

/COMMON ERRORS IN ALGEBRA I/
A FOREWARNING TO BEGINNING TEACHERS

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CHAPTER I

INTRODUCTION

"The only dumb question that you can ask, is the one that
you don't ask." ---Orlando V. Clark, 1985

Oy! Do students think that they are asking "dumb" questions. They even start out by saying "This is probably a dumb question, but . . ." And you may even agree with them and try to hide the grimace or fit of laughter. It is their "release," from their peers, for a request of knowledge.

Every day, experienced teachers contemplate ways to make better students of the children and/or young adults who have found their way into the classroom. You will make errors in your plan, and they will make errors in their work and then find "dumb" mistakes and want to ask "dumb" questions.

The goal of this report is to forewarn the beginning teacher of common error patterns that will occur in an Algebra I class, give insight to why these errors occur, according to research, and to give suggestions of how to decrease the frequency of these errors.

CHAPTER II

SEARCH OF LITERATURE

Whoever said "To err is human" was probably a mathematics student. Algebra starts with nonalgebraic formations of problems; that is, the equations are given. It is then possible to develop skills that allow the student to solve the equation (Wollman, 1983).

But equations involve variables! This makes algebra different from elementary mathematics. These variables cause some students more trouble than others. Hence, errors develop with beginning algebra students working with literal symbols, or variables.

WHY ERRORS OCCUR

Confusion can begin with the linear order of the alphabet, as well as whole numbers. There are many similarities and differences. The letters e and π actually represent certain values. In open sentences letters and numbers appear together, and by solving the open sentence, the letter used is actually serving as a temporary numeral. On the other hand, numerals represent a single number but letters can represent many different numerals (Wagner, 1983).

The reason we use letters, rather than numerals, to represent generalized elements is because letters look like

abbreviated names and thus are easy for young students to interpret naively, and as teachers we know that letters will be used to represent numerical variables later on (Wagner, 1983).

While research into common errors in Algebra I is minimal, at best, there are numerous reports as to why errors occur and differing ways to classify them. When classifying errors according to pupils' individual difficulties, one should acknowledge that errors are also a result of other conditions in the educational process, such as the teacher, the curriculum, the environment, and the interactions of these conditions (Radatz, 1979). Allowing for individual differences among the students, the teacher and the curriculum in your classroom should be constant. Your instruction then becomes the most important phase of the process for the learning of algebra (Bright, 1981).

But what about errors? Bright (1981) classifies errors into three types, operator, applicability, and execution.

Operator errors are the ones that students most often refer to as "dumb errors" or "dumb mistakes." They reflect incorrect knowledge, such as,

$$7 \times 8 = 54.$$

Applicability errors involve the misuse of the rules of algebra, such as,

$$3(x + 1) = 3x \quad \text{or} \quad (xy^2)^5 = x^2y^7.$$

Execution errors include partial executions, such as,

$$3(x + 1) = 3x + 1$$

and the copying of a problem incorrectly.

But from where do these errors come? Students of college algebra possess many of the same error patterns as do the students of high school algebra. These error patterns appear as incorrect generalizations and are transferred into situations where the task is perceived as being difficult or complicated. "Mathematics educators have long stated the importance of developing and 'understanding' mathematics." (Rachlin, 1981)

While reviewing the research on the origins of conceptual difficulties (the operator errors students experienced in mathematics), special attention was paid first to the definition of the mathematical concepts or terms. Stress was placed on the need to simplify the meaning of the definitions of important concepts. Different methods of a concept should be presented but too many can lead to confusion. The need for students to know when to apply a concept in familiar and unfamiliar situations is of great concern. When they can do this, they should retain the concept.

Literature on the origins of conceptual difficulties is divided into two sections, external factors and internal factors. The main emphasis was on the external factors. The external factors are the curriculum, the language of

instruction, and the teaching methods. Concern was expressed that curriculum for primary schools prescribes topics that are too difficult for many school children, given their level of maturity. It was argued that the curriculum may not apply to the environment, especially in developing countries, and this policy leads to conceptual difficulties (Clements, K., 1984). Teachers must be alert to the types of questions to ask so that the problem is posed in the proper social perception of mathematics as a tool to help make intelligent decisions in their daily affairs (McGinty, 1980).

Even when their textbook is written in their native language, the learning of mathematical concepts, symbols, and vocabulary is a "foreign language" problem for many students (Radatz, 1979). Too often a nation's political policy on the language of instruction results in the unfortunate avoidance of many important mathematical expressions and vital terminology. This also hinders the students acquisition of mathematical concepts (Clements, K., 1984).

A language problem comes about when combining terms, applying definitions, understanding concepts, and then there is the similiarity of notations that carry several meanings. When combined with algebra and the idea of a "variable," concepts of symbolism generate language problems (Bright, 1981).

Classroom experiments with different teaching methods have produced disappointing results. There is no evidence that children of today absorb mathematical concepts more readily than did children in the past. In particular, the early promises of programmed learning and individualized mastery-learning scheme, have not materialized. It seems that human interaction is the indispensable key to the successful teaching and learning of mathematical concepts (Clements, K., 1984, p. 125).

Other studies of differing teaching methods have concurred with Clements' comments, that there is no one method better than the other methods.

Radatz (1979) classifies errors of five different types:

1. Errors of perseveration, in which single elements of a task or problem predominate. For example:

$$9 \times 60 = 560$$

$$5 \times 13 = 63$$

$$41 + 7 = 47$$

2. Errors of association, involving incorrect interactions between single elements. For example:

$$66 + 12 = 77$$

$$56 + 15 = 67$$

$$3 \times 9 = 36$$

3. Errors of interference, in which different operations or concepts interfere with each other. For example:

$$3 \times 9 = 12$$

$$18 \div 6 = 12$$

4. Errors of assimilation, in which incorrect hearing causes mistakes in reading and writing, and so on. Such errors are often classified as errors resulting from lack of attention and concentration. For example:

When a student is being asked six more than a number, but replies six times a number.

5. Errors of negative transfer from previous tasks, in which one can identify the effect of an erroneous impression obtained from a set of exercises or word problems. For example:

$$\text{Since } x^3 x^4 = x^7, \text{ then } (x^3)^4 = x^7.$$

The classification of errors by Bright and Radatz are quite similar. There are other studies that classify errors in similar methods, ranging from three to eight groupings. A sharp separation of the possible causes of a given error is often difficult because there is such a close interaction among causes.

Literature regarding math anxieties states that students with high mathematical anxiety tend to score lower on mathematics achievement tests than students with low mathematical anxiety. Instead of trusting their own methods of mastering the material, the highly anxious student needs to rely heavily on a well-structured, controlled plan for learning.

Conversely, most students with a low anxiety have mathematics confidence. Students high in confidence tend to

have more interactions with their teachers concerning mathematics than students low in confidence. Unfortunately, some students seemed to be apathetic about learning, whether they were low or high in anxiety (Clute, 1984).

A study by Marshall (1983) on errors caused by sex differences, concluded that boys and girls both make errors, but not necessarily the same ones. Marshall states:

Sex differences are present in childrens' selection of multiple-choice responses, and types of errors committed can be identified. It has been demonstrated that the distributions of boys' and girls' errors differ. One important feature of the statistical analysis is that several item types were evaluated. The finding of sex-by-distractor interaction applied equally to (a) arithmetic word problems and computations with whole numbers, fractions, and decimals and to (b) items on measurements, probability, geometry, and graphs.

Girls' errors generally are more likely than boys' to be due to the misuse of spatial information, the use of irrelevant rules, or the choice of incorrect operations. Girls also make relatively more errors of negative transfer and key word association. Boys seem more likely than girls to make errors of perseverance and formula inference. Both sexes make language related errors, but both errors are not the same (pp. 334-335).

One drawback to this study and most others that deal with errors, is that the results come only from those that could not solve the given problems.

Kraus (1982) states that age, gender, and grade had no significant difference in the students' ability to solve types of word problems. What was significant was difficulty related to the cognitive level that the problem required.

Ewing (1982) states, from a number of studies on thought processes, there are three general types of information that have come to light. First, the mind seems to have a system which stores and retrieves information. Second, there are factors that interfere or assist in mental activity. These factors are motivation and interest, experience, variations in perception, identifying concepts, previous information, and the ability to break a thought into steps. The third is the cognitive processes themselves, which most thinking involves. Where the errors occur is in how the student views the idea or objective. Can he do it from different perspectives (pp. 8-10)?

The curriculum of mathematics, from elementary school to graduate school, follows a path of increasing abstraction. As the curriculum becomes more abstract, the symbols used become more obscure. For many students, unfamiliarity with mathematical symbols and the abstract concepts to which they refer breeds contempt for mathematics.

Students need to develop a better understanding of the basic concepts of variables and equations. More specifically, they should be able to distinguish between different ways in which letters can be used in equations.

As students progress from year to year in mathematics, the concepts they are learning become increasingly abstract and ambiguous. It is important that we, as teachers, stay

aware of the difficulties that our students are having trying to understand variables, constants, and the conceptual pitfalls to which our students can succumb (Rosnick, 1981).

A study by Quintero (1983) provides evidence that the meaning of concepts and relationships in a two-step word problem is a major source of difficulty in solving the problem and that there are different levels of understanding this meaning. The finding that the concept of ratio, which is not common to all types of word problems, was the key source of difficulty in solving the problems shows the importance of having precise categories to characterize the word problems children are asked to solve. The evidence also suggests that the method of solving two-step problems can also be a source of difficulty (p. 111).

ERROR PATTERNS

Calvert (1983) states that most research on error analysis deals with arithmetic computations. One relevant study on error analysis in algebra conducted by Bright and Harvey, focused on fractional and percent transformations and equivalent linear systems. Regarding fractions and percents, they concluded that most student errors are due to their not being able to distinguish between the three forms of rational numbers (fractions, decimals, and percent). Secondly, errors occur in algebra when students do not realize when equations are equivalent and therefore, many misuse the addition and/or multiplication properties of equality, make coefficient errors, or leave incomplete solutions after some correct steps (p. 9).

One incident of an error pattern, involving fractions at the elementary level, involved one very shy and quiet little girl that made the same error over and over. She did produce some correct answers. When her procedure was discussed with the class, a pattern developed that the class expanded into other relationships of fractions that they were subtracting. They came up with the discovery that

$$\frac{x}{x+y} - \frac{x-y}{x} = \frac{y^2}{x+y}$$

for all natural numbers x and y .

By naming the discovery after the student who had made the error, it gave her more confidence in doing her work

and the project was enjoyable to the class (Haddad, 1980).

Calvert (1983) quotes DeVincenzo as she asserts that "the tendency for students to make the same error in arithmetic and algebra for addition and subtraction of fraction problems implies that remediation in arithmetic will probably also produce 'correct' solutions for algebra addition and subtraction of fraction problems" (p. 11).

Most research involving error patterns in algebra are centered around word problems and translating them into symbols. One such study (Travis, 1981) used the following word problem.

"Three more than twice a certain number is 57. Find the number."

The error patterns found in this study were:

$x + (3 + 2x) = 57$	$x^3(x2) = 57$
$3x + 2 = 57$	$3x + 2x = 57$
$x + 3 = 57$	$2(x + 3) = 57$
$x^2 + 3 = 57$	$3 + 2x^2 = 57$

The errors seem to be in the meaning of twice a number and the use of exponents. There are students that will read x as "x-two". Is this because several notations carry similar meanings or the students' lack of concern?

Other studies involved incorrect cancellation involving fractional polynomials. Such as,

$$\frac{3x + 3}{3} = 3x + 1 \quad \text{or} \quad x + 3.$$

Short cut errors, especially division of fractions where cancellation is used, is a producer of errors (Laursen, 1978).

Another type of error involving word problems is the reversal error. A reversal error is when the student takes the statement "there are five times as many nickels as quarters" and writes the equation " $5N = Q$."

A high percentage of reversal errors are observed in translations from pictures to equations, data tables to equations, and equations to sentences. This shows that the reversal error is not a result of ordering the words in a problem in a particular way. When teaching methods were changed, the reversal errors continued to exist, which suggests that the student's own intuitive method will take over in later problem-solving situations (Clements, J., 1982).

CORRECTING ERRORS

"A problem is a 'great' problem if it is very difficult, it is just a 'little' problem if it is just a little difficult. Yet some degree of difficulty belongs to the very notion of a problem: where there is no difficulty, there is no problem."

---George Polya, 1981, p.117

Studies on correcting errors are more numerous than those on error patterns. Though when one looks for those just involving algebra, the numbers tumble.

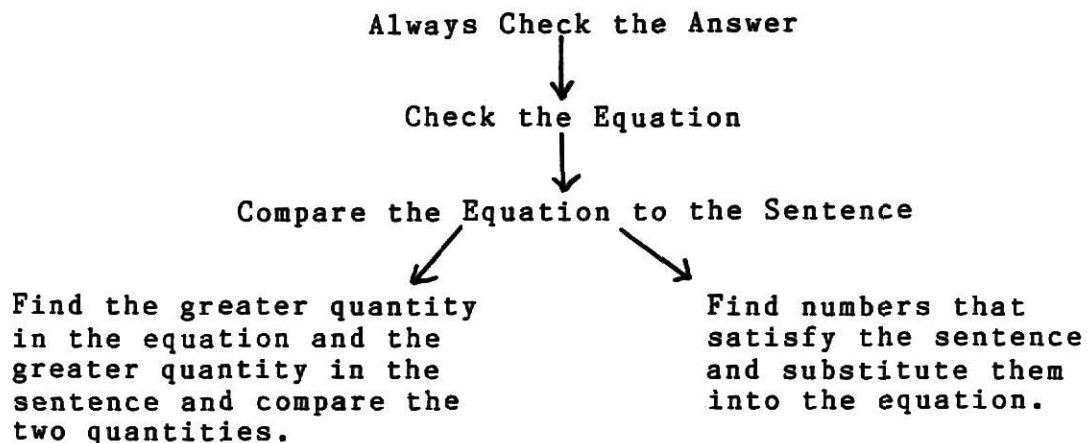
Most textbooks and articles on teaching methods throughout this century have lists for effective mathematics teaching, like the following six criteria.

1. The instruction needs to be developmental, so that new learning is related to similar knowledge.
2. The instruction needs to be well sequenced.
3. The instruction needs to be focused on what students must be able to do as a result of learning.
4. The instruction needs to promote mental activity by the students.
5. The instruction needs to be cumulative.
6. The instruction needs to be comprehensive. (Clements, K., 1984)

If we are going to curb errors it is important to ascertain if the children know which particular operation or sequence of operations should be used to resolve a given situation. One study produced a survey of frequent comments that students find the study of mathematics boring, and that teachers find the students lack of motivation to learn mathematics one of the most difficult problems they face (Clements, K., 1984)

"If you find a particular problem is troublesome, get some help at once, because the problem won't go away. It will appear again and again in future problems," warns John Saxon (1981). Algebra is a skill and is best learned through repeated practice over an extended period of time.

A starting point is the ability of the student to compute with the given equation. Difficulty most often occurs in going from a sentence to an equation. The "meaning" of the sentence is exemplified by these computations, turning the words into an equation. This comes about by symbolizing. But after solving, the students must ask themselves questions regarding the answer they have found (Wollman, 1983). The following chart is one way to help determine if they have symbolized correctly.



When followed, this chart is invaluable to the student as it aids him or her in determining whether or not he or she has in fact gone from sentence to equation correctly.

Another study (Smith, 1977) indicates that there are three variables that help students' achievement if they are correlated positively. The variables pertain to the lesson's objectives which are carefully planned and skillfully executed, the percentage of relevant examples per lesson,

and the average number of "OKs" per minute of lecture or discussion. "OK" seems to be a positive reassurance that the student is doing his or her work correctly.

Another study (Weber, 1978) showed results that simple tasks are most efficiently learned in an authoritarian or highly directed learning environment, while non-teacher-directed learning environments provide higher levels of cognitive learning. It concluded that when the teacher does structure learning, through lecture and feedback, high levels of both learner involvement and achievement are produced.

Some instruction on word problems should give attention to helping students build a plan or outline for the general structure of word problems and the specific structures found within problem categories. Literature in mathematics problem-solving suggests that learners store information in memory which helps them solve stereotyped algebra word problems (Herring, 1981). Along with the study (Berger, 1984) that suggests that detailed side-by-side comparisons of the structure of problems from the same category is a useful approach. Side-by-side comparison is the showing of two problems of the same type, so that students can see that the structure of both problems is similar. Diagrams and pictures play an important role in helping students to organize information about a problem and to generate a structural representation of the problem.

The clearest lesson drawn from this study is that an appreciation of problem structure is a crucial part of expertise in problem-solving.

Finally, it has been found that students do learn more use more of what was learned when teachers gave written comments on tests. Instead of making answers only right or wrong, make comments indicating correct procedures or good starts on problems even if the answer is wrong. This practice does encourage students toward future attempts. Emphasize having students analyze their errors rather than rushing to erase them. This approach shows students how to learn from their mistakes (Morris, 1981).

The literatures just reviewed are teaching suggestions that have had positive results. What follows are suggestions of ideas that have worked for individual teachers in their own situation.

Sometimes student errors will appear after being introduced to a new term or rule of algebra. The idea of cancellation is one such term. Alice Artzt (1986) calls it "cancel fever." Students will reduce rational expressions as follows:

$$\frac{x^2 + 6}{x + 3} = \frac{\overset{x}{\cancel{x^2}} + \overset{2}{\cancel{6}}}{\cancel{x} + \cancel{3}} = x + 2$$

$$\frac{2y + x}{y + 2x} = \frac{\cancel{2y} + \cancel{x}}{\cancel{y} + \cancel{2x}} = 1$$

Kay Laursen (1978) states, "Students use the term cancellation to justify the eliminating of terms and factors in additions, subtractions, multiplications, and divisions. They are tempted to apply the so-called universal law of cancellation: 'When any two things look alike, cross them out' in all variations of computations and manipulations. Algebra students tend to use the word cancellation to justify any procedure that they don't understand."

Some possible remedies: be sure to revisit theorems and definitions, and define terms carefully. Alice Artzt (1986) went so far as to offer her students a party if their performance on the exam indicated that the class was free of "cancel fever." She said that it was worth every cent that it cost her to buy candy for the entire class.

She used side-by-side comparisons of the correct way to cancel or reduce problems involving rational expressions and the incorrect method. She even made her students do the problems the incorrect way, so that they could see the comparison. She also used this method while working with the distributive property. By doing the problems this way, the incorrect method, the error patterns were slowed.

In the literature there have been suggestions made by teachers and others for curing errors. A study by Begel (1973) indicates that although student's attitudes in early grades are positive and increasing, a peak is attained during the junior high school years, followed by a steady

decline continuing through high school and college. Begel has had some success in slowing this decline by introducing mathematical historical passages into the students' studies of algebra. One historical passage could be the following:

"Symbolic algebra made its first appearance in western Europe in the sixteenth century, but did not become prevalent until the middle of the seventeenth century. It is not often realized that much of the symbolism of our elementary algebra textbooks is only about three hundred years old (Eves, 1964, p. 158)."

Literature (Kieran, 1982) provides some evidence that the spontaneous generation of erroneous schemes will creep back into the work of students unless they have continuous repetition of the correct procedure. The emphasis placed on the importance of the equal sign reflects the view that an equivalence notion of equality is a key factor underlying the initial learning of algebra and the solving of equations. But do students really understand the meaning and importance of the equal sign or do they just have it there to separate expressions? Perhaps that is why students are confused when we ask questions, such as:

1. When is a sum equal to a product?
2. When is a difference equal to a sum?
3. When is a sum equal to a quotient?
4. When is a wrong right?

Henry (1973) suggested the preceding questions as a

means to get students involved in rather simple mathematical questions. He hopes that they will respond with more questions. Henry notes that students who will answer questions such as these will receive added practice in developing algebraic manipulative skills, in learning to attack a problem, and in being alert for new questions that arise as each question is answered.

Let students think that they are making a new discovery. Such as the square of 25, which is 625. If you break down the parts, notice that 5 times 5 is 25, OK, the last two digits of the square, but what about the 6? It is 3 times 2. Three is one more than two. Try others, such as, 65×65 or 35×45 or 84×86 . There are two keys.

1. The tens digit in both numbers must be the same.
2. The units must add up to ten.

Let the students discover this (Arpaia, 1974).

Another illustration (Nicolai, 1974) comes from solving linear equations, such as,

$$2x + 3y = 4 \quad \text{and} \quad 5x + 6y = 7$$

The solution is the ordered pair $(-1, 2)$. It will always be, as long as the coefficients and constants are consecutive integers. Try some others.

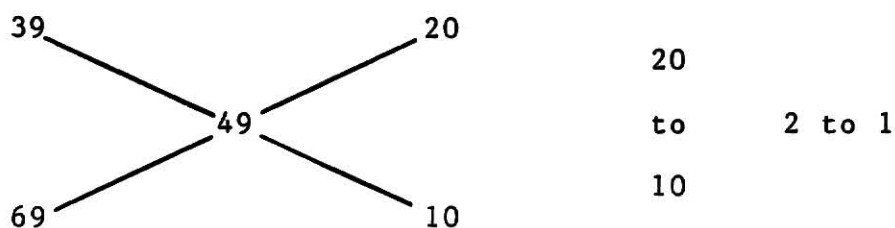
$$4x + 3y = 2 \quad \text{and} \quad x + 0y = -1$$

By letting the students think that it's a new discovery, it will be met with great enthusiasm.

Allen Weiner (1980) suggests that you let students use non-algebraic methods. He questions, "Wouldn't it be more challenging and thought provoking to try to solve the problem without using algebra?" Margaret Maxfield (1974) agrees, thinking back to the very ancient and more recently to our great-great-grandparents. They used to solve mixture problems by a simple method they called alligation, which was removed from school textbooks about 1900.

An example: If peanuts cost 39 cents a pound and filberts cost 69 cents a pound, in what proportion should they be mixed, if the mixture is to cost 49 cents a pound?

To solve, write the prices, 39 and 69 at the left corners of a square, and write the given mixture price, 49, in the center of the square. Now calculate the absolute difference along each diagonal, $69 - 49 = 20$ and $49 - 39 = 10$ and enter them at the right end of the diagonals.



These two numbers give us the ratio of 2-parts peanuts to 1-part filberts. Be sure to check your answer.

Weiner also gives a mixture problem. Candy worth 70 cents a pound is to be mixed with candy worth 40 cents a pound to obtain a mixture worth 50 cents a pound. How much of each kind should be used to have a mixture of 42 pounds?

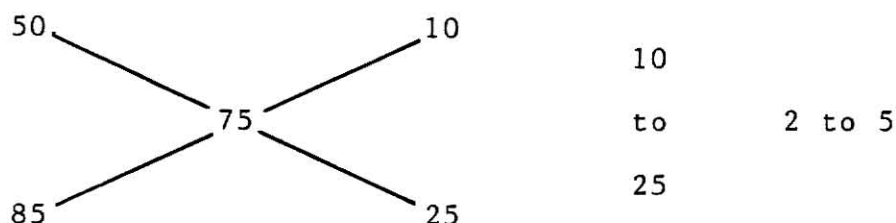
Solution: Suppose that the 42 pounds contained only the expensive 70 cent candy. The value would be $42 \times \$.70 = \29.40 versus the mixture priced 50 cent candy of $42 \times \$.50 = \21.00 . A difference of \$8.40 or 30 cents per pound ($8.40 \div 42$). Hence, $8.40 \div .30 = 28$, or 28 pounds of the 40 cent candy and 14 pounds at 70 cents.

Two more problems with solutions.

Alan can do a job in 6 days. Alan and Bob together can do the job in 2 days. How long will it take Bob alone to do the job?

Solution: Suppose Alan and Bob work 6 days. They will do three jobs in that time. Alan will have done one job, where as Bob will have done two jobs. Hence, Bob alone can do the job in 3 days.

I made 50s on some tests and 85s on the others. If my average grade was 75, what was the ratio of 50s to 85s?



The formal algebraic proof is a good one for guided discovery because it is straightforward. But sometimes an arithmetic approach will require more thought, originality, and even more insight into the problem than an algebraic one. Weiner contends that it is characteristically sound to let students try arithmetic solutions as well as algebraic ones.

Some literature on error patterns involving the use of opposites or negative signs deals with different methods of reading "- A". Van Engen (1972) said that the first "minus" must refer to an operation and the second must refer to the inverse of A.

When teaching algebra students the rules of multiplication of negatives, Sarver (1986) uses a monetary example to illustrate that a negative times a negative is a positive. He gives monetary values to students, as integers. Some students are in the black, as +2, +4, or +1, while others are in the red, as -3, -4, or -2. By taking the sum of the class, they now have a total value. He then has three students leave the room. They become negative people, since they are no longer in the classsroom. He tries to get these three to all have the same value and to be in the red, such as, -4. The class sum would then increase by 12, the positive product of these three negative people. $-3 \times -4 = +12$. This is his approach to $-a \times -b = ab$.

When schools fail to motivate students or when they are perceived not to relate to the real world, then it becomes necessary to examine other teaching alternatives. These last few ideas of letting students make discoveries, non-algebraic methods of solving problems, alligations, and giving integers monetary values are examples of other teaching alternatives. Ewing (1976) suggests games as another alternative. Games can be used successfully in achieving educational objectives as a supplement to other teaching methods when properly played in the classroom. The teacher must be sure that they are designed correctly.

Games have been a way of reinforcing mathematical skills. Tom Giambrone (1980) uses a game that he calls "I have . . ., Who has . . .?" He has cards that have two different algebraic expressions on each card. The students must communicate the first of these as "I have . . ." and then "Who has . . ." the second expression. Whoever holds the "Who has . . ." expression, then would read "I have . . ., Who has . . .?" which is the second expression on his or her card. The students earn points by correct responses. Examples of the cards follow:

$$n + 2$$

$$4n$$

$$4n$$

$$n - 7$$

$$\begin{array}{c} n - 7 \\ 2n + 3 \end{array}$$

$$\begin{array}{c} 2n + 3 \\ n + 2 \end{array}$$

To make the game more difficult, as the students level of achievement increases, the student could read "Who has four more than twice a number divided by two" in place of "Who has two more than a number."

Another game, presented by George C. Lippold (1982) entitled "Pass the answer" is used to enhance evaluation of equations. By having the class form teams of four or five students, the game is then played somewhat like a relay. Each team member is given a different equation where the answer to the first equation must be substituted into the second equation and so on.

For example, Team 1 has the following equations:

$$\begin{array}{lll} 3a + 5 = b & 2b - 12 = c & (c + 4)/2 = d \\ d - 13 = e & e/3 + 11 = f & \end{array}$$

To start, the teacher must give each first player a value for a, such as, $a = 1$, 7 , or -3 . When the first player has his or her answer, he or she passes it on to the next player. The first team finished, "correctly" of course, is the winner.

Games help to break the daily routine and provide students a way to reinforce needed skills, without the pressure of just another daily assignment.

CHAPTER III

ERROR PATTERNS OF 1985-1986

In a search for error patterns in algebra, data, mostly from chapter test, were collected from two classes of Algebra I students. Forty-three students started the year. These students consisted of two seniors, nine juniors, and thirty-two sophomores. With transfers and withdrawals, the school year was finished with thirty of these students still enrolled in these two classes of Algebra I. One was a senior, seven juniors, and twenty-two sophomores, of which only five were male students.

The textbook used for the course was Algebra Structure and Method Book 1 published by the Houghton Mifflin Company, copyrighted in 1981, along with supplemental materials associated with this book.

Error patterns developed from the very beginning chapter and continued on into the year. A topic will be named, common error patterns that occurred will be identified, and an example of that error will be given.

ORDER OF OPERATIONS

When getting started with simplifying expressions, error patterns started to show up in the student's work. The following examples show the three error patterns that were most frequent.

1. Incorrect order of operation, such as adding before dividing.
2. Working only from left to right.
3. Placing additional grouping symbols into the problem.

Problem: $13 + 8 - 7 \times 2 + 4 \div 2$

Error Pattern 1.

$$21 - 14 + 4 \div 2$$

$$21 + 18 \div 2$$

$$39 \div 2$$

$$19 - 1/2$$

Error Pattern 2.

$$21 - 7 \times 2 + 4 \div 2$$

$$15 \times 2 + 4 \div 2$$

$$30 + 4 \div 2$$

$$34 \div 2$$

$$17$$

Problem: $(66 + 11 \div 11 - 4) \div (9 - 6)$

Error Pattern 3.

$$(66 + 11) \div (11 - 4)$$

$$77 \div 11 - 4 \div 3$$

$$7 - 4 \div 3$$

$$3 \div 3$$

$$1$$

Error Pattern ??

$$(77 \div 11 - 4) \div 3$$

$$77 \div 7 \div 3$$

$$11 \div 3$$

$$3 \text{ r } 2$$

INTEGERS, OPPOSITES, AND ABSOLUTE VALUES

The processes of addition and subtraction of numbers are usually thought of only in terms of addition, once integers are introduced. This leads to the terms of "the opposite of a number" and "absolute value" being introduced. This leads to two more error patterns.

- 4 Denying the negative, forgetting about the negative sign.
5. Taking the opposite of everything in an absolute value.

Problem: $(-8 + 6) + -2$

Error Pattern 4.

$$-2 + -2$$

4

Problem: $| -2 | + | 3 | + | -6 |$

Error Pattern 5.

$$2 + -3 + 6$$

$$-1 + 6$$

5

One additional example. On the following True or False question, approximately 65% of the responses were the incorrect answer of True.

-a is always negative, True or False?

THE DISTRIBUTIVE PROPERTY

$$A(B + C) = AB + AC \quad \text{or} \quad BA + CA = (B + C)A$$

Two common error patterns that occurred once the Distributive Property was introduced are:

6. The failure to distribute it all the way through the parentheses.
7. The failure to distribute a negative (or opposite) after the first multiplication.

Problem: $3(2 + x)$

Error Pattern 6.

$$3(2 + x) = 6 + x$$

Problem: $-3(x + 2)$

Error Pattern 7.

$$-3(x + 2) = -3x + 6$$

TRANSFORMATIONS

By now, students have begun to solve equations using the transformations of addition, subtraction, multiplication and division. They do a pretty good job when only one transformation is needed, but watch out when two or more transformations are needed.

Multiple Transformations

8. Doing only one transformation, failure to complete the problem.
9. Applicability error, misinterpreting the rule, or using the rule incorrectly.

Problem:

$$2x + 5 = 11$$

Error Pattern 8.

$$2x + 5 - 5 = 11 - 5$$

$$2x = 6$$

Problem:

$$6 - x = -2$$

$$6 - x - 6 = -2 - 6$$

$$-x = -8$$

Problem: $5x + 5 = 25$

Error Pattern 9.

$$5x + 5 = 25$$

$$\begin{array}{r} -5 \quad -5 \quad -5 \\ \hline x \quad \quad = 20 \end{array}$$

Another illustration of an error pattern involving transformations, in which at least one conceptional error has occurred.

Problem:

$$\begin{array}{r} 2x + 5 = 11 \\ \quad +5 \quad +5 \\ \hline 2x \quad = 16 \end{array}$$

$$(1/2)(2x) = (1/2)(16)$$

$$x = 32$$

EXPONENTS

The rules of exponents:

$$x^a x^b = x^{a+b} \qquad (x^a)^b = x^{ab} \qquad (xy)^a = x^a y^a$$

If it were possible to keep from having problems that combine these rules, life would be easy. Perhaps no error patterns would form. But we do combine the rules within problems; therefore, we have another error pattern.

10. When to multiply and when to add exponents.

Problem: $(3x^2)^5$

Problem: $(x^4 y^5)^7$

Error Pattern 10.

$$15x^{10}$$

$$x^{11} y^{12}$$

POLYNOMIALS

When combining like terms, through addition and subtraction of polynomials, several error patterns will repeat. This comes from learning NOT the rules, but the error patterns achieved earlier. By this time in the school year, it would be a good idea to review and examine the previous errors. Perhaps this will keep the "I don't understand" bug from striking. The most common error pattern is that they will add the exponents together, when adding like terms. The second most common error pattern is to combine anything that has the same variable, no matter what the degree of the term.

11. Adding together the like exponents.

12. Adding together all terms with the same variable.

Problem: Simplify: $(3x^2 + 2x + 1) + (6x^2 - 4x + 7)$

Error Pattern 11.

$$9x^4 - 2x^2 + 8$$

Error Pattern 12.

$$(5x^3 + 1) + (2x^3 + 7) \\ 7x^6 + 8$$

MULTIPLYING POLYNOMIALS

When multiplying a monomial by another monomial, be sure to review the rules of exponents. The confusion over when to add exponents and when to multiply them will lead to error patterns.

Problem: $(3x^2)(2x^4)^3$

Error Pattern 10.

$$\text{Solution A, } 3x^2 8x^{12} = 24x^{24}$$

$$\text{Solution B, } 3x^2 6x^7 = 18x^9$$

"What middle term?" is a common reply when multiplying a binomial by a binomial. The most common error is that the student will multiply the first terms and the last terms only. They will do this regardless of whether you chose to have them work the problem by either or both of the following methods.

$$(x + 3)(x + 2)$$

$$x(x + 2) + 3(x + 2)$$

$$x^2 + 2x + 3x + 6$$

$$x^2 + 5x + 6$$

$$\begin{array}{r} x + 3 \\ x + 2 \end{array}$$

$$\begin{array}{r} x^2 + 3x \\ 2x + 6 \\ \hline \end{array}$$

$$x^2 + 5x + 6$$

13. No middle term.

Error Pattern(s) 13.

Problem: $(x + 3)(x + 6) = x^2 + 18$

Problem: $(x + 4)(x - 4) = x^2 - 16$ Correct!!

Problem: $(x + 2)^2 = x^2 + 2^2 = x^2 + 4^2$

Problem: $(x + 3)(x - 5) = x^2 - 15$

Problem: $(x - 4)(x - 7) = x^2 - 28$

One method often used to show the students how to get all four products is by drawing loops above and below the parentheses. Why not make it fun, as Tom Massey did with "George," which can be found in the Appendix on page 47.

FACTORING

Why not end the year right here? It is nearly impossible to go beyond the chapter on factoring, if the students cannot factor correctly.

There are six guidelines for factoring.

1. Look for the greatest monomial factor.
2. Look for the difference of squares.
3. Look for a trinomial square.
4. Look for a pair of binomials.
5. If four or more terms are involved, look for ways to group terms in pairs or in a group of three terms that is a binomial square.
6. Make sure that each factor is prime.

From these guidelines, several error patterns develop.

14. Does the student divide correctly?
15. Does the student recognize the greatest monomial factor?
16. When they recognize the difference of squares, do they think of it as a trinomial square?
17. Do trinomial squares actually become the squares of binomials?
18. Does the student look at the linear term to find factors, instead of the constant?
19. When a polynomial of four or more terms is to be factored, do they look to break it down into pairs or trinomial squares? Will they look to rearrange it?
20. Will they factor the polynomial out as far as possible, so that all terms are prime?

Error Pattern 14.

Problem: $\frac{3a + 6}{3} = a + 6 \text{ or } 3a + 2$

Both solutions are errors of the distributive property.

Error Pattern 15.

Problem: $5x^6 + 35x^4 + 15x^2 = 5x(x^5 + 7x^3 + 3x)$

Error Pattern 16.

Problem: $x^2 - 16 = (x - 4)^2$

Error Pattern 17.

Problem: $x^2 + 16x + 64 = x(x + 16) + 8$
 $= (x + 8)(x + 2)$

Error Pattern 18.

Problem: $x^2 - 4x - 5 = (x - 4)(x + 1)$

The student MUST know the difference between the definitions of factors of a product and terms of an expression.

Error Pattern 19.

Problem: $x^2 - y^2 + 6x + 9 = (x + y)(x - y) + 3(2x + 3)$

Problem: $3x + 5y - xy - 15 = \text{"prime"}$

Error Pattern 20.

Problem: $x^2 + 6x + 9 - y^2 = (x + 3)^2 - y^2$

Do the students stop there or do they see the difference of two squares? They must learn to factor completely.

FRACTIONS

Once passed the chapter on factoring, the students get to apply what they have learned by reducing fractions. This is where you must be careful with the term cancellation or "cancel fever" will show up in their work.

21. Improper cancellation.

Error Pattern 21.

Problem: $\frac{x^2 - y^2}{x + y} = x - y$ Correct!!

Now may we see some work?

$$\frac{x^2 - y^2}{x + y} \quad \frac{\overset{x}{\cancel{x^2}} - \overset{y}{\cancel{y^2}}}{\cancel{x} + \cancel{y}} = x - y$$

Not quite the procedure that we had hoped for is it.

A couple more examples of error pattern 21.

Problem: $\frac{x^2 + 2x + 1}{x + 1}$

$$\frac{\cancel{x^2} + \cancel{2x} + \cancel{1}}{\cancel{x} + \cancel{1}} = x^2 + 3$$

Problem: $\frac{x^2 + x - 20}{x - 5}$

$$\frac{\cancel{x^2} + \cancel{x} - \cancel{20}}{\cancel{x} - \cancel{5}} = x^2 - 4$$

They will want to cancel out anything that looks the same.
A guideline to working with the operations of fractions can be found in the Appendix on page 48.

GRAPHING

Let's jump into a topic that is a change of pace, working with linear equations. Two error patterns develop.

22. Getting the student to pick a value for one variable and solving the equation to determine the second variables value.
23. Incorrect plotting of an ordered pair when trying to graph the equation of a line.

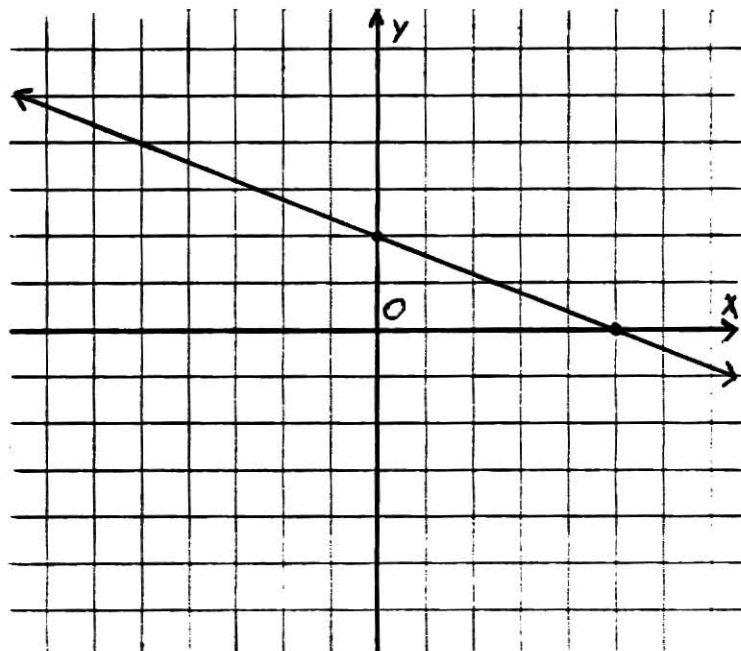
Error Pattern 22.

Problem: $x + 2y = 11$ If the student lets $x = 2$, they will try to make $y = 2$, also.

Error Pattern 23.

Problem: Graph: $x + 3y = 11$

Once the student has correctly found a solution, such as the ordered pair $(5,2)$, they will plot this point as two separate points, where x is 5 and y is 2, as shown on the graph below. They will then draw the line through these two incorrect locations.



WORD PROBLEMS

The thought of a word problem strikes fear into the best of students. The terror of attempting the word problem, why is it so great? The word problem is the only situation that is remotely like the everyday world. Problem-solving, we must find a plan to go about living in this wonderful world. The expressions and equations that have preceded word problems were direct and precise. Is anything like that in this world?

Translating a word problem into an equation is the most enjoyable aspect of algebra, once the student can do it with confidence. Hence, we must remove the translation error patterns.

24. Interpretation errors, not being able to show one's own understanding of the concept.

25. Reversal errors, where two variables are needed to solve the problem, and the student has them reversed.

Error Pattern 24.

Problem: Write the following in words: $-5 < 2$

Solution: -5 is less than 2

This is an error of interpreting the instructions.

Problem: Symbolize: Six less than a number.

Solution: $6 - n$

This is an error of misinterpreting the concept of "less than."

Error Pattern 25.

Problem: Write an equation using the variables Q and N to represent the following statement:

There are four times as many nickels as quarters in Ralph's pocket.

Solution: $4N = Q$

The common reversal error.

Unfortunately, there is one more error pattern that is perhaps the worst of all. That is the pattern of the student that will not attempt a word problem, at any cost.

CHAPTER IV

ANALYSIS AND CONCLUSION

Two pieces of literature not yet mentioned deal with the difficulties involving algebra. Sleeman (1982) mentions that the difficulties of learning algebra have been greatly underestimated and that pupils have a great facility for inferring their own rules and then using them consistently in inappropriate situations.

Kulm (1973) states that the difficulties come from the difficulty of the symbolism and the student's inexperience with the symbolism of algebra. As a teacher, you must be aware of the significance of the symbolism and make a conscious effort and practice to develop the facility (absence of difficulty) with the symbolism.

Remember, you have the experience, let the teaching method that you choose transmit it to the student. Also remember that errors will develop; it is your task to slow them down. The types of errors can be classified in many ways. In this report, types of errors are classified as:

1. Conceptual errors, where the student has learned the rules of arithmetic incorrectly.
2. Applicability errors, where the student misuses the rules or definitions of algebra.

3. Translation errors, combining the errors involved with word problems, into one classification, that come about in going from sentence to equation.
4. Execution errors, where the student fails to complete the task.

Do what you have to do to develop the correct procedures in your classroom. These procedures start with being very authoritarian, making them do the work in the method that you want. Later, you can give them choices about the method that they may prefer. Discussion and homework, day after day, can get boring. A change of pace may be in store, before you hear "Can we have a day off?" Class can be enjoyable and still accomplish the goal that has been set, to increase one's knowledge of algebra.

Try the ideas of others, such as, "Pass the answer," "I have . . ., Who has . . .?", or "|25| and Lose," which can be found in the Appendix on page 49. Just be sure that you are the one that says when the game is to be played. Perhaps, you would like to give them a history lesson or maybe bribe(reward) them into doing well on the next exam by throwing a party as a reward. Whatever you choose, remember that error patterns will happen. You can slow them down by (a) being authoritative, (b) carefully introducing definitions and rules, (c) reviewing often, (d) giving credit for partial work and effort, and (e) making class enjoyable and

fun by taking a change of pace every once in awhile. Remember that the mind stores information, how it will be returned, no one knows.

Just as importantly, as a teacher, you must pay attention to the sequence of the work and encountered tasks, from which the student is apt to abstract (invent) functional invariances. This suggests that no matter how carefully we plan a sequence of examples, we can never know all the intermediate steps and abstracted structures that a student will generate while solving an exercise. Indeed, the student may well produce illegal steps in his solution and from these invent illegal algebraic "principles" (Sleeman, 1984). If you can manage to decrease the number of "inventions" of errors, you will be on your way to the Teachers Hall of Fame.

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APPENDIX

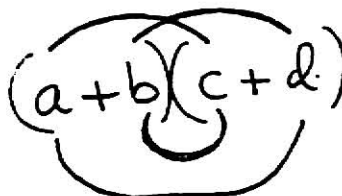
"George" Helps Students Multiply Binomials

Multiplication of binomials is a process not readily mastered by all eighth- and ninth-grade students. A device that was a successful supplement to the theory of the process is "George."

The binomials to be multiplied were arranged as shown below.

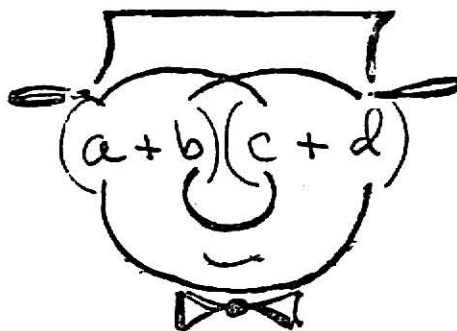
$$(a+b)(c+d)$$

Then lines were drawn, indicating which products were to be added.



A diagram showing the binomial multiplication process. The expression $(a+b)(c+d)$ is enclosed in a large, hand-drawn oval. Inside this oval, there are four curved lines connecting the terms: a line from 'a' to 'c', a line from 'a' to 'd', a line from 'b' to 'c', and a line from 'b' to 'd'. These lines represent the products to be added.

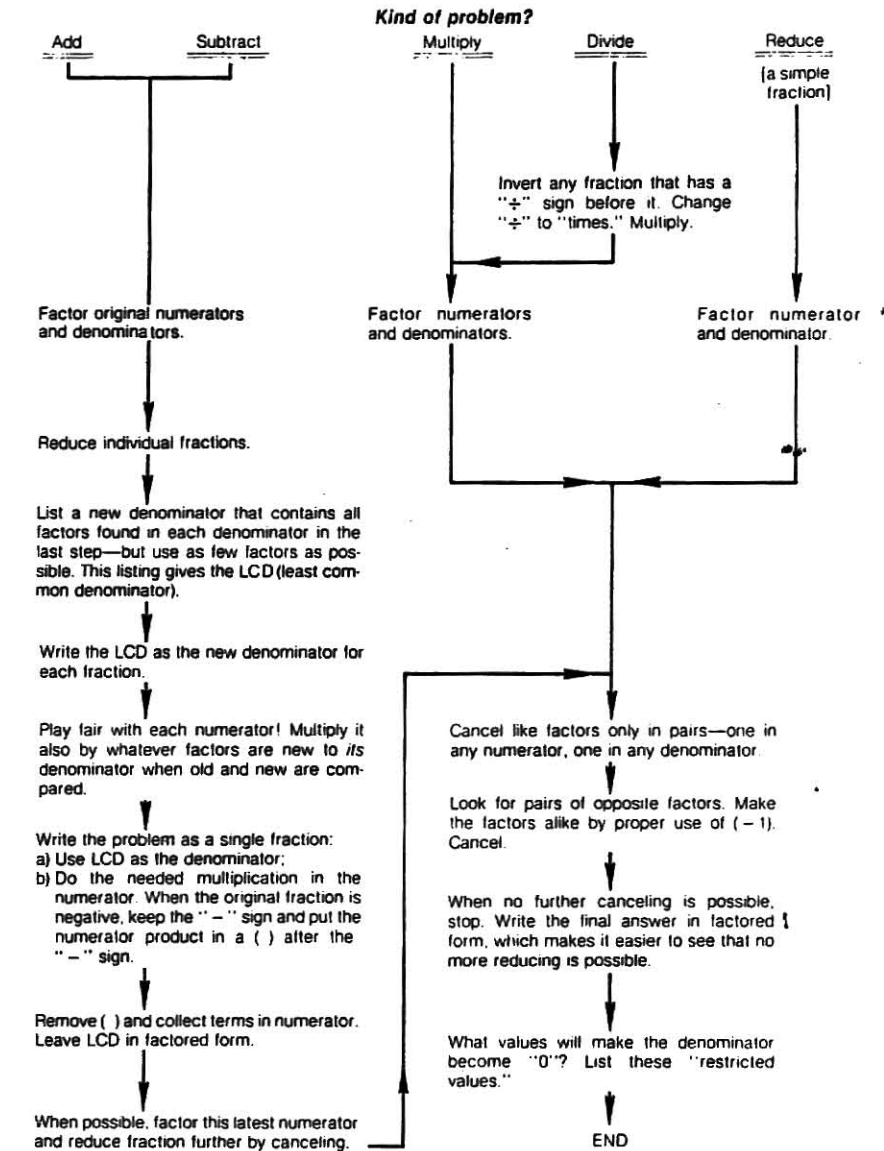
A few more lines were added.



Thus George was introduced to the students as an aid for their multiplication of binomials. He became a familiar aid for students in their computations with binomials. As the school year progressed, students would often ask, "Is it still OK for George to help me?"

—TOM E. MASSEY, P. K. Yonge Laboratory School
University of Florida, Gainesville, Florida

Schema: Operations with Fractions



From the *Mathematics Teacher*, April 1986

|25| AND LOSE

Items needed:

One deck of playing cards for each group.

Instructions:

Deal out 4 cards to everyone, putting the remaining cards in a pile, face down, near the middle of the group. The black numbered cards are positive integers, while the red ones are negative integers. Aces count as ones. Jacks are zero in value. Queens change the direction of play. Kings double the count.

As each player lays down a card, he must state the sum and then take the top card of the remaining cards.

Scoring:

One point is given to anyone stating an incorrect sum, or anyone that fails to pick up a card. Five points are given to the player that equals or exceeds the absolute value of 25.

Winning:

The person with the lowest score after several hands.

Created by:

Robert L. Clark, 1985

COMMON ERRORS IN ALGEBRA I
A FOREWARNING TO BEGINNING TEACHERS

by

Robert L. Clark

B.S., Fort Hays State University, 1973

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Mathematics

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1986

ABSTRACT

This report is a forewarning to beginning teachers of the error patterns that will endanger their students. Since the mind works in mysterious ways, errors that occur will be unexpected and at times they will seem like "inventions" of new rules. Error patterns come about from either conceptional errors, applicability errors, translation errors, or execution errors. No one teaching method is any better than the others in eliminating error patterns. You can slow them down by being authoritative, carefully introducing definitions and rules, reviewing often, giving credit for work and effort, and making class enjoyable. This can be accomplished by trying the ideas of other teachers who have had success in reducing error patterns with these ideas. As teachers, we must teach the algebraic skills that are needed to keep this a wonderful world in which to live.