AN IN-CORE LINEAR PROGRAMMING MODEL OF THE HIGH-TEMPERATURE GAS-COOLED REACTOR

by

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ACRONYMS

AGR	advanced gas-cooled reactor
BOL	beginning of life
CR	conversion ratio
EGCR	experimental gas-cooled reactor
EOL	end of life
FP	fission product
GA	general atomic
HTGR	high-temperature gas-cooled reactor
LHS	left hand side
LP	linear programming
MWe	megawatts electric
MWD/MT	megawatt days/metric tonne
MWt	megawatts thermal
NSSS	nuclear steam supply system
OB	objective function
RHS	right hand side
TP	total power

NOMENCLATURE

a_{ij}	j-th activity constant for the i-th constant
b_i	requirement constant for the i-th constraint
$\underset{\sim}{\bar{A}}$	constant burnup matrix
$\underset{\sim}{B}(t)$	time dependent state transition matrix
c_j	price constant of the j-th decision variable
$\underset{\sim}{C}$	price constant vector
$\underset{\sim}{C}'$	transpose of price constant vector
$\underset{\sim}{C}_B$	price constant basis vector
$\underset{\sim}{C}_B'$	transpose price constant basis vector
$\underset{\sim}{E}_o$	(m+1) × (m+1) update matrix
En_{max}	maximum enrichment allowed
$\underset{\sim}{e}_i$	unit vector in i-th column
f	thermal utilization factor
FR_g	formation rate of the g-th fissile nuclide
I	I-135 nuclides per unit volume
$\underset{\sim}{I}_m$	identity matrix
I_∞	steady state concentration of I-135
k_{eff}	effective multiplication factor
k_∞	infinite multiplication factor
ℓ	number of inequality constraints
m	number of equality constraints
n	number of variables
N_{39}	number of Np-239 atoms per unit volume
N_{F23}	concentration of U-233 fission product pairs per unit volume

N_{F25}	concentration of U-235 fission product pairs per unit volume
N^{M}_{pq}	concentration of p-th species in the q-th region of M-th point in time
$\underset{\sim}{P}_{o}$	requirement vector
$\underset{\sim}{P}_{j}$	j-th column vector
P	Pm-149 nuclides per unit volume
P_1	fission-to-resonance nonleakage probability
P_2	resonance-to-thermal nonleakage probability
P_{ℓ}	resonance escape probability for the ℓ-th fertile species
PFAC	load factor
P_r	radial power peaking factor
P_{wtq}	power peaking weighting factor for the q-th region
P_{avg}	average power density
r	branching ratio
REC_g	recoverable energy per fission of the g-th nuclide
V_q	volume of the q-th region
x_j	j-th decision variable
$\underset{\sim}{X}$	decision variable vector
X	Xe-135 nuclides per unit volume
X_{∞}	steady state concentration of Xe-135
$\underset{\sim}{X}_{B}$	basis decision vector
Y^{I}	fission yield of I-135 (actually of Te-135)
Y^{X}	direct fission yield of Xe-135
Y^{P}	fission yield of Pm-149 (actually of Nd-149)
z	value of objective function
$\underset{\sim}{\alpha}^{j}$	update vector of the j-th column

NOMENCLATURE (CONTINUED)

α_p	capture-to-fission ratio of the p-th nuclide
$\bar{\theta}$	search constant for leaving vector
θ	fluence or flux time
λ^I	decay constant of I-135
λ^X	decay constant of Xe-135
λ^P	decay constant for Pm-149
λ_{39}	decay constant for Np-139
λ_{13}	decay constant for Pa-233
$\bar{\sigma}^X$	average thermal microscopic absorption cross section for Xe-135
$\bar{\sigma}^S$	average thermal microscopic absorption cross section for Sm-149
σ_{39}	microscopic absorption cross section for Np-239
σ_{13}	microscopic absorption cross section for Pa-233
δ_{23}^{fp}	effective fission product microscopic absorption cross section per fission of U-233
δ_{25}^{fp}	effective fission product microscopic absorption cross section per fission of U-235
σ_{B10}^a	microscopic absorption cross section of B-10
σ_g^f	microscopic fission cross section for the g-th nuclide
σ_{ap}	microscopic absorption cross section for the p-th species
Σ_{fF}	macroscopic fission cross section of the fuel
Σ_{aF}	macroscopic absorption cross section of the fuel
Σ_{aM}	macroscopic absorption cross section of the moderator
Σ_{aP}	macroscopic absorption cross section of the poisons
Σ_{aC}	macroscopic absorption cross section of the parasitic poisons
Σ_{aB10}	macroscopic absorption cross section of B-10

NOMENCLATURE (CONTINUED)

$\bar{\phi}$	average thermal flux
ϕ	thermal flux
ϕ_q	average thermal flux in the q-th region
ϕ_{max}	maximum thermal flux in the core
$\underset{\sim}{\eta}$	Eta vector
η	average number of neutrons liberated directly by fission for each thermal neutron absorbed in the fuel.
ε	fast fission factor
ν_g	average yield of neutrons per fission of the g-th species

Subscript (p)	Species
23	U-233
28	U-238
02	Th-232
25	U-235
26	U-236
49	Pu-239
40	Pu-240
41	Pu-241
24	U-234

1.0 INTRODUCTION

The possibility of gaining simultaneously energy and new fuel is of great importance for the economics of power stations, especially in this day and age of energy shortages. Many feel that the High-Temperature Gas-Cooled Reactor concept as developed by General Atomic (GA) will be the major thermal reactor of the future (1,2,3,4). The main reason for this projection is that the light water reactor (LWR) has essentially reached the point of diminishing returns with regard to technical and economic improvement (4). Since it is apparent that the large utilities are starting to accept the HTGR design as feasible and more economical than LWR, it is only proper to begin the development of HTGR fuel management techniques.

The major goal of fuel management is the complete administration and control of technical and economic factors to optimize the use of nuclear fuel that results in the highest performance at a minimum cost (5). Indeed, the value of the fuel involved is usually about twice the capital costs of the plant (6). Many methods have been developed to optimize the combinations of fuel inputs to meet operational needs (7,8,9,10,11,12). The two major optimization methods used are usually "dynamic programming" or "linear programming." All of these studies have either used a boiling water reactor (BWR) or a pressurized water reactor (PWR) as the reference reactor. To date no studies have been made of the HTGR core using linear programming (LP) as a means to optimize. Therefore linear programming was used in this work as the optimization technique to find the optimum combination of U-233 fuel produced and U-235 fuel used to breed U-233.

The main purpose of this work was to develop a versatile LP code which has incorporated into it the appropriate burnup equations and operational

constraints to simulate a HTGR core. This code can serve as a preliminary projector of fuel needs and fuel loading patterns for a nuclear utility company.

This paper can be divided into 4 major parts. The first part deals with the theory of optimization techniques and linear programming. The second part briefly describes the HTGR. The third part details the model formulation by presenting the derivations of the burnup equations and the formulation of the model constraints. The fourth major part presents and discusses the results.

2.0 INTRODUCTION TO OPTIMIZATION THEORY

According to Webster (13), optimum is defined as the amount or degree of something that is most favorable to some end. In a broad sense everyone optimizes to a certain degree. Many decisions are made in the course of a day-to-day living routine in order to accomplish specific tasks. Usually there are numerous ways to complete these tasks. Although some options will generally be better than others, the final decision should be the best-or optimal-way to realize the objective.

For example, driving through city traffic presents a problem to most people. An attempt is usually made to find the shortest possible route from point A to point B without concern for time required to traverse this route. Alternatively, one would seek out the quickest, though not necessarily the shortest, route between A and B. As a compromise, one might attempt to find the shortest path from A to B subject to the auxiliary condition (constraint) that the transit time not exceeds some prescribed value. These are three similar, but different, optimization problems.

In a classical sense, optimization can be defined as the art of obtaining best policies to satisfy certain objectives, at the same time satisfying fixed requirements (14). To apply this definition, the appropriate equations must be written to establish mathematical cause-and-effect relationships, commonly known as a mathematical model. The mathematical model which is considered in this work is one that deals with the quantitative analysis of the HTGR core system. The system consists of transmutation equations (burnup equations) which predict depletion and formation due to neutron bombardment and radioactive decay of nuclear species in the core of the HTGR. The best policy in this case is to get the most productivity for the

least expense, in other words, an optimum combination between U-235, U-233 and Th-232, and still satisfy all necessary requirements to keep the reactor operational. In order to attain the best policy in an optimal manner, the policy must be stated in such a way as to relate certain variables, which are sometimes referred to as policy variables or decision variables, to a final objective. The decision variables are combined in the proper manner to meet this objective. The combination of terms may be defined as an objective function (sometimes called a profit function, performance index, or return function) which may represent some quantity, such as profit or cost, that is to be optimized. More specifically, a particular point is sought in a closed region (i.e., a domain plus its boundaries) that causes the objective function to be either a maximum or minimum.

In many instances the objective of an ordinary optimization problem cannot be stated easily, since it involves the minimization or maximization of some function of dissimilar entities (14,15). To give an example of dissimilar entities, one may consider a safety feature as opposed to a piece of metal in the manufacturing of automobiles. In determining a relationship between the two, value theory (14) may be used by assigning a weighting factor to the degree of safety incorporated into the automobile relative to the cost of the metal used. U-235 and U-233 may also be considered as two dissimilar entities. In analyzing the relationship between U-235 and U-233, one may note that U-235 may be purchased at a specified cost while U-233 is produced by a nuclear conversion process where a neutron is captured by the Th-232 nucleus and by beta decay is transformed into U-233. Thus the value of U-233 is normally expressed in terms of its neutronic worth relative to that of U-235 by the U-233 indifference value (16).

To keep the optimum solution of the objective function within the physical limits of the system being investigated, auxiliary conditions, commonly referred to as constraints, must be formulated. When analyzing a reactor core the constraints are formulated in such a manner as to insure that the desired power level is obtained while thermal-hydraulic, power peaking, and criticality requirements are not violated.

While formulating the constraints there are several important factors to keep in mind. Notice that one need not distinguish between the two types of inequality constraints, since a ($\leq$) constraint can be converted to a ($\geq$) constrain by simply multiplying both sides of the expression by a minus sign. By letting ℓ represent the inequality constraints, m represent the equality constraints, and n represent the number of variables (columns), one may also observe that the number of equality constraints must be less than the number of variables; otherwise the variables will be either uniquely determined by the contraints (if $m-\ell = n$) or overspecified ($m-\ell > n$). It is also important to point out that the number of inequality constraints is unrestricted.

There are two main techniques of optimization which are frequently used. These are simultaneous and sequential (14,15). Sequential techniques obtain the desired optimum by means of a step-by-step procedure (sometimes called a stagewise procedure). The stages are traversed systematically in such a manner that information obtained at a given stage is used to determine further information at the next stage, and so on. Dynamic programming is a powerful sequential optimization technique (14,15). In sequential optimization techniques the optimal values of certain independent variables are determined at each stage, in contrast to simultaneous techniques, where all the optimal independent variables are obtained at once. The simultaneous optimization technique used in this work is linear programming, which employs

iterative algorithms that gradually converge to the desired optimum conditions. All the variables are evaluated during each iteration, even though none of these variables may attain its optimal value. Systematic procedures are then applied to successive iterations to move closer to the desired optimum.

2.1 Linear Programming Theory

Detailed LP (Linear Programming) theory and analysis may be found in many references (14,15,17,18); therefore only a brief description of the theory and method will be given here.

The general linear programming problem may be stated in many forms, but the one used here is the most commonly used.

The LP problem is to find a vector $(x_1,x_2,\ldots,x_j,\ldots,x_n)$ which minimizes or maximizes the linear form (i.e., the objective function),

$$z = c_1 x_1 + c_2 x_2 + \ldots c_j x_j + \ldots + c_n x_n \qquad (2.1.1)$$

subject to the linear constraints

$$x_j \geq 0 \qquad j = 1,2,\ldots,n \qquad (2.1.2)$$

and

$$\begin{aligned}
&a_{11} x_1 + a_{12} x_2 + \ldots + a_{1j} x_j + \ldots + a_{1n} x_n \ (\geq,=, \text{ or } \leq)\ b_1 \\
&a_{21} x_1 + a_{2j} x_j + \ldots + a_{2n} x_n + \ldots + a_{2n} x_n \ (\geq,=, \text{ or } \leq)\ b_2 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&a_{i1} x_1 + a_{i2} x_2 + \ldots + a_{ij} x_j + \ldots + a_{in} x_n \ (\geq,=, \text{ or } \leq)\ b_i \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&a_{d1} x_1 + a_{d2} x_2 + \ldots + a_{dj} x_j + \ldots + a_{dn} x_n \ (\geq,=, \text{ or } \leq)\ b_d
\end{aligned} \qquad (2.1.3)$$

where a_{ij}, b_i, and c_j are constants which are determined by and depend on the technology of the problem. The x_j's are the decision variables; d

represents the number of rows, i.e., $d = m + \ell$; and n represents the number of variables. It is also useful to note that for each constraint, only one of the signs ($\geq, =, \leq$) holds.

Before the appropriate iterative algorithm is applied to the problem, the inequalities must be changed to equalities by adding slack variables for the "less than" constraints ($\leq$) and artificial variables for the "greater than" constraints ($\geq$). For the reader to understand this technique better an example is given in the next section.

Assume for the present that Eq. (2.1.3) are all equalities. The result, given in matrix notation, will be a m×n matrix $\underset{\sim}{A}$, where

$$\underset{\sim}{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{pmatrix} . \qquad (2.1.4)$$

It will also be helpful to reformulate the remainder of the problem in matrix notation as follows: Let

$$\underset{\sim}{X} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_j \\ \vdots \\ x_n \end{pmatrix}, \quad \underset{\sim}{C} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_j \\ \vdots \\ c_n \end{pmatrix}, \quad \text{and} \quad \underset{\sim}{P}_o = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_i \\ \vdots \\ b_m \end{pmatrix} . \qquad (2.1.5)$$

The LP problem now becomes:

Extremize $$z = \underset{\sim}{C}' \underset{\sim}{X} ,$$

subject to

$$\underset{\sim\sim}{A}\underset{\sim}{X} = \underset{\sim}{P}_o , \quad \underset{\sim}{X} \geq 0$$

where $\underset{\sim}{C}'$ is the transpose of $\underset{\sim}{C}$.

Finally, it is convenient to decompose the m×n matrix $\underset{\sim}{A}$ into the vectors

$$\underset{\sim}{P}_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{i1} \\ \vdots \\ a_{m1} \end{pmatrix}, \quad \underset{\sim}{P}_2 = \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{i2} \\ \vdots \\ a_{m2} \end{pmatrix}, \ \ldots, \quad \underset{\sim}{P}_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{ij} \\ \vdots \\ a_{mj} \end{pmatrix}, \ \ldots, \quad \underset{\sim}{P}_n = \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{in} \\ \vdots \\ a_{mn} \end{pmatrix}, \qquad (2.1.6)$$

The LP problem may now be written in vector form as:

Extremize $$z = \underset{\sim}{C}' \underset{\sim}{X} , \qquad (2.1.7)$$

subject to the constraints

$$x_1 \underset{\sim}{P}_1 + x_2 \underset{\sim}{P}_2 + \ldots + x_j \underset{\sim}{P}_j + \ldots + x_n \underset{\sim}{P}_n = \underset{\sim}{P}_o , \qquad (2.1.8)$$

and the non-negative requirements

$$\underset{\sim}{X} \geq 0 . \qquad (2.1.9)$$

In the above problem, there are m equations and n unknowns. There are usually many sets of positive values of the n unknowns unless n = m in which case a unique solution exists. It will be assumed here that n > m; therefore to solve this problem n - m variables are set to zero, and the object

is to find values for the other m variables that satisfy m equations. A systematic way, i.e., simplex or revised simplex algorithms, is then used to see whether there are other ways to assign values that will improve the objective function. The discussion of this process will be made somewhat easier if some new terminology is introduced.

A solution with just as many variables permitted to be nonzero as there are equations, with all other variables being forced to be zero, is called a basic solution. The variables that are permitted to be nonzero are called basic variables, and the collection of these variables is called a basis. Thus there must be exactly as many variables in a basis as there are problem constraint equations. If the values picked for the basic variables satisfy the constraints and are non-negative, they represent a point in the feasible region, and a basic feasible solution exists.

Since the digital computer IBM S/370 Model 158 which uses the product form of the inverse/revised simplex method (19) was used in analyzing the LP Model formulated in this work, a description of revised simplex algorithm will be given here.

Suppose that $\underset{\sim}{P}_1, \underset{\sim}{P}_2, \ldots, \underset{\sim}{P}_m$ of Eq. (2.1.8) form a set of basis vectors, and that $x_1, x_2, \ldots, x_m$ are the corresponding independent variables where the remaining independent variables are equal to zero. It can be written

$$\underset{\sim}{P}_o = x_1 \underset{\sim}{P}_1 + x_2 \underset{\sim}{P}_2 + \ldots + x_m \underset{\sim}{P}_m \tag{2.1.10}$$

or

$$\underset{\sim}{P}_o = \underset{\sim}{B}\, \underset{\sim}{X}_B \tag{2.1.11}$$

where $\underset{\sim}{X}_B$ is an m-dimensional vector whose components are $x_1, x_2, \ldots, x_m$, and $\underset{\sim}{B}$ is an m×m matrix whose columns are the basis vectors $\underset{\sim}{P}_1, \underset{\sim}{P}_2, \ldots, \underset{\sim}{P}_m$. If Eq. (2.1.11) is solved for $\underset{\sim}{X}_B$ as

$$\underset{\sim}{X}_B = \underset{\sim}{B}^{-1} \underset{\sim}{P}_o \ . \tag{2.1.12}$$

The corresponding objective function can be expressed as

$$z = \underset{\sim}{C}_B' \underset{\sim}{X}_B = \underset{\sim}{C}_B' \underset{\sim}{B}^{-1} \underset{\sim}{P}_o \tag{2.1.13}$$

where $\underset{\sim}{C}_B'$ is an m-dimensional vector whose components are the cost coefficients corresponding to the variables $x_1, x_2, \ldots, x_m$. Furthermore, an expression analogous to Eq. (2.1.12) can be written as

$$\underset{\sim}{X}_j = \underset{\sim}{B}^{-1} \underset{\sim}{P}_j \tag{2.1.14}$$

where $\underset{\sim}{P}_j$ is an m-dimensional, nonbasis vector, and $\underset{\sim}{X}_j$ contains the coefficients needed to express $\underset{\sim}{P}_j$ in terms of the basis vectors.

Equations (2.1.12), (2.1.13), and (2.1.14) are fundamental to the development of the revised simplex algorithm. The computational procedure is similar to the basic simplex algorithm, except that the operations are performed on the (m+1) x (m+1) matrix $\underset{\sim}{D}^{-1}$, where

$$\underset{\sim}{D}^{-1} = \begin{bmatrix} 1 & \underset{\sim}{C}_B' \underset{\sim}{B}^{-1} \\ 0 & \underset{\sim}{B}^{-1} \end{bmatrix} \tag{2.1.15}$$

Rather than calculate $\underset{\sim}{D}^{-1}$ directly each time the basis is changed, it is possible to calculate a new inverse from an old inverse by means of the expression

$$\underset{\sim}{D}_i^{-1} = \underset{\sim}{E}_i \underset{\sim}{D}_{i-1}^{-1} \ , \tag{2.1.16}$$

where $\underset{\sim}{D}_i^{-1}$ and $\underset{\sim}{D}_{i-1}^{-1}$ are the next and old inverse matrices, respectively, and $\underset{\sim}{E}_i$ is an (m+1) x (m+1) matrix that is calculated as a part of the ith vector, transformation procedure. It can be shown that $\underset{\sim}{D}_i^{-1}$ (14,15) can be written as

$$\underset{\sim}{D}_i^{-1} = \underset{\sim}{E}_i \, \underset{\sim}{E}_{i-1} \cdots \underset{\sim}{E}_1 \, . \tag{2.1.17}$$

This is known as the product form of the inverse.

The revised simplex method procedure may be summarized in three basic steps:

Step 1. Determination of the entering vector $\underset{\sim}{P}_j$.

$$(z_j - c_j) = \underset{\sim}{C}_B' \, \underset{\sim}{B}^{-1} \, \underset{\sim}{P}_j - c_j$$

$$= (1, \underset{\sim}{C}_B' \, \underset{\sim}{B}^{-1}) \begin{pmatrix} -c_j \\ \underset{\sim}{P}_j \end{pmatrix} \, , \tag{2.1.18}$$

so, for a maximization problem, that the vector having the most negative $(z_j - c_j)$ should enter the solution. Otherwise, if all $(z_j - c_j) \geq 0$, the optimal solution is attained.

Step 2. Determination of the leaving vector $\underset{\sim}{P}_l$.

Given the entering vector $\underset{\sim}{P}_j$ and the current basic solution $x_k = (\underset{\sim}{B}^{-1} \underset{\sim}{P}_o)_k$ $k = 1,2,\ldots,m$, then

$$\underset{\sim}{\alpha}^j = \underset{\sim}{B}^{-1} \underset{\sim}{P}_j \, . \tag{2.1.19}$$

The leaving vector must correspond to

$$\bar{\theta} = \min_k \frac{(\underset{\sim}{B}^{-1} \underset{\sim}{P}_o)_k}{\alpha_k^j} \, , \quad \alpha_k^j > 0 \, . \tag{2.1.20}$$

If all $\alpha_k^j \leq 0$, the problem has no bounded solution.

Step 3. Determination of the next basic solution.

Let the identity matrix $\underset{\sim}{I}_m = (\underset{\sim}{e}_1, \underset{\sim}{e}_2, \ldots, \underset{\sim}{e}_m)$ where $\underset{\sim}{e}_i$ is a unit column-vector with a one-element at the ith place and zero-elements elsewhere.

Let $\underset{\sim}{e}_\ell$ be the unit vector representing the leaving variable x_ℓ. Given x_j as the entering variable, then $\underset{\sim}{M}^{-1}_{\text{next}}$ can be computed from the formula

$$\underset{\sim}{M}^{-1}_{\text{next}} = \underset{\sim}{E}_o \, \underset{\sim}{M}^{-1}_{\text{current}} \tag{2.1.21}$$

where

$$\underset{\sim}{E}_o = \begin{pmatrix} 1 & 0 & \cdots & 0 & -(z_j - c_j)/\alpha^j_k & 0 & \cdots & 0 \\ 0 & \underset{\sim}{e}_1 & \cdots & \underset{\sim}{e}_{\ell-1} & \underset{\sim}{\eta} & \underset{\sim}{e}_{\ell+1} & \cdots & \underset{\sim}{e}_m \end{pmatrix}, \tag{2.1.22}$$

and

$$\underset{\sim}{\eta} = \begin{pmatrix} -\alpha^j_1/\alpha^j_\ell \\ -\alpha^j_2/\alpha^j_\ell \\ \vdots \\ +1/\alpha^j_\ell \\ \vdots \\ -\alpha^j_m/\alpha^j_\ell \end{pmatrix} \tag{2.1.23}$$

where the values of $\underset{\sim}{\alpha}^j$ are given by $(\underset{\sim}{B}^{-1}_{\text{current}} \, \underset{\sim}{P}_j)$.

The next basic solution is given by

$$\begin{pmatrix} x_o \\ \underset{\sim}{x}_B \end{pmatrix} = \underset{\sim}{M}^{-1}_{\text{next}} \begin{pmatrix} 0 \\ \underset{\sim}{P}_o \end{pmatrix}. \tag{2.1.24}$$

Go to Step 1.

2.2 A Linear Programming Example

To help understand the theory presented in the last section a simple example will be given here in which two types of power plants are to be

built. Call the number of one type x_1 and the number of the other x_2. The plants are to be built at the same time and operated for a fixed length of time. Assume the following relationships between the plants can be stipulated:

1. No more than eight plants may be built. This can be represented algebrically as

$$x_1 + x_2 \leq 8 \ . \tag{2.2.1}$$

2. No less than two plants may be built, which is, in equation form,

$$x_1 + x_2 \geq 2 \ . \tag{2.2.2}$$

3. No more than ten units of material may be stored. Type 1 plant uses one unit of this material and type 2 plant produces three units, therefore,

$$-x_1 + 3\ x_2 \leq 10 \ . \tag{2.2.3}$$

4. Type 2 plant produces twice as much energy as type 1 plant, and the manager of the two has decided on a policy of maximum energy production. So,

$$x_1 + 2\ x_2 = \text{maximum } (=z)$$

5. Implicitly, we are buying plants, therefore,

$$x_1 \geq 0 \text{ and } x_2 \geq 0 \ .$$

Now, the linear programming model is formulated. Items 1, 2, and 3 show how the variables are related, and item 5 demonstrates the non-negativity requirements. Item 4 states what is to be optimized. The formulation equations may be given as a formal LP problem statement as Eqs. (2.1.1), (2.1.2), and (2.1.3) as follows:

The LP problem is to find a vector (x_1, x_2) which maximizes

$$z = x_1 + 2x_2$$

subject to the linear constraints

$$x_1, x_2 \geq 0$$

and

$$x_1 + x_2 \leq 8$$

$$x_1 + x_2 \geq 2$$

$$-x_1 + 3x_2 \leq 10 .$$

This problem may be solved in many different ways, but the method used here is the revised simplex/product form of the inverse. It will be helpful to present the problem in the form of the so called starting tableau. Before the tableau is presented it is necessary to briefly discuss the transformation of the inequality equations to equality equations.

As noted earlier in the theory section, the transformations can be made by introducing dummy variables. The "less than" constraints can be changed to equations by adding to the left-hand side (LHS) of each such constraint a non-negative variable, commonly called "slack variable." The "greater than" constraints can be changed to equations by subtracting from the LHS of each constraint a non-negative variable which represents the surplus of the LHS over the right-hand side (RHS). This variable is commonly known as the "surplus variable." For convenience, the surplus variable is usually regarded as a negative "slack variable" (15). Since the negative "slack variables" cannot provide a starting (feasible) solution, "artifical variables" must be added so a starting basis may be attained. The "artificial variables" are assigned values in the objective function which prevent them

from being in the final solution. Usually this is accomplished by giving the "artificial variables" a cost coefficient value of $-\infty$ for maximization problems and a value of $+\infty$ for minimization problems.

Now adding the slack and artificial variables the LP problem may be stated as:

Maximize

$$z = x_1 + 2\,x_2$$

subject to

$$\begin{aligned} x_1 + x_2 + x_3 &= 8 \\ x_1 + x_2 - x_4 + x_5 &= 2 \\ -x_1 + 3x_2 + x_6 &= 10 \end{aligned}$$

and

$$x_j \geq 0 \qquad j = 1,2,\ldots,5 \ .$$

The starting tableau is given as:

Cost coefficients			1	2	0	0	$-\infty$	0
Basis	P_0	c	P_1	P_2	P_3	P_4	P_5	P_6
P_3	8	0	1	1	1	0	0	0
P_5	2	$-\infty$	1	1	0	-1	1	0
P_6	10	0	-1	3	0	0	0	1

From the starting tableau, the starting basic solution is given by

$$X_B = \begin{pmatrix} x_3 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \\ 10 \end{pmatrix} .$$

Since

$$\underset{\sim}{C}_B' = (0, -\infty, 0)$$

and

$$\underset{\sim}{B}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underset{\sim}{C}_B' \underset{\sim}{B}^{-1} = (0, -\infty, 0)$$

and

$$\underset{\sim}{M}^{-1} = \begin{pmatrix} 1 & \underset{\sim}{C}_B'\underset{\sim}{B}^{-1} \\ 0 & \underset{\sim}{B}^{-1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\infty & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

According to the prescribed theory, the problem is solved using the revised simplex method.

First Iteration:

Step 1. Computations of $(z_j - c_j)$ for $\underset{\sim}{P}_1$, $\underset{\sim}{P}_2$, and $\underset{\sim}{P}_4$.

$$(z_j - c_j) = (1, \underset{\sim}{C}_B'\underset{\sim}{B}^{-1}) \begin{pmatrix} -c_1 & -c_2 & -c_4 \\ \underset{\sim}{P}_1 & \underset{\sim}{P}_2 & \underset{\sim}{P}_4 \end{pmatrix}$$

$$= (1, 0, -\infty, 0) \begin{pmatrix} -1 & -2 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & -1 \\ -1 & 3 & 0 \end{pmatrix}$$

$$= (-1-\infty, -2-\infty, +\infty)$$

where $(1, \underset{\sim}{C}_B'\underset{\sim}{B}^{-1})$ is obtained directly from the top row of $\underset{\sim}{M}^{-1}$. Hence, $\underset{\sim}{P}_2$ enters the solution, since it has the most negative $z_j - c_j$.

Step 2. Determination of the leaving vector given the entering vector $\underset{\sim}{P}_2$.

$$\underset{\sim}{\alpha}^2 = \begin{pmatrix} \alpha_3^2 \\ \alpha_5^2 \\ \alpha_6^2 \end{pmatrix} = \underset{\sim}{B}^{-1}\underset{\sim}{P}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}.$$

Hence, for k = 3, 5, and 6

$$\bar{\theta} = \min\left\{\frac{(\underset{\sim}{B}^{-1}\underset{\sim}{P}_o)_k}{\alpha_k^j}\ ,\ \alpha_k^j > 0\right\} = \min\left\{\frac{8}{1}\ ,\ \frac{2}{1}\ ,\ \frac{10}{3}\right\} = 2$$

which corresponds to $\underset{\sim}{P}_5$. Thus $\underset{\sim}{P}_5$ leaves the solution.

Step 3. Determination of the new solution.

$$\underset{\sim}{\eta} = \begin{pmatrix} -\alpha_3^2/\alpha_5^2 \\ +\ 1/\alpha_5^2 \\ -\alpha_6^2/\alpha_5^2 \end{pmatrix} = \begin{pmatrix} -1/1 \\ +1/1 \\ -3/1 \end{pmatrix}$$

Then

$$\underset{\sim}{E}_o = \begin{pmatrix} 1 & 0 & -(z_j - c_j)/\alpha_\ell^j & 0 \\ 0 & \underset{\sim}{e}_{\ell-1} & \underset{\sim}{\eta} & \underset{\sim}{e}_{\ell-1} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 2 + \infty & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{pmatrix}$$

and

$$\underset{\sim}{M}^{-1}_{\text{next}} = \underset{\sim}{E}_o \, \underset{\sim}{M}^{-1}_{\text{current}}$$

$$= \begin{pmatrix} 1 & 0 & 2+\infty & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -\infty & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{pmatrix}$$

This directly gives $\underset{\sim}{C}'_B \underset{\sim}{B}^{-1} = (0,2,0)$. The new solution is then

$$\begin{pmatrix} x_o \\ \underset{\sim}{x}_B \end{pmatrix} = \begin{pmatrix} x_o \\ x_3 \\ x_2 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 8 \\ 2 \\ 10 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 2 \\ 4 \end{pmatrix} .$$

Second Iteration:

Step 1. Computations of $(z_j - c_j)$ for $\underset{\sim}{P}_1$, $\underset{\sim}{P}_4$, and $\underset{\sim}{P}_5$. Note that $\underset{\sim}{P}_5$ may be dropped from the iterative procedure since it was inserted as an artificial variable.

$$\{z_j - c_j\} = (1, 0, 2, 0) \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ -1 & 0 \end{pmatrix} = (1, -2) .$$

Therefore $\underset{\sim}{P}_4$ should enter the solution.

Step 2. Determination of the leaving vector given the entering vector $\underset{\sim}{P}_4$.

$$\underset{\sim}{\alpha}^4 = \begin{pmatrix} \alpha_3^4 \\ \alpha_2^4 \\ \alpha_6^4 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} .$$

Hence, for k = 3, 2, and 6,

$$\bar{\theta} = \min \left\{ \frac{6}{1} , - , \frac{4}{3} \right\} = \frac{4}{3}$$

This corresponds to the leaving vector $\underset{\sim}{P}_6$.

Step 3. Determination of the new solution.

$$\underset{\sim}{\eta} = \begin{pmatrix} -\alpha_3^4/\alpha_6^4 \\ \alpha_2^4/\alpha_6^4 \\ +\ 1/\alpha_6^4 \end{pmatrix} = \begin{pmatrix} -1/3 \\ -(-1)/3 \\ 1/3 \end{pmatrix} = \begin{pmatrix} -1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$$

and

$$\underset{\sim}{E}_o = \begin{pmatrix} 1 & 0 & 0 & -(-2)/3 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 1/3 \end{pmatrix}, \text{ therefore}$$

$$\underset{\sim}{M}^{-1}_{next} = \begin{pmatrix} 1 & 0 & 0 & 2/3 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 1/3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 2/3 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 0 & 1/3 \\ 0 & 0 & -1 & 1/3 \end{pmatrix}$$

and

$$\begin{pmatrix} x_o \\ \underset{\sim}{x}_B \end{pmatrix} = \begin{pmatrix} x_o \\ x_3 \\ x_2 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 2/3 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 0 & 1/3 \\ 0 & 0 & -1 & 1/3 \end{pmatrix} \begin{pmatrix} 0 \\ 8 \\ 2 \\ 10 \end{pmatrix} = \begin{pmatrix} 20/3 \\ 14/3 \\ 10/3 \\ 4/3 \end{pmatrix}.$$

Third Iteration:

Step 1. Computation of $(z_j - c_j)$ for $\underset{\sim}{P}_1$ and $\underset{\sim}{P}_6$.

$$\{z_j - c_j\} = (1,\ 0,\ 0,\ 2/3) \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 1 & 0 \\ -1 & 1 \end{pmatrix} = (-5/3,\ 2/3)$$

and $\underset{\sim}{P}_1$ should enter the basis.

Step 2. Determination of the leaving vector given the entering vector $\underset{\sim}{P}_1$.

$$\underset{\sim}{\alpha}^1 = \begin{pmatrix} \alpha_3^1 \\ \alpha_2^1 \\ \alpha_4^1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1/3 \\ 0 & 0 & 1/3 \\ 0 & -1 & 1/3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4/3 \\ -1/3 \\ -4/3 \end{pmatrix}.$$

Hence, for k = 3, 2, and 4,

$$\bar{\theta} = \min\left\{\frac{14}{4}\ ,\ -\ ,\ -\right\} = \frac{14}{4}\ .$$

Step 3. Determine the new solution.

$$\underset{\sim}{\eta} = \begin{pmatrix} + 1/\alpha_3^1 \\ -\alpha_2^1/\alpha_3^1 \\ -\alpha_4^1/\alpha_3^1 \end{pmatrix} = \begin{pmatrix} 3/4 \\ 1/4 \\ 1 \end{pmatrix},$$

and

$$\underset{\sim}{E}_o = \begin{pmatrix} 1 & 5/4 & 0 & 0 \\ 0 & 3/4 & 0 & 0 \\ 0 & 1/4 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix},$$

and

$$\underset{\sim}{M}^{-1}_{next} = \begin{pmatrix} 1 & 5/3 & 0 & 0 \\ 0 & 3/4 & 0 & 0 \\ 0 & 1/4 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 2/3 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 0 & 1/3 \\ 0 & 0 & -1 & 1/3 \end{pmatrix} = \begin{pmatrix} 1 & 5/4 & 0 & 1/4 \\ 0 & 3/4 & 0 & -1/4 \\ 0 & 1/4 & 0 & 1/4 \\ 0 & 1 & -1 & 0 \end{pmatrix},$$

and

$$\begin{pmatrix} x_o \\ \underset{\sim}{x}_B \end{pmatrix} = \begin{pmatrix} x_o \\ x_1 \\ x_2 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 & 5/4 & 0 & 1/4 \\ 0 & 3/4 & 0 & -1/4 \\ 0 & 1/4 & 0 & 1/4 \\ 0 & 1 & -1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 8 \\ 2 \\ 10 \end{pmatrix} = \begin{pmatrix} 25/2 \\ 14/4 \\ 18/4 \\ 6 \end{pmatrix}.$$

Fourth Iteration:

Step 1. Computation of $(z_j - c_j)$ for $\underset{\sim}{P}_3$ and $\underset{\sim}{P}_6$.

$$\{z_j - c_j\} = (1,\ 5/4,\ 0,\ 1/4)\begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} = (5/4,\ 1/4)$$

Since all $(z_j - c_j) > 0$, the solution is optimal; therefore

$$x_1 = 14/4,\ x_2 = 18/4,\ x_4 = 6$$

and

$$z = 25/2\ .$$

One may observe from the final optimal solution given above that the optimum number of plants to be built is given in terms of fractional values instead of integer values. An integer optimum would be required in this particular case; since the plants cannot be functional in fractional parts. It may be noted that the outstanding computation problem of LP has been that of finding the optimum integer solution to a linear program (17). The method of finding the optimum integer solution is referred to as "Integer Linear Programming" (14,15,17,18). No detailed description is given here; since fractional values are permissible in this work.

A geometrical interpretation of the above solution may help the reader to better visualize how the solution is obtained in two dimensional space. The graphical solution to the example problem is shown in Fig. 2.2.1.

THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

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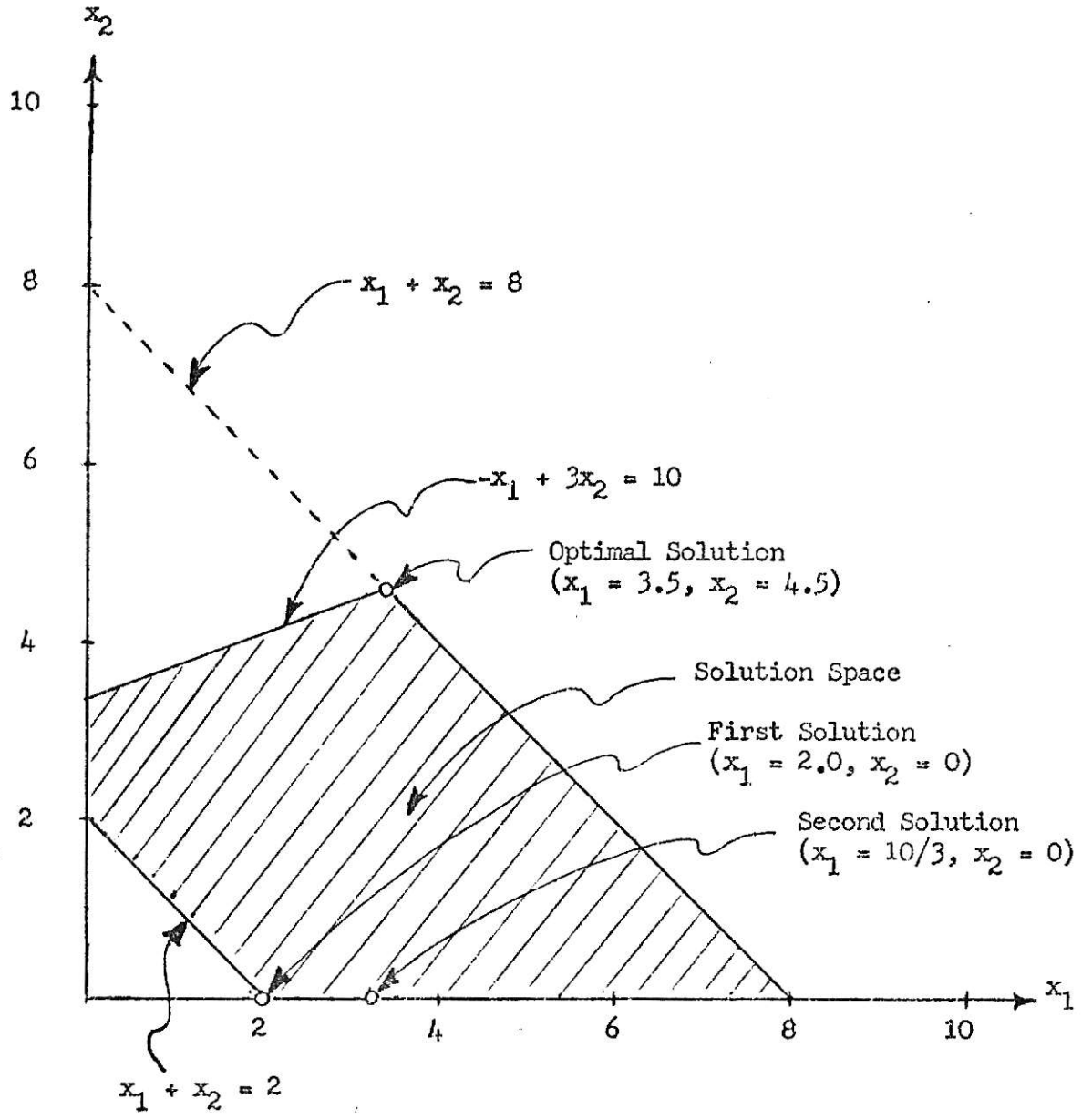


Figure 2.2.1 Graphical Solution to the Example Problem

3.0 DESCRIPTION OF HTGR

3.1 Past and Present HTGR Designs

In any study of a system, one must know the current design basis to simulate, accurately, the system under investigation. In this presentation a brief discussion of past HTGR designs will serve as an introduction to the current design criteria.

The first gas-cooled power reactor was built at Calder Hall in Great Britain and achieved full-power operation in the fall of 1956 (3). The Calder Hall plant served as a prototype for the Magnox reactor plants built by the British in their nuclear power program. Interest in gas-cooled reactors in the United States dates from the design study project of the Daniels reactor in 1945 (3). Though this plant was never built, many of its design features are similar to those of contempory advanced gas-cooled reactors. Beginning in 1956, development work at Oak Ridge National Laboratory and at General Atomic (GA) led to the construction of the 30-MWe Experimental Gas-Cooled Reactor (EGCR) and the 40-MWe Peach Bottom HTGR. The EGCR was similar to the British AGR (Advanced Gas-Cooled Reactor), although cooled by helium. Due to technical and programmatic difficulties, however, construction was never completed (3). The Peach Bottom Nuclear Power Plant, the first HTGR in the U.S., achieved criticality in March 1966 and was placed in commercial operation in June 1967 (2,4). The second HTGR in the U.S. was the 330 MWe Fort St. Vrain Nuclear Generating Station (2) now in the final stages of construction for the Public Service Company of Colorado with GA as the prime contractor. A full-power operation license was granted to Public Service Company of Colorado on December 21, 1973, and initial criticality was reached on January 31, 1974 (4).

The current design criteria now being considered by GA is the 1160 MWe Nuclear Steam Supply System (NSSS) (4) which is patterned after the Fort St. Vrain design. Some pertinent design characteristics are given in Table I (2). The 1160 MWe HTGR core shown in Fig. 3.1.1 consists of 493 columns of eight elements, one stacked on top of the other to complete the core arrangement. The core arrangement is shown in an isometric cross section view in Fig. 3.1.2. The columns of fuel elements are generally arranged in groups of seven for refueling purposes (2). Each group, called a refueling region, rests on a single hexagonal graphite support block and is located directly below a refueling penetration which houses a control rod drive assembly. Within the center column of each region, two parallel channels through the individual top reflector and fuel elements are provided for insertion of the two control rods which comprise a control rod pair that moves as a unit. A third channel is provided within the center column for the insertion of reserve shutdown absorber material. The side reflector elements are also supported and located on graphite blocks which fit together with the hexagonal blocks under the core (20). Each of these graphite blocks is, in turn, supported by three graphite posts which have spherical ends to permit them to rock slightly in accommodating differential expansion between the parts of the structure. The top ends of the posts are located in spherical seats in the core support blocks and their bottom ends rest in spherical seats in carbon basis located on the metal core support floor.

3.2 Fuel Element Design

The fuel element proposed by GA for the HTGR is a graphite prismatic block containing interspersed coolant channels, fuel holes, and burnable poison

TABLE I

HTGR Characteristics

Power level, MW thermal	2900
MW electrical	1160
Efficiency, %	40
Inlet gas temperature, °F	640
Outlet gas temperature, °F	1430
Core configuration	
Active height, ft.	20.8
Equivalent Diameter, ft.	27.6
Active volume, cc x 10^7	35.71
Reflector thickness, ft.	3 to 4
Number of control rods	73 pairs
Fuel Management	
Fuel lifetime at 80% capacity factor	4
Fraction of core replaced each cycle	1/4
Number of refueling regions	73
Fuel elements	
Number of elements	3944
Number of columns	493
Element height, in.	31.2
Element width, in.	14.2 (across flats)
Fuel exposure	
Average MWD/MT	90000
Peak fast fluence, 10^{21} nvt	8

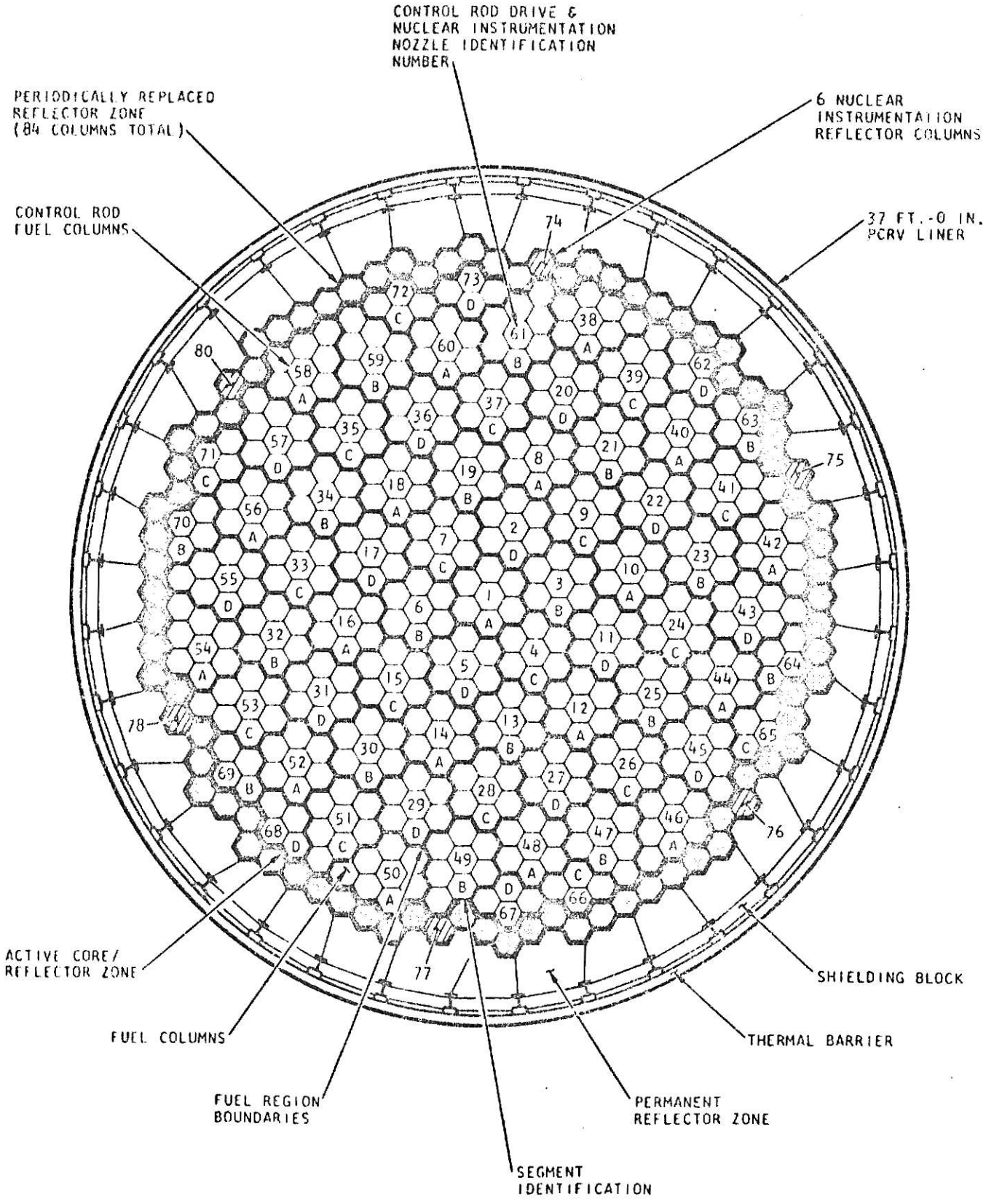


Figure 3.1.1 Core Configuration Region and Segment Identification

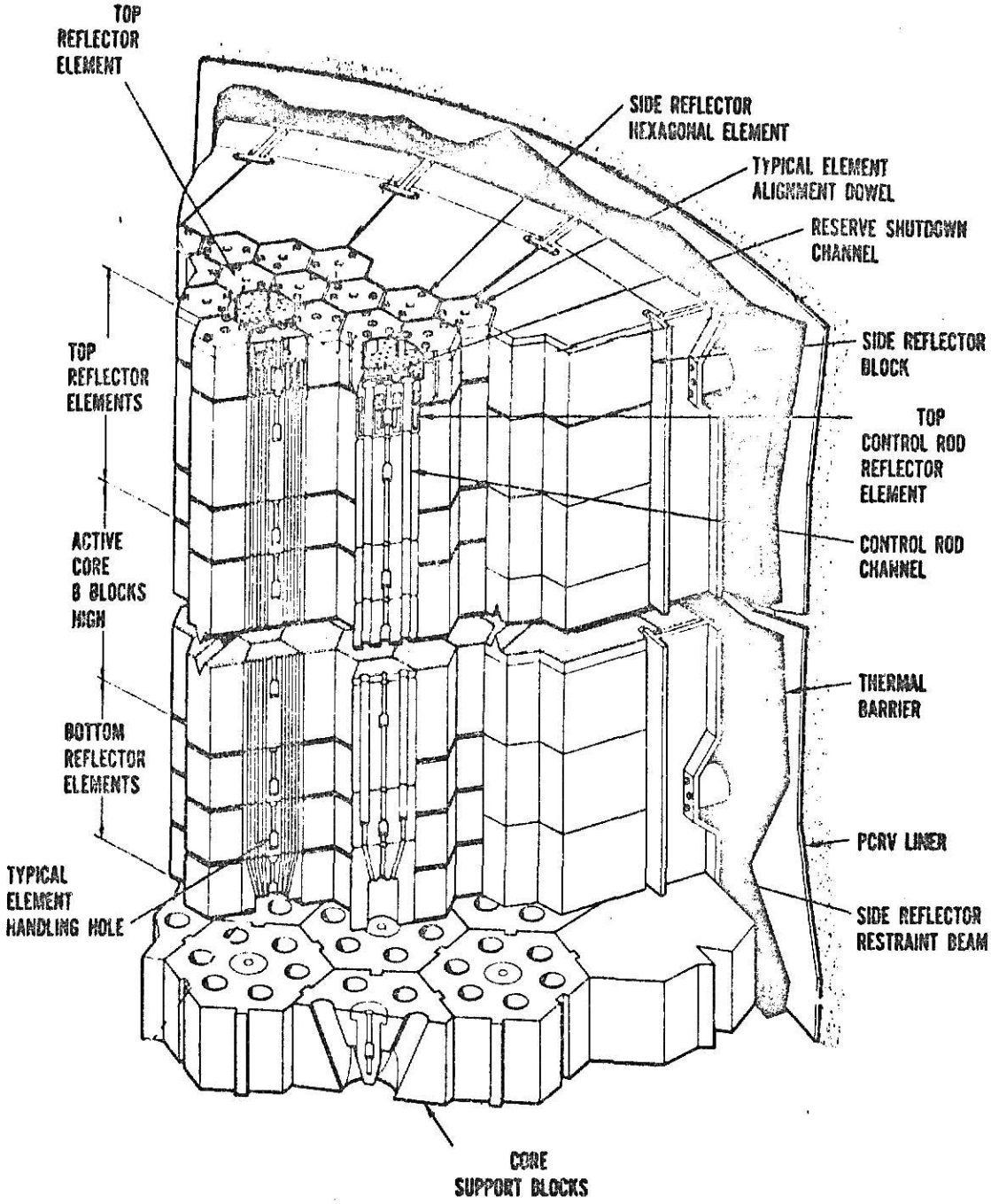


Figure 3.1.2 Core Arrangement, Elevation Section

holes as shown in Fig. 3.2.1. The fuel holes are filled with rods containing coated fuel particles bonded together by a carbonaceous matrix (1). The fuel element structural (and moderator) material is conventional nuclear grade needle-coke graphite (20).

Pyrolytic carbon coated particles of uranium and thorium carbide are the basic fuel materials for the HTGR. The ceramic particle coatings, applied by pyrolytic techniques (4), are multilayered to ensure a high degree of fission-product confinement. A porous inner layer ("buffer" zone) accommodates the expansion of the irradiated fuel and provides storage space for gaseous fission products, thereby minimizing the buildup of internal pressure due to fuel burnup. The outer layer acts as a fission-product retention barrier and provides structural strength. The particle coatings functions as miniature pressure vessels.

There are two principle types of fuel particles in the large HTGR (1, 21,16,2). They are distinguished from one another by the fact that the fertile thorium particles have only a single pyrolytic carbon outer coating and the fissile uranium particles have an additional silicon carbide coating. GA has given these fuel particles names. As shown in Fig. 3.2.2, the names are TRISO and BISO which are acronyms that denote the type of coating, i.e., TRISO contains three types of coating layers and BISO contains two types of coating layers. The design of these particles allows the separation of the discharged uranium particle which contains primarily ^{235}U and ^{236}U, from the discharged thorium particles which contains primarily bred ^{233}U (1). The important fuel particle parameters are listed in Table II.

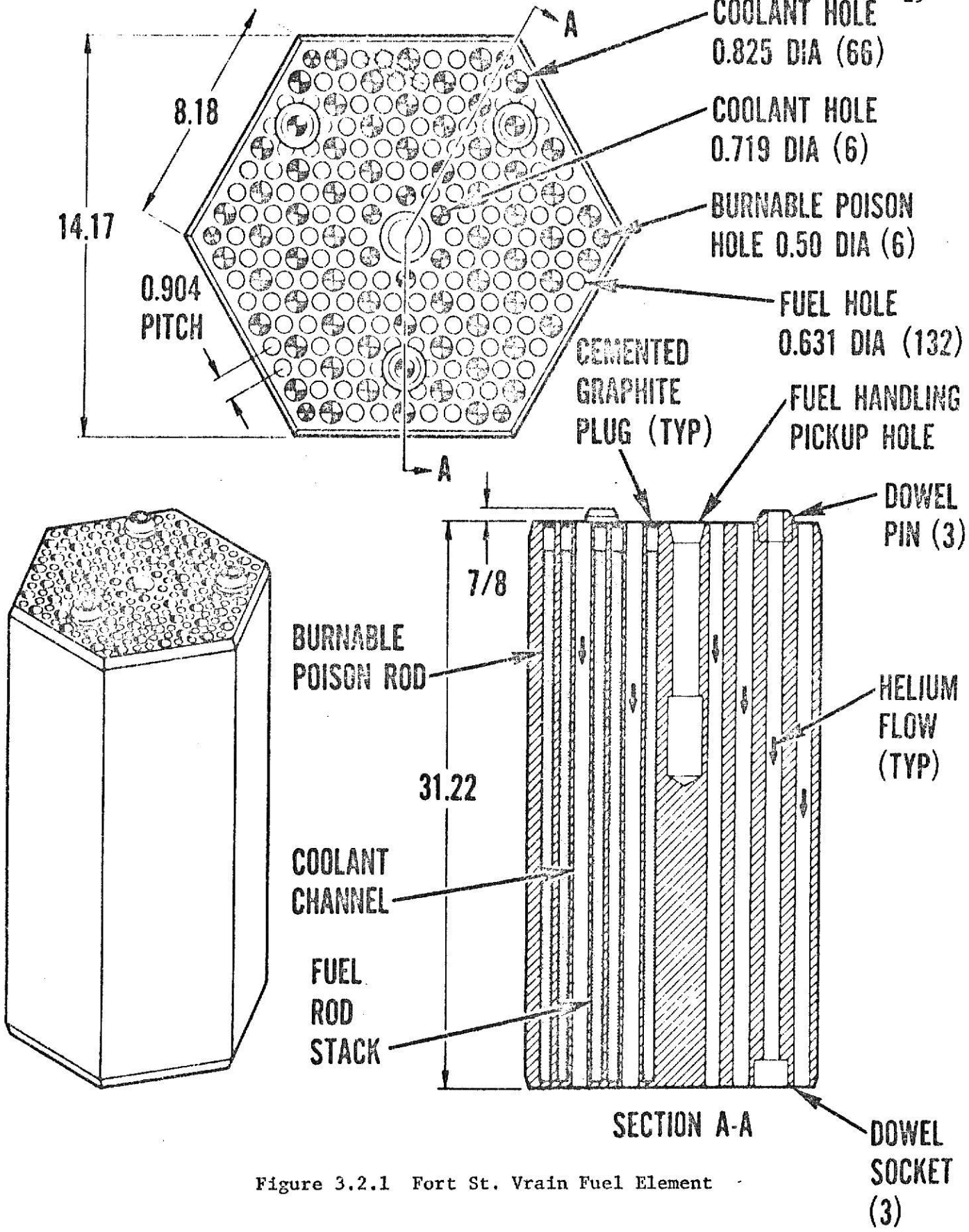


Figure 3.2.1 Fort St. Vrain Fuel Element

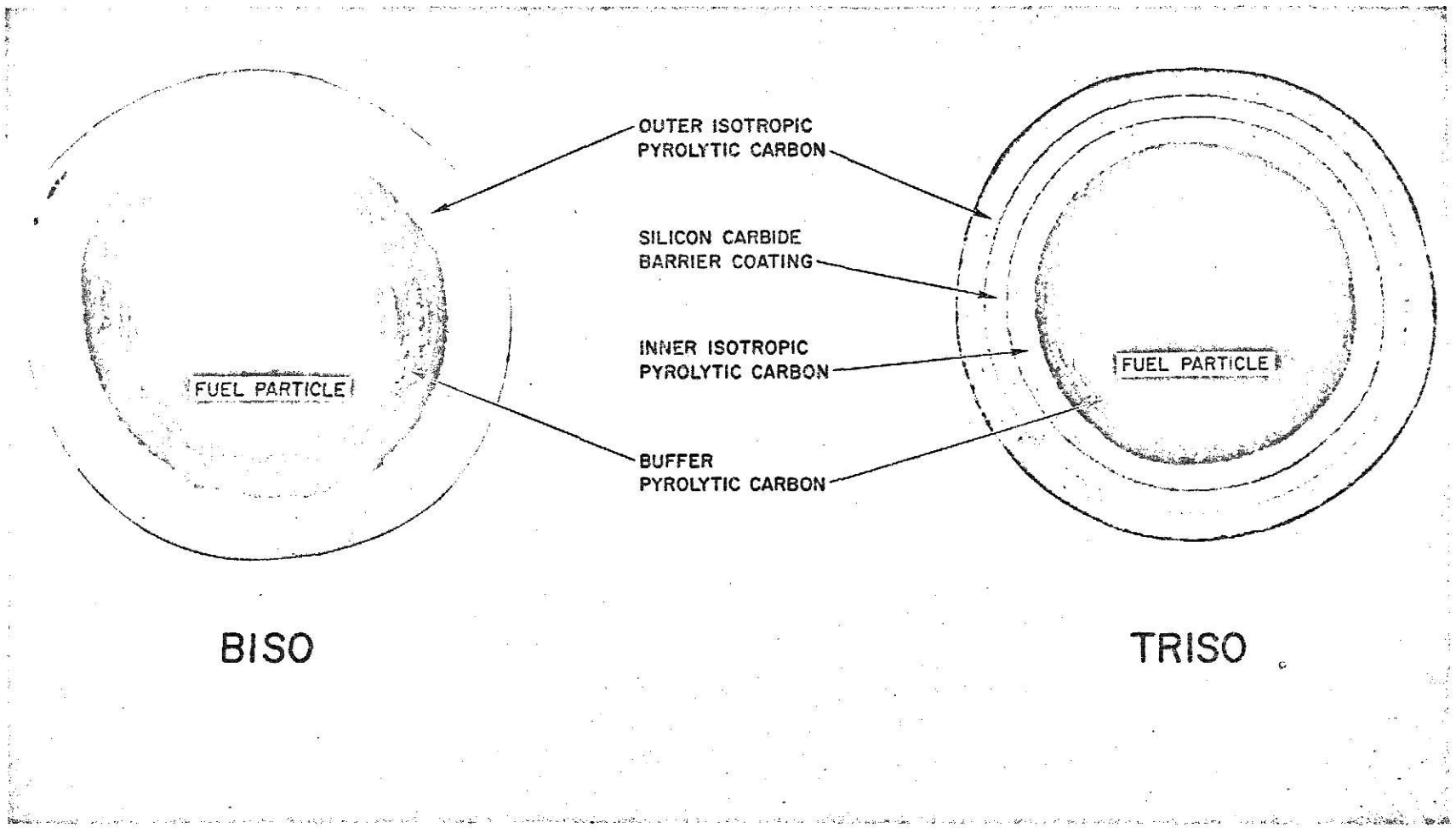


Figure 3.2.2 GA Coated Particles

TABLE II

Fuel Particle Parameters

Parameters	TRISO	Average Density (gm/cc)	BISO	Average Density (gm/cc)	Reference
Particle Composition	UC_2	11.28	ThC_2	8.96	(22)
Average Fuel Particle Dia.	460μ		610μ		(22)
Average Coating Thickness					
Outer isotropic carbon layer	30μ	1.95	50μ	1.95	(23)
Buffer pyrolytic carbon	45μ	1.50	30μ	1.50	(23)
Inner isotropic carbon layer	30μ	1.95	___	___	(23)
Silicon carbide	25μ	3.22	___	___	(23)

4.0 MODEL FORMULATION

When developing a model to simulate any type system, all of the essential constituents of the system must be examined. The essential components of the HTGR or any nuclear power producing reactor are the coolant, moderator, and fuel.

Helium gas is used as the coolant in the HTGR. The Helium gas is circulated through the reactor in order to remove the heat released in fission and in the decay of fission products. In this analysis, the coolant is treated somewhat as a "black box" that is, the coolant's interaction with the rest of the core system is not treated in detail, but only as a specific thermohydraulic limit that constrains the fissile loading of the fuel, i.e., total power requirements.

The moderator that is used in the HTGR core is graphite which also serves as the structural component for the HTGR (2), i.e., containment for the fuel and coolant. The main function of the moderator is to slow down the fast neutrons formed in fission to velocities at which they can be captured more readily by fertile and fissionable materials. In this paper no detailed study was made of the moderating material. Only the gross behavioral characteristics of the moderator are approximated by such factors as resonance escape probability, fast non-leakage probability, etc.

The reactor fuel consists of a mixture of fissionable and fertile materials. The fissile materials (U-235, U-233, Pu-239 and Pu-241) create the energy required to operate the reactor. Most of this energy is potentially created when the fissile material is split by a slowly moving neutron. The fertile materials, Th-232 and U-238, consume neutrons in order to produce additional fissionable material. The major fertile material in the HTGR is Th-232 while

U-238 is the minor fertile material. It may be noted that the lower the enrichment of uranium, the more dominant role U-238 plays as a fertile material. The production of fissile material by the fertile material is commonly called conversion, if the conversion ratio (CR), i.e., the formation of the fissile material divided by the depletion of the fissile material, is less than unity; or breeding if the CR is greater than unity.

The fuel is the main component in any reactor, thus it will be examined here in some detail. The detailed neutron balance can be found in many references (24,25,26); so it will not be presented here. It may, however, serve the reader to briefly define the major properties of the neutron cycle.

Those fast neutrons which have energies greater than about 1 MeV may cause a limited amount of fertile material to fission. To account for this high energy fissioning, the quantity, ϵ, usually called the "fast-fission factor," is defined as the ratio of the net rate of production of fast neutrons to the rate of production of fast neutrons by thermal fission. As the fast neutrons undergo scattering collisions, they are degraded in energy and also tend to diffuse toward the outer surface of the reactor where they can escape. The fraction of the fast neutrons which do not escape from the reactor as they degrade from fission to the so called resonance energy region depends upon the size and moderating properties of the reactor (26,24,27). This fraction can be denoted as P_1, "fission-to-resonance nonleakage probability." The resonance neutrons, i.e., those neutrons which have degraded to the kilovolt energy region, may be captured in fertile material or they may escape resonance absorption by undergoing elastic collisions with the moderator which degrades them to energies below the resonance energy region. The quantity p is defined as the fraction of the resonance neutrons which are not captured but are degraded to lower energies and is called the

"resonance escape probability." The fraction p is a function of the relative proportions and physical arrangement of the moderator and fertile material (26). The resonance absorption is very important in producing new fissionable material, i.e., Pu-239 from U-238 and U-233 from Th-232. Some neutrons diffuse to the outer surfaces of the reactor and escape, but the fraction P_2 that remains in the reactor and become thermal neutrons create the necessary critical chain reaction. The fraction P_2 is usually called the "resonance-to-thermal nonleakage probability." The thermal neutrons are eventually either absorbed in the fissile material, fertile material, or captured by the parasitic material in the core. This will be discussed more thoroughly when the reactivity equation is developed later in this chapter.

The HTGR has two types of fertile materials, i.e., Th-232 and U-238. This complexity can be simplified somewhat by putting a very high enrichment constraint on the uranium input, thus decreasing the importance of U-238. Since the enrichment is allowed to vary in this analysis, two sets of physical parameters for each type of fertile material must be established. These parameters will be specified later in this chapter.

To help distinguish between the various isotopes and their associated properties, the subscripting nomenclature used by Pigford and Benedict (26) is used in this paper. One can derive the subscript for each heavy nuclide and its associated properties by using the atomic number of the nuclear species minus 90 as the first digit in the subscript and the last digit in the subscript can be represented by the last digit of the mass number of the species being considered, i.e., N_{02}, σ_{02}, and p_{02} represent the number of atoms per unit volume of Th-232, absorption cross section of Th-232 and the resonance escape probability of Th-232, respectively.

In order to achieve an optimum policy of operation for any nuclear reactor, an adequate representation of the core characteristics must be formulated. Since the principle products of a power reactor are thermal energy and isotope production resulting from the irradiation of the fuel, the main consideration in simulating a nuclear reactor must be given to the formulation of the equations which predict the formation and depletion of the major nuclides in the core of the reactor. These equations are commonly called transmutation equations or "burnup" equations.

4.1 Burnup Equations

The composition of the fuel will change during irradiation, because of the depletion of initial fissionable material, buildup of fission products, and the buildup of nuclides in fissionable and fertile material. The change in composition of the fuel is proportional to the number of nuclides present, i.e., concentration per unit volume, and to the concentration of the neutrons, i.e., the neutron flux. There are many nuclide chain reactions taking place in a nuclear reactor, and to simulate them all theoretically would be an insurmountable task; therefore reactor designers usually only consider the major chains which contain the most significant fissile and fertile materials.

The significant parts of the nuclide chains in a thermal neutron spectrum associated with Th-232 and U-238 are shown in Fig. 4.1.1. The horizontal arrows indicate neutron capture events while the verticle arrows indicate beta decay processes. The numbers on the decay arrows indicate the half-lives for radioactive decay. From Fig. 4.1.1, one can see how the various species from the two major nuclide chains are related.

*The branching ratios shown are for a typical thermal neutron spectrum

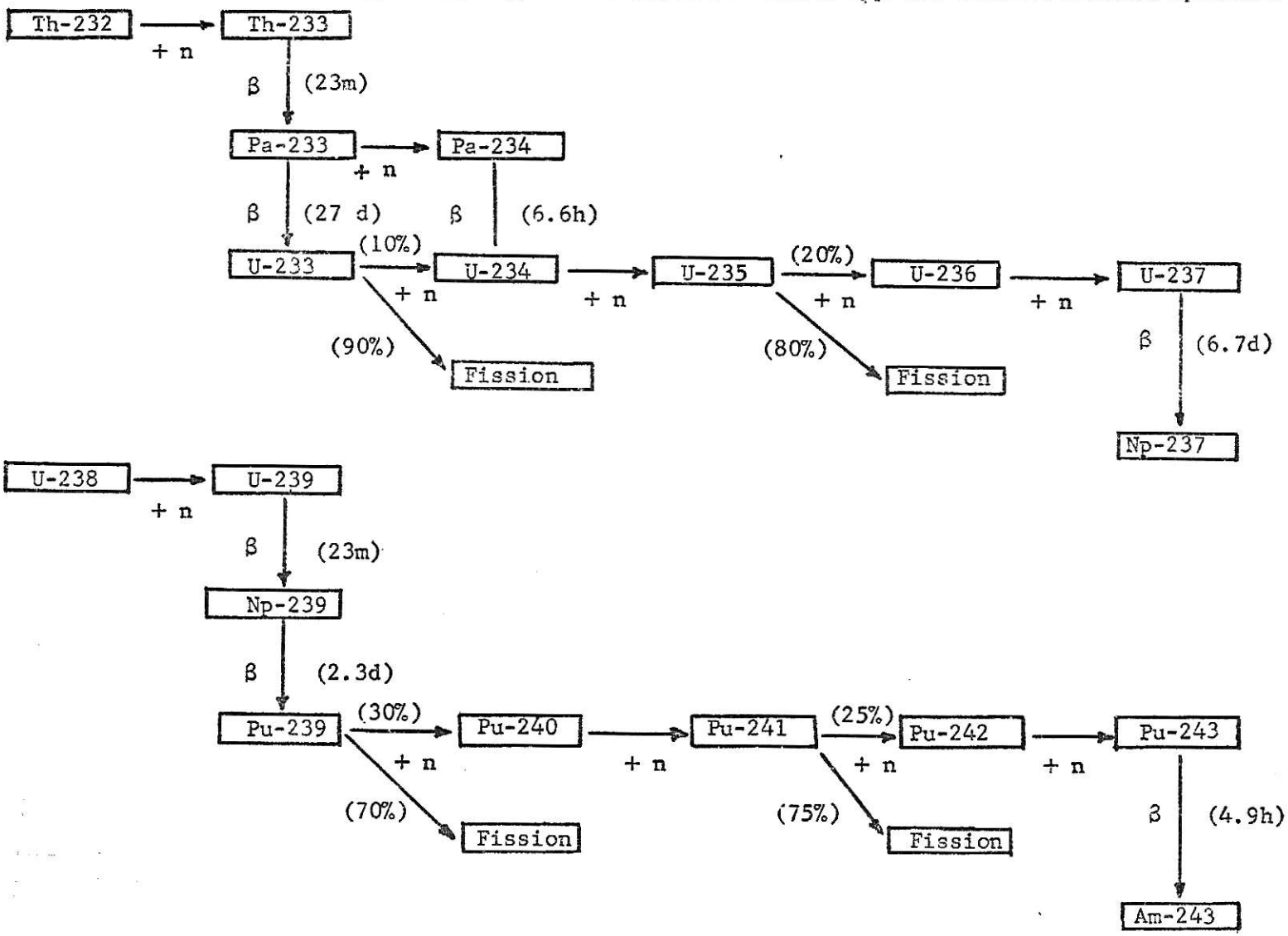


Figure 4.1.1 Transmutation Schemes for Th-232 and U-238*

Some major assumptions are made while deriving the transmutation equations which pertain to Fig. 4.1.1, and it may help the reader to ascertain the reasoning behind these assumptions by presenting the following explanation.

Through the capture of a neutron (as shown in Fig. 4.1.1), U-238 at first transmutates into U-239 which is in turn transformed into Pu-239 by double beta-decay. It may be noted that some of the nuclides produced with a mass number 239 are lost because Np-239 with an absorption cross-section of about 60 barns (25) can also absorb neutrons, being transformed into Np-240. Since the latter and its successors are not significantly fissile, the transmutation into Np-240 represents a genuine loss of neutrons. Since the conversion of Np-239 into Np-240 is neglected in this work, the following simple example will be given to demonstrate the significance of this conversion.

If the concentration per unit volume of Np-239 nuclei is denoted by N_{39}, $\lambda_{39}N_{39}$ is the number of nuclei changed per unit time into Pu-239 by beta-decay, where λ_{39} represents the decay constant of Np-239. The quantity, $\sigma_{39}N_{39}\phi$, represents the number per unit time changed into Np-240 where σ_{39} is the neutron capture cross section of Np-239 and ϕ represents the neutron flux. Therefore the fraction of the Np-239 nuclei decaying can be given as

$$r = \frac{\lambda_{39}N_{39}}{\lambda_{39}N_{39} + \sigma_{39}N_{39}\phi} = \frac{\lambda_{39}}{\lambda_{39} + \sigma_{39}\phi} \qquad (4.1.1)$$

To gain insight into the magnitude of this loss, consider the flux of the HTGR as being $\phi = 10^{14}$ n·cm^{-2}·sec^{-1}, $\sigma_{39} = 60 \times 10^{-24}$ cm^2, and $\lambda_{39} = 3.5 \times 10^{-6}$ sec^{-1}. With these values the fraction, r, is found to be .998; only a .2% loss due to neutron capture in Np-239, which can be neglected in most cases.

Another assumption made in this work is that the portion of neutrons lost through the capture of a neutron is Pa-233 is not large enough to make

a significant change in the neutron cycle. In the HTGR, the average neutron capture cross section, σ_{13}, for Pa-233 is 70 b (28). If the capture reaction takes place, Pa-233 changes into the isotope Pa-234 as shown in Fig. 4.1.1. Since protactinium 233 has a half-life of 27 days, the capture reaction probability is correspondingly high. For the loss in this case Eq. (4.1.1) is also valid. To illustrate this loss an example can be given similar to the one given above. By letting $\phi = 10^{14}$ n·cm^{-2}·sec^{-1}, $\lambda_{13} = 3 \times 10^{-7}$ sec^{-1}, and $\sigma_{13} = 70 \times 10^{-24}$ cm^2, the value of Eq. (4.1.1) can be computed as

$$r = .977$$

which implies only a 2.3% loss due to neutron capture in Pa-233. It is interesting to note that the neutron loss in the capture of Pa-233 is only lost to U-233 and not to U-235. It may also be useful to note that for higher flux levels the loss due to capture in Pa-233 becomes more appreciable. Assume a flux level of 5×10^{14} n·cm^{-2}·sec^{-1}; therefore Eq. (4.1.1) takes the value of .896 which represents a 10.4% loss to Pa-234, a sizable loss which in most cases cannot be neglected. With the above assumptions in mind the "burnup" equations will first be presented in matrix notation and then the corresponding equations will be explicitly derived.

The burnup dependent isotopes in a nuclear reactor are transformed, as stated previously, during the irradiation through neutron capture, fission, and decay. The burnup equations can be written in matrix notation as

$$\frac{d}{dt} \underset{\sim}{N}(t) = \underset{\sim}{\bar{A}} \underset{\sim}{N}(t) \qquad (4.1.2)$$

where the atomic densities of the burnup dependent nuclides, expressed as components of the vector $\underset{\sim}{N}(t)$, satisfy the initial condition:

$$\underset{\sim}{N}(0) = \underset{\sim}{N}^{o} \qquad (4.1.3)$$

and $\bar{\bar{A}}$ is a constant nxn matrix, dependent mainly on the cross sections of the materials considered where n represents the number of state variables in the system. The solution of Eq. (4.1.2) found after applying the boundary condition of Eq. (4.1.3) can be written as:

$$\underset{\sim}{N}(t) = \underset{\sim}{B}(t)\ \underset{\sim}{N}^{o} \tag{4.1.4}$$

where $\underset{\sim}{B}(t)$ is the state transition matrix, and $\underset{\sim}{N}^{o}$ represents the initial concentration of the nuclides.

The elements of the state transition matrix are often time-dependent, and the matrix $\underset{\sim}{B}(t)$ must be evaluated by numerical techniques (27,29,9,30). An analytical solution is possible when both the neutron flux and the microscopic cross sections are constant during the fuel irradiation (30). Since this work is somewhat of a survey of the HTGR fuel cycle, the neutron flux level and the microscopic cross sections are held constant throughout the life of each cycle.

To evaluate the state transition matrix as given in Eq. (4.1.4), it is necessary to define the species being analyzed. From Fig. 4.1.1, the major nuclides may be given in matrix notation as

$$\underset{\sim}{N} = \begin{pmatrix} N_{23} \\ N_{28} \\ N_{02} \\ N_{25} \\ N_{26} \\ N_{49} \\ N_{40} \\ N_{41} \\ N_{24} \end{pmatrix} \tag{4.1.5}$$

where the nomenclature used has been defined previously. Each row in the transition matrix $\underset{\sim}{B}(t)$ represents the coefficients of the burnup equation for the

corresponding element in Eq. (4.1.5). To develop an expression for the transition matrix $\underset{\sim}{B}(t)$, the burnup equations for each of the elements in Eq. (4.1.5) must be derived.

In most derivations of burnup equations, a useful notation commonly referred to as "flux time" or fluence is used (26,7,8,9,10,30). The fuel is exposed to a thermal neutron flux $\phi(t)$ which is usually a function of time. Since the flux is assumed in this analysis to be a constant value throughtout the life of the cycle, the flux will only be a function of the radial distance in the core, averaged over each region considered; therefore it is represented as a constant flux, $\bar{\phi}$, and the expression for "flux time" can be given by

$$\theta \equiv \int_0^t \bar{\phi}\, dt' \quad . \tag{4.1.6}$$

Therefore Eq. (4.1.4) can be written in terms of "flux time" as

$$\underset{\sim}{N}(\theta) = \underset{\sim}{B}(\theta)\ \underset{\sim}{N}^{o} \tag{4.1.7}$$

With above definitions in mind, the individual burnup equations can be developed and solved as follows.

Uranium 233

The rate of change for the number of atoms of U-233 per unit volume can be given as

$$\frac{dN_{23}}{dt} = \underbrace{N_{02}\,\sigma_{02}\,\phi}_{\left\{\begin{array}{l}\text{Gain due to the}\\ \text{absorption of}\\ \text{thermal neutrons}\\ \text{in Th-232.}\end{array}\right\}} + \underbrace{N_{23}\,\sigma_{23}\,\eta_{23}\,\varepsilon\,P_1(1-p_{02})\phi}_{\left\{\begin{array}{l}\text{Gain due to the absorption}\\ \text{of resonance neutrons in}\\ \text{Th-232 from U-233 fission.}\end{array}\right\}}$$

$$+ \underbrace{N_{25}\,\sigma_{25}\,\eta_{25}\,\varepsilon\,P_1(1-p_{02})\phi}_{\left\{\begin{array}{l}\text{Gain due to the}\\ \text{absorption of resonance}\\ \text{neutrons in Th-232 from}\\ \text{U-235 fission.}\end{array}\right\}} - \underbrace{N_{23}\,\sigma_{23}\,\phi}_{\left\{\begin{array}{l}\text{Loss due to the absorption}\\ \text{of thermal neutrons in U-233.}\end{array}\right\}} \tag{4.1.8}$$

where N_{ij} represents the number of atoms per unit volume, σ_{ij} represents the absoroption cross, N_{ij} represents the number of neutrons produced per number of neutrons absorbed, p_{ij} represents the resonance escape probability where the ij subscripting pertains to the nuclide being considered, as defined previously, and ϕ represents the thermal flux. In developing Eq. (4.1.8) it is assumed that resonance capture by Th-232 from the fission of Pu-239 and Pu-241 can be neglected; since the concentrations of these fissile materials are generally very low in the HTGR.

In notation defined earlier, Eq. (4.1.8) can be written in terms of "flux time" as

$$\frac{dN_{23}}{d\theta} = N_{02}\,\sigma_{02} + N_{23}\,\sigma_{23}(L_1-1) + N_{25}\,\sigma_{25}\,L_2 \qquad (4.1.9)$$

where $L_1 = \eta_{23}\varepsilon\, P_1(1-p_{02})$ and $L_2 = \eta_{25}\varepsilon\, P_1(1-p_{02})$.

From the derivation given in Appendix A, the solution to Eq. (4.1.9) can be given as

$$N_{23}(\theta) = X' \exp(R_1\theta) + Y' \exp(R_2\theta) + Z' \exp(-\sigma_{25}\theta) \qquad (4.1.10)$$

where all constants are defined in Appendix A.

To prevent the burnup equations of the nuclides at the end of the nuclide chain, i.e., Pu-239, Pu-240, and Pu-241, given in Fig. 4.1.1 from being too complex, a more approximate form of Eq. (4.1.10) is derived by assuming that the transformation of Th-232 can be represented by a simple exponential depletion term; therefore Eq. (4.1.8) can be solved by the use of an integrating factor as given in Appendix B. It was found, by comparing the results from the two derivations, that the amount of U-233 after burnup was approximately 5% higher when using Eq. (4.1.10) than found when using Eq. (B-8) as given in Appendix B. The solution to Eq. (4.1.9) can be given as

$$N_{23}(\theta) = N_{23}^{o}\{\exp[-\sigma_{23}(1-L_1)\theta]\}$$

$$+ \frac{N_{02}^{o}\,\sigma_{02}}{\sigma_{23}(1-L_1) - \sigma_{02}}\{\exp(-\sigma_{02}\theta) - \exp[-\sigma_{23}(1-L_1)\theta]\}$$

$$+ \frac{N_{02}^{o}\,\sigma_{25}\,L_2}{\sigma_{23}(1-L_1) - \sigma_{25}}\{\exp(-\sigma_{25}\theta) - \exp[-\sigma_{23}(1-L_1)\theta]\} \qquad (4.1.11)$$

where all constants have previously been defined.

Thorium 232

The rate of change of the number of atoms of Th-232 per unit volume is given as

$$\frac{dN_{02}}{dt} = -N_{02}\,\sigma_{02}\,\phi - N_{23}\,\sigma_{23}\,\eta_{23}\,\varepsilon\,P_1(1-p_{02})\phi$$

{Loss due to the absorption of thermal neutrons in Th-232} {Loss due to the absorption of resonance neutrons in Th-232 from U-233 fission}

$$- N_{25}\,\sigma_{25}\,\eta_{25}\,\varepsilon\,P_1(1-p_{02})\phi \qquad (4.1.12)$$

{Loss due to absorption of resonance neutrons in Th-232 from U-225 fission}

Equation (4.1.12) can be rewritten as

$$\frac{dN_{02}}{d\theta} + N_{02}\,\sigma_{02} + N_{23}\,\sigma_{23}\,L_1 = N_{25}\,\sigma_{25}\,. \qquad (4.1.13)$$

The solution to Eq. (4.1.13) was attained in Appendix A and can be given as

$$N_{02}(\theta) = X\exp(R_1\theta) + Y\exp(R_2\theta) + Z\exp(-\sigma_{25}\theta)\,. \qquad (4.1.14)$$

where all constants are defined in Appendix A.

Uranium 234

The net rate of accumulation of U-234 is given by

$$\frac{dN_{24}}{dt} = -\sigma_{24} N_{24} \phi + \frac{N_{23} \sigma_{23} \alpha_{23} \phi}{1 + \alpha_{23}} \qquad (4.1.15)$$

{Loss due to the absorption of thermal neutrons in U-234} {Gain due to the capture of thermal neutrons in U-233}

where $\alpha_{23} = \frac{\sigma_{23}^{c}}{\sigma_{23}^{f}} = \frac{\text{capture cross section of U-233}}{\text{fission cross section of U-233}}$,

σ_{24} represents the absorption cross section of U-234 and N_{24} represents the number of U-234 atoms per unit volume.

By substituting Eq. (4.1.11) into Eq. (4.1.15) and using the appropriate integrating factor, the solution to Eq. (4.1.15) at $N_{24}(0) = N_{24}^{o}$ can be given in "flux time" notation as

$$N_{24}(\theta) = N_{24}^{o} \exp(-\sigma_{24}\theta)$$
$$+ N_{23}^{o} \left\{\frac{C11\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{24}\theta)\}}{\sigma_{24} - \sigma_{23}(1-L_1)}\right\}$$
$$+ N_{02}^{o} \left\{\frac{C11 \cdot C16}{C15}\left\{\frac{\exp(-\sigma_{02}\theta)-\exp(-\sigma_{24}\theta)}{\sigma_{24} - \sigma_{02}} - \frac{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{24}\theta)}{\sigma_{24} + \sigma_{23}(L_1-1)}\right\}\right\}$$
$$+ N_{25}^{o} \left\{\frac{C11 \cdot C17}{C15}\left\{\frac{\exp(-\sigma_{25}\theta)-\exp(-\sigma_{24}\theta)}{\sigma_{24} - \sigma_{25}} - \frac{\exp[\sigma_{23}(L_1-1)\theta]-\exp(-\sigma_{24}\theta)}{\sigma_{24} + \sigma_{23}(L_1-1)}\right\}\right\}$$

(4.1.16)

where $C11 = \frac{\alpha_{23}}{\alpha_{23} + 1}$; $C15 = \kappa_{23} \sigma_{23}$; $\kappa_{23} = \eta_{23} \epsilon P_1 (1-p_{02})$;

$$C16 = \frac{C15 \cdot \sigma_{02}}{\sigma_{23}(1-L_1) - \sigma_{02}} \quad ; \quad C17 = \frac{C15 \cdot \sigma_{25} L_2}{\sigma_{23}(1-L_1) - \sigma_{25}} \quad .$$

Uranium 235

The net rate of accumulation of U-235 can be given by

$$\frac{dN_{25}}{dt} = N_{24}\,\sigma_{24}\,\phi \qquad - \; N_{25}\,\sigma_{25}\,\phi \tag{4.1.17}$$

$$\left\{\begin{matrix}\text{Gain due to the}\\ \text{neutron capture}\\ \text{in U-234}\end{matrix}\right\} \quad \left\{\begin{matrix}\text{Loss due to the}\\ \text{neutron absorption}\\ \text{in U-235}\end{matrix}\right\}$$

where it is assumed that the absorption cross section is equal to the capture cross section of U-234, i.e., no fission reaction in U-234.

Equation (4.1.17) can be given in terms of "flux time" as

$$\frac{dN_{25}}{d\theta} = N_{24}\,\sigma_{24} - N_{25}\,\sigma_{25} \tag{4.1.18}$$

If Eq. (4.1.16) is substituted into Eq. (4.1.18), the solution to Eq. (4.1.18) at $N_{25}(0) = N_{25}^{o}$ can be given by using an integrating factor as

$$\begin{aligned} N_{25}(\theta) = {} & N_{25}^{o}\left\{C91[\exp(-\sigma_{25}\theta)-\exp(-\sigma_{24}\theta)] - C92\{\exp[\sigma_{23}(L_1-1)\theta]-\exp(-\sigma_{25}\theta)\}\right. \\ & \left. - C93[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{25}\theta)] - (C94-1)\exp(-\sigma_{25}\theta)\right\} \\ & + N_{24}^{o}\{C81[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{25}\theta)]\} \\ & + N_{23}^{o}\left\{C82\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{25}\theta)\} - C83[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{25}\theta)]\right\} \\ & + N_{02}^{o}\left\{C84[\exp(-\sigma_{02}\theta)-\exp(-\sigma_{25}\theta)] - C85[\exp(-\sigma_{25}\theta)-\exp(-\sigma_{25}\theta)]\right. \\ & \left. - C86\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{25}\theta)\} + C87[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{25}\theta)]\right\} \end{aligned} \tag{4.1.19}$$

where $C81 = \dfrac{\sigma_{24}}{\sigma_{25}-\sigma_{24}}$; $C82 = \dfrac{\sigma_{24}\cdot C89}{\sigma_{25}-\sigma_{23}(1-L_1)}$; $C83 = \dfrac{\sigma_{24}\cdot C89}{\sigma_{25}-\sigma_{24}}$;

$C84 = \dfrac{\sigma_{25}\cdot C88}{\sigma_{25}-\sigma_{02}}$; $C85 = \dfrac{\sigma_{25}\cdot C88}{\sigma_{25}-\sigma_{24}}$; $C86 = \dfrac{\sigma_{24}-C90}{\sigma_{25}-\sigma_{23}(1-L_1)}$

$C87 = \dfrac{\sigma_{24}\cdot C90}{\sigma_{25}-\sigma_{24}}$; $C88 = \dfrac{C11\cdot C16}{C16(\sigma_{24}-\sigma_{02})}$; $C89 = \dfrac{C11}{\sigma_{24}-\sigma_{24}(1-L_1)}$

$$C90 = \frac{C11 \cdot C16}{C15[\sigma_{24} + \sigma_{23}(L_1-1)]} ; \quad C901 = \frac{C11 \cdot C17}{C15(\sigma_{25} - \sigma_{24})} ;$$

$$C902 = \frac{C11 \cdot C17}{C15[\sigma_{24} + \sigma_{23}(L_1-1)]} ; \quad C91 = \frac{\sigma_{24} \cdot C901}{\sigma_{25} - \sigma_{24}} ;$$

$$C92 = \frac{\sigma_{24} \cdot C902}{\sigma_{25} + \sigma_{23}(L_1-1)} ; \quad C93 = \frac{\sigma_{24} \cdot C902}{\sigma_{25} - \sigma_{24}} ; \quad C94 = \sigma_{24} \cdot C901 \cdot \theta .$$

Uranium 236

The net rate of accumulation of U-236 can be given by

$$\frac{dN_{26}}{dt} = \frac{N_{25}\,\sigma_{25}\,\sigma_{25}\,\phi}{1 + \alpha_{25}} - N_{26}\,\sigma_{26}\,\phi \qquad (4.1.20)$$

{Gain due to the neutron capture in U-235} {Loss due to neutron absorption in U-236}

where N_{26} represents the number of U-236 atoms per unit volume and σ_{26} represents the microscopic absorption cross section for U-236.

The solution to Eq. (4.1.20) subject to $N_{26}(0) = N_{26}^{o}$ can be attained by substituting Eq. (4.1.19) into Eq. (4.1.20) and using the appropriate integrating factor. The solution is given as

$$\begin{aligned} N_{26}(\theta) = {} & N_{26}^{o} \exp(-\sigma_{26}\theta) \\ & + N_{25}^{o} \left(D15[\exp(-\sigma_{25}\theta)-\exp(-\sigma_{26}\theta)] + D17[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{26}\theta)]\right. \\ & \qquad \left. - D18\{\exp[\sigma_{23}(L_1-1)\theta]-\exp(-\sigma_{26}\theta)\}\right) \\ & + N_{24}^{o} \{D19[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{26}\theta)] - D20[\exp(-\sigma_{25}\theta)-\exp(-\sigma_{26}\theta)]\} \\ & + N_{23}^{o} \left(D21[\exp(-\sigma_{25}\theta)-\exp(-\sigma_{25}\theta)] + D22\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{26}\theta)\}\right. \\ & \qquad \left. - D23[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{26}\theta)]\right) \end{aligned} \qquad (4.1.21)$$

where $D11 = \frac{\sigma_{25}\,\alpha_{25}}{1+\alpha_{25}}$; $D12 = \sigma_{26} - \sigma_{25}$; $D13 = \sigma_{26} - \sigma_{24}$; $D14 = \sigma_{26} + \sigma_{23}(L_1-1)$;

$$D15 = \frac{D11 \cdot (C92-C93+1)}{D12} \; ; \; D17 = \frac{D11 \cdot C93}{D13} \; ; \; D18 = \frac{D11 \cdot C92}{D14} \; ;$$

$$D19 = \frac{D11 \cdot C91}{D13} \; ; \; D20 = \frac{D11 \cdot C81}{D12} \; ; \; D21 = \frac{D11 \cdot (C83-C82)}{D12} \; ;$$

$$D22 = \frac{D11 \cdot C82}{D14} \; ; \; D23 = \frac{D11 \cdot C83}{D13} \; .$$

Uranium 238

Although U-238 will only represent a small percentage of the nuclides in the initial loading of the 1160 MWe HTGR core, it must be considered in this analysis since the enrichment was allowed to vary in the LP iterative scheme.

The rate of change of the number of atoms of U-238 per unit volume can be given as

$$\frac{dN_{28}}{dt} = - N_{28}\,\sigma_{28}\,\phi - N_{25}\,\sigma_{25}\,\eta_{25}\,\varepsilon\,P_1(1-p_{28})\phi$$

$$\left\{\begin{array}{l}\text{Loss due to the}\\ \text{absorption of thermal}\\ \text{neutrons in U-238}\end{array}\right\} \quad \left\{\begin{array}{l}\text{Loss due to the absorption of}\\ \text{resonance neutrons in U-238 from}\\ \text{U-235 fission}\end{array}\right\}$$

$$- N_{23}\,\sigma_{23}\,\eta_{23}\,\varepsilon\,P_1(1-p_{28})\phi \qquad (4.1.22)$$

$$\left\{\begin{array}{l}\text{Loss due to the absorption of resonance}\\ \text{neutrons in U-238 from U-233 fission}\end{array}\right\}$$

where all terms have been defined earlier in this Chapter.

It may be noted by the reader that Pu-239 and Pu-241 were not included in the development of Eq. (4.1.22). These two fissile nuclides are neglected since their concentration in the HTGR is so low. In studies that have been made concerning proposed cross-progeny fuel cycles (32,31), Pu-239 can be used as the primary fissile feed material. If cross-progeny fuel cycles

are considered in future work using this model, another term containing Pu-239 resonance capture must be added to Eqs. (4.1.12) and (4.1.22).

Equation (4.1.22) can be rewritten using notation defined by Eq. (4.1.6) as

$$\frac{dN_{28}}{d\theta} = -N_{28}\,\sigma_{28} - N_{25}\,\sigma_{25}\,\kappa_{25} - N_{23}\,\sigma_{23}\,\kappa_{23} \qquad (4.1.23)$$

where $\kappa_{25} = \eta_{25}\,\varepsilon\,P_1(1-p_{28})$; $\kappa_{23} = \eta_{23}\,\varepsilon\,P_1(1-p_{28})$; N_{28} represents the number of U-238 atoms per unit volume, σ_{28} represents the microscopic absorption cross section for U-238, and p_{28} represents resonance escape probability in U-238. The use of the appropriate integrating factor and the substitution of Eq. (4.1.19) and (4.1.11) into Eq. (4.1.23) yields the solution to Eq. (4.1.23) at $N_{28}(0) = N_{28}^{o}$ as

$$\begin{aligned}
N_{28}(\theta) = {} & N_{28}^{o}\exp(-\sigma_{28}\theta) \\
& + N_{25}^{o}\left[C13\cdot C93\left[\frac{\exp(-\sigma_{24}\theta)-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{24}} - \frac{\exp(-\sigma_{25}\theta)-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{25}}\right]\right. \\
& \quad + C13\left[\frac{\exp(-\sigma_{25}\theta)-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{25}}\right] \\
& \quad - C92\left\{\frac{\exp[\sigma_{23}(L_1-1)\theta]-\exp(-\sigma_{28}\theta)}{\sigma_{28}+\sigma_{23}(L_1-1)} - \frac{\exp(-\sigma_{25}\theta)-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{25}}\right\} \\
& \quad \left. + C17\left\{\frac{\exp(-\sigma_{25}\theta)-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{25}} - \frac{\exp[\sigma_{23}(L_1-1)\theta]-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{23}(L_1-1)}\right\}\right] \\
& + N_{23}^{o}\left[C13\cdot C82\left\{\frac{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{23}(1-L_1)} - \frac{\exp(-\sigma_{25}\theta)-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{25}}\right\}\right. \\
& \quad \left. - C13\cdot C83\left[\frac{\exp(-\sigma_{24}\theta)-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{24}} - \frac{\exp(-\sigma_{25}\theta)-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{25}}\right]\right.
\end{aligned}$$

$$+ C15 \left\{\frac{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{23}(1-L_1)}\right\}\Bigg)$$

$$+ N_{24}^{o}\left\{C13 \cdot C81\left[\frac{\exp(-\sigma_{24}\theta)-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{24}} - \frac{\exp(-\sigma_{25}\theta)-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{25}}\right]\right\}$$

$$+ N_{02}^{o}\left[C16\left\{\frac{\exp(-\sigma_{02}\theta)-\exp(-\sigma_{28}\theta)}{\sigma_{28}-\sigma_{02}} - \frac{\exp[\sigma_{23}(L_1-1)\theta]-\exp(-\sigma_{28}\theta)}{\sigma_{28}+\sigma_{23}(L_1-1)}\right\}\right] \qquad (4.1.24)$$

where $C13 = \sigma_{25}\,\kappa_{25}$ and $\kappa_{25} = \eta_{25}\,\varepsilon\,P_1(1-p_{28})$.

Plutonium 239

The net rate of accumulation for Pu-239 can be given as

$$\frac{dN_{49}}{dt} = N_{28}^{o}\,\sigma_{28}\,\phi + N_{25}\,\sigma_{25}\,\eta_{25}\,\varepsilon\,P_1(1-p_{28})\phi$$

{Gain from neutron capture in U-238} {Gain from resonance capture in U-238 from U-235 fission}

$$+ N_{23}\,\sigma_{23}\,\eta_{23}\,\varepsilon\,P_1(1-p_{28})\phi - N_{49}\,\sigma_{49}\,\eta_{49}\,\varepsilon\,P_1(1-p_{28})\phi$$

{Gain from resonance capture in U-238 from U-233 fission.} {Gain from resonance capture in U-238 from Pu-239 fission}

$$- N_{49}\,\sigma_{49}\,\phi \qquad (4.1.25)$$

{Loss due to neutron absorption in Pu-239}

where N_{49} represents the number of atoms of Pu-239 per unit volume, σ_{49} represents the microscopic absorption cross section for Pu-239, and η_{49} represents the number of neutrons produced per number of neutrons absorbed in Pu-249.

It was assumed in the development of Eq. (4.1.25), that the concentration of U-238 remains constant.

It may also be noted that the Pu-239 resonance capture term was included in Eq. (2.1.25). This can be done since the addition of that term doesn't complicate the derivation.

Equation (4.1.25) can be written as

$$\frac{dN_{49}}{d\theta} = N_{28}^{o}\,\sigma_{28} + N_{25}\,\sigma_{25}\,\kappa_{25} + N_{23}\,\sigma_{23}\,\kappa_{23} - N_{49}\,\sigma_{49}\,\gamma \qquad (4.1.26)$$

where $\gamma = 1 - \kappa_{49}$.

By substituting Eqs. (4.1.19) and (4.1.11) into Eq. (4.1.26), the solution to Eq. (4.1.26) at $N_{49}(0) = N_{49}^{o}$, can be given as

$$\begin{aligned}
N_{49}(\theta) = {} & N_{49}^{o}\exp(-\gamma\,\sigma_{49}\theta) \\
& + N_{28}^{o}\cdot C18[1-\exp(-\gamma\,\sigma_{49}\theta)] \\
& + N_{25}^{o}\,\big(C19[\exp(-\sigma_{25}\theta)-\exp(-\gamma\,\sigma_{49}\theta)] - C91\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\gamma\,\sigma_{49}\theta)\} \\
& \quad + C27[\exp(-\sigma_{25}\theta)-\exp(-\gamma\,\sigma_{49}\theta)] - C28\{\exp[\sigma_{23}(L_1-1)\theta]-\exp(-\gamma\,\sigma_{49}\theta)\} \\
& \quad + C196[\exp(-\sigma_{24}\theta)-\exp(-\gamma\,\sigma_{49}\theta)] - C197[\exp(-\sigma_{25}\theta)-\exp(-\gamma\,\sigma_{49}\theta)]\big) \\
& + N_{23}^{o}\,\big(C24\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\gamma\,\sigma_{49}\theta)\} - C241[\exp(-\sigma_{25}\theta)-\exp(-\gamma\,\sigma_{49}\theta)] \\
& \quad - C242[\exp(-\sigma_{24}\theta)-\exp(-\gamma\,\sigma_{49}\theta)] + C243[\exp(-\sigma_{25}\theta)-\exp(-\gamma\,\sigma_{49}\theta)] \\
& \quad + C244\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\gamma\,\sigma_{49}\theta)\}\big) \\
& + N_{02}^{o}\,\big(C20[\exp(-\sigma_{02}\theta)-\exp(-\gamma\,\sigma_{49}\theta)] - C26\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\gamma\,\sigma_{49}\theta)\}\big)
\end{aligned} \qquad (4.1.27)$$

where $C18 = \dfrac{\sigma_{28}}{\gamma\,\sigma_{49}}$; $C19 = \dfrac{C13}{\gamma\,\sigma_{49} - \sigma_{25}}$; $C191 = \dfrac{C13\cdot C92}{C194}$; $C192 = \gamma\,\sigma_{49} - \sigma_{24}$;

$C193 = \gamma\,\sigma_{49} - \sigma_{25}$; $C194 = \gamma\,\sigma_{49} - \sigma_{23}(1-L_1)$; $C195 = \gamma\,\sigma_{49} - \sigma_{02}$;

$$C196 = \frac{C13 \cdot C93}{C192} \; ; \; C197 = \frac{C13 \cdot C93}{C193} \; ; \; C24 = \frac{C13 \cdot C82}{C194} \; ; \; C241 = \frac{C13}{C193} \; ;$$

$$C242 = \frac{C13 \cdot C83}{C192} \; ; \; C243 = \frac{C13}{C193} \; ; \; C244 = \frac{C15}{C194} \; ; \; C245 = \frac{C13 \cdot C81}{C192} \; ;$$

$$C246 = \frac{C13 \cdot C81}{C193} \; ; \; C20 = \frac{C16}{C195} \; .$$

Plutonium 240

The net rate of change for Pu-240 can be given by

$$\frac{dN_{40}}{dt} = \frac{\alpha_{49} N_{49} \sigma_{49} \phi}{1 + \alpha_{49}} - N_{40} \sigma_{40} \phi \qquad (4.1.28)$$

{Gain from the neutron capture in Pu-239} {Loss due to neutron absorption in Pu-240}

Equation (4.1.28) can also be written as

$$\frac{dN_{40}}{d\theta} = C29 \cdot N_{49} - \sigma_{40} N_{40}$$

where $C29 = \frac{\alpha_{49} \sigma_{49}}{1 + \alpha_{49}}$.

By using the appropriate integrating factor and substituting Eq. (4.1.27) into Eq. (4.1.28), the solution to Eq. (4.1.28) at $N_{40}(0) = N_{40}^{o}$ can be given as

$$N_{40}(\theta) = N_{40}^{o} \exp(-\sigma_{40}\theta)$$

$$+ N_{49}^{o} \, C43[\exp(-\gamma \sigma_{49}\theta) - \exp(-\sigma_{40}\theta)]$$

$$+ N_{28}^{o} \, \{C44[1-\exp(-\sigma_{40}\theta)] - C441[\exp(-\gamma \sigma_{49}\theta) - \exp(-\sigma_{40}\theta)]\}$$

$$+ N_{23}^{o} \, \big(C46[\exp[-\sigma_{23}(1-L_1)\theta] - \exp(-\sigma_{40}\theta)] + C461[\exp(-\sigma_{25}\theta) - \exp(-\sigma_{40}\theta)]$$

$$- C462[\exp(-\sigma_{24}\theta) - \exp(-\sigma_{40}\theta)] + C463[\exp(-\gamma \sigma_{49}\theta) - \exp(-\sigma_{40}\theta)]\big)$$

$$+ N_{24}^{o} \{C464[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{40}\theta)] - C465[\exp(-\sigma_{25}\theta)-\exp(-\sigma_{40}\theta)]$$

$$+ C466[\exp(-\gamma\ \sigma_{49}\theta)-\exp(-\sigma_{40}\theta)]\}$$

$$+ N_{02}^{o} \{C467[\exp(-\sigma_{02}\theta)-\exp(-\sigma_{40}\theta)] - C468\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{40}\theta)\}$$

$$+ C469[\exp(-\gamma\ \sigma_{49}\theta)-\exp(-\sigma_{40}\theta)]\} \qquad (4.1.29)$$

where $C26 = \dfrac{C16}{\gamma\ \sigma_{49} + \sigma_{23}(L_1-1)}$; $C27 = \dfrac{C19 \cdot C17}{C13}$; $C28 = \dfrac{C26 \cdot C17}{C16}$;

$C30 = C27 + C19 - C197$; $C31 = C28 + C91$; $C32 = C191 + C28 + C197 - C19 - C27 - C196$; $C33 = C24 + C244$; $C34 = C243 - C241$;

$C35 = C241 - C242 - C24 - C243 - C244$; $C36 = C246 - C245$;

$C37 = C26 - C20$; $C38 = \sigma_{40} - \gamma\ \sigma_{49}$; $C39 = \sigma_{40} - \sigma_{25}$;

$C40 = \sigma_{40} - \sigma_{23}(1-L_1)$; $C41 = \sigma_{40} - \sigma_{24}$; $C42 = \sigma_{40} - \sigma_{02}$;

$C43 = \dfrac{C29}{C38}$; $C44 = \dfrac{C29 \cdot C18}{\sigma_{40}}$; $C441 = \dfrac{C29 \cdot C18}{C38}$; $C45 = \dfrac{C30 \cdot C29}{C39}$;

$C451 = \dfrac{C29 \cdot C31}{C40}$; $C452 = \dfrac{C29 \cdot C18}{C41}$; $C453 = \dfrac{C29 \cdot C32}{C38}$;

$C46 = \dfrac{C29 \cdot C33}{C40}$; $C461 = \dfrac{C29 \cdot C34}{C39}$; $C462 = \dfrac{C29 \cdot C242}{C41}$;

$C463 = \dfrac{C29 \cdot C35}{C38}$; $C464 = \dfrac{C29 \cdot C245}{C40}$; $C465 = \dfrac{C29 \cdot C246}{C39}$;

$C466 = \dfrac{C29 \cdot C36}{C38}$; $C467 = \dfrac{C29 \cdot C20}{C42}$; $C468 = \dfrac{C29 \cdot C26}{C40}$;

$C469 = \dfrac{C29 \cdot C37}{C38}$; $C47 = C441 - C44$; $C48 = C51 - C45 - C452 - C453$;

$C49 = C462 - C46 - C461 - C463$; $C50 = C465 = C464 - C466$;

$C501 = C468 - C467 - C469$.

Plutonium 241

The net rate of change for Pu-241 can be given by

$$\frac{dN_{41}}{dt} = N_{40}\,\sigma_{40}\,\phi - N_{41}\,\sigma_{41}\,\phi \qquad (4.1.30)$$

{Gain from the neutron absorption in Pu-240} {Loss from the neutron absorption in Pu-241}

Equation (4.1.30) can be rewritten as

$$\frac{dN_{41}}{d\theta} = N_{40}\,\sigma_{40} - N_{41}\,\sigma_{41} \qquad (4.1.31)$$

and solved by using the appropriate integrating factor.

The solution to Eq. (4.1.31) at $N_{41}(0) = N_{41}^{o}$ can be given as

$$\begin{aligned}
N_{41}(\theta) = {} & N_{41}^{o}\exp(-\sigma_{41}\theta) \\
& + N_{40}^{o}\{C52[\exp(-\sigma_{40}\theta)-\exp(-\sigma_{41}\theta)]\} \\
& + N_{49}^{o}\{C56[\exp(-\gamma\,\sigma_{49}\theta)-\exp(-\sigma_{41}\theta)] - C56[\exp(-\sigma_{40}\theta)-\exp(-\sigma_{41}\theta)]\} \\
& + N_{28}^{o}\{C57[1-\exp(-\sigma_{41}\theta)] - C571[\exp(-\gamma\,\sigma_{49}\theta)-\exp(-\sigma_{41}\theta)] \\
& \quad + C572[\exp(-\sigma_{40}\theta)-\exp(-\sigma_{41}\theta)]\} \\
& + N_{25}^{o}\big(C58[\exp(-\sigma_{25}\theta)-\exp(-\sigma_{41}\theta)] + C581[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{41}\theta)] \\
& \quad + C582[\exp(-\gamma\,\sigma_{49}\theta)-\exp(-\sigma_{41}\theta)] - C583\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{41}\theta)\} \\
& \quad + C584[\exp(-\sigma_{40}\theta)-\exp(-\sigma_{41}\theta)]\big) \\
& + N_{23}^{o}\big(C59\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{41}\theta)\} + C591[\exp(-\sigma_{25}\theta)-\exp(-\sigma_{41}\theta)] \\
& \quad - C592[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{41}\theta)] + C593[\exp(-\gamma\,\sigma_{49}\theta)-\exp(-\sigma_{41}\theta)] \\
& \quad + C594[\exp(-\sigma_{40}\theta)-\exp(-\sigma_{41}\theta)]\big)
\end{aligned}$$

$$+ N^{o}_{24} \{C60[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{41}\theta)] - C601[\exp(-\sigma_{25}\theta)-\exp(-\sigma_{41}\theta)]$$

$$+ C602[\exp(-\gamma\ \sigma_{49}\theta)-\exp(-\sigma_{41}\theta)] + C603[\exp(-\sigma_{40}\theta)-\exp(-\sigma_{41}\theta)]\}$$

$$+ N^{o}_{02} \{C61[\exp(-\sigma_{02}\theta)-\exp(-\sigma_{41}\theta)] - C611\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{41}\theta)\}$$

$$+ C612[\exp(-\gamma\ \sigma_{49}\theta)-\exp(-\sigma_{41}\theta)] + C613[\exp(-\sigma_{40}\theta)-\exp(-\sigma_{41}\theta)]\} \qquad (4.1.32)$$

where $C52 = \dfrac{\sigma_{40}}{\sigma_{41} - \gamma\ \sigma_{49}}$; $C53 = \dfrac{\sigma_{40}}{\sigma_{41} - \sigma_{25}}$; $C53 = \dfrac{\sigma_{40}}{\sigma_{41} - \sigma_{40}}$;

$C531 = \dfrac{\sigma_{40}}{\sigma_{41} - \gamma\ \sigma_{49}}$; $C54 = \dfrac{\sigma_{40}}{\sigma_{41} - \sigma_{24}}$; $C55 = \dfrac{\sigma_{40}}{\sigma_{41} - \sigma_{23}(1-L_1)}$;

$C56 = C52 \cdot C43$; $C561 = C531 \cdot C43$; $C57 = \dfrac{\sigma_{40} \cdot C44}{\sigma_{41}}$;

$C571 = C52 \cdot C441$; $C572 = C531 \cdot C47$; $C58 = C53 \cdot C45$;

$C581 = C54 \cdot C452$; $C582 = C52 \cdot C453$; $C583 = C55 \cdot C451$;

$C584 = C531 \cdot C48$; $C59 = C55 \cdot C46$; $C591 = C53 \cdot C461$;

$C592 = C54 \cdot C462$; $C593 = C52 \cdot C463$; $C594 = C531 \cdot C49$;

$C60 = C54 \cdot C464$; $C601 = C53 \cdot C465$; $C602 = C52 \cdot C466$;

$C603 = C531 \cdot C50$; $C61 = \dfrac{\sigma_{40}\ C467}{\sigma_{41} - \sigma_{02}}$; $C611 = C55 \cdot C468$;

$C612 = C52 \cdot C469$; $C613 = C631 \cdot C501$.

Plutonium 242 is not considered in this work since the concentration of Pu-241 is usually very low in the HTGR when using U-235 or U-233 as the major fissile materials. The formation of U-237 and its radioactive-decay product Np-237 by capture of neutrons in U-236 was also neglected because of the low cross section of U-236.

Fission Products

Burnout of fission products by absorption of neutrons will be neglected in this analysis. The rate of formation of fission products from U-235 can be given by

$$\frac{dN_{F25}}{dt} = \frac{N_{25}\,\sigma_{25}\,\phi}{1 + \alpha_{25}} \tag{4.1.33}$$

With N_{25} given by Eq. (4.1.19), the solution of (4.1.33), subject to $N_{F25} = N^{o}_{F25}$ at $t = 0$, is

$$\begin{aligned} N_{F25}(\theta) = N^{o}_{F25} &+ N^{o}_{25}\left\{E11\left[\frac{1-\exp(-\sigma_{24}\theta)}{\sigma_{24}} - \frac{1-\exp(-\sigma_{25}\theta)}{\sigma_{25}}\right] + D16\,\frac{1-\exp(-\sigma_{25}\theta)}{\sigma_{25}}\right. \\ &\left. - E12\left\{\frac{1-\exp[-\sigma_{23}(1-L_1)\theta]}{\sigma_{23}(1-L_1)} - \frac{1-\exp(-\sigma_{25}\theta)}{\sigma_{25}}\right\}\right\} \\ &+ N^{o}_{24}\left\{E13\left[\frac{1-\exp(-\sigma_{24}\theta)}{\sigma_{24}} - \frac{1-\exp(-\sigma_{25}\theta)}{\sigma_{25}}\right]\right\} \\ &+ N^{o}_{23}\left\{E14\left\{\frac{1-\exp[-\sigma_{24}(1-L_1)\theta]}{\sigma_{23}(1-L_1)} - \frac{1-\exp(-\sigma_{25}\theta)}{\sigma_{25}}\right\}\right. \\ &\left. + E15\left[\frac{1-\exp(-\sigma_{24}\theta)}{\sigma_{24}} - \frac{1-\exp(-\sigma_{25}\theta)}{\sigma_{25}}\right]\right\} \end{aligned} \tag{4.1.34}$$

where $D16 = \dfrac{\sigma_{25}}{1 + \alpha_{25}}$; $E11 = D16 \cdot C93$; $E12 = D16 \cdot C92$; $E13 = D16 \cdot C81$;

$E14 = D16 \cdot C82$; $E15 = D16 \cdot C83$.

The rate of formation of fission products from U-233 is

$$\frac{dN_{F23}}{dt} = \frac{N_{23}\,\sigma_{23}}{1 + \alpha_{23}} \tag{4.1.35}$$

With N_{23} given by Eq. (4.1.10), the solution of Eq. (4.1.35) subject to $N_{F23} = N^{o}_{F23}$ at $t = 0$ is:

$$N_{F23}(\theta) = N^{o}_{F23}$$

$$+ N^{o}_{23}\ E30 \left\{ \frac{[1 - \exp(R_1\theta)][R_1^2 + R_1(P_1 + P_3) + P_1P_3]}{-R_1(R_1 - R_2)(R_1 + P_3)} \right.$$

$$\left. + \frac{[1 - \exp(R_2\theta)][R_2(P_6 - P_1P_6) + P_1P_6 - P_1P_6P_5]}{-R_2(R_2 - R_1)(R_2 + P_3)} \right\}$$

$$+ N^{o}_{25}\ E30 \left\{ \frac{[1 - \exp(R_1\theta)][R_1(P_6 - P_1P_6) + P_1P_6 - P_1P_6P_3]}{-R_1(R_1 - R_2)(R_1 + P_3)} \right.$$

$$+ \frac{[1 - \exp(R_2\theta)][R_2(P_6 - P_1P_6) + P_1P_6 - P_1P_6P_3]}{-R_2(R_2 - R_1)(R_2 + P_3)}$$

$$\left. + \frac{[1 - \exp(-P_3\theta)][P_1P_6 - P_3P_6]}{P_3(P_3 + R_1)(P_3 + R_2)} \right\}$$

$$+ N^{o}_{02}\ E30 \left\{ \frac{[1 - \exp(R_1\theta)][R_1P_1 + P_1P_3]}{-R_1(R_1 - R_2)(R_1 + P_3)} + \frac{[1 - \exp(R_2\theta)][R_2P_1 + P_1P_3]}{-R_2(R_2 - R_1)(R_2 + P_3)} \right\}$$

(4.1.36)

where $E30 = \dfrac{\sigma_{23}}{1 + \alpha_{23}}$ and all other constants are defined in Appendix A.

4.2 Conversion Ratio

Since in the large HTGR, the conversion ratio is relatively high due to the high burnup attained, it may be appropriate at this time to discuss briefly the conversion ratio and some of its characteristics.

The most significant nuclear advantage of the U-235 (Th-232) U-233 cycle over the U-235 (U-238) Pu-239 cycle in thermal reactors is the potential of a higher conversion ratio (32). The importance of a high CR in assuring good utilization of resources is directly related to the burnup needs. In a converter reactor, CR units of bred fuel are produced for each unit of fuel

consumed, and the net consumption of nuclear fuel is, then, proportional to (1-CR). Hence, other things being equal, a reactor with a conversion ratio of .6 which is a typical CR for the small HTGR 330 MWe Fort St. Vrain Nuclear Generating Station (20) would consume twice as much fuel per unit energy developed as a reactor having a conversion ratio of .8 (32) which is the typical CR for the proposed large HTGR with a generating capacity of 1160 MWe (1,3).

An expression for the CR of the 1160 MWe HTGR may be obtained by considering the following equation.

$$CR = \frac{\text{Formation rate of U-233 + U-235 + Pu-239 + Pu-241}}{\text{Destruction rate of U-233 + U-235 + Pu-239 + Pu-241}}$$

$$= \frac{FR}{DR} \qquad (4.2.1)$$

The DR of Eq. (4.2.1) can be attained by simple exponential depletion rates of the fissile materials. The expression for DR can be given as

$$DR = \exp(-\sigma_{23}\theta) + \exp(-\sigma_{25}\theta) + \exp(-\sigma_{49}\theta) + \exp(-\sigma_{41}\theta) \qquad (4.2.2)$$

The FR of Eq. (4.2.1) is a much more complex expression than the expression for the destruction rates. The FR for the individual fissile materials can be extracted from the burnup equations.

First consider U-233 FR, where from Eq. (4.1.11) the gain, i.e., formation, terms can be extracted as

$$FR_{23} = \frac{N_{02}^{o}\,\sigma_{02}(\exp\{[\sigma_{23}(1-L_1) - \sigma_{02}]\theta\} - 1)}{\sigma_{23}(1-L_1) - \sigma_{02}}$$

$$+ \frac{N_{25}^{o}\,\sigma_{25}L_2\{\exp(-\sigma_{23}\theta) - \exp[-\sigma_{23}(1-L_1)\theta]\}}{\sigma_{23}(1-L_1) - \sigma_{25}}$$

$$+ N_{23}^{o}\exp(\sigma_{23}L_1\theta) \qquad (4.2.3)$$

The FR for U-235 can be extracted from Eq. (4.1.19) and given as

$$FR_{25} = N_{25}^{o} \left(C91[\exp(-\sigma_{25}\theta)-\exp(-\sigma_{24}\theta)] - C92\{\exp[\sigma_{23}(L_1-1)\theta]-\exp(-\sigma_{25}\theta)\} - C93[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{25}\theta)] - C94\ \exp(-\sigma_{25}\theta)\right)$$

$$+ N_{24}^{o} \{C81[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{25}\theta)]\}$$

$$+ N_{23}^{o} \left(C82\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{25}\theta)\} - C83[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{25}\theta)]\right)$$

$$+ N_{02}^{o} \left(C84[\exp(-\sigma_{02}\theta)-\exp(-\sigma_{25}\theta)] - C85[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{25}\theta)] - C86\{\exp[-\sigma_{23}(1-L_1)\theta]-\exp(-\sigma_{25}\theta)\} + C87[\exp(-\sigma_{24}\theta)-\exp(-\sigma_{25}\theta)]\right) . \quad (4.2.4)$$

The FR for Pu-239 can be extracted from Eq. (4.1.27) by using the following simple formula:

$$FR_{49} = N_{49}(\theta) \quad (4.2.5)$$

where γ is redefined as γ_{FR} and $\gamma_{FR} = -\kappa_{49}$.

The FR for Pu-241 can be extracted from Eq. (4.1.32) and attained in a similar manner as Eq. (4.2.5) by using the following formula:

$$FR_{41} = N_{41}(\theta) - N_{41}^{o} \exp(-\sigma_{41}\theta) \quad (4.2.6)$$

where all terms have been defined previously.

Now the expression for the conversion ratio for the HTGR system can be given as

$$CR = \frac{FR_{23} + FR_{25} + FR_{49} + FR_{41}}{\exp(-\sigma_{23}\theta) + \exp(-\sigma_{25}\theta) + \exp(-\sigma_{49}\theta) + \exp(-\sigma_{41}\theta)} . \quad (4.2.7)$$

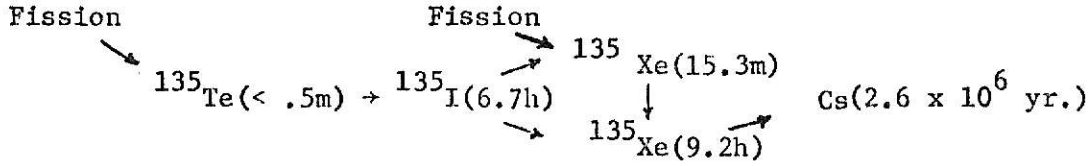
4.3 Poison Equations

During the operation of a reactor a number of light nuceli (atomic weight between 60 and 160) arise as fission products from the heavy nuclei as a result of fission (25,26,29,33). The majority of the fission products produced in the HTGR is attained by the fissioning of U-233 and U-235. The expressions for the fission products of these two fissile nuclides are given by Eqs. (4.1.34) and (4.1.36). The fission products influence the operation of a reactor in many ways (25), but the main effect on the reactor's steady-state operation is that many of these fission products have a large absorption cross section for thermal neutrons and thus severely affect the neutron economy. This is usually called "poisoning."

The two most important fission product poisons will be treated in an individual analysis.

Xenon 135

Xe-135 is formed as the result of the decay of I-135, and also is produced directly from fission. The I-135 is not formed in fission but appears as the result of the decay of Te-135 (tellurium-135). These processes and their half-lives are summarized below:



Since Te-135 decays so rapidly to I-135, it is possible to assume that I-135 is produced directly in fission. The concentrations of I-135 and of Xe-135 in a reactor are given by:

$$\frac{dI}{dt} = \phi \Sigma_f Y^I - \lambda^I I \tag{4.3.1}$$

and

$$\frac{dX}{dt} = \phi \Sigma_f Y^X + \lambda^I I - (\lambda^X + \phi \bar{\sigma}_a^X) X \tag{4.3.2}$$

where I, X = the concentration (atoms per cubic unit) of the I-135 and Xe-135, respectively

$\phi\Sigma_f$ = thermal fission rate (fission per cubic unit per second),

Y^I = fission yield of I-135 (actually of Te-135),

Y^X = direct fission yield of Xe-135,

λ^I = decay constant of I-135 (decays per second),

λ^X = decay constant of Xe-135 (decays per second),

$\bar{\sigma}_a^X$ = average thermal absorption cross section of Xe-135.

In an operating reactor the quantities, Σ_f and ϕ, are usually functions of time (24,33), and the solutions to Eqs. (4.3.1) and (4.3.2) depend upon the nature of these functions. Since the thermal flux, ϕ, is assumed constant throughout the life of the reactor, flux can be treated as a constant value in Eqs. (4.3.1) and (4.3.2). The macroscopic fission cross section, Σ_f, is actually the product of the microscopic fission cross section and the atom concentration per cubic unit of fissile materials in the core. The microscopic fission cross section is assumed constant in this analysis; therefore the only quantity in Eqs. (4.3.1) and (4.3.2) that varies with respect to time is the fissile nuclide concentration. The problem of solving Eqs. (4.3.1) and (4.3.2) using the time dependent expressions of the fissile nuclides in the HTGR core, i.e., U-235, U-233, Pu-239, and Pu-241, would prove to be a very difficult task; so it is assumed in this work that the concentrations of I-135 and Xe-135 quickly rise to their equilibrium values in an operating reactor (24), i.e., the thermal flux saturates the I-135 and Xe-135 since

their half lives are so short. This is especially true in the HTGR since the thermal flux is generally much higher than the conventional light water reactor. The equilibrium concentrations of I-135 and Xe-135 can be found by placing the time derivatives in Eq. (4.3.1) and (4.3.2) equal to zero. Thus from Eq. (4.3.1)

$$I_{\infty} = \frac{\phi \, Y^{I} \Sigma_{f}}{\lambda^{I}} \tag{4.3.3}$$

and from Eq. (4.3.2)

$$X_{\infty} = \frac{\lambda^{I} I_{\infty} + Y^{X} \Sigma_{f} \phi}{\lambda^{X} + \bar{\sigma}_{a}^{X} \phi}$$

$$= \frac{(Y^{I} + Y^{X}) \Sigma_{f} \phi}{\lambda^{X} + \bar{\sigma}_{a}^{X} \phi} . \tag{4.3.4}$$

The macroscopic fission cross section, Σ_f, in Eq. (4.3.4) contains the three major fissile nuclides in the HTGR core, i.e., U-233, U-235, and Pu-239 (Pu-241 is not considered here since it has a very low concentration); therefore the corresponding fission yield values must be used. The appropriate yields are given in Table III and the decay constants and absorption cross sections are given in Table IV. Eq. (4.3.4) will be expanded into an appropriate LP expression in the "Constraint Equations" section of this chapter.

No detailed analysis is made in this work of the end of life (EOL) characteristics of Xenon-135, where the Xenon concentration initially increases after shutdown because the EOL concentration of I-135 decays into Xe-135 (11). The buildup of Xe-135 creates a negative amount of reactivity,

Table III

Fission Product Yields (Atoms per fission) from Thermal Fission[a]

Fissionable Isotope	^{233}U	^{235}U	^{239}Pu
Yield of ^{135}I (direct)	.051	.061	.055
Yield of ^{135}Xe (direct)		.003	
Yield of ^{149}Pm (indirect)	.0066	.0113	.019

[a]Taken from Reference (24).

Table IV

Constants for Fission Product Poisoning Calculations[b]

Isotope	λ (sec^{-1})	$\bar{\sigma}_a$ (barns)
^{135}I	2.87×10^{-5}	
^{135}Xe	2.09×10^{-5}	$374. \times 10^{3}$
^{149}Pm	3.56×10^{-6}	
^{149}Sm	(stable)	$106. \times 10^{2}$

[b]Decay constants from Ref. (24) and cross sections from Ref. (28)

after shutdown of the reactor, which depends on the magnitude of the flux in steady state operation. If the flux is large enough, the buildup of Xe-135 can create enough negative reactivity to prevent the reactor from starting up, i.e., the negative reactivity created by Xe-135 is greater than the excess reactivity of the system. Therefore the Xe-135 must be allowed to reach its peak concentration (about 12 hrs.), and then decay below the excess reactivity level of the reactor, before the reactor can be functional.

The post-shutdown buildup of xenon is of little importance when refueling a reactor since the time it takes to refuel the reactor is usually much longer than the time it takes the negative reactivity created by Xe-135 to subside.

Samarium-149

Samarium-149 is a stable isotope (24,33) arising from beta decay of promethium-149, which is formed directly from fission, and also from the beta decay of the fission product Neodymium-149.

The decay chain for ^{149}Sm can be written as

$$\overset{\text{Fission}}{\searrow} \; ^{149}Nd(2\text{ hr}) \rightarrow ^{149}Pm(54\text{ hr}) \rightarrow ^{149}Sm\text{ (stable)} \; . \qquad (4.3.5)$$

Because the Nd-149 decays comparatively rapidly to Pm-149, the Pm-149 may be assumed to be produced directly in fission with the yield (cf. Table III). The time dependent concentration P of the promethium in atoms/cubic unit can be given by the equation

$$\frac{dP}{dt} = Y^P \Sigma_f \phi - \lambda^P P \; , \qquad (4.3.6)$$

where λ^P is the decay constant for Pm-149 and is given in Table IV. Since Sm-149 is stable, it disappears only as the result of neutron capture. The rate equation for Sm-149 can be given as

$$\frac{dS}{dt} = \lambda^P P - \bar{\sigma}_a^S \phi S \tag{4.3.7}$$

where S is the atom density of Sm-149 and $\bar{\sigma}_a^S$ is its average thermal absorption cross section (cf. Table IV).

Although it takes Sm-149 somewhat longer to reach a steady-state value than it does for Xe-135 since the absorption cross section of Sm-149 is much less than that of Xe-135 and the half-life of Pm-149 is longer than those of I-135 and Xe-135, the concentration of Sm-149 can also be treated as an equilibrium value in an operating reactor (24).

With the time derivatives of Eqs. (4.3.6) and (4.3.7) equal to zero, the equilibrium concentrations can be given as

$$P_\infty = \frac{Y^P \Sigma_f \phi}{\lambda^P} \tag{4.3.8}$$

and

$$S_\infty = \frac{Y^P \Sigma_f}{\bar{\sigma}_a^S} \tag{4.3.9}$$

where the macroscopic fission cross section, Σ_f, is the same as given in Eq. (4.3.4). The use of Eq. (4.3.9) in the LP scheme will be discussed more thoroughly in the "Constraint Equations" section of this chapter.

Permanent Poisons

Many other fission products in addition to Xe-135 and Sm-149 are formed in a reactor, and like these isotopes some are stable and some are radioactive. However, no fission products with absorption cross sections comparable to those

of Xe-135 and Sm-149 are produced with yields sufficiently large to warrant individual treatment (24). The majority of these additional fission products have far smaller cross sections, and once they appear in a reactor there is little probability that they will be removed by neutron absorption during the lifetime of the reactor (24,33).

The fluence dependent expressions for the fission product production of the two major fissile nuclides under study in this analysis, i.e., U-235 and U-233, are given by Eqs. (4.1.34) and (4.1.36). The macroscopic poison cross section due to the accumulation of gross fission products can be given as

$$\Sigma_{23}^{fp} = \delta_{23}^{fp} N_{F23} \tag{4.3.10}$$

and

$$\Sigma_{25}^{fp} = \delta_{25}^{fp} N_{F25} \tag{4.3.11}$$

where N_{F23} is given by Eq. (4.1.36), N_{25} is given by Eq. (4.1.34), δ_{23}^{fp} and δ_{25}^{fp} (cf. Table V) represent the effective cross section of fission products resulting from one average fission of U-233 and U-235, respectively. The discussion of inserting Eqs. (4.3.10) and (4.3.11) into the LP model will be given later in this chapter.

Table V

Gross Fission Product Cross Sections[c]

Isotope	barn/ fission
^{233}U	16.3
^{235}U	19.3

[c]Taken from Reference (33).

4.4 Model Description

The state of the fuel is, in general, defined by the isotopic composition of the fuel and the spatial distribution of isotopes in each fuel assembly. To represent this situation in a LP model would mean that each fuel element would have its isotopic composition and position designated by its own identification indices. Since the HTGR has 3944 fuel elements, it would present an unreasonable computation problem to identify each element. Therefore to incorporate a reasonable spatial distribution into the model, the standard procedure is to divide the core into a number of regions or zones (9,10,11). In this work the core is divided into four equal volume concentric regions with a prescribed average flux level for each region. One of the main reasons for considering only four regions is that GA specifies that one-fourth of the core be removed at each refueling (16,2,4). With this specification the matrix generator which is developed later in this chapter can remove one section, i.e., one region, from the core at each refueling.

Now to give the burnup equations, developed in Section 4.1, spatial dependency relative to the regions, the state variables in the burnup equations must be properly indexed. This can be done by defining the state variables, N_{pq}^{M}, as the concentration per unit volume of the species p in region q where M represents the irradiation history of the fuel, i.e., the "beginning of life" (BOL) condition on the "end of life" (EOL) condition. Table VI summarizes the indexing.

It is also convenient to give a group index in place of the species index, p, that are listed in Table VI, i.e., the fissile nuclides are represented by g, the fertile nuclides are represented by ℓ, and the poison nuclides are represented by j. This indexing is summarized in Table VII.

Table VI

Indexing Used on State Variables

Species	Reference Equation	p	q	M
U-233	(4.1.10)	23	1 → Region 1	0 → BOL
U-238	(4.1.24)	28	2 → Region 2	1 → EOL
Th-232	(4.1.14)	02	3 → Region 3	
U-235	(4.1.19)	25	4 → Region 4	
U-236	(4.1.21)	26		
Pu-239	(4.1.27)	49		
Pu-240	(4.1.29)	40		
Pu-241	(4.1.32)	41		
U-234	(4.1.16)	24		
(U-235)FP*	(4.1.34)	F25		
(U-233)FP*	(4.1.36)	F23		

*FP represents "fission products"

Table VII

Group Indexing

Species	Number of Nuclides	p
U-233 U-235 Pu-239 Pu-241	A*	g
U-238 Th-232 U-234 Pu-240	B*	ℓ
U-236 (U-233)FP (U-235)FP	C*	j

*A, B, and C represent the total number of each group, i.e., A equals 4, B equals 4 and C equals 3.

To conform to some of the optimum standards set by GA, the fuel management parameters specified by GA for the 1160 MWe HTGR are used in this model. These parameters represent the lowest fuel cycle costs consistent with the current thermal and metallurgical performance limits (2). These parameters are given in Table VIII.

The model developed in this work is based on the parameters presented in Table VIII.

Perhaps the most difficult part of developing any LP model is to make the appropriate simplifying assumptions which do not severely deter the model from reality. Usually, the assumptions are needed to change the nonlinear relationships into linear relationships. The major assumptions used in the development of the equations used in this model are listed below:

(1) homogeneous mixture of fuel and moderator in each region,

(2) only the thermal neutron energy group is considered,

(3) the power level is assumed constant throughout life,

(4) the resonance escape, p, and fast fission factor, ϵ, are assumed constant throughout life,

(5) the neutron leakage terms are assumed constant throughout life,

(6) the average flux levels for each region are assumed constant throughout life.

To make sure the reactor meets the power producing and material requirements while maintaining the operational necessities imperative of a nuclear reactor, the appropriate "constraint equations" must be developed.

Table VIII

Fuel Management Parameters

Fuel Cycle	Thorium/Uranium
Fuel lifetime (nominal)	4 years at 80% capacity factor
Replaceable reflector lifetime (nominal)	8 years
Average power density	8.4 w/cc
Refueling interval	1 year at 80% capacity factor
Average C/Th ratio	250

4.5 Constraint Equations

The constraints which contain the neutronic and thermal-hydraulic performance criteria of a linear reactor model are probably the most difficult constraints to formulate realistically (9). This is generally true since the behavior of the reactor is the result of the nonlinear interactions of the neutron population and the core materials. If the assumptions presented in Section 4.4 are used, the nonlinear equations can be simplified and put into the form of linear expressions.

The major constraints used in this work are listed below:

(1) reactivity constraints,

(2) total power constraints,

(3) power peaking constraints,

(4) volume (nuclei) constraints,

(5) enrichment constraint.

Reactivity Constraints

The reactivity constraints are actually composed of two parts as suggested by Brown (9). The localized reactivity constraint which keeps the reactivity distribution, in each region of the core, uniform, i.e., four localized reactivity constraints for the four individual regions, and the overall reactivity constraints which keep the reactor critical. The overall reactivity constraints will be considered first.

The various material parameters which affect the neutronics of a reactor can be approximately related by the "six-factor formula" (24,9):

$$k_{eff} = \eta f \epsilon p P_1 P_2 \qquad (4.5.1)$$

where η is the number of fission neutrons produced for every neutron that is absorbed in the fuel, f (thermal utilization factor) is the ratio of the

number of neutrons absorbed in the fuel to the total number of neutron absorptions in fuel, moderator, cladding, etc., and ε, p, P_1 and P_2 are defined in Section 4.0. Since ε, p, P_1, and P_2 are assumed constant in this analysis, a new term can be defined as

$$P_{all} = \varepsilon p P_1 P_2 \ , \tag{4.5.2}$$

and Eq. (4.5.1) can be rewritten as

$$k_{eff} = \eta f \ P_{all} \ . \tag{4.5.3}$$

For an operating reactor, the terms η and f will vary because the composition of the fuel will change due to interactions with the neutrons. It may help the reader to analyze each of the terms separately.

As stated previously, η is defined as the average number of neutrons liberated directly by fission for each thermal neutron absorbed in the fuel. Thus if the fuel consists of the mixture of fissile and fertile isotopes, η is given by

$$\eta = \frac{\nu \ \Sigma_{fF}}{\Sigma_{aF}} \tag{4.5.4}$$

where ν is the average number of neutrons liberated per fission, Σ_{fF} is the macroscopic fission cross section of the fuel, and Σ_{aF} is the macroscopic absorption cross section of the fuel.

The thermal utilization, f, can be more precisely defined as the fraction of the thermal neutrons which are absorbed by the fuel in an infinite homogeneous mixture of the fuel, moderator, and poisons. It follows from its definition that f can be given by

$$f = \frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aM} + \Sigma_{aP}} \tag{4.5.5}$$

where Σ_{aM} represents the total macroscopic absorption cross section for the moderator, and Σ_{aP} represents the total macroscopic absorption cross section for the poison absorbers in the system, i.e., fission products, U-236, and burnable poisons. It is assumed in this analysis that control rod poisoning which is used to regulate the reactivity in the system can be separated from fuel management calculations. The burnable poison, B-10 (2), is treated in this analysis as though it was smeared uniformly throughout the core. The quantity, Σ_{aP}, can be given as

$$\Sigma_{aP} = \Sigma_{aC} + \Sigma_{aB10} \tag{4.5.6}$$

where Σ_{aC} represents the total macroscopic absorption cross section for parasitic poison in the system, and Σ_{aB10} is the macroscopic absorption cross section of the burnable poison used in the system. The macroscopic absorption cross section for the burnable poison can be given as

$$\Sigma_{aB10} = N_{B10}\, \sigma^{a}_{B10} \tag{4.5.7}$$

where N_{B10} is the B-10 concentration per unit volume and σ^{a}_{B10} is the microscopic absorption cross section for B-10. Σ_{aC} represents a more complicated expression; since it contains the macroscopic absorption cross section for the fission products of U-233 and U-235, and the macroscopic absorption cross section of Xe-135 and Sm-149, where all of the associated fluence dependent concentrations have been defined previously in Sections 4.1 and 4.3. Since Eqs. (4.3.9) and (4.3.4) are dependent on the fissile concentrations they can be rewritten as

$$N_S = \frac{\sum_{g}^{A} N_g\, \sigma^{f}_{g}\, Y^{P}_{g}}{\bar{\sigma}^{S}_{a}} \tag{4.5.8}$$

and

$$N_X = \frac{\phi \sum_g^A N_g \sigma_g^f (Y_g^I + Y_g^X)}{\lambda^X + \bar{\sigma}_a^X \phi} \tag{4.5.9}$$

where N_g is the fissile concentration per unit volume and σ_g^f is its associated microscopic fission cross section. Eqs. (4.5.8) and (4.5.9) can be expanded into regions, and the total macroscopic absorption cross sections for Xe-135 and Sm-149 can be given as

$$\Sigma_{TX} = \left[\sum_q^R \frac{\phi_q \sum_g^A N_{gq} \sigma_g^f (Y_g^I + Y_g^X)}{\lambda^X + \bar{\sigma}_a^X \phi_q}\right] \bar{\sigma}_a^X \tag{4.5.10}$$

and

$$\Sigma_{TS} = \left[\sum_q^R \frac{\sum_g^A N_{gq} \sigma_g^f Y_g^P}{\bar{\sigma}_a^S}\right] \bar{\sigma}_a^S , \tag{4.5.11}$$

respectively, where R represents the number of regions. The fission product concentrations of U-233 and U-235, and the parasitic poison, U-236, can be given in a similar manner as

$$\Sigma_{TP} = \sum_q^R \sum_j^C N_{jq} \sigma_j^a \tag{4.5.12}$$

where N_j represents the concentration of the jth group, σ_j^a represents the associated absorption cross section. The indices used in Eq. (4.5.12) are defined in Table VII. The total macroscopic absorption parasitic poison cross section, Σ_{aC}, can now be given as

$$\Sigma_{aC} = \Sigma_{TX} + \Sigma_{TS} + \Sigma_{TP} \tag{4.5.13}$$

The total macroscopic absorption cross section for the moderator can be given as

$$\Sigma_{aM} = \sum_{q}^{R} N_{cq}\, \sigma_c^a \tag{4.5.14}$$

where N_{cq} represents the concentration of graphite per unit volume in the qth region, and σ_c^a represents the microscopic absorption cross section for the graphite used in the HTGR.

The total macroscopic absorption cross section for the fuel can be given as

$$\Sigma_{aF} = \sum_{q}^{R} \sum_{p}^{D} N_{pq}\, \sigma_p^a \tag{4.5.15}$$

where D = A + B, i.e., the total number of fissile and fertile nuclides.

The total macroscopic fission cross section for the system can be given as

$$\Sigma_{fF} = \sum_{q}^{R} \sum_{g}^{A} N_{gq}\, \sigma_g^f \quad . \tag{4.5.16}$$

By inserting Eqs. (4.5.6), (4.5.14) and (4.5.15) into Eq. (4.5.5), and inserting Eqs. (4.5.16) and (4.5.15) into Eq. (4.5.4), the effective multiplication factor, k_{eff}, can be given as

$$\begin{aligned} k_{eff} = {} & \left(\sum_{q}^{R} \sum_{g}^{A} \nu_g\, N_{gq}\, \sigma_g^f\right) P_{all} \\ & \div \left\{ \sum_{q}^{R} \left[\sum_{p}^{D} N_{pq}\, \sigma_p^a + N_{cq}\, \sigma_c^a + \sum_{j}^{C} N_{jq}\, \sigma_j^a \right.\right. \\ & + \sum_{g}^{A} N_{gq}\, \sigma_g^f\, Y_g^P + \frac{\phi_q\, \bar{\sigma}_a^X}{\lambda^X + \bar{\sigma}_a^X\, \phi_q} \sum_{g}^{A} N_{gq}\, \sigma_g^f\, (Y_g^I + Y_g^X) \\ & \left.\left. + N_{B10q}\, \sigma_{B10}^a \right] \right\} . \end{aligned} \tag{4.5.17}$$

where N_{cq} is the concentration of carbon per unit volume in the qth region, and σ_c^a is the associated microscopic absorption cross section. To sustain

a critical reaction in a reactor the RHS of Eq. (4.4.17) must be at least equal to one.

To incorporate Eq. (4.5.17) into the LP model, it must represent two end points in the irradiation history of one cycle, i.e., "beginning of life" (BOL) and "end of life" (EOL).

Since the control rods are assumed to absorb any excess reactivity at the BOL, the RHS of Eq. (4.5.17) can be greater than one; therefore the BOL constraint for the model can be given as

$$0 < \left(\sum_q^R \sum_g^A \nu_g \, N^o_{gq} \, \sigma^f_g\right) P_{all} - \left\{\sum_q^R \left[\sum_p^D N^o_{pq} \, \sigma^a_p + N^o_{cq} \, \sigma^a_c + \sum_j^C N^o_{jq} \, \sigma^a_j + \sum_g^A N^o_{gq} \, \sigma^f_g \, Y^P_g + \frac{\phi_q \, \bar{\sigma}^X_a}{\lambda^X + \bar{\sigma}^X_a \, \phi_q} \sum_g^A N^o_{gq} \, \sigma^f_g \, (Y^I_g + Y^X_g) + N^o_{B10q} \, \sigma^a_{B10}\right]\right\} \tag{4.5.17}$$

To get the maximum performance out of the fuel, the EOL reactivity constraint is set equal to zero. Therefore the EOL reactivity constraint for the model can be given as

$$0 = \left(\sum_q^R \sum_g^A \nu_g \, N^1_{gq} \, \sigma^f_g\right) P_{all} - \left\{\sum_q^R \left[\sum_p^D N^1_{pq} \, \sigma^a_p + N^1_{cq} \, \sigma^a_c + \sum_j^C N^1_{jq} \, \sigma^a_j + \sum_g^A N^1_{gq} \, \sigma^f_g \, Y^P_g + \frac{\phi_q \, \bar{\sigma}^X_a}{\lambda^X + \bar{\sigma}^X_a \, \phi_q} \sum_g^A N^1_{gq} \, \sigma^f_g \, (Y^I_g + Y^X_g) + N^1_{B10q} \, \sigma^a_{B10}\right]\right\} \tag{4.5.18}$$

The total reactivity constraints only constrain the overall system to be critical while the local reactivity in each region may be violated. Therefore, "localized reactivity" constraints are also required to assure a reasonable loading pattern.

To have flexibility in the LP model while using the "localized reactivity" constraints and still prevent severe flux tilting as a result of an azimuthal reactivity imbalance (20,34) the infinite multiplication factor is used only to constrain the BOL of each region. The infinite multiplication factor can be given as (24,5)

$$k_\infty = \eta f p \varepsilon \ . \tag{4.5.19}$$

Since it is desirable to have the RHS of Eq. (4.5.19) greater than one, Eq. (4.5.19) can be expanded into a LP constraint by the use of Eqs. (4.5.4) and (4.5.5), and given as follows:

$$\begin{aligned} 0 < & \left(\sum_g^A \nu_g \, N_g^o \, \sigma_g^f\right) p\varepsilon \\ & - \Big[\sum_p^D N_p^o \, \sigma_p^a + N_c^o \, \sigma_c^a + \sum_j^C N_j^o \, \sigma_j^a \\ & + \sum_g^A N_g^o \, \sigma_g^f \, Y_g^P + \frac{\phi \, \bar{\sigma}_a^X}{\lambda^X + \bar{\sigma}_a^X \, \phi} \sum_g^A N_g^o \, \sigma_g^f \, (Y_g^I + Y_g^X) \\ & + N_{B10}^o \, \sigma_{B10}^a \Big] \ . \end{aligned} \tag{4.5.20}$$

Total Power Constraints

The power in a reactor is proportional to the product of the flux and the macroscopic fission cross sections. With the assumptions given in Section 4.4 the total reactor power can be given as

$$TP = \left(\sum_{q}^{R} \phi_q \, V_q \sum_{g}^{A} REC_g \, N_{gq} \, \sigma_g^f \right) F_{PC} \qquad (4.5.21)$$

where TP represents the total power in megawatts thermal (MWt); A only includes the fissile isotopes of U-233, U-235, and Pu-239; ϕ_q is the average flux in the qth region; V_q is the active volume in the qth region; REC_g is the recoverable energy per fission of the gth isotope (MeV/fission); σ_g^f is the microscopic fission cross section for the gth species; F_{PC} is the factor used to convert MeV per sec to megawatts (MW).

Since the power is considered to have an average value throughout the life of the reactor, total power must be multiplied by an averaging factor, i.e., load factor, to set a lower limit for the constraints. The "total power" must be satisfied at BOL and EOL; therefore a constraint must be formulated for each point in time.

The BOL total power constraint can be given in a linear equation as

$$TPOW \cdot PFAC < \left(\sum_{q}^{R} V_q \, \phi_q \sum_{g}^{A} REC_g \, N_{gq}^{o} \, \sigma_g^f \right) F_{PC} \qquad (4.5.22)$$

where PFAC is the load factor and TPOW is the total power producing requirements of the HTGR in MWt.

Similarly, the EOL total power constraint can be given as

$$TPOW \cdot PFAC < \left(\sum_{q}^{R} V_q \, \phi_q \sum_{g}^{A} REC_g \, N_{gq}^{1} \, \sigma_g^f \right) F_{PC} \qquad (4.5.23)$$

All factors defined here are given in Table IX.

Power Peaking Constraints

The "power peaking" constraints serve two purposes in this analysis. This constraint can be used to control the distribution of the macroscopic fission cross section which, if not constrained, leads to excessive peak-

to-average power ratios (9). The "power peaking" constraint is also used in this work as a "reactivity distribution" constraint to assure a fuel loading pattern consistent with the flux shape assumption. This can be done since the "out-in" refueling policy is used in this work. When using the "out-in" refueling policy, the fuel is consistently moved inward to the higher burnup regions of the core. Since the fission products and U-236 buildup according to exposure to the neutron flux, the highest accumulation of poisons will be in the center region; therefore the "power peaking" constraint must have the least restriction in the center region with the restriction being more severe as the regions extend outward. This method was used by Howe (34) where the core was divided into zones and the fission distribution was adjusted by adjusting the power density to conform to the prescribed flux shape. Prescribed weighting factors are used to vary the "power peaking" constraint. The choice of the weighting factors is somewhat of an arbitrary one, chosen to accommodate this particular loading and refueling pattern.

The "power peaking" constraints are established in each region in this model and at the BOL and EOL time histories. The BOL "power peaking" constraint can be given as an inequality by

$$P_r\ P_{wtq}\ P_{avg} > \left(\phi_q \sum_g^A REC_g\ N^o_{gq}\ \sigma^f_g\right) F_{PP} \qquad (4.5.24)$$

where P_r is the radial power peaking factor, P_{wtq} is the weighting factor for the qth region, P_{avg} is the average power density (w/cc), and F_{PP} is the factor used to convert (MeV/sec) to watts.

The EOL "power peaking" constraint can be given in a similar manner as

$$P_r\ P_{wtq}\ P_{avg} > \left(\phi_q \sum_g^A REC_g\ N^1_{gq}\ \sigma^f_g\right) F_{PP}\ . \qquad (4.5.25)$$

Volume Constraints

To calculate the maximum space permissible for the fuel in the HTGR, one has to consider both sizes of fuel particles where the TRISO contains the uranium and the BISO contains the thorium. It may be noted that the fuel volume to particle volume is different for each type fuel particle; therefore it is necessary to make somewhat of a detailed analysis to obtain a linear inequality to relate the mixture of the two different types of fuel particles. This can be done by taking the associated fuel particles parameters given in Table II and the dimensions of the fuel element given in Fig. 3.2.1 and applying the appropriate volume ratios for each type of fuel particle. It may be noted that the volume constraint equations are only used for each region at BOL.

The equation for the maximum thorium concentration in terms of fuel particle spheres can be given as

$$NTC_{max} = \frac{S_{max}^{BISO} \cdot \left\{\frac{cm^3\text{-}ThC_2}{BISO\text{-}sphere}\right\} \cdot \rho_{ThC_2} \cdot N_{av}}{AWT_{ThC_2}}$$

$$= \frac{466 \times 10^9 \times 4.72 \times 10^{-5} \times 8.96 \times 6.024 \times 10^{24}}{256}$$

$$= 4630 \times 10^{27} \frac{\text{maximum Th atoms}}{\text{region}} \qquad (4.5.26)$$

where S_{max}^{BISO} is the maximum number of BISO sphere in each region, ρ_{ThC_2} is the density of ThC_2 in grams per cm^3, N_{av} is Avagadro's Number, and AWT_{ThC_2} is the atomic weight of ThC_2.

The equation for the maximum uranium concentration in terms of fuel particle spheres can be given as

$$NU_{max} = \frac{S_{max}^{TRISO} \cdot \left\{\frac{cm^3\text{-}UC_2}{TRISO\text{-}sphere}\right\} \cdot \rho_{UC_2} \cdot N_{av}}{AWT_{UC_2}}$$

$$= \frac{109 \times 10^{10} \times .41 \times 10^{-5} \times 11.28 \times 6.024 \times 10^{24}}{262}$$

$$= 1160 \times 10^{27} \frac{U \text{ atoms}}{region} \qquad (4.5.26)$$

where S_{max}^{TRISO} is the maximum number of TRISO spheres in each region, ρ_{UC_2} is the density of UC_2 in grams per cm^3, and AWT_{UC_2} is the atomic weight of UC_2.

Now the LP inequality for each region scaled by 10^{-27} can be given by

$$1 > \frac{1}{4630} N_{02}^{o} + \frac{1}{1160} N_{25}^{o} + \frac{1}{1160} N_{28}^{o} . \qquad (4.5.27)$$

For the burnable poison B-10 the equation for maximum B-10 concentration can be given as

$$NB10_{max} = 550 \times 10^{27} \frac{B\text{-}10 \text{ atoms}}{region} . \qquad (4.5.28)$$

The LP constraint scaled by 10^{-27} can be given as

$$1 > \frac{1}{550} N^{o} \qquad (4.5.29)$$

where N_{B10}^{o} is the initial load of B-10 in each region.

Enrichment constraint

GA specifies that the optimum enrichment for the 1160 MWe power reactor is 93 percent enriched (2,4). In this analysis the enrichment was allowed to vary to find its own optimum value. Enrichment can be defined as the atom percent of U-235 in uranium when it is increased above the abundance as found in nature (35). Thus, 93 percent enriched uranium will have 93 percent of its atoms U-235 and 7 percent of them U-238. Therefore, the maximum enrichment constraint can be given for each region as

$$\frac{N^o_{25q}}{N^o_{25q} + N^o_{28q}} < En_{max} \qquad (4.5.30)$$

where N^o_{25q} is the initial nuclide concentration of U-235 for the qth region, N^o_{28q} is the initial nuclide concentration of U-238 for the qth region, and En_{max} is the maximum enrichment allowed. Equation (4.5.30) can also be given in LP form as

$$N^o_{25q}(1 - En_{max}) - N^o_{28q}\, En_{max} < 0 \qquad (4.5.31)$$

where En_{max} is .93.

4.6 Objective Function

The objective function is formulated on the basis that the U-233 indifference value can be used to equate the cost relationship between U-233 and U-235. From results based on 20 year levelized fuel cycle cost calculations, the U-233 indifference value was found to be about $16.7/gm U-233 (16). The method used to determine the U-233 price or indifference value in a HTGR can be generally described as follows: Consider first a power reactor fueled initially with enriched uranium which converts thorium to U-233. At the end of the year, it is assumed that the reactor operator may either (1) sell the U-233 and start again with enriched uranium, or (2) recycle the U-233 back into the reactor thereby reducing the enriched uranium requirements. The value of the U-233 as a fuel in HTGRs is then defined as being equal to the selling price which would yield equal average fuel cycle costs for these two cases. Consequently, from an economic viewpoint it would make no difference to the reactor operator as to which cycle to follow, and hence the term "indifference value." Since the price of highly enriched uranium is approximately $12 per gram of U-235 (16), the

comparative cost ratio between U-233 and U-235 can be given as 1.39. Using this cost ratio the objective function for the HTGR can be given as

Minimize:

$$OB = \sum_{q}^{R} (N^{o}_{25q} - 1.39\ N^{1}_{23q}) \qquad (4.6.1)$$

where N^{o}_{25q} is the initial load concentration in the qth region, and N^{1}_{23q} is the after burn concentration (EOL) of U-233 in the qth region.

It may be noted by the reader that Eq. (4.6.1) serves two purposes, i.e., it minimizes the amount of U-235 loaded into the system while it simultaneously maximizes the amount of U-233 produced by the system.

4.7 Fuel Cycle Design Parameters

In any core physics design work the physical parameters of the core must be known to do an accurate analysis of the reactor. Many of the HTGR parameters to date are still under investigation and some dimensions that will be used in the final design are still being analyzed; therefore it is the purpose of this section to give representative design parameters for the large HTGR.

As stated previously, the flux is a function of time, but for this work it is assumed to have a constant average value for each region. Since there is no data available on maximum to average flux ratios for a four region reactor core, the appropriate ratios must be attained through an analytical expression. The procedure to determine the expression for each region is given in Appendix C. From Appendix C, a general expression for a four region two-dimensional average flux level can be given by

$$\phi_k = \phi_{max}(.2918)[r\ J_1(.1452r)]_{R_{k-1}}^{R_k} \qquad (4.7.1)$$

where all factors are defined in Appendix C.

From Eq. (4.7.1) the average flux level for each region can be calculated and given as

$$R_1: \quad \bar{\phi}_1 = .88\ \phi_{max}$$

$$R_2: \quad \bar{\phi}_2 = .66\ \phi_{max}$$

$$R_3: \quad \bar{\phi}_3 = .48\ \phi_{max}$$

$$R_4: \quad \bar{\phi}_4 = .31\ \phi_{max}\ .$$

The other physical parameters used in this work are listed in Table IX. Most of the values given in Table IX are considered to be typical values for the 1160 MWe HTGR.

4.8 Fuel Cycle Technology

The following types of fuel cycles have been considered for the HTGR (21):

(1) Reference thorium cycle with recycle
(U-235 - U-233/Th-232/U-233).

(2) Thorium cycle without recycle
(U-235/Th-232/U-233).

(3) Plutonium makeup cycle
(Pu-239 - U-233/Th-232/U-233)

(4) Uranium cycle
(U-235/U-238/Pu-239).

With cycles 2 and 4 the spent fuel can be either stored or sold, depending on the cost and availability of reprocessing and on the value of the recoverable fuel.

In this work only 1 and 2 of the above fuel cycles are investigated in which the fuel cycles will likely involve at least two and possibly three modes of operation over the lifetime of the plant:

Table IX

HTGR Core Physics Parameters

Microscopic cross sections, barns (T_n = 1634°F)		Reference
σ_a (U-233)	183	(28)
σ^a(Th-232)	1.59	(28)
σ^a(U-238)	9.33	(28)
σ^a(U-235)	156	(28)
σ^a(U-236)	10.9	(28)
σ^a(U-234)	26.3	(28)
σ^a(Pu-239)	614	(28)
σ^a(Pu-240)	508	(28)
σ^a(Pu-241)	451	(28)
σ^a(FP of U-235)	19.3/fission	(33)
σ^a(FP of U-233)	16.3/fission	(33)
σ^a(Sm-149)	1.06×10^4	(28)
σ^a(Xe-135)	3.74×10^5	(28)
σ^a(graphite)	5.90×10^{-6}	(28)
σ^a_a(B-10)	5.49×10^2	(28)
Capture to fission ratio		
α_{25}	.224	(28)
α_{23}	.150	(28)
α_{49}	.620	(28)
α_{41}	.440	(28)
Average yield of neutrons per fission		
ν_{25}	2.43	(28)
ν_{23}	2.50	(28)
ν_{49}	2.89	(28)
ν_{41}	3.03	(28)
Miscellaneous		
Fast fission factor	1.015	(20)
Resonance escape probability (Th-232)	.70	(20)
Resonance escape probability (U-238)	.986	(20)
Fission-to-resonance nonleakage	.98	(20)
Total nonleakge	.97	(3)
ϕ_{max}	2.33×10^{14} $n/cm^2 \cdot sec$)	(3)
REC 23	193 MeV/fission	(24)
REC 25	204 MeV/fission	(24)
REC 49	213 MeV/fission	(24)
P_r (radial power peaking factor)	1.6	(3)

(1) Nonrecycle Operation. Fuel charged to the reactor consists of highly enriched uranium and thorium. The spent fuel removed from the core is either sold or placed in storage awaiting reprocessing and recycle.

(2) Initial recycle operation. An interim period for the early HTGR's when the stored U-233 is used exclusively to fuel the reactor.

(3) Recycle operation. The fuel removed from the core is reprocessed and the U-233 is fed back into the reactor along with highly enriched uranium makeup.

These three modes of operation are schematically represented in Fig. 4.8.1. These particular modes of operation are considered in this work and the results will be presented in Section 5.0.

Since the distribution of nuclear fuel composition in a reactor depends among other things, on the schedule for loading and unloading fuel and on the way the fuel is moved through the reactor, the fuel loading schedule for the reactor must be examined. Some of the possible fuel loading schemes that can be used by a utility to extend burnup of the fuel are "batch", rondelay" (scatter load), "continuous", and "out-in". Descriptions of these fuel loading schemes can be found in the literature (26,10,5,6); therefore a detailed account will not be given here.

The fuel loading scheme that is used in this analysis is the so called "out-in" (outside to center loading) fuel shuffling technique. In this procedure, the fresh fuel is charged near the outer edge and moved progressively toward the center from which it is discharged. Although considerably

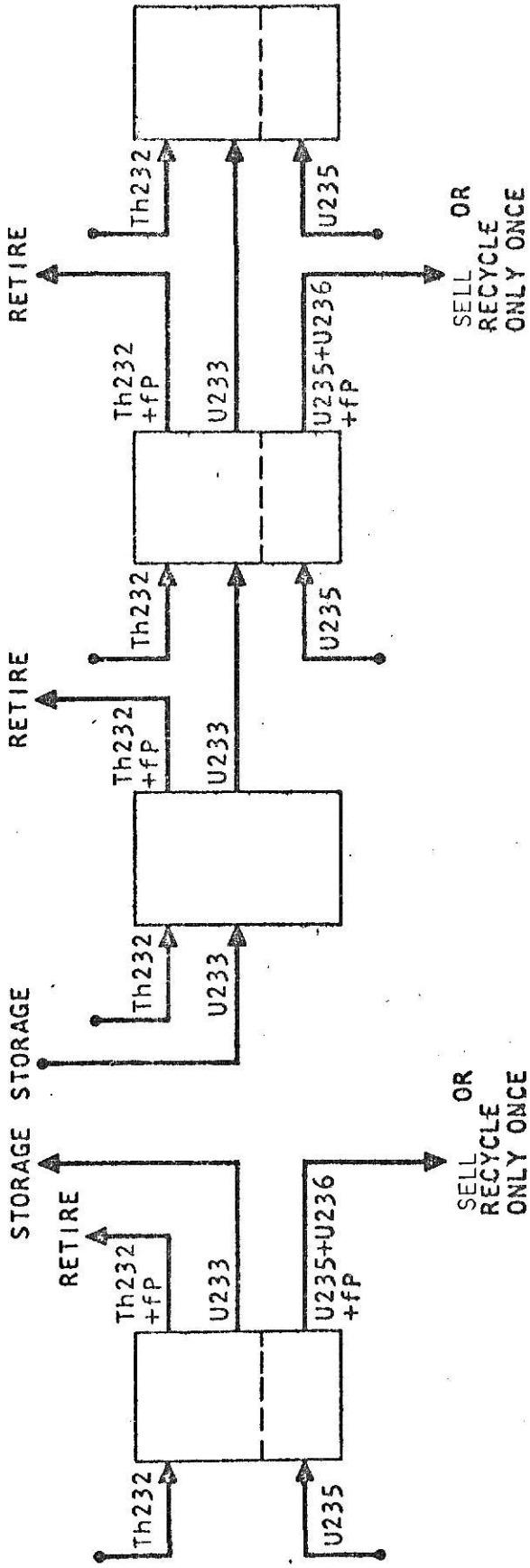


Figure 4.8.1 U-235/Th/U-233 Fuel Cycle Modes

better burnup is attained than in batch irradiation, the neutron utilization is not as efficient as in using other possible shuffling schemes because the highest reactivity, i.e., fresh fuel, is introduced in a region of low neutron importance, whereas the lowest (or possibly negative) reactivity occurs where the flux is highest (3). The advantage of outside to center loading is that it leads to a fairly uniform power density distribution in the radial direction.

4.9 Matrix Generator

A computer program was developed, based on the different fluence levels for the four regions considered and nth number of fuel cycles. The computer program is used to calculate all of the coefficients for the burnup and constraint equations developed in Section 4.0, and to prepare a data set on magnetic tape in the correct tabular form for MPS-360. Once the data set is complete, the MPS-360 will begin execution, i.e., it will start an iterative scheme similar to the one discussed in Section 2.0 (the product form of the inverse/revised simplex), until an optimum solution is reached. The matrix generator code is listed in Appendix D.

The code is extremely powerful since it can generate the number of cycles to find the total optimum of the system, i.e., multiple cycle optimum combination of all state variables that satisfy the operational restrictions of all regions and cycles. The computer code has nine subroutines which represent segments of the fuel cycle operation. In order to help the reader visualize how the fuel cycles are being simulated by using these subroutines, it is necessary to explain the purpose of each subroutine in the matrix generator code.

Probably the most important part of the code is the BURN subroutine which is used to calculate the coefficients of the burnup equations derived in Section 4.0. There are four sets of burnup equations as shown in Fig. 4.9.1 which correspond to the four different flux levels for each region. The BOL and EOL "power peaking," "local reactivity," "volume," and "enrichment constraints" are also located in the BURN subroutine. Actually the BURN subroutine represents the spatial dependency of the system, i.e., all of the coefficients have different values depending on the position in which they are located in the core. In Fig. 4.9.1, the BURN subroutine is presented as BURN1, BURN2, BURN3, and BURN4. The numbers represent the position in the core, i.e., number 1 represents the center region, number 2 represents the next region toward the outer edge of the core, etc.

The PART subroutine is used to generate the total core requirements, i.e., the localized requirements are met in the BURN partition and the "total power" and "total reactivity" constraints are met in the PART partitions. The PART subroutine generates constant coefficients independent of the flux level. The matrix generator produces four segments of PART for each cycle which corresponds to the four regions considered in this analysis. The four segments are located side by side, as shown in Fig. 4.9.1. Actually, the segments add the effects of the nuclide concentrations of the four different regions in terms of "total power" requirements and overall "reactivity" requirements.

The CONST subroutine was developed to look at the individual concentrations of the poisons, i.e., Xe-135, Sm-149, gross fission product accumulation and the total carbon content of the core.

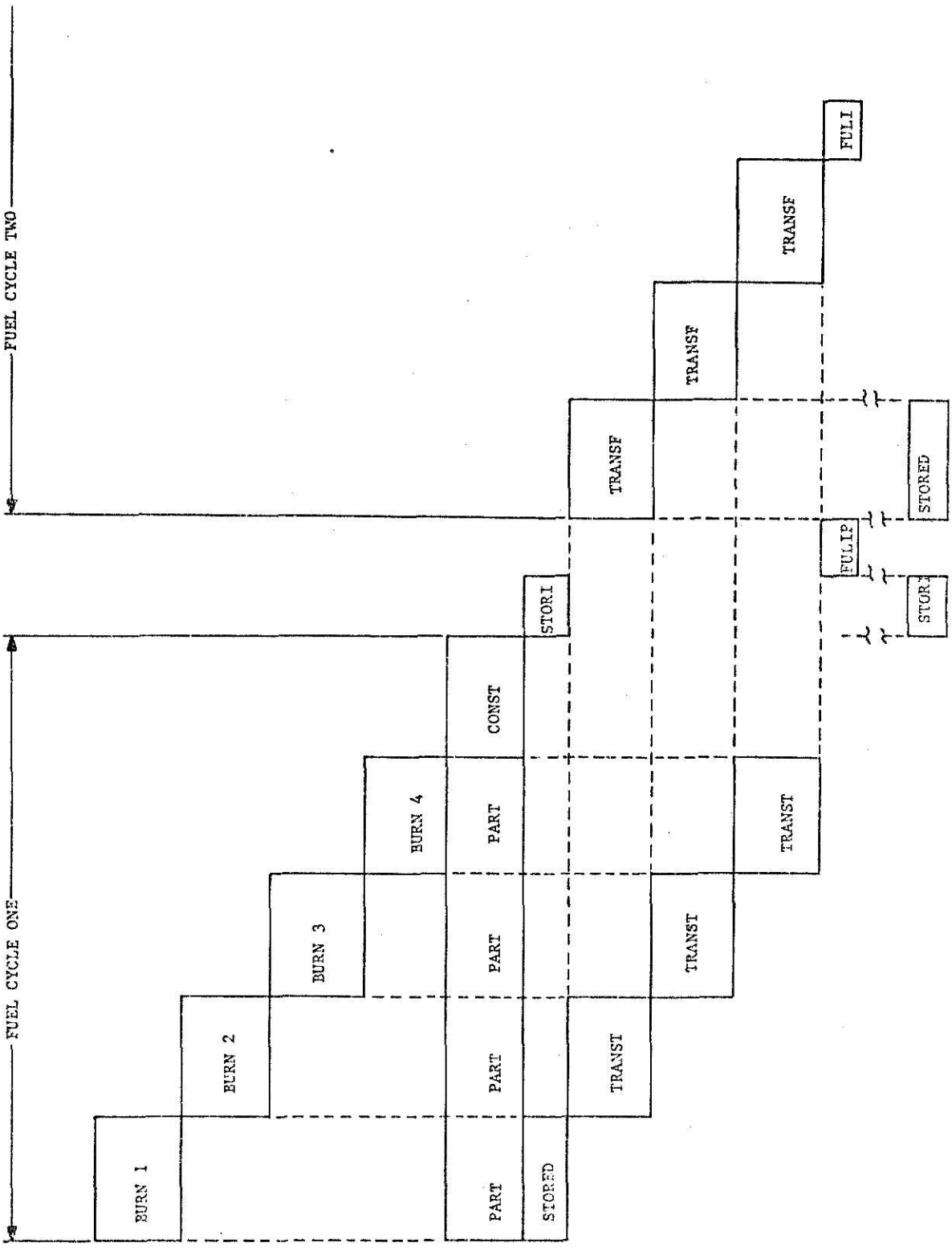


Figure 4.9.1 Schematic of the Matrix Generator

To expand the model into coupled cycles, i.e., concentrations in cycle 1 are dependent on concentrations in cycle 2, 3, and 4, it was necessary to develop "transfer" partitions to transfer the nuclide concentrations from one cycle to the next cycle and into the appropriate regions. Since an "out-in" refueling policy is used in this analysis, the transfers are always made toward the center of the next core (next cycle), e.g., the EOL concentration of region 2, cycle 1 is transferred to region 1, cycle 2. This transfer procedure is depicted for a two cycle case in Fig. 4.9.1. The TRANST subroutine establishes the necessary unit coefficients to take the EOL concentrations of regions 2, 3, and 4 of the nth cycle and, by using the subroutine TRANSF, make them available as BOL concentrations in regions 1, 2, and 3, respectively, of cycle n+1.

At the EOL of each cycle the fuel is removed from the center region for storage and eventual reprocessing and recycle operations (2). To simulate this storage area in the LP iterative scheme, the subroutine, STOREO was developed to remove the EOL concentrations of region 1, as shown in Fig. 4.9.1. These concentrations are then moved to an accumulative storage area for possible reprocessing and recycle operations. The subroutine STORI was developed to produce the accumulative storage area.

The FULIP subroutine was developed to keep an account of the fresh fuel loaded into region 4 of the next cycle, while the subroutine FULI actually transferred the fresh makeup fuel into region 4.

Figure 4.9.1 only represents 2 cycles, although the number of cycles can be extended to n cycles by changing one input card, i.e., change NCYC in the matrix generator code to the desirable number.

5.0 RESULTS AND DISCUSSION

In this analysis the matrix generator that is discussed in Section 4.9 was used to simulate three different fuel loading situations which are given as follows:

Case 1 - Three cycle case with no recycle capabilities, i.e., thorium cycle without recycle (U-235/Th/U-233),

Case 2 - Six cycle case with no recycle capabilities,

Case 3 - Six cycle case with U-233 recycle capabilities in the fourth refueling event, i.e., for the first three years the cycle can be given as (U-235/Th/U-233), and for the next three years the cycle can be given as (U-235 - U-233/Th-232/U-233).

The cases described above were solved using IBM Mathematical Programming System (IBM-MPS) and the optimum mass balance was attained for each case and presented in Tables X, XI, and XII for Case 1, Case 2, and Case 3, respectively. The optimum power producing characteristics for each case are also given in Table XIII.

Before an analysis of the results is given, it may help the reader to have some idea of the magnitude of each problem (Case 1, Case 2, and Case 3) solved by MPS. Case 1 required the smallest matrix which consisted of 348 rows and 701 variables (including slack variables). The optimum solution was found after 441 iterations which represented 2.26 minutes of execution time on the S/370 Model 158 computer. Although Case 1 and 2 required the same size matrix, i.e., 732 rows and 1442 variables (including slack variables), each case required a different execution time and number of iterations; since Case 3 had to consider U-233 feed after three years of operation. Case 2

Table X

Mass Balance, Thorium 3 Cycle Without Recycle

Fueling event	1	2	3	
Time[a], years	0	1	2	3
Fresh makeup, kg-atoms				
Th-235	35500.	36500.	10700.	
U-235	1780.	520.	152.	
U-238	132.	38.7	11.3	
Recycle feed, kg-atoms				
U-233		0.	0.	0.
Discharge, kg-atoms				
Th-232		8270.	7060.	6520.
U-233		125.	177.	204.
U-234		4.64	12.0	18.6
U-235		270.	193.	155.
U-236		28.2	41.9	49.9
U-238		24.3	19.5	16.5
Pu-239		2.76	3.20	3.32
Pu-240		.726	1.18	1.37
Pu-241		.326	.587	.706

[a]At .8 load factor

Table XI

Mass Balance, Thorium 6 Cycle Without Recycle

Fueling event	1	2	3	4	5	6	
Time[a], years	0	1	2	3	4	5	6
Fresh makeup (kg-atoms)							
Th-232	24300.	3870.	410.	12800.	36500.	13100.	
U-235	1800.	466.	490.	486.	520.	187.	
U-238	133.	34.7	36.4	36.1	38.7	13.9	
Recycle feed (kg-atoms)							
U-233	0.	0.	0.	0.	0.	0.	
Discharge (kg-atoms)							
Th-232		8720.	10600.	3070.	768.	3480.	49.1
U-233		126.	192.	187.	191.	204.	187.
U-234		4.67	12.9	17.4	21.3	22.3	21.0
U-235		268.	188.	161.	146.	141.	148.
U-236		26.3	40.8	52.1	59.4	57.0	60.0
U-238		24.2	18.64	17.8	17.1	16.0	17.5
Pu-239		2.76	3.26	3.26	3.28	3.34	3.27
Pu-240		.724	1.19	1.37	1.46	1.46	1.46
Pu-241		.324	.577	.724	.794	.776	.800

[a]At .8 load factor

Table XII

Mass Balance, Thorium 6 cycle With U-233 Recycle

Fueling event	1	2	3	4	5	6	
Time[a], years	0	1	2	3	4	5	6
Fresh makeup (kg-atoms)							
Th-232	24300.	4220.	1010.	237.	1780.	54300.	
U-235	1780.	464.	486.	0.	0.	0.	
U-238	132.	34.5	36.1	1510.	1836.	2590.	
Recycle feed (kg-atoms)							
U-233	0.	0.	0.	409.	499.	706.	
Discharge (kg-atoms)							
Th-232		8640.	10600.	3520.	376.	3820.	641.
U-233		125.	192.	189.	189.	206.	190.
U-234		4.67	12.9	21.1	21.1	22.5	21.2
U-235		269.	188.	160.	147.	140.	147.
U-236		28.1	40.8	36.2	59.7	56.8	59.5
U-238		24.2	18.6	17.6	17.3	15.8	17.2
Pu-239		2.76	3.26	3.27	3.28	3.33	3.28
Pu-240		.724	1.19	1.37	1.46	1.47	1.48
Pu-241		.3235	.577	.722	.800	.772	.800

[a]At .8 load factor

Table XIII

Optimum Power Producing Characteristics

Case 1		
Cycle 1:	2320	MWt
Cycle 2:	2406	MWt
Cycle 3:	2320	MWt
Case 2		
Cycle 1:	2320	MWt
Cycle 2:	2320	MWt
Cycle 3:	2260	MWt
Cycle 4:	2260	MWt
Cycle 5:	2369	MWt
Cycle 6:	2320	MWt
Case 3		
Cycle 1:	2320	MWt
Cycle 2:	2320	MWt
Cycle 3:	2260	MWt
Cycle 4:	2260	MWt
Cycle 5:	2408	MWt
Cycle 6:	2810	MWt

required 1584 iterations to achieve an optimum solution with an execution time of 29.8 minutes while Case 3 required 1660 iterations with an execution time of 33.46 minutes to attain the optimum solution. Some unusual problems were encountered while solving Case 2 and Case 3. A discussion of these problems is given in Appendix E.

The most obvious peculiarity is found when comparing the last cycle mass balance results of each case. This oddity can be illustrated by comparing the mass balance results of Case 1 with the mass balance results of Case 2. This comparison can be accomplished by only comparing the fresh makeup feed for the first three cycles of each case as follows:

	Case 1	Case 2
	(kg-atoms)	(kg-atoms)
Cycle 1		
Th-232	35500.	24300.
U-235	1780.	1800.
U-238	132.	133.
Cycle 2		
Th-232	36500.	3870.
U-235	520.	466.
U-238	38.7	34.7
Cycle 3		
Th-232	10700.	410.
U-235	152.	490.
U-238	11.3	36.4

It may be noted that the only difference in the two cases given above is that Case 2 is a coupled 6 cycle situation and Case 1 is a coupled 3 cycle

situation, i.e., Case 2 is Case 1 extended by 3 cycles. When comparing Cases 1 and 2, the initial input concentrations of the first cycle of both cases is approximately the same with the exception of Th-232. The larger input concentration of Th-232 in Case 1 can be attributed to the less restrictive overall reactivity requirements of Case 1. It may be noted that the fuel loaded into the system of Case 1 has to meet only three overall reactivity requirements whereas the fuel loaded into the system of Case 2 has to meet four overall reactivity requirements; therefore a heavier loading of Th-232 can exist in the initial core of Case 1, i.e., the objective function is improved by using Th-232 to breed U-233. One can also deduce from the comparison of Case 1 and Case 2 that less fissile loading was needed in Case 1, Cycle 3 than was needed in Case 2, Cycle 3. The main reason for the smaller fissile loading can be ascribed to the larger fertile (Th-232) loading of the fuel in the initial two cycles of Case 1. Since a larger fertile loading was present in Case 1, more U-233 was bred in the first 3 cycles of Case 1 than was bred in the first 3 cycles of Case 2; therefore there was less U-235 needed to meet the operational requirements of the third cycle of Case 1.

It may be noted that the total power constraint had to be lowered in the third and fourth years of operation to attain a feasible solution for the 6 cycle cases, i.e., Case 2 and Case 3. The requirement to lower the power requirements for these two years to reach the feasible region can be explained by the interaction of the shuffling schemes used on this model with the coupling effects of the system. As stated previously, the "out-in" shuffling technique was used in the coupling procedure generated by the "matrix generator" as MPS input data. This shuffling technique is very

restrictive since the fuel loaded into region 4 (outside region) must eventually traverse the complete core, region by region, i.e., the fuel loaded into the initial core has a restrictive effect on any fuel loaded into the core within the next four years, or the fuel loaded into the core at any point in time has a restrictive effect on any fuel loaded four years hence. If Cycle 3 of Case 2 is taken as an example, the total power produced as given by Table XIII is only 2260 MWt. This total power produced is about 2.5% lower than most of the other power producing cycles. This low power value may be caused by constraints on either side of cycle 3, i.e., to satisfy the "power peaking" and reactivity constraints in Cycle 2 and Cycle 5, the total power requirements for Cycle 3 must be 2260 MWt. To determine exactly what is causing the lower power requirements of Cycle 3 and 4, a detailed and complex analysis should be made of the interaction of the state variables with the cycle and regional constraints. This analysis is beyond the scope of this work.

It may also be pointed out that in analyzing the results it was found that one of the main restrictive constraints is the "power peaking" constraint in region 1 (center region). This constraint actually dictated the fissile and fertile fuel loaded in the other regions since the fuel loaded in the outer regions must eventually pass through the center region and meet its "power peaking" constraint.

GA predicts that after 8 years the accumulated U-233 would furnish all of the fissile requirements of the HTGR for 3 more years, resulting in good neutronics and a high conversion ratio (39). This situation was simulated on a smaller scale with Case 3. In Case 3, the U-233 was held out of the system for 3 years and then it was allowed to be used as recycle feed for

in the 4th refueling event, 409 kg-atoms was needed to refuel the reactor, and zero amount of U-235 was taken into the system. It may also be noted that 409 kg-atoms actually surpasses the amount of U-233 produced in the first two years of operation, i.e., 317 and 318 kg-atoms produced in the first two years of operation in Case 3 and in Case 2, respectively. Also it may be interesting to note that if the amount produced for 8 years is extrapolated from Case 2, the total amount of U-233 can be given as 1461 kg-atoms for 8 years of operation. If 450 kg-atoms is taken as the average amount of U-233 to refuel the reactor each year, i.e., 1350 kg-atoms for 3 years, then from the optimum results given in this work, GA' prediction of 8 years operation, 3 years independent operation on U-233 recycle can be assumed theoretically correct. Actually, after 8 years of operation the next 3 years of only using U-233 recycle is not entirely independent of U-235; since U-235 will be in the system in varied amounts for at least four more years.

It may be noted from the results given in Table XII of Case 3, that after 3 years of operation, U-233 was allowed to enter the system in unrestricted amounts. This forced all of the U-235 fresh makeup to zero. This action can be attributed to two inherent characteristics of U-233 and the objective function maximizing the EOL concentration of U-233. To get a better representation of the interaction of U-233 and U-235 mixture, via, recycle, the objective function should be changed. This change could quity simply be accomplished by putting a negative value on the BOL concentration of U-233.

Probably the factor that creates the most inaccuracy in the results is the assumption of a constant resonance escape probability, p. One may note that in all three cases, the concentration of Th-232 varies drastically, and

since the value of p is essentially independent of concentration of Th-232, the value of p should change accordingly. A remedy for this inaccuracy would be to do an iterative updating procedure on p, i.e., after the optimal solution is reached, the regional value of p can be found by applying a homogeneous calculation with the concentration of Th-232. A more detailed description of this procedure will be given in the "Suggestions for Further Study" chapter of this paper.

In summary, the model attained in this work is a good beginning for future HTGR fuel management techniques. The two main constraints that really control the model are the "power peaking" constraint in region 1 and the EOL overall reactivity constraint. These constraints could lose their dominant influence on the problem by changing the schuffling technique, and relaxing the EOL overall reactivity constraint. It may also be noted that studies which were done by General Atomic (1) and Oak Ridge National Laboratory (3) are in lose agreement with this work.

6.0 SUGGESTIONS FOR FURTHER STUDY

There are several suggestions which may improve this model. It would be desirable to update such parameters as "resonance escape probability" and "fast fission factor," since they are concentration dependent. One can update these parameters for the least expense by finding the initial optimum basis and saving the basis on magnetic tape. Therefore, after updating the new concentration dependent values, new activity constants could be inserted, via "matrix Generator", and a new optimum basis could be attained by updating the old basis in a minimum number of iterations.

The core may also be divided into four axial regions. This method can serve as a technique to include spatial distribution in the axial direction. The axial direction should be included into HTGR fuel management schemes; since an axial exponential power distribution is inherent in the HTGR core design (1,6,20).

More elaborate techniques such as perturbation theory may be used to update such critical parameters as flux shape and macroscopic cross section values. This may be done by simulating a stagewise process at which EOL concentrations and spatial characteristics of the core can be taken from the optimum basis by READCOMM (40) which is a subroutine that enables a user to augment MPS with procedures written in the FORTRAN language. Before leaving the MPS system, the final basis can be saved by appropriate JCL (Job Control Language) and MPS control parameters (19). The pertinent EOL data can be taken by READCOMM and with the use of a diffusion code such as C2D2G (29), the average thermal and fast flux values could be determined for each region. This code can also compute the X-Y power distributions.

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APPENDICES

APPENDIX A

Derivation of Th-232 and U-233 Transmutation Equations

The net rate of accumulation with respect to time of Th-232 is given by

$$\frac{dN_{02}}{dt} = - N_{02}\,\sigma_{02}\phi \quad - \quad N_{23}\,\sigma_{23}\,\phi\,\eta_{23}\,\varepsilon\,P_1(1-p_{02})$$

$$\left\{\begin{matrix}\text{Absorption of}\\ \text{thermal neutrons}\\ \text{in Th-232}\end{matrix}\right\} \quad \left\{\begin{matrix}\text{Absorption of resonance}\\ \text{neutrons in Th-232 from}\\ \text{U-233 fission}\end{matrix}\right\}$$

$$- N_{25}\,\sigma_{25}\,\phi\,\eta_{25}\,\varepsilon\,P_1(1-p_{02}) \qquad \text{(A-1)}$$

$$\left\{\begin{matrix}\text{Absorption of resonance}\\ \text{neutrons in Th-232 from}\\ \text{U-235 fission}\end{matrix}\right\}$$

where all notation is defined in Chapter 4.0 of this paper.

The net rate of accumulation with respect to time of U-233 can be given by

$$\frac{dN_{23}}{dt} = N_{02}\,\sigma_{02}\,\phi \quad + \quad N_{23}\,\sigma_{23}\,\phi\,\eta_{23}\,\varepsilon\,P_1(1-p_{02})$$

$$\left\{\begin{matrix}\text{Absorption of thermal}\\ \text{neutrons in Th-232}\end{matrix}\right\} \quad \left\{\begin{matrix}\text{Absorption of resonance}\\ \text{neutrons in Th-232 from}\\ \text{U-233 fission}\end{matrix}\right\}$$

$$+ N_{25}\,\sigma_{25}\,\phi\,\eta_{25}\,\varepsilon\,P_1(1-p_{02}) - N_{23}\,\sigma_{23}\,\phi\,. \qquad \text{(A-2)}$$

$$\left\{\begin{matrix}\text{Absorption of resonance}\\ \text{neutrons in Th-232 from}\\ \text{U-235 fission}\end{matrix}\right\} \quad \left\{\begin{matrix}\text{Absorption of thermal}\\ \text{neutrons in U-233}\end{matrix}\right\}$$

In notation defined in Chapter 4.0, Eqs. (A-1) and (A-2) may be written as

$$\frac{dN_{02}}{d\theta} + N_{02}\,\sigma_{02} + N_{23}\,\sigma_{23}\,L_1 = N_{25}\,\sigma_{25}\,L_5 \qquad \text{(A-3)}$$

and

$$\frac{dN_{23}}{d\theta} + N_{23}\,\sigma_{23}(1-L_1) - N_{02}\,\sigma_{02} = N_{25}\,\sigma_{25}\,L_2 \qquad \text{(A-4)}$$

Noting that Eqs. (A-3) and (A-4) are coupled differential equations, the equations can be solved using Laplace transform techniques. To simplify equations (A-3) and (A-4), it is assumed that the net change in nuclide concentrations of U-235 can be represented by an exponential decrease due to neutron capture. Therefore the fluence dependent concentration of U-235 may be given as

$$N_{25}(\theta) = N_{25}^{o}\, e^{-\sigma_{25}\theta} \qquad \text{(A-5)}$$

where all terms are defined in Chapter 4.0 of this paper. Substituting Eq. (A-5) into Eqs. (A-3) and (A-4), and letting $P_1 = \sigma_{02}$, $P_2 = L_1\,\sigma_{23}$, $P_3 = \sigma_{25}$, $P_4 = \sigma_{23}(1-L_1)$, $P_5 = \sigma_{23}$ and $P_6 = L_2\,\sigma_{25}$, the Laplace transforms may be given as

$$\bar{N}_{02}(s+P_1) + \bar{N}_{23}P_2 = N_{02}^{o} - \frac{P_6\,N_{25}^{o}}{s+P_3} \qquad \text{(A-6)}$$

and

$$\bar{N}_{02}(-P_1) + \bar{N}_{23}(s+P_4) = N_{23}^{o} + \frac{P_6\,N_{25}^{o}}{s+P_3} \qquad \text{(A-7)}$$

where $\bar{N}_{02} \equiv \mathcal{L}\{N_{02}(\theta)\}$

$\bar{N}_{23} \equiv \mathcal{L}\{N_{23}(\theta)\}$

and $\mathcal{L}\{N_{23}(\theta)\}$ represents the Laplace transform of the original function $N_{23}(\theta)$.

Noting that Eqs. (A-6) and (A-7) are linear equations, the equations may be transformed into second order determinants and solved using Cramer's rule (36) as follows:

$$\bar{N}_{02} = \frac{\begin{vmatrix} N_{02}^{o} - \dfrac{P_6 N_{25}^{o}}{s + P_3} & P_2 \\ N_{23}^{o} + \dfrac{P_6 N_{25}^{o}}{s + P_3} & s+P_4 \end{vmatrix}}{\begin{vmatrix} s+P_1 & P_2 \\ -P_1 & s+P_4 \end{vmatrix}}$$

$$= \frac{(s+P_4)[N_{02}^{o} - (s+P_3)^{-1}(P_6 N_{25}^{o})] - P_2[N_{23}^{o} + (s+P_3)^{-1}(P_6 N_{25}^{o})]}{(s+P_1)(s+P_4) + P_1 P_2} \qquad \text{(A-8)}$$

Rearranging Eq. (A-8), $\bar{N}_{02}$ can be given as

$$\bar{N}_{02} = \frac{s^2 N_{02}^{o} - (sP_6 N_{25}^{o})(s+P_3)^{-1} + P_4 N_{02}^{o} - (P_4 P_6 N_{25}^{o})(s+P_3)^{-1} - P_2 N_{23}^{o} - (P_2 P_6 N_{25}^{o})(s+P_3)^{-1}}{s^2 + (P_1+P_4)s + P_1 P_4 + P_1 P_2} .$$

(A-9)

The denominator of Eq. (A-9) may be factored using the quadratic equation (36), and the roots may be given as

$$R_1 = .5\{-(P_1+P_4) + [(P_1+P_4)^2 - 4(P_1 P_4 + P_1 P_2)]^{1/2}\} \qquad \text{(A-10)}$$

and

$$R_2 = .5\{-(P_1+P_4) - [(P_1+P_4)^2 - 4(P_1 P_4 + P_1 P_2)]^{1/2}\} . \qquad \text{(A-11)}$$

Substituting Eqs. (A-10) and (A-11) into Eq. (A-9), the following expression is obtained:

$$\bar{N}_{02} = \frac{s^2 N_{02}^{o} + s[N_{02}^{o}(P_4+P_3) - N_{25}^{o} P_6 - N_{23}^{o} P_2] + N_{02}^{o} P_3 P_4 - N_{25}^{o} P_6 P_5 - N_{23}^{o} P_2 P_3}{(s-R_1)(s-R_2)(s+P_3)}$$

(A-12)

Using the Partial fractions method (36), Eq. (A-12) can be written as the sum of rational functions having the form $\frac{A}{(as+b)^r}$, $\frac{A_s + B}{(as^2+bs+c)^r}$ where $r = 1,2,3,\ldots$, as

$$\bar{N}_{02} = \frac{X}{(s-R_1)} + \frac{Y}{(s-R_2)} \quad \frac{Z}{(s+P_3)} \tag{A-13}$$

Now by determining the inverse Laplace transform of each of the partial fractions, the inverse Laplace transform of $\bar{N}_{02}$, i.e., $\mathcal{L}^{-1}\{\bar{N}_{02}\}$, can be found. In order to do this, the constants, X,Y,Z must be found by clearing the fractions and equating like powers of s on both sides of the resulting equation. This was done as follows: Letting $s = R_1$,

$$X = \frac{R_1^2 N_{02}^o + R_1[N_{02}^o(P_4+P_3) - N_{25}^o P_6 - N_{23}^o P_2] + D}{(R_1-R_2)\ (R_1+P_3)} \tag{A-14}$$

where $D = N_{02}^o P_3 P_4 - N_{25}^o P_6 P_5 - N_{23}^o P_2 P_3$.

Letting $s = R_2$,

$$Y = \frac{R_2^2 N_{02}^o + R_2[N_{02}^o(P_4+P_3) - N_{25}^o P_6 - N_{23}^o P_2] + D}{(R_2-R_1)\ (R_2+P_3)} \quad . \tag{A-15}$$

Letting $s = -P_3$,

$$Z = \frac{P_3^2 N_{02}^o - P_3[N_{02}^o(P_4+P_3) - N_{25}^o P_6 - N_{23}^o P_2] + D}{(P_3+R_1)\ (P_3+R_2)} \quad . \tag{A-16}$$

Noting that the inverse Laplace transform of $\bar{N}_{02}$ can be attained by determining the inverse Laplace transform of each expression in Eq. (A-13), the inverse Laplace transform can be given as follows:

$$\mathcal{L}^{-1}\{\bar{N}_{02}\} = N_{02}(\theta)$$

and $$\mathcal{L}^{-1}\{\bar{N}_{02}\} = \mathcal{L}^{-1}\left\{\frac{X}{s-R_1}\right\} + \mathcal{L}^{-1}\left\{\frac{Y}{s-R_2}\right\} + \mathcal{L}^{-1}\left\{\frac{Z}{s+P_3}\right\}$$

therefore

$$N_{02}(0) = \mathcal{L}^{-1}\left\{\frac{X}{s-R_1}\right\} + \mathcal{L}^{-1}\left\{\frac{Y}{s-R_2}\right\} + \mathcal{L}^{-1}\left\{\frac{Z}{s+P_3}\right\}$$

$$= N_{02}^{o}\left\{\frac{e^{R_1\theta}[(P_4+P_3)P_1 + R_1^2 + P_3P_4]}{(R_1-R_2)(R_1+P_3)} + \frac{e^{R_2\theta}[(P_4+P_3)R_2 + R_2^2 + P_3P_4]}{(R_2-R_1)(R_2+P_3)}\right\}$$

$$- N_{25}^{o}\left\{\frac{e^{R_1\theta}(R_1R_6+P_6P_5)}{(R_1-R_2)(R_1+P_3)} + \frac{e^{R_2\theta}(R_2P_6+P_6P_5)}{(R_2-R_1)(R_2+P_3)} + \frac{e^{-P_3\theta}(P_6P_5-P_3P_6)}{(P_3+R_1)(P_3+R_2)}\right\}$$

$$- N_{23}^{o}\left\{\frac{e^{R_1\theta}(R_1P_2+P_2P_3)}{(R_1-R_2)(R_1+P_3)} + \frac{e^{R_2\theta}(R_2P_2+P_2P_3)}{(R_2-R_1)(R_2+P_3)}\right\}. \qquad \text{(A-17)}$$

The value of N_{23} can be obtained in a similar manner. Using Cramer's rule,

$$\bar{N}_{23} = \frac{\begin{vmatrix} s+P_1 & N_{02}^{o} - \dfrac{P_6 N_{25}^{o}}{s+P_3} \\ -P_1 & N_{23}^{o} + \dfrac{P_6 N_{25}^{o}}{s+P_3} \end{vmatrix}}{s^2 + (P_1+P_4)s + P_1P_4 + P_1P_2}$$

$$= \frac{N_{23}^{o}(s+P_1)(s+P_3) + N_{25}^{o}[sP_6 + P_1P_6 - P_1P_6(s+P_3)] + N_{02}^{o}P_1(s+P_3)}{(s-R_1)(s-R_2)(s+P_3)}$$

(A-18)

where all constants have been defined previously. Now expanding Eq. (A-18) in a partial fraction expansion, the following can be attained:

$$s^2N_{23}^o + s[N_{25}^o(P_6-P_1P_6) + P_1N_{02}^o + N_{23}^o(P_1+P_3)] + N_{23}^oP_1P_3$$

$$+ N_{25}^oP_1P_3 + N_{25}^o(P_1P_6-P_1P_6P_3) + N_{02}^oP_1P_3$$

$$= X'(s-R_2)(s+P_3) + Y'(s-R_1)(s+P_3) + Z'(s-R_1)(s-R_2) \qquad \text{(A-19)}$$

The constant X' can be obtained by letting $s = R_2$ as,

$$X' = \frac{R_1^2N_{23}^o + R_1[N_{25}^o(P_6-P_1P_6) + P_1N_{02}^o + N_{23}^o(P_1+P_3)] + D'}{(R_1-R_2)\ (R_1+P_3)} \qquad \text{(A-20)}$$

where $D' = N_{23}^oP_1P_3 + N_{25}^o(P_1P_6-P_1P_6P_3) + N_{02}^oP_1P_3$

Letting $s = R_2$

$$Y' = \frac{R_2^2N_{23}^o + R_2[N_{25}^o(P_6-P_1P_6) + P_1N_{02}^o + N_{23}^o(P_1+P_3) + D']}{(R_2-R_1)\ (R_2+P_3)} \qquad \text{(A-21)}$$

Letting $s = -P_3$

$$Z' = \frac{P_3^2N_{23}^o - P_3[N_{25}^o(P_6-P_1P_6) + P_1N_{02}^o + N_{23}^o(P_1+P_3)] + D'}{(P_3+R_1)\ (P_3+R_2)} \qquad \text{(A-22)}$$

and since

$$N_{23}(\theta) = \mathcal{L}^{-1}\left\{\frac{X'}{s-R_1}\ \frac{Y'}{s-R_2}\ \frac{Z'}{s+P_3}\right\}, \qquad \text{(A-23)}$$

the solution may be given as

$$N_{23}(\theta) = e^{R_1\theta}\ X' + e^{R_2\theta}\ Y' + e^{-P_3\theta}\ Z'\ . \qquad \text{(A-24)}$$

APPENDIX B

Derivation of the Approximate Transmutation Equation for U-233

Equations (A-3) and (A-4) can be decoupled by assuming that the net change in nuclide concentration of Th-232 can be represented by an exponential decrease due to neutron capture in the thermal region; therefore an expression similar to Eq. (A-5) can be written for Th-232 as

$$N_{02}(\theta) = N_{02}^{o}\, e^{-\sigma_{02}\theta} \tag{B-1}$$

where all terms are defined in Chapter 4.0 of this paper. Substituting Eq. (A-5) and (B-1) into Eq. (A-4), a simplified expression for the fluence dependent concentration of U-233 may be attained and given as

$$\frac{dN_{23}}{d\theta} + N_{23}\sigma_{23}(1-L_1) - N_{02}^{o}\sigma_{02}e^{-\sigma_{02}\theta} = N_{25}^{o}\sigma_{25}L_2 e^{-\sigma_{25}\theta} \,. \tag{B-2}$$

Equation (B-2) is now a first order linear differential equation. One can see that Eq. (B-2) can be written in the form

$$\frac{dy}{dx} + P \cdot y = Q(x) \tag{B-3}$$

Equation (B-3) can be solved using an appropriate "integrating factor" (37) which is an expression such that the differential equation becomes exact if it is multiplied by that factor. The factor $e^{\int P\, dx}$ is said to be an "integrating factor" of Eq. (B-3).

The method discussed above may be applied to Eq. (B-2) by rearranging terms as follows:

$$\frac{dN_{23}}{d\theta} + \sigma_{23}(1-L_1)N_{23} = N_{02}^{o}\sigma_{02}e^{-\sigma_{02}\theta} + N_{25}^{o}\sigma_{25}L_2 e^{-\sigma_{25}\theta} \tag{B-4}$$

Noting the analogy between Eqs. (B-3) and (B-4), the integrating factor for Eq. (B-4) may be attained by applying the boundary conditions, $N_{23} = N_{23}^{o}$ at $\theta = 0$; therefore the integrating factor can be given as

$$\exp \int_0^{\theta} \sigma_{23}(1-L_1)d\theta' = \exp[\sigma_{23}(1-L_1)\theta] \ . \tag{B-5}$$

Eq. (B-4) is now multiplied by the integrating factor with the following results:

$$d\{N_{23} \exp[\sigma_{23}(1-L_1)\theta]\} = (N_{02}^{o}\sigma_{02} \exp\{[\sigma_{23}(1-L_1) - \sigma_{02}]\theta\} + N_{25}^{o}\sigma_{25}L_2 \exp\{[\sigma_{23}(1-L_1) - \sigma_{25}]\theta\})d\theta \ . \tag{B-6}$$

The solution to Eq. (B-6) may be attained by integrating both sides of the equation and applying the boundary conditions, $N_{23} = N_{23}^{o}$ at $\theta = 0$, as

$$\begin{aligned} N_{23} \exp[\sigma_{23}(1-L_1)\theta] - N_{23}^{o} &= \int_0^{\theta} N_{02}^{o}\sigma_{02} \exp\{[\sigma_{23}(1-L_1) - \sigma_{02}]\theta'\}d\theta' \\ &+ \int_0^{\theta} N_{25}^{o}\sigma_{25}L_2 \exp\{[\sigma_{23}(1-L_1) - \sigma_{25}]\theta'\}d\theta' \\ &= \frac{N_{02}^{o}\sigma_{02}[\exp\{[\sigma_{23}(1-L_1) - \sigma_{02}]\theta\} - 1]}{\sigma_{23}(1-L_1) - \sigma_{02}} \\ &+ \frac{N_{25}^{o}\sigma_{25}L_2[\exp\{[\sigma_{23}(1-L_1) - \sigma_{25}]\theta\} - 1]}{\sigma_{23}(1-L_1) - \sigma_{25}} \end{aligned} \tag{B-7}$$

By rearranging Eq. (B-7) and multiplying both sides of the expression by $\exp[-\sigma_{23}(1-L_1)\theta]$, the solution to Eq. (B-4) may be given as

$$\begin{aligned} N_{23}(\theta) &= N_{23}^{o}\{\exp[-\sigma_{23}(1-L_1)\theta]\} \\ &+ \frac{N_{02}^{o}\sigma_{02}\{\exp(-\sigma_{02}\theta) - \exp[-\sigma_{23}(1-L_1)\theta]\}}{\sigma_{23}(1-L_1) - \sigma_{02}} \\ &+ \frac{N_{25}^{o}\sigma_{25}L_2\{\exp(-\sigma_{25}\theta) - \exp[-\sigma_{23}(1-L_1)\theta]\}}{\sigma_{23}(1-L_1) - \sigma_{25}} \ . \end{aligned} \tag{B-8}$$

Appendix C

Average Flux Value

For the common case of a reflected cylindrical reactor in which the fuel is distributed uniformly, the two-dimensional flux distribution in the core can be represented approximately by (5)

$$\frac{\phi}{\phi_{max}} = J_o\left(2.405\ \frac{r}{R_1}\right) \tag{C-1}$$

where ϕ_{max} is the flux at the center of the core where it is assumed to have its maximum value; J_o is the zero-order Bessel function of the first kind, r is the radial coordinate, and R_1 is the effective radius of the reactor, including an allowance for the reflector.

The two-dimensional average flux value can be given as

$$\bar{\phi} = \frac{1}{\pi R^2} \int_0^R \phi_{max}\ J_o\left(\frac{2.405r}{R_1}\right)\ 2\pi r\ dr \tag{C-2}$$

where R is the actual radius of the core. Since (24)

$$\int J_o(z)\ z dz = z\ J_1(z)\ , \tag{C-3}$$

the integration of Eq. (C-2) can be done as follows:

$$\bar{\phi} = \frac{2\pi R_1\ \phi_{max}}{\pi R^2\ (2.405)} \int_0^R J_o\left(\frac{2.405r}{R_1}\right)\left(\frac{2.405r}{R_1}\right)dr$$

$$= \frac{2R_1\ \phi_{max}}{R^2\ (2.405)}\left[r\ J_1\left(\frac{2.405r}{R_1}\right)\right]_0^R$$

$$= \frac{2R_1\ \phi_{max}}{R\ (2.405)} J_1\left(\frac{2.405R}{R_1}\right) \tag{C-4}$$

where J_1 is the first order Bessel function. From Eq. (C-4) the maximum to average flux ratio can be given as

$$\frac{\phi_{max}}{\bar{\phi}} = \frac{(R/R_1)(2.405)}{2\ J_1[2.405(R/R_1)]} \qquad (C\text{-}5)$$

For a reflected cylindrical reactor, R/R_1 may be taken roughly as 5/6 (5); therefore Eq. (C-5) can be given as

$$\frac{\phi_{max}}{\bar{\phi}} = \frac{1.002}{J_1(2.004)}$$

$$= \frac{1.002}{.576}$$

$$= 1.74\ . \qquad (C\text{-}6)$$

The value obtained from Eq. (C-6) is in close agreement to the value given by WASH 1085 (3) which is

$$\frac{\phi_{max}}{\bar{\phi}} = 1.61\ . \qquad (C\text{-}7)$$

Therefore Eq. (C-1) can be used as an approximate radial flux distribution by assuming $R_1 = 6R/5$. The expression for the radial dependent flux can now be given as

$$\phi(r) = \phi_{max}\ J_o(.145r) \qquad (C\text{-}8)$$

where R is given in Table I and r is the variable radius in feet.

Since the core is divided into four equal volume regions, Eq. (C-8) must be integrated over that particular part of the core to get the average to maximum flux ratio. The radial dimensions of the regions are given below.

$R_1 = 6.88$ ft. and $0 \rightarrow R_1$: 6.88 ft.

$R_2 = 9.72$ ft. and $R_1 \rightarrow R_2$: 2.84 ft.

$R_3 = 11.9$ ft. and $R_2 \rightarrow R_3$: 2.18 ft.

$R_4 = 13.8$ ft. and $R_3 \rightarrow R_4$: 1.90 ft.

For illustration purposes the average flux for region 2 can be found by integrating Eq. (C-8) from R_1 to R_2 as

$$\bar{\phi}_2 = \frac{\phi_{max}}{\pi(R_1^2 - R_2^2)} \int_{R_1}^{R_2} J_o(.1452r)2\pi r \, dr$$

$$= \phi_{max} \, (.0424) \int_{R_1}^{R_2} J_o(.1452r) r \, dr$$

$$= \phi_{max} \, (.2918)[r \, J_1(.1452r)]_{R_1 = 6.88 \text{ ft}}^{R_2 = 9.72 \text{ ft}}$$

$$= \phi_{max} \, (.2918)[9.72 \, J_1(1.41) - 6.88 \, J_1(.999)]$$

$$= .66 \, \phi_{max} \qquad \text{(C-9)}$$

where the values for the first order Bessel functions are taken from Ref. (38).

A general expression for a four region two-dimensional reactor flux can be written as

$$\bar{\phi}_k = \phi_{max} \, (.2918)[r \, J_1(.1452r)]_{R_{k-1}}^{R_k} \qquad \text{(C-10)}$$

where k represents the region under investigation. From Eq. (C-10) the average flux level for each region can be calculated and given as

$$R_1: \quad \bar{\phi}_1 = .88 \, \phi_{max}$$

$$R_2: \quad \bar{\phi}_2 = .66 \, \phi_{max}$$

$$R_3: \quad \bar{\phi}_3 = .48 \, \phi_{max}$$

$$R_4: \quad \bar{\phi}_4 = .31 \, \phi_{max} \, .$$

APPENDIX D

Computer Program to Generate a Linear Programming (IBM-MPS) Input Matrix

This program represents all of the burnup equations and constraint equations presented in Chapter 4.0. This program iterates over 4 regions and "NCYC" cycles to produce the required matrix input for the IBM-MPS.

The following is a list of the input data required by this program.

Card 1 : T1 - time in seconds of cycle duration.

FLUXM - maximum neutron flux, in units neutrons per cm^2 per sec.

PFAC - average load factor for the reactor.

Card 2 : ALP25 - capture to fission ratio of U-235.

SIG25 - absorption cross section of U-235, in cm^2.

ETA25 - number of neutrons produced to number of neutrons lost ratio in U-235.

ALP23 - capture to fission ratio of U-233.

SIG23 - absorption cross section of U-233, in cm^2.

Card 3 : ETA23 - number of neutrons produced to number of neutrons lost ratio in U-233.

SIG28 - absorption cross section of U-238, in cm^2.

SIG24 - absorption cross section of U-234, in cm^2.

SIG26 - absorption cross section of Th-232, in cm^2.

SIG49 - absorption cross section of Pu-249, in cm^2.

Card 4 : ALP49 - capture to fission ratio of Pu-249.

ETA49 - number of neutrons produced to the number of neutrons lost ratio in Pu-239.

SIG40 - absorption cross section for Pu-240 in cm^2.

SIG41 - absorption cross section for Pu-241, in cm^2.

ALP41 - capture to fission ratio of Pu-241.

Card 5 : ETA41 - number of neutrons produced to the number of neutrons lost ratio in Pu-241.

EPSI - fast fission factor for Th-232.

P11 - fast-to-resonance non-leakage probability.

FAC - scaling factor used in equation, 10^{20}.

Card 6 : YIEP23- fraction yield of Pm-149 (indirect) from U-233 fission.

YIEP25- fraction yield of Pm-149 (indirect) from U-235 fission.

YIEI23- fraction yield of I-135 (direct) from U-233 fission.

YIEI25- fraction yield of I-135 (direct) from U-235 fission.

YIEX23- fraction yield of Xe-135 (direct) from U-233 fission.

Card 7 : YIEX25- fraction yield of Xe-135 (direct) from U-235 fission.

SIGXE - absorption cross section of Xe-135, barns.

SIGSA - absorption cross section of Sm-149, barns.

SIGC - absorption cross section of Carbon, barns.

SIGBIO- absorption cross section of B-10, barns.

Card 8 : SIGCR - effective absorption cross section of control rod, barns.

ENW23 - average number of neutrons produced per fission of U-233.

ENW25 - average number of neutrons produced per fission of U-235.

ENW49 - average number of neutrons produced per fission of Pu-239.

ENW41 - average number of neutrons produced per fission of Pu-241.

Card 9 : PKKK - total neutron leakage (fraction of whole).

ALMP - decay constant for Pm-149, sec^{-1}.

ALMI - decay constant for I-135, sec^{-1}.

ALMX - decay constant for Xe-135, sec^{-1}.

SIFP4 - fission product cross section for U-235, barns/fission

Card 10: SIFP1 - fission product cross section for U-233, barns/fission

Card 11: ENMAX - maximum enrichment allowed in uranium, fraction.

PAVG - average power produced, watts/cc.

PPEAK - power peaking factor.

TPOW - total power produced, MWt.

VREG - active volume of core, cc.

Card 12: REC25 - recoverable energy from fission of U-235, MeV/fission.

REC23 - recoverable energy from fission of U-233, MeV/fission.

REC49 - recoverable energy from fission of Pu-239, MeV/fission.

REC41 - recoverable energy from fission of Pu-241, MeV/fission.

COFAC - conversion factor for MeV to watts.

Card 13: YIEP49- fraction yield of Pm-149 (indirect) from Pu-239 fission.

YIEI49- fraction yield of I-135 (direct) from Pu-239 fission.

YIEX49- fraction yield of Xe-135 (direct) from Pu-239 fission.

Card 14: FLUF1 - fraction of maximum flux level in region 1.

FLUF2 - fraction of maximum flux level in region 2.

FLUF3 - fraction of maximum flux level in region 3.

FLUF4 - fraction of maximum flux level in region 4.

Card 15: P(1) - resonance escape probability for U-238

P(2) - in the k-th region.

⋮

P(k)

Card 16: P(1) - resonance escape probability for Th-232

P(2) - in the k-th region.

⋮

P(k)

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$JOB            YFC,TIME=3,PAGES=40
      DIMENSION  COEF(50),B(55)          ,MID(2),ID(2)
      COMMON/SC/ SCAL1,SCAL2,SCAL3,SCAL4
      COMMON/BAD/ Z1,Z2,Z11,Z3,Z4,Z33
      COMMON/BBD/REC25,REC23,REC49,REC41,COFAC
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      COMMON /PA/YIEP23,YIEP25,YIEI23,YIEI25,YIEX23,YIEX25,
     1 SIF23,SIF25,SIF49, SIF41,KCH,
     2  YIEX49,YIEI49,YIEP49,
     2 ENW23,ENW25,ENW49,ENW41,PKKK ,ALMP,ALMI,ALMX
      COMMON/PB/SIG28,SIG02,SIG26,SIG40,SIG24 ,SIG41,SIG23,SIG25,SIG49
     1,S17,S18,S19,S20,PAVG,PPEAK,VREG ,S13,S14,S15,S16
      COMMON /BU/C(10),P(5),PT(5),GAMA,EKPA25,EKPA49,
     1EKPA23,EL1,EL2,EL3,C11,C12,C13,C14,C15,C16,C17,C18,C19,C192,C193,
     2C194,C195,C20,C21,C22,C23,C25,C26,C27,C28,C89,C82,C24,C29,C69,C691
     3 ,C70,C701,C71,C72,C73,C74,C75,C76,C77,C78,C80,C81,C88,C90,C901,
     4C902,C91,C92,C93,D11,C83,C84,C85,C86,C87,C241,C242,C243,C244
      COMMON/BU/C245,C246,C196,C197,C191,C30,C31,C32,C33,C34,C35,C36,C37
     1,C38,C39,C40,C41,C42,C43,C44,C441,C45,C451,C452,C453,C46,C461,
     3C462,C463,C464,C465,C466,C467,C468,C469,C47,C48,C49,C50,C501,C52,
     4C53,C531,C54,C55,C56,C561,C57,C571,C572,C58,C581,C582,C583,C584
      COMMON/BU/C59,C591,C592,C593,C594,C60,C601,C602,C603,C61,C611,C612
     2,C613,D12,D13,D14,D15,D16,D17,D18,D19,D20,D21,D22,D23,A,B1,X11,X12
     3,P1,P2,P3,P4,P5,P6,B11,B12,B13,B14,B15,B16,B17,B18,B19,B20,B21,
     4B22,ALP25,ETA25,ALP23,ETA23, A148,A128,
     5 ALP49,ETA49,ALP41,ETA41,EPSI,P11,FAC
      COMMON /CP/SIGXE,SIGSA,SIGB10,SIGCR,SIFP4,SIFP1,SIGC
      DO 10 I=1,10
 10   C(I) =1.0
  61  FORMAT(5E15.0)
      READ(5,61) Y1,FLUXM,PFAC
      READ(5,61) ALP25,SIG25,ETA25,ALP23,SIG23,ETA23,SIG28,SIG24,
     1 SIG26,SIG02,SIG49,ALP49,ETA49,SIG40,SIG41,ALP41,ETA41,EPSI,
     2P11,FAC
      READ(5,61) YIEP23,YIEP25,YIEI23,YIEI25,YIEX23,YIEX25,
     1 SIGXE,SIGSA,SIGC,SIGB10,SIGCR,                          ENW23,
     2 ENW25,ENW49,ENW41,PKKK,ALMP,ALMI,ALMX ,SIFP4,SIFP1
      READ(5,61) ENMAX,PAVG,PPEAK,TPOW,VREG,REC25,REC23,REC49,REC41,
     1 COFAC ,YIEP49,YIEI49,YIEX49
C
      Z1     =        YIEP23/ (SIGSA         )
      Z2     =      YIEP25 /( SIGSA  )
      Z11 = YIEP49 / SIGSA
C********NCYC IS THE NUMBER OF FUEL CYCLES CONSIDERED.
C
      NCYC = 6
C
C********NREG IS THE NUMBER OF REGIONS CONSIDERED.
C
      NREG = 4
C
C********NUMBER OF FLUX LEVELS MUST EQUAL NREG.
C
C********CALCULATE FISSION CROSS SECTIONS IN BARNS.
      SIF23 = SIG23 * 1E+24 /(1+ ALP23)
      SIF25 = SIG25 * 1E+24 /(1+ ALP25)
      SIF49 = SIG49 * 1E+24 /(1+ ALP49)
      SIF41 = SIG41 * 1E+24 /(1+ ALP41)
C********READ AVERAGE FLUX RATIO FOR EACH REGION.
```

```
      READ(5,61) FLUF1,FLUF2,FLUF3,FLUF4
      FLUXM = FLUXM* PFAC
C********CALCULATE AVERAGE FLUX LEVEL FOR EACH REGION.
      FLUX(1)=FLUF1*FLUXM
      FLUX(2) = FLUF2 * FLUXM
      FLUX(3) = FLUF3 * FLUXM
      FLUX(4) = FLUF4 * FLUXM
      ST441 = ENW41 / (1+ ALP41)
      ST425 = ENW25 / (1+ ALP25)
      ST423 = ENW23 / (1+ ALP23)
      ST449 = ENW49 / (1+ ALP49)
      MDSK = 9
C*********IMAX IS THE NUMBER OF "EQUAL" CONSTRAINTS IN THE BURN PARTITION.
      IMAX = 12
C*********KMAX IS THE NUMBER OF "GREATER THAN" CONSTRAINTS IN THE BURN PARTITION.
      KMAX = 1
C*********MAXC IS THE MAXIMUM NUMBER OF COLUMNS IN EACH REGION.
      MAXC = 25
C*********MAXR IS THE MAXIMUM NUMBER OF ROWS IN EACH REGION.
      MAXR = 18
C*********KONTR IS NUMBER OF "EQUAL" CONSTRAINTS  IN THE  CONST PARTITION.
      KONTR =11
C*********KMTR IS NUMBER OF "GREATER THAN" CONSTRAINTS IN THE CONST PARTITION.
      KMTR = 3
C*********MAXCC IS  THE NUMBER OF COLUMNS IN THE CONST PARTITION.
      MAXCC = 10
C*********MAXRR IS THE NUMBER OF ROWS IN THE CONST PARTITION.
      MAXRR = 14
C*********MROS REPRESENTS THE NUMBER OF ROWS IN THE STORAGE AREA.
      MROS = 5
C*********MROT REPRESENTS THE NUMBER OF ROWS IN THE TRANSFER PARTITION.
      MROT = 11
C*********MINP REPRESENTS THE NUMBER OF ROWS IN THE INPUT PARTITION.
      MINP = 4
C********* NCOLS REPRESENTS THE  NUMBER OF COLUMNS IN THE INPUT PARTITION.
      NCOLS = 4
C********* NRIS REPRESENTS NUMBER OF COLUMNS IN THE IN-STORAGE AREA.
      NRIS = 5
      ICCT = NROG
      IMAX1= IMAX
      MAXC1 = MAXC
      MAXR1 = MAXR
      KMAX1 = KMAX
      MAXRQ = 70
      MAXCQ = 50
      A148 = 238 * ENMAX
      A128 = 235 - 235 * ENMAX
C********* INTER SCALING FACTORS.
      SCAL3=1.E-03
      SCAL4 = 1.E-02
      SCAL1 = 1.E+02
      SCAL2 = 1.E+03
C*********READ IN RESONANCE ESCAPE PROBABILITIES FOR U-238 AND
C*********TH -232 FOR EACH REGION.
      READ(5,62) (P(K),K=1,NROG)
      READ(5,62) (PT(K),K=1,NROG)
      K = 1
C*********CALCULATE CONSTANTS FOR BURN-UP EQUATION.
      PKKK = PT(K) * EPSI *PKKK
      GAMA  =  1-ST449*EPSI*FI1*(1-P(K))
```

```
EKPA25 = ETA25 * EPSI  * P11*(1-P(K))
EKPA49  = ETA49 * EPSI   * P11* (1- P(K))
EKPA23 = ETA23 * EPSI  * P11* (1-P(K))
EL1 =  ETA23  *  EPSI   * P11* (1-PT(K))
EL2 = ETA25 *  EPSI   * P11* (1- PT(K))
EL3 = EPSI * P11*  (1- P(K))
C11 =  SIG23 * ALP23 / (1+ALP23)
C12 =  C11* SIG02 * SIG25 * FAC/ ( SIG23-SIG02)
C13 = EKPA25 * SIG25
C14 = C13 * C12/ (SIG24- SIG02)
C15 = EKPA23 * SIG23
C16 = C15 * SIG02 / ( SIG23 * (1-EL1) - SIG02)
C17 = C15 * SIG25 * EL2 / ( SIG23 * (1-EL1) - SIG25)
C18 = SIG28 / ( GAMA * SIG49)
C19 = C13 / ( GAMA * SIG49 - SIG25)
C192 = GAMA * SIG49 - SIG24
C193 = GAMA * SIG49 - SIG25
C194 = GAMA* SIG49 - SIG23 *(1-EL1)
C195 = GAMA* SIG49 - SIG02
C20   =C16 / C195
C21 = C14 / (( GAMA * SIG49 - SIG25 )* (SIG25-SIG02)*FAC)
C22 = C14 / (( SIG25-SIG24 )* ( GAMA*SIG49 - SIG24)*FAC)
C23 = C14 / (( SIG25 - SIG24 )* ( GAMA * SIG49- SIG25)* FAC)
C25 = C16 / ( GAMA * SIG49 - SIG02)
C26 = C16 / ( GAMA * SIG49 + SIG23* ( EL1-1))
C27 = C19 * C17 / C13
C28 = C26 * C17 / C16
C89 = C11/(SIG24-SIG23* (1- EL1))
C82 = SIG24 * C89 /  ( SIG25- SIG23 * (1- EL1))
C24 = C13 * C82 / C194
C29 = ALP49 * SIG49 / (1 + ALP49)
C69 = C17 /(SIG28+SIG23*(EL1-1))
C691 =-C69 * C13 / C17
C70 = C13 / (SIG28-SIG25)
C701 =-C13 / (SIG28 - SIG24)
C71 = C17 / ( SIG28- SIG25)
C72 = C14/ ((SIG25-SIG02)*(SIG28-SIG25)*FAC)
C73 = C14 / ((SIG25- SIG24)* (SIG28- SIG24)*FAC)
C74 = C16/ (SIG28+ SIG23* (EL1-1))
C75 = C14 / (( SIG25- SIG02)* (SIG28- SIG02)*FAC)
C76 = C14 /((SIG25- SIG24)*(SIG28-SIG25)*FAC)
C77 = C16 / (SIG28- SIG02)
C78 = C15/ (SIG28 - SIG23* (1-EL1))
C80 = C11* SIG25/ (SIG24-SIG23)
C81 = SIG24 /( SIG25- SIG24 )
C88 =  C11 *C16 /(C15*(SIG24-SIG02))
C90 = C11* C16 /(C15 *(SIG24+SIG23*(EL1-1)))
C901 = C11* C17/ (C15 *(SIG24-SIG25))
C902 = C11* C17 /(C15*(SIG24+ SIG23*(EL1-1)))
C91 = SIG24 * C901 / (SIG25 - SIG24)
C92 = SIG24 * C902 / (SIG25+ SIG23* (EL1-1))
C93 = SIG24 * C902/ (SIG25- SIG24)
D11 = ALP25 * SIG25/ (1+ ALP25)
C83 = SIG24 * C89 / (SIG25 - SIG24)
C84 = SIG24 * C88 / (SIG25 - SIG02)
C85 = SIG24 * C88 / (SIG25 - SIG24)
C86 = SIG24 * C90 / (SIG25- SIG23 *(1- EL1))
C87 = SIG24 * C90 /(SIG25- SIG24)
C241 = C13 / C193
C242 = C13*C83/ C192
```

```
C243= C13 / C193
C244 = C15 / C194
C245= C13* C81/ C192
C246= C13 * C81/C193
C196 = C13 * C93/ C192
C197 = C13 * C93 / C193
C191 = C13 * C92/ C194
C30 = C27 +C19 - C197
C31 = C28 + C191
C32 =C191 +C28 + C197  -C19-C27 - C196
C33 = C24 + C244
C34 = C243 - C241
C35 = C241 + C242 -C24 - C243- C244
C36 = C246 - C245
C37 = C26 - C20
C38 =  SIG40 - GAMA* SIG49
C39 = SIG40 - SIG25
C40 = SIG40 - SIG23 * (1-EL1)
C41 = SIG40 - SIG24
C42 = SIG40 - SIG02
C43 = C29 / C38
C44 = C29 * C18 / SIG40
C441 = C29 * C18 / C38
C45 = C30 * C29 / C39
C451 = C29 * C31 / C40
C452 = C29 * C196 / C41
C453 = C29 * C32 / C38
C46 = C29 * C33 / C40
C461 = C29 * C34 / C39
C462 = C29 * C242 / C41
C463 = C29 * C35 / C38
C464 = C29 * C245 / C41
C465 = C29 * C246 / C39
C466 = C29 * C36 / C38
C467 = C29 * C20 / C42
C468 = C29 * C26 / C40
C469 = C29 * C37 / C38
C47 = C441 - C44
C48 = C451 - C45 -C452 - C453
C49 = C462 - C46- C461 - C463
C50 = C465 - C464 - C466
C501 = C468 - C467 - C469
C52 = SIG40 / (SIG41 -GAMA* SIG49)
C53 = SIG40 / (SIG41- SIG25)
C531 = SIG40 / (SIG41- SIG40)
C54 = SIG40 / (SIG41- SIG24)
C55= SIG40 / (SIG41- SIG23*(1- EL1))
C56 = C52 * C43
C561 = C531 * C43
C57 = SIG40 * C44 / SIG41
C571 = C52 * C441
C572 = C531 * C47
C58 = C53 * C45
C581 = C54 * C452
C582 = C52 * C453
C583 = C55 * C451
C584 = C531 * C48
C59 = C55 * C46
C591 = C53 * C461
C592 = C54 * C462
```

```
      C593 = C52 * C463
      C594 = C531 * C49
      C60 = C54 * C464
      C601 = C53 * C465
      C602 = C52 * C466
      C603 = C531 * C50
      C61 = SIG40 * C467 / (SIG41 - SIG02)
      C611 = C55 * C468
      C612 = C52 * C469
      C613 = C531 * C501
      D12 = SIG26 - SIG25
      D13 = SIG26 - SIG24
      D14 = SIG26 + SIG23* (EL1-1)
      D15 = D11 * (C92-C93 + 1)/ D12
      D16 = D11 / ALP25
      D17 = D11 * C93 / D13
      D18 = D11 * C92 / D14
      D19 = D11 * C81 / D13
      D20 = D11 * C81 / D12
      D21 = D11 * (C83 - C82) / D12
      D22 = D11 * C82 / D14
      D23 = D11 * C83 / D13
      A= SIG02 + SIG23*(1-EL1)
      B1= SIG02 * SIG23 * FAC
      X11= .5 *(-A+ SQRT(A**2-(4*B1/FAC)))
      X12 = .5*(-A-SQRT(A**2-(4*B1/FAC)))
      P1 = SIG02
      P2 = EL1 * SIG23
      P3 = SIG25
      P4 = SIG23 * (1- EL1)
      P5 = SIG23
      P6 = EL2 * SIG25
      B11 = P4 + P3
      B12 = P3 * P4
      B13 = P6 * P5
      B14 = P2 * P3
      B15 = (X11-X12) * (X11+P3)
      B16 = (X12- X11) * (X12+ P3)
      B17 = (P3+ X11) * (P3+ X12)
      B18 = P1+ P3
      B19 = P1* P3
      B20 = P1* P6
      B21 = P1* P6 * P3
      B22 = P3 * P6
      KCH = MAXR +1
      DO 5 L=1,MAXRQ
      DO 6 I=1,MAXCQ
      COEF(1) = 0.0
      T(L,I) = 0.
    6 CONTINUE
    5 CONTINUE
      MAROW = ICCT * MAXR
      MAXAL = MAXR + MAXRR
C********SPECIFY COST COEFFICIENTS.
      COEF(2) = -1.39
      COEF(7) = 1.
      DO 7 L= 1,MAXR
      B(L) = 0.0
    7 CONTINUE
 3000 FORMAT('NAME',10X,'PROBLEM'/'ROWS'/' N  PROFIT')
```

```
 801  FORMAT(' ','E',' ROW',I3)
 3001 FORMAT(' ','G',' ROW',I3)
 3003 FORMAT(' ','L',' ROW',I3)
 3004 FORMAT('COLUMNS')
 3010 FORMAT(T1,'    COL',T8,I3,T15,'ROW',T18,I3,T25,E12.5)
 3012 FORMAT('RHS')
 3011 FORMAT(T1,'    LIMITS',T15,'ROW',T18,I3,T25,E12.5)
 3009 FORMAT('ENDATA')
 3005 FORMAT(T1,'    COL',T8,I3,T15,'PROFIT',T25,E12.5,T40,'ROW',I43
     1,I3,T50,E12.5)
 3006 FORMAT(T1,'    COL',T8,I3,T15,'ROW',T18,I3,T25,E12.5,T40,'ROW'
     1,T43,I3,T50,E12.5)
 3007 FORMAT(T1,'    LIMITS',T15,'ROW',T18,I3,T25,E12.5,T40,'ROW',T4
     13,I3,T50,E12.5)
C******* CONSTRUCTION OF ROWS VECTOR
      WRITE(MDSK,3000)
      IXMU = 3* MROT + MROS + MINP
      NRPP = NREG + 2
      NR=101
      IMUST1 = 0
      ICCT1 = ICCT +1
      ICCT2 = ICCT-1
      ICCT3 = ICCT1 + 1
      ICCT4 = ICCT1 + 2
      DO 899 K3=1,NCYC
C********UPDATE ALL PARMETERS.
      IMAX = IMAX1
      KMAX = KMAX1
      MAXR = MAXR1
      IMUST1 = IMUST1 + 1
      DO 802 K1= 1,NRPP
      IF(K1.LE.ICCT) GO TO 799
      IMAX = KONTR
      KMAX = KMTR
      MAXR = MAXRR
      IF(K1.LE.ICCT1) GO TO 799
      IF(IMUST1.EQ.NCYC) GO TO 400
      IMAX = IXMU
      MAXR = IXMU
      KMAX = 0.
      GO TO 799
 400  KMAX = 0
      IMAX = MROS
      MAXR = MROS
 799  IF (IMAX.LT..5) GO TO 850
      DO 800 K=1,IMAX
      WRITE(MDSK,801)NR
 800  NR=NR+1
      IF(IMAX.EQ.MAXR) GO TO 802
 850  IH=IMAX+1.
      KJ=IMAX+KMAX
      IF(KJ.LT..5) GO TO 860
      IF(KMAX.LT..5) GO TO 860
C******* CONSTRUCTION OF G CONTRAINTS
      DO600K=IH,KJ
      WRITE(MDSK,3001)NR
  600 NR=NR+1
C******* CONSTRUCTION OF L CONTRAINTS
 860  IP=KJ+1.
      IF(IP.GT.MAXR) GO TO 802
```

```
      DO605K=IP,MAXR
      WRITE(MDSK,3003)NR
  605 NR=NR+1
  802 CONTINUE
  899 CONTINUE
C
C        UPDATE PARAMETERS.
C
      IMAX = IMAX1
      MAXR = MAXR1
      KMAX = KMAX1
C*******CONTRUCTION OF THE COLUMN SECTION.
      WRITE(MDSK,3004)
C********INIALIZE
      IMUST1 = 0
      NFIL = 0
      MKK = MAXR + MAXRR
      MKK1 = MROT + MAXR
      MK2 = MROT + MAXR + MAXRR
      NC = 100
      MK3 = MINP + MAXR + MAXRR
      NRP1 = NRPP+1
      MROW1 = (MAROW+ MAXRR) * 2+ (ICCT-1)*MROT +MROS+MINP
      DO 833 K3=1,NCYC
      COEF(2) = -1.39
      COEF(7) = 1.
      MAXR = MAXR1
      MAXC = MAXC1
      ICOUT1 = 0
      ICONU1 = ICCT2
C********UPDATE REFERENCE POINT.
      NFIL = IMUST1 *(ICCT*MAXR + MAXRR+ (ICCT-1) * MROT+ MROS+MINP)
      IMUST1 = IMUST1 + 1
      MAX4 = 1
      KC3 = 0
      DO 803 K1 =1,NRP1
      ICOUT1 = ICOUT1 + 1
      IF(IMUST1.EQ.1) GO TO 925
      KC3= 0
      IF(ICOUT1.LT.ICCT) CALL TRANSF
      IF(ICOUT1.EQ.ICCT) CALL FULIP
  925 CONTINUE
      KCH = MAXR1 + KC3 +1
      KC1 = MAXR1 + MAXRR + KC3 + 1
      KC2 = MAXR1 + MAXRR + KC3 + 1
C********INIALIZE ROW NUMBER OF  FIRST FOUR REGIONS.
      IF(K1.GT.ICCT) GO TO 839
      IF(IMUST1.EQ.1) GOTO 572
      NR = NFIL + 100 -ICONU1* MROT-MINP
      ICONU1 = ICONU1 -1
      GO TO 571
  572 NR = NFIL + 100
      NP = NR - MAXR
      NR = NP + ICOUT1 * MAXR
C********GENERATE BURNUP EQUATIONS.
  571 CALL BURN(K1)
C********GENERATE CONSTRAINT EQUATIONS.
      CALL PART(MAXRR,K1,ICCT)
      IF(ICOUT1.NE.1) GOTO 965
C********GENERATE BULK STORAGE AREA.
```

```
      CALL STORCO
      MAXAL = MAXR + MAXRR + MROS
      IF(KC3.GT.0) MAXAL=MAXR + MAXRR+ MROS + MROT
      GO TO 842
 965  CONTINUE
      IF(IMUST1.EQ.NCYC) GO TO 401
      CALL TRANST
      MAXAL = MAXR + MAXRR + MROT
      IF(KC3.GT.0) MAXAL= MAXR + MAXRR + 2*MROT
      IF(ICOUT1.EQ.ICCT) MAXAL =MAXR+MAXRR+      MROT+M1NP
      GO TO 842
 401  MAXAL = MAXR + MAXRR + MROT
      GO TO 842
 839  CONTINUE
C********INIALIZE ROW NUMBER FOR LAST THREE REGIONS.
      IF(ICOUT1.NE.ICCT1) GOTO 840
      MAXR = MAXRR
      MAXC = MAXCC
      COEF(2) = 0.
      COEF(3) = 0.
      COEF(5) = 0.0
      COEF(7) = 0.0
      CALL CONST
      MAXAL = MAXRR
      NR = MAROW + NFIL + 100
      GO TO 842
 840  CONTINUE
      IF(ICOUT1.NE.ICCT3) GO TO 841
      MAXR = MROS
      MAXC = MRIS
      MAXAL = 2* MROS
C********GENERATE ACCUMULATIVE BULK STORAGE AREAS.
      CALL STORI
      CALL STORI
      IF(IMUST1.EQ.NCYC) MAXAL = MROS
      NR = MAROW + MAXRR + NFIL + 100
      GO TO 842
 841  CONTINUE
      IF(ICOUT1.NE.ICCT4) GO TO 842
      IF(IMUST1.EQ.NCYC) GO TO 833
      MAXR = MINP
      MAXC = NCOLS
      MAXAL = MINP
C********GENERATE INPUT FUEL PARTITION.
      CALL FULI
      NR = MAROW + MAXRR + MROS + 3*MROT +NFIL + 100
 842  CONTINUE
      DO 720 N=1,MAXC
      MM= 1
      NC = NC+1
      NOD = 0
      NOW = 0
 555  DO 710 M=MM,MAXAL
      IF(T(M,N).NE.0.) GO TO 711
      GO TO 710
 711  NTR = NR+M
      IF(ICOUT1.EQ.ICCT1.OR.ICOUT1.EQ.ICCT4) GO TO 550
      IF(IMUST1.GT.1) GO TO 502
      IF(ICOUT1.NE.1) GO TO 501
      IF(M.GT.MAXR) NTR= MAROW + NFIL +100+M-MAXR
```

```
      GO TO 550
  501 CONTINUE
      IF(ICOUT1.GT.ICCT) GO TO 502
      IF(M.LE.MAXR) GO TO 550
      IF(M.GT.MAXR.AND.M.LE.MKK) NTR=MAROW+NFIL+100+M-MAXR
      IF(M.GT.MKK) NTR= MAROW+MROS+MAXRR+(K1-2)*MROT+NFIL+100+M-MKK
      GO TO 550
  502 CONTINUE
      IF(ICOUT1.NE.ICCT3) GO TO 503
      IF(M.GT.MROS) NTR= MROW1+  NFIL + 100 + M- MROS
      GO TO 550
  503 IF(ICOUT1.EQ.1) GO TO 506
      IF(ICOUT1.EQ.ICCT) GO TO 509
      IF(M.LE.MROT) GO TO 550
      IF(M.GT.MROT.AND.M.LE.MKK1) GOTO   507
      IF(M.GT.MKK1.AND.M.LE.MK2) GO TO 508
      NTR= NFIL + 100 + MAROW + MAXRR+ MROS+ (K1-2) *MROT+M-MK2
      GO TO 550
  507 NTR = NFIL+ (K1-1)*MAXR+100 + M-MROT
      GO TO 550
  508 NTR = NFIL +100 + MAROW+ M- MKK1
      GO TO 550
  509 IF(M.LE.MINP) GO TO 550
      IF(M.GT.MINP.AND.M.LE.MK3) GO TO 510
      NTR = NFIL + 100 + MAROW+ MAXRR+ MROS+ (K1-2)* MROT +M- MK3
      GO TO 550
  510 NTR = NFIL + 100 + MAROW-MAXR+ M-MINP
      GO TO 550
  506 IF(M.LE.MROT) GO TO 550
      IF(M.GT.MROT.AND.M.LE.MKK1) GO TO 511
      NTR= NFIL+ 100 + MAROW+M-MKK1
      GO TO 550
  511 NTR = NFIL + 100 + M-MROT
  550 CONTINUE
      IF(NOD.GT.0) GO TO 552
      WRITE(MDSK,3005)NC,COEF(N),NTR,T(M,N)
      NOD = NOD + 1
      GO TO 710
  552 CONTINUE
      NOW = NOW+1
      MID(NOW) = M
      ID(NOW) = NTR
      IF(NOW.LT.2) GO TO 710
      WRITE(MDSK,3006)NC,ID(1),T(MID(1),N),ID(2),T(MID(2),N)
      NOW = 0
  710 CONTINUE
      IF(NOW.EQ.0) GO TO 720
      WRITE(MDSK,3010)NC,ID(1),T(MID(1),N)
  720 CONTINUE
  803 CONTINUE
  833 CONTINUE
C******* CONSTRUCTION OF RHS VECTOR FOR MPS-360 INPUT
      WRITE(MDSK,3012)
      IMUST1 = 0
      DO 625 K3=1,NCYC
C********UPDATE REFERENCE ROW NUMBER.
      MAXR = MAXR1
      NFIL = IMUST1*(ICCT*MAXR+MAXRR+(ICCT-1)*MROT+MROS+MINP   )
      IMUST1 = IMUST1 + 1
C********INITIALIZE.
```

```
      MAXR = MAXR1
      NR = 100 +NFIL
      NREG = ICCT + 1
      NTR=0
      B(13) = 0.
      B(14) = 0.
      B(15) = 0.
      B(17) = 0.
      B(18) = 0.
      DO 804 K1=1,NREG
      IF(K1.LE.ICCT) GOTO948
      MAXR = MAXRR
C********GENERATE RHS TOTAL POWER CONSTRAINTS.
      B(13) = TPOW*PFAC*SCAL3
      B(14) = TPOW*PFAC     *SCAL3
      IF(IMUST1.LT.3.OR.IMUST1.GT.4) GO TO 958
      B(13) = TPOW *PFAC * SCAL3 *.9741
      B(14) = TPOW *PFAC * SCAL3 *.9741
      GO TO 958
 948  B(14) = 1.    * SCAL2
      B(15) = 1.    * SCAL2
      PPEAK1 = PPEAK
C********GENERATE RHS POWER PEAKING CONTRAINT.
      IF(K1.GT.1) PPEAK1 = PPEAK* .87
      IF(K1.GT.2) PPEAK1 = PPEAK* .77
      B(17) = PAVG* PPEAK1
      B(18) = PAVG* PPEAK1
 958  DO730M=1,MAXR
      NR = NR +1
      IF(B(M).EQ.0.)GOTO730
      NTR=NTR+1
      MID(NTR)=M
      ID(NTR)=NR
      IF(NTR.LT.2)GOTO730
      WRITE(MDSK,3007)ID(1),B(MID(1)),ID(2),B(MID(2))
      NTR=0
  730 CONTINUE
      IF(NTR.EQ.0)GOTO731
      WRITE(MDSK,3011)ID(1),B(MID(1))
  731 CONTINUE
      NTR = 0
  804 CONTINUE
  625 CONTINUE
  736 WRITE(MDSK,3009)
      STOP
      END

      SUBROUTINE  BURN(K)
C*************************************************************************
C
C        THIS SUBROUTINE CONTAINS THE BURNUP EQUATIONS FOR THE TH-232
C        AND U-238 FUEL CYCLES. THE SUBROUTINE ALSO CONTAINS  LOCALIZED
C        CONSTRAINT EQUATIONS FOR EACH REGION.
C
C*************************************************************************
      COMMON/SC/ SCAL1,SCAL2,SCAL3,SCAL4
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      COMMON/BAD/ Z1,Z2,Z11,Z3,Z4,Z33
      COMMON /PA/YIEP23,YIEP25,YIEI23,YIEI25,YIEX23,YIEX25,
```

```
1 SIF23,SIF25,SIF49, SIF41,KCH,
2  YIEX49,YIEI49,YIEP49,
2 ENW23,ENW25,ENW49,ENW41,PKKK ,ALMP,ALMI,ALMX
 COMMON/BBD/REC25,REC23,REC49,REC41,COFAC
 COMMON/PB/SIG28,SIG02,SIG26,SIG40,SIG24 ,SIG41,SIG23,SIG25,SIG49
1,S17,S18,S19,S20,PAVG,PPEAK,VREG ,S13,S14,S15,S16
 COMMON /CP/SIGXE,SIGSA,SIGB10,SIGCR,SIFP4,SIFP1,SIGC
 COMMON /BU/C(10),P(5),PT(5),GAMA,EKPA25,EKPA49,
1EKPA23,EL1,EL2,EL3,C11,C12,C13,C14,C15,C16,C17,C18,C19,C192,C193,
2C194,C195,C20,C21,C22,C23,C25,C26,C27,C28,C89,C82,C24,C29,C69,C691
3 ,C70,C701,C71,C72,C73,C74,C75,C76,C77,C78,C80,C81,C88,C90,C901,
4C902,C91,C92,C93,D11,C83,C84,C85,C86,C87,C241,C242,C243,C244
 COMMON/BU/C245,C246,C196,C197,C191,C30,C31,C32,C33,C34,C35,C36,C37
1,C38,C39,C40,C41,C42,C43,C44,C441,C45,C451,C452,C453,C46,C461,
3C462,C463,C464,C465,C466,C467,C468,C469,C47,C48,C49,C50,C501,C52,
4C53,C531,C54,C55,C56,C561,C57,C571,C572,C58,C581,C582,C583,C584
 COMMON/BU/C59,C591,C592,C593,C594,C60,C601,C602,C603,C61,C611,C612
2,C613,D12,D13,D14,D15,D16,D17,D18,D19,D20,D21,D22,D23,A,B1,X11,X12
3,P1,P2,P3,P4,P5,P6,B11,B12,B13,B14,B15,B16,B17,B18,B19,B20,B21,
4B22,ALP25,ETA25,ALP23,ETA23, A148,A12B,
5 ALP49,ETA49,ALP41,ETA41,EPSI,P11,FAC
 S13 = ENW23 * SIF23*PKKK- SIG23*1E+24
 S14 = ENW25 * SIF25 *PKKK- SIG25*1E+24
 S15 = ENW49 * SIF49 * PKKK- SIG49*1E+24
 S16 = ENW41 * SIF41 * PKKK-SIG41* 1E+24
 Z33 = (YIEI49+YIEX49)/((ALMX/(FLUX(K) *1E-24))+SIGXE)
 Z3   = (     YIEI23+ YIEX23)/ ((ALMX/(FLUX(K)*1E-24))+ SIGXE)
 Z4    = (      YIEI25 + YIEX25) /((ALMX/(FLUX(K)*1E-24))+ SIGXE)
 S18 = SIF23 * REC23 * FLUX(K) * COFAC /VREG
 S17 = SIF25 * REC25 * FLUX(K) * COFAC /VREG
 S19 = SIF49 * REC49 * FLUX(K) * COFAC   /VREG
 S20 = SIF41 * REC41 * FLUX(K) * COFAC  /VREG
 BS1 = EXP(-SIGB10*1E-24*FLUX(K) *T1)
 C54 = SIG40 * C37 * FLUX(K) * T1
 A11 = EXP( - SIG25 * FLUX(K) *T1)
 A12 = EXP( - GAMA * SIG49 * FLUX(K) * T1)
 A13 = EXP ( SIG23 *( EL1-1) * FLUX(K) * T1)
 A14 = EXP (- SIG02 * FLUX(K)* T1)
 A15 = EXP(- SIG24 * FLUX(K) * T1)
 A16 = EXP(- SIG23 * FLUX(K) * T1)
 A17 = C19 * ( A11- A12)
 A18 = C27 * (A11-A12)
 A19 = C28 * (A13- A12)
 A191 = C191 * (A13- A12)
 A192 = C196* (A15- A12)
 A193 = C197 * (A11- A12)
 A194 = C24 * (A13- A12)
 A195 = C241 * (A11- A12)
 A196 = C242 * (A15- A12)
 A197 = C243* (A11- A12)
 A198 = C244* (A13- A12)
 A20 = C20 * (A14 - A12)
 A21 = C21* (A11- A12)
 A211 = C245 * (A15 - A12)
 A212 = C246 * (A11- A12)
 A22 = C22* (A15- A12)
 A23= C23 * (A11- A12)
 A24 = C25 * (A14- A12)
 A25 = C26 * (A13- A12)
 A26 = EXP(- SIG40 * FLUX(K) * T1)
```

```
      A27 = EXP(-SIG41 * FLUX(K)* T1)
      A28 = EXP(- SIG28*FLUX(K)* T1)
      A30 = EXP(X11*FLUX(K)*Y1)
      A31 = EXP(X12*FLUX(K)* T1)
      A32 = EXP(-SIG26* FLUX(K) * T1)
      C702 =+C70 * C93 * (A11- A28 )+C701* (A15-A28)* C93
      C703 =-C70 * ( A11- A28)
      C704 = + C92 * C70 * (A11- A28)+C691*C92* (A13-A28)
      C705 =-C71 *(A11- A28 ) +C69* (A13-A28)
      C706 = C82 * C691*(A13- A28)+C82*C70*(A11-A28)
      C707 = C701 * C83* (A15- A28)+C70*C83*(A11- A28)
      C708 =-C78 * (A13-A28)
      C709 = C701 * (A15- A28 )* C81 +C70 *C81*(A11-A28)
      C94 = SIG24 *C901* FLUX(K)* T1*A11
      E11 = D16 * C93
      E12 = D16 * C92
      E13 = D16 * C81
      E14 = D16 * C82
      E15 = D16 * C83
      E18 = (1- A11 )/SIG25
      E16 = E11*(((1-A15)/ SIG24)-E18)
      E17 = D16 * E18
      E19 = E12 *(((1-A13)/(SIG23 * (1- EL1)))- E18)
      E20 = E13 * (((1-A15 ) / SIG24 )- E18)
      E21 = E14 *(((1-A13 )/(SIG23* (1-EL1)))-E18)
      E22 = E15 *(((1-A15)/ SIG24)-E18)
      E30 = SIG23 / (1 + ALP23)
      Y11 = A30 *(B11* X11+ X11**2+ B12) /B15
      Y12 = A31 * (B11* X12+ X12**2 + B12) /B16
      Y13 = -A30 * (X11* P6 + B13) /B15
      Y14 = - A31 * (X12* P6 + B13) /B16
      Y15 = -A11 * (B13 - P3 * P6) / B17
      Y16 = - A30 * (X11* P2+ B14)  / B15
      Y17 = -A31* (X12* P2+ B14)  / B16
      Y18 = A30 * (X11**2 + X11*B18+ B19) /B15
      Y19 = A31 *(X12**2+ X12*B18 + B19 ) /B16
      Y20 = A30 * (X11*(P6-B20)+ B20 -B21) /B15
      Y21 = A31 *(X12*(P6-B20)+ B20-B21) /B16
      Y22 = A11* (B20 - B22) / B17
      Y23 = A30 * (X11* P1+ B19) / B15
      Y24 = A31 * (X12* P1 + B19 ) / B16
      E31 = -E30 *(1-A30)*Y18 /(A30* X11)
      E32 = -E30 *(1-A31) * Y19 /(A31 * X12)
      E33 = -E30 *(1-A30)* Y20 / (A30 * X11)
      E34 = - E30 * (1- A31) * Y21 / (A31 * X12)
      E35 = E30 * (1-A11) * Y22/ (A11 * P3)
      E36 = -E30 * (1- A30) * Y23 / (A30 * X11)
      E37 = - E30 * (1- A31) * Y24 / (A31 * X12)
      MAX = KC3 + 1
      MAXR2 = MAXR + KC3
      DO 5 I=MAX,MAXR2
      DO 6 L= 1,MAXC
      T(I,L) = 0.
 6    CONTINUE
 5    CONTINUE
      KC = KC3+1
C********BURNUP EQUATION FOR U-233.
      T(KC,1)= Y18 + Y19
      T(KC,2) = -1.
      T(KC,5)= (Y23 + Y24) * C(3) / C(1)
```

```
      T(KC,7)= (Y20+Y21+Y22) * C(4) / C(1)
      KC = KC +1
C********BURNUP EQUATION FOR U-238
      T(KC,1)=((C706 +C707+ C708)* C(1)/ C(2) ) *SCAL1
      T(KC,3)= A28* SCAL1
      T(KC,4) = -1 * SCAL1
      T(KC,5)=((-C16*((A14 - A28)/ (SIG28- SIG02) - (A13 - A28) / (SIG28
     1 + SIG23 * (EL1- 1 )))) * C(3) / C(2)) * SCAL1
      T(KC,7)=((C702+ C703 + C704 + C705) * C(4)/C(2)) *SCAL1
      T(KC,17)=(C709* C(9)/ C(2) ) * SCAL1
      KC = KC +1
C********BURNUP EQUATION FOR TH-232
      T(KC,1)=((Y16+ Y17) * C(1) / C(4) ) *SCAL1
      T(KC,5)=(Y11+ Y12 ) *SCAL1
      T(KC,6) = -1 * SCAL1
      T(KC,7)=((Y13+ Y14+ Y15) * C(4)/ C(3) ) *SCAL1
      KC = KC +1
C********BURNUP EQUATION FOR U-235
      T(KC,1)=((C82 * (A13-A11) - C83*(A15-A11 )) * C(1) / C(4))*SCAL2
      T(KC,5)=(( C84 * (A14-A11) -C85*(A15-A11)-C86*(A13-A11) +
     1 C87 * (A15- A11)) * C(3) / C(4) ) * SCAL2
      T(KC,7)=(A11 +C91*(A11-A15)- C92 *(A13-A11)+ C93*(A15-A11)+ C94
     1 ) *SCAL2
      T(KC,8) = -1 * SCAL2
      T(KC,17)=((C81*(A15-A11))*C(9) / C(4)) * SCAL2
      KC = KC +1
C********BURNUP EQUATION FOR U-236
      T(KC,1)=((D21 * (A11 -A32)+D22 * (A13-A32)-D23*(A15-A32))*C(1)/
     1 C(5) ) * SCAL2
      T(KC,7)=((D15*(A11-A32)+D17 * (A15-A32)-D18*(A13-A32))* C(4)/C(5)
     1  ) *SCAL2
      T(KC,9)= A32* SCAL2
      T(KC,10) = -1 * SCAL2
      T(KC,17)=((D19*(A15-A32) -D20*(A11-A32)) * C(9)/ C(5)) * SCAL2
      KC = KC +1
C********BURNUP EQUATION FOR PU-239
      T(KC,1)=((A194-A195 - A196+ A197+A198) * C(1)/ C(6))*SCAL2
      T(KC,3)=(C18 * (1- A12  ) * C(2)/ C(6) ) * SCAL2
      T(KC,5)=((A20                  -A25)*C(3)/C(6)) *SCAL2
      T(KC,7)=((A17 + A18-A19- A191+A192-A193) * C(4) / C(6))*SCAL2
      T(KC,11)= A12 * SCAL2
      T(KC,12) = - 1 * SCAL2
      T(KC,17)=(( A211- A212) * C(9) / C(6) ) *SCAL2
      KC = KC +1
C********BURNUP EQUATION FOR PU-240
      T(KC,1)=((C46 * (A13 - A26 ) + C461* (A11- A26) - C462* (A15- A26)
     1 + C463 * (A12- A26 )) * C(1) / C(7)) *SCAL2
      T(KC,3)=(( C44* (1- A26) - C441 * (A12- A26)) * C(2)/ C(6))*SCAL2
      T(KC,5)=((C467 *(A14- A26 ) - C468 * (A13- A26) + C469*(A12- A26))
     1 * C(2)/ C(7)) * SCAL2
      T(KC,7)=((C45* (A11- A26) - C451 * (A13- A26)+ C452 * (A15-A26)
     1 + C453 * (A12- A26)) * C(4)/ C(7) ) * SCAL2
      T(KC,11)=((C43 * (A12- A26)) * C(6) / C(7)) * SCAL2
      T(KC,13)= A26* SCAL2
      T(KC,14) = -1 * SCAL2
      T(KC,17)=((C464 * (A15 -A26 ) - C465 * (A11- A26)+ C466*(A12-A26))
     1 * C(9)/ C(7) ) * SCAL2
      KC = KC +1
C********BURNUP EQUATION FOR PU-241
      T(KC,1)=((C59 * (A11- A27) + C591 * (A11- A27) - C592 * (A15- A27)
```

```
     1 + C593 * (A12- A27) + C594 * (A26 - A27 )) *C(1) / C(8))*SCAL2
      T(KC,3)=((C57* (1- A27) - C571 * (A12- A27)+ C572* (A26-A27))*C(2)
     1  / C(8) ) * SCAL2
      T(KC,5)=((C61 * (A14- A27) - C611 * (A13 - A27 ) + C612 * (A12 -
     1 A27 ) + C613 * (A26 - A27)) * C(3) / C(8)) * SCAL2
      T(KC,7)=((C58 * (A11- A27) + C581 * (A15- A27) + C582 * (A12- A27)
     1 - C583 * (A13 - A27 ) + C584* (A26- A27)) * C(4) / C(8))*SCAL2
      T(KC,11)=((C56 * (A12- A27) - C561 * (A26- A27)) *C(6) / C(8)
     1  ) * SCAL2
      T(KC,13)=(C52 * ( A26- A27) * C(7) / C(8))*SCAL2
      T(KC,15)= A27* SCAL2
      T(KC,16) = - 1 * SCAL2
      T(KC,17)=(( C60 * (A14 - A27 ) - C601 * (A11- A27) + C602 * (A12-
     1 A27) + C603 * (A26- A27)) * C(9) / C(8)) * SCAL2
      KC = KC +1
C********BURNUP EQUATION FOR U-234
      T(KC,1)=((C89* (A13-A15))* C(1)/C(9)) * SCAL2
      T(KC,5)=((C88*(A14- A15)- C90*(A13-A15))* C(3)/C(9))*SCAL2
      T(KC,7)=((C901*(A11-A15)- C902*(A13-A15))* C(4)/C(9))*SCAL2
      T(KC,17)= A15* SCAL2
      T(KC,18) = -1. * SCAL2
      KC = KC +1
C********FISSION PRODUCTS PRODUCTION FOR U-235 FISSION.
      T(KC,1) =((E21 - E22 ) * C(1)/ C(10)) * SCAL2
      T(KC,7)=((E16+E17- E19    ) * C(4) / C(10)) *SCAL2
      T(KC,17) =(E20 * C(9) / C(10)) * SCAL2
      T(KC,19) = 1. * SCAL2
      T(KC,20) =-1.* SCAL2
      KC = KC +1
C********FISSION PRODUCTS PRODUCTION FOR U-233 FISSION.
      T(KC,1) =(E31 + E32) * SCAL2
      T(KC,5) =(( E36 + E37 ) * C(3) / C(10)) * SCAL2
      T(KC,7) =((E33 + E34 + E35) *C(4) / C(10)) * SCAL2
      T(KC,21) = 1. * SCAL2
      T(KC,22) = -1. * SCAL2
      KC = KC+1
C********B-10 CONTROL POISON EQUATION.
      T(KC,24) = -1. * SCAL1
      T(KC,23) = BS1 * SCAL1
C********BOL LOCALIZED REACTIVITY CONSTRAINT.
      KC = KC+1
      T(KC,1) = S13 * SCAL3
      T(KC,3) = -SIG28* 1E+24 * SCAL3
      T(KC,5) = -SIG02 * 1E+24 * SCAL3
      T(KC,7) = S14 * SCAL3
      T(KC,9) = -SIG26* 1E+24 * SCAL3
      T(KC,11) = S15      * SCAL3
      T(KC,13) = -SIG40* 1E+24 * SCAL3
      T(KC,15) = S16* SCAL3
      T(KC,17) = -SIG24 * 1E+24* SCAL3
      KC = KC+1
C********NUCLIDE CONSTRAINT EQUATION( VOLUME CONSTRAINT).
      T(KC,3) =(1./1159 ) * SCAL2
      T(KC,5) =(1. /4634 ) * SCAL2
      T(KC,7) =(1.    /1159 ) * SCAL2
      KC = KC+1
C********B-10 VOLUME CONSTRAINT.
      T(KC,23) = 1.818E-03  * SCAL2
      KC = KC+1
C********ENRICHMENT CONSTRAINT.
```

```
      T(KC,3) =-A14B     * SCAL4
      T(KC,7) = A12B     * SCAL4
C        BOL POWER PEAKING CONSTRAINT EQUATION.
      KC = KC+ 1
      T(KC,1) = S18
      T(KC,7) = S17
      T(KC,11) = S19
      T(KC,15) = S20
C        EOL POWER PEAKING CONSTRAINT EQUATION.
      KC = KC+1
      T(KC,2) = S18
      T(KC,8) = S17
      T(KC,12) = S19
      T(KC,16) = S20
      RETURN
      END

      SUBROUTINE PART(MAXRR,K,ICCT)
C*************************************************************************
C
C        THIS SUBPROGRAM GENERATES THE NECESSARY TOTAL CYCLE CONSTRAINTS.
C
C*************************************************************************
      COMMON/SC/ SCAL1,SCAL2,SCAL3,SCAL4
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      COMMON/BAD/ Z1,Z2,Z11,Z3,Z4,Z33
      COMMON/BBD/REC25,REC23,REC49,REC41,COFAC
      COMMON /CP/SIGXE,SIGSA,SIGB10,SIGCR,SIFP4,SIFP1,SIGC
      COMMON/P8/SIG28,SIG02,SIG26,SIG40,SIG24 ,SIG41,SIG23,SIG25,SIG49
     1,S17,S18,S19,S20,PAVG,PPEAK,VREG ,S13,S14,S15,S16
      COMMON /PA/YIEP23,YIEP25,YIEI23,YIEI25,YIEX23,YIEX25,
     1 SIF23,SIF25,SIF49, SIF41,KCH,
     2  YIEX49,YIEI49,YIEP49,
     2 ENW23,ENW25,ENW49,ENW41,PKKK ,ALMP,ALMI,ALMX
      MAX1 = MAXR+ MAXRR +KC3
      MAX = MAXR + KC3 +1
      DO 5 L = MAX,MAX1
      DO 6 I=1,MAXC
      T(L,I) = 0.0
 6    CONTINUE
 5    CONTINUE
      SS1 =       250
      SS2 =       6000
      SS5= SIFP4
      SS6 = SIFP1
      S11 = SIGB10
      S12 = SIGCR
C********EQUATIONS FOR CARBON  CONCENTRATION.
      T(KCH,5) = SS1 * SCAL4
C********BOL EQUATIONS FOR ZENON   CONCENTRATION.
      KCH = KCH + 1
      T(KCH,2) = Z3 * SIF23 * SCAL2
      T(KCH,8) = Z4* SIF25  * SCAL2
      T(KCH,12) = Z33* SIF49 * SCAL2
      KCH = KCH + 1
C********BOL EQUATIONS SAMARIUM CONCENTRATION.
      T(KCH,2) = Z1 * SIF23  * SCAL2
      T(KCH,8) = Z2 * SIF25  * SCAL2
      T(KCH,12) = Z11 * SIF49* SCAL2
```

```
      KCH = KCH + 1
C********EOL EQUATIONS FOR ZENON CONCENTRATION.
      T(KCH,1) = Z3 * SIF23 * SCAL2
      T(KCH,7) = Z4* SIF25* SCAL2
      T(KCH,11) = Z33* SIF49  * SCAL2
      KCH = KCH + 1
C********EOL EQUATIONS FOR SAMARIUM CONCENTRATION.
      T(KCH,1) = Z1 * SIF23 * SCAL2
      T(KCH,7) = Z2 * SIF25* SCAL2
      T(KCH,11) = Z11 * SIF49 * SCAL2
      KCH = KCH + 1
C********BOL EQUATIONS FOR FISSION PRODUCTS CONCENTRATION.
      T(KCH,19) = SS5  * SCAL4
      T(KCH,21) = SS6  * SCAL4
      KCH = KCH + 1
C********EOL EQUATIONS FOR FISSION PRODUCTS CONCENTRATION.
      T(KCH,20) = SS5  * SCAL4
      T(KCH,22) = SS6   * SCAL4
      KCH = KCH + 1
C********CONTROL ROD POISON CONSTRAINT.
      T(KCH,23) = 1.
      KCH = KCH+1
C********EQUATIONS USED TO ZERO OUT ANY DESIRED CONCENTRATIONS.
      IF(IMUST1.GT.1.AND.ICOUT1.NE.ICCT) GO TO 20
      T(KCH,9) = 1.
      T(KCH,11) = 1.
      T(KCH,13) = 1.
      T(KCH,15) = 1.
      T(KCH,17) = 1.
      T(KCH,19) = 1.
      T(KCH,21) = 1.
      IF(IMUST1.GE.4) GO TO 20
      T(KCH,1) = 1.
 20   KCH = KCH+1
C        CONTROL ROD POISON.
      T(KCH,25) = 1.
      KCH = KCH+1
C        BOL REACTIVITY CONSTRAINT.
      T(KCH,1) = S13
      T(KCH,3) = -SIG28*1E+24
      T(KCH,5) = -SIG02*1E+24
      T(KCH,7) = S14
      T(KCH,9) = -SIG26* 1E+24
      T(KCH,11) = S15
      T(KCH,13) = -SIG40* 1E+24
      T(KCH,15) = S16
      T(KCH,17) = -SIG24* 1E+24
      T(KCH,23) = -SIGB10
      T(KCH,25) = -SIGCR
C        EOL REACTIVITY CONSTRAINT.
      KCH = KCH + 1
      T(KCH,2) = S13
      T(KCH,4) = -SIG28* 1E+24
      T(KCH,6) = -SIG02* 1E+24
      T(KCH,8) = S14
      T(KCH,10) =-SIG26* 1E+24
      T(KCH,12) = S15
      T(KCH,14) = - SIG40* 1E+24
      T(KCH,16) = S16
      T(KCH,18) = -SIG24* 1E+24
```

```
      T(KCH,24) =-SIGB10
C        BOL TOTAL POWER CONSTRAINT.
      KCH = KCH + 1
      T(KCH,1) = S18*VREG*1E-06 * SCAL3
      T(KCH,7) = S17* VREG*1E-06  * SCAL3
      T(KCH,11) = S19* VREG* 1E-06* SCAL3
      T(KCH,15) = S20 *VREG*1E-06 * SCAL3
C        EOL TOTAL POWER CONSTRAINT.
      KCH = KCH+1
      T(KCH,2)= S18* VREG*1E-06 * SCAL3
      T(KCH,8) = S17* VREG *1E-06  * SCAL3
      T(KCH,12) = S19*VREG*1E-06  * SCAL3
      T(KCH,16) = S20 * VREG*1E-06 * SCAL3
      RETURN
      END

      SUBROUTINE CONST
C*************************************************************************
C
C        THIS SUBPROGRAM IS USED TO PRSENT THE ACCUMULATIVE
C        PRODUCTS OF THE REGIONS.
C
C*************************************************************************
      COMMON/SC/ SCAL1,SCAL2,SCAL3,SCAL4
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      COMMON /CP/SIGXE,SIGSA,SIGB10,SIGCR,SIFP4,SIFP1,SIGC
      DO 5 L=1,MAXR
      DO 6 I=1,MAXC
      T(L,I) = 0.0
   6  CONTINUE
   5  CONTINUE
      S12 = SIGCR
      SS3 = SIGSA
      SS4 = SIGXE
      S11 = SIGB10
C********TOTAL CYCLE ACCUMULATION OF CARBON.
      T(1,1) = -1.
C********TOTAL CYCLE ACCUMULATION OF EOL XENON.
      T(2,5) = -1.* SCAL2
C********TOTAL CYCLE ACCUMULATION OF EOL SAMARIUM.
      T(3,3) = -1. * SCAL2
      KB = 4
C********TOTAL CYCLE ACCUMULATION OF BOL XENON.
      T(KB,4) = -1. * SCAL2
      KB =KB +1
C********TOTAL CYCLE ACCUMULATION OF BOL SAMARIUM.
      T(KB,2) = -1. * SCAL2
      KB =KB +1
      T(KB,2) = SS3  * SCAL4
      T(KB,4) = SS4 * SCAL4
C********TOTAL CYCLE BOL MACROSCOPIC POISON CROSS SECTION(1E-03).
      T(KB,6) = -1. * SCAL4
      KB =KB +1
      T(KB,3) = SS3  * SCAL4
      T(KB,5) = SS4  * SCAL4
C********TOTAL CYCLE EOL MACROSCOPIC POISON CROSS SECTION(1E-03).
      T(KB,7) = -1.   * SCAL4
      KB =KB +1
C********TOTAL CYCLE BOL ACCUMULATION OF CONTROL ROD POISONING.
```

```
      T(KB,8) = -1
      KB =KB +1
C********ZERO OUT EOL CONTROL ROD POISONING.
      T(KB,10) = 1
      KB = KB +1
C********TOTAL CYCLE BOL ACCUMULATION OF B-10 CONTROL POISON.
      T(KB,9) = -1
      KB = KB+1
      T(KB,1) = -SIGC  * SCAL1
      T(KB,6) = -1.
      KB = KB + 1
      T(KB,1) = -SIGC  * SCAL1
      T(KB,7) = -1.
      RETURN
      END

      SUBROUTINE STORED
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      MAX2 = MAX1 + MROS
      MAX = MAX1 + 1
      DO 5 I =MAX,MAX2
      DO 6 K= 1,MAXC
      T(I,K) = 0.
 6    CONTINUE
 5    CONTINUE
      T(KC1,2) =1.
      KC1 = KC1+ 1
      T(KC1,4) = 1.
      KC1 = KC1+ 1
      T(KC1,6) =1.
      KC1 = KC1+ 1
      T(KC1,8) =1.
      KC1 = KC1+ 1
      T(KC1,10) =1.
      RETURN
      END

      SUBROUTINE TRANST
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      MAX3 = MAX1 + MROT
      MAX= MAX1 + 1
      DO 5 I=MAX,MAX3
      DO 6 K= 1,MAXC
      T(I,K) = 0.
 6    CONTINUE
 5    CONTINUE
      T(KC2,2) =1.
      KC2 = KC2 + 1
      T(KC2,4) =1.
      KC2 = KC2 + 1
      T(KC2,6) =1.
      KC2 = KC2 + 1
      T(KC2,8) =1.
      KC2 = KC2 + 1
      T(KC2,10) =1.
      KC2 = KC2 + 1
      T(KC2,12) =1.
      KC2 = KC2 + 1
```

```
      T(KC2,14) =1.
      KC2 = KC2 + 1
      T(KC2,16) =1.
      KC2 = KC2 + 1
      T(KC2,18) =1.
      KC2 = KC2 + 1
      T(KC2,20) =1.
      KC2 = KC2 + 1
      T(KC2,22) =1.
      RETURN
      END

      SUBROUTINE TRANSF
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      DO 5 I=1,MAXR
      DO 6 K=1,MAXC
      T(I,K) = 0.
6     CONTINUE
5     CONTINUE
      T(1,1) =-1.
      T(2,3) =-1.
      T(3,5) =-1.
      T(4,7) =-1.
      T(5,9) =-1.
      T(6,11) =-1.
      T(7,13) =-1.
      T(8,15) =-1.
      T(9,17) =-1.
      T(10,19) =-1.
      T(11,21) =-1.
      KC3 = 11
      RETURN
      END

      SUBROUTINE FULIP
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      DO 5 I=1,4
      DO 6 K=1,7
      T(I,K) = 0.
6     CONTINUE
5     CONTINUE
      T(1,1) = -1.
      T(2,3) = -1.
      T(3,5) = -1.
      T(4,7) = -1.
      KC3 = 4
      RETURN
      END

      SUBROUTINE FULI
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      DO 5 I=1,MAXR
      DO 6 K=1,MAXC
      T(I,K) = 0.
6     CONTINUE
5     CONTINUE
      T(1,1) = 1.
```

```
      T(2,2) = 1.
      T(3,3) = 1.
      T(4,4) = 1.
      RETURN
      END

      SUBROUTINE STORI
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      DEX = -1.
      IF(MAX4.GT.2) DEX = 1.
      DO 5 I=MAX4,MAXR
      DO 6 K=1,MAXC
      T(I,K) = 0.
6     CONTINUE
5     CONTINUE
      KD = MAX4
      T(KD,1) =1.*DEX
      KD = KD+ 1
      T(KD,2) =1. *DEX
      KD = KD+ 1
      T(KD,3) =1. *DEX
      KD = KD+ 1
      T(KD,4) =1. *DEX
      KD = KD +1
      T(KD,5) = 1.* DEX
      MAX4 = MAXR +1
      MAXR = MAXR + MAX4
      RETURN
      END
```

APPENDIX E

Discussion of Error Messages

In finding the optimum solutions to the 6 cycle cases, i.e., Case 2 and Case 3, several problems were encountered which merit discussion here. The first problem was an error message reading "Eta Overflow" which indicated that the storage area for the Eta vector had been exceeded. This error was corrected by controlling the number iterations before invert demand is activated. Invert demand has two control parameters which are "Frequency" and "Clock Control." MPS automatically sets these parameters unless specified differently by the control cards. "Frequency" has assigned to it 100, i.e., 100 iterations will take place before Invert Demand is activated unless a certain duration of execution time has expired on the "Clock Control" which also demands an invert. Therefore a control card, XFREQINV = 50, was inserted into the MPS control program compiler to change the frequency specified number of iterations before invert to 50.

Another error message which proved the most bothersome was given as "Non-Zero Basis DJ after Price." This error suggested that the rwo error was too large, but after close scrutiny of the MPS output the row error was found to be sufficiently low; therefore the identical program was resubmitted and the optimum solution was attained.

AN IN-CORE LINEAR PROGRAMMING MODEL OF THE HIGH-TEMPERATURE GAS-COOLED REACTOR

by

JOSEPH SHERIDAN MILLER

B.S., University of Arkansas, 1972

AN ABSTRACT OF A MASTER'S THESIS

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Manhattan, Kansas

1974

ABSTRACT

Linear programming was used as the optimization technique to minimize the amount of U-235 being used in a 1160 MWe HTGR while simultaneously maximizing the production of U-233. A model was developed which coupled n number of cycles to produce the optimum combination of input fuels to produce U-233. The reactor linearization was achieved by the assumption that the flux spatial distribution was a constant for any given operating period. The reactor core was divided into four concentric annular zones, and a "out-in" fuel movement technique was used while the fissile loading of the core was held uniform by adjusting the power peaking constraints.

A matrix generator computer program was developed to produce the necessary input data on magnetic tape for IBM-MPS. Three separate cases were constructed and the optimum solutions were found. Case 1 was a three cycle no U-233 recycle, Case 2 was a six cycle no U-233 recycle, and Case 3 was a six cycle with U-233 recycle in the fourth refueling event.

From the results of Case 2 and Case 1, it was found that the U-233 produced from eight years of operation of a 1160 MWe HTGR was enough U-233 to operate the same reactor solely on U-233 recycle for three years hence.

THIS BOOK WAS BOUND WITH DUPLICATE PAGES NUMBERED 120 - 139.

THIS IS AS RECEIVED FROM CUSTOMER.

```
$JOB          YFC,TIME=3,PAGES=40
      DIMENSION  COEF(50),B(55)          ,MID(2),ID(2)
      COMMON/SC/ SCAL1,SCAL2,SCAL3,SCAL4
      COMMON/BAD/ Z1,Z2,Z11,Z3,Z4,Z33
      COMMON/BBD/REC25,REC23,REC49,REC41,COFAC
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      COMMON /PA/YIEP23,YIEP25,YIEI23,YIEI25,YIEX23,YIEX25,
     1 SIF23,SIF25,SIF49, SIF41,KCH,
     2  YIEX49,YIEI49,YIEP49,
     2 ENW23,ENW25,ENW49,ENW41,PKKK ,ALMP,ALMI,ALMX
      COMMON/PB/SIG28,SIG02,SIG26,SIG40,SIG24 ,SIG41,SIG23,SIG25,SIG49
     1,S17,S18,S19,S20,PAVG,PPEAK,VREG ,S13,S14,S15,S16
      COMMON /BU/C(10),P(5),PT(5),GAMA,EKPA25,EKPA49,
     1EKPA23,EL1,EL2,EL3,C11,C12,C13,C14,C15,C16,C17,C18,C19,C192,C193,
     2C194,C195,C20,C21,C22,C23,C25,C26,C27,C28,C89,C82,C24,C29,C69,C691
     3 ,C70,C701,C71,C72,C73,C74,C75,C76,C77,C78,C80,C81,C88,C90,C901,
     4C902,C91,C92,C93,D11,C83,C84,C85,C86,C87,C241,C242,C243,C244
      COMMON/BU/C245,C246,C196,C197,C191,C30,C31,C32,C33,C34,C35,C36,C37
     1,C38,C39,C40,C41,C42,C43,C44,C441,C45,C451,C452,C453,C46,C461,
     3C462,C463,C464,C465,C466,C467,C468,C469,C47,C48,C49,C50,C501,C52,
     4C53,C531,C54,C55,C56,C561,C57,C571,C572,C58,C581,C582,C583,C584
      COMMON/BU/C59,C591,C592,C593,C594,C60,C601,C602,C603,C61,C611,C612
     2,C613,D12,D13,D14,D15,D16,D17,D18,D19,D20,D21,D22,D23,A,B1,X11,X12
     3,P1,P2,P3,P4,P5,P6,B11,B12,B13,B14,B15,B16,B17,B18,B19,B20,B21,
     4B22,ALP25,ETA25,ALP23,ETA23, A14B,A12B,
     5 ALP49,ETA49,ALP41,ETA41,EPSI,P11,FAC
      COMMON /CP/SIGXE,SIGSA,SIGB10,SIGCR,SIFP4,SIFP1,SIGC
      DO 10 I=1,10
 10   C(I) =1.0
  61  FORMAT(5F15.0)
      READ(5,61) T1,FLUXM,PFAC
      READ(5,61) ALP25,SIG25,ETA25,ALP23,SIG23,ETA23,SIG28,SIG24,
     1 SIG26,SIG02,SIG49,ALP49,ETA49,SIG40,SIG41,ALP41,ETA41,EPSI,
     2P11,FAC
      READ(5,61) YIEP23,YIEP25,YIEI23,YIEI25,YIEX23,YIEX25,
     1 SIGXE,SIGSA,SIGC,SIGB10,SIGCR,                          ENW23,
     2 ENW25,ENW49,ENW41,PKKK,ALMP,ALMI,ALMX ,SIFP4,SIFP1
      READ(5,61) ENMAX,PAVG,PPEAK,TPOW,VREG,REC25,REC23,REC49,REC41,
     1 COFAC ,YIEP49,YIEI49,YIEX49
C
      Z1    =       YIEP23/ (SIGSA        )
      Z2    =      YIEP25 /( SIGSA  )
      Z11 = YIEP49 / SIGSA
C********NCYC IS THE NUMBER OF FUEL CYCLES CONSIDERED.
C
      NCYC = 6
C
C********NREG IS THE NUMBER OF REGIONS CONSIDERED.
C
      NREG = 4
C
C********NUMBER OF FLUX LEVELS MUST EQUAL NREG.
C
C********CALCULATE FISSION CROSS SECTIONS IN BARNS.
      SIF23 = SIG23 * 1E+24 /(1+ ALP23)
      SIF25 = SIG25 * 1E+24 /(1+ ALP25)
      SIF49 = SIG49 * 1E+24 /(1+ ALP49)
      SIF41 = SIG41 * 1E+24 /(1+ ALP41)
C********READ AVERAGE FLUX RATIO FOR EACH REGION.
```

```
      READ(5,61) FLUF1,FLUF2,FLUF3,FLUF4
      FLUXM = FLUXM* PFAC
C********CALCULATE AVERAGE FLUX LEVEL FOR EACH REGION.
      FLUX(1)=FLUF1*FLUXM
      FLUX(2) = FLUF2 * FLUXM
      FLUX(3) = FLUF3 * FLUXM
      FLUX(4) = FLUF4 * FLUXM
      ETA41 = ENW41 / (1+ ALP41)
      ETA25 = ENW25 / (1+ ALP25)
      ETA23 = ENW23 / (1+ ALP23)
      ETA49 = ENW49 / (1+ ALP49)
      MDSK = 9
C*********IMAX IS THE NUMBER OF "EQUAL" CONSTRAINTS IN THE BURN PARTITION.
      IMAX = 12
C*********KMAX IS THE NUMBER OF "GREATER THAN" CONSTRAINTS IN THE BURN PARTITION.
      KMAX = 1
C*********MAXC IS THE MAXIMUM NUMBER OF COLUMNS IN EACH REGION.
      MAXC = 25
C*********MAXR IS THE MAXIMUM NUMBER OF ROWS IN EACH REGION.
      MAXR = 18
C*********KONTR IS NUMBER OF "EQUAL" CONSTRAINTS  IN THE  CONST PARTITION.
      KONTR =11
C*********KMTR IS NUMBER OF "GREATER THAN" CONSTRAINTS IN THE CONST PARTITION.
      KMTR = 3
C*********MAXCC IS  THE NUMBER OF COLUMNS IN THE CONST PARTITION.
      MAXCC = 10
C*********MAXRR IS THE NUMBER OF ROWS IN THE CONST PARTITION.
      MAXRR = 14
C*********MROS REPRESENTS THE NUMBER OF ROWS IN THE STORAGE AREA.
      MROS = 5
C*********MROT REPRESENTS THE NUMBER OF ROWS IN THE TRANSFER PARTITION.
      MROT = 11
C*********MINP REPRESENTS THE NUMBER OF ROWS IN THE INPUT PARTITION.
      MINP = 4
C********* NCOLS REPRESENTS THE NUMBER OF COLUMNS IN THE INPUT PARTITION.
      NCOLS = 4
C********* MRIS REPRESENTS NUMBER OF COLUMNS IN THE IN-STORAGE AREA.
      MRIS = 5
      ICCT = NREG
      IMAX1= IMAX
      MAXC1 = MAXC
      MAXR1 = MAXR
      KMAX1 = KMAX
      MAXRQ = 70
      MAXCQ = 50
      A148 = 238 * ENMAX
      A128 = 235 - 235 * ENMAX
C********* INTER SCALING FACTORS.
      SCAL3=1E-03
      SCAL4 = 1E-02
      SCAL1 = 1E+02
      SCAL2 = 1E+03
C*********READ IN RESONANCE ESCAPE PROBABILITIES FOR U-238 AND
C*********TH -232 FOR EACH REGION.
      READ(5,60) (P(K),K=1,NREG)
      READ(5,62) (PT(K),K=1,NREG)
      K = 1
C*********CALCULATE CONSTANTS FOR BURN-UP EQUATION.
      PKKK = PT(K) * EPSI *PKKK
      GAMA  =  A-ETA49*EPSI*PI*(1-P(K))
```

```
EKPA25 = ETA25 * EPSI  * P11*(1-P(K))
EKPA49  = ETA49 * EPSI   * P11* (1- P(K))
EKPA23 = ETA23 * EPSI  * P11* (1-P(K))
EL1 =  ETA23  *  EPSI   * P11* (1-PT(K))
EL2 = ETA25 *  EPSI   * P11* (1- PT(K))
EL3 = EPSI * P11*  (1- P(K))
C11 =  SIG23 * ALP23 / (1+ALP23)
C12 =  C11* SIG02 * SIG25 * FAC/ ( SIG23-SIG02)
C13 = EKPA25 * SIG25
C14 = C13 * C12/ (SIG24- SIG02)
C15 = EKPA23 * SIG23
C16 = C15 * SIG02 / ( SIG23 * (1-EL1) - SIG02)
C17 = C15 * SIG25 * EL2 / ( SIG23 * (1-EL1) - SIG25)
C18 = SIG28 / ( GAMA * SIG49)
C19 = C13 / ( GAMA * SIG49 - SIG25)
C192 = GAMA * SIG49 - SIG24
C193 = GAMA * SIG49 - SIG25
C194 = GAMA* SIG49 - SIG23 *(1-EL1)
C195 = GAMA* SIG49 - SIG02
C20   =C16 / C195
C21 = C14 / (( GAMA * SIG49 - SIG25 )* (SIG25-SIG02)*FAC)
C22 = C14 / (( SIG25-SIG24 )* ( GAMA*SIG49 - SIG24)*FAC)
C23 = C14 / (( SIG25 - SIG24 )* ( GAMA * SIG49- SIG25)* FAC)
C25 = C16 / ( GAMA * SIG49 - SIG02)
C26 = C16 / ( GAMA * SIG49 + SIG23* ( EL1-1))
C27 = C19 * C17 / C13
C28 = C26 * C17 / C16
C89 = C11/(SIG24-SIG23* (1- EL1))
C82 = SIG24 * C89 /  ( SIG25- SIG23 * (1- EL1))
C24 = C13 * C82 / C194
C29 = ALP49 * SIG49 / (1 + ALP49)
C69 = C17 /(SIG28+SIG23*(EL1-1))
C691 =-C69 * C13 / C17
C70 = C13 / (SIG28-SIG25)
C701 =-C13 / (SIG28 - SIG24)
C71 = C17 / ( SIG28- SIG25)
C72 = C14/ ((SIG25-SIG02)*(SIG28-SIG25)*FAC)
C73 = C14 / ((SIG25- SIG24)* (SIG28- SIG24)*FAC)
C74 = C16/ (SIG28+ SIG23* (EL1-1))
C75 = C14 / (( SIG25- SIG02)* (SIG28- SIG02)*FAC)
C76 = C14 /((SIG25- SIG24)*(SIG28-SIG25)*FAC)
C77 = C16 / (SIG28- SIG02)
C78 = C15/ (SIG28 - SIG23* (1-EL1))
C80 = C11* SIG25/ (SIG24-SIG23)
C81 = SIG24 /( SIG25- SIG24 )
C88 =  C11 *C16 /(C15*(SIG24-SIG02))
C90 = C11* C16 /(C15 *(SIG24+SIG23*(EL1-1)))
C901 = C11* C17/ (C15 *(SIG24-SIG25))
C902 = C11* C17 /(C15*(SIG24+ SIG23*(EL1-1)))
C91 = SIG24 * C901 / (SIG25 - SIG24)
C92 = SIG24 * C902 / (SIG25+ SIG23* (EL1-1))
C93 = SIG24 * C902/ (SIG25- SIG24)
D11 = ALP25 * SIG25/ (1+ ALP25)
C83 = SIG24 * C89 / (SIG25 - SIG24)
C84 = SIG24 * C88 / (SIG25 - SIG02)
C85 = SIG24 * C88 / (SIG25 - SIG24)
C86 = SIG24 * C90 / (SIG25- SIG23 *(1- EL1))
C87 = SIG24 * C90 /(SIG25- SIG24)
C241 = C13 / C193
C242 = C13*C83/ C192
```

```
C243= C13 / C193
C244 = C15 / C194
C245= C13* C81/ C192
C246= C13 * C81/C193
C196 = C13 * C93/ C192
C197 = C13 * C93 / C193
C191 = C13 * C92/ C194
C30 = C27 +C19 - C197
C31 = C28 + C191
C32 =C191 +C28 + C197  -C19-C27 - C196
C33 = C24 + C244
C34 = C243 - C241
C35 = C241 + C242 -C24 - C243- C244
C36 = C246 - C245
C37 = C26 - C20
C38 =  SIG40 - GAMA* SIG49
C39 = SIG40 - SIG25
C40 = SIG40 - SIG23 * (1-EL1)
C41 = SIG40 - SIG24
C42 = SIG40 - SIG02
C43 = C29 / C38
C44 = C29 * C18 / SIG40
C441 = C29 * C18 / C38
C45 = C30 * C29 / C39
C451 = C29 * C31 / C40
C452 = C29 * C196 / C41
C453 = C29 * C32 / C38
C46 = C29 * C33 / C40
C461 = C29 * C34 / C39
C462 = C29 * C242 / C41
C463 = C29 * C35 / C38
C464 = C29 * C245 / C41
C465 = C29 * C246 / C39
C466 = C29 * C36 / C38
C467 = C29 * C20 / C42
C468 = C29 * C26 / C40
C469 = C29 * C37 / C38
C47 = C441 - C44
C48 = C451 - C45 -C452 - C453
C49 = C462 - C46- C461 - C463
C50 = C465 - C464 - C466
C501 = C468 - C467 - C469
C52 = SIG40 / (SIG41 -GAMA* SIG49)
C53 = SIG40 / (SIG41- SIG25)
C531 = SIG40 / (SIG41- SIG40)
C54 = SIG40 / (SIG41- SIG24)
C55= SIG40 / (SIG41- SIG23*( 1- EL1))
C56 = C52 * C43
C561 = C531 * C43
C57 = SIG40 * C44 / SIG41
C571 = C52 * C441
C572 = C531 * C47
C58 = C53 * C45
C581 = C54 * C452
C582 = C52 * C453
C583 = C55 * C451
C584 = C531 * C48
C59 = C55 * C46
C591 = C53 * C461
C592 = C54 * C462
```

```
      C593 = C52 * C463
      C594 = C531 * C49
      C60 = C54 * C464
      C601 = C53 * C465
      C602 = C52 * C466
      C603 = C531 * C50
      C61 = SIG40 * C467 / (SIG41 - SIG02)
      C611 = C55 * C468
      C612 = C52 * C469
      C613 = C531 * C501
      D12 = SIG26 - SIG25
      D13 = SIG26 - SIG24
      D14 = SIG26 + SIG23* (EL1-1)
      D15 = D11 * (C92-C93 + 1)/ D12
      D16 = D11 / ALP25
      D17 = D11 * C93 / D13
      D18 = D11 * C92 / D14
      D19 = D11 * C81 / D13
      D20 = D11 * C81 / D12
      D21 = D11 * (C83 - C82) / D12
      D22 = D11 * C82 / D14
      D23 = D11 * C83 / D13
      A= SIG02 + SIG23*(1-EL1)
      B1= SIG02 * SIG23 * FAC
      X11= .5 *(-A+ SQRT(A**2-(4*B1/FAC)))
      X12 = .5*(-A-SQRT(A**2-(4*B1/FAC)))
      P1 = SIG02
      P2 = EL1 * SIG23
      P3 = SIG25
      P4 = SIG23 * (1- EL1)
      P5 = SIG23
      P6 = EL2 * SIG25
      B11 = P4 + P3
      B12 = P3 * P4
      B13 = P6 * P5
      B14 = P2 * P3
      B15 = (X11-X12) * (X11+P3)
      B16 = (X12- X11) * (X12+ P3)
      B17 = (P3+ X11) * (P3+ X12)
      B18 = P1+ P3
      B19 = P1* P3
      B20 = P1* P6
      B21 = P1* P6 * P3
      B22 = P3 * P6
      KCH = MAXR +1
      DO 5 L=1,MAXRQ
      DO 6 I=1,MAXCQ
      COEF(I) = 0.0
      T(L,I) = 0.
    6 CONTINUE
    5 CONTINUE
      MAROW = ICCT * MAXR
      MAXAL = MAXR + MAXRR
C********SPECIFY COST COEFFICIENTS.
      COEF(2) = -1.39
      COEF(7) = 1.
      DO 7 L= 1,MAXR
      B(L) = 0.0
    7 CONTINUE
 3000 FORMAT('NAME',10X,'PROBLEM'/'ROWS'/' N  PROFIT')
```

```
 801  FORMAT(' ','E',' ROW',I3)
 3001 FORMAT(' ','G',' ROW',I3)
 3003 FORMAT(' ','L',' ROW',I3)
 3004 FORMAT('COLUMNS')
 3010 FORMAT(T1,'    COL',T8,I3,T15,'ROW',T18,I3,T25,E12.5)
 3012 FORMAT('RHS')
 3011 FORMAT(T1,'    LIMITS',T15,'ROW',T18,I3,T25,E12.5)
 3009 FORMAT('ENDATA')
 3005 FORMAT(T1,'    COL',T8,I3,T15,'PROFIT',T25,E12.5,T40,'ROW',T43
     1,I3,T50,E12.5)
 3006 FORMAT(T1,'    COL',T8,I3,T15,'ROW',T18,I3,T25,E12.5,T40,'ROW'
     1,T43,I3,T50,E12.5)
 3007 FORMAT(T1,'    LIMITS',T15,'ROW',T18,I3,T25,E12.5,T40,'ROW',T4
     13,I3,T50,E12.5)
C******* CONSTRUCTION OF ROWS VECTOR
      WRITE(MDSK,3000)
      IXMU = 3* MROT + MROS + MINP
      NRPP = NREG + 2
      NR=101
      IMUST1 = 0
      ICCT1 = ICCT +1
      ICCT2 = ICCT-1
      ICCT3 = ICCT1 + 1
      ICCT4 = ICCT1 + 2
      DO 899 K3=1,NCYC
C********UPDATE ALL PARMETERS.
      IMAX = IMAX1
      KMAX = KMAX1
      MAXR = MAXR1
      IMUST1 = IMUST1 + 1
      DO 802 K1= 1,NRPP
      IF(K1.LE.ICCT) GO TO 799
      IMAX = KONTR
      KMAX = KMTR
      MAXR = MAXRR
      IF(K1.LE.ICCT1) GO TO 799
      IF(IMUST1.EQ.NCYC) GO TO 400
      IMAX = IXMU
      MAXR = IXMU
      KMAX = 0.
      GO TO 799
 400  KMAX = 0
      IMAX = MROS
      MAXR = MROS
 799  IF (IMAX.LT..5) GO TO 850
      DO 800 K=1,IMAX
      WRITE(MDSK,801)NR
 800  NR=NR+1
      IF(IMAX.EQ.MAXR) GO TO 802
 850  IH=IMAX+1.
      KJ=IMAX+KMAX
      IF(KJ.LT..5) GO TO 860
      IF(KMAX.LT..5) GO TO 860
C******* CONSTRUCTION OF G CONTRAINTS
      DO600K=IH,KJ
      WRITE(MDSK,3001)NR
  600 NR=NR+1
C******* CONSTRUCTION OF L CONTRAINTS
 860  IP=KJ+1.
      IF(IP.GT.MAXR) GO TO 802
```

```
      DO605K=IP,MAXR
      WRITE(MDSK,3003)NR
  605 NR=NR+1
  802 CONTINUE
  899 CONTINUE
C
C        UPDATE PARAMETERS.
C
      IMAX = IMAX1
      MAXR = MAXR1
      KMAX = KMAX1
C*******CONTRUCTION OF THE COLUMN SECTION.
      WRITE(MDSK,3004)
C********INIALIZE
      IMUST1 = 0
      NFIL = 0
      MKK = MAXR + MAXRR
      MKK1 = MROT + MAXR
      MK2 = MROT + MAXR + MAXRR
      NC = 100
      MK3 = MINP + MAXR + MAXRR
      NRP1 = NRPP+1
      MROW1 = (MAROW+ MAXRR) * 2+ (ICCT-1)*MROT +MROS+MINP
      DO 833 K3=1,NCYC
      COEF(2) = -1.39
      COEF(7) = 1.
      MAXR = MAXR1
      MAXC = MAXC1
      ICOUT1 = 0
      ICONU1 = ICCT2
C********UPDATE REFERENCE POINT.
      NFIL = IMUST1 *(ICCT*MAXR + MAXRR+ (ICCT-1) * MROT+ MROS+MINP)
      IMUST1 = IMUST1 + 1
      MAX4 = 1
      KC3 = 0
      DO 803 K1 =1,NRP1
      ICOUT1 = ICOUT1 + 1
      IF(IMUST1.EQ.1) GO TO 925
      KC3= 0
      IF(ICOUT1.LT.ICCT) CALL TRANSF
      IF(ICOUT1.EQ.ICCT) CALL FULIP
  925 CONTINUE
      KCH = MAXR1 + KC3 +1
      KC1 = MAXR1 + MAXRR + KC3 + 1
      KC2 = MAXR1 + MAXRR + KC3 + 1
C********INIALIZE ROW NUMBER OF  FIRST FOUR REGIONS.
      IF(K1.GT.ICCT) GO TO 839
      IF(IMUST1.EQ.1) GOTO 572
      NR = NFIL + 100 -ICONU1* MROT-MINP
      ICONU1 = ICONU1 -1
      GO TO 571
  572 NR = NFIL + 100
      NP = NR - MAXR
      NR = NP + ICOUT1 * MAXR
C********GENERATE BURNUP EQUATIONS.
  571 CALL BURN(K1)
C********GENERATE CONSTRAINT EQUATIONS.
      CALL PART(MAXRR,K1,ICCT)
      IF(ICOUT1.NE.1) GOTO 965
C********GENERATE BULK STORAGE AREA.
```

```
      CALL STORED
      MAXAL = MAXR + MAXRR + MROS
      IF(KC3.GT.0) MAXAL=MAXR + MAXRR+ MROS + MROT
      GO TO 842
 965  CONTINUE
      IF(IMUST1.EQ.NCYC) GO TO 401
      CALL TRANST
      MAXAL = MAXR + MAXRR + MROT
      IF(KC3.GT.0) MAXAL= MAXR + MAXRR + 2*MROT
      IF(ICOUT1.EQ.ICCT) MAXAL =MAXR+MAXRR+     MROT+MINP
      GO TO 842
 401  MAXAL = MAXR + MAXRR + MROT
      GO TO 842
 839  CONTINUE
C********INIALIZE ROW NUMBER FOR LAST THREE REGIONS.
      IF(ICOUT1.NE.ICCT1) GOTO 840
      MAXR = MAXRR
      MAXC = MAXCC
      COEF(2) = 0.
      COEF(3) = 0.
      COEF(5) = 0.0
      COEF(7) = 0.0
      CALL CONST
      MAXAL = MAXRR
      NR = MAROW + NFIL + 100
      GO TO 842
 840  CONTINUE
      IF(ICOUT1.NE.ICCT3) GO TO 841
      MAXR = MROS
      MAXC = MRIS
      MAXAL = 2* MROS
C********GENERATE ACCUMULATIVE BULK STORAGE AREAS.
      CALL STORI
      CALL STORI
      IF(IMUST1.EQ.NCYC) MAXAL = MROS
      NR = MAROW + MAXRR + NFIL + 100
      GO TO 842
 841  CONTINUE
      IF(ICOUT1.NE.ICCT4) GO TO 842
      IF(IMUST1.EQ.NCYC) GO TO 833
      MAXR = MINP
      MAXC = NCOLS
      MAXAL = MINP
C********GENERATE INPUT FUEL PARTITION.
      CALL FULI
      NR = MAROW + MAXRR + MROS + 3*MROT +NFIL + 100
 842  CONTINUE
      DO 720 N=1,MAXC
      MM= 1
      NC = NC+1
      NOD = 0
      NOW = 0
 555  DO 710 M=MM,MAXAL
      IF(T(M,N).NE.0.) GO TO 711
      GO TO 710
 711  NTR = NR+M
      IF(ICOUT1.EQ.ICCT1.OR.ICOUT1.EQ.ICCT4) GO TO 550
      IF(IMUST1.GT.1) GO TO 502
      IF(ICOUT1.NE.1) GO TO 501
      IF(M.GT.MAXR) NTR= MAROW + NFIL +100+M-MAXR
```

```
      GO TO 550
  501 CONTINUE
      IF(ICOUT1.GT.ICCT) GO TO 502
      IF(M.LE.MAXR) GO TO 550
      IF(M.GT.MAXR.AND.M.LE.MKK) NTR=MAROW+NFIL+100+M-MAXR
      IF(M.GT.MKK) NTR= MAROW+MROS+MAXRR+(K1-2)*MROT+NFIL+100+M-MKK
      GO TO 550
  502 CONTINUE
      IF(ICOUT1.NE.ICCT3) GO TO 503
      IF(M.GT.MROS) NTR= MROW1+  NFIL + 100 + M- MROS
      GO TO 550
  503 IF(ICOUT1.EQ.1) GO TO 506
      IF(ICOUT1.EQ.ICCT) GO TO 509
      IF(M.LE.MROT) GO TO 550
      IF(M.GT.MROT.AND.M.LE.MKK1) GOTO   507
      IF(M.GT.MKK1.AND.M.LE.MK2) GO TO 508
      NTR= NFIL + 100 + MAROW + MAXRR+ MROS+ (K1-2) *MROT+M-MK2
      GO TO 550
  507 NTR = NFIL+ (K1-1)*MAXR+100 + M-MROT
      GO TO 550
  508 NTR = NFIL +100 + MAROW+ M- MKK1
      GO TO 550
  509 IF(M.LE.MINP) GO TO 550
      IF(M.GT.MINP.AND.M.LE.MK3) GO TO 510
      NTR = NFIL + 100 + MAROW+ MAXRR+ MROS+ (K1-2)* MROT +M- MK3
      GO TO 550
  510 NTR = NFIL + 100 + MAROW-MAXR+ M-MINP
      GO TO 550
  506 IF(M.LE.MROT) GO TO 550
      IF(M.GT.MROT.AND.M.LE.MKK1) GO TO 511
      NTR= NFIL+ 100 + MAROW+M-MKK1
      GO TO 550
  511 NTR = NFIL + 100 + M-MROT
  550 CONTINUE
      IF(NOD.GT.0) GO TO 552
      WRITE(MDSK,3005)NC,COEF(N),NTR,T(M,N)
      NOD = NOD + 1
      GO TO 710
  552 CONTINUE
      NOW = NOW+1
      MID(NOW) = M
      ID(NOW) = NTR
      IF(NOW.LT.2) GO TO 710
      WRITE(MDSK,3006)NC,ID(1),T(MID(1),N),ID(2),T(MID(2),N)
      NOW = 0
  710 CONTINUE
      IF(NOW.EQ.0) GO TO 720
      WRITE(MDSK,3010)NC,ID(1),T(MID(1),N)
  720 CONTINUE
  803 CONTINUE
  833 CONTINUE
C******* CONSTRUCTION OF RHS VECTOR FOR MPS-360 INPUT
      WRITE(MDSK,3012)
      IMUST1 = 0
      DO 625 K3=1,NCYC
C********UPDATE REFERENCE ROW NUMBER.
      MAXR = MAXR1
      NFIL = IMUST1*(ICCT*MAXR+MAXRR+(ICCT-1)*MROT+MROS+MINP   )
      IMUST1 = IMUST1 + 1
C********INITIALIZE.
```

```
      MAXR = MAXR1
      NR = 100 +NFIL
      NREG = ICCT + 1
      NTR=0
      B(13) = 0.
      B(14) = 0.
      B(15) = 0.
      B(17) = 0.
      B(18) = 0.
      DO 804 K1=1,NREG
      IF(K1.LE.ICCT) GOTO948
      MAXR = MAXRR
C********GENERATE RHS TOTAL POWER CONSTRAINTS.
      B(13) = TPOW*PFAC*SCAL3
      B(14) = TPOW*PFAC     *SCAL3
      IF(IMUST1.LT.3.OR.IMUST1.GT.4) GO TO 958
      B(13) = TPOW *PFAC * SCAL3 *.9741
      B(14) = TPOW *PFAC * SCAL3 *.9741
      GO TO 958
 948  B(14) = 1.    * SCAL2
      B(15) = 1.    * SCAL2
      PPEAK1 = PPEAK
C********GENERATE RHS POWER PEAKING CONTRAINT.
      IF(K1.GT.1) PPEAK1 = PPEAK* .87
      IF(K1.GT.2) PPEAK1 = PPEAK* .77
      B(17) = PAVG* PPEAK1
      B(18) = PAVG* PPEAK1
 958  DO730M=1,MAXR
      NR = NR +1
      IF(B(M).EQ.0.)GOTO730
      NTR=NTR+1
      MID(NTR)=M
      ID(NTR)=NR
      IF(NTR.LT.2)GOTO730
      WRITE(MDSK,3007)ID(1),B(MID(1)),ID(2),B(MID(2))
      NTR=0
  730 CONTINUE
      IF(NTR.EQ.0)GOTO731
      WRITE(MDSK,3011)ID(1),B(MID(1))
  731 CONTINUE
      NTR = 0
  804 CONTINUE
  625 CONTINUE
  736 WRITE(MDSK,3009)
      STOP
      END

      SUBROUTINE  BURN(K)
C*************************************************************************
C
C        THIS SUBROUTINE CONTAINS THE BURNUP EQUATIONS FOR THE TH-232
C        AND U-238 FUEL CYCLES. THE SUBROUTINE ALSO CONTAINS  LOCALIZED
C        CONSTRAINT EQUATIONS FOR EACH REGION.
C
C*************************************************************************
      COMMON/SC/ SCAL1,SCAL2,SCAL3,SCAL4
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      COMMON/BAD/ Z1,Z2,Z11,Z3,Z4,Z33
      COMMON /PA/YIEP23,YIEP25,YIFI23,YIEI25,YIEX23,YIEX25,
```

```
1 SIF23,SIF25,SIF49, SIF41,KCH,
2  YIEX49,YIEI49,YIEP49,
2 ENW23,ENW25,ENW49,ENW41,PKKK ,ALMP,ALMI,ALMX
 COMMON/BBO/REC25,REC23,REC49,REC41,COFAC
 COMMON/PB/SIG28,SIG02,SIG26,SIG40,SIG24 ,SIG41,SIG23,SIG25,SIG49
1,S17,S18,S19,S20,PAVG,PPEAK,VREG ,S13,S14,S15,S16
 COMMON /CP/SIGXE,SIGSA,SIGB10,SIGCR,SIFP4,SIFP1,SIGC
 COMMON /BU/C(10),P(5),PT(5),GAMA,EKPA25,EKPA49,
1EKPA23,EL1,EL2,EL3,C11,C12,C13,C14,C15,C16,C17,C18,C19,C192,C193,
2C194,C195,C20,C21,C22,C23,C25,C26,C27,C28,C89,C82,C24,C29,C69,C691
3 ,C70,C701,C71,C72,C73,C74,C75,C76,C77,C78,C80,C81,C88,C90,C901,
4C902,C91,C92,C93,D11,C83,C84,C85,C86,C87,C241,C242,C243,C244
 COMMON/BU/C245,C246,C196,C197,C191,C30,C31,C32,C33,C34,C35,C36,C37
1,C38,C39,C40,C41,C42,C43,C44,C441,C45,C451,C452,C453,C46,C461,
3C462,C463,C464,C465,C466,C467,C468,C469,C47,C48,C49,C50,C501,C52,
4C53,C531,C54,C55,C56,C561,C57,C571,C572,C58,C581,C582,C583,C584
 COMMON/BU/C59,C591,C592,C593,C594,C60,C601,C602,C603,C61,C611,C612
2,C613,D12,D13,D14,D15,D16,D17,D18,D19,D20,D21,D22,D23,A,B1,X11,X12
3,P1,P2,P3,P4,P5,P6,B11,B12,B13,B14,B15,B16,B17,B18,B19,B20,B21,
4B22,ALP25,ETA25,ALP23,ETA23, A148,A128,
5 ALP49,ETA49,ALP41,ETA41,EPSI,P11,FAC
 S13 = ENW23 * SIF23*PKKK- SIG23*1E+24
 S14 = ENW25 * SIF25 *PKKK- SIG25*1E+24
 S15 = ENW49 * SIF49 * PKKK- SIG49*1E+24
 S16 = ENW41 * SIF41 * PKKK-SIG41* 1E+24
 Z33 = (YIEI49+YIEX49)/((ALMX/(FLUX(K) *1E-24))+SIGXE)
 Z3   = (      YIEI23+ YIEX23)/ ((ALMX/(FLUX(K)*1E-24))+ SIGXE)
 Z4     = (       YIEI25 + YIEX25) /((ALMX/(FLUX(K)*1E-24))+ SIGXE)
 S18 = SIF23 * REC23 * FLUX(K) * COFAC /VREG
 S17 = SIF25 * REC25 * FLUX(K) * COFAC /VREG
 S19 = SIF49 * REC49 * FLUX(K) * COFAC   /VREG
 S20 = SIF41 * REC41 * FLUX(K) * COFAC  /VREG
 BS1 = EXP(-SIGB10*1E-24*FLUX(K) *T1)
 C54 = SIG40 * C37 * FLUX(K) * T1
 A11 = EXP( - SIG25 * FLUX(K) *T1)
 A12 = EXP( - GAMA * SIG49 * FLUX(K) * T1)
 A13 = EXP ( SIG23 *( EL1-1) * FLUX(K) * T1)
 A14 = EXP (- SIG02 * FLUX(K)* T1)
 A15 = EXP(- SIG24 * FLUX(K) * T1)
 A16 = EXP(- SIG23 * FLUX(K) * T1)
 A17 = C19 * ( A11- A12)
 A18 = C27 * (A11-A12)
 A19 = C28 * (A13- A12)
 A191 = C191 * (A13- A12)
 A192 = C196* (A15- A12)
 A193 = C197 * (A11- A12)
 A194 = C24 * (A13- A12)
 A195 = C241 * (A11- A12)
 A196 = C242 * (A15- A12)
 A197 = C243* (A11- A12)
 A198 = C244* (A13- A12)
 A20 = C20 * (A14 - A12)
 A21 = C21* (A11- A12)
 A211 = C245 * (A15 - A12)
 A212 = C246 * (A11- A12)
 A22 = C22* (A15- A12)
 A23= C23 * (A11- A12)
 A24 = C25 * (A14- A12)
 A25 = C26 * (A13- A12)
 A26 = EXP(- SIG40 * FLUX(K) * T1)
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      A27 = EXP(-SIG41 * FLUX(K)* T1)
      A28 = EXP(- SIG28*FLUX(K)* T1)
      A30 = EXP(X11*FLUX(K)*T1)
      A31 = EXP(X12*FLUX(K)* T1)
      A32 = EXP(-SIG26* FLUX(K) * T1)
      C702 =+C70 * C93 * (A11- A28 )+C701* (A15-A28)* C93
      C703 =-C70 * ( A11- A28)
      C704 = + C92 * C70 * (A11- A28)+C691*C92* (A13-A28)
      C705 =-C71 *(A11- A28 ) +C69* (A13-A28)
      C706 = C82 * C691*(A13- A28)+C82*C70*(A11-A28)
      C707 = C701 * C83* (A15- A28)+C70*C83*(A11- A28)
      C708 =-C78 * (A13-A28)
      C709 = C701 * (A15- A28 )* C81 +C70 *C81*(A11-A28)
      C94 = SIG24 *C901* FLUX(K)* T1*A11
      E11 = D16 * C93
      E12 = D16 * C92
      E13 = D16 * C81
      E14 = D16 * C82
      E15 = D16 * C83
      E18 = (1- A11 )/SIG25
      E16 = E11*(((1-A15)/ SIG24)-E18)
      E17 = D16 * E18
      E19 = E12 *(((1-A13)/(SIG23 * (1- EL1)))- E18)
      E20 = E13 * (((1-A15 ) / SIG24 )- E18)
      E21 = E14 *(((1-A13 )/(SIG23* (1-EL1)))-E18)
      E22 = E15 *(((1-A15)/ SIG24)-E18)
      E30 = SIG23 / (1 + ALP23)
      Y11 = A30 *(B11* X11+ X11**2+ B12) /B15
      Y12 = A31 * (B11* X12+ X12**2 + B12) /B16
      Y13 = -A30 * (X11* P6 + B13) /B15
      Y14 = - A31 * (X12* P6 + B13) /B16
      Y15 = -A11 * (B13 - P3 * P6) / B17
      Y16 = - A30 * (X11* P2+ B14)  / B15
      Y17 = -A31* (X12* P2+ B14)  / B16
      Y18 = A30 * (X11**2 + X11*B18+ B19) /B15
      Y19 = A31 *(X12**2+ X12*B18 + B19 ) /B16
      Y20 = A30 * (X11*(P6-B20)+ B20 -B21) /B15
      Y21 = A31 *(X12*(P6-B20)+ B20-B21) /B16
      Y22 = A11* (B20 - B22) / B17
      Y23 = A30 * (X11* P1+ B19) / B15
      Y24 = A31 * (X12* P1 + B19 ) / B16
      E31 = -E30 *(1-A30)*Y18 /(A30* X11)
      E32 = -E30 *(1-A31) * Y19 /(A31 * X12)
      E33 = -E30 *(1-A30)* Y20 / (A30 * X11)
      E34 = - E30 * (1- A31) * Y21 / (A31 * X12)
      E35 = E30 * (1-A11) * Y22/ (A11 * P3)
      E36 = -E30 * (1- A30) * Y23 / (A30 * X11)
      E37 = - E30 * (1- A31) * Y24 / (A31 * X12)
      MAX = KC3 + 1
      MAXR2 = MAXR + KC3
      DO 5 I=MAX,MAXR2
      DO 6 L= 1,MAXC
      T(I,L) = 0.
 6    CONTINUE
 5    CONTINUE
      KC = KC3+1
C********BURNUP EQUATION FOR U-233.
      T(KC,1)= Y18 + Y19
      T(KC,2) = -1.
      T(KC,5)= (Y23 + Y24) * C(3) / C(1)
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```
      T(KC,7)= (Y20+Y21+Y22) * C(4) / C(1)
      KC = KC +1
C********BURNUP EQUATION FOR U-238
      T(KC,1)=((C706 +C707+ C708)* C(1)/ C(2) ) *SCAL1
      T(KC,3)= A28* SCAL1
      T(KC,4) = -1 * SCAL1
      T(KC,5)=((-C16*((A14 - A28)/ (SIG28- SIG02) - (A13 - A28) / (SIG28
     1 + SIG23 * (EL1- 1 )))) * C(3) / C(2)) * SCAL1
      T(KC,7)=((C702+ C703 + C704 + C705) * C(4)/C(2)) *SCAL1
      T(KC,17)=(C709* C(9)/ C(2) ) * SCAL1
      KC = KC +1
C********BURNUP EQUATION FOR TH-232
      T(KC,1)=((Y16+ Y17) * C(1) / C(4) ) *SCAL1
      T(KC,5)=(Y11+ Y12 ) *SCAL1
      T(KC,6) = -1 * SCAL1
      T(KC,7)=((Y13+ Y14+ Y15) * C(4)/ C(3) ) *SCAL1
      KC = KC +1
C********BURNUP EQUATION FOR U-235
      T(KC,1)=((C82 * (A13-A11) - C83*(A15-A11 )) * C(1) / C(4))*SCAL2
      T(KC,5)=(( C84 * (A14-A11) -C85*(A15-A11)-C86*(A13-A11) +
     1 C87 * (A15- A11)) * C(3) / C(4) ) * SCAL2
      T(KC,7)=(A11 +C91*(A11-A15)- C92 *(A13-A11)+ C93*(A15-A11)+ C94
     1 ) *SCAL2
      T(KC,8) = -1 * SCAL2
      T(KC,17)=((C81*(A15-A11))*C(9) / C(4)) * SCAL2
      KC = KC +1
C*********BURNUP EQUATION FOR U-236
      T(KC,1)=((D21 * (A11 -A32)+D22 * (A13-A32)-D23*(A15-A32))*C(1)/
     1 C(5) ) * SCAL2
      T(KC,7)=((D15*(A11-A32)+D17 * (A15-A32)-D18*(A13-A32))* C(4)/C(5)
     1  ) *SCAL2
      T(KC,9)= A32* SCAL2
      T(KC,10) = -1 * SCAL2
      T(KC,17)=((D19*(A15-A32) -D20*(A11-A32)) * C(9)/ C(5)) * SCAL2
      KC = KC +1
C*********BURNUP EQUATION FOR PU-239
      T(KC,1)=((A194-A195 - A196+ A197+A198) * C(1)/ C(6))*SCAL2
      T(KC,3)=(C18 * (1- A12  ) * C(2)/ C(6) ) * SCAL2
      T(KC,5)=((A20                -A25)*C(3)/C(6)) *SCAL2
      T(KC,7)=((A17 + A18-A19- A191+A192-A193) * C(4) / C(6))*SCAL2
      T(KC,11)= A12 * SCAL2
      T(KC,12) = - 1 * SCAL2
      T(KC,17)=(( A211- A212) * C(9) / C(6) ) *SCAL2
      KC = KC +1
C********BURNUP EQUATION FOR PU-240
      T(KC,1)=((C46 * (A13 - A26 ) + C461* (A11- A26) - C462* (A15- A26)
     1 + C463 * (A12- A26 )) * C(1) / C(7)) *SCAL2
      T(KC,3)=(( C44* (1- A26) - C441 * (A12- A26)) * C(2)/ C(6))*SCAL2
      T(KC,5)=((C467 *(A14- A26 ) - C468 * (A13- A26) + C469*(A12- A26))
     1 * C(2)/ C(7)) * SCAL2
      T(KC,7)=((C45* (A11- A26) - C451 * (A13- A26)+ C452 * (A15-A26)
     1 + C453 * (A12- A26)) * C(4)/ C(7) ) * SCAL2
      T(KC,11)=((C43 * (A12- A26)) * C(6) / C(7)) * SCAL2
      T(KC,13)= A26* SCAL2
      T(KC,14) = -1 * SCAL2
      T(KC,17)=((C464 * (A15 -A26 ) - C465 * (A11- A26)+ C466*(A12-A26))
     1 * C(9)/ C(7) ) * SCAL2
      KC = KC +1
C********BURNUP EQUATION FOR PU-241
      T(KC,1)=((C59 * (A11- A27) + C591 * (A11- A27) - C592 * (A15- A27)
```

```
     1 + C593 * (A12- A27) + C594 * (A26 - A27 )) *C(1) / C(8))*SCAL2
      T(KC,3)=((C57* (1- A27) - C571 * (A12- A27)+ C572* (A26-A27))*C(2)
     1  / C(8) ) * SCAL2
      T(KC,5)=((C61 * (A14- A27) - C611 * (A13 - A27 ) + C612 * (A12 -
     1 A27 ) + C613 * (A26 - A27)) * C(3) / C(8)) * SCAL2
      T(KC,7)=((C58 * (A11- A27) + C581 * (A15- A27) + C582 * (A12- A27)
     1 - C583 * (A13 - A27 ) + C584* (A26- A27)) * C(4) / C(8))*SCAL2
      T(KC,11)=((C56 * (A12- A27) - C561 * (A26- A27)) *C(6) / C(8)
     1  ) * SCAL2
      T(KC,13)=(C52 * ( A26- A27) * C(7) / C(8))*SCAL2
      T(KC,15)= A27* SCAL2
      T(KC,16) = - 1 * SCAL2
      T(KC,17)=(( C60 * (A14 - A27 ) - C601 * (A11- A27) + C602 * (A12-
     1 A27) + C603 * (A26- A27)) * C(9) / C(8)) * SCAL2
      KC = KC +1
C********BURNUP EQUATION FOR U-234
      T(KC,1)=((C89* (A13-A15))* C(1)/C(9)) * SCAL2
      T(KC,5)=((C88*(A14- A15)- C90*(A13-A15))* C(3)/C(9))*SCAL2
      T(KC,7)=((C901*(A11-A15)- C902*(A13-A15))* C(4)/C(9))*SCAL2
      T(KC,17)= A15* SCAL2
      T(KC,18) = -1. * SCAL2
      KC = KC +1
C********FISSION PRODUCTS PRODUCTION FOR U-235 FISSION.
      T(KC,1) =((E21 - E22 ) * C(1)/ C(10)) * SCAL2
      T(KC,7)=((E16+E17- E19    ) * C(4) / C(10)) *SCAL2
      T(KC,17) =(E20 * C(9) / C(10)) * SCAL2
      T(KC,19) = 1. * SCAL2
      T(KC,20) =-1.* SCAL2
      KC = KC +1
C********FISSION PRODUCTS PRODUCTION FOR U-233 FISSION.
      T(KC,1) =(E31 + E32) * SCAL2
      T(KC,5) =(( E36 + E37 ) * C(3) / C(10)) * SCAL2
      T(KC,7) =((E33 + E34 + E35) *C(4) / C(10)) * SCAL2
      T(KC,21) = 1. * SCAL2
      T(KC,22) = -1. * SCAL2
      KC = KC+1
C********B-10 CONTROL POISON EQUATION.
      T(KC,24) = -1. * SCAL1
      T(KC,23) = BS1 * SCAL1
C********BOL LOCALIZED REACTIVITY CONSTRAINT.
      KC = KC+1
      T(KC,1) = S13 * SCAL3
      T(KC,3) = -SIG28* 1E+24 * SCAL3
      T(KC,5) = -SIG02 * 1E+24 * SCAL3
      T(KC,7) = S14 * SCAL3
      T(KC,9) = -SIG26* 1E+24 * SCAL3
      T(KC,11) = S15     * SCAL3
      T(KC,13) = -SIG40* 1E+24 * SCAL3
      T(KC,15) = S16* SCAL3
      T(KC,17) = -SIG24 * 1E+24* SCAL3
      KC = KC+1
C********NUCLIDE CONSTRAINT EQUATION( VOLUME CONSTRAINT).
      T(KC,3) =(1./1159 ) * SCAL2
      T(KC,5) =(1. /4634 ) * SCAL2
      T(KC,7) =(1.    /1159 ) * SCAL2
      KC = KC+1
C********B-10 VOLUME CONSTRAINT.
      T(KC,23) = 1.818E-03  * SCAL2
      KC = KC+1
C********ENRICHMENT CONSTRAINT.
```

```
      T(KC,3) =-A14B     * SCAL4
      T(KC,7) = A12B     * SCAL4
C        BOL POWER PEAKING CONSTRAINT EQUATION.
      KC = KC+ 1
      T(KC,1) = S18
      T(KC,7) = S17
      T(KC,11) = S19
      T(KC,15) = S20
C        EOL POWER PEAKING CONSTRAINT EQUATION.
      KC = KC+1
      T(KC,2) = S18
      T(KC,8) = S17
      T(KC,12) = S19
      T(KC,16) = S20
      RETURN
      END

      SUBROUTINE PART(MAXRR,K,ICCT)
C*************************************************************************
C
C        THIS SUBPROGRAM GENERATES THE NECESSARY TOTAL CYCLE CONSTRAINTS.
C
C*************************************************************************
      COMMON/SC/ SCAL1,SCAL2,SCAL3,SCAL4
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      COMMON/BAD/ Z1,Z2,Z11,Z3,Z4,Z33
      COMMON/BBD/REC25,REC23,REC49,REC41,COFAC
      COMMON /CP/SIGXE,SIGSA,SIGB10,SIGCR,SIFP4,SIFP1,SIGC
      COMMON/PB/SIG28,SIG02,SIG26,SIG40,SIG24 ,SIG41,SIG23,SIG25,SIG49
     1,S17,S18,S19,S20,PAVG,PPEAK,VREG ,S13,S14,S15,S16
      COMMON /PA/YIEP23,YIEP25,YIEI23,YIEI25,YIEX23,YIEX25,
     1 SIF23,SIF25,SIF49, SIF41,KCH,
     2  YIEX49,YIEI49,YIEP49,
     2 ENW23,ENW25,ENW49,ENW41,PKKK ,ALMP,ALMI,ALMX
      MAX1 = MAXR+ MAXRR +KC3
      MAX = MAXR + KC3 +1
      DO 5 L = MAX,MAX1
      DO 6 I=1,MAXC
      T(L,I) = 0.0
 6    CONTINUE
 5    CONTINUE
      SS1 =       250
      SS2 =       6000
      SS5= SIFP4
      SS6 = SIFP1
      S11 = SIGB10
      S12 = SIGCR
C********EQUATIONS FOR CARBON  CONCENTRATION.
      T(KCH,5) = SS1 * SCAL4
C*********BOL EQUATIONS FOR ZENON   CONCENTRATION.
      KCH = KCH + 1
      T(KCH,2) = Z3 * SIF23 * SCAL2
      T(KCH,8) = Z4* SIF25  * SCAL2
      T(KCH,12) = Z33* SIF49 * SCAL2
      KCH = KCH + 1
C*********BOL EQUATIONS SAMARIUM CONCENTRATION.
      T(KCH,2) = Z1 * SIF23  * SCAL2
      T(KCH,8) = Z2 * SIF25  * SCAL2
      T(KCH,12) = Z11 * SIF49* SCAL2
```

```
      KCH = KCH + 1
C********EOL EQUATIONS FOR ZENON CONCENTRATION.
      T(KCH,1) = Z3 * SIF23 * SCAL2
      T(KCH,7) = Z4* SIF25* SCAL2
      T(KCH,11) = Z33* SIF49  * SCAL2
      KCH = KCH + 1
C********EOL EQUATIONS FOR SAMARIUM CONCENTRATION.
      T(KCH,1) = Z1 * SIF23 * SCAL2
      T(KCH,7) = Z2 * SIF25* SCAL2
      T(KCH,11) = Z11 * SIF49 * SCAL2
      KCH = KCH + 1
C********BOL EQUATIONS FOR FISSION PRODUCTS CONCENTRATION.
      T(KCH,19) = SS5  * SCAL4
      T(KCH,21) = SS6  * SCAL4
      KCH = KCH + 1
C********EOL EQUATIONS FOR FISSION PRODUCTS CONCENTRATION.
      T(KCH,20) = SS5  * SCAL4
      T(KCH,22) = SS6   * SCAL4
      KCH = KCH + 1
C********CONTROL ROD POISON CONSTRAINT.
      T(KCH,23) = 1.
      KCH = KCH+1
C********EQUATIONS USED TO ZERO OUT ANY DESIRED CONCENTRATIONS.
      IF(IMUST1.GT.1.AND.ICOUT1.NE.ICCT) GO TO 20
      T(KCH,9) = 1.
      T(KCH,11) = 1.
      T(KCH,13) = 1.
      T(KCH,15) = 1.
      T(KCH,17) = 1.
      T(KCH,19) = 1.
      T(KCH,21) = 1.
      IF(IMUST1.GE.4) GO TO 20
      T(KCH,1) = 1.
 20   KCH = KCH+1
C        CONTROL ROD POISON.
      T(KCH,25) = 1.
      KCH = KCH+1
C        BOL REACTIVITY CONSTRAINT.
      T(KCH,1) = S13
      T(KCH,3) = -SIG28*1E+24
      T(KCH,5) = -SIG02*1E+24
      T(KCH,7) = S14
      T(KCH,9) = -SIG26* 1E+24
      T(KCH,11) = S15
      T(KCH,13) = -SIG40* 1E+24
      T(KCH,15) = S16
      T(KCH,17) = -SIG24* 1E+24
      T(KCH,23) = -SIGB10
      T(KCH,25) = -SIGCR
C        EOL REACTIVITY CONSTRAINT.
      KCH = KCH + 1
      T(KCH,2) = S13
      T(KCH,4) = -SIG28* 1E+24
      T(KCH,6) = -SIG02* 1E+24
      T(KCH,8) = S14
      T(KCH,10) =-SIG26* 1E+24
      T(KCH,12) = S15
      T(KCH,14) = - SIG40* 1E+24
      T(KCH,16) = S16
      T(KCH,18) = -SIG24* 1E+24
```

```
      T(KCH,24) =-SIGB10
C        BOL TOTAL POWER CONSTRAINT.
      KCH = KCH + 1
      T(KCH,1) = S18*VREG*1E-06 * SCAL3
      T(KCH,7) = S17* VREG*1E-06  * SCAL3
      T(KCH,11) = S19* VREG* 1E-06* SCAL3
      T(KCH,15) = S20 *VREG*1E-06 * SCAL3
C        EOL TOTAL POWER CONSTRAINT.
      KCH = KCH+1
      T(KCH,2)= S18* VREG*1E-06 * SCAL3
      T(KCH,8) = S17* VREG *1E-06  * SCAL3
      T(KCH,12) = S19*VREG*1E-06  * SCAL3
      T(KCH,16) = S20 * VREG*1E-06 * SCAL3
      RETURN
      END

      SUBROUTINE CONST
C*************************************************************************
C
C        THIS SUBPROGRAM IS USED TO PRSENT THE ACCUMULATIVE
C        PRODUCTS OF THE REGIONS.
C
C*************************************************************************
      COMMON/SC/ SCAL1,SCAL2,SCAL3,SCAL4
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      COMMON /CP/SIGXE,SIGSA,SIGB10,SIGCR,SIFP4,SIFP1,SIGC
      DO 5 L=1,MAXR
      DO 6 I=1,MAXC
      T(L,I) = 0.0
   6  CONTINUE
  5   CONTINUE
      S12 = SIGCR
      SS3 = SIGSA
      SS4 = SIGXE
      S11 = SIGB10
C********TOTAL CYCLE ACCUMULATION OF CARBON.
      T(1,1) = -1.
C********TOTAL CYCLE ACCUMULATION OF EOL XENON.
      T(2,5) = -1.* SCAL2
C********TOTAL CYCLE ACCUMULATION OF EOL SAMARIUM.
      T(3,3) = -1. * SCAL2
      KB = 4
C********TOTAL CYCLE ACCUMULATION OF BOL XENON.
      T(KB,4) = -1. * SCAL2
      KB =KB +1
C********TOTAL CYCLE ACCUMULATION OF BOL SAMARIUM.
      T(KB,2) = -1. * SCAL2
      KB =KB +1
      T(KB,2) = SS3  * SCAL4
      T(KB,4) = SS4 * SCAL4
C********TOTAL CYCLE BOL MACROSCOPIC POISON CROSS SECTION(1E-03).
      T(KB,6) = -1. * SCAL4
      KB =KB +1
      T(KB,3) = SS3  * SCAL4
      T(KB,5) = SS4  * SCAL4
C********TOTAL CYCLE EOL MACROSCOPIC POISON CROSS SECTION(1E-03).
      T(KB,7) = -1.   * SCAL4
      KB =KB +1
C********TOTAL CYCLE BOL ACCUMULATION OF CONTROL ROD POISONING.
```

```
      T(KB,8) = -1
      KB =KB +1
C********ZERO OUT EOL CONTROL ROD POISONING.
      T(KB,10) = 1
      KB = KB +1
C********TOTAL CYCLE BOL ACCUMULATION OF B-10 CONTROL POISON.
      T(KB,9) = -1
      KB = KB+1
      T(KB,1) = -SIGC  * SCAL1
      T(KB,6) = -1.
      KB = KB + 1
      T(KB,1) = -SIGC  * SCAL1
      T(KB,7) = -1.
      RETURN
      END

      SUBROUTINE STORED
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      MAX2 = MAX1 + MROS
      MAX = MAX1 + 1
      DO 5 I =MAX,MAX2
      DO 6 K= 1,MAXC
      T(I,K) = 0.
 6    CONTINUE
 5    CONTINUE
      T(KC1,2) =1.
      KC1 = KC1+ 1
      T(KC1,4) = 1.
      KC1 = KC1+ 1
      T(KC1,6) =1.
      KC1 = KC1+ 1
      T(KC1,8) =1.
      KC1 = KC1+ 1
      T(KC1,10) =1.
      RETURN
      END

      SUBROUTINE TRANST
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      MAX3 = MAX1 + MROT
      MAX= MAX1 + 1
      DO 5 I=MAX,MAX3
      DO 6 K= 1,MAXC
      T(I,K) = 0.
 6    CONTINUE
 5    CONTINUE
      T(KC2,2) =1.
      KC2 = KC2 + 1
      T(KC2,4) =1.
      KC2 = KC2 + 1
      T(KC2,6) =1.
      KC2 = KC2 + 1
      T(KC2,8) =1.
      KC2 = KC2 + 1
      T(KC2,10) =1.
      KC2 = KC2 + 1
      T(KC2,12) =1.
      KC2 = KC2 + 1
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```
      T(KC2,14) =1.
      KC2 = KC2 + 1
      T(KC2,16) =1.
      KC2 = KC2 + 1
      T(KC2,18) =1.
      KC2 = KC2 + 1
      T(KC2,20) =1.
      KC2 = KC2 + 1
      T(KC2,22) =1.
      RETURN
      END

      SUBROUTINE TRANSF
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      DO 5 I=1,MAXR
      DO 6 K=1,MAXC
      T(I,K) = 0.
6     CONTINUE
5     CONTINUE
      T(1,1) =-1.
      T(2,3) =-1.
      T(3,5) =-1.
      T(4,7) =-1.
      T(5,9) =-1.
      T(6,11) =-1.
      T(7,13) =-1.
      T(8,15) =-1.
      T(9,17) =-1.
      T(10,19) =-1.
      T(11,21) =-1.
      KC3 = 11
      RETURN
      END

      SUBROUTINE FULIP
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      DO 5 I=1,4
      DO 6 K=1,7
      T(I,K) = 0.
6     CONTINUE
5     CONTINUE
      T(1,1) = -1.
      T(2,3) = -1.
      T(3,5) = -1.
      T(4,7) = -1.
      KC3 = 4
      RETURN
      END

      SUBROUTINE FULI
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      DO 5 I=1,MAXR
      DO 6 K=1,MAXC
      T(I,K) = 0.
6     CONTINUE
5     CONTINUE
      T(1,1) = 1.
```

```
      T(2,2) = 1.
      T(3,3) = 1.
      T(4,4) = 1.
      RETURN
      END

      SUBROUTINE STORI
      COMMON T(70,50),FLUX(4) ,T1,MAXR ,
     1 MAXC,KC1,KC2,MAX1,KC3,MAX4 , MROS,MROT,IMUST1,ICOUT1
      DEX = -1.
      IF(MAX4.GT.2) DEX = 1.
      DO 5 I=MAX4,MAXR
      DO 6 K=1,MAXC
      T(I,K) = 0.
6     CONTINUE
5     CONTINUE
      KD = MAX4
      T(KD,1) =1.*DEX
      KD = KD+ 1
      T(KD,2) =1. *DEX
      KD = KD+ 1
      T(KD,3) =1. *DEX
      KD = KD+ 1
      T(KD,4) =1. *DEX
      KD = KD +1
      T(KD,5) = 1.* DEX
      MAX4 = MAXR +1
      MAXR = MAXR + MAX4
      RETURN
      END
```