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MEASUREMENTS OF TWO PROPERTIES  
OF CASCADE GAMMA-RAYS

by

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## INTRODUCTION

The nucleus of an atom is pictured as having several well defined states. These states are characterized partially by specifying their energy above a ground state and their total angular momentum and parity. A nucleus is said to decay when it spontaneously undergoes a transition from a state to one of lower energy. There are a number of processes by which a transition can occur, all of which involve transfer of quantities of energy, angular momentum, and possibly charge away from the nucleus. The various conservation laws require that these quantities appear associated with other products of the decay which can be detected such as particles and quanta. Knowledge of the transition is obtained from measurements of these decay products and this knowledge implies information concerning the nuclear states themselves. As an example, a gamma-ray is characterized by giving its energy, angular momentum, and polarization; the measurement of these properties yields the energy, angular momentum, and parity differences, respectively, between the initial and final states of the decaying nucleus.

In most instances the mean life time of an excited nuclear state is so short that the transition leading to the state and the one leading from it are "coincident" within the resolving time of the measuring equipment which is usually adjusted to be less than one microsecond. When this happens, the two transitions are said to be in cascade. This thesis is concerned with the measurement of some properties of gamma-rays emitted in cascade and in particular with measurements known as fast-slow coincidence and directional correlation measurements.

Fast-slow coincidence (FSC) experiments give clues to the arrangement of the nuclear states on an energy scale by determining which transitions are in cascade with a particular transition.

Directional correlation measurements can determine the quantized angular momentum of each of the three states involved in a cascade by measuring the variation in the coincidence rate for the two gamma-rays as the angular separation of their detectors is changed.

The objectives of this work were: First, to build and confirm the successful operation of a transistorized circuit which could be combined with available equipment into a dual FSC configuration. Second, to adapt this configuration for directional correlation experiments and to confirm this operation by measuring a well known gamma-gamma directional correlation.

#### DUAL FSC EXPERIMENTS

In a single FSC experiment a multichannel analyzer (MCA) is directed to determine the energy of the events in one detector which are in coincidence with events of a particular energy in the other. Because of the necessity of avoiding extremely high counting rates in each detector the usual coincidence rate is quite small and hence a running time of several hundred minutes is required to obtain statistically accurate data. If the source under investigation emits a large number of gamma-rays with many cascades the acquisition of a time coincidence spectrum for each cascade becomes a lengthy process. It is desirable, therefore, to make more efficient use of the MCA by obtaining two or more coincidence spectra simultaneously.

One of the MCA's in this laboratory is a 512 channel Nuclear Data model 130-A which has the ability to store an analyzed pulse in either of two 256 channel memory groups depending upon external routing signals. The circuit of this thesis was designed to utilize this capability to allow the accumulation of data from two FSC experiments simultaneously.

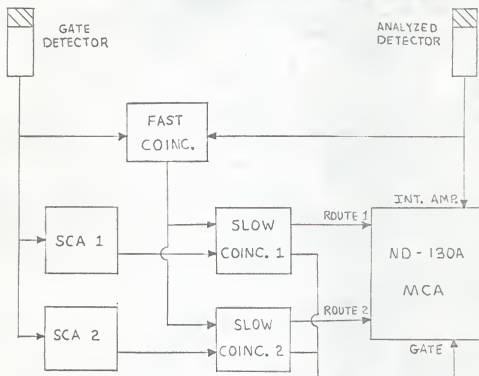
A simplified representation of the dual FSC configuration is shown in Plate I. The fast coincidence circuit generates an output if there is a time coincidence of events in the two detectors. Pulses from the "gate detector" of either of two predetermined heights are selected by the two single channel analyzers (SCA's). If there is a time coincidence corresponding to a pulse of either of these heights, then one of the two slow coincidence circuits delivers a gate signal to the MCA to permit it to process the pulse from the "analyzed detector". The MCA is directed to store the information in a particular half of its memory by a routing signal from the particular slow coincidence circuit which delivered the gate signal to the MCA. Thus the time coincidence spectra for two different energies can be collected during the same running time, one in each half of the MCA memory.

The circuit of this report is referred to as the slow unit and consists of two coincidence circuits with the addition of pulse shaping and delaying circuits to allow the use of input signals from various single channel analyzers and to tailor the output signals to the particular requirements of the Nuclear Data model 130A multichannel analyzer.

EXPLANATION OF PLATE I

A simplified representation of  
the dual FSC configuration.

## PLATE I



## DESIGN AND OPERATION OF THE EQUIPMENT

An SCA generates an output when an amplified input pulse is in the window between its lower and upper level discriminator settings. For several of the SCA's in this laboratory this output appears approximately two microseconds after the input at a time which depends upon the instant when the amplified input pulse first crossed the base line and tripped the lower level discriminator. All of the amplified input pulses have the same rise time so the processing time for each pulse is increased as the base line is raised because a longer time is required for the amplified pulse to attain the height of the base line. Further, the amplified pulses whose heights are near the bottom of the window will take longer to cross the base line than those whose heights are near the top. The resulting variation in processing time is known as "jitter".

These effects contribute to three problems which the slow unit was designed to solve. First, the pulses from the analyzed detector must be delayed until it is decided which of them are to be admitted to the analog-to-digital convertor of the MCA for analysis. Second, the gate signal to the MCA must overlap in time only the pulse to be analyzed so the gate must occur at a definite time after the events of interest in the two detectors. As this time is independent of the energy of the event in the gate detector the start of the gate must not be affected by the jitter of the SCA's. The routing signals need only arrive at the MCA within a "microsecond or two" (14) of the pulse to be analyzed. Third, because the output of the fast coincidence circuit is almost prompt it must be delayed to be coincident with the corresponding output of an SCA.

The first problem was solved by constructing a delaying circuit consisting of a delay line between input and output emitter followers as shown in the schematic diagram in Fig. 3 of Plate III. The overall gain of this circuit is approximately one half. The Technitrol delay line sticks have continuously distributed parameters and can be combined into two stick combinations so that total delays of 1, 2, 3, 4, or 5 microseconds can be easily obtained with the 1, 2, and 3 microsecond sticks on hand. An additional 2 microseconds of delay is obtainable by using the internal amplifier of the MCA.

The solutions of the second and third problems are related. The triggered output of the fast coincidence circuit is sufficiently jitter free that it is delayed electronically and a shaped gate to the MCA is caused to occur at a given time after the start of this fast coincidence output. Exactly how this is accomplished is made apparent by discussing the circuitry of the slow unit. The schematic diagram is shown in Plates II and III.

### Slow Unit Circuitry

An RG-63/U cable from the TRIGGERED OUTPUT of the fast coincidence circuit is terminated at the slow unit input labeled EH. The positive output is differentiated and only the positive going spike corresponding to its leading edge is passed by a diode. This spike triggers a single-shot multivibrator ( $T_1, T_2$ ) whose output pulse width increases as the resistance  $R_A$  increases. This multivibrator's positive output is now differentiated and only the negative going spike corresponding to its trailing edge is passed by a diode and used to trigger another single-shot

## EXPLANATION OF PLATE II

The schematic diagram of the slow unit. (Cont. to Plate III)

Fig. 1. EH input section.

Fig. 2. SCA 1 input section.

Fig. 3. Voltage divider.

PLATE II

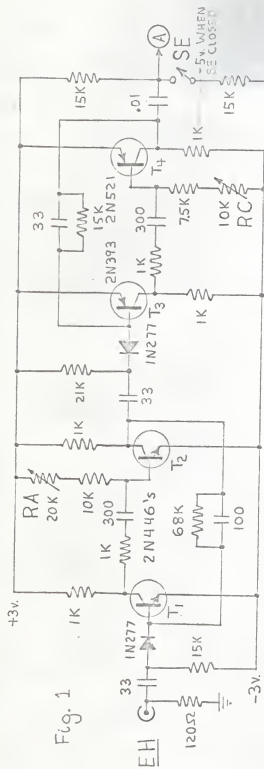


Fig. 1

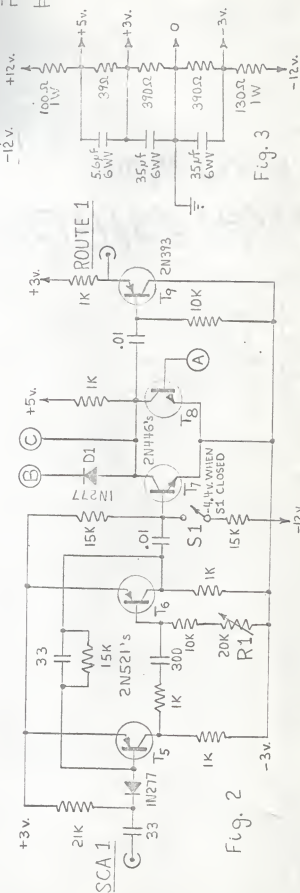


Fig. 2

Fig. 3

### EXPLANATION OF PLATE III

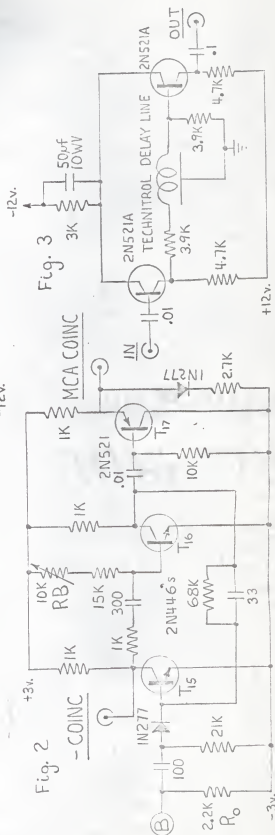
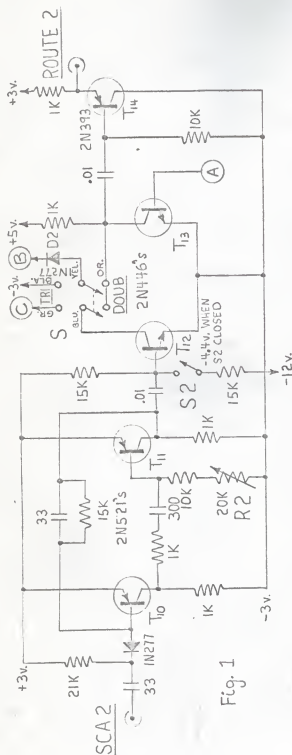
The schematic diagram of the slow unit. (Cont. from Plate II)

Fig. 1. SCA 2 input section.

Fig. 2. MCA COINC output section.

Fig. 3. Delaying circuit for pulses to be analyzed.  
(After ND 130A internal amplifier (14).)

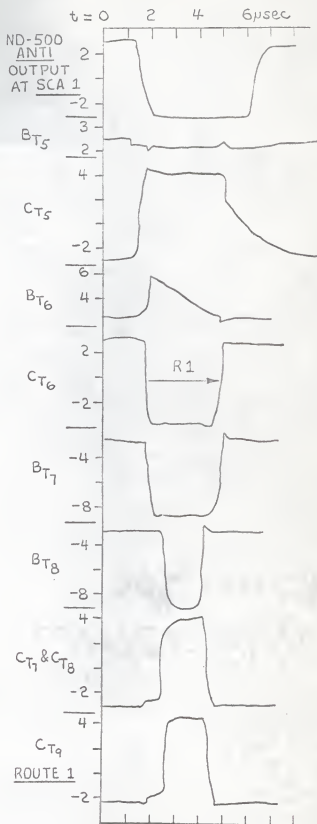
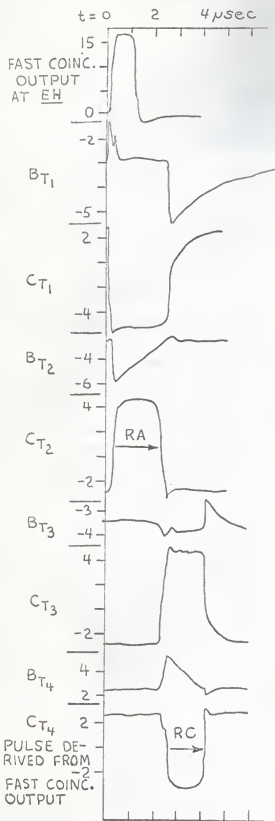
PLATE III



#### EXPLANATION OF PLATE IV

The waveforms seen at various points in the slow unit.  
The voltages were measured with a Tektronix 551 scope with a calibrated 53/54 L preamplifier and a  $\times 10$  attenuator probe. Switches SE, S1, and S2 were open and switch S was in the DOUB position. No input was connected to SCA 2.  $R_{T_1}$  means the base of transistor  $T_1$ , etc. (Cont. to Plate V).

## PLATE IV

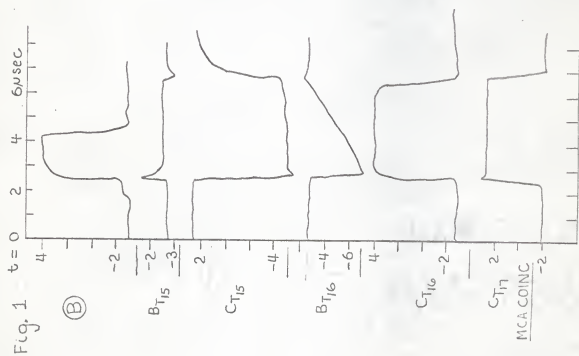
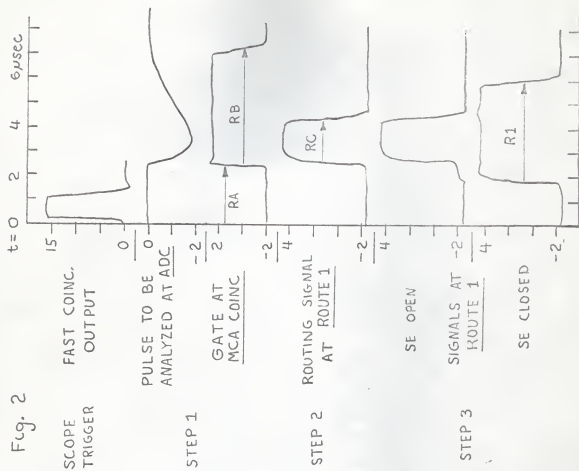


EXPLANATION OF PLATE V

Fig. 1. The waveforms seen at various points in slow unit. (Cont. from Plate IV)

Fig. 2. The waveforms seen during the timing procedure. The pulses from the Triggered Output of the EH fast coincidence circuit were used to trigger the scope in all cases.

## PLATE V



multivibrator ( $T_3, T_4$ ). Thus the start of the output of the second multivibrator is delayed from the start of the fast coincidence output by an amount which can be varied with  $RA$ . The duration of this second multivibrator's output increases as the resistance  $RC$  increases. In the following discussion this negative going rectangular pulse at (A) is referred to as the pulse derived from the fast coincidence output and is applied to the base of  $T_8$  which is one side of a coincidence circuit ( $T_7, T_8$ ).

To permit the use of various SCA's with differently shaped outputs it is desirable to derive a new pulse from them which will have the same shape in all cases. Thus a negative pulse at the slow unit input labeled SCA 1 is differentiated and used to trigger a single-shot multivibrator ( $T_5, T_6$ ) whose output pulse width increases as the resistance  $RI$  increases. This negative going rectangular pulse is referred to as the pulse derived from the SCA 1 output. This pulse is applied to the base of  $T_7$  which is the other side of the coincidence circuit ( $T_7, T_8$ ).

If the base of either  $T_7$  or  $T_8$  is made more negative, then the collector current in that transistor is cut off but their common collector voltage is unaltered. If both bases are made more negative simultaneously, however, the potential of their common collector rises. Thus, if a pulse derived from a fast coincidence output is at the base of  $T_8$  during a time when a wider pulse derived from an SCA 1 output is at the base of  $T_7$ , then the coincidence circuit generates a positive output. The start of this output will be delayed from the start of the fast coincidence output by an amount variable with  $RA$  and its duration can be varied with  $RC$ .

With switch S in the DOUB position the second coincidence circuit ( $T_{12}, T_{13}$ ) operates exactly as above: a pulse at the base of  $T_{12}$  derived from an SCA 2 output overlaps a pulse from (A) derived from a fast coincidence output applied to the base of  $T_{13}$ .

Two diodes,  $D_1$  and  $D_2$ , and a 2200 ohm resistor  $R_0$  form an "OR" network. Thus an output from either slow coincidence circuit appears at (B) and is differentiated and used to trigger still another single-shot multivibrator ( $T_{15}, T_{16}$ ). This multivibrator's output can be made wider by increasing RB and is passed to an emitter follower  $T_{17}$  and then to the slow unit output labeled MCA GOING. This signal is the gate for the MCA. The final diode and resistor are a clipping network to limit the swing of the gate to five volts in accordance with the MCA manufacturer's recommendations (14). The signals at ROUTE 1 and ROUTE 2 are the outputs of the respective coincidence circuits. The emitter followers  $T_9$  and  $T_{14}$  supply the two milliampere current required by the MCA for routing signals.

The switches SE, S1, and S2 are all normally open, the position labeled 0. They are included to simplify the timing procedure which will be discussed later; only their effect on the circuit operation is discussed here. In this paragraph it is assumed as before that the switch S is in the DOUB position. When only SE is closed the bases of  $T_8$  and  $T_{13}$  are both held sufficiently negative that the outputs of the coincidence circuits, and hence the signals at ROUTE 1 and ROUTE 2, reproduce the inverted shape of the pulses derived from the SCA 1 and SCA 2 outputs, respectively. The "OR" network will permit a gate signal for every output from either SCA. With only S1 or S2 closed the signal at ROUTE 1 or ROUTE 2 reproduces the inverted shape of the pulse derived from the fast

coincidence output. There is also a gate for every fast coincidence output. The effects produced when the switches are positioned in ways other than those discussed are less important but can be determined if necessary.

The switch S is included to make the slow unit suitable for use in directional correlation measurements as will be discussed. When the switch is in the DOUB position the unit operates as two double coincidence circuits as described in the preceding paragraphs. When this switch is in the TRI position the anode of diode  $D_2$  is held at minus three volts and the collector of  $T_{12}$  is connected to the common collectors of  $T_7$  and  $T_8$ . Thus there will be signals at MCA COINC and ROUTE 1 only when a pulse derived from a fast coincidence output is simultaneously overlapped by two pulses, one derived from an SCA 1 output and one derived from an SCA 2 output. The unit then operates as a triple coincidence circuit and generates a negative signal at -COINC to operate a scaler.

All required d.c. voltages are obtained from a single external supply by the use of a built-in voltage divider with capacitive filters.

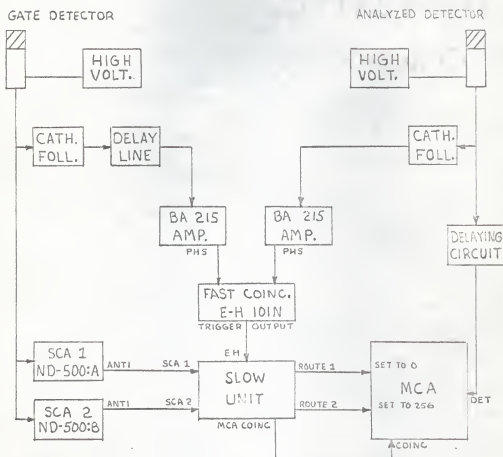
#### Complete Dual FSC Configuration

The dual FSC configuration is assembled as shown in Plate VI. The photomultiplier voltage for the analyzed detector and the gain of the MCA internal amplifier are chosen to display the energy range of interest. The photomultiplier voltage of the gate detector and the gain setting of the SCA amplifiers are chosen so that these amplifiers are not overloaded. The cathode followers in the arms of the fast coincidence section are

EXPLANATION OF PLATE VI

A block diagram of the complete  
dual FSC configuration.

## PLATE VI



necessary to prevent this section from disturbing the critical pulse heights going to the single and multichannel analyzers. The variable delay line in one arm is required to obtain the chance coincidence spectrum as will be discussed. Although it is normally set to introduce no delay, it is always in the circuit. The gain of the following Baird Atomic model 215 amplifier is increased to compensate for the signal attenuation caused by the delay line. The pulse height selector and gain controls of both of these 0-110 volt output non-overloading amplifiers are set to discriminate against detector noise but still to insure that the output pulse heights in the range of interest are considerably higher than the lower level discriminator setting. The latter is required in order for the time jitter in the shaped PHS outputs of these amplifiers to be negligible because these outputs are the inputs to the fast coincidence circuit, an E-R Research Laboratories model 101N coincidence unit.

#### Timing Procedure

It is necessary to delay the pulses from the analyzed detector until a gate signal for the MCA is available. The necessary amount of delay is determined by the response times of the SCA's and the shaping circuits of the slow unit. Because the processing times of the SCA's jitter about the two microsecond points, a delay of three microseconds in the pulses to be analyzed is sufficient. A three microsecond delay can be conveniently obtained; one microsecond in the delay line circuit and two microseconds in the internal amplifier of the MCA. This choice determines the time at which the gate signal to the MCA must occur.

The following steps of the timing procedure are more easily carried out using a dual beam oscilloscope but if necessary they can be done using the external trigger of a single beam model. Diagrams of the waveforms seen are included in Fig. 2 of Plate V. Switch S is in the DOUB position for all steps and switches SE, S1, and S2 are all in their normal open position unless specifically stated otherwise.

The fast coincidence circuit is switched to ignore the gate detector and to trigger an output for every pulse it receives from the analyzed detector. Only one input cable, that from the fast coincidence circuit, is connected to the slow unit and switch S1 is closed so that the slow unit generates a gate at the point labeled MCA COINC for every fast coincidence output. These gate signals for the MCA are observed on one beam of a dual beam scope which is triggered by the positive leading edge of the fast coincidence output. The delayed amplified pulses to be analyzed can be obtained from a point on the front panel of the MCA now labeled ADG and they are observed on the other beam of the scope. The time of the start of the gate signal is adjusted by RA and the width of the gate by RB so that the gate overlaps the pulse to be analyzed. If desired, the energy equivalent of the lower level discriminator setting of the BA 215 amplifier associated with the analyzed detector can easily be determined from a calibrated spectrum which is obtained by the MCA while using these particular gate signals.

The second step of the timing procedure is to adjust the width of the routing signals so that it is between the one to two microsecond limits set by the MCA manufacturer (14). The signals at the slow unit output

labeled ROUTE 1 are observed with switch S1 closed. These positive signals have the inverted shape of the pulses derived from the fast coincidence outputs. The resistance of RC is adjusted so that these pulses have a width slightly greater than one microsecond.

The third step is to adjust the width of the pulses derived from the SCA 1 outputs. These pulses must be wide enough to overlap those derived from the fast coincidence outputs even when the latter are further delayed as they will be to obtain the chance coincidence spectrum. This delay is yet to be discussed but for definiteness, assume that it will be one microsecond.<sup>1</sup> To make the adjustment, the fast coincidence circuit is switched to ignore the analyzed detector and to trigger an output for each input from the gate detector, SCA 1 is connected to the slow unit, switch S1 is opened and resistance R1 is increased to its maximum value. The signals at ROUTE 1 are observed with the scope being triggered by the fast coincidence outputs. The observed signals still have the inverted shape of the pulses derived from the fast coincidence outputs whose width was set in the second step. However, when switch SE is closed the signals at ROUTE 1 have the inverted shape of the pulses derived from the SCA 1 outputs. The duration of these pulses is shortened by decreasing R1 until all of them

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<sup>1</sup>If the delay line could be connected directly to the photomultiplier output of either detector, then a pulse derived from a fast coincidence output would never be further delayed relative to the corresponding pulse derived from an SCA. In this event the extra 1 microsecond width of the latter pulse would not be required in order that it could always overlap the former. The present lumped parameter delay line is not connected to the detector outputs because it would slightly distort the pulses going to the MCA or to the SCA's.

are just sufficiently long to extend the one microsecond beyond the trailing edge of the pulses derived from the fast coincidence outputs. The correct adjustment is easily checked by closing and opening switch SE which alternately displays the forms of the two kinds of pulses. The extent of the time jitter of the SCA outputs can be visualized when the pulses derived from them are displayed in this way.

The adjustment of the width of the pulses derived from the SCA 2 outputs is made by connecting SCA 2 to the slow unit and repeating the third step, manipulating SE and R2 while observing the signals at ROUTE 2.

It may be pointed out that a delay in the pulses to be analyzed of longer than three microseconds or pulses derived from the SCA outputs which last longer than necessary can be used, but either results in unnecessarily increasing the resolving times of the slow coincidence circuits.

#### Window Setting Procedure

As mentioned previously it is necessary to insure that the SCA amplifiers are not overloaded. After this has been checked the use of the individual amplifier outputs is no longer required and the energy ranges of the two windows can be determined simultaneously. The fast coincidence circuit is switched to ignore events in the analyzed detector but to trigger an output for each input from the gate detector. All input connections are made to the slow unit but only the gate output, MCA COINC, is connected to the MCA. Switches SE, S1, and S2 are all open and S remains in the DOUB position. The input to the delaying circuit and the MCA internal amplifier is connected to the output of the cathode follower in the gate detector arm of the fast coincidence section so as not to affect

the height of the pulses going to the SCA's. A calibration spectrum of a convenient source is obtained in one half of the MCA memory with the MCA's COINC switch in the OFF position. When this switch is ON the MCA is permitted to analyze only those pulses which are overlapped by gate signals from the slow unit, that is, only those pulses which are within the window of one of the two SCA's. Thus a profile of the energy range of each of the two windows can be obtained simultaneously in the other half of the memory and can be overlapped with the calibration spectrum for comparison. If the windows are not positioned as desired, their profiles can be erased, the SCA discriminator settings adjusted and new profiles obtained.

#### Quick Checks of Operation

A check on the operation of the routing signals is easily made using one of the arrangements of the window setting procedure. Instead of collecting both window profiles in the first half of the memory one may be routed to second half by connecting either the ROUTE 1 or the ROUTE 2 output to SET TO 256 on the front pannel of the MCA. Further, if the MCA's COINC switch is turned OFF and just the input from one SCA is connected to the slow unit, then the window profile can be routed to the second half leaving a hole in the spectrum collected in the first half of the memory. These tests show two things. The first shows that to every gate there corresponds a routing signal from one or the other of the two ROUTE outputs. The second shows that there is a routing signal corresponding to every pulse from the gate detector which is within the energy range of the SCA window.

### Chance Coincidence Correction

The original data collected in a FSC experiment must be corrected for chance coincidences. These result because the fast coincidence circuit has a finite resolving time within which any two input pulses can trigger an output. In a radioactive source there is a finite probability that two separate nuclei may decay and emit gamma-rays within this resolving time. If one of these gamma-rays loses enough energy in the gate crystal to produce a pulse within the window of an SCA and the other gamma-ray is detected by the analyzed crystal, then the pulse from the latter will be processed and stored by the MCA. But because the gamma-rays were emitted from two separate nuclei and not in cascade from a single nucleus the processed pulse did not correspond to a real coincidence. Thus the data collected is the sum of a real coincidence spectrum plus a chance coincidence spectrum.

The necessary correction is made by subtracting a chance coincidence spectrum from the original data. This chance spectrum is obtained simply by repeating the experiment with one change: the pulses going into one side of the fast coincidence circuit are delayed by an amount greater than the resolving time of this circuit so that the only possible outputs will be due to chance coincidences. This is the reason for the delay line in one arm of the fast coincidence section in Plate VI. As noted previously the amount of this delay is a factor in determining the necessary width of the pulses derived from the SCA outputs. If the resolving time of the fast coincidence is of the order of 0.1 microsecond, then this delay can be about 1 microsecond.

If the half life of the source is not long compared to the running time of the experiment then, of course, the chance coincidence spectrum must be corrected for the decay of the source. This source decay correction can not be avoided because with only one fast coincidence circuit it is not possible to obtain the real plus chance spectrum and the chance spectrum simultaneously.

### THE FSC MEASUREMENTS

The slow unit has been used as an element of a single FSC configuration where all information is stored in a single half of the MCA memory so no routing is required. The results of some of these investigations have been reported in the literature (3), (13), (15). Some of the results (3) so obtained are in agreement with those obtained with an independent fast-slow coincidence package built by Sturup Inc. and purchased for this laboratory. The data presented here demonstrate that identical information is obtained whether the slow unit is used as an element of a dual FSC configuration to accumulate two spectra simultaneously or it is used in a single FSC configuration which obtains the same two spectra in sequence.

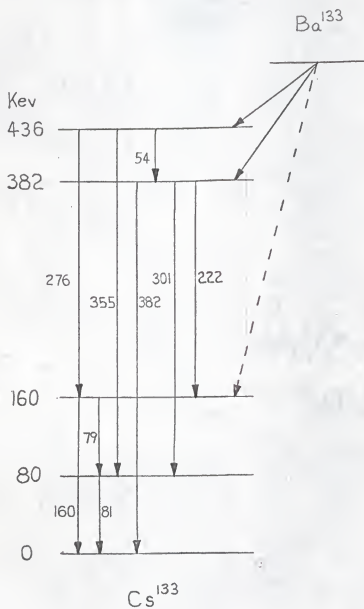
The present measurements were of the coincidence spectra of the 80 and 355 Kev gamma-rays of  $\text{Cs}^{133}$ . From the decay scheme of Plate VII it is seen that gamma-rays of energies 54, 79, 81, 222, 276, 301, and 355 Kev should be coincident with one or the other of the two gamma-rays whose energies are approximately 80 Kev. Only the 81 Kev gamma-ray should be coincident with the 355 gamma-ray.

The gate and analyzed detectors were Harshaw type 16 MB 12/3 3x4" sodium-iodide crystal-photomultiplier combinations operated at 750 and

EXPLANATION OF PLATE VII

The decay scheme of  $\text{Cs}^{133}$ . (From (13).)

## PLATE VII



700 volts, respectively. They were placed about 25 cm from the source with an angular separation of about  $70^\circ$ . An anti-Compton shield of lead faced with copper, cadmium, and aluminum was placed between the crystals to reduce coincidences due to scattering. The resolving time of the fast coincidence circuit was approximately equal to the input pulse width of 0.2 microseconds. The SCA's were sides A and B of a Nuclear Data model ND-500 and the respective window widths were 22 Kev at 80 Kev, and 32 Kev at 355 Kev.

The 80 and 355 Kev coincidence spectra obtained simultaneously are shown in Plate VIII and Fig. 1 of Plate X, respectively. They are almost identical to the two spectra obtained singly and shown in Plate IX and Fig. 2 of Plate X.

In the 80 Kev coincidence spectra only the photopeak of the 355 Kev gamma-rays stands alone. The smaller photopeaks of the 301 and 222 Kev gamma-rays are not resolved from that of the 274 Kev gamma-rays. The peak in the vicinity of 155 Kev is a combination of the Compton distribution and backscatter peaks of primarily the 274 and 355 Kev gamma-rays. The peak at 80 Kev is a composite of the photopeaks of the 79 and 81 Kev gamma-rays. The hump in the vicinity of 54 Kev was the subject of recent research (13). The peak at 30 Kev is due to the cesium x-ray produced in every K-capture disintegration of  $\text{Ba}^{133}$  into  $\text{Cs}^{133}$ .

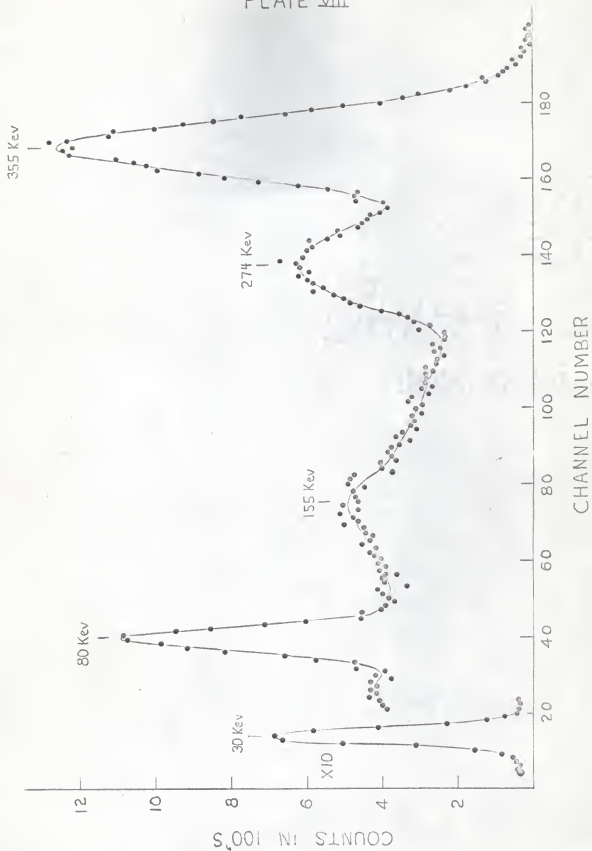
The coincidence spectra of the 355 Kev gamma-rays contains only the x-ray peak and the photopeak of the 81 Kev gamma-rays as expected.

#### EXPLANATION OF PLATE VIII

The spectrum of  $\text{Cs}^{133}$  coincident with 80 Kev.

This data was obtained simultaneously with the coincidence spectrum of the 355 Kev gamma-ray shown in Fig. 1 of Plate X.

## PLATE VIII

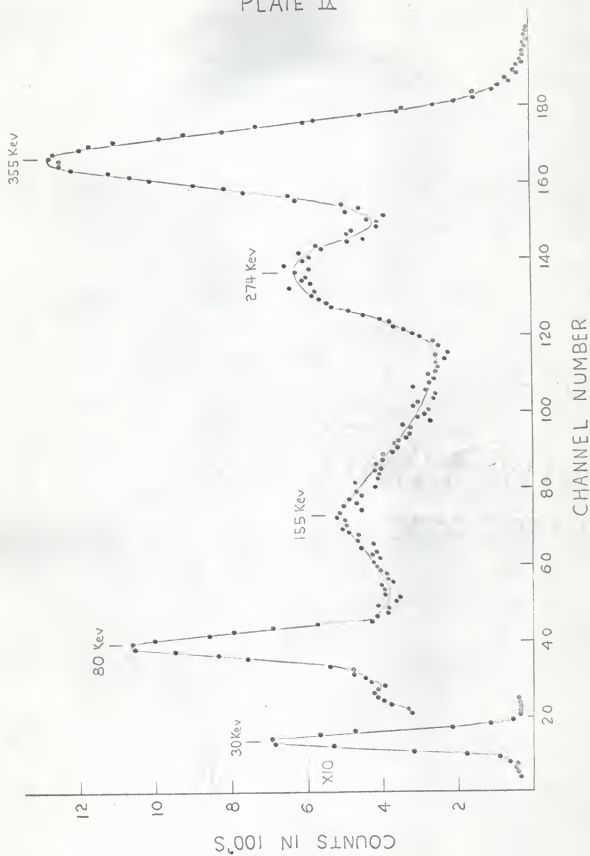


# EXPLANATION OF PLATE IX

The spectrum of Cs<sup>133</sup> coincident with 80 Kev.

This data was obtained by a single FSC configuration with  
no routing.

## PLATE IX



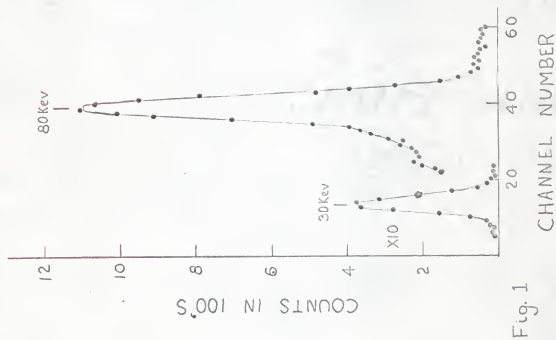
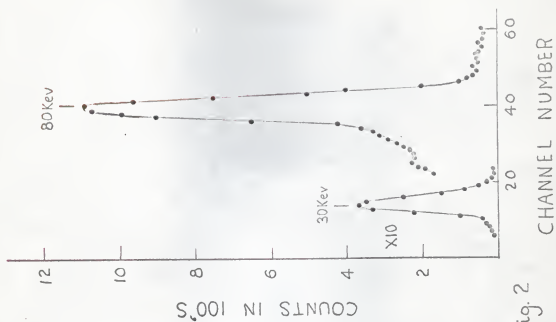
# EXPLANATION OF PLATE X

The coincidence spectrum of the 355 Kev gamma-ray of Cs<sup>133</sup>.

Fig. 1. This data was obtained simultaneously with the 80 Kev. coincidence spectrum shown in Plate VIII.

Fig. 2. This data was obtained by a single FSC configuration with no routing.

## PLATE X



## DIRECTIONAL CORRELATION EXPERIMENTS

### Existence of a Gamma-Gamma Directional Correlation

A general theory has been developed (2) to treat the angular correlations between various pairs of particles or quanta emitted in cascade, such as two alpha-particles, a beta-particle and a gamma-ray, or two gamma-rays. This work concerned only the third type, known as gamma-gamma angular correlations, and it was further restricted to directional measurements in which the detectors were polarization insensitive. Because the plane of polarization of the gamma-rays was not determined, electric and magnetic transitions could not be distinguished and hence the parity differences between the states could not be determined. Thus the following intuitive argument for the existence of the directional correlation need not involve the parity of the states.

The square of the magnitude of the angular momentum vector  $\vec{I}$  of a nuclear state is  $I(I+1)\hbar^2$ , where  $I$  is the angular momentum quantum number of the state. A state of non-zero angular momentum is degenerate. In an extremely strong magnetic field the state would split into  $2I+1$  substates in which the projection of the vector  $\vec{I}$  along the field direction would be quantized so that  $I_z = m\hbar$ , where  $m = I, I-1, \dots -I$ . In the absence of a magnetic field the axis of quantization is arbitrary but a gamma-ray transition can still be considered to occur between a magnetic substate  $m_a$  of state  $I_a$  and a magnetic substate  $m_b$  of state  $I_b$ . Angular momentum is conserved in the transition so  $\vec{I}_a = \vec{I}_b + \vec{L}$ , where  $\vec{L}$  is the vector angular momentum carried off by the gamma-ray. The square of the magnitude of the vector  $\vec{L}$  is  $L(L+1)$ , where  $L$  is the multipole order of the transition.

The projection of the angular momentum along any axis is also conserved in the transition:  $I_{a_z} = I_{b_z} + L_z$ . The projection of the vector angular momentum  $\vec{L}$  along its direction of propagation can only be  $\pm \hbar$ . Accordingly, if the arbitrary axis of quantization is chosen to be along the direction of propagation, then  $L_z = \pm \hbar$  and  $m_a = m_b \pm 1$ . This is a selection rule for the possible gamma-ray transitions between magnetic substates of states  $I_a$  and  $I_b$ .

When single gamma-rays from a source are detected the nuclei which emit them are initially distributed uniformly among the magnetic substates of the initial state so the resulting radiation is isotropic. However, when pairs of gamma-rays emitted in cascade are detected the axis of quantization is chosen along the direction of the first gamma-ray so the nuclei which emit the detected pairs can only be in substates satisfying the selection rule after the first transition. Thus these nuclei are not uniformly distributed among all of the substates of the initial state of the second transition, and the resulting radiation is not isotropic with respect to the direction of the first gamma-ray.

There is a tacit assumption in the preceding argument which requires further discussion. The direction of propagation of the second gamma-ray is related to that of the first only if the orientation of the nucleus is not altered while it is in the intermediate state. This requires that the mean lifetime of the intermediate state be short compared to the interaction time of any process which would perturb it.

In general, this intermediate state has non-zero magnetic dipole and electric quadrupole moments which interact with magnetic or electric fields

present at the nucleus. As an example, the magnetic dipole moment tends to precess with the Larmor frequency about the direction of the magnetic field due to the orbital motion of the electrons. It is required then, that the mean lifetime of the intermediate state be short compared to the inverse of this frequency. This condition is often met because measurements of the splitting of hyperfine structure have shown that this frequency ranges from  $10^7$  to  $10^{10} \text{ sec}^{-1}$  (2) and the mean lifetime for many dipole and quadrupole transitions is less than  $10^{-10} \text{ sec}$ .

There are other perturbations which in some cases can not be ignored. The theory indicates that their effect is to smear out the directional correlation toward isotropy (2). For octupole and higher order transitions where the mean lifetime is between  $10^{-8}$  and  $10^{-5} \text{ sec}$  it has been possible to measure the gyromagnetic ratio of some intermediate states by considering the effect of external magnetic fields upon the measured angular correlation (9), (16), (18).

#### Predictions of the Theory of Angular Correlation

Complete discussions of the theory of angular correlation have been published (2), (10). It is the task of the theory to calculate the relative probability that an isolated nucleus decaying through a cascade will emit two radiations in various directions into two infinitesimal solid angles. Only the results of this theory for one particular case will be presented here.

The case is that of a nucleus which emits two gamma-rays of pure multipole orders  $L_1$  and  $L_2$ . The theory predicts that the relative

probability is a function of the angle  $\theta$  between the directions of the emitted gamma-rays and that this function has the form

$$W(\theta) = 1 + A_2 P_2(\cos \theta) + A_4 P_4(\cos \theta) + \dots + A_j P_j(\cos \theta) + \dots + A_f P_f(\cos \theta),$$

where  $P_j$  is the  $j$ -th order Legendre polynomial. The highest order term contains the largest even  $f$  which is not greater than the smallest of the three numbers  $2I_b$ ,  $2L_1$ , or  $2L_2$ , where  $I_b$  is the angular momentum quantum number of the intermediate state. The coefficients  $A_j$  are products of two factors which are functions of the Clebsch-Gordon and Racah coefficients:

$$A_j = F_j(L_1, I_a, I_b) F_j(L_2, I_c, I_b),$$

where  $I_a$  and  $I_b$  are the angular momentum quantum numbers of the initial and final states, respectively. These  $F_j$  factors have been evaluated numerically and tabulated by Biedenharn and Rose (2).

One example of present interest is one in which  $I_a = 4$ ,  $I_b = 2$ ,  $I_c = 0$ , and both transitions are quadrupole so  $L_1 = L_2 = 2$ . The highest order term contains  $f = 4$ . By consulting the tables for the necessary numerical values of  $F_2$  and  $F_4$  it is found that

$$W(\theta) = 1 + 0.1020 P_2(\cos \theta) + 0.0091 P_4(\cos \theta).$$

A graph of this function for  $\theta$  in the range between  $90^\circ$  and  $180^\circ$  is presented in Fig. 1 of Plate XII. This plot of one quadrant gives all the significant information because both coordinate axes are symmetry axes of a polar plot of  $W(\theta)$ .

Brady and Deutsch (4) first determined that these particular parameters are applicable to the cascade shown in Fig. 1 of Plate XIII which leads from the second excited to the ground state of  $\text{Ni}^{60}$ . This was verified and

#### EXPLANATION OF PLATE XI

- Fig. 1. A graph of the relative probability function  $W(\theta)$  for a pure quadrupole - quadrupole gamma-ray cascade for states with successive angular momentum quantum numbers 4, 2, and 0. (After Steffen (18).)
- Fig. 2. A simplified directional correlation configuration.

## PLATE XI

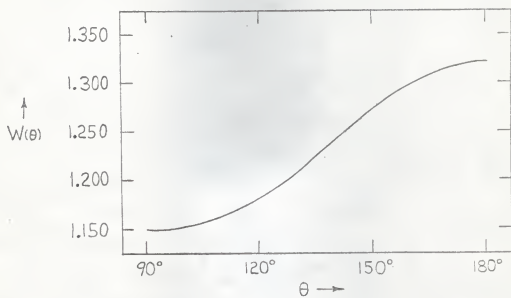


Fig. 1

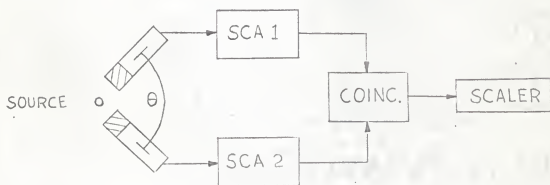


Fig. 2

the form of the theoretical prediction was well confirmed by Colombo, Rossi, and Scotti (6) among others (11), (18). The mean life time of the intermediate state of this cascade is less than  $10^{-11}$  sec. (1).

A useful measure of the extent of this correlation is the anisotropy defined as:

$$\text{Anisotropy} = \frac{W(180^\circ) - W(90^\circ)}{W(90^\circ)}$$

The theory predicts an anisotropy of 0.1667 for this example and Lawson and Frauenfelder (12) have measured it to be  $0.167 \pm .001$ .

### Experimental Considerations

A simplified experimental configuration for making directional correlation measurements is shown in Fig. 2 of Plate XI. The experiment involves obtaining the number of real coincidences in a certain time interval at several different angles so the two scintillation detectors are mounted such that their angular separation is easily varied. The windows of the two SCA's are set to include only the photoelectric peaks of the two gamma-rays of interest in order to eliminate unwanted coincidences between scattered radiation and between other gamma-rays emitted by the source.

In order to obtain statistically accurate data in the available time it is desirable to obtain a high real coincidence rate. This rate is proportional to

$$W(\theta) \propto \frac{W_1}{4\pi} \epsilon_1 \frac{W_2}{4\pi} \epsilon_2 ,$$

Where  $N$  is the disintegration rate or source strength,  $W_1$  and  $\epsilon_1$  are the solid angle subtended at the source and the intrinsic efficiency for the

energy of interest, respectively, for detector number one, and  $W_2$  and  $e_2$  are corresponding quantities for the second detector. The real coincidence rate increases directly as the source strength and the solid angles subtended by the detectors but the usefulness of increasing these factors is limited by the following considerations.

Because of the finite resolving time of the coincidence circuit, chance coincidences also occur at a rate given by  $N_c$  :

$$N_c = 2TN_1N_2 = 2TN^2f_1 \frac{W_1}{4\pi} e_1 f_2 \frac{W_2}{4\pi} e_2 ,$$

where  $T$  is the resolving time,  $N_1$  and  $N_2$  are the respective input rates to each side of the coincidence circuit, and  $f_1$  or  $f_2$  is the fraction of the total spectrum included in the window of SCA 1 or SCA 2. These chance coincidences must be subtracted from the original data. It is apparent that the ratio of the real to the chance coincidence rate is inversely proportional to  $NT$ . Thus for a given resolving time, too large an increase in the source strength does not result in more accurate data because the correction for chance coincidences becomes too large. It is preferable to maintain a real to chance ratio of approximately five to one (10) but this is not always possible.

The theory assumes infinitesimally small detectors and the effect of detectors which subtend a finite solid angle is to decrease the coefficients in the expression for  $W(\theta)$  and smear out the correlation toward isotropy (12). The coefficients become smaller as the solid angles subtended by the detectors are increased so it is not always feasible to obtain higher coincidence rates by using larger detectors or decreasing their distance

from the source. The attenuation factors have been calculated for various geometries by Feingold and Frankel (8). A better procedure is to follow Lawson and Frauenfelder (12) and use the angular resolution of the detectors determined with collimated beams of gamma-rays of the energy range of interest. For regions near 511 Kev it is convenient to use annihilation radiation, in which two gamma-rays are emitted in opposite directions, following a procedure outlined by Church and Kraushaar (5).

Although the theoretical development also assumes a point source the usual experimental approximation is a linear source oriented perpendicularly to the plane of the two detectors. Expressions for source size correction have been developed by Feingold and Frankel (8) but it is usually possible to make the diameter of the source and the solid angles subtended by the detectors sufficiently small that this correction can be neglected.

If the experimental data are to provide accurate results, then care must be taken to minimize the effect of other factors which affect the measured number of coincidences, such as drift in the electronic equipment. For this reason the data are usually accumulated by counting for short time intervals at each of a few angles in turn and then repeating the sequence until sufficient data are obtained. As long as there is no appreciable drift during each sequence the totals of the counts at each angle are still in the correct proportion.

When investigating a cascade between states of unknown angular momenta the data obtained are first corrected for chance coincidences and then a least squares fit is made to a function of the form predicted by theory following a procedure treated by Rose (17). The experimentally determined coefficients are then corrected for the other effects discussed. Because

the theoretical formula involves a set of several parameters it is not possible to determine uniquely all of them with one measurement. If other information is available, it is used to reduce the number of possible sets of parameters until the experimental coefficients are sufficient to distinguish one of those remaining.

#### THE DIRECTIONAL CORRELATION MEASUREMENT

The present work was done with an experimental arrangement different from the one mentioned in the preceding section. Because of the jitter of the available SCA's it was necessary to use an adaptation of the FSC configuration of Plate VI. A diagram of the complete directional correlation configuration is presented in Plate XII.

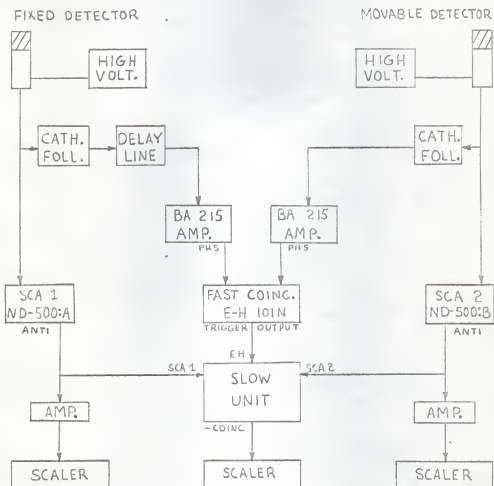
The "fixed" and "movable" detectors were  $1\frac{1}{2} \times 1$ " sodium-iodide crystals mounted on RCA 6342 photomultiplier tubes operated at 830 and 860 volts, respectively. The fixed detector was placed in a stationary holder on a specially constructed circular table top which had a scale graduated in degrees around its outer edge. The movable detector was placed in a holder that could be positioned anywhere along this scale except within  $63^\circ$  on either side of the fixed detector.

The source was prepared by evaporating a solution of  $\text{Co}^{60}$  onto the walls of a  $\frac{3}{32}$ " diameter hole drilled to a depth of  $\frac{1}{2}$ " in the end of a 1" length of  $\frac{5}{16}$ " diameter carbon rod. The carbon walls were thick enough to absorb the 313 Kev beta-rays emitted in the transition to the second excited state of  $\text{Ni}^{60}$ . This source was centered so that the counting rate in the movable detector did not vary more than one percent when it was positioned at each of the angles of interest.

EXPLANATION OF PLATE XII

A block diagram of the complete  
directional correlation configuration.

## PLATE XII



In this experiment the gamma-rays at 1.17 and 1.33 Mev were the only ones present so it was possible to double the coincidence rate compared to that which would normally have been obtained. The SCA's needed only to be used as lower level discriminators and these were both set at about 1 Mev, just above the Compton edge of the 1.17 Mev gamma-ray. Thus the coincidence of either gamma-ray in one detector with the remaining gamma-ray in the other detector could be counted. In the normal arrangement the non-overlapping SCA windows permit coincidence counts only when each gamma-ray is detected in its respective crystal.

The slow unit is easily adjusted for use in this configuration. The previously described procedures for timing and setting the SCA windows needed only slight modification. In the directional correlation configuration each SCA was associated with a different detector so when working with the circuits associated with SCA 1 the fast coincidence circuit was switched to trigger outputs for inputs from the fixed detector and vice versa. Because no routing signals were necessary it was possible to decrease the resolving time of the slow unit by decreasing the width of the pulses derived from the fast coincidence outputs to 0.5 microseconds. The resolving time of the fast coincidence circuit was 0.06 microseconds. A 0.4 microsecond delay was introduced into the fixed detector arm of the fast coincidence section to obtain the number of chance coincidences. Thus the pulses derived from the outputs of SCA 1, associated with the fixed detector, needed to be about 1 microsecond wide so that they could always overlap corresponding pulses derived from the fast coincidence outputs. But the latter pulses were not further delayed relative to the corresponding pulses derived from the outputs of SCA 2 which therefore needed to be

only 0.6 microseconds wide. After the SCA lower level discriminators had been set the MCA was no longer used and so the width of the pulse at -COINC was reduced to 2 microseconds by decreasing RB. Finally the switch S was placed in the TRI position so that the slow unit generated a negative output at -COINC every time there was a time coincidence and both pulses exceeded the lower level discriminator settings of the SCA's.

The data were accumulated during 10 minute intervals at four angles, 180°, 150°, 120°, and 90°, preceded by an interval in which the number of chance coincidences was obtained. This five step sequence was repeated 39 times during a period of six days. The placement of the SCA lower level discriminators and the operation of the other equipment was checked periodically.

The data with chance coincidences subtracted are shown in Plate XIII. The error bars represent only the statistical uncertainty. This uncertainty was so large that none of the corrections previously discussed were applied. The shape of this <sup>N</sup> plot should be compared to the graph of theoretical function in Fig. 1 of Plate XI. The measured anisotropy was  $0.163 \pm .025$ .

The indicated statistical uncertainty could only have been reduced by accumulating more data. The fact that the data strongly indicated the known character of the correlation was deemed sufficient verification of the correct operation of the configuration and the equipment was released for other measurements.

#### EXPLANATION OF PLATE XIII

- Fig. 1. A decay scheme of the 1.33 and 1.17 Mev gamma-ray cascade of  $\text{Ni}^{60}$ .
- Fig. 2. A plot of the measured directional correlation of the  $\text{Ni}^{60}$  cascade. The error bars represent only the statistical uncertainty. The shape of this plot should be compared to the graph of the theoretical function shown in Fig. 1 of Plate XI.

## PLATE XIII

Fig. 1

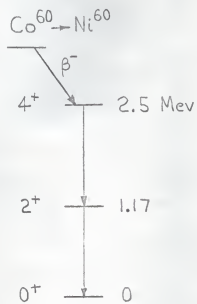
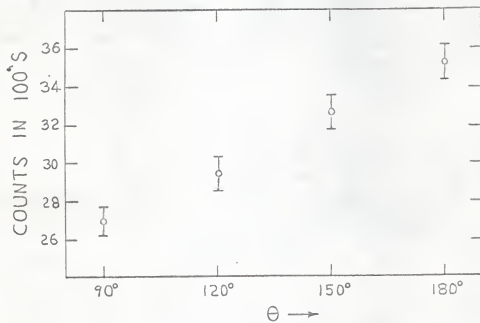


Fig. 2



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MEASUREMENTS OF TWO PROPERTIES  
OF CASCADE GAMMA-RAYS

by

DAVID A. DRAEGERT

B. A., Grinnell College, 1961

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AN ABSTRACT OF A THESIS

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requirements for the degree

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This thesis considers some experimental problems in obtaining the coincidence spectra and directional distribution of gamma-rays emitted in cascade.

Two transistorized slow coincidence circuits incorporating delaying and pulse shaping networks have been built to provide gate and routing signals so that a Nuclear Data model 130A 512 channel analyzer can accumulate the coincidence spectra for two different gamma-rays simultaneously. The transistorized circuitry and its operation in a configuration with two single channel analyzers and a fast coincidence circuit are discussed. The coincidence spectra of the 80 and 355 Kev gamma-rays emitted from  $\text{Cs}^{133}$  have been obtained simultaneously and singly to verify the correct operation of the equipment.

An intuitive argument is given for the existence of a directional correlation between cascade gamma-rays. The theoretical directional probability function applicable to the cascade in  $\text{Ni}^{60}$  is discussed, and the successful adaptation of the fast-slow coincidence configuration for direction correlation experiments has been verified by a measurement of the correlation between the 1.17 and 1.33 Mev gamma-rays of this cascade. Procedures are presented for treating chance coincidences, finite source and detector sizes, and drift in the electronic equipment.