AN APPROXIMATE ELASTIC ANALYSIS OF W SHAPE BEAMS WITH REINFORCED RECTANGULAR WEB OPENINGS

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INTRODUCTION

1. Problem Statement

In construction practice, when openings of some sort are cut in the web of a steel beam to accommodate the passage of utilities or to provide access to equipment, the beam may be weakened in the vicinity of the openings to the extent that reinforcing is required. Although the theory of elasticity provides some basis for estimating the stresses around an opening, it provides little basis for the practical design of reinforcement requirements. The purpose of the study described in this report was to develop an approximate, practical method for determining the amount of the necessary reinforcement at the web openings subjected to combined bending and shear.

The so-called Vierendeel method of analysis was used to calculate theoretical stresses around the holes. An A36 steel W12x45 beam with a 9"x6" rectangular web opening at middepth was used to illustrate the proposed method.

2. Purpose

The primary objectives of this report were:

a. To determine the amount of required reinforcement around rectangular openings in the webs of W shape members.

b. To determine the location of critical stress at the reinforced opening as limited by AISC allowable stresses and/or as limited by a yielding criterion using a proper factor of safety.

3. Scope

The study was limited to W shape steel beams having rectangular openings centered on the neutral axis of the members. However, the proposed method is applicable to any homogeneous and isotropic material within its elastic range. Only horizontal reinforcing bars welded to the web above and below the openings were considered. The horizontal reinforcing bars may be used on both sides or on only one side of the web without any changes in the design method.

LITERATURE REVIEW

In 1924 Timoshenko (17)* developed an approximate method of analysis for determining the stresses around the reinforcement in a circular hole in a uniform tension field; Sobreno and Gurney (6) continued this work independently in 1938. The David W. Taylor Model Test Basin (18) in Washington, D.C. in 1938 published results of an experimental study of various types of small openings reinforced in various ways. In 1949 Reissner and Morduchow (13) developed a method of analysis for the reinforcement of a circular hole for various types of loadings. Joseph and Brock (10) in 1950 developed a mathematical procedure for computing the effects of small circular openings in beams subjected to pure bending. In 1952, Heller (7) considered small circular openings (H/D less than 0.10) in beams subjected to both bending and shear. Later, in 1958, Heller (8) studied the case of square holes with H/D less than 0.25 in members subjected to axial loading only. In 1954 Mantle (11) studied reinforced circular openings with H/D less than 0.25 in a uniform tension field.

In 1958 Worley (19) studied the inelastic behavior of aluminum alloy I-beams with web cutouts unreinforced. His purpose was to determine the shape of the web section cutout that would result in the least reduction in the full plastic load carrying

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*Numbers in parentheses refer to corresponding items in " References ".

capacity per pound of beam weight.

It was noted that with the exception of studies by Heller (8) and Worley (19) in 1958, the past work has been restricted to small circular openings or to other shapes of openings which produce little or no stress concentrations. In most of these cases, the openings are reinforced by a pipe section placed transversely to the plane of the web. This proved effective in reducing the stress level.

Heller, Block and Bart (9) in 1962 investigated the stresses around a rectangular opening with rounded corners in the web of a beam. The effects of varying the corner radius and the aspect ratio of the opening (H/W) on the stress distribution in a beam were included in the study.

In 1963, Segner (14) reported the results of a study of the requirements for reinforcement around large rectangular openings in the webs of W shape beams subjected to both bending and shear. From his tests, it was concluded that : (1) the Vierendeel Truss analogy is an appropriate method of analysis, (2) large deflections should be expected when beams contain openings in the webs, but the deeper the beam is, the less deflection it has.

From then on, the so-called " Vierendeel method of Analysis" has been used by many researchers (1,3,5,12) to study the elastic behavior of W shape beams with rectangular web openings.

THEORETICAL ANALYSIS

1. Introduction

The "Vierendeel Method" (2,20), in which the beam is assumed to act like a Vierendeel Truss in the vicinity of the opening, was used to study the elastic stresses around rectangular openings in this analysis.

Since beams with rectangular openings are used primarily in building construction, design criteria for such beams will be related to the AISC specification (15).

2. Assumptions

a. The vertical shear at the opening is equally distributed to the portions above and below the opening since the opening is centered on the neutral axis of the beam.

b. Points of contraflexure (inflection points) exist at the center of the opening in the portions above and below the opening.

c. The shear force at the opening is less than the AISC allowable shear force (that is, $V/V_c \leq (D-H)/D$)*, therefore, no shear reinforcement are required.

d. The horizontal reinforcing bars are extended beyond the edges of the opening far enough to develop the strength of the bars.

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* The symbols are defined in the section on " Notation ".

3. AISC Allowable Stresses

The AISC allowable stresses for the elastic design of beams are expressed in terms of the specified minimum tensile yield point of the steel, F_y. Although there are many conditions for which the allowable stresses may be slightly modified, the basic maximum allowable bending stress, F_b, in non-compact rolled shapes, built-up members and plate girders is

$$F_b = 0.60\ F_y \tag{1}$$

and the basic maximum allowable shear stress, F_v, in beams is

$$F_v = 0.40\ F_y \tag{2}$$

The many modifications to these basic stresses, found in Section 1.5.1.4 and 1.10.5 of the AISC Specification, can be accounted for by rewriting Eqs. 1 and 2 in the form

$$F_{b1} = 0.6\ C_1\ F_y \tag{3}$$

and

$$F_{v2} = 0.4\ C_2\ F_y \tag{4}$$

in which C_1 and C_2 are coefficients. For compact rolled shapes with a maximum allowable bending stress=$0.66F_y$, $C_1 = 1.10$.

In an actual design, the maximum bending stress, f_b , and the maximum shear stress, f_v , caused by the working loads must be less than F_b and F_v , respectively. The stress f_b is computed according to the principles of elementary beam theory, that is, f_b equals the ratio of the applied bending moment to the section modulus of the beam. The stress f_v is taken as the average shear stress -- the shear force divided by the web area -- and is slightly less than the maximum shear stress calculated from beam theory.

The stresses f_b and f_v that are compared with the allowable stresses given by Eqs. 1 and 2 may occur on different transverse cross sections in a beam. However, when combined high bending and high shear stresses occur on the same cross section in the webs of beams and girders, the AISC allowable stresses may be less than those given by Eqs. 1 and 2. For certain cases of combined stresses, when (4): (a) f_v exceeds $0.6F_v$; and (b) f_b exceeds $0.75\ F_b$, the allowable bending stress is given (15) by the equation

$$F_b = (\ 0.825 - 0.375\ f_v/F_v\ )\ F_y \tag{5}$$

Outside this range, Eqs. 1 and 2 may be used for combined stresses.

4. Yielding Criterion

The theory of yielding for steel that agrees most closely with experimental results is the von Mises theory. For the webs of beams, this theory can be simplified and written in the form

$$f_b^{\ 2} + 3\ f_v^{\ 2} = F_y^{\ 2} \tag{6}$$

where f_b and f_v occur at the same point on a cross section. If F_y is eliminated from Eq. 6 by using the relationships in Eqs. 1,2,3 and 4 (neglecting the range where Eq. 5 should replace Eqs. 1 and 2), the von Mises yielding criterion can be rewritten in terms of the maximum AISC allowable stresses as follows

$$(\ f_b\ /\ F_b\ )^2 + \frac{4}{3}\ (\ f_v\ /\ F_v\ )^2 = 25/9 \tag{7}$$

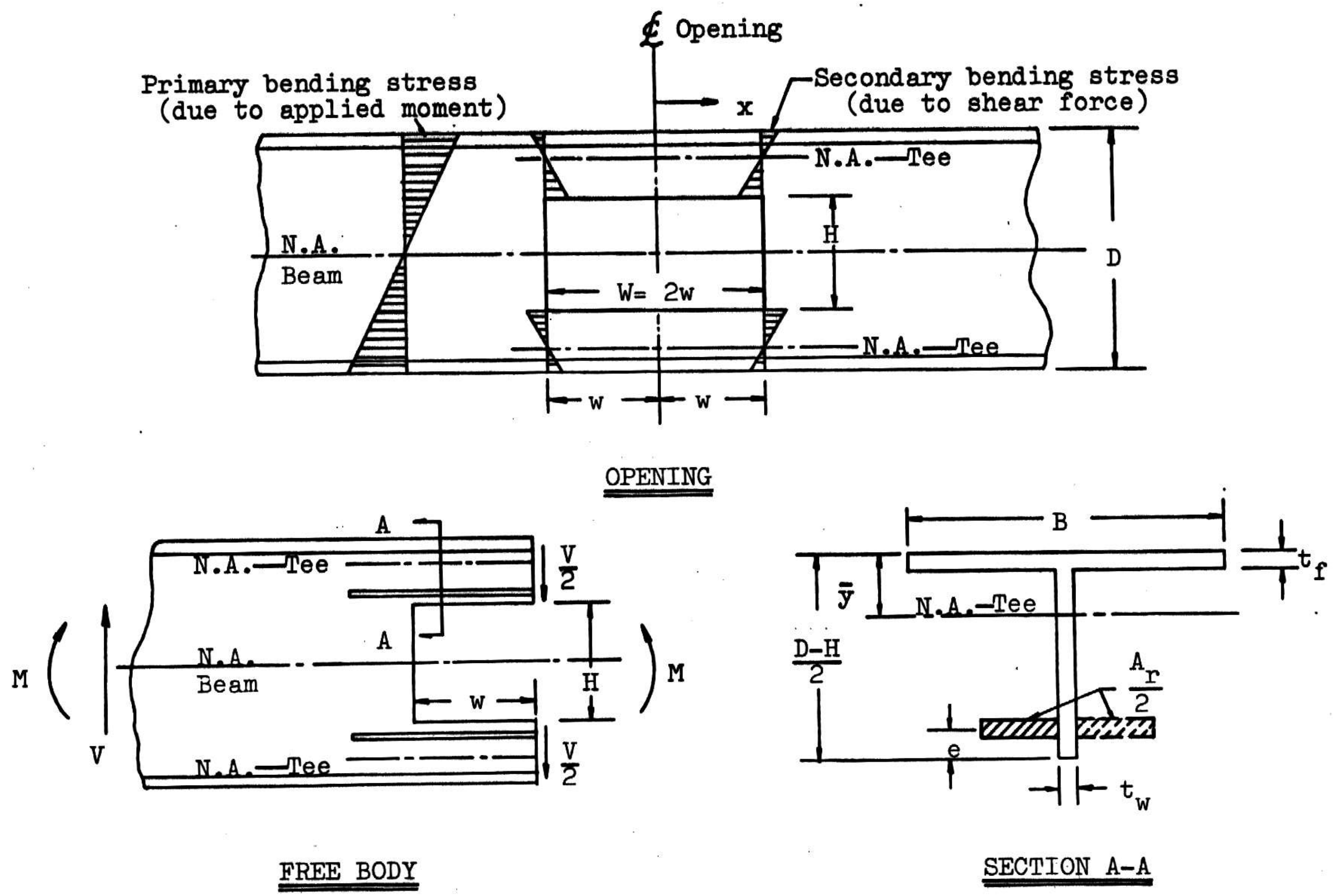

Figure 1. Typical stress distribution and reinforcement

5. Section Properties

The following equations for calculating the section properties of the beam at the opening were developed from Section A-A in Fig. 1 and are explained in detail in Appendix A :

$$\bar{y} = \frac{A_f t_f + \frac{A_w}{4} D\left(1 - 2\frac{H}{D} + \frac{H^2}{D^2} \right) + \frac{A_r}{2} D\left(1 - \frac{H}{D} - 2\frac{e}{D} \right)}{A + A_r - H\,t_w} \tag{8}$$

$$I_R = I + A_r\left(\frac{H}{2} + e \right)^2 - \frac{H^3}{12} t_w \tag{9}$$

$$I_T = \frac{m\,A_r^{\,2} + n\,A_r + p}{A_r^{\,2} + q\,A_r + s} \tag{10}$$

where $m = \frac{A_f}{4}\left(u_1 - 2u_2 - u_3 \right) + \frac{A_n}{3}\left(u_4 + 3u_5 + 3u_6 \right)$

$$n = \frac{1}{6} A_n^{\,2}\left(3u_1 - 6u_2 - 12u_3 + 5u_4 + 12u_5 + 12u_6 \right)$$
$$+ \frac{1}{12} A_f A_t \left(5u_1 - 9u_2 - 18u_3 + 8u_4 + 12u_5 + 24u_6 \right)$$
$$+ \frac{1}{96} A_t^{\,2}\left(5u_1 - 40u_2 - 24u_3 + 72u_4 + 144u_5 + 48u_6 \right)$$

$$p = \frac{1}{3} u_4 A_f^{\,3} + \frac{1}{6} A_f^{\,2} A_t\left(u_1 - 3u_2 + 5u_4 \right) + \frac{1}{48} A_f A_t^{\,2}\left(5u_1 - 28u_2 + 64u_4 \right)$$

$$q = 4A_n + 2A_t$$

$$s = 4A_f\left(A_f + A_t \right) + A_t^{\,2} - 4u_7\left(2A_f + A_t - u_7 \right)$$

in which $u_1 = (D-H)^2$, $u_2 = (D-H)t_f$, $u_3 = (D-H)e$, $u_4 = t_f^2$,

$u_5 = et_f$, $u_6 = e^2$, $u_7 = t_f t_w$;

$A_t = (D-H)t_w$, $A_n = A_f - t_f t_w$

6. Approximate Elastic Solution

Based on the Vierendeel analogy, the total bending stresses, f_b, on the sections above and below the opening can be obtained by adding the primary bending stresses, f_1, caused by the applied bending moment, to the secondary bending stresses, f_2, caused by the shear force at the center of the opening (Fig. 1).

$$f_1 = \pm \frac{M\,y_R}{I_R} \tag{a}$$

$$f_2 = \pm \frac{(\frac{V}{2}\,x)y_T}{I_T} \tag{b}$$

and
$$f_b = \pm \frac{M\,y_R}{I_R} \pm \frac{(\frac{V}{2}\,x)y_T}{I_T} \tag{11}$$

In studying the solution for required reinforcement around a rectangular web opening, two cases were considered critical : (1) Maximum bending stresses occur at extreme fibers of either flange or at the corners of the opening; (2) Bending combined with shear causes yielding at some point in the web.

Case 1. Maximum Bending Stresses Condition

From basic theory and by definition,

$$M = K_1 M_c = \frac{2K_1 I \; F_b}{D} \tag{c}$$

$$V = K_2 V_c = K_2 \; D \; t_w \; F_v \tag{d}$$

substituting Eqs. (c) and (d) into Eq. 11 and letting f_b equal F_b at $x = \pm w$:

$$F_b = \pm \frac{2K_1 I \; F_b \; y_R}{D \; I_R} \pm \frac{K_2 \; D \; w \; t_w F_v \; y_T}{2 \; I_T} \tag{12}$$

$$\text{or} \quad \frac{F_b}{K_2 V_c} = \pm \frac{y_R}{I_R} \; \frac{M}{V} \pm \frac{w}{2} \; \frac{y_T}{I_T} \tag{12a}$$

where $y_R = D/2$, $y_T = \bar{y}$

or $y_R = H/2$, $y_T = -(\frac{D-H}{2} - \bar{y})$, whichever is more critical.

Case 2. Yielding Condition

The Vierendeel analysis and past experimental results(2) have shown that the maximum stress condition causing yielding in the beam occurs : (a) at the boundary of the hole (in which $f_v = 0$) on a cross section through the low-moment edge of the hole, provided the hole is located where $V \neq 0$, (b) at the web-flange interface on a cross section through the high-moment edge of the hole. Thus, both these sections must be investigated in the design.

A. At the Boundary of the Hole :

Since the shear stress, f_v, equals zero, the yielding criterion (Eq. 7) reduces to

$$f_b / F_b = 5 / 3 \qquad \text{or} \qquad f_b = (5/3)F_b \tag{e}$$

substituting $y_R = H/2$ and $y_T = -(\frac{D-H}{2} - \bar{y})$ into Eq. 11, Eq. (e) becomes

$$f_b = \pm \frac{H}{2}\frac{M}{I_R} \pm \frac{Vw(\frac{D-H}{2} - \bar{y})}{2\, I_T} = \frac{5}{3} F_b \tag{13}$$

$$\text{or,} \quad \frac{H}{2I_R}\frac{M}{V} \pm \frac{w}{2I_T}\left(\frac{D-H}{2} - \bar{y} \right) = \frac{5}{3}\frac{F_b}{K_2 V_c} \tag{13a}$$

B. At the Web-Flange Interface :

The average shear stress on the web of the tee-section is

$$f_v = \frac{V}{(D-H)t_w} = \frac{V}{A_t} \tag{f}$$

and the bending stress is obtained by substituting $y_R = \frac{D}{2} - t_f$ and $y_T = \bar{y} - t_f$ into Eq. 11

$$f_b = \pm \frac{M(\frac{D}{2} - t_f)}{I_R} \pm \frac{V\, w(\bar{y} - t_f)}{2\, I_T} \tag{g}$$

Substituting Eqs. (f) and (g) into Eq. 7

$$\left(\frac{M(\frac{D}{2} - t_f)}{F_b I_R} \pm \frac{Vw(\bar{y} - t_f)}{2\, I_T F_b} \right)^2 + \frac{4}{3}\left(\frac{V}{A_t F_v} \right)^2 = \frac{25}{9} \tag{14}$$

Eq. 14 can be rewritten in the form

$$\left(\frac{\frac{D}{2} - t_f}{I_R} \frac{M}{V} \pm \frac{w(\bar{y} - t_f)}{2\, I_T} \right)^2 + \frac{4}{3A_t^{\,2}} \left(\frac{F_b}{F_v} \right)^2 = \frac{25}{9} \left(\frac{F_b}{K_2 V_c} \right)^2$$

(14a)

The area of reinforcement, A_r, which appears in the terms $\bar{y}$, I_R, I_T (Eqs. 8,9,10), can be obtained by solving Eqs. 12,13 and 14 either by trial and error or by computer using a reiterative method.

NUMERICAL EXAMPLE

At a point along a W12x45 beam of ASTM-A36 steel where a rectangular opening is to be centered on the neutral axis of the beam , the moment-shear ratio is 20.0 inches. The opening is to be 6" deep and 9" long (5). Design the necessary reinforcement for $K_2 = 0.25, 0.5$ ($V/V_c \approx 0.5$).

Solution :

<u>W12x45</u> $A = 13.24\ in^2$, $D = 12.06"$, $t_f = 0.576"$

$B = 8.042"$, $t_w = 0.336"$

$I_x = 350.8\ in^4$, $D/A_f = 2.60\ \frac{1}{in}$

$A_f = Bt_f = 4.63\ in^2$

$A_w = Dt_w = 4.06\ in^2$

<u>Opening</u> $H = 6"$, $W = 9"$, $w = W/2 = 4.5"$

<u>A36 Steel</u> $F_b = 22$ ksi, $F_v = 14.5$ ksi

$V_c = A_w F_v = 58.8$ kips

Assume $e = \frac{1}{2}$ in. , then from Eqs. 8, 9 and 10:

$$\bar{\bar{y}} = \frac{5.75 + 2.53\ A_r}{10.91 + A_r}$$

$$I_R = 344.75 + 12.25\ A_r$$

$u_1 = 36.72$, $u_2 = 3.49$, $u_3 = 3.03$,

$u_4 = 0.332$, $u_5 = 0.288$, $u_6 = 0.25$,

$$u_7 = 0.194, \qquad A_t = 2.035, \qquad A_n = 4.436 \text{ in}^2.$$

then,
$$m = 30.9 + 2.9 = 33.8$$
$$n = 200.0 + 86.0 + 2.0 = 288.0$$
$$p = 11.0 + 203.0 + 42.7 = 256.7$$
$$q = 17.7 + 4.1 = 21.8$$
$$s = 123.5 + 4.14 - 8.62 = 119.02$$

$$I_T = \frac{33.8\ A_r^2 + 288.0\ A_r + 256.7}{A_r^2 + 21.8\ A_r + 119.02}$$

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1. Maximum bending stress at the flange:

$$y_R = D/2 = 6.03'', \qquad y_T = \bar{y}$$

a. $K_2 = 0.25$; Eq. 12a becomes

$$\frac{120.6}{12.25\ A_r + 344.75} \pm \frac{5.69A_r^2 + 75.2A_r + 141.0}{33.8A_r^2 + 288A_r + 256.7} = 1.50$$

Try $A_r = 1.0$: $\quad 0.34 + 0.38 = 0.72 < 1.5$

$A_r = 0.5$: $\quad 0.34 + 0.44 = 0.78 < 1.5$

$\underline{A_r = 0}$: $\quad 0.35 + 0.55 = 0.90 < 1.5$ OK.

b. $K_2 = 0.5$;

$$\frac{120.6}{12.25A_r + 344.75} \pm \frac{5.69A_r^2 + 75.2A_r + 141.0}{33.8A_r^2 + 288A_r + 256.7} = 0.75$$

Try $\underline{A_r = 0.75}$: $\quad 0.34 + 0.41 = 0.75$ OK.

2. Maximum bending stress at the edge of the opening:

$$y_R = H/2 = 3.0", \quad y_T = -\left(\frac{D-H}{2} - \bar{y}\right) = -\frac{0.5A_r+27.35}{A_r+10.91}$$

a. $K_2 = 0.25$;

$$\frac{60.0}{12.25A_r+344.75} \pm \frac{1.125A_r^2+73.8A_r+672}{33.8A_r^2+288A_r+256.7} = 1.5$$

Try A_r=1.0; $0.17 + 1.29 = 1.46 < 1.5$

A_r=0.8; $0.17 + 1.44 = 1.61 \nless 1.5$

$\underline{A_r=0.94}$; $0.17 + 1.33 = 1.50$ OK.

b. $K_2 = 0.5$;

$$\frac{60.0}{12.25A_r+344.75} \pm \frac{1.125A_r^2+73.8A_r+672}{33.8A_r^2+288A_r+256.7} = 0.75$$

Try A_r=5.0: $0.15 + 0.42 = 0.57 < 0.75$

A_r=2.5: $0.16 + 0.64 = 0.80 > 0.75$

$\underline{A_r=3.4}$: $0.16 + 0.58 = 0.74$ OK.

3. Yielding at the edge of the opening:

$$y_R = H/2 = 3.0", \quad y_T = -\left(\frac{D-H}{2} - \bar{y}\right) = -\frac{0.5A_r+27.35}{A_r+10.91}$$

a. $K_2 = 0.25$: Eq. 13a becomes

$$\frac{60.0}{12.25A_r+344.75} \pm \frac{1.125A_r^2+73.8A_r+672}{33.8A_r^2+288A_r+256.7} = 2.5$$

Try $A_r = 0.5$: $0.17 + 1.73 = 1.9 < 2.5$

$A_r = 0\ 2$: $0.17 + 2.18 = 2.35 < 2.5$

$\underline{A_r = 0.14}$: $0.18 + 2.29 = 2.47$ OK.

b. $K_2 = 0.5$;

$$\frac{60.0}{12.25A_r+344.75} \pm \frac{1.125A_r^2+73.8A_r+672}{33.8A_r^2+288A_r+256.7} = 1.25$$

Try $\underline{A_r=1.3}$: $0.17 + 1.08 = 1.25$ OK.

4. Yielding at the web-flange interface:

$$y_R = D/2 - t_f = 5.454", \quad y_T = \bar{y} - t_f = \frac{1.954A_r-0.53}{A_r+10.91}$$

a. $K_2 = 0.25$; Eq. 14a becomes

$$\left(\frac{109.08}{12.25A_r+344.75} \pm \frac{4.4A_r^2+46.8A_r-13.0}{33.8A_r^2+288A_r+256.7}\right)^2 + \frac{4\ (22)^2}{3(14.5)^2}\,\frac{1}{2.035^2}$$

$$= \frac{25}{9}\left(\frac{22}{0.25 \times 58.8}\right)^2 = 6.23$$

Try $\underline{A_r=0}$: $(\ 0.316 + 0.051\ )^2 + 0.74 = 0.875 < 6.23$ OK.

b. $K_2 = 0.5$;

$$\left(\frac{109.08}{12.25A_r + 344.75} \pm \frac{4.4A_r^2 + 46.8A_r - 13.0}{33.8A_r^2 + 288A_r + 256.7} \right)^2 = 0.82$$

Try $\underline{A_r = 0.0}$: $(0.316 + 0.051)^2 = 0.135 < 0.82$ OK.

The results of the solution are listed in Table 1.

Table 1. Solution for Reinforcement, A_r (in^2)

Condition / K_2	Maximum Bending Stress		Yielding Condition		Required
	At Flange	At Edge of Opening	At Flange-web Interface	At Edge of Opening	A_r
0.25	0.00	0.94	0.00	0.14	0.94
0.50	0.75	3.40	0.00	1.30	3.40

SUMMARY AND CONSLUSION

This study, based on the Vierendeel method of analysis , provides a relatively simple yet useful method for the design of beams with web openings subjected to combined bending and shear. Although the numerical example illustrates a fundamental approach to the solution, the design procedure could be much simplified if design curves for each W shape member with a specific web opening were developed. However, it would be more practical to conduct this work by taking advantage of a computer.

From the results of the numerical example(Table 1) and other calculations(not presented here), it has been noted that case 1— maximum bending stress at the extreme fibers of either flange or at the corners of the opening— always controls the design. In general, the stresses at the corner of the opening are more critical than those at the extreme fiber of the flange when K_2 is small. The value of K_2 at which the critical stress condition changes from the corner of the opening to the flange of the beam, depends on the aspect ratio(H/W) of the opening, the moment-shear ratio (M/V) at the opening and the ratio of the depth of the opening to the depth of the member (H/D).

SUGGESTIONS FOR FURTHER STUDIES

Although the scope of this study was limited to the beams with moment(horizontal) reinforcement only, it could be extended to investigate beams with both moment and shear reinforcements. The study could also be extended to determine the reinforcement requirements for W shape beams with large web openings other than rectangular, such as circular, elliptical etc.

Moreover, a practical method for determining the type and amount of the reinforcement around eccentric web openings, that is, openings not centered on the neutral axis of the members, should be developed.

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NOTATION

A	area of gross beam
A_f	area of one flange (Bt_f)
A_n	area of the outstretched flange ($A_f - t_f t_w$)
A_r	area of the reinforcement at a given cross section of the beam contains an opening
A_t	net web area of the beam at the opening ($(D-H)t_w$)
A_w	web area of gross beam (Dt_w)
B	flange width of the beam
C_1	coefficient modifying basic AISC allowable bending stress
C_2	coefficient modifying basic AISC allowable shear stress
D	total depth of beam
E	modulus of elasticity
e	distance from the center of the reinforcement in the tee-section to the edge of the opening
F_b	AISC allowable bending stress
F_v	AISC allowable shear stress
F_y	minimum yield point of the steel
f_b	actual bending stress at the critical section near a hole
f_v	actual shear stress at the critical section near a hole
G	total area of the reinforced beam at the opening
H	depth of the rectangular opening
I	moment of inertia of the gross beam
I_R	moment of inertia of the beam at the opening
I_T	moment of inertia of the reinforced tee section

K_1	ratio of the applied moment at the center of the opening to the moment capacity of the gross section (M/M_c)
K_2	ratio of the total vertical shear at the opening to the shear capacity of the gross section (V/V_c)
M	applied moment at the center of the opening
M_c	elastic moment capacity of the gross beam
m,n,p q,s	coefficients for calculating the moment of inertia of the reinforced tee section I_T
t_f	flange thickness of the beam
t_w	web thickness of the beam
V	total applied shear in the section containing the opening
V_c	elastic shear capacity of the gross beam (A_wF_v)
W	total width of rectangular opening
w	half-width of rectangular opening
$\bar{y}$	distance from the top or bottom of the beam to the neutral axis of the tee section
y_R	distance from the neutral axis of the beam to a point of stress in the section containing the opening
y_T	distance from the neutral axis of the tee section to a point of stress in the reinforced tee section

APPENDIX A. DERIVATION OF EQUATIONS

The following expressions were developed from Section A-A in Fig. 1.

1. Neutral Axis of the tee-section:

$$\bar{y} = \frac{\frac{B}{2} t_f^2 + (\frac{D-H}{2} - t_f) \frac{t_w}{2}(\frac{D-H}{2} + t_f) + \frac{A_r}{2}(\frac{D-H}{2} - e)}{Bt_f + ((D-H)/2 - t_f)t_w + A_r/2} \tag{A}$$

$$= \frac{A_f t_f + t_w(\frac{(D-H)^2}{4} - t_f^2) + A_r(\frac{D-H}{2} - e)}{2A_f + A_w - 2t_f t_w + A_r - Ht_w}$$

$$= \frac{A_f t_f + D^2 t_w(\frac{1}{4} - \frac{1}{2}(\frac{H}{D}) + \frac{1}{4}(\frac{H}{D})^2 - (\frac{t_f}{D})^2) + A_r(\frac{D-H}{2} - e)}{A + A_r - Ht_w}$$

$$= \frac{A_f t_f + \frac{A_w}{4} D(1 - 2 \frac{H}{D} + (\frac{H}{D})^2) + \frac{A_r}{2} D(1 - \frac{H}{D} - 2 \frac{e}{D})}{A + A_r - Ht_w} \tag{8}$$

2. Moment of Inertia of the tee-section:

$$I_T = \frac{B}{12} t_f^3 + Bt_f(\bar{y} - \frac{t_f}{2})^2 + \frac{t_w}{12}(\frac{D-H}{2} - t_f)^3 + (\frac{D-H}{2} - t_f)$$

$$(\frac{D-H}{4} + \frac{t_f}{2} - \bar{y})^2 t_w + \frac{A_r}{2}(\frac{D-H}{2} - e - \bar{y})^2 \tag{B}$$

$$= \frac{B}{12} t_f^3 + \frac{t_w}{12}(\frac{(D-H)^3}{8} - \frac{3(D-H)^2}{4} t_f + \frac{3(D-H)}{2} t_f^2 - t_f^3)$$

$$+ Bt_f \left[\frac{\frac{D-H}{4} t_w(\frac{D-H}{2} - t_f) + \frac{A_r}{2}(\frac{D-H}{2} - e - \frac{t_f}{2})}{Bt_f + (\frac{D-H}{2} - t_f) t_w + \frac{A_r}{2}} \right]^2$$

$$+\left(\frac{D-H}{2}-t_f\right)t_w\left[\frac{\frac{D-H}{4}Bt_f+\frac{A_r}{2}\left(\frac{t_f}{2}-\frac{D-H}{4}+e\right)}{Bt_f+\left(\frac{D-H}{2}-t_f\right)t_w+\frac{A_r}{2}}\right]^2$$

$$+\frac{A_r}{2}\left[\frac{\left(\frac{D-H}{2}-t_f\right)\frac{t_w}{2}\left(\frac{D-H}{2}-t_f-2e\right)+Bt_f\left(\frac{D-H}{2}-\frac{t_f}{2}-e\right)}{Bt_f+\left(\frac{D-H}{2}-t_f\right)t_w+\frac{A_r}{2}}\right]^2 \quad \text{------(C)}$$

Letting $G = Bt_f + \left(\frac{D-H}{2} - t_f\right)t_w + \frac{A_r}{2}$, expand Eq.C as follows:

$$G^2 I_T = G^2\left(\frac{B}{12}t_f^3+\frac{t_w}{12}\left[\frac{(D-H)^3}{8}-\frac{3(D-H)^2}{4}t_f+\frac{3(D-H)}{2}t_f^2-t_f^3\right]\right)$$

$$+Bt_f\left(\frac{(D-H)^2}{16}t_w^2\left[\frac{(D-H)^2}{4}-(D-H)t_f+t_f^2\right]+\frac{A_r^2}{4}\left[\frac{t_f^2}{4}\right.\right.$$

$$\left.-\frac{D-H}{2}t_f+\frac{(D-H)^2}{4}-(D-H)e+e^2+et_f\right]+A_rt_w\left[\frac{(D-H)^2}{8}\right.$$

$$\left.\left.-\frac{D-H}{4}t_f\right]\left(\frac{D-H}{2}-e-\frac{t_f}{2}\right)\right)$$

$$+\left(\frac{D-H}{2}-t_f\right)t_w\left(\frac{(D-H)^2}{16}B^2t_f^2+\frac{A_r^2}{4}\left[\frac{t_f^2}{4}+e^2+\frac{(D-H)^2}{16}\right.\right.$$

$$\left.+et_f-\frac{D-H}{4}t_f-\frac{D-H}{2}e\right]+A_rBt_f\left[\frac{D-H}{8}t_f+\frac{D-H}{4}e\right.$$

$$\left.\left.-\frac{(D-H)^2}{16}\right]\right)+\frac{A_r}{2}\left(\left[\frac{(D-H)^2}{4}+t_f^2-(D-H)t_f\right]\frac{t_w^2}{4}\left[t_f^2\right.\right.$$

$$\left.-(D-H)t_f+\frac{(D-H)^2}{4}+4e^2-2(D-H)e+4et_f\right]+B^2t_f^2\left[\frac{(D-H)^2}{4}\right.$$

$$+ \frac{t_f^2}{4} + e^2 - \frac{D-H}{2} t_f - (D-H)e + et_f \Big] + Bt_f t_w \Big[\frac{(D-H)^2}{4} + t_f^2$$

$$- (D-H)t_f - (D-H)e + 2et_f \Big] \left(\frac{D-H}{2} - \frac{t_f}{2} - e \right) \Bigg)$$

$$G^2 I_T = A_r^2 \Bigg(\frac{A_f}{16} \Big[(D-H)^2 - 2(D-H)t_f - 4(D-H)e \Big] + \frac{A_f - t_f t_w}{12} (3e^2$$

$$+ 3et_f + t_f^2) + \frac{A_w}{96}(1 - \frac{H}{D}) \Big[(D-H)^2 - 6(D-H)e - 6(D-H)t_f$$

$$+ 12t_f^2 + 12e^2 + 24et_f \Big] \Bigg)$$

$$+ A_r \Bigg((A_f - t_f t_w)^2 \Big[\frac{1}{8}(D-H)^2 - \frac{1}{4}(D-H)t_f + \frac{5}{24} t_f^2 + \frac{1}{2}e^2$$

$$- \frac{1}{2}(D-H) e + \frac{1}{2} et_f \Big] + A_f A_w (1 - \frac{H}{D}) \Big[\frac{5}{48}(D-H)^2$$

$$- \frac{3}{16}(D-H)t_f + \frac{1}{6} t_f^2 - \frac{3}{8}(D-H)e + \frac{1}{4} et_f + \frac{1}{2} e^2 \Big]$$

$$+ A_w^2 (1 - \frac{H}{D})^2 \Big[\frac{5}{384}(D-H)^2 - \frac{5}{48}(D-H)t_f + \frac{3}{16} t_f^2 + \frac{1}{8} e^2$$

$$- \frac{1}{16}(D-H)e + \frac{3}{8} et_f \Big] - A_w (1 - \frac{H}{D}) t_f t_w \Big[\frac{1}{2}e^2 + \frac{1}{4} et_f$$

$$+ \frac{1}{6} t_f^2 \Big] \Bigg)$$

$$+ \frac{1}{12} A_f^3 t_f^2 + A_f^2 A_w (1 - \frac{H}{D}) \Big[\frac{1}{24}(D-H)^2 - \frac{1}{8}(D-H)t_f + \frac{5}{24} t_f^2 \Big]$$

$$+ A_f A_w^2 (1 - \frac{H}{D})^2 \Big[\frac{1}{3} t_f^2 + \frac{5}{192}(D-H)^2 - \frac{7}{48} (D-H)t_f \Big]$$

$$+ A_w^3 (1 - \frac{H}{D})^3 \Big[\frac{1}{384}(D-H)^2 - \frac{5}{192}(D-H)t_f + \frac{5}{48} t_f^2 \Big]$$

$$- t_f^3 t_w^3 \Big[\frac{5}{24}(D-H)^2 - \frac{5}{24}(D-H)t_f + \frac{1}{12} t_f^2 \Big]$$

$$+ A_f t_f^3 t_w \Big[\frac{1}{4} t_f t_w - \frac{1}{4} A_f \Big] \qquad \text{(D)}$$

Introducing the new notations:

$$(D-H)^2 = u_1 , \qquad (D-H)t_f = u_2 , \qquad (D-H)e = u_3 ,$$

$$t_f^2 = u_4 , \qquad et_f = u_5 , \qquad e^2 = u_6 ,$$

$$t_f t_w = u_7 , \qquad A_w(1 - \frac{H}{D}) = (D-H)t_w = A_t ,$$

$$A_f - t_f t_w = A_f - u_7 = A_n$$

then Eq. D can be written as

$$I_T = \frac{m A_r^2 + n A_r + p}{G^2} = \frac{m A_r^2 + n A_r + p}{A_r^2 + q A_r + s} \tag{10}$$

where the coefficients m, n, p, q, s are defined as before(page 9).

AN APPROXIMATE ELASTIC ANALYSIS OF W SHAPE BEAMS WITH REINFORCED RECTANGULAR WEB OPENINGS

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ABSTRACT

This report presents an approximate, elastic method for determining the amount of the reinforcement necessary around large rectangular openings centered in the webs of W shape beams subjected to varying combinations of bending moment and shear. The method is based on the theory that the beam tends to act like a Vierendeel Truss in the vicinity of the opening.

Based on this theory, two conditions were considered critical : (1) maximum bending stresses occur at extreme fibers of either flange or at the corners of the opening; (2) bending combined with shear causes yielding at some point in the web. Equations for determining the required reinforcement are developed in terms of the moment-shear ratio at the opening. A numerical example on an A36 steel W12x45 beam with 9"x6" rectangular opening at middepth is presented to illustrate the use of the method by a trial and error technique.

It is concluded that the method developed is relatively simple and that the results are satisfactory for design purposes.