PROJECTING FULL-TIME-EQUIVALENCE FOR KANSAS STATE UNIVERSITY USING REGRESSION MODELS WITH TIME SERIES ERRORS

by

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INTRODUCTION

Enrollment projections for colleges and universities have been rather straight-forward in the past because college enrollments were increasing at a very predictable rate; in fact, these increases were well described by a linear trend. Recently, however, this trend has become undependable; colleges have not only experienced decreasing rates in enrollment increases, many have experienced enrollment decreases. While many people were caught unprepared, others, realizing the effect of the population structure, had suspected, if not expected, these declines. Nationally, there is, now, great concern about the enrollment drop and since planning for the future needs of the universities is vital, it is important to be able to project enrollment as accurately as possible. It is necessary, then, to observe factors which will influence college enrollments. The problem at hand is projecting full-time equivalence (FTE) or enrollment for Kansas State University. But first, it is important to look at general trends.

What were the factors that caused or influenced the large increases in college enrollments ten to twenty years ago? Two factors stand out: the increased birth rate of post World War II and the great advances in technology during the last thirty years which called for an increasing proportion of the labor force to be more highly educated. The increased birth rate was only temporary. In fact, it began decreasing in 1958 and has begun to level off at a much lower level than that of the early 1950's. Thus, the college-age population (defined for our purposes as those between the ages of eighteen and twenty-four) is decreasing yearly. Much has been written concerning this and its effect on college enrollment. For instance, Carroll and Morrison (1976) discuss the relationship of the spacing of children, which is increasing as the birth rate decreases, to the college enrollment. Greater spacing allows families greater opportunities to afford to provide a college education for their children because parents would not have to strain under as heavy an economic burden by sending their children to college since there would be fewer of their children in college at any given time. This, one might argue, could counter the decline in the birth rate, at least to some extent.

The second major factor, the advances in technology, made high demands for college-educated persons and because of these demands there were very attractive rewards for persons with a college degree. Thus, a college education became financially beneficial and therefore desirable. In addition, the high demands by technology caused higher education to become more available. G. I. benefits encouraged veterans to enter the college population. Financial loans, grants and scholarships permitted a larger portion of the population in general to take advantage of the college experience. Draft status and employment status were two more factors influencing the decision to attend college. The list could go on but these all stem from the demand for college educated persons. However, it is believed that this demand will not continue to increase but has leveled off. So the real roots of the great increases in college enrollments ten to twenty years ago are in the population structure and the demands of technology.

Previous to these two phenomena, the opportunity to attend college was limited. And during their occurrence, the availability and demand made a college experience seem practically indispensable. Recently, however, many more college-age persons are opting not to attend college, compounding the effect of population decreases on the declines in enrollments and others are delaying the decision to attend. College administrators are, now, beginning to take these declines seriously. Less traditional programs of study are being employed to attract students. Universities are changing philosophies, going from a classical view of higher education to one that is more oriented to career or professional disciplines. Women are being encouraged to take up professional fields of study as pay-offs for women are rising. Non-traditional students (those older than the defined college-age population) are being encouraged to attend, and are in order to keep abreast in a fast moving, highly technological society. So part-time students are increasing. Thus, potential students' decisions to attend or not to attend college are influenced by the degree to which their aspirations would be served.

Since technology does not seem to be making increased demands for college graduates and since college enrollments have continued to increase, an over supply of college educated persons has occurred. Carroll and Morrison (1976) believe the over supply of college educated persons will persist so that the expected lifetime earnings of college and high school students will continue to narrow, so college enrollment in the future will be low and possibly decline further.

Bowen (1974)* is more optimistic, believing that technology will learn to use these people and develop new positions. In addition, Bowen believes higher education to be a growth industry if learning throughout one's life-time is a reality. He estimates a 200% increase by the year 2000.

More specifically, though, is the concern for enrollment at Kansas State University. Kansas State has not experienced the declines other universities have suffered. It was only last year, that is 1977-1978, that full-time-equivalence for both semesters first dropped from the same

*from Carroll and Morrison (1976)

semester a year before. Both KSU enrollment and FTE (1958-1977) are presented in Table I. Factors in a prediction model should explain enrollment increases of Kansas State University. One of these factors is college-age population for Kansas which is presented in Table II. The college-age population was obtained by linear interpolation of the 1960 and 1970 Census of Kansas since yearly figures were not available. For instance, in 1960 there were 44,545 eight-year-olds in Kansas while in 1970 there were 43,870 eighteen-year-olds in Kansas. In ten years this group dropped 675 for various reasons. It was assumed that the yearly rate of decrease was constant over those ten years and continued to be constant. Thus, it was assumed that there were 43,803 nineteen-year-olds in 1971. It can be seen that potential students (college-age population) is decreasing. Previously college-age population increases were reflected in KSU enrollment increases. If enrollment continues to follow the trend of the population, decreases in enrollment can be expected. As of the present though, both FTE and enrollment have increased even while the college-age population has decreased. Again, FTE shows the first real decrease (an indicator of possible future decreases) during the 1977-1978 year. This along with the fact enrollment increased during this time indicates there has been an increase in part-time enrollment. So projections are not simply a function of the college-age population.

Projections of interest from previous research include those of Flora, Hoyt and Anderson. Flora (1976) took into consideration four factors: the changing age structure of the population of Kansas, differing enrollment rates by age and sex, differing patterns of attendance between Regents institutions and other post-secondary education and population shifts in key counties from which Regent institutions recruited. Using three series, Flora obtains estimates of KSU enrollment for 1980 of 19,712, 17,729, and 15,913.

TABLE I

KSU Enrollment and FTE - 1958-1977

	Enrollment		FTE	
	Fall	Spring	Fall	Spring
1958-59	6947	6270	7265	6752
1959-60	6935	6367	7337	6693
1960-61	7265	6982	7896	7326
1961-62	7607	7244	8389	7710
1962-63	8477	8006	9068	8268
1963-64	8652	8286	8932	8441
1964-65	9634	9168	9703	9006
1965-66	10681	10035	10519	9979
1966-67	11285	10446	11231	10399
1967-68	11755	11073	11579	11042
1968-69	12570	11818	12683	11678
1969-70	13149	12345	12943	12107
1970-71	13847	13055	13440	12678
1971-72	14789	13771	14661	13584
1972-73	15158	14223	14922	14044
1973-74	15477	14770	15157	14124
1974-75	16422	15977	15712	14994
1975-76	17901	16798	16831	15822
1976-77	18220	17252	17171	16134
1977-78	19045	17632	17089	15813
1978-79	18293		16954	

TABLE II

College-Age Population of Kansas
(18-24 years of age)

1958	192039
1959	191364
1960	190689
1961	196181
1962	201827
1963	205279
1964	206696
1965	217849
1966	229823
1967	240889
1968	250538
1969	260754
1970	274258
1971	287566
1972	292782
1973	297488
1974	298788
1975	298125
1976	297801
1977	293720
1978	295531*
1979	294934*
1980	289102*

*Estimated by a linear trend, using births in 1960, 1961 and 1962 and the 1970 Census report of 10, 9 and 8 year-olds, respectively.

Hoyt (1977) used a cohort survival ratio to estimate freshman, sophomore, junior, senior and graduate students. The cohort survival ratio is the ratio of the number of students at a certain level one year to the number of students at the preceding level one year before. Thus, by knowing the number of freshmen at KSU one year Hoyt estimates the number of sophomores the next year knowing that on the average the sophomore class is a certain proportion of the previous year's freshman class. There are also cohort survival rates for sophomore to junior classes and junior to senior classes. Hoyt's projections are given in Table III.

TABLE III

Hoyt's Projections

	Projections		Actual	
Fall Semester	Enrollment	FTE	Enrollment	FTE
1977	18,494	16,653	19,045	17,089
1978	18,506	16,605	18,293	16,954
1979	18,370	16,408		
1980	18,384	16,385		

Anderson (1976) also used a cohort survival method to estimate high school seniors. Kansas, following the national trend, is experiencing a decline in the birth rate. And though an echo effect of the Post-war baby boom is expected, Anderson (1977) projects a continuing decrease of high school seniors (x) through 1990 with one two-year period of small increases from 1985-1987. Using total head count (Y) enrollment for all Kansas institutions of higher education from 1963 to 1976, Anderson used regression techniques to fit a linear trend through the data obtaining

$$Y = 3{,}935\ (x) + 64{,}208$$

Using this regression equation, Anderson projected enrollments of 123,233 for 1977 and 127,168 for 1978 (the corresponding actual figures for fall semesters were 118,862 and 124,237). The equation indicates an expected head count increase of 3,935 per year. Even Anderson believes this to be unlikely to continue mainly because the decline in high school seniors for the years 1976-1992 are incongruent with this. Thus, in order to project the number of students in all Kansas institutions of higher education, Anderson used regression techniques using six-year periods of high school students as the independent variable. Having two estimates of high school seniors, he obtains the following models:

$$Y = .7420X - 43,572 \qquad S_e = 6,487$$

$$Y = .7048X - 38,481 \qquad S_e = 3,960$$

where S_e is the standard error of the estimate.

Then to get the equations to the actual 1976 enrollment figures, a constant was added to each equation (15,609 and 8,777 respectively) resulting in these final equations:

$$Y = .7420X - 27,963$$

$$Y = .7048X - 30,104$$

Computing the portion of all Kansas students which went to KSU for the years 1963-1976, Anderson found K-State's share to decrease from 14.29 to 13.52%, then increase to 15.37%. Again using regression techniques with these percentages, Anderson found two models with the coefficient on X representing the increase in percentages of KSU's percentage of Kansas college students. Anderson estimated K-State's share to increase from .1362 to .14% per year. Thus Anderson projects K-State's head count enrollment to be 18,261 in 1977 (the corresponding actual figure was 19,045). The standard error for his estimate was 460. Table IV indicates Anderson's projections for K-State head counts.

TABLE IV

Anderson's Projections for KSU (head count)

1978	18,180
1979	18,048
1980	17,909

Though Anderson mentions the decrease of the proportion of FTE to head count, he doesn't attempt to project FTE.

Since linear regression using time itself is no longer appropriate, it was decided to attempt to construct a time series model to project Kansas State University's enrollment. Time series allows the forecaster to regress the number of students (or FTE) on various factors with a measure of variation taken into account; specifically, a time series auto-regressive process is added to the model.

MODELING FTE FOR KSU

Two methodologies could be considered in modeling enrollment. First, a model could be constructed to fit, and later predict, the number of students (or FTE) for all six state institutions of Kansas; these predictions could, then, be used to project enrollment (or FTE) for Kansas State University (or any of the other state institutions) by taking an appropriate proportion of the total which, of course, is a time series in itself. A measure of competition among the institutions would be a necessity. Since there is competition among the institutions for students because there are only a limited number of potential students this would seem an appropriate methodology if an appropriate measure of competition could be found. This would prevent gross overestimates or underestimates of the number of students at the six institutions which might occur if each institution made its own projections and these were combined.

The second procedure involves modeling enrollment for KSU separately based entirely on its own particular past trends. It is this procedure that is to be discussed and the models will be regression models with time series errors.

Type I Models

It was decided to model FTE rather than enrollment because as FTE seems to be leveling off, if not decreasing, enrollment has continued to increase indicating that there is a greater proportion of part-time students. It seems, then, that FTE may be a more appropriate and useful tool. While enrollment figures are used in determining the building needs of the University, FTE is used in determining the budget and the faculty size. As it can be seen in Table I, as FTE at KSU increases, the difference between first and second semesters tends to increase indicating, possibly, a proportional difference. By making the transformation, $Z_t = F_t/\pi_t$ where F_t represents FTE at the particular time t and π_t represents Kansas' college-age population at that same time t, it was found that Z_t increased linearly over time up to a certain point, leveled off and then increased again. Z_t is presented in Table V and graphically in Figure I. This being the case, projecting Z_t would permit a projection of F_t through projections of the college-age population of Kansas. It is believed that the increases of Z_t from 1958-65 were due to demands by technology which leveled off after 1965 at a level much higher than that of previous demands but these demands should not continue to grow demanding a larger proportion of the population. The increases of Z_t after 1973 are believed to have been due to factors such as returning veterans and high unemployment rates.

An economic factor considered is the ratio of annual incomes of high school graduates to those of college graduates. As this ratio increases, college enrollments should drop while they should increase as it decreases because as the ratio decreases, or as the difference between high school and college graduates widens, there would be greater incentive to attend college. Table VI provides the annual median incomes for both high school and college graduates as provided by Chuck Nelson of the Census Bureau in Washington D. C., along with this ratio. The ratio should be a fluctuating process and so future values should not be difficult to estimate for short periods into the future. A stationary auto-regressive process describing the series is given by:

$$R_t = 0.68808 + 0.3985(R_{t-1} - 0.68808) - 0.3101(R_{t-2} - 0.68808) + e_t$$

where R_t is the ratio of incomes of high school graduates to college graduates.

TABLE V

$Z_t = F_t/\pi_t$ for 1958-77

Semester	Z_t
Fall, 1958	0.0378
Spring, 1959	0.0352
Fall, 1959	0.0383
Spring, 1960	0.0350
Fall, 1960	0.0414
Spring, 1961	0.0384
Fall, 1961	0.0428
Spring, 1962	0.0393
Fall, 1962	0.0449
Spring, 1963	0.0410
Fall, 1963	0.0435
Spring, 1964	0.0411
Fall, 1964	0.0469
Spring, 1965	0.0436
Fall, 1965	0.0483
Spring, 1966	0.0458
Fall, 1966	0.0489
Spring, 1967	0.0452
Fall, 1967	0.0481
Spring, 1968	0.0458
Fall, 1968	0.0506
Spring, 1969	0.0466
Fall, 1969	0.0496
Spring, 1970	0.0464
Fall, 1970	0.0490
Spring, 1971	0.0462
Fall, 1971	0.0510
Spring, 1972	0.0472
Fall, 1972	0.0510
Spring, 1973	0.0480
Fall, 1973	0.0510
Spring, 1974	0.0475
Fall, 1974	0.0526
Spring, 1975	0.0502
Fall, 1975	0.0565
Spring, 1976	0.0531
Fall, 1976	0.0577
Spring, 1977	0.0542
Fall, 1977	0.0582
Spring, 1978	0.0538

Figure I

$Z_t = F_t/\pi_t$ vs. Semester 1958-77

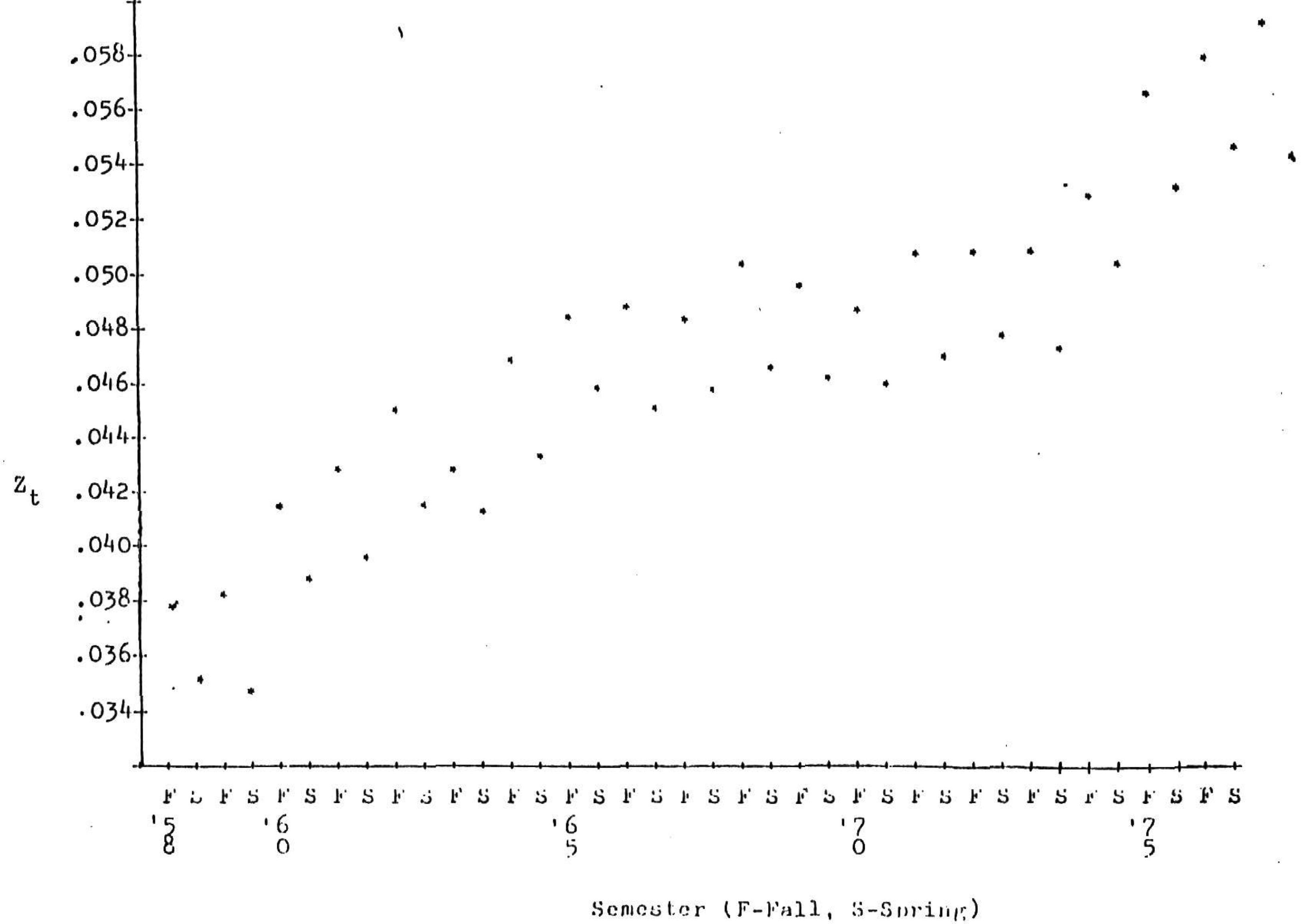

TABLE VI

Median Annual Incomes of Household Heads by Education and Income Ratio of High School Graduates to College Graduates

	College Graduates	High School Graduates	Ratio (hs/c)
1976	$23134	$15886	.68660
1975	$21961	$14729	.67069
1974	$20124	$13941	.69275
1973	$19367	$13188	.68095
1972	$18252	$12163	.66639
1971	$16726	$11269	.67374
1970	$15780	$10861	.68828
1969	$14654	$11100	.70902
1968	$13551	$ 9520	.70253
1967	$12672	$ 8822	.69618
1966	$11603	$ 8045	.69336
1965	na	na	.68425*
1964	$10599	$ 7157	.67525
1963	$ 9709	$ 6845	.70502
1962	na	na	.69625*
1961	$ 9264	$ 6302	.68027
1960	na	na	.68538*
1959	na	na	.69048*
1958	$ 8143	$ 5667	.69559

na indicates that this information was not available

* indicates that the ratio was interpolated since large changes are considered to be unlikely from year to year and should follow the existing trend.

A related factor which could be considered is suggested by Dresch (1974). Dresch believes "intersected shifts in the composition of economic activity" to be an important factor to college enrollments. His argument considers the effect of the great increases of college educated persons of the 1960's during which the proportion of college educated persons was forced up disproportionately. He believes that the economy will permit a certain proportion of the labor force to be college educated though he realizes that technology will allow varying proportions from time to time. However, as more persons were taking advantage of a college education during the 1960's, the supply of college educated persons caught and surpassed the demand. Thus, the relative incomes of college educated persons to the non-college educated persons decreased, reducing the incentive to attend college. A measure that he suggests, then, is the proportion of the persons entering the labor force each year which are college educated. If this proportion is high, then it would seem as though a college education has its rewards but the future may bring a decrease in enrollments due to saturation. As saturation occurs, the proportion of the new entrants to the labor force which are college graduates should decrease (perhaps slowly because these persons who were unable to find positions in their studied fields would still get into the labor force somewhere, in all probability, but their lack of success in getting started in their professional careers would decrease the incentive to attend college). Presented in Table VII are these proportions provided by Dresch (1974) with estimates to the year 2005.

One more variable to be considered is the unemployment rate. As the unemployment rate increases, it would seem as though college enrollments would increase as people attempt to gain new skills for a competitive job market. Unemployment rates over the last twenty-four years are presented

TABLE VII

The Proportion of New Entrants to the Labor Force
Having a College Education 1958-80

Year	Percent
1958	10.8
1959	11.8
1960	12.8*
1961	13.8
1962	14.8
1963	15.8
1964	16.8
1965	17.8*
1966	18.9
1967	20.0
1968	21.1
1969	22.3
1970	23.5*
1971	24.6
1972	25.7
1973	26.8
1974	28.0
1975	29.2*
1976	29.9
1977	30.7
1978	31.4
1979	32.1
1980	32.9**

* Dresch's values; others are linear interpolations

** Dresch's projection for 1980

in Table VIII. One problem with using unemployment rates is that they are very difficult to predict which is necessary for a predictive model. George Johnson of the Council of Economic Advisors in Washington, D. C., provides estimates of the unemployment rate on a national scale for short term forecasts. Under expected conditions, he estimates unemployment rates to be 6.0%, 6.0% and 6.0% for 1978, 1979 and 1980, respectively, but under recessive conditions, he believes 6.0%, 8.0% and 7.0% to be more appropriate. However, the goals of the present administration is to lower the rate to 4.0 percent by 1983, so under conditions allowing this, 6.0%, 5.0%, 5.0% and 5.0% could be used for 1978-81. For the purpose of modeling a constant unemployment rate of 6.0% was used for 1978 through 1980.

Using these factors, Type I models take the following form:

$$F_t = \sum_{j=1}^{2} \alpha_j \delta_{tj} + B_1(\tau_t \pi_t) + B_2(A_t \pi_t) + B_3(\rho_t \pi_t) + B_4(\nu_t \pi_t) + v_t ,$$

where

F_t is the FTE for KSU at time t,

α_1 is the intercept if t is odd; that is, if the semester is fall,

α_2 is the intercept if t is even or the semester is spring,

δ_{tj} allows for different intercepts for the different semesters (Recall that π_t is the college-age population of Kansas),

$\delta_{tj} = \pi_t$ if both t and j are even

$= \pi_t$ if both t and j are odd

$= 0$ otherwise,

τ_t is a segmented polynomial approximating Z_t between 1958 and 1965. This reaction of Z_t is believed to be the result of increased demands by technology for college educated persons and it is believed that that demand will not continue to increase but will level off. Thus, the segmented polynomial approximating the technological demand is defined as:

$$\tau_t = (t + 1)/2 \text{ for } t \text{ odd and } t < 16$$
$$= t/2 \text{ for } t \text{ even and } t < 16$$
$$= 8 \text{ for } t \geq 16 ,$$

A_t is the accumulation of proportions of persons entering the labor force with a college education for the four years immediately preceding time t,

ρ_t is the accumulation of the ratio of annual incomes of high school graduates to those of college graduates for the ten years previous to time t excluding the immediately preceding year,

ν_t is the accumulation of unemployment rates for the four years immediately preceding time t,

v_t is a stationary auto-regressive model on the errors of the regression part of the model.

The estimates of the parameters are obtained by first regressing the FTE on the independent variables by least squares and obtaining the residuals. A stationary auto-regressive process is then fit to these residuals.

After the auto-regressive process part of the model is determined, the parameters of the linear model part of the model are then re-estimated. It is assumed the e_t's in the auto-regressive part of the model are distributed NID $(0,\sigma^2)$.

Estimates were obtained through implementation of the Statistical Analysis Systems. Fitting this model produced the following with a standard deviation of 209. The estimate of the standard deviation is based on the limiting distribution. Variances increase with projected semesters.

TABLE VIII

Seasonally Adjusted National Unemployment Rates (1954-1977)

1954	5.6
1955	4.4
1956	4.2
1957	4.3
1958	6.8
1959	5.5
1960	5.5
1961	6.7
1962	5.6
1963	5.7
1964	5.2
1965	4.5
1966	3.8
1967	3.9
1968	3.6
1969	3.5
1970	4.9
1971	5.9
1972	5.6
1973	4.9
1974	5.6
1975	8.5
1976	7.7
1977	7.0

These were also obtained by the SAS program, Autoreg, in order to construct confidence intervals for projections.

$$F_t = 0.21076\delta_{t1} + 0.20756\delta_{t2} + 0.000794(\tau_t\pi_t) + 0.000125(A_t\pi_t) - 0.029207(\rho_t\pi_t) + 0.000219(\nu_t\pi_t) + v_t ,$$

where $v_t = 0.4782v_{t-1} - 0.3002v_{t-4} + e_t$.

Model IA

Projections to Fall, 1980, are presented in Table IX.

TABLE IX

FTE Projections for KSU using Model IA

Semester	FTE	Actual	95% Confidence Limits
Fall, 1977	17011*	17089	16472-17550
Spring, 1978	15939*	15813	15304-16574
Fall, 1978	17022	16954	16557-17487
Spring, 1979	16062		15515-16609
Fall, 1979	17076		16466-17686
Spring, 1980	16260		15625-16895
Fall, 1980	17108		16308-17908

*these projections were based on Fall, 1958, to Spring, 1977 while the other projections were based on Fall, 1958, to Spring, 1978, using the actual FTE figures for Fall, 1977, and Spring, 1978. The model used to project Fall, 1977, and Spring, 1978, varied slightly, therefore, from Model IA.

A similar model with one modification was then constructed. In this case the income ratio was separated into two distinct parts. The first part was the accumulation of ratios for the three years prior to the immediately preceding year to time t and the second part was the accumulation of the ratios of the six years prior to these. Thus, the second model obtained is as follows having a standard deviation of 212:

$$F_t = 0.174586\delta_{t1} + 0.171366\delta_{t2} + 0.000809(\tau_t\pi_t) + 0.000127(A_t\pi_t) - 0.030171(\rho 1_t\pi_t) - 0.019941(\rho 2_t\pi_t) + 0.000190(\nu_t\pi_t) + v_t,$$

where $v_t = 0.489741v_{t-1} - 0.295468v_{t-4} + e_t$. Model IB

The reason for dividing the accumulation of income ratios into two parts was to account for two factors: the effect it may have on a potential student's decision to attend college and the effect it may have on a potential student's parents' desire for their son or daughter to attend college.

Based on this model, projections to Fall, 1980, are presented in Table X.

TABLE X

FTE Projections for KSU Using Model IB

Semester	FTE	Actual	95% Confidence Limits
Fall, 1977	16731*	17089	15902-17560
Spring, 1978	15657*	15813	14751-16563
Fall, 1978	16940	16954	16399-17481
Spring, 1979	15958		15307-16609
Fall, 1979	16933		16171-17695
Spring, 1980	16096		15273-16919
Fall, 1980	16860		15760-17960

*these projections, again, were based only on the Fall, 1958, to Spring, 1977, semesters while others included the actual Fall, 1977, and Spring, 1978, semesters.

A second modification was made to the original model. This was the exclusion of unemployment rates because predicted future rates were little more than educated guesses. The model obtained had a standard deviation of 239 and is presented as follows:

$$F_t = 0.207318\delta_{t1} + 0.204061\delta_{t2} + 0.000412(\tau_t\pi_t) + 0.000161(A_t\pi_t) - 0.027966(\rho_t\pi_t) + v_t ,$$

where $v_t = 0.82376v_{t-1} - 0.23785v_{t-2} + 0.14224v_{t-3} - 0.39813v_{t-4} + 0.18960v_{t-5} + e_t .$

Model IC

Projections using Model IC to Fall, 1980, are presented in Table XI.

TABLE XI

FTE Projections for KSU Using Model IC

Semester	FTE	Actual	95% Confidence Limits
Fall, 1977	16677*	17089	16169-17185
Spring, 1978	15607*	15813	14921-16293
Fall, 1978	16893	16954	16401-17385
Spring, 1979	15879		15217-16541
Fall, 1979	17042		16270-17814
Spring, 1980	16171		15344-16998
Fall, 1980	17171		16183-18159

*these projections were based only on Fall, 1958, to Spring, 1977, semesters, while others included the actual Fall, 1977, and Spring, 1978, semesters.

Type II Models

Two final models were examined without A_t. Since the proportion of new entrants to the labor force each year having a college education is dependent upon the college enrollment, then projections of this factor used in the previous models are related to college enrollments. Even though these projectionswere obtained from an independent model, it might be apprpriate to construct a model excluding this variable. This is done in the following two models. Also, a measure of the difference between fall and spring semesters is substituted for the two separate intercepts. And added to the model is the college-age population broken down into five distinct parts: high school seniors in Kansas the school year before, two years before, three years before, four years before and the remainder. Anderson (1974) provides the number of high school seniors each year and these are presented in Table XII.

Thus, these final two models follow the form:

$$F_t = B_1(\delta_t \pi_t) + B_2(\tau_t \pi_t) + B_3(\nu_t \pi_t) + B_4(\rho_t \pi_t) + B_5 X_{1t} + B_6 X_{2t} + B_7 X_{3t} + B_8 X_{4t} + B_9 \varepsilon_t + v_t ,$$

where

δ_t = -1 for fall semesters (t is odd)

= +1 for spring semesters (t is even),

X_{1t} is the number of high school seniors the immediately previous school year,

X_{2t} is the number of high school seniors two years before,

X_{3t} is the number of high school seniors three years before,

X_{4t} is the number of high school seniors four years before,

ε_t is the remaining part of the college age population in Kansas

$$\varepsilon_t = \pi_t - X_{1t} - X_{2t} - X_{3t} - X_{4t} ,$$

v_t is an auto-regressive process. However, in this case, v_t depends on t and so the auto-regressive process of this model is non-stationary. That is, the auto-regressive process used will vary depending upon the semester. There will be less chance of error in prediction if the prediction is for a spring semester with the previous fall semester known than in prediction of a fall semester with the immediately previous spring semester known which is

TABLE XII

High School Seniors in Kansas (1951-1981)

1951	18537
1952	19052
1953	19383
1954	19616
1955	20641
1956	20401
1957	21017
1958	21883
1959	23970
1960	25915
1961	25034
1962	24452
1963	28984
1964	33780
1965	33122
1966	32096
1967	32876
1968	34254
1969	33861
1970	34720
1971	36198
1972	35634
1973	35397
1974	34719
1975	34253
1976	35072
1977	34660
1978	34660
1979	32624
1980	30876
1981	29337

reasonable because spring semesters usually show a decline in FTE from the immediately previous fall semester while fall semesters are less predictable.

The resulting model is:

$$F_t = -0.001613(\delta_t\pi_t) + 0.002086(\tau_t\pi_t) + 0.000289(\nu_t\pi_t) - 0.060918(\rho_t\pi_t) + 0.359087\chi_{1t} + 0.367986\chi_{2t} + 0.326125\chi_{3t} + 0.456049\chi_{4t} + 0.433826\varepsilon_t + v_t,$$

$$\text{where } v_t = 0.574486v_{t-1} - 0.403543v_{t-4} + e_t \text{ for } t \text{ odd}$$

$$= 0.761632v_{t-1} - 0.146313v_{t-4} + e_t \text{ for } t \text{ even} .$$

Model IIA

For this model the standard deviation for projecting one semester ahead is 298 if that semester were a fall semester and 126 if it were spring. In constructing confidence intervals, it is necessary to find v_t. For the conditions where v_t depends upon t such that:

$$v_t = Av_{t-1} - Bv_{t-4} + e_t \quad \text{for } t \text{ odd}$$

$$= Cv_{t-1} - Dv_{t-4} + e_t \quad \text{for } t \text{ even},$$

and supposing n is even, as there are 40 observations in the present case, then,

$$v_{n+1} = Av_n - Bv_{n-3} + e_{n+1},$$

$$v_{n+2} = Cv_{n+1} - Dv_{n-2} + e_{n+2}$$

$$= C(Av_n - Bv_{n-3} + e_{n+1}) - Dv_{n-2} + e_{n+2}$$

$$= ACv_n - Dv_{n-2} - BCv_{37} + e_{n+2} + Ce_{n+1},$$

$$v_{n+3} = Av_{n+2} - Bv_{n-1} + e_{n+3}$$

$$= A(Cv_{n+1} - Dv_{n-2} + e_{n+2}) - Bv_{n-1} + e_{n+3}$$

$$= AC(Av_n - Bv_{n-3} + e_{n+1}) - Bv_{n-1} - ADv_{n-2} + e_{n+3} + Ae_{n+2}$$

$$= A^2Cv_n - Bv_{n-1} - ADv_{n-2} + e_{n+3} + Ae_{n+2} + ACe_{n+1} ,$$

$$
\begin{aligned}
v_{n+4} &= Cv_{n+3} - Dv_n + e_{n+4} \\
&= C(Av_{n+2} - Bv_{n-1} + e_{n+3}) - Dv_n + e_{n+4} \\
&= CA(Cv_{n+1} - Dv_{n-2} + e_{n+2}) - Dv_n - BCv_{n-1} + e_{n+4} + Ce_{n+3} \\
&= C^2A(Av_n - Bv_{n-3} + e_{n+1}) - Dv_n - BCv_{n-1} - ACDv_{n-2} + e_{n+4} \\
&\quad + Ce_{n+3} + ACe_{n+2} \\
&= (A^2C^2-D)v_n - BCv_{n-1} - ACDv_{n-2} - ABC^2v_{n-3} + e_{n+4} + Ce_{n+3} \\
&\quad + ACe_{n+2} + AC^2e_{n+1} ,
\end{aligned}
$$

and

$$
\begin{aligned}
v_{n+5} &= Av_{n+4} - Bv_{n+1} + e_{n+5} \\
&= A(Cv_{n+3} - Dv_n + e_{n+4}) - D(Av_n - Bv_{n-3} + e_{n+1}) + e_{n+5} \\
&= AC(Av_{n+2} - Bv_{n-1} + e_{n+3}) - ADv_n - ABv_n + B^2v_{n-3} \\
&\quad + Ae_{n+4} + e_{n+5} - Be_{n+1} \\
&= A^2C(Cv_{n+1} - Dv_{n-2} + e_{n+2}) - A(B+D)v_n - ABCv_{n-1} + B^2v_{n-3} \\
&\quad + e_{n+5} + Ae_{n+4} + ACe_{n+3} - Be_{n+1} \\
&= A^2C^2(Av_n - Bv_{n-3} + e_{n+1}) - A(B+D)v_n - ABCv_{n-1} - A^2CDv_{n-2} \\
&\quad + B^2v_{n-3} + e_{n+5} + Ae_{n+4} + ACe_{n+3} + A^2Ce_{n+2} - Be_{n+1} \\
&= [A^3C^2-A(B+D)]v_n - ABCv_{n-1} - A^2CDv_{n-2} - B(A^2C^2-B)v_{n-3} \\
&\quad + e_{n+5} + Ae_{n+4} + ACe_{n+3} + A^2Ce_{n+2} + (A^2C^2-B)e_{n+1} .
\end{aligned}
$$

The variances for constructing confidence intervals can be found for the i^{th} projection by finding

$$\sum_{j=1}^{i} \alpha_{n+j}^2 \sigma_{j\,(\mathrm{mod}\ 2)}^2$$

where σ_1^2 is the variance for fall semesters and σ_0^2 is the variance for spring semesters; the α's are the coefficients to their respective e_t's.

Projections to Fall, 1980, using this model are presented in Table XIII.

TABLE XIII

FTE Projections for KSU Using Model IIA

Semester	FTE	Actual	95% Confidence Limits
Fall, 1977	16998*	17089	16413-17583
Spring, 1978	15963*	15813	15518-16408
Fall, 1978	16903	16954	16319-17487
Spring, 1979	15938		15493-16383
Fall, 1979	16859		16193-17525
Spring, 1980	15984		15445-16523
Fall, 1980	17032		16364-17700

*these projections were based on Fall, 1958, to Spring, 1977, semesters while others included the actual Fall, 1977, and Spring, 1978, semesters.

The final model follows the same basic form as the previous one but excludes unemployment rates. The model is as follows:

$$F_t = -0.001541(\delta_t\pi_t) + 0.001843(\tau_t\pi_t) - 0.109826(\rho_t\pi_t) + 0.712758\chi_{1t} + 0.648620\chi_{2t} + 0.595548\chi_{3t} + 0.741488\chi_{4t} + 0.758968\varepsilon_t + v_t ,$$

where $v_t = 0.681807v_{t-1} - 0.720811v_{t-4} + e_t$ for t odd

$= 0.784334v_{t-1} - 0.142368v_{t-4} + e_t$ for t even .

Model IIB

The standard deviation for a projection one semester ahead for this model is 326 if the projection were for a fall semester and 122 if it were for spring.

Projections, again, to Fall, 1980, are presented in Table XIV.

TABLE XIV

FTE Projections for KSU Using Model IIB

Semester	FTE	Actual	95% Confidence Limits
Fall, 1977	16883*	17089	16244-17522
Spring, 1978	15723*	15813	15168-16278
Fall, 1978	16466	16954	15827-17105
Spring, 1979	15640		15083-16197
Fall, 1979	16516		15773-17259
Spring, 1980	15794		15163-16425
Fall, 1980	17389		16591-18187

*these projections were based on Fall, 1958, to Spring, 1977, semesters while others included the actual Fall, 1977, and Spring, 1978, semesters' FTE.

While comparisons with Anderson cannot be made directly, Table XV indicates that these five models provide a better fit to the data because the estimated variances of the models and one-step-ahead projections are smaller.

TABLE XV

Variances of Models for One-Step-Ahead Projections

Model	Variance	Standard Deviation
Anderson	211600	460
IA	43681	209
IB	44521	211
IC	57121	239
IIA	88804	298
IIB	106276	326

Also, Table XVI presents projections for all five models along with those of Hoyt and Anderson. Overall, these five models projected FTE for Fall, 1977 more closely than Anderson's projected enrollment for Fall, 1977 and more closely than Hoyt projected either enrollment or FTE for that semester. While Hoyt projected enrollment for Fall, 1978 to increase over that of Fall, 1977, Anderson projected a decrease correctly. Hoyt did, however, correctly project a decrease in FTE just as all five of theses models did. Only Model IIB projected a large decrease for Fall, 1978. Overall, though, these models seem to indicate that FTE

TABLE XVI

Comparisons of Projections

	FTE							Enrollment		
	Actual	Models					Hoyt	Actual	Anderson	Hoyt
		IA	IB	IC	IIA	IIB				
Fall, 1977	17089	17011	16731	16677	16998	16883	16653	19045	18261	18494
Spring, 1978	15813	15939	15657	15607	15963	15723	——	17632	——	——
Fall, 1978	16954	17022	16940	16893	16903	16466	16605	18293	18180	18506
Spring, 1979		16062	15958	15879	15938	15640	——		——	——
Fall, 1979		17076	16933	17042	16859	16516	16408		18048	18370
Spring, 1980		16260	16096	16171	15984	15794	——		——	——
Fall, 1980		17109	16860	17171	17032	17389	16385		17909	18384

for KSU can be expected to remain fairly constant over the next few years or, perhaps, decline slightly.

DISCUSSION

It would be most useful if the proportion of college-age students who attend one of the six state institutions could be projected. This would permit an estimate of the enrollment for these institutions from the college-age population. Then, if the proportion of those going to KSU could be projected, an estimate of K-State's enrollment would be available. And then, if the proportion of those who would attend KSU on a part-time basis could be projected, FTE for K-State would essentially be estimated. But each of those proportions, themselves, are time series variables, not constants, because they will vary from year to year, and not linearly. Perhaps each could be projected if models could be constructed involving variables influencing these proportions. While the proportion of students at all six state institutions who attend KSU is not independent of the proportion who attend any of the other five state institutions, the estimated proportion attending KSU would be that proportion not attending one of the other five. Thus, if the proportion of students attending all six state institutions who attend each of the institutions could be determined, projections could be made for each institution without gross over or underestimates of the enrollment or FTE for all six state schools. This procedure <u>may</u> be more desireable from the standpoint of being more broadly applicable and perhaps <u>may</u> provide a better fit and more reliability.

It also might be useful to split the income ratio into two groups by sex. Presently, the financial benefits of a college education are increasing for females but are decreasing for males. This could be a factor which has helped K-State's enrollment up in the last few years when the college-age population began to decrease.

Perhaps the age structure of students should be examined. Older students (older than the defined college-age population) have increased, especially on a part-time basis. High school graduates are tending more to delay their decision to attend college. Differential rates for varying age groups would help to account for the older students. Prediction of enrollment by prediction of the various classifications (freshman, sophomore, junior, senior and graduate) by using the population age structure may also be valuable.

These and the factors used are general. If possible, it would be more than appropriate to include variables which apply specifically to Kansas State. For example, measures of the agricultural economy such as the relative ratio of annual income of farmers to non-farmers could be used. Perhaps, even a lag on K-State's basketball record from year to year might have an influence on KSU's share of the college-age population.

Whatever variables are used, or could be used, they should be explanatory in nature so that possible future changes which might influence changes in enrollment can be predicted. In addition, as with the models presented, a time series process can be used which will help account for cyclic fluctuations.

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PROJECTING FULL-TIME-EQUIVALENCE FOR KANSAS STATE UNIVERSITY USING REGRESSION MODELS WITH TIME SERIES ERRORS

by

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ABSTRACT

Models with regressive components and autoregressive time series components were employed to project full-time-equivalence for Kansas State University. The autoregressive process of a model was modeled on the residuals of a regression model and then with the autoregressive process as part of the model, the parameters of the regressive component were re-estimated.

Independent variables were based on population and economic influences including the college-age population of Kansas, the demand by industry for college educated persons and the unemployment rate. Two types of models were used. One employed two separate intercepts for the fall and spring semesters. Only one intercept was in the model at any given time. The other type estimates the difference between fall and spring semesters.

Whereas in the past full-time-equivalence for KSU continued to increase, based on these models, it is projected to level off. And though direct comparisons cannot be made to previous models for projecting enrollment, these models have smaller one-step-ahead projection variances indicating a closer fit.