

STUDY OF A RECURSIVE METHOD FOR MATRIX INVERSION
VIA
SIGNAL PROCESSING EXPERIMENTS

by

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CHAPTER I

INTRODUCTION

The main objective of this report is to discuss a recursive matrix inversion formula suggested by Graupe [1] for real symmetric matrices. The use of this formula is explained in connection with two problems of interest in the following areas of communication and signal processing: (i) digital filtering via nonrecursive filters, and (ii) pattern classification via a least-squares minimum distance classifier.

In chapter II a sequential regression formula is used to arrive at an algorithm which could be used in connection with signal processing applications. The need to invert a matrix recursively is pointed out with respect to this algorithm. Illustrative examples related to the matrix inversion formula are also included.

Chapter III briefly reviews some basic ideas associated with pattern classification via a least-squares minimum distance classifier. Again, in connection with this classifier, the need to invert a real symmetric matrix recursively is identified. With the aid of examples it is demonstrated that Graupe's formula can be used effectively.

The data used in this study is included in the form of appendices. Some of the computer programs used to carry out the computations involved in Graupe's are also included in an appendix.

CHAPTER II

ADAPTIVE FILTERING

The purpose of this chapter is to introduce the adaptive digital filter (ADF) via a sequential regression formulation. We shall restrict our attention to nonrecursive filters. It will be shown that the formulation leads to a recursive relation which involves the computation of the inverse of a symmetric matrix.

2.1. Sequential Regression Formulation

Consider a time-varying nonrecursive digital filter transfer function $H_k(Z)$ given by

$$H_k(Z) = a_{0,k} Z^m + a_{1,k} Z^{m-1} + \dots + a_{m,k} \quad (2.1-1)$$

It follows that the corresponding input-output relation can be expressed as

$$g(k) = a_k^T x_k \quad (2.1-2)$$

where

$$a_k^T = [a_{0,k} \ a_{1,k} \ \dots \ a_{m,k}]$$

$$x_k^T = [x(k) \ x(k-1) \ \dots \ x(k-m)]$$

Equation (2.1-2) can alternately be written in the form

$$g(k) = \alpha_k^T \beta_k \quad (2.1-3)$$

where

$$\alpha_k^T = [a_{0,k} \ a_{1,k} \ \cdot \ \cdot \ \cdot \ a_{m,k}]$$

and

$$\beta_k^T = [x(k) \ x(k-1) \ \cdot \ \cdot \ \cdot \ x(k-m)]$$

We now define a cost function $R(\hat{\alpha}_r)$, such that

$$R(\hat{\alpha}_r) = \sum_{k=1}^r [y(k) - \hat{g}(k)]^2 \quad (2.1-4)$$

where

$$\hat{g}(k) = \hat{\alpha}_r^T \beta_k$$

$y(k)$ is the k -th data point of the desired output and r is an index that denotes the current iteration; see Fig. (2.1-1).

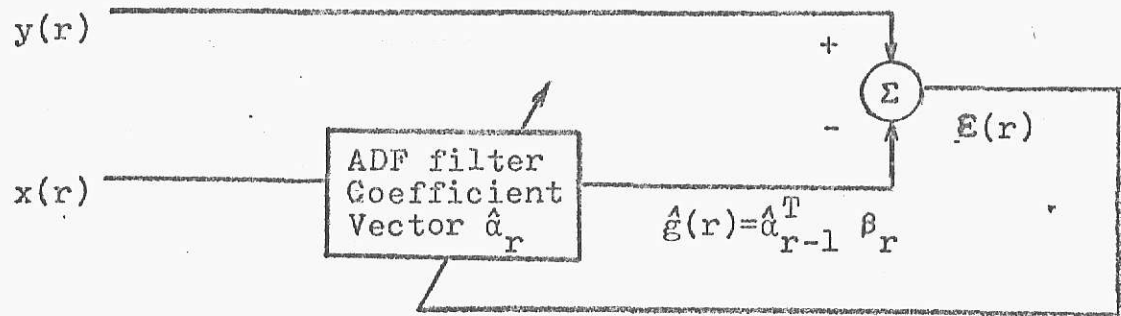


Fig. (2.1-1) ADF (Adaptive Digital Filter) Configuration.

We seek to minimize $R(\hat{\alpha}_r)$ with respect to $\hat{\alpha}_r$. This requires that the gradient of $R(\hat{\alpha}_r)$ be equal to zero; i.e., $\nabla_{\hat{\alpha}_r} R(\hat{\alpha}_r) = 0$. Thus, assuming the $\nabla_{\hat{\alpha}_r} y(k) = 0$, from (2.1-4) we obtain

$$\nabla_{\hat{\alpha}_r} R(\hat{\alpha}_r) = -2 \sum_{k=1}^r [y(k) - \hat{g}(k)] \nabla_{\hat{\alpha}_r} \hat{g}(k) \quad (2.1-5)$$

Now, from (2.1-3) we have

$$\nabla_{\hat{\alpha}_r} \hat{g}(k) = \beta_k \quad (2.1-6)$$

Substitution of (2.1-6) in (2.1-5) results in

$$\nabla_{\hat{\alpha}_r} R(\hat{\alpha}_r) = -2 \sum_{k=1}^r [y(k) - \hat{\alpha}_r^T \beta_k] \beta_k \quad (2.1-7)$$

Since the quantity $\hat{\alpha}_r^T \beta_k$ is a scalar, (2.1-7) can alternately be expressed as

$$\nabla_{\hat{\alpha}_r} R(\hat{\alpha}_r) = -2 \sum_{k=1}^r [y(k) \beta_k - \beta_k B_k^T \hat{\alpha}_r] \quad (2.1-8)$$

Thus the condition $\nabla_{\hat{\alpha}_r} R(\hat{\alpha}_r) = 0$ causes (2.1-8) to yield

$$\hat{\alpha}_r = P_r^{-1} \left[\sum_{k=1}^r y(k) \beta_k \right] \quad (2.1-9)$$

where

$$P_r = \sum_{k=1}^r \beta_k \beta_k^T$$

and $\hat{\alpha}_r^T = [\hat{\alpha}_{0,r} \hat{\alpha}_{1,r} \dots \hat{\alpha}_{m,r}]$ is the desired estimate of the filter coefficient vector associated with the r -th iteration.

2.2 Recursive Computation

From (2.1-9) it is clear that $\hat{\alpha}_r$ is obtained using the data pertinent to r iterations. However, this is actually not necessary. We now show that $\hat{\alpha}_r$ can be computed recursively. First, (2.1-9) is rewritten in the form

$$P_r \hat{\alpha}_r = \sum_{k=1}^{r-1} y(k) \beta_k + y(r) \beta_r \quad (2.2-1)$$

Next, addition and subtraction of $\beta_r B_r^T \hat{\alpha}_{r-1}$ to the right-hand

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side of (2.2-1) results in

$$\begin{aligned} P_r \hat{a}_r &= \left[\sum_{k=1}^{r-1} \beta_k \beta_k^T + \beta_r \beta_r^T \right] \hat{a}_{r-1} + \beta_r [y(r) - \beta_r^T \hat{a}_{r-1}] \\ &= P_r \hat{a}_{r-1} + \beta_r [y(r) - \beta_r^T \hat{a}_{r-1}] \end{aligned} \quad (2.2-2)$$

Finally, premultiplying by P_r^{-1} we obtain

$$\hat{a}_r = \hat{a}_{r-1} + P_r^{-1} \beta_r [y(r) - \beta_r^T \hat{a}_{r-1}] \quad (2.2-3)$$

That is, [see Fig. (1)]

$$\hat{a}_r = \hat{a}_{r-1} + P_r^{-1} \beta_r \mathcal{E}(r) \quad (2.2-4)$$

where $\mathcal{E}(r)$ is the error associated with the r -th iteration.

Thus from (2.2-4) we conclude that $\hat{a}_r$ can be computed recursively using the relation

$$\hat{a}_k = \hat{a}_{k-1} + P_k^{-1} \beta_k \mathcal{E}(k), \quad k=1, 2, \dots, r \quad (2.2-5)$$

2.3. Recurrence Relation for P_k^{-1}

We now show that P_k^{-1} in (2.2-5) can be computed in a recursive manner. From the definition of P_r in (2.1-9), we observe that it is real-symmetric, and can be expressed as

$$P_k = P_{k-1} + H_k H_k^T \quad (2.3-1)$$

where H_k is an $(m \times 1)$ vector*. Premultiplication by P_k^{-1} and postmultiplication by P_{k-1}^{-1} of both sides of (2.3-1) results in

$$P_k^{-1} P_k P_{k-1}^{-1} = P_k^{-1} P_{k-1} P_{k-1}^{-1} + P_k^{-1} H_k H_k^T P_{k-1}^{-1}$$

which yields

$$P_{k-1}^{-1} = P_k^{-1} + P_k^{-1} H_k H_k^T P_{k-1}^{-1} \quad (2.3-2)$$

Now, postmultiplying (2.3-2) by H_k we obtain

$$P_{k-1}^{-1} H_k = P_k^{-1} H_k (1 + H_k^T P_{k-1}^{-1} H_k) \quad (2.3-3)$$

Postmultiplication of both sides of (2.3-3) by

$$(1 + H_k^T P_{k-1}^{-1} H_k)^{-1} H_k^T P_{k-1}^{-1}$$

leads to

$$P_{k-1}^{-1} H_k (1 + H_k^T P_{k-1}^{-1} H_k)^{-1} H_k^T P_{k-1}^{-1} = P_k^{-1} H_k H_k^T P_{k-1}^{-1} \quad (2.3-4)$$

Again, from (2.3-2) it follows that

$$P_{k-1}^{-1} - P_k^{-1} = P_k^{-1} H_k H_k^T P_{k-1}^{-1} \quad (2.3-5)$$

Substituting (2.3-5) in (2.3-4) we obtain

$$P_{k-1}^{-1} H_k (1 + H_k^T P_{k-1}^{-1} H_k)^{-1} H_k^T P_{k-1}^{-1} = P_{k-1}^{-1} - P_k^{-1}$$

which yields

$$P_k^{-1} = P_{k-1}^{-1} - P_{k-1}^{-1} H_k (1 + H_k^T P_{k-1}^{-1} H_k)^{-1} H_k^T P_{k-1}^{-1} \quad (2.3-6)$$

In (2.3-6) it is important to note that the quantity

$$\gamma = (1 + H_k^T P_{k-1}^{-1} H_k) \quad (2.3-7)$$

is a scalar. Thus (2.3-6) can be written as

$$P_k^{-1} = P_{k-1}^{-1} - \frac{1}{\gamma} P_{k-1}^{-1} H_k H_k^T P_{k-1}^{-1} \quad (2.3-8)$$

From (2.3-8) it is clear that P_k^{-1} can be computed recursively, provided some initial P_{k-1}^{-1} is known. However, even this is not necessary according to the following argument by Graupe [1].

First, from (2.3-1) we have (with $k=1$)

$$P_1 = P_0 + H_1 H_1^T \quad (2.3-9)$$

However, by definition

$$P_1 = H_1 H_1^T \quad (2.3-10)$$

Clearly, for (2.3-9) and (2.3-10) to be satisfied, the condition to be satisfied is that

$$P_0 = 0 \quad (2.3-11)$$

Thus, to avoid the singularity problem, we let

$$P_0 = \epsilon I \quad (2.3-12)$$

where $\epsilon \rightarrow 0$ and I is the ($m \times m$) identity matrix; i.e.

$$P_0^{-1} = \frac{1}{\epsilon} I \quad (2.3-13)$$

In (2.3-13) $1/\epsilon$ can be chosen almost arbitrarily between 10 and the highest number that can be stored in the computer. Thus, with the initial value of P_0^{-1} as given by (2.3-13), successive P_k^{-1} can be computed recursively using (2.3-8). It has been stated that the P_k^{-1} computed with this initial guess converges

to the exact value of the inverse.

2.4. Numerical Examples

The purpose of this section is to present two numerical examples which employ the recurrence relation in (2.3-8) starting with an initial guess.

Example 2.4-1 A sequence of 2000 gaussian random numbers (see Appendix I) are grouped into (1x8) vectors, H_k^T , to obtain a set of 250 vectors. Compute the inverse of correlation matrix

$$P_{250} = \sum_{k=1}^{250} H_k H_k^T$$

using the recurrence relation in (2.3-8), and compare the same with the exact inverse obtained via the IBM subroutine INVERS.

Solution: Let $\hat{P}_k$ denote the exact inverse of the k-th iteration, then the corresponding "deviation matrix" is given by

$$D_k = P_k^{-1} - \hat{P}_k$$

The related mean square value is

$$\|D_k\|^2 = \frac{1}{64} \sum_i \sum_j d_{ij}^2$$

where d_{ij} denotes an element of D_k . Hence the root-mean square (RMS) error is given by

$$e_{\text{rms}} = \sqrt{\|D_k\|^2} \quad (2.4-1)$$

The results obtained using the above data are summarized in Table (2.4-1) for three values of ϵ , where ϵ defines the

Table 2.4-1. RMS error versus number of vectors processed.

Mean=0.0

Variance=0.50

NUMBER OF VECTORS	RMS ERROR		
	$\epsilon = 0.0001$	$\epsilon = 0.01$	$\epsilon = 1.0$
10	0.279	13.782	48.228
20	0.011	0.026	1.442
30	0.008	0.011	0.754
40	0.007	0.006	0.451
50	0.005	0.005	0.399
60	0.005	0.003	0.251
70	0.003	0.002	0.168
80	0.002	0.002	0.148
90	0.002	0.001	0.113
100	0.002	0.001	0.092
110	0.002	0.001	0.087
120	0.001	0.001	0.075
130	0.001	0.001	0.069
140	0.001	0.001	0.063
150	0.001	0.001	0.061
160	0.001	0.001	0.056
170	0.001	0.001	0.051
180	0.001	0.001	0.047
190	0.001	0.001	0.044
200	0.001	0.000	0.040
210	0.001	0.000	0.037
220	0.001	0.000	0.037
230	0.001	0.000	0.036
240	0.001	0.000	0.035
250	0.001	0.000	0.034

initial guess; see (2.3-13).

Similar results are presented in Tables (2.4-2) through (2.4-4) for the cases when the variances of the data sequences are 2, 20, and 50 respectively for three values of ϵ , namely 0.0001, 0.01, and 1.0.

From the above results it is apparent that values of $\epsilon \leq 0.01$ work satisfactorily for all three data sequences regardless of their variances which vary from 0.5 to 50.

Example 2.4-2 Using the sequence of 2000 numbers* given in Appendix II, compute the inverse of the correlation matrix

$$P_{250} = \sum_{k=1}^{250} H_k H_k^T$$

where the H_k 's are (8x1) vectors. Use the recurrence relation in (2.3-8), and compare the same with the exact inverse obtained via the IBM subroutine INVERS.

Solution: As in the case of Example 2.4-1 we denote the exact inverse at the k-th iteration by $\hat{P}_k$. Then, using the same notation as in Example 2.4-1, we obtain the results summarized in Table 2.4-5. We observe that even here, a value of ϵ less than or equal to 0.01 works satisfactorily.

The solution to Example 2.4-1 and 2.4-2 is accomplished by the computer program listed in Appendix III.

*Data obtained from Scandia Laboratories, Albuquerque, N.M.

Table 2.4-2. RMS error versus number of vectors processed.

Mean=0.0

Variance=2

NUMBER OF VECTORS	RMS ERROR		
	$\epsilon=0.0001$	$\epsilon=0.01$	$\epsilon=1.0$
10	0.190	0.073	2.176
20	0.009	0.000	0.010
30	0.005	0.000	0.004
40	0.004	0.000	0.002
50	0.003	0.000	0.002
60	0.003	0.000	0.001
70	0.002	0.000	0.001
80	0.001	0.000	0.001
90	0.001	0.000	0.000
100	0.001	0.000	0.000
110	0.001	0.000	0.000
120	0.001	0.000	0.000
130	0.001	0.000	0.000
140	0.001	0.000	0.000
150	0.001	0.000	0.000
160	0.000	0.000	0.000
170	0.000	0.000	0.000
180	0.000	0.000	0.000
190	0.000	0.000	0.000
200	0.000	0.000	0.000
210	0.000	0.000	0.000
220	0.000	0.000	0.000
230	0.000	0.000	0.000
240	0.000	0.000	0.000
250	0.000	0.000	0.000

Table 2.4-3. RMS error versus number of vectors processed.

Mean=0.0

Variance=20

NUMBER OF VECTORS	RMS ERROR		
	$\epsilon=0.0001$	$\epsilon=0.01$	$\epsilon=1.0$
10	0.058	0.002	0.000
20	0.002	0.000	0.000
30	0.001	0.000	0.000
40	0.000	0.000	0.000
50	0.000	0.000	0.000
60	0.000	0.000	0.000
70	0.000	0.000	0.000
80	0.000	0.000	0.000
90	0.000	0.000	0.000
100	0.000	0.000	0.000
110	0.000	0.000	0.000
120	0.000	0.000	0.000
130	0.000	0.000	0.000
140	0.000	0.000	0.000
150	0.000	0.000	0.000
160	0.000	0.000	0.000
170	0.000	0.000	0.000
180	0.000	0.000	0.000
190	0.000	0.000	0.000
200	0.000	0.000	0.000
210	0.000	0.000	0.000
220	0.000	0.000	0.000
230	0.000	0.000	0.000
240	0.000	0.000	0.000
250	0.000	0.000	0.000

Table 2.4-4. RMS error versus number of vectors processed.

Mean=0.0

Variance=50.00

NUMBER OF VECTORS	RMS ERROR		
	$\epsilon=0.0001$	$\epsilon=0.01$	$\epsilon=1.0$
10	0.060	0.001	0.000
20	0.004	0.000	0.000
30	0.000	0.000	0.000
40	0.000	0.000	0.000
50	0.000	0.000	0.000
60	0.000	0.000	0.000
70	0.000	0.000	0.000
80	0.000	0.000	0.000
90	0.000	0.000	0.000
100	0.000	0.000	0.000
110	0.000	0.000	0.000
120	0.000	0.000	0.000
130	0.000	0.000	0.000
140	0.000	0.000	0.000
150	0.000	0.000	0.000
160	0.000	0.000	0.000
170	0.000	0.000	0.000
180	0.000	0.000	0.000
190	0.000	0.000	0.000
200	0.000	0.000	0.000
210	0.000	0.000	0.000
220	0.000	0.000	0.000
230	0.000	0.000	0.000
240	0.000	0.000	0.000
250	0.000	0.000	0.000

Table 2.4-5. RMS error versus number of vectors processed.

Mean=5.45

Variance=16.37

NUMBER OF VECTORS	RMS ERROR		
	$\epsilon=0.0001$	$\epsilon=0.01$	$\epsilon=1.0$
10	9.222	2.295	17.425
20	1.300	0.039	1.270
30	0.419	0.028	1.247
40	0.372	0.030	0.568
50	0.161	0.023	0.434
60	0.110	0.017	0.288
70	0.102	0.015	0.261
80	0.082	0.016	0.270
90	0.064	0.009	0.187
100	0.059	0.007	0.161
110	0.053	0.005	0.146
120	0.051	0.004	0.143
130	0.031	0.003	0.061
140	0.029	0.002	0.053
150	0.023	0.002	0.040
160	0.023	0.001	0.038
170	0.012	0.001	0.014
180	0.011	0.001	0.014
190	0.011	0.001	0.014
200	0.011	0.001	0.014
210	0.010	0.001	0.014
220	0.008	0.004	0.016
230	0.006	0.007	0.021
240	0.006	0.010	0.023
250	0.007	0.009	0.022

CHAPTER III

PATTERN CLASSIFICATION

In this chapter it is demonstrated that the matrix inversion recurrence relation given by (2.3-8) can be used to design a least-square pattern classifier on an adaptive basis. This demonstration is carried out via three examples. Before presenting these examples, a brief discussion related to the notion of a pattern recognition system, and a specific least-squares classifier will be presented in Sections 3.1 through 3.3. The pertinent material is essentially taken from [2].

3.1. Introduction

A pattern recognition system mainly consists of two parts:
1) feature selection; 2) classifier design.

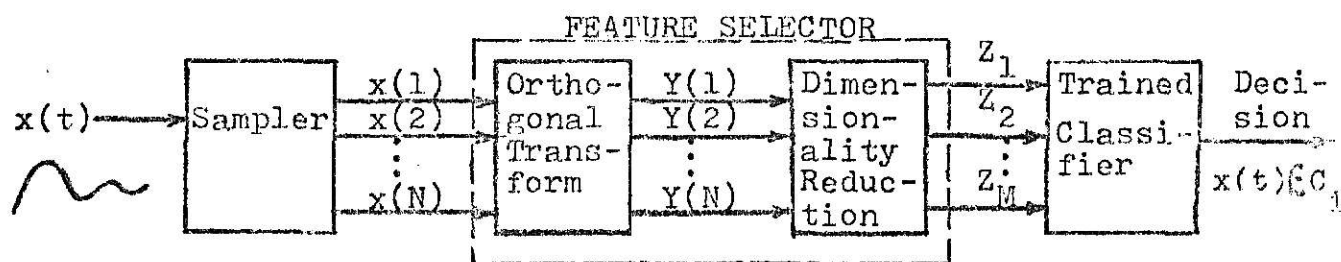


Fig. (3.1.1) A pattern recognition system

In the feature selection part the sampled values of $x(t)$ go through the process of transformation and dimensionality reduction¹. The reduced output vector is referred to as "pattern

¹This dimensionality reduction must be achieved such that the corresponding increase in classification error is relatively small.

vector" which is used as an input to the second part of the system, the trained classifier. The "classifier" is a decision making device that is trained to classify an incoming signal, $x(t)$, as belonging to one of the K classes denoted by $C_1, C_2, \dots, C_K$. In designing classifiers one is concerned with the concept of "training" — i.e., a process in which one uses a set of pattern vectors whose true classification is known (called the training set) to find a discriminant function, $g(z)$, which represents the classifier. The general form of $g(z)$ is as follows:

$$g(z) = W_1 Z_1 + W_2 Z_2 + \dots + W_d Z_d - \theta \quad (3.1-1)$$

By means of the discriminant function in (3.1-1) and an appropriate decision rule, one is able to perform the desired classification of patterns.

3.2.. Minimum-Distance Classifier

In a minimum distance classifier, the discriminant function and the decision rule are such that it would classify the given pattern Z as belonging to C_i if Z is closest to its mean vector $\bar{Z}_i$ for $i=1, 2, \dots, K$. The design of such classifiers is based on the assumption that the pattern classes in the feature space cluster around their respective means $\bar{Z}_i$, for $i=1, 2, \dots, K$. However, this may not be a reasonable assumption in many applications and this leads to the idea of "least-squares mapping technique".

The least-squares mapping technique consists of mapping the given patterns into a decision space wherein the patterns belonging to C_i are made to cluster around a preselected point V_i , $i=1, 2, \dots, K$. This mapping is achieved by the transformation matrix A , and A is chosen such that the overall mean-square mapping error is minimized. Next, a minimum distance classifier which uses this mapping technique is described. It is referred to as a "least-squares minimum distance classifier".

3.3. Least-Squares Minimum Distance Classifier

For the purposes of discussion, it suffices to consider the case where three classes are present. The extension to the general K -class case is straightforward.

Suppose we are given a set of d -dimensional pattern vectors $\{Z\}$. We first augment Z with a new component equal to '-1' to obtain $\hat{Z}$. Now, we seek a transformation matrix, A , which maps $Z \in C_k$ into V_k for $k=1, 2, 3$ in the least-square sense, and we take V_k to be the vertices of the three unit vectors as shown in Fig. (3.3-1).

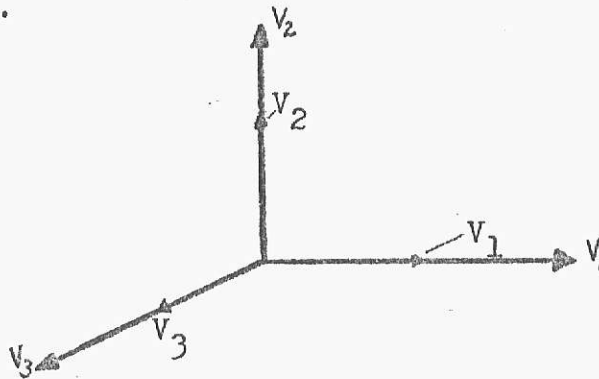


Fig. (3.3-1) Decision space for a 3-class classifier.

The transformation matrix which yields the minimum mean square mapping error is given by [2]:

$$A = \left[\frac{1}{N_i} \sum_{j=1}^{N_i} v_i z_{ij}^T \right] \left[\frac{1}{N_i} \sum_{j=1}^{N_i} z_{ij} z_{ij}^T \right]^{-1} \quad (3.3-1)$$

where N_i denotes the number of patterns in class i , and z_{ij} denotes the pattern belonging to class i , for each class. If we suppose that the a priori probability of C_k is P_k , $k=1, 2, 3$, then A becomes

$$\begin{aligned} A &= \left[\frac{P_1}{N_1} \sum_{j=1}^{N_1} \hat{z}_{1j} \hat{z}_{1j}^T + \frac{P_2}{N_2} \sum_{j=1}^{N_2} \hat{z}_{2j} \hat{z}_{2j}^T + \frac{P_3}{N_3} \sum_{j=1}^{N_3} \hat{z}_{3j} \hat{z}_{3j}^T \right] \\ &= \left[\frac{P_1}{N_1} \sum_{j=1}^{N_1} v_1 \hat{z}_{1j}^T + \frac{P_2}{N_2} \sum_{j=1}^{N_2} v_2 \hat{z}_{2j}^T + \frac{P_3}{N_3} \sum_{j=1}^{N_3} v_3 \hat{z}_{3j}^T \right] \end{aligned}$$

That is

$$\begin{aligned} A &= \left[\sum_{i=1}^3 \frac{P_i}{N_i} \sum_{j=1}^{N_i} v_i \hat{z}_{ij}^T \right] \left[\sum_{i=1}^3 \frac{P_i}{N_i} \sum_{j=1}^{N_i} \hat{z}_{ij} \hat{z}_{ij}^T \right]^{-1} \\ &= \left[\sum_{i=1}^3 \sum_{j=1}^{N_i} \frac{P_i}{N_i} v_i \hat{z}_{ij}^T \right] \left[\sum_{i=1}^3 \sum_{j=1}^{N_i} \frac{P_i}{N_i} \hat{z}_{ij} \hat{z}_{ij}^T \right]^{-1} \end{aligned}$$

Now, by definition

$$S_{v\hat{z}} = \sum_{i=1}^3 \sum_{j=1}^{N_i} \frac{P_i}{N_i} (v_i \hat{z}_{ij}^T) \equiv E(v \hat{z}^T)$$

$$S_{\hat{z}\hat{z}} = \sum_{i=1}^3 \sum_{j=1}^{N_i} \frac{P_i}{N_i} (\hat{z}_{ij} \hat{z}_{ij}^T) \equiv E(\hat{z} \hat{z}^T)$$

where $S_{v\hat{z}}$ and $S_{\hat{z}\hat{z}}$ are called crosscorrelation and autocorrelation matrices respectively. Hence A can be expressed as

$$A = (S_{V\hat{Z}})(S_{\hat{Z}\hat{Z}})^{-1} \quad (3.3-2)$$

We also note that V is a (3×1) matrix and $\hat{Z}^T$ is a $[1 \times (d+1)]$ matrix. Hence $S_{V\hat{Z}}$ is a $3 \times (d+1)$ matrix and $S_{\hat{Z}\hat{Z}}$ is a $(d+1) \times (d+1)$ matrix. Thus the transformation matrix A is a $[3 \times (d+1)]$ matrix.

Having the transformation matrix, we can map Z into the decision space. At this stage we proceed to construct the discriminant function such that it satisfies the minimum distance criteria.

Let L be the transformed pattern of $\hat{Z}$ via transformation matrix A . That is, $L = A\hat{Z}$.

We want to have $\hat{Z}$ classified as belonging to C_{i_0} if L is closest to V_{i_0} . Thus, it is necessary to find the distances between L and the preselected vectors V_i for $i=1, 2, 3$ and select the minimum of the three, i.e.

$$D_i^2 = \|L - V_i\|^2 = \|L\|^2 - 2V_i^T L + \|V_i\|^2, \quad i = 1, 2, 3 \quad (3.3-3)$$

where D_i is the distance between L and V_i . Now, since $\|V_i\|^2 = 1$ for $i=1, 2, 3$, (3.3-3) simplifies to

$$D_i^2 = \|L\|^2 + 1 - 2V_i^T L \quad \text{for } i=1, 2, 3 \quad (3.3-4)$$

It can be observed that D_i^2 , and consequently D_i , is minimum when $V_i^T L$ is maximum. Therefore, it is sufficient for us to design our classifier such that it would only check on the value of $V_i^T L$. Let $b_i = V_i^T L$ for $i=1, 2, 3$, where $L = A\hat{Z}$ and

$$v_1^T = [1 \ 0 \ 0]$$

$$v_2^T = [0 \ 1 \ 0]$$

$$v_3^T = [0 \ 0 \ 1]$$

We can write this in matrix form as follows

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \hat{Z} = A \hat{Z} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1d} & \theta_1 \\ a_{21} & a_{22} & \dots & a_{2d} & \theta_2 \\ a_{31} & a_{32} & \dots & a_{3d} & \theta_3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_d \\ -1 \end{bmatrix}$$

Thus the discriminant functions are

$$g_1(z) = a_{11} z_1 + a_{12} z_2 + \dots + a_{1d} z_d - \theta_1$$

$$g_2(z) = a_{21} z_1 + a_{22} z_2 + \dots + a_{2d} z_d - \theta_2$$

$$g_3(z) = a_{31} z_1 + a_{32} z_2 + \dots + a_{3d} z_d - \theta_3$$

To classify a given pattern, the classifier computes $g_1(z)$, $g_2(z)$ and $g_3(z)$ and assigns the pattern to C_{i0} , if $\text{Max}[g_i(z)] = g_{i0}$ for $i=1, 2, 3$.

3.4. Adaptive Considerations

We observe that the classifier described in Section 3.3 is completely defined by the transformation matrix A , which is given by

$$A = S_{v\hat{Z}} S_{\hat{Z}\hat{Z}}^{-1} \quad (3.4-1)$$

Equation (3.4-1) implies that there is a matrix inversion involved in the computation of A . Moreover, the autocorrelation matrix $S_{\hat{Z}\hat{Z}}$ is a symmetric matrix and hence can be computed recursively using (2.3-8). This aspect is illustrated in what follows via two examples.

Example 3.4-1. Consider a 2-dimensional training set to be as follows:

$$\begin{aligned} c_1: & \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \end{bmatrix} \\ c_2: & \begin{bmatrix} 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \end{bmatrix} \\ c_3: & \begin{bmatrix} 6 \\ -1 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 1 \end{bmatrix}, \begin{bmatrix} 10 \\ 1 \end{bmatrix} \end{aligned}$$

Assuming that $P_1 = P_2 = P_3 = 1/3$, compute and print $S_{\hat{Z}\hat{Z}}$, $S_{\hat{Z}\hat{Z}}^{-1}$, $S_{\hat{V}\hat{Z}}$ and A , as the above training set vectors are processed.

Solution: The solution to this example is obtained using the following steps:

Step 1 - Augment the training set vectors by adding a component -1.

Step 2 - Use Eq. (2.3-8) to compute the inverse autocorrelation matrix, $S_{\hat{Z}\hat{Z}}^{-1}$ with $\epsilon = 0.01$.

Step 3 - Compute the crosscorrelation matrix $S_{\hat{V}\hat{Z}}$.

Step 4 - Multiply the results obtained in steps 2 and 3 to obtain the transformation matrix A as defined in (3.4-1).

Steps 2 through 4 are accomplished by the computer program listed in Appendix III, and the corresponding results are summarized in Table 3.4-1. From this table it is apparent that the

Table 3.4-1. Pertaining to Example 3.4-1.

# OF VECT.	$S_{\hat{z}\hat{z}}$ AUTOCORRELATION MATRIX	$S_{\hat{z}\hat{z}}^{-1}$ INVERSE-AUTOCORRELATION MATRIX	$S_{v\hat{z}}$ CROSSCORRELATION MATRIX	$S_{\hat{z}\hat{z}}^{-1}$ TRANSFORMATION MATRIX	A
1	0.0 0.0 0.0 0.0 9.000 -3.000 0.0 -3.000 1.000	100.00 0.0 0.0 0.0 10.090 29.970 0.0 29.970 90.010	0.0 3.000 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	0.0 0.300 -0.100 0.0 0.0 0.0 0.0 0.0 0.0	0.0 0.300 -0.100 0.0 0.0 0.0 0.0 0.0 0.0
2	0.500 -1.500 0.500 -1.500 9.000 -3.000 0.500 -3.000 1.000	3.920 0.588 -0.196 0.588 20.178 59.941 -0.196 59.941 180.020	-0.500 3.000 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	-0.001 0.300 -0.100 0.0 0.0 0.0 0.0 0.0 0.0	-0.001 0.300 -0.100 0.0 0.0 0.0 0.0 0.0 0.0
3	1.667 -3.000 1.000 -3.000 9.000 -3.000 1.000 -3.000 1.000	1.492 0.447 -0.149 0.447 30.224 89.925 -0.149 89.925 270.025	-1.000 3.000 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	-0.000 0.300 -0.100 0.0 0.0 0.0 0.0 0.0 0.0	-0.000 0.300 -0.100 0.0 0.0 0.0 0.0 0.0 0.0
4	3.500 -4.500 1.500 -4.500 9.000 -3.000 1.500 -3.000 1.000	0.798 0.359 -0.120 0.359 40.252 119.916 -0.120 119.916 360.028	-1.500 3.000 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	-0.000 0.300 -0.100 0.0 0.0 0.0 0.0 0.0 0.0	-0.000 0.300 -0.100 0.0 0.0 0.0 0.0 0.0 0.0
5	6.000 -6.000 2.000 -6.000 9.000 -3.000 2.000 -3.000 1.000	0.499 0.300 -0.100 0.300 50.270 149.910 -0.100 149.910 450.030	-2.000 3.000 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	-0.000 0.300 -0.100 0.0 0.0 0.0 0.0 0.0 0.0	-0.000 0.300 -0.100 0.0 0.0 0.0 0.0 0.0 0.0

Table 3.4-1. Pertaining to Example 3.4-1. (cont.)

# OF VECT.	S_{zz} AUTOCORRELATION MATRIX		S_{zz}^{-1} INVERSE-AUTOCORRELATION MATRIX		S_{vz} CROSSCORRELATION MATRIX		A TRANSFORMATION MATRIX					
6	9.167	-0.833	0.833	0.516	-1.738	-6.213	-1.667	2.500	-0.833	-0.027	-0.383	-2.083
	-0.833	11.667	-3.333	-1.738	7.593	26.716	0.833	0.833	-0.167	0.017	0.427	1.240
	0.833	-3.333	1.000	-6.213	26.716	95.073	0.0	0.0	0.0	0.0	0.0	0.0
7	13.000	3.571	-0.143	0.566	-2.075	-7.319	-1.429	2.143	-0.714	-0.028	-0.383	-2.084
	3.571	13.571	-3.571	-2.075	8.796	31.075	1.571	1.429	-0.286	0.017	0.427	1.240
	-0.143	-3.571	1.000	-7.319	31.075	110.779	0.0	0.0	0.0	0.0	0.0	0.0
8	15.875	7.625	-0.875	0.479	-1.473	-5.280	-1.250	1.875	-0.625	-0.060	-0.210	-1.491
	7.625	16.375	-3.875	-1.473	5.250	19.032	2.125	2.000	-0.375	0.053	0.234	0.578
	-0.875	-3.875	1.000	-5.280	19.032	70.042	0.0	0.0	0.0	0.0	0.0	0.0
9	18.111	11.444	-1.444	0.337	-0.822	-2.980	-1.111	1.667	-0.556	-0.088	-0.093	-1.074
	11.444	20.000	-4.222	-0.822	2.461	9.194	2.556	2.556	-0.444	0.085	0.103	0.113
	-1.444	-4.222	1.000	-2.980	9.194	35.478	0.0	0.0	0.0	0.0	0.0	0.0
10	21.200	13.800	-2.000	0.261	-0.669	-2.352	-1.000	1.500	-0.500	-0.088	-0.094	-1.080
	13.800	20.500	-4.300	-0.669	2.209	8.155	3.000	2.800	-0.500	0.085	0.102	0.111
	-2.000	-4.300	1.000	-2.352	8.155	31.329	0.0	0.0	0.0	0.0	0.0	0.0

Table 3.4-1. Pertaining to Example 3.4-1. (cont.)

# OF VECT.	$S_{\hat{z}\hat{z}}$ AUTOCORRELATION MATRIX		$S_{\hat{z}\hat{z}}^{-1}$ INVERSE-AUTOCORRELATION MATRIX		$S_{v\hat{z}}$ CROSSCORRELATION MATRIX		A TRANSFORMATION MATRIX					
11	22.545	12.000	-2.364	0.067	-0.048	-0.025	-0.909	1.364	-0.455	-0.116	-0.006	-0.751
	12.000	18.727	-3.818	-0.048	0.275	0.934	2.727	2.545	-0.455	0.072	0.143	0.263
	-2.364	-3.818	1.000	-0.025	0.934	4.505	0.545	-0.091	-0.091	0.043	-0.136	-0.508
12	24.750	11.000	-2.750	0.060	-0.017	0.105	-0.833	1.250	-0.417	-0.114	-0.010	-0.765
	11.000	17.167	-3.500	-0.017	0.208	0.680	2.500	2.333	-0.417	0.066	0.159	0.322
	-2.750	-3.500	1.000	0.105	0.680	3.667	1.083	-0.083	-0.167	0.048	-0.149	-0.554
13	27.769	10.769	-3.154	0.056	-0.004	0.164	-0.769	1.154	-0.385	-0.111	-0.016	-0.786
	10.769	15.923	-3.308	-0.004	0.201	0.651	2.308	2.154	-0.385	0.058	0.173	0.370
	-3.154	-3.308	1.000	0.164	0.651	3.668	1.615	0.0	-0.231	0.053	-0.156	-0.581
14	31.571	10.643	-3.571	0.053	0.006	0.210	-0.714	1.071	-0.357	-0.106	-0.022	-0.807
	10.643	14.857	-3.143	0.006	0.201	0.654	2.143	2.000	-0.357	0.052	0.182	0.399
	-3.571	-3.143	1.000	0.210	0.654	3.801	2.143	0.071	-0.286	0.055	-0.159	-0.590
15	36.133	10.600	-4.000	0.051	0.014	0.245	-0.667	1.000	-0.333	-0.101	-0.028	-0.823
	10.600	13.933	-3.000	0.014	0.206	0.676	2.000	1.867	-0.333	0.046	0.189	0.417
	-4.000	-3.000	1.000	0.245	0.676	4.007	2.667	0.133	-0.333	0.055	-0.160	-0.591

final value of the transformation matrix A is given by

$$A = \begin{bmatrix} -0.101 & -0.028 & -0.823 \\ 0.046 & 0.189 & 0.417 \\ 0.055 & -0.160 & -0.591 \end{bmatrix}$$

It is found that this value for A is exactly the same as that obtained directly (see [2]) i.e., without resorting to the recurrence relation in (2.3-8).

Example 3.4-2. Consider the training set given in Table 3.4-2. Assuming that $P_1 = P_2 = P_3 = 1/3$, compute and print $S_{\hat{z}\hat{z}}$, $S_{\hat{z}\hat{z}}^{-1}$, $S_{v\hat{z}}$ and A as the training set vectors are processed.

Locate the boundaries corresponding to the final value of the transformation matrix A.

Solution: Using the four steps cited in the solution to the previous example, we obtain the results that are summarized in Table 3.4-3. From this table it is apparent that the final value of A is given by

$$A = \begin{bmatrix} 0.032 & -0.002 & 0.321 \\ -0.025 & 0.029 & 0.377 \\ -0.007 & -0.026 & -1.660 \end{bmatrix}$$

The corresponding boundaries are located in Fig. 3.4-1, from which it is apparent that the position of the boundaries separate the patterns of the three classes C_1 , C_2 , and C_3 adequately.

Table 3.4-2. Training set for Example 3.5-2.

CLASS #	z_1	z_2	CLASS #	z_1	z_2	CLASS #	z_1	z_2
1	44	61	2	19	59	3	20	40
1	40	59	2	13	58	3	22	45
1	41	55	2	11	56	3	18	42
1	39	53	2	20	56	3	24	42
1	44	53	2	16	55	3	14	41
1	43	50	2	10	53	3	20	41
1	41	50	2	15	50	3	17	40
1	43	46	2	12	50	3	20	38
1	40	45	2	6	48	3	24	38
1	44	44	2	11	46	3	9	39
1	40	42	2	8	44	3	4	36
1	39	40	2	11	43	3	15	36
1	42	40	2	3	41	3	10	35
1	36	36	2	5	37	3	19	34
1	43	36	2	8	36	3	10	31
1	39	33	2	5	34	3	15	31

Table 3.4-3. Pertaining to Example 3.4-2.

# OF VECT.	S_{zz} AUTOCORRELATION MATRIX	S_{zz}^{-1} INVERSE-AUTOCORRELATION MATRIX	S_{yz} CROSSCORRELATION MATRIX	A TRANSFORMATION MATRIX
1	1936.000 2684.000 -44.000 2684.000 3721.000 -61.000 -44.000 -61.000 1.000	65.783 -47.437 0.778 -47.437 34.235 1.078 0.778 1.078 99.982	44.000 61.000 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	0.009 0.012 -0.000 0.0 0.0 0.0 0.0 0.0 0.0
2	1768.000 2522.000 -42.000 2522.000 3601.000 -60.000 -42.000 -60.000 1.000	0.673 -0.513 -2.542 -0.513 0.444 5.110 -2.542 5.110 199.836	42.000 60.000 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	0.053 -0.019 0.001 0.0 0.0 0.0 0.0 0.0 0.0
3	1739.000 2433.000 -41.667 2433.000 3409.000 -58.333 -41.667 -58.333 1.00	0.432 -0.256 3.025 -0.256 0.211 1.595 3.025 1.595 218.811	41.667 58.333 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	0.028 0.003 0.298 0.0 0.0 0.0 0.0 0.0 0.0
4	1684.500 2341.500 -41.000 2341.500 3259.000 -57.000 -41.000 -57.000 1.000	0.562 -0.317 4.968 -0.317 0.236 0.450 4.968 0.450 229.123	41.000 57.000 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	0.030 0.000 0.192 0.0 0.0 0.0 0.0 0.0 0.0
5	1734.800 2339.600 -41.600 2339.600 3169.000 -56.200 -41.600 -56.200 1.000	0.189 -0.080 3.919 -0.080 0.088 2.015 3.919 2.015 276.198	41.600 56.200 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	0.012 0.012 0.113 0.0 0.0 0.0 0.0 0.0 0.0

Table 3.4-3. Pertaining to Example 3.4-2. (cont.)

# OF VECT.	S_{ZZ}^A AUTOCORRELATION MATRIX	S_{ZZ}^{-1} INVERSE-AUTOCORRELATION MATRIX	S_{VZ} CROSSCORRELATION MATRIX	A TRANSFORMATION MATRIX
6	1753.833 2308.000 -41.833 2308.000 3057.500 -55.167 -41.833 -55.167 1.000	0.171 -0.035 5.243 -0.035 0.062 1.939 5.243 1.939 326.194	41.833 55.167 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	0.013 0.010 0.098 0.0 0.0 0.0 0.0 0.0 0.0
7	1734.428 2271.143 -41.714 2271.143 2977.857 -54.429 -41.714 -54.429 1.000	0.199 -0.038 6.215 -0.038 0.061 1.706 5.215 1.706 352.076	41.714 54.429 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	0.013 0.009 0.035 0.0 0.0 0.0 0.0 0.0 0.0
8	1756.625 2234.500 -41.875 2234.500 2870.125 -53.375 -41.875 -53.375 1.000	0.198 -0.016 7.424 -0.016 0.043 1.650 7.424 1.650 398.925	41.875 53.375 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	0.015 0.008 0.018 0.0 0.0 0.0 0.0 0.0 0.0
9	1739.222 2186.222 -41.667 2186.222 2776.222 -52.444 -41.667 -52.444 1.000	0.218 -0.025 7.776 -0.025 0.039 1.013 7.776 1.013 377.256	41.667 52.444 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	0.014 0.006 -0.147 0.0 0.0 0.0 0.0 0.0 0.0
10	1758.900 2161.200 -41.900 2161.200 2692.200 -51.600 -41.900 -51.600 1.000	0.202 -0.007 8.127 -0.007 0.032 1.393 8.127 1.393 412.529	41.900 51.600 -1.000 0.0 0.0 0.0 0.0 0.0 0.0	0.014 0.006 -0.147 0.0 0.0 0.0 0.0 0.0 0.0

Table 3.4-3. Pertaining to Example 3.4-2. (cont.)

# OF VECT.	S_{zz}^A AUTOCORRELATION MATRIX			S_{zz}^{-1} INVERSE-AUTOCORRELATION MATRIX			S_{vz}^A CROSSCORRELATION MATRIX			A TRANSFORMATION MATRIX		
11	1744.454	2117.454	-41.727	0.215	-0.014	8.256	41.727	50.727	-1.000	0.012	0.004	-0.279
	2117.454	2607.818	-50.727	-0.014	0.029	0.880	0.0	0.0	0.0	0.0	0.0	0.0
	-41.727	-50.727	1.000	8.256	0.880	389.433	0.0	0.0	0.0	0.0	0.0	0.0
12	1725.833	2071.000	-41.500	0.223	-0.023	8.077	41.500	49.833	-1.000	0.011	0.003	-0.413
	2071.000	2523.833	-49.833	-0.023	0.027	0.367	0.0	0.0	0.0	0.0	0.0	0.0
	-41.500	-49.833	1.000	8.077	0.367	353.878	0.0	0.0	0.0	0.0	0.0	0.0
13	1728.769	2040.923	-41.538	0.233	-0.019	8.749	41.538	49.077	-1.000	0.011	0.003	-0.413
	2040.923	2452.769	-49.077	-0.019	0.024	0.398	0.0	0.0	0.0	0.0	0.0	0.0
	-41.538	-49.077	1.000	8.749	0.398	383.368	0.0	0.0	0.0	0.0	0.0	0.0
14	1697.857	1987.714	-41.143	0.199	-0.030	6.726	41.143	48.143	-1.000	0.007	0.002	-0.626
	1987.714	2370.143	-48.143	-0.030	0.024	-0.111	0.0	0.0	0.0	0.0	0.0	0.0
	-41.143	-48.143	1.000	6.726	-0.111	271.994	0.0	0.0	0.0	0.0	0.0	0.0
15	1707.933	1958.400	-41.267	0.184	-0.019	6.692	41.267	47.333	-1.000	0.007	0.002	-0.623
	1958.400	2298.533	-47.333	-0.019	0.019	0.117	0.0	0.0	0.0	0.0	0.0	0.0
	-41.267	-47.333	1.000	6.692	0.117	282.314	0.0	0.0	0.0	0.0	0.0	0.0

Table 3.4-3. Pertaining to Example 3.4-2. (cont.)

# OF VECT.	$S_{\hat{z}\hat{z}}$ AUTOCORRELATION MATRIX			$S_{\hat{z}\hat{z}}^{-1}$ INVERSE-AUTOCORRELATION MATRIX			$S_{v\hat{z}}$ CROSSCORRELATION MATRIX			A TRANSFORMATION MATRIX		
16	1696.250	1916.438	-41.125	0.196	-0.022	7.010	41.125	46.438	-1.000	0.007	0.002	-0.656
	1916.438	2222.938	-46.438	-0.022	0.018	-0.095	0.0	0.0	0.0	0.0	0.0	0.0
	-41.125	-46.438	1.000	7.010	-0.095	284.510	0.0	0.0	0.0	0.0	0.0	0.0
17	1617.706	1869.647	-39.824	0.032	0.004	1.438	38.706	43.706	-0.941	0.038	-0.003	0.428
	1869.647	2296.941	-47.176	0.004	0.015	0.835	1.118	3.471	-0.059	-0.036	0.006	-1.228
	-39.824	-47.176	1.000	1.438	0.835	97.454	0.0	0.0	0.0	0.0	0.0	0.0
18	1537.222	1807.667	-38.333	0.016	0.005	0.879	36.556	41.278	-0.889	0.036	-0.003	0.340
	1807.667	2356.222	-47.778	0.005	0.015	0.930	1.778	6.500	-0.111	-0.034	0.005	-1.164
	-38.333	-47.778	1.000	0.879	0.930	78.948	0.0	0.0	0.0	0.0	0.0	0.0
19	1462.684	1744.947	-36.895	0.012	0.005	0.682	34.632	39.105	-0.842	0.035	-0.003	0.274
	1744.947	2397.263	-48.211	0.005	0.016	0.973	2.263	9.105	-0.158	-0.033	0.006	-1.110
	-36.895	-48.211	1.000	0.682	0.973	72.897	0.0	0.0	0.0	0.0	0.0	0.0
20	1409.550	1713.700	-36.050	0.011	0.006	0.684	32.900	37.150	-0.800	0.036	-0.004	0.324
	1713.700	2434.200	-48.600	0.006	0.017	1.032	3.150	11.450	-0.200	-0.035	0.006	-1.161
	-36.050	-48.600	1.000	0.684	1.032	75.680	0.0	0.0	0.0	0.0	0.0	0.0

Table 3.4-3. Pertaining to Example 3.4-2. (cont.)

# OF VECT.	$S_{\hat{z}\hat{z}}$ AUTOCORRELATION MATRIX	$S_{\hat{z}\hat{z}}^{-1}$ INVERSE-AUTOCORRELATION MATRIX	$S_{\hat{v}\hat{z}}$ CROSSCORRELATION MATRIX	A TRANSFORMATION MATRIX
21	1354.619 1674.000 -35.095 1674.000 2462.333 -48.905 -35.095 -48.905 1.000	0.010 0.006 0.006 0.006 0.018 1.082 0.656 1.082 76.767	31.333 35.381 -0.762 3.762 13.524 -0.238 0.0 0.0 0.0	0.037 -0.004 0.343 -0.035 0.006 -1.182 0.0 0.0 0.0
22	1297.591 1622.000 -33.955 1622.000 2478.091 -49.091 -33.955 -49.091 1.000	0.009 0.006 0.006 0.006 0.018 1.095 0.574 1.095 74.086	29.909 33.773 -0.727 4.045 15.318 -0.273 0.0 0.0 0.0	0.035 -0.004 0.266 -0.034 0.006 -1.115 0.0 0.0 0.0
23	1250.956 1584.087 -33.130 1584.087 2479.043 -49.130 -33.130 -49.130 1.000	0.008 0.005 0.005 0.005 0.019 1.106 0.532 1.106 72.797	28.609 32.304 -0.696 4.522 16.826 -0.304 0.0 0.0 0.0	0.036 -0.004 0.292 -0.034 0.006 -1.147 0.0 0.0 0.0
24	1204.833 1543.083 -32.250 1543.083 2479.917 -49.167 -32.250 -49.167 1.000	0.007 0.005 0.005 0.005 0.019 1.113 0.485 1.113 71.239	27.417 30.958 -0.667 4.833 18.208 -0.333 0.0 0.0 0.0	0.035 -0.004 0.270 -0.034 0.006 -1.132 0.0 0.0 0.0
25	1158.080 1492.880 -31.200 1492.880 2472.880 -49.120 -31.200 -49.120 1.000	0.006 0.004 0.004 0.004 0.019 1.082 0.404 1.082 66.599	26.320 29.720 -0.640 4.880 19.400 -0.360 0.0 0.0 0.0	0.034 -0.005 0.148 -0.033 0.007 -1.022 0.0 0.0 0.0

Table 3.4-3. Pertaining to Example 3.4-2. (cont.)

# OF VECT.	$S_{\hat{\hat{z}}\hat{\hat{z}}}$ AUTOCORRELATION MATRIX	$S_{\hat{\hat{z}}\hat{\hat{z}}}^{-1}$ INVERSE-AUTOCORRELATION MATRIX	$S_{\hat{\hat{z}}\hat{\hat{z}}}$ CROSSCORRELATION MATRIX	A TRANSFORMATION MATRIX
26	1118.192 1454.923 -30.423 1454.923 2459.154 -49.000 -30.423 -49.000 1.000	0.006 0.004 0.004 0.004 0.019 1.062 0.358 1.062 63.795	25.308 28.577 -0.615 5.115 20.423 -0.385 0.0 0.0 0.0	0.034 -0.005 0.140 -0.033 0.007 -1.023 0.0 0.0 0.0
27	1079.148 1414.074 -29.593 1414.074 2439.778 -48.815 -29.593 -48.815 1.000	0.005 0.003 0.003 0.003 0.019 1.019 0.301 1.019 59.526	24.370 27.519 -0.593 5.222 21.296 -0.407 0.0 0.0 0.0	0.033 -0.006 0.092 -0.032 0.008 -0.985 0.0 0.0 0.0
28	1044.928 1380.464 -28.929 1380.464 2418.678 -48.607 -28.929 -48.607 1.000	0.005 0.002 0.002 0.002 0.019 0.989 0.264 0.989 56.591	23.500 26.536 -0.571 5.429 22.071 -0.429 0.0 0.0 0.0	0.033 -0.006 0.099 -0.032 0.008 -1.000 0.0 0.0 0.0
29	1009.207 1337.103 -28.034 1337.103 2393.241 -48.345 -28.034 -48.345 1.000	0.005 0.002 0.002 0.002 0.018 0.930 0.204 0.930 51.603	22.690 25.621 -0.552 5.345 22.724 -0.448 0.0 0.0 0.0	0.032 -0.007 -0.007 -0.032 0.009 -0.905 0.0 0.0 0.0
30	976.400 1298.700 -27.267 1298.700 2359.100 -47.967 -27.267 -47.967 1.000	0.004 0.001 0.001 0.001 0.017 0.848 0.153 0.848 45.781	21.933 24.767 -0.533 5.333 23.200 -0.467 0.0 0.0 0.0	0.032 -0.008 -0.052 -0.031 0.009 -0.871 0.0 0.0 0.0

Table 3.4-3. Pertaining to Example 3.4-2. (cont.)

# OF VECT.	$S_{ZZ}^{\wedge\wedge}$ AUTOCORRELATION MATRIX			$S_{ZZ}^{\wedge\wedge-1}$ INVERSE-AUTOCORRELATION MATRIX			$S_{VZ}^{\wedge}$ CROSSCORRELATION MATRIX			A TRANSFORMATION MATRIX		
31	946.968	1266.097	-26.645	0.004	0.000	0.121	21.226	23.968	-0.516	0.032	-0.008	-0.044
	1266.097	2324.806	-47.581	0.000	0.016	0.786	5.419	23.613	-0.484	-0.031	0.009	-0.888
	-26.645	-47.581	1.000	0.121	0.786	41.563	0.0	0.0	0.0	0.0	0.0	0.0
32	918.156	1231.844	-25.969	0.004	-0.000	0.087	20.563	23.219	-0.500	0.032	-0.008	-0.068
	1231.844	2288.281	-47.156	-0.000	0.016	0.722	5.406	23.038	-0.500	-0.031	0.009	-0.873
	-25.969	-47.156	1.000	0.087	0.722	37.272	0.0	0.0	0.0	0.0	0.0	0.0
33	902.454	1222.394	-25.788	0.004	-0.000	0.088	19.939	22.515	-0.485	0.032	-0.008	-0.045
	1222.394	2283.061	-47.121	-0.000	0.016	0.744	5.242	23.212	-0.485	-0.031	0.010	-0.826
	-25.788	-47.121	1.000	0.088	0.744	38.273	0.606	1.394	-0.030	-0.001	-0.000	-0.069
34	890.147	1215.559	-25.676	0.004	-0.000	0.090	19.353	21.853	-0.471	0.032	-0.008	-0.013
	1215.559	2275.470	-47.059	-0.000	0.017	0.764	5.088	22.529	-0.471	-0.031	0.010	-0.777
	-25.676	-47.059	1.000	0.090	0.764	39.187	1.235	2.676	-0.059	-0.001	-0.001	-0.150
35	873.971	1202.428	-25.457	0.004	-0.001	0.088	18.800	21.229	-0.457	0.032	-0.007	0.028
	1202.428	2260.857	-46.914	-0.001	0.017	0.774	4.943	21.886	-0.457	-0.030	0.012	-0.676
	-25.457	-46.914	1.000	0.088	0.774	39.468	1.714	3.800	-0.086	-0.002	-0.003	-0.293

Table 3.4-3. Pertaining to Example 3.4-2. (cont.)

# OF VECT.	$S_{\hat{z}\hat{z}}$ AUTOCORRELATION MATRIX		$S_{\hat{z}\hat{z}}^{-1}$ INVERSE-AUTOCORRELATION MATRIX		$S_{v\hat{z}}$ CROSSCORRELATION MATRIX		A TRANSFORMATION MATRIX					
36	865.694	1197.028	-25.417	0.005	-0.001	0.090	18.278	20.639	-0.444	0.032	-0.006	0.088
	1197.028	2247.055	-46.778	-0.001	0.017	0.785	4.806	21.278	-0.444	-0.030	0.013	-0.617
	-25.417	-46.778	1.000	0.090	0.785	39.925	2.333	4.861	-0.111	-0.002	-0.005	-0.413
37	847.594	1180.189	-25.108	0.005	-0.001	0.083	17.784	20.081	-0.432	0.032	-0.006	0.108
	1180.189	2231.757	-46.622	-0.001	0.017	0.791	4.676	20.703	-0.432	-0.029	0.014	-0.495
	-25.108	-46.622	1.000	0.083	0.791	39.894	2.649	5.838	-0.135	-0.003	-0.007	-0.555
38	835.816	1170.710	-24.974	0.005	-0.001	0.083	17.316	19.553	-0.421	0.033	-0.005	0.153
	1170.710	2217.263	-46.474	-0.001	0.018	0.798	4.553	20.158	-0.421	-0.029	0.016	-0.420
	-24.974	-46.474	1.000	0.083	0.798	40.074	3.105	6.763	-0.158	-0.004	-0.009	-0.677
39	821.795	1158.128	-24.769	0.005	-0.001	0.079	16.872	19.051	-0.410	0.033	-0.005	0.187
	1158.128	2201.436	-46.308	-0.001	0.018	0.800	4.436	19.641	-0.410	-0.028	0.017	-0.326
	-24.796	-46.308	1.000	0.079	0.800	39.952	3.462	7.615	-0.179	-0.004	-0.011	-0.804
40	811.250	1148.175	-24.650	0.005	-0.001	0.078	16.450	18.575	-0.400	0.033	-0.004	0.244
	1148.175	2182.500	-46.100	-0.001	0.018	0.792	4.325	19.150	-0.400	-0.028	0.018	-0.248
	-24.650	-46.100	1.000	0.078	0.792	39.398	3.875	8.375	-0.200	-0.004	-0.014	-0.941

Table 3.4-3. Pertaining to Example 3.4-2. (cont.)

# OF VECT.	S_{zz}^{AA} AUTOCORRELATION MATRIX			S_{zz}^{-1} INVERSE-AUTOCORRELATION MATRIX			S_{vz}^A CROSSCORRELATION MATRIX			A TRANSFORMATION MATRIX		
41	805.512	1142.415	-24.634	0.005	-0.001	0.081	16.049	18.122	-0.390	0.033	-0.002	0.316
	1142.415	2164.488	-45.902	-0.001	0.018	0.787	4.220	18.683	-0.390	-0.028	0.019	-0.201
	-24.634	-45.902	1.000	0.081	0.787	39.046	4.366	9.098	-0.220	-0.004	-0.016	-1.062
42	788.262	1123.571	-24.262	0.005	-0.001	0.070	15.667	17.690	-0.381	0.033	-0.002	0.298
	1123.571	2149.167	-45.738	-0.001	0.018	0.787	4.119	18.238	0.381	-0.027	0.021	-0.079
	-24.262	-45.738	1.000	0.070	0.787	38.634	4.476	9.810	-0.238	-0.005	-0.018	-1.167
43	770.302	1100.791	-23.791	0.005	-0.002	0.051	15.302	17.279	-0.372	0.032	-0.003	0.243
	1100.791	2129.325	-45.512	-0.002	0.018	0.773	4.023	17.814	-0.372	-0.026	0.023	0.078
	-23.791	-45.512	1.000	0.051	0.773	37.330	4.465	10.419	-0.256	-0.006	-0.019	-1.272
44	757.909	1088.045	-23.591	0.005	-0.002	0.047	14.955	16.886	-0.364	0.032	-0.003	0.266
	1088.045	2110.386	-45.295	-0.002	0.018	0.760	3.932	17.409	-0.364	-0.025	0.025	0.150
	-23.591	-45.295	1.000	0.047	0.760	36.485	4.705	11.000	-0.273	-0.007	-0.021	-1.369
45	743.289	1071.644	-23.289	0.005	-0.002	0.037	14.622	16.511	-0.356	0.032	-0.003	0.258
	1071.644	2090.711	-45.067	-0.002	0.018	0.745	3.844	17.022	-0.356	-0.025	0.026	0.241
	-23.289	-45.067	1.000	0.037	0.745	35.400	4.822	11.533	-0.289	-0.007	-0.022	-1.454

Table 3.4-3. Pertaining to Example 3.4-2. (cont.)

# OF VECT.	S_{ZZ}^{AA} AUTOCORRELATION MATRIX	S_{ZZ}^{-1} INVERSE-AUTOCORRELATION MATRIX	S_{VZ} CROSSCORRELATION MATRIX	A TRANSFORMATION MATRIX
46	734.987 1062.391 -23.196	0.005 -0.002 0.037	14.304 16.152 -0.348	0.032 -0.002 0.307
	1062.391 2070.391 -44.826	-0.002 0.017 0.725	3.761 16.652 -0.348	-0.025 0.027 0.275
	-23.196 -44.826 1.000	0.037 0.725 34.323	5.130 12.022 -0.304	-0.007 -0.024 -1.539
47	721.468 1046.383 -22.915	0.005 -0.002 0.028	14.000 15.809 -0.340	0.032 -0.002 0.296
	1046.383 2046.787 -44.532	-0.002 0.017 0.692	3.681 16.298 -0.340	-0.025 0.028 0.344
	-22.915 -44.532 1.000	0.028 0.692 32.404	5.234 12.426 -0.319	-0.007 -0.025 -1.601
48	711.125 1034.271 -22.750	0.006 -0.002 0.026	13.708 15.479 -0.333	0.032 -0.002 0.321
	1034.271 2024.167 -44.250	-0.002 0.016 0.662	3.604 15.958 -0.333	-0.025 0.029 0.377
	-22.750 -44.250 1.000	0.026 0.662 30.864	5.438 12.813 -0.333	-0.007 -0.026 -1.660

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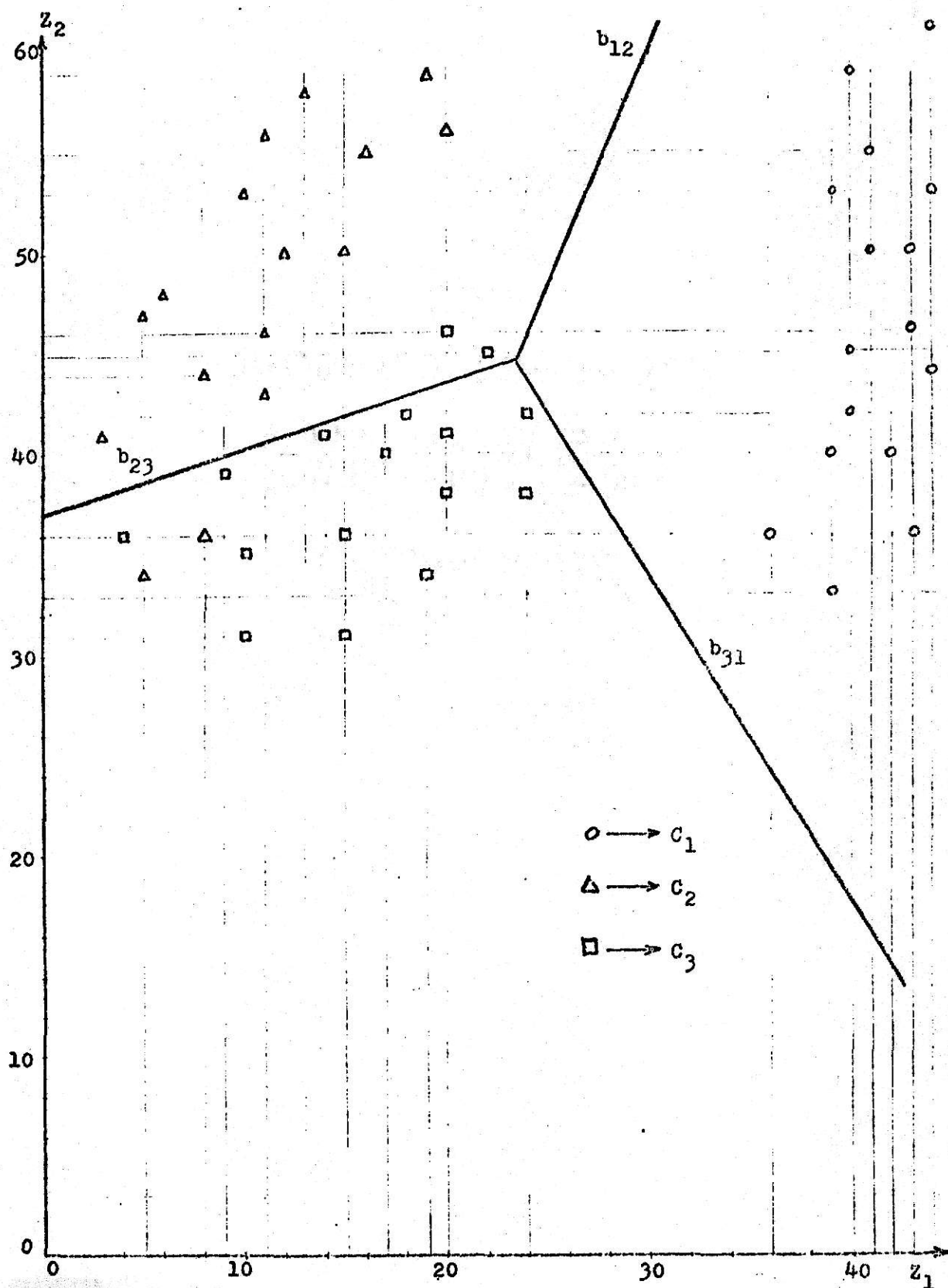


Fig. 3.4-1. Boundaries corresponding to Example 3.4-2.

CHAPTER IV

CONCLUDING REMARKS

From the results presented in Chapters II and III it could be concluded that Graupe's recursive method of inverting real-symmetric matrices (e.g. autocorrelation matrices) can be used effectively in adaptive filtering and pattern classification applications. In each case, an initial guess of the form $P_0^{-1} = \frac{1}{\epsilon}$ was considered, and values of $\epsilon \leq 0.01$ were found to be satisfactory.

In this report, Graupe's algorithm was studied entirely on an empirical basis. Future efforts should consider the convergence properties (e.g., rate of convergence) of the algorithm¹. References that should be used to carry out such a study are listed in the bibliography [3-5].

¹ Perhaps an optimum value of ϵ could be determined which depends on the data variance.

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APPENDIX I

DATA RELATED TO EXAMPLE 2.4-1, TABLES 2.4-1 TO 2.4-4.

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MEAN= 0.0

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INDEX	GROUPS OF (1X8) DATA VECTORS							
1	0.43	-0.53	-0.73	-0.10	0.62	-0.02	-0.21	0.37
2	-0.28	-0.53	-0.28	0.71	0.31	-0.10	0.54	-0.46
3	0.80	0.09	0.60	-0.65	-0.06	0.70	0.57	-0.04
4	-0.16	0.21	-0.52	0.56	-0.57	-0.02	0.68	0.14
5	-0.49	0.19	-0.11	0.07	-0.29	0.87	-0.34	0.56
6	0.50	-0.11	0.39	-0.86	-0.30	0.47	0.32	-0.80
7	-1.02	-0.33	-0.96	0.49	-0.17	0.40	0.46	-0.43
8	0.12	-0.60	-0.63	-0.14	-0.47	-0.23	-0.61	-0.37
9	0.10	-0.56	0.46	-0.01	0.71	0.69	-0.61	0.39
10	0.09	-0.34	-0.17	0.12	-0.19	-0.71	0.77	-1.17
11	0.78	-0.09	1.55	-0.01	0.75	0.17	-0.66	0.44
12	-0.67	0.06	1.16	-0.16	-0.04	-0.56	-0.11	-0.80
13	-0.57	-0.52	-0.28	-0.10	-0.14	-0.28	0.22	-0.38
14	0.24	0.02	0.28	-0.09	0.39	-0.38	0.64	-0.84
15	-0.05	-0.94	-0.12	-0.39	-0.05	0.25	-0.06	0.33
16	-0.27	-1.05	-0.41	-0.26	-0.01	-0.76	0.43	0.11
17	0.76	-0.45	0.94	0.54	-0.08	-0.32	0.41	-0.01
18	0.20	0.74	-0.51	-0.25	0.18	-0.30	-0.32	0.46
19	0.75	-0.13	0.80	-0.90	0.18	-0.09	0.03	-0.00
20	-0.43	0.31	0.40	0.33	-0.14	-0.09	-0.23	0.10
21	-0.17	0.41	-0.14	0.18	0.13	-0.66	-0.29	-0.24
22	0.23	-0.44	0.22	-0.11	-0.07	0.29	0.13	0.46
23	-0.58	-0.41	0.03	0.20	0.19	-0.12	-0.15	0.18
24	-0.91	-0.11	0.35	0.34	0.32	-0.78	0.16	-0.07
25	-0.09	-0.19	0.73	-0.45	-0.75	0.94	-0.14	-0.83
26	0.06	-1.28	-0.29	1.08	0.41	0.13	-0.24	0.92
27	-0.29	-0.02	-0.15	0.16	0.72	0.42	0.15	0.14
28	-0.96	-0.05	-0.31	-0.47	0.12	-0.59	-0.16	-0.67
29	-0.27	-0.00	0.30	0.42	-0.98	0.23	-0.88	-0.01
30	-0.05	-0.02	-0.29	-0.92	0.35	-0.02	0.31	0.08
31	-0.13	1.27	0.49	0.24	-0.46	0.28	0.19	-0.38
32	-0.32	0.23	-0.33	0.64	-0.49	0.23	-0.44	0.07
33	0.05	-0.28	-0.68	0.52	-0.31	-0.04	0.24	-0.56
34	-1.35	0.60	0.48	-1.38	1.00	0.58	-0.48	0.21
35	0.16	0.23	0.20	-0.32	0.88	-0.30	0.20	0.37
36	0.76	-0.01	0.04	-0.07	-0.26	-0.57	0.08	0.90
37	-0.38	0.23	-0.46	0.12	0.52	0.39	0.47	-0.20
38	-0.09	0.31	-0.24	-0.03	-0.87	-0.31	0.66	0.05
39	-0.68	0.10	0.24	0.74	0.50	-0.05	-0.55	0.14
40	-0.48	-0.53	0.04	0.15	0.57	-0.23	0.15	0.56
41	-0.32	-0.23	0.21	-0.05	-0.25	0.26	-0.45	-0.21
42	-0.52	1.37	0.30	0.38	-0.53	0.56	-0.04	-0.16
43	0.60	1.13	-0.14	0.16	-1.35	0.26	0.18	0.18
44	0.71	-0.08	0.44	-0.45	-0.27	-0.02	-0.50	-0.20
45	-1.01	-0.40	-0.91	0.28	0.13	0.33	-0.76	0.19
46	0.12	0.53	0.36	0.08	-0.23	0.43	0.68	-0.16
47	-0.74	-0.92	-0.20	-0.30	-0.42	-0.61	0.14	0.74
48	0.09	-0.40	-0.53	-0.13	-0.51	0.82	0.41	-0.13
49	-0.70	-0.53	-0.09	0.34	-0.08	0.34	0.28	-0.27
50	0.56	-0.46	-0.32	0.84	0.29	1.05	0.58	-0.20

MEAN= 0.0

VARIANCE= 0.50

INDEX	GROUPS OF (1X8) DATA VECTORS							
51	0.03	0.18	-0.18	-0.90	0.54	-0.92	0.08	0.57
52	0.41	-0.24	0.89	-0.62	0.59	0.04	1.60	-0.47
53	-0.14	-0.38	0.43	0.39	-0.14	-0.46	-0.06	-0.83
54	1.06	0.65	0.01	-0.60	-0.72	0.18	0.39	-0.53
55	0.18	0.20	0.66	0.12	-0.22	-0.90	-0.24	-0.57
56	-0.59	-0.87	0.95	0.30	-0.72	-0.11	-0.16	-0.48
57	0.41	0.31	0.91	-0.31	0.20	-0.32	-0.05	0.25
58	0.87	0.11	0.10	-0.49	-0.02	1.07	0.87	0.10
59	0.96	0.38	0.58	-1.00	0.54	0.18	0.41	0.05
60	0.35	0.48	0.39	-0.09	0.31	0.13	-0.12	0.10
61	0.22	0.29	0.58	-0.03	0.20	0.01	-0.44	-0.27
62	-0.76	0.17	0.21	-0.58	-0.35	-0.04	-0.23	0.43
63	0.10	-0.03	-0.17	-0.00	0.59	-0.42	-0.20	0.20
64	0.47	0.06	0.47	-0.06	0.93	-0.01	-0.04	-0.99
65	0.02	-0.19	-0.29	-0.51	0.11	0.29	-0.96	-0.65
66	-0.58	0.57	0.58	0.39	0.04	0.61	-0.16	-0.77
67	0.87	1.22	-0.35	-0.14	-0.34	0.55	0.17	1.11
68	0.02	-0.39	-0.05	0.65	0.39	0.23	0.33	-0.50
69	0.56	0.61	0.54	0.01	0.66	-0.24	-0.88	-0.62
70	-0.33	-0.92	-0.53	0.15	-0.61	0.26	-0.17	-0.27
71	0.01	0.38	-0.21	0.84	0.01	0.34	0.28	-0.45
72	0.26	-0.12	-0.44	-0.21	-0.55	0.61	0.75	-0.17
73	0.11	-0.06	0.32	-0.24	-0.38	-0.32	0.56	0.54
74	-0.78	-0.07	0.12	-0.03	-0.54	-0.33	-0.51	-0.31
75	0.77	-0.30	0.51	0.68	0.40	0.19	0.86	0.75
76	-0.56	0.63	-0.44	-0.39	0.35	0.36	-1.04	1.23
77	-0.10	0.57	-0.71	0.50	-0.26	-0.21	0.60	-1.39
78	-0.17	-0.12	-0.22	0.10	-0.00	0.08	0.07	0.36
79	0.41	-0.06	0.06	-0.87	0.06	0.36	-0.32	-0.01
80	-0.20	0.10	-0.31	-0.65	-0.17	-0.76	-0.28	0.48
81	-0.31	0.23	0.22	0.48	0.26	0.34	0.00	-0.19
82	0.75	0.19	-0.30	-0.73	0.75	-0.75	-0.18	0.49
83	0.66	-0.66	-0.32	-0.55	-0.76	0.60	-0.52	-0.01
84	-0.40	-0.45	-0.28	0.27	0.16	0.01	-0.23	-0.20
85	-0.27	-0.32	-0.15	-0.55	0.43	0.08	0.51	-0.08
86	0.24	-0.41	-0.36	-0.25	-0.53	0.43	-0.51	-1.17
87	0.79	0.15	-0.87	-0.11	0.21	-0.84	-0.49	0.51
88	-0.92	-0.28	0.38	0.12	-0.42	0.40	0.37	0.45
89	-0.74	-0.34	-0.07	-0.83	0.50	0.76	-0.13	-1.33
90	0.03	-0.66	-0.16	-0.22	1.40	-0.15	0.32	-0.38
91	0.55	0.62	0.16	-0.31	-1.29	-0.21	0.02	-0.20
92	0.49	-0.64	-0.07	0.14	-1.16	-1.32	-0.24	0.21
93	1.05	-0.57	0.21	-0.13	0.23	-0.36	0.35	-0.18
94	-0.12	0.69	-0.94	0.11	0.31	0.29	0.58	0.63
95	0.17	1.00	0.51	0.10	-0.02	-1.27	-0.24	0.12
96	0.09	0.73	0.12	-0.91	-0.80	-0.41	-0.80	-0.22
97	0.83	-0.76	-0.55	0.81	-0.13	-0.04	0.18	0.11
98	1.04	0.39	0.53	-0.01	-0.08	-0.52	0.51	-0.41
99	-0.08	-0.45	-0.57	-0.12	0.79	-0.76	0.49	-0.30
100	0.63	-0.93	-0.30	-0.28	0.39	-0.63	-0.58	-0.06

MEAN= 0.0

VARIANCE= 0.50

INDEX	GROUPS OF (1X8) DATA VECTORS							
***	*****							
101	-1.15	-0.17	-0.12	0.24	1.15	-1.02	0.61	-0.70
102	-0.23	0.19	0.52	0.68	0.51	0.17	0.35	-0.74
103	0.54	0.00	0.19	0.28	-0.13	0.57	0.45	0.81
104	-0.15	0.64	-0.09	0.27	0.18	-1.21	0.20	0.94
105	-0.12	-0.54	0.76	-0.59	0.35	1.44	-0.09	-0.36
106	-0.20	-0.17	-0.22	-0.08	-0.69	-0.37	-0.13	-0.61
107	0.28	-0.39	-0.49	-0.46	-1.52	0.47	0.39	-0.31
108	0.01	-0.82	0.02	-0.49	-0.70	-0.41	0.77	0.52
109	0.15	-0.62	-0.15	0.07	0.19	0.09	-0.20	-0.13
110	-0.11	-0.42	-0.46	-0.04	0.57	0.08	-0.70	-0.27
111	0.90	0.65	0.67	-0.09	0.87	-0.31	-0.44	0.08
112	0.35	-0.06	0.26	-0.33	0.52	-0.49	-0.62	-0.57
113	-0.08	0.33	-0.12	-1.03	0.94	-0.82	-0.44	0.24
114	-1.21	0.18	-0.93	0.55	0.55	-1.21	0.91	0.52
115	0.64	0.34	-0.11	-1.35	-0.21	-0.03	0.21	-0.25
116	-0.37	0.68	0.39	0.50	0.08	0.32	-0.21	1.42
117	0.44	0.07	1.11	0.36	0.84	-0.04	-0.07	0.02
118	0.26	-0.63	-0.38	-1.56	0.00	-0.49	-0.57	1.03
119	-0.25	0.44	-0.05	0.00	-0.01	0.57	0.60	0.44
120	0.09	1.15	-0.34	-0.23	-0.27	0.26	-0.26	0.25
121	0.46	-0.65	-1.20	-0.01	0.65	-0.31	0.65	0.43
122	0.03	0.41	-0.08	-0.61	-0.33	-0.02	-0.38	-0.01
123	0.48	0.71	-0.42	0.22	0.22	0.74	-1.03	-0.60
124	0.48	-0.92	-0.18	0.21	0.72	0.58	-0.51	-0.34
125	0.22	-0.09	-0.31	-0.39	0.61	0.11	-0.03	0.25
126	-0.64	0.06	0.24	-0.36	-0.70	-0.04	0.23	0.66
127	0.08	0.38	0.03	0.05	0.22	0.73	-0.43	-0.62
128	0.57	-0.26	0.61	-0.42	-0.19	0.53	0.28	-0.10
129	-0.02	-0.49	-1.48	-0.54	-0.53	-0.02	-0.34	-0.29
130	0.50	-0.45	-0.17	-0.06	-0.35	-0.31	-0.98	1.29
131	0.31	0.22	-0.43	-0.75	0.26	-0.22	0.15	-0.36
132	0.08	-0.12	-0.21	-0.37	-0.27	-0.64	-1.14	0.73
133	0.49	0.40	-0.44	0.31	-0.01	0.04	-0.05	-0.43
134	0.21	-0.36	-1.08	0.54	-0.54	0.43	0.22	-0.37
135	-0.09	-0.03	-0.08	0.06	0.57	-0.35	-0.04	-0.09
136	-0.22	0.24	-0.39	0.10	0.25	0.32	-0.51	-0.11
137	-0.01	-1.18	-0.95	0.90	-0.09	0.04	-0.90	1.07
138	-0.28	0.08	-1.22	0.20	-0.51	0.42	-0.41	-0.07
139	0.64	-0.12	-0.15	-0.26	0.41	0.59	0.26	0.46
140	-0.08	-0.43	-0.18	-0.25	0.10	0.66	0.91	-0.36
141	0.75	0.03	-0.26	-0.52	-0.01	0.73	0.35	0.47
142	-0.21	-0.38	0.15	0.66	-1.01	-0.58	0.38	0.43
143	-0.27	0.70	-0.39	0.53	-0.97	-0.65	0.29	-0.48
144	0.24	0.13	-0.33	-1.17	0.05	0.13	-0.62	-0.78
145	-0.50	-0.22	-0.62	-0.20	-0.03	-0.13	0.47	0.01
146	0.18	0.01	0.79	-0.33	0.20	-0.32	-0.16	-0.10
147	-0.05	-0.32	1.45	0.69	-0.32	0.18	0.80	0.36
148	0.50	0.16	-0.09	-0.42	0.34	-0.02	-0.85	0.38
149	-0.00	-0.69	0.22	0.10	-0.40	0.69	-0.31	-0.04
150	0.12	-0.00	-0.06	0.48	-0.12	0.44	0.22	-0.43

MEAN= 0.0

VARIANCE= 0.50

INDEX	GROUPS OF (1X8) DATA VECTORS							
***	*****							
151	0.04	-0.42	0.61	-0.54	-0.39	0.32	0.04	0.72
152	0.44	-0.08	-0.72	-0.23	-0.28	-0.54	-0.05	-0.12
153	-0.51	0.38	-0.99	0.15	0.64	0.47	0.26	0.04
154	0.37	0.83	-0.15	0.35	0.78	-0.05	-0.74	0.20
155	0.26	0.63	-0.66	0.59	-0.43	0.53	0.26	0.84
156	-1.18	0.15	-0.96	-0.87	0.45	-0.68	0.05	0.96
157	0.24	0.74	0.50	0.19	0.33	-0.57	0.44	0.03
158	0.19	-0.73	-0.23	0.51	0.14	0.47	0.23	0.54
159	0.34	0.60	-0.10	-0.16	0.31	-0.44	-0.28	-0.01
160	0.38	0.60	-0.06	-0.59	0.26	-0.69	0.20	0.37
161	0.48	0.63	-0.05	0.46	0.42	-0.66	0.48	-0.35
162	-0.19	-0.45	-0.54	0.23	1.22	0.25	0.87	-0.16
163	0.05	0.23	0.04	0.46	0.07	0.16	-0.35	0.41
164	0.38	-0.97	0.23	-0.62	-0.09	0.19	-0.86	-0.64
165	0.47	-0.19	0.59	-0.27	-0.33	0.44	-0.37	0.68
166	-0.51	0.44	0.16	0.26	-0.74	0.03	0.43	-0.15
167	-0.87	0.28	0.00	0.19	0.12	-0.42	-0.68	0.36
168	-0.94	0.18	-1.34	-0.23	-0.34	0.19	0.13	-0.31
169	0.45	0.03	0.18	0.24	-0.68	0.48	0.14	-0.15
170	-0.01	-0.83	1.13	0.33	0.52	0.57	-0.35	-0.19
171	-0.16	0.48	0.04	0.78	-0.31	-0.44	0.97	0.07
172	-0.82	0.31	0.47	0.34	0.75	0.57	-0.09	-0.39
173	-0.81	0.03	0.48	0.25	0.13	0.22	-0.25	-1.08
174	-0.46	0.50	0.61	-0.29	-0.75	0.61	-0.20	-0.01
175	-0.09	0.09	0.40	-0.01	0.04	-0.14	0.26	0.80
176	0.98	0.16	-0.07	0.33	0.43	0.09	0.23	0.35
177	-0.58	-0.43	0.72	0.47	-0.66	0.40	-0.79	0.13
178	-0.60	-0.31	0.35	1.13	-0.31	-0.59	-0.89	-0.38
179	0.11	0.37	-0.16	-0.43	-0.58	0.23	-0.78	-0.69
180	0.72	-0.51	0.27	-0.49	-0.55	-0.02	-0.65	0.68
181	-0.10	-0.10	-0.33	-0.81	-0.30	0.77	-0.71	0.24
182	0.33	-0.03	0.10	0.36	0.10	0.71	-0.15	0.46
183	-0.32	0.06	-0.39	-0.24	0.08	-1.08	-0.18	0.33
184	-0.36	-0.46	0.25	0.11	0.56	-0.50	-0.49	-0.15
185	-0.13	0.26	0.07	0.66	0.47	0.58	0.22	-0.01
186	-0.44	0.10	0.12	-0.35	0.23	-0.23	-0.25	-0.24
187	-0.12	-1.09	0.45	-0.19	-0.73	0.17	-1.60	0.63
188	0.50	0.56	0.12	0.39	-0.49	-0.09	-0.03	0.60
189	0.60	0.91	0.66	0.61	-0.61	1.09	-0.74	-0.35
190	0.86	-0.17	0.65	-0.27	-0.18	-0.17	0.07	-0.23
191	-0.06	1.17	-0.38	-0.02	-0.27	-0.26	0.71	-0.55
192	0.54	0.31	-0.38	0.59	0.56	0.44	0.47	0.68
193	0.32	0.10	-0.79	-0.69	-0.29	0.04	0.15	0.94
194	-0.04	0.40	0.44	-0.13	-0.37	0.15	0.57	-0.77
195	0.63	-0.41	0.86	-0.49	-0.27	-0.11	0.00	-0.46
196	-0.98	0.03	0.52	0.47	0.44	-0.64	-0.73	-0.16
197	-0.21	-0.43	-0.54	-0.51	0.19	-0.32	0.66	0.14
198	0.12	0.58	-0.75	-0.19	-0.60	0.97	-0.02	-0.09
199	-0.31	-0.07	0.43	1.16	-0.50	0.34	0.53	-0.35
200	0.68	0.97	-0.45	-0.22	0.42	0.40	-0.39	0.84

MEAN= 0.0

VARIANCE= 0.50

INDEX	GROUPS OF (1X8) DATA VECTORS							
201	-0.24	-0.42	-0.34	-0.08	0.07	0.27	0.52	-0.03
202	0.10	0.11	-0.18	-0.70	-0.11	0.05	0.56	0.54
203	-0.12	-0.08	-0.43	-0.83	0.31	-0.13	0.03	0.03
204	0.28	-0.11	-0.03	-0.23	0.24	0.33	0.74	-0.08
205	0.47	-0.63	0.06	-1.17	0.11	0.05	0.48	-0.79
206	-0.37	0.23	-0.60	-0.40	0.35	0.64	-0.44	-0.12
207	0.44	0.33	0.54	-0.70	-0.11	-0.29	0.27	-0.57
208	0.06	0.54	-0.47	-0.82	-0.35	-0.11	-0.08	-0.17
209	-0.32	0.71	0.41	-1.01	0.55	-0.23	0.31	0.09
210	-0.03	-0.79	-0.25	-0.06	0.53	0.48	0.23	-0.32
211	-0.38	-0.60	0.08	-0.21	-0.50	-0.87	-0.01	-0.40
212	-0.22	0.15	-0.03	0.28	1.40	-0.68	-0.11	-0.17
213	0.64	-0.17	-0.54	0.13	0.14	-0.31	-0.26	0.37
214	0.38	0.28	0.11	-0.91	0.17	-0.17	-0.68	-0.31
215	-0.33	0.38	-0.04	1.40	0.39	0.36	-0.56	0.29
216	0.18	-0.00	-0.94	-0.21	-0.26	-0.61	-0.09	0.16
217	0.10	0.48	-1.03	0.01	0.14	0.04	0.52	-0.71
218	0.59	0.20	-0.27	0.79	0.52	-0.57	0.08	-0.84
219	-0.66	-0.47	-0.60	0.37	0.81	0.15	0.38	-0.24
220	0.02	-0.19	0.53	0.49	-0.08	0.34	-0.28	0.08
221	0.30	-1.58	0.67	0.39	-0.21	-0.41	0.42	-0.39
222	-1.13	-0.77	-0.13	0.30	-0.65	0.53	-0.24	-0.17
223	-0.11	-0.42	-0.33	-0.04	0.00	-0.23	-0.46	0.24
224	0.54	0.85	-0.36	0.11	0.20	0.92	0.08	-0.17
225	0.51	-0.10	0.32	-0.00	-0.65	0.59	0.17	0.57
226	0.45	0.60	-0.23	0.35	0.38	-0.60	1.11	0.97
227	0.06	0.80	0.02	0.91	-0.27	-0.55	0.69	-0.50
228	-0.50	0.37	-0.86	0.41	0.31	-0.12	0.23	0.16
229	0.45	-0.33	-0.32	0.60	-0.44	-0.73	-0.47	-0.07
230	-0.40	-0.93	-0.32	-0.29	-0.11	0.26	0.37	0.31
231	0.10	-0.58	-0.81	0.48	-0.26	0.95	0.57	0.28
232	0.64	-0.40	-0.22	-0.36	0.02	-0.04	-0.07	-0.67
233	-0.60	-0.03	-0.02	-1.06	-0.83	-0.60	-0.26	-0.06
234	0.05	-0.11	0.35	0.62	0.11	0.39	-0.69	-0.89
235	0.27	0.21	-1.06	0.41	0.77	-0.48	0.94	-0.67
236	-0.28	0.32	-0.19	0.05	0.07	-0.57	0.42	-0.42
237	0.10	0.06	-0.01	0.27	0.44	-0.28	-0.44	-0.16
238	0.57	0.30	0.54	0.33	-0.20	0.51	-0.33	1.12
239	0.30	0.41	-0.48	0.95	-0.42	-0.59	0.53	-0.10
240	-0.51	-0.25	-0.53	-0.12	-0.29	-0.47	-0.55	-0.34
241	0.79	0.68	-0.56	1.35	0.61	0.99	0.24	-0.61
242	0.39	0.57	-1.01	-0.41	-0.30	-0.10	0.19	0.59
243	0.46	0.78	1.17	0.86	0.92	-0.13	0.11	-0.76
244	0.68	0.16	0.02	-0.11	-0.31	-0.49	0.28	-0.18
245	-0.77	0.10	-0.40	-0.09	0.43	-0.05	0.03	0.34
246	0.77	-0.06	-0.04	-0.34	0.07	0.29	0.64	-0.23
247	0.50	1.05	0.15	-0.11	0.54	0.14	-0.09	1.10
248	0.57	0.30	-0.29	-0.68	0.26	0.62	0.66	-0.18
249	-0.33	0.55	0.71	-0.29	-0.34	0.35	0.67	-0.07
250	0.97	0.65	0.19	-0.72	0.66	0.93	-0.25	0.40

MEAN= 0.0

VARIANCE= 2.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
1	-1.39	1.03	-1.39	-1.71	1.51	0.64	0.76	0.82
2	-1.82	0.52	-2.85	-2.35	-0.10	1.55	0.75	2.32
3	1.33	-0.02	-2.12	1.77	0.48	1.41	0.28	0.31
4	-1.24	-1.16	1.01	-0.39	-3.06	-1.35	0.55	-3.26
5	3.17	2.58	4.73	0.12	-3.02	-0.26	0.77	-0.44
6	3.27	0.64	1.26	2.25	-1.68	1.13	0.15	0.74
7	-0.23	2.47	-1.22	-1.04	0.64	-4.73	-0.12	-1.78
8	1.35	-0.47	-0.48	-0.81	-0.34	0.62	-2.84	1.95
9	2.73	-0.72	1.66	-0.09	1.08	3.62	-0.79	1.89
10	2.61	1.15	5.41	0.06	0.60	0.74	-2.78	1.99
11	1.69	-1.38	-1.02	0.60	-2.08	-3.59	-1.60	2.21
12	0.68	5.12	0.57	-1.51	-1.25	-0.89	-2.05	2.49
13	0.28	-1.88	-1.61	2.66	2.79	5.27	-0.92	-1.20
14	1.19	-0.94	-1.38	0.09	-0.26	-0.64	0.98	1.08
15	-1.89	-0.59	1.49	1.71	-0.69	-0.18	-1.14	3.29
16	1.75	0.60	1.18	-1.51	1.20	-0.88	-2.08	1.37
17	0.81	0.11	-0.08	1.37	-0.89	1.69	-0.63	-0.71
18	0.00	1.37	-0.11	-0.68	0.75	2.00	-1.59	-1.00
19	-1.98	1.84	1.29	1.28	1.82	2.50	2.24	-3.55
20	3.56	2.98	2.35	0.20	2.02	-2.35	0.07	3.59
21	1.34	2.24	-0.75	-0.96	1.05	3.88	1.10	0.37
22	2.06	3.07	2.20	-1.24	2.63	1.67	0.52	2.76
23	-1.59	2.93	-0.60	0.30	-3.56	-0.54	-0.44	0.69
24	-0.89	-0.74	1.07	-1.38	2.20	-0.30	5.40	0.13
25	-1.14	-0.48	-0.61	0.67	1.61	-3.15	1.25	-2.97
26	2.35	-0.83	-1.43	1.41	-1.63	1.36	-1.69	-0.65
27	-1.70	-0.34	-1.18	-0.22	-3.82	1.68	1.80	-2.98
28	1.40	-1.57	-1.67	-1.25	0.75	2.27	-3.10	-1.98
29	-1.63	2.95	1.32	-0.74	1.78	-0.42	0.83	4.28
30	1.89	0.67	1.99	2.27	-1.03	-3.93	-1.22	1.77
31	-1.32	1.03	0.53	0.72	2.03	4.18	-2.04	-1.56
32	-0.56	-2.50	-2.84	3.59	-1.34	-1.63	-0.42	0.23
33	-1.14	1.52	-3.93	-2.20	0.57	3.09	-1.17	1.11
34	1.67	-1.44	-6.54	2.33	-0.56	0.80	2.92	-2.97
35	0.55	0.05	-0.45	2.12	1.00	-0.05	3.06	1.93
36	0.21	-0.53	-1.47	0.13	0.94	0.98	2.44	-0.22
37	-0.65	-3.75	-1.39	-0.69	-1.03	0.37	-1.74	-3.47
38	0.68	3.85	-2.01	-1.39	-1.22	0.55	-0.26	4.12
39	-3.09	-0.28	2.87	-1.02	0.08	-0.02	4.08	2.51
40	-1.28	-0.67	1.46	-2.61	2.58	-0.88	4.47	1.66
41	-3.17	4.11	-2.04	-3.23	1.98	2.42	-3.87	3.52
42	1.94	-0.47	2.57	0.09	1.98	2.32	-1.76	0.04
43	0.75	-2.96	-0.51	0.30	0.28	3.29	-1.97	-0.82
44	-0.03	-1.91	-1.06	0.35	2.59	1.78	-1.31	-1.12
45	-1.70	2.13	-0.89	1.20	-1.39	0.24	1.41	-0.89
46	2.44	2.61	0.20	-2.21	0.05	1.12	-0.58	1.80
47	3.09	1.00	0.42	1.08	-1.40	0.88	-0.09	2.93
48	-3.04	0.74	-0.03	-1.97	-0.03	1.96	0.08	2.43
49	-1.24	-0.71	1.06	1.58	-2.14	0.83	-0.87	0.27
50	1.18	0.09	-0.12	-1.31	1.98	1.94	-1.72	0.40

MEAN= 0.0

VARIANCE= 2.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
51	-3.08	0.61	-1.35	2.32	-1.97	1.73	-3.29	-3.24
52	2.70	0.29	-2.44	-2.59	-0.29	-1.33	-0.35	-2.68
53	-0.79	-3.41	0.81	-1.03	-1.28	1.20	0.28	-1.98
54	1.15	0.97	-3.38	1.80	-1.22	1.77	-2.19	-1.18
55	-2.76	-1.14	1.18	1.01	-0.43	-1.15	1.45	-0.32
56	2.17	-2.26	0.70	-2.50	-1.19	-3.11	0.39	-1.46
57	-1.35	-0.95	1.37	0.22	2.20	0.33	-2.15	-0.64
58	1.37	0.24	-2.59	0.13	1.44	-0.37	1.02	0.09
59	3.05	-3.23	-2.99	-1.83	0.23	3.25	-2.89	0.68
60	0.38	2.08	-1.61	-2.70	0.28	1.63	-2.68	3.09
61	4.08	-0.35	-2.27	0.48	-2.71	1.24	0.85	-2.73
62	-1.17	0.90	3.25	-3.34	-1.03	2.52	2.91	1.18
63	1.35	-0.69	-0.85	0.78	-2.99	-2.08	-3.30	2.77
64	0.34	4.32	1.63	-0.19	1.12	-2.10	-2.58	3.98
65	0.50	-0.61	-3.10	-1.30	-1.00	0.92	2.23	-1.22
66	0.53	-2.03	1.15	2.40	-1.64	-0.58	0.47	-0.89
67	1.14	-0.49	-1.41	-0.15	-1.10	-0.14	1.21	-1.07
68	-0.97	1.46	-0.57	-3.97	0.32	-1.31	1.70	-7.80
69	-1.09	3.28	-0.14	1.88	0.32	-1.63	1.13	0.86
70	1.47	2.43	2.10	4.34	-3.38	1.34	2.72	0.87
71	1.42	2.35	2.33	4.39	-1.10	2.07	-2.34	0.19
72	1.46	4.50	2.78	-1.04	-1.12	-1.00	3.16	0.76
73	0.31	0.32	3.63	-0.99	2.26	-3.41	-1.59	2.53
74	0.65	-2.71	1.10	-0.50	-1.26	3.29	2.64	-2.53
75	-0.81	-1.16	1.38	-0.62	0.02	-2.45	-0.97	-0.48
76	-1.36	-1.57	-1.32	1.59	1.81	-0.17	-1.21	0.65
77	-2.30	-0.49	-0.80	5.11	-0.19	0.58	1.13	0.79
78	3.07	1.54	3.15	-1.13	1.73	4.25	1.24	-0.10
79	-0.55	1.97	0.73	-0.17	-2.74	-0.70	0.33	-0.05
80	-2.45	-1.75	0.13	-3.06	2.11	-1.82	-0.40	-3.13
81	2.08	-2.16	1.58	-0.84	-2.02	1.35	0.26	4.63
82	3.73	0.19	5.26	-0.56	0.58	-0.76	1.51	-0.83
83	3.20	-3.25	1.38	0.74	1.61	0.33	-1.44	2.45
84	-0.79	-1.02	2.14	2.00	-3.23	-0.94	0.59	-1.57
85	-1.55	0.32	1.75	-1.83	-0.23	-2.12	-1.17	1.05
86	1.63	0.24	0.42	2.23	4.30	-0.75	0.47	0.27
87	-0.56	0.18	0.33	1.10	0.08	-0.37	2.71	4.04
88	2.60	-0.40	-0.30	-0.24	-1.20	-0.55	2.76	0.32
89	-0.19	1.95	2.73	1.15	-1.83	-0.81	-0.18	-0.95
90	-0.23	0.68	1.63	4.22	-2.12	-0.72	3.10	0.20
91	-2.83	-0.75	0.59	-0.07	1.66	0.19	-0.21	-0.29
92	-1.26	1.11	-0.19	3.23	5.19	0.37	1.11	1.54
93	-2.84	-0.28	-0.48	-0.92	-1.82	4.27	-1.75	1.64
94	-0.85	-3.48	-0.11	2.42	-3.67	0.34	0.41	-0.03
95	1.40	-1.03	1.15	2.22	-2.64	-0.97	0.81	0.47
96	0.61	-1.48	-2.52	-0.59	2.95	-1.19	0.63	-0.90
97	-0.50	0.63	-6.90	0.97	-3.19	0.12	1.10	-0.18
98	0.77	0.75	2.21	-0.17	2.65	1.41	-2.60	2.57
99	-2.89	-1.66	1.01	0.96	-1.83	1.15	-1.26	1.31
100	-2.77	-1.17	3.70	-0.70	1.07	-0.23	2.33	-2.01

MEAN= 0.0

VARIANCE= 2.00

INDEX	GROUPS OF [1X8] DATA VECTORS							
101	-2.17	1.70	-1.51	-2.18	-0.95	-0.26	1.37	-3.42
102	1.63	0.39	-2.42	-0.55	1.83	-0.49	-2.94	1.00
103	1.31	0.35	-2.83	-2.84	7.09	1.53	2.61	-2.76
104	1.58	-0.96	-0.53	1.90	-1.44	-1.75	-2.79	1.22
105	1.16	-0.09	-1.32	0.62	-0.09	-1.87	2.08	0.92
106	0.73	-1.58	1.00	-3.72	-1.13	-0.37	0.41	-1.73
107	1.00	-1.99	2.64	1.83	1.14	-0.82	1.40	1.24
108	-1.32	0.14	-0.21	0.21	-3.59	-0.74	2.27	-2.22
109	0.47	-1.73	0.67	2.39	2.39	-1.70	-1.79	1.84
110	-2.93	-0.16	1.48	-0.68	0.78	0.76	-1.57	1.38
111	3.18	0.31	-1.59	-0.05	1.29	1.11	0.13	6.34
112	-0.48	1.14	3.67	-0.77	1.63	-0.22	0.51	0.68
113	-1.23	-0.23	-0.53	0.12	1.48	1.23	0.77	0.35
114	-4.35	3.66	-2.00	-4.43	-3.44	3.93	2.13	3.31
115	0.86	0.27	-0.52	0.53	0.57	-1.70	1.78	-0.49
116	-0.91	-2.97	-1.90	1.96	3.21	0.83	-1.08	-1.09
117	5.06	1.42	0.07	0.80	-1.81	-2.06	0.76	-4.55
118	1.47	2.88	-0.42	0.02	-0.80	-2.11	-1.50	-0.91
119	3.02	0.86	-3.14	0.57	-2.04	-2.23	3.35	1.76
120	0.41	0.83	0.08	-0.61	2.16	1.39	-1.49	-0.53
121	0.35	-3.78	-0.53	5.45	1.51	-0.58	-0.82	0.12
122	1.53	-1.50	-0.78	3.69	1.71	0.31	0.56	-0.31
123	-5.33	-0.89	1.53	-2.93	0.46	0.51	1.86	-1.08
124	0.47	-0.49	-3.38	-1.46	1.47	0.48	2.28	-2.63
125	3.62	1.16	-1.33	1.05	-3.56	-1.33	-0.98	-4.61
126	2.84	-0.49	1.90	-2.44	1.08	-0.46	-0.71	0.13
127	-1.18	0.00	-1.49	1.02	-2.92	-2.48	-1.71	-0.45
128	2.26	0.10	-1.29	0.39	0.15	1.09	3.22	1.60
129	1.88	3.25	-3.32	-1.39	-0.01	0.69	-0.71	0.23
130	-1.62	-3.08	0.64	-3.36	0.31	0.78	-0.30	-0.60
131	-3.55	0.54	-1.20	-0.56	0.81	-0.19	1.65	-2.94
132	-1.20	1.58	1.35	1.95	3.19	2.23	0.35	-2.84
133	0.14	-0.51	0.49	1.13	-0.84	-1.51	-3.01	-0.34
134	-0.84	-4.28	0.44	-0.06	0.42	1.05	-1.21	-1.49
135	-3.43	1.72	-2.62	-0.69	-1.34	-1.64	2.94	-0.34
136	-0.92	-1.04	-0.46	0.22	-2.40	-1.12	2.65	-0.94
137	-4.62	-1.13	-2.90	1.61	0.94	-0.95	-0.88	4.68
138	-1.82	-1.08	0.28	-1.56	2.38	-2.66	-2.45	2.45
139	-1.81	2.56	-0.73	-2.35	-0.37	-1.81	3.15	-1.66
140	0.10	3.23	-1.72	-1.80	-1.62	-1.95	1.13	0.30
141	-1.38	2.40	1.52	1.05	2.34	-0.61	0.67	2.27
142	-3.55	-2.48	1.18	-0.85	3.21	0.65	1.00	2.23
143	-1.71	2.03	-0.54	1.44	-1.29	0.29	-0.70	0.10
144	0.86	-0.60	0.58	0.89	-1.48	2.75	0.78	1.86
145	0.84	-0.92	-1.26	-3.55	0.35	0.50	0.65	-2.54
146	-3.05	2.51	0.13	-2.93	-2.09	-0.02	2.12	0.83
147	-6.11	-0.84	-1.04	-0.30	-1.10	1.65	2.37	-0.05
148	-1.65	-1.53	3.43	-2.71	1.02	-0.03	0.62	-3.24
149	-0.94	-0.10	-2.24	-1.19	-2.02	-0.62	2.06	-2.78
150	2.69	0.90	0.13	-0.81	3.47	-1.66	-0.09	1.28

MEAN= 0.0

VARIANCE= 2.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
151	-4.03	-1.07	-1.24	3.41	1.21	-0.70	-0.63	2.89
152	1.59	1.43	-0.16	2.40	-1.11	-1.29	-0.37	-2.00
153	2.26	-2.13	-0.42	1.12	2.22	-0.97	-2.10	2.57
154	0.68	-2.31	2.19	-1.47	-1.10	-1.29	-0.61	0.55
155	3.55	0.34	-4.14	-0.42	-1.36	-1.79	3.29	-0.10
156	-0.43	1.29	0.80	1.21	3.13	-0.03	0.82	-3.43
157	1.46	-2.02	-2.79	2.40	-1.92	-1.55	-0.83	-1.49
158	-2.10	-0.13	-0.71	0.09	1.20	-3.89	-0.46	1.67
159	-2.39	2.41	1.26	-2.79	2.18	3.39	-0.85	2.00
160	3.29	-0.95	1.30	0.74	-3.27	2.75	-2.81	-2.53
161	-0.36	-0.75	-2.36	1.63	-3.45	0.65	0.86	2.02
162	1.37	-1.55	-1.55	2.83	-0.65	-0.47	-0.62	-0.39
163	1.17	0.12	1.96	1.29	-1.17	-4.15	-4.07	-1.81
164	-0.25	1.70	-1.63	-2.03	2.69	-1.94	-2.27	1.71
165	-2.18	-1.35	-2.13	-2.18	-1.36	0.61	1.98	-1.88
166	-1.93	-1.57	4.67	-0.21	0.38	-0.03	1.88	1.39
167	-0.79	-1.53	-1.03	0.84	-0.40	-1.43	0.64	1.46
168	-4.05	0.25	0.98	3.92	4.03	-1.12	1.46	2.28
169	-1.02	3.21	-1.10	-4.03	3.35	-0.65	-2.47	-2.19
170	-0.99	-1.20	-1.07	-0.03	1.27	-1.57	4.07	-2.00
171	0.75	-1.52	1.28	-1.15	-0.51	-3.43	2.28	-1.19
172	0.89	1.70	0.15	-0.43	1.72	0.23	-0.64	2.18
173	0.14	-2.09	-0.26	-2.91	-2.34	-0.14	0.51	2.08
174	3.20	0.57	2.25	0.35	7.02	1.91	2.94	2.45
175	2.78	1.13	-2.10	0.31	-4.51	0.84	1.84	3.25
176	1.57	1.04	0.87	-2.07	2.78	1.10	2.44	-1.58
177	0.28	-0.24	1.38	2.16	-1.40	-0.86	1.91	1.93
178	-0.38	0.73	1.62	-2.06	2.64	1.41	-2.52	1.73
179	-1.71	-0.58	-2.19	0.24	-3.39	2.38	0.34	-0.23
180	-3.01	3.28	0.14	0.00	0.21	0.49	1.69	-0.00
181	2.42	-0.25	-1.18	4.18	1.15	2.19	0.74	0.37
182	-0.71	0.30	2.05	-4.26	-0.88	-2.06	-1.30	-3.15
183	2.30	-1.63	0.03	3.62	3.84	2.19	-1.24	-0.63
184	0.15	3.41	0.97	-1.22	-1.00	-2.60	0.13	1.90
185	-0.45	2.89	1.07	2.17	0.31	-1.99	2.00	2.39
186	-0.80	-1.75	0.53	0.75	3.47	-1.51	-0.40	-1.21
187	1.80	-1.05	-2.45	-0.54	0.19	-0.72	-1.89	-0.94
188	2.05	0.45	0.35	-4.73	2.16	2.83	2.74	-0.86
189	0.67	2.18	-0.89	1.12	-2.91	-0.39	0.69	-1.00
190	-1.66	-4.39	0.06	-2.03	2.69	4.06	-0.83	0.58
191	-2.22	0.19	1.38	-1.24	-1.35	-1.37	-0.62	5.83
192	-2.31	3.37	-2.72	0.47	2.68	1.79	-1.47	4.72
193	0.77	0.61	-0.03	-1.98	-1.51	-0.03	1.81	3.19
194	-0.27	-0.64	1.64	-1.61	-0.23	-0.35	-1.57	-0.81
195	-0.74	-2.93	0.50	0.52	2.23	-0.74	-0.41	2.69
196	0.07	-0.80	2.76	3.37	1.57	-2.74	0.49	-0.37
197	0.87	7.19	0.62	-0.11	-2.50	-1.89	2.35	-0.04
198	0.35	2.51	0.28	1.03	-0.29	4.26	0.36	1.65
199	1.23	-1.40	5.93	-2.26	1.92	2.16	-2.28	0.63
200	0.19	-3.08	-0.20	-1.01	-2.18	2.26	-0.36	0.87

MEAN= 0.0

VARIANCE= 2.00

INDEX	GROUPS OF {1X8} DATA VECTORS							
201	1.96	0.92	4.08	-2.29	1.12	-0.98	-4.68	-1.68
202	-0.78	-1.95	4.96	-1.13	1.52	-5.11	-0.03	-1.07
203	2.69	-0.22	2.67	1.42	-3.27	0.33	-3.22	-1.35
204	-0.94	-2.46	-0.61	-1.69	0.44	-0.22	-5.04	1.45
205	1.04	-1.21	-0.67	-5.50	0.36	-0.30	-0.28	-0.74
206	-0.67	-1.01	0.70	0.93	0.20	0.55	4.25	0.05
207	0.63	1.60	-0.30	-1.44	-0.35	-1.23	-2.23	1.76
208	-0.34	-1.95	-1.47	-3.66	2.43	0.82	1.46	0.36
209	-0.90	1.81	-0.60	-0.76	-1.78	-0.85	-1.45	1.79
210	3.67	0.33	0.50	-1.81	2.74	-1.78	-1.78	0.00
211	0.07	-0.94	0.04	2.15	-0.31	-1.52	-0.32	2.96
212	1.00	-0.54	0.23	0.09	-1.23	2.38	0.14	-1.40
213	-2.84	-3.02	-0.74	-3.06	1.69	0.38	-1.20	-3.10
214	-0.74	3.07	1.34	-4.34	2.15	-1.03	-1.14	1.79
215	0.00	-0.82	-1.32	1.21	-0.15	2.46	1.52	-0.77
216	0.08	0.77	1.47	0.54	0.49	-0.54	-0.00	1.18
217	2.21	1.29	3.93	0.60	-0.22	0.37	-0.52	-2.41
218	1.09	2.20	2.24	0.34	1.42	-2.36	1.18	2.41
219	-0.58	-1.06	-1.38	0.72	-0.89	-0.28	2.29	1.59
220	3.91	-1.03	-4.72	-1.30	-3.43	1.07	0.03	-0.91
221	-2.75	-0.25	2.40	-0.78	1.48	2.14	3.59	-1.13
222	0.16	0.72	0.20	-0.77	-0.44	-0.62	0.18	2.86
223	-0.67	-2.65	0.87	1.70	0.50	1.25	0.99	-0.96
224	1.47	-0.93	-1.37	-2.43	0.01	0.19	-4.76	1.34
225	1.28	-0.64	3.67	-2.21	-0.20	-1.33	2.13	-0.27
226	1.46	-2.35	2.20	-0.67	1.56	-0.86	0.85	0.15
227	-1.27	-0.60	0.42	-0.87	1.00	0.05	2.62	-1.43
228	3.77	2.07	0.53	0.14	1.82	-2.16	0.63	-3.08
229	1.30	1.11	-3.25	-0.67	-0.27	0.98	0.09	1.17
230	-1.99	-2.03	1.26	1.64	2.43	3.92	0.20	-0.72
231	0.62	2.10	-1.73	-3.98	-0.39	-2.89	0.17	-2.82
232	-0.19	-1.04	1.99	-2.57	1.00	1.01	1.20	0.84
233	0.31	0.01	-1.38	0.83	-1.72	0.06	-1.52	0.20
234	-3.20	-3.32	0.37	-0.85	-0.84	-1.27	1.24	-0.77
235	-2.00	2.45	-0.57	-0.63	-0.66	1.46	-1.35	1.88
236	-1.40	-5.25	-2.00	2.43	0.54	0.70	-0.06	-1.91
237	1.31	1.05	0.30	3.28	0.44	-1.08	0.30	-0.18
238	0.83	0.80	-1.47	4.88	-1.24	4.56	-1.06	-0.98
239	-0.13	-2.56	-1.12	0.18	1.19	-1.93	-0.94	-0.34
240	1.12	-3.56	1.57	2.13	1.44	1.91	-2.14	-0.33
241	3.30	1.24	0.79	-0.31	-0.78	-1.46	-1.45	-0.99
242	1.10	1.31	-3.25	-0.19	-1.78	2.40	0.33	-0.35
243	1.23	0.08	3.64	-2.55	2.15	1.95	0.40	1.52
244	2.39	-0.98	-2.31	1.55	2.71	3.65	-4.03	-1.41
245	-0.72	-4.42	-0.93	3.07	-2.39	-2.06	0.23	-1.20
246	-1.39	-0.79	0.01	0.96	0.55	3.27	0.39	-1.89
247	1.08	-0.63	0.71	2.17	1.23	0.11	-2.34	0.47
248	3.39	3.50	-0.64	-0.33	3.35	-1.10	1.24	1.84
249	-1.75	-0.94	0.16	-1.60	-1.38	0.10	-3.67	-1.84
250	0.36	-0.49	3.34	-2.69	2.74	2.16	-1.86	-2.60

MEAN= 0.0

VARIANCE=20.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
1	-15.82	2.99	-9.28	-6.40	-5.84	-4.62	-1.43	15.01
2	11.35	28.76	55.78	-4.98	43.49	26.80	6.98	4.20
3	12.10	26.95	10.56	-13.12	12.43	0.41	-1.37	1.11
4	13.47	32.12	-2.91	-1.29	-2.01	0.76	-14.45	-14.75
5	-37.51	-1.22	-42.60	0.05	-2.77	-7.63	19.77	-23.83
6	-13.80	1.48	-26.47	20.43	7.16	19.78	33.32	13.39
7	-28.39	-5.27	-12.50	-10.63	4.82	27.51	-1.78	36.44
8	25.77	-6.93	21.35	-3.58	7.23	-19.90	6.52	-55.14
9	-4.29	-28.96	17.11	-28.91	11.44	-17.92	-9.75	-1.83
10	-31.55	3.16	-3.19	-17.07	-25.53	-2.02	-18.57	15.92
11	-8.98	-16.03	-17.52	-18.54	33.36	-27.68	-7.91	17.62
12	10.47	27.99	-3.85	-3.79	5.13	9.65	-5.73	-17.19
13	33.82	-10.24	39.85	-3.28	26.81	-5.50	-40.00	-28.98
14	-11.90	23.81	-24.38	-7.48	15.44	-8.61	-38.69	-38.22
15	-59.66	24.67	25.48	-6.87	-11.95	-35.15	-9.76	-5.37
16	-2.42	26.87	-8.53	8.09	-18.33	19.42	38.80	-10.91
17	26.84	24.95	-4.39	-33.06	13.33	19.63	-40.96	-15.30
18	-24.85	-6.57	19.95	-0.80	0.06	30.01	2.98	-19.01
19	-10.44	6.84	6.51	14.39	-1.10	-24.91	2.66	-2.52
20	-2.92	19.72	-2.69	2.07	6.87	-0.59	-9.90	13.72
21	4.75	6.60	-25.59	31.75	-19.00	-12.52	-2.65	-50.78
22	-0.39	-18.00	19.81	-7.04	-1.67	3.85	-3.58	3.53
23	8.68	0.47	15.56	15.25	-4.11	-6.95	19.36	-3.82
24	18.99	16.53	3.69	8.13	20.71	19.60	-21.82	6.68
25	17.58	-15.29	6.22	21.15	19.83	-11.95	24.94	-5.41
26	-8.53	-0.45	5.20	-16.18	-19.74	-57.09	11.64	19.42
27	-32.30	15.57	22.64	-14.21	-10.96	28.72	30.33	20.71
28	13.30	7.31	-19.41	-3.71	3.22	-9.98	-26.96	-2.01
29	15.30	9.29	1.07	-34.86	19.80	11.33	51.80	10.79
30	0.73	-3.94	46.11	21.77	15.84	-22.82	-1.36	-21.35
31	-3.38	2.13	-2.25	-4.27	-31.65	12.11	-34.41	1.08
32	10.01	2.05	18.02	-23.47	-5.63	-19.35	24.68	-22.37
33	7.92	30.34	-11.06	-26.29	-9.07	-32.68	27.94	7.83
34	17.39	-18.47	-7.44	-3.19	15.07	-3.33	7.41	-48.79
35	5.45	10.16	10.89	-4.65	23.90	13.21	15.12	7.29
36	-20.87	10.76	5.98	-1.14	-5.83	1.49	23.09	-4.38
37	5.45	17.85	-0.16	16.88	3.21	26.03	23.36	15.72
38	-8.54	-14.03	-25.47	18.94	-12.05	11.37	-12.04	-12.89
39	24.18	29.65	-7.95	-5.44	-14.59	-17.97	28.93	-10.65
40	-9.36	3.43	14.46	13.29	-7.37	2.55	-1.72	-18.06
41	-2.12	1.82	23.77	24.64	6.65	-2.55	-11.94	4.44
42	-7.07	-20.62	2.02	-21.84	4.48	31.28	-9.71	-3.64
43	2.81	10.61	11.25	-16.63	3.17	28.56	-2.99	-2.76
44	-25.44	10.06	-6.53	-30.20	19.74	-6.18	0.25	46.61
45	-24.78	-27.74	30.73	-13.01	-8.77	11.59	12.03	4.00
46	-8.20	-8.26	5.05	-15.54	-5.33	6.41	-15.60	-11.06
47	-28.66	-16.97	58.47	11.76	-32.92	-17.20	29.38	-19.03
48	40.88	0.67	13.01	18.40	-14.49	5.31	-1.01	-0.39
49	-32.55	-20.83	30.71	13.93	26.97	-21.55	5.29	-15.60
50	-21.93	-46.92	-26.40	-12.14	28.51	26.76	0.28	-5.13

MEAN= 0.0

VARIANCE=20.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
51	-0.22	16.92	3.71	-30.26	-32.85	14.77	-23.98	30.54
52	19.61	5.23	23.07	9.09	19.92	7.01	4.52	30.96
53	-15.41	-27.46	13.72	15.45	3.86	8.01	-2.18	-24.36
54	-18.25	13.38	17.68	-21.65	16.00	2.31	-32.06	-15.87
55	18.12	-37.73	-43.01	27.31	33.36	14.44	-33.07	-4.04
56	-19.26	13.76	13.67	11.88	-7.01	-11.07	6.82	10.64
57	-23.37	2.37	9.76	41.57	-8.10	-29.70	-0.30	7.73
58	12.83	2.64	7.29	25.93	7.14	-16.91	-2.62	6.74
59	-3.64	-10.91	-11.71	-25.52	35.73	-8.17	-7.87	-12.79
60	-5.74	-3.74	14.79	-3.25	-25.29	1.06	-24.11	-11.33
61	-6.43	-21.33	-11.18	2.28	21.10	-4.70	-19.29	10.65
62	1.31	-9.13	12.42	0.59	11.94	-0.92	-41.40	12.68
63	24.50	7.37	7.62	21.23	4.25	16.94	21.60	-25.70
64	-9.80	-17.28	-3.56	13.71	-4.92	-6.60	-18.25	35.03
65	5.41	-48.56	-19.08	7.58	1.45	13.01	11.07	-5.59
66	-22.81	8.06	3.08	-7.65	20.39	40.28	-18.41	11.97
67	32.55	-12.88	4.97	-22.43	-11.37	19.76	25.35	7.23
68	-21.47	3.14	-31.39	12.76	-35.89	15.97	-5.62	-0.27
69	22.16	-9.33	-23.97	-12.54	22.95	33.44	20.71	-10.99
70	10.47	-15.78	-10.73	-8.80	-17.51	16.70	36.36	14.59
71	-9.50	-21.67	-9.65	13.50	-0.24	-29.71	-6.62	16.00
72	29.27	-12.47	1.31	3.90	11.78	-1.26	3.79	-7.22
73	13.81	26.36	-15.82	11.94	-24.40	26.58	-0.38	-35.55
74	-8.84	-10.66	0.99	7.13	28.24	-1.66	12.91	-9.45
75	-11.65	-8.99	13.77	19.02	6.73	18.54	-24.32	-9.39
76	-7.60	5.89	4.54	17.14	-11.89	11.73	-20.03	-15.84
77	10.36	-11.48	-24.54	-8.99	9.41	2.44	17.81	-29.27
78	-10.75	-26.57	28.23	-9.84	7.65	15.18	1.23	9.86
79	16.10	35.15	-14.90	4.14	19.86	-25.49	2.27	21.06
80	-62.06	28.21	8.30	22.46	3.09	-15.06	-7.06	43.89
81	21.81	7.14	39.56	-5.34	14.36	-8.99	45.29	17.85
82	14.74	-13.52	21.00	-9.72	-29.30	17.25	-8.66	22.50
83	3.75	0.75	-5.33	18.84	-30.85	-31.81	3.12	-2.65
84	-4.11	4.49	2.59	-10.13	6.73	8.36	12.68	1.95
85	-1.83	-7.77	26.79	-27.09	20.47	22.30	12.03	15.81
86	-2.36	18.50	9.30	17.48	27.41	-5.47	-6.78	18.48
87	-18.69	17.82	32.16	-6.87	24.58	-10.42	-11.74	9.48
88	-3.77	44.74	-42.60	29.37	29.01	32.00	-30.80	28.35
89	-10.57	13.78	47.04	15.74	-42.27	6.30	-11.94	-5.38
90	-12.07	-0.52	23.13	1.78	-12.22	-2.98	-3.13	-32.19
91	-21.23	16.36	7.68	-22.99	16.17	-31.31	-32.32	7.45
92	-11.02	-1.04	-37.26	-29.04	-40.05	5.85	32.49	-26.91
93	5.59	1.80	10.33	53.17	16.15	13.01	-16.64	4.26
94	15.63	-20.57	-27.52	-6.83	1.79	-25.27	12.31	-19.53
95	6.13	6.36	-8.77	40.48	-26.09	-4.48	31.37	1.27
96	-15.87	-2.86	28.60	44.64	-30.46	-0.09	12.57	26.18
97	-3.35	-33.72	6.64	15.19	-14.29	-7.55	-12.06	-25.26
98	-29.27	-11.66	7.36	-18.36	-20.54	17.65	-10.48	-13.53
99	-6.60	-2.17	-27.20	13.54	27.80	20.05	-10.66	-19.08
100	-8.32	-10.72	5.00	40.41	27.79	4.19	39.42	-2.40

MEAN= 0.0

VARIANCE=20.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
101	-7.38	-2.77	5.98	-8.21	16.43	-5.41	11.80	-3.94
102	3.24	-3.79	-2.23	-2.79	-9.22	35.79	18.51	-4.19
103	10.56	0.76	2.40	26.19	-32.15	12.31	-28.42	36.41
104	21.64	-14.61	1.92	8.27	-15.32	-11.31	3.05	-2.64
105	3.49	4.64	-1.66	-7.01	-1.70	29.45	4.93	-1.78
106	23.14	13.06	-26.30	9.86	25.75	19.14	-10.72	-1.50
107	7.63	-34.85	10.03	-11.57	24.05	2.28	48.10	17.74
108	-16.01	-24.54	9.37	-21.78	-9.77	3.40	13.45	-4.54
109	-40.74	-41.48	13.73	28.77	1.33	7.03	-2.65	11.22
110	20.45	8.86	-14.84	-10.39	-5.63	-2.29	-8.18	24.52
111	34.60	-18.99	-14.31	10.26	-13.60	-40.04	-11.09	14.91
112	-11.25	9.50	17.34	40.26	-45.57	-41.68	20.63	1.91
113	9.93	-51.14	-17.85	-10.85	15.50	17.32	-20.97	-14.94
114	5.16	13.63	42.15	-33.56	-33.35	21.49	-3.86	3.88
115	41.48	18.34	-0.63	-18.32	4.90	-4.64	3.98	-2.09
116	-14.08	17.50	15.85	-15.60	-12.72	3.46	-25.41	-13.31
117	-34.50	5.67	13.60	-15.89	30.83	-9.67	0.00	-30.27
118	7.27	-2.63	14.67	-9.63	12.58	-19.51	12.24	-53.42
119	-21.75	7.12	21.09	12.69	29.55	18.47	3.34	-23.24
120	5.48	9.47	14.88	20.62	-21.21	8.82	-14.67	-20.19
121	-44.02	18.94	-1.92	3.67	17.31	-23.95	30.25	-4.75
122	16.79	50.07	32.72	-8.62	22.16	-35.30	-9.88	2.62
123	-25.07	-2.62	20.82	33.29	-9.64	-20.70	-3.03	1.45
124	17.45	-4.59	-15.56	-1.08	-1.05	4.39	2.84	11.27
125	-8.64	-1.32	5.58	-22.19	4.95	24.48	-20.23	11.61
126	23.71	1.73	6.29	-20.51	5.39	4.13	19.77	22.01
127	21.51	-20.90	8.59	-6.52	-2.63	27.84	14.88	-18.02
128	-8.18	5.30	-5.48	9.32	-2.25	20.17	17.14	31.07
129	41.64	-5.12	6.11	-3.45	-16.27	-14.37	-21.43	-11.19
130	-41.97	-17.64	-14.61	24.68	12.28	-31.23	-8.80	-25.27
131	-32.00	-17.72	-5.62	-36.74	0.43	-25.90	47.07	-11.65
132	-21.41	9.17	15.13	1.81	-14.78	6.17	18.21	29.22
133	-23.17	-22.45	-10.33	-50.13	3.67	9.50	16.64	-23.15
134	29.75	1.96	0.01	16.97	-7.18	28.62	34.41	10.79
135	44.38	16.93	8.20	-7.37	-30.31	-11.93	23.53	30.56
136	7.76	16.99	-3.73	6.39	-8.67	12.38	-23.95	-24.30
137	-13.09	-23.32	6.25	-12.21	14.76	6.08	3.99	5.73
138	-11.13	30.52	0.17	-13.66	-22.99	-26.30	-0.61	0.19
139	20.67	-26.95	20.05	-8.41	15.11	19.76	-5.72	-21.39
140	49.34	-1.21	-12.06	33.06	-13.90	-31.19	0.68	20.52
141	1.90	2.28	5.86	0.29	7.00	-14.63	-9.37	-14.55
142	5.40	-41.95	15.85	-17.19	-25.15	-6.02	16.15	-7.07
143	6.86	0.62	-0.07	-9.86	26.68	19.16	-10.71	42.50
144	13.31	-15.46	0.14	11.82	19.52	5.45	-17.92	13.68
145	-28.21	24.34	18.50	-2.62	10.40	-42.62	-13.47	-13.99
146	9.33	-25.47	-2.94	-3.64	36.35	-0.52	14.69	-20.99
147	12.95	9.67	-2.17	-21.72	-5.60	-23.72	-1.41	12.23
148	-10.31	-15.73	2.86	-27.33	21.59	-27.69	-9.74	-14.82
149	6.59	32.86	14.17	-10.93	-5.06	-7.89	1.72	17.40
150	10.66	9.99	13.79	-23.00	31.49	-19.80	5.01	-11.57

MEAN= 0.0

VARIANCE=20.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
151	28.95	-9.82	3.76	-13.99	8.27	21.11	-7.83	-2.21
152	-31.52	-12.05	6.11	-14.39	-17.59	-0.61	15.21	-14.98
153	36.28	-2.15	2.86	-14.66	10.64	-0.45	-33.82	9.64
154	19.39	-25.59	36.05	14.74	-29.71	6.13	-22.89	-28.81
155	4.84	12.15	-12.28	3.33	18.30	-16.34	-19.98	29.20
156	19.66	5.61	19.88	0.64	11.69	-23.33	-13.05	3.20
157	10.88	-10.69	-25.42	-3.79	27.49	9.70	-30.08	-7.28
158	-34.47	-2.21	-6.16	-0.44	22.75	-32.73	20.98	-2.71
159	-9.36	25.59	19.70	20.23	14.48	13.92	12.15	-23.55
160	13.25	7.22	14.19	7.76	-0.28	-5.82	35.46	9.72
161	-19.62	-22.77	19.33	11.66	-4.50	12.57	-37.06	-23.36
162	-0.93	-9.86	-22.84	21.47	18.85	13.63	6.63	16.73
163	-3.65	0.49	9.71	-33.27	-13.19	21.90	-1.45	-30.46
164	-0.76	2.80	19.02	-3.04	-23.60	21.90	30.74	34.58
165	14.78	11.62	-12.89	41.70	-15.35	38.16	15.23	11.40
166	-29.99	1.46	-43.99	10.48	8.61	55.21	-15.95	19.51
167	-8.05	6.86	27.75	12.82	5.29	17.59	-10.76	18.46
168	-12.37	-17.65	4.38	-21.74	11.73	9.83	-37.19	7.78
169	-15.91	22.47	27.91	-3.66	-15.04	16.45	-23.19	26.99
170	-11.65	1.73	20.38	-23.43	2.02	-18.00	3.26	-24.37
171	7.46	14.69	-16.18	-31.50	-0.08	-9.00	-25.80	13.23
172	-11.57	-24.14	40.26	1.65	-4.28	7.98	1.65	19.32
173	-1.70	-20.22	-8.26	5.56	-33.58	-2.53	17.66	13.43
174	-15.90	20.98	-19.72	-10.25	-10.92	-16.00	-5.76	-4.91
175	-27.14	13.99	7.92	-16.23	-19.29	12.12	-16.57	43.84
176	-8.38	-6.66	-23.32	17.13	18.35	6.34	-2.73	39.20
177	-12.59	-6.44	-31.40	59.37	-0.96	11.20	-32.22	-19.20
178	-12.75	9.19	5.71	-19.97	19.79	-8.78	-53.00	7.89
179	-1.82	14.75	-49.96	-11.38	37.65	-9.05	6.95	20.29
180	7.22	24.78	3.62	-5.30	9.64	14.91	-0.33	-2.58
181	-18.58	-6.20	28.48	-12.23	12.80	7.64	-2.81	-1.18
182	-12.20	16.36	-33.34	17.39	-15.84	-6.34	-28.47	4.03
183	-6.61	-13.02	20.19	-26.93	-19.26	-2.50	14.74	12.57
184	-14.73	0.18	11.09	-15.65	-20.41	3.71	-1.16	3.98
185	10.33	-9.49	-18.60	40.77	17.72	-3.20	15.87	-22.22
186	-4.25	12.50	-6.86	-28.16	32.18	1.31	-2.15	13.51
187	8.51	0.67	8.36	-32.89	19.99	1.77	16.31	-29.30
188	35.63	29.55	29.08	-3.90	18.18	-17.29	4.80	8.88
189	24.15	13.69	-2.67	28.35	23.79	8.67	-6.17	27.58
190	-18.89	-12.40	35.15	-6.62	-6.15	4.17	15.94	-13.67
191	13.52	25.83	24.57	20.74	-14.62	33.75	23.16	-35.33
192	-31.57	-17.11	-12.40	19.94	15.43	21.94	-32.47	22.12
193	-27.13	13.33	-13.70	0.52	-18.99	33.26	21.06	38.21
194	13.86	-8.33	22.69	12.02	19.17	52.25	-44.19	52.49
195	18.44	-7.55	-21.20	23.95	26.93	-16.55	23.79	24.47
196	13.64	-9.81	-3.35	25.86	21.33	31.37	-2.97	33.70
197	-13.51	-0.57	18.30	7.28	-20.61	-19.44	-32.42	-20.31
198	24.02	34.70	25.75	17.74	-41.85	-4.45	-52.55	1.99
199	-6.73	10.53	-18.95	26.76	14.63	0.86	8.69	-19.88
200	1.25	1.46	26.23	3.88	5.88	1.02	-16.68	13.61

MEAN= 0.0

VARIANCE=20.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
201	-4.98	2.00	3.32	18.63	48.92	-19.42	-16.63	-17.89
202	1.59	-13.30	14.34	40.55	40.78	4.05	-19.13	4.83
203	-12.01	-29.92	-18.66	-0.84	-1.51	-4.02	-12.13	41.60
204	-18.73	26.69	-13.66	3.99	-0.92	0.89	-3.62	31.87
205	-11.56	31.03	-8.63	-15.42	-0.40	3.31	-1.56	15.16
206	-3.45	-2.34	16.46	10.45	17.06	27.78	6.07	-8.99
207	32.52	-38.90	9.65	-28.86	-11.51	-1.18	-28.68	-41.07
208	23.68	35.88	-13.03	-3.81	30.94	20.97	6.22	-41.53
209	-23.23	16.53	12.44	-4.89	1.43	-21.24	-37.22	-10.84
210	-21.08	-2.41	-31.89	17.44	-3.01	-23.28	33.05	10.52
211	17.15	-26.42	-4.01	12.72	-25.35	19.38	-10.94	42.10
212	-1.50	-0.96	-1.87	30.47	-8.55	-8.73	-17.17	-16.59
213	-10.00	-31.50	-3.45	60.23	24.41	16.92	-13.60	-6.01
214	-21.31	-23.52	-26.72	11.52	10.57	20.37	11.81	13.38
215	-8.42	-22.47	-9.64	33.89	-13.04	-12.36	11.07	1.10
216	-4.28	6.16	9.82	-3.15	10.61	1.78	16.23	-3.31
217	-21.86	36.92	13.68	-30.06	-21.45	7.80	-0.70	-20.33
218	5.86	4.34	3.98	-17.31	27.81	10.24	-7.66	29.58
219	-14.08	-17.06	2.76	-15.36	-4.57	13.63	7.36	-14.05
220	5.34	7.25	12.03	5.31	38.42	22.50	-3.60	-11.69
221	11.17	-8.18	33.84	14.24	-6.16	1.39	-8.52	-43.81
222	-9.57	11.16	10.21	20.56	18.70	-5.18	4.65	29.12
223	10.15	39.81	43.18	14.99	30.04	7.33	27.93	6.63
224	-2.63	12.31	-25.23	25.87	-15.11	43.44	33.34	8.26
225	-20.89	23.18	7.02	3.13	0.28	7.70	-7.06	-6.46
226	2.41	-13.05	21.97	-3.69	-26.76	4.62	18.74	2.00
227	14.29	-1.95	1.63	-5.08	0.81	18.74	-17.23	-6.84
228	-18.20	11.33	8.04	28.52	0.01	14.60	17.07	26.55
229	31.95	1.00	3.24	26.61	-12.12	-3.28	13.67	-18.26
230	-28.21	21.70	9.25	-1.25	-18.56	9.63	-35.40	-1.79
231	-11.67	-12.04	8.10	-5.55	-22.27	-22.13	41.89	-24.48
232	-11.37	14.31	-38.16	-16.75	13.78	25.97	-2.43	13.20
233	-0.31	-24.71	-7.52	14.68	6.62	-1.55	3.68	10.77
234	8.57	-14.59	42.06	18.28	33.28	-0.14	12.24	7.77
235	-37.72	-0.77	12.61	3.56	10.80	14.72	-44.71	23.73
236	-12.14	-8.74	26.16	-19.92	-23.80	-12.45	-15.15	38.18
237	12.34	-23.96	-15.25	17.34	-13.48	22.91	12.97	10.65
238	22.75	8.10	10.39	4.90	-1.22	5.30	-8.34	0.67
239	6.12	1.96	5.14	-7.73	-9.97	39.27	-7.04	7.77
240	-10.51	32.17	-28.99	8.99	17.28	-10.65	8.91	11.49
241	-40.11	-6.74	30.04	-15.41	37.57	-19.93	11.53	11.36
242	4.34	-40.26	-15.74	28.61	-32.06	-4.04	32.86	-13.10
243	29.87	6.16	15.70	-29.43	5.40	1.55	-15.08	-22.34
244	3.53	7.05	-13.61	-20.00	-12.99	-18.63	19.75	23.15
245	-7.66	-3.06	18.36	-13.57	-30.23	-20.05	9.37	2.91
246	3.32	10.35	-26.35	-0.59	-9.26	-18.17	5.83	-3.52
247	3.52	1.83	-25.71	28.45	46.93	-8.46	-18.85	3.38
248	-0.03	25.90	2.30	3.09	35.39	13.60	-12.64	3.15
249	-0.31	-42.92	-0.29	12.86	-6.87	-7.45	-23.51	15.31
250	-10.29	-10.07	-11.43	7.30	-2.80	-7.08	0.58	-40.60

MEAN= 0.0

VARIANCE=50.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
***	*****							
1	-35.00	-51.33	19.72	-72.31	-30.86	-51.99	20.29	73.19
2	-61.44	-9.19	29.03	-56.28	67.65	11.30	-17.48	-59.30
3	35.16	78.09	1.71	-50.08	4.23	-105.85	41.91	41.54
4	22.36	-53.15	-107.17	-29.89	-28.55	-42.10	-21.47	24.55
5	17.76	-16.58	7.47	-21.86	111.91	-36.13	22.46	-11.45
6	-111.08	-75.88	-49.29	-2.18	-31.83	23.39	53.78	-17.64
7	-46.58	55.29	77.62	-47.02	-167.19	-2.21	102.56	-45.17
8	-21.16	113.24	-6.71	17.45	-101.59	-1.61	-1.12	4.77
9	32.77	-65.68	52.79	65.06	7.55	-13.48	72.83	-68.99
10	-17.23	54.85	11.20	19.64	2.81	23.52	4.47	-67.62
11	-3.57	-38.85	123.60	55.02	-23.23	20.71	73.89	7.71
12	41.33	-10.45	-4.94	-54.99	22.00	39.41	-88.83	55.83
13	-14.96	26.39	30.67	13.46	-18.92	40.96	-3.61	-24.43
14	-4.87	-42.02	85.51	-15.81	46.59	-13.31	-90.39	15.75
15	-10.80	20.67	14.64	-18.97	61.11	37.91	30.93	25.20
16	-15.18	0.77	73.16	27.80	67.21	5.97	90.41	-47.25
17	-0.44	84.62	-83.87	-51.67	-92.52	52.19	-81.86	-2.77
18	-49.00	58.53	-1.36	16.46	-75.50	37.89	44.19	57.46
19	56.73	-91.15	75.76	-43.99	10.83	24.41	48.65	32.28
20	134.31	21.90	102.57	-9.19	-40.94	23.07	-38.41	-29.49
21	1.34	-15.99	-65.85	-5.32	61.77	-4.80	63.08	70.99
22	25.38	-18.50	25.58	91.46	11.54	52.13	133.22	-67.47
23	-25.99	100.70	-18.06	-45.03	-49.07	105.19	2.07	3.97
24	14.82	-72.05	-141.70	9.04	-27.46	15.70	49.71	34.13
25	65.37	-0.44	59.74	-22.49	-31.06	-155.00	106.22	-28.16
26	-6.74	51.87	-58.65	-15.82	-17.29	54.41	-48.31	-34.06
27	16.05	-28.79	-41.81	-47.59	-43.58	55.26	16.18	-34.75
28	-48.68	-56.10	-84.65	-92.49	32.65	-41.13	-20.22	-31.41
29	16.66	-43.72	67.90	71.80	-46.01	76.59	22.56	-75.20
30	-70.36	94.66	-79.07	-30.37	-36.99	69.72	24.60	-67.29
31	7.84	45.38	-70.49	-25.18	-47.72	-50.39	15.98	41.13
32	18.84	-55.23	-1.28	61.19	-35.61	27.58	26.78	48.91
33	-19.79	29.15	-16.37	52.58	-8.08	64.96	-12.92	54.85
34	59.53	-15.14	-60.67	22.80	27.44	-76.93	-23.04	7.82
35	-25.61	48.21	-29.10	45.70	-36.47	13.25	-23.52	56.61
36	42.36	5.55	33.40	-4.90	42.77	-53.17	15.74	-99.93
37	81.01	-6.81	81.93	-55.18	7.73	-14.88	24.80	-12.98
38	7.93	-2.53	71.55	-81.30	1.00	39.46	33.75	-83.72
39	-59.30	-45.29	57.35	-9.44	65.15	71.18	-27.33	-13.30
40	-3.12	1.24	44.41	-15.77	-57.23	-8.39	-28.37	147.08
41	-5.93	-26.61	-62.21	-26.46	-73.58	12.07	-39.29	-3.72
42	49.83	7.49	42.59	32.32	-141.31	-56.11	-30.00	-16.90
43	131.76	39.87	13.88	34.40	-17.84	48.40	29.56	56.38
44	-42.59	56.85	6.73	53.59	-60.60	86.93	19.47	64.95
45	-5.62	44.76	26.23	13.74	-27.01	20.80	-30.18	-42.37
46	10.24	-60.07	27.45	-11.33	-74.48	-38.65	60.68	33.26
47	-77.43	28.69	65.47	-47.80	-10.45	19.91	22.13	-9.34
48	-51.05	-2.64	-4.63	28.17	-42.33	7.79	-15.75	-71.34
49	6.95	-67.72	22.23	40.41	72.45	-13.67	-22.88	-3.91
50	-35.84	-80.23	-48.88	-37.26	-23.53	16.85	-69.19	-8.23

MEAN= 0.0

VARIANCE=50.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
51	-11.85	-3.83	-62.87	18.99	12.32	-39.30	-24.59	64.54
52	46.49	-2.21	35.33	-67.02	22.56	-20.82	-59.02	-36.78
53	-43.22	10.97	-49.20	28.56	-50.22	33.64	7.61	-13.35
54	-13.42	72.03	-11.38	-20.47	-63.44	35.39	5.38	83.64
55	53.48	67.31	-46.14	-90.26	-24.53	-54.23	-35.63	3.02
56	-24.96	-16.87	-48.39	-56.99	9.09	26.11	114.65	-6.36
57	-81.14	5.82	-13.06	-46.82	-70.00	87.73	-13.70	-45.70
58	-47.49	-28.29	14.93	14.05	-19.23	-8.03	-90.60	83.85
59	43.56	17.12	-59.34	-50.52	4.00	-49.86	64.03	31.10
60	9.60	-71.62	-30.80	33.27	-57.76	73.58	-19.74	-5.10
61	18.20	141.83	5.65	-10.73	138.09	-26.38	-11.82	24.06
62	36.94	-6.21	5.06	41.30	-15.88	11.58	67.86	-32.59
63	33.40	20.59	22.52	13.18	72.91	-101.21	-50.62	-26.22
64	25.16	50.55	13.12	28.75	47.03	-53.41	-37.18	-7.99
65	-70.21	44.01	-18.08	-38.16	-50.93	-83.71	-11.75	20.91
66	64.88	13.29	-15.99	-13.73	-28.40	-30.77	5.76	9.32
67	-52.00	-47.28	74.46	-24.14	57.20	16.73	45.42	56.06
68	-53.28	-1.37	-91.65	-45.54	-1.55	-29.87	-12.69	9.97
69	78.64	-12.66	-59.24	-4.12	37.93	40.75	-38.48	19.87
70	-38.68	5.19	26.78	23.96	68.21	39.91	-1.89	-15.42
71	12.35	38.51	-78.54	62.52	81.87	28.96	-22.84	-47.06
72	49.30	73.61	-70.10	-14.61	-78.50	-80.79	28.76	-26.22
73	89.75	46.84	-42.82	-83.60	-70.33	-28.01	-17.02	45.92
74	1.27	44.51	-41.64	29.38	48.95	-51.36	69.90	-31.81
75	1.46	2.96	-11.47	-1.85	21.93	10.47	-130.41	-10.58
76	57.88	8.52	-47.23	-53.45	141.18	-31.18	12.15	58.44
77	-61.89	-2.50	6.16	48.39	-0.71	35.02	77.63	-25.93
78	9.73	56.25	-96.24	-22.48	-11.18	-29.60	46.13	-14.85
79	40.32	21.08	-49.33	7.76	-47.64	36.27	47.72	-59.50
80	47.46	28.33	1.96	12.93	-17.51	-6.02	62.47	-61.05
81	-1.28	14.32	-87.29	16.86	71.79	-45.16	-29.52	-20.67
82	-38.31	-34.61	-12.01	-56.61	12.82	30.19	1.81	10.46
83	-46.06	-32.14	32.88	16.33	48.18	-18.64	-13.46	81.18
84	93.05	58.05	52.47	-40.47	20.43	19.67	4.75	40.31
85	96.60	72.31	51.82	46.80	-127.83	56.45	36.52	36.68
86	-17.84	-3.05	-63.97	-48.02	45.96	3.03	-38.08	19.12
87	-82.68	68.30	-39.84	48.89	-65.62	20.74	11.05	-13.54
88	-30.35	22.69	-70.71	-38.64	30.01	20.90	13.96	37.52
89	-43.28	-3.55	-1.49	13.22	-24.57	64.84	100.75	71.13
90	-3.87	25.92	22.89	28.30	13.22	63.90	1.49	86.13
91	5.45	47.40	-42.50	30.42	-64.05	-70.60	-153.75	31.33
92	2.25	-2.76	57.43	-56.59	-63.79	22.66	-34.88	55.58
93	104.11	111.76	27.76	-8.89	-93.43	55.02	-11.82	-42.32
94	28.61	77.30	73.56	-2.66	89.61	-12.19	-54.51	-13.51
95	43.33	-69.83	49.91	-64.08	-72.09	-17.65	67.15	-9.19
96	15.85	-43.30	61.90	-19.31	-35.95	-50.04	33.23	69.48
97	63.75	-6.77	64.58	5.46	-9.40	-48.01	23.80	-28.66
98	-45.40	-73.91	-86.94	34.10	0.14	-50.25	-31.06	45.19
99	-94.02	-8.41	12.40	-9.60	-64.75	-145.42	-1.26	-10.11
100	-64.53	26.08	17.68	-1.80	-61.49	-72.20	-6.73	4.24

MEAN= 0.0

VARIANCE=50.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
101	10.65	15.87	-66.02	-18.68	152.50	-19.25	-17.39	-112.91
102	-0.91	-2.71	-83.63	-36.41	19.79	24.74	46.34	87.25
103	-31.62	56.68	-30.05	-81.15	32.97	-28.89	16.03	3.56
104	-63.91	30.36	-50.23	20.92	84.61	31.13	20.26	-15.15
105	-30.20	54.66	60.93	-106.37	67.29	-83.71	-10.38	-20.06
106	37.09	-34.09	108.51	60.81	-26.42	37.76	54.17	37.66
107	55.53	50.43	-2.44	-53.72	-53.93	6.92	44.00	-43.16
108	-57.29	-55.44	-16.81	-26.12	-72.66	-14.91	-60.91	36.31
109	-33.79	34.62	-29.54	-82.58	59.95	33.62	69.82	74.89
110	-6.41	56.94	-135.54	50.75	-13.50	-36.19	-34.05	71.42
111	42.44	-2.15	70.25	-52.32	-50.45	87.01	7.68	-75.29
112	30.34	-56.33	92.93	-67.95	41.69	-35.45	-24.93	-16.39
113	24.87	-19.26	-12.44	-22.31	5.50	7.73	48.21	96.94
114	-56.40	-4.62	9.22	-41.59	-66.45	77.91	7.17	-86.48
115	-45.89	-26.08	-36.99	-51.94	68.42	-13.60	-67.96	-17.81
116	-76.02	2.69	53.98	70.45	2.68	44.53	-97.11	-48.23
117	-79.21	18.02	-12.78	-100.58	78.93	63.64	-0.20	-78.91
118	12.11	56.23	-82.19	58.80	-10.27	5.04	102.84	-61.02
119	-84.47	53.66	-99.17	22.41	-22.34	30.07	-57.90	-145.74
120	-1.38	46.63	-58.65	-35.92	85.30	-49.94	-52.35	15.77
121	-21.04	71.46	-55.55	7.64	5.24	26.33	49.58	22.34
122	24.13	14.50	65.22	-23.09	30.04	70.21	77.95	22.78
123	-48.29	62.06	58.72	-54.26	-47.72	43.03	-37.15	15.94
124	29.27	0.47	30.04	87.93	14.55	-43.88	84.36	0.63
125	24.39	16.07	-15.75	27.33	59.43	70.81	22.55	-24.31
126	4.66	-4.82	-23.56	37.75	29.48	-51.59	57.50	-10.06
127	37.64	24.12	11.68	-56.96	17.29	0.28	19.30	-107.79
128	-59.07	39.23	-4.95	-82.99	15.44	37.72	-61.99	81.33
129	82.08	-23.16	31.62	-66.51	66.50	-77.93	-6.28	6.14
130	28.69	-76.73	126.48	66.32	-36.95	64.66	16.50	115.44
131	-1.67	14.85	-15.29	39.31	-2.32	-73.19	36.43	-41.92
132	8.59	87.90	-88.63	-73.69	12.95	-30.14	33.60	82.89
133	-172.98	128.77	11.56	51.13	1.45	5.12	38.98	38.68
134	-28.77	-26.23	40.35	-12.40	-94.25	-6.06	-20.06	74.30
135	8.78	-40.77	2.81	9.56	18.44	47.63	-10.73	-11.44
136	-42.75	-28.51	-45.98	90.83	32.09	-22.47	96.14	30.29
137	-15.78	46.86	-0.93	5.24	-10.72	-55.04	-19.37	-1.67
138	7.27	21.68	-56.98	26.61	32.59	-88.75	-127.13	-58.50
139	43.98	-17.72	-109.03	28.79	-45.41	-12.26	52.79	8.63
140	61.92	14.93	47.98	-14.42	97.87	35.74	0.62	48.55
141	-21.32	6.11	69.14	-29.17	4.99	-33.41	46.38	10.08
142	-38.18	-57.99	-40.48	108.35	68.55	41.61	70.15	-7.94
143	-71.06	108.99	-25.80	21.98	-18.88	22.13	52.01	-56.70
144	-52.40	-6.61	68.26	-64.45	-14.73	-30.51	-27.96	-37.35
145	-14.61	-18.47	-3.22	22.89	-26.41	-5.00	110.32	-101.07
146	59.88	-40.25	14.84	7.81	-61.35	10.00	-3.08	100.96
147	-11.35	-35.64	27.51	14.16	-26.97	-74.19	11.92	17.57
148	-60.71	-68.29	-60.14	-34.25	-130.69	3.77	85.41	47.60
149	-20.64	-51.89	6.99	36.42	20.06	-44.80	-2.55	-60.13
150	-73.56	-50.10	33.97	-100.00	17.87	-8.57	78.13	-6.79

MEAN= 0.0

VARIANCE=50.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
151	48.12	23.40	25.88	80.31	-94.68	23.78	-42.47	-43.55
152	-38.03	-45.06	-112.22	101.17	-25.03	-36.41	65.72	-21.59
153	-14.42	-69.15	-75.23	86.43	119.42	-77.81	-17.22	57.92
154	36.51	37.46	41.92	59.91	81.23	10.89	38.79	43.81
155	-17.65	-38.91	-5.69	-54.57	2.99	-8.96	-36.17	34.92
156	-9.33	-61.92	37.02	16.82	27.27	73.95	37.97	-19.94
157	29.06	4.75	-24.88	47.92	-3.34	20.96	-108.70	128.07
158	65.08	-2.57	13.70	-37.46	-46.28	-6.60	3.89	77.77
159	-33.67	2.45	-42.18	84.53	-8.96	-47.42	105.82	-22.01
160	-49.63	6.14	-37.42	-12.31	1.20	-40.15	27.17	27.56
161	-65.21	94.02	-16.96	95.88	-23.23	26.52	-51.99	25.31
162	-12.84	-45.18	-25.72	-17.10	10.34	63.93	-1.57	20.07
163	-24.93	-77.53	91.40	-77.40	-55.53	-16.59	58.50	10.66
164	16.08	-15.90	-10.55	-61.21	-28.24	-53.72	58.31	-4.09
165	-72.22	-23.96	23.52	55.31	-65.23	-136.13	-22.08	-25.34
166	-22.25	-15.38	-51.31	-54.01	-23.91	-2.49	-2.58	-4.28
167	33.56	46.16	20.04	34.64	-11.71	8.53	-3.12	7.93
168	-37.21	46.98	-7.36	45.11	-36.05	-91.75	6.39	-39.39
169	-66.97	23.43	-28.43	-48.78	145.65	8.01	6.02	-48.90
170	11.84	11.83	-88.06	26.79	25.97	-30.87	25.85	-20.28
171	-33.19	-51.50	-81.25	-104.33	-2.52	52.93	-54.05	44.80
172	65.51	19.78	47.15	-18.34	2.77	20.76	96.42	-4.84
173	25.53	11.98	-97.80	-91.40	-15.59	33.93	7.31	-170.36
174	14.43	-88.55	-11.04	-49.67	-15.02	53.78	58.72	47.07
175	-50.19	104.51	-37.47	-19.34	-2.94	-108.38	-19.28	-53.73
176	-50.77	-22.52	-22.02	-76.57	13.22	-91.20	-46.61	-23.94
177	-19.72	-33.30	-59.62	-97.54	-23.95	-83.36	6.80	35.29
178	60.54	108.49	-95.17	-58.41	-71.88	26.46	-28.96	72.77
179	7.57	-60.82	76.38	14.63	12.02	-50.40	-23.82	37.34
180	-11.03	-4.90	10.13	45.42	20.31	-2.62	52.30	-72.19
181	72.30	62.57	11.14	57.79	-54.43	-118.86	79.48	-56.97
182	-24.85	-22.07	-15.49	-24.43	-19.60	-37.81	-62.20	-68.18
183	15.09	-22.49	85.30	72.58	-32.65	51.87	-42.67	-107.00
184	9.70	-2.38	-31.41	-27.36	-0.98	11.50	68.16	-24.59
185	-23.43	-75.39	-10.53	99.60	17.98	52.42	-49.65	27.88
186	-16.74	-55.21	3.01	77.28	66.80	35.95	34.00	-0.78
187	-52.64	-55.50	14.28	-70.50	38.07	-126.58	-0.83	38.27
188	-63.56	110.06	-21.62	30.09	-75.64	26.16	25.96	-6.14
189	68.09	27.81	0.36	102.88	-31.74	105.49	94.42	-85.18
190	-90.12	34.06	-64.68	121.71	112.34	-77.25	84.65	-0.03
191	79.39	65.16	-61.66	10.39	-0.83	39.26	-23.29	-1.86
192	-5.81	-42.58	64.48	-57.24	-78.66	16.35	-49.30	8.16
193	21.87	-2.83	-31.17	42.64	21.42	65.35	36.68	28.86
194	80.01	20.75	6.46	-66.14	42.00	47.59	64.74	9.07
195	-13.83	-35.53	-17.54	-59.74	-74.35	-25.61	14.95	47.61
196	7.95	-85.32	51.90	-14.35	14.95	57.08	17.39	43.31
197	-37.09	7.69	19.86	-56.13	2.47	57.00	52.13	45.00
198	18.64	-20.16	-8.59	-111.25	80.80	35.46	49.27	51.51
199	-7.29	-32.55	-128.35	44.10	-7.49	3.80	-11.13	-38.33
200	52.71	6.86	15.64	33.77	80.18	23.35	1.02	-25.69

MEAN= 0.0

VARIANCE=50.00

INDEX	GROUPS OF (1X8) DATA VECTORS							
201	-33.80	34.38	-21.54	-18.42	-13.60	-44.57	-34.22	38.24
202	0.78	-13.65	15.19	11.35	53.73	61.37	-86.75	-47.69
203	24.01	-0.90	-19.09	-3.08	-25.25	2.50	-26.51	-34.66
204	-46.53	-91.05	30.69	62.11	-107.95	40.15	-23.41	-123.85
205	106.75	2.23	19.62	30.76	-51.80	-14.35	-47.38	-16.41
206	1.42	115.27	-47.22	26.68	-64.22	0.32	-18.33	-13.54
207	5.06	-65.60	-14.77	23.71	-102.63	95.49	-106.20	-16.39
208	85.24	-4.05	22.07	45.68	125.55	-117.51	69.11	23.85
209	-40.45	-63.76	-31.64	-33.59	62.89	-27.34	-12.34	106.03
210	-54.43	41.60	79.19	59.74	1.97	-122.70	-70.47	-21.04
211	10.86	48.37	9.63	99.48	-114.62	57.77	-75.19	-8.53
212	23.02	-7.14	-35.24	9.47	55.68	25.37	3.57	-7.60
213	-50.39	11.41	49.67	63.54	5.46	-8.55	-54.12	-69.43
214	-41.78	-9.65	-30.58	-64.48	-22.71	17.33	-18.16	4.80
215	116.43	-34.01	29.09	-50.78	-86.24	14.33	-8.49	-36.06
216	41.80	-75.31	83.77	-71.51	-42.56	-6.21	-45.03	6.80
217	1.93	-97.26	38.54	-2.85	-99.09	67.03	2.30	32.21
218	64.39	-13.50	98.46	29.02	36.74	-4.61	-86.41	-11.00
219	46.75	-37.72	18.63	-102.06	57.53	-9.82	-81.10	-74.00
220	16.59	66.42	-95.89	-22.27	5.83	-37.26	-1.68	92.04
221	41.50	-14.76	59.98	92.22	74.24	-25.60	31.22	35.95
222	-60.95	55.07	41.33	65.25	55.33	36.48	49.78	-43.45
223	26.82	62.24	53.23	120.63	-58.33	60.32	-18.02	2.67
224	-77.61	-6.93	0.75	32.20	-24.15	107.23	8.61	-26.86
225	43.33	-16.10	-61.01	-26.22	50.45	38.55	9.73	-33.21
226	57.23	21.05	23.01	-130.80	8.04	-34.39	15.42	82.44
227	-18.35	90.85	-42.10	-7.71	-8.80	49.74	55.76	-81.07
228	34.19	-20.37	-1.28	-33.11	42.51	2.26	10.84	75.08
229	-17.59	73.72	-49.43	16.81	-45.46	84.50	10.73	-50.28
230	-6.10	-90.56	-31.49	65.88	19.88	-42.21	35.50	-58.31
231	-13.76	23.13	57.63	37.94	-63.89	-16.55	-84.76	49.79
232	27.00	101.11	23.01	56.80	30.79	-27.18	30.02	72.88
233	83.75	129.70	-30.29	-3.69	-38.48	137.23	59.92	9.76
234	-75.91	45.25	2.83	80.28	15.86	37.99	35.02	-40.72
235	-34.42	34.07	27.44	32.55	-63.61	86.45	-64.60	70.26
236	75.81	32.49	-51.39	26.94	-34.29	-106.08	91.14	-8.48
237	-77.65	26.85	-28.57	37.29	-53.63	21.74	-17.69	21.90
238	-27.22	3.46	50.97	-12.59	70.96	-18.76	38.99	10.22
239	-105.32	-1.33	-107.69	1.14	82.06	33.73	41.26	55.32
240	5.63	-51.21	100.53	2.31	22.25	-9.44	-80.80	6.02
241	-26.80	40.15	-169.29	-35.26	134.10	13.05	2.89	11.14
242	14.98	-40.91	-62.07	62.26	60.20	12.52	22.40	-30.48
243	-51.47	-58.07	77.25	-31.30	43.11	50.31	-42.19	-120.02
244	41.45	-125.00	-96.23	-92.10	-74.57	-12.26	-10.79	41.36
245	61.31	-55.38	22.56	3.66	49.72	-63.86	-3.34	-47.52
246	-74.32	37.13	-11.30	29.84	-91.43	6.84	-39.75	62.15
247	2.14	-61.14	7.27	-39.76	-5.45	61.17	-39.94	-30.76
248	-41.72	-13.88	-66.67	18.71	0.24	10.45	-23.84	-27.45
249	111.67	-34.75	-78.02	-20.94	18.22	-33.98	88.63	-79.09
250	29.89	12.59	-21.71	115.14	-58.94	-60.81	27.55	13.15

APPENDIX II

DATA RELATED TO EXAMPLE 2.4-2, TABLE 2.4-5.

MEAN= 5.45

VARIANCE=16.37

INDEX	GROUPS OF (1X8) DATA VECTORS							
1	-0.99	-5.94	-13.86	-17.82	-15.84	-9.57	-0.99	5.94
2	11.55	16.50	17.49	15.18	11.55	7.92	4.62	0.66
3	-4.62	-10.56	-13.86	-11.22	-4.29	1.98	6.60	10.23
4	13.53	16.17	16.50	11.55	5.94	1.65	-1.65	-5.23
5	-8.91	-13.86	-15.51	-12.21	-6.93	0.99	8.58	14.85
6	21.45	23.76	20.13	14.52	7.59	0.66	-5.94	-13.20
7	-17.49	-19.47	-17.49	-10.56	-1.98	6.60	15.51	23.43
8	28.05	26.73	20.13	10.23	0.66	-6.60	-11.88	-13.20
9	-14.19	-13.86	-10.89	-6.93	-0.99	5.28	11.22	15.84
10	18.15	17.49	13.20	7.59	0.99	-6.27	-12.21	-13.20
11	-11.88	-8.58	-3.63	0.99	4.62	11.88	10.15	20.79
12	20.13	15.84	7.26	-1.98	-11.55	-16.17	-16.83	-14.52
13	-9.57	-4.95	1.65	6.60	13.20	19.80	19.80	16.83
14	11.88	3.30	-4.29	-10.56	-12.87	-10.89	-8.58	-4.29
15	1.32	3.96	8.25	14.19	17.16	16.50	12.54	5.28
16	-1.65	-8.91	-13.53	-14.19	-11.88	-8.58	-3.30	1.98
17	8.25	12.87	18.15	19.47	16.50	12.21	3.63	-5.28
18	-13.20	-19.47	-20.79	-16.50	-9.24	-0.33	7.92	16.83
19	19.47	18.81	16.17	11.22	7.26	1.65	-4.95	-10.23
20	-14.85	-16.17	-13.86	-11.55	-6.60	-0.33	5.61	14.19
21	18.15	17.82	15.18	11.88	7.92	2.64	-4.29	-13.20
22	-17.16	-18.81	-16.83	-9.24	0.0	10.89	22.11	29.04
23	28.71	21.45	11.55	0.99	-8.91	-16.50	-22.11	-24.42
24	-21.45	-15.84	-8.58	0.66	11.88	23.43	30.36	30.03
25	23.76	13.53	2.97	-4.95	-10.89	-15.51	-18.48	-17.49
26	-11.88	-4.95	0.99	8.91	16.50	21.12	22.11	19.14
27	11.55	0.99	-8.25	-14.85	-19.14	-20.13	-17.49	-9.57
28	0.33	10.89	21.45	26.40	26.40	21.45	11.55	3.63
29	-2.64	-9.90	-15.18	-19.47	-19.47	-14.52	-8.91	-0.33
30	9.57	18.15	23.10	20.46	14.52	8.25	3.96	2.31
31	-0.33	-7.92	-14.19	-15.84	-13.53	-5.94	3.96	13.86
32	23.43	26.40	22.77	13.53	0.66	-6.93	-12.54	-15.84
33	-17.82	-19.14	-15.51	-8.58	0.99	14.19	25.74	32.67
34	32.01	24.09	12.21	-0.99	-11.55	-18.81	-22.77	-22.77
35	-21.12	-15.18	-6.93	3.30	17.16	26.73	29.04	25.08
36	15.18	6.93	-1.98	-11.88	-20.13	-25.41	-25.08	-18.81
37	-11.22	-0.99	11.22	23.43	30.36	30.03	21.12	10.23
38	1.98	-6.60	-14.85	-22.11	-26.40	-26.40	-20.13	-8.58
39	3.63	14.85	23.43	25.08	20.79	14.85	7.59	1.65
40	-3.30	-10.23	-17.16	-21.45	-21.78	-15.84	-1.65	12.54
41	22.11	25.74	22.11	16.17	10.23	2.97	-4.62	-12.87
42	-20.13	-22.11	-19.80	-13.20	-1.98	11.22	22.44	27.39
43	22.77	10.89	-0.33	-8.58	-13.86	-17.82	-20.46	-16.83
44	-10.56	-1.65	9.57	21.45	30.03	30.36	24.09	12.54
45	-1.98	-11.88	-18.15	-21.78	-22.44	-19.80	-14.85	-7.92
46	0.66	12.87	25.74	35.64	36.63	27.72	13.53	-1.98
47	-13.86	-23.76	-30.03	-29.37	-25.41	-16.83	-6.93	4.62
48	20.79	33.33	39.93	37.95	25.41	13.20	3.30	-10.56
49	-22.77	-31.02	-34.98	-32.67	-22.77	-7.26	8.58	23.43
50	34.65	36.96	32.34	22.11	10.89	0.99	-9.24	-19.47

MEAN= 5.45

VARIANCE=16.37

INDEX	GROUPS OF (1X8) DATA VECTORS								
51	-26.73	-27.39	-21.78	-12.87	0.66	14.19	23.76	27.06	
52	24.42	16.17	6.93	-1.32	-10.56	-17.82	-22.77	-25.08	
53	-22.11	-12.87	0.0	14.52	27.39	33.00	29.37	21.78	
54	10.23	0.33	-6.93	-13.20	-17.82	-21.45	-23.10	-19.80	
55	-10.56	1.32	14.52	24.42	26.40	21.45	14.52	7.26	
56	-0.33	-9.24	-16.17	-18.48	-17.82	-12.54	-1.98	8.58	
57	20.46	27.06	24.75	19.14	10.56	0.99	-7.92	-15.84	
58	-21.45	-23.43	-21.12	-14.19	-3.96	7.26	17.49	24.42	
59	25.74	21.78	15.18	7.59	-0.66	-11.55	-17.16	-19.14	
60	-17.82	-12.54	-3.63	5.28	13.86	16.17	13.53	9.24	
61	4.29	0.66	-1.65	-6.93	-13.53	-19.80	-21.12	-16.50	
62	-6.93	3.96	15.84	23.43	23.76	19.47	11.55	3.96	
63	-0.66	-7.26	-11.55	-14.85	-17.16	-15.84	-10.89	-0.99	
64	10.89	16.83	16.83	11.88	2.64	-3.63	-7.92	-11.55	
65	-14.52	-14.85	-12.21	-7.26	-1.65	2.97	9.90	17.16	
66	21.12	19.80	14.52	8.91	3.63	-2.31	-9.57	-16.17	
67	-20.13	-19.14	-15.18	-6.93	1.65	7.26	12.21	14.19	
68	11.55	6.60	3.30	0.0	-4.95	-9.24	-12.21	-12.21	
69	-8.25	0.66	12.54	21.45	24.42	19.14	10.23	2.31	
70	-4.62	-11.88	-17.49	-21.12	-21.45	-20.13	-14.52	-5.61	
71	4.62	16.50	24.75	25.08	19.14	9.57	0.99	-6.27	
72	-13.20	-18.15	-19.14	-16.17	-10.89	-1.65	9.57	19.14	
73	25.41	23.76	16.50	6.93	-1.65	-12.87	-21.78	-24.42	
74	-23.10	-18.48	-11.22	-1.32	9.24	19.47	25.74	24.42	
75	16.17	9.57	-0.99	-10.23	-18.81	-26.40	-27.39	-22.11	
76	-12.54	0.0	11.55	22.77	30.36	31.02	25.74	15.84	
77	3.96	-8.25	-21.12	-28.71	-31.35	-29.70	-21.78	-9.90	
78	0.99	11.22	17.49	19.80	20.46	18.31	13.86	6.27	
79	-0.66	-8.91	-14.19	-15.84	-14.52	-9.24	0.0	8.91	
80	14.85	15.84	12.21	6.60	0.33	-7.92	-16.17	-21.45	
81	-20.79	-17.82	-12.54	-2.31	4.62	12.21	19.14	21.12	
82	21.45	18.81	10.89	1.32	-7.59	-14.85	-18.48	-16.17	
83	-9.90	-2.97	4.29	11.88	14.19	13.20	11.55	7.92	
84	2.31	-4.95	-12.87	-18.15	-20.13	-18.15	-12.87	-4.29	
85	3.96	11.55	16.83	19.80	19.80	16.83	11.22	4.29	
86	-2.97	-8.58	-11.55	-9.90	-5.28	-0.66	5.61	8.58	
87	6.93	3.30	0.99	-1.98	-7.26	-11.88	-14.85	-16.17	
88	-13.86	-8.25	0.99	8.91	16.50	20.46	21.12	18.81	
89	14.85	9.57	4.29	-1.65	-6.60	-11.55	-13.20	-9.57	
90	-4.29	1.65	6.27	7.26	5.61	0.33	-2.97	-5.94	
91	-8.91	-9.90	-10.23	-8.91	-4.95	-0.33	8.58	18.48	
92	25.08	27.72	24.75	20.13	11.88	0.66	-10.23	-18.81	
93	-24.75	-24.75	-21.12	-14.52	-2.97	7.92	16.17	20.79	
94	19.80	15.51	9.24	2.31	-2.31	-10.23	-15.51	-16.83	
95	-16.17	-8.91	2.31	14.85	25.08	28.38	25.41	18.48	
96	7.26	-3.30	-10.56	-13.20	-14.85	-14.19	-9.24	-2.97	
97	6.60	14.52	17.49	18.81	15.84	8.91	-0.33	-10.23	
98	-17.49	-22.11	-20.46	-11.88	-1.65	9.57	20.79	27.06	
99	27.72	25.74	20.46	11.88	2.31	-7.59	-15.51	-20.13	
100	-20.13	-15.84	-8.91	2.64	11.88	17.16	19.80	17.49	

MEAN= 5.45

VARIANCE=16.37

INDEX	GROUPS OF (1X8) DATA VECTORS							
101	13.20	7.59	-1.98	-7.92	-9.24	-8.91	-6.27	-2.31
102	2.97	10.89	18.15	22.77	22.11	18.81	12.54	3.63
103	-3.63	-10.56	-15.18	-14.52	-8.25	-1.32	2.97	6.60
104	7.92	10.23	15.18	16.83	12.87	6.93	0.66	-3.63
105	-3.63	-0.99	0.99	5.28	8.91	10.23	10.89	11.55
106	13.86	14.19	9.57	2.97	-5.28	-7.92	-5.94	-3.30
107	0.99	4.95	10.56	15.18	16.83	15.84	11.55	5.61
108	-0.66	-4.95	-7.59	-7.26	-2.31	3.30	7.92	13.86
109	18.48	21.45	21.45	17.82	11.88	1.65	-7.92	-14.52
110	-17.82	-11.22	-0.33	8.58	18.15	22.77	24.42	24.09
111	21.78	17.16	7.92	-1.65	-11.55	-19.14	-18.81	-10.89
112	0.33	11.55	21.12	24.75	24.42	22.11	18.81	13.53
113	4.95	-5.61	-12.21	-11.88	-5.61	1.32	6.93	12.87
114	17.16	20.13	22.11	21.12	15.51	6.27	-2.64	-11.55
115	-15.18	-10.23	-0.99	10.23	17.49	19.80	22.44	24.75
116	25.41	22.11	13.86	3.30	-7.92	-17.49	-19.80	-13.86
117	-1.32	9.90	21.12	30.03	33.99	34.65	29.04	18.81
118	6.60	-4.62	-13.53	-19.14	-18.48	-10.23	0.0	10.56
119	19.80	28.38	35.64	36.63	30.69	18.81	5.28	-6.60
120	-13.86	-14.19	-6.60	3.96	12.54	21.78	29.37	33.33
121	33.00	26.07	15.18	3.30	-10.56	-19.47	-22.44	-18.48
122	-5.94	7.92	21.45	33.66	41.53	44.22	41.25	30.03
123	13.53	0.0	-6.60	-9.57	-12.21	-9.90	-1.32	7.92
124	16.17	25.08	32.34	34.65	30.36	19.80	8.58	-1.98
125	-8.58	-10.56	-9.57	-2.97	4.95	10.89	21.78	33.33
126	41.25	42.90	35.97	20.13	3.96	-6.27	-10.89	-13.20
127	-11.22	-4.29	3.30	11.22	21.12	31.02	36.96	36.96
128	28.71	16.83	7.26	-0.99	7.92	2.31	-3.96	-5.94
129	-3.63	2.31	9.90	15.51	19.47	24.09	26.40	26.07
130	23.43	19.47	14.19	8.25	1.98	-3.30	-5.61	-1.65
131	6.60	13.20	18.15	19.47	20.46	23.76	25.08	21.78
132	17.49	13.20	10.23	5.28	0.0	-5.61	-6.60	-1.98
133	5.61	14.52	22.11	26.07	29.37	30.69	29.70	26.73
134	20.46	14.52	6.60	-2.97	-9.24	-11.88	-7.92	0.99
135	11.22	21.12	29.04	33.99	35.64	33.99	28.71	21.45
136	13.53	7.59	1.65	-1.98	-4.62	-5.61	-2.31	4.62
137	12.21	19.14	25.41	27.06	26.40	26.07	22.11	18.81
138	29.70	7.26	1.65	-0.66	-1.65	0.0	5.28	12.21
139	18.15	25.74	31.68	32.34	30.03	24.09	15.18	7.92
140	0.66	-2.97	-3.30	-1.65	1.32	4.95	11.22	17.16
141	24.75	32.01	33.66	30.03	23.10	14.19	7.26	1.32
142	-1.32	1.98	5.28	8.25	12.87	14.85	19.14	25.74
143	29.04	30.36	27.39	18.48	10.56	3.30	-1.65	-2.64
144	-1.32	3.96	9.57	8.91	20.13	24.75	29.04	30.69
145	29.04	25.41	17.16	7.59	0.0	-5.94	-7.26	-3.63
146	3.63	12.21	20.79	29.70	32.34	32.01	29.37	24.09
147	20.13	16.17	7.59	1.65	-1.98	-2.64	-0.33	2.31
148	5.94	11.22	17.49	25.08	29.04	30.69	29.37	25.08
149	20.13	14.85	6.60	-2.97	-6.93	-6.93	-2.97	3.96
150	12.21	22.11	32.01	38.94	39.60	33.00	24.42	14.52

MEAN= 5.45

VARIANCE=16.37

INDEX	GROUPS OF (1X8) DATA VECTORS							
151	3.96	-3.96	-11.22	-15.84	-12.54	-4.62	4.95	14.52
152	26.40	35.97	40.59	39.27	33.33	24.75	16.50	10.56
153	4.29	-1.98	-7.59	-9.57	-3.96	5.28	13.86	23.10
154	30.36	33.99	32.67	27.39	19.14	10.89	4.62	0.33
155	-3.30	-5.94	-6.27	-0.66	8.58	21.45	34.32	40.26
156	41.25	34.65	22.44	12.87	4.62	-0.99	-3.30	-5.94
157	-4.62	-1.32	2.64	8.58	16.83	26.73	33.33	33.33
158	29.37	22.44	14.52	10.23	6.27	-0.99	-3.96	-2.97
159	0.33	8.25	14.85	22.44	31.35	34.32	32.67	25.41
160	14.52	6.27	-0.99	-6.60	-10.23	-10.23	-5.28	3.96
161	15.84	28.71	38.28	42.90	40.59	33.00	22.77	12.54
162	3.96	-3.96	-9.57	-11.22	-10.23	-5.28	3.30	16.17
163	31.35	41.58	42.24	35.31	24.42	16.50	8.91	0.66
164	-6.27	-11.22	-11.55	-6.27	0.0	9.90	22.77	34.98
165	43.23	44.22	34.32	22.44	12.87	5.28	-2.64	-10.23
166	-13.86	-13.20	-7.59	3.63	15.51	27.06	35.64	37.29
167	33.33	29.95	19.80	12.87	8.25	0.66	-6.27	-10.23
168	-10.89	-3.63	10.23	23.43	34.65	37.29	33.99	28.38
169	21.45	15.18	7.59	-0.66	-6.27	-8.58	-6.93	-1.65
170	8.58	21.78	33.66	39.27	34.65	23.43	2.64	3.96
171	-2.64	-7.59	-10.56	-6.27	1.65	12.21	23.76	34.32
172	41.91	41.58	34.98	23.43	9.24	0.33	-4.95	-9.90
173	-11.88	-11.22	-7.26	1.32	12.21	25.41	38.61	48.18
174	47.19	36.30	20.46	5.94	-4.29	-12.21	-16.83	-17.82
175	-15.18	-7.92	0.0	12.54	30.36	44.55	52.14	49.50
176	36.96	22.77	9.90	-1.65	-11.88	-19.47	-22.44	-20.13
177	-11.88	0.99	14.85	31.02	43.23	47.52	43.89	34.65
178	21.78	10.23	-1.32	-11.55	-17.49	-16.17	-9.90	-0.66
179	12.21	24.42	32.34	35.31	32.01	25.74	18.81	11.22
180	1.98	-7.59	-15.18	-17.16	-14.19	-3.96	10.89	26.07
181	39.27	43.89	37.95	29.04	18.15	9.24	2.97	-2.64
182	-7.26	-11.88	-14.19	-11.22	-3.63	8.91	24.09	34.32
183	36.96	32.01	22.77	13.20	5.61	-2.97	-8.25	-9.90
184	-9.24	-2.97	6.60	16.50	27.39	32.67	31.68	27.72
185	20.13	11.38	1.65	-8.58	-14.85	-17.49	-14.52	-6.93
186	3.96	17.16	27.72	33.00	32.67	27.39	21.45	15.84
187	8.58	0.0	-5.28	-8.91	-8.58	-4.29	2.97	13.20
188	22.77	27.39	26.07	20.13	12.87	8.25	5.61	0.33
189	-5.61	-8.91	-9.24	-6.27	1.98	10.56	20.13	27.72
190	30.69	29.04	22.11	14.52	6.93	-1.32	-6.27	-9.57
191	-9.90	-6.60	0.66	10.89	20.46	24.42	22.77	15.84
192	8.25	3.96	1.65	-1.65	-5.94	-7.92	-6.60	-2.64
193	2.64	9.24	19.47	27.72	30.69	26.73	20.13	13.86
194	8.58	5.61	0.33	-6.27	-9.57	-9.90	-6.93	-0.99
195	6.93	14.85	22.77	25.41	21.12	15.18	10.23	4.95
196	0.99	-1.65	-2.97	-2.64	0.99	10.23	20.46	27.72
197	31.68	26.73	19.14	11.88	4.29	-2.97	-9.90	-14.52
198	-14.85	-12.54	-5.94	3.96	14.52	26.40	33.00	31.68
199	25.74	17.82	9.90	3.30	-3.96	-9.57	-11.22	-9.90
200	-4.62	6.27	17.49	27.06	33.00	31.35	24.09	13.86

MEAN= 5.45

VARIANCE=16.37

INDEX	GROUPS OF (1X8) DATA VECTORS								
201	5.61	-5.28	-13.53	-15.84	-15.18	-10.56	-2.64	7.26	
202	17.82	27.72	33.99	32.34	23.76	17.49	6.93	-2.31	
203	-10.89	-18.15	-18.15	-13.86	-4.95	7.59	20.13	32.01	
204	39.60	39.60	33.33	22.77	10.23	-1.65	-13.53	-20.79	
205	-23.10	-21.12	-13.20	-2.31	7.59	17.82	24.42	27.72	
206	30.03	27.72	23.10	14.52	5.28	-2.64	-8.25	-9.57	
207	-6.60	-0.99	8.58	16.83	20.46	20.79	17.82	13.86	
208	8.58	1.65	-6.93	-13.86	-14.95	-13.20	-6.93	4.62	
209	13.20	22.44	28.71	28.71	27.06	22.77	15.18	6.60	
210	-1.98	-7.92	-10.23	-9.90	-5.61	-0.66	5.94	14.52	
211	19.80	20.79	19.47	13.86	5.94	-1.98	-10.23	-14.52	
212	-14.85	-11.88	-4.95	1.98	8.25	14.19	19.14	23.10	
213	25.08	23.76	19.14	9.90	0.99	-6.27	-9.90	-7.92	
214	-1.98	5.28	11.88	13.86	11.55	5.61	1.65	-1.65	
215	-5.28	-7.26	-8.25	-8.91	-8.91	-4.95	2.64	11.22	
216	20.13	25.41	27.39	25.74	19.80	13.53	6.93	0.33	
217	-4.29	-7.26	-7.59	-4.95	-1.32	1.65	4.29	5.28	
218	3.63	1.32	-1.65	-5.94	-8.58	-11.88	-13.86	-13.20	
219	-9.90	-0.99	8.58	18.48	25.41	27.06	22.44	16.17	
220	8.25	-0.66	-10.23	-18.15	-23.76	-24.75	-21.73	-16.50	
221	-4.95	8.91	19.80	25.41	24.09	18.48	10.23	3.63	
222	-1.32	-9.24	-13.86	-14.85	-13.20	-6.27	3.96	14.85	
223	23.43	27.06	25.08	18.81	9.24	-0.33	-8.58	-13.53	
224	-16.17	-15.51	-11.88	-3.63	6.60	13.53	17.16	16.83	
225	14.19	8.91	-0.33	-10.56	-16.83	-17.82	-11.88	-2.97	
226	5.61	16.83	26.40	31.02	33.00	32.01	26.40	16.50	
227	5.28	-5.28	-13.53	-19.14	-19.14	-14.19	-7.26	3.63	
228	12.54	18.81	21.45	18.48	14.85	9.57	1.98	-3.63	
229	-6.60	-7.92	-5.94	-2.31	2.64	11.55	20.46	26.73	
230	28.38	24.09	16.50	7.26	-0.66	-6.60	-8.91	-7.59	
231	-2.31	2.97	7.26	10.23	11.22	14.85	20.46	22.77	
232	18.81	12.21	4.29	-1.98	-1.32	1.32	4.29	-1.32	
233	11.22	12.21	10.89	10.56	11.88	10.56	7.26	0.99	
234	-8.25	-11.55	-10.89	-7.92	-3.63	0.33	6.93	12.87	
235	16.83	18.81	12.87	3.63	-3.63	-9.90	-12.21	-10.23	
236	-5.94	-0.33	5.28	13.20	17.82	18.15	16.50	12.54	
237	6.60	-3.30	-13.20	-20.13	-24.09	-18.81	-8.25	1.32	
238	10.89	16.83	19.14	18.81	15.51	10.23	0.33	-9.24	
239	-16.17	-20.13	-17.82	-8.58	0.99	8.91	16.83	18.48	
240	18.48	17.16	15.18	11.22	1.65	-10.23	-19.14	-20.79	
241	-12.87	-3.96	3.63	10.89	15.18	16.83	16.50	12.54	
242	6.60	-0.66	-7.92	-15.18	-18.81	-15.51	-8.58	-0.66	
243	6.60	11.55	15.18	19.47	21.12	15.51	4.62	-8.58	
244	-22.11	-29.70	-29.04	-21.78	-8.91	1.65	11.22	18.15	
245	22.77	24.42	21.45	13.20	1.98	-9.24	-21.78	-30.03	
246	-30.36	-22.77	-10.23	1.32	10.89	20.13	26.73	25.41	
247	13.48	5.28	-7.59	-16.83	-22.11	-20.13	-13.20	-5.61	
248	1.65	9.90	16.83	22.77	22.11	14.52	3.30	-10.56	
249	-24.75	-33.33	-34.65	-27.39	-13.86	0.66	12.87	21.12	
250	28.05	30.03	27.06	17.82	2.64	-9.57	-16.83	-22.77	

APPENDIX III

THIS APPENDIX PROVIDES A LISTING OF FORTRAN PROGRAMS USED TO OBTAIN SOLUTIONS TO EXAMPLES IN CHAPTER II AND III, RESPECTIVELY.

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0001      DIMENSION P(8,8),CV(8,8),R(8,8),Q(8,8),CO(8,8)
0002      DIMENSION A(2000),B(8),C(8),D(8),H(8),RMS(75),AEPS(10)
0003      21 FORMAT (6E13.4)
0004      22 FORMAT('1',//////////////////////////////////////
0005      23 FORMAT ('*****',3X,'*****')
0006      24 FORMAT (3X,13,2X,8(F7.2,2X))
0007      32 FORMAT('1',15X,'VARIANCE=',F7.2,10X,'MEAN=',F6.2,/,16X,
0008      33 FORMAT(2X,3(5X,4HEPS=F9.6,10X),/,2X,3(5X,'*****',
0009      34 FORMAT(1X,3('# OF VECT.',2X,'RMS DEVIATION',3X),/,1X,
0010      35 FORMAT(/,1X,3(3X,13,9X,F6.3,7X))
0011      M=2000
0012      K=8
0013      L=10
0014      JJ=M/K
0015      NUM=K*K
C*****
C      FOR EX. 2.4-1 THE FOLLOWING BLOCK CAN BE USED TO GENERATE
C      GAUSSIAN RANDOM NUMBERS WITH MEAN=0 AND VARIANCE=0.5.
C      IX=999
C      AMN=0.0
C      STA=0.5
C      DO 30 I=1,M
C      CALL GAUSS (IX,STA,AMN,V)
C      30 A(I)=V
C      AND THE SUBROUTINE TO BE USED FOR THIS PURPOSE IS
C      SUBROUTINE GAUSS (IX,S,AM,V)
C      A=0.0
C      DO 50 I=1,12
C      CALL RANDU(IX,IY,Y)
C      IX=IY
C      50 A=A+Y
C      V=(A-6.0)*S+AM
C      RETURN
C      END
C*****
0016      READ (5,21) (A(I),I=1,M)
0017      PRINT 22
0018      PRINT 23
0019      II=0
0020      DO 9 J=1,JJ
0021      WRITE(6,24) J,(A(I+II),I=1,K)
0022      9 II=II+K
0023      EPS=0.0001
0024      INDEX=0
0025      DO 101 NN=1,3
0026      N=0
C*****
C      INITIALIZATIONS NECESSARY FOR THE COMPUTATION OF
C      AUTOCORRELATION AND INVERCE-AUTOCORRELATION MATRICES.
0027      DO 10 I=1,K
0028      DO 11 J=1,K
0029      CV(I,J)=0
0030      11 P(I,J)=0

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0031      10 P(I,I)=1/EPS
C*****
0032      II=0
0033      100 NO=0
0034      99 N=N+1
0035      NO=NO+1
0036      DO 12 I=1,K
0037      12 H(I)=A(I+II)
0038      II=II+K
C*****
C      COMPUTATION OF AUTOCORRELATION MATRIX.
0039      DO 13 I=1,K
0040      DO 14 J=1,K
0041      CV(I,J)=H(I)*H(J)+CV(I,J)
0042      14 CO(I,J)=CV(I,J)/N
0043      13 CONTINUE
C*****
C      COMPUTATION OF INVERSE-AUTOCORRELATION.
0044      SCALAR=0
0045      DO 15 J=1,K
0046      C(J)=0
0047      DO 16 I=1,K
0048      B(I)=H(I)*P(I,J)
0049      16 C(J)=C(J)+B(I)
0050      15 SCALAR=C(J)*H(J)+SCALAR
0051      SCALAR=SCALAR+1
0052      DO 17 J=1,K
0053      D(J)=0
0054      DO 18 I=1,K
0055      B(I)=P(J,I)*H(I)
0056      18 D(J)=D(J)+B(I)
0057      17 CONTINUE
0058      DO 19 J=1,K
0059      DO 20 I=1,K
0060      R(J,I)=D(J)*C(I)/SCALAR
0061      P(J,I)=P(J,I)-R(J,I)
0062      20 Q(J,I)=P(J,I)*N
0063      19 CONTINUE
C*****
0064      IF(NO.NE.L) GO TO 99
0065      CALL INVERS (CO,K)
C*****
C      THIS BLOCK COMPUTES THE RMS ERROR.
0066      SUM=0
0067      DO 41 J=1,K
0068      DO 42 I=1,K
0069      CO(I,J)=(CO(I,J)-Q(I,J))**2
0070      42 SUM=SUM+CO(I,J)
0071      41 CONTINUE
0072      AVG=SUM/NUM
0073      INDEX=INDEX+1
0074      RMS(INDEX)=SQRT(AVG)
C*****
0075      IF(JJ.NE.N) GO TO 100
0076      AEPS(NN)=EPS
0077      101 EPS=EPS*100
C*****

```

```
      C      COMPUTATION OF MEAN AND VARIANCE FOR THE 2000 DATA POINTS.
0078      AVG=0
0079      DO 103 I=1,M
0080      103  AVG=AVG+A(I)
0081      AMN=AVG/M
0082      SUM=0
0083      DO 104 I=1,M
0084      104  SUM=SUM+(AMN-A(I))**2
0085      VAR=SUM/2000
0086      STA=SORT(VAR)
C*****
0087      PRINT 32,STA,AMN
0088      WRITE(6,33) (AEPS(I),I=1,3)
0089      PRINT 34
0090      DO 130 I=1,25
0091      J=L*I
0092      130  WRITE(6,35) J,RMS(I),J,RMS(I+25),J,RMS(I+50)
0093      STOP
0094      END
```

```
0001      SUBROUTINE INVERS (C,N)
0002      DIMENSION C(N,N),A(1024)
0003      K=0
0004      DO 10 J=1,N
0005      DO 10 I=1,J
0006      K=K+1
0007      A(K)=C(I,J)
0008 10      CONTINUE
0009      EPS=1. E-7
0010      CALL SINVA(A,N,EPS,IER)
0011      IF (IER.EQ.-1) STOP
0012      K=0
0013      DO 20 J=1,N
0014      DO 20 I=1,J
0015      K=K+1
0016      C(I,J)=A(K)
0017      IF (I.EQ.J) GO TO 20
0018      C(J,I)=C(I,J)
0019 20      CONTINUE
0020      RETURN
0021      END
```

```

0001      DIMENSION P(3,3),Q(3,3),R(3,3),CV(3,3),CO(3,3),A(3,3)
0002      DIMENSION B(3),C(3),D(3),H(3),V(3),SVZ(3,3),VZ(3,3)
0003      21 FORMAT(4X,I2)
0004      22 FORMAT('1',//,' # OF VECT.',8X,'AUTOCORRELATION MATRIX',7X,
C' INVERSE OF AUTOCORRELATION',9X,'CROSSCORRELATION MATRIX',5X
C,' TRANSFORMATION MATRIX',/, '*****',8X,'*****')
C,'*****',7X,'*****',9X,
C,'*****',5X,'*****')
0005      23 FORMAT(/,16X,3(F9.3),5X,3(F8.3),9X,3(F8.3),3X,3(F8.3))
0006      24 FORMAT(4F2.0)
0007      K=3
0008      EPS=.01
0009      NN=48
0010      DO 102 JJ=1,3
0011      N=0
0012      DO 11 I=1,K
0013      DO 12 J=1,K
0014      CV(I,J)=0
0015      VZ(I,J)=0
0016      12 P(I,J)=0
0017      P(I,I)=1/EPS
0018      V(I)=0
0019      11 CONTINUE
0020      101 PRINT 22
0021      M=0
0022      100 READ(5,24) CLASS,{H(I),I=1,K}
0023      V(CLASS)=1
0024      N=N+1
0025      M=M+1
0026      DO 17 I=1,K
0027      DO 16 J=1,K
0028      CV(I,J)=H(I)*H(J)+CV(I,J)
0029      VZ(I,J)=V(I)*H(J)+VZ(I,J)
0030      SVZ(I,J)=VZ(I,J)/N
0031      16 CO(I,J)=CV(I,J)/N
0032      17 CONTINUE
0033      SCALAR=0
0034      DO 20 J=1,K
0035      C(J)=0
0036      DO 30 I=1,K
0037      B(I)=H(I)*P(I,J)
0038      C(J)=C(J)+B(I)
0039      30 CONTINUE
0040      SCALAR=C(J)*H(J)+SCALAR
0041      20 CONTINUE
0042      SCALAR=SCALAR+1
0043      DO 50 J=1,K
0044      D(J)=0
0045      DO 40 I=1,K
0046      B(I)=P(J,I)*H(I)
0047      D(J)=D(J)+B(I)
0048      40 CONTINUE
0049      50 CONTINUE
0050      DO 70 J=1,K
0051      DO 60 I=1,K
0052      R(J,I)=D(J)*C(I)/SCALAR
0053      P(J,I)=P(J,I)-R(J,I)
0054      Q(J,I)=P(J,I)*N

```

```
0055      60 CONTINUE
0056      70 CONTINUE
0057          DO 71 J=1,K
0058          DO 72 I=1,K
0059              SUM=0
0060          DO 73 L=1,K
0061              73 SUM=SUM+SVZ(J,L)*Q(L,I)
0062              72 A(J,I)=SUM
0063          71 CONTINUE
0064              PRINT 21,N
0065              DO 110 J=1,K
0066              WRITE(6,23) (CG(J,I),I=1,K),(Q(J,I),I=1,K),(SVZ(J,I),I=1,K)
                  C,(A(J,I),I=1,K)
0067              V(J)=0
0068      110 CONTINUE
0069              IF(N.EQ.NN)GO TO 888
0070              IF(M.EQ.8) GO TO 101
0071              GO TO 100
0072      888 NN=15
0073      102 CONTINUE
0074          STOP
0075          END
```

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STUDY OF A RECURSIVE METHOD FOR MATRIX INVERSION
VIA
SIGNAL PROCESSING EXPERIMENTS

by

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B.S., Kansas State University, 1975

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the
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MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
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1977

This report discusses some aspects pertaining to a recursive method for inverting real symmetric matrices. The use of this method is explained in connection with two problems of interest in the following areas of communication and signal processing: (i) digital filtering via nonrecursive filters, and (ii) pattern classification via a least-squares minimum distance classifier.