MATHEMATICAL AND EXPERIMENTAL ANALYSIS
OF A HYPERBOLIC PARABOLOID

by 45

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TABLE OF CONTENTS

SYNOPSIS

This report presents two methods, mathematical and experimental for the analysis of a hyperbolic paraboloid roof shell structure. Using the first method, two loading conditions were considered, namely snow loading and uniform loading. There are three results under each loading condition; they are the deflection contours, the bending moment and the normal force distribution. For the second method, the only result that obtained is the deflection value under a snow load condition.

Mathematically, the first step is to set up the governing equations. This has been expressed as two, coupled and fourth - order partial differential equations in which the unknown functions are the vertical displacement component and a stress function. Boundary conditions are expressed in terms of these two functions and the equations are solved by the method of finite differences. By experimental analysis, a 1/15 scale model was made of plaster resign. Nine points were established with 0.001 dial gauges to measure the deflection values, under the snow loading condition.

The computations in this report are generally expressed in matrix form and have been wholly performed on a 360 computer so that tediousness and possible inaccurracy of hand computations were minimized. A comparison of the two solutions indicates that the results are reasonably close to each other.

INTRODUCTION

The newest form of shell roof construction - hyperbolic paraboloid shell structure has been greatly developed during recent years, not only because of its economical use of construction material, but also because of the simplicity of its' structural action and its' inherent beauty.

The hyperbolic paraboloid structure studied in this report is one of the most well known types - the umbrella type. It is composed of four equalhyperbolic paraboloid elements, each one with a square plan dimension and distributed in the space as shown in Fig. 1. Shell structures built in this shape may either have stiffening beams along the valleys and exterior edges or they may be self - stiffening, i.e. without edge beams. For research purposes the latter one is studied in this report. Both experimental and numerical analysis have been attempted. The essential of the experimental work is to verify the calculated result of the theoretical work. Because of limitation of time, technique and money, an experimental structural model one-fifteenth of the actual size was made of a different material from the prototype. The disadvantage is that its behavior is not compatible with the prototype. Also, because of the poor character of the model materials, using strain gauges for measurements, the only result obtained from the laboratory work was displacement values. Thus, ideal laboratory experimental result is difficult to obtain.

Theoretical analysis in hyperbolic paraboloid structure are commonly obtained by membrane theory. In general, the actual loading and construction details of such shells as built are far from ideal, and therefore, the membrane state of stress should also include bending - moments and shear force. Formula derived herein for the stresses and displacements in hyperbolic paraboloid shell, under the snow and uniform loading condition, are the higher power of differential equations. These equations are difficult to integrate, hence an approximate method - the finite difference method was introduced to solve this particular form of roof structure. A comparison between the numerical results of these two analyses will be the conclusion of this report.

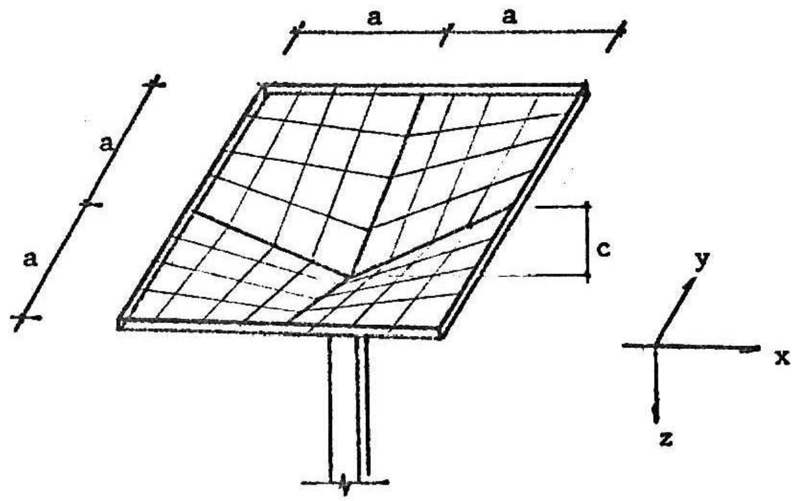


Fig. 1

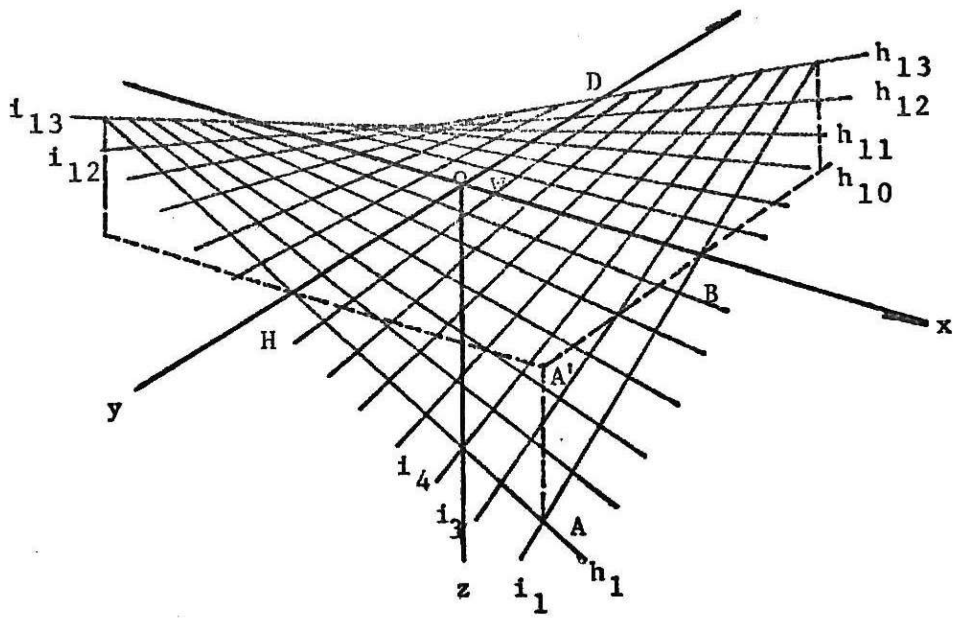


Fig. 2

HYPERBOLIC PARABOLOIDICAL SURFACE DEFINITION AND UMBRELLA FORM EXPLANATION

Surface Definition: Assume two straight nonparallel, non-intersecting lines HOD and ABC (Fig. 2) in space, which will be provisionally named directrix. Straight lines h_n that intersect both directrix, being at the same time parallel to one plane $\overline{xoz}$, named director plane, define the surface. They will be called the first system of generators. The two directrix determine in their turn a second directrix plane $\overline{yoz}$, parallel to them. The surface can be also considered as created by a second system of generators i_n parappel to this plane and intersecting every generator h_n of the first system. The hyperbolic paraboloid surface contains, therefore, two system of straight lines h_n and i_n, each system being parallel to a director plane and both planes forming an arbitrary angle w. (Fig. 2) Every point of the surface is the intersection of two straight lines contained on the surface. Taking as coordinate axis the two generators passing by the crown of the hyperbolic and the hyparbolic axic or intersection of both director planes, the equation of the surface, in these birectangular coordinates, will be:

$$z = k\,xy \ldots\ldots\ldots \qquad (1)$$

k being a constant which represents the unitary slope or warping of the hyperbolic paraboloid surface. (Fig. 2, k = AA'/(OB.OH), xoy = w can be any angle : xoz and yoz are right angle).

Umbrella type explanation: The umbrella type is constructed of four hyperbolic paraboloidical segments. Each segment, just as quadrant ABOH show in Fig. 2 join together along the two slope valley edge and make the other edges horizontally, just as shown in Fig. 1.

THEORETICAL ANALYSIS

Basic Assumption:

1. The material is linearly elastic, homogenous, and isotropic.
2. Points of the shell lying initially on a normal to the middle surface of the shell remain in this normal after deformation.
3. Displacement are small compared to the shell thickness.

Membrane stress equation:

We describe the surface of the shell by the system of rectangular coordinate as Fig. 3, where z is a function of x and y. Consider a small element in shell ABCD, and its projection in XY plane EFGH as shown.

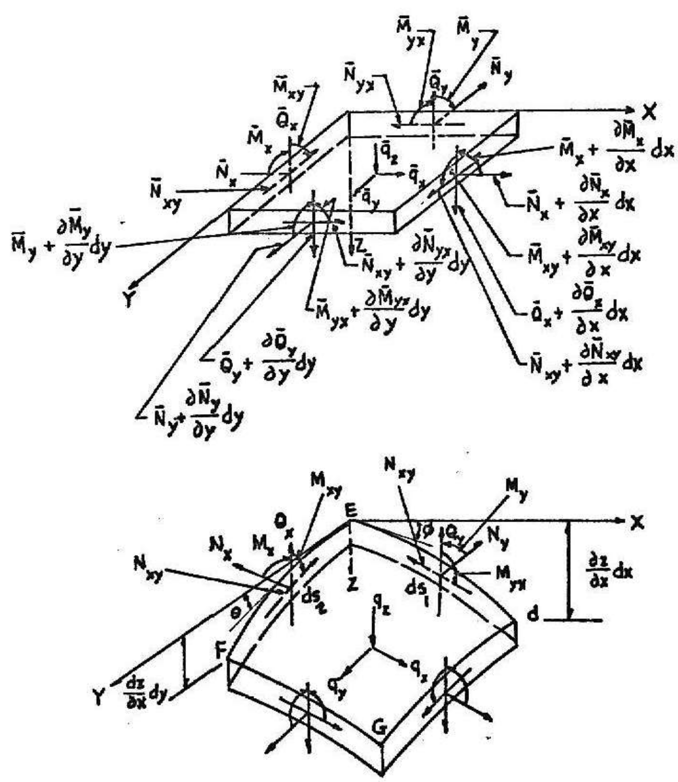


Fig. 3

The relationship between the normal force N_x, N_y, N_{xy}, of the curved element and its projectional force $\overline{N}_x$, $\overline{N}_y$, $\overline{N}_{xy}$ as shown in Fig. 3 is:

$$N_x \, ds_2 \cos\theta = \overline{N}_x dy \qquad \text{since : } dy = ds_2 \cos\phi$$

We have

$$N_x = \overline{N}_x \frac{\cos\phi}{\cos\theta} = \overline{N}_x \qquad \text{since } \phi \text{ and } \theta \text{ are small angle}$$

Similarly

$$N_y = \overline{N}_y \frac{\cos\theta}{\cos\phi} = \overline{N}_y \qquad N_{xy} = \overline{N}_{xy}$$

For the equilibrium condition, summing all force in X direction $\Sigma F_x = 0$ gives:

$$(\overline{N}_x + \frac{\partial \overline{N}_x}{\partial x} dx) dy - \overline{N}_x dy + (\overline{N}_{xy} + \frac{\partial N_{xy}}{\partial y} dy) dx - \overline{N}_{xy} dx +$$

$$\overline{x} \, dx \, dy = 0$$

Collecting terms we obtain:

$$(\frac{\partial \overline{N}_x}{\partial x} + \frac{\partial \overline{N}_{xy}}{\partial y}) \, dx \, dy + \overline{x} \, dx \, dy = 0$$

Since dx dy is not necessarily zero, the conditions $\Sigma F_x = 0$ necessarily is:

$$(\frac{\partial \overline{N}_x}{\partial x} + \frac{\partial \overline{N}_{xy}}{\partial y}) + \overline{x} = 0 \qquad \ldots\ldots\ldots \qquad (2)$$

Similarly from $\Sigma F_y = 0$, we obtain:

$$\frac{\partial \bar{N}_{xy}}{\partial x} + \frac{\partial \bar{N}_y}{\partial y} + \bar{y} = 0 \qquad \ldots\ldots\ldots \qquad (3)$$

In the condition for z components all five stress results appear. The force

$N_x \, dy/\cos\theta$ has a vertical component

$$N_x \frac{dy}{\cos\theta} \sin\theta = \bar{N}_x \tan\theta \, dy = \bar{N}_x \frac{\partial z}{\partial x} dy$$

and the shear force N_{xy} on the same side of the shell element gives:

$$N_{xy} \frac{dy}{\cos\phi} \sin\phi = \bar{N}_{xy} \tan\phi \, dy = \bar{N}_{xy} \frac{\partial z}{\partial x} dy$$

Similar expression are obtained for N_y and N_{yx}. The equation involves their differential increments:

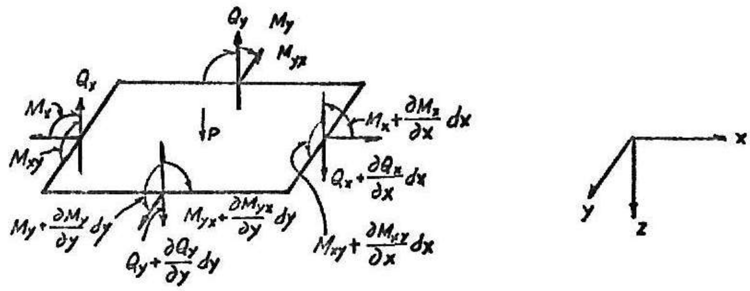
$$\frac{\partial}{\partial x} (\bar{N}_x \frac{\partial z}{\partial x}) + \frac{\partial}{\partial y} (\bar{N}_{yx} \frac{\partial z}{\partial x}) + \frac{\partial}{\partial y} (\bar{N}_{xy} \frac{\partial z}{\partial x}) + \frac{\partial}{\partial x} (\bar{N}_y \frac{\partial z}{\partial y}) +$$

$$\frac{\partial \bar{\theta}_x}{\partial x} + \frac{\partial \bar{\theta}_y}{\partial y} = 0 \qquad (4)$$

where Z for snow loading condition is $P(1 - y/a)(1 - x/a)$, for uniform condition Z is a constant.

Bending Moment and shear force equation:

The equilibrium of the element of the shell requires the summations of the forces in x, y, and z direction as well as the moment about axis to be zero. Since the stress components vary from point to point in the shell the resulting moments must also be a function of x and y. From the projectional element, taking moments of all forces acting on the element with respect to the x axis and neglecting higher-order term, we obtain the equation of equilibrium:



$$\frac{\partial \overline{M}_{xy}}{\partial x} dx\ dy + \frac{\partial \overline{M}_y}{\partial y} dx\ dy + \overline{Q}_y\ dx\ dy = 0$$

Simplify, so we get:

$$\frac{\partial \overline{M}_{xy}}{\partial x} + \frac{\partial \overline{M}_y}{\partial y} + \overline{Q}_y = 0 \qquad \ldots\ldots\ldots \qquad (5)$$

Similarly, the equation of equilibrium for moment with respect to the y axis gives:

$$\frac{\partial \overline{M}_{xy}}{\partial y} + \frac{\partial \overline{M}_y}{\partial x} + Q_x = 0 \qquad \ldots\ldots\ldots \qquad (6)$$

Substitute Equation 5 and 6 into 4:

$$\frac{\partial M_x}{\partial x^2} + 2\frac{\partial M_{xy}}{\partial x \partial y} + \frac{\partial M_y}{\partial y^2} + N_x\frac{\partial^2 z}{\partial x^2} + 2\bar{N}_{xy}\frac{\partial^2 z}{\partial x \partial y} + \bar{N}_y\frac{\partial^2 z}{\partial x^2} + \bar{Z} = 0 \tag{7}$$

Governing Equation:

Assume the mid-plane displacement u, v, and w with respect to positive x, y, and z direction. The unit elongation in x and y directions is e_x and e_y. The shearing strain is denoted by r_{xy}. From any shell book we can get the following relationship:

$$M_x = -D\ \left(\frac{\partial^2 W}{\partial x^2} + \frac{v\partial^2 W}{\partial y^2}\right) \tag{8}$$

$$M_y = -D\ \left(\frac{\partial^2 W}{\partial y^2} + \frac{v\partial^2 W}{\partial x^2}\right) \tag{9}$$

$$M_{xy} = D(1-v)\ \frac{\partial^2 W}{\partial x \partial y} \tag{10}$$

$$\epsilon_x = \frac{\bar{N}_x - v\bar{N}_y}{Et} = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial x}\ \frac{\partial z}{\partial x} \tag{11}$$

$$\epsilon_y = \frac{\bar{N}_y - v\bar{N}_x}{Et} = \frac{\partial v}{\partial y} + \frac{\partial w}{\partial y}\ \frac{\partial z}{\partial x} \tag{12}$$

$$r_{xy} = \frac{\bar{N}_{xy}}{Gx} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial y}\ \frac{\partial z}{\partial x} + \frac{\partial w}{\partial x}\ \frac{\partial z}{\partial y} \tag{13}$$

In which $D = Et^3/12(1-v)$, E is the modulus of elasticity, for the model material-plastic resign the values is 4×10^5 lb/in^2, v is the poission's ratio, assume as zero, t is the thickness of the shell. For convenience, introduce a stress function, where is in such a definition:

$$\frac{\partial^2 \psi}{\partial y^2} = \overline{N}_x \tag{14}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \overline{N}_y \tag{15}$$

$$\frac{\partial^2 \psi}{\partial x \partial y} = -\overline{N}_{xy} \tag{16}$$

Stress function satisfied Equation 2 and 3 must also satisfy equation 4 or 7. Substitute Equation 14, 15, 16 into Equation 7 yields:

$$-D \left(\frac{\partial^4 W}{\partial 1_x{}^4} + \frac{\partial^4 W}{\partial x^2 \partial y} + \frac{\partial^4 w}{\partial y^4}\right) + \left(\frac{\partial^2 \psi}{\partial y^2}\right) \frac{\partial^2 z}{\partial x^2} - 2\, \frac{\partial^2 \psi}{\partial x \partial y} \frac{\partial^2 z}{\partial x \partial y} -$$

$$\frac{\partial^2 \psi}{\partial x^2} \frac{\partial^2 z}{\partial y^2} + \overline{Z} = 0$$

From equation (1)

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2} = 0 \qquad \frac{\partial^2 z}{\partial x \partial y} = k = \frac{c}{ab}$$

then equation can be simplified as follows:

$$-D \left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \frac{2c}{ab}\frac{\partial^2 \psi}{\partial x \partial y} = \overline{Z} \tag{17}$$

From equations 11, 12, and 13 eliminate displacement components u and v, substitute the stress function from equation 13, 14, 15 yielding:

$$\frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial x^4} + \frac{\partial^4 \psi}{\partial y^4} = Eh \left(\frac{2\partial^2 z}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 z}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 z}{\partial y^2}\frac{\partial^2 w}{\partial x^2}\right)$$

similarly to simplicity this equation:

$$\frac{\partial^4 \psi}{\partial x^4} + \frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial y^4} = 2Eh \frac{\partial^2 z}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} \tag{18}$$

5. Boundary condition:

i. Moment along the free edge y = a is zero.

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} - \frac{v\partial^2 w}{\partial x}\right)_{y=a} = 0 \tag{19}$$

ii. Shearing force along the free edge y = a is zero.

$$V_y = \left(Q_y - \frac{\partial M_{yx}}{\partial x}\right)_{y=a} = -D\left[\frac{\partial^2 w}{\partial y^3} - (2-v)\frac{\partial^3 w}{\partial x^2 \partial y}\right] = 0 \tag{20}$$

iii. Twist moment at x = a and y = a is zero.

$$M_{xy} = D(1-v) \left.\frac{\partial^2 w}{\partial x \partial y}\right|_{\substack{x=a \\ y=a}} = 0 \tag{21}$$

iiii. Stress function along edge is zero (assume)

$$\psi = 0 \tag{22}$$

iiiii. Slope of the stress function at y = a is zero.

$$\frac{\partial \psi}{\partial y} = 0 \tag{23}$$

6. Nondimensionalized governing equation:

For convenience a change of nondimensionalized equation forms is needed:

$$x = x/a \quad y = y/a \quad W = w/t \quad T = \psi/Et^3 \quad Q = 12\bar{Z}(a/t)^4/E$$

Substituting into equations 17 and 18 will yield:

$$\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - 24(1-v)\frac{c}{t}\frac{\partial^2 T}{\partial x \partial y} = Q \tag{24}$$

$$\frac{\partial^4 T}{\partial x^4} + 2\frac{\partial^4 T}{\partial x^2 \partial y^2} + \frac{\partial^4 T}{\partial y^4} - 2\frac{c}{t}\frac{\partial^2 W}{\partial x \partial y} = 0 \tag{25}$$

APPLICATION OF FINITE DIFFERENCE EQUATION IN BENDING ANALYSIS

The use of finite difference equations to the solution of difficult structural problems is in large measure comparable to the technique now to surmount mathematical difficulties in the solution of complicated differential equations. The technique here consists of replacing the derivatives of differential equation by its central difference equivalent. The problem is thus reduced to the simple task of solving a system of simultaneous linear algebraic equations.

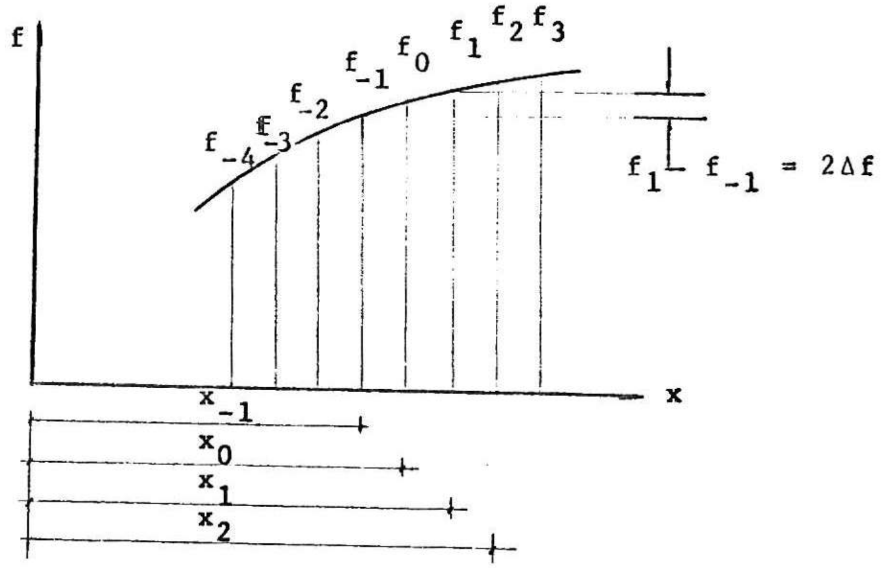


Fig. 5

In this light then, it is evident from geometrical consideration of Fig. 6 that if f is a function of x, then:

$$\frac{d\ f_o}{d\ x} \doteq \frac{f_1 - f_{-1}}{2h} \tag{26}$$

In which f_o represents the coordinate at $x = 0$, and means approximately equal to. By repeating this process, it follows that:

$$\frac{d^2 f_o}{dx^2} = \frac{d}{dx}\left(\frac{df_o}{dx}\right) = \frac{f_2 - f_o}{h^2} - \frac{f_o - f_2}{h^2}$$

$$\frac{d^3 f_o}{dx^3} = \frac{f_3 - 3f_1 + 3f_{-1} - f_3}{h^3} \qquad (27)$$

$$\frac{d^4 f_o}{dx^4} = \frac{f_4 - 4f_2 + 6f_o - 4f_{-2} + f_{-4}}{h^4} \qquad (28)$$

If f is function of both x and y, and belongs to a domain D, we then can divide the domain into a square mesh as Fig. 6 with $x = y = h$ and from equation 26 and 27, it follows:

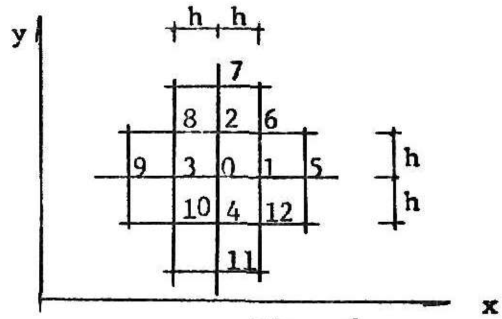


Fig. 6

$$\frac{\partial f_o}{\partial x} \doteq \frac{1}{2h}(f_1 - f_3) \qquad (29) \qquad\qquad \frac{\partial f_o}{\partial x} \doteq \frac{1}{2h}(f_2 - f_4) \qquad (30)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{h^2}\,(f_1 - 2f_o + f_3) \tag{31}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{1}{h^2}\,(f_2 - 2f_o + f_4) \tag{32}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{1}{4h^2}\,(f_8 + f_{12} - f_{10} - f_6) \tag{33}$$

$$\frac{\partial^4 f}{\partial x^4} = \frac{1}{h^4}\,(f_5 - 4f_1 + 6f_o - 4f_3 + f_9) \tag{34}$$

$$\frac{\partial^4 f}{\partial y^4} = \frac{1}{h^4}\,(f_7 - 4f_2 + 6f_o - 4f_4 + f_{11}) \tag{35}$$

$$\frac{\partial^4 f}{\partial y^2 \partial x^2} = \frac{1}{h^4}\,(f_6 + f_8 + f_{10} + f_{12} - 2f_1 - 2f_2 - 2f_3 - 2f_4 + 4f_o) \tag{36}$$

From these expressions of the derivative, the finite difference form of the governing equations and the boundary equations can be written as following:

$$\begin{aligned} &20W_c + 2(W_{re} + W_{se} + W_{sw} + W_{nw}) - 8(W_n + W_s + W_w + W_e) \\ &W_{se} + W_{ss} + W_{nn} + W_{ww} - \frac{6c}{n^2 h}\,(T_{ne} + T_{se} + T_{sw} + T_{nw}) = \theta/n^4 \end{aligned} \tag{37}$$

$$20T_c - 2(T_{ne} + T_{se} + T_{sw} + T_{nw}) - 8(T_n + T_s + T_n + T_e)$$

$$T_{nn} + T_{ss} + T_{ww} + T_{ee} - \frac{c}{2n^2h}(W_{ne} + W_{se} + W_{sw} + W_{nw}) = 0 \tag{38}$$

$$[W_s - 2(1+v)W_c + W_n + vW_e + vW_s]_{y=1} = 0 \tag{39}$$

$$[W_{ss} + W_{nn} - (2-v)(-W_{ne} - W_{nw} + W_{se} + W_{sw}) - 2(3-v)$$

$$(W_n - W_s)] = 0 \tag{40}$$

$$[W_{ne} + W_{se} + W_{nw} + W_{sw}] = 0 \tag{41}$$

$$[T_{y=1}] = 0 \tag{42}$$

$$T_n = T_s = 1_{y=1} \tag{43}$$

Equation 37 through 43, and the summetry of internal boundaries, are the necessary and sufficient equations that must be expressed at every point of the grid to the generate nondimensional finite-difference equations.

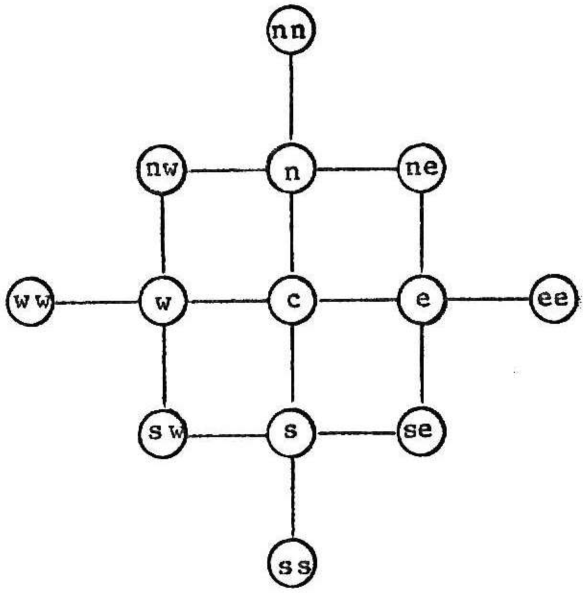


Fig. 7. Notation of the finite difference method.

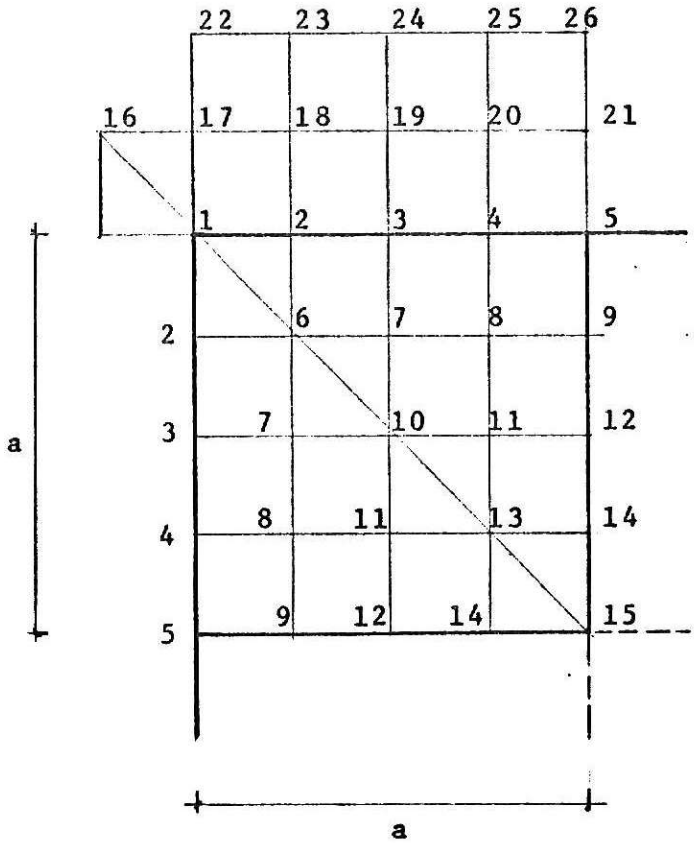


Fig. 8. Grade length is a/4

Numerical Calculation:

In using the finite difference equation, we must first divide the hyperbolic paraboloid surface into a net with many points. We use herein a grade length of a/4 and 25 points were included. Due to symmetry, only half of the quadrant i.e. 15 points is considered. Each point has two unknowns T and W, in addition to the auxiliary boundary points, as shown in Fig. 6, so that totally there are 47 unknowns. By boundary conditions 42, 43.

$$T_i = o\big|_{Y=1} \qquad T_1 = T_2 = T_3 = T_4 = T_5 = T_i = 0$$

$$T_n = T_s\big|_{Y=1} \qquad T_9 = T_{21},\ T_8 = T_{20},\ T_7 = T_{19},\ T_6 = T_{18} = T_{16}$$

$$T_2 = T_{17}$$

$$w = 0\Big|_{\substack{X=1\\Y=1}}$$

12 unknowns can be eliminated. The remaining 35 unknowns will be solved by the governing equations and the boundary equations, 37, 38, 39, 40, 41. For example, consider point 1, according to the notation of Fig. 8 we can obtain the equations as follows:

$$-2W_1 + W_2 + W_{17} = 0$$

$$-6W_2 + W_3 + 2W_6 + 6W_{17} - W_{22} = 0$$

$$20W_1 + 2(W_{16} + W_6 + 4W_{18}) - 16(W_2 + W_{17}) + 2(W_2 + W_{22}) +$$

$$15(T_{16} - T_6) = Q/n^4 = 0.0039Q$$

$$6T_6 - 1.25W_6 + 2T_{16} - 1.25W_{16} + 2.5W_{18} + 2W_{22} = 0$$

Where $n = 4$ $c/h = 40$

Repeating the same process in every point, we get 35 equations, written in the matrix form:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{135} \\ \vdots & & & \vdots \\ a_{351} & a_{352} & \cdots & a_{3535} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{35} \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_{35} \end{bmatrix}$$

or: symbolically, $[a_{ij}][x_i] = [c_i]$

where x_i is the variable of F and W, and a_{ij} is its coefficient.

$x_1 = w_1$ $w_2 = w_2$ $x_3 = w_3$ $x_4 = W_4$ $x_5 = W_5$ $x_6 = F_6$ $x_7 = W_6$

$x_8 = F_7$ $x_9 = W_7$ $x_{10} = F_8$ $x_{11} = W_8$ $x_{12} = F_9$ $x_{13} = W_9$ $x_{14} = F_{10}$

$x_{15} = W_{10}$ $x_{16} = F_{11}$ $x_{17} = W_{12}$ $x_{18} = F_{12}$ $x_{19} = W_{12}$ $x_{20} = F_{13}$ $x_{21} = W_{13}$

$x_{22}=F_{14}$ $x_{23}=W_{14}$ $x_{24}=F_{15}$ $x_{25}=W_{16}$ $x_{26}=W_{17}$ $x_{27}=W_{18}$ $x_{28}=W_{19}$

$x_{29}=W_{20}$ $x_{30}=W_{21}$ $x_{31}=W_{22}$ $x_{32}=W_{23}$ $x_{33}=W_{24}$ $x_{34}=W_{25}$ $x_{35}=W_{26}$

A computer program was written in 360 system (see Appendix) for solving those simultaneous equations. The matrix A, C and the solution of the equation are listed in the following Table I, II, III, IV, V.

Example of Calculation:

1. Deflection:

$$w_1 = W_1\ t = 0.0314\ Q\ t = 0.0314\ \frac{12Z(\frac{a}{t})^4 \cdot t}{E} = 0.0314\ \frac{Za^4}{E}$$

$$w_2 = W_2\ t = 0.0286\ Q\ t = 0.0286\ \frac{12Za^4/t^4 \cdot t}{E} = 0.0286\ \frac{Za^4}{D}$$

2. Moment Calculation:

From Equation 21: $M_y = -D\ (\frac{\partial^2 w}{\partial y^2} - \frac{v\partial^2 w}{\partial x^2})$

since $v = 0$ $\qquad M_y = -D(\frac{\partial^2 w}{\partial y^2}) = -D(\frac{\partial^2 Wt}{\partial (aY)^2})$

$$= -D(\frac{t}{a^2})\ \frac{\partial^2 W}{\partial^2 Y^2}$$

Change to finite difference form:

$$M_{yi} = -D(t/a^2)(1/(1/4)^2)(W_n - 2W_c + W_s)$$

Take point 6 as an example: (W's value show on Fig. 9)

$$M_{y6} = -\frac{Et^3}{12}\frac{t}{a^2}\frac{1}{1/16}(0.0286 - 0.0528 + 0.0218)\frac{12Z(\frac{a}{t})^4}{E}$$

$$= 0.0385\ Za^2$$

$$M_{y7} = 0.0340\ Za^2$$

3. Normal force calculation:

From equation 14:

$$N_y = \frac{\partial^2\psi}{\partial y^2} = \frac{\partial^2 TEt^3}{\partial^2(ay)^2} = \frac{Et^3}{a^2}\frac{\partial^2 T}{\partial Y^2}$$

Change to finite difference form:

$$N_{yi} = \frac{Et^3}{a^2}\frac{12Z(\frac{a}{t})^4}{E}\frac{1}{1/16}(T_s - 2T_c + T_n)$$

$$= (T_s - 2T_c + T_n)\ 19z\ \ a^2z/t$$

Take point 6 as an example:

$$N_{y6} = 19z\ (0 - 0.00046 + 0.000429)\ \alpha^2 z/t = -0.00595\ a^2z/t$$

TABLE I

THE MATRIX A

ROW 1	-2.0000000E 00	1.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	1.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 2	0.0000000E-01	-2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 3	0.0000000E-01	0.0000000E-01	-2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 4	0.0000000E-01	0.0000000E-01	0.0000000E-01	-2.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 5	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	-2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	1.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 6	0.0000000E-01	-6.0000000E 00	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	-2.0000000E 00	6.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	-1.0000000E 00	0.0000000E-01

	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 7	0.0000000E-01	2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	-6.0000000E 00	0.0000000E-01
	3.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	-2.0000000E 00	6.0000000E 00	-2.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	-1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 8	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	-6.0000000E 00	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	-2.0000000E 00	6.0000000E 00
	-2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	-1.0000000E 00	0.0000000E-01	0.0000000E-01	
ROW 9	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	2.0000000E 00	0.0000000E-01	-6.0000000E 00	0.0000000E-01
	2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	-2.0000000E 00
	6.0000000E 00	-2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	-1.0000000E 00	0.0000000E-01	
ROW 10	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	4.0000000E 00	0.0000000E-01
	-6.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	-4.0000000E 00	6.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	-1.0000000E 00	
ROW 11	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	-2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 12	2.0000000E 01	-1.6000000E 01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	2.0000000E 00	-1.6000000E 01	4.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 13	-8.0000000E 00	2.2000000E 01	-8.0000000E 00	1.0000000E 00
	0.0000000E-01	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	3.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01

	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	3.0000000E 00	-8.0000000E 00	2.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 14	1.0000000E 00	-8.0000000E 00	2.0000000E 01	-8.0000000E 00
	1.0000000E 00	0.0000000E-01	2.0000000E 00	0.0000000E-01
	-8.0000000E 00	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	-8.0000000E 00
	2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	
ROW 15	0.0000000E-01	1.0000000E 00	-8.0000000E 00	2.1000000E 01
	-8.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	2.0000000E 00	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	2.0000000E 00
	-8.0000000E 00	2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	1.0000000E 00	0.0000000E-01	
ROW 16	0.0000000E-01	0.0000000E-01	2.0000000E 00	-1.6000000E 01
	2.0000000E 01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	4.0000000E 00	0.0000000E-01
	-8.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	4.0000000E 00	-8.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	
ROW 17	2.0000000E 00	-1.6000000E 01	4.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 01	0.0000000E-01
	-1.6000000E 01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	1.5000000E 01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 18	0.0000000E-01	3.0000000E 00	-8.0000000E 00	2.0000000E 00
	0.0000000E-01	0.0000000E-01	-8.0000000E 00	-1.5000000E 01
	2.2000000E 01	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	1.0000000E 00	0.0000000E-01	-8.0000000E 00	1.5000000E 01
	3.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 19	0.0000000E-01	0.0000000E-01	2.0000000E 00	-8.0000000E 00
	2.0000000E 00	0.0000000E-01	1.0000000E 00	0.0000000E-01
	-8.0000000E 00	0.0000000E-01	2.1000000E 01	0.0000000E-01
	-8.0000000E 00	-1.5000000E 01	2.0000000E 00	0.0000000E-01
	-8.0000000E 00	1.5000000E 01	2.0000000E 00	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 20	0.0000000E-01	0.0000000E-01	0.0000000E-01	4.0000000E 00
	-8.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01

	2.0000000E 00	0.0000000E-01	-1.6000000E 01	0.0000000E-01
	2.0000000E 01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	4.0000000E 00	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	1.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 21	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	1.5000000E 01	2.0000000E 00	0.0000000E-01
	-1.6000000E 01	-3.0000000E 01	4.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 01	0.0000000E-01
	-1.6000000E 01	0.0000000E-01	2.0000000E 00	1.5000000E 01
	2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 22	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.5000000E 01
	3.0000000E 00	0.0000000E-01	-8.0000000E 00	-1.5000000E 01
	2.0000000E 00	0.0000000E-01	-8.0000000E 00	-1.5000000E 01
	2.3000000E 01	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	-8.0000000E 00	1.5000000E 01	3.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 23	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	4.0000000E 00	0.0000000E-01
	-8.0000000E 00	0.0000000E-01	2.0000000E 00	0.0000000E-01
	-1.6000000E 01	0.0000000E-01	2.0000000E 01	0.0000000E-01
	4.0000000E 00	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 24	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	1.5000000E 01	2.0000000E 00	0.0000000E-01
	-1.6000000E 01	-3.0000000E 01	4.0000000E 00	0.0000000E-01
	2.2000000E 01	0.0000000E-01	-1.6000000E 01	1.5000000E 01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 25	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	6.0000000E 00	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	-1.6000000E 01	0.0000000E-01	2.5000000E 01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 26	-1.2500000E 00	0.0000000E-01	2.5000000E 00	0.0000000E-01
	0.0000000E-01	2.2000000E 01	0.0000000E-01	-1.6000000E 01
	0.0000000E-01	2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	2.0000000E 00	-1.2500000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01

	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 27	0.0000000E-01	-1.2500000E 00	0.0000000E-01	1.2500000E 00
	0.0000000E-01	-8.0000000E 00	0.0000000E-01	2.3000000E 01
	1.2500000E 00	-8.0000000E 00	0.0000000E-01	1.0000000E 00
	0.0000000E-01	-8.0000000E 00	0.0000000E-01	3.0000000E 00
	-1.2500000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 28	0.0000000E-01	0.0000000E-01	-1.2500000E 00	0.0000000E-01
	1.2500000E 00	1.0000000E 00	0.0000000E-01	-8.0000000E 00
	0.0000000E-01	2.2000000E 01	0.0000000E-01	-8.0000000E 00
	0.0000000E-01	2.0000000E 00	1.2500000E 00	-8.0000000E 00
	0.0000000E-01	2.0000000E 00	-1.2500000E 00	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 29	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	2.0000000E 00
	0.0000000E-01	-1.6000000E 01	0.0000000E-01	2.1000000E 01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	4.0000000E 00
	0.0000000E-01	-8.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	1.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 30	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	2.0000000E 00	-1.2500000E 00	-1.6000000E 01
	0.0000000E-01	4.0000000E 00	2.5000000E 00	0.0000000E-01
	0.0000000E-01	2.0000000E 01	0.0000000E-01	-1.6000000E 01
	0.0000000E-01	2.0000000E 00	0.0000000E-01	2.0000000E 00
	-1.2500000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 31	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	3.0000000E 00
	-1.2500000E 00	-8.0000000E 00	0.0000000E-01	2.0000000E 00
	1.2500000E 00	-8.0000000E 00	0.0000000E-01	2.3000000E 01
	1.2500000E 00	-8.0000000E 00	0.0000000E-01	-8.0000000E 00
	0.0000000E-01	3.0000000E 00	-1.2500000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 32	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	4.0000000E 00	0.0000000E-01	-8.0000000E 00
	0.0000000E-01	2.0000000E 00	0.0000000E-01	-1.6000000E 01
	0.0000000E-01	2.0000000E 01	0.0000000E-01	4.0000000E 00
	0.0000000E-01	-8.0000000E 00	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 33	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	2.0000000E 00	-1.2500000E 00	-1.6000000E 01
	0.0000000E-01	4.0000000E 00	2.5000000E 00	2.2000000E 01

	0.0000000E-01	-1.6000000E 01	0.0000000E-01	2.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 34	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	6.0000000E 00
	0.0000000E-01	-8.0000000E 00	0.0000000E-01	-1.6000000E 01
	0.0000000E-01	2.5000000E 01	0.0000000E-01	-8.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 35	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	4.0000000E 00	0.0000000E-01	8.0000000E 00
	0.0000000E-01	-3.2000000E 01	0.0000000E-01	2.0000000E 01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	

TABLE II

0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.3900000E-02
0.3900000E-02	0.3900000E-02	0.3900000E-02	0.3900000E-02
0.3900000E-02	0.3900000E-02	0.3900000E-02	0.3900000E-02
0.3900000E-02	0.3900000E-02	0.3900000E-02	0.3900000E-02
0.3900000E-02	0.3900000E-02	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	

TABLE III

W AND F VALUE LIST FOLLOWING

1 X= 0.03141201	2 X= 0.02862203	3 X= 0.02431312	4 X= 0.01930500
5 X= 0.01701302	6 X=-0.00023000	7 X= 0.02638200	8 X=-0.00042900
9 X= 0.02175000	10 X=-0.00056500	11 X= 0.01671500	12 X=-0.00054700
13 X= 0.01363000	14 X=-0.00084800	15 X= 0.01715000	16 X=-0.00112500
17 X= 0.01165500	18 X=-0.00117500	19 X=-0.00885300	20 X=-0.00152000
21 X= 0.00620000	22 X=-0.00161000	23 X= 0.00330300	24 X=-0.00175300
25 X= 0.03542100	26 X= 0.03398700	27 X= 0.03089200	28 X= 0.02662400
29 X= 0.02234200	30 X= 0.02038200	31 X= 0.03791400	32 X= 0.02865200
33 X= 0.02543000	34 X= 0.02425000	35 X= 0.02529800	

Uniform loading condition:

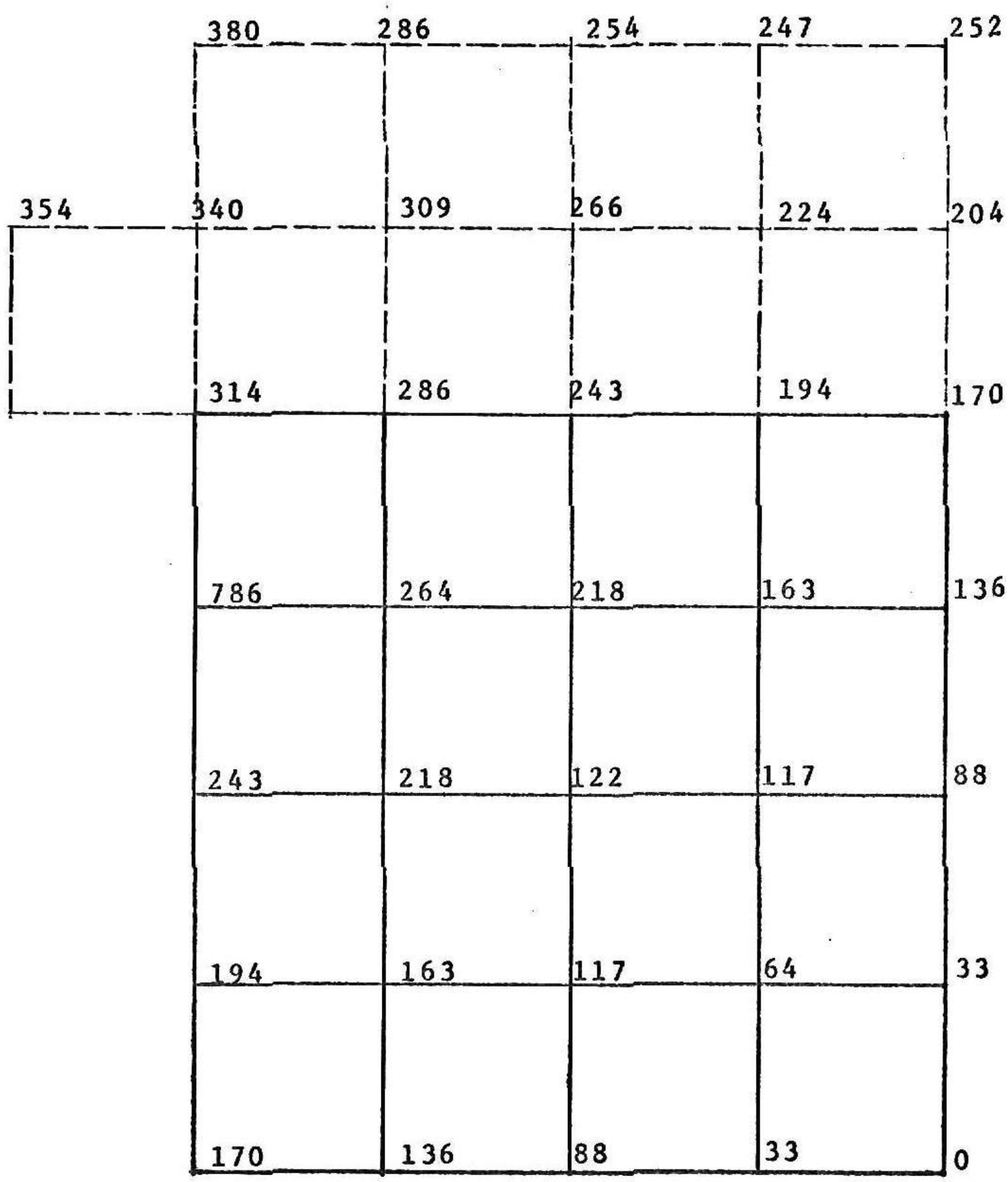


Fig. 9. W's value of various point

W (·) 10^{-4} Q

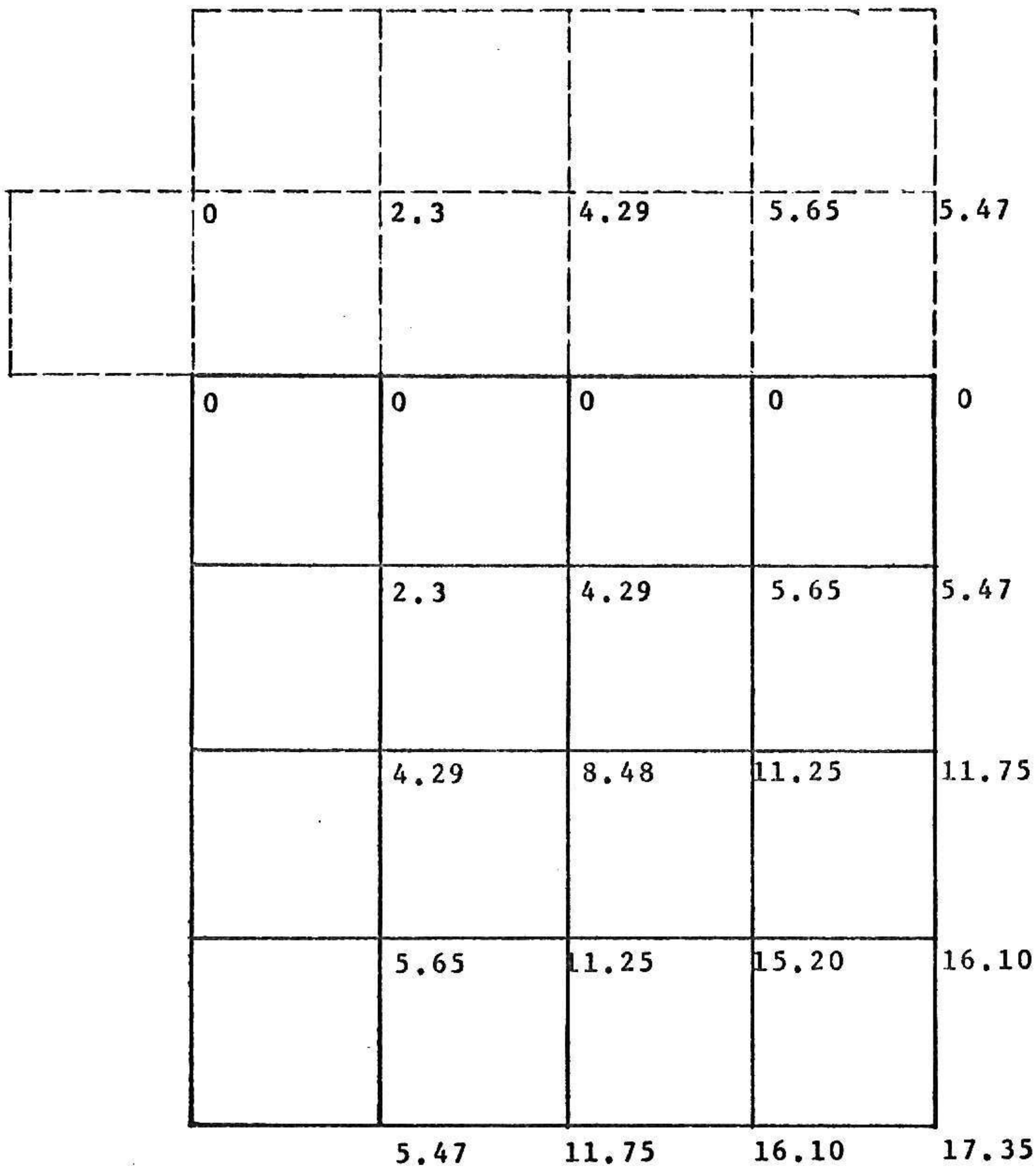


Fig. F's value of various point

$F = (\quad)\ 10^{-4}\ Q$

0	0	0	0	0
.024	.0385	.034	.024	.0225
.010	.017	.014	.014	.012
.04	-.045	-.042	-.042	-.035
-.077	-.086	-.093	-.094	-1.02

Coefficient of M_x

$M_x = (\quad) Z a^2$ in.-lb/in.

0	.088	.665	.0217	.021
0	-.006	-0.002	.001	.015
0	-0.012	-.027	-.032	-.037
0	-.0296	-.0436	-.0586	-.06
0	.007	-0.002	-.0346	-.048

Coefficients of N_x

$N_x = (\quad) Z a^2 / t$ lb/in.

Fig. 10

Uniform loading Condition:

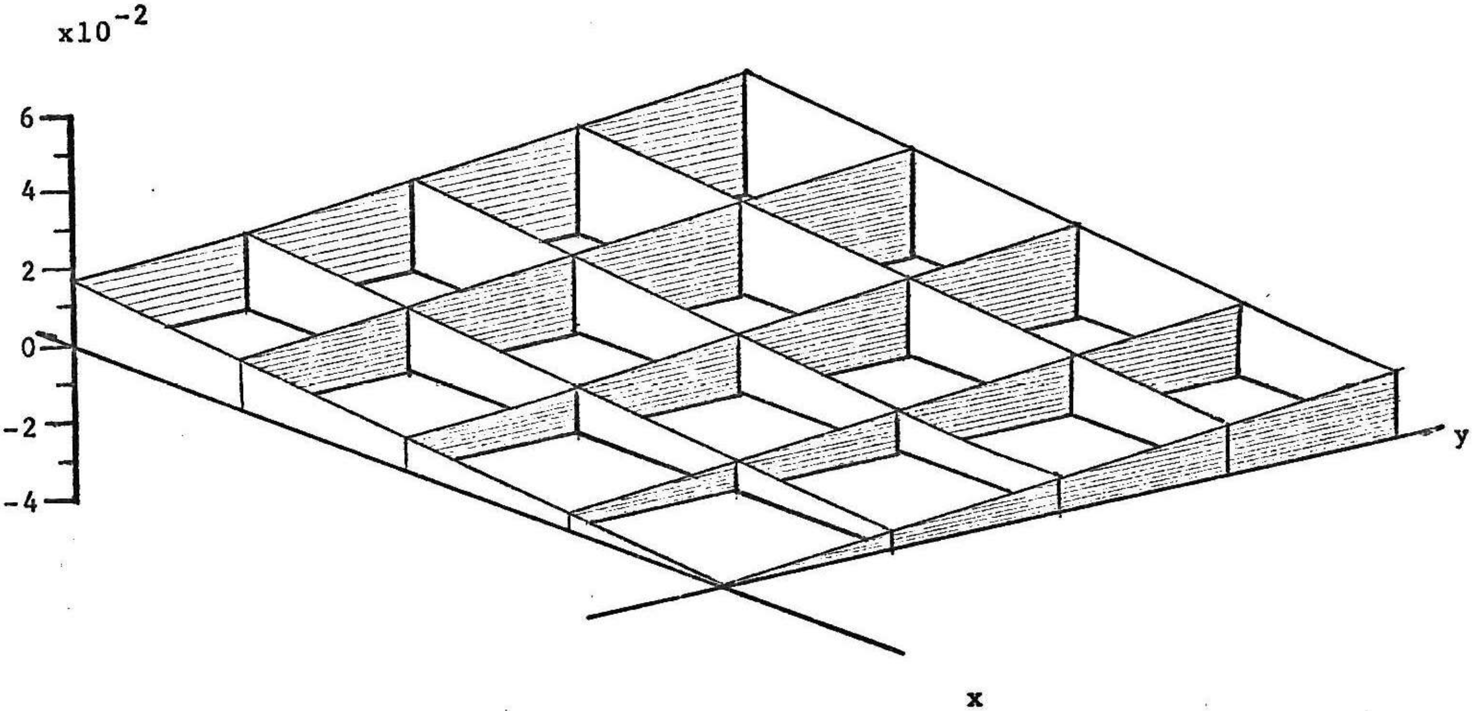


Fig. 11. Magnitude of Deflection Coef.

$w = (\quad) Z a^4 / D$ in

Positive bending moment
causes the bottom fiber
of the shell tension

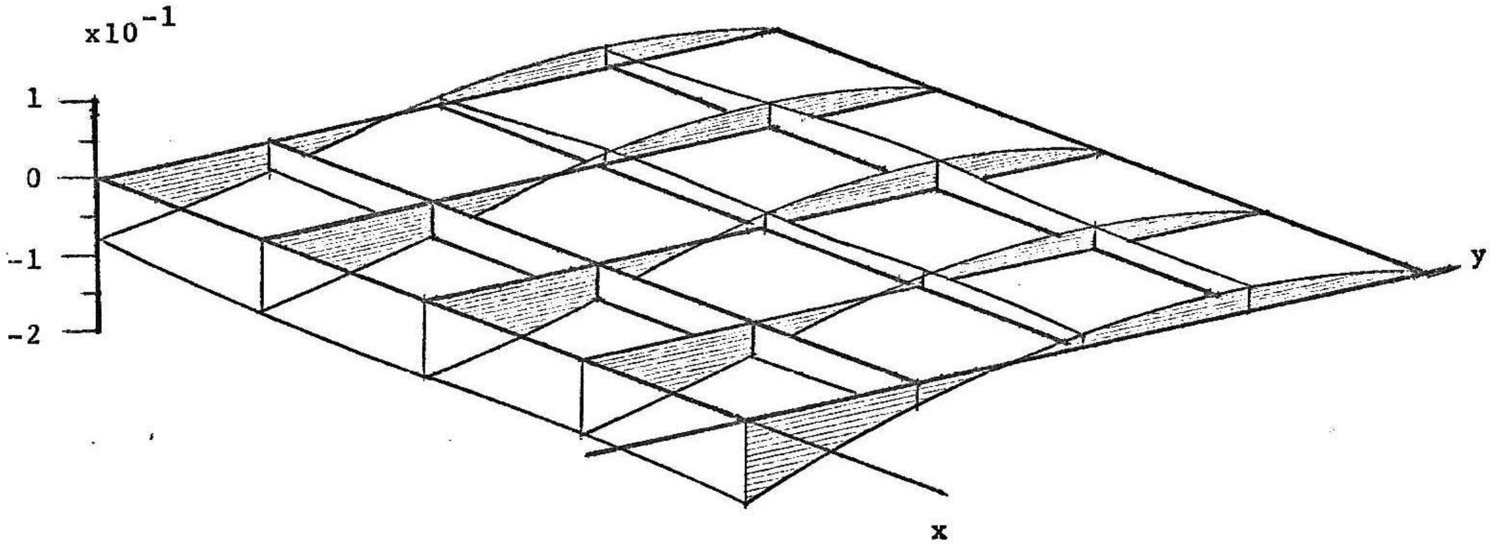


Fig. 12. Distribution of Bending Moment

M_x = ( ) Z a^2 in-lb/in

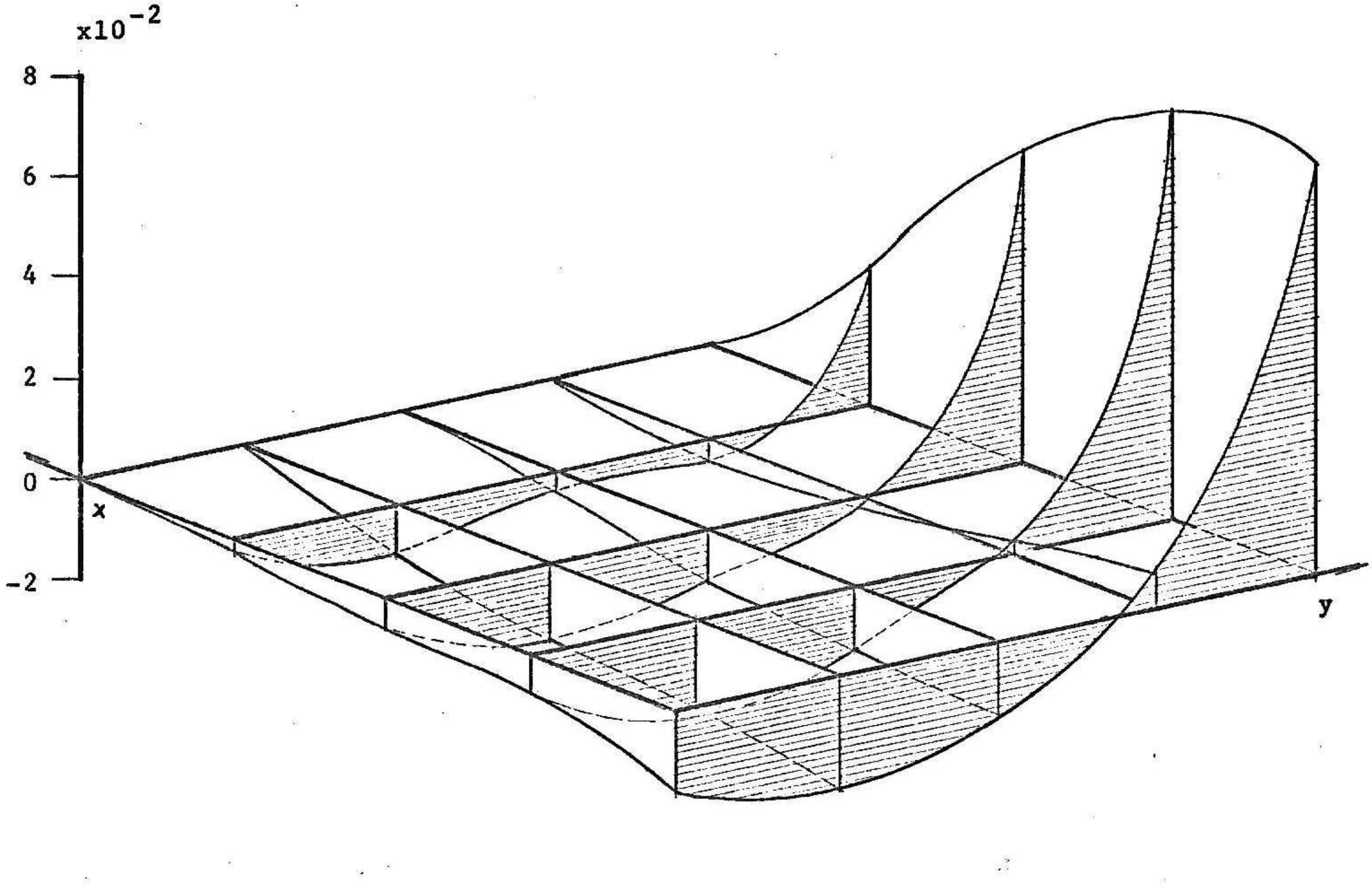


Fig. 13. Distribution of normal force

$N_x = (\quad)\ Z\ a^2\ /\ t$ lb/in

TABLE I

THE MATRIX A

ROW 1	-2.0000000E 00	1.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	1.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 2	0.0000000E-01	-2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 3	0.0000000E-01	0.0000000E-01	-2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 4	0.0000000E-01	0.0000000E-01	0.0000000E-01	-2.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 5	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	-2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	1.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 6	0.0000000E-01	-6.0000000E 00	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	-2.0000000E 00	6.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	-1.0000000E 00	0.0000000E-01

	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 7	0.0000000E-01	2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	-6.0000000E 00	0.0000000E-01
	3.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	-2.0000000E 00	6.0000000E 00	-2.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	-1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 8	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	-6.0000000E 00	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	-2.0000000E 00	6.0000000E 00
	-2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	-1.0000000E 00	0.0000000E-01	0.0000000E-01	
ROW 9	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	2.0000000E 00	0.0000000E-01	-6.0000000E 00	0.0000000E-01
	2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	-2.0000000E 00
	6.0000000E 00	-2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	-1.0000000E 00	0.0000000E-01	
ROW 10	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	4.0000000E 00	0.0000000E-01
	-6.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	-4.0000000E 00	6.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	-1.0000000E 00	
ROW 11	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	-2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 12	2.0000000E 01	-1.6000000E 01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	2.0000000E 00	-1.6000000E 01	4.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 13	-8.0000000E 00	2.2000000E 01	-8.0000000E 00	1.0000000E 00
	0.0000000E-01	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	3.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01

	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	3.0000000E 00	-8.0000000E 00	2.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 14	1.0000000E 00	-8.0000000E 00	2.0000000E 01	-8.0000000E 00
	1.0000000E 00	0.0000000E-01	2.0000000E 00	0.0000000E-01
	-8.0000000E 00	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	-8.0000000E 00
	2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	
ROW 15	0.0000000E-01	1.0000000E 00	-9.0000000E 00	2.1000000E 01
	-8.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	2.0000000E 00	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	2.0000000E 00
	-8.0000000E 00	2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	1.0000000E 00	0.0000000E-01	
ROW 16	0.0000000E-01	0.0000000E-01	2.0000000E 00	-1.6000000E 01
	2.0000000E 01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	4.0000000E 00	0.0000000E-01
	-8.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	4.0000000E 00	-8.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	
ROW 17	2.0000000E 00	-1.6000000E 01	4.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 01	0.0000000E-01
	-1.6000000E 01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	1.5000000E 01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 18	0.0000000E-01	3.0000000E 00	-8.0000000E 00	2.0000000E 00
	0.0000000E-01	0.0000000E-01	-8.0000000E 00	-1.5000000E 01
	2.2000000E 01	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	1.0000000E 00	0.0000000E-01	-8.0000000E 00	1.5000000E 01
	3.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 19	0.0000000E-01	0.0000000E-01	2.0000000E 00	-8.0000000E 00
	2.0000000E 00	0.0000000E-01	1.0000000E 00	0.0000000E-01
	-8.0000000E 00	0.0000000E-01	2.1000000E 01	0.0000000E-01
	-8.0000000E 00	-1.5000000E 01	2.0000000E 00	0.0000000E-01
	-8.0000000E 00	1.5000000E 01	2.0000000E 00	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 2C	0.0000000E-01	0.0000000E-01	0.0000000E-01	4.0000000E 00
	-8.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01

	2.0000000E 00	0.0000000E-01	-1.6000000E 01	0.0000000E-01
	2.0000000E 01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	4.0000000E 00	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	1.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	1.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 21	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	1.5000000E 01	2.0000000E 00	0.0000000E-01
	-1.6000000E 01	-3.0000000E 01	4.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 01	0.0000000E-01
	-1.6000000E 01	0.0000000E-01	2.0000000E 00	1.5000000E 01
	2.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 22	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.5000000E 01
	3.0000000E 00	0.0000000E-01	-8.0000000E 00	-1.5000000E 01
	2.0000000E 00	0.0000000E-01	-8.0000000E 00	-1.5000000E 01
	2.3000000E 01	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	-8.0000000E 00	1.5000000E 01	3.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 23	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	4.0000000E 00	0.0000000E-01
	-8.0000000E 00	0.0000000E-01	2.0000000E 00	0.0000000E-01
	-1.6000000E 01	0.0000000E-01	2.0000000E 01	0.0000000E-01
	4.0000000E 00	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 24	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	2.0000000E 00	0.0000000E-01
	0.0000000E-01	1.5000000E 01	2.0000000E 00	0.0000000E-01
	-1.6000000E 01	-3.0000000E 01	4.0000000E 00	0.0000000E-01
	2.2000000E 01	0.0000000E-01	-1.6000000E 01	1.5000000E 01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 25	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	1.0000000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	6.0000000E 00	0.0000000E-01	-8.0000000E 00	0.0000000E-01
	-1.6000000E 01	0.0000000E-01	2.5000000E 01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 26	-1.2500000E 00	0.0000000E-01	2.5000000E 00	0.0000000E-01
	0.0000000E-01	2.2000000E 01	0.0000000E-01	-1.6000000E 01
	0.0000000E-01	2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	2.0000000E 00	-1.2500000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01

	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 27	0.0000000E-01	-1.2500000E 00	0.0000000E-01	1.2500000E 00
	0.0000000E-01	-8.0000000E 00	0.0000000E-01	2.3000000E 01
	1.2500000E 00	-8.0000000E 00	0.0000000E-01	1.0000000E 00
	0.0000000E-01	-8.0000000E 00	0.0000000E-01	3.0000000E 00
	-1.2500000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 28	0.0000000E-01	0.0000000E-01	-1.2500000E 00	0.0000000E-01
	1.2500000E 00	1.0000000E 00	0.0000000E-01	-8.0000000E 00
	0.0000000E-01	2.2000000E 01	0.0000000E-01	-8.0000000E 00
	0.0000000E-01	2.0000000E 00	1.2500000E 00	-8.0000000E 00
	0.0000000E-01	2.0000000E 00	-1.2500000E 00	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 29	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	2.0000000E 00
	0.0000000E-01	-1.6000000E 01	0.0000000E-01	2.1000000E 01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	4.0000000E 00
	0.0000000E-01	-8.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	1.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 30	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	2.0000000E 00	-1.2500000E 00	-1.6000000E 01
	0.0000000E-01	4.0000000E 00	2.5000000E 00	0.0000000E-01
	0.0000000E-01	2.0000000E 01	0.0000000E-01	-1.6000000E 01
	0.0000000E-01	2.0000000E 00	0.0000000E-01	2.0000000E 00
	-1.2500000E 00	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 31	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	3.0000000E 00
	-1.2500000E 00	-8.0000000E 00	0.0000000E-01	2.0000000E 00
	1.2500000E 00	-8.0000000E 00	0.0000000E-01	2.3000000E 01
	1.2500000E 00	-8.0000000E 00	0.0000000E-01	-8.0000000E 00
	0.0000000E-01	3.0000000E 00	-1.2500000E 00	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 32	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	4.0000000E 00	0.0000000E-01	-8.0000000E 00
	0.0000000E-01	2.0000000E 00	0.0000000E-01	-1.6000000E 01
	0.0000000E-01	2.0000000E 01	0.0000000E-01	4.0000000E 00
	0.0000000E-01	-8.0000000E 00	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 33	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	2.0000000E 00	0.0000000E-01	0.0000000E-01
	0.0000000E-01	2.0000000E 00	-1.2500000E 00	-1.6000000E 01
	0.0000000E-01	4.0000000E 00	2.5000000E 00	2.2000000E 01

	0.0000000E-01	-1.6000000E 01	0.0000000E-01	2.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 34	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	1.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	6.0000000E 00
	0.0000000E-01	-8.0000000E 00	0.0000000E-01	-1.6000000E 01
	0.0000000E-01	2.5000000E 01	0.0000000E-01	-8.0000000E 00
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	
ROW 35	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	4.0000000E 00	0.0000000E-01	8.0000000E 00
	0.0000000E-01	-3.2000000E 01	0.0000000E-01	2.0000000E 01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	0.0000000E-01
	0.0000000E-01	0.0000000E-01	0.0000000E-01	

TABLE IV

0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.2440000E-02	0.4880000E-02	0.7320002E-02	0.9760000E-02
0.9760000E-03	0.1460000E-01	0.9520002E-02	0.2196000E-01
0.2928000E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	

TABLE V

W AND F VALUE LIST FOLLOWING

1 X= 0.00015150	2 X= 0.00012751	3 X= 0.00010345	4 X= 0.00008510
5 X= 0.00007810	6 X=-0.00000048	7 X= 0.00010382	8 X=-0.00000076
9 X= 0.00007920	10 X=-0.00000081	11 X= 0.00005970	12 X=-0.00000071
13 X= 0.00005200	14 X=-0.00000120	15 X= 0.00005450	16 X= 0.00000133
17 X= 0.00000355	18 X=-0.00000132	19 X= 0.00002720	20 X=-0.00000154
21 X= 0.00001552	22 X=-0.00000159	23 X= 0.00000760	24 X=-0.00000168
25 X= 0.00019856	26 X= 0.00017500	27 X=-0.00015120	28 X= 0.00012813
29 X= 0.00013008	30 X= 0.00010402	31 X= 0.00019909	32 X= 0.00016932
33 X= 0.00015205	34 X= 0.00013864	35 X= 0.00013508	

Snow loading Condition:

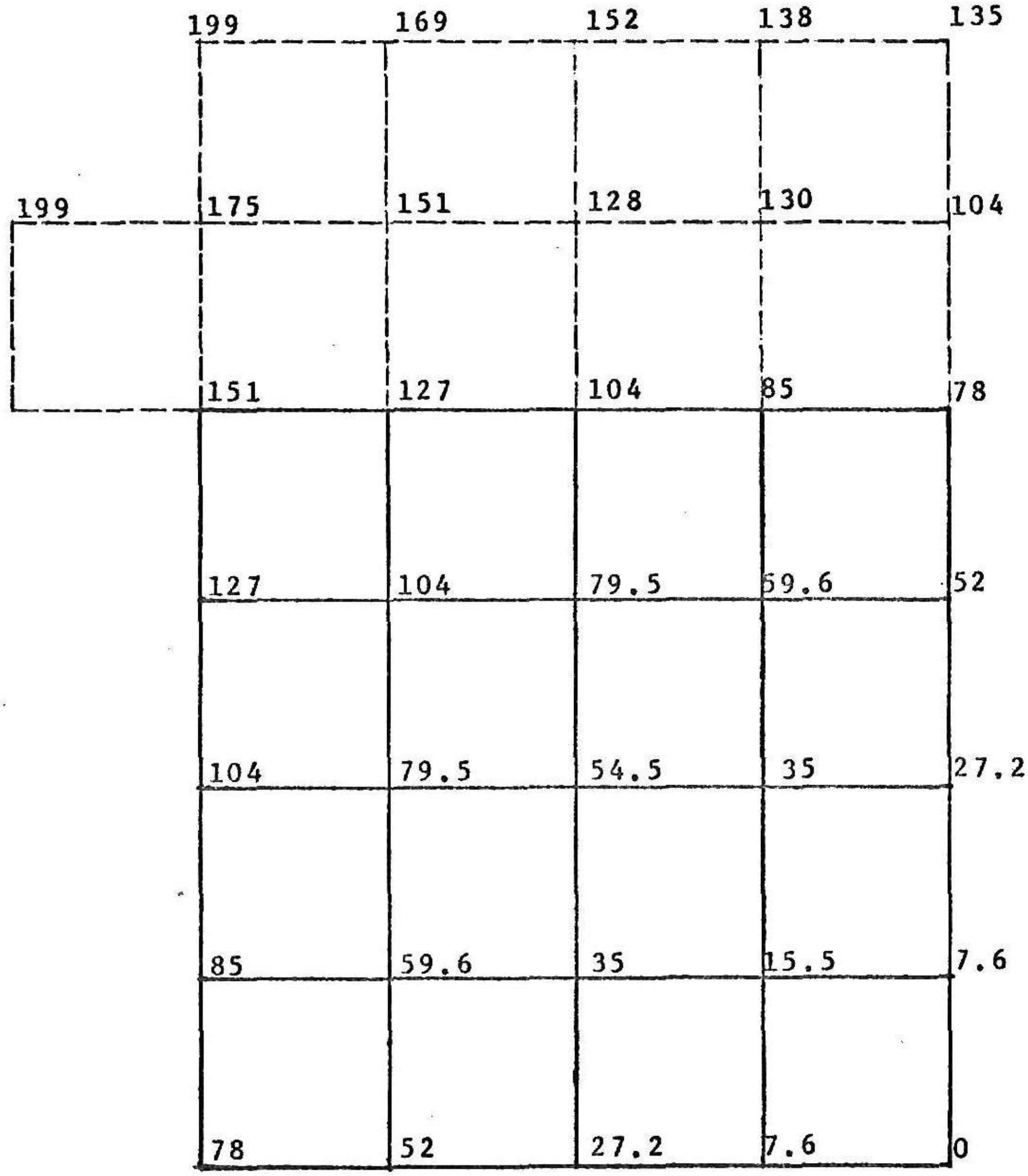


Fig.14. W's value of various point

W ( ) 10^{-5} Q

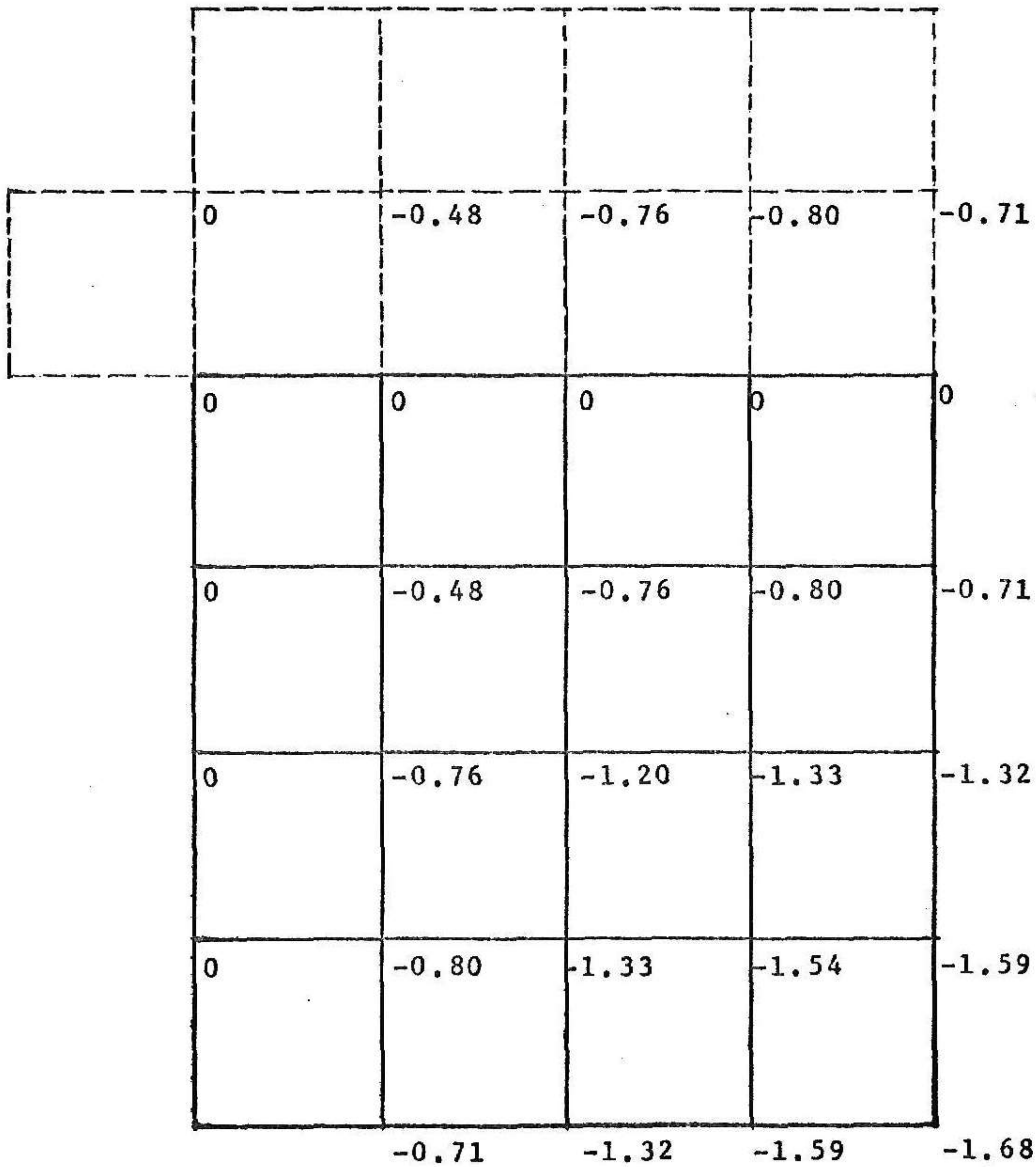


Fig. 14. F's value of various point

F ( ) 10^{-5} Q

0	0	0	0	0
1.6	2.4	0.8	1.3	1.92
-6.4	-7.4	-9.6	-8.3	-8.3
-19.2	-19.7	-17.1	-18.6	-19.2
-22.4	-24.4	-25.2	-25.2	-24.8

Coefficients of M_x in each point

$M_x = (\quad) 10^{-4} Z a^2$ in.-lb/ in.

0	17.6	29.2	31	28
0	-3.8	-6.2	-5.18	-1.9
0	-4.6	-6.0	-6.1	-6.5
0	-2.5	-2.7	-3.1	-3.5
0	1.92	0.2	-2.0	-3.4

Coefficients of N_x on each point

$N_x = (\quad) 10^{-4} Z a^2 / t$ lb/in.

Snow loading Condition:

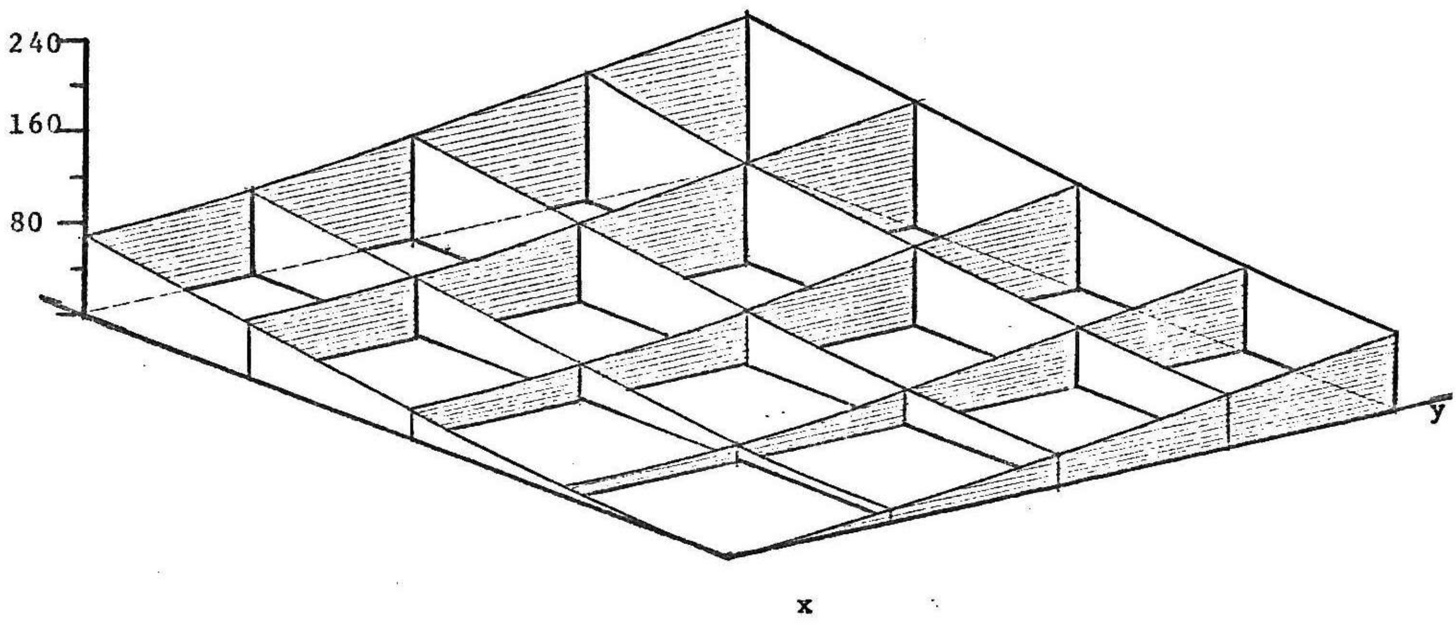


Fig. 16. Magnitude of Deflection Coef.

$w = (\quad)\ Z\ a^4 / D$ in.

Positive bending moment causes the bottom fiber of the shell tension

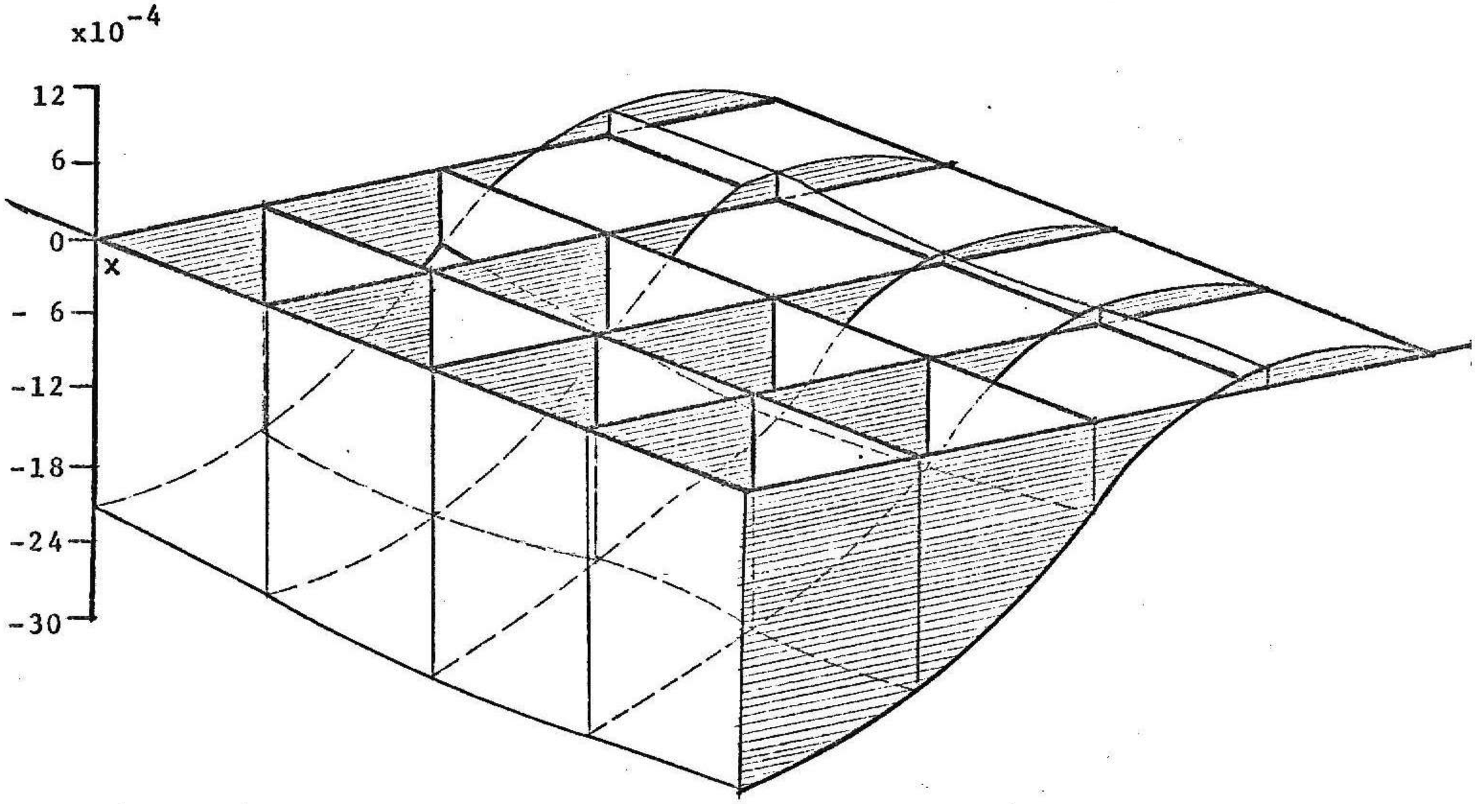


Fig. 17. Distribution of Bending Moment

M_x = ( ) Z a^2 in-lb / in.

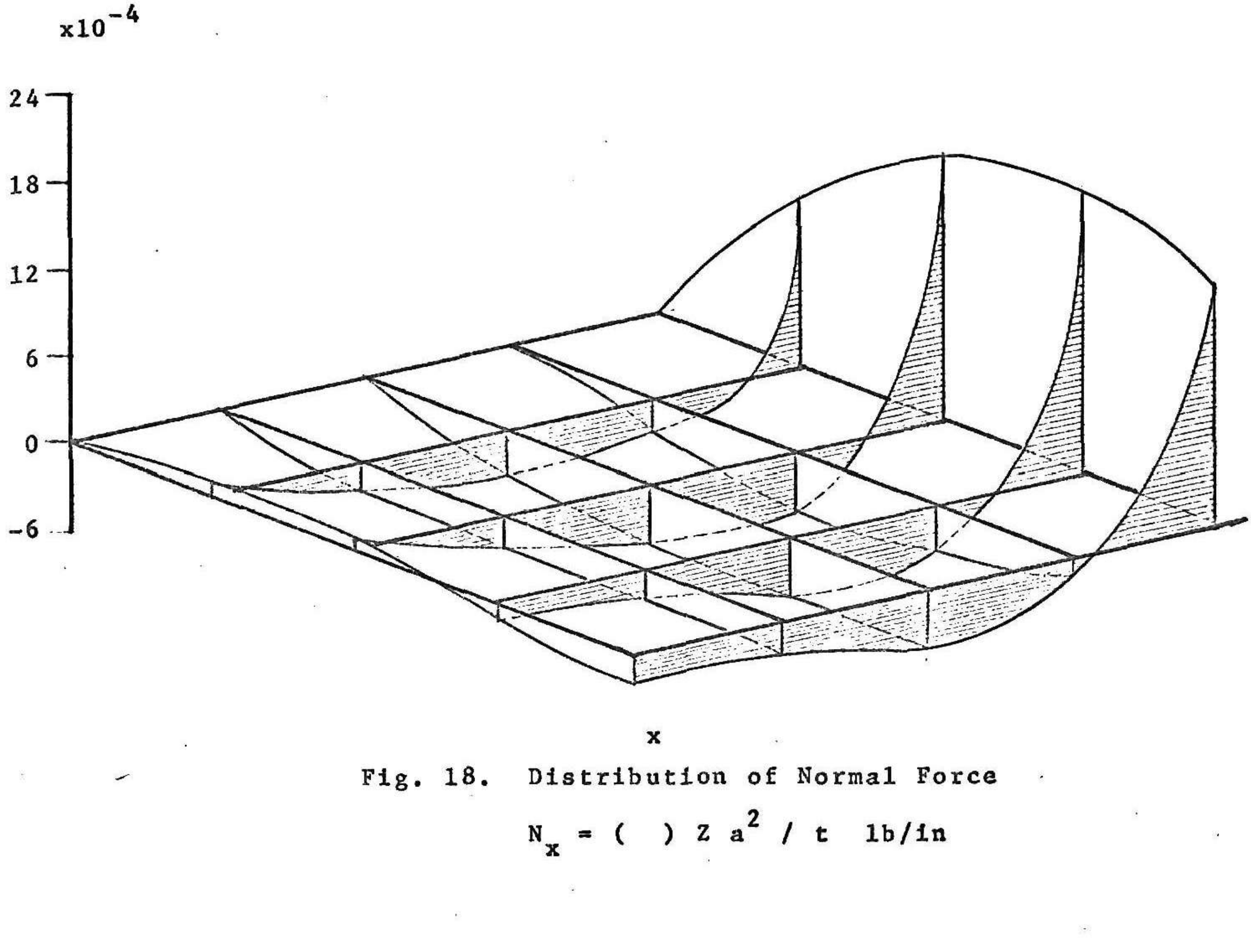


Fig. 18. Distribution of Normal Force

$N_x = (\quad) Z a^2 / t$ lb/in

EXPERIMENTAL ANALYSIS

1. Test Specimen and its material:

A 1/15th model was made 2'-6" square in plane with rise along the valley 8.0 in. The shell membrane, with an average thickness of 6.5/32 in. was supported on a 1.5"x1.5" column, details of the model are shown in Fig. 19. Material used for the shell membrane and the column is polyester resign. Because of the plastic behavior, it was hard to get the accurate values of modulus of elasticity. An approximate value of modulus of elasticity was obtained from the beam loading deflection test, details are shown as follows:

Beam dimension: length : 24 in. depth : 13/32 in. width : 12/16 in.

Concentrate load p were put at the center of the beam:

1. $p_1 = 1$ lb. $\delta_1 = 180/1000$ in.
2. $p_2 = 2$ lb. $\delta_2 = 80/1000$ in.

From the formula:

$$E = \frac{p\ L^3}{48\ E}$$

E is 4.0×10^5 lb/in^2 in average.

2. Test details and loading condition:

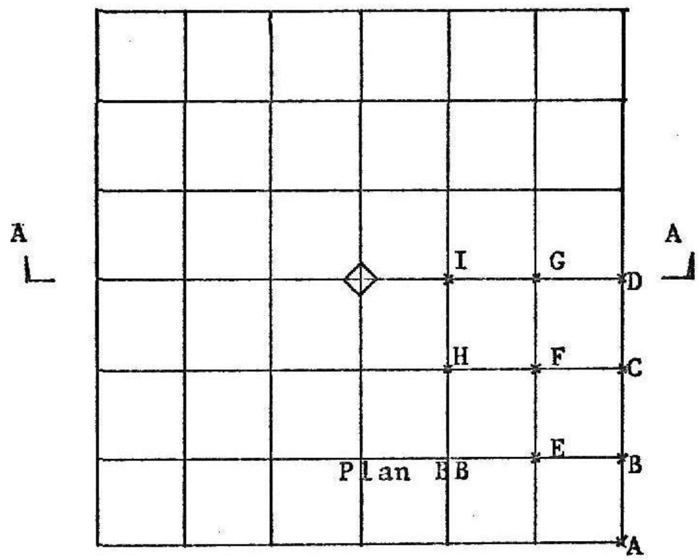
Snow loading was applied using sand and water and put on the top of the shell surface as shown in Fig. 20. In order to measure the vertical deflection, 0.0001 in. dial gauges were mounted under the shell surface as shown in Fig. 19. To minimize slippage of the indicator point along the shell face, the dial gauge were mounted in a normal direction to the surface. Four loading conditions were considered, these were:

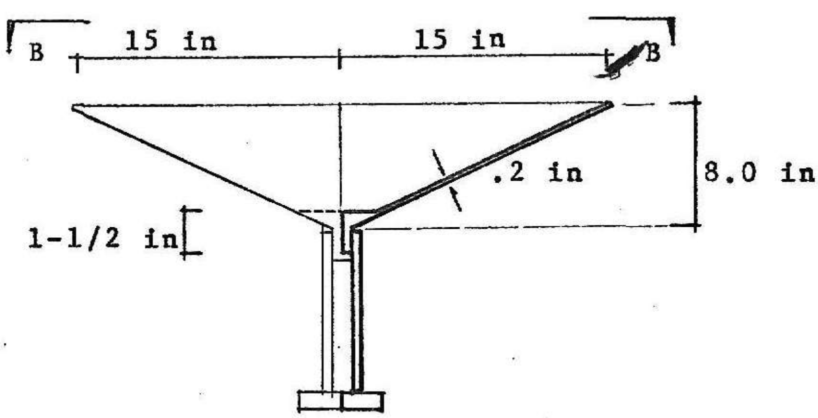
Sand: i. 1/2 height of the rise.

ii. full height of the shell rise.

Water: iii. 1/2 height of the rise.

iiii. full height of the shell rise.





Half Elevation Half Section AA

* mark is the point where the dial gauge were mounted

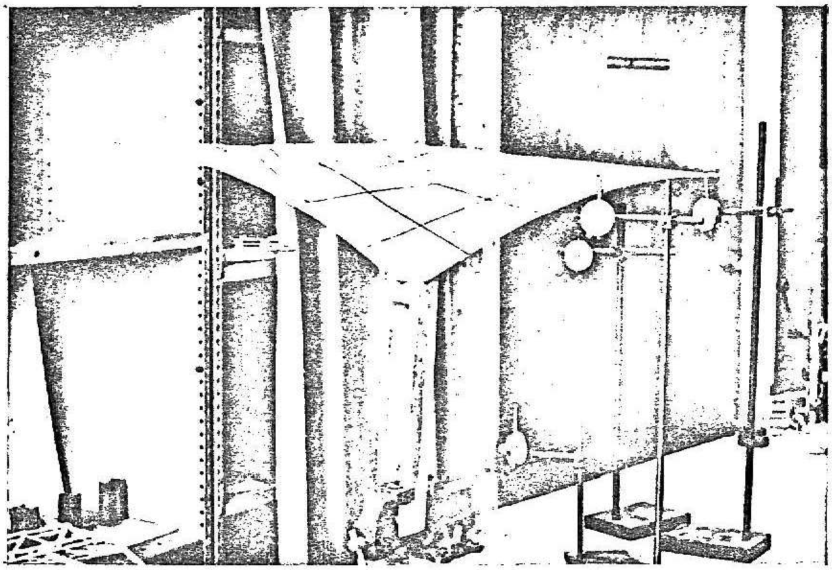


Fig. 19

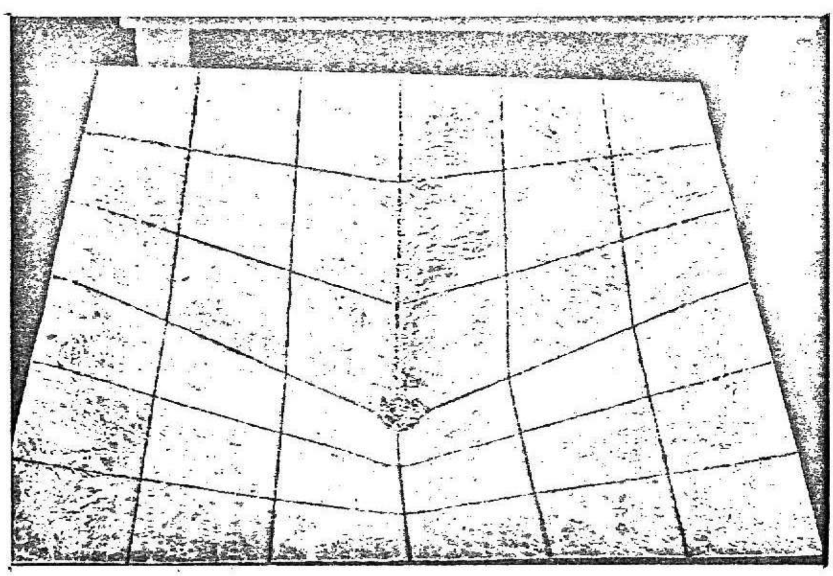


Fig. 20

Resulting deflections:

The deflection for the various loading condition are given in the following table:

Loading condition	Deflection in (10^{-4}) in. at point								
	A	B	C	D	E	F	G	H	I
i	0	-5	-10	-15	10	7	5	14	5
ii	60	43	37	30	28	17	20	10	8
iii	-3	-12	-19	-25	-5	-4	-2	20	12
iii	159	124	104	87	90	68	45	22	18

Positive denote a downward deflection

The deflection profile of the edges are in Table II. This shows the relationship between the loading and the deflection is in linear behavior. i.e. $\delta \propto p$ where: δ is the deflection and p is loading

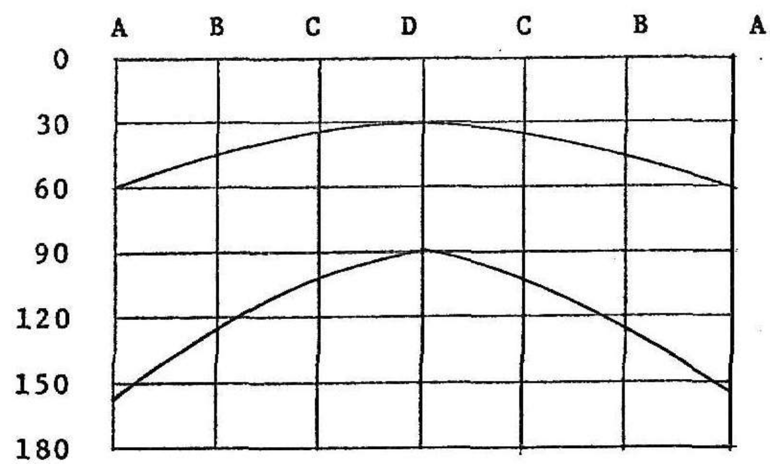


Table II deflection profile along the membrane edge
(for loading condition ii. iiii)

COMPARISON AND DISCUSSION

Brittle-Coating method and electrical strain gauge method were used to find the bending moment and normal force distribution, but both failed. The former failed because there was no suitable way to hang the loading under the shell surface. (model is too small). The latter because of the poor character of material for using the electrical strain gauges. (Expansion coefficient is large)

A comparison between experimental and theoretical deflection magnitudes are shown in the following table:

Indicator point	Experimental result in.	Theoretical result in.	Difference in.
A	0.06	0.0831	0.0231
B	0.043	0.0690	0.0260
C	0.037	0.0510	0.0140
D	0.030	0.0430	0.0130

The maximum difference between these two sets of result is large to 56%. (base on the experimental result.) There are several reasons which are explained as follows:

1. We can hardly get accurate values of Young's modulus of this model material because of its plastic behavior.
2. Non-uniform thickness, because of the construction method-painting layer by layer, it is hard to obtain the uniform thickness.

3. Consideration of only projectional force while the slope of the shell membrane is neglected.
4. The finite difference method is only a very approximate method for solving higher order differential equations.

CONCLUSIONS

The following conclusions can be made from the work completed in this report:

1. From Fig. 11 and 16, it is obviously shown that the maximum vertical deflection occurred at the very corner on the shell surface for both uniform and snow loading conditions.

2. Bending moment to be developed here is of very small magnitude which indicates a disposition towards membrane behavior. Fig. 12 and 17 show that the bending moment has the distinct positive and negative zones. The boundary between the zones is the zero bending-moment contour. This is almost a straight line, which is parallel to the coordinate axis and divides the shell membrane quadrant equally.

3. Maximum normal force N_x occured at the midlength along the free edge, as is shown in Fig. 13 and 18. It penetrates a short distance into the shell. The remainder is compressive force.

ACKNOWLEDGEMENT

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NOTATION

a,b = horizontal dimension of one quadrant of the shell.

c = maximum vertical dimension or rise of the shell.

$D = \frac{E\ t^3}{12\ (1-v)}$ Bending rigidity of shell.

E = Young's modulus.

G = shear modulus.

M_x, M_y = Bending moments per unit length perpendicular to x and y axes.

M_{xy} = twisting moment per unit length perpendicular to the x axes.

N_x, N_y = normal force per unit length parallel to the x and y axes.

$\overline{N}_x, \overline{N}_y$ = projected element forces equivalent to N_x and N_y, respectively.

N_{xy} = shearing force in the direction of y-axis per unit length

$\overline{N}_{xy}$ = projected element force equivalent to N_{xy}

Q = nondimensional loading term = $12\ \overline{Z}\ (a/t)^4/E$

t = thickness of the shell surface

u, v, w, = components of displacement parallel to the x, y, and z axes respective.

W = nondimensional displacement component parallel to the z-axis.

x, y, z, = cartesian shell coordinates.

X, Y, = x/a, y/a.

e_x, e_y = unit elongation in x and y direction.

r_{xy} = shear force strain

θ, ϕ = tangent angle with respect to the x and y axes

Z = vertical load.

APPENDIX

```
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      311 FORMAT(//,' TABLE II ',//)
      128 FORMAT(' THE MATRIX CT ',//)
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         3,//)
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          WRITE(3,104)
          DO 1C5 I=1,N
      105 WRITE(3,106) I,(A(I,J),J=1,N)
          DO 1C7 I=1,N
          DO 1C7 J=1,N
      107 B(I,J)=A(I,J)
          DO 108 I=1,N
      108 INDEX (I,1)=0
          II=0
      109 AMAX=-1.
          DO 110 I=1,N
          IF(INDEX(I,1)) 110,111,110
      111 DO 112 J=1,N
          IF (INDEX(J,1)) 112, 113,112
      113 TEMP=ABS(A(I,J))
          IF (TEMP-AMAX) 112,112,114
      114 IROW = I
          ICOL = J
          AMAX = TEMP
      112 CONTINUE
      110 CONTINUE
          IF(AMAX) 225, 115, 116
      116 INDEX(ICOL,1)=IROW
          IF (IROW-ICOL) 119,118,119
      119 DO 120 J=1,N
          TEMP=A(IROW,J)
          A(IROW,J)=A(ICOL,J)
      120 A(ICOL,J)=TEMP
          II=II+1
          INDEX (II,2)=ICOL
      118 PIVOT= A (ICOL,ICOL)
          A(ICOL,ICOL)=1.0
          PIVOT=1./PIVOT
          DO 121 J=1,N
      121 A(ICOL,J)=A(ICOL,J)*PIVOT
          DO 122 I=1,N
          IF (I-ICOL) 123,122,123
      123 TEMP=A(I,ICOL)
          A(I,ICOL)=0.0
          DO 124 J=1,N
      124 A(I,J)= A(I,J)-A(ICOL,J)*TEMP
      122 CONTINUE
          GO TO 109
```

```
          IROW=INDEX(ICOL,1)
          DO 126 I= 1,N
          TEMP = A(I,IROW)
          A(I,IROW) = A(I,ICOL)
      126 A(I,ICOL) = TEMP
          II= II-1
      225 IF(II) 125,127,125
      127 WRITE(3,128)
          DO 129 I = 1,N
      129 WRITE(3,106)I,(A(I,J),J = 1,N)
          DO 130 I= 1,N
          DO 130 J=1,N
          C(I,J)=0.
          DO 130 K= 1,N
      130 C(I,J)=C(I,J)+B(I,K)*A(K,J)
          WRITE (3,131)
          DO 132 I=1,N
      132 WRITE (3,106) I,(C(I,J),J=1,N)
          WRITE (3,444)
          READ(1,102)(CT(I),I=1,N)
          WRITE (3,102)(CT(I),I=1,N)
          DO 298 I=1,N
          X(I)=0.
      298 CONTINUE
          DO 299 I = 1,N
          DO 299 J = 1,N
          XX  =A(I,J)*CT(J)
          X(I)=X(I)+XX
      299 CONTINUE
          WRITE(3,999)(I,X(I),I=1,N)
          GO TO 134
      115 WRITE  (3,133)
      134 STOP
          END
```

MATHEMATICAL AND EXPERIMENTAL ANALYSIS
OF A HYPERBOLIC PARABOLOID

by

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It is the purpose of this report to investigate and present the structural characteristics of a hyperbolic paraboloid shell-umbrella type, by both experimental and theoretical analysis.

When designing shell structures of this shape, one considers the effects due to symmetrical surface load (uniform dead load), and also the effects due to the snow load (horizontal projection of the hyperbolic paraboloid). Both loading conditions have been attempted and are reported herein.

In the experimental method, a laboratory structural model, one-fifteenth actual size was made of plaster resign. Deflection magnitudes were measured using 0.001 dial gauges for several points under four snow loading conditions.

Mathematically, governing equations were written using membrane theory. These equations are higher power of differential equations and were solved by the method of finite difference, which reduced the problem to a simple task of solving a system of simultaneous linear algebraic equations. The completed procedure in mathematical analysis of this shell structure was as follows:

1. Calculation of deflection.
2. Calculation of bending moment.
3. Calculation of normal force.

A comparison between the numerical result and experimental analysis in terms of the deflection magnitudes have been outlined in this report.