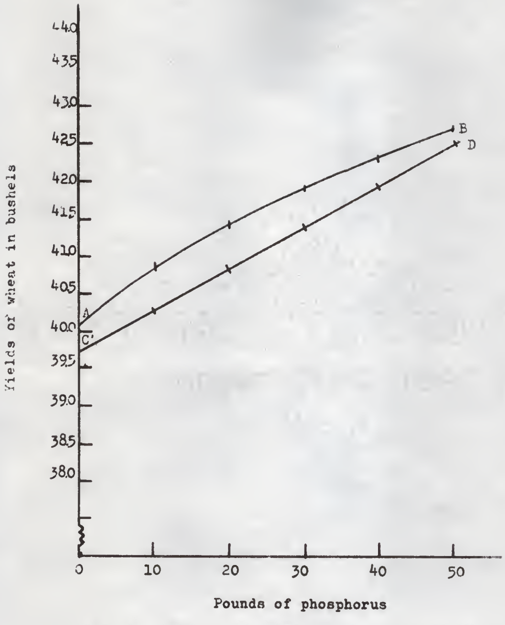

Fig. 5. A comparison of derived Cobb-Douglas function (AB) and linear function (CD) when nitrogen is kept constant at zero level phosphorus used from zero to 50 pounds.

This table has two parts, (a) and (b). Part (a) presents Cobb-Douglas function which is one of the 6 columns in Table 5. Each of the 6 columns with the first figures, shows similar relationship for different technical situations. It is found from this table that total yield ranges from 40.04314 to 41.24100 bushels of wheat and incremental marginal yields show a gradual decrease from 0.24597 to 0.06133. The average marginal yields range from 0.06015 to 0.01533.

Part (b) of the table shows a linear function. This is one of 6 columns shown in Table 5 with the second figures. There is nothing special in this case because the total product increases from 39.71229 to 41.49229 bushels with an average marginal yield of .0178 bushels for each additional pound of nitrogen.

Exact Marginal Yield of Wheat

Table 8 shows the exact marginal yields at different levels of fertilizer use. These figures were obtained by partial derivatives of predicted yields (y') with respect to nitrogen and phosphorus. This table has 11 rows and 6 columns. Each column or row has first figures and second figures calculated for nitrogen and phosphorus respectively derived from Cobb-douglas function. Exact marginal product is very important in marginal analysis. The role of the average marginal products shown in Table 6 and 7 is similar to the exact marginal product. Most often agronomical experiments provide discontinuous input-output figures which are used to calculate average marginal yield. Whenever it is possible to obtain the exact marginal

Table 8. Exact marginal yields of wheat for specified nutrient combinations Based on Cobb-Douglas function

Pounds of Nitrogen (N_2)	Pounds of phosphorus (P_2O_5)					
	0	10	20	30	40	50
0	.07338 .22679	.07482 .16511	.07571 .13025	.07675 .10779	.07748 .09211	.07811 .08046
10	.05274 .22820	.05377 .16613	.05442 .13106	.05516 .10846	.05569 .09268	.05614 .08096
20	.04121 .22927	.04174 .16691	.04252 .13168	.04310 .10897	.04351 .09311	.04387 .08337
30	.03384 .23011	.03450 .16753	.03492 .13216	.03539 .10937	.10573 .09346	.03602 .08164
40	.02872 .23081	.02929 .16803	.02964 .13256	.03004 1.0970	.030328 .093740	.03058 .08188
50	.02496 .23142	.02543 .16848	.02574 .13291	.02609 .10999	.02634 .09399	.02655 .08210
60	.02207 .23196	.02250 .16887	.02277 .13322	.02308 .11025	.02331 .09421	.02350 .08229
70	.01979 .23244	.02018 .16922	.02042 .13350	.02070 .11047	.02690 .09440	.02107 .08246
80	.01794 .23285	.01829 .16952	.01851 .13374	.01876 .11067	.01894 .09457	.01909 .08261
90	.01640 .23324	.01673 .16981	.01693 .13396	.01716 .11086	.01732 .09729	.01746 .08275
100	.01512 .23359	.01541 .17006	.01560 .13416	.01581 .11102	.01596 .09487	.01609 .08287

Marginal yield calculated from partial derivatives of yield (Y') with respect to nitrogen and phosphorus.

(1) $\frac{dy'}{dN} = 0.61763\ N^{-.98167}\ P^{.05664}$ (2) $\frac{dy'}{dP} = 1.90845\ N^{.01833}\ P^{-.94336}$

(First figure for nitrogen, second figure for phosphorus) product, decision making is simplified. It has been explained under input-output relationship in the basic logic that optimum level of fertilization is reached when the marginal product is equal to the inverse price ratio of the input and output. Thus these marginal product figures, particulary exact marginal products shown in this table are very important for determining the optimum level.

The table presented here shows 6 columns and 11 rows with the first figures of exact marginal product for nitrogen and second figures of exact marginal product of wheat (in bushels) for phosphorus based on Cobb-douglas function. If these figures for zero to 50 pounds of phosphorus are observed in any row of the Cobb-douglas function, it is found that there is a decreasing rate of increase (or diminishing return). If one sees the 11 rows for 11 different fixed levels of nitrogen, the same relation holds true; all these Cobb-douglas functions show a diminishing return. Thus the rows explain the wheat response for each additional pound of phosphorus at specified levels.

Now one may again observe the exact marginal yield obtained and shown in six different columns. The first figures are for nitrogen and the second figures are for phosphorus, and both are from Cobb-douglas function. In different columns, phosphorus has been kept fixed at zero to 50 pounds. These 6 columns thus show the response of wheat for nitrogen in terms of exact marginal yield of wheat. The first figures show a diminishing return. From this table showing exact marginal products, one row and one column are shown under Tables 9 and 10 for simpler presentation.

It is possible to calculate the exact marginal product for each pound of nitrogen (N_2) and phosphorus (P_2O_5) at each unit, tenth, hundreth, or even thousandth of actual levels of nitrogen and phosphorus. This table has, however, been prepared for every ten units of the two fertilizers in different combinations.

In case of linear production function, exact marginal product of wheat is equal to the average marginal product of wheat. The exact marginal product is always the same and it is .05546 bushels of wheat for each pound of phosphorus. In case of nitrogen, the exact marginal product is .0178 bushels per pound. Table 6 and 7 may thus be used for presenting exact marginal products and for this reason are not shown in Table 8.

Exact Marginal Yield of Wheat in Response to Phosphorus

Table 9 shows the first figures of row one of Table 8. As the quantity of phosphorus is increased from zero to 50 pounds, the exact marginal yield of wheat is decreased from .22679 to .08046.

Table 9. Exact marginal yield of wheat per pound of phosphorus (P_2O_5) when nitrogen (N_2) is held constant at zero level of fertilizer*

Rate of Phosphorus (P_2O_5) (Pounds)	Bushels of Wheat	Remarks
0	.22679	Nitrogen
10	.16511	(N_2) is
20	.13025	kept at
30	.10779	zero level
40	.09211	of fertili-
50	.08046	zer.

*Based on Cobb-douglas function

As already indicated in Table 7, the exact marginal products presented in this table show a diminishing return in wheat response to phosphorus.

Exact Marginal Yield of Wheat in Response to Nitrogen

This table is obtained from the first figures of column two of Table 8. This table shows the wheat response to each additional input of the 10th pound of nitrogen. As is evident from the table, there is evidence of diminishing returns to nitrogen.

Table 10. Exact marginal yield of wheat per pound of nitrogen (N_2) when phosphorus (P_2O_5) is kept constant at zero level*

Rate of N_2 (nitrogen) in Pounds	Bushels of Wheat	Remarks
0	.07338	Phosphorus (P_2O_5) is kept at zero level of fertilizer use
10	.05274	
20	.04121	
30	.03384	
40	.02872	
50	.02496	
60	.02207	
70	.01979	
80	.01794	
90	.01640	
100	.01512	

* Original equation is $Y' = 33.695\ N^{.01833}\ P^{.05664}$

Partial derivative used for Table 10 and 11 are (1) and (2) respectively.

(1) $\frac{dy'}{dP} = 1.99051\ P^{-.94336}$

(2) $\frac{dy'}{dN} = .70348\ N^{-.98167}$

The exact marginal product of wheat from nitrogen at zero and 100 pound level decreases from .07338 to .01512, respectively.

Derived Demand for Phosphorus

In the previous tables marginal products of wheat are shown for zero, 10, 20, 30, 40, 50, pounds of phosphorus for different levels of nitrogen as shown under 11 rows. The marginal products for different levels of nitrogen from zero to 100 at an interval of 10 pounds were also shown. When one gets the price of wheat, one can convert the marginal product into marginal value product or, in other words, the marginal product curve when converted into marginal value product curve is exactly similar in curvature.

Table 11. Derived demand schedules for phosphorus

Pounds of Phosphorus (P_2O_5) when nitrogen is at zero level of fertilizer use	Marginal Yield of Wheat*	Value of Marginal Yield of Wheat		
		$1.75	$2.00	$2.25
0	.22679	.39688	.45358	.51027
10	.16511	.28894	.33022	.37150
20	.13025	.22794	.26050	.29306
30	.10779	.18863	.21558	.24253
40	.09211	.16119	.18422	.20725
50	.08046	.14081	.16092	.18104

* (1) $Y' = 33.695\ N^{.01833}\ P^{.05664}$

(2) $\frac{dy'}{dp} = 1.090845\ N^{.01833}\ P^{-.94336}$

(2a) $\frac{dy'}{dp} = 1.99051\ P^{-.94336}$

For illustrating this case, row one of the Table 8 (first figures) or the marginal yields under Table 9, are multiplied by price of wheat at $1.75, $2.00, and $2.25. These products are shown in Table 11. If the marginal unit cost is known, one can find how much input could be used economically. The curve obtained for value marginal product is the demand curve for nutrients at different prices. Three demand curves for phosphorus are shown in figure 6.

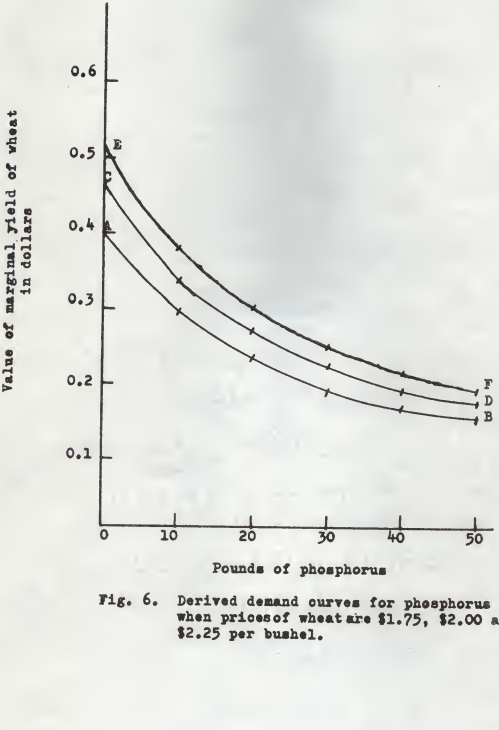

Fig. 6. Derived demand curves for phosphorus when prices of wheat are \$1.75, \$2.00 and \$2.25 per bushel.

Derived Demand for Nitrogen

Table 12 is similar to Table 11. The table is presented to illustrate the marginal value product of wheat in response to nitrogen when phosphorus (P_2O_5) is kept constant at different levels.

Table 12. Derived demand schedules for nitrogen

Pounds of Phosphorus (P_2O_5) when nitrogen is at zero level	Marginal Yield*	Value of Marginal Yield of Wheat		
		$1.75	$2.00	$2.25
0	.07338	.12842	.14676	.16511
10	.05274	.09230	.10548	.11867
20	.04121	.07212	.08242	.09272
30	.03384	.05922	.06788	.07614
40	.02872	.05026	.05744	.06462
50	.02496	.04368	.04992	.05616
60	.02207	.03862	.04414	.04966
70	.01979	.03463	.03958	.04453
80	.01794	.03140	.03588	.04037
90	.01640	.02870	.03280	.03690
100	.01512	.02646	.03024	.03402

*(1) $Y' = 33.695\ N^{.01833}\ P^{.05664}$

(2) $\frac{dy'}{dN} = 0.61763\ N^{-.98167}\ P^{.05664}$

(2a) $\frac{dy'}{dN} = .70348\ N^{-.98167}$

The table shows column number two of Table 8 (all first figures). The marginal products multiplied by the price of wheat gives the value marginal products. There is the same relationship for the demand of nitrogen (N_2) as for the demand of phosphorus, above.

Least Cost Combination for Forty-Two Bushels of Wheat

Table 13 shows an isoquant for 42 bushels of wheat and different combinations of nitrogen, from zero to 100 pounds, at increments of 10 pounds. The corresponding phosphorus requirements are also calculated and are shown in the same table. Table 13 also provides the incremental amounts of nitrogen (ΔN), which are 10 in all stages, and the incremental amounts of phosphorus (ΔP), which are 5.445 to 1.063 (shown in gradual fall). The average marginal rate of substitutions at these stages range from .5445 to .1063. These figures show diminishing rates of substitution. The average marginal rate of substitution was calculated by the relation $\Delta P/\Delta N$. Different inverse price ratios for phosphorus (P_2O_5) and nitrogen (N_2), that is P_N/P_p, were considered which are shown as 2.0, 1.5, and 1.33333. With the available average marginal rates of substitution and inverse price ratios, it is possible to find out the least-cost combination. If $\Delta P/\Delta N$ is equal to P_N/P_p the least-cost situation is obtained. As the exact marginal rate of substitutions are not calculated in this factor-factor relationship, the average marginal rate of substitution which is nearest to the inverse price ratio identifies the least-cost combination. In this table, for all price situations, the least-cost combination is associated with the highest average marginal rate of substitution, namely, 0.5445. The combination for the least cost, therefore, is 10 pounds of nitrogen and 27.6 pounds of phosphorus.

Table 13. Isoquant for forty-two bushels of wheat and least-cost combination

Pounds of Nitrogen N	ΔN	Pounds of Phosphorus P	ΔP	$\frac{P}{N}$ = Avrg. Rate of Sub.	(1) $\frac{P_N}{P_P}$	(2) $\frac{P_N}{P_P}$	(3) $\frac{P_N}{P_P}$	*Least Cost Combination Nitrogen	*Least Cost Combination Phosphorus
0	0	33.04500							
10	10	27.60000	5.44500	.54450	2.00000	1.50000	1.33333	10 lbs.	27.6 lbs.
20	10	22.87000	4.73000	.47300	2.00000	1.50000	1.33333		
30	10	20.09500	2.77500	.27750	2.00000	1.50000	1.33333		
40	10	17.46000	2.63500	.26350	2.00000	1.50000	1.33333		
50	10	15.58250	1.87750	.18725	2.00000	1.50000	1.33333		
60	10	13.78250	1.70000	.18050	2.00000	1.50000	1.33333		
70	10	12.63250	1.15000	.11500	2.00000	1.50000	1.33333		
80	10	11.50250	1.13000	.11300	2.00000	1.50000	1.33333		
90	10	10.42000	1.08250	.10825	2.00000	1.50000	1.33333		
100	10	9.35700	1.06300	.10625	2.00000	1.50000	1.33333		

(1) $Y' = 33.695\ N^{.01833}\ P^{.05664}$

* $\frac{\Delta P}{\Delta N} = \frac{P_n}{P_p}$ (When it is not possible to make them equal, it is nearest to equal.)

In finding out the least-cost combination, the following ratio was used:

$$\frac{\Delta P}{\Delta N} = \frac{P_n}{P_p}$$

However, it is not possible to find out the exact cost combination without use of calculus when the ratio becomes as follows:

$$\frac{dP}{dN} = \frac{P_n}{P_p}$$

The principle is the same, however. The calculus was used in finding out the exact marginal yields, optimum profit point, but not in finding out the least-cost combination.

Isoquant for Forty-four Bushels of Wheat and Least Cost Combinations

This table shows the equal product line (isoquant) for wheat at 44 bushels. As one looks at the nitrogen (N_2) and phosphorus (P_2O_5) combinations, one finds 11 posible combinations which can produce 44 bushels of wheat. There are actually unlimited numbers of combinations for getting 44 bushels of wheat but these numbers were only calculated. These combinations give 11 points through which one can draw an isoquant for 44 bushels of wheat. It is seen from the table that nitrogen was increased in 10 pounds increments, from zero to 100 pounds. The corresponding quantities of phosphorus required and the incremental quantities of phosphorus are also shown. The ratio between the incremental quantities of phosphorus and nitrogen thus gives the average marginal rate of substitution. The last three columns show the inverse price ratio between the two nutrients. The least-cost combination is attained when the marginal rate of substitution equals the inverse price ratio (already explained under basic logic). In case it is not possible to find a stage when both figures are equal (in case of average marginal rate of substitution) the nearest figure to the inverse price ratio will identify the least-cost point of the combination of the two nutrients. In this table, it is observed that 10 pounds of nitrogen and 93.04 pounds of phosphorus is the least-cost combination under specified price relationships. It may be observed that the cost of fertilization any other level is higher than at the least-cost combination indicated above.

Table 14. Yield Isoquant for forty-four bushels of wheat and least cost combination of nutrients

Pounds of Nitrogen N	ΔN	Pounds of Phosphorus P	ΔP	$\frac{\Delta P}{\Delta N}$	$\frac{P_n}{P_p}$	$\frac{P_n}{P_p}$	$\frac{P_n}{P_p}$	*Least Cost Combination Nitro.	Phosphorus
0		107.1600							
10	10	93.0375	14.1225	1.41225	2.00000	1.50000	1.33333	10	93.0375 lbs.
20	10	83.3800	9.6575	.96575	2.00000	1.50000	1.33333		
30	10	77.3850	5.9950	.59950	2.00000	1.50000	1.33333		
40	10	71.6800	5.7050	.57050	2.00000	1.50000	1.33333		
50	10	67.5625	4.1175	.41175	2.00000	1.50000	1.33333		
60	10	63.9100	3.6525	.36525	2.00000	1.50000	1.33333		
70	10	61.1375	2.7725	.27725	2.00000	1.50000	1.33333		
80	10	56.4725	2.6650	.26650	2.00000	1.50000	1.33333		
90	10	55.1250	1.3475	.13475	2.00000	1.50000	1.33333		
100	10	53.9000	1.1650	.11650	2.00000	1.50000	1.33333		

(1) Equation used in this table: $Y' = 33.695\ N^{.01833}\ P^{.05664}$

* $\frac{\Delta P}{\Delta N} = \frac{P_n}{P_p}$ (In case they are not equal, the nearest to equal is used.)

With the help of partial derivatives for nitrogen with respect to phosphorus, one could calculate the exact marginal rates of substitution and find out the level at which the marginal rate of substitution would be just equal to inverse price ratio. The average marginal rate of substitution is used and an approximation of the least-cost combination is attained as shown in Table 13. The principle is the same for both the exact and average marginal rate of substitution of the two nutrients. Under price ratio 2.0 least-cost combination is the same as for 1.5 and 1.33333. The least-cost combination for all these price ratios are 10 pounds of nitrogen and 93.0375 pounds of phosphorus.

Least-Cost Combination for Forty-six Bushels of Wheat

This table illustrates the same method for determining the isoquant for wheat at 46 bushels and different levels of nitrogen and phosphorus under which the least-cost combination is attained. As different isoquants differ in their combination of the two nutrients, this isoquant will have a different least-cost combination than that for 44 bushels of wheat. The table shows that the average marginal rates of substitution ranges from 1.41 to 0.12 for 44 bushels of wheat but in the isoquant for 46 bushels of wheat the average marginal rates of substitution between two input nutrients are 2.225 to .4835. The least-cost combination is attained in the same manner as in the previous two tables. When the inverse price ratio is 2.00, the least cost combination is 20 pounds of nitrogen and 214.95 pounds of phosphorus. Under inverse price ratio, 1.5 the least cost combination is 30 pounds of nitrogen and 199.11 pounds of phosphorus. The third situation is when the inverse price ratio of the two nutrients is 1.33 and we find that the least-cost combination is attained when nitrogen is 40 pounds and phosphorus is 187.14 pounds.

It may be seen from the least-cost combination shown in Table 13, 14, and 15 and optimum level of combination in Tables 16, 17, and 18 that the two concepts are distinctly different, as explained under basic logic.

Table 15. Yield Isoquant for forty-six bushels of wheat and least-cost combination of nutrients(1)

Pounds of Nitrogen N	ΔN	Pounds of Phosphorus P	ΔP	$\frac{\Delta P}{\Delta N}$	$\frac{P_n}{P_p}$	$\frac{P_n}{P_p}$	$\frac{P_n}{P_p}$	*Least Cost Combination: Nitrogen	*Least Cost Combination: Phosphorus
0		265.1250	-----	-----	-----	-----	-----	--	-----
10	10	232.8750	22.2500	2.22500	2.00000	1.50000	1.33333	--	-----
20	10	214.9450	17.9300	1.79300	2.00000	1.50000	1.33333	20	214.94500
30	10	199.1050	15.8400	1.58400	2.00000	1.50000	1.33333	30	199.10500
40	10	187.1350	11.9700	1.19700	2.00000	1.50000	1.33333	40	187.13500
50	10	178.6000	8.5350	.85350	2.00000	1.50000	1.33333	--	-----
60	10	170.3300	8.2700	.82700	2.00000	1.50000	1.33333	--	-----
70	10	162.3950	7.9350	.79350	2.00000	1.50000	1.33333	--	-----
80	10	157.2875	5.1075	.51076	2.00000	1.50000	1.33333	--	-----
90	10	152.3175	4.9700	.49700	2.00000	1.50000	1.33333	--	-----
100	10	147.4825	4.8350	.48350	2.00000	1.50000	1.33333	--	-----

(1) Equation used in this table: $Y' = 33.695\ N^{.01833}\ P^{.05664}$

* $\frac{\Delta P}{\Delta N} = \frac{P_n}{P_p}$ (In case they are not equal, the nearest to equal is used.)

This confirms that the least-cost combination is not necessarily the optimum level of fertilization.

The isoquant for 46 bushels of wheat and least-cost combinations are graphically represented in figure 7. Similar isoquants and least-cost combinations could graphically be represented for 42 and 44 bushels of wheat.

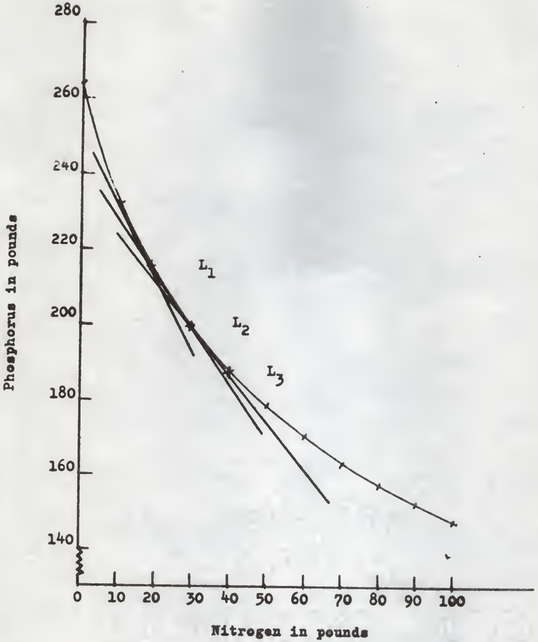

Fig. 7. An isoquant for 46 bushels of wheat showing the least-cost combinations. Three combinations (L_1, L_2 and L_3) indicating three least-cost combinations under three price situations.

Profit Maximization With Phosphorus

Table 16 shows the optimum level of phosphorus fertilization at different prices of phosphorus and wheat.

Table 16. Optimum level of phosphorus (P_2O_5) when nitrogen (N_2) is kept constant at zero level and per acre returns under different price situations

No.	Price of Wheat	Price of Phos. (P_2O_5)	$\frac{P_p}{Py'}$	Code Phosphorus (P_2O_5)	Pounds of Phosphorus (P_2O_5)	Predicted Yield	Per acre Return over fertilizer cost
1	1.75	.05	.02857	89.890	199.725	45.34265	69.36339
2	2.00	.05	.02500	104.550	236.375	45.73274	79.64673
3	2.25	.05	.02222	117.330	268.325	45.03147	90.15456
4	1.75	.10	.05714	43.119	82.7975	43.49408	67.83489
5	2.00	.10	.05000	49.665	99.1625	43.84552	77.77479
6	2.25	.10	.04444	56.276	115.6900	44.15478	87.77926
7	1.75	.15	.08571	28.046	45.1150	42.44679	67.51463
8	2.00	.15	.07500	32.314	55.7850	42.79120	77.21465
9	2.25	.15	.066667	36.615	66.5375	43.10398	87.00332

(1) $\frac{dy'}{dp} = 1.90845\ N^{.01833}\ p^{-.94336} = \frac{P_p}{Py'}$

(2) $\frac{dy'}{dp} = 1.99051\ p^{-.94336} = \frac{P_p}{Py'}$

(3) $Y' = 35.14389\ P^{.05664}$

It has been explained under basic logic that the optimum level is reached when the marginal product becomes equal to the inverse price ratio of the factor and product, that is $\frac{dY'}{dN} = \frac{PN}{Py'}$. At this stage net return is at its maximum.

The first three cases are presented with the same price for phosphorus but with different prices for wheat. The wheat price increases are shown at three levels, \$1.75, \$2.00, and \$2.25. It is found that the level of fertilization rises (199.73, 236.38, and 268.38 pounds of phosphorus) with increase in the price of wheat. The predicted yields of wheat are 45.34, 45.73, and 46.03

bushels; the per acre returns are \$69.36, \$79.65, and \$90.15, respectively. Cases 4, 5, 6, 7, 8, 9 show similar relationships with respect to price of wheat when the price of phosphorus is kept fixed at different levels of \$.10 and \$.15 per pound.

Cases 1, 4, and 7, show that the price of wheat is the same but price of phosphorus changes. It is logical that with the increase in resource price, the level of fertilization will come down. Phosphorus applications in these situations are 199.73, 82.80, and 45.12 pounds. The same relationships are seen in cases 2, 5, 8, and 3, 6, and 9. Any quantities above or below the optimum level of phosphorus shown in this table will bring down the net return.

Thus with an increase in the price, more fertilizers may be used profitably. But when the price of input rises, less input is used for optimum return. Any of the 11 rows could be used as optimum for the purpose of finding out the optimum level. This table is presented as an illustration and in this case, nitrogen is kept fixed at zero level of fertilizer use. The soil contains available nitrogen, both from original soil nutrients and also from residual quantities from previous years' fertilization. It will be seen from Table 14 that there is practically no need for the use of nitrogen fertilizers. The experiment designed shows only zero to 50 pounds of phosphorus whereas the optimum level verified in this table is beyond the experimental limit in all cases except case number 7. This tells us that the experiment might have been designed with phosphorus from 25 to 275 pounds and wheat response observed. This is based on prices of wheat and phosphorus and other assumptions made in this economic analysis.

Profit Maximization with Nitrogen

Table 17 shows the same relationship of optimum level of nitrogen fertilization. In this table only five cases are shown. The first cases show that with the increase in price of wheat the level of fertilization increases from 7.25 through 12.00, through 16.67 pounds, and the net returns are $69.66, $79.45, and $89.85 respectively. It is essential to know how much fertilizer may be used so that net return may be maximized. Any quantity more or less than this quantity attained at the optimum level will bring down the net return from use of fertilizer on wheat production.

Case numbers 4 and 5 are very interesting from an economic and practical point of view. It may be seen that the optimum level of fertilizer at $.15 and $.20 with price of wheat at $1.75, is zero for all practical purposes. When the price of nitrogen is $.10 but the wheat price is held constant, we can attain optimum level with 7.25 pounds of nitrogen.

Table 17. Optimum level of nitrogen (N_2) when phosphorus (P_2O_5) is kept at zero level and per acre return under different price situations

No. **	Price of Wheat	Price of Nitrogen (N_2)	$\frac{Pn}{Py'}$	Coded Nitrogen (N_2)	Pounds of Nitrogen (N_2)	Predicted Yield	Per acre Return over fertilizer cost
1	1.75	.10	.05714	12.901	7.2525	40.22078	69.66112
2	2.00	.10	.05000	14.782	11.9550	40.32441	79.45332
3	2.25	.10	.04444	16.668	16.6700	40.40884	89.25289
4	1.75	.15	.08571	8.5385	*-3.65375	≠39.91759	≠69.85578
5	1.75	.20	.11429	6.3676	*-9.08100	≠39.70267	≠69.47967

* In practice no fertilizer should be used and such a negative figure indicates zero level of fertilizer use.

≠ In practice nitrogen is zero and thus the predicted yields and net returns are a bit higher.

(1) $\frac{dy'}{dN} = 0.61763\ N^{-.98167}\ P^{.05664} = \frac{Pn}{Py'}$

(2) $\frac{dy'}{dN} = 0.70348\ N^{-.98167} = \frac{Pn}{Py'}$

(3) $y' = 38.37861\ N^{.01833}$

Soil is rich in available nitrogen. Theoretically it may be said that the optimum level would be attained even if the level of nitrogen were lowered by 3.65 pounds for \$.15, and 9.08 pounds for \$.20. In other words, even if 3.65 and 9.08 pounds of nitrogen were taken away from the soil, the net return would still be optimum.

From the practical point of view, the functional relationship does not provide any range below zero level of nitrogen and thus this extrapolation has only theoretical significance but it is of no practical improtance. It is neither sound to extrapolate nor possible to take away nitrogen from the soil, but it indicates that if the price of wheat remains at \$1.75, it would be economical to exploit the nitrogen already existing in the soil, when the prices are at levels shown under cases 4 and 5. As a practical application of this situation, nitrogen is costly and cannot be used as fertilizer. Thus zero level or no application of fertilizer presents the real solution.

When the optimum level of fertilizer is used at zero level the predicted yield and net return will be higher than shown in cases 4 and 5.

Now when one looks into the problem of nitrogen use as fertilizer, one finds that only 16.67 pounds of nitrogen may be used as maximum quantity under case 3, for achieving optimum level of application. The experiment, however, had nitrogen levels from zero to 100 pounds. It may be recommended that an experiment conducted in this soil would need zero to 20 pounds of nitrogen at all possible units between the two extremes. The levels between 25 to 100 pounds were practically unnecessary.

Wheat response to nitrogen and optimum level of production is shown graphically. Total products are obtained from table 7 and the inverse price ratio of nitrogen and wheat is taken from table 17.

Profit Maximization with Nitrogen and Phosphorus Fertilizer

Table 18 is presented to show the optimum level of combination of both phosphorus and nitrogen. The optimum level attained in this table, as shown under 9 cases, are similar to those in Tables 16 and 17. The only difference is that in this case, simultaneous determination of both phosphorus and nitrogen are involved to arrive at optimum level of fertilization.

Cases 1, 2, and 3 show the same prices for nitrogen and phosphorus but different prices for wheat. The prices of nitrogen and phosphorus have a constant ratio of 2.00 in all three cases. The prices of wheat are \$1.75, \$2.00 and \$2.25 in 3 cases, respectively. It is found that the optimum levels of nitrogen and phosphorus in pounds are (1) 11.64 and 201.43, (2) 17.33 and 236.58, (3) 22.94 and 271.26 with increased predicted yields and per acre returns as shown in the last two columns. It may be seen from the optimum level

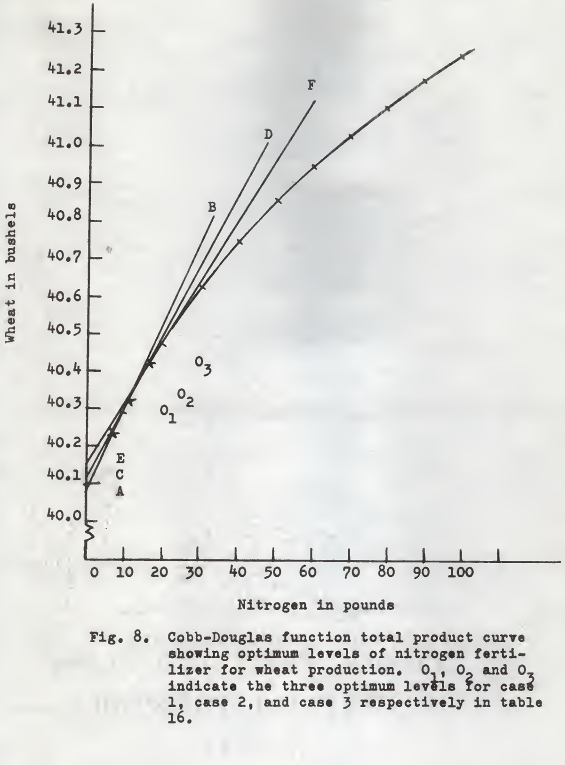

Fig. 8. Cobb-Douglas function total product curve showing optimum levels of nitrogen fertilizer for wheat production. O_1, O_2 and O_3 indicate the three optimum levels for case 1, case 2, and case 3 respectively in table 16.

that the relationship shown in Tables 16 and 17 holds true in this table also. The optimum level of both nitrogen (N_2) and phosphorus (P_2O_5) are increasing when the price of wheat is increasing. The quantity of nitrogen is very small and thus it is less important than is the case with phosphorus. It may be recalled in this connection that Table 16, which was illustrated for phosphorus alone, shows about the same predicted yield and per acre return.

Case numbers 4, 5, and 6 as well as case numbers 7, 8, and 9 show the same relationship with different situations of price ratios between nitrogen and phosphorus.

Taking case numbers 1, 4, and 7, we find that the price of wheat remains the same, but the prices of nitrogen and phosphorus and the ratios of prices of the two nutrients varied.

Table 18. Optimum level of nutrient combinations under specified conditions and optimum yield and per acre returns over fertilizer cost.

No.	Price of Wheat	Price of Nitro. (N_2)	Phos phorus (P_2O_5)	$\frac{Ph}{P_p}$	Nitrogen (N_2) ≠	Phosphorus (P_2O_5) ≠	Yield	Per acre Return over fertilizer cost
1	$1.75	$.10	$.05	2.00	(14.656) 11.640	(90.57261) 201.43153	45.69345	68.72796
2	2.00	.10	.05	2.00	(16.931) 17.3275	(104.63139) 236.57973	46.18000	78.73826
3	2.25	.10	.05	2.00	(19.176) 22.940	(118.50576) 271.26440	46.62040	89.03868
4	1.75	.15	.10	1.50	(9.286) *-1.785 0.000	(43.03996) 82.59990	43.43622	67.75339
5	2.00	.15	.10	1.50	(10.732) 1.830	(49.74207) 99.35518	43.90863	77.60724
6	2.25	.15	.10	1.50	(12.189) 5.4725	(56.49516) 116.23790	44.33049	87.29893
7	1.75	.20	.15	1.33	(6.758) *-8.105 0.000	(27.84242) 44.60605	42.13223	67.04023
8	2.00	.20	.15	1.33	(7.8068) *-5.283 0.000	(32.16339) 55.40848	42.58846	76.86565
9	2.25	.20	.15	1.33	(8.842) * .282 0.000	(36.55193) 66.37983	42.99482	86.78138

* In practice negative figures represent zero levels or no fertilizer application.

≠ Coded numbers are first figures shown in brackets, pounds of nitrogen and phosphorus are second figures.

(1) $\frac{dy'}{dN} = 0.61763\ N^{-.98167}\ P^{.05664} = \frac{Pn}{Py'}$

(2) $\frac{dy'}{dp} = 1.090845\ N^{.01833}\ P^{-.94336} = \frac{P_p}{Py'}$

(3) $.32363\ \frac{P}{N} = \frac{Pn}{Py'} \cdot \frac{Py'}{P_p} = \frac{Pn}{P_p} = \frac{dy'}{dN} \cdot \frac{dp}{dy'} = \frac{dp}{dn}$

The result is that the level of the use of nitrogen and phosphorus have fallen from (1) 11.64 and 201.43, (4) 0.00 and 82.60 to (7) 0.00 and 44.61, thus showing that with increased price of nutrients, the optimum level of fertilization is at a lower level.

Situations considered in 9 cases in Table 18, show that the optimum level of fertilization under different prices of wheat, different prices of nitrogen, and phosphorus differ. The specified levels of nutrients maximize per acre returns under the conditions considered. Any use of fertilizer above or below these levels will decrease the per acre returns.

SUMMARY AND CONCLUSIONS

Economic analysis involved in this work is aimed at the diminishing return area of the production function. The linear function derived does not include any situation under which the law of diminishing return can operate. The analysis was therefore based on linear funtion to a limited extent, but for derivation of least-cost combinations and optimum profit situations, Cobb-Douglas function was used. This function shows an exponent of less than one and may explain the problems within the rational zone of the function. Although the Cobb-Douglas function does not show a high coefficient of determination, it was used to present the technique.

Predicted total products were calculated for both Cobb-Douglas and linear functions. For different combinations of nitrogen and phosphorus 16 functional relationships were derived with the Cobb-Douglas function. Each row and each column in Table 5 represent a different functional relationship. The linear function, however, has the same slope over the entire range of inputs. The difference is only in yields at the y intercept under different specified conditions of the two nutrients, thus different rows and columns represent different linear functions in respect to nitrogen and phosphorus.

In the Cobb-Douglas function wheat responses to phosphorus are more effective than those to nitrogen. The exact marginal yields of wheat are shown in relevant tables. A few examples of average marginal yields of wheat are also shown.

Derived demand schedules are shown for nitrogen and phosphorus when prices of wheat vary.

Isoquants for 42, 44, and 46 bushels of wheat were calculated. The basis of the isoquants were the average marginal rates of substitution for nitrogen and phosphorus. Each isoquant was scheduled from 11 points, or 11 combinations, of the two elements. Three price ratios for nitrogen and phosphorus were considered and least-cost combinations computed. There was only one least cost point for 42 bushels of wheat under three price ratios. There was also a single least-cost point for 44 bushels of wheat for all three price ratios. The isoquant for 46 bushels of wheat had three least-cost combinations for three different price ratios.

The optimum levels of wheat production and net returns were calculated for different price situations of wheat, nitrogen, and phosphorus. Five situations were shown and the optimum levels of production were presented and net returns calculated. The optimum level of nitrogen is low. There is practically no need of nitrogen fertilizer. The soil was rich in available nitrogen. Residual nitrogen was one of the causes for this type of optimum level of nitrogen. If the nitrogen was free of cost or very low in price, it might be economic to use more nitrogen. The optimum levels of phosphorus obtained are the real achievements of this study. The response of phosphorus was much more effective than nitrogen. Nine price situations for wheat and phosphorus were considered. The optimum levels of nitrogen and phosphorus were calculated for both the nutrients considered together. The optimum levels of phosphorus were calculated and net returns were shown. Nine situations showed nine different optimum levels.

There are price situations for nitrogen, which make it unprofitable to use nitrogen at all. It was found in Tables 17 and 18 that optimum levels of nitrogen are less than zero. These situations are shown only for theoretical interest. The realistic situations are shown under nine different optimum levels of phosphorus shown in Table 16. The same thing is true for situations when optimum level of nitrogen is less than zero (Table 17).

In consideration of rising prices of wheat, optimum levels of nitrogen and phosphorus were higher. When prices of nitrogen and phosphorus rise, the optimum levels of fertilization goes down as shown by Tables 16, 17, and 18.

From the optimum levels of phosphorus, most cases show levels of fertilization beyond those considered in the experiment. The phosphorus used in the experiment was only zero, 25 and 50 pounds per acre. Optimum levels found in this analysis were as high as 268 pounds. This indicates a basis for recommendation that for optimum levels of fertilization, the experiment should be designed with higher levels of phosphorus from 25 to 275 pounds of phosphorus.

Optimum levels of nitrogen only ranged from zero to 20 pounds. This agronomic experiment was designed with zero, 25, 50, and 100 pounds of nitrogen. This indicates that considerably more nitrogen was used than was necessary. A level of nitrogen within zero to 25 pounds would be adequate.

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APPENDICES

Appendix 1

Equations Used in Calculating Predicted Yield Figures in Table 5

Cobb-Douglas Functions With Specified Quantity of Nitrogen and Phosphorus

$Y' = a\,P^{b'}$ N_2 fixed, P variable

1. $N = 0$ pounds $Y' = 35.14389\ p^{.05664}$
2. $N_2 = 10$ pounds $Y' = 35.3629\ p^{.05664}$
3. $N_2 = 20$ pounds $Y' = 35.52801\ p^{.05664}$
4. * $N_2 = 25$ pounds $Y' = 35.59877\ p^{.05664}$
5. $N_2 = 30$ pounds $Y' = 35.65942\ p^{.05664}$
6. $N_2 = 40$ pounds $Y' = 35.76724\ p^{.05664}$
7. $N_2 = 50$ pounds $Y' = 35.86159\ p^{.05664}$
8. $N_2 = 60$ pounds $Y' = 35.94583\ p^{.05664}$
9. $N_2 = 70$ pounds $Y' = 36.01996\ p^{.05664}$
10. $N_2 = 80$ pounds $Y' = 36.08398\ p^{.05664}$
11. $N_2 = 90$ pounds $Y' = 36.14463\ p^{.05664}$
12. $N_2 = 100$ pounds $Y' = 36.19854\ p^{.05664}$

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* Equation number 4 was not used. As the original experiment provides 25 pounds of nitrogen, this is shown as the corresponding production function.

Appendix 2

Equations Used in Calculating Predicted Yields in Table 5

Cobb-Douglas Functions With Specified Quantities Of Nitrogen and Phosphorus

$Y' = a\,N^{b_2}$ P_2O_5 fixed N_2 variable

1. $P_2O_5 =$ 0 pounds $Y' = 38.37861\ N^{.01833}$

2. $P_2O_5 =$ 10 pounds $Y' = 39.13000\ N^{.01833}$

3. $P_2O_5 =$ 20 pounds $Y' = 39.59836\ N^{.01833}$

* 4. $P_2O_5 =$ 25 pounds $Y' = 39.92521\ N^{.01833}$

5. $P_2O_5 =$ 30 pounds $Y' = 40.13914\ N^{.01833}$

6. $P_2O_5 =$ 40 pounds $Y' = 40.52329\ N^{.01833}$

7. $P_2O_5 =$ 50 pounds $Y' = 40.85350\ N^{.01833}$

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* Equation number 4 was not used in the table number 5. As the original experiment provides 25 pounds of phosphorus, this production function refers to the same.

Appendix 3

Equations used in Various Tables

(1) $Y' = 33.695\ N.01833\ P.05664$

(2) $Y = 37.88078 + .44499\ N + 1.38652P$

(3) $Y = 38.32577 + 1.38652\ P$

(4) $Y = 39.26730 + .44499\ N$

(5) $\frac{dy'}{dn} = 0.61763\ N^{-.98167}\ P^{.05664}$

(6) $\frac{dy'}{dp} = 1.90845\ N^{.01833}\ P^{-.94336}$

(7) $\frac{dy'}{dp} = 1.99051\ P^{-.94336}$

(8) $\frac{dy'}{dn} = 0.70348\ N^{-98167}$

(9) $.32363\ \frac{P}{N} = \frac{PN}{Py'},\quad \frac{Py'}{Pp} = \frac{PN}{Pp} = \frac{dy'}{dN}\quad \frac{dP}{dy'}, = \frac{dP}{dN}$

(Equation number 9 was based on equations 5 and 6.)

Appendix 4

Coding of the Original Data

Linear Production Function		Cobb Douglas Function	
N_2 or P_2O_5 in Pounds	Coded Number	N_2 or P_2O_5 in Pounds	Coded Number
0	1.0	0	10
10	1.4	10	14
20	1.8	20	18
30	2.2	30	22
40	2.6	40	26
50	3.0	50	30
60	3.4	60	34
70	3.8	70	38
80	4.2	80	42
90	4.6	90	46
100	5.0	100	50

ECONOMICS OF FERTILIZER USE
ON WHEAT PRODUCTION

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This economic analysis was based on one agronomic experiment conducted in Manhattan, Kansas. Information available was discontinuous. Nitrogen used ranged from 0 to 100 pounds and phosphorus ranged from 0 to 50 pounds with increments of 25 pounds. Production functions derived are based on four levels of nitrogen and three levels of phosphorus. Two functional relationships were derived:

(1) Linear production function:

$$Y = 37.88078 + .44499N + 1.38652P$$

(2) Cobb-Douglas function:

$$Y = 33.695\ N^{.01833}\ P^{.05664}$$

Of the two functions, linear function shows much better correlation determination than the Cobb-Douglas function. Since linear function is not reliable for predicting yields beyond the range covered by the experimental data, Cobb-Douglas function was derived and used in extrapolation for finding out optimum levels of production and least-cost combination of nutrients. The linear function was used for intrapolations. Only these two functional relationships were considered. Square root and quadratic functions might have shown better fit, but lack of time and other limitations prevented their use. Sixty observations were used (two types of preparations were not significantly different and thus pooled together) in deriving these production functions. Thus despite other limitations, the analysis may be accepted as based on good foundations.

With one Cobb-Douglas function as shown above, six functions were derived for nitrogen and 11 functions for phosphorus. The total products and marginal products are shown in Tables 5 and 7 respectively. According to Cobb-Douglas functions, total product ranges from 40.04314 to 43.88909 bushels. The exact marginal product per pound of nitrogen ranged from .01512 to .07811 bushels and per pound of phosphorus it ranged from 0.08046 to 0.23359 bushels for different specified levels of nitrogen and phosphorus as shown in Table 8. The exact marginal products have been illustrated in Tables 9 and 10. The average marginal products are shown in Tables 6 and 7.

The exact marginal product schedule was used for deriving demand curves for phosphorus and nitrogen under different prices of wheat. They are shown in Tables 11 and 12.

Isoquants for 42, 44, and 46 bushels of wheat were calculated. The average marginal rates of substitution were calculated for three yield situations, and 11 points or nutrient combinations were calculated for each isoquant. The least-cost combination for the 42 bushel isoquant was 10 pounds of nitrogen and 27.6 pounds of phosphorus for inverse price ratio of 2.0, 1.5, and 1.33. The least-cost combination of the two nutrients for 44 bushels of wheat was 10 pounds of nitrogen and 93.0375 pounds of phosphorus for the three price ratios. The least-cost combinations also were calculated for the isoquant of 46 bushels of wheat. It was found that the least-cost combination for price ratio of 2.00 was 20 pounds of nitrogen, and 214.9450 pounds of phosphorus. With price ratios of 1.50 and 1.33333, the least-cost combinations were not the same. For price ratio of 1.50, least-cost combination was 30 pounds of nitrogen and 199.105 pounds of phosphorus. The least-cost combination for price ratio of 1.33333 was 40 pounds of nitrogen and 187.135 pounds of phosphorus.

The optimum combinations were calculated for nitrogen and phosphorus separately and also in combination. The wheat response to nitrogen was small, due to available soil nitrogen in the experimental plot. Five situations were shown in which optimum nitrogen levels were 7.2525, 11.955, and 16.670 pounds. In the other two cases no nitrogen was applicable.

Optimum levels of phosphorus are interesting. Nine cases were calculated. The price of wheat per bushel was assumed at $1.75, $2.00, and $2.25. The price per pound of phosphorus was assumed to be $.05, $.10 and $.15. The optimum levels for phosphorus decreased with a rise in the price of phosphorus, but rose with an increase in the price of wheat. The optimum level of phosphorus for three rising prices of wheat with constant price of phosphorus at $.05 per pound are 199.725, 236.375, and 268.325 pounds. The corresponding per acre returns above fertilizer cost were $69.36, $79.65, and $90.15, respectively. The same relationship may be seen in cases 4, 5, and 6, and 7, 8, and 9 (Table 16). When the price of wheat is constant, for example, cases 1, 4, and 7, the optimum levels of phosphorus in pounds are 199.725, 82.7975 and 45.115 and the per acre returns above fertilizer cost are $69.36, $67.83, and $67.51, respectively.

Optimum levels were also calculated for nitrogen and phosphorus combined. As the response to nitrogen was small, the situation with both nutrients was nearly the same as the situation with phosphorus considered alone. When we take three prices of wheat and constant prices of nitrogen and phosphorus, as shown in any three cases 1, 2, 3, or 4, 5, 6, or 7, 8, 9, we find that optimum levels of both nitrogen and phosphorus are lower. Cases 4, 7, 8 and 9 are such where cost of nitrogen does not allow for any use of nitrogen. It is only under situations in 1, 2, 3, 5 and 6 that some nitrogen may profitably be used. Per acre returns from combined use of both nutrients are almost the same as those of phosphorus.

The total product and marginal products worked out in this thesis are useful for any time period, provided other conditions remain the same. The price situations may change but it is possible to plug in new price ratios and find new least-cost combinations and optimum levels. The marginal rate of substitutions will also hold true as long as the assumptions and prevailing conditions of the experiment do not change. The study is based on one year experiment which was conducted and designed for agronomic purposes. For this reason, limitations are numerous.

The analysis shows that use of nitrogen from zero to 100 pounds and phosphorus from zero to 50 pounds was not appropriate. The optimum levels of phosphorus needed extrapolation, which indicates that the use of phosphorus ought to have been from 25 to 275 pounds and use of nitrogen from 0 to 25 pounds.