CLOSED LOOP PERFORMANCE OF HYDRAULIC SERVOSYSTEMS

DESIGNED BY THE MAXIMUM POWER METHOD

by

CHARLES KENNETH KNIFFIN

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Approved by:

Ralph O. Turnquist
Major Professor

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NOMENCLATURE

A = piston area, in.^{2}
A_V = servovalve area, in.^{2}
b = width of hydraulic actuating piston, in.
C_d = discharge coefficient
C_L = viscous damping constant of load, lb_f-in/sec.
C_p = viscous damping constant of piston, lb_f-in/sec.
d = diameter of servovalve spool, in.
E = system error, volts
F_A = force delivered by actuator at maximum power point. lb_f.
f = viscous damping const lb_f-in/sec.
F = force reflected to actuating piston rod, lb_f.
h = a small increment of time, sec.
KO = servoamplifier gain
K1 = partial derivative of flow with respect to X
K2 = negative partial derivative of flow with respect to ΔP
K3 = design constant
L = length of cylinder, in.
m_L = mass of load, lb_f-sec^2/in.
m_p = mass of piston, lb_f-sec^2/in.
P = pressure, lb_f/in.^{2}
ΔP = pressure drop across servovalve, lb_f/in.^{2}
$(\Delta P)_0$ = pressure drop across servovalve at some initial steady state operating point, lb_f/in.^{2}
P_d = drain pressure. lb_f/in.^{2}
P_L = pressure on left side of actuating piston, lb_f/in.^{2}
Pr = pressure on right side of actuating piston, lb_f/in.^{2}
Ps = supply pressure, lb_f/in.^{2}
Q = flow rate, in.^{3}/sec.
R = amplitude of input signal, volts
T1 = time constant, sec.
T3 = time constant, sec.
V_L = volume of compressed fluid within the lines, in.^{3}
V_V = volume of compressed fluid within the servovalve, in.^{3}
X = distance traveled by valve spool, in.
X_{max} = maximum spool travel, in.
X_0 = spool position at some steady state operating point, in.
Y = distance traveled by actuating piston, in.
Y_A = actuator velocity at point of maximum power, in/sec.
β = fluid bulk, modulus, lb_f/in.^{2}
ξ = damping ratio
ρ = fluid density, lb_f-sec^2/in.^{4}
ω = frequency of input signal, radians/sec.
ω_n = natural frequency, radians/sec.

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CHAPTER I

INTRODUCTION

Hydraulic actuating systems are used extensively where the design applications call for quick response and a corresponding large amount of force. In such applications the closed-loop servo-type hydraulic system is usually employed.

In a masters report Gupta (1) discussed a design procedure for determining the critical dimensions of a four-way zero-lapped servovalve and a loaded double-ended, double-acting cylinder so that it could follow a sinusoidal input of predetermined design frequency and design amplitude. These critical dimensions are the valve spool diameter, d, the maximum spool travel, X_{max}, and the area of the actuating piston. This design procedure is known as the "Maximum Power Method." Gupta used this design procedure and studied the response characteristics of a system so designed within the framework of an open-loop system in which the servo-valve spool was assumed to be driven by a sinusoidal input at the predetermined design frequency and at a peak-to-peak amplitude of $2X_{max}$.

This masters report is intended to be a logical extension of Gupta's work in that the hydraulic elements designed by the maximum power technique are placed in a closed loop servo-system. The requirements placed upon the additional elements in the closed loop system are studied along with the response characteristics of the complete system.

The closed-loop system was first approximated by a completely linear model; later the non-linearities of fluid compressibility and flow across the servovalve are included in the mathematical model. The author's conclusions are based upon studies of this nonlinear model.

CHAPTER II

THE MAXIMUM POWER DESIGN METHOD

The maximum power design method is based upon the premise that a load consisting of inertia and viscous dumping is to be driven sinusoidally at a predetermined design frequency and design amplitude. This design technique can be used to arrive at the dimensions of a four-way, zero-lapped servo-valve and double-ended, double-acting cylinder as shown in Figure 1.

If one were to plot the velocity of the load ($\dot{Y}$) as a function of the force (F) required to drive the load, for sinusoidal motion, one would obtain a plot similar to Figure 2.a. Since power is the product of force and velocity, lines of constant power would be hyperbolas such as those plotted in Figure 2.a. Obviously the point on the load force-velocity locus corresponding to the maximum power required by the load is point "A". Figure 2.b. is a plot of servovalve-cylinder characteristics for various values of spool travel. Comparing these plots with the plots of constant power superimposed upon them it becomes obvious that the point at which maximum power is delivered by the servovalve-cylinder is point "B", which is generated when the spool is at its maximum displacement from the null position.

The central idea underlying the maximum power technique is that these two points ("A" & "B"), which correspond respectively to the maximum power required by the load and the maximum power delivered by the actuator, should be equated and the servovalve and actuating cylinder dimensions determined by the coordinates of this point. The design equations used by Gupta, which are based upon this idea are:

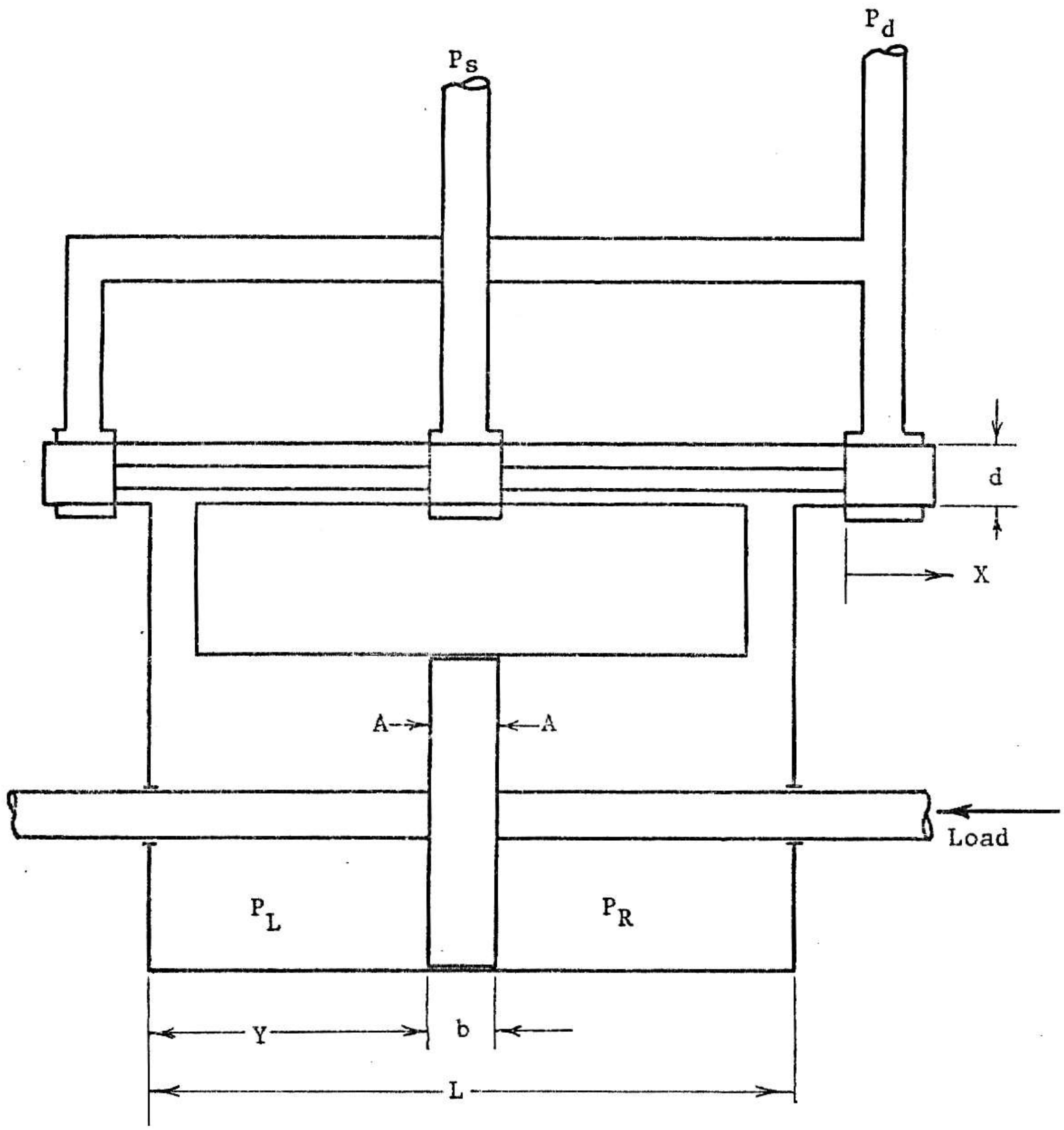

Figure 1: Four-way Zero-Lapped Servovalve and Double-Ended Double-Acting Cylinder.

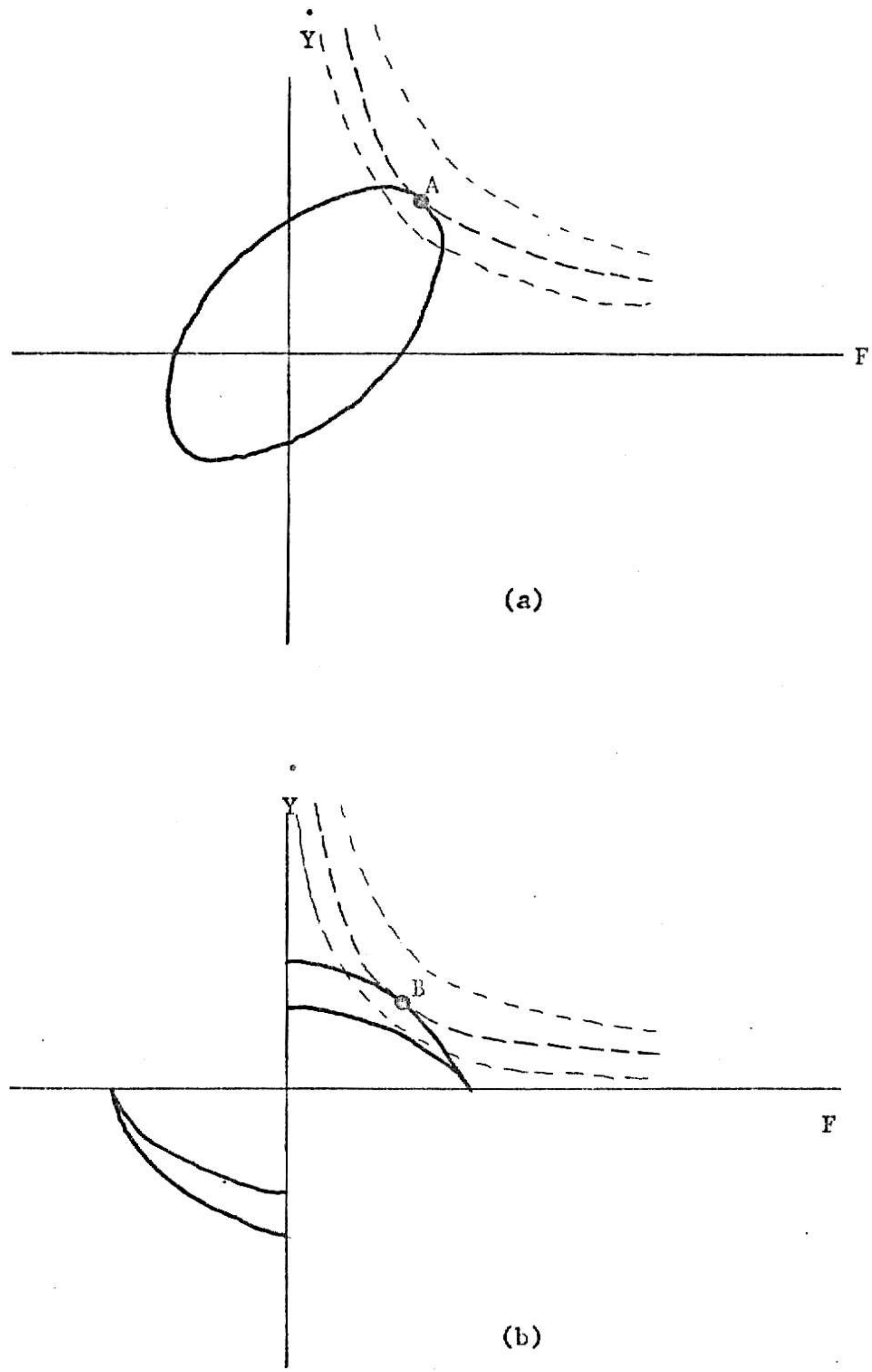

Figure 2: Velocity-Force characteristics for (a) a load driven sinusoidally, (b) a servovalve-cylinder. Lines of constant power are shown in dashes.

$$A = \frac{3\ F_A}{2\ P_S} \tag{1}$$

$$Av = \pi\ d\ X_{max} = \frac{A\dot{Y}_A\sqrt{6}}{C_d\sqrt{\frac{2}{\rho}\ P_S}} \tag{2}$$

The following assumptions were made in formulating these design equations:

1. Supply pressure, P_S, is constant.
2. Drain pressure, P_d, is zero.
3. The flow discharge coefficient, C_d, is constant.
4. Leakage is negligible.
5. Valve body, lines, and cylinder have rigid walls.
6. Line pressure drops are negligible.
7. Hydraulic fluid is incompressible.

In his master's report Gupta demonstrated the maximum power method by applying it to an example problem in which it was desired to drive a load consisting of inertia* and viscous damping** at a frequency of five hertz (31.41 radians/second) with a peak-to-peak amplitude of one half inch. Using design equations (1) and (2) Gupta obtained the following results:

$$A = 1.284\ \text{in.}^2$$

$$d = 0.25\ \text{in.}$$

$$X_{max} = 0.0098\ \text{in.}$$

These same dimensions for the cylinder and servovalve will be used throughout this report so that comparisons may be made between Gupta's open loop results and the closed loop results obtained in this study.

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*mass was $m_p + m_L = 1.555\ \frac{\text{lbf.} - \text{sec}^2}{\text{in.}}$

**damping was $C_p + C_L = 100\ \frac{\text{lbf.} - \text{sec.}}{\text{in.}}$

CHAPTER III

PLACING LINEAR MODELS OF THE OPEN-LOOP ELEMENTS IN A CLOSED-LOOP SYSTEM

The first step in studying the closed-loop response of the servovalve and cylinder designed by the maximum power method was to approximate their characteristics, and those of the additional elements used to make a closed loop system, by linear models. The block diagram of the complete closed loop system is shown in Figure 3.

In Figure 3 the servoamplifier is represented as a constant gain and the servovalve torque motor is represented by a first order transfer function. This choice for the servovalve was made based upon the recommendations of the Moog Company, a leader in the field of servovalves (2). In reference number three Moog also advises that a second order transfer function may be used to represent the servovalve. Throughout this report either a first or a second order representation is adopted for this element in the hopes that the analytic results of this report may, at some future time, be compared to experimental results obtained from the servosystem in the Mechanical Engineering Laboratory, which uses a Moog servovalve.

The servovalve-cylinder with load is represented by the following transfer function which is taken from D'Azzo and Houpis (3):

$$\frac{K3}{S(S + \frac{1}{T3})} \tag{3}$$

The quantities K3 and T3 are defined by the relationships: (2)

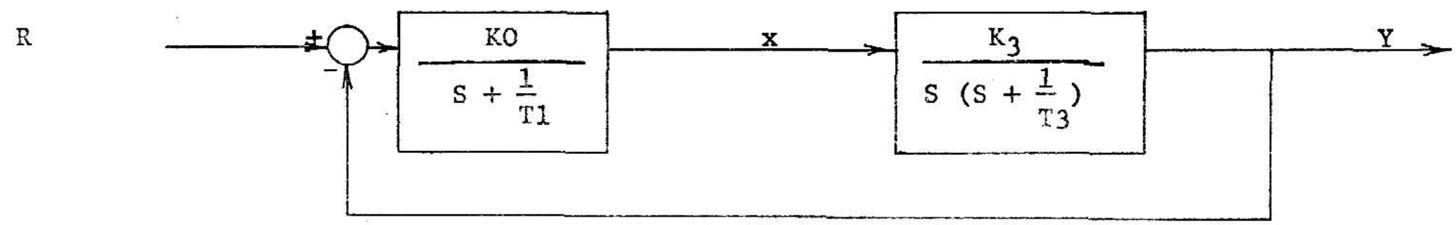

Figure 3: Linear Model of Servovalve and Cylinder

$$K3 = \frac{AK1}{m\ K2} \tag{4}$$

$$T3 = \frac{m\ K2}{A^2 + cK2} \tag{5}$$

The quantities K1 and K2 used in these formulas are determined when the equation governing the flow rate to or from the actuating piston is linearized using the Taylor's series expansion. The flow equation is:

$$Q = C_d\ \pi\ d\ X\ \sqrt{\frac{2(\Delta P)}{\rho}} \tag{6}$$

The constants K1 and K2 are defined to be:

$$K1 = \frac{\partial Q}{\partial X} = C_d\ X\ d\sqrt{\frac{2(\Delta P)_0}{\rho}} \tag{7}$$

$$K2 = -\frac{\partial Q}{\partial \Delta P} = -\ C_d\ \pi\ d\ X_0\sqrt{\frac{1}{2\rho(\Delta P)_0}} \tag{8}$$

In the derivation of K1 and K2 assumed initial steady state operating point values of $(\Delta P)_0$ and X_0 must be used. The value of X_0 so used was assumed to be small so that a good approximation could be made to the average value of X at steady state conditions (X = 0). This was necessary for the simple fact that letting $X_0 = 0$ would make K2 = 0. The value $(\Delta P)_0$ was assumed to be the average pressure difference along the fluid flow path when X_0 is very small.

The maximum power design technique determines the values of A, X_{max}, and d used for the servovalve and cylinder. Therefore for a given design incorporating these values, a mathematical model can be constructed. As indicated previously the same example used by Gupta was used here.

Primary interest was focused on the values of K0 and T1 which resulted in an amplitude ratio of unity when the rest of the closed-loop system was defined as above. This is of importance because an amplitude ratio of unity means that the output of the system exactly follows the input. A digital computer program was written which solved for K0 as T1 was varied so that the amplitude ratio remained unity when the system is excited _at the design frequency_ (see appendix). The results of this analysis are tabulated in Figure 4 for different assumed values of $(\Delta P)_0$ and X_0. Figure 4 indicates that the values of K0 and T1 required are sensitive to the assumed quantities $(\Delta P)_0$ and X_0.

Figure 5 portrays the effect of varying the input frequency on the amplitude ratio. The values of K0, T1, $(\Delta P)_0$, and X_0 were picked so as to make the amplitude ratio unity at the design frequency of 31.41 radians per second.

When using a completely linearized model for a servovalve cylinder system great care must be taken in deriving the transfer functions and in interpreting any results so obtained, however the results may be used as a guide in establishing trends. The most significant trends which are obtained from this completely linearized analysis is that an amplitude ratio of one is possible at the design frequency and that the design frequency is the largest frequency on the frequency response curve for which an amplitude ratio of one is obtained.

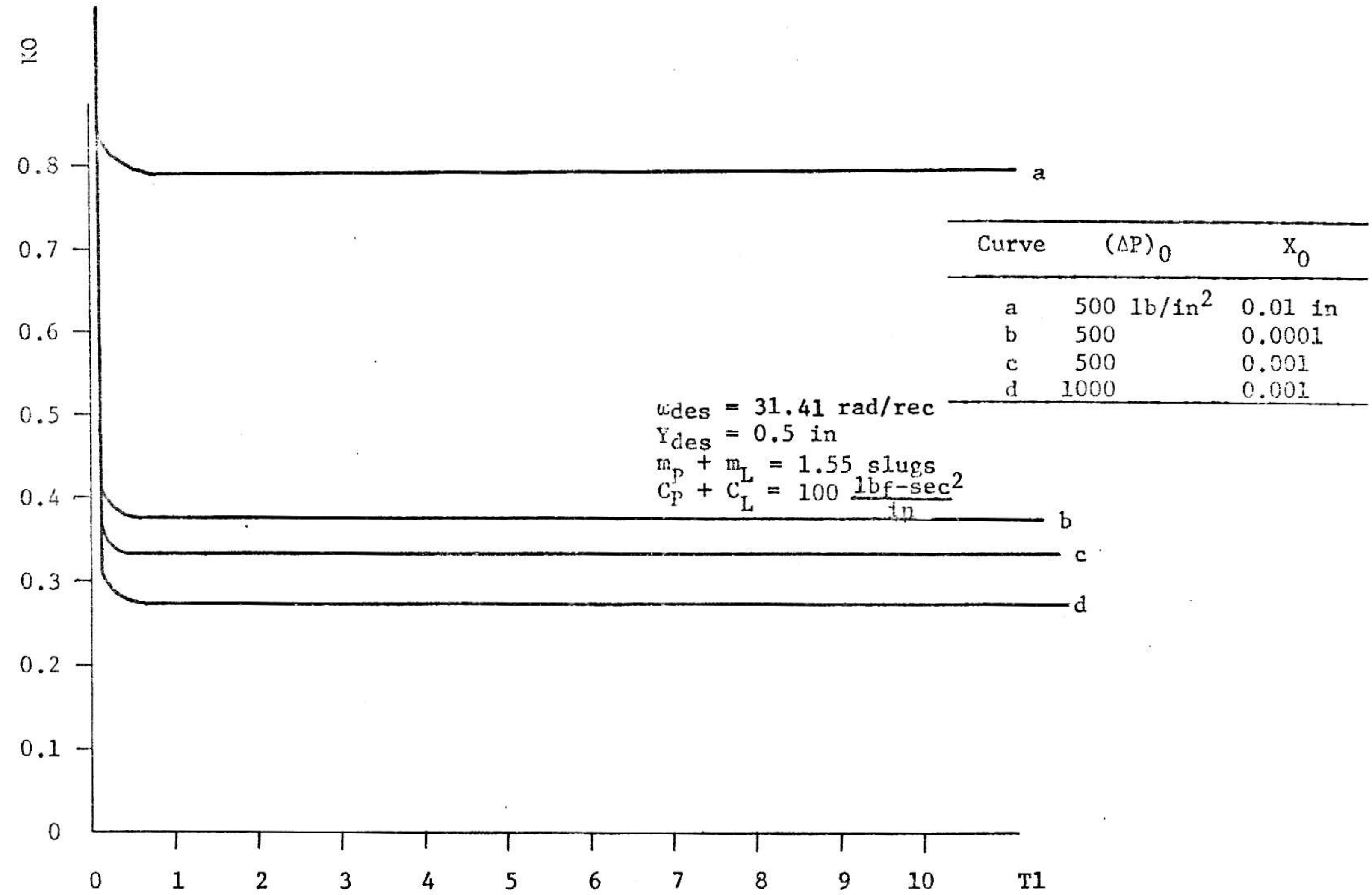

Figure 4: **Values of K0 versus T1 (of servoamplifier and servovalve torque motor) required for amplitude ratio of one, assuming various values of $(\Delta P)_0$ and X_0.**

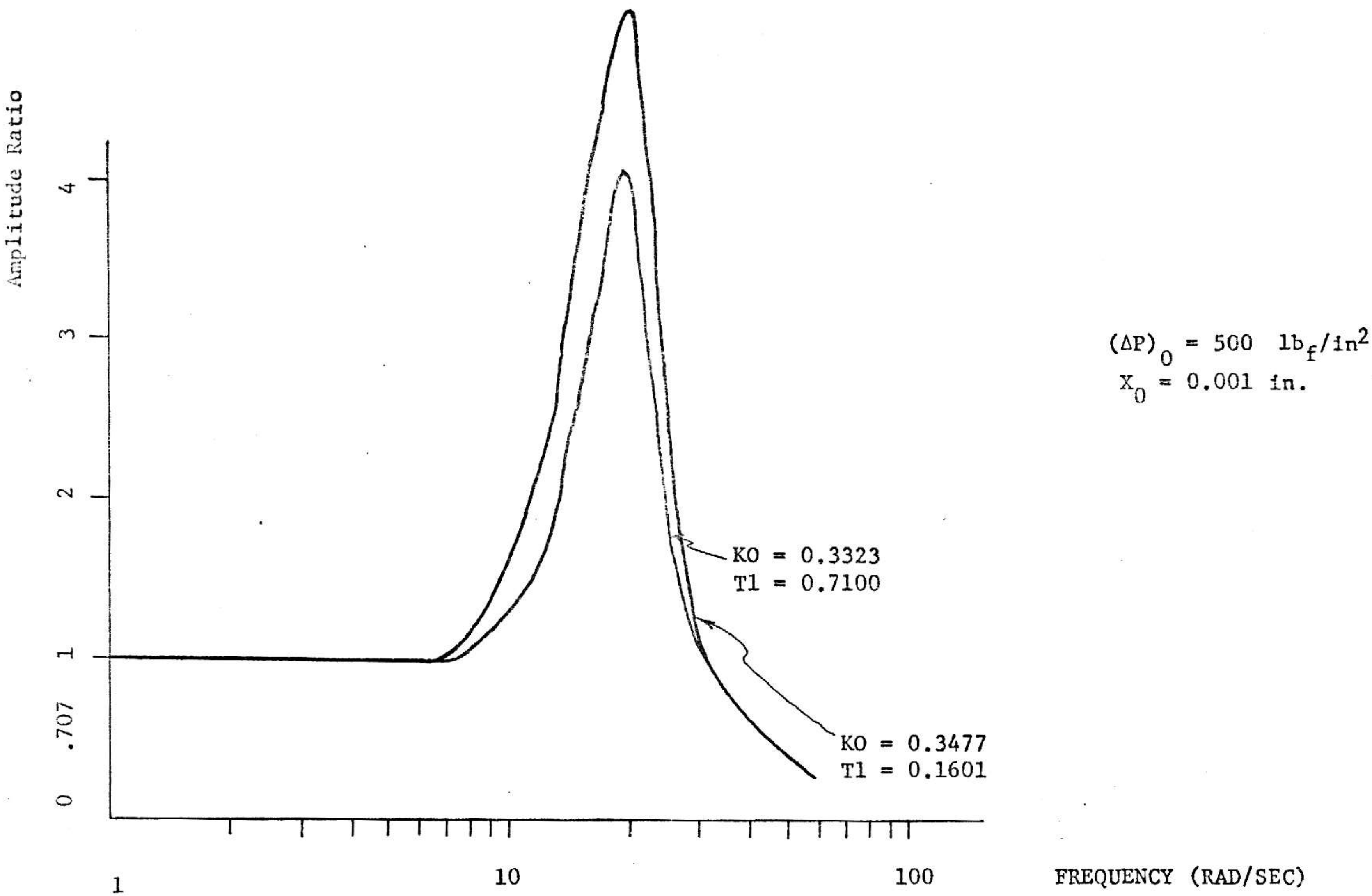

Figure 5: Frequency Response Curve of linear system with different values of KO and T1 (see Figure 4).

CHAPTER IV

REPRESENTING THE SERVOVALVE AND CYLINDER BY NONLINEAR EQUATIONS IN A CLOSED LOOP SYSTEM

The next step in this study was to replace the linear model of the servovalve and cylinder with equations which incorporated the nonlinear effects of fluid compressibility and servovalve flow as a function of pressure drop. The derivation of these nonlinear equations is involved and, for the sake of brevity, not included here. The reader is referred to the masters report by Gupta for this derivation. The dynamic nonlinear equations for the servovalve and cylinder are listed in the appendix along with an explanation of the digital computer programs used in the calculations.

The graphs on the following pages are the results of the computer study made. The nonlinear equations were used in a closed loop system with either a first order or a second order linear model to represent the servoamplifier and servovalve torque motor necessary for a closed loop system, see Figure 6. This approach was used because quite often a designer will choose either a first or second order servosystem model for these elements (2) as indicated previously. A range of parameter values was assumed for the first and second order models so that analytic results could be related to experimental results for Moog servovalves. Some typical parameter values for Moog servovalves are first order time constants (T1) ranging from 0.0013 to 0.0029 seconds and second order natural frequencies ranging from 110 Hz. to 240 Hz. A value of 1.3 was used as a damping ratio on all second order systems because this value corresponds to the particular Moog servovalve in the Mechanical Engineering Laboratory.

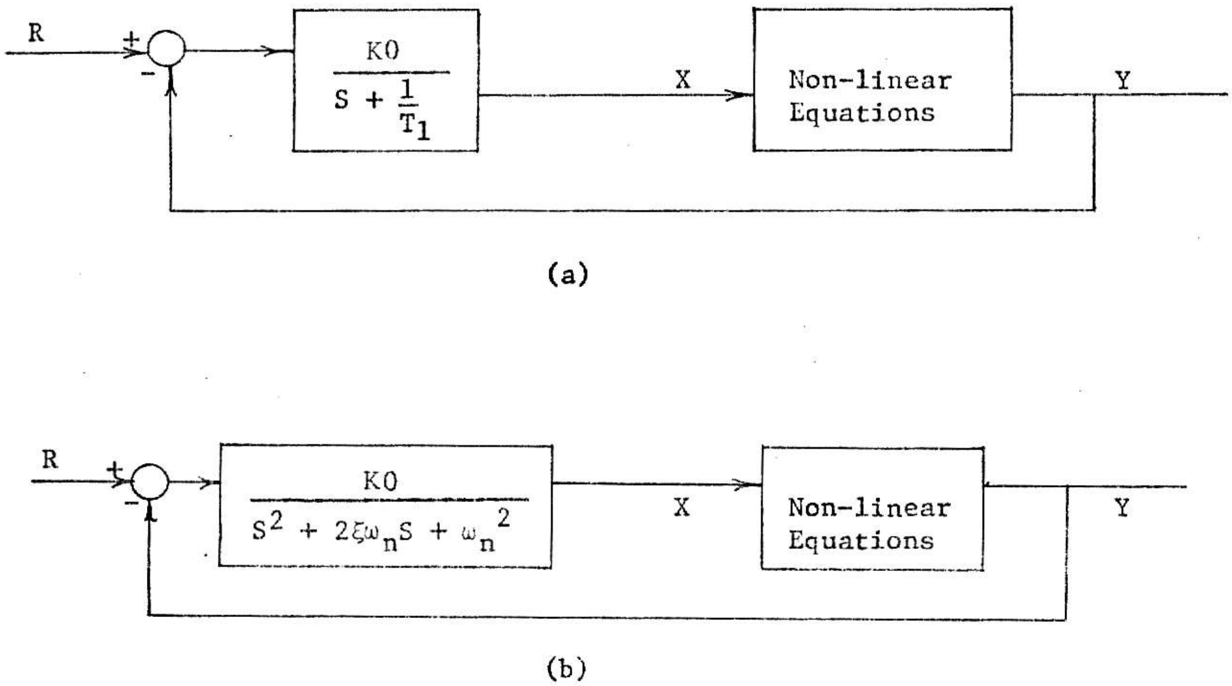

Figure 6: Servovalve and cylinder represented by nonlinear equations in a closed loop system with the servoamplifier and servotorque motor represented by a (a) first order transfer function, (b) second order transfer function.

In Figures 7 through 19 the author refers to either first or second orde systems according to whether the servoamplifier and servotorque motor are represented by either a first or second order transfer function.

Figure 7 is a graph of amplitude ratio versus servoamplifier gain (KO) for a first order transfer function with time constant T1 = 0.0005 sec. Figure 7 indicates that for KO = 38 the amplitude ratio of this system is one It also indicates that for large values of KO the amplitude ratio approaches a constant value, or the system becomes saturated. Figure 8 is a portion of the frequency response of this system when KO is specified to be 38. This graph indicates that the bandwidth of this system corresponds to approximatel 47 radians per second.

Figure 9 is a plot of amplitude ratio versus servoamplifier gain (KO) for a first order system having time constant T1 = 0.00081 sec. Figure 9 als indicates that the system became saturated for large values of KO. A value of KO = 20.0 gives an amplitude ratio of one. Figure 10 shows a frequency response for this system with KO specified to be 20.0. This frequency response curve is typical of those for the other systems. A complete curve was not formed for each case due to the computer time necessary.

Figures 11 and 12 are similar to Figures 7, 8, 9, and 10. Figure 11 shows amplitude ratio as a function of servoamplifier gain for a first order system with T1 = 0.0081. Figure 12 shows a portion of the frequency response for this system with KO specified to be 1.70 (corresponding to an amplitude ratio of unity).

The three first order cases referred to above provide a range of values covering the typical values for first order systems recommended by Moog.

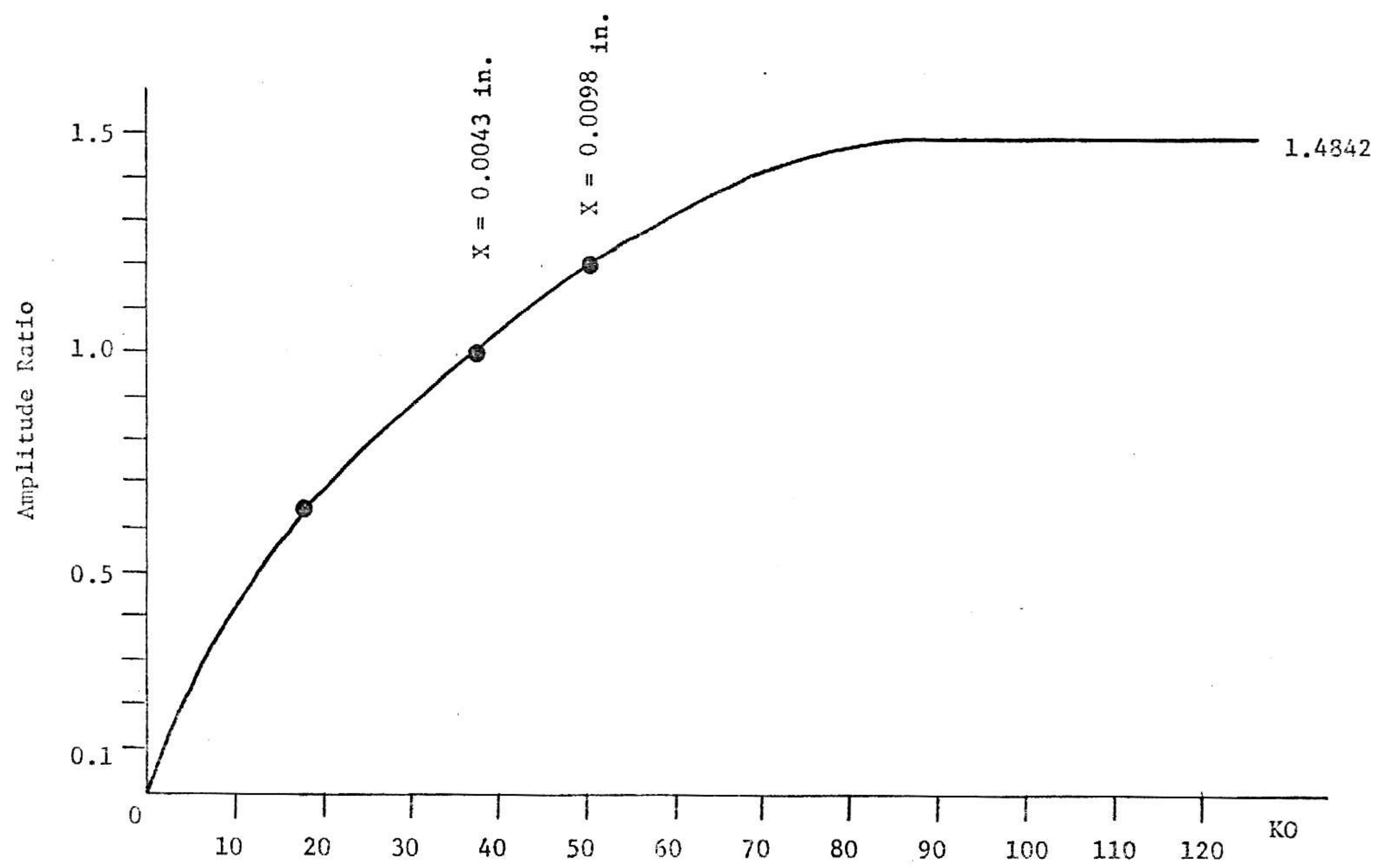

Figure 7: Amplitude ratio versus KO for a first order system with T1 = 0.0005 sec.

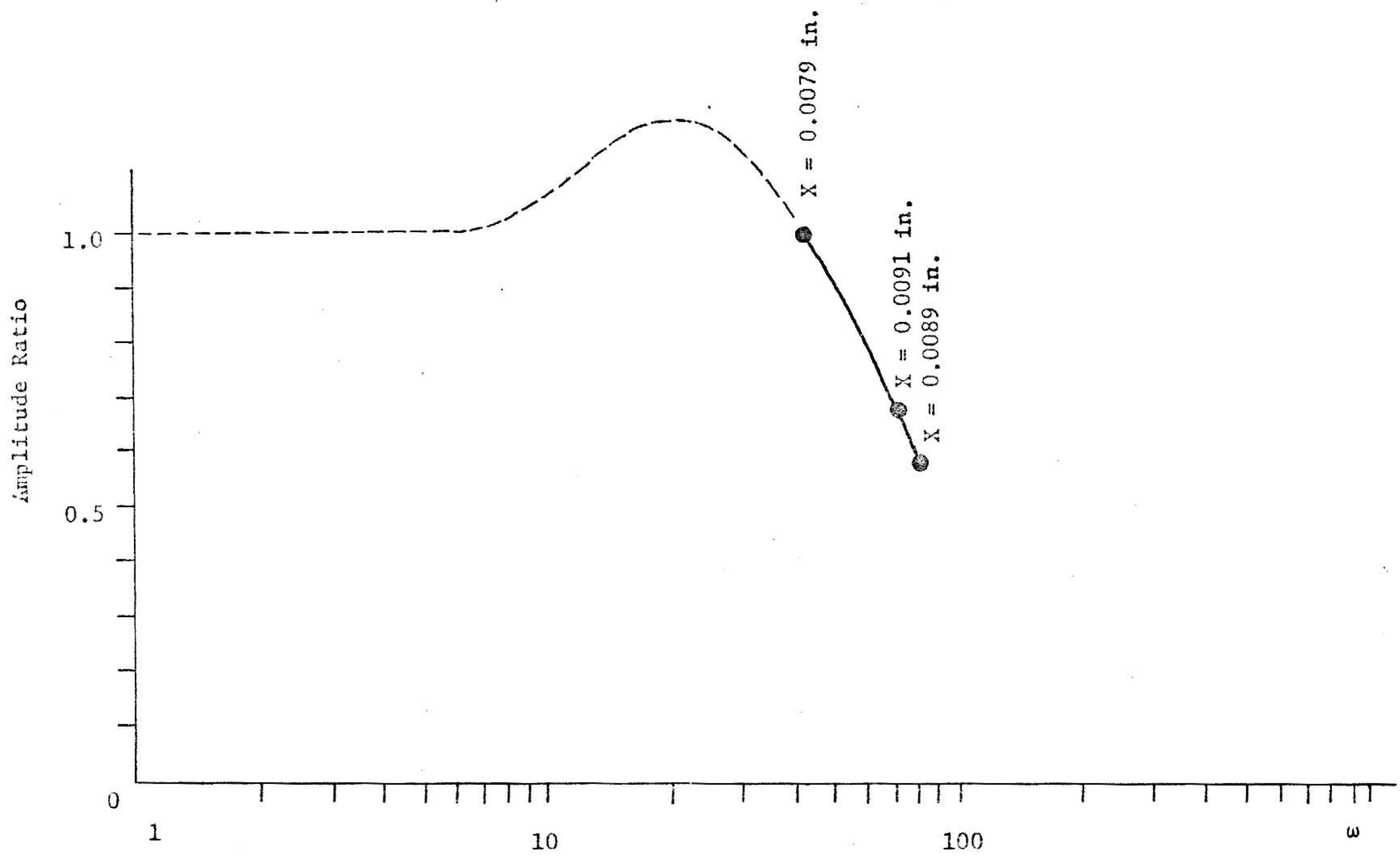

Figure 8: Frequency response curve for a first order system having KO = 38.0, and T1 = 0.0005 sec.

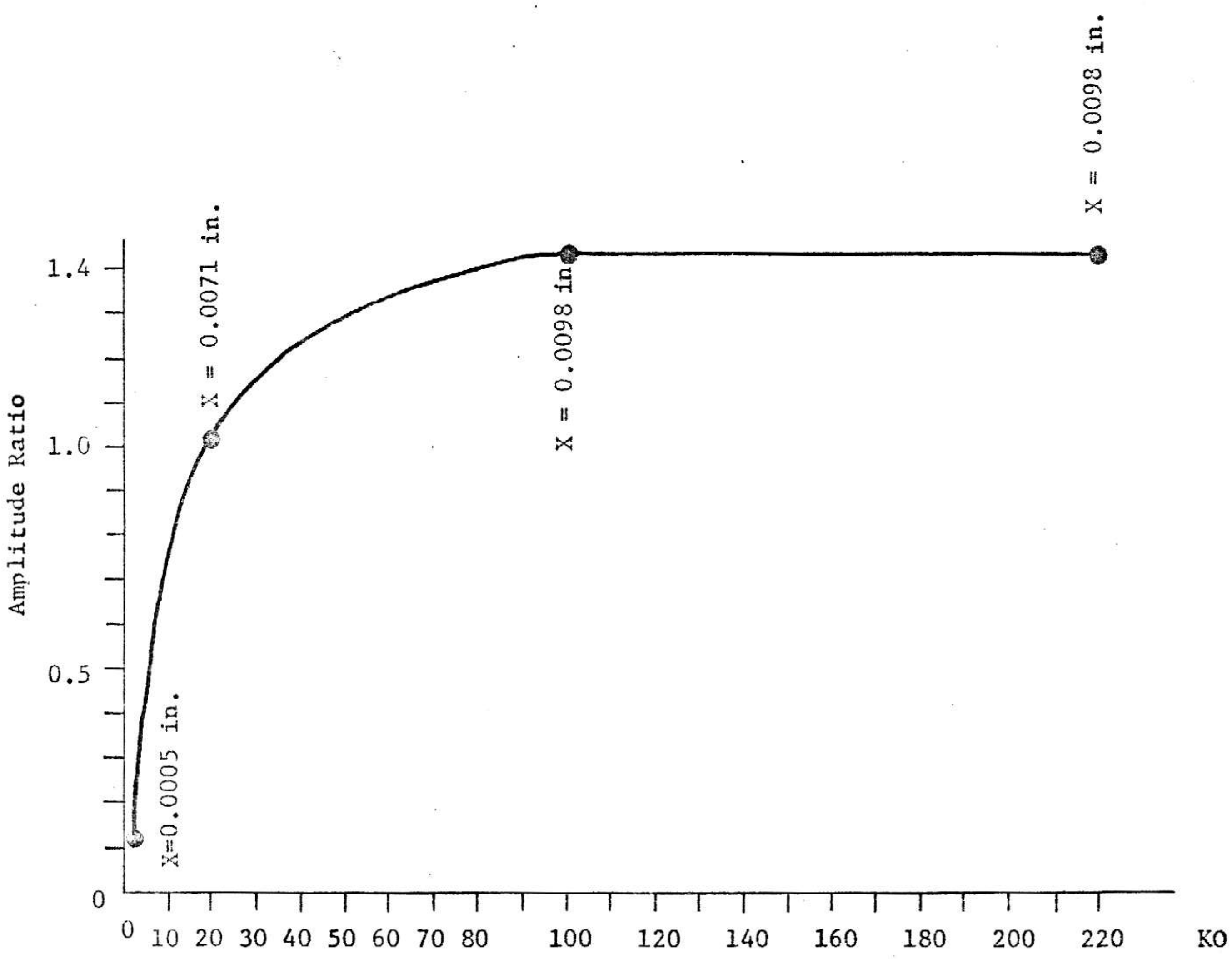

Figure 9: Amplitude ratio versus KO for a first order system having T1 = 0.00081 sec.

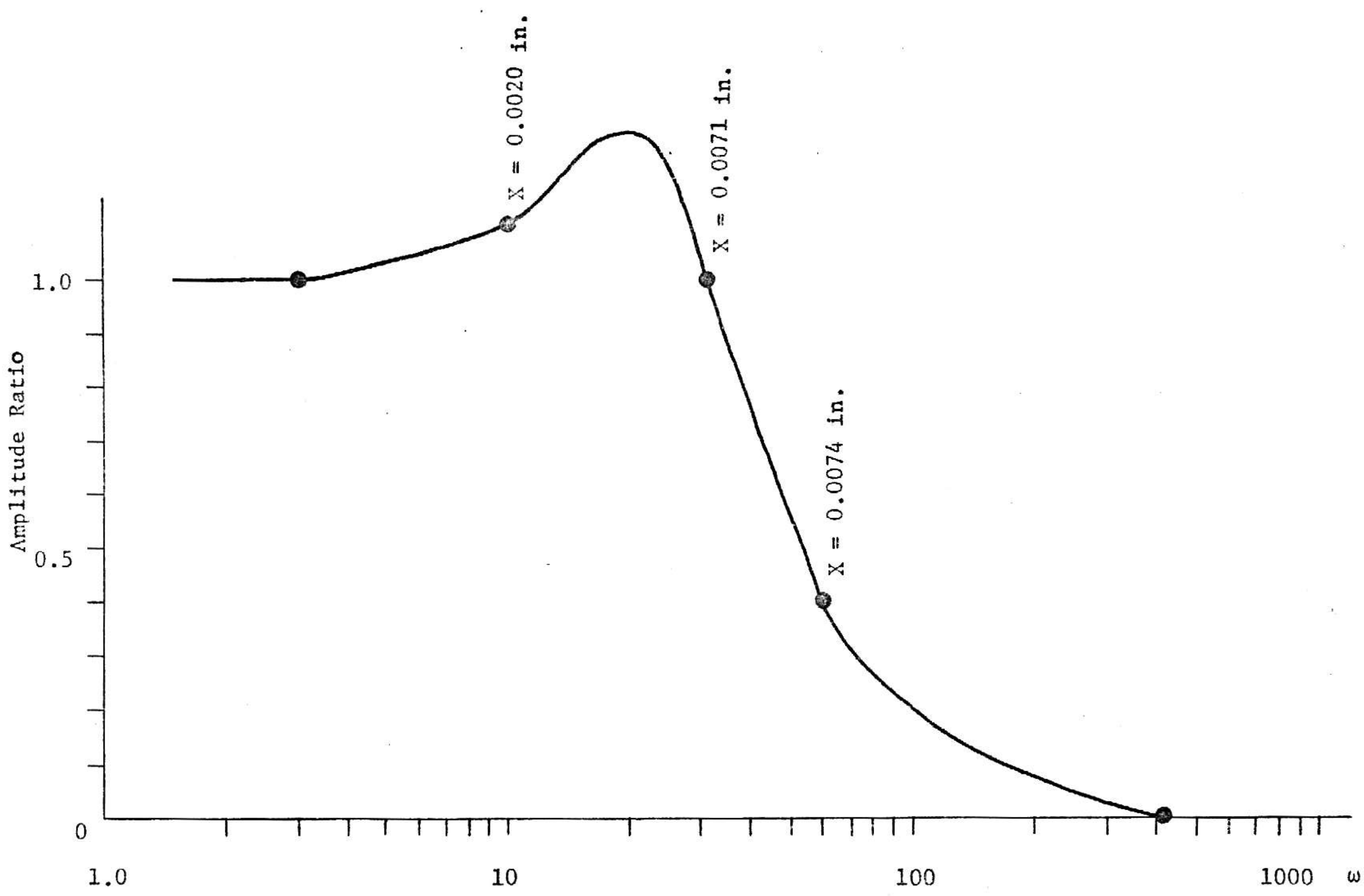

Figure 10: Frequency response for first order system in which KO = 20.0, and T1 = 0.00081 sec.

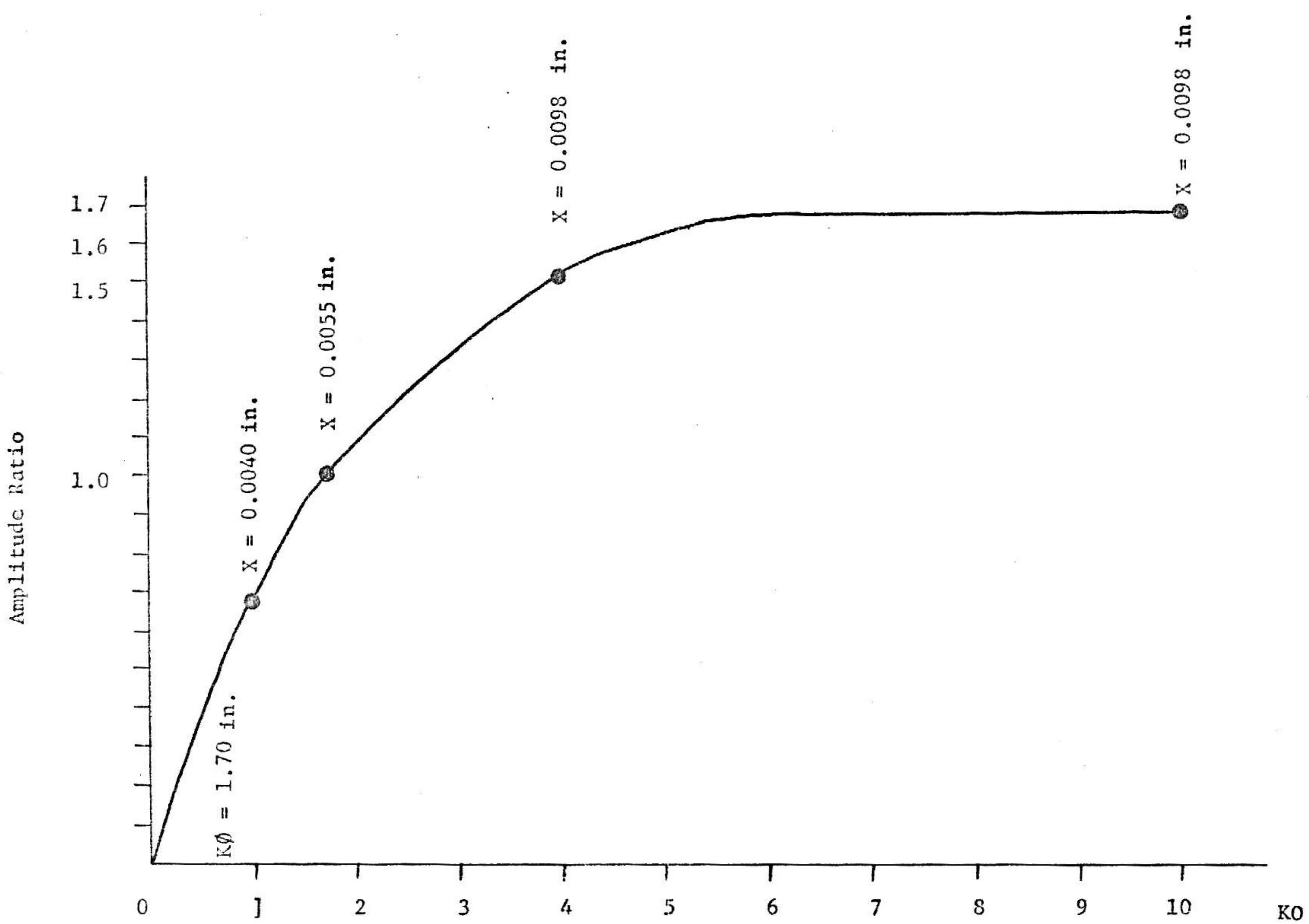

Figure 11: Amplitude ratio versus KO for a first order system having T1 = 0.0081.

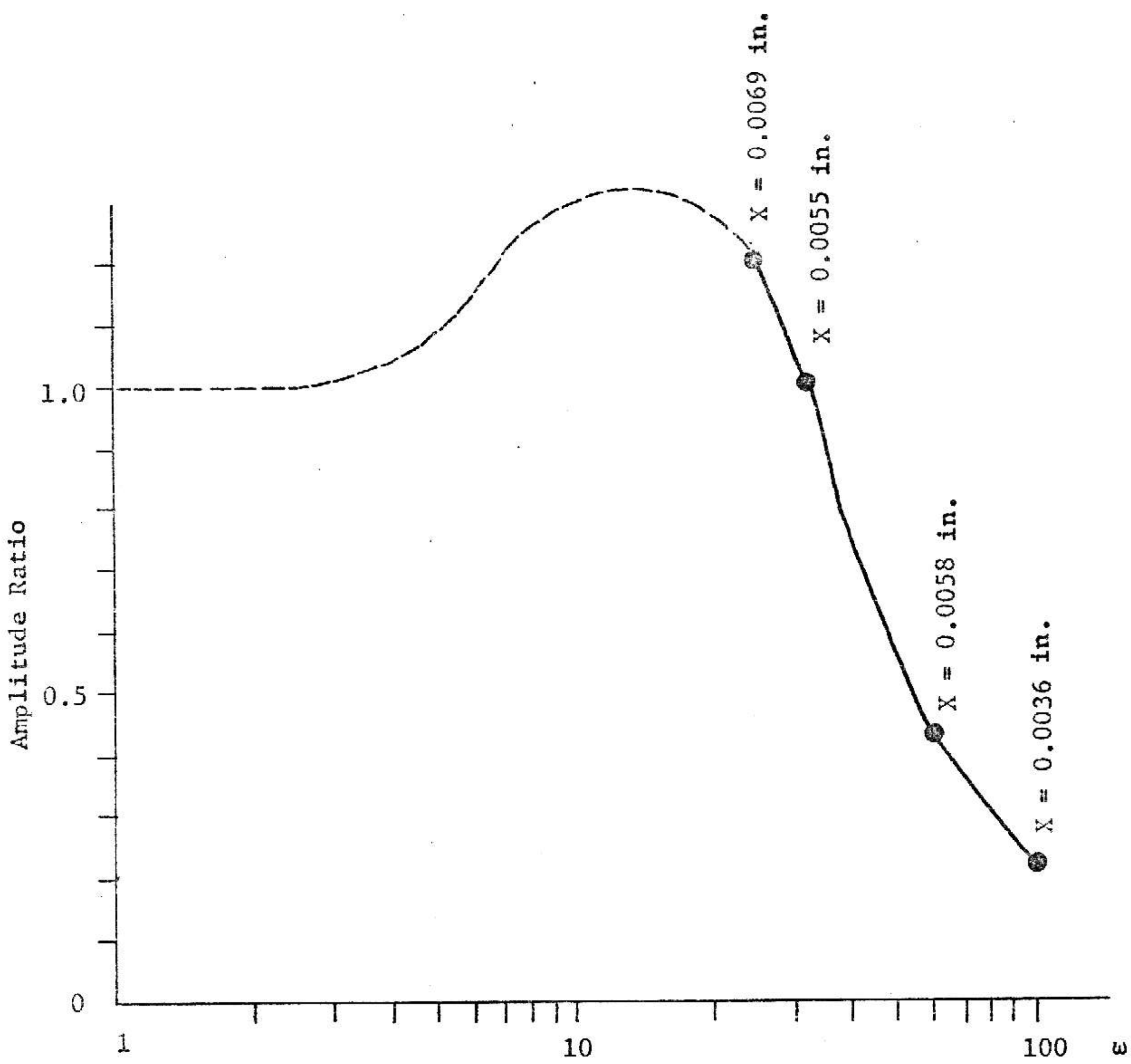

Figure 12: **Frequency response for a first order system having KO = 1.70, and T1 = 0.0081 sec.**

Figures 13 through 18 refer to the second order system shown in Figure 6.b. The value of damping ratio $\xi = 1.3$ was adopted because it corresponded to Moog specifications for the particular servovalve in the mechanical engineering laboratory. Three values of natural frequency ($\omega_n = 350$ radians per second, $\omega_n = 735$ radians per second, and $\omega_n = 1470$ radians per second) were used to cover the range of typical values already referred to.

Figures 13, 15, and 17 are plots of amplitude ratio versus servo-amplifier gain for these three cases. They all exhibit the same general shape that the first order cases did.

Figures 14, 16, and 18 are frequency response curves for each second order case. On each graph the value of KO was picked so as to give an amplitude ratio of unity at the design frequency of 31.14 radians per second.

Beside each calculated point on the graphs just referred to is annotated the maximum value which the spool travel, X, obtained. From this it is seen that the spool travel never obtained its maximum design value, X_{max}, except for large amplitude ratios. This suggests the possibility of modifying the design equations, based on the maximum power design method, so that a smaller value of X_{max} would result when the design calculations are made. Another way of viewing this could be as a built-in safety factor to off-set whatever inadequate assumptions might have been made in deriving the original design equations. Conditions which would violate these assumptions in an actual system are:

(1) Variable supply pressure, P_s

(2) drain pressure, P_d, greater than zero

(3) variable flow discharge coefficients, C_d

(4) expansion of supply lines

(5) sizeable line pressure drops.

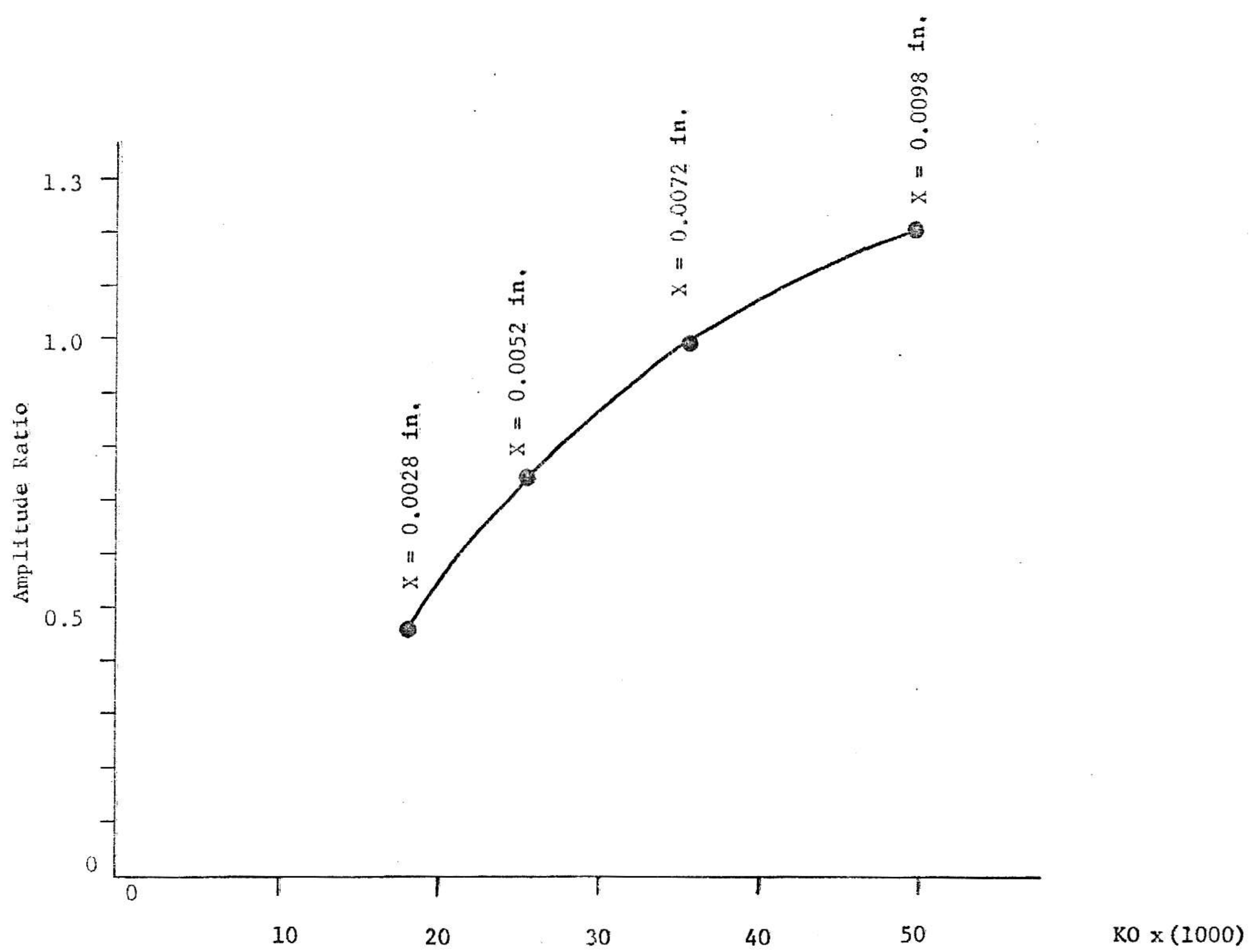

Figure 13: Amplitude ratio versus KO for a second order system having ω_n = 1470.0, and ξ = 1.3.

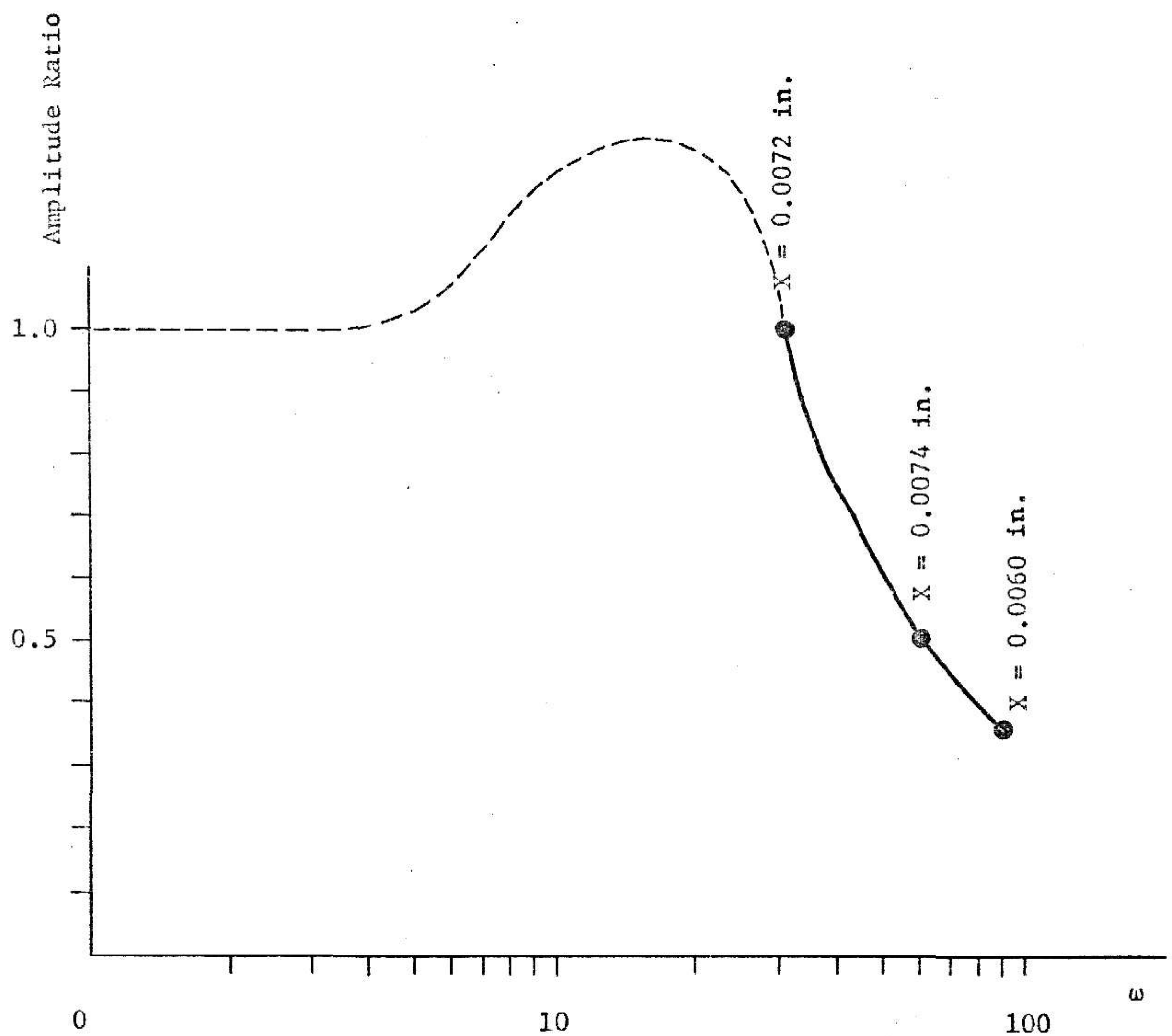

Figure 14: **Frequency response curve for a second order system having ω_n = 1470.0, ξ = 1.3. and KO = 35.000.0.**

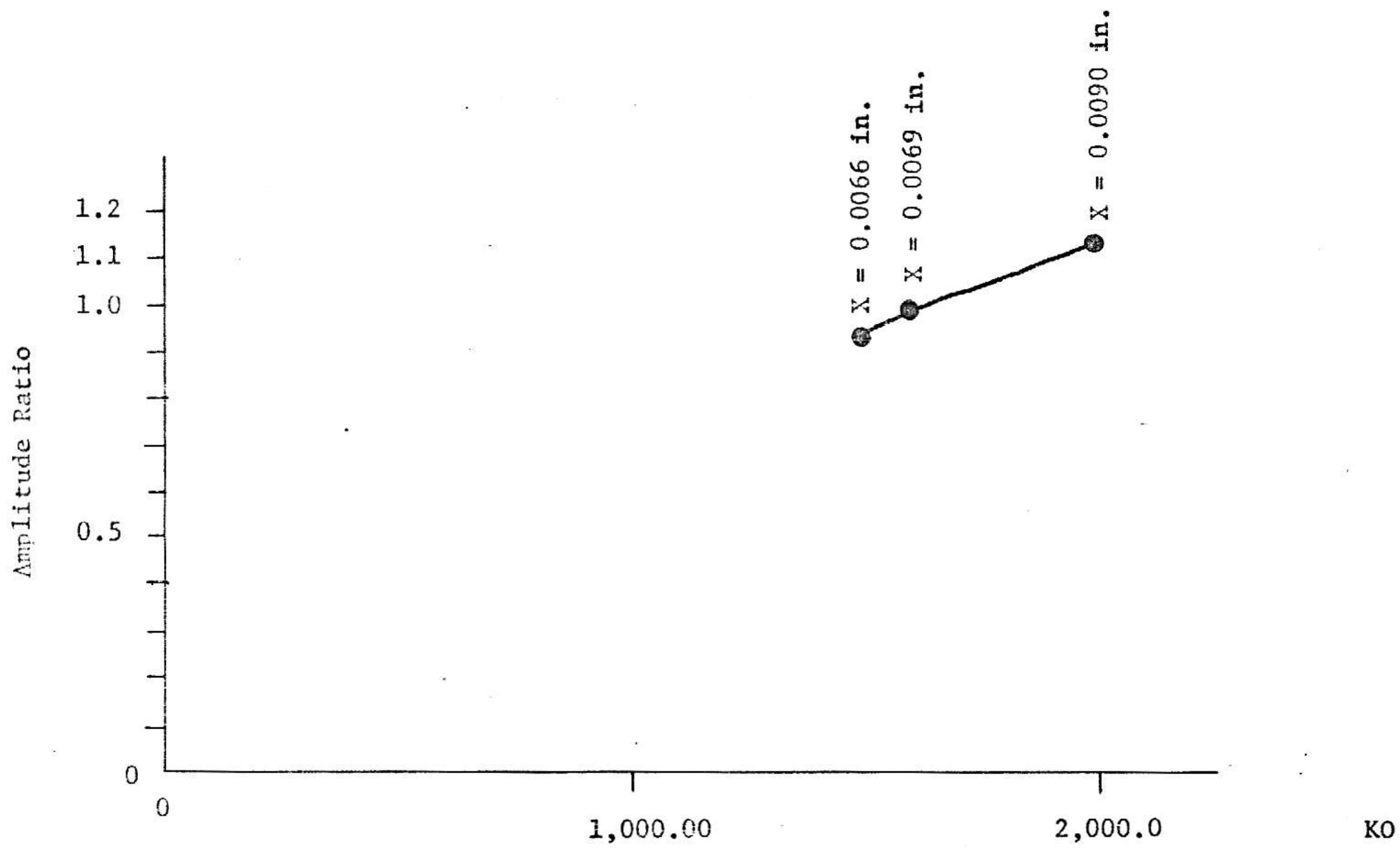

Figure 15: Amplitude ratio versus KO for a second order system having ω_n = 350.0, and ξ = 1.3 .

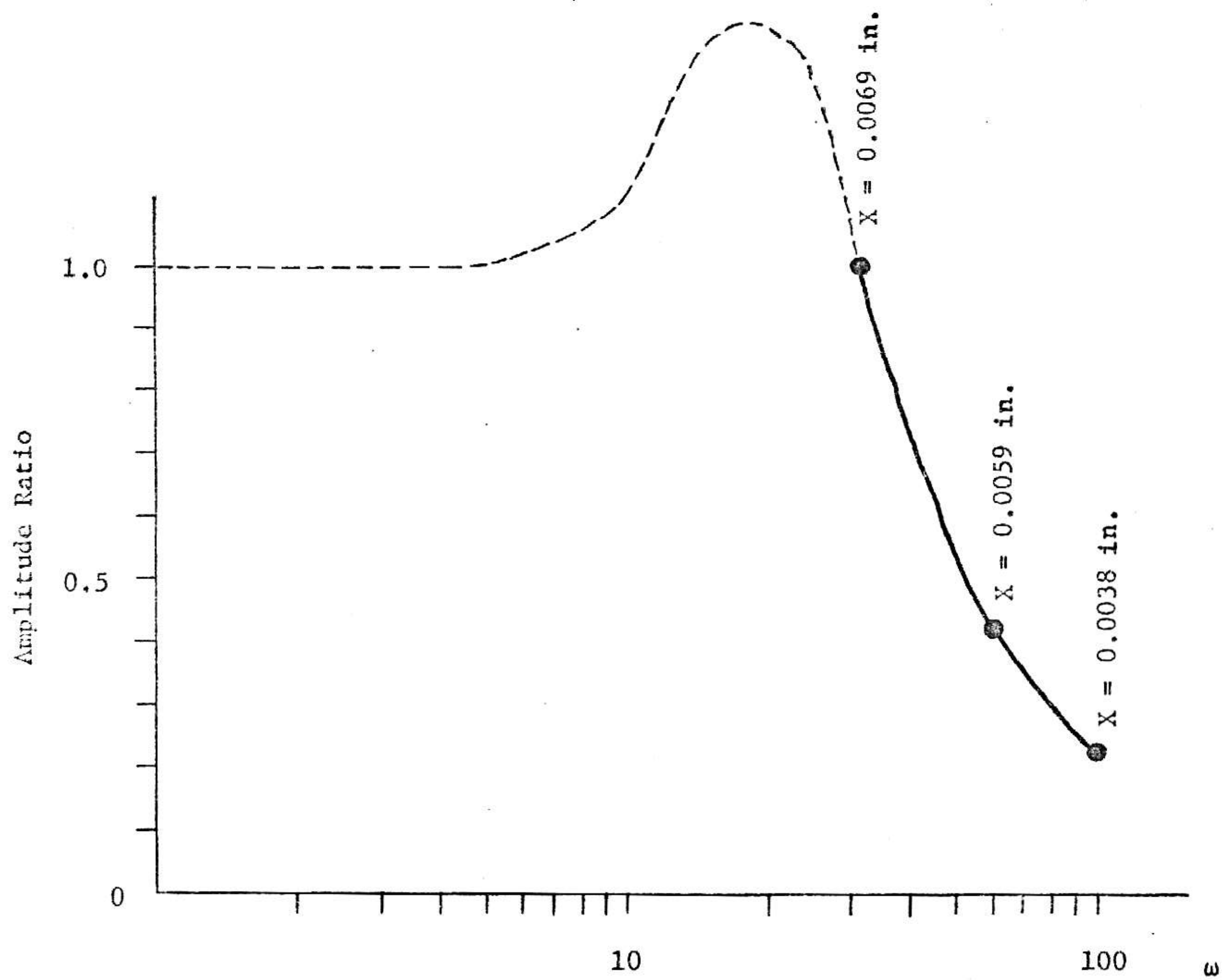

Figure 16: Frequency response for a second order system having $\omega_n = 350.0$, $\xi = 1.3$, and KO = 1650.0.

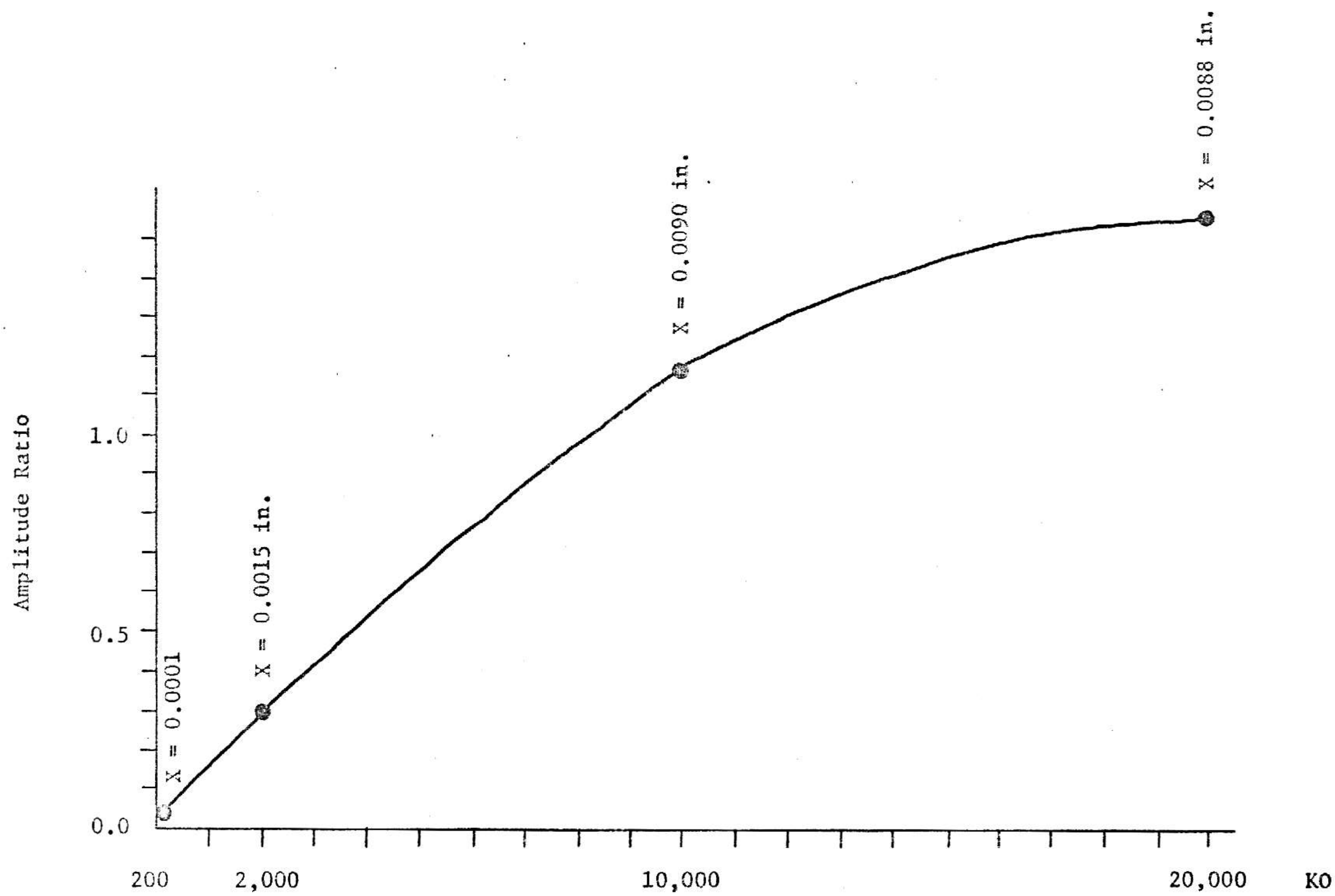

Figure 17: Amplitude ratio versus KO for a second order system having $\omega_n = 735.0$, and $\xi = 1.3$.

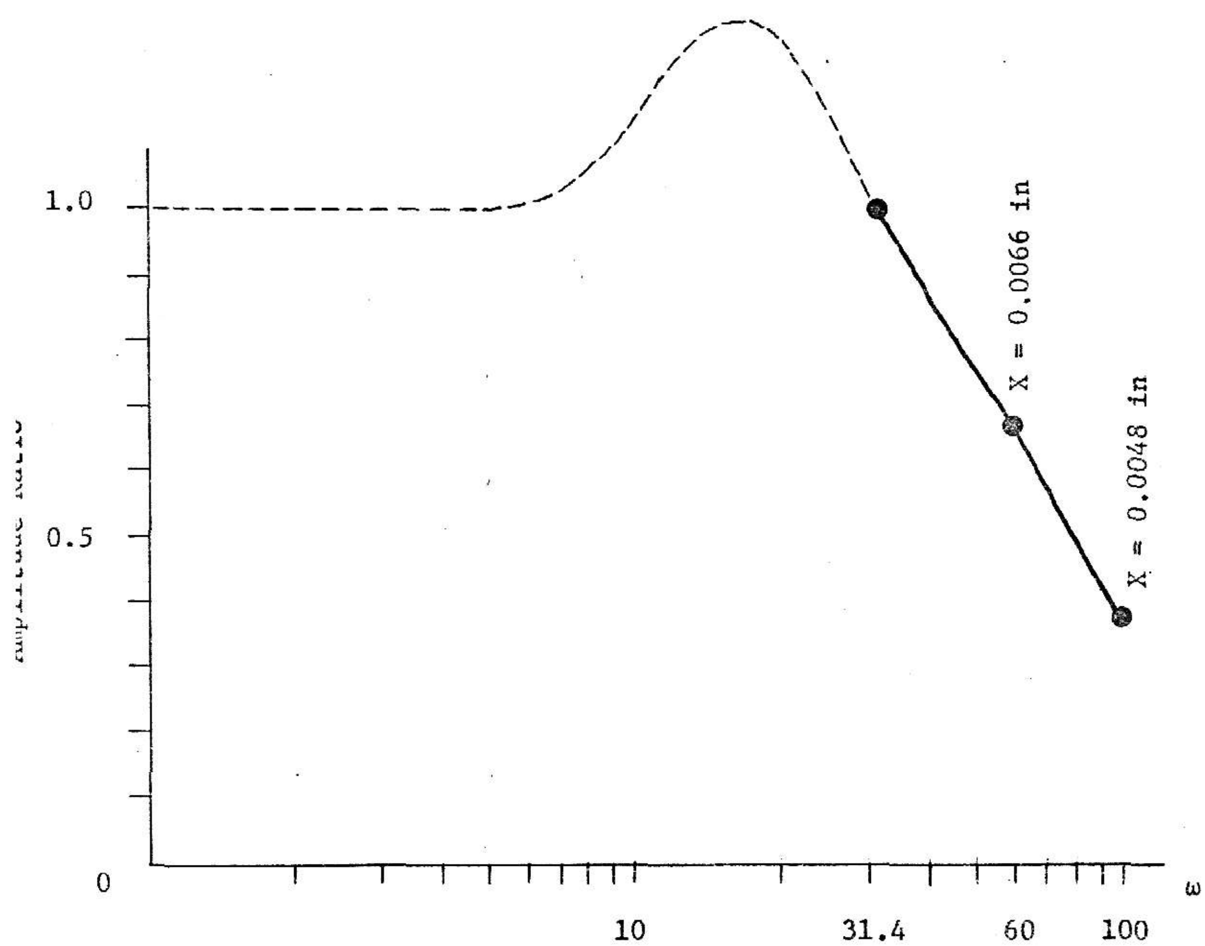

Figure 18: **Frequency response for a second order system having ω_n = 735.0, ξ = 1.3, and KO = 7880.0.**

Gupta obtained similar results in his report. He took the servovalve and cylinder as elements of an open-loop system and assumed that the servovalve spool was sinusoidally driven at the design frequency with a peak-to-peak amplitude of $2X_{max}$. When this was done he consistently obtained maximum values for Y which exceeded the design value for Y.

Figure 19 illustrates how the amplitude ratio varies as the compressed volume of fluid between the servovalve and cylinder is increased. To construct this graph the case of a first order model for the servoamplifier and servovalve torque motor in the closed loop system was used. The constants for the first order element are shown in Figure 10. For compressed volumes exceeding 270 $in.^3$ the amplitude ratio drops off linearly. This graph can also be interpreted as showing the effects of changing the value of the fluid bulk modulus of a system since increasing the compressed volume has the same effect as lowering the fluid bulk modulus. In a real system the fluid bulk modulus is lowered whenever air is entrained in hydraulic fluid (4). For example, if the entrained air is 10% of the total volume, the bulk modulus is reduced by approximately 50%.

In all six cases of first and second order systems studied a point was reached at which the amplitude ratio was constant for all greater values of gain. In light of the relationship indicated in Figure 19 this would seem to indicate that if the compressed volume of fluid is too large, or the value of bulk modulus too small, then it is not possible to obtain an amplitude ratio of unity.

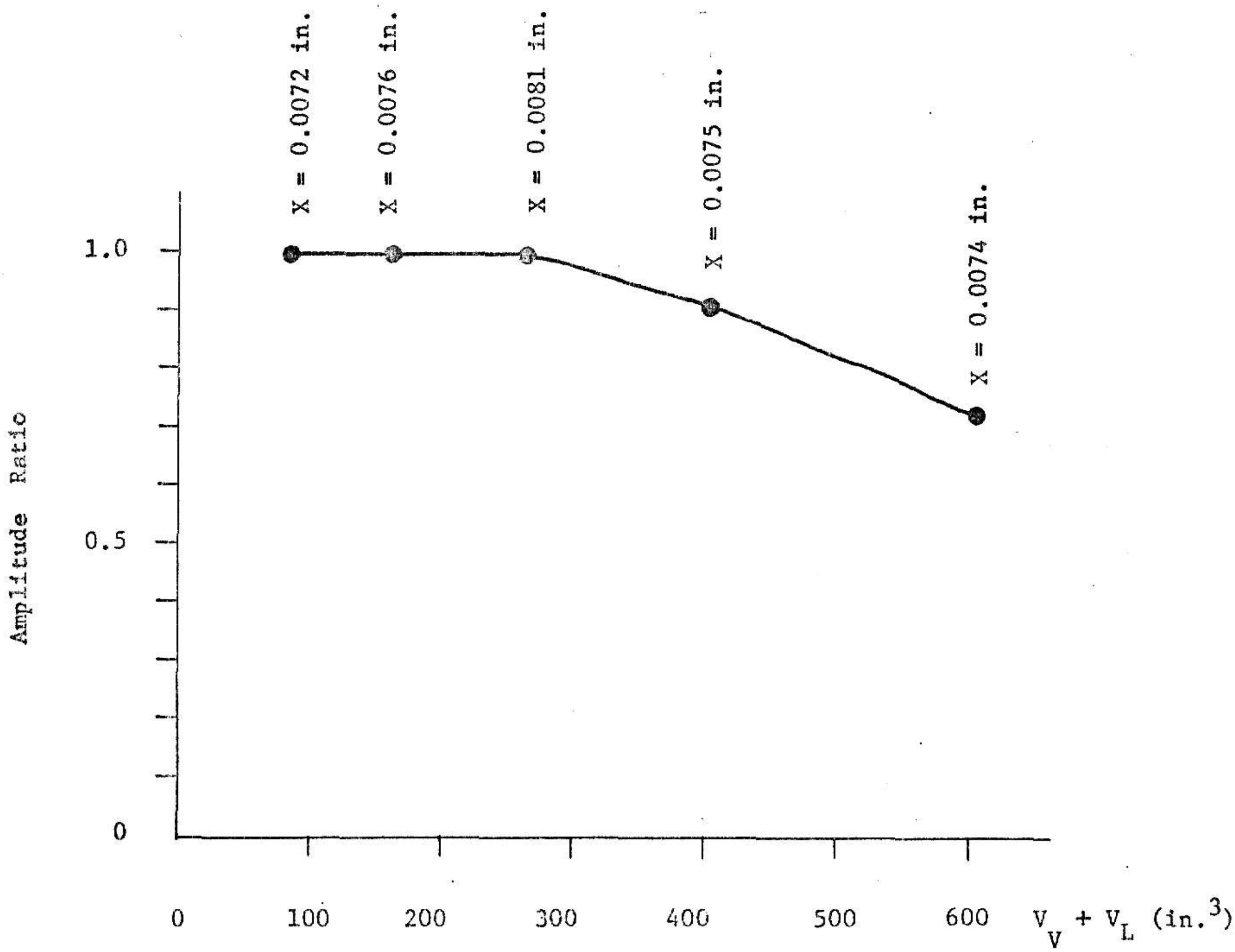

Figure 19: Amplitude Ratio as a function of compressed volume for a first order system where ω = 31.41 radians/second, T1 = 0.00081 seconds, K0 = 20.0, and β = 220,000 lb/in^2.

CHAPTER V

CONCLUSIONS

When the servovalve and cylinder for a hydraulic servosystem were designed using the maximum power method, and the closed loop performance of this system studied, the following conclusions were reached:

1. The maximum value of spool travel X_{max}, which is calculated using the design method is never attained for the amplitude ratio of unity, with an input amplitude equal to the design amplitude, regardless of input frequency. This can either be viewed as an overdesign or as a margin of safety against inadequate assumptions used in the design method, as discussed in Chapter IV.

2. The size of the compressed volume of fluid between the servovalve and cylinder is critical in determining closed loop system performance. The compressed volume should be as small as possible. The effect of an increase in compressed volume can also be obtained by lowering the fluid bulk modulus as was discussed in Chapter III. As an example of this, the largest compressed volume allowable with a value of β = 220,000 lb/in^2 was 270 in.^{3} (for which an amplitude ratio of unity was obtained); this is equivalent to stating that for a compressed volume of 100 in^3 the smallest allowable value of fluid bulk modulus is 8,150 lbs/in^2 (which allows for an amplitude ratio of one). It is possible for the compressed volume to place a limit on the amplitude ratio obtainable from a given system.

3. Linearized models for the servovalve and cylinder bear only faint resemblence to the more accurate non-linear mathematical model, therefore results obtained from linear models should be interpreted with care.

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APPENDIX

ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE

```
C     THIS PROGRAM CALCULATES KO AS A FUNCTION OF T1 FOR A COM-
C     PLETELY LINEARIZED SYSTEM.  DP IS THE CHANGE IN PRESSURE
C     OPPOSING FLUID FLOW.  X IS A SMALL VALUE OF SPOOL TRAVEL
C     NEAR THE STEADY STATE POSITION (X=0).  F IS THE VALUE
C     OF THE VISCOUS DAMPING CONSTANT FOR THE SYSTEM.
      REAL L,KI,N,N1,K2,K3,L1,T1,KO,M
      C=0.625
      P=3.14
      D=0.25
      X=0.001
      DP=200
      RO=0.0000813
      A=1.234
      M=1.55
      W=31.41
      F=100
      R=0.25
      L=((2*DP)/RO)
      L1=SQRT(L)
      KI=C*P*D*L1
      N=(1/(2*RO*DP))
      N1=SQRT(N)
      K2=C*P*D*X*N1
      K3=(A*KI)/(M*K2)
      T3=(1/K3)/((A/KI)+((F*K2)/(A*KI)))
      DO 33 J=1,1000,10
      T1=J*(0.01)
      KO=(((W/(T1*T3))-(W**3))**2+((((1/T1)+(1/T3))**2)*(W**4))
     C)/(2*K3*((1/T1)+(1/T3))*(W**2))
      Z=(KO*K3-((1/T1)+(1/T3))*(W**2))**2+((W/(T1*T3))-(W**3))**2
      Z1=SQRT(Z)
      Z2=((-1)*KO*(W**2))**2+((KO*W)/T3)**2
      Z3=SQRT(Z2)
      X1=(R*Z3)/Z1
      WRITE(6,22) KO,T1,X1
   22 FORMAT(3F10.4)
   33 CONTINUE
      RETURN
      END
```

APPENDIX

SUMMARY OF SERVOVALVE AND CYLINDER EQUATIONS

The motion of the loaded hydraulic cylinder is governed by the equation:

$$A(P_L - P_r) = (m_p + m_L)\,\ddot{Y} + (C_p + C_L)\,\dot{Y} \tag{A.1}$$

The following equations apply for the servovalve-cylinder combination but only when the conditions on X, P_L, and P_r are as specified in each of the five cases listed. Also the first six assumptions listed on page 5 apply.

<u>X = 0</u>

$$\frac{(-A\dot{Y})\ (\beta)}{(V_V + V_L + AY)} = \dot{P}_L \tag{A.2}$$

$$\frac{(A\dot{Y})\ (\beta)}{(V_V + V_L + A(L - b - Y))} = \dot{P}_r \tag{A.3}$$

<u>$X > 0\ ,\ P_S > P_L$</u>

$$\frac{\beta\ \left((C_d)(\pi)(d)(X)\sqrt{\frac{2}{\rho}\ (P_S - P_L)} - A\dot{Y}\right)}{(V_V + V_L + AY)} = \dot{P}_L \tag{A.4}$$

$P_r > 0$

$$\frac{\beta((-C_d)(\pi)(d)(X)\sqrt{\frac{2}{\rho} P_r} + A\dot{Y})}{(V_V + V_L + A\ (L-b-Y))} = \dot{P}_r \tag{A.5}$$

$P_r < 0$

$$\frac{\beta((C_d)(\pi)(d)(X)\sqrt{\frac{2}{\rho} |P_r|} + A\dot{Y})}{(V_V + V_L + A\ (L-b-Y))} = \dot{P}_r \tag{A.6}$$

$\underline{X > 0,\ P_S < P_L}$

$$\frac{\beta((-C_d)(\pi)(d)(X)\sqrt{\frac{2}{\rho} (P_L-P_S)} - A\dot{Y})}{(V_V + V_L + AY)} = \dot{P}_L \tag{A.7}$$

$P_r > 0$

$$\frac{\beta((C_d)(\pi)(d)(X)\sqrt{\frac{2}{\rho} P_r} + A\dot{Y})}{(V_V + V_L + A\ (L-b-Y))} = \dot{P}_r \tag{A.8}$$

$P_r < 0$

$$\frac{\beta((C_d)(\pi)(d)(X)\sqrt{\frac{2}{\rho}\ |P_r|} + A\dot{Y})}{(V_V + V_L + A\ (L-b-Y))} = \dot{P}_r \tag{A.9}$$

$\underline{X<0,\ P_S>P_r}$

$$\frac{\beta((-C_d)(\Pi)(d)(X)\sqrt{\frac{2}{\rho}\ (P_S - P_r)} + A\dot{Y})}{(V_V + V_L + A\ (L-b-Y))} = \dot{P}_r \tag{A.10}$$

$P_L > 0$

$$\frac{\beta((C_d)(\pi)(d)(X)\sqrt{\frac{2}{\rho}\ P_L} \quad -A\dot{Y})}{(V_V + V_L + AY)} = \dot{P}_L \tag{A.11}$$

$P_L < 0$

$$\frac{\beta((-C_d)(\pi)(d)(X)\sqrt{\frac{2}{\rho}\ |P_L|} \quad -A\dot{Y})}{(V_V + V_L + AY)} = \dot{P}_L \tag{A.12}$$

$\underline{X < 0,\ P_S < P_r}$

$$\frac{\beta((C_d)(\pi)(d)(X)\sqrt{\frac{2}{\rho}(P_r - P_S)} + A\dot{Y})}{(V_V + V_L + A\ (L-b-Y)} = \dot{P}_r \qquad (A.13)$$

PL > 0

$$\frac{\beta((C_d)(\pi)(d)(X)\sqrt{\frac{2}{\rho}\,PL} - A\dot{Y})}{(V_V + V_L + AY)} = \dot{P}_L \qquad (A.14)$$

PL < 0

$$\frac{\beta((-C_d)(\pi)(d)(X)\sqrt{\frac{2}{\rho}\,|P_L|} - A\dot{Y})}{(V_V + V_L + AY)} = \dot{P}_L \qquad (A.15)$$

The above equations make up the block labeled "nonlinear equations" in Figures A.1 and A.2. These equations were used in conjunction with the differential equations obtained from the transfer function representations of the servoamplifier and servotorque motor.

Figure A.1 shows a closed-loop system in which a first order model is used to represent the servoamplifier and servovalve motor. The relationship between X and the error signal E is given by:

$$\frac{X}{E} = \frac{KO}{S + \frac{1}{T1}} \qquad (A.16)$$

$$SX + \frac{X}{T1} = (KO)E \qquad (A.17)$$

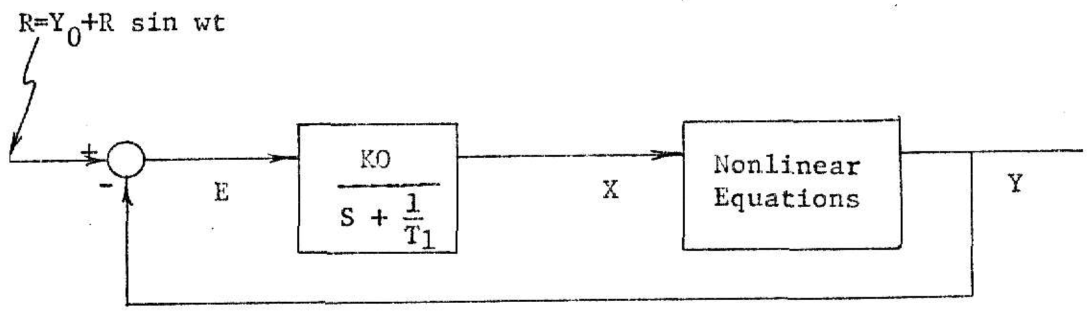

Figure A1: Closed loop system in which a first order system is used to represent servoamplifier and servovalve torque motor.

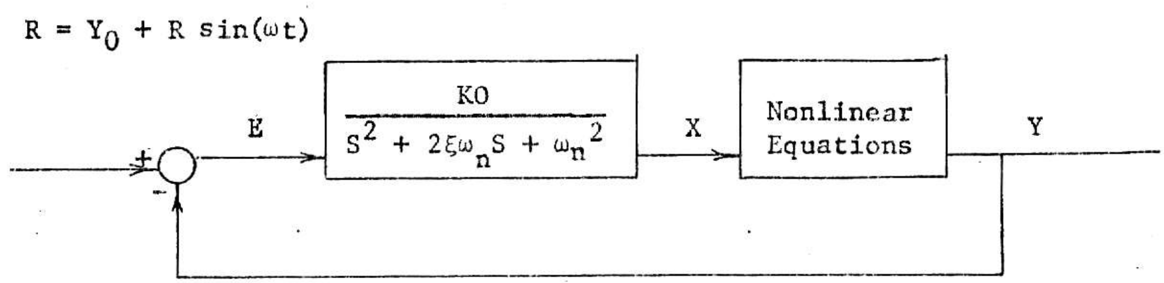

Figure A.2: Closed loop system in which a second order model is used to represent the servoamplifier and servotorque motor.

When this equation is converted to the time domain it becomes:

$$\frac{dX}{dt} = (KO)E - \frac{X}{T1} \tag{A.18}$$

To simulate this system on a digital computer it is first made a discrete system by dividing the time domain solution into small increments of time. Initial values of P_L, P_r, Y, X, and time are assumed and then as time is slowly incremented $\dot{X}$, $\dot{P}_r$, $\dot{P}_L$, and $\dot{Y}$ are calculated at each discrete point. A recurrence formula of the type:

$$X_S = X_{S-1} + h \left.\frac{dX}{dt}\right|_s \tag{A.19}$$

is then used where h = a small increment of time.

By using this relationship a time domain solution for Y(t) can be derived. Figure A.3 hows the typical results for y compared with the assumed sinusoidal input to the closed loop system. The computer program which does this for the complete closed loop system (with first order model) is on the following pages. The computer symbols are listed in Table I.

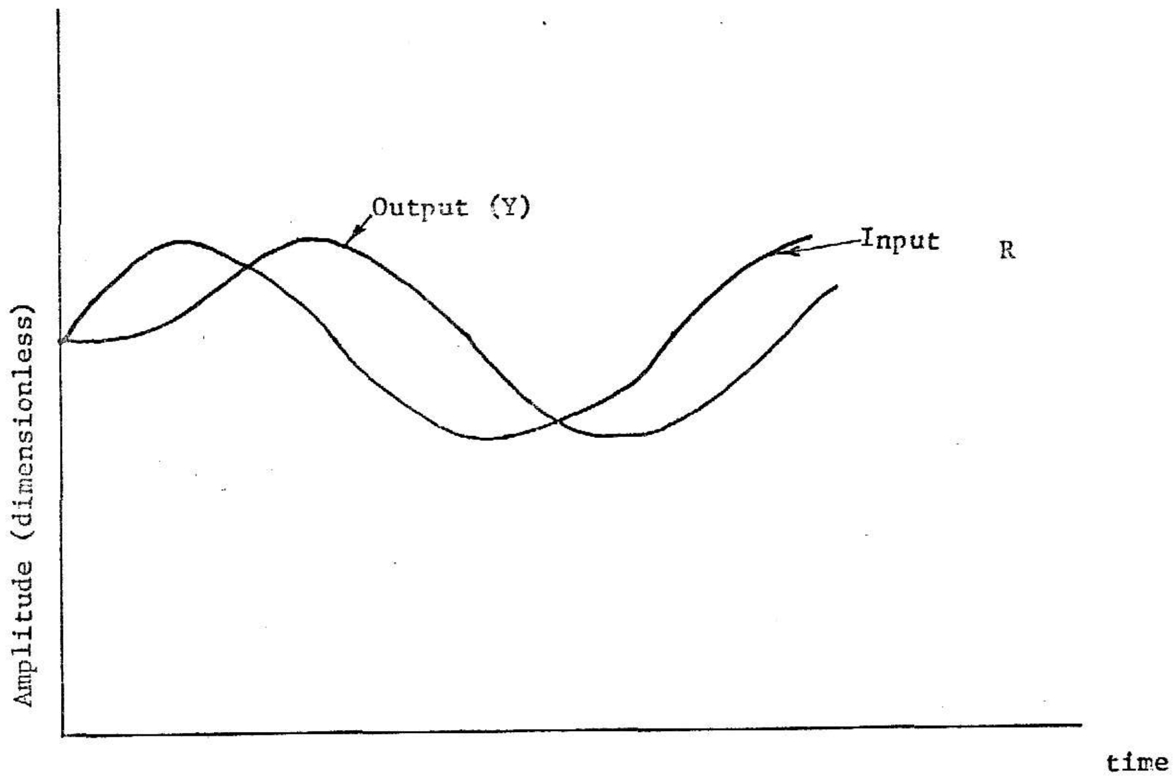

Figure A.3: Plot of typical output (Y) of computer program as compared to input ($Y_0 + R \sin \omega t$).

TABLE A1

Digital Computer Symbols

A = area of actuator piston
B = fluid bulk modulus
BE = thickness of actuator piston
BL = damping coefficient of load
BP = damping coefficient of actuating piston
CD = discharge coefficient
D = spool diameter
DDX = second derivative of X
DDY = second derivative of Y
DX = first derivative of X
DY = first derivative of Y
E = damping ratio
H = differential time increment
KO = gain
L = length of actuating cylinder
ML = mass of load
MP = mass of piston
PI = π
PL = pressure on left side of actuating piston
PR = pressure on right side of actuating piston
PS = supply pressure
R = amplitude of forcing function
RO = fluid density
T = time
T1 = time constant
VL = compressed volume of fluid in lines
W = compressed volume of fluid in valve
W = design frequency
WN = natural frequency
X = spool displacement
Y = actuating piston displacement

```
      KO=9500.0
      WN=730.0
      E=0.5
      VV=80.0
      VL=20.0
      H=0.001
      W=31.14
      T1=0.00081
      R=0.25
      PS=1000.0
      B=220000
      CD=0.625
      PI=3.1416
      D=0.25
      RO=0.0000813
      A=1.284
      L=4.0
      BE=0.5
      BP=10.0
      BL=90.0
      ML=1.49
      MP=0.065
      DX=0.0
      X=0.0
      Y=1.75
      DY=0.0
      PL=500.0
      PR=500.0
      DO 99 J=1,500,1
      T=J*H
      DX=(KO*((1.75+(R*SIN(W*T)))-Y))-(X/T1)
      IF(X.GT.0.0) GO TO 11
      IF(X.LT.0.0) GO TO 22
      IF(X.EQ.0.0) GO TO 33
   11 CONTINUE
      IF(X.GT.0.0098) X=0.0098
      IF(PS.GT.PL) GO TO 44
      DPL=B*(((-1)*CD*PI*D*X*SQRT((2/RO)*(PL-PS)))-(A*DY))/(VV+
     CVL+(A*Y))
      IF(PR.GE.0.0) DPR=B*((CD*PI*D*X*SQRT((2/RO)*PR))+(A*DY))/
     C(VV+VL+A*(L-BE-Y))
      IF(PR.LT.0.0) DPR=B*((CD*PI*D*X*SQRT((2/RO)*(-1)*(PR)))+
     C(A*DY))/(VV+VL+(A*(L-BE-Y)))
      DDY=((A*PL)-(A*PR)-((BP+BL)*DY))/(MP+ML)
      PL=PL+(DPL*H)
      PR=PR+(DPR*H)
      Y=Y+(DY*H)
      IF(Y.GT.3.5) Y=3.5
      IF(Y.LT.0.0) Y=0.0
      DY=DY+(DDY*H)
      X=X+(DX*H)
      WRITE(6,12) PL,PR,DY,Y,X,T
   12 FORMAT (1H ,6F14.4)
      GO TO 99
   44 CONTINUE
      DPL=B*((CD*PI*D*X*SQRT((2/RO)*(PS-PL)))-(A*DY))/(VV+VL+(A
     C*Y))
      IF(PR.GE.0.0) DPR=B*(((-1)*CD*PI*D*X*SQRT((2/RO)*PR))-(A*
     CDY))/(VV+VL+(A*(L-BE-Y)))
```

```
     CDY))/(VV+VL+(A*(L-BE-Y)))
      DDY=((A*PL)-(A*PR)-((BP+BL)*DY))/(MP+ML)
      PL=PL+(DPL*H)
      PR=PR+(DPR*H)
      Y=Y+(DY*H)
      IF(Y.GT.3.5) Y=3.5
      IF(Y.LT.0.0) Y=0.0
      DY=DY+(DDY*H)
      X=X+(DX*H)
      WRITE(6,45) PL,PR,DY,Y,X,T
   45 FORMAT(1H ,6F14.4)
      GO TO 99
   22 CONTINUE
      IF(X.LT.-0.0098) X=-0.0098
      IF(PS.GE.PR) GO TO 55
      DPR=B*(CD*PI*D*X*SQRT((2/RO)*(PR-PS))+(A*DY))/(VV+VL+A*(L
     C-BE-Y))
      IF(PL.GE.0.0) DPL=B*((CD*PI*D*X*SQRT((2/RO)*PL))-(A*DY))/
     C(VV+VL+(A*Y))
      IF(PL.LT.0.0) DPL=B*(((-1)*CD*PI*D*X*SQRT((2/RO)*(-1)*PL)
     C)-(A*DY))/(VV+VL+(A*Y))
      DDY=((A*PL)-(A*PR)-((BP+BL)*DY))/(MP+ML)
      PL=PL+(DPL*H)
      PR=PR+(DPR*H)
      Y=Y+(DY*H)
      IF(Y.GT.3.5) Y=3.5
      IF(Y.LT.0.0) Y=0.0
      DY=DY+(DDY*H)
      X=X+(DX*H)
      WRITE(6,23) PL,PR,DY,Y,X,T
   23 FORMAT (1H ,6F14.4)
      GO TO 99
   55 CONTINUE
      DPR=B*(((-1)*CD*PI*D*X*SQRT((2/RO)*(PS-PR)))+(A*DY))/(VV
     C+VL+(A*(L-BE-Y)))
      IF(PL.GE.0.0) DPL=B*((CD*PI*D*X*SQRT((2/RO)*PL))-(A*DY))/
     C(VV+VL+(A*Y))
      IF(PL.LT.0.0) DPL(B*((-1)*CD*PI*D*X*SQRT((2/RO)*(-1)*(PL
     C)))-(A*DY)/(VV+VL+(A*Y))
      DDY=((A*PL)-(A*PR)-((BP+BL)*DY))/(MP+ML)
      PL=PL+(DPL*H)
      PR=PR+(DPR*H)
      Y=Y+(DY*H)
      IF(Y.GT.3.5) Y=3.5
      IF(Y.LT.0.0) Y=0.0
      DY=DY+(DDY*H)
      X=X+(DX*H)
      WRITE(6,56) PL,PR,DY,Y,X,T
   56 FORMAT (1H ,6F14.4)
      GO TO 99
   33 CONTINUE
      DPL=((-1)*B*A*DY)/(VV+VL+(A*Y))
      DPR=(A*DY*B)/(VV+VL+(A*(L-BE-Y)))
      DDY=((A*PL)-(A*PR)-((BP+BL)*DY))/(MP+ML)
      PL=PL+(DPL*H)
      PR=PR+(DPR*H)
      Y=Y+(DY*H)
      IF(Y.GT.3.5) Y=3.5
      IF(Y.LT.0.0) Y=0.0
```

```
      X=X+(DX*H)
      WRITE(6,34) PL,PR,DY,Y,X,T
   34 FORMAT (1H ,6F14.4)
   99 CONTINUE
      RETURN
      END
```

Figure A.2 shows a closed-loop system in which a second order model is used to represent the servoamplifier and servovalve torque motor. The relationship between X and the error signal e is given by:

$$\frac{X}{E} = \frac{KO}{S^2 + 2\xi\omega s_n + \omega_n^2} \qquad (A.20)$$

$$S^2X + 2\xi\omega_n SX + \omega_n^2 X = (KO)E \qquad (A.21)$$

when this equation is converted to the time domain it becomes:

$$\frac{d^2X}{dt^2} + 2\xi\omega_n \frac{dX}{dt} + \omega_n^2 X = (KO)E \qquad (A.22)$$

To simulate this system on a digital computer it is first made a discrete system by dividing the time domain solution into small increments of time. Initial values of P_L, P_r, Y, X, X, and time are assumed and then as time is slowly incremented $\dot{X}$, $\dot{P}_r$, $\dot{P}_L$, $\ddot{Y}$, and $\dot{Y}$ are calculated at each discrete point. A recurrence formula of the type:

$$X_S = X_{S-1} + h \left.\frac{dX}{dt}\right|_S \qquad (A.23)$$

is then used where h = a small increment of time.

By using this relationship a time domain solution for Y(t) can be derived. The computer program which performs the computations for the complete closed loop system (with second order model) is on the following pages.

```
      KO=9500.0
      WN=730.0
      E=0.5
      VV=80.0
      VL=20.0
      H=0.001
      W=31.14
      T1=0.00081
      R=0.25
      PS=1000.0
      B=220000
      CD=0.625
      PI=3.1416
      D=0.25
      RO=0.0000813
      A=1.284
      L=4.0
      BE=0.5
      BP=10.0
      BL=90.0
      ML=1.49
      MP=0.065
      DX=0.0
      X=0.0
      Y=1.75
      DY=0.0
      PL=500.0
      PR=500.0
      DO 99 J=1,500,1
      T=J*H
      DDX=KO*(1.75+(R*SIN(W*T))-Y)-(2*E*WN*DX)-((WN**2)*X)
      IF(X.GT.0.0) GO TO 11
      IF(X.LT.0.0) GO TO 22
      IF(X.EQ.0.0) GO TO 33
    1 CONTINUE
      IF(X.GT.0.0098) X=0.0098
      IF(PS.GT.PL) GO TO 44
      DPL=B*(((-1)*CD*PI*D*X*SQRT((2/RO)*(PL-PS)))-(A*DY))/(VV+
     CVL+(A*Y))
      IF(PR.GE.0.0) DPR=B*((CD*PI*D*X*SQRT((2/RO)*PR))+(A*DY))/
     C(VV+VL+A*(L-BE-Y))
      IF(PR.LT.0.0) DPR=B*((CD*PI*D*X*SQRT((2/RO)*(-1)*(PR)))+
     C(A*DY))/(VV+VL+(A*(L-BE-Y)))
      DDY=((A*PL)-(A*PR)-((BP+BL)*DY))/(MP+ML)
      PL=PL+(DPL*H)
      PR=PR+(DPR*H)
      Y=Y+(DY*H)
      IF(Y.GT.3.5) Y=3.5
      IF(Y.LT.0.0) Y=0.0
      DY=DY+(DDY*H)
      X=X+(DX*H)
      DX=DX+(H*DDX)
      WRITE(6,12) PL,PR,DY,Y,X,T
    2 FORMAT (1H ,6F14.4)
      GO TO 99
    4 CONTINUE
      DPL=B*((CD*PI*D*X*SQRT((2/RO)*(PS-PL)))-(A*DY))/(VV+VL+(A
     C*Y))
      IF(PR.GE.0.0) DPR=B*(((-1)*CD*PI*D*X*SQRT((2/RO)*PR))-(A*
     CDY))/(VV+VL+(A*(L-BE-Y)))
```

```
      IF(PR.LT.0.0) DPR=B*((CD*PI*D*X*SQRT((2/RO)*(-1)*PR))-(A*
     CDY))/(VV+VL+(A*(L-BE-Y)))
      DDY=((A*PL)-(A*PR)-((BP+BL)*DY))/(MP+ML)
      PL=PL+(DPL*H)
      PR=PR+(DPR*H)
      Y=Y+(DY*H)
      IF(Y.GT.3.5) Y=3.5
      IF(Y.LT.0.0) Y=0.0
      DY=DY+(DDY*H)
      X=X+(DX*H)
      DX=DX+(H*DDX)
      WRITE(6,45) PL,PR,DY,Y,X,T
   45 FORMAT(1H ,6F14.4)
      GO TO 99
   22 CONTINUE
      IF(X.LT.-0.0098) X=-0.0098
      IF(PS.GE.PR) GO TO 55
      DPR=B*(CD*PI*D*X*SQRT((2/RO)*(PR-PS))+(A*DY))/(VV+VL+A*(L
     C-BE-Y))
      IF(PL.GE.0.0) DPL=B*((CD*PI*D*X*SQRT((2/RO)*PL))-(A*DY))/
     C(VV+VL+(A*Y))
      IF(PL.LT.0.0) DPL=B*(((-1)*CD*PI*D*X*SQRT((2/RO)*(-1)*PL)
     C)-(A*DY))/(VV+VL+(A*Y))
      DDY=((A*PL)-(A*PR)-((BP+BL)*DY))/(MP+ML)
      PL=PL+(DPL*H)
      PR=PR+(DPR*H)
      Y=Y+(DY*H)
      IF(Y.GT.3.5) Y=3.5
      IF(Y.LT.0.0) Y=0.0
      DY=DY+(DDY*H)
      X=X+(DX*H)
      DX=DX+(H*DDX)
      WRITE(6,23) PL,PR,DY,Y,X,T
   23 FORMAT (1H ,6F14.4)
      GO TO 99
   55 CONTINUE
      DPR=B*(((-1)*CD*PI*D*X*SQRT((2/RO)*(PS-PR)))+(A*DY))/(VV
     C+VL+(A*(L-BE-Y)))
      IF(PL.GE.0.0) DPL=B*((CD*PI*D*X*SQRT((2/RO)*PL))-(A*DY))/
     C(VV+VL+(A*Y))
      IF(PL.LT.0.0) DPL(B*((-1)*CD*PI*D*X*SQRT((2/RO)*(-1)*(PL
     C)))-(A*DY)/(VV+VL+(A*Y))
      DDY=((A*PL)-(A*PR)-((BP+BL)*DY))/(MP+ML)
      PL=PL+(DPL*H)
      PR=PR+(DPR*H)
      Y=Y+(DY*H)
      IF(Y.GT.3.5) Y=3.5
      IF(Y.LT.0.0) Y=0.0
      DY=DY+(DDY*H)
      X=X+(DX*H)
      DX=DX+(H*DDX)
      WRITE(6,56) PL,PR,DY,Y,X,T
   56 FORMAT (1H ,6F14.4)
      GO TO 99
   33 CONTINUE
      DPL=((-1)*B*A*DY)/(VV+VL+(A*Y))
      DPR=(A*DY*B)/(VV+VL+(A*(L-BE-Y)))
      DDY=((A*PL)-(A*PR)-((BP+BL)*DY))/(MP+ML)
      PL=PL+(DPL*H)
```

```
      Y=Y+(DY*H)
      IF(Y.GT.3.5) Y=3.5
      IF(Y.LT.0.0) Y=0.0
      DY=DY+(DDY*H)
      X=X+(DX*H)
      DX=DX+(H*DDX)
      WRITE(6,34) PL,PR,DY,Y,X,T
   34 FORMAT (1H ,6F14.4)
   99 CONTINUE
      RETURN
      END
```

CLOSED LOOP PERFORMANCE OF HYDRAULIC SERVOSYSTEMS

DESIGNED BY THE MAXIMUM POWER METHOD

by

CHARLES KENNETH KNIFFIN

B. S., Kansas State University, 1968

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1973

This report is a study of how a four-way zero-lapped servovalve and a loaded double-ended, double-acting cylinder designed by the maximum power design method respond when placed within a closed loop system.

The closed loop system is first approximated by a completely linear model; then the servovalve and cylinder are represented by nonlinear equations which account for fluid compressibility and flow across the valve due to pressure drop. The other element in the closed loop system is represented by both first and second order transfer functions.

Computer analyses are made of the response of the closed loop system and results are obtained based upon these analyses. The conclusions reached specify what requirements are placed upon the parameters of the system so that an input of predetermined frequency and amplitude can be reproduced by the closed loop system. Conclusions are also reached concerning how either an increase in the compressed volume of fluid or a decrease in the fluid bulk modulus affect the ability of the closed loop system to reproduce a given sinusoidal input of predetermined frequency and amplitude.